# Three Essays on Search: Optimal Policy with Heterogeneous Workers

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# DEDICATION

To the soul of my Father, the first to teach me.

To my beloved Mother and Brother, for their prayers for me.

To my Husband, for care and support all the time.

And to my Child with hope for bright future.

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# ABSTRACT

The present thesis contributes to the theoretical analysis of the human capital investment and participation decision of heterogeneous workers in the search and matching framework. Its aim is to characterise the equilibrium and to identify the efficiency. The first chapter of this thesis contributes to study equilibrium search and matching to consider the participation decision of heterogeneous workers who have different inherent ability level. The productivity investment decision is endogenous and wages are determined by the Nash bargain among participants. In steady-state equilibrium investment decision reveals the hold-up problem. Given overall labor market condition, equilibrium is determined by free entry condition, optimal productivity investment decision, the participation constraint plus the steady state conditions. In this model I also show that heterogeneity is not the cause of multiplicity. The second chapter utilises the previous analysis to identify those government policies that can achieve efficiency in an economy. I show that the number of labor market participants and job creation and productivity investment are inefficient owing to externalities resulting from participation decision, productivity investment and market tightness condition. Therefore, in my first best policy scenario, I find the government should subsidise training, tax labour market participation and subsidise job creation. The last chapter considers an

integrated framework where social security benefits are chosen to induce optimal search by unemployed workers given the income tax structure imposed by the government. As tax policy distorts the willingness of workers to find employment, it provides a simple rule which identifies the link between optimal social security benefits paid and the tax system. Specifically in the case of a universal, linear income tax scheme, optimal social security benefits should be paid at a flat rate; i.e. all receive the same benefit level regardless of earnings which is the main contribution of the paper.

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# Chapter 1

# Productivity Investment and Labor Force Participation in Search Equilibrium

## 1.1 Introduction

In this paper, I construct a theoretical model that focuses on the role of productivity investment incentives in the standard search and matching model. The model follows directly from insights presented by Lockwood (1986) and is an extension of the basic matching model developed by Pissarides (1990) where individuals need to invest in their productivity before participating in the labor market.

Many economists view the skills of the labor force as a prime contributor to economic performance. Therefore, not surprisingly policy makers are often interested in issues of worker training. For instance, training of less skilled workers was one of the principle policy initiatives of the first Clinton administration and the labour government in Britain has similarly made training a key policy issue.

A number of OECD countries have experienced increased returns in skills investment over the last years. Many policy makers believe that low educated workers can also benefit from the changes in the demand for skills if they invest more in their education. Rosenstein-Rodan (1943), not only pointed out the importance of market demand , but also of skills, and noted that training of workers was a prerequisite for industrialisation, though unlikely to happen.

While it is important to understand the optimisation problem of human capital investment of an agent, it is necessary to pay attention to its subsequent interaction with labor market. Beside considering heterogeneity of workers and human capital investment and the cost related to this investment, we must also analyse how the resulting change in the supply of heterogeneous workers affects the labour market equilibrium. The interdependence becomes more complicated when we recognise there is friction in the labour market. We live in a world where information is imperfect and costly. Individual decision-making at entry (or exit) point of the market, the choice of lowest acceptable productivity and the choice of search intensity margins may be socially inefficient. What is common in all these margins, as Pissarides (1983) makes clear, is that they are all due external factors that work through the matching technology. When individuals choose to enter a market, accept a match or search more intensively they ignore the impact of their decisions on the matching probabilities of other agents in the market. Lockwood (1986) assumes heterogeneous workers in their inherent skills to show the other external effect that work through match acceptance probabilities rather than matching probabilities. In particular, he shows that the presence of high-skill workers lowers the equilibrium acceptance probabilities of low skill workers below what is socially desirable.

An aggregate increase in return to education is a standard property of matching models. With human capital investment, a more productive workforce raises the labour demand. However a tighter labour market increases the incentives to schooling. This mechanism can give rise to multiple equilibria (see e.g. Laing et al.,1995; Burdett and Smith (2002)). They argue that if workers' decision to invest in education and firms' job creation decisions interact positively, then the economy can get stuck in a bad equilibrium. As a result the human capital investment will be insufficient and also unemployment will be high. On the other hand Acemoglu (1996), argue that frictional unemployment can create a hold up problem whereby workers are generally paid a share less than their marginal productivity. Charlot et al.(2005) show that equilibrium is unique in their framework since the productivity effect of schooling dominates the wage effect. Clearly the presence of matching unemployment is a common explanation of weakness of incentives for education investment.

Since Becker (1964), provided the labour market is perfectly competitive, the market outcome is socially optimal. However, in the presence of unemployment, models analysing matching models with ex post Nash bargaining and ex ante costly human capital investment Laing et al.(1995) and Acemoglu (1996), workers under-invest in education which is the result of hold up phenomenon. Clearly search frictions play an important role in that. Though Cole et al.(2001) show that in the frictionless environment with two sided investment there can also be over investment.

My argument is based on workers' heterogeneity and self selection in education. The introduction of heterogeneity makes the model more realistic and in addition allows one to gain insights into the worker behaviour that do not follow from models of homogeneous labor market. When workers differ according to the ability level and the cost of productivity level that he or she is going to attain, the worker must decide to invest in his/her productivity and then participate in the labor market. This is an integral part of participation decision.

It is well established in the literature that a core topic in labor economics is 'self-selection'. The starting point of this topic in economics is Roy's (1951) "Thoughts on the Distribution of Earnings". Self-selection means in theory, that rational individuals make optimising decisions about what markets to participate in- job, education, crime, etc. In the present paper heterogeneity of the labor force in their abilities and education cost give rise to self-selection. Where the cost of productivity investment is not constant and it depends on ability of the worker and also the amount of investment on productivity.

Alternative specifications of the matching process have been considered in the literature: search may be undirected (see, eg. Acemoglu, 1999 ; Albercht and Vroman, 2002) or skilled workers may poach on unskilled jobs (see, e.g. Gautier, 2002). Charlot and Decreuse (2005), consider two separate matching sectors where educated workers direct their search towards high productivity occupations. Adopting those different specifications would, of course, alter our results.

My argument also is related to intra/infra marginal decision. And individual decisions are characterised into two classes: intramarginal decision of resource allocation and inframarginal decision of economic organisation. Intramarginal decisions involve the extent to which resources are allocated and inframarginal decisions are about what activities to engage. Considering the two types of decisions, I analyse a theoretical model that focuses on equilibrium incentives for productivity investment in matching framework with one-sided heterogeneity. There are two risk neutral groups: workers and employers. There are two types of workers;

low ability and high ability workers while employers are identical. Distribution of abilities is exogenous.

This paper argues human capital investment decision of heterogeneous workers in equilibrium search framework. I further demonstrate how it can be used to illustrate efficiency in the next chapter. The technology that I assume is such that workers invest on their education to achieve productivity and then search for a job. Of course education is costly, those who invest in education and search for job are participant in the labor market. In this paper, the choice of investment in human capital is intramarginal decision since it involves deciding the quantity of resources devoted to acquire human capital. Once he/she has chosen his investment decision in human capital, he then searches for the job.

The standard search model features exogenous labor supply-i.e., a fixed size of the labor force. Rather than consider flows between search unemployment and employment of a fixed labor force, I examine the participation decisions of worker. If labor force participation is fully endogenous, just as is labor demand in the form of vacancy creation, then another condition which I name it labor force participation constraint impinges the basic framework equations. Therefore workers reach different participation decisions on the basis of comparison between labor market and non market returns. Here I consider a labor force participation decision, a margin which is absent in most of models of labor market. This paper exploits the basic insights of above models; i.e. self selection at the individual level alters the equilibrium composition of groups. My contribution to this literature is then to highlight the equilibrium characteristics of heterogeneous workers with endogenous productivity investment and free entry of firms which drives job creation.

I use a matching process in the spirit of Pissarides (1990) with a Nash bargaining approach to wage-setting. Given overall labor market condition, equilibrium is determined by free entry condition, optimal productivity investment decision, the participation constraint plus the steady state conditions. Equilibrium can take one of the following forms. The first type of equilibrium is one in which it is beneficial for both types of workers to be active in the labor market. In the second type of equilibrium, there is only the willingness of high ability workers to invest in their productivities and search for job. Third, for small changes in the economic environment multiple equilibria can exist. Finally I show that heterogeneity is not the cause of multiplicity in this model.

I now turn in the next section to the presentation of my model. This is followed, in Section 3, by an analysis of reduce form free entry condition and the equilibrium's definition and existence. In section 4, I extend the model to the two types case of the workers and illustrate the equilibrium. Finally, in section 5, I summarise my results and conclude.

## 1.2 The Model

The model is an extension of the Lockwood (1986), Pissarides (2000) matching framework. The economy is composed of two risk-neutral groups: workers and firms. All firms are identical whose number is endogenously determined by a standard free entry condition but workers differ in their abilities. This heterogeneity in ability implies different workers invest in different productivity levels.

Throughout time is continuous and I shall only consider steady states. There is turnover - all workers die according to a Poisson process with parameter $\phi > 0$. $\phi$ also describe the inflow of new workers into the economy and so ensures the steady state number of workers is one. Workers differ in their inherent ability level: when first entering the economy, each is endowed with an ability $a$ which is considered as a random draw from the population distribution $G(\cdot)$. Assume $G$ is continuous (no mass points) and has support $[0, \overline{a}]$. Given ability $a$, a worker initially chooses an education, or productivity level, $x \geq 0$. I then let $F(x)$ describe the steady state distribution of productivity levels among all active unemployed workers who enter the labour market and seek employment. Of course $F(\cdot)$ is endogenously determined by equilibrium behaviour.

The model has a standard hold-up structure: given beliefs on market wages, an individual who is born with ability $a$ first selects a productivity level $x$. The cost of obtaining that productivity level depends on ability $a$. After investing in her productivity, she then enter the labour market and search for a job. If she does so and contacts a firm, her wage is then determined by Nash bargaining. Of course expectations are rational: the negotiated wage is consistent with her original beliefs. As firms are identical, $w = w^*(x)$ will denote the equilibrium negotiated wage.

At any moment in time a worker is either non-participant, employed or unemployed. A non-participant(inactive) realises there is no gain to trade with any firm. An employed worker with productivity level $x$ produces output with flow value $x$. An unemployed worker enjoys flow income $b > 0$ while unemployed, where $b$ is independent of $a$ and $x$.

There are matching frictions - it takes time for unemployed workers to find vacancies. Assume a worker contacts a vacancy according to a Poisson process with arrival rate $m(\theta)$. Similarly let $\frac{m(\theta)}{\theta}$ denote the rate at which a vacancy contacts an unemployed. Search is random in that given a contact, the worker's productivity $x$ is considered by the firm as a random draw from $F(.)$.

Let $U$ denote the measure of active unemployed workers who participate in the labor market and $V$ denote the measure of vacancies. Let $M(U,V)$ denote the matching function which describes the flow number of contacts between active unemployed workers and vacancies over time. Assume $M$ is increasing in both arguments, concave and has constant returns. By defining market tightness $\theta = \frac{V}{U}$ and $m(\theta) = M(1,\theta)$, then symmetry implies

$$\frac{M(U,V)}{U} = M(1,\theta) \equiv m(\theta)$$
$$\frac{M(U,V)}{V} = \frac{U}{V}M(1,\frac{V}{U}) \equiv \frac{m(\theta)}{\theta}.$$

Note that by assumption on $M$, $m(\theta)$ is increasing in $\theta$ and $\frac{m(\theta)}{\theta}$ is decreasing in $\theta$. For simplicity assume there is no on-the-job search. There are job destruction shocks: each match is destroyed at exogenous rate $\delta$ after which the laid-off worker becomes active unemployed. As it is standard in the Pissarides (2000) framework, equilibrium market tightness is determined by a free entry condition. All choose strategies to maximise expected lifetime payoff.

### 1.2.1 Worker's Payoffs and Job Search Strategies

Before describing optimal productivity choice, I first describe the expected lifetime value of being unemployed with productivity $x$ in a market with tightness $\theta$, which I denote $V_U(x,\theta)$. Below Nash bargaining will yield a negotiated wage outcome which I denote $w = w^N(x,\theta)$. Standard turnover arguments imply the value of being an active worker with productivity $x$ satisfies

$$(r + \phi)V_U(x, \theta) = b + m(\theta)[V_E(x, \theta) - V_U(x, \theta)] \tag{1.1}$$

where $V_E(x, \theta)$ describes the value of being employed with productivity $x$ and tightness $\theta$ is given by

$$(r + \phi)V_E(x, \theta) = w^N(x, \theta) + \delta[V_U(x, \theta) - V_E(x, \theta)], \tag{1.2}$$

an active job seeker enjoys flow payoff $b$ and finds employment at rate $m(\theta)$ with associated gain $V_E(x, \theta) - V_U(x, \theta)$. While employed, the worker negotiates wage $w = w^N(x, \theta)$ as determined below. At rate $\delta$ the job is exogenously destroyed and the worker returns to the pool of unemployed workers. Substituting out $V_E(.)$, these equations imply:

$$V_U(x, \theta) = \frac{(r + \phi + \delta)b + m(\theta)w^N(x, \theta)}{(r + \phi)(r + \phi + m(\theta) + \delta)} \tag{1.3}$$

Clearly, the worker with productivity $x$ will only enter the labour market if and only if $V_E(x, \theta) \geq V_U(x, \theta)$. As education is costly, those who choose not to enter the labour market, the non-participants, will choose $x = 0$.

### 1.2.2 Firm's Payoffs and Optimal Strategies

This section characterises firm's behaviour for the given market tightness. As all firms are identical, if an active unemployed job seeker is acceptable to one employer, he or she is acceptable to all employers. For an active job seeker with ability $a_i$, let $x^*(\theta)$ denote her/his optimal productivity investment. Standard argument implies:

$$rJ_V(\theta) = -k + \frac{m(\theta)}{\theta}\int_{\underline{x}}^{\overline{x}} max[J_F(x^*(\theta)) - J_V(\theta), 0]dF(x). \tag{1.4}$$

Posting a vacancy induces a flow cost $k > 0$. With $\frac{m(\theta)}{\theta}$ a firm holding a vacancy contacts an active job seeker. An employer's expected discounted return when employing an active unemployed worker with productivity $x(\theta)$ is

$$rJ_F(x, \theta) = x - w(x, \theta) + (\delta + \phi)[J_V(\theta) - J_F(x, \theta)] \tag{1.5}$$

The revenue to the firm associated with the filled job is equal to $x - w(x, \theta)$. Accordingly,

$$J_F(x, \theta) = \frac{x - w(x, \theta) + (\delta + \phi)J_V(\theta)}{(r + \delta + \phi)}. \tag{1.6}$$

Clearly, the return of vacancy to an employer depends on the output of the match which entirely determined by the productivity of the worker minus the wage. Free entry requires that new vacancies are created until the capital of holding one is driven to zero, i.e., $J_V = 0$. So,

$$J_F(x) = \frac{x - w(x, \theta)}{(r + \delta + \phi)}. \tag{1.7}$$

Employers by using a "reservation $x$ strategy", hire any worker such that $J_F(x) \succeq J^V$. Lets define $x^R$ where $J_F(x) = J_V$ so, there exist a level of reservation productivity, $x^R$, which is equal to unemployment income, $x^R = b$. Returning to equation (1.4) and imposing free entry condition and reservation productivity strategy, cost of posting vacancy can be expressed as

$$k = \frac{m(\theta)}{\theta} \int_b^{\bar{x}} \frac{x - w(x,\theta)}{r + \phi + \delta} dF(x) \tag{1.8}$$

Equation (1.8) denotes the free entry equilibrium condition which is one of the key equation of the equilibrium model.

### 1.2.3 Wage Determination

Clearly any match between active unemployed worker and firm is consummated whenever the joint surplus is positive. The joint surplus between a worker with productivity $x$ and a firm, given $\theta$ is written as

$$S(x) = [J_F(x,\theta) - J_V(\theta)] + [V_E(x,\theta) - V_U(x,\theta)]$$

Following Diamond and Pissarides, we suppose that the total surplus from the match is divided up according to a generalised Nash bargain, so that the worker receives $\beta \in (0,1)$ percent of the surplus.[1] [2] Then the wage is implicitly determined by

$$\beta S(x,\theta) = V_E(x,\theta) - V_U(x,\theta)$$

$$(1-\beta) S(x,\theta) = J_F(x,\theta) - J_V(\theta)$$

[1]Nash showed that the unique outcome which is consistent with his axioms has worker's bargaining power equal to $\frac{1}{2}$. The solution generalised Nash is $\beta \in (0,1)$ by relaxing his symmetry axiom.

[2]For the case that $\beta = 0$, workers do not get return from their investment in productivity and clearly no one would acquire productivity. When $\beta = 1$, firms will not post any vacancy and the return to productivity would be zero. I do not take into account those cases.

where $\beta \in (0,1)$ is the worker's bargaining power which is exogenous. By substituting out $V_E(x,\theta) - V_U(x,\theta)$ and $J_F(x,\theta) - J_V(\theta)$ from (1.3), (1.2) and (1.7) into above equations, and by imposing the free entry condition $J_V(\theta) = 0$, I obtain the equilibrium wage:

$$w^N(x) = \frac{\beta(r + \phi + m(\theta)x + \delta) + (r + \phi + \delta)(1 - \beta)b}{(\beta m(\theta) + r + \phi + \delta)}. \tag{1.9}$$

The slope of wage equation is positive and less than one which implies that an employer makes more profit the greater the productivity level of the worker it hires.

### 1.2.4 Productivity Investment Decision

In this section the equilibrium market outcome is taken as given. Specifically as each worker is small, he/she takes the market tightness parameter $\theta$ as given. Also he/she anticipates the equilibrium wage that is negotiated by a worker of productivity $x$. Below this is denoted $w = w^N(x,\theta)$. Thus given innate ability $a$ market tightness $\theta$ and wage function $w^N(x,\theta)$, the worker first chooses the optimal level of productivity $x$ to maximise the expected value of lifetime utility. Let $C(x;a)$ denote the cost of investing to productivity $x$ given initial ability $a$. Assume $C(.)$ is strictly increasing, convex and twice differentiable in $x$. Also $C(a;a) = 0$, $C(.)$ is decreasing in $a$ (it is less costly for a higher ability to achieve a given productivity level) and $C_{xa} < 0$ so that higher ability types face a lower marginal cost to achieving a higher productivity level $x$. For ease of exposition assume the Inada condition $C_x(a,a) = 0$.

