A pricing kernel approach to valuing interest rate options

Xiaoquan Liu*

Jing-Ming Kuo†

Jerry Coakley‡

June 2, 2009

*Essex Finance Centre and Essex Business School, University of Essex, Colchester CO4 3SQ, UK. Email: liux@essex.ac.uk. Phone: +44 (0)1206 873849. Financial support from the ESRC (grant number RES-000-22-1951) is gratefully acknowledged.

†Durham Business School, University of Durham, Durham DH1 3LB, UK. Email: jing-ming.kuo@durham.ac.uk. Phone: +44 (0)191 3345229.

‡Corresponding author. Essex Finance Centre and Essex Business School, University of Essex, Colchester CO4 3SQ, UK. Email: jcoakley@essex.ac.uk. Phone: +44 (0)1206 872455.

Abstract

This paper investigates parametric pricing kernels for interest rate options within the intertemporal CAPM framework. The usual GMM estimation produces problematic pricing kernels that either fail statistical robustness tests or are inconsistent with economic theory in terms of being hump-shaped and having negative segments. Adopting the second Hansen-Jagannathan (HJ) distance, the four-term polynomial pricing kernels clearly dominate the nonlinear iso-elastic pricing kernels. The preferred pricing kernel has two significant state variables, the real interest rate and maximum Sharpe ratio. It is always strictly positive and everywhere monotonically decreasing in market returns in conformity with economic theory.

JEL code: C11, G12, G13

Keywords: Pricing kernels, Simulation-based Bayesian approach, LIBOR options

1 Introduction

The London Interbank Offer Rate (LIBOR) is the most commonly cited base rate in global money markets. The overnight LIBOR serves as the benchmark for short-term interbank loans, the 3-month LIBOR for short and medium term corporate loans while the 6-month LIBOR is widely used in interest rate and currency swaps. Hence the hedging of LIBOR exposure is a pervasive problem and the pricing of hedging instruments such as LIBOR options becomes an important issue.

The focus of this paper is on the pricing of options written on 6-month LIBOR futures. One widely adopted approach to pricing options involves specifying the dynamics of the underlying asset and solving for the closed-form solution[1]. Alternatively, a more general method for evaluating the prices of risky assets, including derivatives, is to employ an asset pricing kernel, also known as a stochastic discount factor. The pricing kernel is a strictly positive and random variable that succinctly summarizes investor risk and time preferences with respect to financial assets. It is used to compute today's asset price by discounting, state by state, the corresponding payoffs at future dates (Harrison and Kreps (1979), Hansen and Jagannathan (1991, 1997), Bansal and Viswanathan (1993), and Chapman (1997)).

[1]Examples include Bakshi et al. (1997), Bates (1996), Pan (2002) for index options and Hull and White (1990), Heath et al. (1992), and Singleton and Umantsev (2002) for interest rate options.

The first contribution of this paper to the literature is that it applies to LIBOR options the parametric pricing kernel approach that has been successfully employed in pricing stock index options (Rosenberg and Engle (2002), Jones (2006), and Brennan et al. (2007)) and stock portfolios (Lettau and Ludvigson (2001), Dittmar (2002), and Vaden (2004)). To our knowledge, this is the first study that utilizes this approach for interest rate options. It requires no assumptions on the dynamics of interest rates or on the term structure. We evaluate two functional forms for market returns, a nonlinear power function and a linear Chebyshev polynomial approximation. Both are popular choices in the equity and option pricing literature (Brennan (1979), Chapman (1997), Rosenberg and Engle (2002), and Brennan et al. (2007)). The use of these functional forms ensures comparability between our results and those of previous studies.

Secondly, our study contributes to the asset pricing literature that examines the important role of pricing kernels within the intertemporal CAPM framework of Merton (1973). In this respect it is closely related to Brennan et al. (2004) and Nielsen and Vassalou (2006). They show that the investment opportunity set can be captured by the intercept and slope of the instantaneous capital market line. In the options market, there is strong empirical evidence that market volatility is priced with a negative risk premium (Coval and Shumway (2001) and Bakshi and Kapadia (2003)). Based on these findings, we consider three non-wealth-related state variables for our pricing kernel, namely the real interest rate, the maximum Sharpe ratio, and volatility. We use an exponential affine function with time-varying innovations to ensure that the pricing kernel is nonlinear in these state variables and hence capable of pricing nonlinear payoffs (Chapman (1997) and Dittmar (2002)).

Using monthly moneyness-based portfolio returns on LIBOR options from January 2000 to December 2006, our results indicate that, among the state variables considered, only the coefficients of the real interest rate and the maximum Sharpe ratio are statistically significant regardless of the functional form. Volatility is not priced for our sample in contrast to that of Brennan et al. (2007) who use the same functional forms for index options in the US and UK markets. They find that all candidate state variables are priced in the US market while only the real interest rate and volatility are priced in the UK. Their and our results are contrary to modern asset pricing theory which suggests that a unique pricing kernel is able to price all financial assets (Cochrane (2005)). Our findings indicate that interest rate options can be priced by means of a parsimonious stochastic discount factor as interest rates were less volatile than stocks under the inflation targeting regime that operated in the UK over our sample period.

The third contribution is that we adopt the second Hansen-Jagannathan (HJ) distance in evaluating candidate pricing kernels. The motivation for this is that the usual GMM estimation produces problematic pricing kernels that either fail statistical robustness tests or are inconsistent with economic theory in terms of say producing hump-shaped pricing kernels. In contrast with the first HJ distance, this measure restricts our focus to the family of true *positive* pricing kernels. This positivity constraint guarantees that the pricing kernels are arbitrage free which is essential for correctly pricing contingent claims. The first HJ distance, which measures the deviation of a candidate pricing kernel to the family of true pricing kernels, has been widely applied (Jagannathan and Wang (1996), Buraschi and Jackwerth (1999), Dittmar (2002), Lettau and Ludvigson (2001), among others). By contrast, the second HJ distance has rarely been applied in the literature mainly due to the difficulty in deriving a reliable posterior distribution for the test statistic.

