Do Job Destruction Shocks Matter in the Theory of Unemployment?

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Do job destruction shocks matter in the theory of unemployment?

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Abstract.
The current DMP approach to labor markets presumes job destruction shocks are small. We relax that assumption and also allow unfilled jobs, like unemployment, to evolve as a state variable. Calibrating an otherwise standard DMP framework, we identify a remarkable, (almost) perfect, fit of the empirical facts as reported in Shimer (2005, 2012). The results, however, are also consistent with the insights of Davis and Haltiwanger (1992): that unemployment volatility is driven by large but infrequent job separation shocks. The approach not only provides an important synthesis of two literatures which, in other contexts, have appeared contradictory, it also identifies a more traditional view of the timing and progression of recessions.

JEL Codes: E24, E32, J41, J63, J64.

Keywords: search, job destruction, ins-and-outs of unemployment.

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1 Introduction

What explains the wide variation in unemployment over the cycle - is it due to large variations in job destruction rates or due to large variations in the job finding rates of the unemployed - the so-called ins and outs of unemployment? This important issue has a long history (see Darby et al (1986), Davis and Haltiwanger (1992), Shimer (2012), Elsby et al (2009), Fujita and Ramey (2009)) and has a major impact on how we understand unemployment dynamics over the cycle. The current mainstream approach adopts the Shimer (2005, 2012) and Hall (2005) view that job destruction shocks are small and nearly acyclic. Given that, an equilibrium theory of unemployment must then explain how small productivity shocks generate large variations in unemployment.\(^1\) The view that job destruction shocks are small is, however, highly controversial - see the recent survey Elsby et al (2014). For example Kennan (2006) notes there are steep increases in new UI claims during recessions, especially at the start of a recession. Here we analyse an alternative equilibrium framework, one with job destruction shocks which, when calibrated to the Shimer (2005) data, yields an (almost) perfect fit. Furthermore using simulated data, a second calibration test finds unemployment is much more highly correlated with variations in worker job finding rates than with job separation rates, a result consistent with Shimer (2012). Indeed a third calibration test, performed in Elsby et al (2014), confirms this model yields a remarkable fit of the data. But different to the standard approach, the results are also fully consistent with Davis and Haltiwanger (1992): that unemployment volatility is driven by large but infrequent job destruction shocks.

Our approach is powerful for we consider only a very natural and seemingly minor variation of the standard Diamond/Mortensen/Pissarides framework: namely we allow vacancies to exhibit stock dynamics with a vacancy creation process analogous to the Diamond (1982) coconut model. This is not an entirely novel idea - for related work see Diamond and Fudenberg (1989), Caballero and Hammour (1994), Fujita and Ramey (2005, 2007). This variation, however, changes the dynamic properties of the DMP frame-

\(^1\)The dominant explanation is that there must be small surplus (e.g. Hagendorn and Manovskii (2008)), though others have argued that sticky wages might play a role: see Costain and Reiter (2008), Hall and Milgrom (2008), Mortensen and Nagypal (2008), Hagendorn and Manovskii (2013), Sargent and Lyungqvist (2014) for important contributions to this debate.
work in a fundamental way. It is critically important when the stated aim is to understand unemployment dynamics.

The search literature invariably simplifies the equilibrium matching approach by assuming vacancies are not a state variable. This can be done in a variety of ways: by making a free entry of vacancies assumption (so that vacancies evolve as a jump variable), or that vacancies are a one period "recruitment effort" choice, or that unfilled vacancies die at the end of the period. This simplification, however, comes at a cost: ceteris paribus, vacancy creation rates increase with unemployment (as it is easier for firms to fill vacancies). The difficulty is that vacancies will then tend to covary positively with unemployment, an outcome which is strongly counterfactual. To avoid this outcome Shimer (2005) points out, for the free entry case, it is necessary to assume job destruction shocks are small. Otherwise big jumps in unemployment will generate similarly big jumps in vacancy creation.

Allowing vacancies, like unemployment, to evolve as a stock variable is not only a natural assumption, it is also a game-changer: we show "stock-stock" matching yields countercyclical vacancies even with very large job destruction shocks. To illustrate suppose, say, a financial crisis causes a relatively short-lived spell of high job destruction rates. Newly laid-off workers then try to find work with the existing vacancy stock. Some are lucky and quickly find work. Others are less lucky and, crowded out by the rising tide of unemployed job seekers, are left chasing work. The free entry of vacancies assumption would imply an instantaneous surge in vacancy creation. With instead a relatively inelastic vacancy creation process, oversampling by the rising tide of job seekers depletes the existing vacancy stock. Job finding rates then plummet as many unemployed workers chase too few vacancies. These dynamics would seemingly capture market behavior in a very natural way.

