Information Aggregation and Optimal Structure of the Executive[*]

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Abstract

We provide a novel model of executives in parliamentary democracies that accounts for key features of these institutions: decision-making authority is assigned to individual ministers; and policy relevant information is aggregated through communication between politicians. Politicians hold idiosyncratic biases and have private information relevant either to all policies or to a subset of them. When their information is relevant to all policies and communication takes place in private all decisions should be centralised to a single politician. A government that holds cabinet meetings, where any information communicated to one minister is made available to all, outperforms one where communication is private: a multi-member cabinet can then be optimal. We study the optimal form of authority allocation and find (i) that centralisation is non-monotonic in the degree of ideological divergence between politicians; and (ii) the cabinet need not be single peaked around the most moderate politician, and in fact may not even be ideologically connected. In a large cabinet, however, all power should be centralised to the most moderate politician. In the case where uncertainty is policy specific, and a single politician is informed on each policy, power should never be fully decentralised. In fact numerical simulations show that the optimal executive structure is no less centralized than in the common-state case. Our model provides a justification for centralised authority and the use of cabinet meetings to enhance the quality of policies implemented.

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1. Introduction

The relationship between the President, Congress, and its committees, has been the subject of numerous theoretical studies, but the allocation of decision-making authority in a parliamentary democracy is less well understood. This might reflect the fact that such authority is not clearly codified in these systems. For example, central governance in the United Kingdom is underpinned by several piecemeal and unwritten conventions: the role of the Prime Minister and Cabinet have no constitutional status and both arose due to historical circumstance, practice, and convention. Even when the role of those with executive authority is codified– as in Germany, where article 65 of the Basic Law defines the status of the chancellor, cabinet and ministers– the constitution is typically silent about how the executive should be structured and the relations between its decision-makers.

In the absence of constitutional guidelines that define a clear game form, we propose a new model that incorporates common and distinguishing features of executive governance in parliamentary democracies. The first is that decision-making authority over different policies is assigned to individual ministers who head state departments. The second is the existence of a cabinet: the set of ministers who hold executive office meet at a designated time and place where policy-relevant information, obtained through discussion and debate both outside and inside the chamber, is shared. Though we restrict to the set of institutions that allow for assignment of authority to individuals with collective deliberation over outcomes, we nevertheless address two core aspects of optimal executive structure in parliamentary democracies.

First, and central to our investigation, is the role of the cabinet. It has been been studied historically as an efficient means of rationalising parliamentary activity Cox (1987), and more formally as a system of incentives operated by a Prime Minister (Dewan and Myatt, 2007; Indridason and Kam, 2008; Dewan and Myatt, 2010). To our knowledge, however, no model exists that assesses the role of cabinet meetings. There are several practical justifications for such meetings: they are necessary for a collective decision-making body that votes, though few real world cabinets have explicit voting procedures; they are one of several bodies that help coordinate executive activities;[1] and they enable collective responsibility, by which the policies implemented by ministers are those of the government and have the support of all ministers. We assess the impact of cabinet meetings on the quality of policies that are implemented by the executive. A key principle of cabinet government is that

[1]see for example Saalfeld (2000), pages 60-63.

significant policy issues that fall within the remit of a minister's portfolio should be discussed in cabinet. We contrast the quality of policy outcomes obtained when this principle is upheld with that when cabinet meetings are replaced by informal modes of communication, such as private conversations between policymakers and politicians.

Second, we address the extent to which power in cabinet should be centralised. In the United Kingdom, for example, a longstanding controversy exists "about whether monocratic control is exercised by the premier" reflecting "normative anxieties about Britain's unbalanced constitution"(Dunleavy and Rhodes, 1990). Similarly in Germany, key constitutional principles that guide the allocation of authority often conflict with one another so the "question of whether an issue falls under a departmental minister's competence or the Chancellor's right to determine the government's policy is sometimes ambiguous"(Saalfeld (2000), page 51). In the absence of clear constitutional guidelines, we study how the allocation of power affects the quality of the final decisions implemented.

We relate this quality to the amount of policy relevant information that decision-makers have at their disposal. As in classic models of information going back to Condorcet, and more recently Feddersen and Pesendorfer (1996), in our model there is uncertainty over the best policy to pursue. This is related to corresponding uncertainty about underlying fundamentals: different prognosis about economic growth might affect the optimal policies on public spending, defence, and immigration, for example. We assume that information about such fundamentals is dispersed amongst the set of politicians who are the players in our game. Differences in a politician's private information might reflect differences in his information sources: a politician forms a viewpoint based on his local understanding, research, or the views of interest groups. We impose a coarse information structure that arguably provides a more accurate depiction of the information obtained from such sources. For example a politician may observe whether a national recession has had an impact on his constituency or not that impacts his preferences over economic and social policy.

Specifically we develop a game-theoretic model where there are a set of politicians and issues to be decided. The game has three stages: first, politicians receive private information that is relevant either to all policies or to a subset of them; next, politicians use simultaneous cheap talk messages to convey their information to ministers; who then, finally, implement policies. Politicians–we might think of these as members of a governing majority–share a common goal of implementing well informed policies. But they hold different beliefs as to the best policy to pursue. These differences stem from the variation in their information sources as well as idiosyncratic biases that reflect their world-view, or those of their constituents, and can prohibit the truthful revelation of private information to a decision-maker. We ask whether variation in the executive structure–how centralised it is, who has authority, and whether cabinet meetings are held–impacts on the strategic communication of this policy relevant information.

Communication may be affected by the way authority is allocated: a politician may be truthful when the issue is decided by one minister, while not doing so when it is decided by another. It may also be that a politician is truthful when a minister has control over one issue but not when she has more authority. To explore this issue, and with the restriction that the power to finally determine each policy is in the hands of a single minister, we allow for all allocations: at one extreme all politicians could share authority; at the other all power could be centralised to a single minister; alternatively power might be shared, either equally or unequally, between a cabinet of ministers.[2]

The structure of communication can also have consequences. We assume that at cabinet meetings any information available to one minister, and upon which he consequently bases his decision, is made available to all ministers. This contrasts with a situation where communication with ministers is private and creates different incentives for strategic communication. Notably, without cabinet meetings a politician could convey a different message to each decision maker, whilst this is not possible with them. A politician may then choose to be truthful only when talking in confidence with a minister. Or, by the same token, truthful communication may be made possible by the fact that information is shared. We contrast a world with cabinet meetings to one where communication between a minister and a politician takes place only in private.

We first show that under all circumstances of our model (any allocation of authority, communication structure, and whether a minister's private information is specific to all policies or a subset of them) ex-ante welfare depends on two features: the moderation of decision-makers that minimizes aggregate ideological loss; and their information that reduces the aggregate residual variance of decisions implemented. Our first main result shows that in the absence of cabinet meetings and when politicians private information is relevant to the entire set of policies to be implemented, the optimal assignment involves fully centralised power exercised by a unique individual. With cabinet

[2]In the United Kingdom, for example, decision-making authority has been centralized in a cabinet dominated by the prime minister since the late 19th century, whereas during Parliament's previous "golden age" power as more equally dispersed amongst individual members of Parliament (see Cox (1987) for a discussion of the emergence of centralized authority in Victorian England).

meetings it may instead prove optimal for decision-making authority to be shared between ministers. A politician may be unwilling to communicate truthfully to a single leader who is ideologically distant, while being truthful when power is shared with another cabinet member whose ideology is intermediate. This apparently innocuous observation leads to a powerful normative result: cabinet meetings outperform private communication as a form of information aggregation and so leads to more informed policies being implemented. This central result holds whether private conversations can be held outside of cabinet or not.

Next we characterize the optimal degree of centralisation in a cabinet and show, surprisingly, that it is nonmonotonic in the ideological divergence between politicians. When it is small then ideological disagreement does not prohibit information aggregation and so it is better that a moderate executive leader emerges. When it is large then politicians are unable to communicate truthfully with another and so full centralisation to a moderate leader is desirable. For intermediate values, however, information aggregation is sensitive to the allocation of authority. Then it is better that power is shared in a multi-member cabinet.

Analyzing the allocation of authority further we find that such a cabinet need not be ideologically connected: when two politicians with different ideology share power then there may be a politician with intermediate ideology who has none. The finding that the optimal allocation of authority may involve such "holes" contrasts with the connected-winning coalitions in Axelrod (1970). A similar prediction arises in models of complete information where bargaining takes place over policies and portfolio allocation (Austen-Smith and Banks, 1988) and so our results can be seen as providing a corresponding and novel information aggregation rationale for non-connected executive coalitions.

We then consider the limit case of a large cabinet and show that all decision making authority should be concentrated to politicians who are ideologically close to the most moderate one. In an appendix we perform numerical simulations–randomly drawing ideology profiles and calculating the optimal policy assignment–of an intermediately sized parliament. We find that fully centralized authority is fairly frequent, and that, when it is optimal for authority to be shared, a single minister (perhaps a Prime Minister) should be assigned a large share (on average, at least 80 %) of decisions.

Our results where information is relevant to all policies and widely dispersed provides a novel account for the stylized fact that in parliamentary democracy the diverse preferences of an assembly sit alongside fairly centralized decision-making authority.[3] A challenging case is where uncertainty is specific to each policy and expertise is widely dispersed. Does this lead to decentralized decision-making authority assignments? Surprisingly not. We find that full decentralization is *never* the optimal decision-making authority assignment. In fact, all policy decisions should be granted to the most moderate politician, unless the policy expert has "intermediate" ideology, (i.e., neither she not too moderate, nor too extreme). Numerical simulations reveal that the optimal decision-making authority assignment is no less centralized than in the common-state case.

2. Literature Review

Our paper relates to a broad literature on the politics of information aggregation, that builds on the contributions by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1996, 1997, 1998) who study situations where, as in our model, players share common interests. Following the seminal work by Crawford and Sobel (1982) and Gilligan and Krehbiel (1987), communication games are now a canonical framework to study the politics of information aggregation. Most of this literature has focused on the aggregation of information in committees where a single outcome is determined by voting. In our model, information is aggregated through communication. While Coughlin (2001); Doraszelski, Gerardi, and Squintani (2003); Austen-Smith and Feddersen (2006) explore the consequences of allowing committee members to communicate before they vote,[4] we instead consider communication when a set of policy outcomes are allocated to individuals with decision-making rights that better fits the cabinet setting. Our study of optimal executive structure from an information aggregation perspective relates to work by Dragu and Board (2013) who show that the imposition of judicial review can lead to more informed outcomes.

We analyze our game using the multi-player communication model by Galeotti, Ghiglino, and Squintani (2009) who build on Morgan and Stocken (2008). Its key feature is a coarse information structure that has the implication that a message sent to a decision-maker is either truthful or not. This contrasts with information aggregation with a continuous signal space and a single policy dimension where information can be conveyed only when the message space is partitioned, as in

[3]In the United Kingdom, for example, decision-making authority has been centralized in a cabinet since the late 19th century, whereas during Parliament's previous "golden age" the power to initiate policy rested with individual members (see Cox (1987) for a discussion of the emergence of centralized authority in Victorian England).