Notable exceptions include Hansen et al. (1995), Wang and Zhang (2005), and Li et al. (2008). Hansen et al. (1995) develop an asymptotic distribution for the sample estimate of the second HJ distance under the assumption that the distance is nonzero in population. However, the asymptotic theory no longer holds when the true distance is zero. Wang and Zhang (2005) propose a simulation-based Bayesian approach that facilitates statistical inference for the second HJ distance in finite samples. Bayesian methods provide us with the full posterior density of the model parameters, and subsequently the full posterior of the second HJ distance, resulting in inference that takes account of parameter uncertainty and is valid in finite samples (Koop (2003)). More recently, Li et al. (2008) also adopt the second HJ distance as the yardstick for comparing alternative asset pricing models but they employ a different econometric framework.

We follow Wang and Zhang (2005) and use Bayesian econometrics to provide a robustness test for the second HJ distance in estimating parametric linear and nonlinear pricing kernels. Our results indicate that the linear multi-factor pricing kernels outperform the non-linear models in terms of smaller pricing errors. In addition, unlike the linear pricing kernels obtained from the GMM, those obtained via the second HJ distance conform neatly with economic theory by being strictly positive and decreasing in market returns. These findings underline the inherent advantage of the second HJ distance over competing statistical measures in evaluating pricing kernels for derivatives. The hypothesis of a zero second HJ distance is accepted for both functional forms. The 4-term generalized Chebyshev polynomial model with two state variables has the smallest second HJ distance and so emerges as the preferred functional form for pricing interest rate options.

The rest of the paper proceeds as follows. Section 2 discusses the parametric functional forms of the pricing kernel, the state variables, and the second HJ distance. Section 3 describes the data and the empirical results. Finally, Section 4 concludes.

2 Methodology

2.1 The state variables

The importance of including non-wealth-related state variables in pricing kernels has been widely stressed in the literature. The main reason is that such variables enhance the ability of pricing kernels in capturing time-varying investment opportunities (Garcia et al. (2003), Vaden (2004), Santa-Clara and Yan (2008), among others). In particular, Nielsen and Vassalou (2006) postulate that the intercept and slope of the instantaneous capital market line are sufficient to describe the innovations in the investment opportunity set in the context of portfolio hedging. Supportive empirical evidence is given in Brennan et al. (2004). Their simple ICAPM with aggregate wealth and two state variables, the real interest rate and the maximal Sharpe ratio, dominates the Fama-French three-factor model and the CAPM. In addition, market volatility is also included as a state variable on the basis of the negative volatility premium documented in Coval and Shumway (2001), Bakshi and Kapadia (2003), Eraker and Polson (2003), among others.

In our paper, the real interest rate r, the maximal Sharpe ratio η, and the volatility σ are selected as the candidate state variables for pricing LIBOR options. This particular state variable set $X \equiv (r, \eta, \sigma)$ facilitates comparison with the results in Brennan et al. (2007), who adopt the same state variable set for pricing stock index options in the US and the UK markets.

Hence, this study may be able to shed light on whether the state variable set is distinct for pricing options on different financial assets.

We assume that the real interest rate and the maximum Sharpe ratio follow correlated Ornstein-Uhlenbeck processes. With further specifications, the time series of these two state variables can be estimated from panel data on UK nominal zero-coupon government bond yields via the Kalman filter (see Brennan et al. (2004) for details).

2.2 The pricing kernels

The pricing kernel approach has been widely employed in the asset pricing literature (Breeden (1979), Epstein and Zin (1989), Cochrane (1996), Abel (1990), among others). Cochrane (2005) argues that the projected pricing kernels onto the asset return space have the same pricing implications as the true pricing kernels. As a result, the portfolio choice problem for any investor can be solved by the Euler equation

$$E\left[m_{t+1}\tilde{R}_{i,t+1}|\Omega_t\right] = 1 \tag{1}$$

where m_{t+1} is the pricing kernel, a function of state variable set X; $\tilde{R}_{i,t+1}$ is the gross return on an asset or portfolio i at time $t+1$; and Ω_t is the information available at time t. The pricing kernel is also known as the stochastic discount factor since it varies over time and across states and can be applied to compute the expected discounted return that should always be equal to unity.

Motivated by Rosenberg and Engle (2002) and Brennan et al. (2007), two basic forms of the pricing kernel are evaluated. These are a power function and a Chebyshev polynomial expansion in aggregate wealth growth. They both are augmented by an exponential affine function of the innovations in the state variables. The use of these functional forms in the pricing kernels also provides the basis for a comparison between linear and nonlinear forms of pricing kernels.

Our choice of proposed functional forms builds on theoretical developments. Under the assumptions of CRRA agents and bivariate normal distribution of the underlying asset returns and aggregate wealth growth, Rubinstein (1976) and Brennan (1979) demonstrate that the Black-Scholes option pricing model implies, in a discrete time setting, a power function: $m^* = k(\tilde{R}^{-\gamma})/R_f$. Here R_f is the riskfree interest rate, k is a constant, and γ is the risk aversion factor. In a continuous time setting, Bick (1987) uses the same projected pricing kernel but with continuously compounding interest rate in the Black-Scholes framework. More generally, Dybvig (1981) indicates that the projected pricing kernel implied by the Black-Scholes model is equivalent to a power function of the gross return on aggregate wealth discounted by the continuously compounded interest rate $m^* = k(R_W^{-\gamma})e^{-r}$ where $k = (E[R_W^{-\gamma}])^{-1}$.