Productivity shocks in the model generate similar dynamics: higher productivity generates higher vacancy creation rates which, in turn, cause the vacancy stock to gradually increase over time along with a declining unemployment stock. To identify the relative importance of job destruction shocks, we calibrate this stock-stock model of unemployment and vacancy dynamics to the original Shimer (2005) data. Figure 1 describes the magni-

\[2\text{Menzio and Shi (2011) is an important caveat. Although "vacancies directed to the unemployed" increase with unemployment, vacancies which are "directed to the employed" do not. The composite variable "total vacancies" has complex dynamics.}\]
tude of log-deviations in (i) job separation rates and (ii) labor productivity according to the Shimer (2005) data at business cycle frequencies.

![Figure 1: U.S. Separation Rates and Labor Productivity](image)

Although the job separation process is less persistent than the productivity process it has much greater variance. Modelling these data as AR1 processes finds the job separation and productivity innovations are strongly negatively correlated with a cross correlation of -0.6: a positive [large] job separation innovation is thus correlated with a [smaller] negative productivity innovation, but low productivity is more persistent. The long-run variance of these processes, measured as $\frac{\sigma^2}{(1 - \rho)}$, finds the long-run variance of separation shocks is 7 times that of the productivity shocks.

The calibrated stock-stock model yields an amazing fit of the measured volatility and persistence of unemployment, vacancies and job finding rates as described in Shimer (2005). Furthermore the vacancy stock is almost perfectly negatively correlated with the unemployment stock as found in the data. We perform a second calibration test. Shimer (2012) finds that variations in unemployment are more highly correlated with variations in worker job finding rates. Our simulated data yields the exact same result.

The question, then, is what drives unemployment volatility? In contrast to the current literature, it is large but infrequent job destruction shocks.
Furthermore rather than a perfectly elastic vacancy creation condition (free entry), an inelastic vacancy creation process ensures unemployment is highly persistent following a large job destruction shock. The theory is thus perfectly consistent with the empirical insights of Davis and Haltiwanger (1992).

The next section describes the model and section 3 characterises equilibrium. Section 4 calibrates the model and evaluates the impact of replacing the free entry of vacancies assumption with so-called “Diamond entry”. Given those insights, section 5 decomposes the results by type of productivity shock and section 6 quickly relates the model to the data on new business start-ups and employment change at existing firms over the cycle. Section 7 concludes.

2 Model

The model uses a conventional equilibrium unemployment framework with discrete time and an infinite time horizon; e.g. Pissarides (2000). All firms and all workers are equally productive, there are no sorting dynamics through on-the-job search. All firms pay the same (Nash bargained) wage and each worker-firm match survives until it is hit by a job destruction shock. The essential difference is that vacancies evolve here as a stock variable with a less than perfectly elastic new vacancy creation process. Vacancies are thus a relevant state variable.

There is a fixed measure $F$ of firms who create vacancies. In every period, each firm has one new (independent) ”business opportunity”. Given that opportunity, the firm compares its investment cost $x$ against its expected return. Its expected return depends on the state of the aggregate economy at time $t$, denoted $\Omega_t$ which is described in detail below. We let $J_t = J(\Omega_t)$ denote the expected return of a business opportunity in state $\Omega_t$. The investment cost $x$ is considered as an idiosyncratic random draw from an exogenous cost distribution $H$. For tractability we assume this investment cost captures all of the idiosyncratic features associated with any given business venture - in other words, highly profitable opportunities correspond to low realised values of $x$. Should the firm decide to invest, it pays the upfront cost $x$ and then holds an unfilled job with expected value $J_t$; i.e. each new investment generates one new vacancy.

Following Diamond (1982), each firm invests in its business opportunity if and only if it has positive value; i.e. when $x \leq J_t$. This requires no recall of a business opportunity should the firm not immediately invest in it. As
investment occurs whenever \( x \leq J_t \) then, at the aggregate level, \( i_t = FH(J_t) \) describes total period \( t \) new vacancy creation. We refer to this investment process as Diamond-entry and note that a higher aggregate return \( J_t \) yields greater vacancy creation rate \( i_t \).

To describe how the firm goes about hiring a worker, we adopt the standard matching framework (but without free entry). There is a unit measure of equally productive and infinitely lived workers. All workers and firms are risk neutral and have the same discount factor \( 0 < \beta < 1 \). Workers switch between being employed and unemployed depending on their realised labour market outcomes.

