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Abstract

This paper considers the efficiency of money creation by banks and the principles of the central-bank issuance to improve over it. The ability to issue deposit liability as a means of payment enlarges banks' lending capacities and subjects them to fiercer competition. In circumstances where banks issue too much money, interest-rate policy may help. In circumstances of a credit crunch, quantitative-easing policy helps, under which the central bank lends its issues to all banks. These issues are unbacked by taxation and purely nominal. The optimal quantity of the central bank's lending is unique and implements the first-best allocation. (JEL: D53, E40, E58)

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1 Introduction

Banks create money by circulating deposit liability, which is accepted as a means of payment.[1] This function of banks is critical to real economic activities; e.g., very often, the efficient users of production factors (such as capital, labor) cannot use their own promises to pay, to obtain some of the factors, and thus, they need a means of payment to put the factors into efficient production. Do banks create too much or too little money in equilibrium? If they do, how can the authority, i.e., the central bank (CB hereafter), improves economic efficiency with its ability to print the outside means of payment? Addressing these questions, this paper is among the first attempts to build the analysis of monetary policy on a general-equilibrium examination of private issuance. This approach may better our understanding of central banking because the money that cycles the real economy is issued by both the private banks and the CB. This paper endogenizes both interest-rate policy and quantitative-easing policy as an optimal response by the CB to certain situations.

The acceptance of deposit liability as a means of payment, the paper maintains, comes from the belief that the liability is equivalent to some already-accepted means of payment, often called *cash* (such as gold or the CB's issues).[2] A key factor that underpins

[1] Deposit liability may be issued to exchange for the depositors' cash (which can thus be lent out for another round). It may also be directly lent out to borrowers, both historically and nowadays. Historically, this lending could be done by banks issuing their own notes to a borrower. Nowadays, when most banks do not print notes of their own, it is done by banks putting a number into a borrower's deposit account (possibly after having him open one), with a click of mouse. This is usually how lending is done when the amount lent is big, e.g., mortgages; only rarely do banks disgorge hundred thousands of dollars of greenback to a mortgage borrower.

[2] In case of the cash being the CB's issues, a private bank needs to sustain this belief of its deposit

this belief is that deposit liability can be converted to cash on demand. Banks commit to this conversion on demand, therefore, in order to get their liabilities circulated as money. A widespread disbelief in their ability to do so may trigger bank run.

All this thinking leads to a model in which entrepreneurs cannot use their promises to pay for hiring workers, who, however, accept payment either with the CB's issues, or with bankers' liabilities, which the bankers commit to redeem with the real good on demand. As a result, to hire workers, entrepreneurs have to borrow either bankers' notes or the CB's. While the CB's issuance is a policy decision, bankers' issuance varies with the market conditions that they face, which is affected by the CB's policy decision.

The paper considers the efficiency of private money-issuance in the basic model, where bank run and bank insolvency are assumed to be costless. This assumption allows for a least-fettered issuance, with which banks possess an unlimited capacity. Bertrand competition with such a capacity leads to a unique real allocation, which is the one that would arise if money were not needed for the real economic activities. Therefore, the least-fettered issuance supplies the real sectors with the exact quantity of money that they want.[3] However, if banks' wealth is below a threshold, they fall insolvent in equilibrium, and this insolvency spurs all their depositors to run to them for redemption, that is, bank runs arise.[4]

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liability equivalent to the CB's issues, whereas the CB obviously faces no such need, although both banks' liability functions as money.

[3] This result is in line with the insights of the Banking School in the old debate about systems of free banking (see Goodhart 1988 Chapter 4) and with those of Hayek (1976).

[4] Jacklin and Bhattacharya (1988) and Chari and Jagannathan (1988) also consider bank run that is triggered by concerns about bank assets instead of by mis-coordination. Goldstein (2010) reviews empirical evidence for both points of view.

Bank run is taken as a grave concern to banks in the extension, where it is assumed to be prohibitively costly. This generates a rich setting to consider central banking. In the basic model, banks always issue too much because, due to some real friction, their issues draw more than the first-best number of workers to entrepreneurs. Now, in the extension, required to maintain solvency (as to avoid bank run), banks limit their issuance within a proportion to their wealth. Depending on the aggregate level of their wealth, they may issue too much or too little. For both situations, this paper considers how the CB improves efficiency by printing purely nominal claims. In the situation of a credit crunch, with banks issuing too little money, interest-rate policy, as bounded by 0 from below, helps nothing. But a policy under which the CB lends its issues to *all* banks at a zero interest rate, improves efficiency, because it enlarges the quantity of money supply and thereby eases a shortage of money which constrains the real economic activities. The optimal quantity of the CB's lending is unique and implements the first-best allocation. In the situation where banks issue too much money, this type of quantitative-easing policy may only make things worse. A proper interest-rate policy, however, implements the first-best allocation if the workers' nominal wage is sticky.[5] In that case, a high policy rate pushes up the interest rate of bank lending, thereby diminishing the real economic activities.

This paper is conceptually based on Kiyotaki and Moore (2001). Unlike the literature focusing on financial intermediation,[6] this paper considers banks' function of

[5]The introduction of sticky wages would not affect the analysis of the situation of a credit crunch.

[6]For a good survey of the literature, see Gorton and Winton (2003). Note that this role of money creation is different from the role of financial intermediation between investors and capital users. In this paper, the real resource that workers "lend" to entrepreneurs is their labor and cannot be deposited; rather, bankers issue money to combine workers with entrepreneurs into production.

creating money in a general equilibrium.[7] Brunnermeier and Sannikov (2013), as in this paper, consider financial intermediaries creating money by issuing deposit liability and emphasize how the low wealth of intermediaries constrains their capacity of this issuance. They consider money's value-storing role[8] – namely, as an asset with a risk profile different from capital – whereas this paper focuses on money's role as means of payment. Moreover, the two papers complement one another by embracing a different facet of monetary policy: theirs considers long-term government bonds, this one the quantitative-easing policy. Hart and Zingales (2011, 2013) (HZ hereafter), like this paper, also consider private issuance of money in a transaction role. In their paper, the issuance of money must be collateralized by the real investment. With this link, they find that private issuance entails investment inefficiency. In contrast, this paper considers uncollateralized bank lending and emphasizes the role of banks' wealth. Furthermore, HZ consider fiscal policy (for the reason explained below), whereas this paper considers the CB's issuance of purely nominal claims.

In considering the CB's issuance, this paper examines both interest-rate policy and quantitative-easing policy. Moreover, this examination is built on a general-equilibrium analysis of private banks circulating their liabilities as a means of payment. These two features together distinguish this paper's model – though it is limited in some respects – from other studies of monetary policy in which money's transaction role is

[7] With a search-matching framework, the money-creation role of banks is considered by Cavalcanti et. al. (1999), Cavalcanti and Wallace (1999), Wallace and Zhu (2007), and Araujo and Minetti (2011), among others.

[8] In the papers by Champ, Smith and Williamson (1996) and Freeman (1999a), money – including fiat money and banknotes – plays a similar role, as the saving instruments across different locations.

explicitly modeled.[9] Stein (2012), as in this paper, considers a variety of policies and regulations based on an analysis of private banks' issuance of money, which is defined as the riskless liabilities of the banks. Different from this paper, however, he does not consider the situation in which banks issue too little money and a quantitative-easing policy is optimal.

In this paper, quantitative-easing policy works by expanding banks' lending capacities, reminiscent of the "credit policy" as classified by Reis (2009). In his model, as in HZ's, the government-provided money is essentially sovereign debt, backed by the government's power of taxation. Therefore, the monetary policy is intertwined with fiscal policy, as it is in other studies on unconventional monetary policy, such as Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). In the present paper, by contrast, what the CB issues is unbacked by taxation and purely nominal, but still circulates in the finite-period economy. Moreover, unlike in those papers, in the present paper the quantitative easing policy works not by transferring wealth to banks, but by intensifying competition between them, which may very likely squeeze their profit.

The rest of the paper is organized as follows. Section 2 sets up the basic model, with costless bank run and bank default. The model is analyzed in Section 3. Section 4, assuming that bank run is prohibitively costly, introduces the central bank. Section 5 presents some discussion. Section 6 concludes. All proofs are relegated to Appendix.

[9] This paper's model of money in advance, where money is also privately issued, is reminiscent of cash-in-advance models, especially, those on working capital, such as Christiano and Eichenbaum (1992; 1995) and Fuerst (1992). For a summary of this literature and other relevant ones, see Walsh (2010). In addition, Freeman (1996b) endogenizes the discount-window function of the CB, and Williamson (2012, 2013), and Williamson and Wright (2011) consider monetary policy based on Lagos and Wright (2005). Williamson (2013), in particular, addresses quantitative-easing policy.

2 The Basic Model

The economy has one storable good, corn, which is used as the numeraire. There are three dates, $t = 0$, 1, and 2. Production occurs at $t = 0$ and yields output at $t = 2$, when all the consumption occurs. There are N bankers, N^2 entrepreneurs and N^3 workers, where N is a large number. Thus, bankers are in perfect competition and each serves a large number of entrepreneurs; and there are more workers than entrepreneurs can hire. All agents are risk-neutral and protected by limited liability. For convenience, bankers are referred to as females, entrepreneurs as males.

Workers either produce w kilograms (kg) of corn in autarky, or are hired by entrepreneurs, who each have h units of human (or physical) capital. If an entrepreneur hires L workers at $t = 0$, then his project yields at $t = 2$

$$y = \widetilde{A}h^{1-\alpha}L^{\alpha},$$

where $0 < \alpha < 1$. Without losing any generality, normalize $h = 1$. Productivity, $\widetilde{A}$, is subject to a common shock. At $t = 0$, it is common knowledge that $\widetilde{A} = \overline{A}$ with probability q and $\widetilde{A} = \underline{A}$ with probability $1 - q$. The realization of $\widetilde{A}$ is publicly known at $t = 1$. Let $A_e \equiv q\overline{A} + (1 - q)\underline{A}$ denote the mean. Assume:

$$0 < \underline{A} < A_e\alpha. \tag{1}$$

That is, the negative shock is really severe.