[4]A different, normative, approach consists in devising optimal mechanisms for optimal decisions in committees (see, e.g.Gerardi, McLean, and Postlewaite (2009) or Gershkov and Szentes (2009))

the seminal paper by Crawford and Sobel (1982). Extensions of that paper have shown, however, that in multiple dimensions the decision-maker can extract all information from an informed agent Battaglini (2002). Dewan and Hortalla-Valve (2011) extend that framework to provide insights into a Prime Minister's control over ministers who are perfectly informed. In our model with coarse information, by contrast, no player can be perfectly informed and so the quality of final policies depends on the number of politicians communicating truthfully. Such multi-player communication is, we believe, the relevant aspect of information aggregation in parliaments and cabinets.

Imposing this structure on the message space provides tractability and substantive new political insights as witnessed by recent papers by Patty and Penn (2013) on small networks, Dewan and Squintani (2012) on factions in political parties, and Gailmard and Patty (2009) on delegation and transparency with sequential decision-making. Our paper significantly extends the multi-player communication model in considering the possibility that players have specific information about some decisions but not others. Moreover in studying the question of the optimal assignment of decision-making rights we derive an entirely new set of theoretical results.

A related paper by Dewan and Squintani (2012) adapts the multi-player communication model to analyze the formation of party factions. In their model each party politician is endowed with some say over the party manifesto. They can choose voluntarily to join a faction. Doing so involves delegating authority to the faction leader who makes the final choice. After factions have formed the party politicians communicate their information to faction leaders in private. The authors study a situation where communication can be made only within factions and one where the factional structure does not prohibit intra-party communication. They analyze the welfare consequences of factionalism in each case. The key features of that model–private communication and voluntary delegation of authority–are relevant to party structures. Here, in analyzing the optimal structure of the executive in a parliamentary democracy our main focus is public communication. The optimal allocation of authority is that which would be chosen by a welfare maximizing prime minister, and does not arise through a process of voluntary delegation.

Recent and related work by Patty (2013) complements ours in looking at how the exclusion and inclusion of cabinet members affects strategic communication. While sharing the same broad motivation and some modeling choices, the two papers answer distinct questions. Our focus is on the optimal assignment of executive authority and on the comparison between public and private communication. We we show that cabinet meetings are optimal and present novel insights as to the degree to which power should be centralised in a cabinet and to whom it should be allocated. Patty (2013), by contrast, considers the optimal size and composition of cabinet meetings with an exogenous allocation of decision-making authority. He does not make the restriction that cabinet meetings are limited to those who hold executive authority; indeed in an interesting finding he shows that it may be optimal for some individuals with executive authority to be excluded while others without such authority should be included.

The result that cabinet meetings yield higher welfare than private conversations can be related to Farrell and Gibbons (1989) and Goltsman and Pavlov (2011) who compare public communication and private communication in a much simpler game with a single expert and two decision makers, but critically do not consider the possibility of reassignment of decision-making authority.

More broadly, our results on the deliberative value of collective meetings provide a new angle on the study of cabinet governance that typically have focussed on the cabinet as a system of incentives, managed strategically by a Prime Minister as in citations mentioned earlier. And we provide an information aggregation justification for centralisation of decision-making rights in an assembly (either to a single leader or a cabinet with public information) that contributes to a broad rational choice literature on why majorities adopt restrictive procedures, that looks at the role of committees (Gilligan and Krehbiel, 1987), political parties (Cox and McCubbins, 1993) and cabinets (Cox, 1987). Recent contributions to this debate include Diermeier and Vlaicu (2011) and Diermeier, Prato, and Vlaicu (2013).

3. Model

We consider the following information aggregation and collective decision problem. Suppose that a set $\mathcal{I} = \{1, ..., I\}$ of politicians form a Parliamentary majority who provide consent for a governing executive. They are faced with the collective task of choosing an assignment $a : \mathcal{K} \to \mathcal{I}$ of policy decisions that grants decision-making authority over a set of policies $\mathcal{K} = \{1, ..., K\}$. For each $k \in \mathcal{K}$, the decision $\hat{y}_k$ is a policy on the left-right spectrum $\Re$. For simplicity we think of the assignment as granting complete jurisdiction over policy k, though of course other interpretations, such as, for example, the assignment of agenda-setting rights could also be incorporated. The important element is that decision-making authority over each policy is granted to a unique individual.

In a fully-decentralized executive each policy decision is assigned to a different politician so that $a(k) \neq a(k')$ for all k, k' in $\mathcal{K}$. At the opposite end of the spectrum, all decisions are centralized to a single leader so that $a(k) = a(k')$ for all k, k' in $\mathcal{K}$. We let the range of a be denoted by $a(\mathcal{K}) \subseteq \mathcal{I}$, which we term as the set of politicians with decision-making authority. We refer to such politicians as *ministers*. We let a_j denote the number of policies that minister j takes under assignment a. Our specification thus allows us to capture important elements of the executive body: its size– beyond the extremes of full decentralization and the leadership of one, there are a range of possibilities; and its balance–amongst the set of ministers some may have more authority than others.

Politicians are ideologically differentiated, and care about all policy choices made. For any policy decision $\hat{y}_k$, their preferences also depend on unknown states of the world θ_k, uniformly distributed on $[0,1]$. Specifically, were she to know the vector of states $\theta = (\theta_k)_{k\in\mathcal{K}}$, politician i's payoff would be

$$u_i(\hat{\mathbf{y}}, \theta) = -\sum_{k=1}^{K} (\hat{y}_k - \theta_k - b_i)^2 .$$

Hence, each politician i's ideal policy is $\theta_k + b_i$, where the bias b_i captures ideological differentiation, and we assume without loss of generality, that $b_1 \leq b_2 \leq ... \leq b_I$. The vector of ideologies $\mathbf{b} = \{b_1, ..., b_I\}$ is common knowledge.

Each politician i has some private information on the vector θ. Specifically, we make two opposite assumptions on politicians' information. Firstly, for some of our analysis we assume that uncertainty over all policies is captured by a single *common* state that represents the underlying economic and social fundamentals. For example, an underlying economic recession will influence policy choices of all ministries, from the Home office immigration policy, to the fiscal policy of the Chancellor of the Exchequer. We represent these fundamentals by a single uniformly distributed state of the world θ, so that $\theta_k = \theta$ for all k, and each politician i's signal s_i is informative about θ. Conditional on θ, s_i takes the value equal to one with probability θ and to zero with probability $1-\theta$. Secondly, and in an alternative specification we say that the politician's information is *policy specific*. Each policy has its own underlying set of circumstances over which politicians may be informed. Thus the random variables θ_k are identical and independently distributed across $k \in \mathcal{K}$, and each politician k receives a signal $s_k \in \{0,1\}$ about θ_k only, again with $\Pr(s_k = 1|\theta_k) = \theta_k$. In the case of policy specific information, we take $\mathcal{K}$=$\mathcal{I}$ so that each politician is informed on a single issue. This specification allows us to explore a situation where expertise on policies varies and is widely dispersed amongst the set of politicians.

In our set-up, politicians can communicate their signals to each other before policies are executed. We allow for such communication to take the form either of private conversations or general meetings. We might think of the former as taking place over dinner, or via a secure communication network, with no leakage of information transmitted. Hence, each politician i may send a different message $\hat{m}_{ij} \in \{0, 1\}$ to any politician j. In a meeting, by contrast, a politician is unable to communicate privately with a decision-maker as all communication is available to those who exercise authority. Hence, each politician i sends the same message $\hat{m}_i$ to all decision makers. A pure communication strategy of player i is a function $m_i(s_i)$.

Communication between politicians allows information to be transferred. Up to relabelling of messages, each communication strategy from i to j may be either *truthful,* in that a politician reveals her signal to j, so that $m_{ij}(s_i) = s_i$ for $s_i \in \{0, 1\}$, or *"babbling"*, and in this case $m_{ij}(s_i)$ does not depend on s_i. Hence, the communication strategy profile $\mathbf{m}$ defines the truthful communication network $\mathbf{c}(\mathbf{m})$ according to the rule: $c_{ij}(\mathbf{m}) = 1$ *if and only if* $m_{ij}(s_i) = s_i$ for every $s_i \in \{0, 1\}$, which provides us with the communication structure within the set of politicians $\mathcal{I} = \{1, ..., I\}$.

The second strategic element of our model involves the final policies implemented. Conditional on her information, and after communication has taken place, each assigned decision-maker implements her preferred policy. We denote a policy strategy by i as $y_{i,k} : \{0, 1\}^{\mathcal{I}} \to \Re$ for all policies k such that $i = a(k)$. Given the received messages $\hat{\mathbf{m}}_{-i,i}$, by sequential rationality, politician i chooses $\hat{y}_{i,k}$ to maximize expected utility, for all k such that $i = a(k)$. So,

$$y_{i,k}(s_i, \hat{\mathbf{m}}_{i,-i}) = b_i + E[\theta_k | s_i, \hat{\mathbf{m}}_{-i,i}], \tag{1}$$

and this is due to the quadratic loss specification of players payoffs.

Given an assignment a, an equilibrium then consists of the strategy pair $(\mathbf{m}, \mathbf{y})$ and a set of beliefs that are consistent with equilibrium play. We use the further restriction that an equilibrium must be consistent with some beliefs held by politicians off the equilibrium path of play. Thus our equilibrium concept is pure-strategy Perfect Bayesian Equilibrium. Fixing policy assignment a, then, regardless of the communication mode adopted, there may be multiple equilibria $(\mathbf{m}, \mathbf{y})$. For example, the strategy profile where all players "babble" is always an equilibrium. Equilibrium multiplicity makes the ranking of decision-making authority assignments a not well defined: Given the same assignment a, different equilibria may yield different payoffs to the politicians, so that the politicians' initial collective decision over assignments a is impossible. To avoid this issue, we assume that, for any assignment a, politicians coordinate on the equilibria $(\mathbf{m}, \mathbf{y})$ that give them the highest payoffs.[5] This equilibrium selection is standard in games of communication.

We focus on the *optimal assignment of authority*, defined as the assignments a that induce the equilibria $(\mathbf{m}, \mathbf{y})$ with the largest joint payoffs:

$$\mathcal{W}(\mathbf{m}, \mathbf{y}; a) = -\sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}} E[(y_{a(k),k} - \theta_k - b_i)^2].$$

Our notion of welfare is ex-ante Utilitarian: assume that the collective decision on the optimal assignment a, by politicians in $\mathcal{I}$ maximizes the sum of their expected payoffs. But for some of our results, we can invoke the weaker principle of Pareto optimality.

We first provide a useful derivation for our main results. We first say that a politician j's *moderation* is $\left|b_j - \sum_{i \in \mathcal{I}} b_i / I\right|$, the distance between b_j and the average ideology $\sum_{i \in \mathcal{I}} b_i / I$. We note that politicians' moderation does not depend on the assignment a, nor on the equilibrium $(\mathbf{m}, \mathbf{y})$. Second, we let $d_{j,k}(\mathbf{m})$ denote politician j's *information* on the state θ_k given the equilibrium $(\mathbf{m}, \mathbf{y})$. Specifically, $d_{j,k}(\mathbf{m})$ consists in the number of signals on θ_k held by j, including her own, after communication has taken place and before she makes her choice.

With common value information, each politician's information coincides with the number of politicians communicating truthfully with her, plus her own signal. Later on in our analysis we will adopt a specification with policy specific knowledge, each politician j may hold at most one signal on each θ_k, either because s_k is her own signal ($j = k$), or because s_k was communicated by k to j given the equilibrium communication structure $\mathbf{c}(\mathbf{m})$.