Each period is characterised by the number \( v_t \) of vacancies (currently unfilled jobs) and the number \( u_t \) of unemployed workers (so that \( 1 - u_t \) describes the number employed). The hiring process is frictional: the number \( m_t \) of new job-worker matches in period \( t \) is described by a matching function \( m_t = m(u_t, v_t) \), where \( m(.) \) is positive, increasing, concave and homogenous of degree one.

While unemployed a job seeker enjoys per period payoff \( z > 0 \). In period \( t \), each job-worker match produces the same market output \( p = p_t \), where aggregate productivity \( p_t \) evolves according to an exogenous AR1 process (described below). Job destruction is also an exogenous, stochastic process. \( \delta_t \) describes the probability that any given job-worker match is destroyed. In the event of such a job destruction shock, the worker becomes unemployed and the job’s continuation payoff is zero. This job destruction parameter, \( \delta_t \), also evolves according to an exogenous AR1 process (described below).

We next describe the sequence of events within each period \( t \). Each period has 5 stages:

**Stage I [new realisations]:** given \((p_{t-1}, \delta_{t-1})\) from the previous period, new values of \( p_t, \delta_t \) are realised according to

\[
\begin{align*}
\ln p_t &= \rho_p \ln p_{t-1} + \varepsilon_t \\
\ln \delta_t &= \rho_\delta \ln \delta_{t-1} + (1 - \rho_\delta) \ln \delta + \eta_t
\end{align*}
\]

where \((\varepsilon_t, \eta_t)\) are white noise innovations drawn from the Normal distribution with mean zero, covariance matrix \( \Sigma \), \( \delta > 0 \) is the long-run average job destruction rate while long-run productivity \( p \) is normalised to one;

**Stage II [bargaining and production]:** the wage \( w_t \) is determined by Nash bargaining. Production takes place so that a job match yields one period profit \( p_t - w_t \) while the employed worker enjoys payoff \( w_t \). Each unemployed worker enjoys payoff \( z \);
Stage III [vacancy investment]: firms invest in new vacancies $i_t$;

Stage IV [matching]: let $u_t, v_t$ denote the stock of unemployed job seekers and vacancies at the start of this stage. Matching takes place so that $m_t = m(u_t, v_t)$ describes the total number of new matches;

Stage V [job destruction]: each vacancy and each filled job is independently destroyed with probability $\delta_t$.

3 Markov Dynamics and Equilibrium.

This section describes the (Markov) equilibrium dynamics. As $u_t$ is defined as the number unemployed in period $t$ immediately prior to the matching stage (stage IV), then $u_t$ evolves according to:

$$u_t = u_{t-1} + \delta_{t-1}(1 - u_{t-1}) - (1 - \delta_{t-1})m_{t-1}$$

where $m_{t-1} = m(u_{t-1}, v_{t-1})$. The second term describes the stock of employed workers in period $t - 1$ who become unemployed through a job destruction shock. The last term describes the match outflow where such matches are also subject to the period $t - 1$ job destruction shock.

The vacancy stock dynamics are given by

$$v_t = (1 - \delta_{t-1})[v_{t-1} - m_{t-1}] + i_t,$$

where the first term describes those vacancies which survive (unfilled) from the previous matching event, while $i_t$ describes new vacancy creation.

To determine equilibrium new vacancy creation $i_t$ we restrict attention to Markov equilibria. Once $(p_t, \delta_t)$ are realised, define the intermediate stock of vacancies

$$\tilde{v}_t = (1 - \delta_{t-1})[v_{t-1} - m_{t-1}]$$

which is the number of surviving vacancies carried over from the previous matching event. When bargaining occurs in stage II, let $\Omega_t = \{p_t, \delta_t, u_t, \tilde{v}_t\}$ denote the corresponding state space. As described below, any standard Nash bargaining procedure yields a wage rule of the form $w_t = w^N(\Omega_t)$. Stage III then determines optimal investment $i_t = i(\Omega_t)$. As the matching and separation dynamics ensure $\Omega_t$ evolves as a first order Markov process, then $\Omega_t$ is indeed a sufficient statistic for optimal decision making in period $t$. 

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We next characterise the Bellman equations describing optimal behaviour. In period $t$ and at the start of stage II with state vector $\Omega_t$ (i.e. prior to production and matching but after new $p_t, \delta_t$ have been realised) let:

$J_t = J(\Omega_t)$ denote the expected value of a vacancy;

$J^F_t = J^F(\Omega_t)$ denote the expected value of a filled job;

$V^U_t = V^U(\Omega_t)$ denote the worker’s expected value of unemployment;

$V^E_t = V^E(\Omega_t)$ denote the worker’s expected value of employment.

