# Author's Accepted Manuscript

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www.elsevier.com/locate/eer

PII: S0014-2921(15)00068-9

DOI: http://dx.doi.org/10.1016/j.euroecorev.2015.05.003

Reference: EER2708

To appear in: European Economic Review

Received date: 8 October 2013 Revised date: 7 January 2015 Accepted date: 6 May 2015

Cite this article as: Ben Etheridge, A test of the household income process using consumption and wealth data, *European Economic Review*, http://dx.doi.org/10.1016/j.euroecorev.2015.05.003

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# A Test of the Household Income Process Using Consumption and Wealth Data\*

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29th May 2015

#### Abstract

The evolution of household income can be explained almost equally well by rival models. However, rival models have very different implications for other household behaviours. I therefore test between two prominent models in the UK using panel data on consumption and wealth, as well as income, over 1991-2006. To operate the test, I show that long-lived income shocks transmit far less than one-for-one through to consumption, and particularly so for younger households. I then compare these estimates of transmission with estimates of households' ability to smooth shocks, captured by the data on wealth. Conditional on the suitability of the consumption model, my estimates provide evidence against the restricted income process ('RIP') and in favour of an alternative heterogeneous income process ('HIP'). This finding also explains why cross-sectional consumption inequality grew slowly over the period even though the variance of long-lived shocks was high. Finally, I conclude it is important to consider mean reversion of shocks when constructing life-cycle consumption models.

JEL Classification: D12, D31, D91, E21

**Keywords**: Income risk, Consumption inequality, Wealth

<sup>\*</sup>This paper is a revised version of my dissertation chapter 3. Thanks to Richard Blundell and Hamish Low for invaluable advice and encouragement. Further thanks to Tom Crossley, Luigi Pistaferri, James Banks, Melvyn Coles, and to two anonymous referees for comments. Thanks finally to Zoe Oldfield and Cormac O'Dea for the financial wealth data. Financial assistance was provided by the ESRC through the Institute for Fiscal Studies PhD scholarship. The British Household Panel Survey (BHPS) is produced by the ESRC UK Longitudinal Studies Centre, together with the Institute for Social and Economic Research at the University of Essex. The Family Expenditure Survey (FES) is produced by the Office for National Statistics. Data for both the BHPS and FES were supplied by the ESRC data archive. All errors are my own.

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# 1 Introduction

Households experience differing successes as they progress through life. Some households prosper and earn high incomes, while others suffer. For many households, incomes fluctuate throughout life both up and down. Precisely how incomes fluctuate strongly affects both household welfare and how households behave. The nature of the income process determines, for example, precautionary saving and the cost of risk. How incomes fluctuate is the subject of this paper.

The nature of the income process has been debated using panel data on income at least since Lillard and Willis (1978). In its present format, the debate centers around which of two parsimonious models fit the data best: the first model features a common growth path and a stochastic unit root (permanent) component. This model is favoured by MaCurdy1982, and more recently hryshko2008rip, and is often called the restricted income process (RIP). Alternatively, the second model is characterized by a household-specific growth path and a stochastic component that is persistent but mean-reverting. This model is favoured by Guvenen2009 and termed the heterogeneous income process (HIP).

Of course, these models are parsimonious characterizations of more complex underlying structural processes. But it is important to discern the appropriate parsimonious model because these models are used in so many applications and they give such different predictions for behaviour.<sup>3</sup> However, the debate seems unlikely to be settled using panel income data alone, because both competing models fit these data almost equally well.<sup>4</sup> Given that the models, however, imply very different household behaviours, it seems sensible to select between them by examining how income shocks affect a decision variable. A good decision variable for this purpose is non-durable consumption. Household non-durable consumption choices have little friction and theories of household consumption have been extensively developed and tested. Papers pursuing this strategy include primiceri2009heterogeneous, guvenen2007learning and Guvenen2010.

In this paper, I test the nature of the income process in the UK using data on incomes, food consumption and household wealth.<sup>5</sup> I use data from the British Household Panel Survey (BHPS) over 1991-2006. I find that the consumption and wealth data imply rejection of the RIP model and favour the HIP. To operate the test, I first estimate how much of long-lived income shocks transmit through to consumption.

<sup>&</sup>lt;sup>1</sup>To be precise, most of this literature examines the process for wages or individual earnings. My paper concerns household incomes, for which the debates apply equally.

<sup>&</sup>lt;sup>2</sup>See also baker1999earnings.

<sup>&</sup>lt;sup>3</sup>Not only do the models have different implications for, say precautionary saving, they also affect the design of optimal social insurance. See, for example Farhi2011 who solve for the optimal dynamic income taxes in terms of the persistence of income fluctuations.

<sup>&</sup>lt;sup>4</sup>See, for example, Meghir2010 for a discussion.

<sup>&</sup>lt;sup>5</sup>Of course I only observe food expenditure, not consumption. As is common in the literature I ignore the distinction between consumption and expenditure in most of this paper.

The estimation is similar to BPP2008 and kaplan2008much. The key idea behind the test is simply as follows: under the RIP model, and with incomplete markets, permanent income shocks should transmit through to consumption almost completely. If long-lived shocks are instead mean-reverting then they transmit far less. Similarly, if each household has its own specific growth path then the transmission estimator is biased towards 0, because the income fluctuations look more variable to the econometrician than to the household.<sup>6</sup> I estimate the transmission of long-lived shocks to be 0.42 (42%). Therefore, either because of mean-reversion or because of household-specific growth paths, or both, the HIP process is favoured.

To be accurate, my test must take account of households' life-cycle saving behaviour. My test therefore incorporates data on household wealth. Wealth holdings affect how these long-lived shocks transmit through to consumption. For example, even a permanent shock does not transmit completely when households are able to self-insure by accumulating assets. In calibrated models, the transmission coefficient is typically around 0.8, averaged over the life-cycle. On a theoretical level, the transmission coefficient under RIP with self-insurance is given approximately by the share of human capital in discounted life-time wealth. I therefore describe my test more precisely as follows: if long-lived shocks transmit through to consumption as much as this human capital share then we fail to reject the RIP. But, because income transmits less than this share then we favour the HIP.

In a related way, I provide further evidence to support HIP by examining the life-cycle pattern of transmission. As households approach retirement, transitory shocks look more like permanent shocks (and vice versa). Therefore mean-reverting shocks (a feature of HIP) should transmit close to the human capital share for older households, and only much less for the young. I estimate the transmission coefficient for older households to be 0.49, which cannot be distinguished from their human capital share (0.56). But the transmission estimate for the younger group is 0.34, and their human capital share is 0.83, which is significantly higher. These estimates echo those from BPP2008, discussed by kaplan2008much. Even though BPP2008's transmission estimate is overall higher than mine (at 0.64), they estimate it to be constant over the life-cycle and puzzlingly lower than the human capital share for the young.

My test shows the power of using consumption data in particular to identify income dynamics over and above using income data alone. This power can be illustrated using a simple example. Suppose that long-lived shocks are very slowly mean-reverting but not permanent, say with a persistence rate of 97% per year. Income panel data alone are unlikely to reject a unit-root, permanent process. Abstracting

<sup>&</sup>lt;sup>6</sup>If households slowly learn their earning then this should push the size of the consumption response to income shocks upwards. However, Guvenen2010 conclude that heterogeneity in income growth is mostly foreseen and that any learning effect is negligible.

<sup>&</sup>lt;sup>7</sup>See carroll2009MPC and kaplan2008much.

<sup>&</sup>lt;sup>8</sup>See blp2004income.

from the life-cycle, suppose further that households live forever and the interest rate is 3%. Then the mean-reverting shock has half the net present value of a permanent shock.<sup>9</sup> Therefore the consumption response to the mean-reverting shock will also be half. Consumption data should therefore clearly reject the unit root, permanent model.

However, precisely because I use the consumption and wealth data, my test is conditional on the model of intertemporal choice: In this case, a model of self-insurance. This model precludes some insurance provided by richer market structures, for example those modelled in KruegerPerri06, Attanasio2007 and Broer2013. The presence of extra insurance in the US is the explanation favoured by BPP2008, and modelling and testing environments with richer market structures remains an active area of research. However, I note that the income data includes all public and private transfers, so should capture many of the insurance mechanisms available. Therefore, my approach focuses on the effects on consumption of self-insurance after additional informal insurance. Nevertheless, it should be remembered that my test is primarily a rejection of RIP conditional on the self-insurance framework.

I then investigate the income data alone to put my results into context. I confirm, similarly to Guvenen2009 in the US, that the income panel alone in first differences can't distinguish between RIP and HIP. However, when estimating a variety of income models I conclude that, in terms of magnitudes, mean reversion is more important than heterogeneity in trends in explaining the results from the consumption data. Finally I examine the income data in levels. In particular, I examine growth in the variance of log incomes over the life-cycle. Here I find that both mean reversion and heterogeneity in trends is important in explaining the data. I conclude from these exercises, that both salient aspects of the HIP process - mean reversion and heterogeneity in trends - are important in fitting the income process, but mean reversion is particularly important when thinking about short-term consumption responses.

In addition to identifying the income process, my results help explain the evolution of cross-sectional inequality. Consumption inequality grew quite slowly over the period, as documented by BlundellEth2010, implying that consumption risk was low.<sup>10</sup> Meanwhile, using a variety of income models, I estimate the quantity of long-lived shocks from the income panel to be large.<sup>11</sup> If consumption risk identifies permanent income risk then the cross-sectional evidence contradicts that from the income panel prima facie. But these two pieces of evidence are reconciled by the HIP model. Quantitatively, I estimate the variance of long-lived shocks to be around 0.02-0.025 (standard deviation of shocks of around 14-16% per year) in a benchmark model. The implied contribution of long-lived shocks to consumption risk is

<sup>&</sup>lt;sup>9</sup>The net present value of a shock  $\zeta$  with persistence  $\rho$  and rate of return r is  $\zeta \frac{1+r}{r+(1-\rho)}$ .

<sup>&</sup>lt;sup>10</sup>An extensive literature uses the evolution of the cross-sectional variance of log consumption to identify idiosyncratic risk. attanasio2001intertemporal use these moments to identify the variance of shocks to the marginal utility of wealth. BlundellPreston98 use these moments to identify the variance of permanent income shocks. In effect, they assume that all shocks to marginal utility come from income and that permanent income shocks transmit fully into consumption.

<sup>&</sup>lt;sup>11</sup>BlundellEth2010 also report estimates of the variance of permanent shocks using the RIP model.

around  $0.42^2 \times 0.0225 = 0.004$  (standard deviation of 6.5% per year). This quantity lines up well with the estimate from the growth in consumption inequality of around 0.005 (7% per year).

To the best of my knowledge, this paper is the first to test the nature of the income process using data on incomes, consumption and wealth simultaneously. This paper is closest to Guvenen2010, who test the the income process in the US using an income and consumption panel and also estimate when households learn about their specific income trend. However, while their paper is fully structural, my test is simpler and arguably more transparent. It also incorporates the wealth data. Moreover, my test doesn't require full estimation of all parameters, such as the discount rate and risk aversion, upon which results may depend. A secondary contribution of the current paper is to use the expenditure data in the BHPS. The BHPS is an internationally-important data used for work on income uncertainty, such as by Postel2010, but the expenditure data have been little used. I show that the expenditure data can be used to tackle important research questions such as the present one.

This study also fits into a long literature examining consumption and income dynamics using microdata, going back to hall1982sensitivity. Besides the papers already mentioned, similar estimations to mine are performed on Russian data by Gorodnichenko2010 and Spanish data by Casado2011. They find a comparable transmission of permanent shocks to BPP2008. More recently, Blundell2012 use data on income, consumption and assets in the PSID to examine the transmission of wage shocks, focusing on labour supply responses. However, they do not test the nature of the underlying wage process. Similarly, Krueger2011 use panel data on income, consumption and wealth to quantify the relevant importance of income and wealth shocks. They use a different methodology and also do not test the nature of the underlying income process. My paper also relates to a small literature which uses decision variables to test for advance information. An example is Heckman2005, who examine educational choice. Finally, my findings relate to the growing literature looking in more detail at structural shocks to firms and to workers, such as productivity and employment risks. See for example, altonji2009modeling, Lise2006 and Low2009.

This paper proceeds as follows. Section 2 discusses the key features of the food panel data in the BHPS and describes treatment of the income data. Section 3 describes the model of income and consumption dynamics which nests both RIP and HIP specifications. Section 4 describes the test of the RIP-self insurance process against the alternatives. Section 5 describes the procedure for taking it to the data, including how I infer changes to total household non-durable consumption from consumption of food. Section 6 presents the results. Section 7 concludes. An online appendix gives further details, including some validation exercises using the expenditure data.

<sup>&</sup>lt;sup>12</sup>Some exceptions include, for example, Guariglia 2002.

<sup>&</sup>lt;sup>13</sup>Through most of the paper I refer to the null hypothesis as the 'RIP'. I also emphasize that the test is conditional on the self-insurance consumption model being correct. Hence, I also sometimes refer to the null as 'RIP-self insurance' to re-emphasize this point.

# 2 Data from the BHPS

The analysis uses data from the BHPS and FES. Other papers, for example BlundellEth2010, give extensive details on both these data sets. Here I describe just those features of the BHPS survey, and my treatment, that are particularly important for this specific analysis. A brief discussion of the FES data is contained in appendix A.

Despite its status as the main UK household panel survey, the BHPS has limited data on consumption. The survey only contains questions about food consumed within the home (food 'in') and about energy use and small durables purchases such as TVs and kitchen appliances. Within this set, only for food does consumption plausibly equal expenditure. I therefore focus on these responses. In comparison, the PSID survey for the US includes food purchased outside the house (food 'out'). Food 'in' has a much lower income elasticity than food 'out' because high income households substitute towards restaurant meals, so the signal from changes in food consumption to total consumption and living standards is weaker than in the equivalent US analysis. Consequently any hypothesis test will have lower power than those in, say, BPP2008. However, an advantage of the BHPS data is that it covers a period over which the PSID had reduced coverage. An aim of this paper is to examine the effect of income shocks on total (non-durable) consumption. Section 5, therefore, discusses in detail how I infer changes to total consumption from my food measure.

