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Wage Inequality: A Structural Decomposition *

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Abstract

The objective of this paper is to study why some workers are paid more than others. To do so we construct and quantitatively assess an equilibrium search model with on-the-job search, general human capital accumulation and two sided heterogeneity. In the model workers differ in abilities and firms differ in their productivities. The model generates a simple (log) wage variance decomposition that is used to measure the importance of firm and worker productivity differentials, frictional wage dispersion and workers' sorting dynamics. We calibrate the model using a sample of young workers from the UK. We show that heterogeneity among firms generates great deal of wage inequality. Among low skilled workers job ladder effects are small, most of the impact of experience on wages is due to learning-by-doing. High skilled workers are much more mobile. Job ladder effects have sizeable impact.

Keywords: Job search, human capital accumulation, wage inequality, turnover.
JEL: J63, J64, J41, J42.

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1 Introduction

A major focus of Dale Mortensen's work was on addressing the following question - Why are workers, similar or otherwise, paid differently? Of course, several partial answers have long been known. Differentials in worker abilities, for example, have long been recognized as an important source of wage inequality. Human capital theory, pioneered by Becker (1964), explains why years of education and other relevant fixed factors play a role in explaining wage differences. Following early work by Mincer (1974), many have also argued that learning-by-doing implies workers accumulate more human capital while working. This implies that time in employment may well play an important role in explaining wage differences among workers. Standard wage regressions confirm the importance of the above factors, but also show that a large proportion of wage inequality is still left unexplained. Mortensen (2003) made a significant contribution by showing that when labour market frictions are suitably modelled wage inequality is a natural equilibrium outcome.

As Mortensen (2003) demonstrated on the job search plays a central role in explaining wage differences. If workers search while employed, the longer a worker is employed the greater the probability she will find and accept a higher paying job and this in itself can lead to inequality (e.g. Burdett (1978), for early work in this area). In such a framework search frictions can generate wage differentials among the employed even when workers and firms are identical (see, for example, Burdett and Mortensen (1998), and Hornstein, Krusell and Violante (2011)). Of course, difference in the productivities of firms generate further wage differentials. Several different theories reach this conclusion (see, for example, Lentz and Mortensen (2008), Postel-Vinay and Robin (2002), and Bartelsman and Doms (2000)).

In this paper we assess the extent to which each of the factors mentioned above contribute to overall wage inequality. We do this in the context of an equilibrium search model based on the Burdett and Mortensen (1998) framework (henceforth B/M)), but allowing for human capital accumulation as analysed in Burdett, Carrillo-Tudela and Coles (2011). The main innovations relative to Burdett et al. (2011) is that here we (i) allow firms to differ in their productivities and (ii) use the resulting model as a measuring tool to assess the relative contributions of the aforementioned factors on overall wage inequality. We use UK data and compare the relative contributions of these factors across low, medium and high skilled workers. Differentiating across skill groups is important since we find that low skilled workers have a much more tenuous job ladder than medium and high skilled workers. As shown below this difference has an important effect on the nature of wage inequality between these workers.

In the framework developed, employed workers of different abilities enter the labour market, accumulate general human capital through learning-by-doing as well as engaging in on-the-job search.[1] Firms post wage rates and in equilibrium differentiate their pay policies as an optimal reaction to workers' on-the-job search. In equilibrium more productive firms offer higher paying jobs, wages increase over time as workers becomes more productive and move from less to more productive jobs when the opportunity arises. These dynamics generate positive sorting among

[1] Our approach abstracts from life-cycle effects on job search and wages. For recent work on this issue, see Bowlus and Liu (2013) and Menzio, Telyukova and Visschers (2014).

workers and firms. In equilibrium more productive workers end up employed in more productive firms; and more productive firms end up employing a more productive workforce.

As argued by Bagger, Fontaine, Postel-Vinay and Robin (2014), an important issue is to explain why wages, on average, increase with experience.[2] Here we focus on decomposing the cross-sectional wage distribution. Our model provides a simple variance decomposition that relates (log) wages to differences in worker abilities, firm productivities and differences in pay policies. We also capture the effects of general human capital accumulation and sorting on wage variation. Thus the model is able to encompass a similar variance decomposition as analysed by Abowd, Kramarz and Margolis (1999) and Postel-Vinay and Robin (2002). At the same time the model captures the effects of labour market experience on wage variation as is the focus in the traditional applied labour literature (see Rubinstein and Weiss, 2007, for an insightful survey). This decomposition is a key element in our quantitative analysis and allows us to decompose overall wage inequality into its constituent parts.[3]

Another contribution of this study is that we address the issues raised by Hornstein et al. (2007, 2011). They define the Mm ratio as the ratio of the average wage earned to the lowest wage paid in the market among equally productive workers. For plausible parameter values, they explain why most search models generate a reservation wage that is too large to match the observed Mm ratio in the US economy. Within the framework developed here work experience is valuable. Hence, unemployed workers are willing to accept low starting wages.[4] Hornstein et al. (2007, 2011) approach is important for it not only provides relevant information for calibrating the model, it provides a coherent empirical framework for analysing wage dispersion. Indeed our calibration approach follows closely their methodology.[5]

To quantitatively assess the factors behind wage inequality in our model we use labour market histories of a sample of young workers drawn from the British Household Panel Survey (BHPS). We evaluate the model on young workers as it is precisely at this stage of a worker's labour market history that job mobility is most common. We divide the sample into the three skill (educational) groups mentioned above and analyze them separately. We highlight two main results.

First, our variance decomposition shows that the contribution of labour market experience in accounting for wage inequality is sizeable and it increases across skill groups. For high skilled workers it accounts for 27 percent of overall wage inequality, whereas for low skilled workers it accounts for only 19 percent. This difference follows as low skilled workers are more likely to move from one job to another via unemployment whereas high skilled workers are more likely

[2]Bagger et al. (2014) use the offer-matching framework to structurally estimate how wages evolve at firms when workers' productivity also evolves stochastically.

[3]A different approach uses statistical models, based on Mincer's (1974) original work, to try to identify the impact of experience on wages. Prominent examples include Topel (1991), Altonji and Williams (2007), Dustmann and Meghir (2005), among many others. The evidence from this body work, however, remains hotly debated. There are a few other papers which have investigated learning-by-doing within a search environment. Bunzel et al. (2000) provide an interesting early example. Fu (2009), Yamaguchi (2010) and Bagger et al. (2014) provide more recent examples. Manning (2003), Rubinstein and Weiss (2007), Barlevy (2008) and Bowlus and Liu (2013) estimate a wage process similar to the one identified here but do not consider equilibrium.

[4]Indeed some college interns would seem to work for no pay in return for job experience.

[5]Also see Ortego-Marti (2012) and Tjaden and Wellschmied (2014) for related work on the Mm approach.

to move from job to job via on-the-job search. Low skilled workers are also more likely to spent long periods of time in unemployment, which hinders their ability to accumulate human capital.

Second, the variance decomposition shows that wage dispersion among equally productive workers is large for all skill groups. Most of this dispersion arises due to firm productivity differentials. Indeed an important conclusion from our work is that firm heterogeneity has a very large impact on overall wage inequality. We find that the contribution of firm productivity differentials in explaining wage inequality decreases across skill groups, while the importance of worker ability differences in explaining wage inequality increases across skill groups.

The rest of the paper is outlined as follows. Section 2 describes the model. Section 3 defines and characterizes the equilibrium. Section 4 describes the data, the calibration procedure and present the main results. All proofs are relegated to an Appendix.

2 The Model

Time is continuous with an infinite horizon and we only consider steady state. There is a continuum of firms and workers, each of measure one. Any worker's life is described by the exponential distribution with parameter $\phi > 0$. To keep the population of workers constant, ϕ also describes the inflow of new labour market entrants.

A worker when entering the labour market has initial ability ε which is considered a random draw from an exogenous distribution $A(.)$ with support $[\underline{\varepsilon}, \overline{\varepsilon}]$. Learning-by-doing implies a worker's ability increases at rate ρ when working, where $0 < \rho < \phi$. Assuming an unemployed worker's productivity remains constant through time, a type ε worker with x years experience has productivity $y = \varepsilon e^{\rho x}$.

Firms have a constant returns to scale technology and are ex-ante heterogeneous with fixed productivity parameter p. Let $\Gamma(.)$ denote the exogenous distribution of productivities across firms which, for ease of exposition we assume is differentiable [no mass points] with connected support $[\underline{p}, \overline{p}]$.

A worker with productivity y who is employed at a firm with productivity p generates flow revenue yp.[6] We assume a firm pays each of its employees the same piece rate θ, and so a worker y employed at firm p paying piece rate θ earns flow wage $w = \theta yp$, the firm enjoys corresponding flow profit $(1-\theta)yp$. Given the firm's p and θ, however, it is convenient to define $z = p\theta$ as its corresponding wage rate paid, where zy describes the wage paid to any employee y. We let $F(z)$ denote the fraction of firms which offer wage rate no greater than z, with support denoted $[\underline{z}, \overline{z}]$. $F(.)$, of course, is endogenously determined.

Each unemployed and employed worker receives job offers according to an exogenous Poisson process with parameters λ_u and λ_e, respectively. Conditional on receiving a job offer, random matching implies $F(z)$ describes the probability the offered wage rate is no greater than z. If a worker rejects a job offer, there is no recall.

Each employed worker is displaced into unemployment at rate $\delta > 0$. While unemployed, a

[6] An important simplification is that the worker's rate of human capital accumulation ρ is independent of the workers productivity and that of her employer.

worker with productivity y enjoys flow payoff by, where b denotes home productivity.

Workers are risk neutral, discount the future at rate $r \geq 0$ and maximize expected discounted lifetime income. Firms do not discount the future and so maximize steady state flow profit.

Optimal Search Strategies

For a given F, consider optimal worker behavior. Let $W^U(y)$ denote the maximum expected lifetime payoff of an unemployed worker with productivity y, let $W^E(y, z)$ denote the maximum expected lifetime payoff of a worker with productivity y employed at a firm paying wage rate z. The Bellman equation implies $W^E(.)$ is defined recursively by:

$$(r+\phi)W^E(y,z) = zy + \rho y \frac{\partial W^E}{\partial y} + \lambda_e \int_z^{\overline{z}} [W^E(y,z') - W^E(y,z)]dF(z') + \delta \left[W^U(y) - W^E(y,z)\right],$$

where the second term describes the increase in value through learning-by-doing. It is immediate that $W^E(.)$ must be strictly increasing in z. Hence any employee y quits to any outside offer $z' > z$. Thus all employees adopt the same quit strategy and so the rate an employee leaves a firm paying wage rate z is

$$q(z) = \phi + \delta + \lambda_e(1 - F(z)).$$

As there is no human capital accumulation while unemployed, the Bellman equation describing $W^U(y)$ is

$$(r+\phi)W^U(y) = by + \lambda_u \int_{\underline{z}}^{\overline{z}} \max[W^E(y,z') - W^U(y), 0]dF(z').$$

As $W^E(y, z)$ is strictly increasing in z, an unemployed worker accepts job offer z' if and only if $W^E(y, z') \geq W^U(y)$. Thus an unemployed worker with productivity y adopts a reservation wage strategy, where the worker's reservation wage rate z_R solves $W^E(y, z_R) \geq W^U(y)$.