In light of the theoretical links connecting the Black-Scholes model with the pricing kernel approach, we first assume that the pricing kernel is expressed as a power function of the aggregate wealth return, R_w, augmented by an exponential affine function of the innovations in the state variables discussed above,

$$m_{t+1} = \beta(R_{w,t+1})^{-\gamma} \exp^{b_1 \Delta r_{t+1} + b_2 \Delta \eta_{t+1} + b_3 \Delta \sigma_{t+1}} \tag{2}$$

where β, b_1, b_2, and b_3 are constants and γ is the relative risk aversion parameter. This iso-elastic function captures the decreasing marginal utility of wealth.

The second functional form of the pricing kernel is a Chebyshev polynomial in aggregate market returns. Chapman (1997) discusses the benefit of approximating pricing kernels by means of polynomials. Such an approach combines linearity in the functional form with nonlinearity in the state variables. Hence it is capable of pricing nonlinear payoffs while retaining linear interpretation. Our second candidate pricing kernel is expressed as the sum of Chebyshev polynomials augmented by an exponential affine function of the innovations in the state variable as follows,

$$m_{t+1} = \wp^n(R_{w,t+1}) \exp^{b_1 \Delta r_{t+1} + b_2 \Delta \eta_{t+1} + b_3 \Delta \sigma_{t+1}} \tag{3}$$

where $\wp^n(R_{w,t+1})$ consists of n-term Chebyshev polynomials. We follow Brennan et al. (2007) and Chapman (1997) use both 3- and 4-term polynomial approximations.

2.3 The second HJ distance and Markov Chain Monte Carlo (MCMC) Bayesian inference

Following Wang and Zhang (2005), we estimate the pricing kernel parameters by minimizing the second HJ distance $\text{Dist}_{HJ2}(\theta)$ and obtain the p-values from a Markov Chain Monte Carlo (MCMC) simulation-based Bayesian approach. Let z_t be a matrix of size $t \times n+l+k$ composed of n asset returns r_t, l state variables s_t, and k factors f_t which include all other information like polynomial terms, thus $z_t = (r_t', f_t', s_t')'$. According to Hansen and Jagannathan (1997), the second HJ distance is defined as

$$\text{Dist}_{HJ2}(\theta) = \sqrt{\min_{\theta} \max_{\lambda \in R^{\tilde{n}}} E\left[m_t(\theta)^2 - ([m_t(\theta) - \lambda' r_t]^+)^2 - 2\lambda' 1_n\right]} \tag{4}$$

where $m_t(\theta) = g(\theta, f_t, z_{t-1})$ is the candidate pricing kernel, $R^{\tilde{n}}$ is the space of $\tilde{n}$ real numbers, and the assets return, r_t, can be scaled by $H(z_t)$.

For the data-generating process in the MCMC simulation, z_t is assumed to follow a VAR, hence $z_t = C + Az_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0_m, \Omega)$. Under the assumption of independent non-informative prior distributions for unknown parameters, z_0, B and Ω, we have $p(\Phi) = p(z_0)p(B)p(\Omega)$, where $\Phi = (z_0', vec(B)', vech(\Omega)')'$, $p(z_0) \propto$ constant, $p(B) \propto$ constant, $p(\Omega) \propto |\Omega|^{-(m+1)/2}$, $m = n + k + l$ and B is the matrix of parameters including C and A in the VAR system. Note that $vech(\Omega)$ is the vector converting the upper triangle of matrix Ω.

The MCMC simulation method is applied to tackle the difficulty in deriving the posterior distribution of the unknown parameter set Φ. We carry out $S = 10,000$ simulations and discard the first $S_0 = 1000$ simulations to approximate the posterior distribution for the second HJ distance. More specifically, we carry out the procedures below to compute the p-values.

In the first stage, we choose an arbitrary $z_0^{(0)}$, and perform the simulations ($j = 1, ..., S$):

1. Obtain the jth sample of unknown parameters from their conditional posterior distributions

- Draw $\Omega^{(j)}$ from $IW\left(T\hat{\Omega}(z_0^{(j-1)}), T-1, m\right)$, where IW is the inverted Wishart distribution;

- Draw $vec(B^{(j)})$ from the truncated normal distribution

$$N\left(vec(\hat{B}(z_0^{(j-1)})), \Omega^{(j)} \otimes [X(z_0^{(j-1)})'X(z_0^{(j-1)})]^{-1}\right).$$

  We limit the norm of the eigenvalues of parameter matrix A to be less than unity to ensure that the VAR is stationary.

- Draw $z_0^{(j)}$ from $N\left([A^{(j)}]^{-1}(z_1 - C^{(j)}), [A^{(j)}]^{-1}\Omega^{(j)}[A^{(j)\prime}]^{-1}\right)$ where

$$\hat{\Omega}(z_0) = \frac{1}{T}[Z - X(z_0)\hat{B}(z_0)]'[Z - X(z_0)\hat{B}(z_0)]$$

$$X(z_0) = ((1, z_0')', (1, z_1')', ..., (1, z_{T-1}')')'$$

$$\hat{B}(z_0) = [X(z_0)'X(z_0)]^{-1}X(z_0)'Z$$

$$Z = (z_1, ..., z_T)'.$$

2. Obtain the jth sample with unconditional mean $\tilde{\mu}(\nu_t)$ and variance $\tilde{\Sigma}(\nu_t)$

$$\tilde{\mu}(\nu_t)^{(j)} = \tilde{C}^{(j)} + \tilde{A}^{(j)}\mu(z_t)^{(j)}$$

$$\tilde{\Sigma}(\nu_t)^{(j)} = \tilde{A}^{(j)}\Sigma^{(j)}\tilde{A}^{(j)\prime} + D\Omega^{(j)}D'$$

where

$$\mu(z_t)^{(j)} = \left(I_m - A^{(j)}\right)^{-1} C^{(j)}$$

$$vec(\Sigma(z_t)^{(j)}) = \left(I_{m^2} - A^{(j)} \otimes A^{(j)}\right)^{-1} vec(\Omega^{(j)}).$$