The specifics of the BHPS questions about food expenditures are as follows: the first wave of the BHPS asks 'Thinking about your weekly food bills approximately how much does your household usually spend in total on food and groceries?' Respondents give exact answers. Respondents include all of food, bread, milk, soft drinks etc. They also include take-aways eaten in the home. Respondents exclude pet food, alcohol, cigarettes and meals out. From wave 2 onwards spending information is collected according to 12 bands. For these waves I impute consumption to be the mid-point of each interval. For the bottom interval (£0-£10) I assign £5 spending. For the top band (£160+) I assign £180. The alternative is to estimate moments of interest using maximum likelihood by assuming an underlying distribution such as the normal. As a robustness check, figure 1 shows a comparison of the cross-sectional mean and variance using both the midpoints and maximum likelihood estimates, together with estimates from the FES. It shows that both treatments of the BHPS data give similar results in these dimensions. Appendix C discusses the use of midpoints in more detail, including analysis of the autocovariances and a validation using data from the PSID.

An advantage of the BHPS over the PSID is that it poses fewer problems of timing. Whereas there is concern that income and consumption may refer to different time periods in the PSID, all relevant questions in the BHPS ask about current circumstances.<sup>14</sup> Income and consumption observations should

<sup>&</sup>lt;sup>14</sup>See hall1982sensitivity for a detailed discussion of the PSID data.

therefore be synchronized. Almost all interviews are carried out between September 1 and December 1 in each survey year (less than 10% carry on into the new year). While the gap between interviews could be a minimum of 9 months and a maximum of 15 months within this main period, I neglect this variation in timing and consider that all first differences indicate yearly changes in variables.

The income concept used in the main analysis is household disposable income net of taxes and transfers. I obtain the measures for total household labour income and household net disposable income from an auxiliary data set (see \*Levy2008 for more documentation, and Jenkins2010 for a discussion). For both these two variables I use current measures (usual monthly income at the time of interview) rather than annual incomes. Net disposable income is defined as the sum of earned income, asset income and transfers (public and private) minus state taxes (income tax and national insurance contributions). Capital gains, or the drawing down of capital, is excluded in this definition. Pension income, which is often derived from the drawing down of capital is included in the definition, but because my sample consists of heads of working age, its contribution to income is small. As for other income concepts: wages are defined as usual earnings in the current job divided by usual hours. I remove wages and earnings that have been imputed by the BHPS compilers.

As for wealth, the BHPS has comprehensive information on housing wealth for most years. However, comprehensive information on financial wealth is available for 1995, 2000 and 2005 only. While the value of the first house and the value of all mortgages are reported exactly, the value of second homes and other financial wealth are reported in bands only. For these banded data I use imputations on the value of each type of asset (see e.g. \*Banks2002 for a description of the procedure). The data on financial wealth come from derived datasets reported in \*Banks2002 and Crossley2010. Data on pension wealth over 1991-2001 come from another auxiliary, derived dataset (see Disney, Emmerson, and Tetlow, 2009).

The sample selection proceeds as follows: I use only the core BHPS sample and ignore the low-income booster sample. Following BPP2008, I take only households headed by a stable and heterosexual couple in the BHPS, but allow for entry and exit of children. Naturally, this makes the discussion relevant to couples only. I exclude households with heads aged less than 25 or more than 65 and take only those heads born between 1940 and 1969. Finally, I exclude responses from Northern Ireland in the BHPS because they are not represented in the FES. I form an unbalanced panel by selecting households for whom the first difference of income appears at least 5 times over the course of the survey (16

<sup>&</sup>lt;sup>15</sup>The data for 1995 do not account for student loans and credit card debts. I ignore this consideration and treat the data as comparable across waves.

<sup>&</sup>lt;sup>16</sup>Here I select on a dynamic aspect of the data. This may cause differential selection between the BHPS and the FES. Nevertheless, one would think that including only stable couples in the BHPS (and all couples in the FES) would result in lower estimates for permanent risk in the BHPS. Therefore, sample selection does not weaken the motivation behind this paper.

<sup>&</sup>lt;sup>17</sup>The dynamics of income and consumption for unstable households are potentially more interesting and important. See, for example, voena2010yours for an analysis of the effect of divorce on consumption, savings and labour supply.

years). Therefore a household appears in the covariance matrices with a minimum of 6 appearances for income, though households could conceivably appear 9 times and still be dropped from the sample. Food expenditure is almost always observed.

I trim the top and bottom 0.5% of the distribution of all income variables.<sup>18</sup> I do not trim the food consumption distribution: since expenditures are assigned to 12 bands there is not the same chance of reporting implausibly high or low observations through mis-coding, or omission of a component. Such trimming of the levels of income does not theoretically make a difference to the central estimates, but improves precision. In addition I trim the most egregious changes in income level: households for whom residual income either grows by more than 1000% or falls by more than 90%. The initial sample comprises 116,111 household-year observations with 96,787 income observations. The final sample comprises 17,732 observations with 17,055 income observations.

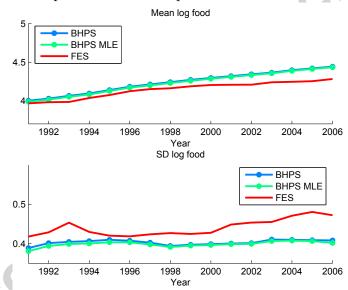


Figure 1: Comparison of food expenditures in the FES and BHPS

Notes: 'BHPS' gives the statistics using the imputation described above. 'BHPS MLE' gives maximum likelihood estimates using the observed bands assuming that food is distributed normally.

 $<sup>^{18}</sup>$ By taking logs of all the income and expenditure variables I also remove any negative observations. These comprise 0.6% of the initial sample themselves.

#### 3 The Model

#### 3.1Models of Income Dynamics

I use an encompassing model of income dynamics which nests both the HIP and RIP specifications. Income is assumed to be composed of four parts: First, it contains a deterministic component, common across the cohort. This component reflects the lifetime shape of the wage profile and life-cycle labour supply, among other income sources. Second, it contains a 'trend' component that is specific to the individual/household. Third, it contains a stochastic highly-persistent component evolving as an AR(1) process. Finally, it contains a stochastic short-lived 'transitory' component. This transitory component might include measurement error, which I do not attempt to identify separately.

Formally:

$$lnY_{it} = g_{Z,t} + \beta_i t + lnP_{it} + v_{it}$$

$$lnP_{it} = \rho lnP_{it-1} + \zeta_{it}$$

$$v_{it} = \epsilon_{it} + \theta \epsilon_{it-1}$$
(2)

$$v_{it} = \epsilon_{it} + \theta \epsilon_{it-1} \tag{2}$$

where  $g_{Z,t}$  is the deterministic component, depending on observable characteristics (Z) such as cohort, education, demographic variables and time.  $P_{it}$  is persistent income for household i at time t,  $\zeta_{it}$  is the innovation to the persistent income component,  $\beta_i$  is the idiosyncratic heterogeneous trend and  $v_{it}$ is the transitory income component. Under the HIP specification the transitory component is usually modeled as a pure white-noise error. Under the RIP specification it is often modeled more complexly. Here I have followed e.g. Meghir 2004 and captured the short-lived persistence by modeling it as a MA(1). In this process,  $\epsilon_{it}$  is the time-t innovation to transitory income (including measurement error) and  $\theta$  is the moving average parameter governing duration of the transitory shock. This choice as the best RIP specification finds empirical support, and is discussed further in section 6.3. I make the usual assumptions that  $\epsilon_{it}$  and  $\zeta_{it}$  represent genuine time-t innovations to the household and that households can perfectly distinguish transitory from persistent shocks.

An important point to note is that, in this paper we assume this heterogeneous trend is known to the household, so doesn't affect consumption growth at all. When Guvenen2010 estimate how much households learn about their individual growth they find very low learning and conclude that assuming that households completely foresee their own trend is a good approximation to the true process. Of course households may also get advance information about near-term fluctuations. As kaplan2008much discuss, the estimator under the null is robust to advance information of this form.

To sum up, under the RIP specification then  $Var\beta_i = 0$  and  $\rho = 1$ . The HIP specification could be

completely general but commonly imposes  $\theta = 0$ . The general model is similar but not identical to those used by e.g. MoffittGott2002, a commonly used specification in the labour literature. Such models are less used in the macro literature because of their complexity. When testing the processes, I call the RIP model the null and the HIP model the alternative model.

As a final point, note that the HIP specification has more parameters than RIP. Therefore, it may seem that the test is loaded in favour of the HIP process. Nevertheless, given that several authors (notably MaCurdy1982 and hryshko2008rip) have concluded in favour of the RIP, and that the unit-root process is a workhorse of quantitative work, a rejection of RIP, even conditional on the consumption model, is a noteworthy result.

# 3.2 Transmission coefficients

The analysis centres around the transmission of the shocks  $\zeta$  and  $\epsilon$  through to consumption. In line with kaplan2008much let  $x_{it}$  be a shock to log income (of either type). I define the transmission of  $x_{it}$  into log non-durable consumption (given by  $c_{it}$ ) as  $\lambda_{x_t} \equiv \frac{\text{Cov}(\Delta c_{it}, x_{it})}{\text{Var}(x_{it})}$ .

# 3.3 Solution for consumption

I now derive an approximate solution to the standard household's optimization problem along the lines of blp2004income. Defining  $c_{it}$  to be household log consumption, net of predictable components (depending mainly on demographic variables), then appendix B shows that the approximate solution for observed consumption changes is:

$$\Delta c_{it} \approx \Gamma_t + \pi_{it} \left( h_{\rho t} \zeta_{it} + \alpha_t \epsilon_{it} \right) + \xi_{it} + \Delta \nu_{it}$$
(3)

where  $\Gamma_t$  is a constant reflecting saving due to the discount rate, interest rates and the precautionary motive, and is constant across households within the cohort.  $h_{\rho t}$  captures the annuitization value of persistent income shocks. It is not observable but can be theoretically characterized. It depends on the interest rate, growth in incomes, and, in particular on the degree of persistence of the shock, and the age of the household (which we capture here by t).  $\pi_{it}$  captures the transmission of permanent shocks into consumption.  $\pi_{it}\alpha_t$  captures the transmission of transitory shocks into consumption, and  $\xi_{it}$  is an idiosyncratic shock to consumption due to, say, idiosyncratic portfolio returns.  $\nu_{it}$  is measurement error; here it is modeled as classical, but we could, for example, impose an MA(1) structure.

The income process and equation 3, together with the assumptions that all shocks are uncorrelated and unforeseen, provide all the covariance restrictions for growth moments implied by the model. The covariance restrictions are most accessibly summarized in tables 1 and 2. The vertical axis of table 1

gives lagged and current consumption and income changes, while future and current consumption and income changes are given on the horizontal axis. Table 2 goes into more detail with the covariances of consumption changes with current and future income changes. The covariance of consumption changes with past income changes is zero, while the autocovariance of consumption changes is non-zero only at one lag/lead; and then only because of measurement error. All covariances of variables at more than two periods' distance are zero under the RIP process. Importantly, in contrast, the covariance of consumption changes with future income changes is always non-zero when  $\rho < 1$  because a current shock to income implies mean reversion stretching into the future.

Table 1: Theoretical Consumption and Income Covariance Moments: Part a

	$\Delta c$	$\Delta c_{+1}$	$\Delta y$ $\Delta y_{+1}$
$\begin{array}{c} \Delta c \\ \Delta c_{-1} \end{array}$	$\pi^2 h_\rho^2 \sigma_\zeta^2 + \psi^2 \sigma_\epsilon^2 + \sigma_\xi^2 + 2\sigma_\nu^2 - \sigma_\nu^2$	$-\sigma_{\nu}^{2}$ $0$	See Table Part b
$\Delta y$	$\phi h_{\rho} \sigma_{\zeta}^2 + \psi \sigma_{\epsilon}^2$	0	$\frac{\frac{2\rho}{1+\rho}\sigma_{\zeta}^{2} + g(\theta)\sigma_{\epsilon}^{2} + \sigma_{\beta}^{2}}{-\frac{(1-\rho)^{2}}{1-\rho^{2}}\sigma_{\zeta}^{2} - (1-\theta)^{2}\sigma_{\epsilon}^{2} + \sigma_{\beta}^{2}}{-\frac{\rho(1-\rho)^{2}}{1-\rho^{2}}\sigma_{\zeta}^{2} - (1-\theta)^{2}\sigma_{\epsilon}^{2} + \sigma_{\beta}^{2}}$ $-\frac{\rho(1-\rho)^{2}}{1-\rho^{2}}\sigma_{\zeta}^{2} - (1-\theta)^{2}\sigma_{\epsilon}^{2} + \sigma_{\beta}^{2}$
$\Delta y_{-1}$	0	0	$-\frac{(1-\rho)^2}{1-\rho^2}\sigma_{\zeta}^2 - (1-\theta)^2\sigma_{\epsilon}^2 + \sigma_{\beta}^2 \qquad -\frac{\rho(1-\rho)^2}{1-\rho^2}\sigma_{\zeta}^2 - \theta\sigma_{\epsilon}^2 + \sigma_{\beta}^2$

Notes:  $\pi$  captures the share of human capital wealth in life-time wealth

 $\sigma_{\zeta}^2$  is the variance of persistent shocks,  $\sigma_{\epsilon}^2$  the variance of transitory shocks

 $\sigma_{\varepsilon}^2$  is the variance of heterogeneous growth on consumption

 $\sigma_{\nu}^{2}$  is the variance of measurement error on consumption

 $\theta$  is the MA(1) coefficient

 $h_{\rho}$  is the annuity value of persistent income shocks.

I define  $g(\theta) \equiv 2(1 - \theta + \theta^2)$  to save space in the table

Table 2: ...Theoretical Consumption and Income Covariance Moments: Part b

	$\Delta y$	$\Delta y_{+1}$
$\Delta c$	$\pi h_{\rho} \sigma_{\zeta}^2 + \psi \sigma_{\epsilon}^2$	$\pi h_{\rho}(\rho - 1)\sigma_{\zeta}^{2} - (1 - \theta)\psi\sigma_{\epsilon}^{2}$
$\Delta c_{-1}$	$\pi h_{\rho}(\rho - 1)\sigma_{\zeta}^{2} - (1 - \theta)\psi\sigma_{\epsilon}^{2}$	$\pi h_{\rho}(\rho - 1)\rho\sigma_{\zeta}^{2} - \psi\theta\sigma_{\epsilon}^{2}$

Notes: See table 1

Examining equation 3 in more detail, we can therefore think of  $h_{\rho t}\zeta_{it}$  as the shock to 'permanent' income. Or, alternatively, we can think of  $\pi_{it}h_{\rho t}$  as the approximate theoretical transmission coefficient on the persistent income shock. In other words, under the general model, as long as all shocks are uncorrelated then  $\lambda \approx \pi_{it}h_{\rho t}$ . Crucially  $h_{\rho t} < 1$  if  $\rho < 1$  and  $h_{\rho t} = 1$  if  $\rho = 1$ , i.e. if shocks are permanent.

