ISSN 1755-5361

University of Essex

Department of Economics

Discussion Paper Series

No. 770 August 2015

Information Inertia

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August 11, 2015

Abstract

We study how aversion to ambiguity about the predictability of future asset values and cash flows affects optimal portfolios and asset prices. We show that optimal portfolios do not always react to new information even though there are no information processing costs or other market frictions. Moreover, the equilibrium price of the market portfolio does not always incorporate all available public information that is worse than expected. This informational inefficiency leads to price underreaction consistent with momentum.

Keywords: Ambiguity Aversion, Knightian Uncertainty, Informational Efficiency, Information Inertia, Inattention to News, Public Information, Momentum, Predictability.

JEL Classification: D80, D81, G10, G11, G12.

*We would like to thank Kerry Back, James Dow, Simon Hayley, Christian Heyerdahl-Larsen, Sujoy Mukerji, Emre Ozdenoren, Han Ozsoylev, Martin Schneider, seminar participants at the Wharton Brown Bag Seminar, Oxford University, London Business School, the University of Utah, Queen Mary University of London, and participants at the European workshop on general equilibrium theory at the University of Exeter, the Cass 2012 Behavioral Finance Conference, the 2012 NBER Summer Asset Pricing Meeting, the Econometric society European meeting 2012, and the 7th meeting of the Finance Theory Group for helpful comments and suggestions. Ganguli thanks ESRC grant ES/K00168X/1 for research support. Illeditsch gratefully acknowledges support from Cynthia and Bennett Golub.

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There is a vast amount of empirical research which studies the predictability of cash flows and discount rates for many asset classes around the world.[1] The economic and statistical significance of the predictability results vary from study to study and the strength of these results as well as the theoretical underpinnings and interpretations are widely debated. In this paper, we study how aversion to ambiguity about the predictability of future asset values and cash flows affects optimal portfolios and equilibrium asset prices. We show that optimal portfolios do not always react to new information and prices do not reflect all available information about an asset in equilibrium. We refer to this phenomenon as information inertia.

Consider a linear regression model with a signal $\tilde{s}$ that predicts future excess returns $\tilde{r}_e$. Specifically,

$$\tilde{r}_e = \alpha + \beta\tilde{s} + \varepsilon, \qquad \beta = R\frac{\sigma_e}{\sigma_s}, \qquad \tilde{\varepsilon} \sim \mathcal{N}\left(0, \sigma_e^2(1-R^2)\right),$$

where R denotes the correlation between $\tilde{r}_e$ and $\tilde{s}$.[2] Investors do not know the correlation R and thus know neither the economic significance β nor the explanatory power R^2 of the predictor $\tilde{s}$. We focus on predictors with strictly positive β.

Suppose investors have mean-variance preferences over excess returns and are averse to ambiguity about the correlation R in the sense of Gilboa and Schmeidler (1989). Hence, they consider a family of linear regression models described by the interval $[R_a, R_b]$ and evaluate the outcome of investment decisions under the regression model that yields the lowest expected utility. This "max-min" formulation of preferences is a commonly used representation of decision-making under ambiguity in asset markets, as discussed in Epstein and Schneider (2010).[3]

[1]For a review of this literature see Cochrane (2005) or Koijen and Nieuwerburgh (2011) and the references therein.

[2]The variance of $\tilde{r}_e$ and $\tilde{s}$ is σ_e^2 and σ_s^2, respectively.

[3]Gilboa and Schmeidler (1989) axiomatize this max-min or multiple prior preference representation which implies behavior that is consistent with experimental evidence (Ellsberg (1961)) and more recent portfolio choice experiments (Ahn, Choi, Gale, and Kariv (2011) and Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010)). We consider different representations in Section IV and the internet appendix. For a discussion of different preferences specifications that describe aversion to ambiguity see Backus, Routledge, and Zin (2004), Epstein and Schneider (2010), and Hansen and Sargent (2010b).

We show that risky portfolios do not always react to the predictor $\tilde{s}$ when investors are averse to ambiguity. Intuitively, ambiguity about the economic significance and the explanatory power of the predictor $\tilde{s}$ implies ambiguity about the conditional Sharpe ratio of the asset. An ambiguity averse investor seeks robust decision rules and considers the worst case scenario for the Sharpe ratio. For a long position, if the signal is lower than expected, the worst case scenario for the expected excess return is a high economic significance (β_b) because in this case the investor will significantly revise the expected value of the asset downwards. On the other hand, the worst case scenario for the volatility is a low explanatory power (R_a^2) because in this case less risk is resolved by the predictor. We show that there is a range of low signals where neither the mean nor the volatility effect on the Sharpe ratio dominates and investors ignore the signal when choosing a long position. Hence, risky portfolios exhibit information inertia.

The information inertia result for risky portfolios is economically sizable. For instance, suppose ambiguity aversion is parameterized with a 99% confidence interval around an R^2 of 0.09. Then 16% of predictors below their mean will not affect a long stock position if the unconditional Sharpe ratio is 0.4. This probability is 12% and 20% when the unconditional Sharpe ratio is 0.3 and 0.5, respectively. This is the case even though the predictor $\tilde{s}$ is statistically significant ($R_a > 0$). In contrast, the portfolios of standard expected utility maximizers will always respond to statistically significant predictors unless there are transaction costs or other costs to acquire or process information.

We also study the effects of ambiguity aversion on the equilibrium price of the market portfolio when there is predictability about its future cash flows. We show that the price of the market portfolio fails to incorporate all available information in equilibrium. This informational inefficiency has an interesting asymmetry. While signals that convey information that is better than expected are always reflected in the stock price some signals that convey information that is worse than expected are not.

Moreover, the reaction of the equilibrium price to information depends on the unconditional risk premium of the market portfolio. The price of a very risky market portfolio is more likely to underreact to signals that are worse than expected whereas the price of a market portfolio that is less risky tends to overreact to this signal. However, the most striking result is that a market portfolio with intermediate risk shows no reaction to many signals that are worse than expected even though there are no information processing costs or other market frictions. For instance, if the unconditional risk premium is between 5% and 10%, then 20% to 39% of bad signals are not reflected in the price when ambiguity aversion is parameterized with a 99% confidence interval around an R^2 of 0.09.

This paper may also shed some light on the documented momentum of asset prices in the United States and other developed countries.[4] Specifically, assets that have performed well in the past tend to continue to perform well. Most of the papers in the literature rely on behavioral explanations for this phenomenon (e.g. Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Hong and Stein (1999)). We provide an explanation for momentum that is based on investors who are averse to ambiguity. Moreover, prices underreact to good news and the underreaction to bad news increases with the risk of the asset and ambiguity aversion which is consistent with the findings in Zhang (2006).[5]

In addition, we study the effects of investor heterogeneity on optimal portfolios and asset prices when all investors receive the same signal about future cash flows. There is a range of signals for which investors' portfolios and the price of the market portfolio do not react to new information in equilibrium even when investors differ with respect to their aversion to risk and ambiguity. This is no longer true if there are some investors who are ambiguity neutral because their portfolios always respond to information. We show that in this case equilibrium portfolios and the price of the

[4]For a review of the literature on momentum strategies see Jegadeesh and Titman (2011), Moskowitz, Ooi, and Pedersen (2012), and the reference therein.

[5]Kelsey, Kozhan, and Pang (2011) study the effects of ambiguity aversion on the profitability of momentum strategies. In contrast to our paper, in their model prices always react to information and there is momentum even without ambiguity averse investors.

market portfolio do not react a lot to these signals.

We also consider two additional models of preferences that are less parsimonious but allow for a distinction between ambiguity and ambiguity aversion: the KMM model of Klibanoff, Marinacci, and Mukerji (2005) and the GHTV model of Gajdos, Hayashi, Tallon, and Vergnaud (2008). With these alternative preference representations, risky portfolios and the equilibrium price always respond to new information. However, we show numerically that neither risky portfolios nor equilibrium prices react a lot if ambiguity aversion is sufficiently large.

This paper complements recent work on optimal portfolios and equilibrium asset prices when investors process public signals. Epstein and Schneider (2008) show that investors react more to bad signals than to good signals when there is ambiguity about the precision of these signals. Illeditsch (2011) shows that this ambiguity leads to risky portfolios that are sensitive to news but insensitive to changes in the stock price—a phenomenon referred to as portfolio inertia. We are the first to show that ambiguity aversion leads to information inertia for risky portfolios and equilibrium prices without relying on information processing costs or other market frictions. Moreover, the economic mechanism that leads to information inertia is novel because it does not occur at the kink in investors' utility in contrast to the portfolio inertia results in Illeditsch (2011).[6]

This paper contributes to the literature on optimal portfolio choice with ambiguity. We know from Dow and Werlang (1992), Cao, Wang, and Zhang (2005), Epstein and Schneider (2007), Easley and O'Hara (2009), and Campanale (2011) that ambiguity leads to portfolio inertia of the risk-free portfolio. Epstein and Wang (1994), Epstein and Schneider (2010), and Illeditsch (2011) show that portfolio inertia can also arise for risky portfolios. Garlappi, Uppal, and Wang (2007) characterize optimal portfolios with multiple assets and ambiguity aversion. Uppal and Wang (2003), Benigno and Nistico (2012), and Boyle, Garlappi, Uppal, and Wang (2012) show that ambiguity aversion leads to under-diversified portfolios. In contrast, we show that

[6]We are not aware of any work with multiple prior preferences that leads to qualitatively different results than standard expected utility that do are not due to the kink in utility.

ambiguity averse investors use the unconditional asset distribution when contemplating a long (short) position with moderate risk instead of relying on a signal that conveys bad (good) news.

Our work is also related to a large literature that studies the informational efficiency of prices when there is asymmetric information. For instance, prices do not fully reveal private information in equilibrium, (i) if it is costly to acquire information (Grossman (1976) and Grossman and Stiglitz (1976)), (ii) if there are noise traders (Grossman and Stiglitz (1980)), (iii) if informed investors anticipate how their trades will impact prices (Kyle (1985) and Back, Cao, and Willard (2000)), (iv) if there is ambiguity (Caskey (2009), Condie and Ganguli (2011), and Condie and Ganguli (2014)).[7] What is striking in this paper is that a costless informative public signal is not always incorporated in the price when an investor is averse to ambiguity.

Our paper is also related to recent literature on portfolio choice and asset pricing when there is ambiguity about the predictability of future asset returns/cash flows. Hansen and Sargent (2010a) study the price of risk when investors who seek robust decision rules find it difficult to differentiate between i.i.d. consumption growth and one with a persistent component (long run risk of Bansal and Yaron (2004)).[8] Chen, Ju, and Miao (2011) solve a dynamic consumption and portfolio choice problem when there is ambiguity about whether stock returns are IID or predictable. Ju and Miao (2012) and Collard, Mukerji, Sheppard, and Tallon (2011) explain many asset pricing puzzles by introducing ambiguity into a dynamic representative agent model in which consumption and dividends follow a hidden state regime-switching process and a hidden state model with a persistent latent state variable, respectively. The first paper considers the robust control approach and the other three papers consider the recursive smooth ambiguity model to describe preferences.[9] Our focus in this paper

[7] Mele and Sangiorgi (2011), Ozsoylev and Werner (2011), and Tallon (1998) study the effects of ambiguity aversion on asset prices in the presence of private information and noise traders.

[8] For a survey of learning models when investors seek robust decision rules see Hansen and Sargent (2007).

[9] Strzalecki (2011) and Maccheroni, Marinacci, and Rustichini (2006) provide axiomatic foundations for the robust control model and Klibanoff, Marinacci, and Mukerji (2005), Nau (2006), Klibanoff, Marinacci, and Mukerji (2009), and Hayashi and Miao (2011) provide axiomatic foundations for the smooth ambiguity model and its dynamic extension.

is on the parsimonious and tractable Gilboa and Schmeidler (1989) preference model which is a good description of ambiguity averse behavior as shown by Ahn, Choi, Gale, and Kariv (2011) and Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010).[10]

There is a growing literature in macroeconomics that imposes an exogenous constraint or cost on the ability of investors to process information in order to explain why macroeconomic variables exhibit inertia.[11] These ideas have also been used in finance to explain information inertia of portfolios (Abel, Eberly, and Panageas (2007)), excess correlation (Peng and Xiong (2006)), financial contagion (Mondria (2010) and (Mondria and Quintana-Domeque 2012)), and portfolio under-diversification (Nieuwerburgh and Veldkamp 2010), among others.[12] In contrast, we derive inertia from a rational choice model with multiple prior utility. Moreover, information inertia affects investors' utility and thus leads to a welfare loss that depends on risk aversion and the signal.

There is an extensive literature on optimal portfolio choice when there is Bayesian model uncertainty about the predictability of future returns (e.g. Keim and Stambaugh (1986), Barberis (2000), and Xia (2001) among others).[13] In all these papers investors hedge against model uncertainty but their portfolios always react to new information. We also study portfolios and equilibrium prices when there is Bayesian model uncertainty, which is a special case of the KMM model studied in Section IV and show that there is no information inertia. Balduzzi and Lynch (1999), Balduzzi and Lynch (2000), Lynch and Tan (2010), and Lynch and Tan (2011) study the effects of transaction costs on optimal portfolios when there is return predictability. While in these papers transaction costs lead to state dependent portfolio adjustment we derive state dependent adjustment of portfolios from a rational choice model.

The rest of the paper is organized as follows. In Section I, we introduce the

[10] Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010) consider α-maxmin preferences to model ambiguity aversion which in their experiment is observationally equivalent to using maxmin preferences with a smaller set of beliefs (see Ravanelli and Svindland (2014)).

[11] See Sims (2003), Sims (2010), and the references therein.

[12] See Veldkamp (2011) and the reference therein for an overview of this literature.

[13] For a comprehensive survey of static and dynamic portfolio choice models when returns are predictable see Wachter (2012).

model. In Section II, we solve for optimal portfolios and discuss the information inertia results. In Section III we solve for equilibrium portfolios and stock prices and discuss the informational inefficiency of prices and momentum, in Section IV we show that our results are quantitatively robust to a different preference model, and in Section V we discuss the economic significance of our information inertia results. We conclude in Section VI.

I Information Structure and Preferences

Suppose there are two dates 0 and 1. Investors can invest in a risk-free asset and a risky asset. Let p denote the price of the risky asset, $\tilde{d}$ the future value or dividend of the risky asset, and θ the number of shares invested in the risky asset. There is no consumption at date zero. The risk-free asset is used as numeraire, so the risk-free rate is zero. Hence, future wealth $\tilde{w}$ is given by

$$\tilde{w} = w_0 + \left(\tilde{d} - p\right)\theta, \tag{1}$$

in which w_0 denotes initial wealth.

Suppose investors receive a signal $\tilde{s}$ about the future value $\tilde{d}$ of the asset. The joint distribution of $\tilde{d}$ and $\tilde{s}$ is normal:

$$\begin{pmatrix} \tilde{d} \\ \tilde{s} \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} \bar{d} \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_d^2 & \beta \\ \beta & 1 \end{pmatrix}\right), \tag{2}$$

where $\beta = R\sigma_d$. Investors do not know the correlation between $\tilde{d}$ and $\tilde{s}$ and are ambiguity averse in the sense of Gilboa and Schmeidler (1989). Hence, they consider a family of joint distributions described by $R \in [R_a, R_b]$ with $R_a > 0$ and $R_b < 1$ when making decisions.[14]

We follow Gilboa and Schmeidler (1993) and determine the family of conditional

[14]There is no ambiguity about the marginal distribution of the signal and hence there is no loss in generality by normalizing the mean and the variance of the signal to zero and one, respectively.

dividend distributions given the signal by applying Bayes rule for each correlation. Hence, standard normal-normal updating for each $R \in [R_a, R_b]$ leads to

$$\tilde{d} \mid \tilde{s} = s \ \sim \mathrm{N}_R \left(\mu(s, R), \sigma^2(R)\right), \tag{3}$$

where $\mu(s, R) = \bar{d} + \beta s$ denotes the conditional mean and $\sigma(R) = \sigma_d\sqrt{1 - R^2}$ the conditional volatility of $\tilde{d}$ given s.

The utility of a multiple prior or MEU investor who holds θ shares of the risky asset is

$$\min_{R\in[R_a,R_b]} \mathrm{E}_R \left[u \left(w_0 + \left(\tilde{d} - p\right) \theta\right) \mid \tilde{s} = s\right] \tag{4}$$

where $u(\cdot)$ denotes the Bernoulli utility function of the investor.

Suppose investors have CARA utility over future wealth $\tilde{w}$, that is, $u(\tilde{w}) = -e^{-\gamma\tilde{w}}$ with $\gamma > 0$ and let $\mathrm{CE}(\theta)$ denote the certainty equivalent of an MEU investor. Hence, the investor's utility given in equation (4) is equal to $u\left(\mathrm{CE}(\theta)\right)$ with

$$\mathrm{CE}(\theta) = \min_{R\in[R_a,R_b]} \overline{\mathrm{CE}}(\theta, R), \tag{5}$$

where $\overline{\mathrm{CE}}(\theta, R)$ denotes the certainty equivalent of a standard expected utility or SEU investor with belief R. Specifically,

$$\overline{\mathrm{CE}}(\theta, R) = \mathrm{E}_R\left[\tilde{w} \mid \tilde{s} = s\right] - \frac{1}{2}\gamma \mathrm{Var}_R\left[\tilde{w} \mid \tilde{s} = s\right]. \tag{6}$$

The assumption of CARA utility and normal beliefs lead to mean-variance preferences over future wealth with ambiguity aversion about the mean and variance described by the interval $[R_a, R_b]$. Investors are more averse to ambiguity (in the sense of Gilboa and Schmeidler (1989)) if the interval $[R_a, R_b]$ is large and hence we measure the degree of aversion to ambiguity by the size of the interval.[15]

[15]The preference model of Gilboa and Schmeidler (1989) does not allow for a distinction between ambiguity and aversion to ambiguity. We consider in Section IV and the internet appendix preference models that allow for a distinction between ambiguity and ambiguity aversion.

