

# Trading in Networks: Theory and Experiments

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## Abstract

We propose a model of posted prices in networks. The model maps traditional concepts of market power, competition and double marginalization into networks, allowing for the study of pricing in complex structures of intermediation, such as supply chains, transportation and communication networks and financial brokerage.

We provide a complete characterization of equilibrium prices. Our experiments complement our theoretical work and point to *node criticality* as an organizing principle for understanding pricing, efficiency and the division of surplus in networked markets.

**JEL Classification:** C70, C71, C91, C92, D40.

**Keywords:** Intermediation, competition, market power, double marginalization.

## 1 Introduction

Supply, service and trading chains are a defining feature of the modern economy. They are prominent in agriculture, in transport and communication networks, in international trade, in markets for bribes and in finance. Goods and services pass through individuals or firms located along these chains. The routing of economic activity, the earnings of individuals and the efficiency of the system depend on the prices set by these different intermediaries. The aim of this paper is to understand how the network structure of chains shapes market power and thereby determines prices and efficiency.

To fix ideas, consider pricing in a transport network. A tourist wants to travel on the Eurostar from London to Paris to see the Louvre. The first leg of the journey is from Home to St. Pancras Station, using one of a number of different services, such as taxi companies, bus services and the Underground. Once at St. Pancras Station, the only service provider to Paris Nord Station is Eurostar. Upon arriving at Paris Nord, there are a number of alternatives (bus, Metro and taxi) to get to the Louvre. The network consists of alternative paths, each comprised of local transport alternatives in London and in Paris and a common node (the Eurostar Company). Each of the service providers sets a price, and the traveler picks the cheapest ‘path’. Section 2 of this paper develops a number of other applications for which pricing in networks is important.

These examples motivate the following model. There is a source node,  $\mathcal{S}$ , and a destination node,  $\mathcal{D}$ . A path between the two is a sequence of interconnected nodes, each occupied by an intermediary. The source node, the destination node and all the paths between them, together, define a network. The passage of goods from source to destination generates *value*. Intermediaries simultaneously post a price to get a share of this value; the prices determine a total cost for every path between  $\mathcal{S}$  and  $\mathcal{D}$ . We assume that the good moves along a least-cost path and an intermediary earns payoffs only if she is located on it. Posted prices are the norm in transport and communication networks. We characterize the Nash equilibria of the pricing game.

A node is said to be critical if it lies on all paths between  $\mathcal{S}$  and  $\mathcal{D}$ . Our main finding is that *criticality* of nodes defines market power and, consequently, pricing, earnings and the efficiency of economic activity in networked markets. We now elaborate on the scope of this finding and locate it in the context of the literature.

In the benchmark model, intermediaries know the value. We prove existence and provide a complete characterization of Nash equilibrium (Theorem 1). For a given network, there typically exist multiple equilibria: a) they range from efficient to inefficient (where trade breaks down completely) and b) in every efficient equilibrium, all the surplus goes either to  $\mathcal{S}$  and  $\mathcal{D}$  or all of it goes to the intermediaries. The presence of critical traders is *sufficient* but *not necessary* for intermediation rents; non-critical intermediaries may extract rents because intermediaries on competing paths mis-coordinate and price themselves out of contention. In the presence of critical traders, there exist equilibria in which the entire surplus accrues to these traders, but there also exist equilibria in which it is captured by the non-critical intermediaries. Standard equilibrium refinements do not help us in this situation: either they are too demanding and we face non-existence problems, or they are insufficiently restrictive.

To gain a deeper understanding of the relation between networks and market power, we take the model to the laboratory. Our experiments highlight the ability of human subjects to coordinate on efficient outcomes. They show that critical traders set high prices and extract most of the sur-

plus. Thus, our theoretical work and experiments, taken together, establish that the presence of critical intermediaries is *both necessary and sufficient* for large surplus extraction by intermediaries and that most of the surplus does accrue to critical traders.

In markets with multiple vertically related firms, double marginalization is a major concern for policy and regulation; see, e.g., Lerner (1934), Tirole (1993) and Spulber (1999).<sup>1</sup> In our benchmark model, the number of intermediaries per se has no impact on the efficiency of trade because the value is perfectly known to all intermediaries. We extend our benchmark model to a setting in which value is uncertain. We prove existence and provide a complete characterization of equilibrium in this model (Theorem 2). As in the benchmark model, there typically exist multiple equilibria. However, the new model also exhibits important differences. Intermediaries who set positive prices and lie on a least-cost path all set the same price; this price and the efficiency of trade are falling in the number of intermediaries. The multiplicity of equilibrium motivates an experimental investigation. Our experiments highlight the impact of the length of trading chains, especially the number of critical intermediaries, on prices and the efficiency of trade.