Optimal productivity of worker, denoted $x^* \equiv x^*(a,\theta)$ conditional on being active is then given by;

$$x^*(a,\theta) = \arg\max_{x \geq a}[V_U(x,\theta) - C(x;a)] \tag{1.10}$$

The RHS is the sum of value of being active unemployed minus the direct cost of investment in the productivity. The Inada condition ensures the necessary condition for optimal $x^*$ is given by;

$$\frac{\partial V_U(x^*,\theta)}{\partial x} = \frac{\partial C(x^*;a)}{\partial x} \tag{1.11}$$

Of course this condition describes the optimal investment choice for active workers - those who will choose to enter the labour market and search for employment. Not all workers, however, will choose to be active. In equilibrium, there is a critical ability $a^c$ where those with ability $a < a^c$ will choose not to be active. Specifically by staying out of the labour market, each worker can always generate payoff $b/(r+\phi)$.

Thus only workers whose participation in the labour market exceeds $b/(r+\phi)$ will be active labour market members. I therefore formally define $a^c$ as follows.

**DEFINITION of** $a^c(\theta)$:

$$V_U(x^*,\theta) - C(x^*,a^c) = \frac{b}{r+\phi} \quad (\textit{Active Constraint}) \tag{1.12}$$

where $x^* = x^*(a^c,\theta)$ is the optimal productivity choice of an active participant with ability $a = a^c$. Note that $a^c$ depends on market tightness $\theta$ - below we shall show that $a^c$ is decreasing in $\theta$; i.e. higher market tightness leads to more workers choosing to become active. Claim 1 now establishes that a worker with ability $a$

is active in the labour market if and only if $a \geq a^c(\theta)$.

**CLAIM 1.** For any $\theta$:

*($I$) Individuals with $a \geq a^c(\theta)$ are active, and choose $x = x^*(a, \theta)$.*

*($II$) Individuals with $a < a^c(\theta)$ are inactive, choose $x = a$ at zero cost and enjoy $\frac{b}{r+\phi}$.*

Proof. As an active worker of ability $a$ solves the program

$$\max_{x \geq a}[V_U(x, \theta) - C(x; a)],$$

the Envelope theorem implies this payoff is strictly increasing in $a$. As worker $a = a^c$ is indifferent to participating, then all those with $a > a^c$ strictly prefer to participate (and invest to $x^*(a, \theta)$ while all those with $a < a^c(\theta)$ strictly prefer not to participate (and so choose $x = a$) and hence for these workers there is no return to education, and so these discouraged workers would drop out of the labor market process. □

It is straightforward that the optimal level of productivity $x^*(a)$ in (1.11) satisfies

$$\frac{\partial C(x^*, a)}{\partial x} = \frac{m(\theta)\beta}{(r + \phi)(r + \phi + \delta + \beta m(\theta))} \tag{1.13}$$

The marginal productivity cost with different ability level is shown in Figure (1.1) where $a_2 > a_1$. There is marginal cost function for each level of ability. For the larger ability level the marginal cost curve will shift down. Consequently, for the given optimal productivity, for instance $x^*$, the marginal cost function for the worker with lower ability is higher than the worker with higher ability. This implies that the one with bigger $a$ select higher productivity. It is not rational for

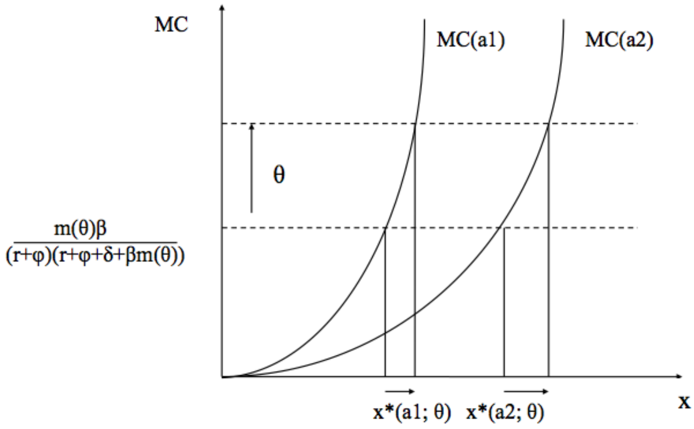


Figure 1.1: Marginal Productivity Cost

the worker with the very low level of ability to invest in her productivity, so she does not select productivity.

### 1.2.5 The Reduced Form Free Entry Condition

The following claim and proposition help us to identify a solution to the free entry condition.

**PROPOSITION 1.** $a^c(\theta)$ is continuous and strictly decreasing in $\theta$.

Proof: The above has established that for any active worker:

$$V_U(x,\theta) = \frac{(r+\phi+\delta)b + m(\theta)w^N(x,\theta)}{(r+\phi)(r+\phi+m(\theta)+\delta)}$$

with

$$w^N(x,\theta) = \frac{\beta(r+\phi+m(\theta)+\delta)x + (1-\beta)(r+\phi+\delta)b}{(\beta m(\theta)+r+\phi+\delta)}$$

Clearly $V_U$ is increasing in $x$. Some algebra also establishes that $V_U$ is increasing and continuously differentiable with $\theta$. Now $a^c$ is defined by

$$V_U(x^*, \theta) - C(x^*, a^c) = \frac{b}{r + \phi} \quad (ActiveConstraint) \tag{1.14}$$

with $x^* = x^*(a^c, \theta)$ given by

$$\frac{\partial V_U(x^*, \theta)}{\partial x} = \frac{\partial C(x^*; a^c)}{\partial x}. \tag{1.15}$$

Totally differentiating equation (1.14) w.r.t $\theta$ and using (1.15) implies:

$$\frac{da^c}{d\theta} = \frac{\frac{\partial V_U}{\partial \theta}}{\frac{\partial C}{\partial a^c}} < 0$$

as required. □

**CLAIM 2.** The Nash wage $w^N(x, \theta)$ is continuous and strictly increasing in $\theta$.

Proof. Trivial by differentiating the above solution for $w^N(.)$ w.r.t $\theta$. We now identify a solution to the free entry condition. Substituting out $w^N(.)$ in the equation for $V_U(.)$ yields

$$V_U(x, \theta) = \frac{b(r + \phi + \delta) + m(\theta)\beta x}{(r + \phi + \delta + \beta m(\theta))(r + \phi)}. \tag{1.16}$$

As $V_U$ is linear in $x$ while $C(.)$ is convex, then for $a \geq a^c$, the first order condition for $x^*(a, \theta)$:

$$\frac{m(\theta)\beta}{(r + \phi + \delta + \beta m(\theta))(r + \phi)} = \frac{\partial C(x^*; a)}{\partial x} \tag{1.17}$$

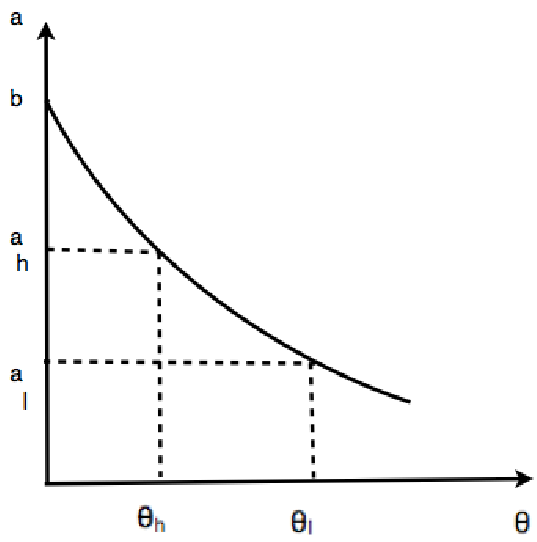


Figure 1.2: Critical Ability and Market Tightness

describes a global maximum. Note further for $a > a^c$ this equation implies $x^*(a, \theta)$ is a continuous and strictly increasing function of $a$ and $\theta$. $\square$

Also inserting the expression for $w = w^N(x, \theta)$ into equation (1.7) gives:

$$J_F(x, \theta) = \frac{(r + \phi + \delta)(x - b)}{(r + \phi + \delta + \beta m(\theta))}. \tag{1.18}$$

Thus the free entry equation which specifies market tightness is defined by inserting the wage equation into the $J_F$ and then substitute it out and rearranging the terms. The next step is to show that for $a > a^c$, that $x^*(a, \theta)$ is a continuous and increasing function of $\theta$.

**CLAIM 3:** $x^*(a,\theta)$ is a continuous and increasing function of $\theta$ and is strictly increasing in $a$

Proof. For types $a > a^c$ who are active, their optimal investment choice $x^*(.)$ is given by the first order condition:

$$\frac{\partial V_U(x^*,\theta)}{\partial x} = \frac{\partial C(x^*;a)}{\partial x}.$$

Equation(1.17) implies

$$\frac{\partial V_U(x,\theta)}{\partial x} = \frac{\beta m(\theta)}{(r+\phi+\delta+\beta m(\theta))(r+\phi)} \tag{1.19}$$

and so $\partial V_U/\partial x$ is a continuous, increasing function of $\theta$. As $C(.)$ is twice differentiable and strictly convex in $x$, the implicit function theorem implies $x^*$ is a continuous increasing function of $\theta$. Also as $C_{xa} < 0$ by assumption, then $x^*$ must strictly increase in ability. $\square$

Claim 3 establishes that investment by active workers increases as market tightness increases, and does so continuously. Furthermore, comparing workers who are active, higher ability types invest to a strictly higher productivity level.

Given $F(x(a)) = \frac{G(a)-G(a^c)}{1-G(a^c)}$ where

$$\begin{cases} a = a^c, & F(x(a)) = 0 \\ a = \overline{a}, & F(x(a)) = 1 \end{cases}$$

and identifying a market equilibrium reduces to finding a $\theta$ which solves the equation.

Now, it is apparent that equation (1.8) can be written as

$$k = \frac{m(\theta)}{\theta} \frac{(1-\beta)}{r+\phi+\delta+\beta m(\theta)} \int_{b}^{\bar{a}} (x^*(a;\theta) - b) \frac{dG(a)}{1-G(a^c)} \tag{1.20}$$

where $x^*(a;\theta)$ is the optimal productivity choice of ability $a$ worker with market tightness $\theta$. Throughout the rest of this part $\Phi(\theta)$ corresponds to the right hand side of (1.20).

### 1.2.6 Steady State Turnover

The steady state number of active unemployed workers, $U$, is the difference between the flow of workers who transits from employment to unemployment and the flow that transits in the opposite direction. First, let us determine the steady state number of unemployed workers, given $a^c$. All those with ability less than $a^c$ are never employed. Hence, $G(a^c)$ are never employed. Hence, $G(a^c)$ are non participant (inactive unemployed). Those with ability at least as great as $a^c$ are sometimes unemployed (maybe active in the labor market). $U$ then is the number of workers with ability at least as great as $a^c$ who are unemployed in a steady state. To determine $U$ note

$$(\phi + m(\theta))U = \delta(1 - G(a^c) - U) + \phi(1 - G(a^c)) \tag{1.21}$$

so,

$$U = \frac{(\delta+\phi)(1-G(a^c))}{\phi+\delta+m(\theta)}. \tag{1.22}$$

## 1.3 Decentralised Equilibrium

### 1.3.1 Definition

*A market equilibrium is defined as follows:*

*ME1: worker participate in the labor market if and only if* $a \geq a^c$ *where:*

$$V_U(x^*(a^c,\theta),\theta) - C(x^*(a^c,\theta),a^c) = \frac{b}{r+\phi} \tag{1.23}$$

*ME2: active participants choose optimal productivity choice* $x^*$ *where:*

$$\frac{m(\theta)\beta}{(r+\phi+\delta+\beta m(\theta))(r+\phi)} = \frac{\partial C}{\partial x}(x^*(a,\theta),a) \tag{1.24}$$

*ME3: free entry condition:*

$$k = \frac{m(\theta)}{\theta}\frac{(1-\beta)}{r+\phi+\delta+\beta m(\theta)}\int_b^{\bar{a}}(x^*(a;\theta)-b)\frac{dG(a)}{1-G(a^c)} \tag{1.25}$$

### 1.3.2 Existence and Characterisation

**CLAIM 4.** It follows that a market equilibrium defined by a triple equations (1.23), (1.24) and (1.25) exists if $k < \Phi(\underline{\theta})$.[3]

[3] I assume that $m(\theta)$ and $\frac{m(\theta)}{\theta}$ satisfy the standard properties:
i) $m(\theta)$ is increasing in $\theta$,
ii) $\frac{m(\theta)}{\theta}$ is decreasing in $\theta$,
iii) $\lim_{\theta\to 0} m(\theta) = 0$ and $\lim_{\theta\to\infty} m(\theta) \to \infty$

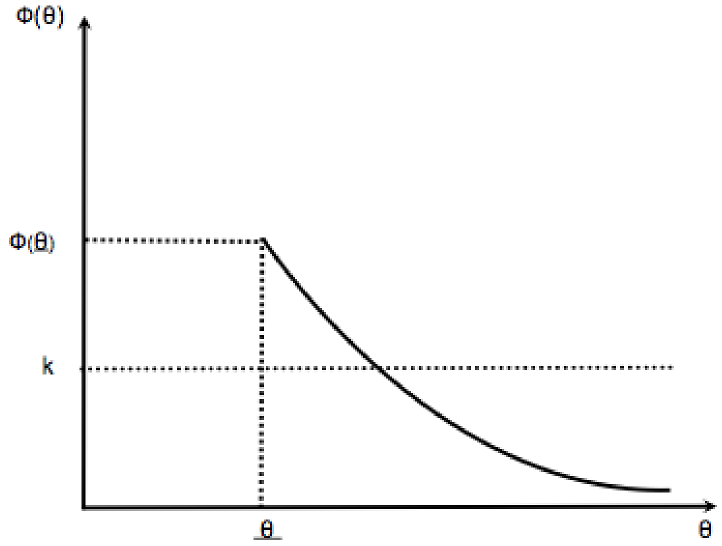


Figure 1.3: Existence of Equilibrium

Proof.

*Case I:* $\theta \to 0$

In this case, $m(\theta) \to 0$ which means it is impossible for the workers to get job and hence nobody will be in the market as $a^c \to \infty$. The cost of posting vacancy is zero. Clearly the right hand side of (1.25) will be higher than the left hand side, i.e. $\frac{b}{r+\phi} = \frac{b}{r+\phi} - C(x;a)$

*Case II:* $\theta$ *is finite*

Consider $\theta = \underline{\theta}$ subject to $a^c(\underline{\theta}) = \overline{a}$. At $\theta = \underline{\theta}$, $\Phi(\theta)$ will be equal to

$$\Phi(\underline{\theta}) = \frac{m(\underline{\theta})}{\underline{\theta}} \frac{(1-\beta)}{r+\delta+m(\underline{\theta})}[x^*(\overline{a};\underline{\theta}) - b)]$$

Now suppose $\theta < \underline{\theta}$, then $a^c > a$ and therefore no one choose investment in productivity.

*Case III:* $\theta \to \infty$

Suppose that $\theta$ approaches infinite, then the $m(\theta) \to \infty$ which implies $a^c \to \underline{a}^c$. As $\theta \to \infty$, $x^*(a;\theta)$ is given by

$$\frac{1}{r+\phi} = C_x(x^*;a)$$

by substituting $x^*$ into (1.23) we have

$$\frac{b}{r+\phi} = \frac{w(x^*(a^c,\infty))}{r} - C(x^*(a^c,\infty);a^c)$$

it is obvious the result is just a finite number. Finally by plugging $\theta \to \infty$ into $\Phi(\theta)$ it shows $\Phi(\theta) \to 0$. Given $k > 0$, then by the intermediate-value and Fixed point theorem, there must exist a value $\theta^* \in [\underline{\theta}, +\infty)$, equilibrium exist if $k < \Phi(\theta)$, (see Figure 1.3). $\square$

For the unique solution of $\theta$, $\Phi(\theta)$ should be monotonically decreasing in $\theta$, which I cannot prove analytically. However to bring out the equilibrium issue clearly I show the model for the two types of workers in the next section.

## 1.4 Two Types Case

Now suppose there are two types of entrants, where $i \in l, h$ denote a worker's type. Those with low-ability, $a_l$ and those with high-ability, $a_h$ where $a_h > a_l > 0$. Assuming fraction $\eta_i$ of entrants are type $i$, the distribution of ability across entrants, denoted $G(a)$ is

$$\begin{cases} G(a) = 0 & \text{for } a < a_l \\ G(a) = \eta_l & \text{for } a_l \leq a < a_h \\ G(a) = 1 & \text{for } a \geq a_h. \end{cases}$$

In steady state, let $1-\pi$ denote the proportion of active agents who are high ability ($\pi$ will be endogenously determined and depending on the investment choice of workers).

$E_i$ denotes the number of employed workers with ability $a_i$. Let $U = U_l^A + U_h^A$ the number of active unemployed job seekers. Random search implies $(1-\pi)\frac{m(\theta)}{\theta}$ is the rate at which a firm contacts an active job seeker with high ability.

### 1.4.1 Determining $V_U(x, \theta)$

Again, the first step is to write the value of unemployment and the value of vacancy. In this case, I proceed by writing the return to be active unemployment explicitly:

$$V_U(x,\theta) = \frac{(r+\phi+\delta)b + m(\theta)w^N(x,\theta)}{(r+\phi)(r+\phi+m(\theta)+\delta)}. \tag{1.26}$$

The arguments behind the definition of critical ability, $a^c$ and optimal productivity investment are unchanged. Optimal productivity of type $i$ worker, denoted $x^* \equiv x^*(a_i, \theta)$ conditional on being active is then given by;

$$x^*(a_i,\theta) = \arg\max_{x \geq a_i} [V_U(x,\theta) - C(x; a_i)] \tag{1.27}$$

and the necessary condition for optimal $x_i^*$ is given by;

$$\frac{\partial V_U(x_i^*, \theta)}{\partial x} = \frac{\partial C(x_i^*; a)}{\partial x} \tag{1.28}$$

The active constraint is given by

$$V_U(x_i^*, \theta) - C(x_i^*, a^c) = \frac{b}{r + \phi} \quad (\textit{Active Constraint}) \tag{1.29}$$

where $x_i^* = x^*(a_i^c, \theta)$ is the optimal productivity choice of an active participant with ability $a = a^c$. Note that $a^c$ depends on market tightness $\theta$ - it is decreasing in $\theta$; i.e. higher market tightness leads to more (low ability) workers choosing to become active. Claim 4 now establishes that a worker with ability $a_i$ is active in the labour market if and only if $a_i \geq a^c(\theta)$.

