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COMPETING FOR LOYALTY: THE DYNAMICS OF RALLYING SUPPORT

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Abstract. We consider a class of dynamic collective action problems in which either a single principal or two competing principals vie for the support of members of a group. We focus on the dynamic problem that emerges when agents negotiate and commit their support to principals sequentially. A danger for the agents in this context is that a principal may be able to succeed by exploiting competition among members of the group. Would agents benefit from introducing competition between opposing principals? We show that when principals' policies provide value to the agents, competition actually reduces agents' welfare. JEL codes: D70, D72, C78.

keywords: contracting externalities, political economy, corporate takeovers, vote buying.

1. Introduction

Collective action problems can make groups weak and ineffective. This is particularly problematic when an external principal can exploit the incentives of individual members to free ride on each other, leading the group to devastating outcomes with little compensation to only some of its members. The free-riding problem appears in central applications throughout economics, from corporate governance (corporate takeovers) to public economics (public good provision), political economy (vote buying), and industrial organization (exclusive deals).

Consider for example exclusive deals. In the classical version of this problem, an incumbent firm attempts to exclude rivals by signing exclusive contracts with buyers. While preventing entry hurts buyers, an incumbent using discriminatory offers can induce individual buyers to sign these contracts without being fully compensated, by turning buyers against one another (Rasmusen, Ramseyer, and Wiley Jr (1991), Segal and Whinston

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(2000)). But the incumbent can also achieve a similar result even without using complex contracts if it can contract with buyers sequentially. The key point of the previous papers, in fact, is that sequential contracting strengthens the incumbent's ability to exclude by exacerbating the free-rider problem among buyers.

A similar logic applies to vote buying. Dal Bo (2007) shows that a single principal can lead a group to her preferred decision at a negligible cost by using "pivotal" contracts, which allow the principal to commit to bribes that are conditional on the entire vote profile. But Rasmusen and Ramseyer (1994) and Genicot and Ray (2006) show that the principal can also exploit the group without using complex contracts if she can approach voters sequentially.

A common feature of the situations in which sequential contracting is pervasive is the irreversibility of agents' actions. In the exclusive deals setup, for instance, the irreversibility comes from the fact that buyers sign a contract with the incumbent. To varying degrees, this feature is at the core of a number of key problems in politics and economics. A party notable who publicly supports a presidential candidate cannot switch his support to the other candidate without facing a reputational loss. A firm that chooses between two alternative technologies (Blu-ray or HD-DVD, Boeing or Airbus) can only switch technologies at a substantial adjustment cost. A shareholder that sells its shares to a raider attempting a takeover simply cannot undo this action.

The reason why sequential contracting can be problematic for members of the group is that it can allow principals to create and exploit an intertemporal competition among members of the group: "take this now, otherwise I'll find others to support me and you'll get nothing". Suppose for example that a principal is trying to induce members of a legislature to vote for an unpopular fiscal austerity measure. A legislator might want to support the legislation even in exchange of a small benefit when the alternative is to get the outcome without receiving any compensation. The key point here is that a legislator's outside option declines in the number of individuals who have effectively been bribed (Genicot and Ray (2006)).

But what if two incumbent firms were to compete for exclusive contracts, or two lobbies were to compete for committee members' votes, or if the incumbent management were to fight a raider's attempt to take over the firm? Would agents benefit from competition between opposing principals in these contexts?

On the face of it, the answer to this question seems straightforward: since a single principal can exploit agents by picking the group apart – the logic goes – introducing competition must naturally increase agents' welfare. We show, however, that this intuition can be misleading. As it turns out, a key consideration is whether the principal's policy provides value to the agents or not relative to the status quo. In a *public good* environment, agents prefer the principal's policy to the status quo. This is the case of a presidential candidate with high changes of winning the general election, a party line favored by backbenchers, or a value-creating raider attempting a corporate takeover. In a *costly actions* environment, instead, the success of the principal leads to a loss for the agents. This can be the case with coalitions for war, exclusive deals, or unpopular legislative policies.

Our main result is that competition between principals generally improves agents' welfare with costly actions, but *reduces* agents' welfare with public goods. In fact, when two leaders stand behind equally good alternatives, agents are always better off when facing a single alternative than two alternatives. In general, with public goods, agents are always better off facing the best alternative by itself than two alternatives, even if the difference in the value of the two alternatives is arbitrarily small. Moreover, when the number of agents is sufficiently large, agents can be better off with a monopoly of the worst alternative than when this alternative competes with an arbitrarily better alternative. With costly actions, instead, the results reverse, and competition is indeed beneficial.

The superiority of monopoly to competition depends crucially on the nature of the free-riding effect. In a public good setting – as it is the case with an efficient raid – agents would benefit from not trading with the principal. In this context free-riding opportunities give agents bargaining power against the principal. We show that competition between principals reduces the value of free-riding in expectation, and thus diminishes the bargaining power of agents vis a vis the monopolistic setting.[1]

Consider the problem of the pivotal follower in each case. In the single alternative case, if the pivotal follower refuses an offer by the leader, there is a relatively high chance that some other uncommitted follower will come to the negotiating table tomorrow and the game will end. This relatively high free-riding ability gives the pivotal follower a considerable bargaining power against the leader and protects him from getting fully expropriated. Now consider the problem of a pivotal follower under competition. The number of uncommitted followers with competing principals is at most as large as in the single alternative case – if one principal was completely shut out – and is generally lower. In fact, if the battle between the leaders is very close, a follower could be pivotal when both leaders need only one additional commitment to win. But in this case there are no free riding possibilities, and as a result, the pivotal follower has no bargaining power. The example is extreme, but the logic is similar in all other cases in which a leader is

[1]With negative externalities (*costly actions* in our setup) competition is indeed beneficial to followers.

not shut out; competition between the leaders gives the pivotal follower fewer free-riding possibilities, and results in a lower equilibrium payoff for the followers.[2]

A concern in this context is the possible multiplicity of equilibria. We show that this is not an issue in our case, as our results holds for all Markov Perfect equilibria (MPE). In fact, we establish our main result using an induction argument that exploits general equilibrium properties.

In the main part of the paper we present a stripped down model that isolates the key aspects of the problem. In this benchmark model, followers' payoffs in terminal nodes are unaffected by whether they supported the eventual winner, the eventual loser, or remained uncommitted. An indirect consequence of this simplifying assumption is that transfers from principals to agents are positive in the costly actions model, but negative with public goods, a feature that can be unappealing in some applications. We show, however, that in an expanded version of the model where we allow payoffs to depend on whether each follower backed a winning or losing candidate, the direction of the cash transfer is unrelated to whether the decision involves a public good or a costly action, and, concomitantly, on whether competition is hurtful or beneficial for followers. In fact, under reasonable assumptions on parameters, most relevant applications involve positive transfers. We illustrate this with a modified version of the celebrated paper on corporate takeovers by Grossman and Hart (1980).

The reversal of the direction of the transfers does not alter the main free-riding logic at the core of our results. When the terminal payoff of an uncommitted follower is positive (public goods) meeting the leader is bad news for followers, and followers have an incentive to free ride. In the benchmark model this is just because of the cash transfer. In the expanded model this happens because of the change in total compensation, even when cash transfers go in the opposite direction. The key point that holds in all versions of the model is that competition hurts followers because it reduces free riding opportunities.

2. Literature Review

Our paper contributes to two strands of literature. One is the literature on *contracting with externalities*, developed largely within industrial organization. These papers explore problems in which a single principal contracts with a group of agents in the presence of

[2]The intuition for the reversal of the result with costly actions is the mirror image of the public good case. Since in this case followers receive positive transfers from the principal/s in equilibrium, by refusing an offer followers risk not receiving an offer at all (not being compensated for a change that will happen anyways). Because this outside option determines their bargaining power vis a vis the principal, followers are better off when the chances of this happening are smaller (i.e., under competition).

externalities among agents (e.g., corporate takeovers, exclusive contracts). The second is the literature on *vote buying* and interest group influence developed in political economy. In these papers, two lobbyists compete offering transfers to members of a committee to obtain their support in favor or against a bill. The paper is also more loosely connected to the literature on innovation races.

The seminal paper in the literature on contracting with externalities is Grossman and Hart (1980), on corporate takeovers. The main point of the paper is that externalities across shareholders can prevent takeovers that add value to the company. The idea is that since shareholders that do not sell can capture the increase in value brought by the *raider*, no shareholder will tender his shares at a price that would allow the raider to profit from the takeover. Free-riding is also at the heart of the papers by Rasmusen, Ramseyer, and Wiley Jr (1991) and Segal and Whinston (2000), which study a problem in which an incumbent firm may be able to exclude rivals by signing exclusive deals with buyers. This hurts the buyers, but the incumbent is able to pull this off when it can turn buyers against one another exploiting the externalities that exist among them. These papers show that the incumbent's ability to deal with buyers sequentially strengthens its ability to exclude, as it exacerbates the free-rider problem among buyers.

A fundamental reason why the principal can exploit the agents in the Rasmusen et al and Segal papers – as opposed to say in efficient takeovers – is that by signing an exclusive contract a buyer imposes a *negative* externality on other buyers.[3] This assumption is maintained in Genicot and Ray (2006), who study optimal sequencing of offers for a single principal interacting with a group of agents who impose negative externalities on each other by contracting with the principal. As in Segal and Whinston (2000), here too a subset of the agents are *exploited* by the principal, accepting low offers because they anticipate that if they were to reject this offer, "other agents must succumb to the sequential onslaught" (Genicot and Ray (2006)).[4]

The literature on contracting with externalities is closely related to our monopolistic model (Section 4.1), where the externality is due to the fact that agents make a collective decision. The key innovation of our paper is to extend the model of sequential contracting with

[3]The distinction between positive and negative externalities, as well as *increasing* or *decreasing* externalities among agents in contracting models was first introduced in a static setting by Segal (1999), which offers a model of contracting with externalities that unifies various applications. The focus of this paper is on how the type of externalities among agents affect efficiency. Segal (2003) examines "robust" bilateral contracting with externalities (in the sense of implementation assuming the worst equilibrium for the principal) under various assumptions about the bargaining technology.

[4]This is also the case in the single-principal vote buying model by Dal Bo (2007), which also imposes negative externalities among agents. In this context, the principal can exploit agents even in a static setting, by using *pivotal* contracts. Relatedly, Segal (2003) and Segal and Whinston (2000) show that the principal can exploit agents in a static setting if she can make discriminatory offers.

externalities to a competitive setup and compare the welfare of agents under monopoly and competition.[5] In fact, the result that monopoly is preferred to competition depends crucially on the nature of the free-riding effect introduced by Grossman and Hart. In a public good setting – as it is the case with an efficient raid – agents would benefit from not trading with the principal. In this context free-riding opportunities give agents bargaining power against the principal. Our main result establishes that competition between principals reduces the value of free-riding in expectation, and thus diminishes the bargaining power of agents vis a vis the monopolistic setting.[6]

Our model of competition between principals is related to the literature on vote buying, where two lobbyists make offers to agents to win their support (Myerson (1993), Groseclose and Snyder (1996), Dekel, Jackson, and Wolinsky (2008, 2009), Morgan and Várdy (2011)). Relative to this literature, the fundamental innovation is to introduce sequential "vote buying" with irreversible actions by followers. The closest to this are the Dekel, Jackson and Wolinsky papers, which also have an infinite horizon, two parties competing for votes, and – in some versions of the model – partial irreversibility. However, here (as in all the previous vote buying literature) agents are not considered strategic actors, and are assumed to vote sincerely at each point in time for the alternative giving them a higher payoff. In our model, instead, agents are fully strategic and forward looking, as in the sequential bargaining setup of Gul (1989) and Iaryczower and Oliveros (2013).

Because of the dynamic nature of our game, the competition between principals resembles the models of innovation races pioneered by Harris and Vickers (1985, 1987). In contrast with this literature, in our model steps forward for the principals require interactions with agents that are forward-looking and strategic.

3. The Basic Model

Two leaders, A and B, compete to gather the support of a majority of members (followers) in a group of size n. The first leader to obtain the commitment of $q \equiv (n+1)/2$ members wins, and implements her preferred alternative. There is an infinite number of periods, $t = 1, 2, \ldots$. In each period t before a leader won, any one of the $k(t)$ uncommitted followers at time t meets leader $\ell = A, B$ with probability $\pi_\ell / k(t)$. Say that at the time of

[5]The literature on non-cooperative coalitional bargaining games with externalities considers general bargaining problems with multiple agents (Bloch (1996), Ray and Vohra (1999), Ray and Vohra (2001), Gomes (2005), Gomes and Jehiel (2005)). These papers aim to establish efficiency, existence and uniqueness properties in this class of games, and are for the most part unrelated to the industrial organization literature on contracting with externalities.

[6]Our results also show that with negative externalities (*costly actions* in our setup) competition is indeed beneficial to followers.

the meeting, ℓ needs m_ℓ additional followers to win. In the meeting with an uncommitted follower i, leader ℓ offers i an amount $p_\ell(m_\ell, m_{-\ell})$ to secure i's support (possibly $-\infty$). Follower i can accept or reject ℓ's offer. If he accepts, he commits his support for ℓ and receives $p_\ell(m_\ell, m_{-\ell})$. If he rejects the offer, i remains uncommitted.

Leader ℓ gets a payoff of $\overline{v}_\ell > 0$ if and when she wins, and $\underline{v}_\ell$ if and when her opponent $j \neq \ell$ wins, $\overline{v}_\ell > \underline{v}_\ell$. In any period before a leader wins, leaders get a payoff of zero (a normalization). In the basic model, we assume that leaders provide value to the agents; i.e., followers get a payoff of $w_\ell > 0$ if and when alternative ℓ wins.[7] As with leaders, we normalize followers' payoffs in any period before an alternative wins to zero. Leaders and followers have discount factor $\delta \in (0,1)$. The solution concept is Markov Perfect equilibrium (MPE). We let $W(\vec{m})$ denote the continuation value of an uncommitted follower in state $\vec{m} \equiv (m_A, m_B)$, $W_{out}(\vec{m})$ denote the continuation value of a committed follower in state $\vec{m}$, and $V_\ell(\vec{m})$ denote the continuation value of leader ℓ in state $\vec{m}$. It will also be useful to define $\vec{m}^A \equiv (m_A - 1, m_B)$ and $\vec{m}^B \equiv (m_A, m_B - 1)$.

As a benchmark, we also consider the case in which there is only one alternative to the status quo. The model is the same as before, with $\pi_\ell = 0$ for some $\ell = A, B$. Because there is a single leader, the state is just the number m of additional followers the leader needs to win. The price is $p(m)$, the value of an uncommitted follower $w(m)$, the value of a committed follower $w_{out}(m)$, and the leader's value $v(m)$.

