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The Condorcet Jur(ies) Theorem*

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Abstract

Should two issues be decided jointly by a single committee or in separately by different committees? Similarly, should two defendants be tried together in a joint trial or tried separately in severed trials? Multiplicity of issues or defendants introduces novel strategic considerations. As in the standard Condorcet Jury Theorem, we consider large committees with common values and incomplete information. Our main result is that the joint trial by a single committee can aggregate information if and only if the severed trials by separate committees can aggregate information. Specifically, suppose that either for the joint trial or for the severed trials there exists an sequence of equilibria that implements the optimal outcome with probability approaching one as the number of voters goes to infinity. Then a sequence of equilibria with similar asymptotic efficiency exists for the other format. Thus, the advantage of either format cannot hinge on pure information aggregation with many signals.

1 Introduction

In United States law, the decision to join multiple related counts or defendants in a single trial before one jury or to sever these decisions into different trials before different juries. A large body of legal scholarship studies the many rules and precedents that govern when joinder and severance. Not just an academic topic, joinder and severance have important implications in practice for particular agents in the courts:

> A basic understanding of the law regarding joinder and severance is essential for any lawyer practicing in the federal criminal courts. Whether a defendant is tried singly or jointly with co-defendants can play a vital role in whether that defendant is convicted or acquitted. Likewise, an acquittal may turn upon whether or not a defendant is tried for one offense at a time or for multiple offenses jointly (Decker 1977–1978).

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For the court system in general, in *Richardson v. Marsh* (1987) the Supreme court points out the frequent incidence of joint trials, and argues for their potential efficiency and coordination advantages.

> Joint trials play a vital role in the criminal justice system, accounting for almost one-third of federal criminal trials in the past five years. . . . It would impair both the efficiency and the fairness of the criminal justice system to require, in all these cases of joint crimes where incriminating statements exist, that prosecutors bring separate proceedings, presenting the same evidence again and again, . . . Joint trials generally serve the interests of justice by avoiding inconsistent verdicts and enabling more accurate assessment of relative culpability – advantages which sometimes operate to the defendant's benefit. Even apart from these tactical considerations, joint trials generally serve the interests of justice by avoiding the scandal and inequity of inconsistent verdicts (*Richardson v. Marsh* 1987, 209–210).

In this paper, we scrutinize the Court's argument that joinder "generally serves the interests of justice." In particular, we examine how joinder or severance aggregates private information in the standard environment of the Condorcet Jury Theorem, where information is independently distributed and the size of juries becomes large. A large literature on the Jury Theorem studies when large juries can aggregate information to reach the optimal outcome. These insights are limited to settings with a single issue or defendant. This paper takes first steps in developing our theoretical knowledge of joint versus severed trials or committees in environments with common values. Our main result is that joinder will aggregate information if and only if severance will aggregate information. That is, under the classic assumptions of the Condorcet Jury Theorem, neither format enjoys an informational advantage over the other.[1]

The equivalence of the two formats with common values contrasts with earlier findings for private-value environments. In a prior paper, we studied private-value elections for multiple issues and found joint elections can be sharply worse than separate elections. For example, the simultaneous election of two issues can fail to enact an overwhelming Condorcet winner (Ahn and Oliveros 2012, Example 1). The inefficiency is related to the wedge between the unconditional belief that the second issue will pass and the conditional belief that it will pass when a voter is pivotal for the first election. Holding a separate election for each issue eliminates this wedge and provides a potential solution. In fact, with private values it is straightforward to show that separate elections always yield limit equilibria that implement the Condorcet winner.

In contrast, this paper proves that with common values the performance of joint and severed elections converge. Specifically, information aggregates in the joint trial if and only if information aggregates in the severed trials. This undercuts information aggregation, at least in the standard environment of the Condorcet Jury Theorem, as an argument for superiority of either format. The design of joint or separate elections has important implications for the aggregation of *preferences*

[1] For finite committees, the relationship is ambiguous. Examples at the end of the paper show that joinder can strictly outperform severance, and vice versa.

but not for the aggregation of *information*. While the current paper and our prior work introduce this contrast, they are limited to extreme environments of purely common or purely private values. The disparity between pure private and common value environments suggests the importance of further work to understand more realistic settings with mixed values where payoffs have both private and common components.