II Portfolio Choice

We are interested in the sensitivity of optimal portfolios to changes in the signal and hence for the remainder of this section we fix the price p and determine the optimal portfolio for the risky asset as a function of the signal.

Let $\bar{\theta}(s,R)$ denote the optimal portfolio of an SEU investor with belief R and $\theta(s)$ the optimal portfolio of an MEU investor. The optimal portfolio of an SEU investor is

$$\bar{\theta}(s,R) = \frac{\mathrm{E}_R\left[\tilde{d} \mid \tilde{s}=s\right]-p}{\gamma \mathrm{Var}_R\left[\tilde{d} \mid \tilde{s}=s\right]} = \frac{\lambda(s,R)}{\gamma\sigma(R)}. \tag{7}$$

where $\lambda(s,R)=\frac{\mu(s,R)-p}{\sigma(R)}$ denotes the conditional Sharpe ratio of the asset.

For the SEU investor an increase in the signal will always lead to an increase in the demand for the asset and hence optimal portfolios always react to news. This is no longer true when investors are averse to ambiguity as the next theorem shows.

Theorem 1 (Portfolio Choice)**.** *Let $\lambda_d = \frac{\bar{d}-p}{\sigma_d}$ denote the unconditional Sharpe ratio of the asset. The optimal portfolio for an MEU investor with risk aversion γ and ambiguity aversion described by $[R_a, R_b]$ who receives the signal s is*

$$\theta(s) = \begin{cases} \frac{\lambda(s,R_a)}{\gamma\sigma(R_a)} & s \geq s_1 \equiv -R_a \max(\lambda_d,0) - \frac{1}{R_a}\min(\lambda_d,0) \\ \max\left(\frac{\lambda_d}{\gamma\sigma_d},0\right) & s_1 > s \geq s_2 \equiv -R_b \max(\lambda_d,0) - \frac{1}{R_b}\min(\lambda_d,0) \\ \frac{\lambda(s,R_b)}{\gamma\sigma(R_b)} & s_2 > s \geq s_3 \equiv -\frac{1}{R_b}\max(\lambda_d,0) - R_b\min(\lambda_d,0) \\ \min\left(\frac{\lambda_d}{\gamma\sigma_d},0\right) & s_3 > s \geq s_4 \equiv -\frac{1}{R_a}\max(\lambda_d,0) - R_a\min(\lambda_d,0) \\ \frac{\lambda(s,R_a)}{\gamma\sigma(R_a)} & s < s_4. \end{cases} \tag{8}$$

Suppose the unconditional Sharpe ratio is positive ($\lambda_d > 0$). The left graph of Figure 1 shows that the optimal portfolio does not always react to signals that convey news that is worse than expected. Specifically, there is a range of bad signals for which investors do not adjust their long position in the risky asset and there is another range of bad signals for which investors do not hold the risky asset. We briefly discuss the intuition for information inertia of the risk-free portfolio next and then focus on the intuition for risky portfolios for the remainder of this section.

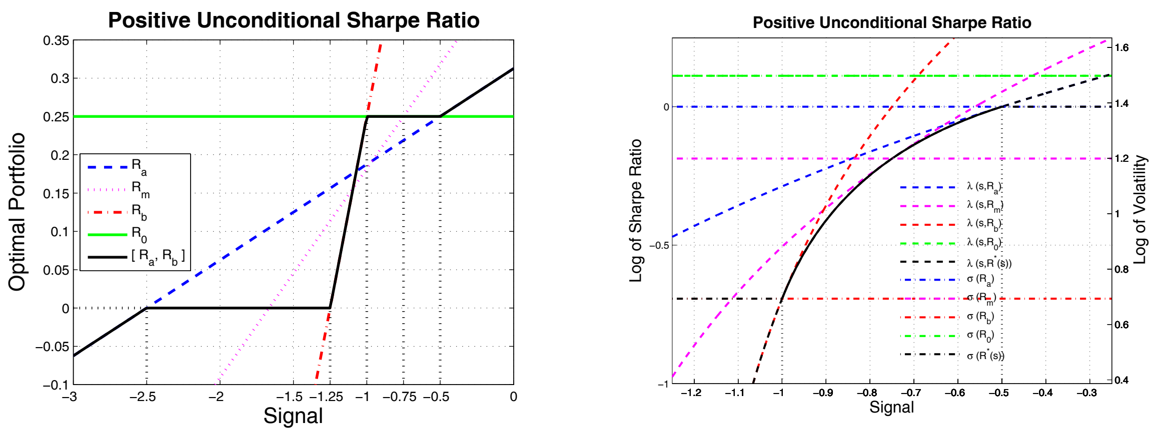


Figure 1: Portfolio Choice

The left graph shows the optimal portfolio and the right graph shows the investors' perceived (log of the) conditional Sharpe ratio and volatility as a function of the signal. Red lines represent an SEU investor with belief $\beta_b = R_b\sigma_d = 4$, purple lines represent an SEU investor with belief $\beta_m = \sigma_d(R_a+R_b)/2 = 3$, blue lines represent an SEU investor with belief $\beta_a = R_a\sigma_d = 2$, green lines represent an SEU investor with belief $R_0 = 0$, and black lines represent an MEU investor with ambiguity aversion ($[R_a, R_b]$). In the right graph dashed lines represent the Sharpe ratio and chain-dotted lines the volatility. The parameters are $\bar{d} = 100$, $p = 95$, $\sigma_d^2 = 20$, and $\gamma = 1$.

Why does the risk-free portfolio exhibit information inertia? To answer this question consider first an SEU investor with belief R and recall that the conditional Sharpe ratio is

$$\lambda(s, R) = \frac{\mu(s, R) - p}{\sigma(R)} = \frac{\lambda_d + Rs}{\sqrt{1 - R^2}}. \tag{9}$$

An SEU investor would buy the asset if the conditional Sharpe ratio is positive and sell short the asset if the conditional Sharpe ratio is negative. There is only one signal realization for which the conditional Sharpe ratio is zero and thus an SEU investor would refrain from holding the asset. In contrast, MEU investors only buy the asset if there is no ambiguity that the conditional Sharpe ratio is positive and sell short the asset if there is no ambiguity that the conditional Sharpe ratio is negative. There is a range of bad signals ($-\frac{\lambda_d}{R_a} < s < -\frac{\lambda_d}{R_b}$) for which the conditional Sharpe ratio is positive for some R and negative for others and thus the risk-free portfolio exhibits information inertia.[16]

[16]This form of inertia also appears in Condie and Ganguli (2011) and Illeditsch (2011).

The risk-free portfolio is the only portfolio that perfectly hedges against ambiguity by making utility independent of the unknown parameter R. All other portfolios are exposed to ambiguity. The next propositions shows that all these portfolios can be determined by evaluating the optimal portfolio of an SEU investor at the belief R that minimizes her utility. Put differently, an MEU investor behaves distinctly different from an SEU investor at a kink of her utility which in this model only occurs at the risk-free portfolio.[17]

Proposition 1 (Characterization of Optimal Portfolios)**.** *Let* $\lambda_d > 0$*. Then*

$$\theta(s) = \begin{cases} 0 & \text{if} \quad -\frac{\lambda_d}{R_a} \leq s \leq -\frac{\lambda_d}{R_b} \\ \bar{\theta}(s, R^*(s)) & \text{otherwise}, \end{cases} \tag{10}$$

where

$$R^*(s) = \underset{R \in [R_a, R_b]}{\text{argmin}}\ \overline{CE}(\theta(s), R). \tag{11}$$

We show in the next proposition that the belief $R^*(s)$ that minimizes utility at the optimal risky portfolio can be determined from the conditional Sharpe ratio $\lambda(s, R)$ and therefore does not require prior knowledge of the optimal portfolio $\theta(s)$.

Proposition 2 (Robust Sharpe Ratio)**.** *Let* $\lambda_d > 0$*. If* $s > -\frac{\lambda_d}{R_b}$*, then* $\bar{\theta}(s, R) > 0$ *for all* $R \in [R_a, R_b]$ *and*

$$R^*(s) = \underset{R \in [R_a, R_b]}{\text{argmin}}\ \lambda(s, R) = \begin{cases} R_a & \text{if} \quad s > -R_a \lambda_d \\ -\frac{s}{\lambda_d} & \text{if} \quad -R_b \lambda_d < s \leq -R_a \lambda_d \\ R_b & \text{if} \quad -\frac{\lambda_d}{R_b} < s \leq -R_b \lambda_d. \end{cases} \tag{12}$$

If $s < -\frac{\lambda_d}{R_a}$*, then* $\bar{\theta}(s, R) < 0$ *for all* $R \in [R_a, R_b]$ *and*

$$R^*(s) = \underset{R \in [R_a, R_b]}{\text{argmax}}\ \lambda(s, R) = R_a. \tag{13}$$

Propositions 1 & 2 show that an investor who is averse to ambiguity about the correlation R chooses a portfolio that is robust to changes in the conditional Sharpe ratio $\lambda(s, R)$. Specifically, an MEU investor with a long position considers the belief

[17] Illeditsch (2011) considers a model where kinks also occur away from certainty.

R that minimizes $\lambda(s, R)$ and an MEU investor with a short position considers the belief R that maximizes $\lambda(s, R)$.

To gain intuition suppose there is no ambiguity that the conditional Sharpe ratio is positive ($s > -\frac{\lambda_d}{R_b}$) and thus the MEU investor is long in the asset. If the signal conveys good news ($s > 0$), then an increase in the correlation R always increases the conditional Sharpe ratio because a more informative signal raises the conditional mean and reduces the conditional volatility. Hence, the MEU investor behaves like an SEU investor with belief R_a and thus the portfolio reacts moderately to news (blue dashed line in Figure 1). However, if the signal conveys bad news ($s < 0$), then an increase in the correlation R decreases the conditional mean and volatility and thus the effects on the conditional Sharpe ratio are unclear. For moderately bad news ($-R_a\lambda_d \leq s < 0$), the volatility effect dominates and the portfolio reacts moderately (blue dashed line in Figure 1). Whereas for worse news ($-\frac{\lambda_d}{R_b} < s \leq -R_b\lambda_d$) the mean effect dominates and thus the portfolio reacts strongly to news (red chain-dotted line in Figure 1).[18]

There is a range of bad signals ($-R_b\lambda_d < s < -R_a\lambda_d$) for which neither the conditional mean nor volatility dominates and the conditional Sharpe ratio is minimized in the interior. Hence, small changes in the correlation change both the mean and volatility but leave the Sharpe ratio unchanged. The interior minimizer depends on the signal, that is $R^* = -\frac{s}{\lambda_d}$, because a change in the signal affects only the mean directly, the correlation changes in order to have a counterbalancing indirect effect on volatility. We know from Propositions 1 & 2 that in this case the MEU investor behaves like an SEU investor with belief $R^* = -\frac{s}{\lambda_d}$ but the portfolio does not react to news and hence coincides with the portfolio of an SEU investor who thinks the correlation between the signal and the asset is zero (green solid line in Figure 1).[19]

Why are there risky portfolios that do not react to news even though there is

[18] If $s < -\frac{\lambda_d}{R_a}$, then there is no ambiguity that the conditional Sharpe ratio is negative and thus the MEU investor is short the asset. In this case the worst case scenario is always a low correlation.

[19] The utility of the SEU investor with belief $R = 0$ is strictly greater than the utility of the SEU investor with belief $R^* = -\frac{s}{\lambda_d} \geq R_a > 0$.

no ambiguity about the fact that the signal is informative ($R_a > 0$)? To answer this question suppose $s = -0.75$ in which case the optimal portfolio of the MEU investor and an SEU investor with belief $R_m = (R_a + R_b)/2$ coincide (left graph of Figure 1). An increase in the signal raises the Sharpe ratio perceived by the SEU and MEU investor and thus makes the asset more attractive. The perceived risk for the SEU investor does not change and thus her demand for the asset increases (dotted line). However, an increase in the signal also increases the risk perceived by the MEU investor because in this case the perceived correlation between the asset and signal decreases and thus less risk is resolved by the signal. The increase in the Sharpe ratio is exactly offset by the increase in the volatility and thus the MEU investor does not change her portfolio (black solid line). Formally,

$$d \ln \theta(s) = d \ln \lambda\left(s, R^*(s)\right) - d \ln \sigma\left(R^*(s)\right) = 0, \qquad \forall s \in (-R_b\lambda_d, -R_a\lambda_d). \tag{14}$$

The right graph of Figure 1 shows the (log) of the conditional Sharpe ratio and volatility when the signal conveys bad news. There is a range of signals for which both the Sharpe ratio and volatility are strictly increasing in the signal. Moreover, the Sharpe ratio increases with the signal at exactly the same rate as the volatility increases with the signal in this range. Hence, any change of the portfolio due to changes in the Sharpe ratio is exactly offset by a change in risk.

Suppose the unconditional Sharpe ratio is zero ($\lambda_d = 0$). There is no information inertia because the MEU investor behaves like an SEU investor with belief R_a. Intuitively, investors will long the asset when news is good and they will short the asset when news is bad. But there is no confusion about the interpretation of the signal when news is good (bad) and investors are long (short) the asset because the worst case scenario for the conditional Sharpe ratio and volatility of the asset is a signal with a low correlation R. Suppose the unconditional Sharpe ratio is negative ($\lambda_d < 0$). In this case the risk-free portfolio and a portfolio consisting of a short position in the asset exhibits information inertia. The intuition is similar to the case where the unconditional Sharpe ratio is positive and thus omitted.

We determine the quantitative significance of the information inertia results in Section V and conclude this section with a summary of the portfolio choice results.

Model Predictions 1 (Information Inertia of Portfolios)**.** *If investors are averse to ambiguity about the predictability of future asset values, then*

(i) there is a range of bad signals over which investors do not adjust their long position in the asset when the unconditional Sharpe ratio is positive,

(ii) there is a range of good signals over which investors do not adjust their short position in the asset when the unconditional Sharpe ratio is negative, and

(iii) there is a range of good and bad signals over which investors do not hold the asset when the unconditional Sharpe ratio is not zero.

III Equilibrium Price and Portfolio

In this section, we determine the price of the risky asset in equilibrium when the economy is populated by investors who all receive the same signal about its future cash flows but may differ with respect to risk and ambiguity aversion. We show that there is a range of bad signals over which investors do not adjust their risky portfolios in equilibrium and thus prices do not always incorporate public information that is worse than expected. Moreover, we show that this informational inefficiency leads to price underreaction consistent with momentum.

Suppose there are H investors who all receive the signal $\tilde{s}$ about the future value of the dividend $\tilde{d}$. Investors may differ with respect to their initial wealth, and their aversion to risk and ambiguity. Let w_{0h} denote investor h's initial wealth, $\gamma_h > 0$ her risk aversion coefficient, and $[R_{ah}, R_{bh}]$ the interval that represents her ambiguity aversion with $0 < R_{ah} \leq R_{bh} < 1 \,\forall\, h \in \{1, \ldots, H\}$.

An equilibrium in this economy is defined as follows:

Definition 1 (Equilibrium)**.** *The signal-to-price map $p(s)$ is an equilibrium if and* *only if (i) each investor chooses a portfolio θ_h to maximize*

$$\min_{R_h \in [R_{ah}, R_{bh}]} \mathrm{E}_{R_h} \left[u_h \left(w_{0h} + \left(\tilde{d} - p(s) \right) \theta_h \right) \mid \tilde{s} = s \right], \qquad \forall s \in \mathbb{R} \tag{15}$$

and (ii) markets clear, that is, $\sum_{h=1}^{H} \theta_h = 1$ and investors consume the liquidating dividend $\tilde{d}$ at date 1.

A Homogeneous Ambiguity Aversion

We know that if all investors are standard expected utility maximizers, then there exists a representative investor (SEU-RI) with these preferences.[20] We show in the next proposition that this is still true when all investors have the same aversion to ambiguity and we determine the utility of the ambiguity averse representative investor (MEU-RI) in equilibrium.[21]

Proposition 3 (MEU-RI and Equilibrium Utility)**.** *Assume that all investors have the same ambiguity aversion $[R_a, R_b]$. Then there exists a representative investor with initial wealth $w_0 = \sum_{h=1}^{H} w_{0h}$ and aggregate risk tolerance $1/\gamma \equiv \sum_{h=1}^{H} 1/\gamma_h$. Moreover, the utility of the MEU-RI in equilibrium is*

$$\min_{R \in [R_a, R_b]} \mathrm{E}_R \left[u\left(\tilde{d}\right) \mid \tilde{s} = s \right] = u \left(\mu\left(s, R^*(s)\right) - \frac{1}{2} \gamma \sigma^2 \left(R^*(s)\right) \right), \tag{16}$$

where

$$R^*(s) = \begin{cases} R_a & \text{if} \quad s \geq -\gamma \sigma_d R_a \\ -\frac{s}{\gamma \sigma_d} & \text{if} \quad -\gamma \sigma_d R_b < s < -\gamma \sigma_d R_a \\ R_b & \text{if} \quad s \leq -\gamma \sigma_d R_b. \end{cases} \tag{17}$$

For the remainder of this subsection we consider a representative investor (MEU-RI) with initial wealth w_0, risk aversion γ, and ambiguity aversion $[R_a, R_b]$.[22] Her equilibrium utility is determined by minimizing the equilibrium utility of an SEU-RI over her belief R. The utility of the SEU-RI is strictly increasing in the posterior mean of the dividend and strictly decreasing in the residual variance of the dividend.