**CLAIM 5.** For any $\theta$:
*(I) Individuals with $a_i \geq a^c(\theta)$ are active, and choose $x = x^*(a, \theta)$.*
*(II) Individuals with $a_i < a^c(\theta)$ are inactive, choose $x = a_i$ at zero cost and enjoy* $\frac{b}{r+\phi}$.

Proof. As an active worker of ability $a_i$ solves the program

$$\max_{x \geq a} [V_U(x, \theta) - C(x; a)],$$

the Envelope theorem implies this payoff is strictly increasing in $a_i$. As worker $a = a^c$ is indifferent to participating, then all those with $a_i > a^c$ strictly prefer to participate (and invest to $x^*(a_i, \theta)$ while all those with $a_i < a^c(\theta)$ strictly prefer not to participate (and so choose $x = a_i$) and hence for these types there is no return to education, therefore they would drop out of the labor market process.$\square$

### 1.4.2 Steady State Turnover

In order to solve the entry condition for equilibrium market tightness, I first describe steady state turnover. Suppose type $i$ have ability $a_i > a^c(\theta)$ and so are active labour market participants. Recall that $E_i$ was defined as the number of type $i$ workers who are employed and $U_i$ the number who are (active) unemployed. Steady state implies

$$(m(\theta) + \phi)U_i = \delta E_i + \phi \eta_i \tag{1.30}$$

where the LHS describes the flow of $i$ type workers out of unemployment, while the inflow is composed of employed workers who lose their jobs and those new market entrants who are type $i$. As $E_i + U_i = \eta_i$ this implies the number of active type $i$ unemployed worker is

$$U_i = \frac{(\phi + \delta)\eta_i}{m(\theta) + \phi + \delta}. \tag{1.31}$$

Similarly for the pool of employed workers. Note then that if both $a_l, a_h > a^c(\theta)$ then the fraction of (active) unemployed workers who are type $i$ is

$$\frac{U_i}{U_l + U_h} = \eta_i$$

As $\pi$ denotes the fraction of active unemployed workers who are low ability, then $\pi = \eta_l$ in this case. Conversely $a_h > a^c(\theta) > a_l$ implies low ability types are not active in the labour market. As $U_l = 0$, this implies $\pi = 0$ : all active unemployed workers are high ability. This effect plays an important part in what follows.

### 1.4.3 Wage Determination

I now determine the equilibrium wage function $w^N(x,\theta)$. Again consider type $i$ with ability $a_i > a^c(\theta)$ and so are active labor market participants. Suppose such a worker invests to productivity $x$ and so enjoys value $V_U(x,\theta)$. Let $J_F(x,\theta)$ denote the firm's value of employing a worker with productivity $x$ and $J_V(\theta)$ denote the value of a vacancy. Given $\beta \in (0,1)$ describes the worker's bargaining power, Nash bargaining implies the negotiated wage satisfies

$$\beta[J_F(x,\theta) - J_V(\theta)] = (1-\beta)\left[V_E(x,\theta) - V_U(x,\theta)\right].$$

Of course a free entry equilibrium implies $J_V(\theta) = 0$ while

$$J_F(x,\theta) = \frac{x - w^N(x,\theta)}{r+\delta+\phi} \tag{1.32}$$

as the job is closed only in the event that the worker dies or the job is destroyed. By also substituting out $V_E(x,\theta) - V_U(x,\theta)$ using the above, equilibrium Nash wage agreement is:

$$w^N(x,\theta) = \frac{\beta(r+\phi+m(\theta)+\delta)x + (1-\beta)(r+\phi+\delta)b}{(\beta m(\theta)+r+\phi+\delta)} \tag{1.33}$$

The wage is thus a weighted of the worker's productivity $x$ and the worker's flow value of unemployment $b$, where the weight depend on $\theta$ and the worker's bargaining power $\beta$. Same as what I discussed in the first part of this chapter a rise in the productivity of the worker makes the size of the surplus to be shared between a firm and a worker with productivity $x$, bigger which causes the rise in the Nash bargaining wage.

### 1.4.4 The Value of a Vacancy

Consider now a firm participating in the labor market. Market tightness determines which types of workers are active in the labour market as $a^c = a^c(\theta)$. Recall that $\pi$ denotes the fraction of active unemployed workers who are type $i = l$ and that for types $a_i > a^c(\theta)$, their optimal productivity choice is $x^*(a_i, \theta)$. In a free entry equilibrium with random search, the expected value of a vacancy is:

$$rJ_V(\theta) = -k + \frac{m(\theta)}{\theta}[\pi J_F(x^*(a_l, \theta), \theta) + (1-\pi)(J_F(x^*(a_h, \theta), \theta)] \qquad (1.34)$$

where $k$ is the flow cost of the vacancy. Free entry requires that new vacancies are created until the capital of holding one is driven to zero, i.e., $J_V = 0$. So, free entry condition:

$$k = \frac{m(\theta)}{\theta}[\pi J_F(x^*(a_l, \theta), \theta) + (1-\pi)(J_F(x^*(a_h, \theta), \theta)] \qquad (1.35)$$

It is immediately obvious that by applying the Proposition 1, claim 2 from the first part of the chapter and also knowing for $a_i > a^c$, $x^*(a_i, \theta)$ is continuous and strictly increasing function of $a_i$ and $\theta$, we can find a reduced form free entry condition as follows: inserting the expression for $w = w^N(x, \theta)$ into equation (1.32) gives:

$$J_F(x, \theta) = \frac{(r + \phi + \delta)(1-\beta)(x^* - b)}{(r + \phi + \delta + \beta m(\theta))} \qquad (1.36)$$

Thus the free entry equation which specifies market tightness is defined by inserting the wage equation into the $J_F$ and then substitute it out and rearranging the terms. Thus identifying a market equilibrium reduces to finding a $\theta$ which solves the free entry condition equation,

$$k = \frac{m(\theta)}{\theta} \frac{1-\beta}{r+\phi+\delta+\beta m(\theta)} [\pi(\theta)x^*(a_l,\theta) + (1-\pi(\theta))x^*(a_h,\theta) - b] \tag{1.37}$$

where

$$\pi(\theta) = \begin{cases} 0 & \text{if} \quad a_l < a^c(\theta) < a_h, \\ \eta_l & \text{if} \quad a^c(\theta) < a_l. \end{cases}$$

Note that if $a^c(\theta) > a_h$ then there are no active labour market participants and there is no trade.

## 1.5 Decentralised Equilibrium

### 1.5.1 Definition

*A market equilibrium is defined as follows:*

*ME1': worker participate in the labor market if and only if $a_i \geq a^c$ where:*

$$V_U(x^*(a^c,\theta),\theta) - C(x^*(a^c,\theta),a^c) = \frac{b}{r+\phi} \tag{1.38}$$

*ME2': active participants choose optimal productivity choice $x^*$ where:*

$$\frac{m(\theta)\beta}{(r+\phi+\delta+\beta m(\theta))(r+\phi)} = \frac{\partial C}{\partial x}(x^*(a_i,\theta),a_i) \tag{1.39}$$

*ME3': free entry condition:*

$$k = \frac{m(\theta)}{\theta} \frac{1-\beta}{r+\phi+\delta+\beta m(\theta)} [\pi(\theta)x^*(a_l,\theta) + (1-\pi(\theta))x^*(a_h,\theta) - b] \qquad (1.40)$$

*ME4': the proportion of active workers who are type i consistent with steady state turnover; when low-type are active then* $\pi(\theta) = \eta_l$ *and when low-type is inactive* $\pi(\theta) = 0$

### 1.5.2 Existence and Characterisation

There are three types of possible equilibria. I define

$$\Omega(\theta) = \frac{m(\theta)}{\theta} \frac{1-\beta}{r+\phi+\delta+\beta m(\theta)} [\pi(\theta)x^*(a_l,\theta) + (1-\pi(\theta))x^*(a_h,\theta) - b] \qquad (1.41)$$

which describes the expected return to creating a vacancy. Identifying an equilibrium requires finding a $\theta$ which solves $\Omega(\theta) = k$. Note that $m(\theta)$ is a continuous function of $\theta$ by assumption. The next step is to show that for $a_i > a^c$, that $x^*(a_i,\theta)$ is a continuous and increasing function of $\theta$.

**CLAIM 6:** $x^*(a_i,\theta)$ is a continuous and increasing function of $\theta$ and is strictly increasing in $a_i$.
Proof follows directly from Claim 3.
Claim 6 establishes that investment by active workers increases as market tightness increases, and does so continuously. Furthermore, comparing workers who are active, higher ability types invest to a strictly higher productivity level.

**LEMMA 1.** As $\theta \to 0$, $x^*(a_i, \theta) \to a_i$, for all $a_i > a^c$, $a^c(\theta) \to b$.

Proof. As $\theta \to 0$, equation(**??**) implies $\frac{\partial V_U(x,\theta)}{\partial x} = 0$. Hence as $\theta \to 0, x^*(a_i, \theta) \to a_i$, thus by (1.17), $V_U(x^*, \theta) \to \frac{b}{r+\phi}$ and (1.33) implies $a^c \to b$.

**PROPOSITION 2.** $\exists \ f$ s.t $\theta_i = f(a_i)$ and is strictly decreasing with $\theta_i$ with $\theta_i = 0$ at $a_i = b$.

Proof. Since $a^c(\theta_i)$ is a function of single variable and from Proposition 1 it is strictly decreasing in $\theta$, the Inverse Function Theorem implies there exists $f(a_i) = [a^c]^{-1}(a_i)$ and

$$f'(a_i) = \frac{1}{(a^c(\theta))'} = \frac{d\theta}{da} < 0$$

from Lemma 1 the proof is completed. □

It is now straightforward, using Lemma 1 and proposition 2, to identify Market Equilibrium. Lets define $\theta_l$ and $\theta_h$ where

$$\begin{aligned} a_l &= a^c(\theta_l) \\ a_h &= a^c(\theta_h). \end{aligned}$$

Note that at market tightness $\theta = \theta_i$, workers with ability $a_i$ are indifferent between being active in the labour market and not participating. As Proposition 1 establishes that $a^c(\theta)$ is a strictly decreasing function of $\theta$, then these definitions imply $\theta_h < \theta_l$. This then implies three possible scenarios as depicted in Figure 1.4: (i) If $\theta < \theta_h$ then no workers are active in the labour market. With no loss of generality we suppose $\Omega(.) = 0$ in this region; i.e. the expected return to creating a vacancy is zero.

(ii) If $\theta \in (\theta_h, \theta_l)$ then types $i = h$ are active as $a_l < a^c(\theta) < a_h$. As this implies $\pi(\theta) = 0$, the expected return to a vacancy is:

$$\Omega(\theta) = \frac{m(\theta)}{\theta} \frac{1-\beta}{r+\phi+\delta+\beta m(\theta)} [x^*(a_h, \theta) - b].$$

Note Claim 6 implies $\Omega(.)$ is continuous in this range. Its slope is ambiguous, however, as $x^*(a_h, .)$ is an increasing function.

(iii) if $\theta > \theta_l$, then all types are active as $a^c(\theta) < a_l$. As this implies $\pi(\theta) = \eta_l$, the expected return to a vacancy is:

$$\Omega(\theta) = \frac{m(\theta)}{\theta} \frac{1-\beta}{r+\phi+\delta+\beta m(\theta)} [\eta_l x^*(a_l, \theta) + \eta_h x^*(a_h, \theta) - b].$$

Again Claim 6 implies $\Omega(.)$ is continuous in this range. Furthermore in the first part of this paper I proved that $\Omega(\theta) \to 0$ as $\theta \to \infty$ [see proposition 1].

Of course $\Omega(\theta)$ is not continuous in $\theta$ at $\theta_l, \theta_h$. Clearly $\Omega(.)$ increases by a discrete amount at $\theta_h$ as $\Omega = 0$ for $\theta < \theta_h$. At $\theta_l$, however, it is easy to see that $\Omega(.)$ decreases by a discrete amount. The discontinuity is caused by low types switching to being active and, by Claim 4, their productivity $x^*(a_l, .) < x^*(a_h, .)$. To be more precise lets have a look at the each regions in details. A critical step is to note that the nature of equilibrium depends on the continuity of the right hand side of (1.40), i.e. $\Omega(\theta, a_i)$. Clearly as $\pi(\theta)$ is not continuous at $a^c = a_i$ for $i = l, h$ then

$\Omega(.)$ is not continuous at that points.

**CLAIM 7.** *$\exists$ $\theta_l$ s.t. $l$ types drop out if $\theta < \theta_l$.*

**CLAIM 8.** *$\exists$ $\theta_h$ s.t. $h$ types drop out if $\theta < \theta_h$.*

**CLAIM 9.** *Given the above claims there exist three types of equilibrium:*

*Region 1:* The first is an equilibrium in which it is not beneficial for low/high ability worker to invest in their productivity since $\theta \leq \theta_h$ i.e. $a_l, a_h < a_c$ which implies $\theta = 0$ is equilibrium and clearly everyone are inactive in this case. This is "autarchic equilibria" where workers do not participate to the labor market and as a result, firms do not post vacancies. The main concern of this paper is to focus on "non-autarchic" equilibria.

*Region 2:* The second is the region that $\theta_h \leq \theta < \theta_l$, which shows that only high-ability type are active as $a_l < a^c < a_h$ and accordingly, $\pi(\theta) = 0$; the proportion of active low ability workers is zero. Lets show the equilibrium equations for this case are as follows:

$$\frac{m(\theta)\beta}{(r+\phi+\delta+\beta m(\theta))(r+\phi)} = \frac{\partial C(x^*(a_h,\theta);a_h)}{\partial x} \tag{1.42}$$

$$V_U(x^*(a_l,\theta),\theta) - C(x^*(a_l,\theta);a_l) < \frac{b}{r+\phi} \tag{1.43}$$

$$V_U(x^*(a_h,\theta),\theta) - C(x^*(a_h,\theta);a_h) > \frac{b}{r+\phi} \tag{1.44}$$

$$k = \frac{m(\theta)}{\theta}\frac{1-\beta}{r+\phi+\delta+\beta m(\theta)}(x^*(a_h,\theta)-b) \tag{1.45}$$

from the participation constraint for the above region it is clear that it is not beneficial for low ability type to invest to her/his productivity. One can show that the LHS of (1.44) is increasing in market tightness. For the proof please look at Appendix 1A. The low ability type takes life as leisure and accordingly they will not participate in the labour market.

Region 3: The third is where $\theta_l \leq \theta$ which implies $a_l, a_h > a^c$. This region illustrates a labor market that both types are active and participate in the labor market as critical ability is lower even from the ability of low ability worker. I term such a steady state equilibrium a "Joint Type Equilibrium". This requires

$$\frac{m(\theta)\beta}{(r+\phi+\delta+\beta m(\theta))(r+\phi)} = \frac{\partial C(x^*(a_l,\theta);a_l)}{\partial x} \tag{1.46}$$

$$V_U(x^*(a_l,\theta) - C(x^*(a_l,\theta);a_l) > \frac{b}{r+\phi} \tag{1.47}$$

$$k = \frac{m(\theta)}{\theta}\frac{1-\beta}{r+\phi+\delta+\beta m(\theta)}[\eta_l x^*(a_l,\theta) + (1-\eta_l)x^*(a_h,\theta) - b)] \tag{1.48}$$

$\Omega(.)$ and $k$ are illustrated on the vertical axis and market tightness is shown on the horizontal axis in Figure 1.4. The place that the critical ability meets the ability of high type corresponds to the market tightness in the region 2 and accordingly, $\Omega(.) = \Omega(\theta, a_h)$. The place that the critical ability meets the ability of low type corresponds to market tightness in the region 3 with $\Omega(.) = \Omega(\theta, a_{lh})$. Of course if the critical ability is really high even higher than the ability of high type then no one participate and region 1 represent that on the Figure 1.4.

Of course, as set above, at the point where $\theta = \theta_l$ subject to $a^c = a_l$, the gap between the two graphs illustrates those workers that are indifferent being active or inactive.

In order that this type of Equilibrium occurs, it must be worthwhile for just

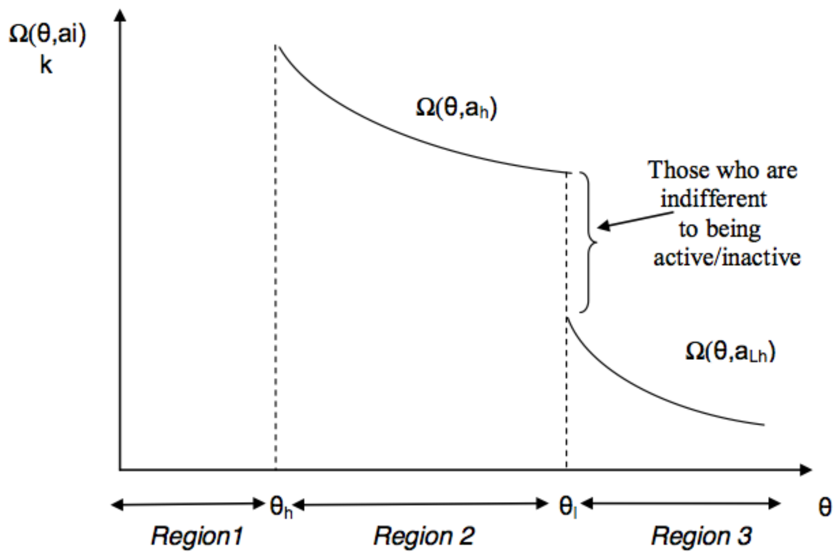


Figure 1.4: The Equilibrium Value of Market Tightness

high-ability active unemployed to participate in the labor market; $V_U(x^*(a_l,\theta),\theta) - C(x^*(a_l,\theta);a_l) < \frac{b}{r+\phi}$ must hold. Similarly, Joint Type equilibrium requires $V_U(x^*(a_l,\theta) - C(x^*(a_l,\theta);a_l) > \frac{b}{r+\phi}$. Mixed Strategy arises because at $\theta = \theta_l$ there exist some workers with $a^c = a_l$ who are indifferent being active or inactive.