3. Compute the value of the second HJ distance for the jth sample.

In the second stage, we compute the posterior cumulative probability distribution of the second HJ distance and the p-values can be derived as follows,

$$prob(\mathrm{Dist}_{HJ2} \leq Y) \approx \frac{1}{S - S_0} \sum_{j=S_0+1}^{S} I[\mathrm{Dist}_{HJ2}^{(j)} \leq Y] \tag{5}$$

where $I[.]$ is 1 when $\mathrm{Dist}_{HJ2}^{(j)} \leq Y$ is true and 0 otherwise.

3 Data and empirical results

The data used in this paper are settlement prices for 6-month LIBOR futures options traded on the London International Financial Futures and Options Exchange (LIFFE) from January 2000 to December 2006. We exclude options whose prices are below 5 pence or have less than 14 day to maturity to avoid potential stale prices and microstructure issues. We calculate monthly returns for all the options as long as they are traded for two consecutive months. We group the option returns into five put and call portfolios according to their moneyness. The moneyness classes are chosen so that options are approximately evenly distributed.

Summary statistics for the option return portfolios are reported in Table 1.

[Table 1 around here]

All the call option portfolios have positive 1-month returns while put option returns tend to be negative except in one case but the returns are less negative for deep in-the-money (ITM) put portfolios. According to the Jarque-Bera test, the null hypothesis of normal distribution is rejected at the 5% level for all the portfolios except the most ITM call portfolio. This pattern is consistent with the index options market as put options, especially out-of-the-money (OTM) put options, are often overpriced as a precaution against extreme event like market crashes.

In order to have comparable results with previous studies, we follow Brennan et al. (2007) and employ a set of instrumental variables in the GMM estimation. They include a constant, the real interest rate, the maximum Sharpe ratio, and the volatility. Table 2 provides summary statistics for

[Table 2 around here]

bond yields, the state variables and their innovations. The data for inferring the state variables consist of UK government bond yields of different maturities from January 1996 to December 2006 available from the Bank of

England website. In Panel A, we can see a slow increase in monthly yields with increasing maturity and a rather stable and small standard deviation. In Panel B, we tabulate summary statistics for the state variables and their innovations. We notice that the average return from the nominal interest rate, taken as the midpoint between LIBOR and LIBID, is very close to the return for holding the market. This is due to a sharp correction in the market at the turn of the century.

Our main results are summarized in Tables 3 and 4. Table 3 presents the empirical results from the GMM estimation for comparison with the findings

[Table 3 around here]

in the extant literature like Brennan et al. (2007). Panel A gives the parameter values for the iso-elastic power pricing kernel. We first include all three candidate state variables in the pricing kernel. The risk aversion parameter γ is 5.21, close to the estimate of 4.05 in Bliss and Panigirtzoglou (2004) for 4-week UK index options with a power utility function. The coefficient for the real interest rate is 44.83 and significant. The positive coefficient is consistent with previous evidence of a negative risk premium associated with interest rate risk in Brennan et al. (2004) and Brennan et al. (2007). The coefficient for the maximum Sharpe ratio is -3.35, in line with the findings in Nielsen and Vassalou (2006) and Brennan et al. (2004). This implies a positive risk premium for this state variable. The coefficient for volatility risk is 10.81. However, this parameter is statistically insignificant, indicating that volatility risk is not priced. In addition, for iso-elastic power kernels the test for over-identifying restrictions are all rejected pointing to a lack of statistical robustness for this functional form (Hansen and Singleton (1982)).

The coefficient for volatility is invariably insignificant for the polynomial pricing kernels reported in Panels B and C. Interestingly, the value for the over-identifying test J_T is greatly reduced and now the over-identifying restrictions are accepted implying improved overall robustness. In addition, the polynomial pricing kernel with two state variables, the real interest rate and maximum Sharpe ratio, has the lowest value for the J_T test[2].

The market-related component of the pricing kernels are shown in Figure 1.

[Figure 1 around here]

We observe a high degree of variation in the scale and shape of the pricing kernels. Consistent with the literature, large sections of the polynomial pricing kernels are negative. There is a clear hump in the 3-term polynomials, while the 4-term polynomials exhibit an N-shape against market returns. They indicate that investors are actually risk seeking in the positive slope regions and they will pay to acquire fair gambles in wealth. This is not only

[2] As real interest rate and maximum Sharpe ratio are theoretically motivated together in Brennan et al. (2004) and Nielsen et al. (2006), we pair them in the empirical tests.

contrary to economic theory but also counter-intuitive.

Table 4 summarizes the empirical results when coefficients are obtained by

[Table 4 around here]

minimizing the second HJ distance. Panel A summarizes parameter values for the iso-elastic power pricing kernel. We first include all three state variables in the pricing kernel. The coefficients for the real interest rate, maximum Sharpe ratio, and volatility are 47.52, -4.05, and 10.53, respectively, similar in magnitude to the values in Table 3. With three state variables in the pricing kernel the second HJ distance is 0.28 with a p-value of 0.47. Therefore the null hypothesis of zero second HJ distance is rejected.

As volatility is consistently insignificant from the GMM results, we test the pricing kernel without volatility. We notice that the parameter for risk aversion increases slightly from 3.70 to 4.36. The coefficients for the real interest rate and maximum Sharpe ratio have the same sign and similar magnitude as in the previous specification. However, when we remove volatility from the pricing kernel the second HJ distance drops significantly from 0.28 to 0.07 and the p-value increases dramatically from 0.47 to 0.96. Taken together, these statistics indicate that the null hypothesis of zero second HJ distance can now be accepted[3].