As all workers are risk neutral and income and learning-by-doing are both proportional to y, the solution to the above Bellman equations takes the separable form:

$$W^U(y) = \alpha^U y, \text{ and } W^E(y,z) = \alpha^E(z)y,$$

where α^U and $\alpha^E(.)$ are determined below. The unemployed worker's reservation wage rate z_R is now given by $\alpha^E(z_R) = \alpha^U$ and so is independent of worker productivity y. Proposition 1 determines α^U and $\alpha^E(.)$.

Proposition 1: *Given $F(.)$, optimal job search implies*
(i) $\alpha^E(.)$ is the solution to the initial value problem:

$$\frac{d\alpha^E}{dz} = \frac{1}{q(z) + r - \rho},$$

with $\alpha^E(z) = (\overline{z} + \delta\alpha^U)/(r + \phi + \delta - \rho)$ at $z = \overline{z}$, *(ii)* (α^U, z_R) *satisfy the pair of equations:*

$$\rho\alpha^U = b - z_R + (\lambda_u - \lambda_e)\int_{z_R}^{\overline{z}} \frac{1 - F(z)}{q(z) + r - \rho} dz, \tag{1}$$

$$(r + \phi)\alpha^U = b + \lambda_u \int_{z_R}^{\overline{z}} \frac{1 - F(z)}{q(z) + r - \rho} dz. \tag{2}$$

Further $\overline{z} > b(r + \phi - \rho)/(r + \phi)$ *implies a unique solution exists for* $\alpha^U, \alpha^E(.)$ *and that* $\alpha^U > 0$ *and* $z_R < \overline{z}$*.*

Using (1) and (2), the reservation wage rate z_R is given by:

$$(r + \phi)z_R = b(r + \phi - \rho) + [\lambda_u(r + \phi - \rho) - (r + \phi)\lambda_e]\int_{z_R}^{\overline{z}} \frac{1 - F(z')}{q(z') + r - \rho} dz'. \tag{3}$$

Given this characterisation of optimal worker behavior, we now consider optimal firm behavior.

Firm Profits

Consider now the optimal wage setting strategy of a firm p when F describes the distribution of wage rate offers made by all other firms and unemployed workers adopt reservation wage rate z_R given by (3). As a firm with productivity $p < z_R$ can only make negative profit, it cannot be active in the labour market. Hence define $p_0 = \max\{z_R, \underline{p}\}$ which describes the lowest productivity firm which is active in the labour market. As $1 - \Gamma(p_0)$ describes the measure of active firms, those with $p \geq p_0$, then

$$\Gamma_0(p) = \frac{\Gamma(p) - \Gamma(p_0)}{1 - \Gamma(p_0)} \tag{4}$$

describes the distribution of firm productivities across active firms.

To characterise equilibrium, we first need to define three steady-state objects conditional on worker type ε: (a) U_ε is the fraction of type ε workers who are unemployed, (b) $N_\varepsilon(y)$ is the fraction of unemployed type ε workers with productivity no greater than y, and (c) $H_\varepsilon(y, z)$ is the joint distribution function describing the probability that an employed type ε worker has current ability no greater than y, employed at wage rate no greater than z.

We now compute $\Omega(z; p)$ defined as steady state flow profit of a firm p which pays wage rate $z \geq z_R$. The standard way of doing this is to integrate over the profits generated by those workers employed at the firm. With a zero discount rate, however, Burdett et al. (2011) show steady state flow profit is more easily obtained by integrating over the inflow of new hires times the expected lifetime profit of each hire. Consider then a firm with productivity p which pays wage rate $z \geq z_R$. If it hires a new employee with productivity y, the expected profit from that hire is $(p - z)y/(q(z) - \rho)$ as the employee leaves the firm at rate $q(z)$ and productivity y grows at rate ρ as long as the employment relationship survives. The steady state flow profit of a firm p is therefore:

$$\Omega(z; p) = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left[\begin{array}{c} \frac{\lambda_u U_\varepsilon}{\lambda_u U_\varepsilon + \lambda_e(1 - U_\varepsilon)} \int_{y'=\varepsilon}^{\infty} \frac{(p-z)y'}{q(z)-\rho} dN_\varepsilon(y') \\ + \frac{\lambda_e(1-U_\varepsilon)}{\lambda_u U_\varepsilon + \lambda_e(1-U_\varepsilon)} \int_{z'=\underline{z}}^{z} \int_{y'=\varepsilon}^{\infty} \frac{(p-z)y'}{q(z)-\rho} dH_\varepsilon(y', z') \end{array} \right] dA(\varepsilon),$$

where the first term describes the profits generated by recruiting new workers from the unemployment pool, the second by attracting workers employed at firms paying a wage rate below z.

For $p \in [p_0, \overline{p}]$, define maximal profit as $\overline{\Omega}(p) = \max_{z \geq z_R} \Omega(z;p)$. Let $F_p(z)$ denote the distribution of wage rates offered by firms with productivity p. We now formally define equilibrium.

A Market Equilibrium is a set $\{z_R, U_\varepsilon, N_\varepsilon(.), H_\varepsilon(.,.), F(.), F_p(.)\}$ for all $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$ and $p \in [p_0, \overline{p}]$ such that:
(i) the productivity distribution of active firms Γ_0 is given by (4) with $p_0 = \max\{z_R, \underline{p}\}$;
(ii) the constant profit condition is satisfied for each firm type $p \in [p_0, \overline{p}]$; i.e.,

$$\begin{aligned} \Omega(z;p) &= \overline{\Omega}(p) \quad \text{for } z \text{ where } dF_p(z) > 0; \\ \Omega(z;p) &\leq \overline{\Omega}(p) \quad \text{for } z \text{ where } dF_p(z) = 0, \end{aligned}$$

(iii) where aggregation implies offer distribution

$$F(z) = \int_{p_0}^{\overline{p}} F_p(z) d\Gamma_0(p);$$

(iv) $U_\varepsilon, N_\varepsilon(.)$ and $H_\varepsilon(.,.)$ are consistent with steady state turnover, and
(v) z_R solves the conditions in Proposition 1.

Given a Market Equilibrium exists it is simple to show (and has been shown many times in the literature) that
(a) $\underline{z} = z_R$, and
(b) for each $p \in [p_0, \overline{p}]$, $F_p(.)$ must be continuous [no mass points].
Standard turnover arguments further imply unemployment rate $U_\varepsilon = U$ where

$$U = \frac{\phi + \delta}{\phi + \delta + \lambda_u}$$

is the same for all ability types. Given this simplification, Lemma 1 now solves for the market distributions $N_\varepsilon(.)$ and $H_\varepsilon(.)$.

Lemma 1: *A Market Equilibrium implies*

$$N_\varepsilon(y) = 1 - \frac{\lambda_u \delta}{(\phi + \lambda_u)(\phi + \delta)} \left(\frac{y}{\varepsilon}\right)^{-\left(\frac{\phi(\phi+\delta+\lambda_u)}{\rho(\phi+\lambda_u)}\right)} \quad \text{for all } y \geq \varepsilon,$$

$$\begin{aligned} H_\varepsilon(y,z) &= \frac{(\phi+\delta)F(z)}{q(z)} \left[1 - \left(\frac{y}{\varepsilon}\right)^{-\frac{q(z)}{\rho}}\right] \\ &\quad - \frac{\delta \lambda_u F(z)}{\lambda_u \delta + \lambda_e(1-F(z))(\phi+\lambda_u)} \left[\left(\frac{y}{\varepsilon}\right)^{-\left(\frac{\phi(\phi+\delta+\lambda_u)}{\rho(\phi+\lambda_u)}\right)} - \left(\frac{y}{\varepsilon}\right)^{-\frac{q(z)}{\rho}}\right] \end{aligned} \tag{5}$$

for all $z \in [\underline{z}, \overline{z}]$ and $y \geq \varepsilon$.

A Market Equilibrium requires that a firm of productivity $p \in [p_0, \overline{p}]$ chooses $z \leq p$ to maximize $\Omega(z;p)$. Given $N_\varepsilon(.)$ and $H_\varepsilon(.,.)$ identified in Lemma 1 the next result solves for $\Omega(z;p)$.

Proposition 2: *Steady state profits are given by*

$$\Omega(z;p) = \widetilde{\varepsilon} l(z)(p-z), \tag{6}$$

where $\widetilde{\varepsilon} = E\{\varepsilon\}$ *and*

$$l(z) = \frac{\phi(\phi+\delta+\lambda_u)(\phi+\delta-\rho)(\phi+\delta+\lambda_e-\rho)}{(q(z)-\rho)^2(\phi+\delta+\lambda_e)[\phi(\phi+\delta+\lambda_u)-\rho(\phi+\lambda_u)]}.$$

As $\phi > \rho$ by assumption, a little algebra establishes that $l(.) > 0$ and is increasing in z for all $z \geq \underline{z}$. As $l'(.) > 0$, it is straightforward to show that equilibrium implies more productive firms offer a strictly higher z. We let $z = \zeta(p)$ denote the equilibrium wage rate offer strategy of firm p in a market equilibrium and the above implies $\zeta(.)$ is strictly increasing. The offer distribution F thus solves $F(z) = \Gamma_0(\zeta(p))$. It is now straightforward to obtain a closed form solution for ζ.

Proposition 3: *A Market Equilibrium implies*

$$\zeta(p) = p - [q(p)-\rho]^2 \int_{\underline{z}}^{p} \frac{dx}{[q(x)-\rho]^2}. \tag{7}$$

Note, the offered $\zeta(p)$ described in (7) is derived for a given z_R and p_0. Showing an equilibrium exists requires showing that z_R solves the conditions in Proposition 1 given ζ satisfies Proposition 3 and that $p_0 = \max\{\underline{p}, z_R\}$. Given p_0 and noting that in equilibrium $F(\zeta(p)) = \Gamma_0(p)$, (3) and (7) imply that z_R solves $T(z_R;p_0) = 0$, where

$$\begin{aligned} T(z_R;p_0) \equiv\ & (r+\phi)z_R - b(r+\phi-\rho) \\ & - [\lambda_u(r+\phi-\rho)-(r+\phi)\lambda_e] \int_{p_0}^{\overline{p}} \left[\frac{p_0 - z_R}{(\phi+\delta+\lambda_e-\rho)^2} + \int_{p_0}^{x} \frac{ds}{(q(s)-\rho)^2}\right] \beta(x)dx, \end{aligned} \tag{8}$$

and

$$\beta(x) = \frac{2(q(x)-\rho)\lambda_e(1-\Gamma_0(x))\Gamma_0'(x)}{q(x)+r-\rho} > 0.$$

Denote $z_R(p_0)$ the solution to $T(z_R;p_0) = 0$ for any p_0. Since $p_0 = \max\{\underline{p}, z_R(p_0)\}$, however, there are two possible cases. First $p_0 = \underline{p}$ if and only if $\underline{p} > z_R(\underline{p})$. Otherwise, $p_0 > \underline{p}$ and some firms will not be active in the labour market. Given these results, we can now establish existence and uniqueness of equilibrium.