As there are more workers than entrepreneurs can hire, entrepreneurs hire workers at a real wage of w, what they earn in autarky.

Bankers each have G units of corn, where the unit is defined as N kg and used wherever bankers are concerned. If there is no friction, bankers are irrelevant to corn production. What makes bankers important is the following friction of payment.

Friction 1: *entrepreneurs cannot use their promises to pay for hiring workers.*[10]

Workers accept payment with corn, and with bankers' promises to pay if these promises *can* be converted into corn on demand. At $t = 0$, bank can thus lend out either corn, or the paper notes that certify their promises to pay certain amount of corn; these notes they commit to redeem on demand. They lend out only the notes, and keep the corn as the reserve ready to meet the demand for redemption at $t = 1$ when their loans to the entrepreneurs (which mature at $t = 2$) yield nothing yet. This modeling of notes-lending captures two ways in which banks in real life issue deposit liability and create money. It obviously captures the direct lending of deposit liability,[11] which Withers (1920, page 24) regarded as "epoch-making." It also captures the essence of the cycles of loan-deposit-loan that macroeconomics textbooks typically describe: Bankers may lend only corn to entrepreneurs, who pass it on to the workers hired; the workers, feeling unsafe to store it in their basements, deposit it back to bankers and take the notes that certify the bankers' liabilities to them; then, bankers lend out corn for another round. The amount certified on a note is the note's *face value* or *par value*, which, this paper assumes, cannot be contingent on the realization of $\widetilde{A}$. Denote by D the aggregate face value of a banker's notes.

[10] Following Kiyotaki and Moore (2001), this friction can be interpreted in two ways. Directly, it represents a borrowing constraint, arising because workers do not trust entrepreneurs' promises. Alternatively, this friction captures a *resale constraint*: suppose there are many, say K, types of goods and each is equally needed for subsistence and produced by N/K entrepreneurs; the workers of an entrepreneur trust his promise to pay them with his product, but they cannot use this promise for exchanging other goods, nor can they easily bring his product around for that purpose. This modified version of the model will deliver results qualitatively the same as the present version.

[11] For more of its present relevance, see footnote 1.

Bankers need to take care of two matters: liquidity and solvency. If a banker issues $D > G$, she may have the liquidity problem: She is unable to meet the demand for redemption at $t = 1$ if she exhausts her reserve, G. In that case, the banker suspends redemption at $t = 1$ and reopens for redemption at $t = 2$ after receiving payments from the debtor entrepreneurs. As such, G currently represents both bankers' wealth and reserve, but it is the wealth that G really captures, and the reserve plays no role in this paper, all being clarified in Section 5. In addition to this liquidity problem, bankers may become insolvent, unable to redeem all of their notes at par, if they have issued too much, with a too big D. Anticipating this possibility of insolvency, workers discount bankers' notes at $t = 0$, with a factor of $\delta \leq 1$. In the basic model, assume that bank illiquidity and insolvency are costless to handle, therefore do not concern bankers; this assumption is made to study the least fettered issuance in a competitive circumstance.

Besides the friction of payment, the economy suffers another friction.

Friction 2: *entrepreneurs are unable to commit on the scale of their projects in terms of the number of workers they hire.*

This friction can be regarded as "real," in the sense that it is unrelated to a means of payment. Due to this friction, the contract between a banker and an entrepreneur is not conditional on the scale of the latter's project, and as follows. At $t = 0$, the entrepreneur borrows the banker's notes of overall face value E, whereby he owes the banker a debt of $E(1+r)$, which he repays at $t = 2$ either with corn or with the banker's notes. Thus, r is the interest rate charged by the banker.

Assume that bankers' choices of (D, r) are publicly observed and that no bankers issue notes at $t = 1, 2$.

The timing is as follows.

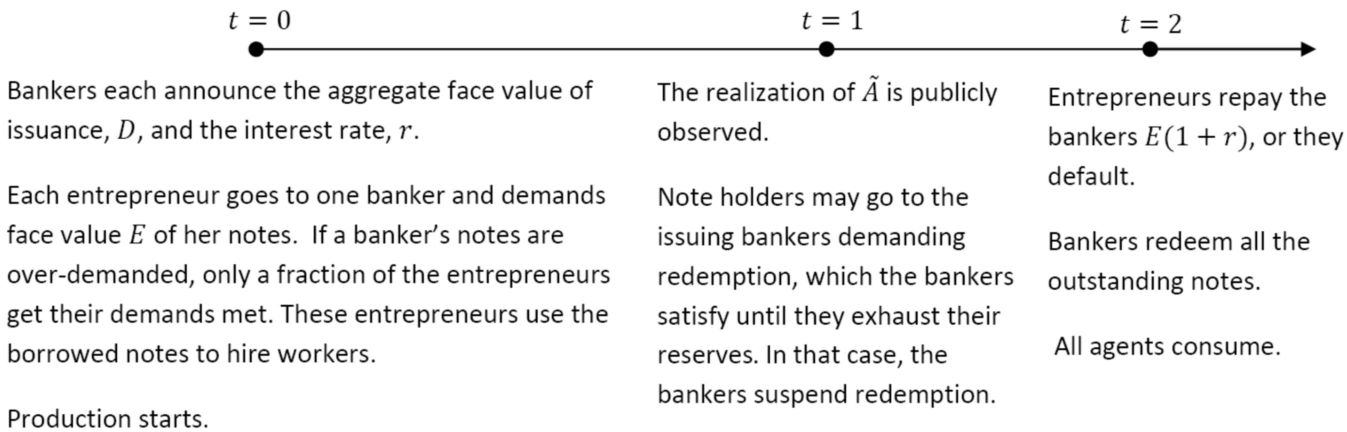

Figure 1: Timing of Events

Passing on to the equilibrium, this paper figures out two benchmark allocations.

The First-Best and Second-Best Allocations

Efficiency concerns the number of workers allocated to entrepreneurs. Define the first-best allocation as the number of workers at which the social surplus of each project is maximized. Due to universal risk neutrality and the opportunity cost of labor being w, the social surplus is $A_e L^\alpha - wL$, to maximize which, the first-best allocation is

$$L^{FB} = (\frac{A_e \alpha}{w})^{\frac{1}{1-\alpha}}. \tag{2}$$

The second-best allocation is defined as the allocation of the competitive equilibrium if Friction 1 were absent, but Friction 2 stayed – that is, if entrepreneurs could hire workers with their promises to pay, but their wage offer could not be contingent on the scale of the projects. The equilibrium allocation is as follows.

Lemma 1 *The second-best number of workers drawn to entrepreneurs is:*

$$L^{SB} = (\frac{q\overline{A}\alpha + (1-q)\underline{A}}{w})^{\frac{1}{1-\alpha}}. \tag{3}$$

Obviously, $L^{SB} > L^{FB}$.[12] This wedge between the two allocations allows for a circumstance where bankers issue too much money, as will be shown.

3 The Analysis of Competitive Equilibrium

This section analyzes bank issuance in competitive equilibrium: first the demand of bankers' notes, then the supply, and, finally, the meeting of the two.

3.1 The Demand Side of the Market for Notes

Consider the entrepreneurs who jointly borrow all of a banker's issues, D, at an interest rate of r, where (D, r), as will be shown, determines δ, the discount factor of the banker's notes. If the entrepreneurs each borrow face value E of the notes, then, given that the notes are worth δE and the real wage is w, they each hire L workers, where

$$L = \delta E / w. \tag{4}$$

At $t = 2$, these entrepreneurs have the duty to repay the banker in total a value of $D(1+r)$ (as each one's debt to the banker is $1+r$ times of what he borrows), with either corn or her notes. The latter means can be cheaper – thus preferred – only if the entrepreneurs default: If they did not default and the banker's notes were discounted, altogether their demand for the notes would be $D(1+r)$ (in face value), but the outstanding notes, due to possible redemption at $t = 1$, was no more than D, below the demand. Therefore, either the entrepreneurs need to outlay a real value of $E(1+r)$ to repay their loans by

[12] This is because of Friction 2; if entrepreneurs could compete by posting both wage and scale, then the equilibrium allocation would conform to the first-best. A full-fledged analysis can be found in Wang (2010).

any means, or they default. Thus, the decision problem of the entrepreneurs is:

$$\max_{E} q(\overline{A}L^{\alpha} - E(1+r)) + (1-q)\max(\underline{A}L^{\alpha} - E(1+r), 0), s.t.(4). \tag{5}$$

Lemma 2 *Entrepreneurs all default in the bad state: For any (w, δ, r), the solution to problem (5) satisfies $\underline{A}L^{\alpha} < E(1+r)$.*

Therefore, the entrepreneurs' borrowing maximizes their profit in the good state, $\overline{A}L^{\alpha} - E(1+r)$, and, thus, is

$$E(\delta, r) = (\frac{\overline{A}\alpha}{1+r})^{\frac{1}{1-\alpha}}(\frac{\delta}{w})^{\frac{\alpha}{1-\alpha}}. \tag{6}$$

Then, the amount of the labor they hire and their profit are, respectively,

$$L(R) = (\frac{\overline{A}\alpha}{wR})^{\frac{1}{1-\alpha}} \tag{7}$$

$$V(R) = q(1-\alpha)(\frac{\overline{A}^{\frac{1}{\alpha}}\alpha}{wR})^{\frac{\alpha}{1-\alpha}}, \tag{8}$$

where

$$R \equiv \frac{1+r}{\delta}. \tag{9}$$

So defined, R can be regarded as the *real* gross interest rate of borrowing: To obtain a means of payment that is worth 1, an entrepreneur borrows notes of face value $1/\delta$, then in a debt of $(1+r)/\delta$. Naturally, real variables L and V depend only on the real interest, R, and inversely: at a higher real interest rate, the entrepreneurs employ fewer workers and obtain less profit.