[3]We also test the pricing kernel incorporating three state variables, the real interest rate, the maximum Sharpe ratio and the expected inflation. The results show that inflation risk is not priced in interest rate options.

When we remove the maximum Sharpe ratio from the pricing kernel and include only the real interest rate, the second HJ measure increases to 0.33 with a lower p-value. Removing all three state variables, the second HJ distance further increases to 0.55, the highest among all the specifications with the lowest p-value. The results from this panel emphasize the importance of incorporating non-wealth-related state variables in asset pricing kernels. They also demonstrate that interest rate options can be priced by a more parsimonious pricing kernel without volatility risk. This may be due to the stable interest rates and monetary conditions associated with the inflation targeting regime operated by the Bank of England over the course of our sample period.

In Panel B and Panel C, we tabulate parameter estimates for 3-term and 4-term polynomial pricing kernels, respectively. There is reasonably good consistency of the sign and magnitude of the parameter estimates between these two panels and also with Panel A for iso-elastic functional form. Specifically, in Panel B with third degree polynomial expansions, the second HJ measure drops significantly from 0.27 to 0.02 when volatility is removed from the pricing kernel while the p-value goes up significantly from 0.47 to 0.99. The null hypothesis is accepted only when real interest rate and maximum Sharpe ratio are included in the pricing kernel.

In Panel C when there are four polynomial terms, the pricing kernels are more flexible with the smallest pricing errors as indicated by the lowest second HJ distance and the highest p-values among the three panels. The zero second HJ distance is accepted in all three specifications. However, similar to results in the previous panels, the pricing kernel with only real interest rate and maximum Sharpe ratio has the smallest second HJ distance.

In the top two rows of Panel C, we observe that the second HJ distance is reduced only by 0.002 when we remove volatility from the pricing kernel. In order to test the statistical significance of this reduction, we carry out more simulations as outlined in Section 2.3. Results show that the probability of the second HJ distance without volatility being smaller than the second HJ distance with volatility is 0.73, hence the reduction is statistically robust.

The market-related component of the above pricing kernels are plotted in

[Figure 2 around here]

Figure 2. All the pricing kernels are strictly positive thus offering no arbitrage opportunity. They are predominantly[4] monotonically downward sloping, conforming to economic theories predicting a risk averse representative agent with diminishing marginal utility (Rubinstein (1976) and Lucas (1978)).

The pricing kernels depicted in Figure 2 are in contrast to the empirical pric-

[4]There is one small exception. The three-term Chebyshev polynomial pricing kernel with three state variables has a small hump for low market returns. Note that this is dominated by the four-term Chebyshev polynomial pricing kernel with two state variables which is the preferred kernel and is everywhere monotonically decreasing in market returns.

ing kernels recovered from US and UK index options in Brown and Jackwerth (2004), Liu et al. (2008), and Rosenberg and Engle (2002). Although they adopt different methodologies over different sample periods, the papers all report the pricing kernel to be hump-shaped, which is termed the *pricing kernel puzzle* in Brown et al. (2004). Our results indicate that utilizing information contained in non-wealth-related state variables and adopting a robust econometric methodology can produce empirical results that comply with theoretical predictions.

To summarize our empirical findings, for our sample period the 4-term polynomial approximation with the real interest rate and the maximum Sharpe ratio emerges as our preferred pricing kernel. After employing the second HJ distance as the objective function, even the linear pricing kernels meet the requirements of being arbitrage free and strictly monotonic. Therefore, the second HJ distance not only provides a robust criterion for testing the performance of candidate pricing kernels over contingent claims but also produces pricing kernels that are consistent with economic theory.

The difference in the state variable set between our results and those on index option pricing kernels in Brennan et al. (2007) may be due to the fact that the LIBOR has been more stable than a market-wide index such as the FTSE-100 index over our sample period. It also highlights the fact that a more parsimonious pricing kernel is appropriate for interest rate options despite asset pricing theories predicting a unique pricing kernel for all traded financial assets. Our results also reflect the economic and monetary environment in which the LIBOR options are traded. In 1997, the Bank of England became independent and was authorized to set the base interest rate in order to keep inflation within a low target band. Thus the economy enjoyed stable growth over our sample period. Our result that the expected inflation risk is not a concern for LIBOR option traders bears this out.

4 Concluding remarks

In this paper, we empirically evaluate the parametric pricing kernels that best price LIBOR options within the intertemporal CAPM framework. The usual GMM estimation produces problematic pricing kernels. Although the iso-elastic power kernels are monotonically decreasing in market returns as predicted by economic theory, they reject the over-identifying restrictions thus indicating a lack of statistical robustness for this functional form. Similarly, while the over-identifying restrictions are accepted for both the three- and four-term polynomial pricing kernels implying overall statistical robustness, the corresponding pricing kernels are hump-shaped and frequently negative in contrast to economic theory predictions.

This provides the motivation for applying the second HJ distance as objective function in estimating the pricing kernel parameters. The second HJ distance is particularly important for pricing derivatives and managed portfolios with option-like returns since it is defined as the pricing error over contingent claims. Our results show that the linear Chebyshev polynomial approximation approach to pricing kernels is preferred to the nonlinear iso-elastic power function approach. Our preferred four-term polynomial pricing kernels with two state variables — the real interest rate and maximum Sharpe ratio — are strictly positive and everywhere monotonically decreasing in market returns in conformity with economic theory.

This differs from previous research that examines US and UK index options and finds that volatility, in addition to real interest rate and maximum Sharpe ratio, is also priced. However, the recent extreme volatility in LIBOR during the credit crunch probably implies that our sample period was special insofar as volatility was very low throughout. This may well explain the lack of significance of volatility in our preferred pricing kernels. Extending our study to include recent years would be an interesting extension of the current research.