[20] See Chapter 7 in Back (2010).

[21] Wakai (2007) and Illeditsch (2011) show that there exists a representative investor when investors have the same aversion to ambiguity but differ w.r.t. their CARA coefficient.

[22] We discuss the properties of the equilibrium price when investors have different ambiguity aversion in the next subsection.

Hence, the belief R that minimizes the SEU-RI's utility depends on the nature of the news.

Suppose the signal conveys bad news ($s < 0$), then the worst case for the posterior mean $\mu(s, R)$ is a high correlation because in this case the investor significantly revises the value of the dividend downwards whereas the worst case for the residual variance $\sigma^2(R)$ is a low correlation because in this case there is less risk resolved by the signal. If the signal conveys very bad news ($s \leq -\gamma\sigma_d R_b$), then the mean dominates and the MEU-RI investor behaves like an SEU-RI investor with belief R_b. If the signal conveys moderately bad or good news ($s \geq -\gamma\sigma_d R_a$), then the MEU-RI investor behaves like an SEU-RI investor with belief R_a. There is a range of bad signal values ($-\gamma\sigma_d R_b < s < -\gamma\sigma_d R_a$) for which neither the posterior mean nor the residual variance dominates and utility is minimized in the interior.

The equilibrium price when the representative investor is an SEU investor with belief R is

$$\bar{p}(s, R) = \mathrm{E}_R\left[\tilde{d} \mid \tilde{s} = s\right] - \gamma \mathrm{Var}_R\left[\tilde{d} \mid \tilde{s} = s\right] = \mu(s, R) - \gamma\sigma^2(R). \tag{18}$$

The price of the asset is strictly increasing in the signal and hence it fully incorporates all available information. This is no longer true when the representative investor is averse to ambiguity as the next theorem shows.

Theorem 2 (Equilibrium Price)**.** *Consider an economy with an MEU-RI with risk aversion γ and ambiguity aversion $[R_a, R_b]$ who receives the signal s. There is a unique equilibrium price,*

$$p(s) = \begin{cases} \mathrm{E}_{R_a}\left[\tilde{d} \mid \tilde{s} = s\right] - \gamma \mathrm{Var}_{R_a}\left[\tilde{d} \mid \tilde{s} = s\right] & \text{if} \quad s > -\gamma\sigma_d R_a \\ \mathrm{E}\left[\tilde{d}\right] - \gamma \mathrm{Var}\left[\tilde{d}\right] & \text{if} \quad -\gamma\sigma_d R_b \leq s \leq -\gamma\sigma_d R_a \\ \mathrm{E}_{R_b}\left[\tilde{d} \mid \tilde{s} = s\right] - \gamma \mathrm{Var}_{R_b}\left[\tilde{d} \mid \tilde{s} = s\right] & \text{if} \quad s < -\gamma\sigma_d R_b. \end{cases} \tag{19}$$

Moreover, $p(s) = \bar{p}(s, R^*(s))$ where $\bar{p}(\cdot)$ is given in equation (18) and $R^*(s)$ is given in equation (17).

The left graph of Figure 2 shows the equilibrium price as a function of the signal.

The price reacts moderately to signals that convey good and moderately bad news and it reacts strongly to signals that convey very bad news. There is a range of signals that convey bad news for which the price does not react even though the utility of the RI is sensitive to changes in these signals.

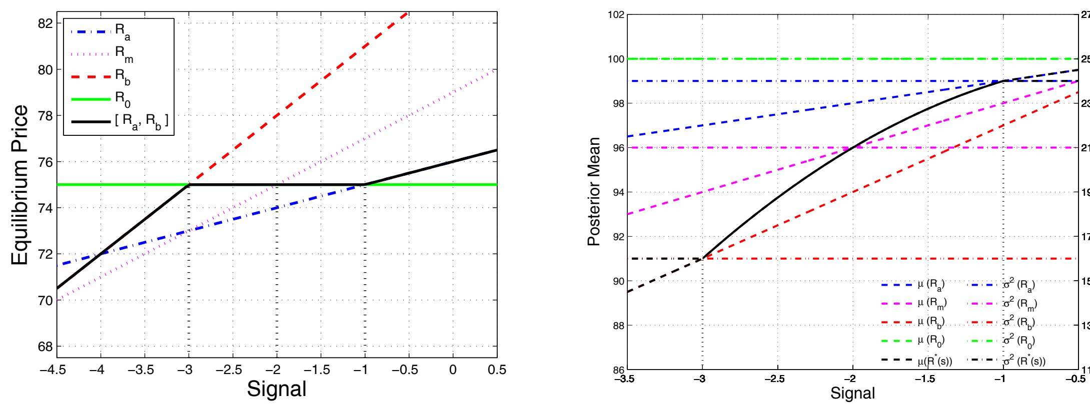


Figure 2: Equilibrium Price

The left graph shows the equilibrium price and the right graph shows the RI's perceived posterior mean and variance as a function of the signal. Red lines represent an SEU-RI economy with belief $\beta_b = R_b\sigma_d = 3$, purple lines represent an SEU-RI economy with belief $\beta_m = \sigma_d(R_a + R_b)/2 = 2$, blue lines represent an SEU-RI economy with belief $\beta_a = R_a\sigma_d = 1$, green lines represent an SEU-RI economy with belief $R_0 = 0$, and black lines represent an MEU-RI economy with ambiguity aversion $[R_a, R_b]$. In the right graph dashed lines represent the mean and chain-dotted lines the variance. The parameters are $\bar{d} = 100$, $\sigma_d^2 = 25$, and $\gamma = 1$.

Why does the price not always incorporate signals that convey bad news? We know from Theorem 2 that the equilibrium price $p(s)$ coincides with the equilibrium price $\bar{p}(s, R^*(s))$ in an economy with an SEU-RI whose belief about the correlation minimizes her utility from consuming the dividend.[23] Consider a two standard deviation bad news surprise ($s = -2$). In this case the equilibrium price is $p = 75$ when there is ambiguity aversion and when there is no ambiguity aversion $\beta_m = 2$ (see left graph of Figure 2). If the signal decreases, then the SEU-RI requires a lower price as compensation for the lower posterior mean in order to hold the market portfolio.

[23]We know from the previous section that an MEU investor behaves differently from an SEU investor only if she holds the risk-free portfolio which is not an equilibrium allocation.

However, the MEU-RI revises the worst case scenario belief about R upwards if the signal drops. The price does not change because the lower posterior mean that would require a drop in the equilibrium price is exactly offset by the lower risk premium that would require an increase in the price. Formally,

$$dp(s) = d\mu(s, R^*(s)) - \gamma d\sigma^2(R^*(s)) = 0, \qquad \forall\, s \in (-\gamma\sigma_d R_b, -\gamma\sigma_d R_a). \tag{20}$$

The right graph of Figure 2 shows the posterior mean and residual variance perceived by the MEU-RI as a function of the signal. The graph shows that there is a range of signals for which both the mean and variance are strictly increasing in the signal. Moreover, the posterior mean increases at the same rate as the residual variance increases in this signal range. Hence, any change in the price due to changes in the posterior mean is exactly offset by a change in the residual variance.

B Heterogeneous Ambiguity Aversion

We show in the next proposition that equilibrium prices still fail to incorporate all available public information when investors are heterogeneous in their ambiguity aversion.[24]

Proposition 4 (Information Inertia)**.** *Let* $1/\gamma \equiv \sum_{h=1}^{H} 1/\gamma_h$ *denote aggregate risk tolerance and let* $[R_a, R_b] \equiv \bigcap_{h=1}^{H} [R_{ah}, R_{bh}] \neq \emptyset$*. Then the equilibrium price is*

$$p(s) = \mathrm{E}\left[\tilde{d}\right] - \gamma \mathrm{Var}\left[\tilde{d}\right] \qquad \forall\, s \in \left[-\gamma\sigma_d R_b, -\gamma\sigma_d R_a\right]. \tag{21}$$

[24] We do not report the equilibrium price outside of the inaction region but provide numerical examples in Figure 3.

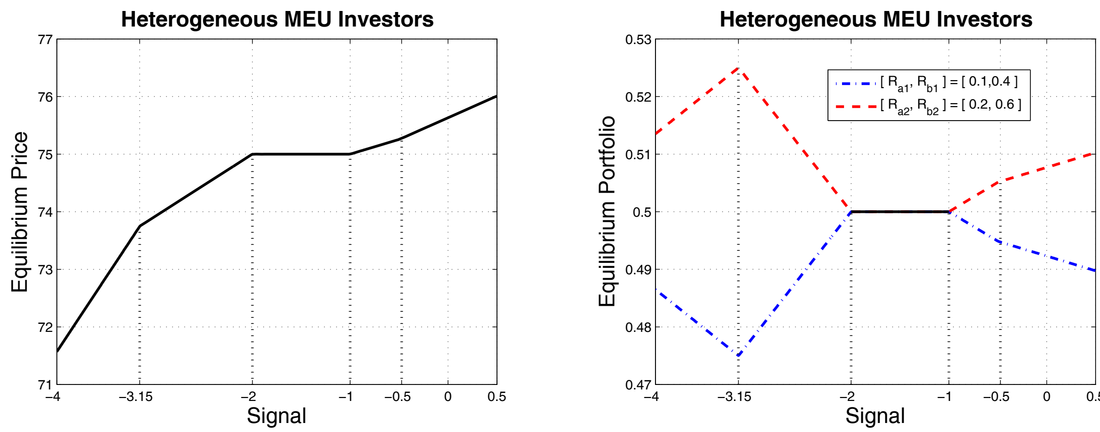


Figure 3: Equilibrium Price and Portfolio

The left graph shows the equilibrium price and the right graph shows the equilibrium portfolios as a function of the signal in an economy populated with heterogenous MEU investors. The blue chain-dotted line shows the optimal portfolio of an MEU investor with ambiguity aversion $[R_{a1}, R_{b1}] = [0.1, 0.4]$ and the red dashed line shows the optimal portfolio of an MEU investor with ambiguity aversion $[R_{a2}, R_{b2}] = [0.2, 0.6]$. There is a range of signals ($[-2, -1]$) for which investors' portfolios and thus the equilibrium price do not react to news. The parameters are $\bar{d} = 100$, $\sigma_d^2 = 25$ and $\gamma_1 = \gamma_2 = 1$.

To gain intuition consider an economy populated by two MEU investors with ambiguity aversion $[R_{a1}, R_{b1}] = [0.1, 0.4]$ and $[R_{a2}, R_{b2}] = [0.2, 0.6]$, respectively. The left graph of Figure 3 shows the equilibrium price and the right graph shows their equilibrium portfolios as a function of the signal. Consider the five different signal regions (i) $(-\infty, -3.15]$, (ii) $[-3.15, -2]$, (iii) $[-2, -1]$, (iv) $[-1, -0.5]$, and (v) $[-0.5, \infty)$. Both MEU investors behave like SEU investors with beliefs $R_{b1} = 0.4$ and $R_{b2} = 0.6$ in the first signal range and thus the equilibrium price reacts a lot to these signals. The equilibrium portfolio of the second MEU investor (red dashed line) is increasing in the signal because her worst case scenario belief ($R_{b2} = 0.6$) is larger than the worst case scenario belief of the second MEU investor (blue chain-dotted line) and thus she puts more weight on the signal. The analysis is similar for the fifth signal range because with good news the worst case scenario for both investors is a low R.

For the other three ranges of signals there is at least one investor who ignores the signal and uses her prior when choosing her optimal portfolio. In other words, there is at least one investor who behaves as if the signal is uninformative even though her utility is negatively affected by it. Consider the second signal range. The first MEU investor still behaves like an SEU investor with belief $R_{b1} = 0.4$ but the second MEU investor does not rely on the signal. Hence her demand, which is increasing for the first range of signals, is now decreasing because neither mean nor variance depends on the signal and the equilibrium price increases with it. The equilibrium price still reacts to signals in the second region because of the first investor but not as much as for the first range of signals. Both investors do not rely on the signals in the third region and hence the equilibrium price does not reflect these signals. The intuition for the fourth signal range is similar to the second. In this case the first investor does not rely on the signal when choosing her optimal portfolio and hence in equilibrium her asset demand decreases with the signal.

There is no information inertia in optimal portfolios of SEU investors and hence the equilibrium price always responds to changes in the signals in their presence. But how much do SEU investors move the price? To answer this question, we consider a unit mass of investors where α denotes the fraction of MEU investors and $1 - \alpha$ denotes the fraction of SEU investors. The left graph of Figure 4 shows that there is a range of signals for which prices do not respond a lot to news if the fraction of MEU investors is sufficiently large.

C Momentum

We show in this subsection that the insensitivity of price to bad news and the underreaction to good and moderately bad news leads to momentum. Moreover, the economic significance of momentum depends on the unconditional risk premium of the asset.

Let $\hat{R}$ denote the correlation between $\tilde{d}$ and $\tilde{s}$ that generates the data with $\hat{R} = (R_a + R_b)/2$.[25] Consider a time series of dividends and stock prices generated from

[25]It is common in the literature (e.g. Hansen and Sargent (2001)) to assume that midpoint of the

the CARA-normal model with representative SEU or MEU investors. Suppose an econometrician regresses future price changes on a constant and the current stock price $p(s)$. Specifically,

$$d - p(s) = \text{constant} + \text{slope} \times p(s) + \text{noise}. \tag{22}$$

The right graph of Figure 4 shows the slope of this predictability regression as a function of the unconditional risk premium of the asset for four different representative agent economies. The green solid line represents an economy in which the belief of the SEU-RI coincides with the data generating belief $\hat{R}$. In this case the stock price incorporates all available information and thus the slope is zero. The blue dashed line represents an economy in which the SEU-RI perceives a higher R than the econometrician and the red chain-dotted line represents an economy in which the SEU-RI perceives a lower R than the econometrician. In this case the stock price incorporates all available information but does so incorrectly from the econometrician's point of view. Hence, the price underreacts in the first economy consistent with momentum and overreacts in the second economy consistent with reversals.

The black solid line represents an economy with an MEU-RI. In this case, there is momentum if the unconditional risk premium of the asset is sufficiently large.[26] If the unconditional risk premium of the stock is large, then the slope is positive because moderate price reactions to news are more likely than strong price reactions to news. For stocks with intermediate risk, the economic significance is larger than in an economy with an SEU-RI with belief R_a because in this case many bad signals are not reflected in the stock price. In contrast, for every homogenous and heterogeneous SEU investors economy with beliefs $R \in [R_a, R_b]$ the slope of the regression in equation (22) lies between the red chain-dotted and blue dashed line and does not depend on the unconditional risk premium.

ambiguity interval generates the data.

[26]This dependence on the unconditional risk premium may help to distinguish our explanation of profitable momentum strategies from other explanations. See Jegadeesh and Titman (2011) and the references therein for review of momentum strategies.

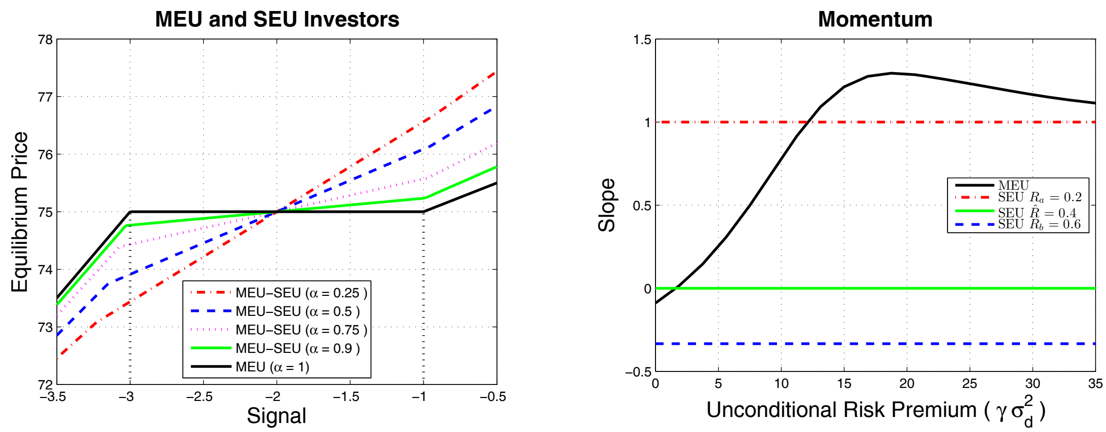


Figure 4: Equilibrium Price and Momentum

The left graph shows the equilibrium price as a function of the signal when there is a unit mass of investors where α denotes the fraction of MEU investors and $1-\alpha$ denotes the fraction of SEU investors. If α increases, then there is a range of signals for which prices do not respond a lot to news. The right graph shows the slope of a regression of future price changes on current prices. If the unconditional risk premium of the asset is sufficiently large, then the slope is positive which is consistent with momentum. The parameters are $\bar{d} = 100$, $\sigma_d^2 = 25$, $R_a = 0.2$, $R_b = 0.6$, and $\gamma = 1$.

We conclude this section with a summary of our results.

Model Predictions 2 (Information Inertia of Equilibrium Portfolios, Informational Inefficiency, and Momentum)**.** *If investors are averse to ambiguity about the predictability of future cash flows, then*

(i) prices do not always incorporate public signals that convey bad news,

(ii) there is a range of bad signals over which investors do not adjust their risky portfolio positions in equilibrium,

(iii) and the informational inefficiency of prices and the resulting underreaction leads to momentum.