Also note, in general, that  $h_{\rho t} > h_{\rho s}$  if t > s, i.e. the transmission of persistent shocks is higher the closer the household is to the end of working life. This is because a persistent labour income shock is similar to a permanent labour income shock near retirement. See the final part of appendix B for more details. Finally, I anticipate the discussion of the test in the next section by emphasizing that  $\pi_{it}$  can be computed directly from data on incomes and wealth.

#### The Test 4

In this section I describe how to test between the RIP and the HIP specifications. I first show that the RIP specification with self insurance imposes a simple moment restriction on the data. I then show that each of the components of the HIP specification that differ from the RIP causes a failures of this moment restriction. In addition, these failures are all in the same direction. Therefore, the data can clearly distinguish between the two processes.

#### The Estimator Under RIP 4.1

Under the null hypothesis of RIP with self-insurance, then  $h_{\rho t} = 1$  and the coefficient on the long-lived shock in equation 3 is  $\pi_{it}$ . Furthermore, under the RIP specification with an MA(1) transitory shock, and dropping subscripts:<sup>19</sup>

$$\operatorname{Cov}\left(\Delta y_{it}, \sum_{k=-2}^{2} \Delta y_{it+k}\right) = \sigma_{\zeta}^{2}$$

$$\operatorname{Cov}\left(\Delta c_{it}, \sum_{k=-2}^{2} \Delta y_{it+k}\right) = \pi \sigma_{\zeta}^{2}$$

$$\operatorname{Cov}\left(\Delta c_{it}, \sum_{k=-2}^{2} \Delta y_{it+k}\right) = \pi \sigma_{\zeta}^{2}$$

this strategy cleanly identifies the risk and transmission parameters. Therefore under the model in equation 3 and the null hypothesis we have the correspondence:

$$\pi = \frac{\operatorname{Cov}\left(\Delta c_{it}, \sum_{k=-2}^{2} \Delta y_{it+k}\right)}{\operatorname{Cov}\left(\Delta y_{it}, \sum_{k=-2}^{2} \Delta y_{it+k}\right)} \equiv \phi \tag{4}$$

Under the RIP model then the share of human capital in life-time wealth equals the panel income data moment restriction, which I define as  $\phi$  for consistency with kaplan 2008 much and BPP 2008.

<sup>&</sup>lt;sup>19</sup>The analysis can of course be adapted to an MA(q) process of any length. In this case the correspondence becomes, for example,  $\operatorname{Cov}\left(\Delta y_{it}, \sum_{k=-(q+1)}^{(q+1)} \Delta y_{it+k}\right) = \sigma_{\zeta}^{2}$ .

This moment in equation 4 is a natural choice to focus on, because, as kaplan 2008 much discuss, identification of the transmission coefficient on permanent shocks to income is best considered as a regression of  $\Delta c_{it}$  on  $\Delta y_{it}$ , with  $\Delta y_{it}$  instrumented by  $\sum_{k=-2}^{2} \Delta y_{it+k}$ . The strategy works because the instrument contains only the time-t permanent shock and other shocks that do not affect time-t consumption growth. Specifically, the instrument holds time-t transitory shocks constant. Of course I could also estimate the transmission coefficients by minimum distance using the scheme in tables 1 and 2, as in BPP 2008. Minimum distance estimation is less transparent, but I perform this procedure too and report results in section 6.

In short we have that, under the self-insurance RIP model:

$$\phi = \lambda_{\zeta} = \pi$$

As both  $\phi$  and  $\pi$  are estimable, this gives us two easy ways to identify the transmission of permanent shocks through to consumption.

## 4.2 The Estimator Under HIP

Under the more general HIP specification then the moments,  $\phi$  and  $\pi$ , need not be equal. To show this, first I consider the moments when there are heterogeneous trends only (and the idiosyncratic shock is permanent). Later I consider when there are no heterogeneous trends but there is an AR(1) process on the long-lived shock.

#### 4.2.1 Heterogeneous trends

When  $\rho = 1$  but  $\text{Var}\beta \neq 0$  then, by a simple Taylor-series approximation

$$\operatorname{Cov}\left(\Delta y_{it}, \sum_{k=-q}^{q} \Delta y_{it+k}\right) \approx \sigma_{\zeta}^{2} + (2q+1)\sigma_{\beta}^{2}$$

while

$$\operatorname{Cov}\left(\Delta c_{it}, \sum_{k=-2}^{2} \Delta y_{it+k}\right) \approx \pi \sigma_{\zeta}^{2}$$

$$\approx \lambda_{\zeta} \sigma_{\zeta}^{2}$$
(5)

Then our panel data estimator is given by:

$$\frac{\operatorname{Cov}\left(\Delta c_{it}, \sum_{k=-2}^{2} \Delta y_{it+k}\right)}{\operatorname{Cov}\left(\Delta y_{it}, \sum_{k=-2}^{2} \Delta y_{it+k}\right)} = \frac{\pi \sigma_{\zeta}^{2}}{\sigma_{\zeta}^{2} + 5\sigma_{\beta}^{2}} < \pi$$

We can sum up this as follows:

$$\phi < \lambda_{\zeta} = \pi \tag{6}$$

i.e. the panel data estimator is biased downwards from the true transmission coefficient, but the transmission coefficient is given (approximately at least) by the human capital share. I demonstrate this result by computing values of  $\phi$ ,  $\lambda_{\zeta}$  and  $\pi$  from simulations according to an exact consumption and saving model which I solve numerically. The left-hand half of table 3 gives these moments when the heterogeneous trend is fixed to account for 0\%, then 40\%, then 80\% of total cross-sectional variance of income at age 65. In computing these moments I have lowered the variance of permanent shocks to keep the final variance of incomes constant across simulations. Table 4 lists the other parameters, such as the discount rate, used to generate these simulation results. The simulations in table 3 corroborate the approximate result given in equation 6, although for higher variances of the heterogeneous trend, the human capital share estimator seems to become biased upwards slightly from the true transmission.

#### 4.2.2Mean-reversion

Now consider the case when  $\rho < 1$  but  $Var\beta = 0$ . In this case it is harder to relate  $\phi$  to  $\pi$ . Nevertheless, by a Taylor-series expansion we have:

$$\operatorname{Cov}\left(\Delta y_{it}, \sum_{k=-2}^{2} \Delta y_{it+k}\right) \approx \left(1 - 3\left(1 - \rho\right) + \mathcal{O}\left(1 - \rho\right)^{2}\right) \sigma_{\zeta}^{2}$$

$$\operatorname{Cov}\left(\Delta c_{it}, \sum_{k=-2}^{2} \Delta y_{it+k}\right) \approx \pi h_{\rho t} \rho^{2} \sigma_{\zeta}^{2}$$

$$\operatorname{Cov}\left(\Delta c_{it}, \sum_{k=-2}^{2} \Delta y_{it+k}\right) \approx \pi h_{\rho t} \rho^{2} \sigma_{\zeta}^{2}$$
$$\approx \lambda \rho^{2} \sigma_{\zeta}^{2}$$

and therefore

$$\phi \equiv \frac{\operatorname{Cov}\left(\Delta c_{it}, \sum_{k=-2}^{2} \Delta y_{it+k}\right)}{\operatorname{Cov}\left(\Delta y_{it}, \sum_{k=-2}^{2} \Delta y_{it+k}\right)} \approx \pi h_{\rho t} \left(1 + (1 - \rho) + \mathcal{O}\left((1 - \rho)^{2}\right)\right)$$

For  $\rho < 1$  we have  $h_{\rho t} < 1$  but  $1 + (1 - \rho) + \mathcal{O}\left((1 - \rho)^2\right) > 1$ . Therefore  $\phi$  could by less than or greater than  $\pi$ . In practice  $h_{\rho t}$  declines away from 1 faster than  $\rho$  when  $\rho$  is less than, but close to, 1. Therefore, for  $\rho$  close to 1,  $\phi \approx \pi h_{\rho t} \equiv \lambda < \pi$ . The performance of the estimator is discussed similarly

by kaplan 2008 much. We can summarize this relationship under mean reversion as follows:

$$\phi \approx \lambda_{\zeta} < \pi$$

Again, I demonstrate this result by computing values of  $\phi$ ,  $\lambda_{\zeta}$  and  $\pi$  from simulations according to an exact consumption and saving model, solved numerically. The right hand side of table 3 gives the value of  $\phi$ ,  $\lambda_{\zeta}$  and  $\pi$  for various choices of  $\rho$ . Again, table 4 lists the other parameters, such as the discount rate, used to generate these simulation results. Note in particular that throughout these simulations, the variance of long-lived shocks is held constant at 0.025 per year.

Table 3: Transmission Coefficients in Simulated Alternative Models

			Het'geneous Trends		AR(1): $\rho =$				
Parameter	Description	Age	0%*	40%*	80%*	0.97	0.95	0.9	0.8
$\phi \ \lambda \ \pi$	Estimator True transmission Human capital share	30	0.86 0.88 0.91	0.85 0.91 0.94	0.66 0.89 0.97	0.64 0.61 0.95	0.52 0.50 0.96	0.34 0.32 0.97	0.19 0.17 0.98
$egin{array}{c} \phi \ \lambda \ \pi \end{array}$	Estimator True transmission Human capital share	55	0.48 0.53 0.57	0.50 0.59 0.63	0.43 0.62 0.70	$\begin{array}{ c c } \hline 0.50 \\ 0.52 \\ 0.64 \\ \hline \end{array}$	0.48 0.49 0.66	0.41 0.41 0.68	0.30 0.28 0.70

Notes: See text for more details of the simulations.

Table 4: Parameter Values for Ta	ble <mark>3</mark>
Description	Value
Variance of permanent shocks	0.015
Coefficient of relative risk aversion	1.5
Interest rate	0.03
Discount rate	0.971
% Growth in incomes p.a.	0.5
Working life (number of periods)	45
Retirement length	10

Notes: One period is one year.

To conclude, the key point to notice is that under either alternative, that  $\rho < 1$  or  $Var\beta > 0$ , then

<sup>\*</sup> This %age gives the share of the variance of income at age 65 that is due to the heterogeneous trend.

 $\phi < \pi$ . This provides the test of the RIP model. Only under the null of RIP is  $\phi = \pi$ .

As a final point, note that I have excluded borrowing constraints in the model. Guvenen2010, on the other hand, model borrowing constraints carefully. Borrowing constraints can bind more often in the HIP model than in the RIP model, because, for example, if a household foresees that its income will grow particularly fast in the future, then it will want to borrow. Nevertheless, kaplan2008much simulate the transmission of income shocks under non-zero borrowing constraints for both the mean-reverting process and heterogeneous trends. They find that the presence of borrowing constraints makes little difference to the true transmission coefficients nor the estimator when income is mean reverting. On the other hand, they find that tight borrowing constraints can substantially bias the estimates of transmission of highly persistent shocks. However, the bias implies that the estimated transmission coefficients are much higher than the true coefficients. The bias therefore doesn't weaken my present argument. I also note that my study is over 1991-2006, a period of particularly loose borrowing restrictions.

# 5 Implementing the Estimator

# 5.1 Using Food Expenditures to Infer Consumption Choices

I do not observe total non-durable consumption, only food consumption. In order to make inference about the response of non-durable consumption to shocks, I form a measure of 'adjusted' food as follows. I begin with a simple specification for food demand:

$$f_{i,t} = \mathbf{W}'_{i,t}\mu + \mathbf{p}'_{t}\Theta + \beta(q_{i,t})c_{i,t} + e_{i,t}$$
(7)

where  $\mathbf{W}_i$  is vector of household fixed effects,  $\mathbf{p_t}$  is a vector of prices,  $\mu$  and  $\Theta$  are vectors of coefficients.  $\beta_{q_{i,t}}$  is the income elasticity of demand for food, for group q, to which household i belongs.  $e_{i,t}$  is an error term uncorrelated with total consumption and reflecting, for example, taste shocks. Appendix A discusses estimation of this equation and gives specification tests. The income elasticity is estimated to be around 0.4 for all relevant groups, principally those separated by age/cohort and education. Using 7 we can define 'adjusted' food as:

$$\tilde{f}_{i,t} = f_{it} - \mathbf{W}'_{i,t}\mu + \mathbf{p}'_{t}\Theta$$
$$= \beta (q_{i,t}) c_{i,t} + e_{i,t}$$

If we assume that the income elasticity does not vary much between consecutive years, then for a group with the same value of  $q_{i,t}$ , and hence the same income elasticity of demand:

$$\Delta c_{i,t} \approx \frac{1}{\beta_{q,t}} \left( \Delta \tilde{f}_{i,t} - \Delta e_{i,t} \right)$$

I use these equations to translate the moments in tables 1 and 2 into moments of food changes. I now absorb variation in taste for food  $(e_{i,t})$  into measurement error  $(\nu_{i,t})$ . The non-zero moments on the left-hand side of the table, for example, then become:

$$\operatorname{Var}(\Delta \tilde{f}_{qt}) = \phi^2 \beta_{q,t}^2 \sigma_{\zeta}^2 + \psi^2 \beta_{q,t}^2 \sigma_{\epsilon}^2 + \sigma_{\xi_{q,t}}^2 + 2\sigma_{\nu_{q,t}}^2$$

$$\operatorname{Cov}(\Delta \tilde{f}_{qt}, \Delta \tilde{f}_{qt+1}) = -\sigma_{\nu_{q,t}}^2$$

$$\operatorname{Cov}(\Delta \tilde{f}_{qt}, \Delta y_t) = \phi \beta_q \sigma_{\zeta}^2 + \psi \beta_q \sigma_{\epsilon}^2$$

for group indexed by  $q.^{20}$ 

This method is styled on and closely relates to that used by BPP2008 (henceforth BPP). It contrasts with other methods of imputing total consumption such as skinner 1987 superior, who regresses consumption on observable features (such as food and durables) that are present in both the panel and the crosssection, and ziliak 1998 does, who uses income and changes in wealth to calculate consumption as a residual. To give further explanation for my treatment of the data it is useful to compare it in detail to BPP's treatment. BPP translate food demands into non-durable consumption by fully inverting equation 7. BPPImput2004 show that this procedure preserves the mean of non-durable consumption and replicates the time-series of the variance up to an intercept shift. I do not replicate this procedure because my definition of food has a far lower income elasticity and so the denominator in the inversion is much closer to zero. When I invert fully, the error in food demands  $(e_{it})$  is magnified by far more. The variance of changes in this imputed 'non-durable' consumption is implausibly large (around 0.4) and dwarfs that from income (around 0.1). However, my procedure ultimately has a similar effect to BPP's. The only substantive difference is that I cannot pool observations of households with different cross-sectional income elasticities of food demand. I can still deploy different elasticities across time when estimating a non-stationary model. And when I estimate on different groups (such as by cohort or education), I deploy different elasticities with each group. However, appendix A shows that the elasticity does not vary significantly across groups or over time.<sup>21</sup> The main drawback of my method is that I cannot assess external validity of the procedure by comparing the distribution of imputed consumption in the BHPS with that from the FES.