References

Abel, A., 1990. Asset prices under habit formation and catching up with the Joneses. American Economic Review 80, 38–42.

Bakshi, G., Cao, C., Chen, Z., 1997. Empirical performance of alternative option pricing models. Journal of Finance 52, 2003–2049.

Bakshi, G., Kapadia, N., 2003. Delta-hedged gains and the negative market volatility risk premium. Review of Financial Studies 16, 527–566.

Bansal, R., Viswanathan, S., 1993. No arbitrage and arbitrage pricing: a new approach. Journal of Finance 48, 1231–1262.

Bates, D., 1996. Jumps and stochastic volatility: exchange rate processes implicit in deutschemark option. Review of Financial Studies 9, 69–108.

Bick, A., 1987. On the consistency of the Black-Scholes model with a general equilibrium framework. Journal of Financial and Quantitative Analysis 22, 259–275.

Bliss, R. R., Panigirtzoglou, N., 2004. Option-implied risk aversion estimates. Journal of Finance 59, 407–446.

Breeden, D. T., 1979. An intertemporal asset pricing model with stochastic consumption and investment opportunities. Journal of Financial Economics 7, 265–296.

Brennan, M., 1979. The pricing of contingent claims in discrete time models. Journal of Finance 34, 53–68.

Brennan, M., Liu, X., Xia, Y., 2007. Option pricing kernels and the ICAPM, working paper, UCLA and University of Essex.

Brennan, M., Wang, A., Xia, Y., 2004. Estimation and test of a simple model of intertemporal asset pricing. Journal of Finance 59, 1743–1775.

Brown, D. P., Jackwerth, J. C., 2004. The pricing kernel puzzle: Reconciling index option data and economic theory, working paper, University of Wisconsin at Madison and University of Konstanz.

Buraschi, A., Jackwerth, J., 1999. Is volatility risk priced in the option market?, working paper, London Business School.

Chapman, D. A., 1997. Approximating the asset pricing kernel. Journal of Finance 52, 1383–1410.

Cochrane, J. H., 1996. A cross-sectional test of an investment-based asset pricing model. Journal of Political Economy 104, 572–621.

Cochrane, J. H., 2005. Asset Pricing. Princeton University Press, Princeton, N.J.

Coval, J., Shumway, T., 2001. Expected option returns. Journal of Finance 56, 983–1009.

Dittmar, R., 2002. Nonlinear pricing kernels, kurtosis preference, and the cross-section of asset returns. Journal of Finance 57, 369–403.

Dybvig, P., 1981. A practical framework for capital budgeting of projects having uncertain returns, working Paper, Washington University.

Epstein, L., Zin, S., 1989. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. Econometrica 59, 937–969.

Eraker, B., M. J., Polson, N., 2003. The impact of jumps in volatility and returns. Journal of Finance 58, 1269–1300.

Garcia, R., Luger, R., Renault, E., 2003. Empirical assessment of an intertemporal option pricing model with latent variables. Journal of Econometrics 116, 49–83.

Hansen, L. P., Heaton, J., Luttmer, E., 1995. Econometric evaluation of asset pricing models. Review of Financial Studies 8, 237–274.

Hansen, L. P., Jagannathan, R., 1991. Implications of security market data for models of dynamic economies. Journal of Political Economy 99, 225–262.

Hansen, L. P., Jagannathan, R., 1997. Assessing specification errors in stochastic discount factor models. Journal of Finance 52, 557–590.

Hansen, L. P., Singleton, K. J., 1982. Generalized instrumental variables estimation of nonlinear rational expectations models. Econometrica 50, 1269–1288.

Harrison, J., Kreps, D., 1979. Martingales and arbitrage in multiperiod securities markets. Journal of Economic Theory 20, 381–408.

Heath, D., Jarrow, R., Morton, A., 1992. Bond pricing and the term structure of interest rates: A new methodology for contingent claims valuation. Econometrica 60, 77–105.

Hull, J., White, A., 1990. Pricing interest-rate-derivative securities. Review of Financial Studies 3, 573–592.

Jagannathan, R., Wang, Z., 1996. The conditional capm and the cross section of expected returns. Journal of Finance 51, 3–54.

Jones, C. S., 2006. Nonlinear factor analysis of S&P 500 index option returns. Journal of Finance 61, 2325–2363.

Koop, G., 2003. Bayesian Econometrics. Wiley-Interscience.

Lettau, M., Ludvigson, S., 2001. Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying. Journal of Political Economy 109, 1238–1287.

Li, H., Xu, Y., Zhang, X., 2008. Evaluating asest pricing models using the second Hansen-Jagannathan distance, forthcoming Journal of Financial Economics.

Liu, X., Shackleton, M., Taylor, S. J., Xu, X., 2008. Empirical pricing kernels obtained from the UK index options market, forthcoming Applied Economics Letters.

Lucas, R. E., 1978. Asset prices in an exchange economy. Econometrica 46, 1429–1445.

Merton, R., 1973. An intertemporal capital asset pricing model. Econometrica 41, 867–888.

Nielsen, L., Vassalou, M., 2006. The instantaneous capital market line. Economic Theory 28, 651–664.

Pan, J., 2002. The jump-risk premia implicit in options: Evidence from an integrated time-series study. Journal of Financial Economics 63, 3–50.

Rosenberg, J., Engle, R., 2002. Empirical pricing kernels. Journal of Financial Economics 64, 341–372.

Rubinstein, M., 1976. The valuation of options and the pricing of uncertain income streams. Bell Journal of Economics and Management Science 7, 407–425.