Our findings also relate to work on strategic models of judgement aggregation, which constitute a prominent class of common-value environments. A recent literature studies judgement aggregation from an axiomatic perspective, see a recent symposium introduced by List and Polak (2010). Some papers consider judgment aggregation with strategic agents. For example, de Clippel and Eliaz (2012) compare strategic equilibria across a setting where voters decide premises and a setting where voters decide the implied conclusions of these premises directly. Bozbay, Dietrich, and Peters (2011) and Bozbay (2012) consider optimal voting rules for judgment aggregation from a mechanism design perspective. While not part of our original motivation, one interpretation of our main result for judgment aggregation is that having different committees deciding separate logical premises is asymptotically equivalent to having a single committee decide all premises simultaneously.

The rest of the paper proceeds as follows. Section 2 introduces the voting model and the two election formats. Section 3 presents examples that illustrate the different strategic considerations in the joint trial and the severed trials. As with a single issue, conditioning on being pivotal for the outcome is essential to equilibrium behavior. As observed by Austen-Smith and Banks (1996), being pivotal often eliminates the strategic incentive to vote sincerely. But deciding multiple issues introduces additional complications, which depend on whether the issues are joined or severed. In some situations, voting sincerely based on one's private signal is efficient and an equilibrium for the joint trial, but fails to be an equilibrium for the severed trials. In other situations, sincere voting is efficient and an equilibrium for the severed trials, but not for the joint trial.

The examples illustrate the strategic subtlety of voting over multiple issues. Nonetheless, Section 4 establishes an equivalence between weak information aggregation in a joint trial and in severed trials. Specifically, suppose that there exists a sequence of equilibria for one format such that the probability of correct verdicts goes to one as the number of voters goes to infinity. Then there exists an analogous asymptotically efficient sequence of equilibria for the other format.

2 Model

There is a set $X = \{1, 2\}$ of two up-down issues to be decided, for example, the passage of two referendums or the guilt of two defendants. The set of possible outcomes is the power set of bundles: $\mathcal{X} = \{\{1, 2\}, \{1\}, \{2\}, \emptyset\}$. In the referendums setting, each bundle corresponds to the set of approved initiatives. In the jury setting, each bundle corresponds to the set of convicted defendants.

Let Ω denote a finite set of states of the world. The prior probability of state $\omega \in \Omega$ is denoted $P(\omega)$. Let $U(A|\omega)$ denoted the common utility for all voters for outcome A when the state of the world is ω. Assume a unique best outcome $A_\omega \in \mathcal{X}$ maximizes $U(A|\omega)$ for each state of the world. We normalize utilities so $U(A_\omega|A) = 1$. For any set $A \in \mathcal{X}$, let $A^1 = A \cap \{1\}$ and $A^2 = A \cap \{2\}$ denote the projections onto the first and second issues.

The finite set S is a set of possible signals.[2] The conditional probability of signal $s \in S$ given ω is denoted $F(s|\omega)$. Given the state of the world ω, each voter receives a conditionally independent signal from the distribution $F(\cdot|\omega)$. The conditional product distribution of the signal profile $\mathbf{s} = (s_1, \ldots, s_I)$ is denoted $F(\mathbf{s}|\omega)$.

We consider two voting games. The first is a joint election on both issues, where a single committee of I voters decides both issues using q-majority rule. The set of possible ballots for each voter is $\mathcal{X} = \{\{1,2\}, \{1\}, \{2\}, \emptyset\}$, where submitting ballot A means voting for every issue in A and voting against every issue outside A. If more than qI of the voters support an issue, that issue passes. The final outcome is the set of issues that are supported by more than q fraction of the voters. Formally, the aggregation rule $\mathcal{F}(A_1, \ldots, A_I)$ is defined by