Our model offers a generalization of the classical models of price com-

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<sup>1</sup>Double marginalization figured prominently in the Microsoft antitrust case in the United States: it was used as an argument against splitting Microsoft into two firms, one specializing in operating systems and the other specializing in software development (Economides (2001)).

petition (a la Bertrand) and the Nash demand game (Nash, 1950) to a setting with multiple price-setting agents, in which coordination, competition and double marginalization are important. In the theoretical literature, there has been considerable recent interest in the study of intermediation in networks. There are, broadly, three protocols for “price” formation: auctions (Kotowski and Leister (2012)), bargaining (Condorelli, Galeotti and Renou (2013), Gofman (2011), Manea (2013), Siedlarek (2012), Bedayo, Mauleon and Vannetelbosch (2016)) and posted prices (Acemoglu and Ozdagler (2007a, 2007b), Blume et al. (2007) and Gale and Kariv (2009)). As we study a model with posted prices, our paper falls in the third strand of work.<sup>2</sup> There are three main differences between our paper and the papers cited above: 1) the generality of our network framework (which encompasses all networks and allows for incomplete information); 2) our complete characterization of equilibrium; and 3) our methodological combination of theory and experiments. To the best of our knowledge, the result on the role of node criticality in shaping pricing and division of sur-

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<sup>2</sup>For an early paper on the relation between price and quantity competition, see Sonnenschein (1968). For models of networks in which traders choose quantities, see Babus and Kondor (2013), Malamud and Rostek (2013) and Nava (2010). Our paper also broadly relates to Ostrovsky (2008), which extends the study of pairwise stability developed in the matching literature to more general environments of trade, such as supply chains. However, our focus on how the structure of supply chains affects market power is very different from the questions studied in Ostrovsky (2008).

plus is novel.<sup>3</sup> Building upon the results in the current paper, Condorelli and Galeotti (2016), show that node criticality is also useful for the analysis of market power in networks, under different trading protocols (including auctions and bilateral bargaining).

We contribute to the economic study of networks. The research on networks has been concerned with the formation, structure and functioning of social and economic networks; for book-length surveys, see Goyal (2007), Jackson (2008), Vega-Redondo (2007) and Bramoullé, Galeotti and Rogers (2016). The problem of ‘key players’ has traditionally been studied in terms of maximal independent sets, Bonacich centrality, eigenvector and degree centrality; see, for example, Ballester, Calvó-Armengol and Zenou (2006), Bramoullé and Kranton (2007), DeMarzo, Vayanos and Zwiebel (2003), Elliot and Golub (2013), Galeotti et al. (2010) and Golub and Jackson

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<sup>3</sup>Acemoglu and Ozdaglar (2007a, 2007b) consider parallel paths between the source and destination pair. This rules out the existence of “critical” traders. Blume et al. (2007) consider a setting with only a single layer of intermediation; this rules out coordination problems and the interaction between coordination and the market power of intermediaries. Finally, Gale and Kariv (2009) study multiple layers of intermediaries and full connectivity across adjacent layers; this rules out “critical” traders.

We should also mention the literature on vertical relation in industrial organization theory; here the focus has been on rich contractual models but within simple two layer networks between a single upstream firm and several downstream retails (see e.g., Segal and Whinston (2003), Nocke and Rey (2014)). By contrast, we allow for a fairly general class of networks but restrict attention to a very simple contractual form: posted prices.

(2010). The contribution of our paper is to show that criticality of nodes, which is very different from “classical” measures of centrality, offers an appropriate measure of market power.

Our paper also contributes to the large body of experimental work on bargaining and trading in markets. Our finding on efficiency in the benchmark model echoes a recurring theme in economics, first pointed out in the pioneering work of Smith (1962), and more recently highlighted in the work of Gale and Kariv (2009). The special case of one critical intermediary can be interpreted as a dictator game; our results on full extraction of surplus stand in contrast to the general message from the research on dictator games; see Engel (2011). The case of two critical intermediaries may be viewed as a symmetric Nash demand game. Our experiments reveal a high frequency of trade and equal division of surplus; these results are consistent with those in the existing literature, such as Roth and Murnighan (1982), Roth (1995), and Fischer et al. (2006). Charness, Corominas-Bosch and Frechette (2007) study efficiency and surplus division with bargaining in two-sided networked markets. To the best of our knowledge, our paper is the first experimental study of chains of intermediation in networks.<sup>4</sup> The

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<sup>4</sup>There is a large sociological literature on exchange. We share with this literature the motivation of how power may emerge in networks, but we are also interested in questions of efficiency, and our formulation in terms of posted prices and our results are quite different. We refer the reader to Easley and Kleinberg (2010) for a survey of this work.



treatments involving a combination of critical and non-critical intermediaries are novel relative to the literature. These treatments provide us a first glimpse into the interaction between market power and competition in supply chains and related environments.

The rest of the paper is organized as follows. In Section 2 we describe the model and discuss how a number of important questions in applications can be studied within our framework. Section 3 analyzes the benchmark model where value is common knowledge, while Section 4 takes up the model with unknown value. Section 5 concludes. All proofs are presented in the Appendix. Supplementary material is presented in online Appendix A; sample instructions of experiments can be found in online Appendix B.