Santa-Clara, P., Yan, S., 2008. Crashes, volatility, and the equity premium: Lesson from S&P 500 options, forthcoming Review of Economics and Statistics.

Singleton, K., Umantsev, L., 2002. Pricing coupon-bond options and swaptions in affine term structure models. Mathematical Finance 12, 427–446.

Vaden, J., 2004. Option trading and the CAPM. Review of Financial Studies 17, 207–238.

Wang, Z., Zhang, X., 2005. Empirical evaluation of asset pricing models: Arbitrage and pricing errors over contingent claims, working paper, Federal Reserve Bank of New York and Cornell University.

Table 1. Summary statistics of the LIBOR option portfolio returns

This table provides summary statistics of the monthly portfolio returns with LIBOR futures options from January 2000 to December 2006. The p-values for the Jarque-Bera test of normality are reported in the parentheses.

moneyness	no.	mean	std	skew	kurt	min	max	JB test
				Call Options				
≤ 0	56	0.089	0.698	2.370	7.967	-0.724	3.676	265.504 (0.000)
≤ 0.01	48	0.129	0.431	2.172	7.802	-0.476	2.348	245.635 (0.000)
≤ 0.02	47	0.052	0.169	1.069	2.786	-0.295	0.761	37.726 (0.000)
≤ 0.03	43	0.030	0.103	0.584	1.378	-0.210	0.404	9.729 (0.000)
> 0.03	50	0.019	0.063	0.299	0.588	-0.124	0.226	1.993 (0.369)
				Put Options				
≤ -0.01	31	-0.095	0.668	3.040	12.357	-0.715	3.631	584.844 (0.000)
≤ 0	34	-0.084	0.665	2.624	7.821	-0.733	2.897	275.443 (0.000)
≤ 0.01	48	0.005	0.398	2.291	6.559	-0.465	1.841	198.659 (0.000)
≤ 0.02	50	-0.009	0.164	1.622	4.362	-0.301	0.702	91.330 (0.000)
> 0.02	58	-0.008	0.098	0.877	2.201	-0.216	0.347	24.084 (0.005)

Table 2. Summary statistics of UK government bond yield and state variables

Panel A provides summary statistics of UK government bond yields of different maturities from January 1996 to December 2006. These data are taken from the Bank of England website. Panel B shows summary statistics of the state variables and the innovations of the state variables used in the pricing kernels. The state variables are real interest rate r, inflation π, maximum Sharpe ratio η, riskfree rate r, and returns on aggregate wealth proxied by FTSE-100 index returns.

Panel A: UK government bond yield (%)									
Maturity (yr)	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	10.0
Mean	5.14	5.23	5.28	5.31	5.32	5.32	5.32	5.31	5.30
Stdev	1.05	1.02	1.01	1.01	1.02	1.03	1.05	1.06	1.10
Max	7.23	7.03	7.13	7.38	7.58	7.75	7.89	8.00	8.18
Min	3.20	3.26	3.42	3.58	3.71	3.82	3.90	3.98	4.02

Panel B: State variables								
	r	Δr	η	$\Delta\eta$	σ	$\Delta\sigma$	r_f	r_W
Mean	0.033	0.000	0.896	0.001	0.158	-0.001	0.048	0.060
Stdev	0.056	0.013	0.796	0.170	0.050	0.014	0.012	0.480
Skew	-0.683	4.263	-0.002	0.755	0.503	1.175	0.342	-0.269

Table 3. GMM parameter estimates of the pricing kernels

Panel A gives the parameters estimated for the iso-elastic power function

$$m_{t+1} = \beta(R_{w,t+1})^{-\gamma} \exp^{b_1 \Delta r_{t+1} + b_2 \Delta \eta_{t+1} + b_3 \Delta \sigma_{t+1}} .$$

Panels B and C present the parameters estimated for the polynomial pricing kernel

$$m_{t+1} = \wp^n(R_{w,t+1}) \exp^{b_1 \Delta r_{t+1} + b_2 \Delta \eta_{t+1} + b_3 \Delta \sigma_{t+1}},$$

where γ is the risk aversion parameter, $R_{w,t+1}$ is portfolio returns, $\wp^n(R_{w,t+1})$ consists of n-term Chebyshev polynomials, and r, η, and σ stand for the real interest rate, maximum Sharpe ratio, and volatility, respectively. J_T is over-identifying test statistic. The p-values are reported in the parentheses.

β	γ	r	η	σ	J_T
Panel A: Iso-elastic power function					
0.533	5.211	44.834	-3.354	10.810	153.960
(0)	(0.120)	(0)	(0)	(0.187)	(0)
0.642	5.869	39.694	-3.039		147.641
(0)	(0.012)	(0)	(0)		(0)
0.901	3.360	11.431			158.935
(0)	(0.034)	(0)			(0)
1.001	4.116				153.137
(0)	(0)				(0)
Panel B: Polynomial approximation (n=3)					
		45.960	-3.481	14.949	48.859
		(0.002)	(0.001)	(0.248)	(0.636)
		47.773	-3.079		39.632
		(0.020)	(0.002)		(0.928)
		36.667			52.468
		(0.002)			(0.572)
Panel C: Polynomial approximation (n=4)					
		44.935	-3.602	12.966	39.383
		(0.029)	(0.010)	(0.346)	(0.901)
		40.945	-3.695		36.888
		(0.025)	(0.005)		(0.955)
		20.794			42.555
		(0.012)			(0.870)

Table 4. Second HJ distance parameter estimates of the pricing kernels

Panel A gives the parameters estimated for the iso-elastic power function

$$m_{t+1} = \beta(R_{w,t+1})^{-\gamma} \exp^{b_1 \Delta r_{t+1} + b_2 \Delta \eta_{t+1} + b_3 \Delta \sigma_{t+1}} .$$

Panels B and C present the parameters estimated for the polynomial pricing kernel

$$m_{t+1} = \wp^n (R_{w,t+1}) \exp^{b_1 \Delta r_{t+1} + b_2 \Delta \eta_{t+1} + b_3 \Delta \sigma_{t+1}},$$

where γ is the risk aversion parameter, $R_{w,t+1}$ is portfolio returns, $\wp^n(R_{w,t+1})$ consists of n-term Chebyshev polynomials, and r, η, and σ stand for the real interest rate, maximum Sharpe ratio, and volatility, respectively. The p-values for the 2nd HJ distance are also reported.