4. Results in the Benchmark Model

4.1. **Monopolistic Leadership.** We begin by solving the model when there is a single alternative. For convenience, we denote the game with a single alternative and initial state m, $\Gamma^s(m)$. We show that in this case there is a unique MPE, in which the leader makes an offer in every meeting until she collects a majority of committed followers, and the uncommitted followers who meet the leader accept these offers. This result allows us to pin down the equilibrium payoff for a follower at the beginning of the game.

Let $\beta_0(m)$ denote the probability that a random follower meets the leader when the leader has to secure the support of m additional followers. Note that in this case there are $\frac{n+1}{2} - m$ committed followers and $\frac{n-1}{2} + m$ uncommitted followers, so $\beta_0(m) \equiv 2/(n-1+2m)$.

[7]In Section 5.1 we consider the costly action case, in which $w_\ell < 0$ for $\ell = A, B$.

Proposition 4.1. *The game $\Gamma^s(q)$ has a unique MPE. In this equilibrium, the payoff of a follower is given by*

$$w(q) = \left(\prod_{m=1}^{q} r(m) \right) \delta^q w \quad \textit{for} \quad r(m) \equiv \frac{1 - \beta_0(m)}{1 - \delta\beta_0(m)}$$

The intuition for the proof can be seen in two steps. First, fix the proposed equilibrium. Since $v > 0$ and $w > 0$, when the leader needs to collect the support of only one additional follower ($m = 1$), the leader and the follower can create and capture a positive surplus by moving forward. Thus, given full information, there is a price at which this transaction occurs. Now consider the situation in which there are m followers remaining. Since in equilibrium there is trade whenever the leader needs to secure the support of $t < m$ additional followers, then in state m there is also a positive surplus for the leader and the selected follower to obtain if they move forward, and then again a price at which this happens. This shows that the proposed strategy profile is an equilibrium.

By the same logic, in any equilibrium there must be a transaction when $m = 1$. Suppose then that in equilibrium there is trade whenever the leader needs to secure the support of $t < m$ additional followers. Recall that in state m, in the proposed equilibrium there is a positive surplus for the follower and the leader. Then if in state m the leader does not make an offer with positive probability or the follower does not accept the offer with positive probability, leader and follower would obtain a lower payoff in this state, and thus the gain from moving forward would be higher. It follows that the leader will make an offer, which the follower will accept.

Proposition 4.1 implies that in equilibrium the leader cannot extract all surplus from the followers. The reason for this is similar to the logic behind under-provision of a public good. Note that since the followers benefit from implementing the alternative to the status quo when $w > 0$, the leader actually charges them to move on. By rejecting the offer, however, a follower can rely on others to pay the bill. This generates an outside option that gives each follower some bargaining power over the leader. Since the cost of deferring implementation of the proposal decreases with δ, the value of the outside option is increasing in δ, and so is the followers' equilibrium payoff. In fact, as δ approaches 1, $r(m) \to 1$ and $\overline{w} \to w$.

Note also that $w(m)/w(m-1) = r(m)\delta < 1$. Thus the equilibrium payoff of uncommitted followers increases as the leader gets closer to achieving the majority. This is not just due to the effect of being closer to completion. Note that the value of committed followers is given by $w_{out}(m) = \delta^m w$, so that $w_{out}(m)/w_{out}(m-1) = \delta$. It follows that the rate of growth of the value for uncommitted followers (as the leader gets closer to achieving a majority) exceeds that of the committed followers.

4.2. **Competition and Main Result.** Consider now the competitive game $\Gamma^c(q,q)$, with two alternatives to the status quo, A and B, and initial position $\vec{m} = (q,q)$. Similarly to the monopolistic case, in equilibrium a follower i will commit his support to a leader ℓ in state $\vec{m}$ if and only if the leader and follower jointly benefit from this trade, taking the joint continuation strategy of the remaining followers and both leaders as given.

Recall that we defined $\vec{m}^A$ as the state that is one step from $\vec{m}$ in the direction of A, $\vec{m}^A \equiv (m_A - 1, m_B)$ and similarly $\vec{m}^B \equiv (m_A, m_B - 1)$. Follower i accepts an offer $p_\ell(\vec{m})$ from ℓ only if $p_\ell(\vec{m}) + \delta W_{out}(\vec{m}^\ell) \geq \delta W(\vec{m})$, and accepts with probability one if this inequality is strict. Thus in equilibrium, if ℓ makes an offer, she offers

$$p^*_\ell(\vec{m}) \equiv -\delta[W_{out}(\vec{m}^\ell) - W(\vec{m})]. \tag{1}$$

The offer by ℓ has to compensate i from the outside opportunities of refusing to commit his support for ℓ, taking him – along with the entire group – back to a position $\vec{m}$. Leader ℓ is willing to make this offer if $p^*_\ell(\vec{m}) \leq \delta[V_\ell(\vec{m}^\ell) - V_\ell(\vec{m})]$, or substituting $p^*_\ell(\vec{m})$ from (1), if the surplus for i and ℓ for making ℓ one step closer to the goal is nonnegative; i.e.,

$$S_\ell(\vec{m}) \equiv [V_\ell(\vec{m}^\ell) - V_\ell(\vec{m})] + [W_{out}(\vec{m}^\ell) - W(\vec{m})] \geq 0. \tag{2}$$

Given that whenever a transfer is made, it is determined by (1), we can consider, without loss of generality, a transformed game in which each leader ℓ chooses in each state $\vec{m}$ whether to offer $p^*_\ell(\vec{m})$ to a follower or pass. A MPE is then fully characterized by the probability $\gamma_\ell(\vec{m})$ that leader $\ell = A, B$ makes an offer in each state $\vec{m}$, and the probability $\alpha_\ell(\vec{m})$ that an uncommitted follower accepts an offer $p^*_\ell(\vec{m})$ from leader $\ell = A, B$ in each state $\vec{m}$. A (simplified) behavioral strategy $\sigma = (\vec{\gamma}, \vec{\alpha})$ is a MPE if given continuation values computed using σ, for all uncommitted followers $i \in N$, leaders $\ell = A, B$, and feasible states $\vec{m} \leq (q,q)$, we have (i) $\alpha_\ell(\vec{m}) \in [0,1]$ and (ii) $\gamma_\ell(\vec{m}) > 0 \Rightarrow S_\ell(\vec{m}) \geq 0$ and $\gamma_\ell(\vec{m}) < 1 \Rightarrow S_\ell(\vec{m}) \leq 0$. In our next result we establish existence of a MPE (possibly involving mixed strategies).

Proposition 4.2. *The competitive game $\Gamma^c(q,q)$ has a MPE.*

In the natural extension of the equilibrium in the monopolistic case to the competitive game, leaders make relevant offers in all states $\vec{m}$ such that $m_A, m_B \leq q$. We call this a *fully competitive* equilibrium. Differently than its counterpart in the monopoly game, this strategy profile is not always an equilibrium in this context, as condition (2) is not necessarily satisfied for both leaders in all states when followers anticipate that both leaders will make relevant offers in every state. On the other hand, given any parameter values, if the leaders' value of winning is large enough the strategy profile in which all meetings result in transactions will be an equilibrium (Proposition 4.4 below).

To develop the intuition for our main results we begin our analysis of the competitive environment focusing on a fully competitive equilibrium. This culminates in Theorem 4.5, where we establish the comparison between the competitive and monopolistic environments for a fully competitive equilibrium. We then extend this result to all equilibria in Theorem 4.6.

We start our analysis of fully competitive equilibria solving the three-follower example.

Example 4.3. *Suppose $n = 3$. We solve this game by backward induction. Consider the state $\vec{m} = (1,1)$, reached after one follower has committed for each leader. From (1), leader $\ell = A, B$ offers the follower $p_\ell(1,1) = \delta[W(1,1) - w_\ell]$, which the follower accepts. Thus*

$$W(1,1) = \pi_A (p_A(1,1) + \delta w_A) + \pi_B (p_B(1,1) + \delta w_B).$$

Substituting $p_\ell(1,1)$, we find $W(1,1) = 0$. Because only one follower remains uncommitted there are no free riding possibilities and all surplus is extracted by the leaders. Consider next the state $\vec{m} = (1,2)$. As before, a follower meeting the leader gets a payoff $\delta W(1,2)$, and thus

$$W(1,2) = \frac{1}{2}\delta W(1,2) + \frac{1}{2}(\pi_A \delta w_A + \pi_B \delta W(1,1))$$

Since $W(1,1) = 0$, the follower only profits if he is able to free ride in $\vec{m} = (1,2)$ and A wins: $W(1,2) = (\delta \pi_A/(2-\delta))\, w_A > 0 = W(1,1)$. Proceeding in the same manner we have that $W(2,1) = (\delta \pi_B/(2-\delta))\, w_B$ and

$$W(2,2) = \left(\frac{2\delta^2}{3-\delta}\frac{1}{2-\delta}\right)\left((\pi_A)^2 w_A + (\pi_B)^2 w_B\right)$$

□

We can now easily compare the followers' equilibrium payoff in the example with the payoff of an uncommitted follower in the monopolistic leadership case. Note first that since $w(1) > 0$ from Proposition 4.1, the previous example implies that $W(1,1) = 0 < w(1)$. This situation is fairly special, though, since in the competitive game with $\vec{m} = (1,1)$ only one follower remains uncommitted and there are no free riding possibilities. Now, from the single leader case we have,

$$w(2) = \left(\frac{2\delta^2}{3-\delta}\frac{1}{2-\delta}\right) w$$

Note that the expression in parenthesis is equal to the corresponding expression for $W(2,2)$ above. Thus, if $w = w_A > w_B$ or even $w = w_A = w_B$, followers prefer a monopoly of A to competition between A and B, for any discount factor $\delta < 1$. Moreover, for $\Delta w = w_A - w_B$ small, followers also prefer a monopoly of B to competition if $\pi_A \approx 1/2$.

There are two important lessons to learn from this example. First, since a monopoly can be preferred to competition even for $\delta \to 1$, the reason why followers are better off under single leadership is not rooted on the fact that competition takes more expected time to generate utility than single leadership. In particular it cannot be due to the fact that delay may destroy some surplus. Second, because sometimes a monopoly of the worst leader dominates competition, the reason behind the followers' welfare ordering cannot be due to the "risk" of ending up collectively selecting the worst choice available. We will show that the source of the follower's preferences for single leadership over competition is rooted on how the bargaining power shifts across games.

Consider now a group of n followers. Let $\beta(\vec{m})$ denote the probability that any given uncommitted follower meets with one of the leaders. Note if ℓ has to secure the support of m_ℓ more followers there are $(n+1) - m_A - m_B$ committed followers, and $m_A + m_B - 1$ uncommitted followers. Then $\beta(\vec{m}) = 1/(m_A + m_B - 1)$. As in the examples, eq. (1) implies that the expected payoff of a follower after meeting one of the leaders is $\delta W(\vec{m})$ independently of whether he accepts the proposal or not. This is a crucial property, for it allows us to decouple the system of partial difference equations for $W(\vec{m})$ and $V_\ell(\vec{m})$, $\ell = A, B$. Then

$$W(\vec{m}) = \left(\frac{1}{m_A + m_B - 1}\right)\delta W(\vec{m}) + \left(\frac{m_A + m_B - 2}{m_A + m_B - 1}\right)\delta \sum_{\ell} \pi_\ell W(\vec{m}^\ell), \tag{3}$$

so that letting $C(k) \equiv \frac{k-2}{k-(1+\delta)}$, we have

$$W(\vec{m}) = C(m_A + m_B)\delta \sum_{\ell} \pi_\ell W(\vec{m}^\ell). \tag{4}$$

Equation (4) is a partial difference equation with end points $W(m_A, 0) = w_B$ for $m_A > 0$ and $W(0, m_B) = w_A$ for $m_B > 0$, which we can solve to obtain the particular solution

$$W(\vec{m}) = \sum_{j=A,B} (\delta\pi_j)^{m_j} \times \left[\sum_{l=0}^{m_{-j}-2} \left(\prod_{k=0}^{m_j-1+l} C(m_{-j}+m_j-k)\right) \times \binom{m_j-1+l}{m_j-1} \times (\delta\pi_{-j})^l\right] \times w_j, \tag{5}$$

where we have adopted the convention that for any $f(\cdot)$, $\sum_{k=a}^{b} f(k) = 0$ whenever $b < a$.

As the group moves closer to completion, there are two opposite effects on followers' welfare. On the one hand, getting closer to completion increases the value of an uncommitted follower (since $w_\ell > 0$ and $\delta < 1$). On the other hand, it also results in a lower ability to free ride. The value of an uncommitted follower in each state $\vec{m}$ depends on how the tradeoff between proximity and free riding is resolved, as given by the probability of successful free riding along alternative paths to a terminal state from the initial state $\vec{m}$.

And the key point here is that not all paths to a terminal state have the same effect on the probability of free riding. Thus, the followers' value increases as we move closer to completion while maximizing free riding possibilities. This is achieved by a lopsided win of the leading principal, taking into consideration both the current balance of power and expectations about future performance (see Figure 1). On the other hand, free riding opportunities are minimized in tight races, where a principal wins by a small margin. In fact, as we have seen in example 4.3, the leaders extract all surplus from an uncommitted follower in state $(1,1)$, who has no free riding opportunities.

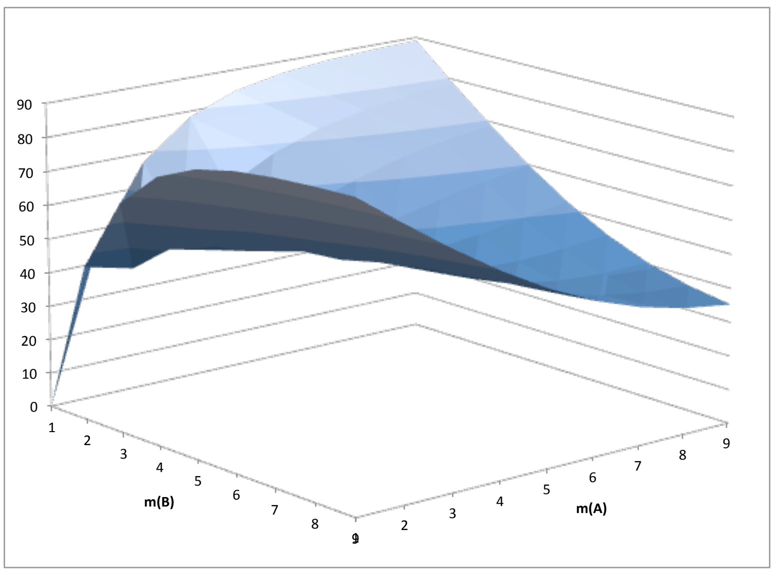

FIGURE 1. $W(\cdot)$ in a CE, for $n = 9$, $\delta = 0.95$ and $\pi_\ell = 0.5, w_\ell = 100$ for $\ell = A, B$.