 $<sup>\</sup>overline{\phantom{a}^{20}\text{I}}$  can also vary the other parameters (such as  $\phi$ ,  $\psi$  etc) by group but I suppress these subscripts in the present discussion.

<sup>&</sup>lt;sup>21</sup>Time-variation in the elasticity is crucial to BPP's argument. That argue that assuming a constant elasticity implies an increase in insurance over time whereas, in fact, insurance stayed constant, while the elasticity varied over time. The evolution of the elasticity over time does not appear so important to my analysis.

In practice, when I remove the predictable components of consumption changes, as discussed in 3.1, I regress on a very similar vector of controls as in the demand equation.<sup>22</sup> Therefore I do not need to impute adjusted food as an intermediate step, but instead perform one regression on observed food demands. Nevertheless methodologically, my analysis is based around a demand specification. And to emphasize, I estimate a demand equation for food in appendix A in order to derive income elasticities.

# 5.2 Implementing the Instrumental Variables Estimator

The estimator of the transmission coefficients in equation 4 has many attractive properties: as well as being intuitive and transparent, as kaplan 2008 much discuss, the estimator on  $\phi$  is robust to advance information of one period. However I improve the estimator by making three adjustments in practice: First, I drop  $\Delta y_{it-2}$  and  $\Delta y_{it-1}$  from equation 5 and exploit that  $\text{Cov}(\Delta c_{it}, \Delta y_{it} + \Delta y_{it+1} + \Delta y_{it+2}) = \phi \sigma_{\zeta}^2$ . I do this primarily because this choice of moments is more efficient. The step is valid because the covariance of  $\Delta c_{it}$  with lagged income changes is zero under the PIH. On the other hand, this covariance is negative when there are liquidity constraints. But I can assess whether there are constraints by looking at the data. Furthermore, using  $\text{Cov}(\Delta c_{it}, \Delta y_{it} + \Delta y_{it+1} + \Delta y_{it+2})$  is robust to habit formation.<sup>23</sup> The choice makes no substantive difference because table 7 in the results section shows that  $\text{Cov}(\Delta c_{it}, \Delta y_{it-1})$  and  $\text{Cov}(\Delta c_{it}, \Delta y_{it-2})$  are insignificant and of opposite sign. Second, I adapt the estimator for the use of the unbalanced panel. When estimating the model I do not require that 6 years of consecutive observations be present. Therefore I identify  $\phi \sigma_{\zeta}^2$  as  $\sum_{k=0}^2 \text{Cov}(\Delta c_t, \Delta y_{it+k})$  and  $\sigma_{\zeta}^2$  as  $\sum_{k=-2}^2 \text{Cov}(\Delta y_t, \Delta y_{it+k})$  (ie. I take the summation outside the covariance operator). Third and finally, in my main estimation, I pool observations over all time periods. This yields reliable results because the income elasticity of food demands is almost constant over the period.

In summary, my estimators for  $\sigma_\zeta^2$  and  $\phi$  in terms of moments of adjusted food and income are:

$$\hat{\sigma_{\zeta}^2} = \sum_{k=-2}^{2} \hat{\text{Cov}}(\Delta y, \Delta y_{+k})$$
(8)

$$\hat{\phi} = \frac{1}{\hat{\beta}} \frac{\sum_{k=0}^{2} \hat{\text{Cov}} \left( \Delta \tilde{f}, \Delta y_{+k} \right)}{\hat{\sigma}_{\zeta}^{2}}$$
(9)

where the sample covariances are taken across individuals and time and  $\hat{\beta}$  is the average income elasticity across time for the relevant group. As mentioned above, I also estimate using other methods and report

<sup>&</sup>lt;sup>22</sup>I do not regress on price in this vector, but this is common across all households so has no effect on idiosyncratic variation.

 $<sup>^{23}</sup>$ Cov  $(\Delta c_{it}, \Delta y_{it-1})$  will be positive if it takes more than one period for consumption to respond fully to permanent income shocks.

the results in table 6.

It is worth discussing briefly the estimation of the transmission of transitory shocks. Likewise I identify the transmission of transitory shocks through the regression of  $\Delta c_{it}$  on  $\Delta y_{it}$ , instrumented by  $\Delta y_{it+1}$ . Mirroring the case for permanent shocks, variation in  $\Delta y_{it+1}$  induces change in time-t transitory income and holds fixed the time-t permanent income shock. The estimator is:

$$\hat{\psi} = \frac{1}{\hat{\beta}} \frac{\hat{\text{Cov}}\left(\Delta \tilde{f}, \Delta y_{+1}\right)}{\hat{\text{Cov}}\left(\Delta y, \Delta y_{+1}\right)}$$
(10)

Identification of the other parameters given in table 1 is less straightforward and requires minimum distance techniques. Of these, the variance of transitory shocks and the MA(1) coefficient can be identified through the income moments alone. The variance of other idiosyncratic shocks to consumption and measurement error on consumption, however, requires fitting the variance of consumption growth.

In summary, estimation of the transmission factors proceeds in the following distinct stages. First I estimate the food demand equation using the FES. Second, I regress food and income in the BHPS on vectors of controls to form residuals. These controls are: demographic characteristics of the household (the logs of number of adults, children under 4, children age between 5 and 11, and children aged between 12 and 18); educational attainment of the head interacted with year, regional dummies and a quartic in the head's age. Finally I estimate the parameters of interest using the covariance restrictions described in equations 8, 9 and 10.

# 5.3 Estimating the Share of Human Capital Wealth in Discounted Lifetime Wealth

The final component of my analysis is the computation of  $\pi$ , the share of human capital in life-time wealth.  $\pi_{it}$  is defined as:

 $\frac{\text{Discounted Labour Income}}{\text{Wealth} + \text{Discounted Labour Income}}$ 

for a household indexed by i at time t. Total household wealth is calculated as the sum of net housing wealth and financial wealth of the head and spouse. I then report versions of total wealth both including and excluding pension wealth. I do this because pension wealth is very illiquid and it is unlikely households can borrow against this in the case of an adverse shock. Moreover, on the practical side, I do not have data on pensions for 2005. I then compute expected future income by the following procedure. As in the rest of the analysis I restrict the sample to households headed by a couple, in order to eliminate multi-tax unit households. I then estimate permanent income by averaging income at time t-1, t and

t+1, to smooth measurement error and transitory shocks. I assume future net income grows at 0.5%pa until the head mandatorily retires at 65, then no labour income thereafter. This value for income growth seems sensible given the patterns shown in Attanasio2010a. I discount this income stream at an interest rate of 3%pa.

Of course, the results depend on these choices in computation. I therefore perform robustness checks against all the main assumptions; they change the results little. In particular, and most importantly, I try altering the discount rate. This is prompted by Kaplan2011, who discuss that estimates of human capital wealth are lower when the stochastic nature of earnings is taken into account: Correcting for income risk raises the effective discount rate, compared to using the straight-forward risk-free rate. This correction therefore drives down the estimate of  $\pi$ , and brings it closer to  $\phi$ . I approach this issue by retaining a constant discount rate but pushing it upwards. When using a constant income growth and discount rate the important statistic is the 'net' discount rate: the interest rate net of income growth. In section 6.1, therefore, I present additional results with a net discount rate of 7%pa, instead of the benchmark 2.5%pa. This corresponds, for example, to income growth of 0.5%pa and a gross discount rate of 7.5%pa. For young households in particular this value seems on the conservative side (ie counterfactually high) because their average income growth is far higher than my benchmark assumption. As discussed, this discount rate does not change the main result that RIP with self insurance is rejected for younger households.

# 6 Results

# 6.1 Results From the Pooled Consumption Model

Table 5 shows the components of the key test. The first column shows estimates of the transmission of persistent shocks ( $\phi$ ) first for the whole sample, then broken down by age group. The second column of table 5 shows estimates of the mean of  $\pi$ , the share of human capital in discounted total wealth. These estimates are computed including pension wealth, so assume that households can borrow against pension wealth if bad shocks strike. The third column of table 5 shows 95% confidence intervals around the central estimates of  $\phi$ , based on asymptotic standard errors. For the sample as a whole the confidence interval around  $\phi$  excludes the central estimate of  $\pi$ , whatever the definition of  $\pi$  chosen. On this basis we reject the null hypothesis of RIP with self insurance. Note that  $\pi$  is estimated extremely precisely, so for this discussion I ignore its sampling error. In short, this transmission of shocks must rule out the RIP because the RIP would imply failure of the budget constraint.

The next two rows give results for the sample of households with heads aged less than 45 and households with heads aged over 45. For the younger group, the estimate of  $\phi$  is lower than for the group as a whole

and the estimates of  $\pi$  are higher. The upper bound on the confidence interval for the younger group is even further away from the central estimate of  $\pi$  than for the sample as a whole. This provides strong evidence against the null of RIP and in favour of the HIP model.

Moving to the third row of table 5 we see that the estimate of  $\phi$  for the older group cannot be distinguished from  $\pi$ . This group therefore does not provide evidence against the null of RIP. Note from table 3 that, when mean reversion is most important in driving the lower transmission of shocks, the gap between  $\phi$  and  $\pi$  reduces with age. As discussed before, the effects of transitory, persistent and permanent shocks become more similar the closer the household gets to the end of career. Therefore, we might not expect the the test for the older group to have the most power.

The final two columns of table 5 show alternative measures of  $\pi$ . The fourth column shows estimates of the mean of  $\pi$  with pension wealth excluded. As discussed in section 5 the benchmark definition of total wealth includes human capital wealth, pension wealth, net housing wealth and financial wealth. Note that this benchmark definition of wealth is very broad. It may be more sensible to exclude pension wealth because it is unlikely that households can borrow against it, so it is unlikely to aid the smoothing of shocks. On this alternative definition,  $\pi$  is even larger and further away from  $\phi$ . For example, the estimate of  $\pi$  for younger households excluding pension wealth is 0.91. Finally, the last column shows estimates when the discount rate is varied. As discussed in section 5.3, the higher the rate used to discount future income, the lower the estimate of  $\pi$ . In this fifth column I use an extremely high net discount rate of 7% pa. This 'net' discount rate captures pure discounting net of income growth. The table shows that for the sample as a whole, the estimate of  $\pi$  now sits inside the confidence interval for  $\phi$ , though for the young sample, and even with this high discount rate,  $\pi$  is outside the confidence interval and RIP is still rejected.

BPP2008 also estimate  $\phi$  to be slightly less than either implied by financial wealth holdings or implied by simulations. They interpret their results in terms of 'partial insurance', i.e. extra insurance over and above that provided by self-insurance. I do not interpret the results in such a light, however, because the income definition used includes all measured contingent transfers, in particular all (public and private) transfers and gifts. Of course, such extra insurance may be important if, for example, measured income is a poor indication of access to resources provided by extended family networks. As meyer2003measuring argue, this may be especially true for the poorest households. As a related point, I do emphasize that my results are more generally contingent on the consumption model being correct.

Figure 2 shows the basic point in more detail. It shows a quadratic polynomial fit of estimates of the transmission of permanent income shocks. In the same graph I plot the age profile of  $\pi$ , the proportion of human capital wealth in lifetime wealth. As discussed above, in the simple self-insurance model, these asset moments provide a first-order theoretical approximation of the transmission coefficient. I present calculations of this asset moment both including and excluding pension wealth.

Table 5: Testing the Null Hypothesis of Restricted Income Process

	$\phi$	$\pi$	95% CI on $\phi$	$\pi_{wopen}$	$\pi_{hi disc}$
All ages	0.42(0.14)	$0.71 \ (0.004)$	$[0.15 \ 0.70]$	0.83 (0.003)	0.65 (0.004)
Young	0.35(0.19)	0.82 (0.003)	$[-0.03 \ 0.73]$	$0.91 \ (0.003)$	0.75(0.004)
Old	0.49 (0.19)	$0.58 \ (0.005)$	$[0.11 \ 0.86]$	$0.74 \ (0.005)$	$0.52 \ (0.006)$

Notes: Young are < 45, old are > 45.

Asymptotic standard errors in parentheses.

See text for details on computation.

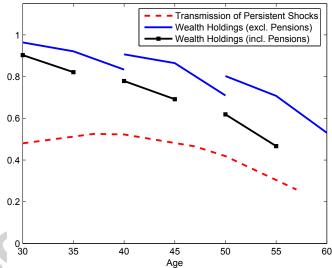
 $\phi$  is the estimated transmission of persistent shocks.

 $\pi$  is the mean share of human capital in life-time wealth including pension wealth.

 $\pi_{wo pen}$  is the mean share of human capital in life-time wealth excluding pension wealth.

 $\pi_{hi\,disc}$  uses future income discounted at 7% net. See text for more discussion.

Figure 2: Age Profile of Transmission Coefficients



Notes: 'Wealth Holdings' is the mean share of human capital wealth in total life-time wealth. These are calculated both including and excluding pension wealth. 'The Transmission of Persistent Shocks' is calculated as a quadratic fit through estimates for each cohort for the first and second halves of the sample period.