Theorem 1: *There exists a unique Market Equilibrium.*

3 Implications

In a Market Equilibrium wages are dispersed as: (i) workers have disperse initial abilities; (ii) productivity differences arise in the cross section as workers differ (by age and thus) by labour market experience; (iii) there are differences in wages paid due to search frictions and labour market sorting. The aim of the calibration is to quantify the relative importance of each of these factors when explaining wage inequality. We focus the discussion by commenting on four particular implications of the model.

The model identifies the following wage equation for worker i with initial ability ε_i, experience x_{it} at date t and employment at firm j paying wage rate z_j:

$$\log w_{ijt} = \rho x_{it} + \log z_j + \log \varepsilon_i. \tag{9}$$

In contrast to a large fraction of the empirical labour literature, this wage equation contains a firm fixed effect. From the worker's perspective, that fixed effect z_j is the realised outcome to a stochastic search process. As the implied quit process is not random (an employee at firm j only quits to an outside offer paying a higher wage rate $z > z_j$), such turnover has important empirical implications both for individual wage dynamics and cross-sectional wage inequality.

1. **Decomposing experience effects on wages**

The classic explanation for why wages increase with experience is that there is learning-by-doing and so more experienced workers (being more productive) earn higher wages. But on-the-job search with no learning-by-doing also predicts that wages, on average, increase with experience (e.g. Burdett, 1978, Burdett and Mortensen, 1998). Conditional on experience, the wage equation (9) implies expected log wage:

$$E(\log w \mid x) = \rho x + E(\log z \mid x) + E(\log \varepsilon). \tag{10}$$

It is straightforward to show $E(\log z \mid x)$ is increasing and concave in experience.[7] One important aim of the calibration is to evaluate how much of the observed impact of experience on wages is due to each of these processes.

2. Equilibrium sorting

Equilibrium implies more productive firms offer higher wage rates z. As equilibrium search

[7]Integration by parts implies

$$E(\log z \mid x) = \log z_R + \int_{z_R}^{\overline{z}} \frac{1 - H(z \mid x)}{z} dz.$$

As $y = \varepsilon e^{\rho x}$, each firm p uses strategy $z = \zeta(p)$ and so $F(\zeta(p)) = \Gamma_0(p)$, it can be shown that

$$H(z \mid x) = F(z) \left[\frac{\lambda_u \delta + \lambda_e (1 - F(z))(\phi + \lambda_u) e^{-\left(\frac{\lambda_u \delta + \lambda_e (1-F(z))(\phi+\lambda_u)}{(\phi+\lambda_u)}\right)x}}{\lambda_u \delta + \lambda_e (1 - F(z))(\phi + \lambda_u)} \right].$$

As $\partial H(z \mid x)/\partial x < 0$ and $\partial^2 H(z \mid x)/\partial x^2 > 0$, it is easily established that $E(\log z \mid x)$ is an increasing and concave function of experience.

also implies $E(\log z \mid x)$ is increasing in x, there is positive assortative matching between the productivity of a firm and the average experience (and thus average productivity) of its workforce. Such sorting increases wage inequality. For example, older workers tend to earn more not only because they are more experienced and so more productive, they are also more likely to be employed at more productive firms which pay higher wage rates. Indeed, a little algebra establishes that the probability a worker with initial ability ε and current productivity y is employed at a firm with productivity no greater than p, $H_\varepsilon(p \mid y)$, is first order stochastically increasing in y; i.e. $\partial H_\varepsilon(. \mid y)/\partial y < 0$. Given $\lambda_u \geq \lambda_e$, the distribution of worker productivities employed at firms with productivity p conditional on these workers having an initial ability of ε, $H_\varepsilon(y \mid p)$, is first order stochastically increasing in p; i.e. $\partial H_\varepsilon(. \mid p)/\partial p < 0$.[8]

Eeckhout and Kircher (2011) develop a theory on how to identify sorting in assignment models. In our model workers' initial abilities, ε, are uncorrelated with firms' productivities, p, as is typically found in regressions using the Abowd et al. (1999) methodology. The sorting allocation highlighted in this paper is driven by experience effects. In our framework workers' productivities are an evolving endogenous variable that correlates with firms' productivities because of on-the-job search. One way of incorporating sorting by types, as in the assignment model, would be to allow workers to choose their search effort as a function of their initial abilities (see Lentz, 2010). Bagger and Lentz (2015) show that the latter type of sorting is an important determinant of wage inequality. Below we show that the sorting generated by our theory also has important implications for wage inequality.

3. Variance decomposition of log wages

As experience effects (x_{it}) and job search outcomes (z_j) are both independent of initial ability ε_i, the wage equation (9) implies the following variance decomposition of log wages:

$$var(\log w) = \rho^2 var(x) + var(\log z) + 2\rho cov(\log z, x) + var(\log \varepsilon). \tag{11}$$

The first term describes the contribution of learning-by-doing and disperse labour market experiences in explaining wage inequality. The second and third terms describe how variations in wage rates across firms affect wage inequality. A perfectly competitive market would imply both of these terms are zero (the law of one price). Search frictions instead generate disperse wage rates z, where the third term describes the added wage inequality generated by sorting (that higher productivity firms tend to employ more experienced workers). The last term attributes the wage dispersion that is otherwise not captured by the model as unobserved dispersion in worker abilities.

As $z = \theta p$, one can further decompose the dispersion in firm wage rates as

$$var(\log z) = var(\log \theta) + var(\log p) + cov(\log p, \log \theta). \tag{12}$$

[8] We omit the derivations of these results as they are similar to the ones found in Burdett et al. (2011). It is important to note that these sorting results rely on the assumed positive complementarity between workers' and firms' productivities. See Lentz (2010) and Eeckhout and Kircher (2011), among others, for an analysis of sorting using a more general production function.

The first term captures the variation in wage rates that arises when there is no firm heterogeneity (e.g. Burdett et al., 2011). The issue then is how important is firm heterogeneity in explaining wage dispersion? An important feature of the model is that there is piece rate compression: although more productive firms pay higher wage rates, they do not increase wages so much that they increase the piece rate paid $\theta = z/p$. This is not entirely surprising as the perfectly competitive case implies perfect piece rate compression - that all firms pay the same wage rate regardless of productivity p. Taking piece rate compression into account, we must add the second and third terms in (12) together and so identify the net effect of firm heterogeneity on frictional wage inequality. It turns out this net effect is large: firm heterogeneity has a large impact on wage dispersion.

4. The Mm ratio

The Mm ratio is defined as the ratio of the average wage earned to the lowest wage paid in the market among equally productive workers. For plausible parameter values, Hornstein et al. (2011) explain why most search models generate a reservation wage that is too high to match the observed Mm ratio in the US economy. That paper does not, however, consider a model with both on-the-job search and learning-by-doing as done in Burdett et. al (2011). With both features present, the on-the-job search framework easily generates empirically relevant Mm ratios: unemployed workers are willing to accept low starting wages as experience is valuable.[9] Here we use information on the Mm ratio to usefully calibrate the model. Using the approximation method described in Hornstein et al. (2011), Lemma 2 relates the Mm ratio to the fundamentals of the model.

Lemma 2: *Given a market equilibrium, the Mm ratio can be well approximated as*

$$Mm \cong \left[1 + \frac{\lambda_u(r+\phi-\rho) - (r+\phi)\lambda_e}{(r+\phi)(r+\phi+\delta+\lambda_e-\rho)}\right] \Big/ \left[\frac{r+\phi-\rho}{r+\phi}\frac{b}{z^M} + \frac{\lambda_u(r+\phi-\rho) - (r+\phi)\lambda_e}{(r+\phi)(r+\phi+\delta+\lambda_e-\rho)}\right], \tag{13}$$

where z^M is the average z earned by employed workers.

4 Quantitative Analysis

We calibrate the model using simulated methods of moments to match salient features of the UK labour market using the British Household Panel Survey (BHPS).

Data

The BHPS is an annual survey of individuals, age 16 years or more, in a nationally representative sample of about 5,500 households. Approximately 10,000 individuals are interviewed

[9]Hornstein et. al (2011) adopt as baseline calibration values $\lambda_u = 0.43$, $\delta = 0.03$, $b/z^M = 0.4$, $r = 0.0041$, $\phi = 0.0021$. In addition, they set $\lambda_e = 0.13$ to quantify the Mm ratio obtained from a model with on-the-job search, but without learning-by-doing. They set $\rho = 0.0017$ to quantify the Mm ratio obtained from a model with learning-by-doing, but without on-the-job search. When using all these parameters values in the model presented here we obtain an $Mm = 1.49$, a ratio which is within the bounds of the estimated Mm ratios, presented in Hornstein et al. (2007).

each year. It started in 1991 and was subsumed by the new and bigger survey "Understanding Society" in 2010. The BHPS contains socioeconomic information, including information about household organization, the labour market, income and wealth, housing, health and socioeconomic values. Using this information one is able to reconstruct the labour market histories of individuals since leaving full-time education. Maré (2006) provides a comprehensive guide on how to derive consistent histories that summarize individual's transitions between employment, unemployment and non-participation; transitions between jobs; occupational and industry changes; actual and potential work experience; wages and hours worked; and several socio economic characteristics that are standard in household survey data.

We construct individual labour market histories following Maré's (2006) procedure, considering only white male workers. To focus on young workers we consider those individuals that were originally sampled in 1991 and were between 16 and 30 years of age at that time. We construct their entire employment history since leaving full-time education using retrospective work history information and follow these workers over time until 2004 (or earlier if they left the sample before). We then stratify the sample of workers into three educational or skill groups. We consider workers to be low skilled if they reported having no qualification, other qualifications, apprenticeship, CSE, commercial qualifications or no O-levels. Medium skilled workers are those who reported having O-level or equivalent qualifications. High skilled workers are those that achieved A-levels, nursing qualifications, teaching qualifications, university degree or higher and other higher qualifications.[10] We further restrict attention to paid (dependent) full-time employment spells in the private sector and unemployment spells that lasted at least one month. To keep the sample as homogeneous as possible we only consider those employment and unemployment spells that occur before an individual reported he became (if at all) self-employed, a civil servant, worked for the central or a local government or the armed forces, long-term sick or entered retirement. We also dropped those individuals that re-entered full-time education or had a spell in government training.