Define R^{FB} (R^{SB}) by $L(R) = L^{FB}$ (L^{SB}); that is, at $R = R^{FB}$ (R^{SB}), entrepreneurs hire the first-best (second-best) number of workers. From (2), (3) and (7),

$$R^{FB} = \frac{\overline{A}}{A_e} \tag{10}$$

$$R^{SB} = \frac{\overline{A}\alpha}{q\overline{A}\alpha + (1-q)\underline{A}}. \tag{11}$$

After all bankers have chosen (D, r), entrepreneurs decide which bankers to go to. In the equilibrium of this subgame, an entrepreneur gets the same expected profit, $\widehat{V}$, from going to any banker who attracts some entrepreneurs.[13] Define $\widehat{R}$ by $V(R) = \widehat{V}$. Then, $\widehat{R}$ can be regarded as the real interest rate that prevails on the notes market, contingent on all bankers' choices of (D, r).

3.2 The Supply Side of the Market for Notes

Given there is a large number of bankers, a single banker is too small to affect $\widehat{V}$, and thus takes it as given when choosing (D, r). To attract entrepreneurs, a banker offers $V(R) \geq \widehat{V}$ or, equivalently, $R = (1 + r)/\delta \leq \widehat{R}$. The interest rate r is chosen by the banker directly. The discount factor, δ, is determined by (D, r). It depends on whether the banker defaults. In the good state, when the entrepreneurs do not default, the banker has no difficulty redeeming her notes. But in the bad state, by Lemma 2, all the entrepreneurs default, which drags the banker to default if she has issued so much that her wealth is not sufficient to cover the loss, as the proposition below states.

Proposition 1 *If a banker chooses* (D, r), *then,*

(i) in the bad state, the value of her loans is

$$\underline{Y} = \frac{\underline{A}(1 + r)}{\overline{A}\alpha} D, \tag{12}$$

and she does not default if and only if

$$D \cdot (1 - \frac{\underline{A}(1 + r)}{\overline{A}\alpha}) \leq G; \tag{13}$$

[13] On the one hand, no entrepreneur goes to a banker offering $V(R) < \widehat{V}$ when he can get $\widehat{V}$ elsewhere. On the other hand, if a banker offers $V(R) > \widehat{V}$, she induces over-demand for her issues (which is never optimal), so an entrepreneur coming to her is served with such a probability l that $l \cdot V(R) = \widehat{V}$.

(ii) the discount factor of her notes at $t = 0$ *is*

$$\delta(D, r) = \left\{ \begin{array}{c} 1, \text{ if (13) is satisfied} \\ q + (1-q)(\frac{G}{D} + \frac{A(1+r)}{\bar{A}\alpha}), \text{ otherwise} \end{array} \right\}. \tag{14}$$

Intuitively, in the bad state, the banker's balance sheet is:

Asset	**Liability**
Reserve (G)	Equity
Loans to the entrepreneurs ($\underline{Y}$)	Liability to the note holders (D)

Table 1: The balance sheet of a banker in the bad state

She does not default if and only if

$$D \leq G + \underline{Y}, \tag{15}$$

which, with $\underline{Y}$ given by (12), is equivalent to (13).

An intuition for (14) is that a banker's notes are not discounted (i.e., $\delta = 1$) if she will never default; otherwise, her notes are discounted in the bad state at a factor of $(G + \underline{Y})/D$, the quotient of the asset value over the liability.

Having examined the matter of bankers' solvency, we now consider the matter of liquidity. On the one hand, *if a banker is known to become insolvent, then she faces a bank run.* The insolvency of the banker means that she cannot redeem *all* her notes at par. Therefore, some of them must be discounted. But if a holder of her notes gets to the banker early enough, before her reserve is depleted, he gets his holding redeemed at par, suffering no discount. Thus, the prospect of insolvency triggers all the note holders to run to the banker for redemption. On the other hand, *if a banker is solvent, then she has no liquidity problems.* At $t = 1$, a note holder, even if he believes all the other note holders are demanding redemption, does not bother himself to go to the banker for redemption (especially if that incurs a transportation cost): Staying home, he knows that the banker will suspend redemption today, and, at $t = 2$, his holding is worth its par value; therefore, he loses nothing. *To bankers, therefore, illiquidity is a problem resulting from doubt about their solvency.*

Now consider bankers' decision problem at $t = 0$. If a banker chooses (D, r), then her net profit is $q(G + Dr) + (1 - q)\max(G + \underline{Y} - D, 0) - G$, where the "max" term is there because the banker may default in the bad state. This net profit, with $\underline{Y}$ given by (12), equals

$$\Pi(D, r) = q(G + Dr) + (1 - q)\max(G - (1 - \frac{\underline{A}(1 + r)}{\overline{A}\alpha})D, 0) - G. \tag{16}$$

Taking the real interest rate prevailing on the market, $\widehat{R}$, as given, each banker chooses (D, r) to maximize $\Pi(D, r)$, subject to the constraint that she can attract entrepreneurs, that is,

$$\frac{1 + r}{\delta(D, r)} \leq \widehat{R}, \tag{17}$$

where $\delta(D, r)$, the discount factor determined by (D, r), is given by (14). The solution to this problem depends on $\widehat{R}$ and is given below.

Proposition 2 *The solution to and the value of a banker's problem are:*

(i) if $\widehat{R} > R^{SB}$, *then* $D = \infty$ *and* $\Pi = \infty$;

(ii) if $\widehat{R} = R^{SB}$, *then* $\Pi = 0$, *and the banker is indifferent with to any quantity of issues,* D, *with* r *determined by* D *through the binding (17);*

(iii) if $R^{SB} > \widehat{R}$, *then lending makes a loss, and thus* $D = 0$ *and* $\Pi = 0$.

By the proposition, the profit margin of lending to entrepreneurs is positive if and only if $\widehat{R} > R^{SB}$. For an intuition, note that a banker wants the interest rate, r, as high as possible, and, thus, constraint (17) is binding. Therefore, all bankers offer $R = \widehat{R}$. What a banker obtains by lending to one entrepreneur, $\widehat{\pi}$, is the difference of the social value of the entrepreneur's project minus his profit from it; that is, $\widehat{\pi} = A_e L^\alpha - wL - V$. With L and V given by (7) and (8) and $R = \widehat{R}$,

$$\widehat{\pi} = (\frac{\overline{A}\alpha}{w})^{\frac{\alpha}{1-\alpha}}(q\overline{A}\alpha + (1-q)\underline{A}) \cdot \widehat{R}^{\frac{-1}{1-\alpha}}(\widehat{R} - R^{SB}). \tag{18}$$

Therefore, the profit margin of lending, $\widehat{\pi}$, is positive if and only if $\widehat{R} > R^{SB}$. If the profit margin of lending is positive, a banker gets $\Pi = \infty$ because she has an infinitely large lending capacity (i.e. $D = \infty$), despite her finite stock of corn. This unlimited capacity is derived from the privilege that bankers can lend out their liabilities.

3.3 The Equilibrium: Bankers' Wealth and Bank Run

In equilibrium, defined below, the prevailing real interest rate, $\widehat{R}$, clears the market for notes. Let $\mathbb{N} \equiv \{1, 2, ..., N\}$.

Definition 1 *A profile* $(\{D_i, r_i, \delta_i, \beta_i, E_i\}_{i\in\mathbb{N}}; \widehat{R})$ *forms an equilibrium if:*

(i) given $\widehat{R}$, *it is optimal for banker* i *to choose* (D_i, r_i), *which determines* $\delta_i = \delta(D_i, r_i)$ *through (14);*

(ii) given bankers' choices of $\{D_i, r_i, \delta_i\}_{i\in\mathbb{N}}$, $N\beta_i$ *entrepreneurs go to banker* i *and each demand her notes of face value* $E_i = E(\delta_i, r_i)$ *given by (6);*

(iii) the market clears: $D_i = \beta_i E_i$[14] *and* $\sum_{i\in\mathbb{N}} N\beta_i = N^2$.

[14] Note that D is in the unit of N kg.

In any equilibrium, bankers neither get an infinitely large profit, nor abstain from lending, which, by Proposition 2, is the case if and only if

$$\widehat{R} = R^{SB}. \tag{19}$$

At this value of $\widehat{R}$, entrepreneurs hire L^{SB} workers. Therefore, on the real side, the equilibrium allocation is unique and is the second-best, namely, the one that would arise if the friction of payment were absent. This friction, therefore, is completely overcome by competition under the least fettered issuance. However, on the nominal side, there is indeterminacy. At $\widehat{R} = R^{SB}$, by Proposition 2, the profit margin of lending is 0, and individual bankers are indifferent to any quantity of issues, although in the aggregate their issues exactly suffice for entrepreneurs to hire L^{SB} workers. This indeterminacy leads to a continuum of equilibria.

Proposition 3 *(i) In any equilibrium, $\widehat{R} = R^{SB}$, the profit margin of bank issuance is 0, and the second-best allocation is implemented.*

(ii) In any equilibrium, a number of bankers default and thus face a bank run at $t = 1$ upon the news of $\widetilde{A} = \underline{A}$, if and only if bankers' wealth is so low as to satisfy

$$G < [(q\overline{A}\alpha + (1-q)\underline{A}) - \underline{A}](\frac{q\overline{A}\alpha + (1-q)\underline{A}}{w})^{\frac{\alpha}{1-\alpha}} \equiv G^{SB}. \tag{20}$$

The profit margin of bank issuance is nullified because bankers engage in Betrand competition with an unlimited capacity, which arises from the privilege that their liabilities are accepted as money. Therefore, this privilege actually harms bankers under sufficient competition. Moreover, since bankers are indifferent to any quantity of issues, the quantity of inside money circulated in the economy is determined by the real sectors, namely, the sectors of entrepreneurs and workers. Lastly, the aggregate quantity of bank issues is fixed at a level exactly sufficient for entrepreneurs to hire the second-best number of workers. If bankers' wealth, G, is small, then it cannot back the issuance of this quantity of money without causing default, which triggers bank run, as was noted.

In the basic model, by the proposition, bankers' issues always draw to entrepreneurs the second-best number of workers, which is bigger than the first-best number. In this sense, bankers always overissue. To consider central banking in a richer setting, in the extension, bank run is introduced as a grave concern to bankers. Then, there arises a circumstance in which bankers issue too little money.