$$\mathcal{F}(A_1, \ldots, A_I) = \{x : \#\{i : x \in A_i\} \geq qI\}.$$

A strategy for juror i is a function $\sigma_i : S \to \Delta\mathcal{X}$ that assigns a distribution over ballots $\sigma_i(s)$ to each signal s. When $\sigma_i(s)$ is a degenerate point mass on the ballot A, we slightly abuse notation and write $\sigma_i(s) = A$. A profile $(\sigma_1, \ldots, \sigma_I)$ of strategies is *symmetric* if $\sigma_i = \sigma_j$ for all i, j. When referring to symmetric strategy profiles, we drop the subscript. The common expected utility for the strategy profile $\boldsymbol{\sigma}(\mathbf{s}) = \sigma_1(s_1), \ldots, \sigma_I(s_I)$ is

$$EU(\boldsymbol{\sigma}) = \sum_{\Omega} \sum_{S} \sum_{\mathcal{X}^I} U\left(\mathcal{F}(A_1, \ldots, A_I) \,|\, \omega\right) \, \boldsymbol{\sigma}(\mathbf{s}) \, F(\mathbf{s}|\omega) \, P(\omega).$$

In the jury setting, this corresponds to a single trial for both defendants, where qI of the jurors must find each defendant guilty to reach a guilty verdict for that defendant. We refer to this game as a *joint trial*. We will study symmetric Nash equilibria of the joint trial.[3] We consider the limit of symmetric equilibria as the number of voters goes to infinity and let the subscript denote the size of the electorate rather than a specific individual voter. We are interested in whether a sequence of strategies (σ_I) will enact the optimal outcome A_ω in large elections. In particular, we say that the *probability of error goes to zero* if, for every ω,

$$\sum_{S} \sum_{\mathcal{X}^I} U\left(\mathcal{F}(A_1, \ldots, A_I) \,|\, \omega\right) \, \boldsymbol{\sigma}_I(\mathbf{s}_I) \, F(\mathbf{s}_I|\omega) \to U(A_\omega|\omega),$$

as I goes to infinity. The probability of error goes to zero if and only if the probability of the optimal outcome A_ω goes to one for every state of the world.

[2]The assumption of finite signals is for expositional convenience and all results would hold with a continuum of signals. We thank a referee for pointing this out.

[3]All symmetry assumptions are for expositional convenience. Suitable analogs of the results hold for possibly asymmetric strategies.

In the second game, a total of $2I$ voters are divided into two disjoint committees of I voters that decide each issue separately using q-majority rule. Let the first $1, \ldots, I$ voters constitute the first committee and the last $I+1, \ldots, 2I$ voters constitute the second committee. The voters in the first committee can either vote up or down on the first issue $\mathcal{X}_1 = \{\{1\}, \emptyset\}$ and the voters in the second committee can vote up or down on the second issue $\mathcal{X}_2 = \{\{2\}, \emptyset\}$. The outcome of the first committee is

$$\mathcal{F}^1(A_1, \ldots, A_I) = \begin{cases} \{1\} & \text{if } \#\{i : A_i = \{1\}\} \geq qI \\ \emptyset & \text{otherwise} \end{cases}.$$

The outcome of the second committee $\mathcal{F}^2(A_{I+1}, \ldots, A_{2I})$ is defined analogously. The outcome of the game is $\mathcal{F}^1(A_1, \ldots, A_I) \cup \mathcal{F}^2(A_{I+1}, \ldots, A_{2I})$. A strategy for a member i of the first committee is a function $\sigma_i^1 : S \to \Delta\{\{1\}, \emptyset\}$ and for a member j of the second committee is a function $\sigma_j^2 : S \to \Delta\{\{2\}, \emptyset\}$. A profile of strategies is *semi-symmetric* if $\sigma_i = \sigma_{i'}$ for all voters i, i' in the first committee and $\sigma_j = \sigma_{j'}$ for all voters j, j' in the second committee. The common expected utility for the strategy profile $(\boldsymbol{\sigma}^1(\mathbf{s}), \boldsymbol{\sigma}^2(\mathbf{s})) = (\sigma_1^1(s_1), \ldots, \sigma_I^1(s_I), \sigma_{I+1}^2(s_{I+1}), \ldots, \sigma_{2I}^2(s_{2I}))$ is

$$EU(\boldsymbol{\sigma}^1, \boldsymbol{\sigma}^2) = \sum_{\Omega} \sum_{S} \sum_{[\mathcal{X}^1]^I \times [\mathcal{X}^2]^I} U\left(\mathcal{F}^1(A_1^1, \ldots, A_I^1) \cup \mathcal{F}^2(A_{I+1}^2, \ldots, A_{2I}^2) \,|\, \omega\right) \boldsymbol{\sigma}(\mathbf{s}) \, F(\mathbf{s}|\omega) \, P(\omega).$$