Armed with these definitions, and given an assignment a and an equilibrium $(\mathbf{m}, \mathbf{y})$, we show in the Appendix that the equilibrium ex-ante welfare $\mathcal{W}(\mathbf{m}, \mathbf{y}; a)$ can be rewritten as:

$$\mathcal{W}(\mathbf{m}, \mathbf{y}; a) = \underbrace{-\sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}} (b_i - b_{a(k)})^2}_{\textit{aggregate ideological loss}} - \underbrace{\sum_{k \in \mathcal{K}} \frac{I}{6[d_{a(k),k}(\mathbf{m}) + 2]}}_{\textit{aggregate residual variance}}. \tag{2}$$

[5]Indeed, it can be easily shown that for any given assignment a each politicians' ranking among the possible equilibria $(\mathbf{m}, \mathbf{y})$ is the same (see Galeotti, Ghiglino, and Squintani (2009), Theorem 2.)

This decomposes the welfare function into two elements– aggregate ideological loss and the aggregate residual variance of the politicians' decisions– and proves useful in the results that follow.[6]

4. Private Conversations in Common State Model

We begin our study of the optimal assignment of decision making in an environment where underlying fundamentals are common to all policies so that politicians' information is relevant to all decisions. Initially we explore the situation where politicians communicate only in private with decision-makers. We first describe the equilibrium communication structure given any policy assignment a. The characterization extends Corollary 1 of Galeotti, Ghiglino, and Squintani (2009) to the case of arbitrary policy assignments. For future reference, for any assignment a, we write $d_j^*(a)$ as the information $d_j(\mathbf{m})$ associated with any welfare-maximizing equilibrium $(\mathbf{m}, \mathbf{y})$.[7] When the state θ is common across policies, and the communication is private, we prove in the Appendix that the profile $\mathbf{m}$ is an equilibrium if and only if, whenever i is truthful to j,

$$|b_i - b_j| \leq \frac{1}{2\left[d_j(\mathbf{m}) + 2\right]}. \tag{3}$$

The fact that truthful communication can arise in our model is a consequence of the binary signal structure. When the direction of policy under truthful revelation is opposite to the bias of the politician relative to the minister (for example a politician with $b_i > b_j$ and $s_i = 0$) he is concerned that by misrepresenting his signal the final policy will overshoot his preferred one. An important consequence of equilibrium condition 3 is that truthful communication from politician i to minister j is independent of the specific policy decisions assigned to j and of the possibility of communicating with any other politician j'. Furthermore, truthful communication from politician i to minister j becomes less likely with an increase in the difference between their ideological positions.[8]

[6]Note that, statistically, the residual variance may be interpreted as the inverse of the precision of the politicians' decisions.

[7]Because there is a single politicy-relevant state θ, we drop the subscript k from the notation $d_{j,k}(\mathbf{m})$, the number of informative signals held by politician j on state θ_k in equilibrium.

[8] A perhaps more surprising effect is that the possibility for i to communicate truthfully with j decreases with the information held by j in equilibrium. To see why communication from i to j is less likely to be truthful when j is well informed in equilibrium, suppose that $b_i > b_j$, so that i's ideology is to the right of j's bliss point. Suppose j is well informed and that politician i deviates from the truthful communication strategy –she reports $\hat{m}_{ij} = 1$ when $s_i = 0$–then she will induce a small shift of j's action to the right. Such a small shift in j's action is always beneficial in expectation to i, as it brings j's action closer to i's (expected) bliss point. Hence, politician i will not be able to truthfully communicate the signal $s_i = 0$. By contrast, when j has a small number of players communicating with her, then i's report $\hat{m}_{ij} = 1$ moves j's action to the right significantly, possibly beyond i's bliss point. In this case, biasing rightwards j's action may result in a loss for politician i and so she would prefer to report truthfully- that is, she will not deviate from the truthful communication strategy.

Having characterized equilibrium communication between politicians and ministers we explore the implication of expression 3 with respect to the assignment of executive authority.

Proposition 1. *Suppose that θ is common across policies, and that communication is private. For generic ideologies* $\mathbf{b}$*, any Pareto optimal assignment involves decision-making authority being centralized to a single leader j: that is $a(k) = j$ for all k.*

The finding that all decisions should be assigned to a single leader and, hence, executive authority should be fully centralized, follows from two different facts. First, truthful communication from politician i to minister j in equilibrium is independent of the specific policy decisions assigned to j, (or to any other politician j'). Second, the stipulation that every politicians' information is relevant for all policies implies that politicians and policies are "interchangeable." As a consequence of these two facts, whoever is the optimal politician to make one policy decision will also be the optimal politician to make all of them. This result holds with our utilitarian welfare criterion and under the weak welfare concept of Pareto optimality. In sum, with the restriction to private conversation between a politician and a minister, the optimal size of the executive is one: leadership by a dominant Prime Minister emerges.

5. Cabinet Meetings vs. Private Conversations in the Common State Model

Having considered communication via private meetings, we now study optimal assignment of decision making authority when there is a cabinet providing a forum for information to be conveyed to the set of ministers. This change to the communication environment affects the strategic calculus of information transmission: it is possible that politician i would not wish to communicate with minister j on a policy if that information is shared with minister j'; conversely, politician i might share information with j *because* minister j' also has access to that information.

The next result characterizes communication equilibria under any policy assignment. It extends Theorem 1 of Galeotti, Ghiglino, and Squintani (2009) to the case of arbitrary policy assignments.

Lemma 1. *Suppose that the state θ is common across policies k, and communication takes place in cabinet meetings. The strategy profile* $\mathbf{m}$ *is an equilibrium if and only if, whenever i is truthful,*

$$\left| b_i - \sum_{j \neq i} b_j \gamma_j(\mathbf{m}) \right| \leq \sum_{j \neq i} \frac{\gamma_j(\mathbf{m})}{2[d_j(\mathbf{m}) + 2]}, \tag{4}$$

where for every $j \neq i$,

$$\gamma_j(\mathbf{m}) \equiv \frac{a_j/[d_j(\mathbf{m})+2]}{\sum_{j'\neq i} a_{j'}/[d_{j'}(\mathbf{m})+2]}.$$

Intuitively, each politician i's willingness to communicate truthfully depends on a weighted average of all the ministers' ideologies. The specific weights are inversely related to the equilibrium information of each politician. Analyzing them reveals that, in contrast to the earlier case, truthful communication from politician i to minister j in equilibrium depends upon the policy assignment. Thus the characterization of the communication structure given by Lemma 1 implies that our earlier result in proposition 1– namely that private conversation leads to fully centralized authority – can be reversed once we allow for public meetings. So formal power-sharing agreements in a multi-member executive that meets in cabinet may be optimal. We illustrate this possibility with a simple example with 4 politicians and a generic set of biases.

Example 1. Suppose that $I = K = 4$. Biases are $b_1 = -\beta$, $b_2 = \varepsilon$, $b_3 = \beta$, and $b_4 = 2\beta$, where ε is a positive quantity smaller than β.[9] We compare four assignments, full decentralization, leadership by politician 2 (the most moderate politician), and two forms of power sharing agreements between politicians 2 and 3: in the symmetric power-sharing agreement, politicians 2 and 3 make two decisions each; in the asymmetric power-sharing agreement, politician 2 makes 3 choices, and 3 makes one choice.

The analysis requires calculating the welfare maximizing equilibria for each of the four assignments and comparing welfare across them. Details are relegated to the Appendix. Here, we note that taking the limit for vanishing $\varepsilon > 0$ the following observations obtain. First, for $\beta < 1/24$, all players are fully informed under any of the four considered assignments; at the same time, for $\beta > 1/18$, there is no truthful communication regardless of the assignment; in both cases the optimal assignment entails selecting the most moderate politician 2 as the unique leader. Second, for $\beta \in (1/24, 1/21)$, politician 1 and 4 are willing to communicate truthfully under any power sharing agreement, but politician 4 is not willing to share information if politician 2 is the single leader. Third, for $\beta \in (1/21, 1/18)$, players 1 and 4 are both willing to talk publicly only when the symmetric power sharing agreement is in place. Finally, for $\beta \in (1/24, 1/18)$, there is no advantage from assigning any choice to player 3 instead of player 2. Our result is summarized as follows.

[9] When $\epsilon = 0$ there is a multiplicity of optimal allocations, which is not generic.

Result 1. *Suppose that* $I = K = 4$*, with* $b_1 = -\beta$*,* $b_2 = \varepsilon$*,* $b_3 = \beta$*, and* $b_4 = 2\beta$*, and compare leadership by* 2*, full decentralization, and power sharing agreements between* 2 *and* 3*, under public communication of information with common state. As* ϵ *goes to zero the following holds: For* $\beta < 1/24$ *or* $\beta > 1/18$*, it is optimal to select 2 as the leader; For* $\beta \in (1/24, 1/21)$*, the optimal assignment is the symmetric power sharing agreement of 2 and 3; For* $\beta \in (1/21, 1/18)$*, the optimal assignment is the asymmetric power sharing agreement where 2 makes 3 choices, and 3 makes one choice.*

The fact that full authority centralization is always optimal when conversations are private though not necessarily when there are public meetings, together with the observation that private and public communication equilibria coincide when all authority is granted to a single leader, provides a striking result: the possibility of cabinet meetings induces a Pareto improvement.

This result, one of the main findings of our paper, holds independently of whether or not private conversations take place alongside cabinet deliberations, in our model. The above argument is, evidently, conclusive when private conversations are ruled out. To assess the opposite case, note that private conversation may always involve babbling in equilibrium. Then, because we always select the Pareto optimal equilibrium of any communication game, it immediately follows that the argument developed above holds also when cabinet discussion may be supplemented with a private exchange of views between politicy-makers. We state our finding formally:

Proposition 2. *Suppose that the state* θ *is common across policies* k*. For generic ideologies* $\mathbf{b}$*, the optimal assignment of decision-making authority when information is exchanged in cabinet meetings Pareto dominates any authority assignment when information is exchanged only privately.*

Proposition 2 bears important consequences for optimal executive structure. It shows that if politicians in $\mathcal{I}$ can assign authority optimally, then imposing a cabinet structure to the executive– a public meeting at a designated time and place where ministers provide the information relevant to their decisions– induces a Pareto improvement over other forms of executive governance. In particular, Cabinet government Pareto dominates an executive where individual ministers implement policy but are not bound to share policy relevant information.

6. Optimal Cabinet Design in Common State Model

We have seen that when conversations with ministers take place in private then the optimal executive is centralised with a single decision-maker. A multi-member executive that meets in cabinet can, however, outperform a single-member executive. This provides a normative justification for cabinet governance. What is the optimal assignment of authority within a cabinet? An analysis of result 1 suggests that it is non-monotonic in the level of disagreement between politicians and that centralisation is optimal in the two polar cases, those with very high or very low agreement. The next result formalizes this intuition for our general setting.

Proposition 3. *Let i^* be the most moderate politician, i.e., $i^* = \arg\min_{j\in I} \left|b_j - \sum_{i\in\mathcal{I}} b_i/I\right|$*

1. *There exists $\Delta > 0$ such that if $\max_{j\in I} |b_j - b_{i^*}| \leq \Delta$ then the optimal allocation of authority is centralised to politician i^*.*
2. *For every profile of ideologies $\{b_1, ..., b_I\}$ there exists $\delta < \infty$ such that, under profile of ideologies $\{\beta b_1, ..., \beta b_I\}$ and for all $\beta \geq \delta$, the optimal allocation of authority is centralised to politician i^*.*

When politicians have similar ideological preferences then there are few strategic constraints on communication in the parliament. In particular, even a relatively extreme politician can communicate with the most moderate one. Since information can then be aggregated regardless of the specific allocation of authority, it is better that power is centralised to a single moderate politician. When disparity in the ideological views of the politicians is large then communication with ministers is not possible and so from a welfare perspective it is desirable that the decision-maker respects the diversity of viewpoints. Again, the most moderate politician is best placed to do so and so power should be centralised. The fact that a multi-member cabinet can sometimes be optimal, as a direct consequence of Proposition 2 and Example 1, completes the proof of the claim.