β	γ	r	η	σ	2nd HJ distance	p-value
Panel A: Iso-elastic power function						
0.652	3.696	47.518	-4.052	10.528	0.275	0.471
0.755	4.356	33.224	-3.331		0.073	0.961
0.811	4.306	10.970			0.334	0.465
0.990	3.803				0.546	0.245
Panel B: Polynomial approximation (n=3)						
		40.219	-3.397	12.468	0.268	0.474
		41.597	-4.004		0.024	0.988
		18.143			0.193	0.585
Panel C: Polynomial approximation (n=4)						
		42.954	-3.083	15.170	0.003	1.000
		46.828	-3.270		0.001	1.000
		26.982			0.060	0.971

Figure 1. Market-related component of the pricing kernels estimated by the GMM

The pricing kernels are either with real interest rate (1 sv), with real interest rate and maximum Sharpe ratio (2 sv), or with real interest rate, maximum Sharpe ratio, and volatility (3 sv), where sv stands for state variables.

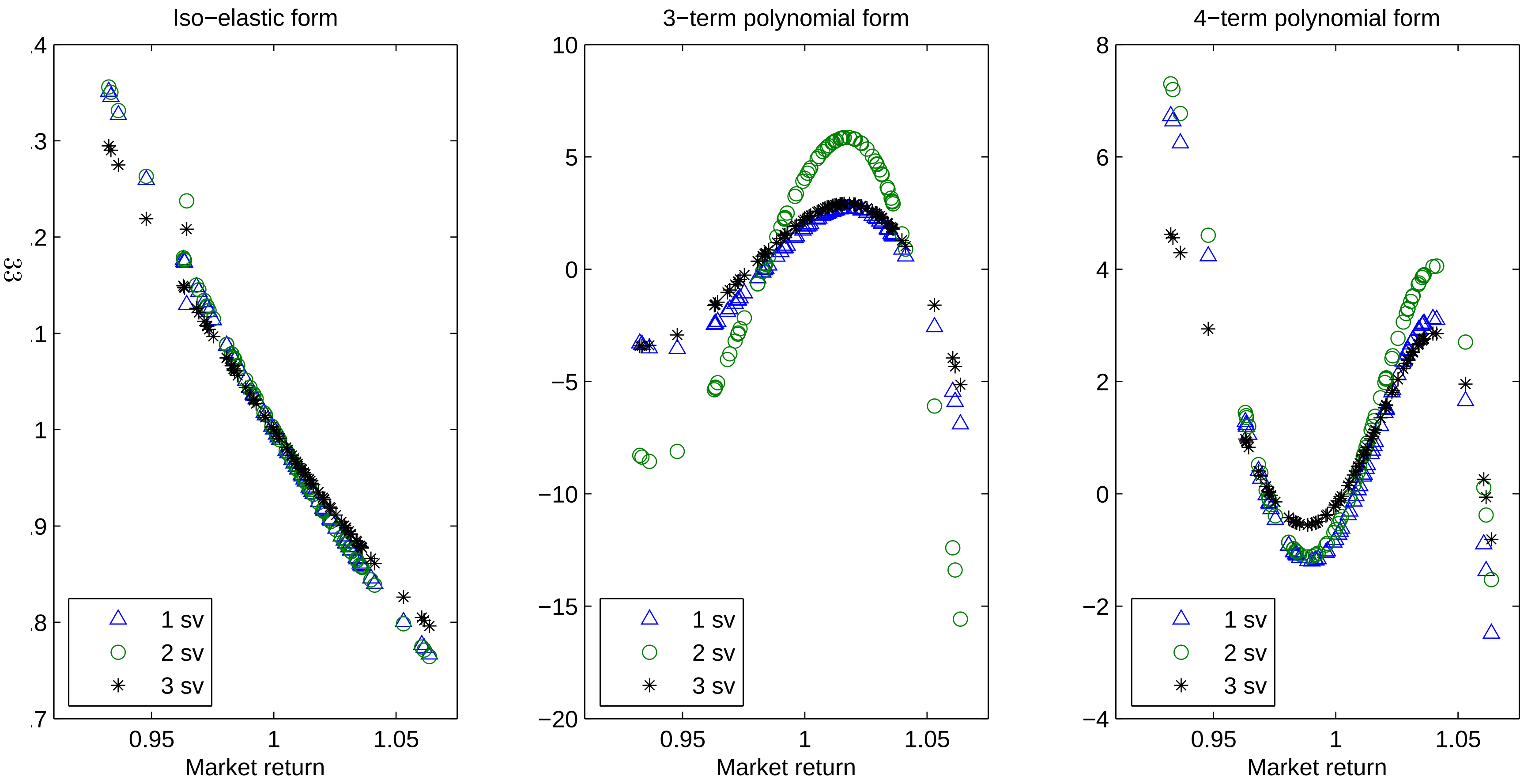

Figure 2. Market-related component of the pricing kernels estimated by minimizing the second HJ distance

The pricing kernels are either with real interest rate (1 sv), with real interest rate and maximum Sharpe ratio (2 sv), or with real interest rate, maximum Sharpe ratio, and volatility (3 sv), where sv stands for state variables.