In the jury setting, this corresponds to having a separate trial for each defendants. We refer to this game as *severed trials*. We will study semi-symmetric Nash equilibria of the severed trials. For a sequence of semi-symmetric strategies (σ_I^1, σ_I^2), we say the the *probability of error goes to zero* if, for all ω:

$$\sum_{S} \sum_{[\mathcal{X}^1]^I \times [\mathcal{X}^2]^I} U\left(\mathcal{F}^1(A_1^1, \ldots, A_I^1) \cup \mathcal{F}^2(A_{I+1}^2, \ldots, A_{2I}^2) \,|\, \omega\right) \boldsymbol{\sigma}(\mathbf{s}) \, F(\mathbf{s}|\omega) \, \to U(A_\omega|\omega),$$

as I goes to infinity. As in the joint trial, the probability of error goes to zero if and only if the probability of both trials reflecting the optimal outcome A_ω goes to one.

3 Examples

The following examples illustrate some of the strategic subtleties in deciding multiple issues with common values. In all the examples, sincere voting is informative and asymptotically efficient. However, sincere voting is incentive compatible in either the joint trial or the severed trials, but not incentive compatible in the other format. Note that we do not mean to suggest that sincere voting is interesting per se, but rather to use sincere voting as a sharp illustration of the distinct strategic considerations in joined and severed trials.

With a single issue, being pivotal for the outcome provides additional information regarding the state of the world. This conditioning often precludes sincere or informative voting from being an equilibrium (Austen-Smith and Banks 1996). Since incentive compatibility is maintained in one format but not the other, the strategic reasoning in the following examples is necessarily distinct from the standard story. In the first example, sincere voting is not an equilibrium in the joint trial because each voter can deviate on both issues simultaneously. This deviation is precluded by severing the trials, where sincere voting is an equilibrium. In the second example, sincere voting is an equilibrium in the joint trial but fails to be an equilibrium in the severed trials. There, the joint trial allows voters to coordinate across issues, while this coordination is not possible in the severed trials.

In the first example, two defendants are accused of the same crime and exactly one is guilty. The sincere strategy profile is efficient and incentive compatible in the severed trials. However, it is not an equilibrium in the joint trial. This is because the space of deviations is larger in the joint trial: when she is pivotal for either defendant, any juror is better off finding both defendants innocent, an option that is not available to her in the severed trials.

Example 1 (Too many actions in joint trial)**.** Let $q = \frac{1}{2}$. Suppose $\Omega = \{\{1\}, \{2\}\}$ and $P(\omega) = \frac{1}{2}$ for every ω.

$$U(A|\omega) = \begin{cases} 1 & \text{if } A = \omega \\ \frac{2}{3} & \text{if } A = \emptyset, \{1,2\} \\ 0 & \text{if } A = \{1,2\} \setminus \omega \end{cases} .$$

Exactly one of the defendants is guilty. Jurors are risk averse in the number of correct verdicts: the marginal utility for deciding at least one of the verdict correctly (which can be guaranteed by finding both innocent) rather than none of them correctly, $U(\emptyset|\omega) - U(\omega^{\complement}|\omega) = \frac{2}{3}$, is greater than the marginal utility for deciding both rather than only one of the verdicts correctly, $U(\omega|\omega) - U(\emptyset|\omega) = \frac{1}{3}$.