### 1.5.3 Multiple Equilibria

Multiple equilibria, can arise if the $\Omega$ function is increasing at $\theta$. Lets look at the active constraint equation at $\theta = \theta_h$

$$\frac{b}{r+\phi} < \frac{(r+\phi+\delta)b + m(\theta)w(x)}{(r+\phi)(r+\phi+m(\theta)+\delta)} - C$$

Substituting $w(x)$ and rearranging, we have

$$C < \frac{m(\theta)\beta(x^*(a,\theta)-b)}{(r+\phi)(\beta m(\theta)+r+\delta+\phi)}$$

where $C = \frac{m(\theta_h)\beta(x^*-b)}{(r+\phi)(\beta m(\theta_h)+r+\delta+\phi)}$, inserting $C$ into the above equation and also finding $x^*$ from the optimal productivity constraint and substituting it in the above equation we have,

$$\frac{m(\theta_h)-\theta_h m'(\theta_h)}{\theta_h m'(\theta_h)} \frac{C_{xx}C}{(C_x)^2} \frac{\beta m(\theta_h)+r+\delta+\phi}{r+\delta+\phi} \leq 1 \tag{1.49}$$

$$\xi_{m(\theta),\theta}\, \xi_{MC,C} \frac{\beta m(\theta_h)+r+\delta+\phi}{C_x(r+\delta+\phi)} \leq 1 \tag{1.50}$$

where $\xi_{m(\theta),\theta}$ is the elasticity of matching with respect to the stock of vacancies and $\xi_{MC,C}$ is the elasticity of marginal cost function with respect to the cost function. Clearly to construct an example lets assume $C(x_i; a_i) = x_i^{\gamma} a_i^{-1}$, then

$$\frac{1-\alpha}{\alpha} \frac{\gamma-1}{\gamma} \frac{\beta m(\theta_h)+r+\delta+\phi}{r+\delta+\phi} \leq 1 \tag{1.51}$$

multiple equilibria i.e; having $\frac{\partial\Omega(\theta)}{\partial\theta} > 0$, requires the following: If the elasticity of arrival rate of vacancy to the worker i.e.$\alpha$ is close to one; the marginal cost of investing in productivity is more elastic with respect to productivity investment i.e. $\gamma$ and also worker's bargaining power goes to zero, i.e; $\beta = 0$ so that workers appropriate nothing, nearly zero, of the surplus. It is important to understand, however, that multiple equilibrium do not occur for all possible parameter configurations.

## 1.6 Is Heterogeneity the Cause of Multiplicity?

By Comparing the equilibrium part in two types case and the first part of the paper, the immediate question is raised as to whether heterogeneity is the cause of multiplicity? To answer this question, suppose workers are homogenous, in order that multiple equilibria occurs, it must be check that $\Omega(.)$ is increasing at $\theta$. That is:

$$\frac{\theta m'(\theta) - m(\theta)}{\theta^2} \frac{(x^*(a_h,\theta) - b)(1-\beta)}{\beta m(\theta) + r + \delta + \phi} + \frac{m(\theta)}{\theta}$$
$$\frac{\beta(r+\delta+\phi)}{r+\phi} \frac{m'(\theta)}{(\beta m(\theta) + r + \delta + \phi)^3} \frac{1-\beta}{C_{xx}} > 0$$

where $C_{xx}$ is the second derivative of productivity cost function w.r.t productivity. As the expression for optimal productivity choice is:

$$\frac{\partial C(x^*)}{\partial x} = \frac{\beta m(\theta)}{(r+\phi)(\beta m(\theta) + r + \delta + \phi)} \tag{1.52}$$

Inserting the above expression for $C_x(x^*(\theta;a))$, gives

$$\frac{m(\theta) - \theta m'(\theta)}{\theta} \frac{x^* - b}{m'(\theta)(r+\delta+\phi)} \frac{(\beta m(\theta) + r + \delta + \phi)C_{xx}}{C_x} \leq 1 \tag{1.53}$$

By looking at above equation one can claim that if optimal productivity choice is close to unemployment benefit $b$ then $\frac{\partial \Omega}{\partial \theta} > 0$. Note that $x^*$ close to $b$ contradict the active search constraint, so heterogeneity is not the cause of multiplicity.

## 1.7 Conclusion

Workers and firms face considerable problem contacting each other and of course these difficulties have consequences on the equilibrium characteristics of the labor market. This paper studies an equilibrium search model that highlights the role of inherent ability and productivity investment in the labor market. My argument is related to the endogenous participation and investment decision of heterogeneous workers who have an inherent ability level. When productivity investment is costly and workers are heterogeneous in ability one can think only the ablest choose to acquire productivity(education). While here I show the important role of critical ability and cost of investing on productivity which makes the result different.

Clearly the active constraint plays an important role in the analysis and makes it possible to examine the interaction between critical ability and market tightness and also the choice of optimal productivity investment. I show here those with ability above critical ability will participate in the labour market. The choice of participation involves an opportunity costs in terms of forgone utility of leisure and direct cost of productivity investment. I prove that the critical ability decreases in market tightness while optimal productivity investment increases in market tightness and ability of the worker.

Accounting for the critical ability and optimal productivity, I derive the outcome of wage bargaining. Embedding the wage bargaining with free entry condition, optimal productivity investment and participation decision, I describe equilibrium characteristics. I show the existence of equilibrium and then extend the model for two types case of workers since it gives a better description of the equilibrium and probability of existence of multiple equilibrium. Equilibrium can take one of the following forms. One in which it is beneficial for both types of workers to be active in the labour market, I call it the "Joint Type Equilibrium", second one is the one that only there is willingness of high ability workers to invest in their productivity and finally the last one is the one where there is no benefit for either types to invest in their productivity and consequently there is no participation of workers in the labour market and no posting of vacancies from the firm side.

Given two types workers does it imply that multiple equilibrium exist? The model could answer the question, it could exist under certain conditions. Finally, it is worth remarking that because of the heterogeneity in the extended part, multiplicity will not raise or -i.e. heterogeneity is not the cause of multiplicity.

In the second chapter of the thesis I focus on the optimal policy applying on the two case types of the first chapter. Allowing the central planner to use different policies such as training subsidies, lump sum participation tax and job creation subsidy, I show conditions under which these policies are efficient in increasing the output of the economy.

## 1.8 Appendix 1A.

Define

$$\Psi_i(\theta) = \frac{1}{r}[\frac{(r+\delta)b + m(\theta)w(x_i^*)}{(r+\delta+m(\theta))}] - C(x_i^*, a_i^*) \tag{1.54}$$

as we proves $x_i^*$ is chosen optimally, the Envelope Theorem implies

$$\frac{d\Psi_i}{d\theta} = \frac{m'(\theta)}{r}\frac{(r+\delta)(w-b)}{(r+\delta+m(\theta))^2} > 0 \tag{1.55}$$

# Chapter 2

# The Efficiency of Productivity Investment in Search Equilibrium Framework

## 2.1 Introduction

This chapter analyses the efficiency in an economy with endogenous productivity investment decision of heterogeneous workers who have different inherent ability level as described in the first chapter of this thesis. In this chapter I show that the economy which I introduced in the first chapter is inefficient. This is due to the fact that workers do not internalise the firm's cost of posting a vacancy, the productivity investment of the other workers and the search intensity is lower in the economy. Therefore, the labor market is overcrowded with low ability workers who have less investment in productivity and reduce the probability that high productivity workers match. So, the goal of this chapter is to demonstrate how the government can achieve efficiency in this economy by observing the productivity investment of the workers. Therefore, assuming the government can observe workersducation, I allow the government to use different policies to get an efficient allocation in the economy. In my first best policy scenario, I find the government should subsidise training, subsidise job creation and tax labour market participation.

First the implementation of endogenous productivity investment allows me to analyse the hold up problem mentioned in the literature. A natural holdup and inefficiencies arise in a market with ex ante investments and trading frictions. Basically, an investment is held up if one party must pay the cost while others share in the payoff. In many situations, investments must be sunk before agents meet each other. For example, workers must complete their education before finding jobs. As agents do not know who their partners will be at the time of investment, related arrangements or contracts are impossible (Acemoglu, 1996). In particular, when wages are determined by ex post bargaining, the equilibrium is always inefficient. One could say to remove the bargaining power from the workers, but this, in turn, depresses wages below their social product and creates excessive entry of firms.

Here, the purpose of this chapter is to show how the planner can achieve efficiency in an economy having endogenous productivity with heterogeneous workers where the government cannot observe whether the worker is high or low ability type but education is observable by the government. Also, it is well established in the literature that private decisions in markets with random job matching generates inefficiencies. Of course these inefficiencies are through externalities. In the previous chapter I derive a set of necessary conditions for market solution. In the present chapter I look for the corresponding allocation that maximises steady-state net output and compare the solution with the decentralised result.

Therefore assuming the education level of the workers is observable by the government, I allow the government to use different policies such as job creation subsidy, training subsidy and tax participation, applying the principle of targeting[1]to get an efficient allocation in the economy.

The implementation of training subsidy policy allows me to analyse the interaction of it with worker bargaining power. Solving the optimal training subsidy policy shows that higher worker's bargaining power leads to implementation of a lower training subsidy in order to restore investment decision efficiency. This result is interesting since it can guide the hold up problem. That is, it is interesting when individuals make ex ante investments before matching with firms, disregarding their ability, when wages are determined by ex post bargaining, the equilibrium is inefficient. Wages increase with productivity investment, creating problem for unemployed active job seekers' investment, also all the bargaining power is controlled by the workers leading to very high wage level and excessive entry of workers. Clearly with ex ante investments, no bargaining solution achieves efficiency, and the efficiency result is more striking.

The labour market participation tax would not seem realistic. But we can think in terms of education policy, such as graduate loans or graduate tax. Should the government subsidise tertiary education[2]and, if so, might it fund students with graduate loans or a graduate tax?

Most of the existing literature focuses on the design of optimal education policies. Since higher education is costly and faces competing imperatives for public spending therefore tertiary education is an important element in national eco-

[1]In several papers in the 1960s such as Bhagwati (1971), the general principle of targeting in economic policy was developed. The main concern in the principle of targeting is the distortions from the usual marginal conditions of Pareto-efficiency which are best tackled by using policy instruments that do directly on the relevant margin.

[2]Tertiary education refers to any further education (FE) pursued beyond the high school level.

nomic performance and a major determinant of a person's life chances. It requires a funding system by which institutions can charge different prices, but many people argue that the tertiary education should be financed from taxation. Higher education in most advanced economies is heavily subsidised by government and they are justified on efficiency grounds by externalities. For this, it is important to know what effect the education subsidy and the taxes that finance them (here we called the participation tax) have on the improvement of the economy.[3]The UK higher education has been subject to a number of changes since 1969s, and the system has moved from one where it was financed by the taxpayer to one where graduates themselves now contribute to the cost of their education.[4]

Heckman, Lochner and Taber (1999) in their general equilibrium micro-simulation model concluded that tuition subsidies could raise the welfare of the least able workers through general equilibrium effects on the wages of the unskilled. Keane and Wolpin (1997) micro-simulation model concluded quite the opposite -that only the most able would benefit from a tuition subsidy. These models are very complex and depend on huge amount of assumptions, it is difficult to draw from them firm conclusions concerning when tuition subsidies might help the unskilled and when they might not. The debate about student finance should be around the development of the loans, fees and bursaries systems. Shackleton (2010) argue that financing higher education by a tax imposed on those graduates -simply because they are graduates- is a bad idea. He thinks this is a tax that would be unrelated

[3]In early 1960s the UK government became concerned that the UK higher education sector was relatively small compared to the rest of the developed world- the UKs Higher education participation rate at around 6 percent, was one of the lowest in the OECD (Barr and Crawford 2005). The government were mainly concerned that the lack of higher education in the workforce would stop economic growth.

[4]I do not discuss here about the distributional consequences of this subsidy.

to the cost of an individual's higher education and only loosely related to its benefits. Also he argues how would the government define a 'degree' for the purpose of the tax and what would happen to those who had higher tertiary education in UK (for example) and then went to live abroad? While graduate loans are a personal debt and individuals can be tracked even when they move overseas. Of course there is no agreement that graduate tax could be imposed on those working in other countries.

The idea behind graduate tax is straightforward. The government is in effect financing the human capital investment by subsidising higher education. The main future benefits to the graduate is in the form of higher earnings. Because of this investment the government is entitled to a dividend from the ensuing income benefits and takes the form of percentage tax on graduates' income over their working lives. In this paper for simplicity I consider a fixed tax when workers enter in the labor market. The labor market participation tax here resembles the graduate tax. [5]

In 1990, the first UK students loan scheme was implemented. Graduate loan offers a plausible solution to the problem the planner could explore alternative devices for cost recovery such as graduate tax. In this paper the participation tax could be considered as graduate tax. Although most studies have been optimistic about the effectiveness of graduate loans as a cost recovery programs. Albrecht and Ziderman 1993, examines those effects and identifies some fundamental flaws in existing programs. They discussed that a graduate tax could bring in significantly more revenue than traditional loan program.

[5] Higher education in most advanced economies is heavily subsidised by the government. The discussion about the efficiency and equity , Johnson (2004) is not discussed here.

How best to finance higher education sector is an ongoing issue in most countries. Barr (2004) list twenty factors against a graduate tax, as to why this approach to funding higher education should not be considered. He emphasises that apart from all these reason there is common sense factor that needs to take into account. But one can ask Why should government subsidy education? Who benefits from higher education? One can answer these questions by discussing about the private return to tertiary education (Steel and Sausman (1997)),Social and Cultural Benefits (Bynner and Egerton (2000)), Social rates of return to higher education (Layard et al.(2002)), education and growth (Bassanini and Scarpetta (2001)).

Booth and Coles (2010) prove how the taxes and social security payments directly affect the participation decision. Their results provide supporting evidence for the observation motivating their paper. Their model shows how the participation decision can lead to under participation and as a result have effect on individuals' decisions to invest in acquiring skills. They apply a tax on labour income which leads to large substitution effects to home production.

The general efficiency condition in here is based on a labor force participation margin and the optimal investment decision which are absent in most specification of search frameworks. Building upon the above findings, this chapter contributes to the existing literature extending the analysis of optimal policy in the search and matching equilibrium framework, with productivity investment and risk neutral workers. I analyse different policies as training subsidy, participation fee entry and job creation subsidy to get efficient allocation in an economy with education investment. I show the key roles of these policies to achieve efficiency.

The rest of the paper is structured as follows. In the second section I present the model and review market equilibrium solution from the first chapter following the social planner problem. In the third part I show that the decentralised solution is inefficient. In the fourth section I focus on the three different policies that can be implemented by the planner that may achieve an efficient allocation. Finally I summarise the main finding of this chapter.

## 2.2 Model

### 2.2.1 Basic Framework

The present model builds on the first chapter. This analysis considers only steady state, where time is continuous and the economy is composed of workers and employers. All employers are identical whose number is endogenously determined by a standard free entry condition. Workers differ in their abilities. Given their ability $a^i$, each entrant type $i$ first invests in education and so determines his/her productivity level $x$. Let $C(x; a_i)$ denote the cost of investing to productivity $x$ given initial ability $a_i$. Assume $C(.)$ is strictly increasing, convex and twice differentiable in $x$. $C(.)$ is decreasing in $a$ (it is less costly for a higher ability to achieve a given productivity level) and $C_{xa} < 0$ so that higher ability types face a lower marginal cost to achieving a higher productivity level $x$. For ease of exposition assume the Inada condition $C_x(a_i, a_i) = 0$.

There is turnover of workers where $\phi$ is the inflow of new entrants and all workers die according to a Poisson process with parameter $\phi$. Thus steady state implies there is a unit measure of workers in the economy. There are two types of entrants: those with low-ability ($a_l$) and those with high-ability ($a_h$). Let $i \in \{l, h\}$

denote a worker's type.The proportion of high ability type is $1-\eta$.

After investing in productivity $x$, a type $i$ entrant decides whether to search or not. I define those who choose to search as active job seekers, all others are inactive (non-participant). Clearly as education is costly, those who choose to be inactive will also choose zero education. Conversely the Inada condition ensures those who are active choose a strictly positive education level. Of those who choose to be active, let $1-\pi$ denote the proportion who are high ability.

To fill a job, an employer must first create a vacancy at flow cost $k$. If $V$ denotes the number of vacancies and $U = U_l^A + U_h^A$ the number of active unemployed job seekers, then the match flow is described by a matching function $M = M(U, V)$ which is increasing in both arguments and has constant returns. Let $\theta = \frac{V}{U}$ denote market tightness. As active job seekers meet vacancies at rate $\frac{M}{U}$, standard arguments imply this job contact rate is $m(\theta) \equiv M(1, \theta)$ and $m(.)$ is an increasing concave function. Similarly $\frac{m(\theta)}{\theta}$ is the rate at which a firm holding a vacancy contacts an active job seeker. Random search implies $(1-\pi)\frac{m(\theta)}{\theta}$ is the rate at which a firm contacts an active job seeker with high ability. Finally, job matches break up at an exogenous rate $\delta$ in which case the worker returns to the pool of unemployed workers. In the previous chapter of this thesis, I derived a set of necessary conditions of market solution. I focused on the part two of chapter one which lays out the model with two types of workers. Lets recall the definition of market equilibrium from previous chapter:

*ME1': worker participate in the labor market if and only if $a_i \geq a^c$ where:*

$$V_U(x^*(a^c, \theta), \theta) - C(x^*(a^c, \theta), a^c) = \frac{b}{r+\phi} \tag{2.1}$$

*ME2': active participants choose optimal productivity choice $x^*$ where:*

$$\frac{m(\theta)\beta}{(r+\phi+\delta+\beta m(\theta))(r+\phi)} = \frac{\partial C}{\partial x}(x^*(a_i,\theta),a_i) \tag{2.2}$$

*ME3': free entry condition:*

$$k = \frac{m(\theta)}{\theta}\frac{1-\beta}{r+\phi+\delta+\beta m(\theta)}[\pi(\theta)x^*(a_l,\theta)+(1-\pi(\theta))x^*(a_h,\theta)-b] \tag{2.3}$$

*ME4': the proportion of active workers who are type $i$ consistent with steady state turnover; when low-type are active then $\pi(\theta)=\eta_l$ and when low-type is inactive $\pi(\theta)=0$.*

In the next section, I compare private and social necessary conditions. I show that private returns are not equal to social returns, since the individuals do not internalize the effect of their productivity investment choice and their participation choice on firm's incentives to post vacancy and on the other workers with different abilities. I show this point formally in the next section. For this purpose, I consider the problem of the benevolent social planner.