Having obtained the expression for $W(\vec{m})$ in terms of the fundamentals, we can write down equilibrium transfers. Note that once a follower is committed, all strategic considerations are brushed aside, as a committed follower just needs to wait for a leader to form a majority. Thus

$$W_{out}(\vec{m}) = \sum_{j=A,B} (\delta\pi_j)^{m_j} \times \left(\sum_{l=0}^{m_{-j}-1} \binom{m_j - 1 + l}{l} \times (\delta\pi_{-j})^l \right) \times w_j \tag{6}$$

From equation (1), expressions (5) and (6) pin down equilibrium transfers $p_\ell(\vec{m})$ in terms of the fundamentals. This in turn allows us to solve for the value of the leaders, which is given, in recursive form, by

$$V_\ell(\vec{m}) = \pi_\ell\left(\delta V_\ell\left(\vec{m}^\ell\right) - p_\ell(\vec{m})\right) + (1-\pi_\ell)\delta V_\ell\left(\vec{m}^{-\ell}\right) \tag{7}$$

Once we write transfers in terms of the primitives, (7) becomes a stand alone difference equation, which we can solve as we did with W and W_{out}. This allows us to prove the next result.

Proposition 4.4. *All else equal, if the leaders' payoff for winning is sufficiently high (either $\overline{v}_\ell$ or $\overline{v}_\ell - \underline{v}_\ell$), the game $\Gamma(q,q)$ has a fully competitive equilibrium.*[8]

We are now ready to state our first main result. In this theorem, we will assume that the sufficient condition for the existence of a fully competitive equilibrium is satisfied, and compare the value of uncommitted followers when there is a single alternative with the value under competition. The theorem establishes that followers are better-off when only the best alternative is available than under competition. This holds for any difference in the value of the alternatives, and in fact also when the two alternatives give followers the same value; i.e., $w_A = w_B = w$. Moreover, for any terminal values $w_A > 0$ and $w_B > 0$, if the initial number of uncommitted followers is sufficiently large, then followers are better-off under a monopoly of the worst alternative (say B) than under competition between A and B.

Theorem 4.5. *Let $W(q,q)$ denote the payoff of an uncommitted follower in a fully competitive equilibrium of the game with two alternatives A and B such that $w_A, w_B > 0$. Then*

$$W(q,q) < \max\{w_A(q), w_B(q)\}.$$

Moreover, (i) for any n, there exists $\varepsilon(n) > 0$ such that if $|w_A - w_B| < \varepsilon(n)$, then $W(q,q) < \min\{w_A(q), w_B(q)\}$, and (ii) there exists $\overline{n}$ such that in a fully competitive equilibrium of the game with $q = (n+1)/2 > \overline{n}$, $W(q,q) < \min\{w_A(q), w_B(q)\}$.

How can competition between alternatives be bad for followers? The key to understand the result is to understand how followers' bargaining power changes across the two games.

Consider the problem of the pivotal follower when there is a single alternative (the q^{th} member to meet the leader). If the pivotal follower refuses an offer by the leader, he will

[8] Note this equilibrium is inefficient, as the worst alternative wins with positive probability. This result is in line with the literature on non-cooperative coalitional bargaining games with externalities, which has shown that inefficiency is the norm in these class of games (Bloch (1996), Ray and Vohra (1999), Ray and Vohra (2001), Gomes (2005), Gomes and Jehiel (2005)).

be able to free ride on others with probability $(q-1)/q = (n-1)/(n+1)$. This relatively high free-riding ability gives the pivotal follower considerable bargaining power against the leader and protects him from getting fully expropriated.

Now consider the problem of a pivotal follower under competition (a follower who could give one of the leaders a win). The number of uncommitted followers in this situation is at most as large as in the single alternative case and generally lower. In fact, if the battle between the leaders is even until the end, the pivotal follower could meet the leader in state $\vec{m} = (1,1)$. But here there are no free riding possibilities; i.e., if the follower rejects an offer he meets one of the leaders again for sure. As we discussed earlier, this gives the pivotal follower no bargaining power, so $W(1,1) = 0$. This situation is extreme, but the logic generalizes to other states in which the number of uncommitted followers is lower than in the single alternative case. In most states, competition between the leaders gives the pivotal follower fewer free-riding possibilities, and results in a lower equilibrium payoff.

To illustrate this point, consider the competitive and monopolistic games with five followers. Using Proposition 4.1 and equation (5), and simplifying, $w(3) > W(3,3)$ if and only if

$$w > \sum_{j=A,B} (\pi_j)^3 \left[1 + \frac{3}{2-\delta}(\delta\pi_{-j})\right] w_j \tag{8}$$

The expected payoff for followers under competition can be divided in three terms. First is the payoff from events in which one principal wins without conceding a single vote. This is captured by the first term in the bracket. This term involves no discounting because reaching terminal nodes does not take additional periods than under monopoly. Second is the payoff from maximally divided wins, where a principal wins conceding two votes. These terms do not contribute to followers' payoffs because they eliminate free riding opportunities (must go through the node $(1,1)$). Third are the (three) events in which one of the principals wins conceding one vote. This is the second term in the bracket. The algebra isolates two conceptually distinct effects. One, captured by the discount factor δ in the numerator, is simply the inefficiency brought by delay, given that these paths waste a period in an unnecessary transaction. The other measures how free-riding opportunities affect bargaining power. This is captured by the $2-\delta$ term in the denominator.

Both delay and free-riding vanish as $\delta \to 1$, because patience eliminates the source of power that free-riding gives to the followers. However, these are two distinct effects. In fact, delay is not necessary for the result. To see this, let $\gamma(\vec{m})$ denote the probability of reaching the node $\vec{m}$ from the initial node (q,q) in a competitive equilibrium. If we let

$w_A = w_B = w$ and rearrange, we can write (8) as

$$(1-\delta)\left[\gamma(2,1)+\gamma\left(1,2\right)+\gamma\left(1,1\right)\right]+\frac{1}{2}\delta\gamma\left(1,1\right)>0$$

Note that as $\delta \to 1$ all terms vanish except $\vec{m} = (1,1)$, in which the remaining follower has no opportunity to free-ride.

This logic is also at the heart of the second result. Consider again the single alternative case. As we noted above, the probability that the pivotal uncommitted follower will be able to free ride after refusing an offer from the leader is $(n-1)/(n+1)$. This probability goes to one as $n \to \infty$. On the other hand, the game with competition can be very close. Thus, even if n is large, the pivotal player can be one of only a few uncommitted followers, and as a result have few free-riding opportunities, and therefore less bargaining power. This effect trickles down all the way through to the beginning of the game, and then even a monopoly of the worst alternative is preferred to competition. Interestingly, this result holds for any difference in the value of the alternatives to the followers, as long as the group is sufficiently large.

Theorem 4.5 shows that if the conditions are such that both leaders compete for support in every state, introducing competition reduces followers' welfare. Differently to the monopolistic leadership case, however, the equilibrium of the two leader game will not necessarily be fully competitive. In fact, for some parameter values any MPE will necessarily involve mix strategies. The question then arises as to whether our result is specific to the fully competitive equilibrium or instead holds for any MPE.

Our next result addresses this issue. Exploiting incentive compatibility conditions that must hold in any MPE, we show that whenever an alternative A provides equal or larger value to followers than a second alternative B ($w_A \geq w_B$), followers are better off when a single principal advocates in favor of A than when principals for A and B compete with one another to attain the support of a majority of the followers. This is the main result of the paper.

Theorem 4.6. *Let $W(q,q)$ denote the payoff of an uncommitted follower in a MPE of the game with two alternatives A and B such that $w_A \geq w_B > 0$. Then (i) $W(q,q) < w_A(q)$. Moreover, (ii) there exists $\varepsilon > 0$ such that if $w_A - w_B < \varepsilon$, $W(q,q) < w_B(q)$.*

The proof is by induction and consists of four steps:

(1) First, we show, within the competitive game, that in any state $\vec{m}$ there is always an alternative $j \in \{A, B\}$ such that followers are better off after moving one step in the direction of j; i.e., $W(\vec{m}) < \max_{j\in\{A,B\}}\{W(\vec{m}^j)\}$.

(2) Second, we show that when a principal j is one step away from winning the competitive game followers would be strictly better off by removing one of the alternatives from consideration; e.g., $W(m_A, 1) < \max\{w_A(m_A), w_B(1)\}$. This step connects the competitive and monopolistic games.
(3) Third, combining steps 1 and 2 we then show that when principal j is two steps away from winning the competitive game followers would be strictly better off removing one of the alternatives from consideration; e.g., $W(m_A, 2) < \max\{w_A(m_A), w_B(2)\}$ for all $m_A \geq 2$.
(4) Fourth, we establish the induction step for $\vec{m} \geq (3,3)$: if $W\left(\vec{m}^\ell\right) \leq w_\ell(m_\ell - 1)$ and $W\left(\vec{m}^\ell\right) \leq w_{-\ell}(m_{-\ell})$ for $\ell = A, B$, then $W(\vec{m}) \leq \max\{w_A(m_A), w_B(m_B)\}$. Iterative application of the induction step covers the entire state space and establishes the result.

Theorem 4.6 makes clear that the comparison between competition and monopoly does not depend on the selection of a fully competitive equilibrium in the competitive game. Thus, in particular, the comparison holds not only when leaders' value of winning is large, but for any feasible parameter configuration. The sketch of the proof of Theorem 4.6 also makes clear that the comparison holds not only in the diagonal (states $\vec{m}$ such that $m_A = m_B$) but in the entire state space. Thus, the presence of an initial advantage would not alter our results.

Corollary 4.7. *Consider any equilibrium of the game* $\Gamma(q_A, q_B)$*, where* $q_A < q_B$*. Then (i)* $W(q_A, q_B) < \max\{w_A(q_A), w_B(q_B)\}$*. Moreover, (ii) there exists* $\overline{n}$ *s.t. if* $\overline{n} < q_A < q_B$*, in a fully competitive equilibrium of* $\Gamma(q_A, q_B)$*,* $W(q_A, q_B) < \min\{w_A(q_A), w_B(q_B)\}$*.*

Note moreover that this allows us to extend the benchmark model to arbitrary q-rules. Suppose for example that B needs to obtain the support of a supermajority $q_B > (n+1)/2$ of members to implement a reform leading to a value of $w_B > 0$ for the followers, while A can block the reform by getting the support of $q_A = n - q_B$ members, leading to $w_A = 0$. Note that the model for an arbitrary non-unanimous q-rule is formally equivalent to introducing initial advantages, with the exception that $w_A = 0$. Thus part (i) of Remark 4.7 applies to this case as stated. However, since $w_A = 0$, part (ii) of the remark does not necessarily hold.[9]

An alternative conceptualization of q-rules, which is occasionally used in the literature, would require both principals $i = A, B$ to obtain $q_i \neq (n+1)/2$ votes to win. This

[9]With a strict public good ($w_A > 0$), $w_A(m) > 0$ for any state $m \geq 1$. In the proof of Theorem 4.5 we show that for any $w_A(q_A) > 0$ we can choose (q_A, q_B) large enough so that the fully competitive equilibrium value $W(q_A, q_B)$ can be sandwiched in $(0, w_A)$. When $w_A = 0$, however, $w_A(q_A) = 0$ as well, and then $W(q_A, q_B) > w_A(q_A)$.

alternative supermajority model is interesting because it brings light to the discussion of delay vs free-riding. With a supermajority, $q_i > (n+1)/2$, there are now multiple states in which there is a single uncommitted follower, and hence more states in which all surplus is extracted from this last surviving uncommitted follower. This effect will trickle down the recursion now with more force, and monopoly will be preferred to competition even for $\delta \to 1$. With *submajorities* $q_i < (n+1)/2$, instead, there is no state that eliminates free-riding opportunities completely. Thus when $\delta \to 1$, the value of free riding opportunities vanishes, and followers are indifferent between monopoly and competition.

5. Applications and Extensions

In Section 4 we presented a bare bones model of dynamic competition with irreversible actions. This simple model allowed us to focus on the core mechanism behind our result. To take the theory to applications, however, we must adapt the model to incorporate various new features. In the next sections we do this, illustrating the results in each case with clear applications of the model. For simplicity, we assume throughout that the leaders place a sufficiently high value on winning, so that the competitive game has a fully competitive equilibrium.

5.1. **Costly Actions.** In the benchmark model we assumed that the alternatives championed by the leaders are a "public good", in the sense that followers prefer the outcome associated with a victory of A or B to the status quo. In some instances, though, it is natural to assume that the actions under consideration are costly for members of the group, in the sense that followers would prefer the status quo to any of the options proposed by the leaders; i.e., $w_A, w_B < 0$. The *costly action* model can naturally arise in multiple settings, as in vote buying (when a committee considers proposals that members find worse than the status quo), exclusive deals (see our example in Section 5.2), and coalitions for war (if decision-makers in each country favor inaction).

Here we extend our analysis to consider the case of $w_A, w_B < 0$. As before, we begin with the case in which there is only one alternative to the status quo. We show that as long as the leader's payoff for winning is sufficiently high, or players are sufficiently patient, the game $\Gamma^s(q)$ has a unique equilibrium. In this equilibrium, the leader makes an offer whenever she meets an uncommitted follower up to the point in which she wins.

Proposition 5.1. *Let $w < 0$. All else equal, if the leader's payoff for winning is sufficiently high or players are sufficiently patient, the equilibrium characterized in Proposition 4.1 remains the unique MPE of the game $\Gamma^s(q)$, and followers' equilibrium payoff is given* *by*

$$w(q) = \left(\prod_{m=1}^{q} r(m) \right) \delta^q w < 0 \quad \textit{with} \quad r(m) \equiv \frac{1-\beta_0(m)}{1-\delta\beta_0(m)}$$

Proposition 5.1 says that if followers are sufficiently patient, the leader can implement a policy that is arbitrarily bad for followers with the support of a majority of the group. This of course is terrible news for those agents who are not compensated.

The reason for this result is that the leader uses uncommitted followers against each other. Upon meeting, both leader and follower know that if the follower rejects the leader's offer, the next follower will accept it. Thus, the follower can only delay the implementation of the costly action for one period, if he forgoes any compensation. To prevent this delay, the leader can offer to compensate the follower *for this differential* (and not for the full cost that implementation will bring to the follower). Note that since the power of the follower stems from his ability to delay implementation, it is decreasing in the followers' discount factor δ. In fact, from Proposition 4.1, $w(q)$ attains an upper bound of zero as $\delta \to 0$, and a lower bound of $w < 0$ as $\delta \to 1$. This implies that as transaction frictions vanish and the time in between meetings vanishes, followers' expected compensation goes to zero. In the costly actions model patience increases the bargaining power of the principal, and allows her to exploit agents, even without using discriminating contracts.

Would followers benefit from competition between principals in this context? As one might anticipate, the answer is yes: with costly actions, competition increases followers' welfare.