Each juror gets a correct signal with three-fifths probability. Let $S = \Omega$ and

$$F(s|\omega) = \begin{cases} \frac{3}{5} & \text{if } s = \omega \\ \frac{2}{5} & \text{if } s \neq \omega \end{cases}$$

First consider severed trials. The sincere strategy profile $\sigma^1(s) = s \cap \{1\}$ and $\sigma^2(s) = s \cap \{2\}$ aggregates information and is also incentive compatible. To see this, consider a juror in the first trial who assumes she is pivotal for the first defendant and whose private signal indicates that the first defendant is guilty. When she is pivotal for the first trial, the other jurors' signals for the first trial cancel each other, so her posterior is based on her private signal, namely that the probability of the state $\omega = \{1\}$ is $\frac{3}{5}$.There are two cases to consider for the second trial. In the first case, the second jury decides the second verdict correctly. Then, conditional on being pivotal, her expected utility after seeing the signal $s = \{1\}$ of convicting the first defendant is $\frac{3}{5} \times 1 + \frac{2}{5} \times \frac{2}{3}$. On the other hand, her expected utility after seeing the signal $\{1\}$ of acquitting is $\frac{2}{5} \times 1 + \frac{3}{5} \times \frac{2}{3}$. The first quantity is larger. Similarly, in the second case where the second jury incorrectly decides the second verdict, the pivotal juror is better off following her signal. Since voting sincerely is better in either case, she should certainly do so.

Now consider the joint trial. The sincere strategy $\sigma(s) = s$ again aggregates information, but is not incentive compatible. To see this, suppose a juror's private signal indicates the first defendant is guilty. Now suppose she is pivotal for some issue. Then half of the other voters submitted the ballot $\{1\}$ and the other half submitted the ballot $\{2\}$, so she is pivotal for both issues. Moreover, the other voters' signals cancel themselves and her posterior based on her private signal $\{1\}$ is that the probability $\omega = (1, 0)$ is $\frac{3}{5}$. Then voting to convict the first defendant alone, i.e. submitting the ballot $\{1\}$, provides an expected utility of $\frac{3}{5}U(\{1\}|\{1\}) + \frac{2}{5}U(\{1\}|\{2\}) = \frac{3}{5}$. On the other hand, voting to acquit both defendants, i.e., submitting the ballot $\emptyset$, provides a strictly greater (sure) expected utility of $\frac{2}{3}$. So the suggested strategy is not incentive compatible.

In the second example, sincere voting is efficient and incentive compatible in the joint trial, but fails to be incentive compatible in the severed trials. In this environment, the two issues are substitutes: it is best to pass one issue or the other, but it is very bad to pass both issues together. The joint trial provides a way for voters to coordinate their votes, but this coordination is broken when the trials are severed.

Example 2 (No coordination in severed trials)**.** Suppose $\Omega = \{\{1\}, \{2\}\}$ and $S = \{\{1\}, \{2\}\}$. Let

$$U(A|\omega) = \begin{cases} 1 & \text{if } A = \omega \\ \frac{3}{4} & \text{if } A^1 \neq \omega^1 \text{ and } A^2 \neq \omega^2 \\ \frac{1}{2} & \text{if } A = \emptyset \\ 0 & \text{if } A = \{1, 2\} \end{cases}$$

Suppose

$$F(s|\omega) = \begin{cases} \frac{3}{5} & \text{if } s = \omega \\ \frac{2}{5} & \text{if } s \neq \omega \end{cases}$$

For example, suppose that a state that faces excess traffic can either build a new highway or a new high speed railway. One of the options is better than the other. Conditional on the state of the world, the best outcome is to build the better option, but even the inferior option is better than doing nothing at all. However, the worst possible outcome would be spending the money to build both the highway and the railway.

The sincere strategy profile $\sigma^*(s) = s$ is an equilibrium of a joint election, and takes the probability of an error to zero. To see that it is an equilibrium, consider a voter who sees the signal $\{1\}$. If she is pivotal for either issue, then she is pivotal for both issues. Moreover, the other voters' signals have canceled and her posterior puts probability $\frac{3}{5}$ that the state is $\{1\}$. Then her expected utility for submitting the ballot $\{1\}$ is

$$\frac{3}{5}U(\{1\}|\{1\}) + \frac{2}{5}U(\{1\}|\{2\}) = \frac{3}{5} \times 1 + \frac{2}{5} \times \frac{3}{4}.$$

Her expected utility for submitting the ballot $\{2\}$ is

$$\frac{3}{5}U(\{2\}|\{1\}) + \frac{2}{5}U(\{2\}|\{1\})) = \frac{3}{5} \times \frac{3}{4} + \frac{2}{5} \times 1.$$

This is strictly worse than submitting $\{1\}$. Finally, when she is pivotal, her (sure) expected utility for submitting the ballot $\{1, 2\}$ is 0, and her (sure) expected utility for submitting the ballot $\emptyset$ is $\frac{1}{2}$. These are both worse as well. So sincere voting is incentive compatible in the single election.