These restrictions leave us with a sample of 1,867 individuals, where 486 are considered low skilled, 658 medium skilled and 723 high skilled. We assume that an individual changed jobs if he changed employer. A change in employer is identified when the worker declared a change in his 2-digit occupation and 2-digit industry. In principle, this could underestimate the number of jobs an individual holds during his working life as he can change jobs within the same employer. However, to be consistent with the theory, we consider job-to-job transitions as employer-to-employer transitions.[11] We consider as our earnings variable the real hourly (gross) wage of these individuals.[12] We trim the wage data by 5 percent on each side to reduce measurement

[10]See Dustmann and Pereira (2008) for a similar classification using the BHPS. The main difference is that we consider those workers with nursing qualifications, teaching qualifications and A-levels as high skilled workers. We do this to have somewhat an even number of workers in each skill group.

[11]Since we do not count spells that are shorter than a month a transition in which the individual changed employer but experienced an intervening spell of unemployment of less than a month is considered a direct job-to-job transition. If the individual experiences an unemployment spell longer than a month, then he is considered unemployed. See Jolivet et al. (2006) for a similar assumption.

[12]Following Dustmann and Pereira (2008), we construct real hourly wages by dividing monthly (gross) earnings by 4.33 weeks and then by the average number of hours worked in a week in full-time jobs. We also take into account overtime hours and use the CPI to deflate nominal wages.

error and to consider all jobs that pay above the national minimum wage, introduced in the UK in 1999. It is worth pointing out that wage data is only available as from 1991, the first wave of the BHPS. Overall there are 12,091 spells in the sample, where 3,434 of those are associated with low skilled workers, 4,382 are associated with medium skilled workers and 4,275 are associated with high skilled workers.

4.1 Calibration

We consider the reference time period as a month. Following Hornstein et al. (2011), we let $r = 0.0041$ and fix $\phi = 0.0021$ so that workers participate in the labour market, on average, for 40 years. We approximate the firm productivity and worker ability distributions using (truncated) Weibull distributions. Let κ_1, κ_2 and $\underline{p}$ describe the shape, scale and location parameters, respectively, of the productivity distribution among firms; and α_1, α_2 and $\underline{\varepsilon}$ describe the shape, scale and location parameters, respectively, of the worker ability distribution.

This parameterization leaves a set of 13 parameters, $\Psi = \{\delta, \lambda_u, \lambda_e, \rho, \underline{p}, \overline{p}, \kappa_1, \kappa_2, b, \underline{\varepsilon}, \overline{\varepsilon}, \alpha_1, \alpha_2\}$ to be estimated. To do this, we minimize the sum of squared distances between a set of simulated moments from the model and their counterparts in the data.

Table 1: Targeted Moments

Moments	Low Skilled		Medium Skilled		High Skilled	
	Data	Model	Data	Model	Data	Model
Average durations (months)						
Unemp spell	12.35	12.15	8.42	8.32	5.87	5.84
Employment spell	41.63	43.77	68.63	68.26	71.00	70.50
Job spell	35.68	37.03	43.26	43.90	45.76	44.58
Returns to experience (%)						
2 years	7.67	6.65	7.92	7.07	9.12	8.19
4 years	14.62	12.86	14.94	13.75	17.35	15.88
6 years	20.85	18.62	21.06	20.06	24.67	23.05
8 years	26.36	23.94	26.27	25.98	31.09	29.71
10 years	31.15	28.81	30.59	31.51	36.61	35.86
Wage Dispersion						
Mm ratio	1.48	1.44	1.52	1.55	1.51	1.45
mean(log w)	0	-5.71E-03	0	9.51E-03	0	-2.14E-02
var(log w)	0.089	0.088	0.082	0.079	0.094	0.098
Skewness(log w)	0.027	-0.114	0.013	-0.095	-0.049	-0.265
Kurtosis(log w)	3.196	2.618	2.987	2.671	3.004	3.013
log($\underline{w}$)	-0.991	-0.954	-0.968	-1.018	-1.287	-1.217
log($\overline{w}$)	1.048	0.887	0.904	0.960	1.024	1.044

Targeted Moments We target 15 moments based on the main characteristics of the labour market to which the model is directly related. Table 1 describes these moments decomposed by skill group (low, medium, high) as described above.[13]

[13]The targeted moments are compared with the corresponding moments from model simulations for different values of the parameters in Ψ until the loss functions described above is minimised. For each model simulation

To identify wage dispersion from the data, we build on the approach of Hornstein et al. (2007). For each skill group, a regression of log wages is run on a dummy for marital status, 8 regional dummies, 8 (one-digit) occupational dummies, 8 (one-digit) industry dummies, dummies for cohort effects and a time trend. The resulting log wage residual is then used as our measure of wage dispersion. The empirical wage experience profile is identified by regressing log wages on a quadratic on experience, a quadratic on tenure, a dummy for marital status, 8 regional dummies, 8 (one-digit) occupational dummies, 8 (one-digit) industry dummies, dummies for cohort effects and a time trend.[14] To obtain the empirical Mm ratio we follow the procedure of Hornstein et al. (2007), which involves controlling for worker fixed effects in the experience regression. In the Appendix, we discuss this procedure in more detail. The calibration requires not only that the model is consistent with the observed log wage residuals across workers (by skill group) but also with the estimated experience effects and the empirical Mm ratio. A minor difficulty with this approach is that the mean log wage residual is zero for each skill group. Below we will find that high skilled workers have longer employment spells, are employed in more productive firms, spend less time in unemployment, etc. The calibration requires each skill group to have the same average log wage residual. It thus compensates by attributing a higher mean worker ability ε to the lowest skill group. Clearly computing mean ability by skill group requires also taking into account the occupational and location dummies used in the original regression. This is not straightforward as one must adopt a theory of how different ability workers select into different occupations (lumberjack or accountant) and locations (Sherwood Forest or London). Fortunately this issue does not otherwise distort our results as equilibrium market behaviour is independent of the assumed distribution of abilities ε. In essence the unobserved worker ability distribution captures the variation in the data (by skill group) that is not otherwise explained by the model.

Key features of the data described in Table 1 find the low skilled group (compared to the high skilled group) has longer average unemployment spells (one year compared to 6 months), shorter average employment spells (3.5 years compared to 6 years) but average job spells which are not so different (3 years compared to 3.75 years). This latter statistic arises as high skilled workers have much higher quit rates. Such turnover provides direct information on the transition parameters δ, λ_u and λ_e. As the parameters that govern worker turnover, human capital accumulation and the firm productivity distribution, determine the shape of the wage-experience profile, information

run, we set $b = 0.4z^M$, following Hornstein et al. (2011), to jointly recover the values of z_R, z^M and b using (8) and (19). Further, the simulated data is constructed such that it has the same structure as the BHPS for consistent measurement. See the Appendix for further details of the simulation procedure.

[14]One potential worry with the above specification is that workers' unobservable characteristics might be biasing the estimated returns to general experience because, for example, more able workers could be more likely to receive outside offers than less able workers in the data. The results of Dustmann and Pereira (2008), however, suggest that any potential bias of this sort is very small. These authors estimate returns to experience for the UK using the BHPS by skill/education categories. When controlling for worker and job match (unobservable) fixed effects, their estimated experience effects hardly change across specification and estimation methods. See also Williams (2009). Furthermore, as suggested by Dustmann and Pereira (2008) and Williams (2009) we incorporate yearly dummies into the wage regressions to control for the presence of a macro trend. We find that the latter is important for high and medium skilled workers as without it the estimated returns to experience nearly double in size. In addition we also incorporate cohort dummies into our wage regressions. In this case, we find that these dummies have a very small impact on the estimated returns to experience across skill groups.

on the average wage-experience profile also helps tie down parameter values. We also incorporate information on the distribution of (residual) log wages.

Table 1 shows the fit of the model is very good and has no difficulty in matching the empirical Mm ratios or the cross-sectional wage distribution. What is of central interest now is understanding how the labour market differs across skill groups and identifying the impact of job search and learning-by-doing on (i) average wage profiles by experience and (ii) wage inequality.

Table 2: Parameter values

	Low Skilled	Medium Skilled	High Skilled
Parameters			
λ_u	0.082	0.121	0.171
δ	0.021	0.012	0.012
λ_e	0.010	0.038	0.039
ρ	0.0018	0.0016	0.0020
$\underline{p}$	3.049	6.950	9.810
κ_1	0.5908	0.5040	0.403
κ_2	0.4018	0.4033	0.403
$\overline{p}$	25.176	25.222	28.349
b	0.808	2.701	3.889
$\underline{\varepsilon}$	0.267	0.089	0.041
α_1	1.222	0.906	1.770
α_2	2.320	1.564	1.563
$\overline{\varepsilon}$	0.560	0.174	0.116
Endog. variables			
z_R	1.405	4.284	6.718
urate	0.217	0.105	0.077
Pr EE	0.004	0.011	0.011

Parameters

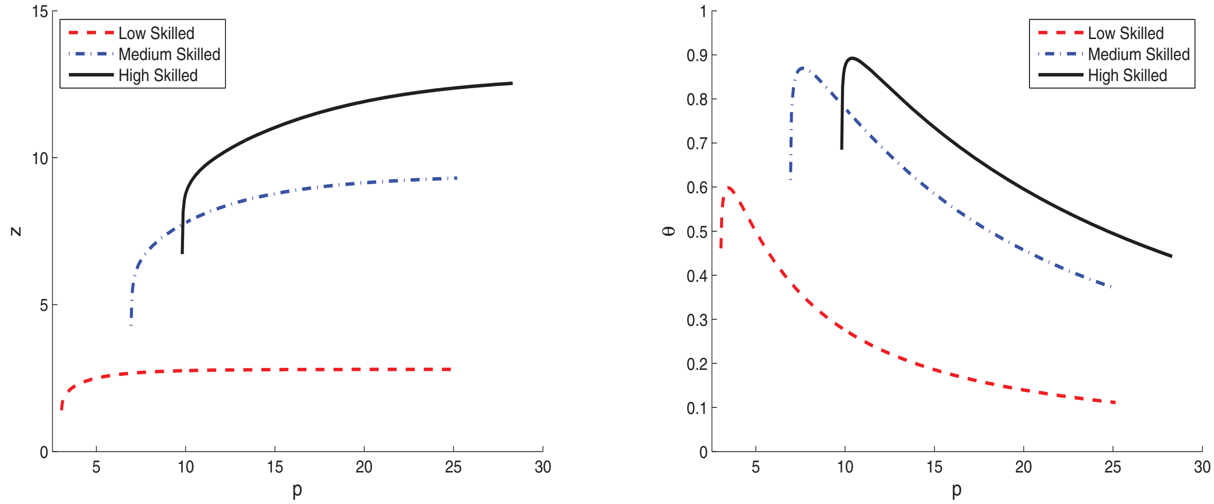
Table 2 describes the calibrated parameter values for each skill group. Those values present a picture that is similar to the one identified in Steward (2007). Relative to the high skilled group, low skilled workers spend more time in unemployment and there is little upward job mobility through job-to-job transitions.[15] As they spend long periods in unemployment, low skilled worker also accumulate less human capital through learning-by-doing and so earned wages are likely to remain low into the longer term. In contrast, high skilled workers can more quickly find work while unemployed, enjoy much greater job security and enjoy a much greater chance of receiving (and quitting to) an attractive outside offer.[16]

[15] The job-to-job transition rate is computed using

$$\Pr EE = (\phi+\delta)\left[\left(1+\frac{\phi+\delta}{\lambda_e}\right)\ln\left(1+\frac{\lambda_e}{\phi+\delta}\right)-1\right],$$

The rate at which a high skilled worker receives a preferred outside offer is three times that of a low skilled worker.