Table 6 shows estimation under alternative econometric specifications. In addition I show estimates of the transmission of the transitory shock, which are all indistinguishable from zero. The first row shows my benchmark results. The second row shows the results from using the unadapted IV estimator. The estimate of the transmission of shocks for the whole sample is almost identical and the confidence interval still does not overlap with the interval around  $\pi$ . The estimates for the specific age groups are more different and the standard errors are larger, particularly for the young group. For both groups the 95% confidence interval for  $\phi$  overlaps with that for  $\pi$  though only just for the young group. Taking the

definition of  $\pi$  without pension wealth a test of equality of  $\pi$  and  $\phi$  has a p-value of 0.057. The bottom row shows minimum distance estimates under the null specification as in BPP2008. The estimates here are very similar to the benchmark estimates and much more precisely estimated. For both the whole sample and the young group,  $\phi$  and  $\pi$  can be clearly distinguished. I rely less on these estimates, however, because identification is less transparent and it is more difficult to describe the behaviour of the estimator under failure of the null.

Table 6: Transmission of Shocks Under Alternative Estimators

	$\phi$	$\phi_{young}$	$\phi_{old}$	$\psi$
benchmark estimator	0.42	0.35	0.49	0.01
	(0.14)	(0.19)	(0.19)	(0.05)
standard IV	0.46	0.51	0.37	0.01
	(0.14)	(0.21)	(0.19)	(0.05)
minimum distance	0.37	0.31	0.44	0.01
	(0.09)	(0.12)	(0.12)	(0.03)

Notes: Asymptotic standard errors in parentheses.

# 6.2 The Covariance Structure of Food and Income Changes

In order to understand the results better it is helpful to look at the panel data moments more closely. The key moments are presented in table 7 in two columns. The left hand column shows the covariances of changes to residual food expenditure with changes to residual disposable income. I now discuss these to assess the basic consumption model and compare it to some simple alternatives. The 3rd row of the left-hand column shows the contemporaneous covariance between consumption and income changes. This covariance is significantly positive indicating that income changes do indeed have traction on food expenditure. The 1st and 2nd rows show the covariances of consumption changes with lags of income changes. These cannot be distinguished from zero, in line with the theoretical counterparts under the permanent income hypothesis.<sup>24</sup>

Rows 4 and 5 of table 7 show the covariances of consumption changes with leads of income changes. These moments in principal can be used to discern between the alternative models. Under the RIP-self insurance model the theoretical covariances corresponding to these rows are negative but likely very small because, say, a positive MA(1) transitory shock induces a small increase in consumption at time t,

 $<sup>^{24}</sup>$ The empirical moment  $\text{Cov}\left(\Delta \tilde{f}, \Delta y_{-1}\right)$  can therefore be used to test two main alternative models. Under the alternative hypothesis of excess sensitivity due to, say, liquidity constraints, this moment should be negative. (See Flavin, 1981). Under the alternative hypothesis of habit formation, this moment should be positive, because consumption takes more than one period to adjust to a permanent income shock. The empirical covariance is insignificant, indicating that neither effect is present and dominant.

then disappears at t+1 and t+2. In the HIP model the negative covariance should be stronger because, say a positive persistent shock induces induces a larger increase in consumption at time t. The empirical moments  $\text{Cov}(\Delta f_t, \Delta y_{t+1})$  and  $\text{Cov}(\Delta f_t, \Delta y_{t+2})$  are insignificant and quantitatively not different from zero. Note however, that when  $\rho$  is close to 1, the data are unlikely to have enough power to discern between the models. This implies that these moments cannot be used alone to detect mean reversion.

The income moments in the right hand column of table 7 display the classic features of the null permanent-transitory model in the RIP. All autocovariances are significantly different from zero, except for the third lag: the key signature of a permanent and MA(1)-transitory process. This is the basis behind the test of MaCurdy1982 and Meghir2004 and the contention by, for example, Guvenen2009. I discuss this debate in more detail in section 6.3 where I examine all the higher-order autocovariances in more detail. But it seems that the autocovariance structure of income changes does not provide evidence alone against the null RIP model of income dynamics.

Table 7: Covariances of Residual Food and Income Changes

	$\Delta f$	$\Delta y$
$\Delta y_{-2}$	-0.0004	
	(0.0009)	
$\Delta y_{-1}$	0.0007	<b>70</b> -
	(0.0008)	
$\Delta y$	0.0033***	0.1110***
	(0.0008)	(0.0026)
$\Delta y_{+1}$	-0.0002	-0.0391***
4.6	(0.0008)	(0.0018)
$\Delta y_{+2}$	0.0002	-0.0064***
	(0.0009)	(0.0014)
$\Delta y_{+3}$		0.0017
		(0.0014)

Notes: Asymptotic standard errors in parentheses.

## 6.3 Results from the Income Data Alone

One motivation for this paper is that income data alone do not have enough power to discern between the RIP and HIP. However, it is worth investigating the income data in detail to put the results into context with the established literature. I first do this by examining the income data in differences, then in levels. By estimating a variety of income specifications, I show first that mean-reversion, and not heterogeneity in trends, likely explains the bulk of my findings from the consumption data. However, I argue, similarly to Guvenen2009 in the US, that heterogeneity in trends is important in explaining some features of the growth in income inequality.

## 6.3.1 Estimation Using First Differences

As discussed, it has been usual, at least since MaCurdy1982, to test the income process by examining the autocovariance structure of residual income changes. These changes are shown by year up to the 3rd lag in table 8. The standard analysis given by, for example, Meghir2010 is that when income is mean reverting, the higher-order autocovariances should be negative. Here, the autocovariances at the 3rd lag are indistinguishable from zero. Moreover, the point estimate on the 3rd lag is greater than zero, providing further evidence against mean reversion. A chi-squared test that all the 3rd-order autocovariances are equal to 0 has a p-value of 0.41, so cannot be rejected. This finding is often used as prima facie evidence in support of RIP. My main results imply, however, that this prima facie evidence should not be taken as conclusive.

I now investigate what results we obtain by estimating two different models using these income data alone. These estimates are shown in the first columns of table 9. Across both models I assume that the variances and parameters are constant over the sample period and estimate by minimum distance using an identity weighting matrix. The first column shows estimates using the permanent and MA(1) transitory decomposition (the RIP process), using the insignificant autocovariance at the 3rd lag to suggest the MA(1) structure. In the second column I allow for an autoregressive component on the 'long-lived' shock, though I restrict  $Var(\beta)$  to be zero. In this case the estimated coefficient on  $\rho$  is not distinguishable from 1.

How do these estimates relate to the results using the consumption data? Focusing on the second column, the high estimate of  $\rho$  is often used as argument for using the unit root, permanent specification. However, as noted previously, even though  $\rho$  is close to 1, the point estimate implies that the transmission of shocks through to consumption is dramatically reduced. As implied by table 3, a coefficient of 0.95 implies a transmission of around 0.5 at age 30 under the chosen parametrization. This transmission is very close to the reduced-form estimates from the consumption data.

## 6.3.2 Estimation Using Data in Levels

The third column of table 9 shows results from the same model as used in column 2, but estimated using data in levels. The estimate of the variance of persistent shocks is half that in the second column. The estimate of  $\rho$  is similar to that in the second column though is estimated much more precisely. This evidence perhaps emphasizes that, when using income data alone it is preferable to use levels moments than first differences. Recall, however, that when using euler equations and consumption data, we must use first differences because only these moments relate to the model directly.

 $<sup>^{25}</sup>$ The autocovariances at lags higher than three periods, although not shown, are similarly indistinguishable from zero.

Table 8: Autocovariances of Income Changes

	0 lags	1 lag	2 lag	$3  \log$
1991	0.1266	-0.0529	-0.0114	0.0011
	(0.0117)	(0.0077)	(0.0062)	(0.0041)
1992	0.1042	-0.0256	-0.0078	0.0021
	(0.0092)	(0.0056)	(0.0052)	(0.0035)
1993	0.1063	-0.0485	0.0051	-0.0060
	(0.0089)	(0.0067)	(0.0038)	(0.0050)
1994	0.1033	-0.0312	-0.0058	0.0044
	(0.0083)	(0.0044)	(0.0057)	(0.0047)
1995	0.0872	-0.0327	-0.0018	0.0022
	(0.0077)	(0.0050)	(0.0037)	(0.0039)
1996	0.0951	-0.0331	-0.0082	0.0005
	(0.0074)	(0.0055)	(0.0044)	(0.0044)
1997	0.0927	-0.0245	-0.0111	0.0040
	(0.0075)	(0.0048)	(0.0041)	(0.0040)
1998	0.1129	-0.0508	-0.0126	0.0065
	(0.0098)	(0.0076)	(0.0052)	(0.0059)
1999	0.1102	-0.0286	-0.0036	-0.0030
	(0.0087)	(0.0059)	(0.0057)	(0.0049)
2000	0.1116	-0.0476	-0.0118	0.0079
	(0.0108)	(0.0090)	(0.0052)	(0.0062)
2001	0.1330	-0.0351	-0.0150	0.0068
-6	(0.0135)	(0.0085)	(0.0066)	(0.0054)
2002	0.1220	-0.0492	0.0007	-0.0063
	(0.0125)	(0.0074)	(0.0061)	(0.0060)
2003	0.1249	-0.0494	0.0007	
	(0.0107)	(0.0098)	(0.0063)	
2004	0.1369	-0.0477		
	(0.0149)	(0.0068)		
2005	0.1177			
	(0.0103)			
Notes	Agreementatio	atandand ann	ong in navont	hogog

Notes: Asymptotic standard errors in parentheses.

Table 9: Estimation from Various Income Models

	$  \sigma_{\beta}^2 = 0 \& \rho = 1$	$\sigma_{\beta}^2 = 0$	$\sigma_{\beta}^2 = 0$ in levels	No restrictions
$Var(\zeta)$	0.020 (0.003)	0.023 (0.004)	0.011 (0.001)	0.022 (0.003)
$\operatorname{Var}(\epsilon)$	0.052 (0.004)	$0.050 \ (0.005)$	0.072 (0.004)	0.057 (0.003)
heta	$0.123 \ (0.027)$	$0.113\ (0.034)$	$0.269 \ (0.028)$	0.138 (0.029)
ho		$0.952 \ (0.126)$	$0.955 \ (0.007)$	0.867 (0.016)
$Var(\alpha)$			0.104 (0.008)	$0.141\ (0.013)$
$Var(\beta) \times 100$				0.019 (0.005)
$Corr(\alpha, \beta)$				$0.723 \ (0.111)$

Notes: Columns 1 and 2 estimated using first differences. Columns 3 and 4 estimated using levels data. See text for more details. Estimation by minimum distance using identity weighting matrix. Asymptotic standard errors in parentheses.

- $\zeta$  is the persistent shock.
- $\epsilon$  is the transitory shock.
- $\theta$  is MA parameter.
- $\rho$  is the persistence parameter.
- $\alpha$  is initial (log) income.
- $\beta$  is the household-specific growth rate.

The fourth column of table 9 shows the estimates using the unrestricted HIP model. This model must be estimated using levels income data because the estimates using differences are very unreliable. This model implies a coefficient on  $\rho$  that is significantly less than 1 and a variance of the heterogeneous effect that is small but economically important in the evolution of inequality. The estimates of  $\sigma_{\zeta}^2$  and  $\rho$  imply that the persistent fluctuations contribute around 0.085 to the variance of log income in the long term. The correlation on the initial endowment  $\alpha$  and  $\beta$  is high but should perhaps be ignored because I pool data from all cohorts, so do not have data on the beginning of everyone's career.

How important are the heterogeneous trends for explaining the results from the consumption data? The estimates of  $\sigma_{\zeta}^2$ ,  $\rho$  and  $\sigma_{\beta}^2$  imply that heterogeneity biases down the estimate of the transmission coefficient,  $\hat{\phi}$ , by less than 10%. This implies that the main force pushing down  $\hat{\phi}$  is likely to be mean reversion.

#### 6.3.3 Examining Growth in the Cross-Sectional Variance

One main reason for modelling income processes is to capture key moments in the cross-section. Here I examine how well different models capture the evolution of the variance of (residual) log income over the life-cycle. These moments are plotted in figure 3 for three cohorts both over time and over age. The

data come from the FES because the sample size is larger, but the overall pattern is similar to that in the BHPS. The data are residuals after regressions on the standard observables for households headed by couples, split by the decade of birth of the head. Using a similar picture for the US, Heathcote2010 argue that the observed profiles and a pure RIP process cannot be reconciled. In short, and in the context of the present application, they argue that if the variance of (log) permanent shocks is estimated to be 0.02 (first column of table 3), then, the variance of log incomes should grow over a 30 year working life by 0.6. This growth is much larger than the observed growth in the variance, which here is around 0.15.

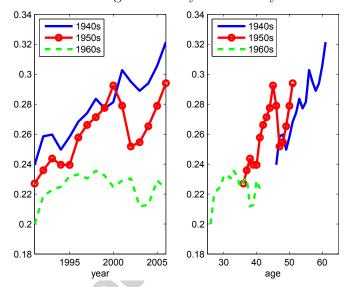


Figure 3: Variance of Log Income by Cohort: by Time and by Age

Notes: Data from the FES. Plots are partially smoothed using a 3-period moving average filter. Underlying data are residual log income for households headed by couples after regressions on age and education of head, region and household size.

How does the HIP resolve this disparity? First, mean reversion in the persistent process immediately brings down the growth in the income cross-section. To illustrate this point I reproduce the life-cycle profile in the left hand side of figure 4 and show on the right hand side results from model simulations. These simulations use an extra estimation performed on the panel data in levels split by the cohort bands. The first simulated line ('mean-reversion' only) shows that growth in the variance of income is brought down substantially compared to the unit-root specification and matches overall growth in inequality in the data. However, with mean reversion alone, all the growth in the cross-section comes at the beginning of the life-cycle. As found by Guvenen2009 for the US and also shown here, much of the growth in income inequality occurs towards the end of working life. The second simulated line ('HIP') shows that the heterogeneous trend is more successful in matching this aspect of the data. Therefore, although it seems that mean reversion is more important in explaining high-frequency consumption responses, heterogeneity in trends is vital in explaining the evolution of incomes.