Let $E[. | \Omega_t]$ denote the expectations operator given period $t$ state vector $\Omega_t$. The timing of the model implies the value functions $J_t, J^F_t$ are defined recursively by:

$$J_t = -c + \beta (1 - \delta_t) E \left\{ \frac{m(u_t, v_t)}{v_t} J^F_{t+1} + [1 - \frac{m(u_t, v_t)}{v_t}] J_{t+1} | \Omega_t \right\}$$

(3)

$$J^F_t = p_t - w_t + \beta (1 - \delta_t) E \{ J^F_{t+1} | \Omega_t \}.$$ (4)

The worker value functions are also defined recursively:

$$V^U_t = z + \beta E \left[ V^U_{t+1} + (1 - \delta_t) \frac{m(u_t, v_t)}{u_t} [ V^E_{t+1} - V^U_{t+1} ] | \Omega_t \right]$$

(5)

$$V^E_t = w_t + \beta E \left[ V^E_{t+1} + \delta_{t+1} [ V^U_{t+1} - V^E_{t+1} ] | \Omega_t \right].$$ (6)

Diamond entry implies the reservation cost rule - invest if and only if cost $x \leq J^P_t$. Equilibrium new vacancy creation $i_t = i(\Omega_t)$ is given by:

$$i_t = FH(J_t),$$ (7)

where $J_t = J(\Omega_t)$.

Assuming workers have bargaining power $\phi \in [0, 1]$, the axiomatic Nash bargaining approach closes the model with

$$(1 - \phi) [ V^E_t - V^U_t ] = \phi [ J_t - J^V_t ].$$

Using the above equations, this condition determines the equilibrium wage $w_t = w(\Omega_t)$. The above thus yields a system of autonomous, first order difference equations determining (i) the evolution of $\Omega_t$ and (ii) the equilibrium value functions with corresponding investment rule $i_t = i(\Omega_t)$. 
4 Calibration and Comparative Dynamics.

The central issue of interest is to compare the dynamic properties of the standard textbook model with free entry where vacancies evolve as a jump variable (“stock-jump” dynamics) against a model of Diamond entry where vacancies evolve as a stock variable (“stock-stock” dynamics). Specifically we assess the extent to which unemployment and vacancy dynamics are consistent with their:

(a) observed volatilities over the business cycle;
(b) observed persistences over the cycle, and
(c) the Beveridge curve - the observed negative covariance between unemployment and vacancies.

4.1 Calibration Parameters.

As the framework is so standard, we simply adopt the calibration parameters as described in Mortensen and Nagypal (2007). Specifically we assume each period corresponds to one month, a standard Cobb-Douglas matching function $m = A u^\gamma v^{1-\gamma}$ and the following Mortensen/Nagypal parameter values.

Table 1: Mortensen/Nagypal Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>elasticity parameter on matching function</td>
</tr>
<tr>
<td>$\phi$</td>
<td>worker bargaining power</td>
</tr>
<tr>
<td>$z$</td>
<td>outside value of leisure</td>
</tr>
<tr>
<td>$\beta$</td>
<td>monthly discount factor</td>
</tr>
</tbody>
</table>

Notice that bargaining is efficient in the sense that the Hosios condition is satisfied. As the productivity process for $p_t$ (described below) ensures its (long run) mean value $\bar{p}$ is equal to one, surplus $(\bar{p} - z)/z = 43\%$ is large. The monthly discount factor implies an annual discount rate of 4%.

We next calibrate the stochastic process for $\{p_t, \delta_t\}$. Figure 1 in the Introduction describes the quarterly measures of aggregate productivity and separation rates as obtained in Shimer (2005). As these data are only recorded quarterly while the model adopts a monthly time structure, we choose the autocorrelation parameters $\rho_p$, $\rho_\delta$ and covariance matrix $\Sigma$ so that the implied process $(p_t, \delta_t)$, when reported at quarterly intervals, matches the first
order autocorrelation and cross correlation implied by the data. Doing this yields:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_p$</td>
<td>productivity autocorrelation</td>
<td>0.978</td>
</tr>
<tr>
<td>$\rho_\delta$</td>
<td>separation autocorrelation</td>
<td>0.925</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>st. dev. productivity shocks</td>
<td>0.0064</td>
</tr>
<tr>
<td>$\sigma_\delta$</td>
<td>st. dev. separation shocks</td>
<td>0.031</td>
</tr>
<tr>
<td>$\rho_{p\delta}$</td>
<td>cross correlation</td>
<td>-0.60</td>
</tr>
</tbody>
</table>

Table 2: $(p_t, \delta_t)$ Stochastic Process

The job destruction innovations are negatively correlated with the productivity innovations. Although the separation process is less persistent than the productivity process it has much greater variance.