### 2.2.2 Social Planner's Problem

Following Hosios (1990), I solve the social planner problem which determines the efficient allocation on the above economy. I assume the planner's discount rate equals to zero. By allowing this assumption I simplify the analysis to compare steady-state solutions rather than having to determine the discounted value of the change in some variable along the convergent path from one solution to another. The Planner chooses $\{E_i, U_i^A, V, x_i\}$ to maximise steady state aggregate net of output minus productivity investment and search cost in the economy, where $E_i$ is the number of type $i$ employed worker , $U_i^A$ is the number of type $i$ active unemployed workers, $V$ is the number of total vacancies in the economy and $x_i$ refers to the flow output produced by low/high ability worker. For simplicity the planner problem is :

$$\max_{E_i, x_i, U_i^A, V} P = \Sigma_{i=l,h}(E_i x_i + [\eta_i - E_i]b) - \Sigma_{i=l,h}(\phi[U_i^A + E_i])C(x_i; a_i) - kV. \quad (2.4)$$

Welfare is the sum of output produce by active low/high ability job seekers net of unemployment benefit to the inactive unemployed workers, minus the sum of investment productivity costs for both types and the total cost of posting vacancies in the economy. The planner should maximises $P$ subject to the steady sate turnover[6] :

$$U_i^A(\phi + m(\theta)) = \delta E_i + \phi(U_i^A + E_i) \quad \textit{for} \;\; i = l, h \quad (2.5)$$

where $\theta = \frac{V}{U_l^A + U_h^A}$. Since $m(\theta)$ is the arrival rate of vacancies and $\phi$ is arrival rate of new entrants, the flow of low/high ability job seekers out of unemployment is $(m(\theta) + \phi)U_i^A$. The corresponding flow into active unemployment is $\delta E_i + \phi(U_i^A + E_i)$ where $E_i$ is the number of low/high ability employed worker in the labor market. Lets define a control $\lambda_i = \frac{E_i + U_i^A}{\eta_i}$ where $0 \leq \lambda_i \leq 1$. $\lambda_i$ defines the proportion of type $i$ who are active. Clearly if both types choose $\lambda_i = 1$, then the number of unemployed workers is same as number of active workers in this economy. Consequently the proportion of active workers who are low/high ability type is equal to the number of workers of low/high type. Also if $\lambda_i = 0$ then no one participate in the labor market ,i.e. $E_i = -U_i^A = 0$. Using the expression for

[6]The detailed solution of Planner problem is in the Appendix 2A.

$\pi_i$ where

$$\pi_i = \frac{\eta_i[\frac{E_i+U_i^A}{E_i+U_i}]}{\eta_l[\frac{U_l^A+E_l}{E_l+U_l]} + \eta_h[\frac{U_h^A+E_h}{U_h+E_h}]}$$

which denotes the proportion active agents who are low ability in the economy, if $\lambda_i = 0$ then $\pi_i = 0$ and for the case that $\lambda_i = 1$ then $\pi_i = \eta_i$. Given $\lambda_i$ and the steady state turnover constraint (2.5) then the planner's problem equation (2.4) reduces to:[7]

$$\max_{x_i,\lambda_i,\theta} \mathbb{P} = \Sigma_{i=l,h}\frac{\lambda_i\eta_i m(\theta)}{\phi+\delta+m(\theta)}x_i + \Sigma_{i=l,h}[\eta_i - \frac{\lambda_i\eta_i m(\theta)}{\phi+\delta+m(\theta)}]b- \tag{2.6}$$

$$k\frac{\theta(\phi+\delta)}{m(\theta)+\phi+\delta}\Sigma_{i=l,h}(\lambda_i\eta_i) - \Sigma_{i=l,h}\phi[\eta_i\lambda_i C(x_i;a_i)].$$

This is the standard optimisation problem solved by the Lagrangian method. The necessary conditions for optimality are described in Appendix 2B.

In the standard matching model, the decentralised allocation is inefficient unless the so-called Hosios Condition holds.[8] To highlight the novel inefficiency, a series of possible optimal productivity investment, labor market participation and vacancy creation decision externalities are explained in the next section. Later I assume the planner has three tools to alter the market outcome.

[7]See Appendix for the complete solution.

[8]This Condition states that without a capital choice, the equilibrium is optimal if and only if the worker's bargaining share is equal to the elasticity of the matching function with respect to the number of vacancies.

## 2.3 Efficiency

### 2.3.1 Socially Efficient Labor Market Tightness

Using the first order conditions presented in the previous part, I solve the efficient labor market tightness $\theta$.

**PROPOSITION 2.1.** The socially efficient labor market tightness is given by:

$$k = \frac{m'(\theta)}{(m(\theta) + \phi + \delta - m'(\theta)\theta)}[\frac{\lambda_l \eta_l}{\lambda_l \eta_l + \lambda_h \eta_h} x_l + \frac{\lambda_h \eta_h}{\lambda_l \eta_l + \lambda_h \eta_h} x_h - b] \tag{2.7}$$

The Hosios rule sets the worker share of the net surplus equal to the elasticity of the matching function with respect to unemployment. It can be written

$$1 - \beta = \frac{m'(\theta)\theta}{m(\theta)}. \tag{2.8}$$

From chapter one I know that the labor market tightness in the decentralised case without policy is given by:

$$k = \frac{m(\theta)(1-\beta)}{\theta(r + \phi + \delta + \beta m(\theta))}[\pi(\theta)x^*(a_l, \theta) + (1 - \pi(\theta))x^*(a_h, \theta) - b] \tag{2.9}$$

If the Hosios condition holds then, given that participation is efficient then the decentralised free entry condition will be equal to the planner solution. (see Appendix 2C)

Consider those individuals that are indifferent to participate in labor market, from the participation constraint lets substitute $x_i - b$ into above market and planner free entry conditions then:

$$k = \frac{m'(\theta)\phi}{m(\theta) - m'(\theta)\theta}[\pi C(x_l; a_l) + (1-\pi)C(x_h; a_h)] \;\; \textit{Planner solution},$$

$$k = \frac{(1-\beta)\phi}{\beta\theta}[\pi C(x_l; a_l) + (1-\pi)C(x_h; a_h)] \;\; \textit{Market solution}.$$

Equating the corresponding social planner and market productivity's investment decision gives $\frac{m'(\theta)\theta}{m(\theta)} = 1 - \beta$. Observe that if the worker is indifferent to participate in the labor market then Hosios Condition will be satisfied. This result extends Hosios'(1990) results, which showed that without a capital choice, the equilibrium is optimal if and only if the worker's bargaining share is equal to the elasticity of the matching function, however, with endogenous capital investment, this bargaining share leads to hold up problems, as shown previously. At the root of excessive of posting vacancies result is the fact that firms create a negative externality when they enter, since they make it harder for the other firms to find workers. Simultaneously, they create positive externality on workers irrespective of their abilities because they increase the probability that workers find employment. Basically increase entry of low ability workers's participation imposes a diseconomy on existing participants and external economy on firms. Hence, firms create more vacancies, leading to further vacancy creation and so on. The balance of these forces is ambiguous in general but depends on the relative share of surplus going to workers and firms and the optimal productivity investment of the workers according to their ability and cost of investment.

### 2.3.2 Socially Efficient Productivity Investment

Using the first order conditions of the planner problem, I can solve the socially efficient productivity levels $x_i$.

**PROPOSITION 2.2.** The socially efficient productivity investment $x_i$ is given by:

$$\frac{\partial C}{\partial x_i} = \frac{m(\theta)}{\phi(\phi + \delta + m(\theta))} \tag{2.10}$$

Proof. solving the necessary condition (2.22) for optimality as described in Appendix 2A complete the proof.

In chapter one I have shown that the solutions for the decentralised case is given by:

$$\frac{\partial C}{\partial x}(x_i^*(\theta), a_i) = \frac{m(\theta)\beta}{(r + \phi + \delta + \beta m(\theta))(r + \phi)} \tag{2.11}$$

[9] Observe that (2.10) represents the marginal cost of productivity investment when the planner choose $x$ optimally. Whereas (2.11) represents marginal cost of productivity investment when the individual chooses her/his productivity optimally. The difference between these solution is in parameter $\beta$ which is the worker bargaining power. Investment in productivity reveals the hold up problem. Hold up arise because workers must invest in productivity before meeting a firm, and firms may reap some of the benefits from larger investments. Therefore, the corresponding social and private marginal investment solutions are equal if and only if worker has got full bargaining power. When individuals make ex ante investments before matching with firms disregard their ability and also wages are determined by ex post bargaining, the equilibrium is inefficient.[10] Wages increase with productivity

[9] As agents optimal expenditure decisions ignore the share to be obtained by their trading partners, agents's search and recruitment expenditures are inefficient, Mortensen(1982a).

[10] This result is related to Acemoglu and Shimer (1999b) findings.

investment, creating hold up problem for unemployed active job seekers' investment, also all the bargaining power is controlled by the workers leading to very high wage level and excessive entry of workers. Clearly with ex ante investments, no bargaining solution achieves efficiency. It is often emphasised that human capital externalities raise output at the aggregate level, here it is clear that the social solution exceed the market solution unless the worker's bargaining power is equal to one.

### 2.3.3 Socially Efficient Participation Level

Using the first order conditions presented in Appendix 2A, I can solve the socially efficient participation level $\lambda_i$.

**PROPOSITION 2.3.** From the social point of view, an individual with productivity $x_i$ will participate in the labor market only if

$$x_i > b + \frac{\phi(\phi+\delta+m(\theta))}{m(\theta)} C(x_i) + \frac{k\theta(\phi+\delta)}{m(\theta)}.$$

The standard search model features a fixed size of the labor force (see, for instance, Pissarides (2000)) while in here I endogenies the labor force participation. With fixed participation, Hosios (1990) showed that the wage rule decentralises the efficient labor market allocation if and only if the bargaining power of the worker equals the elasticity of the number of aggregate matches with respect to the number of individuals searching for the jobs. While the supplies of labour have as yet been endogenous, we are able to determine whether their incentives for entry are efficient. In this case, we can determine the parameter $\lambda_i$ which shows the proportion of high/low ability workers who are active that is those who participate in the labor market. Diamond (1982b) argue that the presence of an additional worker(firm) makes the entry easier(harder) for vacancies to find workers but harder(easier) for workers to find jobs. Observe that labor heterogeneity and cost of investing on productivity make the additional source of inefficiency from those identified by the matching literature. In the chapter one I have shown the participation decision of market solution is given by:

If $x_i > b + \frac{\phi(\phi+\delta+\beta m(\theta)}{\beta m(\theta)} C(x_i^*)$ then participate.

In words, given the efficient optimal productivity investment, efficient labor market participation requires no cost of posting vacancy which can be concluded from equating the numerator of the last part planner participation that is $\frac{k\theta(\phi+\delta)}{m(\theta)}$ to zero.

In the traditional search and matching models we have two traditional externalities. When firms enter the market, they make it harder for other firms to find workers, so a negative externality is happened (congestion externality), but since they increase the probability that workers find employment a positive externality on workers is happened (thick market externality)[see Pissarides (2000)]. These two externalities cancel each other under the Hosios condition. Notice that in my model I find additional externality called "composition externality". It is created by the different types of workers searching for a job with different productivity investment. Therefore, labor market is overcrowded with low productivity workers who search for a job and reduce the probability that high productivity workers match. Clearly these externalities makes the decentralised solution inefficient.

## 2.4 Policy Implications

The next step is to examine whether policy can improve on the decentralised allocation. In this case the government can uses different policy instruments. I study three policy instruments that may allow us to achieve the *First Best* in the economy. I assume that the government apply the principle of targeting[11] and implement the following policies: training subsidy $s$ for those who invest on their education, labour market participation tax $t$ and finally a job creation subsidy $z$. It is also of interest to know more generally how, these policies interact with worker bargaining power to affect efficiency.

### 2.4.1 Optimal Training Subsidy Policy

I first introduce the optimal training subsidy and show that how it interact with worker bargaining power to affect efficiency. Lets introduce $e_i$ which is the difference between the inherent ability $a_i$ and the ex-post productivity $x_i$ of the worker, i.e. $x_i = a_i + e_i$. The productivity investment cost $C(x_i; a_i)$ features the same as

$$C(a_i + e_i; a_i) \equiv \hat{C}(e_i; a_i)$$

So, we can simply define

$$C(x_i; a_i) = C(a_i + e_i; a_i) \equiv \hat{C}(e_i; a_i).$$

[11]The generalisation of the principle of targeting is in line with Dixit, Grossman and Helpman (1996). In their common agency model, more efficient instruments are chosen because the government cares about social welfare.

Introducing the optimal training subsidy per unit of investment $s$ applied by the government leads to

$$[1 - s]\frac{\partial \hat{C}}{\partial e_i}(e_i; a_i) = \frac{\beta m(\theta)}{\phi(\phi + \delta + \beta m(\theta))} \tag{2.12}$$

**PROPOSITION 2.4.** The optimal training subsidy that targets the efficient productivity investment decision level is given by:

$$s^* = 1 - \frac{\beta(\phi + \delta + m(\theta))}{(\phi + \delta + \beta m(\theta))} \tag{2.13}$$

<u>Proof.</u> Substituting (2.10) into (2.12) and rearranging the terms, I find the optimal policy in proposition (2.4). □

**COROLLARY.** The optimal education subsidy rate is given:
If $\beta = 1$ then $s^* = 0$ and
If $\beta < 1$ then $s^* = \frac{(1-\beta)(\phi+\delta)}{\phi+\delta+\beta m(\theta)} > 0$.
Assuming $\beta = 1$ then the optimal education subsidy will be equal to zero and it eliminates the investment decision externality. For the case that worker has some bargaining power but not the full , then optimal training policy is positive. It turns out that a higher worker's bargaining power leads to implement a lower training subsidy in order to restore investment decision efficiency. One way of achieving productivity investment decision efficiency is to raise worker bargaining power so that worker appropriate all of the surplus. This formalises the notation that efficiency requires a solution to the hold up problem. Since firms do not share

in the cost of ex-ante productivity investments, this leads to underinvestment.

### 2.4.2 Participation Tax Policy

I now turn to a formal analysis of the effects of participation fee(tax) entry policy. The reason of introducing this policy is to deter the individuals with low ability to participate in the labor market. Let $t$ be the lump-sum fee entry regardless of skill, so an active unemployed worker value function with this policy is given by:

$$(r+\phi)V_U(x,\theta) = b - t + m(\theta)[V_E(x,\theta) - V_U(x,\theta)] \tag{2.14}$$

and when employed,

$$(r+\phi)V_E(x,\theta) = w(x,\theta) + \delta[V_U(x,\theta) - V_E(x,\theta)]. \tag{2.15}$$

It is simple to show that imposing the participation fee policy the wage will be

$$w^p = \frac{x(r\phi + \delta + m(\theta)\beta) + (b-t)(1-\beta)(r+\phi+\delta)}{(r+\phi+\delta+\beta m(\theta))} \tag{2.16}$$

and accordingly the value of being active unemployed worker substituting (2.17) into (2.14) gives:

$$V_U(x,\theta) = \frac{(b-t)(r+\phi+\delta) + m(\theta)\beta x}{(r+\phi+\delta+\beta m(\theta))(r+\phi)}. \tag{2.17}$$

**PROPOSITION 2.5.** Using the optimal productivity investment policy $s^*$, the optimal labour market tax participation is given by the following condition:

$$t^* = \beta k\theta + \frac{\phi}{\phi+\delta}[(\phi+\delta)(\beta-1) + s(\phi+\delta+\beta m(\theta))C(x;a_i)] \tag{2.18}$$

Proof. Using the decentralised participation constraint and substitution gives,

$$x > b + \frac{t(r+\phi+\delta)}{m(\theta)\beta} + \frac{(r+\phi+\delta+\beta m(\theta))(r+\phi)(1-s^*)}{m(\theta)\beta} \tag{2.19}$$

given the optimal training subsidy policy solution, comparing with the social planner solution, i.e;

$$x > b + \frac{k\theta(\phi+\delta)}{m(\theta)} + \frac{\phi(\phi+\delta+m(\theta)}{m(\theta)}C(x;a_i),$$

rearranging the terms, I find the solution in proposition (2.5). □

If $\beta = 1 \Rightarrow s^* = 0 \Rightarrow t^* = k\theta$, the first thing to note is that if $\beta = 1$ (i.e. worker bargaining power is full) $s^* = 0$ as claimed, so that the optimal labour market participation fee will be equal to $k\theta$. It is also apparent that if $\beta$ is less than one, the optimal fee policy will be $\beta k\theta$. The term $k\theta$ is commonly interpreted as the value of saved hiring costs due to the existence of an additional matched worker, and for the case that $\beta$ is less than one we can see the optimal fee entry is $\beta k\theta$ which often interpreted as the capitalised value built into negotiated wage, shared according the worker's bargaining power. One might conjecture from this that introducing a lump-sum tax entry will be ineffective while it would discourage workers to participate in labor market. It is the interest to know how in equilibrium this type of taxation interact with worker bargaining power to affect efficiency. It turns out that these two policies are closely related.[12]

### 2.4.3 Job Creation Subsidy Policy

Now suppose that some of the fee entry is redistributed as lump-sum payments to firms to subsidised the cost of posting a vacancy. Let this subsidy be $z$. The value of a vacant job is,

$$rJ_V(\theta) = -k + z + \frac{m(\theta)}{\theta}[\pi(J_F(x_l^*(\theta) - J_V) + (1-\pi)(J_F(x_h^*(\theta)) - J_V(\theta)]$$

**PROPOSITION 2.6.** Using the optimal policies $s^*$ and $t^*$, the optimal job creation subsidy $z^*$ is given by,

$$z^* = \Bigg\{[\frac{m'(\theta)}{(m(\theta)+\phi+\delta-m'(\theta)\theta)} - \frac{m(\theta)(1-\beta)}{\theta(r+\phi+\delta+\beta m(\theta))}]\times \tag{2.20}$$

$$[\frac{\lambda_l\eta_l}{\lambda_l\eta_l+\lambda_h\eta_h}(x_l-b) + \frac{\lambda_h\eta_h}{\lambda_l\eta_l+\lambda_h\eta_h}(x_h-b)] - \beta k m(\theta)\frac{1-\beta}{\phi+\delta+\beta m(\theta)}\Bigg\}.$$

Of course this is the solution of the case that $\beta$ is strictly between 0 and 1. Clearly $z^*$ is positive as long as the multiplication of the two bracket on the right hand side of equation(2.20) is greater than $\frac{\beta k m(\theta)(1-\beta)}{\beta m(\theta)+\phi+\delta}$.