However, the associated strategy profile $\sigma^1(s) = [\sigma^*(s)]^1$ and $\sigma^2(s) = [\sigma^*(s)]^2$ is not an equilibrium if the issues are decided separately by disjoint committees. To see this, consider a voter i in the first committee and observes the signal $s_i = \{1\}$. When she is pivotal, the other voters' signals cancel each other and her posterior probability is simply based on her private signal. So, the posterior probability of $\omega = \{1\}$ is $\frac{3}{5}$. With probability approaching one, if $\omega = \{1\}$, then the second committee playing strategy σ^2 will vote against the railway, while if $\omega = \{2\}$ it will support the railway. So, for sufficiently large I, her expected utility for voting for the highway can be made arbitrarily close to

$$\frac{3}{5}U(\{1\}|\{1\}) + \frac{2}{5}U(\{1, 2\}|\{2\}) = \frac{3}{5}.$$

Her expected utility for voting against the highway can be made arbitrarily close to

$$\frac{3}{5}U(\emptyset|\{1\}) + \frac{2}{5}U(\{2\}|\{2\}) = \frac{3}{5} \times \frac{1}{2} + \frac{2}{5} \times 1 = \frac{7}{10}.$$

So, for a sufficiently large I, her best response to this strategy profile is to vote down on issue 1. Hence, the associated strategy (σ^1, σ^2) in the split juries game is not an equilibrium.

4 Asymptotic equivalence

The examples in Section 3 suggest that the strategic considerations are different in joint and severed trials. Nonetheless, we demonstrate that if there exists an efficient sequence of equilibria in either format, then there exists an efficient sequence of equilibria in the other. Therefore, an argument for the superiority of either format cannot hinge on information aggregation with many voters, but must appeal to other considerations.

Proposition 1. *There exists a sequence of symmetric equilibria* (σ_I^*) *in the joint trial such that the probability of error goes to zero if and only if there exists a sequence of semi-symmetric equilibria* $(\sigma_I^{1*}, \sigma_I^{2*})$ *in the severed trials such that the probability of error goes to zero .*

Proposition 1 is a corollary of the following two lemmata. The first adapts an insight of McLennan (1998) for common value elections: if any strategy profile aggregates information, then there exists a Nash equilibrium that aggregates information.

Lemma 1 (McLennan 1998)**.** *The following are true:*

(i) *If there exists a sequence of symmetric strategies* (σ_I) *in the joint trial such that the probability of error goes to zero, then there exists a sequence of symmetric equilibria* (σ_I^*) *such that the probability of error goes to zero.*

(ii) *If there exists a sequence of semi-symmetric strategies* (σ_I^1, σ_I^2) *in the severed trials such that the probability of error goes to zero, then there exists a sequence of semi-symmetric equilibria* $(\sigma_I^{1*}, \sigma_I^{2*})$ *such that the probability of error goes to zero.*

Proof. We will prove the first claim, the proof of the second claim is nearly identical. Consider a fixed I. A straightforward adaptation of the proof of Theorem 2 of McLennan (1998) demonstrates that if $\bar{\sigma}_I$ maximizes the common expected utility of the agents among all symmetric strategy profiles, then it is a symmetric equilibrium. The common expected utility $EU(\sigma_1, \ldots, \sigma_I)$ is a continuous function on the compact space of symmetric strategy profiles, so the maximizing $\bar{\sigma}_I$ exists and is an equilibrium.