The framework is further calibrated to fit the long run turnover means. To ensure comparability of results, we follow Shimer (2005) who argues that (i) the mean job separation probability should equal 3.4% per month, (ii) the average duration of an unemployment spell is 2.2 months and thus the long run unemployment rate equals $u = 7\%$. We also note the average duration of vacancies is around 3 weeks (Blanchard and Diamond (1989)). For the free entry case, Table 3 describes the remaining parameter values so that the model fits these turnover means.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>0.17</td>
</tr>
<tr>
<td>$A$</td>
<td>0.594</td>
</tr>
<tr>
<td>$\bar{\delta}$</td>
<td>0.034</td>
</tr>
</tbody>
</table>

Table 3: Turnover Parameters [free entry]

Before describing the calibration results for the textbook model with free entry, however, we next describe how we calibrate the model with "Diamond entry". For comparability we maintain the parameter values described in Tables 1 and 2. As turnover with Diamond entry is different, however, we have to recalibrate the turnover parameters in Table 3.

When calibrating a DMP framework, it is often found the advertising cost $c$ must be large. This is typically motivated by arguing it reflects previously sunk job creation investments. Here we take the converse case: we instead
attribute all job creation costs to the ex-ante investment process \( x \sim H(\cdot) \) and presume small advertising costs \( c = 0 \) (say jobs are filled by word of mouth recommendations).

Given vacancy creation rule \( i_t = FH(J_t) \), we adopt the functional form

\[
i_t = F J_t^\xi
\]

where \( \xi \) describes the elasticity of new vacancy creation with respect to vacancy value. \( \xi = \infty \) would describe infinitely elastic new vacancy creation (analogous to the free entry case) while \( \xi = 0 \) would imply perfectly inelastic (fixed) new vacancy creation. In what follows we consider two cases. The first follows Fujita and Ramey (2005) which assumes \( H \) is uniform; i.e. \( \xi = 1 \) and so new vacancy creation is neither elastic nor inelastic.

The second calibrates \( \xi \) using the results of Merz and Yashiv (2007). That paper estimates a representative firm structure where the firm’s cost of creating \( i \) new vacancies is \( p_t N_t c(i/N_t) \) where \( p_t \) is aggregate productivity, \( N_t \) is firm employment and \( c(\cdot) \) is a convex function. Note this cost structure implies constant returns to hiring: a firm which has twice the number of employees \( N \) and wishes to create twice as many new vacancies \( i \) has double the hiring costs. If \( J_t \) describes the expected return to a new vacancy, optimality implies the first order condition

\[
p_t c'(i_t/N_t) = J_t,
\]

so that the marginal cost of creating a new vacancy equals its expected return. The estimates in Merz and Yashiv (2007) find \( c(\cdot) \) is close to a cubic. Optimality thus implies the vacancy creation condition:

\[
i_t = \left[ \frac{AN_t}{p_t^{0.5}} \right] J_t^{1/2}.
\]

Comparing equation (10) with (8), noting that variations in aggregate employment \( N_t \) and productivity \( p_t \) are small and both are procyclical, suggests \( \xi = 1/2 \).

With \( c = 0 \) and \( \bar{\delta} = 0.034 \), Table 1d reports the calibrated parameter values for \( A, F \) so that "Diamond entry" with \( \xi = 1 \) or with \( \xi = 1/2 \) fits the long run turnover means.
Table 4: Turnover Parameters [Diamond Entry]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Diamond entry</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>scale parameter on matching function</td>
<td>0.594</td>
</tr>
<tr>
<td>$F(\xi = 1)$</td>
<td>entrepreneurial activity</td>
<td>0.0075</td>
</tr>
<tr>
<td>$F(\xi = 1/2)$</td>
<td>entrepreneurial activity</td>
<td>0.0157</td>
</tr>
</tbody>
</table>

4.2 Results

We report the results in three parts. The first describes the implied volatility of unemployment, vacancies and the vacancy/unemployment ratio for the three calibrated models and compares them to the data. The second describes the persistence and the covariance of unemployment and vacancies over the cycle (Beveridge curve). In all results, Column 1 describes the empirical measures taken from Shimer (2005). The other columns are the equivalent measures using data generated by the calibrated models, where column 2 describes the free entry case, column 3 is the Diamond entry case with $\xi = 1$ and column 4 is the case $\xi = 1/2$.