Proposition 5.2. *Consider any equilibrium of the game with two costly alternatives inducing payoffs $w_A, w_B < 0$ for the followers, and initial position (q, q), where $q \equiv (n+1)/2$. Then $W(q,q) \geq \min\{w_A(q), w_B(q)\}$. Moreover, (ii) there exists $\overline{n}$ such that in a fully competitive equilibrium of the game with $q = (n+1)/2 > \overline{n}$, $W(q,q) \geq \max\{w_A(q), w_B(q)\}$.*

The intuition for this result is straightforward. With costly actions, followers can only cut their loses if they are part of the coalition that supports the leader. Thus, the same logic in Theorems 4.5 and 4.6 now reverses the result: a pivotal agent that refuses the offer with a single alternative will have a much lower chance of being brought back to the table of negotiations than a pivotal follower in a competitive environment, and as a result can demand a higher transfer in exchange of his support. Formally, the proof of this result is identical to that of Theorems 4.5 and 4.6, reversing inequalities due to $W(\cdot) < 0$.[10]

[10]By the same logic, it follows that if one of the leaders proposes a public good and one of them champions a costly action, say $w_A > 0 \geq w_B$, then followers prefer $\Gamma_A^s(q) \gtrsim \Gamma_A^c(q) \gtrsim \Gamma_B^s(q)$.

5.2. **Insiders, Outsiders and Rivals.** In the benchmark model we assumed that the net-of-transfers payoff that followers obtain when an alternative $\ell = A, B$ wins is w_ℓ, independently of who, if anybody, they had decided to support. In applications, however, it can be more natural – or even necessary – to distinguish between the *insider* payoff obtained by an individual who gave her support to a leader when that leader wins, the *outsider* payoff obtained by an uncommitted follower when a leader wins, and – in the competitive model – the *rival* payoff obtained by an individual who gave her support to a principal when the opposite principal wins. Consider for example the following passage from "HRC", the book on Hillary Clinton by Politico's Jonathan Allen and The Hill's Amie Parnes:

> "They carefully noted who had endorsed Hillary, who backed Barack Obama, and who stayed on the sidelines – standard operating procedure for any high-end political organization. But the data went into much more nuanced detail .... For Hillary, whose loss was not the end of her political career, the spreadsheet was a necessity of modern political warfare, an improvement on what old-school politicians called a favor file. It meant that when asks rolled in, she and Bill would have at their fingertips all the information needed to make a quick decision – including extenuating, mitigating, and amplifying factors – so that friends could be rewarded and enemies punished."

Capturing this situation *requires* that we distinguish between the insider payoff z_A obtained by a party boss who supported Mrs Clinton if she succeeds, the rival payoff y_A obtained by a party boss who supported Mr Obama if Mrs. Clinton wins, and the outsider payoff w_A obtained by a party boss who stayed on the sidelines if Mrs Clinton wins. Similarly, consider the exclusive contracting setup. Suppose that two incumbents try to convince buyers to sign exclusive contracts, but only that who obtains more than half of the market can prevail. Then a buyer who signs an exclusive deal with company A obtains a payoff of $z_A > 0$ if A wins, but $y_A < z_A$ if B wins, due to adjustment costs.

A similar distinction can be made in setups in which there is a single principal. Consider for example a real estate developer who wants to buy properties in a block to build a shopping center. Assume, for simplicity, that the shopping center can be built if and only if the developer acquires at least half of the houses in the block (independently of location), and normalize the value of each house if the mall is not built to zero. If an owner does not sell the house and the mall is built, the house has a loss of value $w < 0$. If, on the other hand, the owner does sell the house to the developer, the value of his assets changes by $z < w$! Again, capturing this situation requires that we distinguish between the insider payoff z obtained by an owner who gave her support to the developer when the developer succeeds, the outsider payoff w obtained by an uncommitted owner when the developer succeeds, and the status quo payoff (zero).

The central application we consider in this section is corporate takeovers. The free-riding that is central to our paper is also at the core of the seminal Grossman and Hart (1980) paper. Grossman and Hart (GH) analyze a situation in which a company or individual (the raider) acquires shares of a target company in order to control its board of directors. It is assumed that the raider can improve the value of the company. To capture this, we assume that under the raider's control the value of a share is $w > 0$, and normalize the value of a share under the incumbent management to zero. In GH, the raider buys shares by announcing a tender price p at which he is willing to buy all shares tendered to him, and wins control of the firm if more than 50 % of the shares are tendered to her. It is assumed that in making his tender decision, each shareholder ignores his impact on the outcome of the bid.

In this context, GH show that – since shareholders *that do not sell* can capture the increase in value brought by the raider – any shareholder who thinks that the raid will succeed with certainty will not tender his shares at a price $p < w$. Thus, in equilibrium the potential raider does not attempt the takeover, even when it has a value advantage (for a raid to succeed the raider needs to offer $p \geq w$, but under these conditions the raider makes no profit).

In order to capture the main source of the externality in the GH model within our model, we need to distinguish the payoff of a shareholder who does not sell to the raider if the raider wins ($w > 0$) form the payoff of a shareholder who sells to the raider if the raider wins ($z = 0$). With this amendment, we can consider our version of the GH takeover problem. There are of course structural differences within the two models. In particular, instead of a tender price p at which the raider is willing to buy all shares tendered to him, as in GH, we assume that the raider approaches shareholders sequentially, possibly offering different prices to each shareholder (as in GH and Segal (2003), we assume shareholders are homogeneous). Moreover, differently than GH, we suppose that shareholders are fully aware of the effect of their action on the outcome of the raid attempt.[11]

What does this model say about takeovers? To attack this question while at the same time setting up the framework for other applications, we analyze the general version of the model and return to our takeover application shortly. We denote the values of leaders and followers with a hat, to distinguish them from the analogous quantities in the benchmark model.

[11]Holmstrom and Nalebuff (1992) show that when shareholdings are divisible the free-riding problem does not prevent the takeover process in the GH model.

Assume first that – as in our benchmark model – there is an equilibrium in which the leader makes relevant offers to followers in all states $m \leq q$. Because the equilibrium transfer is such that the follower is indifferent between accepting or rejecting the offer, the equilibrium payoff of a follower meeting the leader in state m is $\delta \hat{w}(m)$, independently of whether he accepts or rejects the leader's offer. As a result, the recursive representation of the value of uncommitted followers is unchanged from the benchmark game, and thus so is its solution, $\hat{w}(m) = \left(\prod_{k=1}^{m} r(m)\right) \delta^m w$. The value of a committed follower, on the other hand, changes in the obvious way, so that now it is given by $\hat{w}_{out}(m) = \delta^m z$. In particular, if $z = 0$ as in the takeover model, we have $w_{out}(m) = 0$. Substituting in the equilibrium transfers, this directly implies that

$$\hat{p}(m) = \left[\prod_{k=1}^{m} r(k)\right] \delta^{m+1} w > 0, \tag{9}$$

where as before $r(m) = (n + 2m - 3)/(n + 2m - 1 - 2\delta)$, so that transfers from the raider to shareholders are positive even as $w > 0$. In fact, $\hat{p}(m) \to w$ for all m as $\delta \to 1$. Thus, as frictions vanish, the raider has to fully compensate shareholders for the increase in the value of the company under her management, as in GH. For any $\delta < 1$, however, $\hat{p}(m) < w$, and the raider can appropriate some of the surplus it generates.[12] Moreover, note that from (9), we have

$$\hat{p}(m-1) - \hat{p}(m) = (1 - r(m)\delta) \left[\prod_{k=1}^{m-1} r(k)\right] \delta^m w > 0$$

Thus, the price curve (intertemporal price discrimination) is strictly decreasing in m, so that shareholders who transact first with the raider obtain a lower price for their shares.[13]

Would shareholders benefit from introducing competition between raiders? In the benchmark model we saw that whenever the principals added value, competition was detrimental to followers' welfare. But while transfers from the principals to the agents were always negative with public goods, in the generalized environment public goods can coexist with

[12] This result is similar to that of Harrington and Prokop (1993), who consider a dynamic version of GH in which the raider can reproach the shareholders who have not sold (taking all offers at the posted price in each period). This setup leads to a "Coase conjecture" result, in which the raider's profit is diluted for large discount factors.

[13] The fact that in the extended model leaders might have to hand out positive transfers to followers (depending on the value of z vs w) suggests that the strategy profile in which the leader makes relevant offers in all states $m \geq 1$ might fail to be an equilibrium for some parameter values. Indeed, the condition for existence is violated when the outsider payoff w is high relative to both the insider payoff z and the leader payoff v. As before, however, this strategy profile is an equilibrium for any w if v is sufficiently large. It is also an equilibrium (ii) whenever $v > 0$ and $z > w$, (iii) whenever $v > 0 > w$, and (iv) whenever $z = 0$ and $v \geq qw$. Part (iv) applies in the case of the corporate takeover. Thus, we predict raids will occur whenever the raider has a positive quality differential with the incumbent management.

positive transfers from the principals to the followers. Thus it is not immediately obvious whether or not competition among principals could be valuable for followers in this environment.

The fact is that our main result is not affected by the direction of transfers: with public goods, competition is still detrimental to followers' welfare. The key is that – as it was the case in the monopolistic environment – in the competitive environment each leader i makes offers to fully extract the joint surplus generated by moving one step in her direction. As a result, the value of a follower when he meets a leader is $\delta\hat{W}(\vec{m})$, independently of whether he accepts or rejects the leader's offer. Because of this, the recursive representation of the followers value $\hat{W}$ is given by (4), exactly as before, and thus $\hat{W}(\vec{m}) = W(\vec{m})$.

This implies that the followers' value only depends on the outsider payoffs w_ℓ, with all the new elements going into the "revised" cash transfer $\hat{p}_\ell(\vec{m})$. It follows that to determine whether we are in a "public good" or a "costly action" model, we only need to consider the outsider payoffs w_ℓ: if $w_\ell < 0$ competition is beneficial for followers, and if $w_\ell > 0$, competition is detrimental for followers. In the case of takeovers, in particular, this says that shareholders would not benefit from competition between raiders.

Now consider the value of cash transfers. The amount leader ℓ will offer to a follower when meeting in a state $\vec{m}$ satisfies

$$\hat{p}_\ell(\vec{m}) = \delta\left[W(\vec{m}) - \hat{W}^\ell_{out}(\vec{m}^\ell)\right], \tag{10}$$

where $\hat{p}_\ell(\vec{m})$ is the cash transfer ℓ proposes in state $\vec{m}$, where $\hat{W}^\ell_{out}(\vec{m}^\ell)$ is the value in state $\vec{m}^\ell$ of a follower who committed his support to leader ℓ, and where we have used the fact that $\hat{W}(\vec{m}) = W(\vec{m})$ for all $\vec{m}$. In the benchmark model we had $W_{out}(\vec{m}^\ell) = \sum_\ell J_\ell(\vec{m})w_\ell$ for some $J_\ell(\cdot)$. In the current setting instead $\hat{W}^\ell_{out}(\vec{m}^\ell) = J_\ell(\vec{m}^\ell)z_\ell + J_i(\vec{m}^\ell)y_i$ for $i \neq \ell$. Thus substituting,

$$\hat{p}_\ell(\vec{m}) = p_\ell(\vec{m}) - \delta\left[J_\ell(\vec{m}^\ell)(z_\ell - w_\ell) + J_i(\vec{m}^\ell)(y_i - w_i)\right] \tag{11}$$

Note that when $J_\ell(\vec{m}^\ell)(z_\ell - w_\ell) < J_j(\vec{m}^\ell)(w_j - y_j)$, prices are higher than in the benchmark model. That is, transfers are higher than in the benchmark model when the weighted gain from committing to the winner is larger than the weighted gain to not committing to the loser. In particular, in the takeover model we have that if one of raiders $j = A, B$ buys a shareholder out, the (ex) shareholder is excluded from any benefits the company can produce, so $z_j = y_j = 0$. Thus prices are higher than in the benchmark model. Moreover, note that in this case we actually have $\hat{W}^\ell_{out}(\vec{m}^\ell) = J_\ell(\vec{m}^\ell)z_\ell + J_i(\vec{m}^\ell)y_i = 0$, so that

$$\hat{p}_\ell(\vec{m}) = \delta W(\vec{m}) > 0$$

As this example illustrates, in the model with insiders, outsiders and rivals the equivalence between the effect of competition and the direction of the transfers breaks down. This might seem disconcerting. Since the intuition for the result in the benchmark model relied heavily on free riding, it can be difficult at first glance to reconcile competition being bad for followers with positive transfers from the leaders to the followers.

Once we separate real from nominal compensation, the main idea is unchanged. In the original model, with no "bonus payments" (no insider/outsider differential) the only possible compensation to/from followers is in cash. Thus when followers receive a positive cash transfer, meeting the leader is good for followers, and when in equilibrium followers give a transfer to the leader, avoiding the leader is good for followers. But when the payoff of insiders, outsiders and rivals differ, the total compensation (or taxation) is composed of both cash and a "bonus" (the insider/outsider differential). In this context, focusing on the cash component of total compensation can be misleading, as this can go in any direction, but total compensation (cash and bonus) will have the same informational content as before: when total compensation is positive, followers want to meet the leader, and when total compensation is negative, they want to avoid them. To see this more clearly, note that

$$\hat{W}^{\ell}_{out}(\vec{m}^{\ell}) = W_{out}(\vec{m}^{\ell}) + J_{\ell}(\vec{m}^{\ell})(z_{\ell} - w_{\ell}) + J_j(\vec{m}^{\ell})[y_j - w_j] \quad \text{for } j \neq \ell.$$

Thus, substituting in (10), we have that

$$\delta[W(\vec{m}) - W_{out}(\vec{m}^{\ell})] = \underbrace{\hat{p}_{\ell}(\vec{m}) + J_{\ell}(\vec{m}^{\ell})(z_{\ell} - w_{\ell}) + J_j(\vec{m}^{\ell})(y_j - w_j)}_{T_{\ell}(\vec{m})}. \tag{12}$$

$T_{\ell}(\vec{m})$ is the total compensation, in both cash and "goodies", that a follower obtains from ℓ in exchange of his support in state $\vec{m}$. Here $\hat{p}_{\ell}(\vec{m})$ is the cash component, and $J_{\ell}(\vec{m}^{\ell})(z_{\ell} - w_{\ell}) + J_j(\vec{m}^{\ell})(y_j - w_j)$ is the discounted net expected increment in goodies, $(z_{\ell} - w_{\ell})$ being the insider's gain (the increment in goodies the follower gets when being with ℓ if ℓ wins, relative to being uncommitted), and $(y_j - w_j)$ is the rival's loss.