4 Extension: Central Banking

In this section, different from the basic model where bank run is assumed to be costless,

Assume: *Bank run is extremely costly to bankers, who, therefore, disallow it.*

The costs of a bank run might be due to the suffering of the note holders when they all run to the banks and possibly queue before their doors overnight. Foreseeing this suffering, workers accept no issues liable to default, which triggers a bank run. This assumption, therefore, imposes upon bankers' decision problem a no-default constraint, which, by Proposition 1(i), is:

$$D \cdot \left(1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha}\right) \leq G. \tag{21}$$

The constraint anchors the lending capacity of a banker (D) to her wealth (G).[15] What

[15] The same purpose could be served by a moral-hazard-related assumption, such as that made by Getler and Kiyotaki (2010), in which the equity value of a banker (i.e., $G + Y - D$; see Table 1) should never fall below αG, with $\alpha \in (0, 1)$; otherwise, the banker will abscond with α fraction of her wealth without being caught.

happens in equilibrium with the assumption, in the absence of central banking, is summarized below.

Proposition 4 *(i) If $G \geq G^{SB}$, then in all equilibria $\widehat{R} = R^{SB}$, the profit margin of bank issuance is 0, and $L = L^{SB}$. If $G < G^{SB}$, there is a unique equilibrium in which the profit margin of bank issuance is positive, and $\widehat{R}$ is determined by G through*

$$G = (\frac{\overline{A}\alpha}{w^{\alpha}})^{\frac{1}{1-\alpha}} \frac{1 - \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}}{\widehat{R}^{\frac{1}{1-\alpha}}} \equiv G(\widehat{R}). \tag{22}$$

(ii) If G increases (from 0) to G^{SB}, the quantity of issues, D, increases to wL^{SB}; the real interest rate, $\widehat{R}$, decreases (from $\overline{A}\alpha/\underline{A}$) to R^{SB}; and the number of workers whom entrepreneurs hire increases to L^{SB}.

If bankers' wealth, G, is beyond G^{SB}, by Proposition 3, bankers are able to issue enough – namely, the second-best quantity – of inside money to nullify the profit margin, without falling insolvent. In equilibrium, therefore, the no-default constraint is not binding and the second-best allocation is implemented. In the case where $G < G^{SB}$, the profit margin of lending is positive because the inadequate wealth of bankers limits their lending capacity and thereby relaxes competition. The positive profit margin drives all bankers to issue as much as possible, until the non-default constraint, (21), is binding. This clears the indeterminacy in the quantity of issues by individual bankers and induces a unique equilibrium. It also shows that, now, the quantity of money circulated is anchored by bankers' wealth (G), thus determined by the supply side (i.e., the banking sector), rather than by the demand side, as was in the basic model. Therefore, the lower the bankers' wealth, the less the inside money circulated in the economy and, as a result, the higher the interest rate of borrowing ($\widehat{R}$) and the fewer the workers entrepreneurs hire.

Given $G(\widehat{R})$ in (22), define $G^{FB} \equiv G(R^{FB})$, the level of wealth at which the real interest rate reaches the first-best value. With R^{FB} given by (10), $G^{FB} = (A_e\alpha - \underline{A})(A_e\alpha/w)^{\frac{\alpha}{1-\alpha}}$. Obviously, $G^{FB} < G^{SB}$. By Proposition 4 (ii), $\widehat{R} > R^{FB}$ and $L < L^{FB}$ if $G < G^{FB}$, while $\widehat{R} < R^{FB}$ and $L > L^{FB}$ if $G > G^{FB}$. This depicts two circumstances. In the former, bankers issue too little money (relative to the first-best), causing a credit crunch, while in the latter, bankers issue too much money. The paper consider how the central bank can improve efficiency in both circumstances.

The central bank (CB hereafter), in this paper, is defined as the unique entity that can costlessly issue any quantity of another means of payment, called "CB notes." CB notes, like the issues of a banker, certify the CB's promises to pay a certain amount of corn, which marks the face values of the notes. However, while a banker commits to redeem her issues with corn, the CB makes no such a commitment to back its notes. CB notes, therefore, are purely nominal, but they still circulate in this finite-period economy. The CB, with its ability to issue another means of payment, can implement two types of policy, which it announces at $t = 0$, before bankers choose (D, r).

One is *interest-rate policy*, under which the CB sets the risk-free interest rate at any $r_p \geq 0$ by offering a saving asset, as follows. If the CB receives a deposit of some bankers' notes of overall face value F at $t = 0$, it issues to the depositors CB notes of face value $F(1 + r_p)$ at $t = 2$. By taking in the notes, the CB becomes a creditor to the issuing bankers and oblige them together to repay $F(1 + r_p + \varepsilon)$ for some $\varepsilon > 0$, either with corn or with CB notes. The issued CB notes are then valued at par at $t = 2$. On the one hand, they can never be valued over par because these debtor bankers can pay the CB with corn. On the other hand, the notes are not valued under par in equilibrium; otherwise, they would be cheaper than corn. The debtor bankers would want to use only CB notes (and not corn) to clear all their debts. Thus, their demand for CB notes would be $F(1 + r_p + \varepsilon)$ in face value, but the total supply of the notes is $F(1 + r_p)$, below the demand. However, bankers can escape the punitive interest rate $1 + r_p + \varepsilon$ by paying their note holders the same interest rate of $1 + r_p$ (or even a little more), thereby keeping them to stay. As a result of the policy rate set at r_p, therefore, the holders of bankers' notes get a net interest rate of r_p from $t = 0$ to $t = 2$ and bankers, if issuing notes of aggregate face value D at $t = 0$, are in a liability of $D(1 + r_p)$ at $t = 2$.

Under the other type of monetary policy, the CB lends a certain quantity of its notes to all bankers, in order to enlarge their lending capacities. How these notes are circulated and priced is explained in detail below. This type of policy can be called as *quantitative-easing policy* because, by enlarging the quantity of money supply, it eases the constraints imposed by a shortage of money on the real economic activities.

4.1 The Optimal Monetary Policy if $G < G^{FB}$

In this circumstance, concerned about insolvency and with low wealth, bankers issue too little money, causing a credit crunch, with symptoms of a high real interest rate of bank lending (i.e., $\widehat{R} > R^{FB}$) and a small scale of entrepreneurs' projects (i.e., $L < L^{FB}$). As presently the interest rate for the note holders is already 0 and reaches the lower bound of the policy rate, no interest-rate policy helps. A proper quantitative-easing policy, however, restores the first-best allocation.

Suppose that the CB lends to all bankers each S units of its notes (i.e., of an aggregate face value of S units corn) at $t = 0$ and obliges them to pay the same value back at $t = 2$, either with CB notes or with corn. Then, at $t = 0$, bankers lend to entrepreneurs both their own issues and the borrowed CB notes. Assume that an entrepreneur borrows only CB notes or only the banker's issues, but not both of them, in order to avoid the problem of which debt of the two is senior.[16]

A loan contract for bankers' issues is as in the basic model: If an entrepreneur borrows from a banker her issues of face value E, then he owes the banker a debt of $E(1+r)$, which he repays at $t=2$ with either corn or the banker's issues.

A loan contract for CB notes is similar. If an entrepreneur borrows CB notes of face value E_s, then he owes the banker a debt of $E_s(1+r_s)$, which he repays at $t=2$ with either corn or CB notes. Assume that even in case of default, the CB notes-borrowing entrepreneurs still want to pay out as much of their debt as possible, therefore, they try to buy as close to $E_s(1+r_s)$ of CB notes as possible. Consider, first, the case in which S is in such a range that entails $r_s>0$ in equilibrium (within which the optimal monetary policy sits). In this case, all the CB notes are lent out, and the aggregate debt of the CB notes-borrowing entrepreneurs is $S(1+r_s)$.

CB notes, as mentioned, are not redeemable. They circulate because the CB notes-borrowing entrepreneurs want to buy them at $t=2$ to repay their loans. The price (or the discount factor) of CB notes at the date, p_2, depends on the strength of these entrepreneurs' output of corn, Y_s, relative to the quantity of the CB notes in circulation, S, and is as follows:

$$p_2 = \min(1, Y_s/S). \tag{23}$$

First, $p_2 \leq 1$ because the notes can never be valued over par. Otherwise, no notes, but corn, would be used to repay debts. Second, if $Y_s \geq S$, then $p_2 = 1$. Suppose, otherwise, the notes are valued under par. Then, these entrepreneurs would want only CB notes

[16] That is, when the entrepreneur defaults, should he first clear the debt with the banker's issues or that with the CB notes?

(and not corn) to repay all their debts, $S(1+r_s)$ in total. Their demand for CB notes in face value would be $\min(Y_s/p_s, S(1+r_s))$,[17] which, given $Y_s \geq S$ and $p_2 < 1$, is bigger than S, the supply of CB notes. This argument also shows that if $p_2 < 1$, then these entrepreneurs default; otherwise, they would buy $S(1+r_s)$ units of CB notes, which are not available. Lastly, if $Y_s < S$, then obviously $p_2 < 1$. As was argued, the CB notes-borrowing entrepreneurs default, and thus spend all their output to buy the notes. Therefore, $p_2 = Y_s/S$.

Then, $p_2 = 1$ in the good state, when entrepreneurs do not default. In the bad state, when entrepreneurs default by Lemma 2, $p_2 = Y_s/S$, which, with a calculation similar to that leading to equation (12), is found equal to $\underline{A}(1+r_s)/(\overline{A}\alpha)$; this term will be confirmed to be smaller than 1 in equilibrium. The discount factor of CB notes at $t = 0$, δ_s, equals the mean of these discount factors at $t = 2$. Thus,

$$\delta_s = q \cdot 1 + (1-q) \cdot \frac{\underline{A}(1+r_s)}{\overline{A}\alpha}.\text{[18]} \tag{24}$$

In the bad state, as CB notes are discounted, there is an inflation, which plays a key part for the CB issuance to improve efficiency. In the absence of the CB's intervention, bankers' notes are made contingent on the realization of $\widetilde{A}$ only through default, which triggers a costly bank run. Disallowing default restrains bankers to issue only non-contingent claims, the quantity of which, when bankers' wealth is low (below G^{FB}), is limited, imposing a constraint on the real economic activities. The CB breaks through

[17] If $Y_s \geq S(1+r_s)p_2$, the entrepreneurs do not default and want only $S(1+r_s)$ of CB notes to clear their debts. If $Y_s < S(1+r_s)p_2$, the entrepreneurs default, but by assumption, they still want to pay out as much of their debts as possible, and thus spend all their output, Y_s, to buy CB notes, inducing a demand of Y_s/p_2.