Combining propositions 2 and 3 provides an answer to the question posed in our introduction: in the absence or ambiguity of constitutional guidelines, to what extent should power be centralised? Our results reveal that when information aggregation is not an important consideration–even relative extremists can communicate to the center, or politicians do not communicate at all–then full centralisation is desirable. But when information aggregation is more sensitive to the allocation of authority then it is better that power is shared in a multi-member cabinet.

Beyond this insight, what does the optimal allocation of authority within a cabinet look like? There are subtle considerations that make it difficult to provide a precise answer. Though note that in the example described in Result 1 the optimal allocation is single peaked around the moderate politician. One might conjecture then that this is a general property of the optimal authority allocation in cabinets.[10] This conjecture, as our next example shows, is incorrect.

Example 2. Suppose $I = |\mathcal{K}| = 7$ and assume that $b_{i+1} - b_i = 0.116$ for all $i \in I$. In this case one can easily show that the optimal allocation of authority a_i^* is $a_3^* = 1, a_4^* = 5, a_6^* = 1$. That is one policy is allocated to politician 3, five are allocated to politician 4, and the remaining one is allocated to politician 6. Optimal equilibrium communication involves only politicians 4 and 5 communicating truthfully; all other politicians babble.

Here the optimal allocation of authority is not single peaked: whereas the most moderate politician 4 has the largest share of authority her immediate neighbour 5 has none, while politician 6 has some. To provide intuition for this surprising result, consider an alternative single-peaked allocation $a_3 = 1, a_4 = 5, a_5 = 1$. This closely resembles a^* except that the policy allocated to politician 6 under a^* is now given to politician 5. This transfer of authority makes the allocation more concentrated toward the moderate politician 4 and so better reflects the diversity of politicians' views. However, under this single peaked allocation less information is aggregated.

To understand why, focus on the incentive for politician 5 to communicate truthfully. Under the alternative and suboptimal allocation a^*, politician 5 has an incentive to misreport a low signal to other cabinet members as doing so would bias their decisions toward her ideal point. Under the optimal allocation, by contrast, her incentive to misreport a low signal to 3 and 4 is offset by its effect on politician 6 who is now included in the cabinet. Misreporting a low signal could bias politician 6's action far away from 5's bliss point. So we see that whereas 5 communicates truthfully in the optimal allocation she does not do so in the single peaked allocation.

An intriguing implication of our finding is that the set of ministers included in the cabinet is not ideologically connected. The prediction that they will be connected arises in classic models of coalition formation Axelrod (1970), though this result can be overturned in non-cooperative bargaining models Austen-Smith and Banks (1990). From our information aggregation creating

[10] To further explore optimal decision-making authority assignments in cabinet governments we have also run simulations for a 7 member parliament. The results show that in most cases centralization of authority in a multi-member cabinet is Pareto superior to other executive forms. The details and the results of this exercise are reported in an online appendix.

"holes" in the cabinet– so that moderate politicians are bye-passed for more extreme ones– provides better incentives for communication and disciplines politicians who otherwise would misrepresent their views.

Till now we have considered a small group of politicians. Since parliament is a large representative body, and those on the government payroll can be represent a significant fraction (in the UK, of around 650 MPs roughly 20% play some role in government) it is interesting to observe the optimal allocation of authority in the limit case as the number of politicians becomes large. Doing so we provide a strong characterization result: all decision making authority should be concentrated to politicians who are ideologically close to the most moderate one.

Proposition 4. *Suppose that biases b_i, $i = 1, ..., I$ are i.i.d. and drawn from a distribution with connected support with mean $\bar{b}$. For every small $\delta > 0$, there exists a possibly large $I_\delta > 0$ so that for all $I > I_\delta$, with at least $1 - \delta$ probability, the fraction of decisions in the optimal assignment concentrated to politicians with biases b such that $|b - \bar{b}| < \delta$ is larger than $1 - \delta$.*

The proof of Proposition 4 consists of two parts. First, we show that when all decisions are allocated to a single politician i and parliament grows large the decision-maker becomes fully informed. Second, we compare the case where all decisions are allocated to the most moderate politician to that where some decisions are allocated to a less moderate one. As the parliament becomes large the aggregate residual variance obtained in each of these assignments vanishes. The difference in the aggregate ideological loss of these assignments is, however, bounded below.

It is important to stress that Proposition 4 does not imply that welfare is equivalent in large parliaments that adopt cabinet meetings and ones where communication with decision-makers is private. In fact, our normative justification for cabinet governance holds for any size majority, including large ones.

7. Policy Specific Information

This section studies optimal assignment of decision making authority when each politician's information is policy specific, so that only politician k receives a signal about θ_k, for each policy k. We begin by characterizing equilibrium communication.

Lemma 2. *Suppose that information is policy specific. The profile* $(\mathbf{m}, \mathbf{y})$ *is an equilibrium if and only if, whenever politician* k *is truthful to* $a(k) \neq k$,

$$|b_k - b_{a(k)}| \leq 1/6.$$

Since each politician has only one signal and that signal is informative of only one policy decision, the amount of information held by politician $a(k) \neq k$ depends only on whether k is truthful or not. Hence, whether k is truthful (or not) does not depend on the communication strategy of any other politician. Further, because each politician is informed on one policy only, and this policy may be assigned to a single policy maker, there is no difference between private conversations and cabinet meetings.

This characterization of information transmission bears the following implication. The possibility that a politician k truthfully communicates her signal to the minister $a(k)$ to whom decision k is assigned is independent of any other assignment. Hence, for all choices k, the optimal assignment $a(k)$ can be selected independently of other assignments. The optimal assignment is to allocate decision k to the politician j who maximizes:

$$-\sum_{i=1}^{I} \frac{(b_j - b_i)^2}{I} - \frac{1}{6(d_{j,k}(\mathbf{m}) + 2)},$$

where $d_{j,k}(\mathbf{m}) = 1$ if $|b_k - b_j| \leq 1/6$ and $d_{j,k}(\mathbf{m}) = 0$, otherwise.

Simplifying the above expression, and using Lemma 2, we see that the optimal selection of $a(k)$ takes a simple form when information is policy specific: policy decision k should be assigned to either the most moderate politician $m^* = \arg\min_m \left| b_m - \sum_{i=1}^{I} b_i/I \right|$, or to the most moderate politician $m(k)$ informed of k, i.e., to $m(k) = \arg\min_{m:|b_m - b_k| \leq 1/6} \left| b_m - \sum_{i=1}^{I} b_i/I \right|$, depending on whether

$$\sum_{i=1}^{I} \frac{\left(b_i - b_{m(k)}\right)^2}{I} - \sum_{i=1}^{I} \frac{(b_i - b_{m^*})^2}{I} > (<) \frac{1}{36}. \tag{5}$$

Because for any j, the quantity $\sum_{i=1}^{I} (b_i - b_j)^2 / I$ is the average ideological loss, whereas the information gain is $1/36$, we may summarize our analysis as follows.

Lemma 3. *When information is policy specific, each decision k is optimally assigned to either the most moderate informed politician $m(k)$ or to the most moderate one m^*, depending on whether the difference in average ideological loss is smaller or greater than the informational gain.*

With the above characterization we now show that although policy specific information might lead one to believe that full decentralization is optimal, this is, in fact, never the case.

Proposition 5. *Despite policy specific information, full decentralization is never optimal for generic ideologies* $\mathbf{b}$. *The most moderate politician m^* is assigned the policies of sufficiently moderate and of sufficiently extreme-bias expert politicians, but not necessarily the policies of intermediate-bias politicians.*

The complete proof of this proposition is provided in the appendix, here we convey the main intuition behind the result. Because moderate policy experts are willing to inform the most moderate politician m^*, it is optimal that she is given authority on these policies. Since extreme policy experts are willing to communicate only with extreme politicians, it is better to let the (uninformed) most moderate politician decide. Only for intermediate case policies k, it is not optimal that the most moderate politician decides, and that the decision is given to $m(k)$.

Our result relates to Dessein (2002) who shows, in a different environment, that it is optimal for a decision-maker to delegate authority to an expert with a small bias and a signal from a continuum. We show that decentralization is not optimal with policy specific information; instead it is better that the most moderate politician is assigned the decisions of moderate experts. The difference arises because our model has binary signals instead of continuous ones. Nevertheless our result that full decentralization is never optimal still hold if signals are continuous: then, the decision of extreme experts should be optimally assigned to the most moderate politician.

8. Conclusion

We have analyzed the optimal structure of the executive, restricting to the set of institutions that allow for assignment of decision-making authority to individuals with collective deliberation over outcomes; two of the core features of cabinet governance found in parliamentary democracies. Our main results focus on the case where uncertainty over policy outcomes is generated by a shock that affects all policies. Then we show that cabinet meetings, that provide a mechanism for information made available to one minister to be available to all decision-makers, can improve the quality of policies implemented: they allow for more policy relevant information to be aggregated than would be the case in their absence. We then analyzed the optimal degree of centralisation of executive power and showed that it is non-monotonic in the ideological divergence in the majority party: For intermediate levels of divergence it is optimal for power to be shared between cabinet ministers; otherwise it is optimal for power to be concentrated to the most moderate politician. Surprisingly, when power is shared in a cabinet of ministers then in the optimal allocation the set of ministers with decision-making authority need not be ideologically connected. In large cabinets, however, authority should be concentrated to the most moderate politicians. Overall our results highlight the desirability of a centralised governing executive that holds cabinet meetings. We ask whether this depends upon the dispersion of policy relevant expertise. Surprisingly, in the case where shocks are policy specific and a single politician is imperfectly informed about its effects the degree of centralisation is no less centralised.

The two informational environments we have considered – perfect correlation and independence– are the classic ones most studied in game theoretical applications, for example with respect to auctions. We thus take them as the natural starting point for our investigation into optimal executive structure. An obvious question is whether our core insights are robust to the imposition of a mixed case. Our preliminary investigations reveal that this is in fact the case: the results developed here are not confined to the two cases we study and the trade-offs we have highlighted are indeed relevant to the mixed case also.[11]

Finally, we highlight several areas of further research. Our work establishes a normative benchmark for evaluating the assignment of decision-making authority to heads of executive departments in parliamentary democracies. In practice the assignment of decision-making rights is carried out by a prime minister who is answerable to the parliamentary majority. As noted by Strøm (2000), this creates a "singular chain" of delegation, from the parliamentary majority to a prime minister and

[11] Due to space limitations we omit the analysis but provide a brief summary here. The full analysis is available upon request. To investigate the case of correlation among policies, we considered the simple environment where, before making a choice, each politician knows that with probability p all policies are perfectly correlated and with the remaining probability they are perfectly independent. The model degenerates to the two environments we have considered for $p \in \{0, 1\}$. Characterising the incentive compatibility conditions for truthful communication for arbitrary allocations and modes of communication reveals that, perhaps unsurprisingly, it is continuous in p. This, in turn, implies that our results derived under the assumption of perfect correlation and of independence hold for p sufficiently high and p sufficiently low, respectively. Moreover, the analysis reveals that optimal allocation in the case of intermediate correlation involves a trade off between politicians' moderation and the information that they hold in equilibrium that is the driving force behind our results.

the heads of departments, that distinguishes parliamentary democracies from presidential ones.[12] Thus our analysis reveals the delegation patterns of a Prime Minister who maximizes the welfare of a parliamentary majority. Since the class of parliamentary democracies is large there are, of course, several variations from this ideal type that could be considered. For example, in some parliamentary democracies the assignment is to a senior executive head shadowed by a junior minister within the same department.[13] In other parliamentary democracies, such as Israel, the cabinet votes over policy rather than delegating the decision to a single minister. And of course the assignment of authority might be part of a prime minister's strategic plan and so her objectives may conflict with those of members of her government and the majority in parliament. A further extension might consider how centralized authority affects the interaction between party elites and voters whose actions jointly determine the ideological composition of the assembly; and how the degree of centralization of decision-making authority responds to party control over nomination of the members of Parliament. All of these substantive applications could be approached within the current modeling framework though we have not done so here.