[16] These job reallocation patterns also reflect that medium and high skilled employed workers face less search frictions than low skilled workers. Using $k = \lambda_e/(\phi+\delta)$ as a measure of the extent of search frictions faced



(a) z-functions

(b) θ-functions

Figure 1: Offered z and θ as functions of firm productivities

Figure 1 depicts the wage rate, z, and piece rate, θ, functions generated by the model. These wage rate functions characterise workers' "job" ladders. Workers of different skills jump into their respective job ladders with their first offer from unemployment and then move up as they receive acceptable outside offers when employed. Job destruction shocks reset this process by bringing workers back to unemployment. From the figure it is immediate that the height of the job ladder is increasing in workers' skills, such that high skilled workers are able to access higher job ladders than medium and low skilled workers. We discuss the returns of climbing these job ladders in the next subsections.

The values of z_r in Table 2 imply $p_0 = \underline{p}$ and hence all firms at the left tail of the productivity distribution find it profitable to participate in the labour market. This arises because unemployed workers are willing to accept a low wage rate to become employed and accumulate work experience. We find that this "foot-in-the-door" effect is more pronounced for low skilled workers.[17]

Furthermore, the values of $\{\underline{p}, \kappa_1, \kappa_2, \overline{p}\}$ imply that high skilled workers face a firm productivity distribution that is to the right of the distribution faced by medium skilled workers; while the latter distribution is, in turn, to the right of the firm productivity distribution faced by low skilled workers.[18] This characteristic implies that on average low skilled workers access jobs

by employed workers and noting that a higher value implies lower search frictions, the estimated transitions parameters imply that low skilled workers face 6 times more search frictions relative to medium and high skilled workers.

[17] If one were to counterfactually set $\rho = 0$, the reservation z_r for low skilled workers would be 28.11 percent higher than the value reported in Table 1; while for medium and high skilled workers it would be 16.71 and 21.17 precent higher, respectively.

[18] This result mainly arises due to differences in the returns to experience, values of the Mm ratio and the extent of search frictions we observed across skill groups. In the case of low and medium skilled workers, for example, to match similar wage-experience profiles and Mm ratios in a context in which medium skilled workers

in firms with lower productivities than medium and high skilled workers. As discussed below, the θ functions depicted in Figure 1 imply that low skilled workers also face stronger piece rate compression than medium and high skilled workers.

Decomposing the experience effect on wages

Recall that the expected log wage conditional on experience is

$$E(\log w \mid x) = \rho x + E(\log z \mid x) + E(\log \varepsilon).$$

For the low skilled group, the calibration finds that 88% of the experience effect on wages is due to learning-by-doing. The rationale is twofold. First learning-by-doing rates are reasonably large (comparable in value to the learning-by-doing rates of the high skilled). Second the low skilled are unlikely to climb the job ladder: their job destruction rate ($\delta = 0.021$ per month) is not only high, it is 5 times greater than the (average) rate at which low skilled workers quit to preferred outside offers ($P(EE) = 0.004$ per month). In other words each unskilled worker, when employed, is far more likely to return to the unemployment pool than climb the job ladder.

For the high skilled group, the job ladder plays a more important role: the calibration finds 69% of the experience effect on wages is due to human capital accumulation. Although learning-by-doing continues to explain the larger part of wage growth by experience, on-the-job search plays a more sizeable role.

This interpretation of the data is entirely consistent with the findings of Bagger et al. (2014) and Menzio, et al. (2014) who suggest that, among young workers, human capital accumulation plays a more important role than job-to-job transitions in explaining average wage differentials. This does not mean, however, that search plays little role in explaining wage inequality.

The variance decomposition of wage inequality by skill group

Table 3: Variance decomposition of log (w)

	var(log w)	var(logε)	ρ^2 var(x)	var(log$z$)	2ρcov(x,log$z$)
Low Skill	**0.088**	**0.016**	**0.015**	**0.055**	**0.002**
	Total=100%	18.2%	17.1%	62.5%	2.3%
Medium Skill	**0.079**	**0.018**	**0.011**	**0.043**	**0.007**
	Total=100%	22.8%	13.9%	54.4%	8.9%
High Skill	**0.098**	**0.030**	**0.018**	**0.042**	**0.008**
	Total=100%	30.6%	18.4%	42.9%	8.2%

Table 3 describes the variance decomposition implied by (11): Column 1 describes the total variation in log wages for each skill group, the remaining columns decompose that variation

face lower search frictions, the calibration adjusts by given low skilled workers a higher value of ρ but lower job productivity relative to medium skilled workers. For high skilled workers, however, higher returns to experience and low search frictions imply both a higher value of ρ and higher job productivity relative to low and medium skilled workers.

using (11). Not surprisingly unobserved worker heterogeneity [column 2] explains a large chunk of the observed wage dispersion, ranging from 18% for the unskilled group to 31% for the high skilled group. Dispersion in labour market experience with learning-by-doing [column 3] also contributes a significant amount to wage inequality, ranging between 14% to 18% depending on skill groups. In all cases, however, the largest contributor to wage inequality is the dispersion in wage rates paid by firms. We discuss this further below. The final column describes the effect of positive sorting on wage inequality. Reflecting that job ladder effects are small for the low skilled, sorting only contributes 2.3% to total wage inequality for these workers. For the higher skill groups, sorting instead contributes around 8-9% to total wage variation.

Table 4: Variance decomposition of log (z)

	var(log$z$)	var(logθ)	var(log$p$)	cov(log$p$,logθ)
Low Skill	**0.055**	**0.020**	**0.060**	**-0.024**
	62.5%	22.7%	68.2%	-27.3%
Medium Skill	**0.043**	**0.014**	**0.045**	**-0.016**
	54.4%	17.7%	57.0%	-20.2%
High Skill	**0.042**	**0.014**	**0.049**	**-0.021**
	42.9%	14.3%	50.0%	-21.4%

Table 4 now uses (12) to decompose the variation in firm wage rates $z = \theta p$ and so identify the impact of firm heterogeneity on wage inequality. The first column reports the variation in firm wage rates as described in the previous table. Note the law of one price would imply zero variation (no firm fixed effects). The second column describes the wage rate dispersion that is attributable to pure non-competitive wage formation when firms are identical (e.g. Burdett et al., 2011). This component accounts for between 14-22% of observed wage inequality. Taking the last two columns together (where the last column describes piece rate compression), the added (net) effect of firm heterogeneity on total wage inequality is very large, being as high as 41% for the low skilled group (68%-27%) and 29% for the high skilled group (50%-21%). Thus firm heterogeneity has a very large impact on overall wage inequality, an effect which is missing in competitive markets.

Note that piece rate compression on its own is an important component of the variation in firm wage rates. For all skill groups, Figure 1.b shows that offered piece rates initially increase and then decrease with firm productivities. This property implies that, except for firms in the left tail of the productivity distribution, there is a positive relation between firm productivity and their level of monopsony power. This is similar to the result obtained by Bontemps et al. (2000). They find that, except for very low productive firms in some sectors, more productive firms have higher values of the monopsony power index $1 - w(p)/p$, where $w(.)$ is an increasing function that associates offered wages to firm of productivity. In our context their monopsony power index can be expressed as $1 - \theta(p)$, which increases with p for high enough productivities. Since piece rate compression is stronger among low skilled workers, our results imply that firms exert higher monopsony power on low skilled workers than on medium and high skilled workers.

5 Further Discussion

The framework developed in this paper is nicely tractable and seems well suited to understand the various causes of wage inequality. The empirical investigation presented yielded several new insights. Three seem particularly worthy of restating. First, heterogeneity among firms generates a lot of wage inequality. Second, among low skilled workers job ladder effects are small, most of the impact of experience on wages is due to learning-by-doing. Third, high skilled workers are much more mobile. Job ladder effects have sizeable impact.

In related work, Postel-Vinay and Robin (2002) estimate their sequential auction model without human capital accumulation to analyse the relative contribution of worker ex-ante heterogeneity (abilities), firm ex-ante heterogeneity (productivities) and search frictions in overall wage inequality using French data. In line with their results, we also find that the importance of firm productivities differentials in explaining wage inequality decreases with skill groups; while the importance of worker ability differentials increases with skill groups.[19] More recently Bagger and Lentz (2015) estimate a version of the Postel-Vinay and Robin (2002) model that allows for sorting between worker initial abilities and firm productivities (see Lentz, 2010). Although our models generate sorting in a different way, we find that the contribution of sorting in explaining wage inequality for medium and high skilled workers is of the same order of magnitude as the one Bagger and Lentz (2015) find when estimating their model on Danish data.

An important assumption in our framework is that firms are restricted to post constant wage rate contracts. This assumption is not innocuous as firms would have an incentive to post wage contracts that depend on their applicants' experience (see Carrillo-Tudela, 2009). Given risk neutrality, these contracts can be characterised by promotion or step contracts as firm would want to backload wages to reduce workers' quit probabilities. Allowing for wage-experience contracts in our framework would have an impact on the variance decomposition results as we will have to take into account the wage growth associated with these contracts as well as its interaction with human capital accumulation. Although allowing for this type of contracts is appealing, the analysis in Carrillo-Tudela (2009) and Burdett and Coles (2010) suggests that such an extension becomes intractable in the presence of firm heterogeneity.

In this paper we have also restricted our analysis to an economy that is in steady state. An important question, however, is to what extent the employment patterns and the relative contributions of worker ability differentials, firm productivity differentials, and frictional wage dispersion change over the business cycle. Recent work by Coles and Mortensen (2012) suggests that such an extension is possible. We leave this important extension for future research.