[18] Alternatively, this equation can be derived from equation (14) by replacing G with 0 (and replacing r with r_s), because CB notes are backed not by bankers' wealth, but only by the loans.

this constraint by offering contingent claims, and they are made contingent through inflation, not through a costly process of default.

The effects of the quantitative-easing policy are summarized below.

Proposition 5 *(i) If the central bank lends to each banker* $S \leq \underline{S}(G) \equiv \frac{\overline{A}\alpha}{q(\overline{A}\alpha - \underline{A})}(G^{SB} - G)$ *units of its notes, then there is a unique equilibrium in which the real interest of bank lending,* $\widehat{R}$*, is determined by* S *through*

$$S = (\frac{\underline{A}}{w^{\alpha}})^{\frac{1}{1-\alpha}} \frac{1-(1-q)\theta}{q\theta^{\frac{1}{1-\alpha}}} - \frac{1-(1-q)\theta}{q(1-\theta)} G \equiv F(\theta), \text{ with } \theta \equiv \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}, \tag{25}$$

and the interest rate of lending CB notes is

$$r_s = \frac{[q\overline{A}\alpha + (1-q)\underline{A}]\widehat{R} - \overline{A}\alpha}{\overline{A}\alpha - (1-q)\underline{A}\widehat{R}}. \tag{26}$$

It satisfies $\underline{A}(1+r_s)/(\overline{A}\alpha) < 1$.

(ii) With S *increased,* $\widehat{R}$ *and* r_s *decrease and the number of workers whom entrepreneurs hire,* L*, increases. At* $S = \underline{S}(G)$*,* $\widehat{R} = R^{SB}$*,* $r_s = 0$*, and* $L = L^{SB}$*.*

(iii) If $G \geq \underline{G} \equiv G(\frac{1}{\alpha}R^{SB})$*, where function* $G(\widehat{R})$ *is given in (22), bankers' profit decreases with* S *always, and if* $G < \underline{G}$*, it increases with* S *for* $S \leq F(\frac{\underline{A}}{\overline{A}\alpha} \cdot \frac{1}{\alpha}R^{SB})$ *and decreases with* S *otherwise.*

(iv) The optimal quantity of the CB's lending is $S = F(\frac{\underline{A}}{\overline{A}\alpha} \cdot R^{FB}) < \underline{S}(G)$ *and implements the first-best allocation.*

Some comments and intuitions are as follows.

First, *the quantitative-easing policy affects the real interest rate,* $\widehat{R}$*, thus efficiency, if and only if* $\widehat{R}$ *is above threshold* R^{SB}. The "if" part is given by result (ii), which says that if $\widehat{R} > R^{SB}$, the (further) issuance of CB notes (i.e., a bigger S) lowers $\widehat{R}$. As for the "only if" part, note that by Proposition 2 (iii), by no means can $\widehat{R}$ be dragged down below R^{SB}, otherwise bankers would stop lending altogether, not a case in equilibrium. At $\widehat{R} = R^{SB}$, any further issuance by the CB – namely, $S > \underline{S}(G)$ – will not increase the quantity of money circulated, but crowd out the inside money or stay in the vaults of bankers.

Second, result (iii) states that while the supply of fiat money always benefits entrepreneurs by lowering the real interest rate, it may make bankers worse off, although it gives them free funding. This is because the policy enlarges the lending capacities of all bankers, thus subjecting them to fiercer competition. Even when the banking sector as whole is worse off from the free CB funding, however, an individual banker has incentives to request it if $r_s > 0$ because she takes prices $\widehat{R}$ and r_s as given and does not take into account the negative effects of her request on these prices.

Third, the first-best is achieved with the optimal monetary policy, which, therefore, helps not only with the friction concerned with the means of payment (i.e., Friction 1), but also with that concerned with the real economy (i.e., Friction 2).

4.2 The Optimal Monetary Policy if $G > G^{FB}$

In this circumstance, bankers issue too much money, making the real interest rate too low and drawing too many workers to entrepreneurs relative to the first-best allocation. Quantitative-easing policy, by Proposition 5, may only further lower the real interest rate and draw still more workers to entrepreneurs, making things worse. Interest-rate policy, as shown below, has real effects and can implement the first-best allocation if and only if the nominal wage to workers is sticky.

Suppose that the CB sets the policy rate at r_p. As a result, the holders of bankers' notes earn a net interest rate of r_p, and bankers, if issuing D at $t = 0$, face a liability of $D(1 + r_p)$ at $t = 2$. With this amount of liability and the bad-state value of the loans, $\underline{Y}$, given by (12), bankers stay solvent in the bad state if and only if

$$D(1 + r_p) \leq G + \frac{\underline{A}(1 + r)}{\overline{A}\alpha} D. \tag{27}$$

The analysis that follows depends on whether the nominal wage at $t = 0$ is flexible or sticky. Let us start with the case of flexible wages. As bankers do not default and the net interest rate of holding notes from $t = 0$ to $t = 2$ is r_p, with flexible wages, at $t = 0$ workers accept a nominal wage (i.e., a payment with the notes of a face value) of $w/(1+r_p)$. By borrowing E, an entrepreneur hires $E(1+r_p)/w$ workers. It follows that with a banker's issues of face value D, $L = D(1 + r_p)/w$ workers are hired and that the discount factor in (4) is now $\delta = 1 + r_p$, which, with (9), implies $R = (1 + r)/(1 + r_p)$. Substitute $R(1 + r_p)$ for $1 + r$ and wL for $D(1 + r_p)$ in (27) and note that the number of workers hired is $L = (\overline{A}\alpha/wR)^{\frac{1}{1-\alpha}}$ by (7). Then, from (27) it follows:

$$w\left(\frac{\overline{A}\alpha}{wR}\right)^{\frac{1}{1-\alpha}} \left(1 - \frac{\underline{\underline{A}}R}{\overline{A}\alpha}\right) \leq G. \tag{28}$$

As all bankers offer $R = \widehat{R}$, this no-default constraint, when binding, is equivalent to (22) in Proposition 4. By Proposition 4(ii), the real interest determined by (22) is above R^{SB} if and only if $G < G^{SB}$. Moreover, if and only if $R > R^{SB}$, by Proposition 2, issuance bears a positive profit margin, which drives bankers to issue as much as possible so long as they stay solvent, that is, until the no-default constraint, (28), is binding. It follows that in equilibrium, if and only if $G < G^{SB}$, constraint (28) is binding and the real interest is the same as given by (22); and if $G \geq G^{SB}$, the real interest rate is R^{SB} (because it can never fall below R^{SB} by Proposition 2). In both cases, the real interest rate, which determines all the other real variables, is the same as given by Proposition 4 – as it is in the absence of the CB. Therefore,

Proposition 6 *With flexible wages, the interest-rate policy has no real effects, but deflates the nominal wage to $w/(1+r_p)$.*

Now, consider the case of sticky wages, where workers, for some reason, do not accept a nominal wage below w.[19] Then, borrowing notes of face value E, an entrepreneurs still hires E/w (rather than $E(1+r_p)/w$) workers. This implies that the number of workers hired with a banker's issues of aggregate face value D is $L = D/w$ and that the discount factor in (4) is $\delta = 1$, whereby the real interest rate is $R = 1 + r$. Substitute R for $1 + r$ and wL (with $L = (\overline{A}\alpha/wR)^{\frac{1}{1-\alpha}}$ by 7) for D in (27), which then becomes:

$$w(\frac{\overline{A}\alpha}{wR})^{\frac{1}{1-\alpha}}[(1+r_p) - \frac{\underline{A}}{\overline{A}\alpha}R) \leq G. \tag{29}$$

This no-default constraint is binding if the profit margin of bank issuance is positive, as before. Given that the workers' real wage is now $w(1 + r_p)$, the profit to a banker from lending to one entrepreneur becomes $A_e L^\alpha - w(1 + r_p)L - V$. With L and V as functions of R given by (7) and (8), it equals $[\overline{A}\alpha/(w^\alpha R)]^{\frac{1}{1-\alpha}}[R/R^{SB} - (1 + r_p)]$.This profit margin never goes under 0. Therefore,

$$R \geq R^{SB}(1 + r_p). \tag{30}$$

[19]Introducing the constraint of sticky wages would not affect the analysis of quantitative-easing policy in Subsection 4.1, where workers are paid either with bankers' notes of overall face value w or with the CB notes of overall face value $w/\delta_s > w$.

The equilibrium real interest rate is pinned down by conditions (29), (30), and that if (30) is not binding – that is, if the profit margin of bank issuance is positive – then the no-default constraint, (29), is binding. The results are summarized as follows.

Proposition 7 *Suppose that the nominal wage to workers is sticky. (i) If the CB sets the policy rate at r_p, then the equilibrium real interest rate of bank lending, R, is as follows. For $G < G^{SB}$, R is the root of*

$$1 + r_p = \frac{Gw^{\frac{\alpha}{1-\alpha}}}{(\overline{A}\alpha)^{\frac{1}{1-\alpha}}} R^{\frac{1}{1-\alpha}} + \frac{\underline{A}}{\overline{A}\alpha} R \equiv \Phi(R) \tag{31}$$

if $r_p \leq r_p^*$, otherwise $R = R^{SB}(1 + r_p)$, where

$$r_p^* \equiv [q(\overline{A}\alpha - \underline{A})/G]^{\frac{\alpha}{1-\alpha}} (q\overline{A}\alpha + (1-q)\underline{A})/w - 1,$$

and $r_p^* > 0$ if $G < G^{SB}$. For $G \geq G^{SB}$, $R = R^{SB}(1 + r_p)$ for all $r_p \geq 0$.