Finally, when worker has full bargaining power the optimal job creation subsidy is equal to the social planner's cost of creating a vacancy. That is ,

[12] Bovenberg and Jacobs(2005) consider optimal tax policy where the government taxes labour income but, as workers also underinvest in education, government offers education subsidies.

$$z^* = (\frac{m'(\theta)}{(m(\theta) + \phi + \delta - m'(\theta)\theta)})[\frac{\lambda_l \eta_l}{\lambda_l \eta_l + \lambda_h \eta_h}(x_l - b) + \frac{\lambda_h \eta_h}{\lambda_l \eta_l + \lambda_h \eta_h}(x_h - b)].$$

The detailed solution is in Appendix 2D.

## 2.5 Conclusion

This chapter analyses efficiency in an equilibrium search model with endogenous productivity investment with one-sided heterogeneity. I have shown that the market solution is not efficient, since workers and firms do not internalize the cost of posting a vacancy of the firms, participation decision and productivity investment of the workers. The market solution implies that the productivity investment of the worker is lower than the planner's solution which reveals the hold up problem. It arises because worker must invest in productivity before meeting a firm and firm reaping some of the benefits from the worker's investment. The decentralised solution implies that workers with low productivity will participate in the labour market, therefore job creation and labour market tightness will not be equal to social planner case. Therefore, the number of workers with low productivity in the economy is high and the job creation is low.

Since the market solution is not efficient, optimal policies are required. Assuming the government observes the worker's eduction, I consider participation labour market tax policy where the government taxes participant workers, as workers underinvest in education, it in addition offers education subsidies. I show that training subsidy for those who decide to participate in the labor market, increases the incentive to invest on productivity. On the other hand, training subsidy for those who are low ability type will increase incentive to be active. The introduction of a participation tax will have a perverse effects; it deters workers incentive to enter to the labor market since it is a kind of tax(fee) required to be paid as soon as he/she enters labor market.

This effect is more obvious for workers with low ability, it reduces the incentive for them to participate in the labour market. Of course the income flow of inactive unemployed workers(unemployment benefit) is important determinant of the participation and investment decision. Not surprisingly taxes on labor market participation and training subsidy distort human capital investment and participation decision at different ability levels. But these distortions are potentially high for those individuals at the participation margin, whose abilities are close to threshold(critical) ability, i.e. who are indifferent to participate in the labour market. The other additional insight from the paper is that the planner, using principle of targeting internalizes the externality by means of the efficient instrument, i.e. the one that aims directly at the source.

From the arguments of proceeding chapter it is clear for the two types case of workers that the degree of inefficiency of equilibrium turns on the degree of worker bargaining power, $\beta$. I remark throughout the paper that one way of achieving efficiency is to raise worker bargaining power so that workers appropriate all of the surplus. One might conjecture that how introducing participation fee $\tau$, job creation subsidy $z$ and training subsidy $s$ in equilibrium they interact with worker bargaining power to affect efficiency. It turns out that these optimal policies are closely related. Applying the principle of targeting shows when workers achieve all the surplus the optimal training subsidy equal to zero and the optimal participation tax will be hence $k\theta$. Therefore the optimal job creation subsidy is positive. If the Hosios condition holds, then given that participation is efficient then the decentralised, free entry condition will be equal to the planner solution.

## 2.6 Appendix 2A

**Social Planner Problem**

Lets assume $r = 0$, Planner maximises steady state flow payoffs. People die at rate $\phi \Rightarrow \phi$ is entry rate.

Lets define $\lambda_i$ the proportion of type $i \in l, h$ who are active; i.e., $\frac{U_i^A + E_i}{\eta_i}$. Clearly if both types choose $\lambda_i = 1$ then the number of unemployed workers is same as number of active unemployed workers in this economy. Consequently the proportion of active workers who are low/high type is equal to the number of workers of low/high type. Also if $\lambda_i = 0$ then $\pi_i = 0$. [13]

The objective function of the social planner is:

$$\max_{\eta_l,\eta_h,\lambda_l,\lambda_h,x_l,x_h,U_l,U_h,V} P = (\eta_l - U_l)x_l + U_l b + (\eta_h - U_h)x_h + U_h b - \phi[\eta_l \lambda_l C(x_l; a_l) + \eta_h \lambda_h C(x_h; a_h)] - kV \tag{2.21}$$

subject to three steady sate turnovers by the following equations:

$$(m(\theta) + \phi)U_i^A = \delta E_i + \phi \eta_i \lambda_i \tag{2.22}$$

$$\phi(U_i - U_i^A) = \phi \eta i(1 - \lambda_i) \tag{2.23}$$

$$U_i + E_i = \eta_i \tag{2.24}$$

[13] $\pi_i = \frac{\eta_i[\frac{E_i + U_i^A}{E_i + U_i}]}{\eta_l[\frac{U_l^A + E_l}{E_l + U_l}] + \eta_h[\frac{U_h^A + E_h}{U_h + E_h}]}$.

and also the market tightness condition:

$$\theta = \frac{V}{U_l^A + U_h^A} \tag{2.25}$$

The first steady state condition; (2.22) is that the flow of low/high ability active unemployed worker out of unemployment equals the flow of low/high ability unemployed worker back into active unemployment. Since $m(\theta)$ is the arrival rate of vacancies and $\phi$ is arrival rate of new entrants, the flow of high/low ability unemployed active workers out of unemployment is $(m(\theta) + \phi)U_i^A$. The corresponding flow into active unemployment is $\delta E_i + \phi\eta_i\lambda_i$. The second steady sate condition is that the flow of inactive workers out of unemployment equals the flow of inactive workers into unemployment. There are $\eta$ high/low ability workers, of whom $1 - \lambda$ are inactive. So the flow into inactive low/high ability unemployment would be $\phi\eta_i(1-\lambda_i)$. Moreover total number of workers having high/low ability is $\eta$ which is sum of low/high employed and unemployed workers in third steady sate condition. Substituting (2.23) and (2.24) into (2.22) gives,

$$U_i^A(\phi + m(\theta)) = \delta E_i + \phi(U_i^A + E_i) \quad \textit{for} \;\; i = l, h \tag{2.26}$$

which implies that

$$U_i^A = \frac{(\phi + \delta)E_i}{m(\theta)} \tag{2.27}$$

Lets define

$$\lambda_i = \frac{E_i + U_i^A}{\eta_i} \tag{2.28}$$

Substituting (2.27) into (2.28) and rearranging in terms of $E_i$ gives,

$$E_i = \lambda_i \eta_i \frac{m(\theta)}{\phi + \delta + m(\theta)} \tag{2.29}$$

Substituting (2.29) into (2.21) the planner problem reduces to

$$\max_{x_i, \lambda_i, \theta} \mathbb{P} = \frac{\lambda_l \eta_l m(\theta)}{\phi + \delta + m(\theta)} x_l + \frac{\lambda_h \eta_h m(\theta)}{\phi + \delta + m(\theta)} x_h + [\eta_l - \frac{\lambda_l \eta_l m(\theta)}{\phi + \delta + m(\theta)}]b+ \tag{2.30}$$

$$[\eta_h - \frac{\lambda_h \eta_h m(\theta)}{\phi + \delta + m(\theta)}]b - k\frac{\theta(\phi + \delta)}{m(\theta) + \phi + \delta}(\lambda_l \eta_l) + (\lambda_h \eta_h) - \phi[\eta_l \lambda_l C(x_l; a_l)] - \phi[\eta_h \lambda_h C(x_h; a_h)]$$

I can rewrite (2.31) function as:

$$\max_{x_i, \lambda_i, \theta} \mathbb{P} = \Sigma_{i=l,h} \frac{\lambda_i \eta_i m(\theta)}{\phi + \delta + m(\theta)} x_i + \Sigma_{i=l,h} [\eta_i - \frac{\lambda_i \eta_i m(\theta)}{\phi + \delta + m(\theta)}]b- \tag{2.31}$$

$$k\frac{\theta(\phi + \delta)}{m(\theta) + \phi + \delta} \Sigma_{i=l,h}(\lambda_i \eta_i) - \Sigma_{i=l,h} \phi[\eta_i \lambda_i C(x_i; a_i)]$$

## 2.7 Appendix 2B

**Social Planner Solution**

Using the Lagrangian Method I can solve this standard optimisation problem as:

This problem satisfies the following first order conditions:

$$\frac{\partial \mathbb{P}}{\partial x_i} = \frac{\partial C}{\partial x_i} - \frac{m(\theta)}{\phi(\phi + \delta + m(\theta))} = 0 \tag{2.32}$$

$$\frac{\partial \mathbb{P}}{\partial \lambda_i} = x_i - b - \frac{\phi(\phi+\delta+m(\theta)}{m(\theta)}C(x_i;a_i) - \frac{k\theta(\phi+\delta)}{m(\theta)} = 0 \tag{2.33}$$

$$\frac{\partial \mathbb{P}}{\partial \theta} = k - \frac{m'(\theta)}{(m(\theta)+\phi+\delta-m'(\theta)\theta)}[\frac{\lambda_l\eta_l}{\lambda_l\eta_l+\lambda_h\eta_h}x_l + \frac{\lambda_h\eta_h}{\lambda_l\eta_l+\lambda_h\eta_h}x_h - b] = 0 \tag{2.34}$$

Rearranging (2.32)

$$\phi\eta_i\lambda_i = (m(\theta)+\phi)U_i^A - \delta(\eta_i\lambda_i - U_i^A) \tag{2.35}$$

$$\eta_i\lambda_i = (\frac{m(\theta)}{\phi+\delta}+1)U_i^A \tag{2.36}$$

## 2.8 Appendix 2C

Consider those individuals that are indifferent to participate in labor market, from the participation constraint lets substitute $x_i - b$ into above market and planner free entry conditions then:

$$k = \frac{m'(\theta)\phi}{m(\theta)-m'(\theta)\theta}[\pi C(x_l;a_l) + (1-\pi)C(x_h;a_h)] Planner solution \tag{2.37}$$

$$k = \frac{(1-\beta)\phi}{\beta\theta}[\pi C(x_l;a_l) + (1-\pi)C(x_h;a_h)] Market solution \tag{2.38}$$

when the worker is indifferent to participate equality holds then (**??**) and (**??**) are satisfied which implies

$$\frac{m'(\theta)\phi}{m(\theta)-m'(\theta)\theta} = \frac{(1-\beta)\phi}{\beta\theta}$$

then

$$\frac{m'(\theta)}{m(\theta)-m'(\theta)\theta} = \frac{(1-\beta)}{\beta\theta}$$

dividing both sides by $m(\theta)$

$$\frac{\frac{m'}{m(\theta)}}{1-\frac{m'\theta}{m(\theta)}} = \frac{1-\beta}{\beta\frac{m(\theta)\theta}{m'(\theta)}}$$

rearranging the terms proves that Hosios Condition,i.e; $1-\beta = \frac{m'(\theta)\theta}{m(\theta)}$ satisfies.

## 2.9 Appendix 2D

**Optimal Policies**

$$(r+\phi)V_U(x,\theta) = b - t + m(\theta)[V_E(x,\theta) - V_U(x,\theta)] \tag{2.39}$$

$$(r+\phi)V_E(x,\theta) = w(x,\theta) + \delta[V_U(x,\theta) - V_E(x,\theta)] \tag{2.40}$$

$$rJ_V(\theta) = -k + \frac{m(\theta)}{\theta}[\pi(J_F(x_l^*(\theta)) - J_V(\theta)) + (1-\pi)(J_F(x_h^*(\theta)) - J_V(\theta)] \tag{2.41}$$

$$rJ_F(x,\theta) = x - w(x,\theta) + (\delta+\phi)[J_V(\theta) - J_F(x,\theta)] \tag{2.42}$$

$$w^N(x,\theta) = \frac{(r+\phi+m(\theta)+\delta)x\beta + (r+\phi+\delta)(1-\beta)(b-t)}{(\beta m(\theta)+r+\phi+\delta)} \tag{2.43}$$

$$V_u(x,\theta) = \frac{(b-t)(r+\phi+\delta) + m(\theta)\beta x}{(r+\phi+\delta+\beta m(\theta))(r+\phi)} \tag{2.44}$$

**Investment Decision**

$$C(x_i; a_i) = C(a_i + e_i; a_i) = \hat{C}(e_i; a_i)$$

where $x_i = a_i + e_i$

*The Planner Solution*

$$\frac{\partial C}{\partial x} = \frac{m(\theta)}{\phi(\phi + \delta + m(\theta))}$$

*The Market with policy Solution*

$$(1 - s)\frac{\partial \hat{C}}{\partial e_i} = \frac{\beta m(\theta)}{\phi(\phi + \delta + \beta m(\theta))}$$

<u>Optimal Training Policy</u>

if $\beta = 1 \Rightarrow s^* = 0$

if $\beta < 1 \Rightarrow s^* = 1 - \frac{\beta(\phi+\delta+m(\theta))}{(\phi+\delta+\beta m(\theta))} > 0$

**Participation Decision**

$V_U(x, \theta) - C(x; a_i) > \frac{b}{r+\phi}$ then participate

*The Planner Solution*

If $x_i > b + \frac{\phi(\phi+\delta+m(\theta)}{m(\theta)} C(x_i^*) + \frac{k\theta(\phi+\delta)}{m(\theta)}$ then participate

*The Market with Policy Solution*

If $x_i > b + \frac{t(r+\phi+\delta)}{m(\theta)\beta} + \frac{(r+\phi+\delta+\beta m(\theta))(r+\phi)(1-s))}{m(\theta)\beta} C(e_i; a_i)$ then participate

<u>Optimal Participation fee</u>

$$t^* = k\theta\beta + [\phi(\phi + \delta + m(\theta))\beta - \phi\beta(\phi + \delta + m(\theta))] \frac{1}{\phi + \delta}$$

if $\beta = 1 \Rightarrow s^* = 0 \Rightarrow t^* = k\theta$

if $\beta < 1 \Rightarrow s^* = 1 - \frac{\beta(\phi+\delta+m(\theta)}{(\phi+\delta+\beta m(\theta)} >)0 \Rightarrow t^* = \beta k\theta$

**Free Entry Condition**

*The Planner Solution*

$k = \frac{m(\theta)(1-\beta)}{\theta(r+\phi+\delta+\beta m(\theta))} [\pi(x_l^*(\theta) - b) + (1 - \pi)(x_h^*(\theta) - b)]$

*The Market with Policy Solution*

$$k = \frac{m(\theta)}{\theta}\frac{1-\beta}{r+\phi+\delta+\beta m(\theta)}[\pi(x_l^*(\theta)+b-t)+(1-\pi)(x_h^*(\theta)+b-t)]+z$$

<u>Optimal Job Creation Subsidy</u>

$$z^* = (\frac{m'(\theta)}{(m(\theta)+\phi+\delta-m'(\theta)\theta)} - \frac{m(\theta)(1-\beta)}{\theta(r+\phi+\delta+\beta m(\theta))})\times \tag{2.45}$$

$$[\frac{\lambda_l\eta_l}{\lambda_l\eta_l+\lambda_h\eta_h}(x_l-b)+\frac{\lambda_h\eta_h}{\lambda_l\eta_l+\lambda_h\eta_h}(x_h-b)] - t^*(\frac{m(\theta)}{\theta}\frac{1-\beta}{\phi+\delta+\beta m(\theta)})$$

if $\beta = 1 \Rightarrow s^* = 0 \Rightarrow t^* = k\theta \Rightarrow z^* = (\frac{m'(\theta)}{(m(\theta)+\phi+\delta-m'(\theta)\theta)})[\frac{\lambda_l\eta_l}{\lambda_l\eta_l+\lambda_h\eta_h}(x_l-b)+\frac{\lambda_h\eta_h}{\lambda_l\eta_l+\lambda_h\eta_h}(x_h-b)]$

if $\beta < 1 \Rightarrow s^* = 1 - \frac{\beta(\phi+\delta+m(\theta)}{(\phi+\delta+\beta m(\theta))} >)0 \Rightarrow t^* = \beta k\theta \Rightarrow$

$$z^* = \left\{[\frac{m'(\theta)}{(m(\theta)+\phi+\delta-m'(\theta)\theta)} - \frac{m(\theta)(1-\beta)}{\theta(r+\phi+\delta+\beta m(\theta))}]\right. \tag{2.46}$$

$$\left.\times[\frac{\lambda_l\eta_l}{\lambda_l\eta_l+\lambda_h\eta_h}(x_l-b)+\frac{\lambda_h\eta_h}{\lambda_l\eta_l+\lambda_h\eta_h}(x_h-b)] - t^*\frac{m(\theta)}{\theta}\frac{1-\beta}{\phi+\delta+\beta m(\theta)}\right\}.$$

$$z^* = (\frac{m'(\theta)}{(m(\theta)+\phi+\delta-m'(\theta)\theta)} - \frac{m(\theta)(1-\beta)}{\theta(r+\phi+\delta+\beta m(\theta))})\times \tag{2.47}$$

$$[\frac{\lambda_l\eta_l}{\lambda_l\eta_l+\lambda_h\eta_h}(x_l-b)+\frac{\lambda_h\eta_h}{\lambda_l\eta_l+\lambda_h\eta_h}(x_h-b)] - \frac{\beta k m(\theta)(1-\beta)}{\phi+\delta+\beta m(\theta)}$$

if $\beta = 1 - \frac{m'(\theta)\theta}{m(\theta)} \Rightarrow s^* = \frac{(m(\theta)-m'(\theta)\theta)(\phi+\delta+m(\theta))}{m(\theta)(\phi+\delta+\beta m(\theta))} \Rightarrow t^* = \frac{(k\theta)(m(\theta)-m'(\theta)\theta)}{m(\theta)} \Rightarrow z^* = \frac{k(m(\theta)-m'(\theta)\theta)}{m(\theta)(\phi+\delta+m(\theta)-m'(\theta)\theta)}$

# Chapter 3

# Optimal Social Security and Taxation with Moral Hazard

## 3.1 Introduction

Social security systems are a common feature of all developed economies; they are provided so as to uphold the standard of living of all citizens, regardless of the causes of their poverty.[1] While economists and policymakers are confronted with challenging issues when faced with structuring and adjusting social security systems, there are many concerns that arise with the system. In particular, how it interacts with the economy or the labour market, the interactive effect these systems have on the searcher behaviour to accept the jobs, how it can be adjusted to discourage greater unemployment and more importantly how it can be implemented to induce optimal search by job seekers, are indisputable.