## 2 The Model

There is a source node,  $\mathcal{S}$ , and a destination node,  $\mathcal{D}$ . A path  $q$  between  $\mathcal{S}$  and  $\mathcal{D}$ , is a sequence of distinct nodes  $\{i_1, \dots, i_l\}$  such that  $g_{\mathcal{S}i_1} = g_{i_1i_2} = \dots = g_{i_l\mathcal{D}} = 1$ . The set of paths is denoted by  $\mathcal{Q}$ . Every node  $i$  is called an *intermediary*; let  $\mathcal{N} = \{1, 2, 3, \dots, n\}$ ,  $n \geq 1$ , denote the set of intermediaries. The nodes  $\mathcal{N} \cup \{\mathcal{S}, \mathcal{D}\}$  and the paths  $\mathcal{Q}$  define a network,  $g$ .

Every intermediary  $i$  simultaneously posts a price  $p_i \geq 0$ . Let  $p = \{p_1, p_2, \dots, p_n\}$  denote the price profile. The network  $g$  and the price profile  $p$  define a cost for every path  $q$  between  $\mathcal{S}$  and  $\mathcal{D}$ :

$$c(q, p) = \sum_{i \in q} p_i. \quad (1)$$

Payoffs arise out of active intermediation: an intermediary  $i$  obtains  $p_i$  only if he lies on a feasible least cost path. A *least-cost* path  $q'$  is one such that  $c(q', p) = \min_{q \in \mathcal{Q}} c(q, p)$ . Define  $c(p) = \min_{q \in \mathcal{Q}} c(q, p)$ . A path  $q$  is feasible if  $c(q, p) \leq v$ , where  $v$  is the value of economic ‘good’ generated by the path. All paths generate the same value  $v$ . If there are multiple least-cost paths, one of them is chosen randomly to be the active path. Given  $g$ ,  $p$  and  $v$ , we denote by  $\mathcal{Q}^v = \{q \in \mathcal{Q} : c(q, p) = c(p), c(p) \leq v\}$  the set of feasible least-cost paths, and intermediary  $i$ ’s payoff is:

$$\pi_i(p, v) = \begin{cases} 0 & \text{if } i \notin q, \forall q \in \mathcal{Q}^v \\ \frac{\eta_i^v}{|\mathcal{Q}^v|} p_i & \text{if } i \in q, q \in \mathcal{Q}^v, \end{cases} \quad (2)$$

where  $\eta_i^v$  is the number of paths in  $\mathcal{Q}^v$  that contain intermediary  $i$ . Implicit in this formulation is the assumption that intermediaries have zero costs.<sup>5</sup>

We consider the case in which intermediaries know the value of  $v$  when they choose their price. In this scenario, we normalize  $v$  to be equal to 1, and, therefore, intermediary  $i$ ’s profit is  $\Pi_i(p) = \pi_i(p, 1)$ . We also examine

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<sup>5</sup>It is possible to extend our analysis to cover heterogenous costs of intermediation. Heterogenous costs will mean that generically there exists a unique path with minimum total ‘cost’ of intermediaries. This path will play a role in the characterization of equilibrium.

the situation in which intermediaries face demand uncertainty when they set their intermediation price. This formulation allows us to study the trade-off between higher prices and lower volume of trade. In this case, we assume that it is common knowledge that  $v$  has a distribution  $F(\cdot)$  on the interval  $[0, 1]$ , with a continuously differentiable density  $f(\cdot)$ . Given network  $g$  and price profile  $p$ , the expected payoff to intermediary  $i$  is:

$$\Pi_i(p) = E_v[\pi_i(p, v)].$$

We study (pure strategy) Nash equilibrium of the posted price game. A price profile  $p^*$  is a Nash equilibrium if for all  $i \in \mathcal{N}$ ,  $\Pi_i(p^*) \geq \Pi_i(p_i, p_{-i}^*)$  for all  $p_i \geq 0$ . An equilibrium is efficient (resp. inefficient) if trade occurs (resp. does not occur) regardless of the realization of  $v$ . When  $v = 1$  is known, an equilibrium  $p^*$  is efficient if  $c(p^*) \leq 1$  (resp.  $c(p^*) > 1$ ); otherwise, the equilibrium  $p^*$  is inefficient. Under demand uncertainty, an equilibrium  $p^*$  is efficient (resp. inefficient) if, and only if,  $c(p^*) = 0$  (resp.  $c(p^*) > 1$ ); when  $c(p^*) \in (0, 1)$ , we say that the equilibrium  $p^*$  is partially efficient.

In principle, nodes that lie on many paths have more opportunities to act as an intermediary. The betweenness centrality of a node  $i \in \mathcal{N}$  is the fraction of paths on which intermediary  $i$  lies.<sup>6</sup> Let  $\eta_i = |\{q \in \mathcal{Q} | i \in q\}|$  and define betweenness centrality of intermediary  $i$  as  $BC_i = \eta_i / |\mathcal{Q}|$ , where

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<sup>6</sup>We consider all paths and not just the shortest paths; in this, we follow Borgatti and Everett (2006).