The key point that (12) illustrates is that the *total* transfer $T_{\ell}(\vec{m})$ is *exactly the same* as the value of the cash transfer in the benchmark model, when no goodies are available. What matters is not the direction of the cash transfers but the total compensation, which is pinned down by $\delta[W(\vec{m}) - W_{out}(\vec{m}^{A})]$, and is therefore unrelated to z_{ℓ} and y_j. This generalizes in a natural way our previous intuition. In a public good environment the total compensation (excluding policy gains) $T_{\ell}(\vec{m})$ will be negative, and followers extract a free rider surplus when they can avoid meeting the leaders. Thus, competition – which reduces the free riding opportunities – is not beneficial for followers. With costly actions, instead, meeting the leaders allows followers to extract a positive total compensation, and therefore competition increases their equilibrium payoff.[14]

The generalized model allows us to extend the insights of this paper to a wide range of applications:

Vote Buying. Consider a national legislature, which is about to vote on a fiscal restraint bill proposed by the executive. We assume that legislators – who understand the dire state of fiscal affairs for the state – privately favor the bill ($w > 0$). Voters, however, oppose it, so that supporting the bill is costly for legislators; i.e., $z < 0 < w$. Then

$$\hat{p}(m) = \delta^{m+1} \left(\left[\prod_{k=1}^{m} r(k) \right] w - z \right) > 0,$$

so that the executive effectively buys legislators' support with a positive transfer. Moreover, as in corporate takeovers, here $\hat{p}(m-1) > \hat{p}(m)$, so that legislators who commit their support early obtain a smaller transfer than those who commit their support in later stages. Now suppose that legislators face two options. The executive's proposal (say A) aims to reduce the deficit by increasing taxes, while an alternative proposal favored by a powerful lobby (B) seeks to reduce public expenditures. We assume that legislators prefer increasing taxes, so that $w_A > w_B > 0$. Our main result implies that in this case legislators would be better off in the absence of the lobbyist. Still, both the executive and the lobbyist offer legislators positive transfers.

Endorsements. Consider the endorsement game we discussed at the top of this section. We argued that capturing this situation requires that we distinguish between the insider payoff z_A obtained by a party boss who supported Mrs Clinton if she succeeds, the rival payoff y_A obtained by a party boss who supported Mr Obama if Mrs. Clinton wins, and the outsider payoff w_A obtained by a party boss who stayed on the sidelines if Mrs Clinton wins. In this example, we assume that for both $j = A, B$, $z_j > w_j$ (supporters are rewarded) and $w_j > y_j$ (enemies are punished). In addition, we assume that both contenders are good candidates, so that $w_j > 0$ for $j = A, B$. In this case, promises flow from the candidates to party members if the cost of supporting the losing candidate is sufficiently large and from members to candidates otherwise, but party members always prefer to have a shoo-in candidate (competition is detrimental for followers' welfare).

[14]The previous remark also clarifies that the implicit assumption that followers cannot single-handedly give their support to a principal, which can be awkward in the benchmark model, is not an issue in most cases of interest. Here followers get a negative total compensation by meeting the leader, which is partially mitigated by the positive transfer. By offering their support unilaterally, followers would only be forgoing surplus. This can be seen most clearly in our application to corporate takeovers, where shareholders would certainly lose by offering their shares to the raider for free. Thanks to David Ahn for pointing this out.

Exclusive Deals. We consider a problem in which two firms compete for the market of a product with increasing returns to scale signing exclusive contracts with buyers. This problem relates to the analysis in Rasmusen, Ramseyer, and Wiley Jr (1991) and Segal and Whinston (2000), with two key differences. First, while in these papers only the incumbent can sign exclusive contracts, and the challenger decides to enter or not after all buyers have made their decision, we allow both firms to sign exclusive contracts. For consistency with this first choice, we assume that both firms are initially competing in the market. Thus, ours is a model where firms take actions to induce exit, as opposed to a model of a single incumbent trying to deter entry.

To illustrate, consider the HD optical disc format war between Blu-ray and HD-DVD, in which Sony (Blu-ray) and Toshiba (HD-DVD) were courting movie studios for their exclusive support. Before the first movie releases in 2006, each format had the informal support of three of the Big Six studios: HD-DVD had Universal Studios, Paramount Pictures, and Warner Brothers Pictures, while Blu-ray Disc had Columbia Pictures, Disney, and 20th Century Fox. Shortly after, however, Sony reportedly put together various deals that reshaped the initial alliances.

> "Warner actually wanted to go HD-DVD. They gave Toshiba the chance to bring another studio into the HD-DVD camp before they turned Blu. Fox was lined up, and told the HD-DVD camp it was going to switch to HD-DVD, which would've also turned Warner exclusively HD-DVD. At the last possible minute, it nixed the deal. [Pittsburgh Post columnist] Don Lindich says it's because Fox received a reported $120 million payout from Sony to stay Blu-ray – Warner then switched and received between $400 and $500 million for its defection."[15]

Following Warner's decision to "go Blu" in January of 2008, Wal-Mart, Best-Buy and Netflix each then decided to phase out HD-DVD. On mid February of 2008, Toshiba announced it would cease developing, manufacturing and marketing HD-DVD players.

The generalized model allows us to capture this application quite naturally. Suppose A is the better technology. Then $y_A \geq y_B$ and $z_A = w_A \geq z_B = w_B$. Note here $z_j = w_j$, unless there is an economic advantage of being an early supporter (which we assume here is not the case). Because of adjustment costs, we have $z_j > y_j$ for $j = A, B$, and because buyers' prices will go up after one of the incumbents is pushed out, we can assume $0 > z_j > y_j$ for $j = A, B$. Then transfers are

$$\hat{p}_\ell(\vec{m}) = p_\ell(\vec{m}) + \delta J_i(\vec{m}^\ell)(w_i - y_i)$$

[15] The Real Reason Warner Went Blu-ray (1/14/08), by Matt Buchanan. (http://gizmodo.com/344680/the-real-reason-warner-went-blu-ray).

Since $0 = J_\ell(\vec{m}^\ell)(z_\ell - w_\ell) < J_j(\vec{m}^\ell)(w_j - y_j)$, transfers from the principals are higher than in the benchmark model, and thus positive. However, since $w_j < 0$, competition increases followers' welfare. What makes this a costly actions model is the fact that a monopolist would increase prices for buyers. In addition to this *price effect* there could be a *growth effect*, in which the size of the market grows markedly after consolidation. In fact, this seems to have been the case in the HD-DVD / Blu-Ray war. This growth effect could well overturn the negative price effect. In this case, we would have $z_j > w_j > 0$, and competition would reduce followers' welfare.

5.3. **Contingent Transfers.** So far we have assumed that leaders offer instantaneous cash transfers in exchange for a commitment of support. Transfers that occurred in the past are sunk, and hence do not affect the incentives in subsequent periods. Alternatively one can assume that the leader and the follower agree on a contingent transfer in exchange for support; a "partnership" offer instead of a buyout. This in fact seems the most appropriate assumption in some applications, as in the case of endorsements by party elders in presidential primaries. In this case candidates negotiate with party elders their support, but they do so in exchange of future promises.

As it was the case with previous extensions, substituting cash for promises can change the conditions for existence of a fully competitive equilibrium, but does not alter the conclusions regarding the ranking of competition and monopoly. Let $\tilde{W}_{out}(\vec{m}'|p_j(\vec{m}))$ denote the value in state $\vec{m}' = (m'_A, m'_B)$ of a committed follower locked with a promise $p_j(\vec{m})$ acquired towards leader j in state $\vec{m} = (m_A, m_B)$. Note that

$$
\begin{aligned}
\tilde{W}_{out}(\vec{m}'|p_A(\vec{m})) &= \sum_t \Pr(j \text{ wins in } t \text{ periods} \,|\, \vec{m}')\, \delta^t [w_j + p_j(\vec{m})] \\
&\quad + \sum_t \Pr(\ell \text{ wins in } t \text{ periods} \,|\, \vec{m}')\, \delta^t w_\ell \\
&= W_{out}(\vec{m}') + \underbrace{\sum_t \Pr(j \text{ wins in } t \text{ periods} \,|\, \vec{m}')\, \delta^t p_j(\vec{m})}_{\tilde{p}_j(\vec{m}', \vec{m})},
\end{aligned} \tag{13}
$$

where $W_{out}(\vec{m}')$ denotes the value of a committed follower in state $\vec{m}'$ in the cash game, and $\tilde{p}_j(\vec{m}', \vec{m})$ gives the expected value of the contingent transfer $p_j(\vec{m})$ in state $\vec{m}'$. Note then that the value function $\tilde{W}_{out}(\vec{m}'|p_\ell(\vec{m}))$ is separable in transfers and the value derived from implementing the alternative. Thus when ℓ meets an uncommitted follower in state $\vec{m}$, she offers a contingent payment $p_\ell(\vec{m})$ such that

$$
\tilde{p}_\ell(\vec{m}^\ell, \vec{m}) + \delta W_{out}(\vec{m}^\ell) = \delta \tilde{W}(\vec{m}) \tag{14}
$$

This implies that the continuation payoff of a follower after he meets one of the leaders is $\delta\tilde{W}(\vec{m})$ no matter what, and therefore the recursive representation of $\tilde{W}(\vec{m})$ is given by (3) as in the "cash" game, so that $\tilde{W}(q,q) = W(q,q)$; i.e., the value of the uncommitted follower at the beginning of the promises game is equal to the value in the cash game. This moreover implies by (14) that the expected value of the payment in the promise game is the same as in the cash game.

Now, to evaluate existence of a fully competitive equilibrium (for our large n results), we need to consider the value of the leader. And in this regard there is in fact a crucial difference with the benchmark cash model. Note that since promises are executed if and only when the leader wins, present exchanges now affect the incentives for future exchanges and must be incorporated on the value function. In particular, the relevant state in the promises game is composed of the number of additional followers that each leader needs in order to win, as before, but now also the stock of promises that a leader brings to the table when meeting another follower.

This difference complicates the algebra, but does not alter our main results. To see this note that after a leader wins, she obtains a payoff composed of a direct benefit $\overline{v}_\ell$ and a transfer from/to all committed followers. These two components are, indeed, additively separable. Moreover, this property still holds recursively, which implies that the value function of the leader in any state – now with the stock of promises as part of the state – is also additively separable in the utility for winning and the promises collected if and when she wins. It follows immediately that Proposition 4.4 extends to this case and a fully competitive equilibrium exists for sufficiently high $\overline{v}$ or $\overline{v} - \underline{v}$. A similar argument holds for the monopoly case, and the welfare comparison in the paper holds.

6. Conclusion

In this paper, we consider a class of dynamic collective action problems in which either a single monopolistic principal or two competing principals vie for the support of members of a group. We focus on the dynamic problem that emerges when agents' decisions to support an alternative are irreversible and agents negotiate and commit their support to the principal sequentially.

A danger for the agents in this context is that a principal may be able to poach agents to her side by exploiting competition among members of the group. Would agents benefit from introducing competition between opposing principals? We show that when the principals' policy provides value to the agents, competition reduces agents' welfare. The key to this result is the concept of free-riding at the core of the literature on contracting with externalities. Competition hurts agents because it reduces their free-riding opportunities.

In the main part of the paper we worked with a stripped down model in which followers' payoffs in terminal nodes are unaffected by whether they supported the eventual winner, the eventual loser, or remained uncommitted. In Section 5.2 we generalize this model, allowing followers' payoffs to depend on whether each follower backed a winning or losing candidate. The generalized model allows us to extend the insights of this paper to a wide range of applications, including corporate takeovers, vote buying, endorsements, and exclusive deals. The key point that holds in all versions of the model is that competition hurts followers when it reduces free riding opportunities. Equivalently, competition reduces agents' welfare if and only if the principals' policy provide value to what Segal (1999) calls *non-traders*; i.e., to agents who did not commit their support in favor or against the principal.

Two issues are left for future work. First, in our analysis we assumed that it is always the leaders who can make proposals to the followers. This assumption is prevalent in the literature on contracting with externalities (see Genicot and Ray (2006), Rasmusen, Ramseyer, and Wiley Jr (1991), Rasmusen and Ramseyer (1994), Segal (1999, 2003), Segal and Whinston (2000), and Möller (2007)) and is invoked here for simplicity. In our sequential model, however, the assumption also carries conceptual grip. This was recognized by Rasmusen, Ramseyer, and Wiley Jr (1991), who note (footnote 4) that "relaxing this assumption raises the danger of extortion by the crucial customers, who could demand all of the second-period monopoly profits for an agreement not to patronize the rival."

This *hold up* problem is also present in our model when followers have sufficient proposal power.[16] Consider for simplicity the monopolistic model when followers have all proposal power. In this case, the principal knows that the pivotal follower will always be able to extract all winning prize v. Anticipating this, her willingness to pay in any state $m > 1$ is zero. When the winning prize v or the discount factor δ are large enough, this hold-up problem induces an equilibrium in mixed strategies in which meetings in $m > 1$ are unsuccessful with positive probability. The endogenous positive probability of a failure of negotiations delays the arrival of the only profitable state for followers, making followers in prior states indifferent between waiting or making the principal a relevant offer.

The hold-up problem in this context is certainly interesting, but basically orthogonal to the free-riding effect we study in this paper. With arbitrary proposal power to the followers, the comparison between monopoly and competition will naturally depend on how hold-up affects the comparison between monopoly and competition, and on the weight

[16]Suppose that in any bilateral meeting, leaders make proposals with probability $\psi \in (0, 1)$, followers with probability $1 - \psi$. By continuity, giving followers little bargaining power (i.e., $\psi \to 1$) does not change our results (a proof of this result is available upon request).

of the two effects. This opens the possibility that if hold-up were to favor competition, competition might be preferred to monopoly when followers have a relatively high ability to make proposals to principals, even with public goods. We leave a full characterization of the net effect with arbitrary proposal power for future research.

The second issue that we have left for future work is a systematic treatment of heterogeneity among followers. The difficulty of introducing heterogeneity among followers is that in addition to tracking the number of followers that each leader needs to win, the state space must now also keep track of the number of agents of each type that remain uncommitted, say $n(\theta)$. Thus, with k types, this requires $k + 2$ dimensions. This is problematic because the value that agents obtain in general depends on combinations of these $k + 2$ dimensions, and thus on the history of transactions in a complicated way. This makes the idea of working with a large number of fully heterogeneous followers a daunting task.

Allowing heterogeneity can be very interesting in many of the applications we considered. Agents could differ in the value they give to each alternative, in their discount factors, or in the probability of meeting each principal. However, to the best of our knowledge there is nothing to suggest that introducing heterogeneity would change our results. To illustrate this, we consider a simple example with three agents, in which we allow the probability of each agent meeting each leader be heterogeneous (see Remark 7.5 in the online Appendix). We show that if the winning prize for the principals is sufficiently large there is a fully competitive equilibrium in the competitive game, and a unique equilibrium in the monopolistic game, such that monopoly is preferred to competition. This is of course just one example of the multiple sources of heterogeneity that are possible here. While we were able to explore other sources of heterogeneity (e.g., in agents' willingness to pay) a general analysis is left for future research.