4.3 Volatility in unemployment and vacancies.

Table 5 reports business cycle volatility measured as the standard deviation of unemployment ($\sigma_u$), of vacancies ($\sigma_v$) and of the vacancy/unemployment ratio ($\sigma_{v/u}$) from trend.\(^3\)

Table 5: Volatility of unemployment and vacancies

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Data</th>
<th>Free Entry</th>
<th>Diamond $\xi = 1$</th>
<th>Diamond $\xi = 1/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_u$</td>
<td>0.19</td>
<td>0.08</td>
<td>0.15</td>
<td>0.17</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>0.20</td>
<td>0.05</td>
<td>0.14</td>
<td>0.17</td>
</tr>
<tr>
<td>$\sigma_{v/u}$</td>
<td>0.38</td>
<td>0.07</td>
<td>0.28</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Notes: $\sigma_u$: standard deviation of variable $u$. To calculate the statistics: simulated data are quarterly averages of monthly series, in logs as deviations from an HP trend with smoothing parameter $10^5$.

Consistent with the arguments in Shimer (2005), the free entry case (column 2) explains only around one third to one half of the observed volatility.

\(^3\)The model-generated data was passed through an HP filter with parameter $10^5$ and the standard deviations are measured as deviations from the trend.
By introducing vacancy stock dynamics into the framework, "Diamond entry" increases volatility in all variables. The case $\xi = 1/2$ generates volatilities which are very close to the data. Indeed if our aim were to exactly fit the data, we would need $\xi \simeq 1/4$; i.e. the job creation process needs to be more inelastic.

To understand why dropping the free entry assumption yields such a huge improvement in model fit, Figures 2 and 3 describe the impulse response function of unemployment and vacancies to a single separation innovation at date zero (holding productivity fixed $p_t = 1$).

![Figure 2: Impulse Response of Unemployment to a Separation Shock](image)

In Figure 2 with free entry of vacancies (FE), the impulse response function of unemployment to a job separation shock yields a relatively small increase in unemployment which quickly recovers to its steady state value. As described in Shimer (2005), the free entry approach yields too little volatility and persistence. Diamond entry with $\xi = 1/2$ (DE) instead generates a much higher unemployment peak and greater persistence. Figure 3, which describes the corresponding impulse response function for vacancies, reveals why.

Free entry of vacancies (FE) implies vacancies instantaneously increase given a rise in unemployment. This vacancy response ensures unemployment quickly recovers to its long run steady state value and the model demonstrates little persistence (the persistence observed is largely due to the separation
process being an AR1 process). This of course reflects the original insights in Shimer (2005).

In contrast with Diamond entry (DE), Figure 3 demonstrates the vacancy stock falls as unemployment increases. The job destruction shock generates a rising tide of unemployed workers, some of whom quickly re-match with the existing vacancy stock. As vacancy creation rates are inelastic (compared to free entry) such oversampling causes the vacancy stock to fall. Unemployed worker job finding rates then plummet as the increasing number of unemployed workers chase few remaining vacancies.

For the same job destruction shock, reducing the vacancy creation elasticity from $\xi = 1$ to $\xi = 1/2$ finds unemployment grows more and is even more persistent. This occurs as vacancy creation rates respond even more slowly to rising unemployment. As identified in Table 5, lower $\xi$ thus generates greater variation and persistence in unemployment, the vacancy stock and job finding rates (as implied by market tightness $\theta = V/U$).

This impulse response function might suggest that job creation rates increase following a job destruction shock. An important property of the calibration, however, is the correlated shock structure: that a large separation shock (in expectation) is followed by a persistent low productivity phase. By reducing new job creation rates, the low productivity phase further reduces the recovery rate of the economy. In the Conclusion we relate this insight to
the New Keynesian macro-approach.

### 4.4 Persistence and covariance of Unemployment and Vacancy stocks.

Table 6 describe the persistence and covariance of unemployment and vacancies over the cycle.