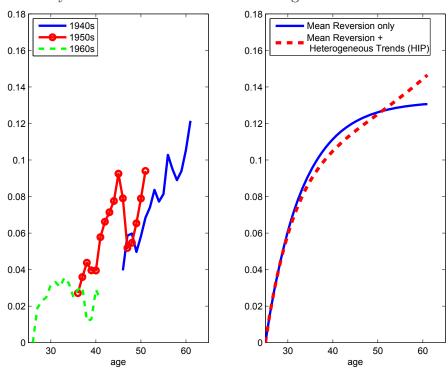


Figure 4: Life-cycle Growth in the Variance of Log Income: Data and Model Fit

Notes: Left hand plot is reproduction of right hand plot in figure 3 normalized to zero at age 26 to show growth. Right-hand plot shows simulations from model fit after estimation on autocovariances of levels data over years, split by cohort band. Estimation by minimum distance using an identity weighting matrix.

#### 6.4 Other Results

Table 10 presents estimates of the transmission parameters for different groups and for different income concepts. The first column shows the estimates of the transmission,  $\phi$ , of the persistent shock. The second column shows estimates of the transmission,  $\psi$ , of the purely transitory shock. The first row repeats the key estimates from table 6. The overall impressions from this table are as follows: first, the transmission of transitory shocks is economically close to zero and insignificant; second, the estimates of  $\phi$  don't vary much over separate groups and there are no significant differences between estimates. The second and third rows separate the sample by the head's education status. Rows four to six of table 10 show the estimates split of cohort: those born in the 1940s, then 1950s, then 1960s. Although the 1960s has the highest transmission coefficient, the point observed before still applies: I can't reject that the transmission is flat and less than 0.5 for all groups. The next two rows of table 10 show results when I split the sample period into two halves. The transmission of permanent shocks is again estimated imprecisely, but it seems, as in Blundell et al., 2008's analysis, that transmission is stable over the survey

<sup>&</sup>lt;sup>26</sup>I define high education as having A-levels or above. i.e. the head is educated until at least 18 years old. This comprises roughly half the sample.

period.

Table 10: Transmission Estimates: Breakdown by Sample

		$\phi$		$\psi$
Income - All groups	0.42	(0.14)	0.01	(0.05)
High Educ	0.30	(0.22)	0.04	(0.08)
Low Educ	0.51	(0.20)	0.01	(0.08)
Born in 1940s	0.43	(0.20)	0.02	(0.10)
1950s	0.25	(0.21)	-0.01	(0.07)
1960s	0.63	(0.33)	0.03	(0.11)
Early period	0.51	(0.20)	-0.01	(0.08)
Late period	0.37	(0.20)	0.03	(0.07)
Head Wage	0.34	(0.14)	0.07	(0.11)
Head Earnings	0.33	(0.15)	-0.07	(0.13)

Notes: Asymptotic standard errors in parentheses.

Var  $(\zeta)$  is the variance of permanent shocks.

The final row of table 10 shows the estimates when replacing household income with head wages and head earnings. The transmission coefficients should now be thought of as estimates from a factor model of consumption changes along the lines of altonji2002dynamic.<sup>27</sup> These estimates are worth looking at because most estimates of income dynamics examine the dynamics of the head wage or head earnings. These estimates imply that head labour supply varies little after shocks to head wages. The role of labour supply in the transmission of shocks is examined further by Blundell et al., 2012.

Finally, it is worth discussing the effect of income risk on consumption risk. This contribution explains the effect of income risk on the evolution of (consumption) inequality. By definition, the effect is given by  $\lambda_{\zeta}^2 \text{Var}(\zeta)$ . For the sake of the following discussion I assume that  $\lambda_{\zeta}$  can be approximated by  $\phi$ . This step is appropriate because, as discussed above, the estimates of  $\sigma_{\beta}^2$  imply that slope heterogeneity doesn't bias down estimates of  $\phi$  by too much. For the variance of income shocks, I take the evidence from table 9 that the estimate of the size of persistent income risk is around 0.02-0.025 across the majority of specifications. Using 0.0225 as an approximation of the variance of income shocks, then the contribution to consumption risk is  $0.42^2 \times 0.0225 = 0.004$ . This small figure is more in line with estimates of shocks to marginal utility given by growth in the cross-sectional inequality of consumption for fixed cohorts. BlundellEth2010 documents the growth of the variance of log consumption to be around 0.005. Note

 $<sup>\</sup>phi$  captures the transmission of persistent shocks.

 $<sup>\</sup>psi$  captures the transmission of transitory shocks.

<sup>&</sup>lt;sup>27</sup>In this case there is no underlying theoretical model of consumption because we are not closing the budget constraint.

that the contribution of income shocks to consumption risk must be less than total consumption risk because of other factors such taste shocks and wealth shocks.

# 7 Conclusions

In this paper, I test the nature of the income process in the UK over 1991-2006. I use data on income, food consumption and wealth from the British Household Panel Survey (BHPS). My main finding is that the consumption and wealth data imply rejection of the 'RIP' model and favour the 'HIP' model. To operate the test, I first estimate how much of long-lived income shocks transmit through to consumption. The estimation is similar to BPP2008 and kaplan2008much. The key idea behind the test is put simply as follows: under the RIP model, and with only self-insurance beyond measured transfers, permanent income shocks should transmit through to consumption almost completely. Specifically, these should transmit by as much as the share of human capital wealth in total discounted lifetime wealth. If, instead, long-lived shocks are mean-reverting then they transmit through to consumption far less. Similarly, if each household has its own specific growth path then the transmission estimator will be biased towards 0. I estimate the transmission of long-lived shocks to be 0.42 (42%), and only 0.34 for younger households. Therefore the HIP process is favoured.

In addition to identifying the income process, my results help explain the evolution of cross-sectional inequality. As discussed by Heathcote2010, it is difficult to reconcile the RIP model estimated in first differences with life-cycle growth in the variance of log incomes. However, the HIP model fits the moments. Moreover, although mean reversion is more important for explaining the joint consumption and income moments, explaining all the growth in the variance of log incomes requires not only mean reversion but also heterogeneity in trends.

Similarly, as BlundellEth2010 document, consumption inequality grew quite slowly over the period, implying that consumption risk was low. Meanwhile, across a variety of income specifications, I estimate the quantity of long-lived shocks from the income panel to be large. If consumption risk identifies permanent income risk then the cross-sectional evidence contradicts that from the income panel prima facie. But these two pieces of evidence are reconciled by the reduced transmission of shocks. Quantitatively, I estimate the variance of long-lived shocks to be around 0.02-0.025 (standard deviation of shocks of around 14-16% per year) across a variety of income models. The implied contribution of long-lived shocks to consumption risk is around  $0.42^2 \times 0.0225 = 0.004$  (standard deviation of 6.5% per year). This quantity lines up well with the estimate from the growth in consumption inequality of around 0.005 (7% per year).

How general are my results? I do not wish to claim that the HIP model is the appropriate one in all economies. I conjecture from my results that the income process may vary across time and across region.

The income models tested here are those used in the macroeconomics literature. They are chosen in part because they are parsimonious. In contrast, the models estimated in the labour economics literature, such as MoffittGott2002, are generally more complex. These models typically include all of a permanent component, a persistent but mean-reverting component and a heterogeneous growth component combined. Each component may become more prominent in different economies. For example, as BlundellEth2010 discuss, the UK economy in the 1990s experienced less structural change than in the 1980s. If so, the permanent component might have been more important in the 1980s than in the '90s. In turn, this implies that the transmission of shocks would be higher in the 1980s than in the '90s. Of course I cannot test this conjecture here and could not test it with any standard data set. Nevertheless, an important corollary of my results is that it is also important to consider how persistent income shocks are when modelling consumption responses.

This paper suggests several further strands of research. First, I note that estimates of consumption risk induced by income risk are more robust than estimates of income risk alone. The estimates of consumption risk are largely robust to the specification of the income process. Future research could follow this path, because quantifying consumption risk remains an important task in its own right.<sup>28</sup> Another interesting area of future research would be to assess the contributions from both the components of income and from other sources. Finally, it is worth repeating that this paper concerns only stable households headed by a couple. Non-stable households are probably more interesting, but are, of course, harder to study. Research on their behaviour and circumstances is needed.

<sup>&</sup>lt;sup>28</sup>For example, the level of consumption risk determines the optimal intertemporal savings distortion. See farhi2009capital.

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# A Using FES Expenditure Data

This appendix gives more details on data from the FES. First I give a brief description of food questions in the dataset, then I give further details of my demand estimation.

## A.1 Comparing BHPS and FES food consumption data

As mentioned in section 2, the food questions in the BHPS are based on recall. In contrast, the FES collects data on expenditure in a diary survey. Each household details all their spending, both home and abroad, over a two week period.<sup>29</sup> Several papers discuss the relative merits and characteristics of recall versus diary methods, such as Battistin2003. I include both food and groceries in 'food' because this gives a closer match to the BHPS data.

Table 11 shows the characteristics of the final samples in the FES and BHPS datasets, pooled across the first half and then the second half of the sample period. There are some levels differences between the datasets: notably, households in the BHPS appear to have more adults and fewer children than those in the FES. However, the trends from the first half to the second half of the period are similar for all measures across both datasets.

# A.2 Estimating the Food Demand Equation

My analysis requires a uniform income elasticity of demand across each group I study. A uniform income elasticity is a controversial claim. At a raw theoretical level, it is well known that the implied log-linear demand function fails adding up (Deaton and Muellbauer, 1980). More generally, most demand analyses estimate a concave elasticity (for example Browning and Meghir, 1991). Nevertheless, I present evidence that any non-linearity does not substantially affect the analysis.

Table 12 gives the results from the estimation of the main food demand equation. I instrument expenditure variables by log income and interactions to remove attenuation bias from measurement error. The main point of this regression is to back out income elasticities. The base elasticity is 0.38 for the low education group, younger than 45 years of age, in year 1991. I allow the elasticity to vary by education, age and allow for a linear effect across time. The coefficients on all these interactions are small and, except for education, insignificant at the 5% level. These estimates indicate that the income elasticity does not vary much across different parts of the income distribution.

<sup>&</sup>lt;sup>29</sup>In addition to this diary, household members perform an interview in which they are asked to recall expenditures on large infrequently-purchased items, such as cars.

Table 11: Comparison of Means, BHPS and FES

	1991-1998 BHPS	1991-1998 FES	1999-2006 BHPS	1999-2006 FES
head's age	40.9416	40.8875	46.7315	47.4973
hh size	3.472	3.411	3.375	3.208
Number of adults	2.416	2.199	2.497	2.228
Number of children	1.040	1.212	0.906	0.980
Compulsory level of education	0.453	0.506	0.346	0.484
Working	0.890	0.934	0.883	0.877
Retired	0.008	0.004	0.040	0.047
Other labour force status	0.101	0.061	0.077	0.076
0 cars	0.072	0.090	0.043	0.068
1 car	0.448	0.466	0.335	0.384
2 cars	0.401	0.369	0.481	0.438
>2 cars	0.078	0.075	0.139	0.109
Homeowner	0.826	0.799	0.869	0.844

Notes: Rows "Compulsory education" and below give proportions.

The means are simple pooled averages, unweighted by the sample sizes in each year.

The results don't change when I allow for a full set of interactions between expenditure and year. When I allow for a quadratic term in total expenditure (and keep the other interactions with the linear term), the coefficient on total expenditure squared is -.0505, and on total expenditure is 0.918, with standard errors of 0.015 and 0.18. The implied elasticity at the 10th centile of the expenditure distribution in 2000 is 0.43, and at the 90th centile is 0.29. The food demand equation does therefore display some curvature, but not much.

Another important consideration is the effect of participation on food demands. An effect of participation on the intercept has a large impact on the implied transmission of income through to consumption. This is because a large fraction of income variation comes through participation effects. If the coefficient on participation in the demand for food is, for example, negative then a positive income shock might imply a large increase in non-durable consumption, even if the increase in food demand is small. However, dummies for male and female participation have small coefficients, and are insignificant at the 10% level. The coefficient on female participation, for example, is -.007 with a p-value of 13%.

The analysis so far depends on the exogeneity of total expenditure. I test for endogeneity in the main equation (other than by measurement error) by including asset income and its interactions in the set of instruments. The exclusion of asset income in the determination of food demands is based on the two-stage budgeting framework. A Sargan test of the over-identifying restrictions has a p-value of

 $0.04\%^{30}$ , providing evidence of misspecification. The estimated income elasticity is slightly lower when instrumenting with asset income alone. The estimated elasticity is 0.37 for the base group in 1999 compared to 0.40 in the main equation. Allowing for joint determination of food and total expenditure, on the other hand, implies larger standard errors on the estimated transmission coefficient.

<sup>&</sup>lt;sup>30</sup>Chi-squared statistic of 20.65 with 4 degrees of freedom



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Variable	Estimate	Variable	Estimate	Variable	Estimate
ln c	0.376***	Age spouse <sup>3</sup>	-0.00176	yr = 1994	-0.0445*
	(0.0129)		(0.00125)		(0.0265)
$\ln c \times > 45$	0.00211	Age spouse <sup>4</sup>	9.07e-05	yr = 1995	-0.0660**
	(0.00178)		(6.81e-05)		(0.0330)
$\ln c \times High education$	0.00210**	Yorkshire	-0.00785	yr = 1996	-0.0541
	(0.000850)		(0.00798)		(0.0401)
$\ln c \times (year-1991)$	0.00263*	North West	-0.0252***	yr = 1997	-0.102**
	(0.00142)		(0.00914)		(0.0484)
$\ln p_{food}$	-0.711***	East Midlands	0.000188	yr = 1998	-0.130**
	(0.196)		(0.0103)		(0.0568)
ln Number Adults	0.435***	West Midlands	-0.0214**	yr = 1999	-0.154**
	(0.0127)		(0.00959)		(0.0649)
$\ln \# \text{ kids aged 0-4}$	0.213***	East Anglia	-0.0442***	yr = 2000	-0.169**
	(0.00676)		(0.00932)		(0.0742)
$\ln \# \text{ kids aged 5-10}$	0.198***	London	-0.0416***	yr = 2001	-0.224***
	(0.00586)	~'0	(0.00891)		(0.0809)
$\ln \# \text{ kids aged } 11\text{-}18$	0.250***	South East	-0.0552***	yr = 2002	-0.248***
	(0.00572)		(0.00937)		(0.0890)
Age head	0.0536***	South West	-0.0337***	yr = 2003	-0.252***
0	(0.0139)		(0.0109)		(0.0975)
Age $head^2$	-0.0116***	Wales	-0.0229**	yr = 2004	-0.285***
9	(0.00324)		(0.0110)		(0.106)
Age head <sup>3</sup>	0.000808***	Scotland	-0.00510	yr = 2005	-0.316***
	(0.000238)		(0.00943)		(0.115)
Age spouse	-0.0182	year = 1992	-0.0275**	yr = 2006	-0.322***
	(0.0238)		(0.0133)		(0.123)
Age spouse <sup>2</sup>	0.0108	yr = 1993	-0.0585***	Constant	0.275
	(0.00835)		(0.0192)		(0.272)
Observations	46,204				
R-squared	0.321				

Standard errors in brackets

Instrumented: ln c and interactions.