IV Parameter Uncertainty and the Smooth Model

We have shown so far that ambiguity aversion leads to portfolios and prices that do not react to news in equilibrium. We study in this section the implications for portfolio choice and asset pricing when investors face Bayesian model or parameter uncertainty and we distinguish between ambiguity and aversion to ambiguity by considering the smooth ambiguity model axiomatized in Klibanoff, Marinacci, and Mukerji (2005). We show that there are signal regions for which portfolios and prices show very low sensitivity to news if aversion to ambiguity is sufficiently high.

Consider the model described in Section I and assume that the correlation between the signal and the dividend is random. Let $\mathbb{P}$ denote the prior distribution for $\tilde{R}$ with support $[R_a, R_b] \subset (0, 1)$.[27] The joint distribution of $\tilde{d}$ and $\tilde{s}$ conditional on knowing the correlation R is normal and given in equation (2). Hence, standard Bayesian updating leads to

$$\tilde{d} \mid \tilde{s} = s, \tilde{R} = R \sim \mathrm{N}_R\left(\mu(s, R), \sigma^2(R)\right). \tag{23}$$

The investor does not learn anything about the correlation R after observing the signal and hence the prior $\mathbb{P}$ coincides with the posterior.[28] Let $u(\cdot)$ denote the function that measures attitudes toward risk and $\phi(\cdot)$ the function that measures attitudes towards ambiguity. The utility of an ambiguity averse investor in the sense of Klibanoff, Marinacci, and Mukerji (2005) who holds θ shares of the risky asset is therefore

$$\mathrm{E}_{\mathbb{P}}\left[\phi\left(\mathrm{E}_{\tilde{R}}\left[u\left(\tilde{w}\right) \mid \tilde{s} = s\right]\right)\right] = \mathrm{E}_{\mathbb{P}}\left[\phi\left(u\left(\overline{\mathrm{CE}}(\theta, \tilde{R}\right]\right)\right], \tag{24}$$

where $\overline{\mathrm{CE}}(\theta, R)$ denotes the certainty equivalent of an SEU investor with dogmatic belief R. If $\phi(\cdot)$ is linear, then investors are neutral to ambiguity and thus we call them BMU investors (investors who face Bayesian model uncertainty) otherwise we call them KMM investors.

For the remainder of this section we assume that investors have constant absolute risk aversion and constant relative ambiguity aversion, that is, $u(w) = -e^{-\gamma w}$ and

[27]The SEU investors of the previous sections have dogmatic priors over the correlation R.

[28]Investors can draw inferences about the correlation R, if they observe a time series of $\tilde{d}$ and $\tilde{s}$. In this case $\mathbb{P}$ in equation (24) below would be the distribution of $\tilde{R}$ conditional on observing the signal $\tilde{s} = s$.

$\phi(u) = -\frac{1}{1+\alpha}(-u)^{1+\alpha}$ with γ positive and α nonnegative.[29] Hence, the certainty equivalent $\overline{\text{CE}}(\theta, R)$ is given in equation (6).

A Portfolio Choice

Let $\theta(s)$ denote the portfolio of a KMM investor that maximizes utility given in equation (24). The properties of $\theta(s)$ are summarized in the next proposition.

Proposition 5 (Portfolio Choice). *For every distribution $\mathbb{P}$ with support $[R_a, R_b] \subset [0,1]$ such that utility given in equation (24) exists, let $\mathbb{Q}(R; s, \theta(s))$ denote the risk and ambiguity adjusted distribution of $\tilde{R}$ conditional on $\tilde{s} = s$. Specifically,*

$$d\mathbb{Q}(R; s, \theta(s)) = \frac{e^{-\gamma(1+\alpha)\left(\sigma_d s R\,\theta(s) + \frac{1}{2}\gamma\sigma_d^2 R^2 \theta(s)^2\right)}}{\mathrm{E}_{\mathbb{P}}\left[e^{-\gamma(1+\alpha)\left(\sigma_d s \tilde{R}\,\theta(s) + \frac{1}{2}\gamma\sigma_d^2 \tilde{R}^2 \theta(s)^2\right)}\right]} d\mathbb{P}(R) \tag{25}$$

The optimal portfolio is unique and implicitly given by

$$\theta(s) = \frac{\lambda^{\mathbb{Q}}(s, \theta(s))}{\gamma\sigma^{\mathbb{Q}}(s, \theta(s))} \tag{26}$$

where

$$\sigma^{\mathbb{Q}}(s, \theta(s)) = \sigma_d \sqrt{1 - \mathrm{E}_{\mathbb{Q}(R; s, \theta(s))}\left[\tilde{R}^2 \mid \tilde{s} = s\right]} \tag{27}$$

$$\lambda^{\mathbb{Q}}(s, \theta(s)) = \frac{\bar{d} - p + \sigma_d s \mathrm{E}_{\mathbb{Q}(R; s, \theta(s))}\left[\tilde{R} \mid \tilde{s} = s\right]}{\sigma^{\mathbb{Q}}(s, \theta(s))}. \tag{28}$$

Why does the Sharpe ratio and volatility depend on the position in the asset? To answer this question consider first the case where $\alpha = 0$. The BMU investor hedges against parameter uncertainty by adjusting its distribution for risk. The risk adjusted probability $\mathbb{Q}$ depends on the position in the asset because the effects of different realizations of $\tilde{R}$ on utility depend on the asset position. For instance, suppose an investor who contemplates a long position in the asset receives a signal that conveys bad news. If the long position is very large, then the investor is more

[29] We choose constant absolute risk aversion for $u(\cdot)$ so that conditional on knowing R investors have mean-variance preferences. The choice of constant relative ambiguity aversion for $\phi(\cdot)$ simplifies the analysis but does not change the qualitative result of this section.

concerned about the residual variance and thus the risk adjusted probability of low correlation states is higher than the actual probability of these states. Similarly, for a moderate long position in the asset, the investor is more concerned about a low posterior mean and thus the risk adjusted probability of high correlation states exceeds the actual probability of these states.[30] A KMM investor is also averse to ambiguity and thus puts additional weight on the states of the world for which R has adverse effects on utility. An increase in risk aversion would also make a BMU investor more concerned about parameter uncertainty. However, an increase in risk aversion has the indirect effect of decreasing the asset position which makes her less concerned about parameter uncertainty. The smooth model alleviates this tension by having an additional parameter that increases the aversion to parameter uncertainty.

The left graph of Figure 5 shows the optimal portfolio as a function of the signal when the unconditional Sharpe ratio is positive and $\tilde{R}$ is uniformly distributed on the interval $[R_a, R_b]$. The black solid line represents an MEU investor, the red solid line represents a BMU investor ($\alpha = 0$), and the other three lines represent KMM investors with different degrees of ambiguity aversion α. There is a range of signals for which risky portfolios become less and less sensitive to news as ambiguity aversion increases. Moreover, the figure shows that there are signal ranges for which asset demand is strictly increasing in α, which is consistent with Gollier (2011) who also finds that an increase in aversion to ambiguity does not always lead to a decrease in asset demand.

Why does the sensitivity to news for some risky portfolios decrease with aversion to ambiguity? The intuition for this result is similar to the intuition for the information inertia result. Asset demand for a KMM investor is increasing in the Sharpe ratio and decreasing in the volatility of the asset. Both the Sharpe ratio and volatility are determined by averaging over $\tilde{R}$ using the risk and ambiguity adjusted probability $\mathbb{Q}$. The risk and ambiguity adjustment depends on the signal and thus the conditional volatility and Sharpe ratio depend on the signal. The right graph of Figure 5 shows

[30]There is no parameter uncertainty for the risk free portfolio and thus $\mathbb{Q}$ and $\mathbb{P}$ coincide.

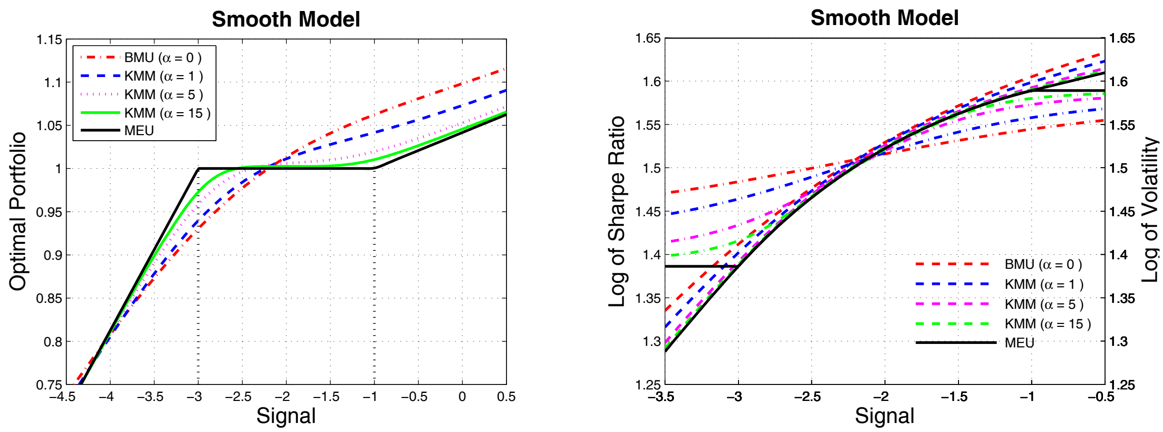


Figure 5: Optimal Portfolio

The left graph shows the optimal portfolio and the right graph shows the risk and ambiguity adjusted (log of the) conditional Sharpe ratio and volatility as a function of the signal. The black lines represent an MEU investor, the red lines represent a BMU investor ($\alpha = 0$), and the other lines represent KMM investors with different degrees of ambiguity aversion α. In the right graph dashed lines represent the Sharpe ratio and chain-dotted lines the volatility. The parameter R is uniformly distributed on the interval $[R_a, R_b] = [0.2, 0.6]$ and $\bar{d} = 100$, $p = 75$, $\sigma_d^2 = 25$, and $\gamma = 1$.

the (log) of the conditional Sharpe ratio and volatility perceived by a KMM investor for different degrees of ambiguity aversion α. If α is sufficiently large, then there is a range of signals for which both the conditional Sharpe ratio and volatility increase at approximately the same rate and thus the portfolio does not react much to these signals.[31]

B Equilibrium Price

Suppose there exists a representative investor with prior $\mathbb{P}$ over the correlation $\tilde{R}$. In equilibrium the representative investor holds the asset ($\theta = 1$) and consumes the liquidating dividend $\tilde{d}$. The properties of the equilibrium price are summarized in the next proposition.

[31]There is also a range of signals for which the risk-free portfolio does not react much to news if α is sufficiently large.

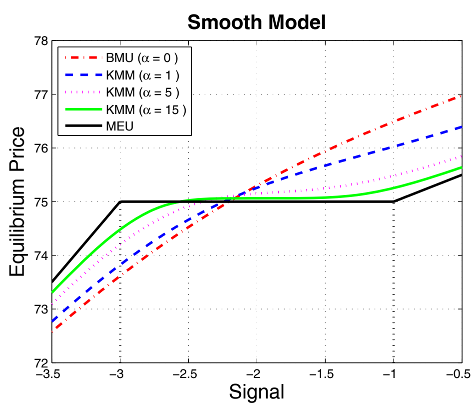
Proposition 6. *The unique equilibrium price is*

$$p(s) = \bar{d} - \gamma\sigma_d^2 + s\sigma_d \,\mathrm{E}_{\mathbb{Q}(R,s)}\left[\tilde{R} \mid \tilde{s} = s\right] + \gamma\sigma_d^2 \mathrm{E}_{\mathbb{Q}(R,s)}\left[\tilde{R}^2 \mid \tilde{s} = s\right], \tag{29}$$

where $\mathbb{Q}(R, s)$ denotes the risk and ambiguity adjusted equilibrium distribution of the correlation $\tilde{R}$ conditional on $\tilde{s} = s$. Specifically,

$$d\mathbb{Q}(R; s) = \frac{e^{-\gamma(1+\alpha)\left(\sigma_d s R + \frac{1}{2}\gamma\sigma_d^2 R^2\right)}}{\mathrm{E}_{\mathbb{P}}\left[e^{-\gamma(1+\alpha)\left(\sigma_d s \tilde{R} + \frac{1}{2}\gamma\sigma_d^2 \tilde{R}^2\right)}\right]} d\mathbb{P}(R). \tag{30}$$

The left graph of Figure 6 shows the equilibrium price as a function of the signal when $\tilde{R}$ is uniformly distributed on the interval $[R_a, R_b]$. The black solid line represents an economy with an MEU-RI the red solid line represents an economy with a BMU-RI ($\alpha = 0$), and the other three lines represent economies with a KMM-RI with different degrees of ambiguity aversion α. There is a range of signals for which the equilibrium price becomes less sensitive to changes in the signal when α increases.



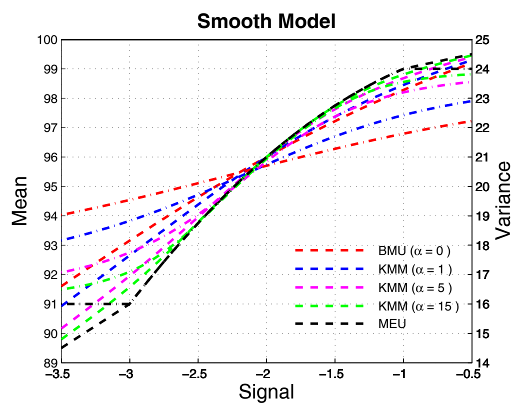


Figure 6: Equilibrium Price

The left graph shows the equilibrium price and the right graph shows the risk and ambiguity adjusted conditional mean and variance as a function of the signal. The black lines represent an MEU investor, the red lines represent a BMU investor ($\alpha = 0$), and the other three lines represent KMM investors with different ambiguity aversion α. In the right graph dashed lines represent the mean and chain-dotted lines the variance. The parameter R is uniformly distributed on the interval $[R_a, R_b]$ and $\bar{d} = 100$, $\sigma_d^2 = 25$, and $\gamma = 1$.

Why does the sensitivity of the equilibrium price to news decrease with aversion to ambiguity? Intuitively, the price increases with the posterior mean and decreases with the posterior variance. The RI hedges against risk and ambiguity and thus both the mean and variance depend on the signal. The right graph of Figure 6 shows that there is a range of signal values for which both the risk and ambiguity adjusted mean and variance increase at approximately the same rate and thus the equilibrium price does not react much to these signals.

V Calibration

In this section we focus on the quantitative significance of information inertia. Suppose $\tilde{s}$ is a standardized predictor of $\tilde{y}$ where $\tilde{y}$ is either the future excess returns or divided growth rate of the market portfolio. The unconditional mean and variance of $\tilde{y}$ are $\bar{y}$ and σ_y^2, respectively. Investors are averse to ambiguity about the correlation between $\tilde{y}$ and $\tilde{s}$ and consider a family of linear regression models described by $R \in [R_a, R_b]$. Specifically,

$$\tilde{y} = \alpha + \beta \tilde{s} + \varepsilon, \qquad \alpha = \bar{y}, \qquad \beta = R\sigma_y, \qquad \tilde{\varepsilon} \sim \mathcal{N}\left(0, \sigma_y^2(1-R^2)\right). \tag{31}$$

We use a confidence interval for the correlation R as a proxy for ambiguity aversion.[32] Let $\hat{R}$ denote the point estimate for the correlation R, $\hat{a}$ the significance level of the confidence interval $[R_a, R_b]$, and T the size of the data sample.[33] The size of the interval is strictly decreasing in the significance level $\hat{a}$ and hence $\alpha = 1 - \hat{a}$ can be interpreted as a measure for ambiguity aversion.

We consider 84 observations of $\tilde{y}$ and $\tilde{s}$, that is, $T = 84$ and three different

[32] Garlappi, Uppal, and Wang (2007) use a confidence interval for expected stock returns as a proxy for ambiguity aversion.

[33] The confidence interval (see Fisher (1921)) for the correlation R is $[R_a, R_b]$ with

$$R_a = \tanh\left(\operatorname{artanh}\left(\hat{R}\right) - \Phi^{-1}\left(1 - \frac{\hat{a}}{2}\right)\frac{1}{\sqrt{T-3}}\right) \tag{32}$$

$$R_b = \tanh\left(\operatorname{artanh}\left(\hat{R}\right) + \Phi^{-1}\left(1 - \frac{\hat{a}}{2}\right)\frac{1}{\sqrt{T-3}}\right). \tag{33}$$

values for the point estimate of the correlation, that is, $\hat{R} \in \{30\%, 40\%, 50\%\}$. These estimates correspond to R^2s in predictive regressions ranging from 0.09 to 0.25.[34] We focus on predictors for excess returns that are statistically significant to determine the probability of information inertia in optimal portfolios and we focus on predictors for dividend growth that are statistically significant to determine the probability of information inertia in equilibrium prices. Hence $R_a > 0$ in both cases. If we allow for insignificant predictors ($0 \in [R_a, R_b]$) then the probability of information inertia is higher.