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7. Appendix

Proof of Proposition 4.1. We begin allowing a MPE in mixed strategies. When the leader meets follower i in state m, she makes an offer $p(m)$ with probability $\gamma_m \in [0,1]$. The follower accepts the offer with probability $\alpha_m \in [0,1]$. Note that the follower i meeting the leader in state m accepts only if $\delta w_{out}(m-1) + p(m) \geq \delta w(m)$, and accepts with probability one if this inequality holds strictly. Note that since i accepts offers $p(m) > -\delta[w_{out}(m-1) - w(m)]$ with probability one, then any such proposal cannot be offered in equilibrium, for L could make a lower offer and still get accepted. Thus, whenever L meets a follower i in state m, she offers

$$p(m) = \begin{cases} -\delta[w_{out}(m-1) - w(m)] & \text{if (16) holds} \\ -\infty & \text{otherwise.} \end{cases} \tag{15}$$

L is willing to make the offer is state m if

$$\alpha_m[\delta v(m-1) - p(m)] + (1-\alpha_m)\delta v(m) \geq \delta v(m),$$

which boils down to

$$p(m) \leq \delta[v(m-1) - V(m)],$$

as before. Thus the leader obtains a non-negative payoff from making an offer if and only if

$$s(m) \equiv [v(m-1) - v(m)] + [w_{out}(m-1) - w(m)] \geq 0 \tag{16}$$

Now suppose that in equilibrium (16) holds strictly in state m. Then the follower meeting the leader in state m must accept all such offers; i.e., $\alpha_m = 1$. This is because since the follower accepts any offer higher than $-\delta[w_{out}(m-1) - w(m)]$, if $\alpha_m < 1$ the leader would increase the offer slightly, getting a discrete gain in payoffs. Thus, if in equilibrium the follower rejects the leader's offer with positive probability in state m, (16) must hold with equality in state m; i.e., if $\alpha_m < 1$, then

$$s(m) = [v(m-1) - v(m)] + [w_{out}(m-1) - w(m)] = 0$$

The value of an uncommitted follower in state m is

$$w(m) = \left(\frac{2}{n+2m-1}\right)\delta w(m) + \left(\frac{n+2m-3}{n+2m-1}\right)\delta[\gamma_m\alpha_m w(m-1) + (1-\gamma_m\alpha_m)w(m)],$$

or equivalently,

$$w(m) = H_m \delta w(m-1),$$

where

$$H_m \equiv \left(\frac{(n+2m-3)\gamma_m\alpha_m}{n+2m-1-2\delta-(n+2m-3)\delta(1-\gamma_m\alpha_m)}\right)$$

Thus

$$w(m) = \left[\prod_{k=1}^{m} H_m\right] \delta^m w \tag{17}$$

The value of a committed follower in state m is

$$w_{out}(m) = \gamma_m \alpha_m \delta w_{out}(m-1) + (1 - \gamma_m \alpha_m) \delta w_{out}(m)$$

or

$$w_{out}(m) = \left(\frac{\gamma_m \alpha_m \delta}{1 - \delta(1 - \gamma_m \alpha_m)} \right) w_{out}(m-1)$$

so that

$$w_{out}(m) = \left[\prod_{k=1}^{m} \left(\frac{\gamma_k \alpha_k}{1 - \delta(1 - \gamma_k \alpha_k)} \right) \right] \delta^m w. \tag{18}$$

The value for the leader in state m is

$$v(m) = \gamma_m \alpha_m (\delta v(m-1) - p(m)) + (1 - \gamma_m \alpha_m) \delta v(m)$$

or

$$v(m) = \left(\frac{\gamma_m \alpha_m \delta}{1 - \delta(1 - \gamma_m \alpha_m)} \right) (v(m-1) + w_{out}(m-1) - w(m)), \tag{19}$$

Now suppose that in equilibrium L makes a relevant offer in every $m > 1$. We will solve for the equilibrium values and then come back and verify that (16) holds for all m to check that this is an equilibrium. First, note that since L makes a relevant offer in every meeting, (17) boils down to

$$w(m) = \left[\prod_{k=1}^{m} \left(\frac{n + 2k - 3}{n + 2k - 1 - 2\delta} \right) \right] \delta^m w = \left[\prod_{k=1}^{m} r(k) \right] \delta^m w \tag{20}$$

and (18) boils down to

$$w_{out}(m) = \delta^m w. \tag{21}$$

Substituting (20) and (21) in (19), we have

$$v(m) = \delta v(m-1) + \left(1 - \delta \prod_{k=1}^{m} r(k) \right) \delta^m w$$

Recursively we have that

$$v(m) = \delta^m v + \left[\sum_{l=1}^{m} \left(1 - \delta \prod_{k=1}^{l} r(k) \right) \right] \delta^m w \tag{22}$$

Then note that

$$v(m-1)-v(m) = \delta^{m-1}(1-\delta)v+\delta^{m-1}w\left\{(1-\delta)\sum_{l=1}^{m-1}\left(1-\delta\prod_{k=1}^{l}r(k)\right)-\delta\left(1-\delta\prod_{k=1}^{m}r(k)\right)\right\}$$

and

$$w(m)-w_{out}(m-1) = -\left[1-\delta\prod_{k=1}^{m}r(k)\right]\delta^{m-1}w$$

so substituting, (16) is

$$s^{*}(m) = (1-\delta)\delta^{m-1}\left[v+w\sum_{l=1}^{m}\left(1-\delta\prod_{k=1}^{l}r(k)\right)\right]\geq 0$$

which is satisfied if and only if

$$v+w\sum_{l=1}^{m}\left(1-\delta\prod_{k=1}^{l}r(k)\right)\geq 0$$

Because this always holds for $v>0$ and $w>0$, it follows that this is an equilibrium.

Next we show that this is the unique equilibrium with an induction argument. First note from (17) and (19) that for all $m\geq 1$, $v(m)$ and $w(m)$ are maximized when $\gamma_m=\alpha_m=1$. Then $s^{*}(1)\geq 0$ implies $s(1)=[v-v(1)]+[w-w(1)]>0$ whenever $\gamma_1\alpha_1<1$. It follows that in state $m=1$ the leader makes a proposal with probability one; i.e., $\gamma_1=1$. But then $\alpha_1=1$ as well. For suppose $\alpha_1\in(0,1)$. Then $s(1)>0$ and the leader would gain by increasing the offer slightly, getting it accepted with probability one. Now suppose that in equilibrium $\gamma_t=\alpha_t=1$ for all $t<m$. Consider the surplus in state m. Note that $v(m-1)$ and $w_{out}(m-1)$ are exactly as in the equilibrium characterized above. Since $v(m)$ and $w(m)$ are maximized when $\gamma_m=\alpha_m=1$, then $s^{*}(m)\geq 0$ implies $s(m)>0$ whenever $\gamma_m\alpha_m<1$. Thus $\gamma_m=1$. As before, then also $\alpha_1=1$, for otherwise $s(m)>0$ and the leader would gain by increasing the offer slightly, getting it accepted with probability one.
■

Proof of Proposition 4.2. Note that the total surplus in the game cannot be higher than $v_A+v_B+n\left(w_A+w_B\right)=\overline{S}$ and any player can guarantee a payoff of zero by declining all offers or not making any offers. Therefore, any value function, given the state, is contained in the compact set $\left[0,\overline{S}\right]$. We are going to prove existence by combining a fixed point argument and a construction of equilibrium node by node.

The proof of existence follows a series of steps. We start first be defining an equilibrium in a suitable way to apply regular fixed point arguments. We then proceed by focusing on a state $\vec{m} \geq (2,2)$, and we assume that there is an equilibrium in the continuation of $\vec{m}$. Equivalently, we assume

$$\left\{W\left(\vec{m}^j\right), W^{out}\left(\vec{m}^j\right)\right\}_{j=A,B} \in \left[0,\overline{S}\right]^4 \text{ and } \left\{V_A\left(\vec{m}^j\right), V_B\left(\vec{m}^j\right)\right\}_{j=A,B} \in \left[0,\overline{S}\right]^4 \tag{23}$$

are well defined values. This assumption allows us to determine the value in states $\vec{m}^A$ and $\vec{m}^B$ for committed and uncommitted followers, as well as for the leaders. Using Kakutani's fixed point theorem we can show that there is an equilibrium in that particular state $\vec{m}$, conditional on the continuation values in states $\vec{m}^A$ and $\vec{m}^B$. The third step is to deal with all end nodes of the form $(1, m_B)$ and $(m_A, 1)$ for $m_A, m_B \geq 2$ and show that a slight modification of the same arguments give existence in all those states. This modification is needed because the continuation values when the leaders win is fixed, but the other continuation value is given by the state in which the losing side actually obtains the support of an extra follower. This methodology leaves us with the final step which is dealing with the nodes $(1,1)$, $(1,2)$ and $(2,1)$. Example 4.3 in the text has in fact already constructed an equilibrium that emerges in these nodes so the existence of equilibrium follows.

Step 1: Definition of equilibrium

First let's define an equilibrium of the game node by node. An equilibrium of node $\vec{m}$ is (i) a set of value functions $\{W(\vec{m}), W^{out}(\vec{m})\} \in \left[0,\overline{S}\right]^2$, $\{V_A(\vec{m}), V_B(\vec{m})\}_{j=A,B} \in \left[0,\overline{S}\right]^2$, (ii) transfers $\{p_j(\vec{m})\}_{j=A,B} \in \left[-\overline{S},\overline{S}\right]^2$, and (iii) proposal probabilities $\{\gamma_j(\vec{m}), \alpha_j(\vec{m})\}_{j=A,B} \in [0,1]^4$ such that the following conditions hold:

(1) Leader $j = A, B$ sets transfers to maximize her utility:

$$p_j(\vec{m}) + \delta W^{out}\left(\vec{m}^j\right) = \delta W(\vec{m})$$

(2) Leaders and followers make and accept proposals optimally. Leader j makes a proposal to the follower in state $\vec{m}$ with probability

$$\gamma_j(\vec{m}) = \left\{ \begin{array}{c} 1 \text{ if } S_j(\vec{m}) > 0 \\ [0,1] \text{ if } S_j(\vec{m}) = 0 \\ 0 \text{ if } S_j(\vec{m}) < 0 \end{array} \right\}$$

where as before $S_j(\vec{m})$ is the surplus $S_j(\vec{m}) = (W^{out}(\vec{m}^j) + V_j(\vec{m}^j)) - (W(\vec{m}) + V_j(\vec{m}))$, and the follower accepts the leader's offer with probability

$$\alpha_j(\vec{m}) = \left\{ \begin{array}{c} 1 \text{ if } p_j(\vec{m}) > \delta W(\vec{m}) - \delta W^{out}(\vec{m}^j) \\ [0,1] \text{ if } p_j(\vec{m}) = \delta W(\vec{m}) - \delta W^{out}(\vec{m}^j) \\ 0 \text{ if } p_j(\vec{m}) < \delta W(\vec{m}) - \delta W^{out}(\vec{m}^j) \end{array} \right\}$$

(3) The value of a committed follower verifies

$$W_{out}(\vec{m}) = \delta \times \frac{\sum_{j=A,B} \pi_j \times (\gamma_j(\vec{m})\,\alpha_j(\vec{m})) \times W^{out}(\vec{m}^j)}{1 - \delta \sum_{j=A,B} \pi_j \times (1 - \gamma_j(\vec{m})\,\alpha_j(\vec{m}))},$$

the value of an uncommitted follower verifies

$$W(\vec{m}) = \frac{\sum_{j=A,B} \pi_j \times (\gamma_j(\vec{m})\,\alpha_j(\vec{m})) \times \delta W(\vec{m}^j)}{\frac{m_A+m_B-1-\delta}{m_A+m_B-2} - \delta \times \sum_{j=A,B} \pi_j \times (1 - \gamma_j(\vec{m})\,\alpha_j(\vec{m}))},$$

and leader j's value verifies

$$V^j(\vec{m}) = \frac{\sum_{j=A,B} \pi_j \times (\gamma_j(\vec{m})\,\alpha_j(\vec{m})) \times (\delta V^j(\vec{m}^j) - p_j(\vec{m}))}{1 - \delta \sum_{j=A,B} \pi_j \times (1 - \gamma_j(\vec{m})\,\alpha_j(\vec{m}))}$$

Step 2: Interior Nodes: $\vec{m} \geq (2,2)$

Let $\left[0,\overline{S}\right]^4 \cup \left[-\overline{S},\overline{S}\right]^2 \cup [0,1]^4 = \Upsilon \subset \mathcal{R}^{10}$ and let $\left(\widehat{W}, \widehat{W}^{out}, \widehat{V}^A, \widehat{V}^B\right) \in \left[0,\overline{S}\right]^4$, $(\widehat{p}_A, \widehat{p}_B) \in \left[-\overline{S},\overline{S}\right]^2$ and $\{\widehat{\gamma}_A, \widehat{\alpha}_A, \widehat{\gamma}_B, \widehat{\alpha}_B\} \in [0,1]^4$ be arbitrary. We now define the correspondences to show existence of equilibrium at $\vec{m}$ whenever (23) holds.

(1) We define leader j's best response $f_{\gamma_j} : \Upsilon \to [0,1]$ for $j = A, B$ as

$$f_{\gamma_j}\left(\widehat{W}, \widehat{V}^j\right) = \left\{ \begin{array}{c} 1 \text{ if } W^{out}(\vec{m}^j) + V_j(\vec{m}^j) > \widehat{W} + \widehat{V}^j \\ [0,1] \text{ if } W^{out}(\vec{m}^j) + V_j(\vec{m}^j) = \widehat{W} + \widehat{V}^j \\ 0 \text{ if } W^{out}(\vec{m}^j) + V_j(\vec{m}^j) < \widehat{W} + \widehat{V}^j \end{array} \right\}$$

and the follower's best response when facing leader j, $f_{\alpha_j} : \Upsilon \to [0,1]$ for $j = A, B$ as

$$f_{\alpha_j}\left(\widehat{W}, \widehat{p}_j\right) = \left\{ \begin{array}{c} 1 \text{ if } \widehat{p}_j > \delta\widehat{W} - \delta W^{out}(\vec{m}^j) \\ [0,1] \text{ if } \widehat{p}_j = \delta\widehat{W} - \delta W^{out}(\vec{m}^j) \\ 0 \text{ if } \widehat{p}_j < \delta\widehat{W} - \delta W^{out}(\vec{m}^j) \end{array} \right\}$$

Note that both f_{γ_j} and f_{α_j} for $j = A, B$ are upper hemicontinuous correspondences with nonempty images. To show that the image is convex take $(x_1, x_2) \in f_{\gamma_j}\left(\widehat{W}, \widehat{V}^j\right)$ which implies that $W^{out}(\vec{m}^j) + V_j(\vec{m}^j) = \widehat{W} + \widehat{V}^j$ and since in that case $f_{\gamma_j}\left(\widehat{W}, \widehat{V}^j\right) = [0,1]$, we have that the convex combination of (x_1, x_2) is also in $[0,1]$. The same applies to f_{α_j}.