(ii) The optimal policy rate, which implements the first-best allocation, is:

$$r_p^{FB} = \left\{ \begin{array}{c} \Phi(R^{FB}) - 1 \ \textit{if } G \leq G_s \\ R^{FB}/R^{SB} - 1 \ \textit{if } G > G_s \end{array} \right\},$$

where $G_s \equiv [(q\overline{A}\alpha + (1-q)\underline{A}) - \underline{A}](A_e\alpha/w)^{\frac{\alpha}{1-\alpha}}$ and satisfies $G^{FB} < G_s < G^{SB}$.

By result (i), with sticky wages, a higher policy rate raises the real interest rate of bank lending and to no limit. If the policy rate is above r_p^*, bankers get 0 profit, as the constraint of non-negative profit margin, (30), becomes binding. Bankers cannot pass all the costs from a high policy rate on to entrepreneurs or keep a fixed profit margin, by charging them a high interest rate r, because the high r diminishes entrepreneurs' demand for bank credit, thus subjecting bankers to fiercer competition. With a higher policy rate, which pushes up the interest rate of bank lending, entrepreneurs get a thinner profit and hire fewer workers, but the workers hired get a better real wage.

5 Discussion: Reserve and the Role of G

This section is intended to clarify that bankers' stocks of corn, G, which, thus far, describe both bankers' wealth and reserve, essentially represent the former. In clarifying this, the paper shows that an increase in bankers' reserve affects nothing of the results. Suppose, here and now, that at $t = 0$, not only bankers have corn, but so do some workers and that these workers deposit G_w units of their corn with each banker in exchange for her notes of overall face value D_w. As a result, for each banker, the wealth stays at G, while the reserve increases to $G + G_w$. The balance sheet of a banker is:

Asset	**Liability**
Reserve $(G + G_w)$	Equity
Loans to entrepreneurs (Y)	Debt to the holders of notes loaned out (D)
	Debt to the initial depositors (D_w)

Table 2: The balance sheet of a banker with increased liquidity coverage

The banker does not default in the bad state if and only if

$$D + D_w \leq G + G_w + \underline{Y}.$$

When she does not default, her notes are valued at par. Therefore,

$$D_w = G_w.$$

Then, in the bad state, the non-default condition becomes

$$D \leq G + \underline{Y},$$

exactly the same as (15), from which all the subsequent results are derived. It is now clear that G represents bankers' wealth because here their reserve is $G + G_w$.

6 Conclusion

Banks create money by circulating their deposit liabilities, which are accepted as a means of payment (when the banks' solvency is not in doubt). This paper analyzes this important money-creation function of banks in a general equilibrium. Building on this analysis, it considers how the central bank can improve economic efficiency by printing purely nominal claims. It shows how the quantitative-easing policy in a circumstance of a credit crunch, and the interest-rate policy in a circumstance of two much money issued, affects efficiency. It determines the optimal monetary policy in each circumstance.

Appendix: The Proofs

Of Lemma 1:

In equilibrium, only one promised wage, denoted by F, prevails on the market, as will be shown. Competitive equilibrium is thus defined as a profile of (F, L), such that:

(a) Given that F prevails on the market, the optimal labor demand of each entrepreneur is L;

(b) Given that each entrepreneur demands L workers, F clears the labor market.

The two conditions are elaborated as follows.

For (a): Given F, each entrepreneur's problem of deciding on labor demand is:

$$\max_L q(\overline{A}L^\alpha - FL) + (1-q)\max(\underline{A}L^\alpha - FL, 0),$$

where the "max" term appears because the entrepreneur might default in the bad state. That is indeed the case at the optimum. Otherwise, the entrepreneur's problem is

$$\max_L q(\overline{A}L^\alpha - FL) + (1-q)(\underline{A}L^\alpha - FL).$$

The solution is $L = (\frac{A_e\alpha}{F})^{\frac{1}{1-\alpha}}$. Then in the bad state his output is $\underline{A}((\frac{A_e\alpha}{F})^{\frac{\alpha}{1-\alpha}}$, which is smaller than $F \cdot (\frac{A_e\alpha}{F})^{\frac{1}{1-\alpha}}$, the wage obligation, because $\underline{A} < A_e\alpha$ as assumed in (1). Hence, he defaults in the bad state, contradictory to what was supposed.

Defaulting in the bad state, entrepreneurs choose L to maximize the profit in the good state, $\overline{A}L^\alpha - FL$. Therefore, given F, the labor demand is:

$$L = (\frac{\overline{A}\alpha}{F})^{\frac{1}{1-\alpha}}. \tag{32}$$

For (b): As there are a lot more workers than entrepreneurs can hire, the labor market is cleared by an expected wage income of w, the output of workers in autarky. In the good state, the workers hired get the promised wage, F. In the bad state, entrepreneurs default and all the output goes to the workers, each obtaining $\frac{\underline{A}L^\alpha}{L} = \underline{A}L^{\alpha-1}$. The expected wage equals w:

$$qF + (1-q)\underline{A}L^{\alpha-1} = w. \tag{33}$$

Equations (32) and (33) together give (3).

Now, show that only one F prevails on the market. If an entrepreneur posts F, then by (32) he hires $L = (\frac{\overline{A}\alpha}{F})^{\frac{1}{1-\alpha}}$ workers, whose wage income is F in the good state and $\underline{A}L^{\alpha-1} = \frac{\underline{A}}{\overline{A}\alpha}F$ in the bad state. Both increase with F. Therefore, workers go only to entrepreneurs who post the highest F, and in equilibrium, only one F prevails.

Q.E.D.

Of Lemma 2:

Suppose, otherwise, an entrepreneur does not default in the bad state. Then, his problem is:

$$\max_E q(\overline{A}L^\alpha - E(1+r)) + (1-q)(\underline{A}L^\alpha - E(1+r)) \text{ s.t. } (4).$$

From the constraint, $E = wL/\delta$. Substitute it into the objective and let $\gamma \equiv w(1+r)/\delta$. Then, the problem becomes

$$\max_{L} A_e L^\alpha - \gamma L.$$

The solution is $L = (A_e\alpha/\gamma)^{\frac{1}{1-\alpha}}$. At this scale, the entrepreneur will default in the bad state: $\underline{A}L^\alpha < E(1+r)|_{E=wL/\delta;\gamma\equiv w(1+r)/\delta} \Leftrightarrow \underline{A}L^\alpha < \gamma L \Leftrightarrow \underline{A}L^{\alpha-1} < \gamma|_{L=(A_e\alpha/\gamma)^{\frac{1}{1-\alpha}}} \Leftrightarrow \underline{A} < A_e\alpha$, which is assumed in (1) – hence a contraction to what was supposed.

Q.E.D.

Of Proposition 1:

(i): As the entrepreneurs all default in the bad state and each hands over his whole output, $\underline{y}$, to the banker, the value of the banker's loans in the bad state, $\underline{Y}$, equals $\underline{y}$ times the number of entrepreneurs she finances, D/E. With $\underline{y} = \underline{A}L^\alpha$, and E and L given by (6) and (7),

$$\frac{\underline{y}}{E} = \frac{\underline{A}(1+r)}{\overline{A}\alpha}. \tag{34}$$

Then, $\underline{Y} = \underline{y} \cdot D/E = D \cdot \underline{y}/E = \underline{A}(1+r)/\overline{A}\alpha \cdot D$, that is (12). With $\underline{Y}$ so given, the condition of no-default in the bad state, $\underline{Y} + G \geq D$, becomes (13).

(ii): By (i), if condition (13) holds true, the banker never defaults and is always able to redeem her notes at par. Therefore, her notes are valued at par, that is, $\delta = 1$, which gives rise to the upper branch of (14).

If condition (13) is violated, at $t = 1$, the banker defaults in the bad state. All the value of her assets, $G + \underline{Y}$, goes pro rata to the note holders and a note of face value 1 is thus worth $(G + \underline{Y})/D$, which, with $\underline{Y}$ given by (12) (note that the derivation of (12) is independent of whether the banker is solvent or not), equals

$$\frac{G}{D} + \frac{\underline{A}(1+r)}{\overline{A}\alpha}.$$

In the good state, when the entrepreneurs do not default and, thus, neither does the banker, the notes are worth their par values. At $t = 0$, the value of a note of face value 1, namely, the discount factor of the notes, equals the mean of its value at $t = 1$ and is as given by the lower branch of (14).

Q.E.D.

Of Proposition 2:

With rearrangement, the profit of a banker choosing (D, r) is:

$$\Pi(D,r) = \begin{Bmatrix} \frac{q\overline{A}\alpha+(1-q)\underline{A}}{\overline{A}\alpha}(r - r^{SB})D, \text{ if } D(1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha}) \leq G \\ qDr - (1-q)G, \text{ otherwise} \end{Bmatrix}, \tag{35}$$

where

$$r^{SB} \equiv \frac{(1-q)(\overline{A}\alpha - \underline{A})}{q\overline{A}\alpha + (1-q)\underline{A}} = R^{SB} - 1. \tag{36}$$

Given D, bankers want r as high as possible. Hence, (17) is binding, which implies

$$r = \widehat{R} \cdot \delta(D, r) - 1. \tag{37}$$

The Proposition is equivalent to the series of following lemmas.

Lemma 3A: if $\widehat{R} \geq \frac{\overline{A}\alpha}{\underline{A}}$, then $D = \infty$, $r = \widehat{R} - 1$, and $\Pi = \infty$.