A Information Inertia of Portfolios

Suppose the signal $\tilde{s}$ predicts future excess returns $\tilde{r}_e$, that is, $\tilde{y} = \tilde{r}_e$, $\bar{y} = \mu_e$, and $\sigma_y = \sigma_e$ in equation (31). Let θ denote the fraction of wealth invested in the risky asset and $1 - \theta$ the fraction of wealth invested in the risk-free asset. Investors are averse to ambiguity in the sense of Gilboa and Schmeidler (1989) and have mean-variance preferences over excess returns. Hence, the optimal portfolio of an MEU investor maximizes

$$\min_{R \in [R_a, R_b]} \left(\mathrm{E}_R \left[\tilde{r}_e \mid \tilde{s} = s \right] \theta - \frac{1}{2} \gamma \, \mathrm{Var}_R \left[\tilde{r}_e \mid \tilde{s} = s \right] \theta^2 \right), \tag{34}$$

where $\mathrm{E}_R \left[\tilde{r}_e \mid \tilde{s} = s \right] = \alpha + \beta s$ and $\mathrm{Var}_R \left[\tilde{r}_e \mid \tilde{s} = s \right] = \sigma_e^2 (1 - R^2)$.

The optimal portfolio of an SEU investors with mean-variance preferences and belief R is

$$\bar{\theta}(s, R) = \frac{\mathrm{E}_R \left[\tilde{r}_e \mid \tilde{s} = s \right]}{\gamma \mathrm{Var}_R \left[\tilde{r}_e \mid \tilde{s} = s \right]}. \tag{35}$$

The optimal portfolio for an MEU investor with risk aversion γ and ambiguity aversion

[34] Koijen and Nieuwerburgh (2011) consider 84 annual observations to study the predictability of the price-dividend ratio for stock market returns and dividend growth. The R^2s of return predictability regressions with significant price-dividend ratio range from 4.82% to 17.72% and the R^2s of cash flow predictability regressions with significant price-dividend ratio range from 4.08% to 24.42%.

$[R_a, R_b]$ is

$$\theta(s) = \begin{cases} \bar{\theta}(s, R_a) & s \geq s_1 \equiv -R_a \max(\lambda_e, 0) - \frac{1}{R_a} \min(\lambda_e, 0) \\ \max\left(\bar{\theta}(s, 0), 0\right) & s_1 > s \geq s_2 \equiv -R_b \max(\lambda_e, 0) - \frac{1}{R_b} \min(\lambda_e, 0) \\ \bar{\theta}(s, R_b) & s_2 > s \geq s_3 \equiv -\frac{1}{R_b} \max(\lambda_e, 0) - R_b \min(\lambda, 0) \\ \min\left(\bar{\theta}(s, 0), 0\right) & s_3 > s \geq s_4 \equiv -\frac{1}{R_a} \max(\lambda_e, 0) - R_a \min(\lambda_e, 0) \\ \bar{\theta}(s, R_a) & s < s_4, \end{cases}$$

where $\lambda_e = \frac{\mu_e}{\sigma_e}$ denotes the unconditional Sharpe ratio.

We know from Section II that there is range of good and bad signals for which investors neither buy nor sell short the asset (information inertia of the risk-free portfolio). Moreover, if the unconditional Sharpe ratio is positive ($\lambda_e > 0$), then there is a range of bad signals for which investors do not change their long position and if the unconditional Sharpe ratio is negative ($\lambda_e < 0$), then there is range of good signals for which investors do not change their short position (information inertia for risky portfolios). The size of the inaction region and the probability of information inertia is given in Proposition 8 in Appendix A.

Table 1 shows the probability of information inertia conditional on bad news for risky portfolios (Risky PF), for the riskfree portfolio (Risk-free PF), and for both (Total).[35] We consider five different values for the unconditional Sharpe ratio, that is, $\lambda_e \in \{0.25, 0.3, 0.35, 0.4, 0.5\}$ and ambiguity aversion decreases from $\alpha = 0.99$ to $\alpha = 0.5$. From Panel B of the Table in which $\alpha = 0.95$, we see that for an asset with unconditional Sharpe ratio of 0.3, the conditional probability of information inertia for a risky portfolio is 9.3% when the R^2 is 0.09 and 8.6% when the R^2 is 0.16. The total probability of information inertia conditional on a bad news surprise is 62.7% and 54.3%, respectively. The probability of information inertia for risky portfolios is increasing in the unconditional Sharpe ratio and decreasing in the R^2 of the predictability regression. The probability of information inertia for the risk free portfolio is increasing in ambiguity aversion and non-monotonic in the unconditional

[35] The unconditional probability is half of the conditional probability.

Sharpe ratio and the R^2 of the predictability regression.

B Information Inertia of Prices

Suppose the signal $\tilde{s}$ predicts future dividend growth $\tilde{r}_e$, that is, $\tilde{y} = \tilde{g}_d$, $\bar{y} = \mu_g$, and $\sigma_y = \sigma_g$ in equation (31). The equilibrium expected excess return with an SEU-RI who has mean-variance preferences and belief R is constant and equal to $\bar{\mu}_e \equiv \gamma\sigma_g^2(1 - R^2)$. This is no longer true when there is ambiguity aversion about R. Specifically, the equilibrium expected excess return with an MEU-RI who has mean-variance preferences and ambiguity aversion $[R_a, R_b]$ is

$$\mu_e(s) = \begin{cases} \gamma\sigma_g^2(1 - R_a^2) & \text{if} \quad s > -\gamma R_a \sigma_g \\ \gamma\sigma_g^2 & \text{if} \quad -\gamma R_b \sigma_g \leq s \leq -\gamma R_a \sigma_g \\ \gamma\sigma_g^2(1 - R_b^2) & \text{if} \quad s < -\gamma R_b \sigma_g. \end{cases} \tag{36}$$

There is a range of signals for which the information does not get incorporated into the price and thus the expected excess return in equilibrium is equal to the unconditional expected excess return $\gamma\sigma_g^2$. The size of the inaction region and the probability of information inertia is given in Proposition 9 in Appendix A.

We report the probability of information inertia in prices conditional on bad news in Table 2.[36] We consider six different values for the unconditional expected excess return, that is, $\gamma\sigma_g^2 \in \{0.02, 0.03, 0.04, 0.05, 0.075, 0.1\}$ and ambiguity aversion decrease from $\alpha = 0.99$ to $\alpha = 0.5$. Panel B of Table 2 shows that for $\alpha = 0.95$ and an unconditional expected excess return of 10%, the conditional probability of information inertia is 29.8% when the R^2 is 9% and 26.7% when the R^2 is 16%. The probability of information inertia is increasing in the unconditional expected excess return and decreasing in the R^2 of the predictability regression.

[36]The unconditional probability is half of the conditional probability.

Table 1: Information Inertia of Optimal Portfolios

This table shows the probability of information inertia conditional on a bad news surprise for different ambiguity aversion (α), explanatory power of the excess return predictability regressions (R^2), and unconditional Sharpe ratios (λ_e). The size of the data sample is fixed at $T = 84$. The probability of information inertia for a long position is decreasing in the R^2 and increasing in λ_e and α. The probability of information inertia for the risk free portfolio is increasing in α and non monotonic in λ_e and the R^2.

	$R^2 = 9\%$			$R^2 = 16\%$			$R^2 = 25\%$		
Sharpe Ratio	**Risky PF**	**Riskfree PF**	**Total**	**Risky PF**	**Riskfree PF**	**Total**	**Risky PF**	**Riskfree PF**	**Total**
Panel A ($\alpha = 0.99$)									
0.25	10.16%	63.97%	74.12%	9.41%	61.50%	70.91%	8.44%	38.35%	46.79%
0.3	12.17%	57.43%	69.60%	11.27%	59.51%	70.78%	10.10%	41.72%	51.82%
0.35	14.18%	51.22%	65.39%	13.11%	55.61%	68.72%	11.73%	43.50%	55.23%
0.4	16.17%	45.38%	61.55%	14.94%	50.90%	65.84%	13.35%	43.85%	57.20%
0.5	20.12%	34.91%	55.03%	18.54%	41.26%	59.80%	16.51%	41.25%	57.76%
Panel B ($\alpha = 0.95$)									
0.25	7.79%	59.87%	67.66%	7.20%	44.05%	51.25%	6.44%	26.39%	32.83%
0.3	9.34%	53.37%	62.71%	8.62%	45.65%	54.28%	7.70%	29.36%	37.07%
0.35	10.88%	46.88%	57.76%	10.04%	45.15%	55.19%	8.95%	31.36%	40.31%
0.4	12.41%	40.79%	53.20%	11.44%	43.08%	54.52%	10.19%	32.42%	42.60%
0.5	15.45%	30.09%	45.54%	14.19%	36.31%	50.51%	12.59%	32.03%	44.63%
Panel C ($\alpha = 0.9$)									
0.25	6.56%	53.62%	60.18%	6.06%	35.42%	41.48%	5.41%	21.26%	26.67%
0.3	7.86%	49.33%	57.19%	7.25%	37.53%	44.79%	6.47%	23.82%	30.29%
0.35	9.16%	43.73%	52.89%	8.44%	37.95%	46.39%	7.52%	25.64%	33.16%
0.4	10.45%	37.89%	48.34%	9.62%	36.96%	46.58%	8.56%	26.73%	35.29%
0.5	13.01%	27.28%	40.29%	11.94%	32.15%	44.09%	10.58%	26.90%	37.47%
Panel D ($\alpha = 0.75$)									
0.25	4.61%	37.92%	42.53%	4.25%	23.37%	27.62%	3.79%	14.15%	17.94%
0.3	5.52%	37.08%	42.61%	5.09%	25.31%	30.40%	4.53%	15.97%	20.50%
0.35	6.43%	34.35%	40.78%	5.92%	26.19%	32.11%	5.27%	17.34%	22.61%
0.4	7.34%	30.49%	37.83%	6.75%	26.11%	32.86%	5.99%	18.24%	24.23%
0.5	9.14%	21.89%	31.03%	8.37%	23.69%	32.06%	7.41%	18.72%	26.13%
Panel E ($\alpha = 0.5$)									
0.25	2.71%	21.74%	24.45%	2.50%	13.18%	15.68%	2.22%	8.04%	10.27%
0.3	3.25%	22.03%	25.28%	2.99%	14.44%	17.43%	2.66%	9.12%	11.78%
0.35	3.78%	21.10%	24.88%	3.48%	15.13%	18.61%	3.09%	9.94%	13.04%
0.4	4.32%	19.27%	23.59%	3.96%	15.28%	19.24%	3.52%	10.52%	14.03%
0.5	5.37%	14.26%	19.63%	4.92%	14.23%	19.15%	4.35%	10.92%	15.26%

Table 2: Information Inertia of Prices

This table shows the probability of information inertia in equilibrium prices conditional on a bad news surprise for different ambiguity aversion (α), explanatory power of the cash flow predictability regressions (R^2), and unconditional expected excess return ($\gamma\sigma_g^2$). The size of the data sample is fixed at $T = 84$. The probability of information inertia is decreasing in the R^2 and increasing in $\gamma\sigma_g^2$ and α.

	Unconditional Expected Excess Return					
	2%	3%	4%	5%	7.5%	10%
$\boldsymbol{R^2}$	**Panel A** ($\alpha = 0.99$)					
9%	8.13%	12.17%	16.17%	20.12%	29.73%	38.81%
16%	7.54%	11.27%	14.94%	18.54%	27.14%	34.99%
25%	6.77%	10.10%	13.35%	16.51%	23.88%	30.27%
$\boldsymbol{R^2}$	**Panel B** ($\alpha = 0.95$)					
9%	6.24%	9.34%	12.41%	15.45%	22.83%	29.82%
16%	5.77%	8.62%	11.44%	14.19%	20.77%	26.76%
25%	5.17%	7.70%	10.19%	12.59%	18.20%	23.03%
$\boldsymbol{R^2}$	**Panel C** ($\alpha = 0.9$)					
9%	5.26%	7.86%	10.45%	13.01%	19.23%	25.13%
16%	4.86%	7.25%	9.62%	11.94%	17.47%	22.51%
25%	4.34%	6.47%	8.56%	10.58%	15.28%	19.32%
$\boldsymbol{R^2}$	**Panel D** ($\alpha = 0.75$)					
9%	3.69%	5.52%	7.34%	9.14%	13.51%	17.66%
16%	3.41%	5.09%	6.75%	8.37%	12.25%	15.78%
25%	3.04%	4.53%	5.99%	7.41%	10.69%	13.51%
$\boldsymbol{R^2}$	**Panel E** ($\alpha = 0.5$)					
9%	2.17%	3.25%	4.32%	5.37%	7.95%	10.39%
16%	2.00%	2.99%	3.96%	4.92%	7.20%	9.27%
25%	1.78%	2.66%	3.52%	4.35%	6.27%	7.92%

VI Conclusion

We study how information about an asset affects optimal portfolios when investors do not know the model that links this information to future asset values. We show that ambiguity averse investors do not always act on information that is worse than expected. Hence, they do not participate in the stock market or rebalance their portfolios as frequently as traditional models would predict which is consistent with the household portfolio choice literature.[37] Our explanation does not rely on information processing costs or other market frictions and it is different from the explanation in Epstein and Schneider (2010) and Illeditsch (2011) who show that aversion to ambiguity leads to risk-free and risky portfolios that do not react to changes in the stock price—a phenomenon which they refer to as portfolio inertia.

We also study the effects of ambiguity aversion on the equilibrium price of the market portfolio when investors receive information about its future cash flows. We show that the price of the market portfolio fails to incorporate all available information in equilibrium. Specifically, signals that convey information that is better than expected are always reflected in the stock price while some signals that convey information that is worse than expected are not. This informational inefficiency leads to price underreaction consistent with momentum.

References

Abel, Andrew, Janice Eberly, and Stavros Panageas, 2007, Optimal inattention to the stock market, *American Economic Review* 97, 244–249.

Ahn, David, Syngjoo Choi, Douglas Gale, and Shachar Kariv, 2011, Estimating ambiguity aversion in a portfolio choice experiment, *Working Paper*.

[37]See Bodie, Detemple, and Rindisbacher (2009), Campbell (2006), and the references therein for a review of this literature.

Back, Kerry, 2010, *Asset Pricing and Portfolio Choice Theory* (Oxford University Press).

Back, K., H. Cao, and G. Willard, 2000, Imperfect competition among informed traders, *Journal of Finance* 55, 2117–2155.

Backus, David K., Bryan R. Routledge, and Stanley E. Zin, 2004, Exotic preferences for macroeconomists, NBER Working Paper 10597.

Balduzzi, Pierluigi, and Anthony W. Lynch, 1999, Transaction costs and predictability: some utility cost calculations, *Journal of Financial Economics* 52, 47–78.

———, 2000, Predictability and transaction costs: The impact on rebalancing rules and behavior, *Journal of Finance* LV, 2285–2309.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* LIX, 1481–1509.

Barberis, Nicholas, Andrei Shleifer, and Robert Vishny, 1998, A model of investor sentiment, *Journal of Financial Economics* 49, 307–343.

Barberis, N. C., 2000, Investing for the long run when returns are predictable, *Journal of Finance* 55, 225–264.

Benigno, Pierpaolo, and Salvatore Nistico, 2012, International portfolio allocation under model uncertainty, *American Economic Journal: Macroeconomics* 4, 144–189.

Bodie, Zvi, Jerome Detemple, and Marcel Rindisbacher, 2009, Life-cycle finance and the design of pension plans, *Annual Review of Financial Economics* 1, 249–286.

Bossaerts, Peter, Paolo Ghirardato, Serena Guarnaschelli, and William Zame, 2010, Ambiguity in asset markets: Theory and experiment, *Review of Financial Studies* 23, 1325–1359.

Boyle, Phelim, Lorenzo Garlappi, Raman Uppal, and Tan Wang, 2012, Keynes meets markowitz: The trade-off between familiarity and diversification, *Management Science* 58, 253–272.

Campanale, Claudio, 2011, Learning, ambiguity and life-cycle portfolio allocation, *Review of Economic Dynamics* 14, 339–367.

Campbell, John Y., 2006, Household finance, *Journal of Finance* LXI, 1553–1604.

Cao, H. Henry, Tan Wang, and Harold H. Zhang, 2005, Model uncertainty, limited market participation, and asset prices, *Review of Financial Studies* 18, 1219–1251.

Caskey, Judson A., 2009, Information in equity markets with ambiguity-averse investors, *Review of Financial Studies* 22, 3595–3627.

Chen, Hui, Nengjiu Ju, and Jianjun Miao, 2011, Dynamic asset allocation with ambiguous return predictability, *Working Paper*.

Cochrane, John H., 2005, *Asset Pricing* (Princeton University Press) 1 revised edn.

Collard, Fabrice, Sujoy Mukerji, Kevin Sheppard, and Jean-Marc Tallon, 2011, Ambiguity and the historical equity premium, *Working Paper*.

Condie, Scott, and Jayant Ganguli, 2011, Ambiguity and rational expectations equilibria, *Review of Economic Studies* 78, 821–845.

Condie, Scott, and Jayant V. Ganguli, 2014, The pricing effects of ambiguous private information, Unpublished manuscript.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 1998, Investor psychology and security market under- and overreactions, *Journal of Finance* LIII, 1839–1885.

Dow, J., and S. Werlang, 1992, Uncertainty aversion, risk aversion, and the optimal choice of portfolio, *Econometrica* 60, 197–204.

Easley, David, and Maureen O'Hara, 2009, Ambiguity and nonparticipation: The role of regulation, *Review of Financial Studies* 22, 1818–1843.

Ellsberg, D., 1961, Risk, ambiguity, and the savage axioms, *Quarterly Journal of Economics* 75, 643–69.

Epstein, Larry G., and Martin Schneider, 2007, Learning under ambiguity, *Review of Economic Studies* 74, 1275–1303.

——— , 2008, Ambiguity, information quality, and asset pricing, *Journal of Finance* LXIII, 197–228.

——— , 2010, Ambiguity and asset markets, *Annual Review of Financial Economics* 2, 315–346.

Epstein, Larry G., and Tan Wang, 1994, Intertemporal asset pricing under Knightian uncertainty, *Econometrica* 62, 283–322.

