

Comments on “Interval type-2 fuzzy sets are generalization of interval-valued fuzzy sets: towards a wide view on their relationship” [2]

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Abstract—This Letter makes some observations about [2] that further support the distinction between an interval type-2 fuzzy set (IT2 FS) and an interval-valued fuzzy set (IV FS), points out that all operations, methods and systems that have been developed and published about IT2 FSs are, so far, only valid in the special case when IT2 FS = IVFS, and suggests some research opportunities.

Index Terms—Interval type-2 fuzzy sets, interval valued fuzzy sets, type-2 fuzzy sets

TYPE-2 fuzzy sets (T2 FSs) have been beset with some unfortunate notation, although, as is explained in [9], this has not prevented their use nor has it led to any theoretically incorrect results. The work presented in [1] addressed the many facets of the notational problems and there are many useful recommendations in [1] to remedy them. Undoubtedly, the statement $J_x \subseteq [0,1]$ in the widely used definition of a T2 FS \tilde{A} [7]

$$\tilde{A} = \{(x,u), \mu_{\tilde{A}}(x,u) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1]\}, \quad (1)$$

has caused much confusion.

Recall that $J_x \subseteq [0,1]$ means: “every element of J_x is also an element of $[0,1]$.” Unfortunately, no formulaic definition was given in [7] for J_x something thought to have been remedied in 2006 [8], where the following formula is given for J_x :

$$J_x = \{(x,u) : u \in [\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)]\} \quad (2)$$

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in which $\underline{\mu}_{\tilde{A}}(x)$ and $\overline{\mu}_{\tilde{A}}(x)$ are the lower and upper membership functions of \tilde{A} , respectively. The intent of [8] was to provide authors with standard background material about interval type-2 fuzzy logic systems that they could use, as is, for background materials in their papers, so that they would not have to re-invent such background materials. However, [8] is not widely known or used, because it is only found as a Standard on the website of the IEEE Computational Intelligence Society.

In [2], four different choices (examples) are given for J_x which conform to the general meaning of $J_x \subseteq [0,1]$, namely: multisets, interval-valued fuzzy set (IVFS), multi-IVFS and combinations of numbers and intervals. Only one of these choices corresponds to the way J_x has been used by the IT2 FS community, namely that of an *interval-valued fuzzy set (IVFS)*. Consequently, [2] points out that the phrase “IT2 FS” is a more general term than the phrase “IVFS” and includes IVFS as a special case.

Although J_x has not been ambiguous to the IT2 FS community since 2006 [8], and was understood by that community to mean (2), it is important to acknowledge that new researchers may become familiar with the T2 literature at different entry time points for them (e.g., the most common seems to be by reading [7]). One cannot expect them to imagine that there are notational problems. Additionally, it is no longer possible for new researchers to be knowledgeable about the very large T2 literature; so, since $J_x \subseteq [0,1]$ has been used from the very beginning one must abide by the general meaning of $J_x \subseteq [0,1]$.

As acknowledged by Nilesh Karnik and Qilian Liang (J_x appeared first in [5] and [6]) the intention was that J_x is meant to be interpreted as (2). Regardless of the “intention,” *math is math* and so, since J_x in $J_x \subseteq [0,1]$ was originally left undefined, it is too late to say “Oh, we meant it to be defined as an IVFS (as in (2)).” Hence, using the interpretation explained in [2] provides a more general definition of the IT2FS according to the mathematical definition of $J_x \subseteq [0,1]$.

It is very important, however, for readers to understand (the following has not been mentioned in [2]) that *all operations, methods and systems that have been developed and published about IT2 FSs are, so far, only valid in the special case when $IT2 FS = IVFS$* ¹. Whether or not the interval type-2 operations, methods and systems developed so far are valid, as is, or are extendable to other cases of IT2 FSs (as defined in [2]) is unknown as of early 2015. One such example is how to compute the centroid of an IT2 FS when $IT2 FS \neq IVFS$.

In conclusion, it is believed that [2] presents a wider view on the relationship between interval-valued fuzzy sets and interval type-2 fuzzy sets. However, there is a need to explore the wider relationship between the already developed IT2 FLS theory and operations (which were developed under the special case of $IT2 FS = IVFS$) and how they extend when employing the other general cases of IT2 FSs presented in [2]. For example, a reviewer of this Letter asked an interesting question that we are paraphrasing as: If one begins with IT2 FSs that are not IV FSs is it possible for their join or meet not to also be IT2 FSs, e.g. could they be IV FSs?

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¹ Join and meet are used for all T2 FSs. They are worked out in detail only for $IT2 FS = IVFS$ in [6] and [7].