(2) We define the leader's optimal transfer correspondence $f_{p_j} : \Upsilon \to \left[-\overline{S}, \overline{S}\right]$ for $j = A, B$ as

$$f_{p_j}\left(\widehat{W}\right) = \delta\left(\widehat{W} - W^{out}\left(\vec{m}^j\right)\right)$$

Note these are continuous functions which are trivially non empty and convex (they are single valued).

(3) We define the committed followers' value function $f_{W^{out}} : \Upsilon \to \left[0, \overline{S}\right]$ as

$$f_{W^{out}}\left(\widehat{\gamma}_A, \widehat{\alpha}_A, \widehat{\gamma}_B, \widehat{\alpha}_B\right) = \delta \times \frac{\sum_{j=A,B} \pi_j \times \left(\widehat{\gamma}_j \times \widehat{\alpha}_j\right) \times W^{out}\left(\vec{m}^j\right)}{1 - \delta \sum_{j=A,B} \pi_j \times \left(1 - \widehat{\gamma}_j \times \widehat{\alpha}_j\right)},$$

the uncommitted followers' value function $f_W : \Upsilon \to \left[0, \overline{S}\right]$ as

$$f_W\left(\widehat{\gamma}_A, \widehat{\alpha}_A, \widehat{\gamma}_B, \widehat{\alpha}_B\right) = \frac{\sum_{j=A,B} \pi_j \times \left(\widehat{\gamma}_j \times \widehat{\alpha}_j\right) \times \delta W\left(\vec{m}^j\right)}{\frac{m_A + m_B - 1 - \delta}{m_A + m_B - 2} - \delta \times \sum_{j=A,B} \pi_j \times \left(1 - \widehat{\gamma}_j \times \widehat{\alpha}_j\right)},$$

and the leaders' value functions $f_{V_j} : \Upsilon \to \left[0, \overline{S}\right]$ for $j = A, B$ as

$$f_{V^j}\left(\widehat{p}_j, \widehat{\gamma}_A, \widehat{\alpha}_A, \widehat{\gamma}_B, \widehat{\alpha}_B\right) = \frac{\sum_{j=A,B} \pi_j \times \left(\widehat{\gamma}_j \times \widehat{\alpha}_j\right) \times \left(\delta V^j\left(\vec{m}^j\right) - \widehat{p}_j\right)}{1 - \delta \sum_{j=A,B} \pi_j \times \left(1 - \left(\widehat{\gamma}_j \times \widehat{\alpha}_j\right)\right)}$$

Note that for any $\delta < 1$ these are continuous functions which are trivially non empty and convex (they are single valued).

Note that the transformation $\Gamma : \Upsilon \to \Upsilon$ defined as

$$\Gamma = \left(\left\{f_{\gamma_j}\right\}_{j=A,B}, \left\{f_{\alpha_j}\right\}_{j=A,B}, \left\{f_{p_j}\right\}_{j=A,B}, f_{W^{out}}, f_W, \left\{f_{V_j}\right\}_{j=A,B}\right)$$

is an upper hemicontinuous correspondence, defined on a non empty, compact convex subset of $\mathcal{R}^{10}$ (i.e. $\Upsilon = \left[0, \overline{S}\right]^4 \cup \left[-\overline{S}, \overline{S}\right]^2 \cup [0,1]^4$) with images that are non-empty and convex. Thus Kakutani's fixed point Theorem applies and there is some

$$x^* = \left\{W^*, W^{out*}, V^{A*}, V^{B*}, p_A^*, p_B^*, \gamma_A^*, \alpha_A^*, \gamma_B^*, \alpha_B^*\right\} \in \Upsilon \quad \text{such that } \Gamma\left(x^*\right) = x^*.$$

Step 3: Boundary Nodes excluding $(1,2)$, $(2,1)$ and $(1,1)$.

Let's consider now $\vec{m} = (1, m_B)$ with $m_B \geq 2$ and $\vec{m} = (m_A, 1)$ with $m_A \geq 2$. Take an arbitrary $\vec{m} = (1, m_B)$ and note that $W\left(\vec{m}^A\right) = W^{out}\left(\vec{m}^A\right) = w_A$, $V_A\left(\vec{m}^A\right) = \overline{v}_A$ and $V_B\left(\vec{m}^A\right) = \underline{v}_B$, so (23) turns into

$$\left\{W\left(\vec{m}^B\right), W^{out}\left(\vec{m}^B\right)\right\} \in \left[0, \overline{S}\right]^2 \text{ and } \left\{V_A\left(\vec{m}^B\right), V_B\left(\vec{m}^B\right)\right\} \in \left[0, \overline{S}\right]^2 \tag{24}$$

Therefore as long as the values are well defined for $\vec{m}^B = (1, m_B - 1)$ a similar argument to that used in Step 2 leads to existence of equilibrium as long as $m_B > 2$. This is because at $(1,2)$, (24) is satisfied for a given set of values.

Step 4: Nodes $(1,2)$, $(2,1)$ and $(1,1)$.

Since in Example 4.3 we showed that there was an equilibrium with trade in all nodes, this step is straightforward. The values for $W(1,1)$, $W(1,2)$, $W(2,1)$ and $W(2,2)$ were presented in the example. Because the equilibrium involves trade in every node we must have that $\left\{\gamma_j(\vec{m}), \alpha_j(\vec{m})\right\}_{j=A,B} = (1,1)$ for any $\vec{m} \in \left\{(1,2), (2,1), (1,1)\right\}$. Note that the explicit form of $W_{out}(\vec{m})$ in the case of trade after every meeting was given by (6) and the transfers are fully determined by $p_j(\vec{m}) = \delta(W(\vec{m}) - W_{out}(\vec{m}^j))$. Finally, expression (7) determines $V^j(\vec{m})$ when considering the transfers and the fact that in the last node we have $V^j(1,1) = \delta\pi_j(\overline{v}_j + w_j) + \delta\pi_{-j}\underline{v}_j$.

Wrap Up: Steps 4, 3 and 2 define a recursion that gives existence. ■

Proof of Proposition 4.4. We will show that for any $j = A, B$ there is a $v^* \in \mathbb{R}_+$ such that if $\overline{v}_j \geq v^*$, when all players play the proposed equilibrium strategies, $S_j(\vec{m}) \geq 0$ for all $\vec{m}$.

Consider the surplus expression (2). Note that (5) and (6) imply that $W_{out}(\vec{m}^j) - W(\vec{m})$ does not depend on $(\overline{v}_A, \underline{v}_A, \overline{v}_B, \underline{v}_B)$, and is therefore a constant. It follows that $\overline{v}_{-j}$ and $\underline{v}_{-j}$ do not affect $S_j(\vec{m})$, and $\overline{v}_j$ and $\underline{v}_j$ enter $S_j(\vec{m})$ only through the term $V_j(\vec{m}^j) - V_j(\vec{m})$. Now, note that having expressed $p_j(\vec{m})$ in terms of the primitives of the model, we can solve (7) as a stand alone partial difference equation, to obtain

$$V_j(\vec{m}) = (\delta\pi_j)^{m_j}\left[\sum_{l=0}^{m_{-j}-1}\binom{m_j-1+l}{l}(\delta\pi_{-j})^l\right]\overline{v}_j + (\delta\pi_{-j})^{m_{-j}}\left[\sum_{l=0}^{m_j-1}\binom{m_{-j}-1+l}{l}(\delta\pi_j)^l\right]\underline{v}_j - H(\vec{m}). \tag{25}$$

where $H(\vec{m})$ is a function of prices $p_j(r,s)$ for $r \leq m_j, s \leq m_{-j}$, which are constant in $(\overline{v}_j, \underline{v}_j, \overline{v}_{-j}, \underline{v}_{-j})$ by (5) and (6). Thus $V_j(\vec{m}^j) - V_j(\vec{m})$ is given by

$$\left\{(\delta\pi_j)^{m_j-1}\left[\sum_{l=0}^{m_{-j}-1}\binom{m_j-2+l}{l}(\delta\pi_{-j})^l\right] - (\delta\pi_j)^{m_j}\left[\sum_{l=0}^{m_{-j}-1}\binom{m_j-1+l}{l}(\delta\pi_{-j})^l\right]\right\}\overline{v}_j$$

$$-(\delta\pi_{-j})^{m_{-j}}\binom{m_A+m_B-2}{m_j-1}(\delta\pi_j)^{m_j-1}\underline{v}_j + H(\vec{m}) - H(\vec{m}^j).$$

We will show that this expression can be made arbitrarily large by increasing $\overline{v}_j$ or reducing $\underline{v}_j$. The last line is a constant. From the second line it follows that all else equal, there is a $\underline{v}^*$ such that if $\underline{v}_j < \underline{v}^*$, then $S_j(\vec{m}) > 0$. Next, after some algebra, the bracket in the first line can be written as

$$(\delta\pi_j)^{m_j-1}\left[(1-\delta)\sum_{l=0}^{m_{-j}-1}\binom{m_j-1+l}{l}(\delta\pi_{-j})^l + \binom{m_A+m_B-2}{m_j-1}(\delta\pi_{-j})^{m_{-j}}\right] > 0.$$

Thus, all else equal, there is a $\overline{v}^*$ such that if $\overline{v}_j > \overline{v}^*$, then $S_j(\vec{m}) > 0$. ■

Proof of Theorem 4.5. The first statement follows as a corollary of Theorem 4.6. Now consider the second part. From expression (5), we have

(26)
$$W(q,q)=\sum_{l=0}^{q-2}\left(\prod_{k=0}^{q-1+l}C(2q-k)\right)\times\binom{q-1+l}{l}\times\left[(\delta\pi_A)^q(\delta\pi_B)^l w_A+(\delta\pi_B)^q(\delta\pi_A)^l w_B\right]$$

On the other hand, with a single alternative, $w(q)=\left(\prod_{m=1}^{q}r(m)\right)\delta^q w$. Now, since $r(k)=\frac{n+2k-3}{n+2k-(1+2\delta)}$ by definition and $n=2q-1$, we have $r(k)=C(q+k)$. Thus

$$w(q)=\left(\prod_{k=1}^{q}C(q+k)\right)\delta^q w=\left(\prod_{k=0}^{q-1}C(2q-k)\right)\delta^q w \tag{27}$$

Suppose without loss of generality that $w_A>w_B$. We want to show that for sufficiently large q the equilibrium payoff of an uncommitted follower in the game with a single alternative yielding value w_B is larger than his (competitive) equilibrium payoff in the game with two alternatives yielding value w_A and w_B. Suppose not. Then making $w=w_B$ in (27), and dividing (26) by (27),

$$U(q)\equiv\sum_{l=0}^{q-2}\left(\prod_{k=q}^{q-1+l}C(2q-k)\right)\times\binom{q-1+l}{l}\times\left[(\pi_A)^q(\delta\pi_B)^l+(\pi_B)^q(\delta\pi_A)^l\left(\frac{w_B}{w_A}\right)\right]\geq\frac{w_B}{w_A}$$

Now, since $\delta\leq 1$, $\prod_{k=q}^{q-1+l}C\left(2q-k\right)\leq 1$, and $w_B/w_A<1$, for any integer q, we have

$$U(q)<\sum_{j=A,B}(\pi_j)^q\sum_{l=0}^{q-2}\frac{\Gamma\left(q+l\right)}{\Gamma\left(q\right)}\frac{(\delta\pi_j)^l}{l!}\equiv\overline{U}(q)$$

where for any integer k, we define $\Gamma\left(k\right)\equiv(k-1)!$. Now define the function

$$F(a,b,c,z)\equiv\sum_{l=0}^{\infty}\frac{\binom{a+l}{l}\binom{b+l}{l}}{\binom{c+l}{l}}z^l,$$

and note that we can write

(28)
$$\begin{aligned}\overline{U}(q)&=\sum_{j=A,B}(\pi_j)^q\left(F\left(q,1,1,\delta\pi_{-j}\right)-\sum_{l=q-1}^{\infty}\frac{\Gamma\left(q+l\right)}{\Gamma\left(q\right)}\frac{(\delta\pi_{-j})^l}{l!}\right)\\&=\sum_{j=A,B}\left\{\left(\frac{\pi_j}{1-\delta\pi_{-j}}\right)^q-(\pi_j)^q\sum_{l=q-1}^{\infty}\frac{\Gamma\left(q+l\right)}{\Gamma\left(q\right)}\frac{(\delta\pi_{-j})^l}{l!}\right\}.\end{aligned}$$

where the equality follows from the fact that $F(a,b,b,z) = (1-z)^{-a}$ (see Property 15.1.8 for hypergeometric functions in Abramowitz and Stegun (2012); p.556). Noting that

$$\left(\frac{\pi_j}{1-\delta\pi_{-j}}\right) = \left(\frac{1-\pi_{-j}}{1-\delta\pi_{-j}}\right) < 1$$

as long as $\delta < 1$, it follows that for any $\varepsilon > 0$ there is a Q such that if $q > Q$, then $\overline{U}(q) < \varepsilon$. Thus, for any π_B/π_A, there is a Q such that $U(q) < \pi_B/\pi_A$ whenever $q > Q$. ■

Proof of Theorem 4.6. Let $\gamma_j(\vec{m})$ be the probability that leader $j = A, B$ makes an offer in state $\vec{m}$, $\alpha_j(\vec{m})$ be the probability that an uncommitted follower accepts an offer from leader $j = A, B$ in state $\vec{m}$, and $\mu_j(\vec{m}) \equiv \gamma_j(\vec{m})\,\alpha_j(\vec{m})$. Then

$$\begin{aligned} W(\vec{m}) = {} & \left(\frac{1}{m_A+m_B-1}\right)\delta W(\vec{m}) \\ & + \left(\frac{m_A+m_B-2}{m_A+m_B-1}\right)\sum_{j=A,B}\pi_j\left(\begin{array}{c}\mu_j(\vec{m})\,\delta W(\vec{m}^j) \\ +(1-\mu_j)\,\delta W(\vec{m})\end{array}\right). \end{aligned} \tag{29}$$

For $j = A, B$, define

$$\xi_j(\vec{m}) \equiv \frac{\delta\pi_j\mu_j(\vec{m})}{\left(\frac{m_A+m_B-1}{m_A+m_B-2}\right)(1-\delta)+\delta\sum_{j=A,B}\pi_j\mu_j(\vec{m})}$$

whenever $\vec{m} \neq (1,1)$, and $\xi_j(1,1) \equiv 0$. Then we can write (29) as

$$W(\vec{m}) = \sum_{j=A,B}\xi_j(\vec{m})\,W\left(\vec{m}^j\right) \tag{30}$$

for all $\vec{m}$ and $j = A, B$. Note in particular that the recursion (30) implies that if $w_A, w_B > 0$ (as we are assuming here), then $W(\vec{m}) \geq 0$ for all $\vec{m}$.