Further lines of enquiry can be addressed within our framework, but would involve more extensive modifications of the model. Here we have assumed that the parliamentary majority assigns "decision-making authority" to ministers, having in mind the fusion of legislative and executive powers found in many parliamentary democracies. Our model could be modified so that the parliamentary majority nominates agenda setters whose proposal needs then to be formally approved by the Parliament. More generally, the parliamentary majority could assign authority to committees rather than to individuals. As these lines of enquiry require significant changes to the model, we defer them to future research.

9. Appendix

Equilibrium beliefs. In our model a politicians' equilibrium updating is based on the standard Beta-binomial model. Suppose that a politician i holds n bits of information, i.e. she holds the private signal s_i and $n-1$ politicians truthfully reveal their signal to her. The probability that l

[12]Though as argued by Patty (2013), the distinction is perhaps not so clear.

[13]In Thies (2001) such arrangements allow ministers with different preferences to enforce the implementation of a compromise policy. Here, and as in the models of Austen-Smith and Banks (1996) and Laver and Shepsle (1996) the minister in charge of an issue always implements his preferred policy.

out of such n signals equal one, conditional on θ is

$$f(l|\theta, n) = \frac{n!}{l!\,(n-l)!}\theta^l (1-\theta)^{(n-l)}.$$

Hence, politician i's posterior is

$$f(\theta|l, n) = \frac{(n+1)!}{l!\,(n-l)!}\theta^l (1-\theta)^{(n-l)},$$

the expected value is

$$E(\theta|l, n) = \frac{l+1}{n+2},$$

and the variance is

$$V(\theta|l, n) = \frac{(l+1)(n-l+1)}{(n+2)^2 (n+3)}.$$

Derivation of equilibrium welfare, expression 2. Assume $(\mathbf{m}, \mathbf{y})$ is an equilibrium. The ex-ante expected utility of each player i is:

$$\begin{aligned}
Eu_i(\mathbf{m}, \mathbf{y}) &= -E\left[\sum_{k=1}^{K} (\hat{y}_k - \theta - b_i)^2; (\mathbf{m}, \mathbf{y})\right] \\
&= -\sum_{k=1}^{K} E\left[(y_{a(k),k} - \theta - b_i)^2; (\mathbf{m}, \mathbf{y})\right] \\
&= -\sum_{k=1}^{K} E\left[(b_{a(k)} + E\left[\theta|\, \Omega_{a(k)}\right] - \theta - b_i)^2; \mathbf{m}\right]
\end{aligned}$$

where $\Omega_{a(k)}$ denotes the equilibrium information of player $a(k)$, and we have dropped the reference to $\mathbf{y}$ in the last equality, as, after substituting $y^{a(k)}$ with $b_{a(k)} + E\left[\theta|\, \Omega_{a(k)}\right]$, the reported expression no longer depends on $\mathbf{y}$. Hence

$$\begin{aligned}
Eu_i(\mathbf{m}, \mathbf{y}) &= -\sum_{k=1}^{K} E\left[(b_{a(k)} - b_i)^2 + \left(E\left[\theta|\, \Omega_{a(k)}\right] - \theta\right)^2 - 2(b_{a(k)} - b_i)\left(E\left[\theta|\, \Omega_{a(k)}\right] - \theta\right); \mathbf{m}\right] \\
&= -\sum_{k=1}^{K} \Big[(b_{a(k)} - b_i)^2 + E\left[\left(E\left[\theta|\, \Omega_{a(k)}\right] - \theta\right)^2; \mathbf{m}\right] \\
&\quad -2(b_{a(k)} - b_i)\left(E[E\left[\theta|\, \Omega_{a(k)}\right]; \mathbf{m}] - E[\theta; \mathbf{m}]\right)\Big],
\end{aligned}$$

by the law of iterated expectations, $E[E\left[\theta|\, \Omega_{a(k)}\right]; \mathbf{m}] = E[\theta; \mathbf{m}]$, and by definition $E\left[\left(E\left[\theta|\, \Omega_{a(k)}\right] - \theta\right)^2; \mathbf{m}\right] = \sigma_k^2(\mathbf{m})$.

Further, note that the equilibrium information $\Omega_{a(k)}$ of player $a(k)$ may be represented as any vector in $\{0,1\}^{d_j(\mathbf{c})}$. Letting l be the number of digits equal to one in any such vector, we obtain

$$\begin{aligned} E\left[\left(E\left[\theta|\,\Omega_{a(k)}\right]-\theta\right)^2;\mathbf{m}\right] &= \int_0^1 \sum_{l=0}^{d_{a(k)}(\mathbf{c})} \left(E\left[\theta|l,d_{a(k)}(\mathbf{c})\right]-\theta\right)^2 f(l|d_{a(k)}(\mathbf{c}),\theta)d\theta \\ &= \int_0^1 \sum_{l=0}^{d_{a(k)}(\mathbf{c})} \left(E\left[\theta|l,d_{a(k)}(\mathbf{c})\right]-\theta\right)^2 \frac{f\left(\theta|l,d_{a(k)}(\mathbf{c})\right)}{d_{a(k)}(\mathbf{c})+1}d\theta, \end{aligned}$$

where the second equality follows from $f(l|d_{a(k)}(\mathbf{c}),\theta)=f(\theta|l,d_{a(k)}(\mathbf{c}))/(d_{a(k)}(\mathbf{c})+1)$.

Because the variance of a beta distribution of parameters l and d, is

$$V(\theta|l,d)=\frac{(l+1)\,(d-l+1)}{(d+2)^2\,(d+3)},$$

we obtain:

$$\begin{aligned} E\left[\left(E\left[\theta|\,\Omega_{a(k)}\right]-\theta\right)^2;\mathbf{m}\right] &= \frac{1}{d_{a(k)}(\mathbf{c})+1}\left[\sum_{l=0}^{d_{a(k)}(\mathbf{c})} V\left(\theta|l,d_{a(k)}(\mathbf{c})\right)\right] \\ &= \sum_{l=0}^{d_{a(k)}(\mathbf{c})} \frac{(l+1)\left(d_{a(k)}(\mathbf{c})-l+1\right)}{\left(d_{a(k)}(\mathbf{c})+1\right)\left(d_{a(k)}(\mathbf{c})+2\right)^2\left(d_{a(k)}(\mathbf{c})+3\right)} \\ &= \frac{1}{6(d_{a(k)}(\mathbf{c})+2)}. \end{aligned}$$

■

Proof of Proposition 1. Fix any assignment a. Any Pareto optimal equilibrium $(\mathbf{m},\mathbf{y})$ maximizes the welfare

$$\mathcal{W}(\mathbf{m},\mathbf{y};\gamma;a)=-\sum_{i\in\mathcal{I}}\gamma_i\sum_{k\in\mathcal{K}}E[(\hat{y}_k-\theta_k-b_i)^2|s_i,\mathbf{m}_{\bar{N}_i,i}],$$

for some Pareto weights γ. Following the same steps in the derivation of expression 2 we obtain that

$$\mathcal{W}(\mathbf{m},\mathbf{y};\gamma;a)=-\sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{I}}\gamma_i(b_i-b_{a(k)})^2-\sum_{k\in\mathcal{K}}\frac{1}{6[d_{a(k),k}(\mathbf{m})+2]}.$$

This decomposition, together with equilibrium condition 3, imply that, as long as j is active under a the equilibrium information $d_j^*(a)$ associated to any Pareto optimal equilibrium $(\mathbf{m},\mathbf{y})$ is independent of the set of policy choices $a^{-1}(i)$ assigned to any player i, including j. Hence, choosing the Pareto-optimal assignment is equivalent to finding the index j that maximizes

$$(6) \qquad -\sum_{i=1}^{I}\gamma_i\left(b_j-b_i\right)^2-\frac{1}{6(d_j(\mathbf{m})+2)},$$

and to assigning all policy choices k to such optimal j. For generic vectors of biases $\mathbf{b}$, the expression (6) has a unique maximizer. ■

Proof of Lemma 1 and Derivation of Expression 3. We first prove Lemma 1 and then derive Expression 3 as a corollary. Consider any $j \in a(\mathcal{K})$, and suppose let $C_j(\mathbf{c})$ be the set of players truthfully communicating with j in equilibrium, i.e. the equilibrium network neighbors of j. The equilibrium information of j is thus $d_j = |C_j(\mathbf{c})|+1$, the cardinality of $C_j(\mathbf{c})$ plus j's signal. Consider any player $i \in C_j(\mathbf{c})$. Let s_R be the vector containing s_j and the (truthful) messages of all players in $C_j(\mathbf{c})$ except i. Let also $y^j_{s_R,s}$ be the action that j would take if he has information s_R and player i has sent signal s; analogously, $y^j_{s_R,1-s}$ is the action that j would take if he has information s_R and player i has sent signal $1-s$. Agent i reports truthfully signal s to a collection of agents J if and only if

$$-\sum_{j\in J}\sum_{k:a(k)=j}\int_0^1\sum_{s_R\in\{0,1\}^{d_j-1}}\left[\left(y^j_{s_R,s}-\theta-b_i\right)^2-\left(y^j_{s_R,1-s}-\theta-b_i\right)^2\right]f(\theta,s_R|s)d\theta\geq 0.$$

Using the identity $a^2-b^2=(a-b)(a+b)$ and simplifying, we obtain:

$$-\sum_{j\in J}\int_0^1 a_j\sum_{s_R\in\{0,1\}^{d_j-1}}\left[\left(y^j_{s_R,s}-y^j_{s_R,1-s}\right)\left(\frac{y^j_{s_R,s}+y^j_{s_R,1-s}}{2}-(\theta+b_i)\right)\right]f(\theta,s_R|s)d\theta\geq 0.$$

Next, observing that

$$y^j_{s_R,s}=b_j+E\left[\theta|s_R,s\right],$$

we obtain

$$-\sum_{j\in J}\int_0^1 a_j\sum_{s_R\in\{0,1\}^{d_j-1}}\left[\left(E[\theta+b_j|s_R,s]-E[\theta+b_j|s_R,1-s]\right)\right.$$
$$\left.\cdot\left(\frac{E[\theta+b_j|s_R,s]+E[\theta+b_j|s_R,1-s]}{2}-(\theta+b_i)\right)\right]f(\theta,s_R|s)d\theta\geq 0.$$