Proof. In this case, a banker chooses $D = \infty$, $r = \widehat{R} - 1$, without default – thus $\delta = 1$ – because $1 + r = \widehat{R} \geq \frac{\overline{A}\alpha}{\underline{A}}$ and, thus, the condition for non-default, (13), is honored for any D; intuitively, at such a high $\widehat{R}$, the bank issue can be solely and fully backed by the loans. At $r = \widehat{R} - 1$, the profit margin $r - r^{SB} = \widehat{R} - R^{SB} > 0$, which implies $\Pi = \infty$. ■

For the remaining cases in which $\overline{A}\alpha/\underline{A} > \widehat{R}$ and thus $1 - (1-q)\underline{A}/(\overline{A}\alpha) \cdot \widehat{R} > 0$, note that if $D(1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha}) > G$ – namely, if a banker chooses to default – then solved from (37), with $\delta(D, r)$ given by (14):

$$r = \frac{qD + (1-q)G}{D[1 - (1-q)\underline{A}/(\overline{A}\alpha) \cdot \widehat{R}]}\widehat{R} - 1. \tag{38}$$

Substitute it into the lower branch of (35), then the profit of a banker who chooses to default is

$$\frac{(\widehat{R}/R^{SB} - 1)}{\overline{A}\alpha - (1-q)\underline{A}\widehat{R}}[qD + (1-q)G]. \tag{39}$$

Lemma 3B: if $\frac{\overline{A}\alpha}{\underline{A}} > \widehat{R} > \frac{\overline{A}\alpha}{q\overline{A}\alpha + (1-q)\underline{A}} (= R^{SB})$, then $D = \infty$, $r = \frac{(q\overline{A}\alpha + (1-q)\underline{A})\widehat{R} - \overline{A}\alpha}{\overline{A}\alpha - (1-q)\underline{A}\widehat{R}}$, and $\Pi = \infty$.

Proof. In this case, by (37) and $\delta \le 1$, $1 + r < \widehat{R} < \frac{\overline{A}\alpha}{\underline{A}}$. Thus, $1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha} > 0$. The non-default condition, (13), is equivalent to $D \le G(1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha})^{-1}$. If a banker chooses not to default, then the quantity of her issues, D, is bounded from above and she gets a finite profit. But if she chooses to issue $D = \infty$, thus to default in the bad state, then her profit is given by (39). It follows that $\Pi = \infty$ because $\widehat{R}/R^{SB} - 1 > 0$ and $\overline{A}\alpha - (1-q)\underline{A}\widehat{R} > 0$ for the range of $\widehat{R}$ given in the condition. With $D = \infty$, $r = \frac{q\widehat{R}}{1-(1-q)\underline{A}/(\overline{A}\alpha)\cdot\widehat{R}} - 1$ by (38). Intuitively, with $\widehat{R} > R^{SB}$, the profit margin of bank issuance is positive and bankers want to issue $D = \infty$, thereby obtaining $\Pi = \infty$, but differently from the case where $\widehat{R} \ge \frac{\overline{A}\alpha}{\underline{A}}$, now with a lower $\widehat{R}$ the issuance of $D = \infty$ cannot be supported by loans only and has to induce default in the bad state. ■

These two lemmas imply result (i) of the proposition. Result (ii) is proved in the following lemma.

Lemma 3C: if $\widehat{R} = R^{SB}$, then the banker obtains $\Pi = 0$ and is indifferent to any quantity of issues, D, with r determined by D through equation (37).

Proof. In this case, if bankers choose not to default, so that $\delta = 1$, then $r = \widehat{R} - 1 = r^{SB}$. Thus, by the upper branch of (35), the coefficient before D, namely, the profit margin of issuance, equals 0. The same is true if bankers choose to default because, then, their profit is given by (39) and the coefficient before D equals 0 too. Therefore, bankers obtain 0 profit always and are indifferent to any D. At any chosen D, the value of r is determined by (37), with $\delta(D, r)$ given by (14). ■

Result (iii) is proved in the following lemma.

Lemma 3D: if $R^{SB} > \widehat{R}$, then lending to entrepreneurs makes a loss and $D = \Pi = 0$.

Proof. In this case, if bankers choose not to default, then $r = \widehat{R} - 1 < r^{SB}$. Hence, the coefficient before D in the upper branch of (35) is negative, and lending makes a loss. If bankers choose to default, then their profit is given by (39). The coefficient before D, as $\widehat{R} < R^{SB}$, is now negative also, and lending makes a loss too. Therefore, the optimal $D = 0$, whereby $\Pi = 0$. ■

The whole proposition is proved. Q.E.D.

Of Proposition 3:

(i): It has been shown in the main text.

(ii): Prove the "if" part by reduction to absurdity. Suppose that in one equilibrium, no bankers default, namely, (13) is honored for all bankers. Then, for all bankers $\delta = 1$ and, thus, by (37), $1 + r = \widehat{R}|_{\text{result } (i)} = R^{SB}$. By (13), each banker issues,

$$D \leq G/(1 - \frac{\underline{A}}{\overline{A}\alpha} R^{SB})|_{(11)} = \frac{G[q\overline{A}\alpha + (1-q)\underline{A}]}{q(\overline{A}\alpha - \underline{A})}.$$

With $\delta = 1$ and $1 + r = R^{SB}$, by (6) the demand by entrepreneurs is $E = (q\overline{A}\alpha + (1 - q)\underline{A})^{\frac{1}{1-\alpha}} w^{\frac{-\alpha}{1-\alpha}}$. If $G < G^{SB}$, then the supply is below the demand – thus not in equilibrium – because $G[q\overline{A}\alpha + (1-q)\underline{A}]/[q(\overline{A}\alpha - \underline{A})] < (q\overline{A}\alpha + (1-q)\underline{A})^{\frac{1}{1-\alpha}} w^{\frac{-\alpha}{1-\alpha}} \Leftrightarrow G < G^{SB}$.

To prove the "only if" part, it suffices to show if $G \geq G^{SB}$, in the symmetric equilibrium, no bankers default. Suppose, for the construction of the equilibrium, that it is the case. Then, $\delta = 1$. By the analysis above, $r = R^{SB} - 1$ and entrepreneurs' demand for notes $E = (q\overline{A}\alpha + (1-q)\underline{A})^{\frac{1}{1-\alpha}} w^{\frac{-\alpha}{1-\alpha}}$, which equals the notes supply, D, in equilibrium. With this value of (D, r), it is straightforward to check back that if $G \geq G^{SB}$, indeed the non-default condition, (13), is honored and, hence, no bankers default in the bad state. Q.E.D.

Of Proposition 4:

Note that as there is no default, $\delta = 1$ and $1 + r = R$.

(i): If $G \geq G^{SB}$, by Proposition 3, there is an equilibrium in which no bankers default in the absence of no-default constraint, that is, the constraint is non-binding. Therefore, its presence affects nothing of the real allocation, which is thus the same as in that equilibrium, given by Proposition 3(i), namely, $\widehat{R} = R^{SB}$, the profit margin of bank issuance is 0, and $L = L^{SB}$.

If $G < G^{SB}$, note, first, that in the equilibrium, $\widehat{R} > R^{SB}$; otherwise, by Proposition 2, $\widehat{R} = R^{SB}$ (because in no equilibria $\widehat{R} < R^{SB}$, which completely discourages bank lending), at which to meet the demand for bankers' issues entails banker default, disallowed here. Second, with $1 + r = R$, no-default constraint (21) becomes

$$D \leq G\left(1 - \frac{\underline{A}}{\overline{A}\alpha} R\right)^{-1}. \tag{40}$$

Third, as bankers do not default, their profit is given by the upper branch of (35), which, as $r - r^{SB} = (\widehat{R} - 1) - (R^{SB} - 1) = \widehat{R} - R^{SB}$, becomes $\frac{q\overline{A}\alpha + (1-q)\underline{A}}{\overline{A}\alpha}(R - R^{SB})D$.

A banker's problem is now

$$\max_{D,R} (R - R^{SB})D, \text{ s.t. (40) and } R \leq \widehat{R}.$$

As $\widehat{R} > R^{SB}$, at the maximum, both constraints are binding. Therefore, the supply of bankers' notes is

$$D = \frac{G}{1 - \underline{A}\widehat{R}/(\overline{A}\alpha)}. \tag{41}$$

Intuitively, taking the positive profit margin (which is in proportion to $R - R^{SB} > 0$) as given, bankers issue as much as possible, until the non-default constraint, (40), is binding.

The equilibrium real interest rate, $\widehat{R}$, is found by equalizing this supply of bankers' notes to the demand, given by (6). With $1 + r = \widehat{R}$ and $\delta = 1$, this equalization gives rise to (22):

$$G = G(\widehat{R}) \equiv (\frac{\overline{A}\alpha}{w^{\alpha}})^{\frac{1}{1-\alpha}} \cdot \frac{1 - \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}}{\widehat{R}^{\frac{1}{1-\alpha}}}.$$

It is straightforward that $G'(\widehat{R}) < 0$, $G(R^{SB}) = G^{SB}$ and $G(\overline{A}\alpha/\underline{A}) = 0$. Therefore, for any given $G < G^{SB}$, equation (22) determines a unique $\widehat{R} \in (R^{SB}, \overline{A}\alpha/\underline{A})$, which, by (41), determines a unique D. Hence, the equilibrium uniquely exists.

(ii): Let $\widehat{R}(G)$ be the inverse function of $G(\widehat{R})$. Then, in the equilibrium $\widehat{R} = \widehat{R}(G)$. As $G(\widehat{R})$ is decreasing, so is $\widehat{R}(G)$. Moreover, $\widehat{R}(G^{SB}) = R^{SB}$ and $\widehat{R}(0) = \overline{A}\alpha/\underline{A}$ because $G(R^{SB}) = G^{SB}$ and $G(\overline{A}\alpha/\underline{A}) = 0$. By (22) and (41) together, $D = (\overline{A}\alpha/\widehat{R})^{\frac{1}{1-\alpha}} w^{\frac{-\alpha}{1-\alpha}}$, which decreases with $\widehat{R}$, therefore increases with G. With $R = \widehat{R}$ for each banker, the number of workers hired in equilibrium, by (7), is $L = (\overline{A}\alpha/w\widehat{R})^{\frac{1}{1-\alpha}}$, which decreases with $\widehat{R}$. Thus, L increases with G. Moreover, $L = L^{SB}$ at $G = G^{SB}$, where $\widehat{R} = R^{SB}$.

Q.E.D.