Fisher, Ronald A., 1921, On the "probable error" of a coefficient of correlation deduced from a small sample, *Metron* 1, 3–32.

Gajdos, T., T. Hayashi, J.-M. Tallon, and J.C. Vergnaud, 2008, Attitude toward imprecise information, *Journal of Economic Theory* 140, 27–65.

Garlappi, Lorenzo, Raman Uppal, and Tan Wang, 2007, Portfolio selection with parameter and model uncertainty: A multi-prior approach, *Review of Financial Studies* 20, 41–81.

Gilboa, Itzhak, and David Schmeidler, 1989, Maxmin expected utility with non-unique prior, *Journal of Mathematical Economics* 18, 141–153.

——— , 1993, Updating ambiguous beliefs, *Journal of Economic Theory* 59, p. 33–49.

Gollier, Christian, 2011, Portfolio choices and asset prices: The comparative statics of ambiguity aversion, *Review of Economic Studies* 78, 1329–1344.

Grossman, Sanford, 1976, On the efficiency of competitive stock markets where trades have diverse information, *Journal of Finance* XXXI, 573–585.

———, and Joseph Stiglitz, 1976, Information and competitive price systems, *American Economic Review* 66, 246–253.

Grossman, S., and J. Stiglitz, 1980, On the impossibility of informationally efficient markets, *American Economic Review* 70, 393–408.

Hansen, Lars Peter, and Thomas Sargent, 2001, Robust control and model uncertainty, *American Economic Review* 91, 60–66.

———, 2007, *Robustness* (Princeton University Press) 1 edn.

———, 2010a, Fragile beliefs and the price of uncertainty, *Quantitative Economics* 1, 129–162.

Hansen, Lars Peter, and Thomas J. Sargent, 2010b, Wanting robustness in macroeconomics, in Benjamin M. Friedman, and Michael Woodford, ed.: *Handbook of Monetary Economics*, vol. 3 of *Handbook of Monetary Economics* . chap. 20, pp. 1097–1157 (Elsevier).

Hayashi, Takashi, and Jianjun Miao, 2011, Intertemporal substitution and recursive smooth ambiguity preferences, *Theoretical Ecnomics* 6, 423–472.

Hong, Harrison, and Jeremy Stein, 1999, A unified theory of underreaction, momentum trading, and overreaction in asset markets, *Journal of Finance* LIV, 2143–2184.

Illeditsch, Philipp Karl, 2011, Ambiguous information, portfolio inertia, and excess volatility, *Journal of Finance* LXVI, 2213–2248.

Jegadeesh, Narasimhan, and Sheridan Titman, 2011, Momentum, *Annual Review of Financial Economics* 3, 493–509.

Ju, Nengjiu, and Jianjun Miao, 2012, Ambiguity, learning, and asset returns, *Econometrica* 80, 559–591.

Keim, Donald B., and Robert F. Stambaugh, 1986, Prediciting returns in the stock and bond markets, *Journal of Financial Economics* 17, 357–390.

Kelsey, David, Roman Kozhan, and Wei Pang, 2011, Asymmetric momentum effects under uncertainty, *Review of Finance* 15, 601–631.

Klibanoff, Peter, Massimo Marinacci, and Sujoy Mukerji, 2005, A smooth model of decison making under ambiguity, *Econometrica* 73, 1849–1892.

———, 2009, Recursive smooth ambiguity preferences, *Journal of Economic Theory* 144, 930–976.

Koijen, Ralph S. J., and Stijn Van Nieuwerburgh, 2011, Predictability of returns and cash flows, *Annual Review of Financial Economics* 3, 467–491.

Kyle, A.S., 1985, Continuos auctions and insider trading, *Econometrica* 53, 1315–1336.

Lynch, Anthony W., and Sinan Tan, 2010, Multiple risky assets, transaction costs, and return predictability: Allocation rules and implications for u.s. investors, *Journal of Financial and Qunatitative Analysis* 45, 1015–1053.

———, 2011, Explaining the magnitude of liquidity premia: The roles of return predictability. wleath shocks, and state-dependent transaction costs, *Journal of Financial* LXVI, 1329–1368.

Maccheroni, Fabio, Massimo Marinacci, and Aldo Rustichini, 2006, Ambiguity aversion, robustness, and the variational representation of preferences, *Econometrica* 74, 1447–1498.

Mele, Antonio, and Francesco Sangiorgi, 2011, Uncertainty, information acquisition and price swings in asset markets, Working paper.

Mondria, Jordi, 2010, Portfolio choice, attention allocation, and price comovement, *Journal of Economic Theory* 145, 1837–1864.

——— , and Climent Quintana-Domeque, 2012, Financial contagion and attention allocation, Forthcoming at Economic Journal.

Moskowitz, Tobias J., Yao Hua Ooi, and Lasse Heje Pedersen, 2012, Time series momentum, *Journal of Financial Economics* 104, 228–250.

Nau, Robert F., 2006, Uncertainty aversion with second-order utilities and probabilities, *Management Science* 52, 136–145.

Nieuwerburgh, Stijn Van, and Laura Veldkamp, 2010, Information acquisition and under-diversification, *Review of Economic Studies* 77, 779–805.

Ozsoylev, Han, and Jan Werner, 2011, Liquidity and asset prices in rational expectations equilibrium with ambiguous information, *Economic Theory* 48, 269–291.

Peng, Lin, and Wei Xiong, 2006, Investor attention, overconfidence, and category learning, *Journal of Financial Economics* 80, 563–602.

Ravanelli, Claudia, and Gregor Svindland, 2014, Ambiguity aversion in standard and extended ellsberg frameworks: alpha-maxmin versus maxmin preferences, Working Paper.

Sims, Christopher A., 2003, Implications of rational inattention, *Journal of Monetary Economics* 50, 665–690.

——— , 2010, Rational inattention and monetary economics, in Benjamin M. Friedman, and Michael Woodford, ed.: *Handbook of Monetary Economics* . , vol. 3 (Elsevier).

Strzalecki, Tomasz, 2011, Axiomatic foundations of multiplier preferences, *Econometrica* 79, 47–73.

Tallon, Jean-Marc, 1998, Asymmetric information, nonadditive expected utility, and the information revealed by prices: an example, *International Economic Review* 39, 329–342.

Uppal, Raman, and Tan Wang, 2003, Model misspecification and underdiversification, *Journal of Finance* LVIII, 2465–2486.

Veldkamp, Laura L., 2011, *Information Choice in Macroeconomics and Finance* (Princeton University Press).

Wachter, Jessica, 2012, Asset allocation, *Annual Review of Financial Economics* 2, 175–206.

Wakai, Katsutoshi, 2007, Aggregation under homogeneous ambiguity: A two-fund separation result, *Economic Theory* 30, 363–372.

Xia, Y., 2001, Learning about predictability: The effects of parameter uncertainty on dynamic asset allocation, *Journal of Finance* LVI, 205–246.

Zhang, X. Frank, 2006, Information uncertainty and stock returns, *The Journal of Finance* 61, 105–137.

A Appendix: Proofs and Additional Results

We first state and prove two results that are of independent interest and will also be useful in proofs of the main results. The first result (Proposition 7) determines the certainty equivalent of the MEU investor and the second result (Theorem 3) determines the MEU optimal portfolio as a function of price.

Proposition 7 (Preferences)**.** *Let $\hat{\theta}_a \equiv -s/(\gamma R_a \sigma_d)$ and $\hat{\theta}_b \equiv -s/(\gamma R_b \sigma_d)$. The certainty equivalent of an investor with risk aversion γ and ambiguity aversion described by $[R_a, R_b]$ who has recieved signal s is*

$$CE(\theta) = \begin{cases} \mathrm{E}_{R_a}\left[\tilde{w} \mid \tilde{s} = s\right] - \frac{1}{2}\gamma \mathrm{Var}_{R_a}\left[\tilde{w} \mid \tilde{s} = s\right] & \text{if} \quad \theta \leq \min\left(\hat{\theta}_a, 0\right) \\ \mathrm{E}\left[\tilde{w}\right] - \frac{1}{2}\gamma \mathrm{Var}\left[\tilde{w}\right] - \frac{s^2}{2\gamma} & \text{if} \quad \min\left(\hat{\theta}_a, 0\right) < \theta \leq \min\left(\hat{\theta}_b, 0\right) \\ \mathrm{E}_{R_b}\left[\tilde{w} \mid \tilde{s} = s\right] - \frac{1}{2}\gamma \mathrm{Var}_{R_b}\left[\tilde{w} \mid \tilde{s} = s\right] & \text{if} \quad \min\left(\hat{\theta}_b, 0\right) < \theta \leq \max\left(\hat{\theta}_b, 0\right) \\ \mathrm{E}\left[\tilde{w}\right] - \frac{1}{2}\gamma \mathrm{Var}\left[\tilde{w}\right] - \frac{s^2}{2\gamma} & \text{if} \quad \max\left(\hat{\theta}_b, 0\right) < \theta \leq \max\left(\hat{\theta}_a, 0\right) \\ \mathrm{E}_{R_a}\left[\tilde{w} \mid \tilde{s} = s\right] - \frac{1}{2}\gamma \mathrm{Var}_{R_a}\left[\tilde{w} \mid \tilde{s} = s\right] & \text{if} \quad \theta > \max\left(\hat{\theta}_a, 0\right). \end{cases} \tag{37}$$

The certainty equivalent CE(θ) is a continuous and concave function of the stock demand θ. Moreover, it is continuously differentiable except for the portfolio $\theta = 0$ if $s \neq 0$.

Proof of Proposition 7. The certainty equivalent CE(θ) of the ambiguity averse MEU investor satisfies

$$\mathrm{CE}(\theta) = \min_{R \in [R_a, R_b]} \overline{\mathrm{CE}}(\theta, R). \tag{38}$$

Note that

$$\frac{\partial \overline{\mathrm{CE}}(\theta, R)}{\partial R} = \theta \sigma_d s + \gamma \theta^2 \sigma_d^2 R. \tag{39}$$

Consider three cases, (i) $s = 0$, (ii) $s > 0$, and (iii) $s < 0$.

(i) $s = 0 \Leftrightarrow \hat{\theta}_a = \hat{\theta}_b = 0$.

Then $\frac{\partial \overline{\mathrm{CE}}(\theta,R)}{\partial R} > 0$ for all $R \in [R_a, R_b]$. Thus the minimum of $\overline{\mathrm{CE}}(\theta, R)$ is attained at R_a and hence,

$$\mathrm{CE}(\theta) = \min_{R \in [R_a, R_b]} \overline{\mathrm{CE}}(\theta, R) = \overline{\mathrm{CE}}(\theta, R_a) \text{ for all } \theta \in \mathbb{R}. \tag{40}$$

$\overline{\mathrm{CE}}(\theta, R_a)$ is continuously differentiable and concave in θ for all $\theta \in \mathbb{R}$ and thus so is CE(θ).

(ii) $s > 0 \Leftrightarrow \hat{\theta}_a < \hat{\theta}_b < 0$.

Suppose $\theta < \hat{\theta}_a < 0$ or $\theta > 0$. Then $\frac{\partial \overline{\mathrm{CE}}(\theta,R)}{\partial R} > 0$ for all $R \in [R_a, R_b]$. Thus, the minimum of $\overline{\mathrm{CE}}(\theta, R)$ is attained at R_a.

Suppose $\hat{\theta}_b < \theta < 0$. Then $\frac{\partial \overline{\mathrm{CE}}(\theta,R)}{\partial R} < 0$ for all $R \in [R_a, R_b]$. Thus, the minimum of $\overline{\mathrm{CE}}(\theta, R)$ is attained at R_b.

Suppose $\hat{\theta}_a \leq \theta \leq \hat{\theta}_b$. Then, since $\frac{\partial^2 \overline{\mathrm{CE}}(\theta,R)}{\partial R^2} > 0$, the minimum is attained when $\frac{\partial \overline{\mathrm{CE}}(\theta,R)}{\partial R} = 0$, i.e. $R^*(\theta) \equiv \underset{R \in [R_a, R_b]}{\operatorname{argmin}} \overline{\mathrm{CE}}(\theta, R) = \frac{-s}{\gamma \sigma_d \theta}$. Note that $R^* \in [R_a, R_b]$ when $\hat{\theta}_a \leq \theta \leq \hat{\theta}_b < 0$ and that

$$\overline{\mathrm{CE}}(\theta, R^*) = \mathrm{E}[\tilde{w}] - \frac{1}{2}\gamma \mathrm{Var}[\tilde{w}] - \frac{s^2}{2\gamma} = \overline{\mathrm{CE}}(\theta, 0) - \frac{s^2}{2\gamma}. \tag{41}$$

Using the above, we get

$$\text{CE}(\theta) = \begin{cases} \overline{\text{CE}}(\theta, R_a) & \text{if } \theta \leq \hat{\theta}_a \\ \overline{\text{CE}}(\theta, 0) - \frac{s^2}{2\gamma} & \text{if } \hat{\theta}_a < \theta \leq \hat{\theta}_b \\ \overline{\text{CE}}(\theta, R_b) & \text{if } \hat{\theta}_b < \theta \leq 0 \\ \overline{\text{CE}}(\theta, R_a) & \text{if } 0 < \theta. \end{cases} \tag{42}$$

as desired.

$\text{CE}(\theta)$ is continuous for all $\theta \in \mathbb{R}$ and $R \in [R_a, R_b]$ and $\overline{\text{CE}}(0, R_a) = \overline{\text{CE}}(0, R_b)$. $\overline{\text{CE}}(\theta, R)$ is continuously differentiable for all $\theta \in \mathbb{R}$ and $R \in [R_a, R_b]$ and the $\frac{\partial^2 \overline{\text{CE}}(\theta,R)}{\partial \theta^2} \leq 0$ for all $\theta \in \mathbb{R}$ and $R \in [R_a, R_b]$. Thus, for any $\theta \neq 0$ there is an open neighborhood for such $\text{CE}(\theta)$ is continuously differentiable and $\frac{\partial^2 \text{CE}(\theta)}{\partial \theta^2}$ exists and is non-positive.

To verify concavity and non-differentiability of $\text{CE}(\theta)$ at $\theta = 0$, we calculate the left derivative $\text{CE}'^{-}(\theta)$ and the right derivative $\text{CE}'^{+}(\theta)$ at $\theta = 0$.

$$\text{CE}'^{-}(0) \equiv \lim_{\theta \uparrow 0} \frac{\partial \text{CE}(\theta)}{\partial \theta} = \bar{d} + R_b \sigma_d s - p \tag{43}$$

$$\text{CE}'^{+}(0) \equiv \lim_{\theta \downarrow 0} \frac{\partial \text{CE}(\theta)}{\partial \theta} = \bar{d} + R_a \sigma_d s - p \tag{44}$$

Thus, $\text{CE}'^{-}(0) > \text{CE}'^{+}(0)$, so $\text{CE}(\theta)$ is concave for all $\theta \in \mathbb{R}$, not differentiable at $\theta = 0$, and continuously differentiable at all $\theta \neq 0$.

(iii) $s < 0 \Leftrightarrow \hat{\theta} > \hat{\theta}_b > 0$.

Using reasoning similar to that for the above case, we get

$$\text{CE}(\theta) = \begin{cases} \overline{\text{CE}}(\theta, R_a) & \text{if } \theta \leq 0 \\ \overline{\text{CE}}(\theta, R_b) & \text{if } 0 < \theta \leq \hat{\theta}_b \\ \overline{\text{CE}}(\theta, 0) - \frac{s^2}{2\gamma} & \text{if } \hat{\theta}_b < \theta \leq \hat{\theta}_a \\ \overline{\text{CE}}(\theta, R_a) & \text{if } \hat{\theta}_a < \theta \end{cases} \tag{45}$$

and that $\text{CE}(\theta)$ is continuous and concave in $\theta \in \mathbb{R}$. Moreover, $\text{CE}(\theta)$ is continuously differentiable at all $\theta \neq 0$.

Finally, combining the above cases provides the desired expression and properties for $\text{CE}(\theta)$. □

Theorem 3 (Optimal Demand)**.** *Optimal demand at price p for an investor with risk aversion γ and ambiguity aversion described by $[R_a, R_b]$ who has recieved signal s is*

$$\theta(s,p) = \begin{cases} \overline{\theta}(s,R_a,p) & \quad\quad p \leq p_1(s) \equiv \mu(s,R_a) - \gamma\sigma^2(R_a)\max\left(\hat{\theta}_a,0\right) \\ \max\left(\overline{\theta}(s,0,p),0\right) & p_1(s) < p \leq p_2(s) \equiv \mu(s,R_b) - \gamma\sigma^2(R_b)\max\left(\hat{\theta}_b,0\right) \\ \overline{\theta}(s,R_b,p) & p_2(s) < p \leq p_3(s) \equiv \mu(s,R_b) - \gamma\sigma^2(R_b)\min\left(\hat{\theta}_b,0\right) \\ \min\left(\overline{\theta}(s,0,p),0\right) & p_3(s) < p \leq p_4(s) \equiv \mu_{R_a}(s) - \gamma\sigma^2(R_a)\min\left(\hat{\theta}_a,0\right) \\ \overline{\theta}(s,R_a,p) & \quad\quad p > p_4(s), \end{cases} \tag{46}$$

where $\mu(s,R) = \bar{d} + R\sigma_d s$ and $\sigma^2(R) \equiv \sigma_d^2(1-R^2)$.

Proof of Theorem 3. Consider three cases: (i) $s = 0$, (ii) $s > 0$, and $s < 0$. For expositional simplicity, we make the dependence on p and s explicit.