$BC_i \in [0, 1]$ . Intermediary  $i$  is said to be *critical* if  $BC_i = 1$ . Let  $\mathcal{BC} = \{i \in \mathcal{N} : BC_i = 1\}$  be the set of critical intermediaries. Observe that criticality is a property of the network *per se*, and is independent of the price profile. For simplicity, we suppress the dependence of  $\mathcal{BC}$  on  $g$ .

The model offers a general framework to study the relation between networks and the pricing behavior of traders. We now discuss a number of applications to illustrate the scope of the model.

## 2.1 Applications

**1. Transportation and communication Networks:** The example we sketched in the introduction falls under the large umbrella of transportation and communication networks (which include airlines, shipping, Internet and cable TV). Traditionally, these sectors have been heavily regulated or under public-sector control. The large-scale privatization in the UK (during the 1980s) was a precursor to a global trend. Now, it is common for a consumer to make a choice among alternative bundles of services provided by a number of distinct service providers. A key policy concern is the nature of market power in these networks.<sup>7</sup>

**2. Supply chains:** Consider a Sony Vaio Laptop. It usually has an Intel processor, a hard drive from Seagate Technology, Hitachi, Fujitsu or Toshiba, RAM from Infineon or Elpida, a wireless chipset from Atheros

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<sup>7</sup>Firms in communication and transportation networks use a rich set of price strategies; discrimination with regard to source and destination is common.

or Intel, an optical drive from Hitachi or Matsushita, and a graphic card from Intel, NVIDIA or AMD. The speakers may be from HP or Sony. The different intermediate input suppliers set prices, and Sony picks the best combination of inputs and prices.

Anderson and Wincoop (2004) show that trade intermediation costs amount to a significant tax on international transactions. Hummels, Jun and Yi (2001) show that production supply chains increasingly traverse the world and decisively shape the pattern and volume of trade. Antras and Costinot (2011) is a recent attempt to understand international trade with intermediaries, whereas Antras and Chor (2013) study the optimal organization of a supply chain. The empirical significance of supply chains motivates a systematic study of strategic pricing in general networks.

**3. Corruption:** The bribing of public officials for access to goods and services and for the granting of licenses and permits is a prominent feature of economic life in many countries. Shleifer and Vishney (1993) and Ades and Di Tella (1999) argue that the level of bribes should be viewed as a function of officials' 'market power.' In some contexts, there is a single line of officials (or committees) that must approve a decision, while in others, there may exist multiple competing chains of decision makers (as on highway tolls; Olken and Barron (2009)). These examples motivate an inquiry into the ways that the network of decision making shapes the power of officials in the market for bribes.

**4. Intermediation in agriculture:** Consider coffee. At the start, there

is a farmer in a developing country who typically works on a small farm. The farmer chooses from among a few intermediaries who process his coffee cherries to obtain beans. These intermediaries then sell the beans to one of the small number of exporting trading firms. The exporters sell to dealers/brokers, who, in turn, sell to roasters (such as Nestle). The roasters then sell to large supermarkets and local stores. Finally, consumers buy the coffee from a retailer.

Such long chains of intermediation are common across the agricultural sector (see, for example, Fafchamps and Minten (1999)). Historically, the market power of intermediaries has been a major concern and has led to large-scale state intervention in this sector. However, by the 1990s, it was felt that state agencies discouraged innovation and the entry of new intermediaries, leading to a very inefficient system (see Bayley (2002) and Meerman (1997)). Recent decades have witnessed a large-scale liberalization of the intermediation sector. The effects of liberalization have, however, been mixed; for a discussion, see Traub and Jayne (2008). This research motivates a theoretical study of the determinants of pricing and division of surplus in intermediation networks.

**5. Financial Intermediation:** Consider the market for municipal bonds in the United States, which is the largest capital market for state and municipal issuers. It has market capitalization of over \$4 trillion, with daily trading volumes of around \$ 10-20 billion. Li and Schürhoff (2014) show that trading of these bonds is organized as a decentralized over-the-counter

(OTC) broker-dealer market. The network of traders has a core-periphery structure, with roughly 20-30 dealer firms at the core and several hundred peripheral dealer firms (around 700 firms trade in municipal bonds in any given month). Bonds move from the municipality through an average of six inter-dealer trades. There is systematic price dispersion across dealers, with dealers in the core maintaining systematically larger margins. These empirical patterns motivate a theoretical study of how traders choose partners and how the ensuing network shapes pricing margins and profitability.

In Examples 1, 2 and 3, a consumer or a firm will choose the path: it is reasonable to suppose that the cheapest path will be picked. In Examples 4 and 5, on the other hand, the agent who owns an object will sell it to the highest bidder downstream and has no interest in the cost of the entire path.