Now suppose some sequence (σ_I) take the error probability to zero, i.e. the common expected utility goes to one. Then the sequence of equilibria $(\bar{\sigma}_I)$ must also take the error probability to zero. If it did not, then there would be a state of the world ω where the probability of an error is strictly positive for arbitrarily large juries. Then the common expected utility of $(\bar{\sigma}_I)$ strictly less than the common expected utility achieved by (σ_I), which would contradict its optimality over all symmetric strategy profiles. $\square$

The standard application of McLennan's observation is to argue for efficiency *within* a fixed voting institution: for example, McLennan (1998) shows that if sincere voting aggregates information, then there exists some equilibrium that also aggregates information. In contrast, we use McLennan's observation to argue *across* institutions: we show that information aggregation under one mechanism implies information aggregation under *another* mechanism. In particular, if (σ_I) aggregates information in a joint trial, then the corresponding strategies in the separated trials where each juror in trial x plays the marginal distribution of $\sigma(s)$ for issue x also aggregates information. Conversely, if (σ_I^1, σ_I^2) achieves full efficiency in the separated trials, then the strategies in the joint trial defined by the product distribution of σ_I^1 and σ_I^2 also achieve full efficiency.

Lemma 2. *There exists a sequence* (σ_I) *of symmetric strategies for the joint trial that takes the probability of error to zero if and only if there exists a sequence* (σ_I^1, σ_I^2) *of semi-symmetric strategies for severed trials that takes the probability of error to zero.*

Proof. We first prove the "only if" direction. Suppose there exists a sequence (σ_I) of symmetric strategies for the joint trial that takes the probability of error to zero. Now consider the following semi-symmetric strategies for the split trials:

$$[\sigma_I^1(s)](\{1\}) = [\sigma_I(s)](\{1,2\}) + [\sigma_I(s)](\{1\})$$
$$[\sigma_I^2(s)](\{2\}) = [\sigma_I(s)](\{1,2\}) + [\sigma_I(s)](\{2\})$$

Without loss of generality, consider a state ω where the optimal outcome is $A_\omega = \{1,2\}$. The vote count on issue 1 in the joint trial when voters use strategy σ_I follows a binomial distribution of I draws with a success probability equal to the probability of including issue 1 in the ballot:

$$\sum_{s\in S} F(s|\omega)\left([\sigma_I(s)](\{1,2\}) + [\sigma_I(s)](\{1\})\right).$$

By construction, this is exactly the distribution of the vote count in the first trial when voters use strategy σ_I^1. By assumption, the probability that the vote count on the first issue is greater then or equal to qI goes to one in the joint trial, so the probability that vote count in the first of the severed trials is greater then or equal to qI also goes to one. Similarly, the probability the vote count on the second issue is greater than or equal to qI also goes to one.

To prove the "if" direction, suppose there exists a sequence (σ_I^1, σ_I^2) of semi-symmetric strategies for severed trials that takes the probability of error to zero. Consider the following symmetric strategies for the unified trial:

$$[\sigma_I(s)](A) = [\sigma_I^1(s)](A \cap \{1\}) \times [\sigma_I^2(s)](A \cap \{2\}).$$[4]

Without loss of generality, consider a state ω where the optimal outcome is $A_\omega = \{1,2\}$. Then the conditional probability that both issues will pass in the severed trials goes to one. The vote count in the first of the severed trials follows a binomial distribution defined by I draws with a success probability of

$$\sum_{s\in S} F(s|\omega)[\sigma_I^1(s)](\{1\}).$$

The vote count on the first issue in the joint trial follows a binomial distribution with success probability of

$$\begin{aligned}
&\sum_{s\in S} F(s|\omega)\left\{[\sigma_I(s)](\{1,2\}) + [\sigma_I(s)](\{1\})\right\}\\
&= \sum_{s\in S} F(s|\omega)\left\{[\sigma_I^1(s)](\{1\}) \times [\sigma_I^2(s)](\{2\}) + [\sigma_I^1(s)](\{1\}) \cdot [\sigma_I^2(s)](\emptyset)\right\}\\
&= \sum_{s\in S} F(s|\omega)\left\{[\sigma_I^1(s)](\{1\}) \times \left([\sigma_I^2(s)](\{2\}) + [\sigma_I^2(s)](\emptyset)\right)\right\}\\
&= \sum_{s\in S} F(s|\omega)[\sigma_I^1(s)](\{1\}).
\end{aligned}$$