<table>
<thead>
<tr>
<th>Serial Persistence</th>
<th>Data</th>
<th>Free Entry</th>
<th>Diamond $\xi = 1$</th>
<th>Diamond $\xi = 1/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{cor}(u_t, u_{t-1})$</td>
<td>0.94</td>
<td>0.85</td>
<td>0.95</td>
<td>0.96</td>
</tr>
<tr>
<td>$\text{cor}(v_t, v_{t-1})$</td>
<td>0.94</td>
<td>0.77</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>$\text{cor}(u_t, v_t)$</td>
<td>-0.89</td>
<td>0.33</td>
<td>-0.93</td>
<td>-0.97</td>
</tr>
</tbody>
</table>

Notes: $\text{cor}(u_t, u_{t-1})$: correlation between $u_t$ and $u_{t-1}$. To calculate the statistics: simulated data are quarterly averages of monthly series, in logs as deviations from an HP trend with smoothing parameter $10^5$.

The first two rows describe the autocorrelation of vacancy and unemployment stocks. Both versions of ”Diamond entry” generate the right degree of persistence, though the free entry case does reasonably well in this dimension. The big difference, however, is how unemployment and vacancies covary over the cycle. Row 3 describes the Beveridge curve: that the covariance of vacancies with unemployment is large and very negative. The free entry case obtains the wrong correlation, the reason being that the job separation process does not imply small shocks. In contrast both ”Diamond entry” models find vacancies and unemployment covary negatively. Indeed the negative covariance would seem a little too strong.

### 4.5 A Second Calibration Test.

Shimer (2012) argues that the observed variation in unemployment is primarily due to variations in worker re-employment rates. To understand that argument, note that steady state unemployment

$$u = \frac{\bar{x}}{\bar{x} + \bar{f}}$$

where $\bar{x}$ is the (steady state) exit rate of employed workers into unemployment, $\bar{f}$ the rate unemployed workers become employed. It turns out for
aggregate data, that the unemployment proxy

$$u_t^P = \frac{x_t}{x_t + f_t},$$

where $x_t$ is the period $t$ exit rate and $f_t$ the job finding rate, is a surprisingly good approximation for actual unemployment $u_t$. This proxy variable can then be decomposed into job separation effects (variations in $x_t$) and job finding effects (variations in $f_t$). For example putting $x_t = \bar{x}$, the sequence $\bar{x}/(\bar{x} + f_t)$ describes the variation in $u_t^P$ due solely to variations in $f_t$. Similarly $x_t/(x_t + \bar{f})$ describes the variation in $u_t^P$ due to variations in $x_t$. Shimer (2012) defines the contribution of the job finding rate to variations in unemployment as the covariance of $u_t$ and $\bar{x}/(\bar{x} + f_t)$ divided by the variance of $u_t$. Column 1, Table 1 in Shimer (2012) reports that 77% of the variation in unemployment is explained by variations in the job finding rate $f_t$, while variations in the job separation rate $x_t$ instead explain only 24%. Thus unemployment variations are much more highly correlated with variations in worker job finding rates.

We repeat this decomposition on simulated data generated by the Diamond entry model with $\xi = 0.5$. For this data, the proxy variable $u_t^P = x_t/(x_t + f_t)$ is indeed very highly correlated with the model generated unemployment $u_t$. Computing the same statistics finds job finding variations, $\bar{x}/(\bar{x} + f_t)$, explain 70% of the unemployment variation, while job separation variations $x_t/(x_t + \bar{f})$ explain only 24%. Thus the simulated data exhibit the same properties. The interpretation however, is very different. As we now show, the unemployment dynamics here are driven by infrequent but large job separation shocks. Unemployment is more highly correlated with job finding rates simply because job destruction shocks are short-lived while the propagation mechanism implies job finding rates fall (endogenously) as more unemployed workers chase fewer vacancies.

5 How Important are Job Destruction Shocks?

We use our structural model (with $\xi = 0.5$) to decompose the separate contributions of job destruction shocks and productivity shocks on the volatility of unemployment, vacancies and job finding rates (market tightness). First

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4 see Figures 4 and 6 in Elsby et al (2009) and Table 1 in Fujita and Ramey (2009) for alternative estimates.

5 with $x_t \equiv \delta_t$ and $f_t \equiv (1 - \delta_t)m(\theta_t)$. 

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we shut down all productivity shocks (i.e. set $p_t = \bar{p} = 1$) and suppose job destruction process

$$\ln \delta_t = \rho \ln \delta_{t-1} + (1 - \rho) \ln \bar{\delta} + \eta_t$$

as previously calibrated. Column 3 in Table 7 describes the model generated volatilities of unemployment, vacancies and market tightness (job destruction only). Second we instead shut down separation shocks by putting $\delta_t = \bar{\delta}$, and suppose productivity follows the AR1 process as previously calibrated. Column 4 describes the results. Column 5 reports the results previously obtained when both shock processes are active.