The latter two examples motivate the following Bid-Ask price variant of our model. Following Gale and Kariv (2009), suppose that every intermediary  $i \in \mathcal{N}$  simultaneously sets a bid and ask  $(b_i, a_i)$ . The source  $\mathcal{S}$  accepts the highest bid, and the destination  $\mathcal{D}$  buys as long as the lowest ask price is not greater than  $v$ . The object passes from intermediary  $i$  to a connected intermediary  $j$  with the highest bid  $b_j$ , subject to the condition that  $b_j \geq a_i$ . We study this alternative model of pricing in online Appendix A. The analysis there establishes that every equilibrium outcome in our model is also an equilibrium outcome of the Bid-Ask model; the converse is not true in general. However, for some important classes of networks –

that include trees and multi-partite networks – the equilibrium outcomes in the two models are equivalent. So, for these networks, our equilibrium characterization result in the benchmark model, Theorem 1, also holds for the Bid-Ask model.

### 3 Complete information: Networks, market power and efficiency

We prove existence and provide a complete characterization of Nash equilibrium for the case in which  $v$  is known. For any given network, there typically exist multiple equilibria with widely varying pricing, efficiency and division of surplus. We take the model to the laboratory. The experiments highlight two points: 1) the ability of human subjects to coordinate on efficient outcomes; and 2) the role of node criticality as an important network property for understanding market power.

We say that trader  $i$  is *essential* under  $p$  if he belongs to every feasible least-cost path. Given price profile  $p$ , for path  $q$ , let  $c_{-j}(q, p) = \sum_{i \in q, i \neq j} p_i$ , be the total cost of all intermediaries other than  $j$ .<sup>8</sup>

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<sup>8</sup>It is worth noting the distinction between essential and critical nodes. Criticality is a property of the network per se, while essentiality is defined by the network and the price profile together. So, a node may be essential even if there are no critical nodes in the network: this point is taken up in the discussion on multiple equilibria below.



## Theorem 1

- A. **Existence:** In every network, there exists an efficient equilibrium.
- B. **Characterization:** An equilibrium  $p^*$  is inefficient ( $c(p^*) > 1$ ); or intermediaries extract all the surplus ( $c(p^*) = 1$ ); or they earn nothing ( $c(p^*) = 0$ ). Moreover,
1.  $p^*$  is an equilibrium in which intermediaries earn nothing if, and only if, no trader is essential.
  2.  $p^*$  is an equilibrium in which intermediaries earn all the surplus if, and only if, (i) if trader  $i$  belongs to the least-cost path, and he sets a positive price then trader  $i$  is an essential trader; and (ii) if trader  $i$  belongs only to non-least-cost paths, and he belongs to path  $q$  then  $c_{-i}(q, p^*) \geq 1$ .
  3.  $p^*$  is an inefficient equilibrium if, and only if, if trader  $i$  belongs to path  $q$  then  $c_{-i}(q, p^*) \geq 1$ .

The argument for the existence of an efficient equilibrium is constructive. First, consider a network with no critical traders. The 0 price profile is a Nash equilibrium, as no intermediary can earn positive profits by deviating and setting a positive price. If an intermediary sets a positive price,  $\mathcal{S}$  and  $\mathcal{D}$  will circumvent him, as there exists a zero cost path without him. Next, consider a network with critical traders. It may be checked that a price

profile in which critical traders set positive prices that add up to 1 and all non-critical traders set 0 price is an equilibrium.

The characterization yields a number of insights. The *first* observation is that in every efficient equilibrium, intermediation costs take on extreme values. The intuition is as follows: if the feasible least-cost path is unique, then intermediaries in that path exercise market power; thus, if intermediation costs are below the value of exchange, an intermediary in that path could slightly increase his intermediation price while guaranteeing that exchange takes place through him. In contrast, when there are multiple feasible least-cost paths, there is price competition among intermediaries on different paths. In that case, whenever intermediation costs are larger than zero, an intermediary demanding a positive price gains by undercutting his price. Price competition drives intermediation costs down to zero.

The *second* observation is on how critical traders have market power. Observe that a critical trader is essential. Hence, the presence of critical traders is sufficient to ensure that intermediaries extract all surplus in every efficient equilibrium.

Criticality dictates that all surplus must accrue to intermediaries, but the theory is permissive about how it is distributed among them. To see this point, consider the Ring with Hubs and Spokes network presented in Figure 1, and suppose that  $\mathcal{S}$  and  $\mathcal{D}$  are located on  $(a_1, d_1)$ . Then, there exists an equilibrium in which all surplus accrues to the critical intermediaries, e.g.,  $A$  and  $D$  charge  $1/2$  and all other intermediaries charge 0. However, there

is also an equilibrium in which the entire surplus is earned by non-critical intermediaries, e.g.,  $A$  and  $D$  charge 0,  $B$  and  $C$  charge  $1/2$ , and  $F$  and  $E$  charge 1.