Instruments are: ln y and interactions.  $Age^2$  is divided by 10,  $Age^3$  by 100 and  $Age^4$  by 1000 for readability of coefficients

# B Approximating the Covariance Structure of Income and Consumption Changes

In this appendix, I derive an expression for changes to the covariance structure of consumption and income in the presence of income shocks of indeterminate duration. The proof follows that in \*blp2004income (henceforth referred to as BLP). My derivation is conceptually very similar and requires only minor technical changes. I give the derivation here in reasonable detail for completeness. I follow the following plan: first I sketch the key ideas; second I present a stripped down version of the model displayed in section 3, and finally I show that the mechanics of the derivation work in the same way to BLP while emphasizing the parts which differ.

#### B.1 Sketch Proof

The proof revolves around equating the consumption account and the income account of the distribution of (the log of) future life-time resources. To derive a relationship between the shocks to consumption and income I then take the following steps:

- 1. I take a Taylor-type expansion of the distribution of future resources around expected resources and period-by-period innovations.
- 2. By taking the difference between expectations at time t and t-1, I generate expressions for innovations to future resources first in terms of (percentage) consumption innovations, then in terms of (percentage) income innovations. To first order, the equality between the two takes a simple and attractive form.
- 3. Finally, BLP show that you can bound the size of the higher-order terms to show that the first-order terms can indeed be approximately equated.

#### B.2 The Model

I now specify a stripped-down version of the model used in section 3. In this subsection I surpress i subscripts for brevity.

Households are born at time t = 0, work until  $t = T_w$  and die at time t = T. The household maximises lifetime utility:

$$V_t(A_t, P_t) = \max_{\{C_k(A_k, P_k)\}_{k=t}^T} \mathbb{E}_t \left( \sum_{k=t}^T \beta^{k-t} \frac{C_k^{1-\gamma}}{1-\gamma} \right)$$

where  $\beta$  is a subjective discount factor, assumed to be common across households. I ignore deterministic changes to consumption needs here for simplicity. These could be re-introduced and would affect the (common) gradient on consumption growth.

We have the law of motion for assets and terminal condition:

$$A_{t+1} = \begin{cases} R(A_t - C_t) + Y_{t+1} & \text{if } t < T_w \\ R(A_t - C_t) & \text{if } t \ge T_w \end{cases}$$

$$A_{T+1} \ge 0$$

For clarity, we also distinguish between beginning-of-period assets  $A_t$  and end-of-period assets  $\mathbb{M}_t \equiv A_t - C_t$ , so that the law of motion, at time t, and before retirement can be written:

$$A_t = \mathbb{M}_{t-1}R + Y_t$$

The life-time budget constraint at time t can be written:

$$\sum_{s=0}^{T-t} \frac{C_{t+s}}{R^s} = \sum_{s=0}^{T_w-t} \frac{Y_{t+s}}{R^s} + \mathbb{M}_{t-1}R$$

Income has both a persistent and a transitory component. The persistent component evolves according to an AR(1) process:

$$\ln Y_t = g_t + \ln P_t + \epsilon_t$$

$$\ln P_{t+1} = \rho \ln P_t + \eta_{t+1}$$

$$\eta_t \sim N(0, \sigma_\eta^2) , \quad \ln P_0 \sim N(0, \sigma_\alpha^2) , \quad \epsilon_t \sim N(0, \sigma_\epsilon^2)$$

such that  $g_t$  is the deterministic component of income, (later assumed common across households with the same observable characteristics),  $\rho$  governs mean reversion as in the income process specified in section 3. In this exposition I have omitted the moving average component to the transitory shock. This could be re-instated and would affect the analysis little.

# B.3 An Approximate Consumption Growth Equation

With CRRA preferences, the standard arguments of log-linearization apply. I now write  $c_{it} \equiv \ln C_{it}$ . Re-instating i subscripts we have that the change to log consumption is approximately a martingale with drift:

$$\Delta c_{it} = v_{it}^C + \Gamma_t + \mathcal{O}\left(\mathbb{E}_{t-1}|v_{it}^C|^2\right) \tag{11}$$

 $v_{it}^C$  is the innovation to consumption. For CRRA preferences,  $\Gamma_t$  is constant across consumption levels and hence across consumers with the same preferences.

## **B.4** Approximating Lifetime Resources

As in BLP I define a function  $F: \mathbb{R}^{N+1} \to \mathbb{R}$  by  $F(\xi) = \ln \sum_{j=0}^{N} \exp(\xi_j)$ . By exact Taylor expansion around an arbitrary point  $\xi^0 \in \mathbb{R}^{N+1}$ 

$$F(\xi) = \mathcal{K} + \sum_{j=0}^{N} \frac{\exp \xi_{j}^{0}}{\sum_{k=0}^{N} \exp \xi_{k}^{0}} (\xi_{j} - \xi_{j}^{0})$$

$$+ \frac{1}{2} \sum_{j=0}^{N} \sum_{k=0}^{N} \frac{\partial^{2} F(\bar{\xi})}{\partial \xi_{j} \partial \xi_{k}} (\xi_{j} - \xi_{j}^{0}) (\xi_{k} - \xi_{k}^{0})$$
(12)

where  $\mathcal{K} = \ln \sum_{j=0}^{N} \exp \xi_j^0$  is constant.

#### B.4.1 Approximating the Consumption Account of Lifetime Resources

We now expand the consumption account of lifetime resources around  $\mathcal{K}_c = \ln \sum_{j=0}^{T-t} \mathbb{E}_{t-1} \frac{C_{it+j}}{R^j}$ , the logarithm of expected discounted expenditures. Again I write  $c_{it} \equiv \ln C_{it}$ . I define:

$$\xi_j = c_{it+j} - j \ln R$$
  
$$\xi_j^0 = \mathbb{E}_{t-1} c_{it+j} - j \ln R$$

Applying the approximation formula in 12, and taking expectations with respect to information set  $\mathcal{I}$ :

$$\mathbb{E}_{\mathcal{I}} \ln \sum_{j=0}^{T-t} \frac{C_{it+j}}{R^j} = \mathcal{K}_c$$

$$+ \sum_{j=0}^{T-1} \theta_{it+j} [(\mathbb{E}_{\mathcal{I}} c_{it+j} - \mathbb{E}_{t-1} c_{it+j})]$$

$$+ \mathcal{O}(\mathbb{E}_{\mathcal{I}} ||v_{it}^T||^2)$$

such that:

$$\theta_{it+j} = \frac{\exp[\mathbb{E}_{t-1}c_{it+j} - j\ln R]}{\sum_{k=0}^{T-t} \exp[\mathbb{E}_{t-1}c_{it+k} - k\ln R]}$$

are the shares of discounted consumption in total lifetime consumption and  $\sum_{j=0}^{c-t+T} \theta_{it+j} = 1$  and  $v_{it}^T$  is the vector of future innovations to consumption. One intuitive interpretation for this expression is

as follows: discounted future consumption is given by expected discounted future consumption plus the percentage deviations of consumption in each period weighted by their share in the whole.

#### B.4.2 Approximating the Income Account of Lifetime Resources

Similarly to above, we now expand the income account around  $\mathcal{K}_y = \ln \sum_{j=0}^{T_w-t} \mathbb{E}_{t-1} \left[ \frac{Y_{it+j}}{R^j} + A_{it} \right]$ , the logarithm of expected discounted incomes. I write  $y_{it} \equiv \ln Y_{it}$ . Letting  $N = T_w - t + 1$ , I define:

$$\xi_j = y_{it+j} - j \ln R$$
  

$$\xi_j^0 = \mathbb{E}_{t-1} y_{it+j} - j \ln R$$
  

$$\xi_N = \xi_N^0 = \ln A_t$$

Applying the approximation formula in 12, and taking expectations with respect to information set  $\mathcal{I}$ :

$$\mathbb{E}_{I} \ln \left( \sum_{j=0}^{T_{w}-t-1} \frac{Y_{it+j}}{R^{j}} + \mathbb{M}_{t-1} R \right)$$

$$= \mathcal{K}_{y}$$

$$+ \pi_{it} \sum_{j=0}^{T_{w}-t-1} \alpha_{t+j} [(\mathbb{E}_{I} y_{it+j} - \mathbb{E}_{t-1} y_{it+j})]$$

$$+ (1 - \pi_{it}) [\mathbb{E}_{I} (\ln A_{it}) - \mathbb{E}_{t-1} (\ln A_{it})]$$

$$+ \mathcal{O}(\mathbb{E}_{I} || \nu_{it}^{R} ||^{2})$$

where  $\nu_{it}^{R}$  is the vector of future innovations to income and:

$$\alpha_{t+j} = \frac{\exp[\mathbb{E}_{t-1}y_{it+j} - j\ln R]}{\sum_{k=0}^{T_w - t - 1} \exp[\mathbb{E}_{t-1}y_{it+k} - k\ln R]}$$

$$\pi_{it} = \frac{\sum_{j=0}^{T_w - t - 1} \exp[\mathbb{E}_{t-1}y_{it+j} - j\ln R]}{\Lambda_{it}}$$

$$\Lambda_{it} = \sum_{j=0}^{T_w - t - 1} \exp[\mathbb{E}_{t-1}y_{it+j} - j\ln R] + \exp\mathbb{E}_{t-1}\ln(A_{it})$$

Intuitively,  $\alpha_{t+j}$  is an annuitization factor for income for which  $\sum_{j=0}^{T_w-t} \alpha_{t+j} = 1$ ,  $\pi_{it}$  is the share of human capital wealth in lifetime wealth, and  $\Lambda_{it}$  is total lifetime wealth. An intuitive explanation for these expressions is similar to that for consumption, except that the picture is complicated by financial wealth. Now, discounted future life-time income is given by expected future life-time income plus the percentage deviations of income in each period weighted by their share in the whole, and weighted by the share of life-time flow income in life-time wealth.

## B.5 Equating Innovations to the Consumption and Income Accounts

Due to the lifetime budget constraint, the distributions of income and consumption accounts can be equated with respect to any information set,  $\mathcal{I}$ . Applying the operator  $\mathbb{E}_t - \mathbb{E}_{t-1}$  to the consumption account:

$$(\mathbb{E}_{t} - \mathbb{E}_{t-1}) \circ \ln \sum_{j=0}^{T-t} \frac{C_{it+j}}{R^{j}}$$

$$= \sum_{j=0}^{T-t} \theta_{it+j} \left[ (\mathbb{E}_{t} - \mathbb{E}_{t-1}) \circ c_{it+j} \right] + \mathcal{O}(\mathbb{E}_{I} || v_{it}^{T} ||^{2})$$

$$= \sum_{j=0}^{T-t} \theta_{it+j} v_{it}^{C} + \mathcal{O}(\mathbb{E}_{I} || v_{it}^{T} ||^{2})$$

$$= v_{it}^{C} + \mathcal{O}(\mathbb{E}_{I} || v_{it}^{T} ||^{2})$$
(13)

Applying the operator  $\mathbb{E}_t - \mathbb{E}_{t-1}$  to the income account and rearranging:

$$(\mathbb{E}_{t} - \mathbb{E}_{t-1}) \circ \ln \sum_{j=0}^{T-t} \frac{Y_{it+j}}{R^{j}}$$

$$= \pi_{it} \sum_{j=0}^{T_{w}-t-1} \alpha_{t+j} [(\mathbb{E}_{t} - \mathbb{E}_{t-1}) \circ y_{it+j})] + (1 - \pi_{it}) [(\mathbb{E}_{t} - \mathbb{E}_{t-1}) \circ \ln A_{it}] + \mathcal{O}(\mathbb{E}_{I} || v_{it}^{T} ||^{2})$$

$$= \pi_{it} \sum_{j=0}^{T_{w}-t-1} \alpha_{t+j} [(\mathbb{E}_{t} - \mathbb{E}_{t-1}) \circ y_{it+j})] + \mathcal{O}(\mathbb{E}_{I} || v_{it}^{T} ||^{2})$$

$$= \pi_{it} (\sum_{j=0}^{T_{w}-t-1} \alpha_{t+j} \rho^{j} \eta_{it} + \alpha_{t} \epsilon_{it}) + \mathcal{O}(\mathbb{E}_{I} || v_{it}^{T} ||^{2})$$

$$= \pi_{it} h_{\rho t} \eta_{it} + \pi_{it} \alpha_{t} \epsilon_{it} + \mathcal{O}(\mathbb{E}_{I} || v_{it}^{T} ||^{2})$$

$$(14)$$

where  $h_{\rho t} < 1$  and:

$$h_{\rho t} = \sum_{j=0}^{T_w - t - 1} \alpha_{t+j} \rho^j$$

$$= \sum_{j=0}^{T_w - t - 1} \frac{\exp[\mathbb{E}_{t-1} y_{it+j} - j \ln R]}{\sum_{k=0}^{T_w - t - 1} \exp[\mathbb{E}_{t-1} y_{it+k} - k \ln R]} \rho^j$$

$$= \frac{1}{B_t} \sum_{j=0}^{T_w - t - 1} \frac{G_{t+j} P_{it-1}^{\rho^j}}{R^j} \rho^j$$

where  $B_t = \sum_{j=0}^{T_w - t - 1} \frac{G_{t+j} P_{it-1}^{\rho^j}}{R^j}$  is the sum of discounted future incomes. In general it is not possible to simplify the  $h_{ot}$  expression further. This is because, for example, the sequence of expected growth rates on income  $G_{t+j}$  can take any form, and because the series  $\sum_{j=0}^{T_w-t-1} \frac{P^{\rho^j}}{R^j} \rho^j$  cannot be simplified for general P. But if the household starts with central income such that  $P_{it-1} = 1$  and income has constant growth over the working career such that  $G_{t+j} = G_t g^j$ , then  $B_t = G_t \frac{\left(1 - \left(\frac{g}{R}\right)^{T_w - t + 1}\right)}{1 - \frac{g}{D}}$  and

$$h_{\rho t} = \frac{G_t}{B_t} \frac{\left(1 - \left(\frac{g\rho}{R}\right)^{T_w - t + 1}\right)}{1 - \frac{g\rho}{R}}$$

$$= \frac{\left(1 - \frac{g}{R}\right)}{\left(1 - \frac{g\rho}{R}\right)} \frac{\left(1 - \left(\frac{g\rho}{R}\right)^{T_w - t + 1}\right)}{\left(1 - \left(\frac{g}{R}\right)^{T_w - t + 1}\right)}$$

$$< 1 \qquad \text{if } \rho < 1$$

$$(15)$$

Putting together equations 13 and 14, and inserting into equation 11 gives:

$$\Delta c_{it} = \Gamma_t + \pi_{it} (h_{\rho t} \eta_{it} + \alpha_t \epsilon_{it}) + \mathcal{O}(\mathbf{E}_I || v_{it}^T ||^2)$$
er:
$$\Delta c_{it} \approx \Gamma_t + \pi_{it} (h_{\rho t} \eta_{it} + \alpha_t \epsilon_{it})$$

or approximating to first order:

$$\Delta c_{it} \approx \Gamma_t + \pi_{it} (h_{\rho t} \eta_{it} + \alpha_t \epsilon_{it})$$

as in equation 3. This expression interprets the shock to consumption in terms of the components of the income process.