Denote

$$\Delta\left(s_R,s\right)=E\left[\theta|s_R,s\right]-E\left[\theta|s_R,1-s\right].$$

Observing that:

$$f(\theta, s_R|s) = f(\theta|s_R, s)P(s_R|s),$$

and simplifying, we get:

$$-\sum_{j\in J} a_j \sum_{s_R\in\{0,1\}^{d_j-1}} \int_0^1 \left[\Delta\left(s_R, s\right)\left(\frac{E\left[\theta|s_R, s\right] + E\left[\theta|s_R, 1-s\right]}{2} + b_j - b_i - \theta\right)\right] \cdot$$
$$\cdot f(\theta|s_R, s)P(s_R|s)d\theta \geq 0.$$

Furthermore,

$$\int_0^1 \theta f(\theta|s_R, s)d\theta = E[\theta|s_R, s],$$

and

$$\int_0^1 P(\theta|s_R, s)E[\theta|s_R, s]d\theta = E[\theta|s_R, s],$$

because $E[\theta|s_R, s]$ does not depend on θ. Therefore, we obtain:

$$\begin{aligned} & -\sum_{j\in J} a_j \sum_{s_R\in\{0,1\}^{d_j-1}} \left[\Delta\left(s_R, s\right)\left(\frac{E\left[\theta|s_R, s\right] + E\left[\theta|s_R, 1-s\right]}{2} + b_j - b_i - E\left[\theta|s_R, s\right]\right)\right] P(s_R|s) \\ = & -\sum_{j\in J} a_j \sum_{s_R\in\{0,1\}^{d_j-1}} \left[\Delta\left(s_R, s\right)\left(-\frac{E\left[\theta|s_R, s\right] - E\left[\theta|s_R, 1-s\right]}{2} + b_j - b_i\right)\right] P(s_R|s) \geq 0. \end{aligned}$$

Now, note that:

$$\begin{aligned} \Delta\left(s_R, s\right) &= E[\theta|s_R, s] - E[\theta|s_R, 1-s] \\ &= E\left[\theta|l+s, d_j+1\right] - E\left[\theta|l+1-s, d_j+1\right] \\ &= \left(l+1+s\right)/\left(d_j+2\right) - \left(l+2-s\right)/\left(d_j+2\right) \\ &= \begin{cases} -1/\left(d_j+2\right) & \text{if } s=0 \\ 1/\left(d_j+2\right) & \text{if } s=1. \end{cases} \end{aligned}$$

where l is the number of digits equal to one in s_R. Hence, we obtain that agent i is willing to communicate to agent j the signal $s = 0$ if and only if:

$$-\sum_{j\in J} a_j \left(\frac{-1}{d_j+2}\right)\left(-\frac{-1}{2(d_j+2)} + b_j - b_i\right) \geq 0,$$

or

$$\sum_{j\in J} a_j \frac{b_j - b_i}{d_j + 2} \geq -\sum_{j\in J} a_j \frac{1}{2\left(d_j + 2\right)^2}$$

Note that this condition is redundant if $\sum_{j\in J} a_j \left(b_j - b_i\right) > 0$. On the other hand, she is willing to communicate to agent j the signal $s = 1$ if and only if:

$$-\sum_{j\in J} a_j \left(\frac{1}{d_j + 2}\right)\left(-\frac{1}{2(d_j + 2)} + b_j - b_i\right) \geq 0,$$

or

$$\sum_{j\in J} a_j \frac{b_j - b_i}{d_j + 3} \leq \sum_{j\in J} a_j \frac{1}{2\left(d_j + 3\right)^2}.$$

Note that this condition is redundant if $\sum_{j\in J} a_j \left(b_j - b_i\right) < 0$. Collecting the two conditions yields:

$$\left|\sum_{j\in J} a_j \frac{b_j - b_i}{d_j + 3}\right| \leq \sum_{j\in J} \frac{a_j}{2\left(d_j + 3\right)^2}. \tag{7}$$

Rearranging condition 7 completes the proof of Lemma 1. ■

Proof of Result 1. Consider a cabinet of 4 politicians, with biases $b_1 = -\beta$, $b_2 = \varepsilon$, $b_3 = \beta$, and $b_4 = 2\beta$. We suppose that $\varepsilon > 0$ is small, so that politician 2 is the most moderate. We compare four assignments, full decentralization, leadership by politician 2, a symmetric power-sharing agreement where politicians 2 and 3 make two decisions each, and an asymmetric power-sharing agreement where politician 2 makes 3 choices, and 3 makes one choice.

Consider leadership by politician 2, first. We calculate $d_2 = 4$ if $2\beta - \varepsilon \leq 1/12$, i.e., $\beta \leq \varepsilon/2 + 1/24$, whereas $d_2 = 3$ if $\beta + \varepsilon \leq 1/10$, i.e. $\beta \leq 1/10 - \varepsilon$, as well as $d_2 = 2$ if $\beta - \varepsilon \leq 1/8$, i.e., $\beta \leq 1/8 + \varepsilon$, and $d_2 = 1$ if $\beta > 1/8 + \varepsilon$.

Consider the symmetric power sharing rule. First note that, if 1 is willing to talk, then so are all other players. Hence, for $2\left(\beta + \varepsilon\right) + 2 \cdot 2\beta \leq \frac{4}{2(3+3)}$, i.e., $\beta \leq 1/18 - \varepsilon/3$, then both $d_2 = 4$ and $d_3 = 4$. Further, for $2\left(2\beta - \varepsilon\right) + 2\beta \leq \frac{4}{2(2+3)}$, i.e., $\beta \leq 1/15 + \varepsilon/3$, then 1 does not talk, but 4 does, and so, $d_2 = 3$ and $d_3 = 3$. Finally, for $\beta - \varepsilon \leq 1/8$, i.e., $\beta \leq 1/8 + \varepsilon$, then both 2 and 3 talk to each other: $d_2 = 2$ and $d_3 = 2$. Of course, $d_2 = 1$ and $d_3 = 1$, if $\beta > 1/8 + \varepsilon$.

Hence, the symmetric power sharing rule dominates the single leader 2 on $(\varepsilon/2 + 1/24, 1/18 - \varepsilon/3]$ in terms of information transmission. It will dominate on a subset, because of the moderation effect, but as $\varepsilon \to 0$, the subset converges to $(1/24, 1/18)$.

Consider now the asymmetric power sharing rule. In this case the condition for 1 to talk (if 4 is talking) becomes: $\frac{3}{4}(\beta+\varepsilon)+\frac{1}{4}2\beta \leq \frac{1}{2(3+3)}$, i.e., $\beta \leq 1/15 - 3\varepsilon/5$. The condition for 4 to talk if 1 is talking becomes, $\frac{3}{4}(2\beta-\varepsilon)+\frac{1}{4}\beta \leq \frac{1}{2(3+3)}$, i.e., $\beta \leq 1/21 + 3\varepsilon/7$. Hence, for $\beta \leq 1/21 + 3\varepsilon/7$, then both $d_2 = 4$ and $d_3 = 4$. Instead, the condition for 1 to talk if 4 does not talk is $\frac{3}{4}(\beta+\varepsilon)+\frac{1}{4}2\beta \leq \frac{1}{2(2+3)}$, i.e., $\beta \leq 2/25 - 3\varepsilon/5$. And the condition for 4 to talk if 1 does not talk is $\frac{3}{4}(2\beta-\varepsilon)+\frac{1}{4}\beta \leq \frac{1}{2(2+3)}$, i.e. $\beta \leq \frac{3}{7}\varepsilon + \frac{2}{35}$. Hence, for $\beta \leq 2/25 - 3\varepsilon/5$, then both $d_2 = 3$ and $d_3 = 3$. The condition for 2 and 3 to each other talk is $\beta \leq 1/8+\varepsilon$; in this case $d_2 = 2$ and $d_3 = 2$. Again, $d_2 = 1$ and $d_3 = 1$, if $\beta > 1/8+\varepsilon$.

Hence, the asymmetric power sharing agreement dominates the single leader 2 informationally on $(\varepsilon/2 + 1/24, 1/21 - 3\varepsilon/7]$. Due to the moderation effect, it also dominates the symmetric power sharing agreement. For $\varepsilon \to 0$, asymmetric power sharing agreement dominates on $(1/24, 1/21)$.

Finally, consider full decentralization. The player who is least likely to speak publicly is 1. Given that all other players speak, he speaks if and only if $(\beta+\varepsilon)+2\beta+3\beta \leq \frac{3}{2[3+3]}$or $\beta \leq \frac{1}{24} - \varepsilon/6$. In this case, all players receive 3 signals, $\frac{1}{24} = 0.04166\,7$. Then, if 1 does not speak, the least likely to speak is 4. This occurs if and only if $\frac{3\beta}{3+3}+\frac{2\beta-\varepsilon+\beta}{2+3} \leq \frac{1}{2(3+3)^2}+\frac{2}{2(2+3)^2}$, i.e. if $\beta \leq \frac{2}{11}\varepsilon + \frac{97}{1980}$ for $\varepsilon \to 0$, this is close to $\frac{97}{1980} \approx 0.04899$. When 1 does not speak publicly, whereas 4 does, the d-distribution is: $3, 2, 2, 2$; which is informationally better than the private communication to 2. But, of course, it is worse in terms of moderation... Further, decentralization is dominated by the symmetric power sharing agreements, for the range $\beta \leq 1/18 - \varepsilon/3$, as $1/18 \approx 0.055556$; because in this range $d_2 = 3$ and $d_3 = 3$ for the asymmetric power sharing agreements. Then, if 1 and 4 do not speak, the least likely to speak is 3 —because 2 is more central. This occurs if and only if $\frac{2\beta}{2+3}+\frac{\beta-\varepsilon}{1+3}+\frac{\beta}{2+3} \leq \frac{1}{2(1+3)^2}+\frac{2}{2(2+3)^2}$, i.e., $\beta \leq \frac{5}{17}\varepsilon + \frac{57}{680} \approx 0.083824$, with the distribution $2, 1, 1, 2$. This is dominated by the asymmetric power sharing agreements, because for $\beta \leq 1/10 - 3\varepsilon/5$, i.e., essentially, $\beta \leq 1/10$, we have $d_2 = 2$ and $d_3 = 2$. Finally, 2's condition to speak if nobody else speaks under decentralization is: $2\beta - \varepsilon + \beta - \varepsilon + \beta + \varepsilon \leq \frac{3}{2(1+3)}$, i.e. $\beta \leq \frac{1}{4}\varepsilon + \frac{3}{32} \approx 0.09375$. Because this yields the distribution $1, 0, 1, 1$, we obtain that it is dominated by the asymmetric power sharing agreements. ■

Proof of Proposition 2. From Proposition 1, we know that all Pareto optimal assignments a under private communication of common value information entails a single leader, i.e., there is j such that $a(k) = j$ for all k. Suppose now that communication is public, and suppose that an assignment a with a unique leader j is selected. Then, because $\gamma_j(\mathbf{m}) = 1$ and $\gamma_{j'}(\mathbf{m}) = 0$ for all $j' \neq j$, condition (4) in Lemma 1 reduces to condition (3). Hence, the set of equilibria under private and public communication coincide under a. But because the optimal assignment under public communication a^* need not entail a single leader, the statement of the result immediately follows in the case that private conversations are ruled out under public communication. Allowing for private conversations does not change the argument, because babbling all private conversations is always possibly part of an equilibrium, and we select the optimal equilibrium in any communication game which follow the assignment and the choice of communication rule. ■

Proof of Proposition 3. Part 1. Define Δ so that $\Delta \leq \frac{1}{2(I+2)}$. Since $\max_{j \in I} |b_j - b_{i^*}| \leq \Delta$ and $\Delta \leq \frac{1}{2(I+2)}$ then, for all $i \in I$,

$$|b_i - b_{i^*}| \leq \frac{1}{2(I+2)}.$$

Next, consider allocation a^* where all policies are given to i^*, i.e., $a^*(k) = i^*$ for all k. Since $|b_i - b_{i^*}| \leq \frac{1}{2(I+2)}$ there exists an equilibrium where all politicians communicate truthfully to player i^*.