The *final* observation is about the multiplicity of equilibria. Consider the ring network with six traders presented in Figure 1, and suppose that  $\mathcal{S}$  is located at  $A$  and  $\mathcal{D}$  is located at  $D$ . The three equilibria described by Theorem 1 are possible in this network: all intermediaries set price 0; all of them set price 1; and intermediaries  $B$  and  $C$  set price 1, while intermediaries  $E$  and  $F$  set price  $1/2$  each. In the last case, note that  $E$  and  $F$  are essential but not critical. Thus, criticality is not necessary for surplus extraction by intermediaries.

This multiplicity motivates an exploration of equilibrium refinements. We consider a number of possible refinements, including strictness, strong Nash equilibrium, elimination of weakly dominated strategies, and coalition proof equilibrium. We find that, in some cases, these refinements are too strong; for example, there does not exist a strict or strong Nash equilibrium in some networks. In other cases, the refinement is not effective; for example, a wide range of outcomes (including those with coordination failure) may be sustained under elimination of weakly dominated strategies and coalition proof. We discuss these refinements in greater detail in online Appendix A. Given the limited usefulness of standard equilibrium refinements, we turn to an experimental investigation of posted prices in

networks.<sup>9</sup>

## 3.1 Posted prices in the Laboratory

### 3.1.1 Experimental Design

We have chosen networks that allow us to examine the roles of coordination, competition and market power. These networks are depicted in Figure 1.

The ring networks with four, six and ten traders allow us to focus on coordination and competition.<sup>10</sup> For every choice of  $\mathcal{S}$  and  $\mathcal{D}$ , there are always two competing paths of intermediaries. In Ring 4, for any non-adjacent pair, there are two paths with a single intermediary each. Ring 6 and Ring 10 allow for situations with a higher (and possibly unequal) number of intermediaries on either path.

The Ring with Hubs and Spokes network allows for a study of the impact of market power: for instance, if  $\mathcal{S}$  is located at  $a_1$  and  $\mathcal{D}$  is located at  $a_2$ ,

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<sup>9</sup>Goyal and Vega-Redondo (2007) consider a cooperative solution concept the kernel in their work. They show that non-critical traders would earn 0, and critical traders would split the surplus equally in allocations in the kernel. Our analysis above reveals that this solution is a Nash equilibrium of the pricing game but that there exist a variety of other equilibria.

<sup>10</sup>We have also run experiments on a ring network with eight traders. The results are in line with those presented in this section, but to simplify exposition, we do not present them.