(i) $s = 0 \Leftrightarrow \hat{\theta}_a = \hat{\theta}_b = 0$, so it follows from the proof of Proposition 7 that $\mathrm{CE}(\theta) = \overline{\mathrm{CE}}(\theta, R_a)$ for all $\theta \in \mathbb{R}$. Thus, it follows that $\theta(s,p) = \overline{\theta}(s,R_a,p)$ for all $p \in \mathbb{R}$.

(ii) $s > 0 \Leftrightarrow \hat{\theta}_a < \hat{\theta}_b < 0$, so it follows from the proof of Proposition 7 that $\mathrm{CE}(\theta)$ is given by (42).

Consider five sub-cases: (a) $p \leq p_1 = \mu(s,R_a)$, (b) $p_1 < p \leq p_2 = \mu(s,R_b)$, (c) $p_2 < p \leq p_3 = \mu(s,R_b) - \gamma\sigma^2(R_b)\hat{\theta}_b$, (d) $p_3 < p \leq p_4 = \mu(s,R_a) - \gamma\sigma^2(R_a)\hat{\theta}_a$, and (e) $p_4 < p$.

(ii)(a) Suppose $p \leq p_1$. We show that $\theta(s,p) = \overline{\theta}(s,R_a,p)$. First, note that

$$\overline{\theta}(s,R_a,p) = \frac{\mu(s,R_a) - p}{\gamma\sigma^2(R_a)} \geq \frac{\mu(s,R_a) - p_1}{\gamma\sigma^2(R_a)} = 0. \tag{47}$$

Moreover, for any $\theta > 0$, $\mathrm{CE}(\theta) = \overline{\mathrm{CE}}(\theta, R_a)$ from (42). Thus, since $\mathrm{CE}(\theta)$ is concave, $\overline{\theta}(s,R_a,p)$ is the local and hence global maximizer of $\mathrm{CE}(\theta)$ for all $p \leq p_1$.

(ii)(b) Suppose $p_1 < p \leq p_2$. We show that $\theta(s,p) = 0$. First, note that since $R_a\sigma_d > 0$,

$$\overline{\theta}(s,0,p) = \frac{\bar{d} - p}{\gamma\sigma_d^2} < \frac{\bar{d} - p_1}{\gamma\sigma_d^2} \leq \frac{\mu(s,R_a) - p_1}{\gamma\sigma_d^2} = 0. \tag{48}$$

Since $\mathrm{CE}(\theta)$ is concave, it suffices to show that $\theta = 0$ is a local maximizer.

Given (42), there exists $\epsilon > 0$ such that

$$\mathrm{CE}(\theta) = \begin{cases} \overline{\mathrm{CE}}(\theta, R_b) & \text{if } -\epsilon < \theta \leq 0 \\ \overline{\mathrm{CE}}(\theta, R_a) & \text{if } 0 \leq \theta < \epsilon. \end{cases} \tag{49}$$

For $-\epsilon < \theta \leq 0$,

$$\mathrm{CE}(0) - \overline{\mathrm{CE}}(\theta, R_b) = \theta \left(p - \bar{d} - R_b\sigma_d s\right) + \frac{1}{2}\gamma(\sigma_d^2(1 - R_b^2)) \geq 0 \tag{50}$$

when $p \leq p_2$.

For $0 \leq \theta < \epsilon$,

$$\mathrm{CE}(0) - \overline{\mathrm{CE}}(\theta, R_a) = \theta \left(p - \bar{d} - R_a\sigma_d s\right) + \frac{1}{2}\gamma(\sigma_d^2(1 - R_a^2)) \geq 0 \tag{51}$$

when $p_1 \leq p$. Combining the above, shows that $\theta = 0$ is a local and hence global maximizer of $\mathrm{CE}(\theta)$ for $p_1 < p \leq p_2$.

(ii)(c) Suppose $p_2 < p \leq p_3$. We show that $\theta(s, p) = \overline{\theta}(s, R_b, p)$. First, note that

$$\overline{\theta}(s, R_b, p) = \frac{\mu(s, R_b) - p}{\gamma\sigma^2(R_b)} < \frac{\mu(s, R_b) - p_2}{\gamma\sigma^2(R_b)} = 0 \tag{52}$$

when $p_2 < p$ and that

$$\overline{\theta}(s, R_b, p) = \frac{\mu(s, R_b) - p}{\gamma\sigma^2(R_b)} \geq \frac{\mu(s, R_b) - p_3}{\gamma\sigma^2(R_b)} = \hat{\theta}_b \tag{53}$$

when $p \leq p_3$.

From (42), $\mathrm{CE}(\theta) = \overline{\mathrm{CE}}(\theta, R_b)$ when $\hat{\theta}_b < \theta \leq 0$. Thus, given concavity of $\mathrm{CE}(\theta)$, $\overline{\theta}(s, R_b, p)$ is a local and hence global maximizer of $\mathrm{CE}(\theta)$ when $p_2 < p \leq p_3$.

(ii)(d) Suppose $p_3 < p \leq p_4$. We show that $\theta(s, p) = \overline{\theta}(s, 0, p)$. First, note that since $R_a\sigma_d > 0$,

$$\overline{\theta}(s, 0, p) = \frac{\bar{d} - p}{\gamma\sigma_d^2} < \frac{\bar{d} - p_3}{\gamma\sigma_d^2} < \frac{\bar{d} - p_2}{\gamma\sigma_d^2} \leq 0. \tag{54}$$

Also, $p_3 = \mu(s, R_b) - \gamma\sigma^2(R_b)\hat{\theta}_b = \bar{d} - \gamma\sigma_d^2\hat{\theta}_b$ and $p_4 = \mu(s, R_a) - \gamma\sigma^2(R_a)\hat{\theta}_a = \bar{d} - \gamma\sigma_d^2\hat{\theta}_a$. Hence,

$$\hat{\theta}_a \leq \overline{\theta}(s, 0) < \hat{\theta}_b \tag{55}$$

when $p_3 < p \leq p_4$.

From (42), $\mathrm{CE}(\theta) = \overline{\mathrm{CE}}(\theta, 0) - \frac{s^2}{2\gamma}$ when $\hat{\theta}_a < \theta \leq \hat{\theta}_b < 0$. Thus, since $\mathrm{CE}(\theta)$ is concave, $\overline{\theta}(s, 0, p)$ is a local and hence global maximizer of $\mathrm{CE}(\theta)$ for $p_3 < p \leq p_4$.

(ii)(e) Suppose $p_4 < p$. We show that $\theta(s, p) = \overline{\theta}(s, R_a, p)$. First, note that

$$\overline{\theta}(s, R_a, p) = \frac{\mu(s, R_a) - p}{\gamma\sigma^2(R_a)} < \frac{\mu(s, R_a) - p_4}{\gamma\sigma^2(R_a)} \leq \hat{\theta}_a = 0. \tag{56}$$

Moreover, for any $\theta < \hat{\theta}_a$, $\mathrm{CE}(\theta) = \overline{\mathrm{CE}}(\theta, R_a)$ from (42). Thus, since $\mathrm{CE}(\theta)$ is concave, $\overline{\theta}(s, R_a, p)$ is the local and hence global maximizer of $\mathrm{CE}(\theta)$ for all $p > p_4$.

Using the above, we get

$$\theta(s,p) = \begin{cases} \overline{\theta}(s, R_a, p) & \text{if } p \leq p_1 \\ 0 & \text{if } p_1 < p \leq p_2 \\ \overline{\theta}(s, R_b, p) & \text{if } p_2 < p \leq p_3 \\ \overline{\theta}(s, 0, p) & \text{if } p_3 < p \leq p_4 \\ \overline{\theta}(s, R_a, p) & \text{if } p_4 < p. \end{cases} \tag{57}$$

as desired.

(iii) When $s < 0 \Leftrightarrow \hat{\theta}_a > \hat{\theta}_b > 0$, then it follows from the proof of Proposition 7 that $\mathrm{CE}(\theta)$ is given by (45). Moreover, $p_1 = \mu(s, R_a) - \gamma\sigma^2(R_a)\hat{\theta}_a$, $p_2 = \mu(s, R_b) - \gamma\sigma^2(R_b)\hat{\theta}_b$, $p_3 = \mu(s, R_b)$, and (d) $p_4 = \mu(s, R_a)$. Thus, using similar reasoning as above, we get

$$\theta(s,p) = \begin{cases} \overline{\theta}(s, R_a, p) & \text{if } p \leq p_1 \\ \overline{\theta}(s, 0, p) & \text{if } p_1 < p \leq p_2 \\ \overline{\theta}(s, R_b, p) & \text{if } p_2 < p \leq p_3 \\ 0 & \text{if } p_3 < p \leq p_4 \\ \overline{\theta}(s, R_a, p) & \text{if } p_4 < p. \end{cases} \tag{58}$$

as desired.

Combining the three cases above provides the desired expression for $\theta(s)$. □

Proof of Theorem 1. Consider three cases: (i) $\lambda = 0$, (ii) $\lambda > 0$, and (iii) $\lambda < 0$

and the expression for demand in Theorem 3. We omit the dependence on p for expositional ease.

(i) Suppose $\lambda = 0$. Then $s_1 = s_2 = s_3 = s_4 = 0$ and from Theorem 3, $\theta(s) = \overline{\theta}(s, R_a)$ if $p \leq p_1 \Leftrightarrow s \geq 0$ and if $p > p_4 \Leftrightarrow s < 0$.

(ii) $\lambda > 0$. Then $s_1 = -R_a\lambda > s_2 = -R_b\lambda > s_3 = -\frac{\lambda}{R_b} > s_4 = -\frac{\lambda}{R_a}$.

Then from Theorem 3 the following holds.

$$\theta(s) = \begin{cases} \overline{\theta}(s, R_a) & \text{if } p \leq p_1 \Leftrightarrow s \geq s_1 \\ \overline{\theta}(s, 0) & \text{if } p_1 < p \leq p_2 \Leftrightarrow s_1 > s \geq s_2 \\ \overline{\theta}(s, R_b) & \text{if } p_2 < p \leq p_3 \Leftrightarrow s_2 > s \geq s_3 \\ 0 & \text{if } p_3 < p \leq p_4 \Leftrightarrow s_4 \leq s < s_3 \\ \overline{\theta}(s, R_a) & \text{if } p > p_4 \Leftrightarrow s < s_4. \end{cases} \tag{59}$$

(iii) $\lambda < 0$. Then $s_1 = -\frac{\lambda}{R_a} > s_2 = -\frac{\lambda}{R_b} > s_3 = -R_b\lambda > s_4 = -R_a\lambda$.

Then from Theorem 3 the following holds.

$$\theta(s) = \begin{cases} \overline{\theta}(s, R_a) & \text{if } p \leq p_1 \Leftrightarrow s \geq s_1 \\ 0 & \text{if } p_1 < p \leq p_2 \Leftrightarrow s_1 > s \geq s_2 \\ \overline{\theta}(s, R_b) & \text{if } p_2 < p \leq p_3 \Leftrightarrow s_2 > s \geq s_3 \\ \theta(s, 0) & \text{if } p_3 < p \leq p_4 \Leftrightarrow s_4 \leq s < s_3 \\ \overline{\theta}(s, R_a) & \text{if } p > p_4 \Leftrightarrow s < s_4. \end{cases} \tag{60}$$

Combining the above cases provides the desired expression. $\square$

Proof of Proposition 1. Suppose $\lambda > 0$. Then optimal demand for the MEU investor is given by (59). Moreover, from the proof of Proposition 7 it follows that

$$R^*(s) = \begin{cases} R_a & \text{if } s > s_1 = -R_a\lambda \text{ or } s \leq s_4 = -\frac{\lambda}{R_a} \\ -\frac{s}{\lambda} & if - \lambda R_b < s \leq -R_a\lambda \\ R_b & \text{if } s \leq s_2 = -R_b\lambda \text{ or } s > s_3 = -\frac{\lambda}{R_b} \end{cases} \tag{61}$$

Hence, it follows that

$$\theta(s) = \begin{cases} \overline{\theta}(s, R^*(s)) & \text{if } s > -\frac{\lambda}{R_b} \text{ or } s < -\frac{\lambda}{R_a} \\ 0 & \text{if } -\frac{\lambda}{R_a} \leq s \leq -\frac{\lambda}{R_b}. \end{cases} \tag{62}$$

$\square$

Proof of Proposition 2. Let $s > -\frac{\lambda}{R_b}$. Then, $s > -\frac{\lambda}{R}$ for all $R \in [R_a, R_b]$ since $-\frac{\lambda}{R}$ is increasing in R. From this it follows that for all $R \in [R_a, R_b]$, $\frac{\bar{d}+R\sigma_d s-p}{\sigma(R)} = \lambda(s, R) > 0$ and hence $\overline{\theta}(s, R) = \frac{\lambda(s,R)}{\gamma\sigma(R)} > 0$.

Moreover, $\frac{d^2}{dR^2}\lambda(s, R) > 0$ for all $R \in [R_a, R_b]$, so $\lambda(s, R)$ is strictly convex in R. Letting R' denote the unique minimizer of $\lambda(s, R)$ over $[R_a, R_b]$, the first-order (Kuhn-Tucker) condition for the constrained minimisation yields

$$0 \leq \frac{s + \lambda R'}{(1 - R'^2)} \tag{63}$$

with equality holding if $R_a < R' < R_b$. This yields the result that $R^*(s)$ as in (61) is unique minimizer of $\lambda(s, R)$ over $[R_a, R_b]$.

An analogous argument shows that if $s < -\frac{\lambda}{R_a}$, then for all $R \in [R_a, R_b]$, $\lambda(s, R) < 0$ and hence $\overline{\theta}(s, R) < 0$. Moreover, in this case, $\frac{d^2}{dR^2}\lambda(s, R) < 0$ for all $R \in [R_a, R_b]$, so $\lambda(s, R)$ is strictly concave in R and the first order condition yields the result that $R^*(s) = R_a$ is the unique maximizer of $\lambda(s, R)$ over $[R_a, R_b]$.

$\square$

Proof of Proposition 3. Using $\theta_h(s, p)$ to denote the demand at price p for investor h who recieves signal s, it follows from Theorem 3 that

$$\theta_h(s, p) = \begin{cases} \frac{\mu(s,R_a)-p}{\gamma_h\sigma^2(R_a)} & p \leq p_1 \\ \max\left(\frac{\bar{d}-p}{\gamma_h\sigma_d^2}, 0\right) & p_1 < p \leq p_2 \\ \frac{\mu(s,R_b)-p}{\gamma_h\sigma^2(R_b)} & p_2 < p \leq p_3 \\ \min\left(\frac{\bar{d}-p}{\gamma_h\sigma_d^2}, 0\right) & p_3 < p \leq p_4 \\ \frac{\mu(s,R_a)-p}{\gamma_h\sigma^2(R_a)} & p > p_4, \end{cases} \tag{64}$$

where p_1, p_2, p_3, p_4 are as in 3 due to homogeneous ambiguity aversion $[R_a, R_b]$ across investors.

Summing up individual demands leads to aggregate demand,

$$\theta(s,p) = \sum_{h=1}^{H} \theta_h(s,p) \begin{cases} \frac{\mu(s,R_a)-p}{\gamma\sigma^2(R_a)} & p \le p_1 \\ \max\left(\frac{\bar{d}-p}{\gamma\sigma_d^2}, 0\right) & p_1 < p \le p_2 \\ \frac{\mu(s,R_b)-p}{\gamma\sigma^2(R_b)} & p_2 < p \le p_3 \\ \min\left(\frac{\bar{d}-p}{\gamma_h\sigma_d^2}, 0\right) & p_3 < p \le p_4 \\ \frac{\mu(s,R_a)-p}{\gamma\sigma^2(R_a)} & p > p_4, \end{cases} \tag{65}$$

with $\frac{1}{\gamma} = \sum_{h=1}^{H} \frac{1}{\gamma_h}$ risk tolerance, wealth $w_0 = \sum_{h=1}^{H} w_0 h$ and ambiguity aversion described by $[R_a, R_b]$.

The representative investor holds the risky asset in equilibrium and consumes the dividend. Thus, using (5) and (6) and Proposition 1 with $\theta(s,p) = 1$ for the RI yields (16) as the representative investor utility in equilibrium. Since $u' > 0$, the equilibrium utility can be computed by solving

$$\min_{R \in [R_a, R_b]} \mu(s,R) - \frac{1}{2}\gamma\sigma^2(R). \tag{66}$$

Since $\mu(s,R) - \frac{1}{2}\gamma\sigma^2(R)$ is strictly convex in R over $[R_a, R_b]$, the following first order (Kuhn-Tucker) condition is necessary and sufficient for the solution to the constrained minimization problem.

$$0 \le R(s + \sigma_s \gamma) \tag{67}$$

with equality if $R \in (R_a, R_b)$. Solving this yields (17) as desired. □

Proof of Theorem 2. We make the dependence of demand on price p explicit for expositional ease. Market clearing requires that $\theta(s,p) = 1$ since there is one unit of the risky asset in aggregate.

Consider three cases: (i) $s > -\gamma R_a \sigma_d$, (ii) $-\gamma R_b \sigma_d \le s \le -\gamma R_a \sigma_d$, and (iii) $s < -\gamma R_b \sigma_d$.