[19] Postel-Vinay and Robin (2002) find that the contribution of worker ex-ante heterogeneity is more important in those occupations that one would consider require higher qualification levels (e.g. "Executives, managers and engineers" or "Supervisors, administrative and sales"); while the importance of firm productivity is more important in those occupations that one would consider require lower qualification levels (e.g. "Sales and service workers" and "Unskilled manual workers").

References

[1] Abowd, John M., Francis Kramarz and David N. Margolis. 1999."High Wage Workers and high Wage Firms." *Econometrica*, 67(2): 251-334.

[2] Altonji, Joseph G. and Nicolas Williams. 2005."Do Wages Rise with Job Seniority? A Reassessment." *Industrial and Labor Relations Review*, 58(3): 370-397.

[3] Bagger, Jesper and Rasmus Lentz. 2015."An Empirical Model of Wage Dispersion with Sorting."Mimeo, University of Wisconsin-Madison, USA.

[4] Bagger, Jesper, Francois Fontaine, Fabien Postel-Vinay and Jean-Marc Robin. 2014. "A Feasible Equilibrium Search Model of Individual Wage Dynamics with Human Capital Accumulation." *American Economic Review*, 104(6): 1551-1596.

[5] Bartelsman, Eric and Mark Doms. 2000. "Understanding Productivity: Lessons from Longitudinal Microdata." *Journal of Economic Literature.* 38: 569-94.

[6] Barlevy, Gadi. 2008, "Identification of Search Models using Record Statistics." *Review of Economic Studies,* 75(1): 29-64.

[7] Becker, Gary S. 1964. *Human Capital: A Theoretical and Empirical Analysis with Special Reference to Education.* Chicago, IL: University of Chicago Press.

[8] Bontemps, Christian, Jean-Marc Robin and Gerard J. van den Berg. 2000. "Equilibrium Search with Continuous Productivity Dispersion: Theory and Nonparametric Estimation." *International Economic Review*, 41(2): 305-358.

[9] Bowlus, Audra and Huju Liu. 2013. "The Contribution of Search and Human Capital to Earnings Growth over the Life Cycle." *European Economic Review*, 64(C): 305-331.

[10] Bunzel, Henning, Bent J. Christensen, Nicholas M. Kiefer and Lars Korsholm. 2000. "Equilibrium Search and Human Capital Accumulation." in H. Bunzel, B.J. Christensen, N.M. Kiefer and D.T. Mortensen, eds. *Panel Data and Structural Labor Market Models*, North-Holland Publishing.

[11] Burdett, Kenneth. 1978. "A Theory of EmployeevSearch and Quits. " *American Economic Review*, 68(1): 212-220.

[12] Burdett, Kenneth and Dale T. Mortensen. 1998. "Wage Differentials, Employer Size, and Unemployment." *International Economic Review*, 39(2): 257-273.

[13] Burdett, Kenneth and Melvyn G. Coles. 2010. "Wage/tenure Contracts With Heterogeneous Firms," *Journal of Economic Theory*, 145(4): 1408-1435.

[14] Burdett, Kenneth, Carlos Carrillo-Tudela and Melvyn G. Coles. 2011. "Human Capital Accumulation and Labour Market Equilibrium." *International Economic Review*, 52(3): 657-677.

[15] Carrillo-Tudela, Carlos. 2009. "An Equilibrium Search Model with Optimal Wage-experience Contracts. " *Review of Economic Dynamics*, 12(1): 108-128.

[16] Coles, Melvyn G. and Dale Mortensen. 2012. "Equilibrium Labour Turnover, Firm Growth and Unemployment" ISER Working Paper No. 2012-07, University of Essex, UK.

[17] Dustmann, Christian and Costas Meghir. 2005. "Wages, Experience and Seniority." *Review of Economic Studies*, 72 (1): 77-108.

[18] Dustmann, Christian and Sonia Periera. 2008. "Wage Growth and Job Mobility in the United Kingdom and Germany." *Industrial and Labor Relations Review*, 61(3): 374-393.

[19] Eeckhout, Jan and Philipp Kircher. 2011. "Identifying Sorting in Theory. " *Review of Economic Studies*, 78(3): 872-906.

[20] Fu, Chao. 2011. "Training, Search and Wage Dynamics." *Review of Economic Dynamics*, 14(4): 650-666.

[21] Hornstein, Andreas, Per Krusell and Giovanni L. Violante. 2011. "Frictional Wage Dispersion in Search Models: A Quantitative Assessment." *American Economic Review.* 101(7): 2873-2898.

[22] Hornstein, Andreas, Per Krusell and Giovanni L. Violante. 2007. "Frictional Wage Dispersion in Search Models: A Quantitative Assessment." Mimeo. Department of Economics, NYU.

[23] Jolivet, Gregory, Fabien Postel-Vinay and Jean-Marc Robin. 2006. "The Empirical Content of the Job Search Model: Labor Mobility and Wage Distributions in Europe and the US." *European Economic Review*, 50: 877-907.

[24] Lentz, Rasmus. 2010. "Sorting by Search Intensity." *Journal of Economic Theory*, 145(4): 1436-1452.

[25] Lentz, Rasmus and Dale T. Mortensen. 2005. "Productivity Growth and Worker Reallocation." *International Economic Review*, 46(3): 731-749.

[26] Manning, Alan. 2003. *Monopsony in Motion: Imperfect Competition in the Labour Market*, Princeton, NJ, USA: Princeton University Press.

[27] Maré, David. 2006. "Constructing Consistent Work-life Histories: A guide for users of the British Household Panel Survey." ISER Working Paper 2006-39, University of Essex, UK.

[28] Menzio, Guido, Irena Telyukova and Ludo Visschers. 2014. "Directed Search over the Life cycle." forthcoming in *Review of Economic Dynamics*.

[29] Mincer, Jacob. 1974. *Schooling, Experience and Earnings.* New York: National Bureau of Economic Research.

[30] Mortensen, Dale T. 2003. *Wage Dispersion: Why are Similar Workers Paid Differently?*, Cambridge, MA, USA: MIT Press.

[31] Ortego-Marti, Victor. 2012. "Unemployment History and Frictional Wage Dispersion. " Mimeo, Department of Economics, University of California Riverside.

[32] Postel-Vinay, Fabien and Jean-Marc Robin. 2002. "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity." *Econometrica*, 70(6): 2295-2350.

[33] Rubinstein, Yona and Yoram Weiss. 2007. "Post Schooling Wage Growth: Investment, Search and Learning." in Eric A. Hanushek and Finis Welch. eds., *Handbook of Economics of Education*, Vol. 1, Amsterdam: Elsevier Science B.V.

[34] Steward, Mark. 2007. "The Interrelated Dynamics of Unemployment and Low-wage Employment." *Journal of Applied Econometrics*, 22: 511-531.

[35] Topel, Robert. 1991. "Specific Capital, Mobility, and Wages: Wages Rise with Job Seniority." *Journal of Political Economy*, 99(1): 145-176.

[36] Tjaden, Volker and Felix Wellschmied. 2014. "Exploring the Causes of Frictional Wage Dispersion. " *American Economic Journal: Macroeconomics*, 6(1): 134-161.

[37] Williams, Nicolas. 2009. "Seniority, experience, and wages in the UK." *Labour Economics*, 16(3): 272-283.

[38] Yamaguchi, Shitaro. 2010. "Job Search, Bargaining, and Wage Dynamics." *Journal of Labor Economics*, 28(3): 595-631.

Appendix

A Proofs

Proof of Proposition 1:

Given the functional forms for W^U and W^E, the Bellman equation describing W^U is equivalent to

$$(r+\phi)\alpha^U = b + \lambda_u \int_{z_R}^{\overline{z}} [\alpha^E(z') - \alpha^U] dF(z') \tag{14}$$

and the Bellman equation for W^E is equivalent to

$$(r+\phi+\delta)\alpha^E(z) = z + \rho\alpha^E(z) + \lambda_e \int_{z}^{\overline{z}} [\alpha^E(z') - \alpha^E(z)] dF(z') + \delta\alpha^U, \tag{15}$$

which is a functional equation for $\alpha^E(.)$. Differentiating (15) with respect to z yields the differential equation describing α^E and evaluating (15) at $z = \overline{z}$ yields its boundary value $\alpha^E(\overline{z})$.

We now solve the conditions for z_R and α^U. First evaluate (15) at $z = z_R$. As $\alpha^E(z_R) = \alpha^U$ one obtains

$$(r+\phi)\alpha^U = z_R + \rho\alpha^U + \lambda_e \int_{z_R}^{\overline{z}} [\alpha^E(z') - \alpha^U] dF(z').$$

Comparing this equation with (14), integrating by parts and using the differential equation for α^E establishes (1) described in Proposition 1. Similarly, $\alpha^E(z_R) = \alpha^U$, integration by parts and using the differential equation for α^E then yields (2) in Proposition 1. Thus (1) and (2) describe a pair of equations for (α^U, z_R).

We now establish that a solution exists and is unique. First note that the equation described by (1) has slope

$$\left[\frac{d\alpha^U}{dz_R}\right]_{eqn(1)} = -\frac{1}{\rho}\left[\frac{r+\phi+\delta+\lambda_u(1-F(z_R))-\rho}{q(z_R)+r-\rho}\right] < 0$$

and implies $\alpha^U = (b-\overline{z})/\rho$ at $z_R = \overline{z}$. On the other hand, the equation described by (2) has slope

$$\left[\frac{d\alpha^U}{dz_R}\right]_{eqn(2)} = -\frac{1}{r+\phi}\left[\frac{\lambda_u(1-F(z_R))}{q(z_R)+r-\rho}\right] < 0,$$

for $z_R < \overline{z}$ and zero otherwise and implies that $\alpha^U = b/(r+\phi)$ at $z_R = \overline{z}$. Note that

$$\left[\frac{d\alpha^U}{dz_R}\right]_{eqn(2)} > \left[\frac{d\alpha^U}{dz_R}\right]_{eqn(1)}$$

for all z_R and hence (2) is always flatter than (1). Continuity of (1) and (2) and the restriction $\overline{z} > b(r+\phi-\rho)/(r+\phi)$ then guarantee there exists a single crossing between these two functions such that $\alpha^U > 0$ and $z_R < \overline{z}$.$\|$

Proof of Lemma 1:

Consider the pool of type ε unemployed workers with productivity no greater that y. It is straightforward to verify that steady-state turnover implies

$$N_\varepsilon(y) = \frac{\phi\,(\phi+\delta+\lambda_u) + \delta\lambda_u H_\varepsilon(y,\overline{z})}{(\phi+\lambda_u)(\phi+\delta)}$$

for all $y \geq \varepsilon$. Next consider the pool of type ε employed workers who have productivity no greater than y and receive a payoff no greater than z. The arguments in Burdett et al. (2011) imply that $H_\varepsilon(.,.)$ satisfies the following partial differential equation,

$$\frac{\partial H_\varepsilon(y,z)}{\partial y} + \frac{q(z)}{\rho y} H_\varepsilon(y,z) = \frac{(\phi+\delta)F(z)N_\varepsilon(y)}{\rho y}, \tag{16}$$

for $z \in [\underline{z}, \overline{z}]$ and $y \geq \varepsilon$. For a given z, integrating over y using the integrating factor $y^{\frac{q(z)}{\rho}}$ and noting that $H_\varepsilon(\varepsilon, z) = 0$ yields

$$H_\varepsilon(y,z) = \frac{(\phi+\delta)F(z)}{\rho} y^{-\frac{q(z)}{\rho}} \int_\varepsilon^y {y'}^{\frac{q(z)}{\rho}-1} N_\varepsilon(y')dy'$$

for all $y \geq \varepsilon$, $z \in [\underline{z}, \overline{z}]$.