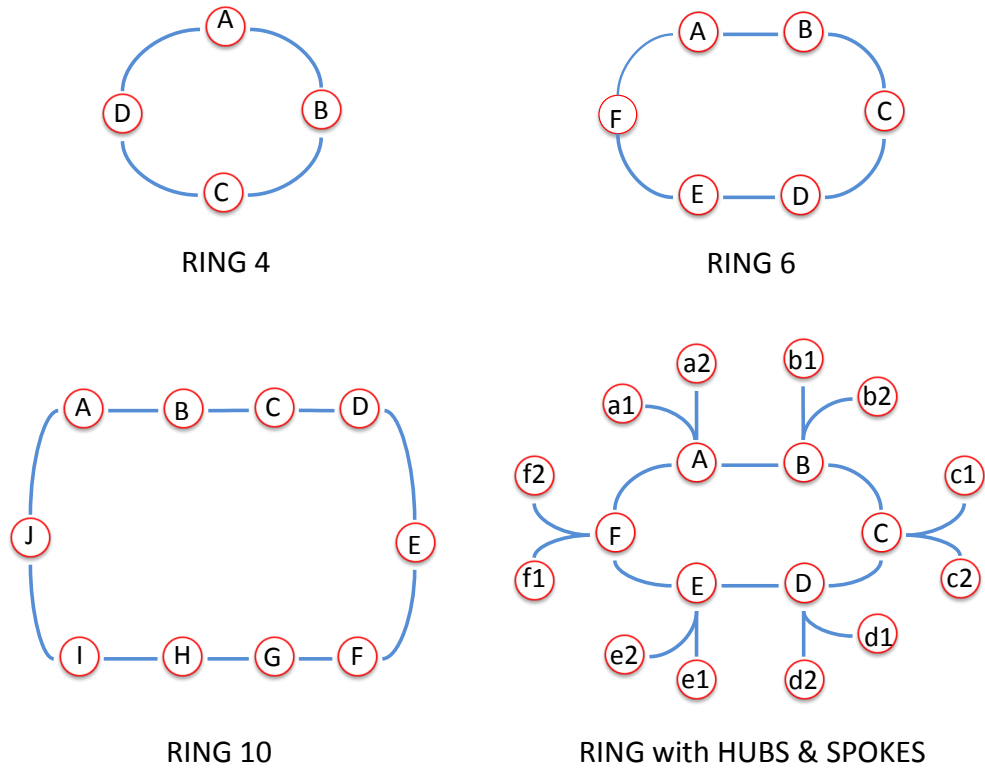


Figure 1: Networks in the benchmark design

intermediary  $A$  is a pure monopoly, while if  $\mathcal{D}$  is  $b_1$ , then the intermediaries  $A$  and  $B$  play a symmetric Nash demand game. This network also creates the space for both market power and competition to come into play. For instance, if  $\mathcal{S}$  is located at  $a_1$  and  $\mathcal{D}$  is located at  $e_1$ , then there are two competing paths: a shorter path (through  $A$ ,  $F$ , and  $E$ ) and a longer path (through  $A$ ,  $B$ ,  $C$ ,  $D$ , and  $E$ ). Traders  $A$  and  $E$  are the only critical intermediaries.

To put these experimental variations into perspective, we summarize the equilibrium analysis for the selected networks. In Ring 4, there is a unique equilibrium that corresponds to the Bertrand outcome. In every

other network, whenever there are at least two intermediaries on every path, there exist both efficient and inefficient equilibria. This observation motivates our first question:

**Question 1:** *How does the efficiency of trade vary with ring size and the presence of critical traders?*

If trading does take place, Theorem 1 predicts an extremal division of trade surplus: either intermediaries earn 0 surplus or they extract all trade surplus. In Ring 4, the intermediation cost is 0 in the unique equilibrium; but in all other Rings, both extremal outcomes are possible in equilibrium. In the Ring with Hubs and Spokes, whenever exchange involves critical traders, equilibrium dictates full surplus extraction by intermediaries. These considerations motivate the second question:

**Question 2:** *Is the division of surplus extremal? How does it vary with the presence of critical traders?*

Finally, we turn to the situation in the Ring with Hubs and Spokes where all three forces of interest – coordination, competing paths and critical traders – are present. Theorem 1 tells us that all surplus must accrue to intermediaries, but it is silent on how the surplus is distributed among them. This observation motivates our third question:

**Question 3:** *What is the division of surplus between critical and non-critical intermediaries?*

Treatment	Session		Total
	1	2	
Ring 4	16 / 240	16 / 240	32 / 480
Ring 6	18 / 180	24 / 240	42 / 420
Ring 10	20 / 120	20 / 120	40 / 240
Ring w. hubs/spokes	18 / 180	24 / 240	42 / 420

Table 1: Treatments in Benchmark Model

### 3.1.2 Experimental procedures

We ran the experiments at the Experimental Laboratory of the Centre for Economic Learning and Social Evolution (ELSE) at University College London (UCL) between June and December 2012. The subjects in the experiment were recruited from the ELSE pool of human subjects consisting of UCL students, across all disciplines. Each subject participated in only one of the experimental sessions. After subjects read the instructions, an experimental administrator read the instructions aloud. Each experimental session lasted around two hours. The experiment was computerized and conducted using the experimental software z-Tree, developed by Fischbacher (2007). Each session used one network treatment, and we ran two sessions for each treatment. Each session consisted of 60 independent rounds. Table 1 provides an overview of the experimental design. In each cell, we report number of subjects/number of group observations.

We employed random matching with random assignment of network positions across rounds. In each round of a treatment, subjects were assigned with equal probability to one of the possible positions of a network. In Ring  $n$ , all nodes were possible positions. In Ring with Hubs and Spokes, each spoke node was a computer-generated agent, and the remaining nodes

were all feasible positions for the human subjects. Groups with one subject per intermediary position were then randomly formed. The position of a subject and the groups formed in each round depended solely on chance and was independent of the subject’s position and the groups formed in previous rounds, respectively.

We deliberately chose the protocol of random matching with random assignment. This procedure anonymized the identity of the subjects throughout a session and, thus, helped avoid “repeated games” effects that arise if the same fixed group of subjects play a game repeatedly. The advantage of using subjects repeatedly under this protocol was that it allowed us to collect a large amount of data from a given number of subjects, while they had an opportunity to learn how to play a game. Other protocols, in which subjects never again meet someone who they have played before require large subject pools or provide fewer observations with less opportunity for subjects’ learning. It is worth emphasizing that, as we only varied the network structure, any experimental difference in subjects’ behavior across treatments will be evidence of network effects because we kept the random matching and assignment protocol constant across all treatments.<sup>11</sup>

For each group, a pair of two non-adjacent nodes was randomly selected

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<sup>11</sup>As we shall see, our findings are in line with existing experimental literature that shows that the random matching protocol is an effective way to minimize the repeated games effects (Duffy and Ochs (2009)). We discuss possible repeated games effects after we report the first finding in the next section.



as  $\mathcal{S}$  and  $\mathcal{D}$ . Each pair of two non-adjacent nodes was equally likely to be selected. All of the subjects in each group were informed of the position of  $\mathcal{S}$  and  $\mathcal{D}$  in the network. All traders were informed that the surplus/value of exchange was 100 tokens. Then, all human subjects in an intermediary role were asked to submit an intermediation price: a real number (up to two decimal places) between 0 and 100. The computer calculated the intermediation costs across different paths. Exchange took place if the least-cost among all paths was less than or equal to 100. If there were multiple feasible least-cost paths then one of them was chosen at random.