(i) Suppose $s > -\gamma R_a \sigma_d$. Then $\hat{\theta}_a < 1$. We need to verify that markets clear when $p(s) = \mu(s,R_a) - \gamma\sigma^2(R_a)$. From Theorem 1, it follows that

$$\theta(s,p) = \overline{\theta}(s,R_a,p) = \frac{\mu(s,R_a) - p(s)}{\gamma\sigma^2(R_a)} = 1 \tag{68}$$

if and only if

$$p(s) = \mu(s, R_a) - \gamma\sigma^2(R_a) \le p_1 = \mu(s, R_a) - \gamma\sigma^2(R_a) \max\left\{\hat{\theta}_a, 0\right\} \tag{69}$$

or

$$p(s) = \mu(s, R_a) - \gamma\sigma^2(R_a) > p_4 = \mu(s, R_a) - \gamma\sigma^2(R_a) \min\left\{\hat{\theta}_a, 0\right\}. \tag{70}$$

Since $\hat{\theta}_a < 1$, $p(s) \le p_1$ and the result follows.

(ii) Suppose $-\gamma R_b \sigma_d \le s \le -\gamma R_a \sigma_d$. Then $\hat{\theta}_b \le 1 \le \hat{\theta}_a$. We need to verify that markets clear when $p(s) = \bar{d} - \gamma\sigma_d^2$. From Theorem 1, it follows that

$$\theta(s, p) = \overline{\theta}(s, 0, p) = \frac{\bar{d} - p(s)}{\gamma\sigma_d^2} = 1 \tag{71}$$

if and only if

$$p(s) = \bar{d} - \gamma\sigma_d^2 > p_1 = \mu(s, R_a) - \gamma\sigma^2(R_a) \max\left\{\hat{\theta}_a, 0\right\} \tag{72}$$

and

$$p(s) = \bar{d} - \gamma\sigma_d^2 \le p_2 = \mu(s, R_b) - \gamma\sigma^2(R_b) \max\left\{\hat{\theta}_b, 0\right\}. \tag{73}$$

Since $\hat{\theta}_a \ge 1$ and $\mu(s, R_a) - \gamma\sigma^2(R_a)\hat{\theta}_a = \bar{d} - \gamma\sigma_d^2\hat{\theta}_a$, we have $p(s) > p_1$.

If $\hat{\theta}_b \le 0$, then $s \ge 0$. So, $p(s) = \bar{d} - \gamma\sigma_d^2 \le \bar{d} + R_b\sigma_d s = \mu(s, R_b) = p_2$. If $0 < \hat{\theta}_b$, then since $\hat{\theta}_b \le 1$ we $p(s) = \bar{d} - \gamma\sigma_d^2 \le \bar{d} - \gamma\sigma_d^2\hat{\theta}_b = \mu(s, R_b) - \gamma\sigma^2(R_b)\hat{\theta}_b = p_2$. So, $p_1 < p(s) \le p_2$

(iii) Suppose $s < -\gamma R_b \sigma_d$. Then $\hat{\theta}_b > 1$. We need to verify that markets clear when $p(s) = \mu(s, R_b) - \gamma\sigma^2(R_b)$. From Theorem 1, it follows that

$$\theta(s, p) = \overline{\theta}(s, R_b, p) = \frac{\mu(s, R_b) - p(s)}{\gamma\sigma^2(R_b)} = 1 \tag{74}$$

if and only if

$$p(s) = \mu(s, R_b) - \gamma\sigma^2(R_b) > p_2 = \mu(s, R_b) - \gamma\sigma^2(R_b) \max\left\{\hat{\theta}_b, 0\right\} \tag{75}$$

and

$$p(s) = \mu(s, R_b) - \gamma\sigma^2(R_b) \le p_3 = \mu(s, R_b) - \gamma\sigma^2(R_b) \min\left\{\hat{\theta}_b, 0\right\}. \tag{76}$$

Since $\hat{\theta}_b > 1$, $p_2 < p(s) \leq p_3$ and the result follows.

Combining the above cases provides the desired result. $\square$

Proof of Proposition 4. Using $\theta_h(s)$ to denote the demand for investor h, it follows from Theorem 3 that

$$\theta_h(s,p) = \begin{cases} \frac{\mu(s,R_{ah})-p}{\gamma_h \sigma^2(R_{ah})} & p \leq p_{1h} \equiv \mu(s,R_{ah}) - \gamma_h \sigma^2(R_{ah}) \max\left(\hat{\theta}_{ah}, 0\right) \\ \max\left(\frac{\bar{d}-p}{\gamma_h \sigma_d^2}, 0\right) & p_{1h} < p \leq p_{2h} \equiv \mu(s,R_{bh}) - \gamma_h \sigma^2(R_{bh}) \max\left(\hat{\theta}_{bh}, 0\right) \\ \frac{\mu_{bh}-p}{\gamma_h \sigma^2(R_{bh})} & p_{2h} < p \leq p_{3h} \equiv \mu(s,R_{bh}) - \gamma_h \sigma^2(R_{bh}) \min\left(\hat{\theta}_{bh}, 0\right) \\ \min\left(\frac{\bar{d}-p}{\gamma_h \sigma_d^2}, 0\right) & p_{3h} < p \leq p_{4h} \equiv \mu(s,R_{ah}) - \gamma_h \sigma^2(R_{ah}) \min\left(\hat{\theta}_{bh}, 0\right) \\ \frac{\mu(s,R_{ah})-p}{\gamma_h \sigma^2(R_{ah})} & p > p_{4h}, \end{cases} \tag{77}$$

where $\hat{\theta}_{ah} \equiv -s/(\gamma_h R_{ah} \sigma_d)$ and $\hat{\theta}_{bh} \equiv -s/(\gamma_h R_{bh} \sigma_d)$.

We first show that there exists an equilibrium. Individual demand given in equation (77) is continuous and non-increasing in p with $\lim_{p \to -\infty} \theta_h(s,p) = \infty$ and $\lim_{p \to \infty} \theta_h(s,p) = -\infty$ for all $h \in \{1, \dots, H\}$. Hence, aggregate demand $\theta(s,p) = \sum_{h=1}^{H} \theta_h(s,p)$ is continuous and non-increasing in p with $\lim_{p \to -\infty} \theta(s,p) = \infty$ and $\lim_{p \to \infty} \theta(s,p) = -\infty$. Hence, there exists an equilibrium because the market clearing condition $\theta(s,p) - 1 = 0$ has always a solution.

We next determine the equilibrium stock price $p(s)$ for all $s \in [-\gamma \sigma_d R_b, -\gamma \sigma_d R_a]$. By assumption we have that $R_a = \max\{R_{a1}, \dots, R_{ah}\}$ and $R_b = \min\{R_{b1}, \dots, R_{bh}\}$. Hence, since $s < 0$,

$$p_1(s) \equiv \max_{h \in \{1,\dots,H\}} p_{1h}(s) = \max_{h \in \{1,\dots,H\}} \left\{\bar{d} + \frac{\sigma_d}{R_{ah}} s\right\} = \bar{d} + \frac{\sigma_d}{R_a} s \tag{78}$$

$$p_2(s) \equiv \min_{h \in \{1,\dots,H\}} p_{2h}(s) = \min_{h \in \{1,\dots,H\}} \left\{\bar{d} + \frac{\sigma_d}{R_{bh}} s\right\} = \bar{d} + \frac{\sigma_d}{R_b} s. \tag{79}$$

We have that $R_b \geq R_a$ and thus (i) $[\hat{s}_b, \hat{s}_a] \neq \emptyset$ and (ii) $p_2(s) \geq p_1(s)$ for all $s \in [\hat{s}_b, \hat{s}_a]$.

It follows from equations (77)-(79) that

$$\theta_h(s,p) = \frac{\bar{d} - p}{\gamma_h \sigma_d^2} \qquad \forall\, p_1(s) \leq p \leq p_2(s), \quad \text{and} \quad \forall h \in \{1, \dots, H\}. \tag{80}$$

Summing over all investors leads to

$$\theta(s,p) = \sum_{h=1}^{H} \theta_h(s,p) = \frac{\bar{d}-p}{\sigma_d^2} \sum_{h=1}^{H} \frac{1}{\gamma_h} = \frac{\bar{d}-p}{\gamma \sigma_d^2} \qquad \forall\, p_1(s) \leq p \leq p_2(s).$$

Imposing the market clearing condition $\theta(s,p)$=1 leads to the price $p(s) = \bar{d} - \gamma\sigma_d^2$. Finally, the desired result follows from noting that $p_1(s) \leq \bar{d} - \gamma\sigma_d^2 \leq p_2(s)$ if and only if $-\gamma\sigma_d R_b \leq s \leq -\gamma\sigma_d R_a$. □

Proof of Proposition 5. For all $\theta \in \mathcal{R}$, the function $\xi(\theta, s, R)$,

$$\xi(\theta, s, R) = \frac{e^{-\gamma(1+\alpha)\left(\sigma_d s R\,\theta + \frac{1}{2}\gamma\sigma_d^2 R^2\theta^2\right)}}{\mathrm{E}_{\mathbb{P}}\left[e^{-\gamma(1+\alpha)\left(\sigma_d s \tilde{R}\,\theta + \frac{1}{2}\gamma\sigma_d^2 \tilde{R}^2\theta^2\right)}\right]} \tag{81}$$

is non-negative and $\mathrm{E}_{\mathbb{P}}\left[\xi(\theta, s, \tilde{R})\right] = 1$ hence $d\mathbb{Q}\,(s, R, \theta)$ as defined in (25) is a conditional probability distribution.

The utility $U(\theta)$ of a KMM investor from holding portfolio θ is as given in (24) with $u(w) = -e^{-\gamma w}, \gamma > 0$ and $\phi(u) = -\frac{1}{1+\alpha}\,(-u)^{1+\alpha}, \alpha \geq 0$. The first-order condition for this investor's optimal portfolio is

$$0 = U'(\theta) = \mathrm{E}_{\mathbb{P}}\left[\phi'(u(\overline{\mathrm{CE}}(\theta, \tilde{R})))u'(\overline{\mathrm{CE}}(\theta, \tilde{R}))\left(\lambda(s, \tilde{R}) + \gamma\theta\sigma(\tilde{R})\right)\sigma(\tilde{R})\right]. \tag{82}$$

The second derivative of (24) with respect to θ is

$$U''(\theta) = \mathrm{E}_{\mathbb{P}}\left[\left(\lambda(s, \tilde{R}) + \gamma\theta\sigma(\tilde{R})\right)^2 \sigma^2(\tilde{R})\left(\phi''(\cdot)u'(\cdot) + \phi'(\cdot)u''(\cdot)\right) - \phi'(\cdot)u'(\cdot)\gamma\sigma^2(\tilde{R})\right] < 0 \tag{83}$$

given $u' > 0$, $u'' < 0$ and $\phi' > 0, \phi'' \leq 0$.

Hence, KMM investor utility is strictly concave in θ and optimal portfolio is unique. Solving for $\theta(s)$ using the first-order condition and $\phi'(u) = (-u)^{\alpha}$ and $u'(w) = \gamma e^{-\gamma w}$ yields

$$\begin{aligned} \theta(s) &= \frac{\mathrm{E}_{\mathbb{P}}\left[\phi'(\cdot)u'(\cdot)\left(\lambda(s,\tilde{R})\sigma(\tilde{R})\right)\right]}{\gamma\mathrm{E}_{\mathbb{P}}\left[\sigma^2(\tilde{R})\right]} \\ &= \frac{(\bar{d}-p)+\sigma_d s \mathrm{E}_{\mathbb{Q}(R;s,\theta(s))}\left[\tilde{R}|\tilde{s}=s\right]}{\gamma\sigma_d^2\left(1-\mathrm{E}_{\mathbb{Q}}\left[(\tilde{R}^2)\right]\right)} \\ &= \frac{\lambda^{\mathbb{Q}(s,\theta(s))}}{\gamma\sigma^{\mathbb{Q}}(s,\theta(s))} \end{aligned} \tag{84}$$

as desired.

□

Proof of Proposition 6. Setting $\theta(s) = 1$ in (25) and using $\mathbb{Q}(R, s)$ to denote $\mathbb{Q}(R; s, 1)$ yields the distribution of $\tilde{R}$ condtional on $\tilde{s} = s$ in (30).

The representative investor holds the risky asset and consumes the dividend in equilibrium. Setting $\theta(s) = 1$ in the first-order condition (82) yields,

$$\lambda^{\mathbb{Q}(s,1)} = \gamma \sigma^{\mathbb{Q}}(s, 1). \tag{85}$$

Using the expressions for $\lambda^{\mathbb{Q}}(s, \theta(s))$ and $\sigma^{\mathbb{Q}}(s, \theta(s))$ from Proposition 5 with $\theta(s) = 1$ and $\mathbb{Q}(R, s)$ for $\mathbb{Q}(R; s, 1)$, solving for $p(s)$ yields (29) as the unique equilibrium price. □

The following results provide the size and probability of information inertia in portfolio choice (Proposition 8) and in equilibrium prices (Proposition 9) that are used for the calculations in Section V.

Proposition 8. *The size of the signal region for which risky portfolios do not react to news is $(R_b - R_a)\lambda_e$. The probability of investors exhibiting information inertia for risky asset positions conditional on $\tilde{s} \leq x$ is*

$$\frac{1}{\Phi(x)} \cdot \begin{cases} 0 & \text{if } \; x < -\lambda_e R_b \\ \Phi(\lambda_e R_b) - \Phi(-x) & \text{if } \; -\lambda_e R_b \leq x \leq -\lambda_e R_a \\ \Phi(\lambda_e R_b) - \Phi(\lambda_e R_a) & \text{if } \; x > -\lambda_e R_a, \end{cases} \tag{86}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal distributed variable.

The size of the signal region for which the risk-free portfolio does not react to news is $\frac{R_b - R_a}{R_b R_a}\lambda_e$. The probability of investors exhibiting information inertia when holding the risk-free portfolio conditional on $\tilde{s} \leq x$ is

$$\frac{1}{\Phi(x)} \cdot \begin{cases} 0 & \text{if } \; x < -\frac{\lambda_e}{R_b} \\ \Phi\left(\frac{\lambda_e}{R_b}\right) - \Phi(-x) & \text{if } \; -\frac{\lambda_e}{R_b} \leq x \leq -\frac{\lambda_e}{R_a} \\ \Phi\left(\frac{\lambda_e}{R_a}\right) - \Phi\left(\frac{\lambda_e}{R_b}\right) & \text{if } \; x > -\frac{\lambda_e}{R_a}. \end{cases} \tag{87}$$

Proof of Proposition 8. The expression for MEU investor optimal portfolio given in sub-section A follows from reasoning similar to that for (8). That is, reasoning similar to that for (59) (resp. (60)) in the proof of Theorem 1, yields that MEU investors are long (resp. short) in the stock, but demand does not react to news, i.e.

$$\theta(s) = \overline{\theta}(s, 0) \geq 0(\text{ resp. } \leq 0), \tag{88}$$

if and only if $\lambda_e > 0$ (resp. $\lambda_e < 0$) and $s_1 > s \geq s_2$, where $s_1 = -R_a\lambda_e$ and $s_2 = -R_a\lambda_e$ (resp. $s_4 \leq s < s_3$, where $s_4 = -R_a\lambda_e$ and $s_3 = -R_b\lambda_e$).

Hence, the size of the signal region for which risky portfolios do not react to news is $(R_b - R_a)\lambda_e$. Moreover, $\tilde{s} \sim \mathrm{N}(0, 1)$, so it follows that the probability of investors exhibiting information inertia for a risky asset position conditional on $\tilde{s} \leq x$ is as given in (86).

Moreover, reasoning similar to that for (59) (resp. (60)) in the proof of Theorem 1, yields that MEU investors hold the riskless portfolio, but demand does not react to news, i.e.

$$\theta(s) = 0, \tag{89}$$

if and only if $\lambda_e > 0$ (resp. $\lambda_e < 0$) and $s_4 \leq s < s_3$, where $s_4 = -\frac{\lambda_e}{R_a}$ and $s_3 = -\frac{\lambda_e}{R_b}$ (resp. $s_1 > s \geq s_2$, where $s_1 = -\frac{\lambda_e}{R_a}$ and $s_2 = -\frac{\lambda_e}{R_a}$).

Hence, the size of the signal region for which riskless portfolios does not react to news $\frac{R_b - R_a}{R_b R_a}\lambda_e$. Moreover, $\tilde{s} \sim \mathrm{N}(0, 1)$, so it follows that probability of investors exhibiting information inertia for a risky asset position conditional on $\tilde{s} \leq x$ is as given in (87). □

Proposition 9. *The size of the signal region for which the stock price does not react to news is $\gamma\sigma_g(R_b - R_a)$. The probability of the equilibrium price exhibiting information inertia conditional on $\tilde{s} \leq x$*

$$= \begin{cases} 0 & \text{if } \; x < -\gamma R_b \sigma_g \\ \frac{\Phi(x) - \Phi(-\gamma R_b \sigma_g)}{\Phi(x)} & \text{if } \; -\gamma R_b \sigma_g \leq x \leq -\gamma R_a \sigma_g \\ \frac{\Phi(-\gamma R_a \sigma_g) - \Phi(-\gamma R_b \sigma_g)}{\Phi(x)} & \text{if } \; x > -\gamma R_a \sigma_g, \end{cases} \tag{90}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal distributed variable.

Proof of Proposition 9. Since the representative investor holds the stock in equilibriun ($\theta(s) = 1$) and consumes the dividend, the expression for equilibrium expected excess return is as given in (36) using reasoning similar to that for Proposition 3. It follows that the size of the signal region for which the stock price does not react to news is $\gamma\sigma_g(R_b - R_a)$. Moreover, $\tilde{s} \sim \mathrm{N}(0, 1)$, so the probability of the equilibrium price exhibiting information inertia conditional on $\tilde{s} \leq x$ is as given in (90). $\square$