#### Conditions Under Which $h_{\rho t}$ Increases With Age **B.6**

I discuss in section 3 that  $h_{\rho t}$  should increase with age in realistic situations. This claim is not true for all income processes: in some pathological cases  $h_{\rho t}$  may even decrease for some ages. This is the case if the sequence of expected incomes  $G_t$  oscillates over time. However, in realistic cases, it can be shown, albeit with the use of simulations, that  $h_{\rho t}$  increases with age.

If the household starts with central income,  $P_{it-1} = 1$ , and income growth and the interest rate are constant, then as shown above in equation 15:

$$h_{\rho t} = \frac{\left(1 - \frac{g}{R}\right)}{\left(1 - \frac{g\rho}{R}\right)} \frac{\left(1 - \left(\frac{g\rho}{R}\right)^{T_w - t + 1}\right)}{\left(1 - \left(\frac{g}{R}\right)^{T_w - t + 1}\right)}$$

where g is the income growth rate, R is the interest rate,  $T_w$  denotes the end of career,  $\rho$  is the coefficient of persistence, and t is the current time period. Clearly if  $\rho = 1$  then  $h_{\rho t} = 1$ . Figure 5 shows  $h_{\rho t}$  over the course of 20 time periods for various other parametrizations: for values of  $\rho \in \{0.95, 0.99\}$ , values of  $R \in \{1.005, 1.02, 1.04\}$ , and with g held fixed at 1.02. As the above equation shows, the relevant parameter is  $\frac{g}{R}$  so varying only one of numerator and denominator is necessary. Note that the combinations of parameters allow for every apparently important case:  $\frac{g}{R} < 1$  and  $\frac{g\rho}{R} < 1$ , or  $\frac{g}{R} > 1$  and  $\frac{g\rho}{R} < 1$ , or  $\frac{g}{R} > 1$  and  $\frac{g\rho}{R} > 1$ . Under all combinations,  $h_{\rho t}$  increases.

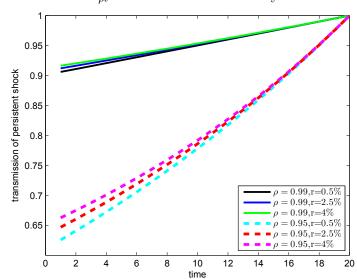


Figure 5: Simulated  $h_{\rho t}$  Over Time for a Variety of Parametrizations

Notes: See text for details on computation.

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# C Taking Midpoints of Food Consumption

The food data in the BHPS are a potentially valuable resource, but have not been used widely. An important contribution of this paper, therefore, is to demonstrate that these data do in fact convey useful economic information. In this section I assess the validity of my treatment of food expenditures as discussed in section 2. I compare to alternative treatments and argue that taking midpoints of consumption yields empirically accurate results. The argument I present has 2 strands: first I show that taking the midpoints corresponds empirically well to performing maximum likelihood estimation using the normal distribution and that the normal is the natural choice for this type of analysis. Second, I perform a validation exercise using PSID data, for which we have point observations of household expenditure. I show that banding these data, then using the midpoints makes little difference to estimates of the relevant variances and covariances.

## C.1 Analysis of the BHPS data

Taking midpoints of the food points is arbitrary and performed for convenience. An alternative is to specify the underlying distribution of expenditures and to estimate the 2nd moments using maximum likelihood. Here I specify an underlying normal distribution, joint across food and income and across time. This assumption has a theoretic rationale. The normal distribution is the natural choice because the maximum likelihood estimator for a cross section of continuous data is just the sample mean, sample variance and the sample correlation. Therefore taking the (non-parametric) covariance matrix of data is akin to estimating the covariance matrix by maximum likelihood under the assumption that the data are normally distributed.

The likelihood function used is

$$LL\left(\mu,\Sigma\right) = \sum_{i=1}^{n} \left(\Phi\left(\tilde{x}_{i}^{U}, \tilde{y}_{i}^{U}, \rho\right) + \Phi\left(\tilde{x}_{i}^{L}, \tilde{y}_{i}^{L}, \rho\right) - \Phi\left(\tilde{x}_{i}^{U}, \tilde{y}_{i}^{L}, \rho\right) - \Phi\left(\tilde{x}_{i}^{L}, \tilde{y}_{i}^{U}, \rho\right)\right)$$

where  $\mu$  is the (2x1) vector of means;  $\Sigma$  the covariance matrix;  $\Phi$  () is the bivariate standard normal cdf for observations ( $\tilde{x}_i, \tilde{y}_i$ ) with correlation coefficient  $\rho$ ;  $\tilde{x}_i = \frac{x_i - \mu_x}{\sigma_x}$ , and  $x_i^U$  and  $x_i^L$  are upper and lower limits of the band containing  $x_i$ . As the number of bands increases, in the limit the log likelihood tends towards the standard likelihood function, and the solution for  $\mu$  and  $\Sigma$  is the sample mean and variance as above. The cdf of the bivariate normal distribution, however, has no analytic expression. I therefore approximate it using the method in Owen1956.<sup>31</sup> The derivatives of the cdf can be expressed

<sup>&</sup>lt;sup>31</sup>This transforms computations of the bivariate normal cdf to a formula of two parameters. I then compute a table (2-dimensional grid) of values of the cdf using numerical integration and then compute all intermediate values using interpolation. It is easy to store enough data in the grid to leave the approximation error of the interpolation negligible.

analytically, however, so the optimization in the maximum-likelihood estimation relies mostly on precise analytical expressions.

The top panel of figure 6 shows the variance of changes in consumption using both midpoints and MLE. The MLE estimates are computed estimating the joint distribution of  $(c_t, c_{t+1})$ , and then using  $\operatorname{Var}(\Delta c_t) = \operatorname{Var}(c_t) + \operatorname{Var}(c_{t+1}) - 2\operatorname{Cov}(c_t, c_{t+1})$ . The figure shows that the variance of changes using the midpoints is slightly higher than using MLE. This is likely because taking midpoints induces extra measurement error. However, both sets of estimates have similar dynamics so it seems this extra measurement error is constant over time.

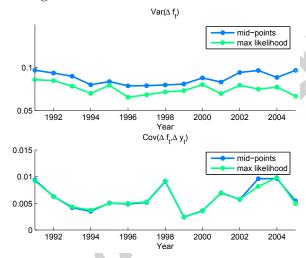


Figure 6: Estimating the Joint Distribution of Food and Income Changes

More importantly, the bottom panel of figure 6 shows the covariance of food changes with income changes. This is the key moment used in the main analysis. I estimate these by performing separate bivariate normal estimations for  $Cov(f_{it}, \Delta y_{it})$  and  $Cov(f_{it+1}, \Delta y_{it})$ , then subtracting. Here the MLE estimates are almost identical to the midpoint estimates. This is likely because the extra (non-standard) measurement error induced by assigning each band to its mid-point is orthogonal to measurement error in income.

Using the midpoints instead of the maximum likelihood estimates comes at no real loss of efficiency. In the first year of the survey, for example, the standard error of  $\text{Var}(\Delta f_{it})$  using the MLE (as derived using the inverse of the hessian) is 0.0031. When using the midpoints, the standard error is 0.0048. More importantly, the standard error on  $\text{Cov}(\Delta f_{it}, \Delta y_{it})$  from the MLE estimates, given by bootstrapped estimates, is almost identical to that when using the midpoints.

As discussed, the normal distribution assumption has a theoretical appeal. However for a finite number of bands, the accuracy of the method depends on the true distribution and it is important to quantify

the error under this approach. For this, I perform parallel computations with the most similar data set for which we observe the panel of food consumptions. For this we turn to the PSID data.

#### C.2 Validation from the PSID

We can test how close this estimator comes to the sample covariance for data distributed as usual by performing a validation exercise with other data sets. Here I pick food data from the Michigan Panel Study of Income Dynamics (PSID). The data were downloaded from the data archive for the BPP paper. The PSID is the standard dataset for studies of the present type. The reader can go to BPP for a description of the dataset.

I perform the following actions on the data. As in BPP I use only households for whom the head is born between 1919 and 1960. My definition of food is food in, to keep comparability with the BHPS data. To remove outliers I first remove households with an annual income less than \$10. I then trim the top and bottom 0.5% of the cross-sectional distributions of food and income. I also remove those observations for which the change in log food consumption is greater than 1.6 or less than -1.6. Like the main analysis, I do not perform this on the income data.<sup>32</sup> I assign expenditures to bands in the following way: I set thresholds for the top and bottom band each to capture 0.075% of the distribution, in line with the proportions in the BHPS. I then set the intervals at equal spaces in log space. The induced distribution of expenditures is similar to that in the BHPS; for example the modal band in both datasets captures around 25% of observations. I then assign midpoints as the geometric mean of the interval thresholds. For the top and bottom band I assign each observation so that all the observations are equally spaced. This assignment is, of course, entirely arbitrary, but in line with that from the BHPS. The results that follow are robust to other sensible assignments.

 $<sup>^{32}</sup>$ I do not perform regressions on household characteristics. These change the size of the variances and covariances, but likely do not affect the accuracy of the approximations, which depend on the shape of the joint distribution of income and food consumption. This joint distribution is not affected so much by the first-stage regressions.

Kernel density estimate

Normal density

kernel = epanechnikov, bandwidth = 0.1047

Figure 7: The Distribution of Food Consumption from the PSID

To assess normality in the underlying data, figure 7 shows kernel density estimates of the cross-sectional distribution and the distribution of consumption changes, accompanied by fitted normal distributions. The cross-sectional distribution is skewed with a long left-hand tail. The distribution of changes is symmetric but clearly leptokurtic.

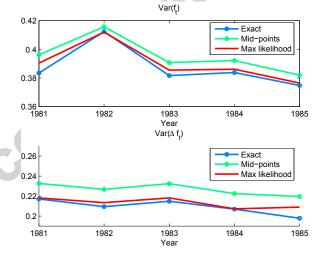


Figure 8: Estimating 2nd Moments of the Food Distribution in the PSID

The top panel of figure 8 shows estimates of the cross sectional variance of food expenditures using the exact data, the imposed midpoints, and maximum likelihood estimates using the imposed bands. I pick 1981-1985 as an example sub-period. Both the midpoints and the maximum likelihood estimates slightly overstate the variance, but they capture the dynamics well. The bottom panel of figure 8 shows estimates of the variance of changes of food expenditure using the three different methods. As for the BHPS, the estimates using mid-points are higher because of the extra measurement error. The

maximum likelihood is closer to the exact variance. Both the approximations (using the midpoint and the maximum likelihood) follow the dynamics of the PSID very closely.

 $Cov(\Delta f_{\downarrow}, \Delta y_{\downarrow})$ 0.024 Mid-points Max likelihood 0.022 0.02 0.018 0.016 0.014 0.012 0.01 0.008 L 1981 1982 1983 1984 1985

Figure 9: Estimating the Joint Distribution of Food and Income Changes in the PSID

Figure 9 shows the covariance of food changes and income changes using the different estimation methods. To repeat, this is the crucial moment for the identification of transmission parameters. Again the midpoints and the bands give almost exactly the same answer. They also capture the level and the dynamics of the precise estimates extremely well. The standard errors on the covariances are almost identical (at 0.0035 in 1981) when using either the precise observations or the midpoints. There is therefore no loss of efficiency when using the midpoints.

# C.3 Concluding Remarks

I conclude that taking midpoints of the banded food data yields empirically accurate results. As a final word I discuss further econometric alternatives. An obvious alternative when using banded data is to identify bounds on the relevant variances and covariances non-parametrically. The advantage of this approach is that it doesn't require imputing food data at all nor does it require placing parametric assumptions on the underlying distribution. Stoye2010, for example, discusses identification of spread parameters using (univariate) banded data. There are several problems with such an approach. First, the top and bottom bands in the data are unlimited. The variance is therefore unbounded without at least some minimal further restrictions on the distribution. Second, even with limits on the top and bottom band, the implied non-parametric bounds on the variance are quite large. They are derived by allocating the observations to the extremities of the observed bands which yield minimal and maximal

variance.<sup>33</sup> We know from all other datasets, however, that food expenditures are smoothly distributed. A simple bounds analysis therefore greatly overstates reasonable ignorance about the exact variance. A more sophisticated approach would allow for including statistical restrictions on the shape of the distribution. However, I know of no econometric theory developed in this area which would be suitable for the present study.

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<sup>&</sup>lt;sup>33</sup>For a univariate distribution the maximum bound is obtained by placing all observations furthest away from the mean band, and the minimum bound by placing all observations closest to the mean band.