Using these formulae we now solve for steady state $N_\varepsilon(.)$ and $H_\varepsilon(.,.)$. In particular, using the above expression for $N_\varepsilon(.)$ and simplifying yields

$$\frac{\partial H_\varepsilon(y,\overline{z})}{\partial y} = \frac{\phi\,(\phi+\delta+\lambda_u)}{\rho(\phi+\lambda_u)} \frac{1-H_\varepsilon(y,\overline{z})}{y}$$

for all $y \geq \varepsilon$. As this differential equation is separable and we have the boundary condition $H_\varepsilon(\varepsilon, \overline{z}) = 0$, integration implies

$$H_\varepsilon(y,\overline{z}) = 1 - \left(\frac{y}{\varepsilon}\right)^{-\left(\frac{\phi(\phi+\delta+\lambda_u)}{\rho(\phi+\lambda_u)}\right)}.$$

Using this and simplifying yields the expression for $N_\varepsilon(.)$. Using the latter to substitute out $N_\varepsilon(.)$ in the above expression for $H_\varepsilon(y,z)$, direct integration and some algebra then establishes (5).‖

Proof of Proposition 2:

Consider a firm with productivity p offering $z \geq z_R$. This firm's steady state profit is given by

$$\Omega(z;p) = \frac{p-z}{(q(z)-\rho)(\lambda_u U + \lambda_e(1-U))} \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left[\lambda_u U \int_{y'=\varepsilon}^{\infty} y' dN_\varepsilon(y') + \lambda_e(1-U) \int_{y'=\varepsilon}^{\infty} \int_{z'=\underline{z}}^{z} y' \frac{\partial^2 H_\varepsilon(y',z')}{\partial y' \partial z'} dz' dy'\right] dA(\varepsilon).$$

Next use the results in Lemma 1 to solve for the integrals in $\Omega(z;p)$. Consider the first integral in the expression in brackets. Using the expression for N_ε one obtains

$$\int_{y'=\varepsilon}^{\infty} y' dN_\varepsilon(y') = \varepsilon N_\varepsilon(\varepsilon) + \int_\varepsilon^\infty \left(\frac{\phi\,(\phi+\delta+\lambda_u)}{\rho(\phi+\lambda_u)}\right) \frac{\lambda_u \delta}{(\phi+\lambda_u)(\phi+\delta)} \left(\frac{y'}{\varepsilon}\right)^{-\left(\frac{\phi(\phi+\delta+\lambda_u)}{\rho(\phi+\lambda_u)}\right)} dy'.$$

Integration and some algebra then establish that

$$\int_{y'=\varepsilon}^{\infty} y' dN_\varepsilon(y') = \frac{\phi\,(\phi+\delta+\lambda_u)\,\varepsilon}{(\phi+\delta)}\left[\frac{(\phi+\delta-\rho)}{\phi\,(\phi+\delta+\lambda_u)-\rho(\phi+\lambda_u)}\right].$$

Next consider the second integral in $\Omega(z;p)$. Integrating over z' implies

$$\int_{y'=\varepsilon}^{\infty}\int_{z'=\underline{z}}^{z} y\frac{\partial^2 H_\varepsilon(y',z')}{\partial y'\partial z'}dz'dy' = \int_{y'=\varepsilon}^{\infty} y'\left[\frac{\partial H_\varepsilon(y',z')}{\partial y'}\right]_{\underline{z}}^{z} dy'.$$

As $F(\underline{z})=0$, (5) implies $H_\varepsilon(y',\underline{z})=0$. (16) then implies $\frac{\partial H_\varepsilon(y',\underline{z})}{\partial y'}=0$ and the previous expression reduces to

$$\int_{y'=\varepsilon}^{\infty}\int_{z'=\underline{z}}^{z} y'\frac{\partial^2 H_\varepsilon(y',z')}{\partial y'\partial z'}dz'dy' = \int_{\varepsilon}^{\infty} y'\frac{\partial H_\varepsilon(y',z)}{\partial y'}dy'.$$

Now (5) implies

$$\frac{\partial H_\varepsilon(.,.)}{\partial y} = \frac{\phi(\phi+\delta+\lambda_u)F(z)}{y\rho[\lambda_u\delta+\lambda_e(1-F(z))(\phi+\lambda_u)]}\Big[\lambda_e(1-F(z))\Big(\frac{y}{\varepsilon}\Big)^{-\frac{q(z)}{\rho}} + \frac{\delta\lambda_u}{\phi+\lambda_u}\Big(\frac{y}{\varepsilon}\Big)^{-\left(\frac{\phi(\phi+\delta+\lambda_u)}{\rho(\phi+\lambda_u)}\right)}\Big].$$

Using this expression and integrating yields

$$\begin{aligned}\int_{y'=\varepsilon}^{\infty}\int_{z'=\underline{z}}^{z} y'\frac{\partial^2 H_\varepsilon(y',z')}{\partial y'\partial z'}dz'dy' &= \Big[\frac{\phi(\phi+\delta+\lambda_u)F(z)\varepsilon}{\lambda_u\delta+\lambda_e(1-F(z))(\phi+\lambda_u)}\Big]\times \\ &\Big[\frac{\lambda_e(1-F(z))}{q(z)-\rho}+\frac{\delta\lambda_u}{\phi(\phi+\delta+\lambda_u)-\rho(\phi+\lambda_u)}\Big].\end{aligned}$$

Substituting out the expression for the integrals in $\Omega(z;p)$ and some additional algebra yields (6) in the text.‖

<u>Proof of Proposition 3:</u>

Consider a firm with productivity p. This firm chooses a $z \geq z_R$ to maximise

$$\Omega(z;p) = \widetilde{\varepsilon}l(z)(p-z), \tag{17}$$

where $l(z)$ is given in the text. Let $z^* = \zeta(p)$ denote the solution to the above maximisation problem (if one exists). Assume the second order condition for a maximum holds. The envelope theorem then implies that $\Omega'(\zeta(p)) = l(\zeta(p))$, which describes a first order differential equation for $\Omega(.)$ in terms of p subject to the boundary condition $\Omega(\zeta(p_0)) = \widetilde{\varepsilon}l(\zeta(p_0))(p_0-\underline{z})$. Noting that $F(\zeta(p)) = \Gamma_0(p)$, its solution is given by

$$\Omega(\zeta(p)) = \widetilde{\varepsilon}l(\zeta(p))(p-\zeta(p)) = \widetilde{\varepsilon}\int_{\underline{z}=z_R}^{p}\frac{\phi(\phi+\delta+\lambda_u)(\phi+\delta-\rho)(\phi+\delta+\lambda_e-\rho)}{(q(x)-\rho)^2(\phi+\delta+\lambda_e)[\phi(\phi+\delta+\lambda_u)-\rho(\phi+\lambda_u)]}dx,$$

where $q(x) = \phi+\delta+\lambda_e(1-\Gamma_0(x))$. Since $\zeta(p) = p - \Omega(\zeta(p))/\widetilde{\varepsilon}l(\zeta(p))$, substituting out for $\Omega(\zeta(p))$ and $l(\zeta(p))$ and some algebra yields (7), the expression for $\zeta(p)$ in the text.

Next we show that (7) indeed satisfies the first order condition of the firm's maximisation problem and then that the second order condition for a maximum is indeed met at $z = z^*$. First note that differentiation of (7) wrt p implies ζ satisfies the differential equation

$$\zeta'(p) = \frac{2(p-\zeta(p))\lambda_e \Gamma_0'(p)}{q(p)-\rho}, \tag{18}$$

given the boundary condition $\zeta(p_0) = z_R$. Noting that the first order condition for a maximum implies that for a given p

$$l'(z)(p-z) - l(z) = 0$$

at $z^* = z$, using the expression for $l(z)$ and $F'(\zeta(p))\zeta'(p) = \Gamma_0'(p)$, some algebra establishes that (18) is indeed obtained from the above first order condition. Hence the function ζ implied by (18) satisfies the first order condition for a maximum given the boundary condition $\zeta(p_0) = z_R$. Further, note that $\zeta'(p) > 0$ for all $p \geq p_0$.

Now let $\Omega(\zeta(\widehat{p}); p) = \widetilde{\varepsilon}(p - \zeta(\widehat{p}))l(\zeta(\widehat{p}))$ denote the steady state profit of a firm of productivity p by offering a $z = \zeta(\widehat{p})$ and let $\Delta(\widehat{p}) = \widehat{p} - p$. For $\widehat{p} \in (p, \overline{p}]$, the second order condition for a maximum requires that offering such a z should not increase profits or that

$$\begin{aligned}
\left[\frac{d\Omega(\zeta(x);p)}{dx}\right]_{x=\widehat{p}} &= \widetilde{\varepsilon}\left[\frac{dl(\zeta(x))}{dz}\zeta'(x)(p-\zeta(x)) - \zeta'(x)l(\zeta(x))\right]_{x=\widehat{p}} \\
&= \widetilde{\varepsilon}\left[\left((x-\zeta(x))\frac{dl(\zeta(x))}{dz} - l(\zeta(x))\right)\zeta'(x) - \Delta(x)\frac{dl(\zeta(x))}{dz}\zeta'(x)\right]_{x=\widehat{p}} \leq 0.
\end{aligned}$$

Since the first order condition implies $(x-\zeta(x))\frac{dl(\zeta(x))}{dz} - l(\zeta(x)) = 0$ for any $x > p_0$, one obtains that

$$\left[\frac{d\Omega(\zeta(x);p)}{dx}\right]_{x=\widehat{p}} = \left[-\varepsilon\Delta(x)\frac{dl(\zeta(x))}{dz}\zeta'(x)\right]_{x=\widehat{p}} \leq 0$$

is always satisfied. For $\widehat{p} \in [p_0, p)$ a similar argument shows that

$$\left[\frac{d\Omega(\zeta(x);p)}{dx}\right]_{x=\widehat{p}} = \left[-\varepsilon\Delta(x)\frac{dl(\zeta(x))}{dz}\zeta'(x)\right]_{x=\widehat{p}} \geq 0$$

is always satisfied.