Table 7: volatility of unemployment and vacancies by shock process ($\xi = 0.5$)

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Data</th>
<th>Job Destruction only</th>
<th>Productivity Shocks only</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_u$</td>
<td>0.19</td>
<td>0.14</td>
<td>0.025</td>
<td>0.17</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>0.20</td>
<td>0.13</td>
<td>0.044</td>
<td>0.17</td>
</tr>
<tr>
<td>$\sigma_{v/u}$</td>
<td>0.38</td>
<td>0.27</td>
<td>0.069</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Notes: $\sigma_u$: standard deviation of variable $u$. To calculate the statistics: simulated data are quarterly averages of monthly series, in logs as deviations from an HP trend with smoothing parameter $10^5$.

Table 7 establishes that the job destruction process is the principal reason for the large volatility in unemployment, vacancies and job finding rates. Productivity shocks by themselves (Column 4) yield only 15% of the observed volatility of unemployment. This latter result occurs as the assumed surplus $(p_t - z)/z$ is not small, and so small productivity shocks are not amplified into large variations in unemployment. The model would thus "fail" the Shimer (2005) critique as currently interpreted. But that interpretation supposes that job destruction shocks are small. Relaxing that restriction not only generates an excellent fit of the data (Tables 2 and 3), the results remain consistent with Shimer (2012) which finds that variations in unemployment are more highly correlated with variations in worker job finding rates.

6 New Business Start-Ups and the Cycle.

In the working paper version of this paper, Coles and Moghaddasi (2011) instead suppose $i_t = FH(i_t)$ describes vacancy creation by new business start-ups. It also showed how to incorporate time-to-build constraints. That
approach was partly motivated by Figure 4 below. Using data contained in the BDS, which is an annual census of firms in the U.S., Figure 4 describes (net) job creation in the U.S by (i) new start-up firms (defined as firms aged less than one year) and (ii) existing firms (defined as firms which existed in the previous census year).

There are two important features of this data. Note first that

- net job creation at pre-existing firms (those firms more than one year old) are, on average, negative while net job creation by new start-ups is positive and large, being around 3 million new jobs a year.

Such job turnover is consistent with the concept of creative destruction, where new firms with better technologies gradually drive pre-existing firms out of business (e.g. Caballero and Hammour (1994), Klette and Kortum (2004), Lentz and Mortensen (2008) among many others). The key business cycle observation, however, is that

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6 The Business Dynamics Statistics (BDS) is available at http://webserver03.ces.census.gov/index.php/bds/bds_database_list. Kane (2010) was the first to aggregate the data as described in Figure 2.
job creation rates by new business start-ups is largely invariant to the cycle whereas there are large, but infrequent, spikes of net employment loss at pre-existing firms.

The model is entirely consistent with this view of the business cycle: the job creation process is inelastic [by start-ups] while employment [at pre-existing firms] is liable to large, but infrequent losses. The working paper version was criticised for ignoring that existing firms are responsible for the large part of gross hires. This version has simply reinterpreted variables more appropriately.

7 Conclusion.

The challenge for any equilibrium theory of unemployment and vacancies is to explain the following facts, that

- (a) vacancies covary negatively with unemployment; e.g. Shimer (2005),
- (b) there is large unemployment volatility even though productivity shocks are small; e.g. Shimer (2005) and
- (c) variations in unemployment are much more highly correlated with worker job finding rates than with employed worker separation rates; e.g. Shimer (2012).

The DMP approach with free entry of vacancies can generate this outcome by assuming small job destruction shocks and small surplus (see Sargent and Lyungqvist (2014) for a recent discussion). Here we have identified an alternative structure which provides a very different view of the timing and progression of recessions. The implied unemployment dynamics are not only perfectly consistent with data properties (a)-(c), they are also consistent with the original job creation/job destruction insights of Davis and Haltiwanger (1992). Furthermore once large job destruction shocks are allowed, small surplus and sticky wage arguments play a less direct role in explaining unemployment dynamics.

The Great Financial Crash of 2007/8 establishes that financial failures play an important role in generating large job destruction shocks. Extending our approach to consider financial failure is an important direction for
future research. There is some work on this issue in frictional labor markets, see for example Hall (2014) and Eckstein et al (2014). Our view of the timing and progression of unemployment dynamics is also complementary to the New Keynesian macro-approach. Here the correlated shock structure implies a large job destruction shock is followed (in expectation) by a persistent low productivity phase. This latter low productivity phase reduces the recovery rate of the economy. A New Keynesian macro approach potentially endogenises the low productivity phase as a low aggregate demand outcome which, here, could be due to persistently high unemployment.

References