We need to show that $W(q,q) < \max\{w_A(q), w_B(q)\}$. The proof follows from three lemmas. Lemma 7.2 establishes the result for $q = 1$ and shows an additional result for all boundary states which is used in Lemma 7.3. The proof for interior states is by induction. Lemma 7.3 establishes the base case, and Lemma 7.4 the induction step. Iterative application of the induction step covers the entire state space and establishes the result. We begin with Lemma 7.1, which establishes an intermediate result that is used in the proof of Lemmas 7.2 and 7.4. ■

Lemma 7.1 (Bound)**.** *In any MPE of the game* $\Gamma(\vec{m})$*,*

$$W(\vec{m}) \leq \max_{j\in\{A,B\}} \left\{\delta r(m_j) W\left(\vec{m}^j\right)\right\}$$

Proof of Lemma 7.1. Note that for all $m_A \geq 2, m_B \geq 2$ we have

$$W(\vec{m}) = \xi_A(\vec{m}) W\left(\vec{m}^A\right) + \xi_B(\vec{m}) W\left(\vec{m}^B\right)$$

Thus we need to show that

$$\sum_{j=A,B} \xi_j(\vec{m}) W\left(\vec{m}^j\right) \leq \delta \max\left\{r(m_A) W\left(\vec{m}^A\right), r(m_B) W\left(\vec{m}^B\right)\right\}$$

(a) Assume first that $W\left(\vec{m}^A\right) \geq W\left(\vec{m}^B\right)$ so it is sufficient if

$$\left[\sum_{j=A,B} \xi_j(\vec{m})\right] W\left(\vec{m}^A\right) \leq \delta \max\left\{r(m_A) W\left(\vec{m}^A\right), r(m_B) W\left(\vec{m}^B\right)\right\}$$

Note that since $r(m) = \frac{n+2m-3}{n+2m-(1+2\delta)}$, then

$$\left[\sum_{j=A,B} \xi_j(\vec{m})\right] = \frac{\delta\left[\pi_A\mu_A(\vec{m}) + \pi_B\mu_B(\vec{m})\right]}{\frac{m_A+m_B-1}{m_A+m_B-2}(1-\delta) + \delta\left[\pi_A\mu_A(\vec{m}) + \pi_B\mu_B(\vec{m})\right]} \leq \delta \min\left\{r(m_A), r(m_B)\right\}. \tag{31}$$

Then it is sufficient if

$$\min\left\{r(m_A), r(m_B)\right\} W\left(\vec{m}^A\right) \leq \max\left\{r(m_A) W\left(\vec{m}^A\right), r(m_B) W\left(\vec{m}^B\right)\right\}$$

which is true when either $r(m_A) W\left(\vec{m}^A\right) \geq r(m_B) W\left(\vec{m}^B\right)$ or the opposite holds.

(b) Suppose instead that $W\left(\vec{m}^A\right) \leq W\left(\vec{m}^B\right)$. Then it is sufficient if

$$\left[\sum_{j=A,B} \xi_j(\vec{m})\right] W\left(\vec{m}^B\right) \leq \delta \max\left\{\begin{array}{c} r(m_A) W\left(\vec{m}^A\right), \\ r(m_B) W\left(\vec{m}^B\right) \end{array}\right\}$$

and again using (31) it is sufficient if

$$\min\left\{r(m_A), r(m_B)\right\} W\left(\vec{m}^B\right) \leq \max\left\{r(m_A) W\left(\vec{m}^A\right), r(m_B) W\left(\vec{m}^B\right)\right\}$$

which is true. This completes the proof. ■

Lemma 7.2 (Boundaries)**.**

$$W(m_A, 1) < w_B(1) \text{ for all } m_A \geq 1 \quad \text{and} \quad W(1, m_B) < w_A(1) \text{ for all } m_B \geq 1$$

Proof of Lemma 7.2. The result for state $\vec{m} = (1,1)$ follows immediately from the fact that $W(1,1) = 0$. Now consider the remaining boundary states (states adjacent to terminal states). Solving the recursion (30) for the boundaries, we obtain

$$W(m_A,1) = \left(\sum_{l=1}^{m_A-1} \xi_B(m_A-l,1)\prod_{k=0}^{l-1}\xi_A(m_A-k,1) + \xi_B(m_A,1)\right) w_B \tag{32}$$

$$W(1,m_B) = \left(\sum_{l=1}^{m_B-2} \xi_A(1,m_B-l)\prod_{k=0}^{l-1}\xi_B(1,m_B-k) + \xi_A(1,m_B)\right) w_A \tag{33}$$

for all $m_A, m_B \geq 1$.

Consider $\vec{m} = (2,1)$. Note that since

$$W(2,1) = \frac{\delta\pi_B\mu_B(2,1)}{2(1-\delta) + \delta(\pi_A\mu_A(2,1) + \pi_B\mu_B(2,1))} w_B$$

for $n \geq 3$,

$$W(2,1) < \frac{\delta}{2-\delta} w_B < \frac{(n-1)\delta}{n+1-2\delta} w_B = r(1)\delta w_B = w_B(1)$$

By the same argument, $W(1,2) < w_A(1)$. Next, consider $W(m_A,1)$ for $m_A \geq 3$. We have

$$\begin{aligned} W(m_A,1) &= \left(\sum_{l=0}^{m_A-3} \xi_B(m_A-(1+l),1)\prod_{k=0}^{l}\xi_A(m_A-k,1) + \xi_B(m_A,1)\right) w_B \\ &= \left(\begin{array}{c} \sum_{l=0}^{m_A-3}\left[\xi_B(m_A-(1+l),1) + \xi_A(m_A-(1+l),1) - 1\right]\prod_{k=0}^{l}\xi_A(m_A-k,1) \\ -\prod_{k=0}^{m_A-2}\xi_A(m_A-k,1) + (\xi_B(m_A,1) + \xi_A(m_A,1)) \end{array}\right) w_B, \end{aligned}$$

and since $(\xi_B(m_A-(1+l),1) + \xi_A(m_A-(1+l),1)) \leq 1$, it follows that

$$W(m_A,1) \leq (\xi_B(m_A,1) + \xi_A(m_A,1)) w_B < \delta r(1) w_B = w_B(1)$$

Analogously, we have that $W(1,m_B) < w_A(1)$. ■

Lemma 7.3 (Base Case)**.**

$$W(m_A,2) < \max\{w_A(m_A), w_B(2)\} \quad \textit{for all } m_A \geq 2$$

and

$$W(2,m_B) < \max\{w_A(2), w_B(m_B)\} \quad \textit{for all } m_B \geq 2$$

Proof of Lemma 7.3. First, note that

$$\begin{aligned} W(2,2) &\leq \xi(4)\max\{w_B(1), w_A(1)\} \\ &< \delta\left(\frac{2}{3-\delta}\right)\max\{w_B(1), w_A(1)\} \\ &< \delta r(2)\max\{w_B(1), w_A(1)\} = \max\{w_B(2), w_A(2)\} \end{aligned}$$

Next consider $W(m_A, 2)$. By successive application of Lemma 7.1

$$\begin{aligned} W(m_A,2) &\leq \max\{\delta r(m_A) W(m_A-1,2), \delta W(m_A,1) r(2)\} \\ &\leq \max\left\{\delta^2 \prod_{j=0}^{1} r(m_A-j) W(m_A-2,2), \delta^2 \prod_{j=0}^{0} r(m_A-j) W(m_A-1,1) r(2), \delta W(m_A,1) r(2)\right\} \\ &\quad .... \\ &\leq \max\left\{\delta^{m_A} \prod_{j=0}^{m_A-2} r(m_A-j) W(1,2), \max_{k\leq m_A-2}\left\{\delta^{k+1}\prod_{j=0}^{k-1} r(m_A-j) W(m_A-k,1) r(2)\right\}, \delta W(m_A,1) r(2)\right\} \end{aligned}$$

Now, we have shown in Lemma 7.2 that $W(m_A, 1) < w_B(1)$ for all $m_A \geq 1$, and $W(1, m_B) < w_A(1)$ for all $m_B \geq 1$. Using these results in the RHS of the expression above, we get

$$W(m_A,2) < \max\left\{\delta^{m_A} \prod_{j=0}^{m_A-2} r(m_A-j) w_A(1),\ \max_{k\leq m_A-2}\left\{\ \delta^{k+1}\prod_{j=0}^{k-1} r(m_A-j) w_B(1)\ \right\} r(2)\ , \delta r(2) w_B(1)\right\}$$

Using (20) we get that

$$\max_{k\leq m_A-2}\left\{\delta^{k+1}\prod_{j=0}^{k-1} r(m_A-j) w_B(1)\right\} = \delta^2 r(m_A) w_B(1),$$

so

$$W(m_A,2) < \max\left\{\delta^{m_A}\prod_{j=0}^{m_A-2} r(m_A-j) w_A(1), \delta r(2) w_B(1)\right\}.$$

Therefore, using equation (20) and Lemma 7.2 one more time, we have

$$W(m_A,2) < \max\{w_A(m_A), w_B(2)\}.$$

By the same logic, $W(2, m_B) < \max\{w_B(m_B), w_A(2)\}$. ■

Lemma 7.4 (Induction Step)**.** *Consider any state* $\vec{m} \geq (3,3)$*. If*

$$W\left(\vec{m}^B\right) \leq \max\{w_A(m_A), w_B(m_B-1)\} \tag{34}$$

and

$$W\left(\vec{m}^A\right) \leq \max\{w_A(m_A-1), w_B(m_B)\} \tag{35}$$

then

$$W(\vec{m}) \leq \max\{w_A(m_A), w_B(m_B)\} \tag{36}$$

Proof of Lemma 7.4. By Lemma 7.1,

$$W(\vec{m}) \leq \max\left\{\delta r(m_A) W\left(\vec{m}^A\right), \delta r(m_B) W\left(\vec{m}^B\right)\right\} \tag{37}$$

Using (34) and (35), and then noting that $w_j(m_j) = \delta r(m_j)\, w_j(m_j - 1)$ for $j = A, B$, and substituting, (37) becomes

$$W(\vec{m}) \leq \max \left\{ \begin{array}{c} \max\{w_A(m_A), \delta r(m_A)\, w_B(m_B)\}, \\ \\ \max\{\delta r(m_B)\, w_A(m_A), w_B(m_B)\} \end{array} \right\} \leq \max\{w_A(m_A), w_B(m_B)\}$$

■

Proof of Proposition 5.1. The proof of Proposition 4.1 implies that the equilibrium characterized in Proposition 4.1 is still the unique MPE if and only if

$$v \geq -w \sum_{l=1}^{m} \left(1 - \delta \prod_{k=1}^{l} r(k)\right)$$

Thus, for any given $w < 0$, there is a $\overline{v}(w)$ such that this condition is satisfied if $v > \overline{v}(w)$. Similarly, for any given $w < 0$ there is a $\overline{\delta}(w) \in (0, 1)$ such that this condition is satisfied if $\delta > \overline{\delta}(w)$. ■

Remark 7.5. *In the benchmark model we assumed that he matching probabilities are uniform among followers. Here we show with an example that allowing matching probabilities to be heterogeneous across followers does not invalidate our results. For simplicity, we focus on a fully competitive equilibrium of the competitive game.*

Example. When followers differ on a dimension that is payoff relevant the state space must be modified accordingly. This is because the identities of the remaining uncommitted followers may matter. The reason is quite intuitive: a follower needs to know what is the probability of being selected again in the future to calculate the value of waiting (rejecting the offer) and this probability depends crucially on the pool of uncommitted followers.

Here we define the state space Ω in terms of the *identities of committed* followers for each leader. Let N be the set of agents, and $\mathcal{N} = 2^{|N|}$. Then

$$\Omega = \left\{(Q, R) \in \mathcal{N}^2 : Q \cap R = \emptyset\right\}$$

The pair $(C_A, C_B) \in \Omega$ describes the set of followers committed to A (C_A) and to B (C_B). Because followers are heterogenous we have now a set of value functions: $W(C_A, C_B) = \{W_k(C_A, C_B) \text{ for } k \in N\}$. In the single leader case we let $w_k(C)$ be the value of uncommitted follower k when the leader j traded with followers in C.

For simplicity, we assume homogeneous payoffs; i.e., $\overline{v}_j = \overline{v}$ and $\underline{v}_j = \underline{v}$ and $w_j = w$ for $j = A, B$. With heterogenous matching probabilities we need to make assumptions about how this matching process changes after trading. We assume that

$$\pi_{ij}\left(C_A, C_B\right) = \frac{\pi_{ij}}{\sum_{j' \in \{A,B\}} \sum_{i' \notin \{C_A \cup C_B\}} \pi_{i'j'}} \text{ if } i \notin \{C_A \cup C_B\} \tag{38}$$

and 0 if $i \in \{C_A \cup C_B\}$.

Assume that there are three followers, $N = \{1, 2, 3\}$, and the matching probabilities are arbitrary and are updated according to (38). We can show that for sufficiently large $\overline{v}$ the competitive game has a fully competitive equilibrium and the monopolistic game has a unique MPE with trade in every state. In this case payoffs at the beginning of each game for follower 1 are given by

$$\begin{aligned} W_1\left(0,0\right) &= \left(\frac{1}{1 - \pi_{1A} - \pi_{1B}}\right)\left(\frac{\pi_{2A}\pi_{3A} + \pi_{3B}\pi_{2B}}{\pi_{1B} + \pi_{1A}}\right) w \quad \text{and} \\ w_1\left(0\right) &= 2\left(\frac{\pi_3}{1 - \pi_1}\right)\left(\frac{\pi_2}{\pi_1}\right) w \end{aligned}$$

Thus a follower prefers monopoly over competition as long as

$$\left(\frac{\pi_2}{1 - \pi_1}\right)\left(\frac{\pi_3}{\pi_1}\right) > \left(\frac{1}{1 - \pi_{1A} - \pi_{1B}}\right)\left(\frac{\pi_{2A}\pi_{3A} + \pi_{2B}\pi_{3B}}{\pi_{1B} + \pi_{1A}}\right)$$

Assume that $\pi_1 = \pi_{1A} + \pi_{1B}$ so the total probability that 1 meets *a* leader in the competitive game is equal to the probability that he meets the leader in the monopolistic game. Now we have that $W_1\left(0,0\right) < \omega_1\left(0\right)$ as long as

$$\left(\pi_{3A} + \pi_{3B}\right)\left(\pi_{2B} + \pi_{2A}\right) > \pi_{2A}\pi_{3A} + \pi_{3B}\pi_{2B}$$

which is true as long as $\pi_{3A}\pi_{2B} + \pi_{3B}\pi_{2A} > 0$. Thus, if every follower has a chance of meeting both leaders, followers are better off under monopoly. ■