[1]Research suggests that they [unemployment benefit schemes] reduce the aggregate poverty rate by almost one percentage point, Moffitt (2014).

This paper, motivated by these concerns, develops an integrated framework where social security benefits are chosen to induce optimal search by unemployed workers GIVEN the income tax structure imposed by government. Here workers are risk neutral and the tax structure reallocates income from the rich to the poor. Generally, tax policy distorts the willingness of workers to find employment. However, here the novelty is that optimal benefit program is co-ordinated with optimal tax policy.

The planner does not observe job offers and the planner objective here is to maximise an unemployed worker welfare and is allowed three policy variables; (i) a constant social security benefit $b$, (ii) a marginal income tax $\tau$ and (iii) a break-even income level $w_0$, where those instruments must satisfy the incentive compatibility of unemployed worker that the worker adapts reservation wage strategy and contribution constraint.

Several points arise. The effects of the marginal tax rate on the reservation wage of unemployed worker depend on the social security benefit and the break-even income threshold. When the break even level of income is higher than the sum of social security benefit and the opportunity cost of employment, an increase in marginal tax rate makes workers less selective during search and decreases the reservation wage. A lower reservation wage increases the probability of a worker accepting a job. Conversely, when this sum is higher than the break even level, unemployment becomes more attractive compared to employment. The reservation wage also rises. The reservation wage change, the critical response of these results, reflects the workersttitude towards unemployment. The imposition of break even income on the worker reservation wage has more definitive effects. It reduces reservation wage of the worker. Clearly the workers respond by adjusting the reservation wage.

Given the underlying policy trade-off is between social security benefits, marginal income tax and break even income level, all policies play an important role. In the expansive literature on optimal benefits, government tax policy on employed worker wage has been ignored. An important feature of the analysis is that the tax system is related to two stands of planner choices. The first consists of the amount of redistribution one wishes to make across the worker population and the second is related to amount of tax income needs to finance government spending. I bring these two strands in the given tax structure. There is double infinity of efficient tax policies.

Much interest has recently been shown in studying the responsiveness of unemployment or unemployment duration to unemployment benefits. Some work in this area has centred on the responsiveness of reservation wages to benefits, Fishe (1982), Feldstein and Poterba (1984) and Shimer and Werning (2006). Fishe (1982) and Feldstein and Poterba (1984) find that one dollar increase in benefits may raise pretax reservation wage by as much as 0.44 dollar. While Feldstein and Poterba (1984) argue this as the evidence of the moral hazard cost of increasing unemployment benefits. While this evidence (moral hazard problem) is close in spirit to the one I adopt here, I first assume the planner observes each worker type and then suggests a practical tax policy in the case when workers type is private information.

Reducing inequality is an important aspect of economic analysis. Most economists assume that equity and efficiency cannot be achieved together- redistributive transfers increase equity but make a loss in efficiency-the so-called leaky bucket, Okun (1970). He asserts that any dollar transferred from a richer individual to a poorer individual, will result in less than a dollar increase in income for the recipient. According to the substantial empirical evidence, government transfers designed to create greater equity can lead to inefficiencies. For instance, to test the effects of welfare program design on labor supply and well-being of recipients (negative income tax experiments, Burtless(1986)) in 1970s the US funded a series of experiments.While Okuns leaky bucket is a reality for many transfers programs, in a number of real world policy situations, equity and efficiency are not inevitably in conflict with each other, Blank (2002). This paper interestingly, unlike actual social security benefit programs, proves that all should receive the same benefit level based on the equity issue.

Following Diamond/Mirrlees there is a large optimal tax policy literature which considers asymmetric information on worker type. As that literature assumes that the planner knows the distribution of workers type across the entire population, however, the worker's type is unknown information to the planner. The optimal tax policy reduces to a mechanism design problem with truth telling constraint in that framework. Also they set out the problem of using taxation and government production to maximise a social welfare function. They show the optimal redistribution- by using subsidising and taxing different goods to alter individual real incomes- occurs when there is a balance between the equity improvements and the efficiency losses from further taxation.

As mentioned earlier, this paper first assumes the planner observes worker's type and then instead assumes there is asymmetric information where the worker's type is private information plus the planner does not know the distribution of workers types. An important feature of optimal social security benefit policy, given tax parameters, is that it is completely independent of worker's type which implies it is a flat rate paid to all. Clearly this is very different to standard social security benefit schemes where benefits paid are frequently positively related to previous earning.[2]

The remainder of the paper proceeds as follows: The next section presents the model that includes how risk neutral workers behave when confronted with constant social security benefit and given tax system. Then I describe the problem of the government in choosing the optimal level of social security benefit subject to incentive compatibility constraint and the contribution constraint. In section (3) I analyse a practical policy application. In section (4) I discuss some important caveats, in section (5) I discuss about available evidence of UK welfare economy focusing on universal tax credit and I conclude in section (6).

## 3.2 The Model

Time is continuous and has an infinite horizon. Workers are risk neutral, have the same discount rate $r > 0$, are initially unemployed and have the same home productivity $d \geq 0$. Unemployed workers search sequentially for employment with random search. Workers are ex-ante heterogenous in that they face differ job search opportunities: each is described by $\theta = (\alpha, F(.))$ where

(i) $\alpha$ describes the rate the unemployed worker receives job offers, and

(ii) conditional on a job offer, the wage offered is a random draw from $F(.)$

[2] World's best places for unemployment pay such as Norway where unemployed receive 87.6 percent of their previous salaries for five hundred days and in Finland they receive 85.1 percent of their previous salaries for one year. In Sweden, Israel, Japan and Germany, the unemployed can claim benefits worth between 66 percent and 90 percent of their last salaries.
This information obtained on 15/07/20015 from the following web site:
$http : //www.forbes.com/2008/06/27/unemployment - benefits - world - forbeslife - cx_m w_0 627worldunemployment.html$

with support denoted $[\underline{w}, \overline{w}]$.

For the moment I assume the Planner observes each worker's type $\theta$ but relax this assumption later.

While a worker is unemployed, he/she receives social security benefit $b$ (which is assumed duration independent) and chooses an optimal job search strategy to maximise expected discounted lifetime earnings. The Planner does not observe job offers. Thus should the worker reject a job offer, he/she remains unemployed and continues to receive benefit $b$ (with no recall of offers). This generates a standard moral hazard problem.

If the worker accepts job offer $w$ then, for simplicity, I assume the job is for life. There is no on-the-job search. Once employed on wage $w$, the worker faces a linear tax code with marginal income tax rate $\tau \in (0, 1)$. Take home pay is then $w_0 + (1 - \tau)[w - w_0]$ where $w_0$ is referred to as the break even income threshold; i.e. if employed on wage $w < w_0$, the worker is a net benefit receiver from the government, while instead earning wage $w > w_0$ implies a net tax payer. As $\theta$ is observed, at this stage I allow these tax parameters to be type specific.

As workers are risk neutral, there is no loss in generality by assuming workers have no savings (i.e. they simply consume all savings at date $t = 0$ and then proceed optimally). In the absence of any insurance motive, the tax system here is a purely redistributive scheme which requires that each worker $\theta$ contributes some amount $C_\theta$ to the public purse, where contributions $C_\theta$ may vary across types (e.g. high earning types may be required to make a bigger contribution). The novelty here is that the optimal benefit program is co-ordinated with optimal tax policy; i.e. for each $\theta$, the Planner chooses a type specific policy $(b, \tau, w_0)$ to maximise worker welfare, subject to the moral hazard problem that job offers are

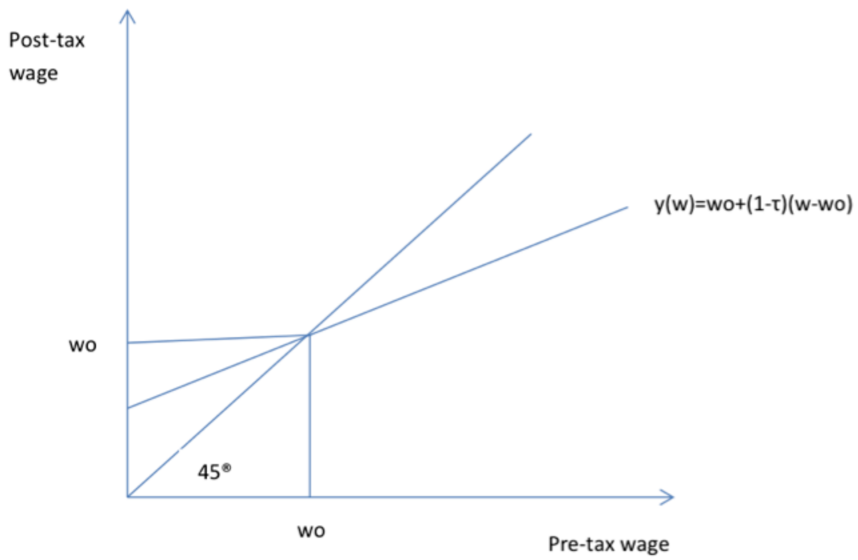


Figure 3.1: Workers Post-Tax Earnings

not observed and that worker $\theta$ must, in expectation, contribute $C_\theta$ to the public purse. Theorem 1 below describes that optimal policy. To establish the result, I first describe worker $\theta's$ optimal job search strategy given tax parameters $(b, \tau, w_0)$.

### 3.2.1 Worker Behaviour

As the worker's type $\theta$ is fixed in this section, for ease of notation I drop reference to $\theta$. As a job is for life and there is no on-the-job search, becoming employed at wage $w$ with tax code $(\tau, w_0)$ implies worker lifetime value:

$$V^e(w) = \frac{w_0 + (1-\tau)[w - w_0]}{r}. \tag{3.1}$$

While unemployed and with random search, the value of being unemployed, denoted $V^u$, satisfies the standard Bellman equation

$$rV^u = b + d + \alpha \int_{\underline{w}}^{\overline{w}} \max\left[V^e(w) - V^u, 0\right] dF(w). \tag{3.2}$$

At rate $\alpha$ the worker receives a job offer which either yields capital gain $V^e(w) - V^u \geq 0$ (and the worker accepts the job), otherwise the worker rejects the job offer (and remains unemployed). Substituting out $V^e(w)$ using (3.1) in (3.2) gives

$$rV^u = b + d + \alpha \int_{\underline{w}}^{\overline{w}} \max\left[\frac{w_0 + (1-\tau)[w - w_0]}{r} - V^u, 0\right] dF(w). \tag{3.3}$$

As $\tau < 1$ implies $V^e(w)$ is increasing in wage $w$, the worker's optimal job search strategy has the reservation wage property[3]: the worker accepts any job offer $w \geq R$ where

$$\frac{w_0 + (1-\tau)[R - w_0]}{r} = V^u. \tag{3.4}$$

Using (3.4) to substitute out $V^u$ in the Bellman equation above and simplifying yields the reservation wage equation:

$$R = \frac{b + d - \tau w_0}{1 - \tau} + \frac{\alpha}{r} \int_R^{\overline{w}} [w - R]\, dF(w). \tag{3.5}$$

(3.5) determines the worker's reservation wage $R = R(b, w_0, \tau)$ as a function of the government's tax parameters $(b, w_0, \tau)$ (and also the worker's type $\theta$). Claim 1 shows how the policy parameters $(b, w_0, \tau)$ distort job search incentives.

[3]for $w < R$ implies $V^e(w) < V^u$ and so $w$ will be rejected, and $w > R$ implies $V^e(w) > V^u$ and so $w$ should be accepted.

**CLAIM 1.** For $\tau \in (0, 1)$, the reservation wage $R(.)$ implies

$$\frac{\partial R}{\partial b} = \frac{1}{(1-\tau)[1+\frac{a(1-F(R))}{r}]} > 0; \tag{3.6}$$

$$\frac{\partial R}{\partial \tau} = \frac{b+d-w_0}{(1-\tau)^2[1+\frac{a(1-F(R))}{r}]} \tag{3.7}$$

$$\frac{\partial R}{\partial w_0} = \frac{-\tau}{(1-\tau)[1+\frac{a(1-F(R))}{r}]} < 0 \tag{3.8}$$

As is standard an increase in $b$ increases the worker's reservation wage $R$. An increase in the break even level of income $w_0$ instead decreases the reservation wage. This occurs for an increase in $w_0$ increases the worker's take-home pay at every wage which, by increasing the return to taking a job offer, decreases the worker's reservation wage. The impact of an increase in the marginal tax rate $\tau$, however, is ambiguous. If the break even level of income is low, $w_0 < b + d$, then an increase in the marginal tax rate increases the worker's reservation wage; the worker substitutes into unemployment. But the converse holds if $w_0 > b+d$, in that case an increase in the marginal tax rate yields the counterintuitive result that the worker's reservation wage decreases and the worker substitutes into employment.

### 3.2.2 The Planner Problem

By assumption the worker's type $\theta$ is observed, the worker is initially unemployed and the Planner requires this worker to pay expected discounted tax $C_\theta$. Given policy parameters $(b, w_0, \tau)$ and the worker's type $\theta$, the worker adopts reservation wage strategy $R = R(b, w_0, \tau)$ as described by (3.5). The worker's expected discounted contribution to the public purse, $C_0$, is then identified recursively by

$$rC_0 = -b + \alpha \int_R^{\overline{w}} \left[ \frac{\tau(w - w_0)}{r} - C_0 \right] dF(w), \tag{3.9}$$

where $R = R(b, w_0, \tau)$. While unemployed the worker receives $b$ from the government. Given reservation wage $R$, the worker exits unemployment at rate $\alpha[1 - F(R)]$ and thereafter pays tax on future realised earnings $w$. Of course conditional on taking a job $w \geq R$, the wage earned is a random draw from the wage distribution $F(.)$ truncated at $R$, and $\left[\frac{\tau(w-w_0)}{r} - C_0\right]$ is the corresponding capital gain in tax receipts enjoyed by the government when the worker accepts a job offer. Rewriting for $C_0$, noting that worker $\theta$'s lifetime tax contribution $C_0$ must equal $C_\theta$, the Planner's tax policy $(b, w_0, \tau)$ must satisfy the contribution constraint:

$$\frac{-b + \frac{\alpha}{r} \int_R^{\overline{w}} \tau(w - w_0) dF(w)}{1 + \frac{\alpha}{r}[1 - F(R)]} = rC_\theta, \tag{3.10}$$

with $R = R(b, w_0, \tau)$. The government's policy is thus to choose $(b, \tau, w_0)$ to maximise $V^u$, the expected lifetime utility of the initially unemployed worker subject to (i) incentive compatibility - that the worker adopts reservation wage strategy $R = R(.)$ and (ii) the contribution constraint - that the policy generates net contribution $C_\theta$. Noting $V^u$ is given by (3.4), the Planner's problem is formally defined as follows.

***Definition of the Planner's Problem:***

$$\max_{b,\tau,w_0,R} \frac{\tau w_0 + (1-\tau)R}{r}$$

subject to

(i) incentive compatibility - that $R = R(b, \tau, w_0)$ is the solution to (3.5), and

(ii) contribution constraint - that (3.10) is satisfied.

Fortunately there is an elegant way to solve this policy problem. The first step is to solve for the optimal level of $b$ using a policy perturbation argument. Suppose a policy optimum exists, denoted $(b^*, \tau^*, w_0^*)$, where the worker adopts corresponding reservation wage $R = R^*$. Consider a policy perturbation where the Planner chooses $b = b^* + db$ where $db \neq 0$ but small. Suppose the Planner in addition chooses policy variations $(d\tau, dw_0)$ so that the worker's reservation wage is unchanged; i.e. $dR = 0$, and the worker's contribution $C_0$ is also unchanged. As such a policy variation remains incentive compatible and meets the contribution constraint, a necessary condition for optimality is it does not increase $V_u$.