So, the probability that the vote count on the first issue in the joint trial will be greater than or equal to qI hereditarily goes to one. Similarly, the probability the second issue passes in the joint trial also goes to one. □

One useful implication of Proposition 1 is that it translates sufficient conditions for information

[4] It is straightforward to verify that this is a well-defined mixed strategy.

aggregation from the severed trials into the joint trial. Any existing sufficient condition for information aggregation in the standard Condorcet Jury Theorem for a single defendant can therefore be imposed for each issue to guarantee information aggregation in the joint trial. For example, if the information structure can statistically distinguish whether the optimal outcome acquits or convicts either defendant, then the optimal joint outcome can be attained at the limit.

The basic logic for establishing Proposition 1 is quite general. A simple extension of the argument shows that the same result holds with the possibility of abstention. Another extension shows that, with three or more defendants, information aggregation under any segregation of defendants into different trials, e.g., five defendants tried in one trial and three defendants tried in another, implies information aggregation for all formats.

We should mention what Proposition 1 leaves open. It only maintains the equivalence of the existence of an asymptotically efficient sequence of equilibria across formats. There could exist an additional inefficient sequence of equilibria in one format, but with no analogous sequence in the other format, leaving miscoordination as a potential disadvantage of one format. In cases where information fails to aggregate, Proposition 1 provides no guidance regarding which environment is superior.

Finally, the result leaves open the welfare comparison for finite juries, which is in fact ambiguous.[5] One case where the joint trial is superior assumes the parameters of Example 2 and compares a joint trial with a single juror with severed trials, both decided by different jurors. The welfare-maximizing strategy for the juror in the joint trial is to vote $\{1\}$ if the signal is $\{1\}$ and to vote $\{2\}$ if the signal is $\{2\}$, yielding an expected utility of $\frac{3}{5} \times 1 + \frac{2}{5} \times \frac{3}{4} = \frac{9}{10}$. On the other hand, in the severed trials, having each juror vote the projection of her signal yields an expected utility of $\frac{3}{5} \times \frac{3}{5} \times 1 + \frac{2}{5} \times \frac{3}{5} \times \frac{1}{2} + \frac{3}{5} \times \frac{2}{5} \times 0 + \frac{2}{5} \times \frac{2}{5} \times \frac{3}{4} = \frac{3}{5}$.[6] So, here the joint trial is superior to severed trials.

On the other hand, the following example shows that sometimes the severed trials can improve welfare. Let $\Omega = S = \{\{1,2\}, \{1\}, \{2\}, \emptyset\}$. Let $F(\omega|s) = \frac{1}{2}$ if $\omega = s$ and $F(\omega|s) = \frac{1}{6}$ if $\omega \neq s$. Let $U(A|\omega) = 1$ if $A^1 = \omega^1$ or $A^2 = \omega^2$ and $U(A|\omega) = 0$ otherwise, that is, the only way to not get full utility is by getting both issues wrong.[7] Then the maximal expected utility in the joint trial with a single juror is $\frac{5}{6}$, since the probability of getting both issues wrong is getting the opposite signal which happens with probability $\frac{1}{6}$. Now suppose each juror in severed trials votes the projection of her signal on her issue. This will yield expected utility $\frac{8}{9}$, since the probability that each juror will get a compatible signal on her issue is $\frac{1}{2} + \frac{1}{6} = \frac{2}{3}$, so the probability that both jurors get the incompatible signal is $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.[8]

[5] We thank a referee whose comments led to the following examples.

[6] In contrast to the case with large juries, the probability of an error on the other issue is nontrivial when juries are small. Ironically, in this example, the possibility of an error by the other jury promotes sincere voting because missing both issues is better than getting only a single issue right.

[7] There is more than one optimal outcome A_ω here, but assuming that getting exactly one issue correct provides utility 0.99 will yield identical predictions.

[8] In both of these examples, the assumption that there is a single juror in each trial is only for convenience. Analogous examples with several jurors in each trial can be constructed.

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