The welfare of any assignment a is determined by expression 2. Consider the assignment a^* in which all decisions are given to player i^*. Because all players are truthful in equilibrium to i^*, the second term of expression 2 is maximized by a^*. The first term is also maximized by a^* by definition of i^*. In fact, supposing that any decision k is allocated to any player $a(k)$, we can write the sum $\sum_{i \in I}(b_i - b_{a(k)})^2$ as $\sum_{i \in I}(b_i - b_{i^*} + b_{i^*} - b_{a(k)})^2 = \sum_{i \in I}[(b_i - b_{i^*})^2 + (b_{i^*} - b_{a(k)})^2 + 2(b_i - b_{i^*})(b_{i^*} - b_{a(k)})^2] = I(b_{i^*} - b_{a(k)})^2 + \sum_{i \in I}(b_i - b_{i^*})^2 + 2(b_{i^*} - b_{a(k)}) \sum_{i \in I}(b_i - b_{i^*})$ and the last term is zero if and only if $a(k) = i^*$.

Part 2. Fix $\{b_1, ..., b_I\}$ and consider the game with $\{\beta b_1, ..., \beta b_I\}$. Consider allocation a^* where the moderate political i^* takes all the decisions, i.e., $a^*(k) = i^*$ for all k, and let m^* be the associated best communication equilibrium. The welfare associated to (a^*, m^*) is

$$\mathcal{W}(a^*, m^*) = -\beta^2 |\mathcal{K}| \sum_{i \in I}(b_i - b_{i^*})^2 - |\mathcal{K}| RV_{i^*}(a^*, m^*) \geq -\beta^2 |\mathcal{K}| \sum_{i \in I}(b_i - b_{i^*})^2 - |\mathcal{K}| \frac{I}{18}$$

where $RV_{i^*}(a^*, m^*)$ is the residual variance of decision maker i^* under communication equilibrium m^*, and the inequality follows because aggregate residual variance can be bound above by the aggregate residual variance obtained in the babbling equilibrium, which is $|\mathcal{K}|I/18$.

Next, consider an arbitrary allocation $a \neq a^*$ and let m be the associated best communication equilibrium. We have that

$$W(a,m) = -\beta^2 \sum_{k\in\mathcal{K}} \sum_{i\in I} (b_i - b_{a(k)})^2 - \sum_{k\in\mathcal{K}} RV_{a(k)}(a,m) \leq -\beta^2 \sum_{k\in\mathcal{K}} \sum_{i\in I} (b_i - b_{a(k)})^2$$

where the inequality follows because, for every $a(k)$, $RV_{a(k)}(a,m)$ can be bounded below by 0. Hence, to prove that for β sufficiently high $W(a^*,m^*) > W(a,m)$ for all $a \neq a^*$, it is sufficient to prove that

$$-\beta^2|\mathcal{K}| \sum_{i\in I} (b_i - b_{i^*})^2 + \beta^2 \sum_{k\in\mathcal{K}} \sum_{i\in I} (b_i - b_j)^2$$

can be made arbitrarily large by increasing β. To see that this holds note that

$$-|\mathcal{K}| \sum_{i\in I} (b_i - b_{i^*})^2 + \sum_{k\in\mathcal{K}} \sum_{i\in I} (b_i - b_{a(k)})^2 = \sum_{k\in\mathcal{K}} \left[\sum_{i\in I} (b_i - b_{a(k)})^2 - (b_i - b_{i^*})^2 \right]$$

which is a positive constant and, since i^* is independent of β, such positive constant is independent of β. ■

Proof of Proposition 4 The proof of proposition 4 proceeds in two steps. The first step shows that if all decisions are allocated to a single agent, the information of this agent approaches infinity as the number of agents I goes to infinity. This is formalised in the following lemma.

Lemma 4. *Suppose that biases b_j, $j = 1, 2, ..., I$ are i.i.d. and drawn from a distribution of connected support. If all decisions are assigned to the same politicians i, then the optimal equilibrium information d_i^I of politician i grows to infinity in probability as I becomes infinite.*

Proof of Lemma 4. Recall that for any I, the optimal equilibrium information d_i^I solves the condition

$$\left| \left\{ j = 1, 2, ..., N : |b_i - b_j| \leq \frac{1}{2\left(d_i^I + 2\right)} \right\} \right| = d_i^I.$$

We now to show that, for $d > 0$,

$$\lim_{I\to\infty} \Pr\left(d_I \leq d\right) = 0.$$

Note, in fact, that:

$$
\begin{aligned}
\Pr\left(d_I \leq d\right) &= \Pr\left(\left|\left\{j=1,2,...,N : |b_i - b_j| \leq \frac{1}{2\left(d+2\right)}\right\}\right| \leq d\right) \\
&= \Pr\left(\times_{j=1}^{I-d}\left\{b_j : |b_i - b_j| > \frac{1}{2\left(d+2\right)}\right\}\right) \\
&= \left(\Pr\left\{b_j < b_i - \frac{1}{2\left(d+2\right)}\right\} + \Pr\left\{b_j > b_i + \frac{1}{2\left(d+2\right)}\right\}\right)^{I-d},
\end{aligned}
$$

and it is now immediate to see that

$$
\lim_{I\to\infty} \Pr\left(d_I \leq d\right) = \lim_{I\to\infty}\left(\Pr\left\{b_j < b_i - \frac{1}{2\left(d+2\right)}\right\} + \Pr\left\{b_j > b_i + \frac{1}{2\left(d+2\right)}\right\}\right) = 0
$$

This concludes the proof of Lemma 4. ■

We now turn to the second step. We compare the expected per-person per-action payoff $W_I^{m_I}$ for assigning all decisions K to the most moderate politician $m_I = \arg\min_i \left(b_i - \sum_{j=1}^{I} \frac{b_j}{I}\right)^2$, to the payoff $W_N^{\alpha\varepsilon}$ for assigning a fraction $\alpha_I \geq \alpha > 0$ of the K actions, such that $\alpha_I K$ is an integer, to a different politician j_I such that $b_{j_I} - E\left[b_j\right] > \delta > 0$, for all I. The remaining fraction $1-\alpha_I$ of actions is assigned to m_I. Hence,

$$
\begin{aligned}
W_I^{m_I} - W_I^{\alpha\varepsilon} &= E\left[\alpha_I\left(\sum_{i=1}^{I}\frac{\left(b_i-b_j\right)^2}{I} - \sum_{i=1}^{I}\frac{\left(b_i-b_{m_I}\right)^2}{I} + \frac{1}{2\left(d_j+2\right)} - \frac{1}{2\left(d_{m_I}+2\right)}\right)\right] \\
&= \alpha_I E\left[\sum_{i=1}^{I}\frac{\left(b_i-b_{m_I}-\left(b_j-b_{m_I}\right)\right)^2}{I} - \sum_{i=1}^{I}\frac{\left(b_i-b_{m_I}\right)^2}{I}\right] + \\
&+ \alpha_I E\left[\frac{1}{2\left(d_j+2\right)} - \frac{1}{2\left(d_{m_I}+2\right)}\right] \\
&= \alpha_I E\left[\sum_{i=1}^{I}\frac{\left(b_i-b_{m_I}\right)^2+\left(b_j-b_{m_I}\right)^2-2\left(b_i-b_{m_I}\right)\left(b_j-b_{m_I}\right)}{I} - \sum_{i=1}^{I}\frac{\left(b_i-b_{m_I}\right)^2}{I}\right] \\
&\quad +\alpha_I E\left[\frac{1}{2\left(d_j+2\right)} - \frac{1}{2\left(d_{m_I}+2\right)}\right]
\end{aligned}
$$

Since, by Lemma 4, $\lim_{I\to\infty}\Pr\left(d_{m_I} \leq d\right) = 0$ for all $d > 0$, it follows that $\lim_{I\to\infty} E\left[\frac{1}{2\left(d_{m_I}+2\right)}\right] = 0$. Further, $\lim_{I\to} m_I = E\left[b_i\right] = \lim_{I\to\infty} E\left[\sum_{i=1}^{I}\frac{b_i}{I}\right]$. Using these facts we have that

$$
\lim_{I\to\infty} W_I^{m_I} - W_I^{\alpha\varepsilon} \geq \lim_{I\to\infty} \alpha \sum_{i=1}^{I}\frac{\left(b_j-b_{m_I}\right)^2}{I} \geq \alpha\delta^2 > 0.
$$

This result implies the as I approaches infinity, all decisions are optimally concentrated to politicians sufficiently close to the most moderate agent m_I. This concludes the proof of proposition 4. ■

Proof of Proposition 5. We first prove that full decentralization is never optimal.

Note that if there is $i > 1$ such that $b_i - b_{i-1} \leq 1/6$, then i is informed of $i-1$'s message or viceversa. For generic assignments of $\mathbf{b}$, it cannot be the case that $\left|\sum_{j=1}^{I} \gamma_j b_j - b_i\right| = \left|\sum_{j=1}^{I} \gamma_j b_j - b_{i-1}\right|$. Supposing without loss of generality that $\left|\sum_{j=1}^{I} \gamma_j b_j - b_i\right| < \left|\sum_{j=1}^{I} \gamma_j b_j - b_{i-1}\right|$, it is therefore welfare superior to assign $a(i-1) = i$ rather than $a(i-1) = i-1$. So suppose that $b_i - b_{i-1} > 1/6$ for all i, so that for all $j \neq i$, $d_{i,j}(\mathbf{m}) = 0$ in any equilibrium $(\mathbf{m}, \mathbf{y})$. Hence, assigning $a(1) = \lfloor (I+1)/2 \rfloor \equiv m^*$ yields higher welfare than $a(1) = 1$ if and only if:

$$\sum_{i=1}^{I} \frac{(b_i - b_1)^2}{I} - \sum_{i=1}^{I} \frac{(b_i - b_{m^*})^2}{I} > \frac{1}{36}.$$

The left-hand side can rewritten as:

$$\begin{aligned} D(\boldsymbol{\Delta}) \quad = \quad & \sum_{i=2}^{I} \left[(i-1)\frac{1}{6} + \sum_{j=2}^{i} \left(\Delta_j - \frac{1}{6} \right) \right]^2 - \\ & \sum_{i=m+1}^{I} \left[(i-m)\frac{1}{6} + \sum_{j=m+1}^{i} \left(\Delta_j - \frac{1}{6} \right) \right]^2 - \sum_{i=1}^{m-1} \left[(m-i)\frac{1}{6} + \sum_{j=i+1}^{m} \left(\Delta_j - \frac{1}{6} \right) \right]^2, \end{aligned}$$

where $\Delta_2 = b_2 - b_1, ..., \Delta_I = b_I - b_{I-1}$.