Of Proposition 5:

(i) The equilibrium is characterized as follows. Entrepreneurs' demand for CB notes, E_s, solves the same problem as the demand for bankers' notes, given by (5), except that r is replaced with r_s and δ with δ_s. Therefore, the real interest rate of borrowing CB notes is $(1+r_s)/\delta_s$, while the real interest rate of borrowing bankers' notes is $1+r$ (since $\delta = 1$ given that bankers do not default). By (8), only the real interest rate of the borrowing concerns the entrepreneurs. In equilibrium, as made clear later, lending of both means of payment bears a positive profit margin and thus both are lent out. Then, the real interest rate of lending the two are equalized, and, as before, equal to $\widehat{R}$:

$$\frac{1+r_s}{\delta_s} = 1 + r = \widehat{R}. \tag{42}$$

As the profit margin of lending is positive, bankers issue notes to the point at which the no-default constraint becomes binding, that is, $D = G/(1 - \frac{\underline{A}}{\overline{A}\alpha}\widehat{R})$; and also they lend out all the CB notes, S. The aggregate value of these means of payment supplied, $\delta_s S + D$, when the market clears, equals the wage payment that entrepreneurs demand to hire workers:

$$wL = \delta_s S + \frac{G}{1 - \underline{A}\widehat{R}/(\overline{A}\alpha)}, \tag{43}$$

where, by (7), the number of workers hired is

$$L = (\frac{\overline{A}\alpha}{w})^{\frac{1}{1-\alpha}} \widehat{R}^{\frac{-1}{1-\alpha}}. \tag{44}$$

These four equations (note 42 has two) together with equation (24) (which settles δ_s), as shown below, determine a unique profile of $(\delta_s, r_s, r, \widehat{R}, L)$ in equilibrium – thus, the equilibrium exists uniquely. Passing on to show that, we derive equations (25) and (26). By (42), $1 + r_s = \widehat{R}\delta_s$. Substituting it into (24) and rearranging, we have:

$$\delta_s = \frac{q}{1 - (1-q)\underline{A}/(\overline{A}\alpha) \cdot \widehat{R}}. \tag{45}$$

Substitute it and (44) into (43)), rearrange, let $\theta \equiv \underline{A}/(\overline{A}\alpha) \cdot \widehat{R}$, and we come to (25):

$$S = (\frac{\underline{A}}{w^{\alpha}})^{\frac{1}{1-\alpha}} \frac{1-(1-q)\theta}{q\theta^{\frac{1}{1-\alpha}}} - \frac{1-(1-q)\theta}{q(1-\theta)} G \equiv F(\theta). \tag{46}$$

Equations (42) and (45) together imply $1 + r_s = \frac{q\widehat{R}}{1-(1-q)\underline{A}/(\overline{A}\alpha)\cdot\widehat{R}}$, from which (26) follows.

Now, come to show that for $S \leq \underline{S}(G)$, equation (46) determines a unique $\widehat{R}$ (which determines a unique r_s, r, L, and δ_s); and that this $\widehat{R}$ decreases from $\widehat{R}(G)$ to R^{SB} when S runs from 0 to $\underline{S}(G)$, where $\widehat{R}(G)$ is the inverse function of $G(\widehat{R})$ given by (22) in Proposition 4, namely, the equilibrium real interest rate without central-bank intervention. Both assertions follow from the following three observations. First, $F'(\theta) < 0$, therefore, θ, and thus $\widehat{R}$, decreases with S. Second, equation $F(\theta) = 0$ is equivalent to (22), thus leading to $\widehat{R} = \widehat{R}(G)$. Therefore, at $S = 0$, $\widehat{R} = \widehat{R}(G)$. And third, $F(\frac{\underline{A}}{\overline{A}\alpha}R^{SB}) = \underline{S}(G)$, therefore, $\widehat{R} = R^{SB}$ at $S = \underline{S}(G)$.

Lastly, $\underline{A}(1+r_s)/(\overline{A}\alpha) < 1 \Leftrightarrow 1 + r_s < \overline{A}\alpha/\underline{A}|_{(26)} \Leftrightarrow q\widehat{R}/[1-(1-q)\underline{A}/(\overline{A}\alpha)\cdot\widehat{R}] < \overline{A}\alpha/\underline{A} \Leftrightarrow \widehat{R} < \overline{A}\alpha/\underline{A}|_{\widehat{R}<\widehat{R}(G)} \Leftarrow \widehat{R}(G) < \overline{A}\alpha/\underline{A}$, which is affirmed by Proposition 4(ii).

(ii): It was shown above that $\widehat{R}$ decreases with S and equals R^{SB} at $S = \underline{S}(G)$. By (26), r_s increases with $\widehat{R}$ and $r_s = 0$ at $\widehat{R} = R^{SB}$. Therefore, r_s decreases with S and equals 0 at $S = \underline{S}(G)$.

(iii): In the unique equilibrium, each banker serves N entrepreneurs and obtains from each of them $\widehat{\pi}$, given by (18), thus $N\widehat{\pi}$ overall. Note that $\widehat{\pi}$ increases with $\widehat{R}$ for $\widehat{R} \in [R^{SB}, \frac{1}{\alpha}R^{SB}]$ and decreases with it for $\widehat{R} > \frac{1}{\alpha}R^{SB}$. The result then follows from the fact that $\widehat{R} \leq \widehat{R}(G) \leq \frac{1}{\alpha}R^{SB}$ for any $S \geq 0$ if $G \geq G(\frac{1}{\alpha}R^{SB})$ and that $\widehat{R} = \frac{1}{\alpha}R^{SB}$ at $S = F(\frac{\underline{A}}{\overline{A}\alpha}\cdot\frac{1}{\alpha}R^{SB})$ if $G < G(\frac{1}{\alpha}R^{SB})$.

(iv): As $G < G^{FB}$, $\widehat{R}(G) > R^{FB} > R^{SB}$. Note that $\widehat{R} = \widehat{R}(G)$ at $S = 0$ and $\widehat{R} = R^{SB}$ at $S = \underline{S}(G)$. Therefore, there is a unique S between 0 and $\underline{S}(G)$ at which $\widehat{R} = R^{FB}$ and this S equals $F(\frac{\underline{A}}{\overline{A}\alpha}\cdot R^{FB})$ by (25).

Q.E.D.

Of Proposition 6:

It is proved in the main text.

Of Proposition 7:

(i): As was said, the real interest rate in equilibrium is determined by three conditions: (29); (30); and if (30) is not binding, then (29) is. Note that (30) is equivalent to

$$1 + r_p \leq R/R^{SB} \tag{47}$$

and (29) is equivalent to

$$1 + r_p \leq \Phi(R), \tag{48}$$

where $\Phi(R)$ is defined in (31):

$$\Phi(R) \equiv \frac{Gw^{\frac{\alpha}{1-\alpha}}}{(\overline{A}\alpha)^{\frac{1}{1-\alpha}}} R^{\frac{1}{1-\alpha}} + \frac{\underline{A}}{\overline{A}\alpha} R.$$

Then in equilibrium, (48) and (47) hold, and one of them must be binding.

The binding constraint is the one that is tighter, namely that with a smaller value on the right hand side. Let $\chi(R) \equiv \Phi(R) - R/R^{SB}$ for $R \geq 0$. Then $\chi(R) = 0$ has two roots: 0 and

$$R^* = \frac{\overline{A}\alpha}{w} [\frac{q(\overline{A}\alpha - \underline{A})}{G}]^{\frac{1-\alpha}{\alpha}}.$$

$R^* \geq R^{SB}$ if and only if $G \leq G^{SB}$. Because $\chi'(0) < 0$, $\chi < 0$ for $R \in (0, R^*)$ and $\chi > 0$ for $R > R^*$. Moreover, let $r_p^* \equiv R^*/R^{SB} - 1 = [q(\overline{A}\alpha - \underline{A})/G]^{\frac{\alpha}{1-\alpha}} (q\overline{A}\alpha + (1-q)\underline{A})/w - 1$, which, by the definition of R^*, also equals $\Phi(R^*) - 1$. Finally, as $r_p \geq 0$, $R \geq R^{SB}$ by (47).

Consider first the case where $G < G^{SB}$. In this case, $R^{SB} < R^*$. By the definition of R^*, for $R \in [R^{SB}, R^*)$, $\chi < 0$, and thus $\Phi(R) < R/R^{SB}$, therefore, (48) is binding. For

$R > R^*$, $\chi > 0$, and thus $\Phi(R) > R/R^{SB}$, therefore, (47) is binding. R increases with r_p, and at $R = R^*$, $r_p = r_p^*$, no matter which of (47) and (48) is binding. Therefore, $R < R^*$ if $r_p < r^*$ and $R > R^*$ if $r_p > r^*$. It follows that if $r_p < r^*$, (48) is binding, giving rise to $1 + r_p = \Phi(R)$ and if $r_p > r^*$, (47) is binding, giving rise to $R = R^{SB}(1 + r_p)$.

Consider then the case where $G \geq G^{SB}$, and thus $R^{SB} \geq R^*$. For all $R \geq R^{SB}$ (namely, all the possible equilibrium values of R), $R \geq R^*$. Then, always, $\chi > 0$, and thus $\Phi(R) > R/R^{SB}$. Therefore, (47) is binding, giving rise to $R = R^{SB}(1 + r_p)$.

(ii): There exists a unique policy rate under which $R = R^{FB}$ and the first-best allocation is implemented: by Proposition 4, at $r_p = 0$ (namely, without intervention by the CB), $R < R^{FB}$ because now $G > G^{FB}$; by result (i), R increases with r_p to infinite; therefore, there exists a unique r_p under which $R = R^{FB}$. To find this r_p, it suffices to find the inverse function of $R(r_p)$ given by result (i). This inverse function is that for $G < G^{SB}$, $r_p = \Phi(R) - 1$ if $R \leq \Phi^{-1}(1 + r_p^*) = R^*$ and $r_p = R/R^{SB} - 1$ otherwise, while for $G \geq G^{SB}$, $r_p = R/R^{SB} - 1$ always. $R^* > R^{FB}$ if and only if $G < G_s$. Therefore, R^{FB} falls in the domain of function $\Phi(R) - 1$ if $G < G_s$, otherwise it is in the domain of function $R/R^{SB} - 1$. Hence, the optimal policy rate, r_p^{FB}, equals $\Phi(R^{FB}) - 1$ if $G < G_s$ and $R^{FB}/R^{SB} - 1$ otherwise. It is straightforward to check $G^{FB} < G_s < G^{SB}$.

Q.E.D.

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