Finally note that a firm with productivity $p = p_0$ will not offer a $z < z_R = \zeta(p_0)$ as doing so will not increase profits. It will strictly decrease profits if $p_0 = \underline{p} > z_R$ and yields the same (zero) profit if $p_0 = z_R$. A firm with $p = \overline{p}$, on the other hand, will not offer a $z > \zeta(\overline{p})$ as doing so does not attract or retain any additional worker, but strictly decreases flow profit $\overline{p} - z$ and hence steady state profits.‖

<u>Proof of Theorem 1:</u>

Step 1: The first step to proof existence is to solve for p_0. Note that for any p_0, $T(z_R; p_0)$ gives the solution to $z_R = z_R(p_0)$ when both (3) and (7) are satisfied. Given $p_0 = \max\{\underline{p}, z_R(p_0)\}$, we have that $p_0 = \underline{p}$ if and only if $\underline{p} > z_R(\underline{p})$. Using (8) the latter condition can be expressed as

$$\underline{p} > z_R(\underline{p}) = \frac{b(r+\phi-\rho) + [\lambda_u(r+\phi-\rho) - (r+\phi)\lambda_e]\int_{\underline{p}}^{\overline{p}}\left[\frac{p}{(\phi+\delta+\lambda_e-\rho)^2} + \int_{\underline{p}}^{x}\frac{ds}{(q(s)-\rho)^2}\right]\beta(x)dx}{(r+\phi) + \frac{[\lambda_u(r+\phi-\rho)-(r+\phi)\lambda_e]}{(\phi+\delta+\lambda_e-\rho)^2}\int_{\underline{p}}^{\overline{p}}\beta(x)dx}.$$

On the other hand, if the above condition does not hold (i.e. $\underline{p} \leq z_R(\underline{p})$), then $p_0 \geq \underline{p}$. In this case (8) implies p_0 satisfies $p_0 = \widetilde{T}(p_0)$, where

$$\widetilde{T}(p_0) = \frac{b(r+\phi-\rho)}{r+\phi} + \frac{[\lambda_u(r+\phi-\rho)-(r+\phi)\lambda_e]}{r+\phi}\int_{p_0}^{\overline{p}}\left[\int_{p_0}^{x}\frac{ds}{(q(s)-\rho)^2}\right]\beta(x)dx.$$

Note that $\widetilde{T}$ is continuous, bounded and strictly decreasing between $p_0 \in [\underline{p}, \overline{p}]$. Since $\widetilde{T}(\underline{p}) > \underline{p}$ and $\overline{p} \in (b, \infty)$, there exists a unique $p_0^* \in (\underline{p}, \overline{p}]$ such that $p_0^* = \widetilde{T}(p_0^*)$.
Step 2: Given a $p_0 \leq \overline{p}$ always exists, z_R is described by the unique solution to (8). $\zeta(.)$ is then characterised by (7) in Proposition 2 and $F(\zeta(p)) = \Gamma_0(p)$ for all $p \in [p_0, \overline{p}]$, where Γ_0 is given by (4). Furthermore, since Proposition 2 implies no firm with productivity $p \in [p_0, \overline{p}]$ will offer a different z, as doing so yields lower steady state profits, this establishes existence of a unique Market Equilibrium.‖

Proof of Lemma 2:

Since frictional wage dispersion concerns wage dispersion that is not driven by difference in abilities, without loss of generality consider the case in which all workers enter with initial productivity $\varepsilon = 1$. Next note that $H(\infty, z)$ describes the distribution of z across employed workers given the offer distribution F. Using integration by parts and $\underline{z} = z_R$, it can be easily shown that the average z earned by employed workers, z^M, is given by

$$z^M = z_R + \int_{z_R}^{\overline{z}}[1 - H(\infty, z)]dz. \tag{19}$$

Putting $y = \infty$ in (5) implies

$$H(\infty, z) = \frac{(\phi+\delta)F(z)}{q(z)}.$$

Since r and ρ are typically of the same order of magnitude (see section 5), we follow Hornstein, et al. (2007) and approximate $H(\infty, z)$ by

$$H(\infty, z) \simeq \frac{(r+\phi-\rho+\delta)F(z)}{q(z)+r-\rho}.$$

Solving for $1 - F(z)$ and using (3) yields

$$\begin{aligned} z_R &\simeq \frac{b(r+\phi-\rho)}{r+\phi} + \frac{\lambda_u(r+\phi-\rho)-(r+\phi)\lambda_e}{(r+\phi)(r+\phi+\delta+\lambda_e-\rho)}\int_{z_R}^{\overline{z}}[1-H(\infty,z)]dz \\ &\simeq \frac{b(r+\phi-\rho)}{r+\phi} + \frac{\lambda_u(r+\phi-\rho)-(r+\phi)\lambda_e}{(r+\phi)(r+\phi+\delta+\lambda_e-\rho)}\left(z^M - z_R\right). \end{aligned}$$

Dividing both sides by z^M we have the expression in the text.‖

B Simulation Procedure

To simulate the model we compute the employment histories of 10,000 workers for each iteration of the minimisation process done in the simulation minimum distance procedure. In simulating the employment histories we assume that all workers start unemployed and experience different types of shocks during their lifetime depending on the worker's employment status.

When unemployed, workers face a retirement and a job offer shock, where both process are Poisson with rates ϕ and λ_u and the shocks are mutually exclusive. What is important here is to capture the time spend in unemployment for each individual. To obtain the unemployment duration, we draw two random numbers, $r1 \in [0,1]$ and $r2 \in [0,1]$, using a uniform distribution and then exploit the fact that the inter arrival time between events in a Poisson process follows an exponential distribution with parameter equal to the rate of the process. That is, the duration until the worker receives a job offer is determined by $tu = -log(1-r1)/\lambda_u$ and the duration until the worker experience a retirement shock is $td = -log(1-r2)/\phi$. Since this a competing risk model the unemployment duration of an individual is given $\min\{td, tu\}$. If the individual retires, $td < tu$, then he leaves the sample. If the individual becomes employed, $td \geq tu$, then a firm productivity is sampled from Γ by, once again, choosing a random number between 0 and 1 and using the inverse of Γ to recover the corresponding productivity p. The latter then allows us to compute the corresponding z and θ.

Given the individual becomes employed in a firm with productivity p, the value of $z(p)$ is computed using (7) and the value of $\theta = z(p)/p$. This individual now faces three shocks: a retirement shock, a job offer shock and a displacement shock. All these shocks follow Poisson process with rates, ϕ, λ_e and δ, respectively. As in the case of unemployed workers, what is important is the duration of the job and the employment spells, where the latter is defined as the sum of job spells that start with the worker transiting from unemployment to employment and end with the worker becoming unemployed or leaving the labour market. We use the same procedure as before to obtain the durations until the worker receives a job offer tj, receives a displacement shock, tu, and receives a retirement shock, td. The job duration until the worker experiences one of these three events in then $\min\{tj, tu, td\}$. If the worker becomes unemployed, $tu = min\{tj, tu, td\}$, then the procedure described above for unemployed workers is repeated. If the worker leaves the labour market, $td = min\{tj, tu, td\}$, then he drops from the sample. If the worker receives an outside offer, $tj = min\{tj, tu, td\}$, a new p' is drawn using the same procedure described above and the values of $z(p)$ and $z(p')$ are compared. If $z(p) \geq z(p')$ the worker stays employed in his current job, while if $z(p) < z(p')$ the worker moves to the new firm and we repeat the process given a the new firm productivity p'. During this procedure we calculate the labour market experience of workers as the sum of employment spells. This information can then be used to compute workers' wages at each point in which an event has occurred taking into account that workers accumulate human capital at rate ρ.

The above procedure generates the full labour market histories of workers for an average life of $1/\phi$ months. However, the BHPS sample is restricted to workers that in 1991 were between 16 and 30 years of age and by 2004 were between 30 and 44 years of age. Hence one needs to create a sample of the simulated data that resembles that of the BHPS in terms of the age structure and has the same variance of actual experience (this is crucial for the variance decomposition exercise). It is only after creating such a sample that we compute the average wage-experience profiles by using an OLS regression on log wages on a constant a quadratic on experience and tenure. Using this sample we also compute all the other moments targeted in the calibration as described in Table 1. In particular, using this sample we compute the Mm ratio in the simulation such that it is consistent with the way we compute the Mm ratio in the data. We now detail such a procedure.

Estimating of the Mean-min ratio: Following Hornstein, et al. (2007) we first estimate the wage equation

$$\log w_{ijt} = \beta X_{it} + \eta_{ijt}, \tag{20}$$

for each year of the sample period and skill group using OLS, where X is a vector of covariates consisting of a quadratic in actual experience, quadratic on tenure, a dummy for marital status, 8 regional dummies, 8 (one-digit) occupational dummies, 8 (one-digit) industry dummies and cohort dummies, where η denotes white noise and is assumed to be normally distributed. The second step is to eliminate unobserved worker heterogeneity from wages by using the individual residuals $\widehat{\eta}_{it}$ and their individual specific mean $\overline{\eta}_i = \sum_{t=1}^{N_i} \widehat{\eta}_{it}/N_i$. The vector $\{\overline{\eta}_i\}_{i=1}^N$ then captures the wage variation due to fixed unobserved individual factors. Finally, we use the estimated distribution of transformed wages, $\widetilde{w}_{it} = \exp(\widehat{\eta}_{it} - \overline{\eta}_i)$, across individuals and time to calculate the Mm ratio for each skill group.

For each skill group, we estimate a set of three Mm ratios using the minimum observed wage, the wage at the first percentile, and fifth percentile. Given that the wage data has already been trimmed by 5 percent on each side when performing the OLS regressions and that the minimum observed wage is still very noisy for the medium and high skilled categories, we use as a target the Mm ratio obtained from averaging the ones obtained for the first and fifth percentile.

As pointed out by Hornstein, et al. (2007) the danger with their approach is that one may underestimate the amount of frictional wage dispersion when controlling for those worker characteristics that also provide information on generate wage dispersion due to productivity differentials among workers. Further, by introducing a polynomial on experience and tenure in (20) one is reducing the effects of on-the-job search and human capital accumulation on wage dispersion. However, in the data this reduction is not very strong and hence the downward bias does not have a mayor impact on the estimated parameters. Indeed, when estimating (20) without controlling for experience or tenure effects, the resulting average Mm ratios are 1.57, 1.48 and 1.54, for low, medium and high skilled workers, respectively. These Mm ratios are only slightly different than the ones used as targets in the simulations, reported in Table 1.