We now show that $D(\boldsymbol{\Delta})$ increases in all its terms Δ_k.

When $k > m$, we obtain:

$$\frac{\partial}{\partial \Delta_k} D(\boldsymbol{\Delta}) = 2\sum_{i=k}^{I} \left[(i-1)\frac{1}{6} + \sum_{j=2}^{i} \left(\Delta_j - \frac{1}{6} \right) \right] - 2\sum_{i=k}^{I} \left[(i-m)\frac{1}{6} + \sum_{j=m+1}^{i} \left(\Delta_j - \frac{1}{6} \right) \right]$$

which is clearly positive because $m > 1$ and $m+1 \geq 2$.

When $k = m$, we have

$$\frac{\partial}{\partial \Delta_k} D(\boldsymbol{\Delta}) = 2\sum_{i=k}^{I} \left[(i-1)\frac{1}{6} + \sum_{j=2}^{i} \left(\Delta_j - \frac{1}{6} \right) \right] > 0$$

Suppose finally that $k < m$,

$$\begin{aligned}\frac{\partial}{\partial \Delta_k} D(\boldsymbol{\Delta}) &= 2\sum_{i=k}^{I}\left[(i-1)\frac{1}{6}+\sum_{j=2}^{i}\left(\Delta_j-\frac{1}{6}\right)\right]-2\sum_{i=1}^{k-1}\left[(m-i)\frac{1}{6}+\sum_{j=i+1}^{m}\left(\Delta_j-\frac{1}{6}\right)\right] \\ &= 2\sum_{i=k}^{I}(i-1)\frac{1}{6}-2\sum_{i=1}^{k-1}(m-i)\frac{1}{6} \qquad (8) \\ &\quad +2\sum_{i=k}^{I}\sum_{j=2}^{i}\left(\Delta_j-\frac{1}{6}\right)-2\sum_{i=1}^{k-1}\sum_{j=i+1}^{m}\left(\Delta_j-\frac{1}{6}\right). \qquad (9)\end{aligned}$$

Because, $k < m$, evidently,

$$\begin{aligned}2\sum_{i=k}^{I}(i-1)\frac{1}{6} &> 2\sum_{i=m+1}^{I}(i-1)\frac{1}{6} > 2\sum_{i=m+1}^{I}(i-m)\frac{1}{6}, \\ &\text{and } 2\sum_{i=1}^{k-1}(m-i)\frac{1}{6} < 2\sum_{i=1}^{m-1}(m-i)\frac{1}{6}.\end{aligned}$$

and hence expression (8) is strictly positive. Further

$$\begin{aligned}2\sum_{i=k}^{I}\sum_{j=2}^{i}\left(\Delta_j-\frac{1}{6}\right) &> 2\sum_{i=k}^{m}\sum_{j=2}^{i}\left(\Delta_j-\frac{1}{6}\right) \\ &= 2\sum_{i=2}^{k}\sum_{j=i}^{m}\left(\Delta_j-\frac{1}{6}\right) = 2\sum_{i=1}^{k-1}\sum_{j=i+1}^{m}\left(\Delta_j-\frac{1}{6}\right)\end{aligned}$$

and hence expression (9) is strictly positive, concluding that $\partial D(\boldsymbol{\Delta})/\partial \Delta_k$ is strictly positive. Hence, we may take $\boldsymbol{\Delta} = \mathbf{1/6}$, so that

$$D(\mathbf{1/6}) = \sum_{i=2}^{I}\left[(i-1)\frac{1}{6}\right]^2-\sum_{i=m+1}^{I}\left[(i-m)\frac{1}{6}\right]^2-\sum_{i=1}^{m-1}\left[(m-i)\frac{1}{6}\right]^2,$$

Noting that for I odd,

$$D(\mathbf{1/6}) = \sum_{i=2}^{I}\left[(i-1)\frac{1}{6}\right]^2-2\sum_{i=m+1}^{I}\left[(i-m)\frac{1}{6}\right]^2=\frac{1}{4}I(I-1)^2\frac{1}{36}\geq\frac{1}{4}\cdot 3\cdot 4\frac{1}{36}>\frac{1}{36}.$$

and for I even,

$$D(\mathbf{1/6}) = \sum_{i=2}^{I}(i-1)^2-\sum_{i=I/2+1}^{I}(i-I/2)^2-\sum_{i=1}^{I/2-1}(I/2-i)^2=\frac{1}{4}I^2(I-2)\frac{1}{36}\geq\frac{1}{4}\cdot 16\cdot 2\frac{1}{36}>\frac{1}{36},$$

we conclude that $a(1) = \lfloor (I+1)/2 \rfloor \equiv m^*$ yields higher welfare than $a(1) = 1$.

Having proved that full decentralization is never optimal, we now show the most moderate politician should be assigned the decision of sufficiently moderate and sufficiently extreme-bias politicians.

Indeed, first note that for any k such that $|b_k - b_{m*}| < 1/6$, the most moderate politician m^* is equally informed as k in equilibrium, and hence it is optimal that m^* is assigned policy k.

Second, recall that, whenever $m(k) \neq m^*$, policy k should be assigned to m^* if and only if inequality is satisfied. Expand the left-hand side of this inequality as follows:

$$\begin{aligned}
& \sum_{i=1}^{I} \frac{b_i^2 + b_{m(k)}^2 - 2b_i b_{m(k)} - b_i^2 - b_{m^*}^2 + 2b_i b_{m^*}}{I} \\
= & \sum_{i=1}^{I} \frac{(b_{m(k)} + b_{m^*} - 2b_i)(b_{m(k)} - b_{m^*})}{I} \\
= & (b_{m(k)} + b_{m^*} - 2\bar{b})(b_{m(k)} - b_{m^*}),
\end{aligned}$$

where $\bar{b}$ is the average of the bias vector $\mathbf{b}$. Take k such that $b_k - b_{m^*} > 0$, the case when $b_k - b_{m^*} < 0$ is analogous. Note that, by construction, $0 < b_k - b_{m(k)} < 1/6$. Hence, if $b_k - b_{m^*}$ is sufficiently large, this is also the case for $b_{m(k)} - b_{m^*}$, so that the above expression is larger than $1/36$ and inequality 9 is satisfied. ■

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10. Supplementary Material for On Line Appendix: Cabinet Simulations

In this appendix we explore further optimal decision-making authority assignments in cabinet governments by running simulations for a 7 member parliament in which players' biases are independent and identically distributed according to a skew normal distribution, a distribution chosen for tractability. Skew normal distributions depend on three parameters which are related with the three usual moments; mean μ, variance σ^2 and skewness γ, where γ controls the asymmetry of the sampled distributions of ideology draws and σ determines the concentration of such sampled distributions draws. The normal distribution is obtained as a special case when $\gamma = 0$, whereas the most extreme skewness is for $\gamma = 1$. Because only difference in ideologies matter for our characterization, we can normalize μ to zero, without loss of generality.

We first analyze cabinets where the θ is the same for all policies. We calculate two statistics that capture the degree of centralization of authority: (i) the average number of decisions allocated to the executive leader – the individual who makes the most decisions; and (ii) the frequency of draws for which a single leader makes all decisions in a cabinet environment. The results shown in table 1 and table 2 confirm a general tendency towards centralized authority, which have been described in large legislatures by Proposition 4. In fact, the average number of decisions made by the leader ranges from 79% to 100%. Interestingly, the fraction of decisions assigned to the leader is U-shaped in the variance of the distribution, and this holds independently of the asymmetry of the distribution, or skewness. Finally, allocating all actions to a single leader is often suboptimal: the frequency with which a single leader is chosen to implement *all* policy decisions may be below 50%. An implication is that in most cases centralization of authority in a multi-member cabinet is Pareto superior to other executive forms.

Next we build on the results in section 8. Having shown there that full decentralization is never optimal, we now explore optimal government in the case of policy specific information. As in the common state case, we discuss numerical results obtained for legislatures with $I = 7$ politicians. The simulation shown in Table 3 and 4 report the leader's average number of assigned decisions and the frequency with which the executive leader makes decisions when information is policy specific. The results show that centralization of executive authority is not smaller (in fact, usually, it is larger) than in the common-state case with evenly distributed expertise for given parameter values γ and σ of the skew normal distribution. Thus, whilst we uncover rich equilibrium behavior allowing for both single leadership and cabinet arrangements, the optimal decision-making authority assignment is no more decentralized than in the common-state case. Further, the fraction of decisions made by the most moderate politician is non-monotonic in the dispersion of politicians' ideologies.

Table 1. The Average Number of Decisions made by the Executive Leader

	$\gamma = 0$	$\gamma = 1/4$	$\gamma = 1/2$	$\gamma = 3/4$	$\gamma = 1$
$\sigma^2 = 10$	7.00	7.00	7.00	6.99	7.00
$\sigma^2 = 1$	6.91	6.93	6.89	6.91	6.88
$\sigma^2 = 0.1$	6.17	6.21	6.18	6.16	6.21
$\sigma^2 = 0.01$	5.58	5.51	5.53	5.68	5.67
$\sigma^2 = 0.001$	6.35	6.35	6.37	6.49	6.35
$\sigma^2 = 0.0001$	7.00	7.00	7.00	7.00	7.00

Table 2. Frequency with which the Executive Leader makes all Decisions

	$\gamma = 0$	$\gamma = 1/4$	$\gamma = 1/2$	$\gamma = 3/4$	$\gamma = 1$
$\sigma^2 = 10$	1.00	1.00	1.00	0.99	1.00
$\sigma^2 = 1$	0.95	0.96	0.93	0.95	0.92
$\sigma^2 = 0.1$	0.57	0.60	0.58	0.61	0.62
$\sigma^2 = 0.01$	0.40	0.36	0.36	0.41	0.45
$\sigma^2 = 0.001$	0.70	0.70	0.74	0.78	0.69
$\sigma^2 = 0.0001$	1.00	1.00	1.00	1.00	1.00

Table 3. Average Number of Decisions Assigned to the Executive Leader when Information is Policy-specific

	$\gamma = 0$	$\gamma = 1/4$	$\gamma = 1/2$	$\gamma = 3/4$	$\gamma = 1$
$\sigma^2 = 10$	6.96	6.95	6.97	6.97	6.97
$\sigma^2 = 1$	6.70	6.73	6.69	6.81	6.74
$\sigma^2 = 0.1$	5.97	5.90	5.90	5.86	5.75
$\sigma^2 = 0.01$	6.59	6.54	6.53	6.57	6.55
$\sigma^2 = 0.001$	7.00	7.00	7.00	7.00	7.00
$\sigma^2 = 0.0001$	7.00	7.00	7.00	7.00	7.00

Table 4. Frequency with which the Executive Leader makes all Decisions

	$\gamma = 0$	$\gamma = 1/4$	$\gamma = 1/2$	$\gamma = 3/4$	$\gamma = 1$
$\sigma^2 = 10$	0.97	0.96	0.97	0.97	0.97
$\sigma^2 = 1$	0.78	0.82	0.80	0.85	0.83
$\sigma^2 = 0.1$	0.38	0.39	0.39	0.37	0.29
$\sigma^2 = 0.01$	0.66	0.64	0.64	0.64	0.66
$\sigma^2 = 0.001$	1.00	1.00	1.00	1.00	1.00
$\sigma^2 = 0.0001$	1.00	1.00	1.00	1.00	1.00