To hold $R$ constant, a first order Taylor expansion on (3.5) implies $(db, d\tau, dw_0)$ must satisfy

$$dR = \frac{1}{1-\tau}db - \frac{\tau}{1-\tau}dw_0 + \frac{b+d-w_0}{(1-\tau)^2}d\tau = 0. \tag{3.11}$$

Hence the constraint $dR = 0$ requires $(dw_0, d\tau)$ satisfy:

$$\tau dw_0 - \frac{b+d-w_0}{(1-\tau)}d\tau = db. \tag{3.12}$$

As a policy perturbation satisfying (3.12) ensures $dR = 0$, equation (3.10) and the constraint $dC_0 = 0$ additionally requires $(db, d\tau, dw_0)$ satisfy

$$-db + \left[\frac{\alpha}{r}\int_R^{\overline{w}}(w - w_0)dF(w)\right]d\tau - \frac{\alpha}{r}\tau[1 - F(R)]dw_0 = 0$$

which I rearrange as:

$$\left[\frac{\alpha}{r}\int_{R}^{\overline{w}}(w-w_0)dF(w)\right]d\tau-\frac{\alpha}{r}\tau[1-F(R)]dw_0=db \tag{3.13}$$

Thus for $db \neq 0$, (3.12) and (3.13) imply $dR = dC_0 = 0$ if and only if $(dw_0, d\tau)$ satisfy:

$$\begin{bmatrix} \tau & -\frac{b+d-w_0}{(1-\tau)} \\ -\frac{\alpha}{r}\tau[1-F(R)] & \frac{\alpha}{r}\int_R^{\overline{w}}(w-w_0)dF(w) \end{bmatrix}\begin{bmatrix} \mathrm{dw}_0 \\ \mathrm{d}\tau \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \end{bmatrix}db.$$

Let $\Delta = \frac{\alpha\tau}{r}\left[\int_R^{\overline{w}}(w-w_0)dF(w)-[1-F(R)]\frac{b+d-w_0}{(1-\tau)}\right]$. Then as long as $\Delta \neq 0$, perturbation $(db, d\tau, dw_0)$ holds $dR = dC_0 = 0$ if and only if:

$$\begin{bmatrix} \mathrm{dw}_0 \\ \mathrm{d}\tau \end{bmatrix}=\begin{bmatrix} \frac{\frac{\alpha}{r}\int_R^{\overline{w}}(w-w_0)dF(w)+\frac{b+d-w_0}{(1-\tau)}}{\Delta} \\ \frac{\tau[1+\frac{\alpha}{r}[1-F(R)]]}{\Delta} \end{bmatrix}db.$$

Consider then the first order impact of this policy perturbation on the objective function; i.e

$$rdV_u = [w_0 - R]d\tau + \tau dw_0.$$

Substituting out $(d\tau, dw_0)$ using the above implies:

$$rdV_u=\left[[w_0-R]\frac{\tau[1+\frac{\alpha}{r}[1-F(R)]]}{\Delta}+\tau\frac{\frac{\alpha}{r}\int_R^{\overline{w}}(w-w_0)dF(w)+\frac{b+d-w_0}{(1-\tau)}}{\Delta}\right]db.$$

The necessary condition for optimality is $dV_u/db = 0$, otherwise a policy variation with $db \neq 0$ exists which strictly increases welfare and satisfies the constraints.

Hence a necessary condition for optimality is

$$\left[ [w_0 - R][1 + \frac{\alpha}{r}[1 - F(R)]] + \frac{\alpha}{r} \int_R^{\overline{w}} (w - w_0) dF(w) + \frac{b + d - w_0}{(1 - \tau)} \right] = 0.$$

Rewriting this equation for $b$ and simplifying yields Lemma 1.

**LEMMA 1.** In the Planner's problem, optimal $b$ satisfies

$$b = \tau w_0 - d + (1 - \tau)R - \frac{\alpha(1 - \tau)}{r} \int_R^{\overline{w}} [w - R] \, dF(w). \tag{3.14}$$

The interpretation for optimal $b$ in Lemma 1 is complex for it applies for arbitrary tax parameters $(\tau, w_0)$. The structure of the optimal policy will become transparent once optimal $(\tau, w_0)$ are also determined.

Given Lemma 1, I now use (3.14) to substitute out optimal $b$ in the Planner's problem and so reduce the dimensionality of the programming problem. This generates, however, a curious property: equation (3.14) describing the optimal choice of $b$ is equivalent to the worker's reservation wage equation (3.5) determining $R = R(b, \tau, w_0)$. The insight for what follows is that the incentive compatibility constraint is not binding at the policy optimum.

Substituting out $b$ using (3.14) in the optimal program, the Planner's problem formally reduces to

$$\max_{\tau, w_0, R} \tau w_0 + (1 - \tau)R \tag{3.15}$$

subject to the contribution constraint

$$\frac{-\left[\tau w_0 - d + (1-\tau)R - \frac{\alpha(1-\tau)}{r}\int_R^{\overline{w}}[w-R]\,dF(w)\right] + \frac{\alpha}{r}\tau\int_R^{\overline{w}}(w-w_0)dF(w)}{[1+\frac{\alpha}{r}[1-F(R)]]} = rC_\theta. \tag{3.16}$$

where optimal $b$ given by (3.14) ensures the worker adopts the optimal reservation wage $R^*$; i.e. (3.14) ensures the incentive compatibility constraint is automatically satisfied.

I solve this latter problem by substituting out the constraint (3.16). Rewriting (3.16) for $w_0$ and simplifying yields

$$w_0 = \frac{-[r+\alpha[1-F(R)]]C_\theta + d - (1-\tau)R + \frac{\alpha}{r}\tau\int_R^{\overline{w}} wdF(w) + \frac{\alpha(1-\tau)}{r}\int_R^{\overline{w}}[w-R]\,dF(w)}{\tau[1+\frac{\alpha}{r}[1-F(R)]]}. \tag{3.17}$$

Substituting out $w_0$ in the objective function and simplifying finds the Planner's problem reduces to

$$\max_{\tau,R}[-rC_\theta + \frac{d+\frac{\alpha}{r}\int_R^{\overline{w}} wdF(w)}{[1+\frac{\alpha}{r}[1-F(R)]]}]. \tag{3.18}$$

As the objective function does not depend on $\tau$, the optimal choice of $\tau$ is not uniquely determined. The necessary condition for optimal $R$ implies (3.21) below and so I identify the following optimal policy.

**THEOREM 1.** For any $\theta$ and contribution $C_\theta$, the optimal policy (b,$\tau, w_0$) is not uniquely determined. For any $\tau \in (0,1)$, the Planner sets social security benefit

$$b = \tau(w_0 - d), \tag{3.19}$$

with $w_0$ given by

$$w_0 = R^* - \frac{rC_\theta}{\tau}. \tag{3.20}$$

where the worker's reservation wage $R^*$ solves

$$R^* = d + \frac{\alpha}{r}\int_{R^*}^{\overline{w}}[w - R^*]dF(w). \tag{3.21}$$

Proof. (3.21) was established in the text. As (3.14) describes optimal $b$, then using (3.21) to substitute out $R = R^*$ in (3.14) yields (3.19). Using (3.21) to simplify (3.17) implies (3.20) satisfies the contribution constraint. This completes the proof of Theorem 1.

Note that (3.21) implies the optimal reservation wage $R^*$ is consistent with worker $\theta$ receiving his/her opportunity cost of employment $d$ while searching and enjoying the full return to job search (i.e. with no taxation on earnings). Given the Planner has three policy instruments $(b, \tau, w_0)$ and only two restrictions, (i) to implement reservation wage $R^*$ and (ii) satisfy budget balance (3.10), it is not then surprising that there is a degree of freedom in the optimal tax policy. Conditional on *any* choice $\tau \in (0, 1)$, the optimal break-even threshold $w_0$ depends on the worker's reservation wage and the required contribution $C_\theta$: the larger the required contribution the lower the break-even threshold. What is remarkable, however, is that conditional on the marginal tax rate $\tau$ and break-even threshold

$w_0$, the optimal social security benefit policy $b$ is independent of the worker's type $\theta$. I now argue how Theorem 2 suggests a very simple and practical tax policy in the case when $\theta$ is private information.

## 3.3 A Practical Policy Application

The previous section made some very strong, though standard assumptions, namely (i) each person's type $\theta = (\alpha, F(.))$ is observed, and (ii) the Planner decides ex-ante how much contribution $C_\theta$ each individual $\theta$ should contribute to the public purse. Neither assumption is particularly palatable. Following seminal work by Diamond/Mirrlees, there is a large literature on optimal tax policy when there is asymmetric information on worker type $\theta$. But an unappealing assumption in that approach is that although the Planner does not know the worker's type $\theta$, he/she knows the distribution across the entire population. With that assumption, optimal tax policy reduces to a mechanism design problem with truth telling constraints. Unfortunately that approach rarely provides practical implications for optimal tax policy.

Suppose in the above framework that there is not only the moral hazard problem described above, but there is asymmetric information: the worker's type $\theta$ is private information. Furthermore suppose the government does not know the distribution of types $G(\theta)$. How then can the government design an efficient tax structure? Theorem 2 provides the answer.

Consider a universal tax scheme, such as pay as you earn in the UK (PAYE) whereby individuals who earn wage $w$ must pay tax $T(w)$ to the government, where the tax schedule $T(.)$ is the same for all. Suppose further the government chooses a linear PAYE scheme with tax parameters $(\tau, w_0)$. On the assumption that the government observes (or can measure) home productivity $d$, Theorem 2 implies the following efficient tax and benefit policy.

**THEOREM 2.** Given any linear, universal tax scheme $(\tau, w_0)$ with $\tau \in (0, 1)$ then the tax policy is efficient if and only if social security benefit $b = \tau(w_0 - d)$. Proof follows directly from Theorem 1.

Theorem 2 turns Theorem 1 on its head. Rather than identifying the optimal tax and benefit policies given an exogenous distribution of contributions $C_\theta$, Theorem 2 instead identifies the unique optimal unemployment benefit policy given (exogenous) tax parameters $(\tau, w_0)$. The assumption that $\theta$ is private information plays no important role for the optimal benefit paid is independent of $\theta$; i.e. optimal social security benefits $b$ is a flat rate paid to all.

Unlike actual social security benefit programs, Theorem 2 implies (with linear, universal tax schemes) that all should receive the same benefit level. This policy prescription is very different to standard unemployment benefit schemes where benefits paid are frequently positively related to (previous) earnings. Of course an important distinction here is that I am considering an equity issue - the efficient transfer of resources from the well paid to the poorly paid. As the tax policy transfers resources from the well paid to the less well paid while employed, why should it be optimal to pay higher social security benefits to the (previously) highly paid? Theorem 2 establishes benefits should be paid at a flat rate.

There is a double-infinity of efficient tax policies. The choice of tax parameters $(\tau, w_0)$ depends on (i) how much redistribution the Planner wishes to make across the worker population and (ii) how much tax needs to be collected to finance government spending.

For example a Laissez-faire policy which sets $\tau = 0$ (and then efficient $b = 0$) implies zero redistribution across workers. Conversely letting $\tau \to 1$ implies perfect redistribution where, in this limit, all employed workers enjoy the same after-tax wage $w_0$ and efficient $b \to w_0 - d$. Away from these two extremes, inverting equation (3.20) implies the expected contribution of worker type $\theta$ to the public purse is

$$C_\theta = \frac{\tau (R^* - w_0)}{r}$$

where $R^*$ is given by (3.21). As high skilled workers (high $\alpha$ and high wage offers) have high reservation wage $R^*$, the tax program ensures they contribute more to the public purse than do workers with low reservation wages. Indeed workers with $R^* > w_0$ are net contributors to the public purse and contribute more as the tax rate $\tau$ increases, while less skilled workers with $R^* < w_0$ are net benefit receivers who receive more the greater the marginal tax rate. Furthermore increasing the marginal tax rate $\tau$ increases linearly the transfer of resources from high $R^* > w$ workers to low $R^* < w_0$ workers.

The choice of $w_0$ determines gross tax revenues, where decreasing $w_0$ implies all contribute more to the public purse and tax revenues increase. For example if the government needs to reduce the deficit by raising taxes, it does not do this by increasing the marginal tax rate but by reducing the break-even income threshold. Of course such a policy change hits all individuals equally - each person's contribution to the public purse increases by $[\tau/r][-dw_0]$. If this is deemed too regressive, the Planner can additionally increase $\tau$ so that the well paid pay more and the poor pay less. In this way, the government can use tax policy to separately address income inequality and the need to balance its budget while $b$ satisfying (3.19) ensures efficient job search.

## 3.4 Some Important Caveats

There are many possible extensions for this approach. For example suppose there is layoff risk: that at rate $\delta$ the worker is laid-off and becomes unemployed. For the case that workers are risk neutral, it is obvious that the analysis goes through: with no requirement that a worker $\theta$ **must** contribute some ex-ante amount $C_\theta$ to the public purse, there is a double infinity of optimal linear tax schemes $(w_0, \tau)$ where the choice of social security benefit paid ensures the unemployed worker adopts the optimal job search strategy $R^*$. Similarly if there is exogenous on-the-job search. Whether the optimal social security benefit remains a flat rate, however, is not immediately obvious.

The approach ignores ex-ante investment in education by workers: it simply assumes workers are born with search options $(\alpha, F(.))$ while, in truth, young people invest in education so as to improve those opportunities. With a linear tax scheme and optimal benefits, (3.4) implies the initially unemployed worker enjoy value

$$V^u = \frac{w_0 + (1-\tau)[R^* - w_0]}{r}. \tag{3.22}$$

This identifies the fundamental trade-off for the government. Investing in education, presumably, improves job search opportunities: job offer rates $\alpha$ improve and the wage offer distribution $F(.)$ is more favourable. In this framework, changes in $R^*$ is the appropriate metric for measuring the return to education. Although raising the marginal tax rate $\tau$ reduces inequality, the cost is it reduces investment incentives - a well-known trade-off.

The approach also does not consider equilibrium wage formation and, in particular, does not consider how changing the tax program affects wage outcomes. This is an important omission. For example, in the extreme case of $\tau = 1$, there is no purpose to a firm paying a wage greater than $w_0$ as the government fully taxes that wage premium. Indeed the optimal policy of the firm is to set wage $w = 0$ as the employed worker then receives $w_0$ in benefits from the government. Clearly such an economic outcome would not be sustainable. An important issue for the government is to assess by how much an increase in the marginal tax rate $\tau$ depresses wage offers by firms? This is an empirical question which goes far beyond the scope of this paper.

Finally the paper does not consider equilibrium job creation and endogenous job search effort and thus the equilibrium arrival rate of job offers $\alpha$. Clearly charging a positive marginal tax rate $\tau$ and a positive break-even income level $w_0$ extracts rents from high skill workers and passes them to low skill workers. One should expect that such a transfer of employment rents depresses job creation in the high skill sector and increases job creation in the low skill sector. Again this is an interesting theoretical question which I leave to future research.

## 3.5 Universal Tax Credit

A prevalent approach to social security benefits is to target welfare payments to individuals or households whose income falls below a certain level. If a claimant's circumstances change so that he/she earns more, whether by working more hours or by finding better paid employment, the worker will lose their entitlement to some if not all of their income-related benefits. When a country's tax structure is not properly co-ordinated with its social security benefit schemes, poverty traps arise should benefits paid be withdrawn too quickly as earnings increase. Such benefit withdrawals can sometimes generate a 100% marginal tax rate and, with no (or little) financial gain to taking work, unemployed workers have little incentive to find employment. Or put differently, they hold reservation wages which are too high relative to their job opportunities.

An interesting feature of recent tax policy in the UK is the move to universal tax credits. This new tax structure aims to properly co-ordinate benefit claims with income tax payments by replacing six means-tested working age benefits [Jobseekers Allowance, Employment and Support Allowance, Income Support, Housing Benefit, Working Tax Credit and Child Tax Credit] into a single tax credit/payment should their income fall below a particular income threshold. The purpose is not only to simplify the benefits system and co-ordinate it properly with income taxation, but also to remove poverty traps and so provide incentives for poorly paid workers to find employment and reduce in-work poverty. Curiously the intended marginal tax rate is still very high at 65%; i.e. additional earnings reduce UC by 65%.

The marginal effective tax rate (METR) is the percentage of an extra unit of income that the claimant loses due to income taxes, National Insurance Contributions (NICs), etc.[4] Therefore, the taper rate of 65% under UC forms the base of the METR; a claimant liable for National Insurance and paying income tax (at the main rate of 20 percent) who earns an additional pound will not only lose 65p

[4] It the income withdrawn through the tax and benefit system.

of UC but will have to pay an additional 20p in income tax as well as 12p in NICS; creating a METR of 76.2 percent. This demonstrates the substantial effect the METR can have on disposable income and work incentives provided to claimants. Around 4 million workers will see a change in their METR, the most significant of these changes are seen in five major groups.[5],[6]

The paper considered here speaks directly to such a tax system but in the case that all face the same marginal tax rate. In such a tax environment, ensuring the unemployed have optimal job search incentives requires all receive the same, flat level of social security benefit.

[5]The most effective way to comprehensively appraise a tax system and how it affects work incentives is to evaluate the tax rate schedule through the calculation of the marginal effective tax rate and the participation tax rate. These tax rates will principally be analysed with the aid of Brewer, Browne and Jin (2011) research and an impact assessment carried out by the Department of Work and Pensions (DWP).

[6]The marginal effective tax rate (METR) is essential to providing strong incentives to enter employment; Council tax support could sabotage these work incentives. After the 65 percent withdrawal of benefits has been applied to net income, around 20 percent of the remainder may then be withdrawn through CTS. Therefore, claimants who pay NICs and tax, facing both withdrawal of UC and council tax rebate could be facing tremendous METRs. Pennycook, M. and Hurrell, A. (2013) "No Clear Benefit: The financial impact of Council Tax Benefit reform on low income households", estimate marginal deduction rates (MDRs) of over 80 percent; they mention that once Council Tax support has been included, the MDRs of most households receiving both UC and Council Tax support (around 620,000 CTB claimants or 9 in 10 of every current claimant in employment) will rise to around 81 per cent. Accordingly, claimants would lose as much as 81 pence of each additional pound they earn through taxes and withdrawn benefits, obviously dis-incentivising work and creating extreme difficulties for claimants to escape poverty; the poverty trap. he Public Accounts Committee found that for up to 225,000 people, the replacement of CTB has meant a weakening of work incentives, and marginal rates of up to 97 per cent.Finch, Corlett and Alakeson (2014), Universal Credit: A policy under review. London: Resolution Foundation p.22) Moreover, due to the authority granted to local councils, CTS schemes could easily raise withdrawal rates, giving rise to METRs such as 97 percent, exacerbating the negative effects.

## 3.6 Conclusion

Despite extensive attention in the economics literature to the optimal unemployment benefits, there has been little attention focused on social security benefit to induce optimal search by unemployed workers given the income tax structure imposed by the government.

In the case of risk neutral workers but with moral hazard and hidden information on the worker's type, this paper has identified a simple rule which identifies the link between optimal social security benefits paid and the tax system. Specifically in the case of a universal, linear income tax scheme, I have shown optimal social security benefits should be paid at a flat rate; i.e. all receive the same benefit level regardless of (expected future) earnings.

Although this benefit policy generates efficient search incentives and thus optimal re-employment rates, the analysis has not considered how high tax rates will generate other inefficiencies in the economy, such as low education investments by workers and reduced job creation by firms.

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