At the end of the round, subjects observed all posted prices in their group, the trading outcome, and the earnings of all the subjects. We assumed that each of  $\mathcal{S}$  and  $\mathcal{D}$  was allocated one half of the net surplus— i.e., one half of 100 minus the intermediation costs. Then, the subjects moved to the next round.

In each round, earnings were calculated in terms of tokens. For each subject, the earnings in the experiment were the sum of his or her earnings over 60 rounds. At the end of the experiment, subjects were informed of their earnings in tokens. The tokens were exchanged for British pounds, with 60 tokens equaling £1. Subjects received their earnings plus £5 show-up fee privately, at the end of the experiment.

Network	minimum distance of buyer-sell pair				
	All ( $\geq 2$ )	2	3	4	5
Ring 4	1.00 (480)	1.00 (480)	--	--	--
Ring 6	1.00 (420)	1.00 (289)	1.00 (131)	--	--
Ring 10	1.00 (240)	1.00 (49)	1.00 (87)	1.00 (69)	1.00 (35)
Ring with Hubs and Spokes	0.95 (420)	1.00 (126)	0.94 (155)	0.90 (109)	0.90 (30)

Note. The number of group observations is reported in parentheses.

Table 2: Frequency of Trading

### 3.1.3 Findings

We start by examining the efficiency of trade in networks. Table 2 reports the relative frequency of trade across different treatments.

Trade occurs with probability 1 in ring networks, regardless of their size and of the distance between  $\mathcal{S}$  and  $\mathcal{D}$ . In Ring with Hubs and Spokes, the frequency of trade is around 0.95. So, market power does not have any significant effect on efficiency of trading. Overall, despite the need for coordination among intermediaries along the same path, the presence of competition between paths and the presence of market power of some intermediaries, traders across all treatments are very successful in coordinating on prices that ensure exchange.

**Finding 1:** *The level of efficiency is remarkably high in all networks. Trading in Rings with four, six, and ten intermediaries occurs with probability 1. In the Ring with Hubs and Spokes, trading occurs with probability around*

0.95.

In Rings, we distinguish trading situations with respect to distances of the two competing paths between  $\mathcal{S}$  and  $\mathcal{D}$ , denoted by  $(d(q), d(q'))$ . In Ring with Hubs and Spokes, we distinguish trading situations with respect to (i) the number of critical intermediaries ( $\#Cr$ ), (ii) the number of intermediation paths ( $\#Paths$ ), and (iii) the distance of each path  $(d(q), d(q'))$ . Figure 2 presents the average intermediation costs, conditional on trading, based on the last 20 rounds, with a 95 percent confidence interval across different trading situations.

In Table 3 of online Appendix A, we report the movement across rounds in average intermediation costs across distinct trading situations in Rings and Ring with Hubs and Spokes. When there are no critical traders (resp. there are only critical traders), there is a clear downward trend (resp. upward trend) in intermediation costs across rounds. The trends across rounds reflect subjects' learning to play the games. We interpret them as suggesting that subjects learned to compete between distinct paths when there are no critical traders, while they learned to coordinate on how to divide surplus with only critical traders. In treatments with both critical and non-critical traders, intermediation costs are stable over time.

We now comment on an issue that is potentially more subtle and that can arise in our setting.<sup>12</sup> In our experiments, subjects know that there

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<sup>12</sup>We are grateful to a referee for drawing our attention to this issue.

is random assignment to locations across rounds. So, for instance, they know that each of them will have a chance to occupy critical nodes. This may lead them to be not overly concerned about the surplus accruing to critical nodes in any specific round. Thus, the relationship between network location and surplus could potentially be due to this repeated game effect. A simple way to investigate this point is to examine the players' behavior and the division of surplus in the last round of the game. If this argument were valid, then the allocation of surplus would be very different in the last round, as non-critical players, for instance, might insist on a fair share of the surplus. But Table 4 in online Appendix A shows us that there is essentially no difference in behavior of prices in the later rounds versus the last round. Thus, we conclude that this type of 'repeated game' effect is not an issue in our experiment.

In Ring 4, intermediation costs are around five percent of the surplus. In the other rings, intermediation costs vary between ten and twenty percent of the surplus. The overall conclusion is that intermediation costs in all ring networks are modest and, between the two efficient equilibria, are much closer to the one with zero intermediation cost, especially in the smaller rings.

In the Ring with Hubs and Spokes, when  $\mathcal{S}$  and  $\mathcal{D}$  are served by a sole critical intermediary, the situation is analogous to the dictator game, widely studied in the experimental literature (for a survey, see Engel (2011)). We find a surplus extraction of 99 percent, which is much higher than that

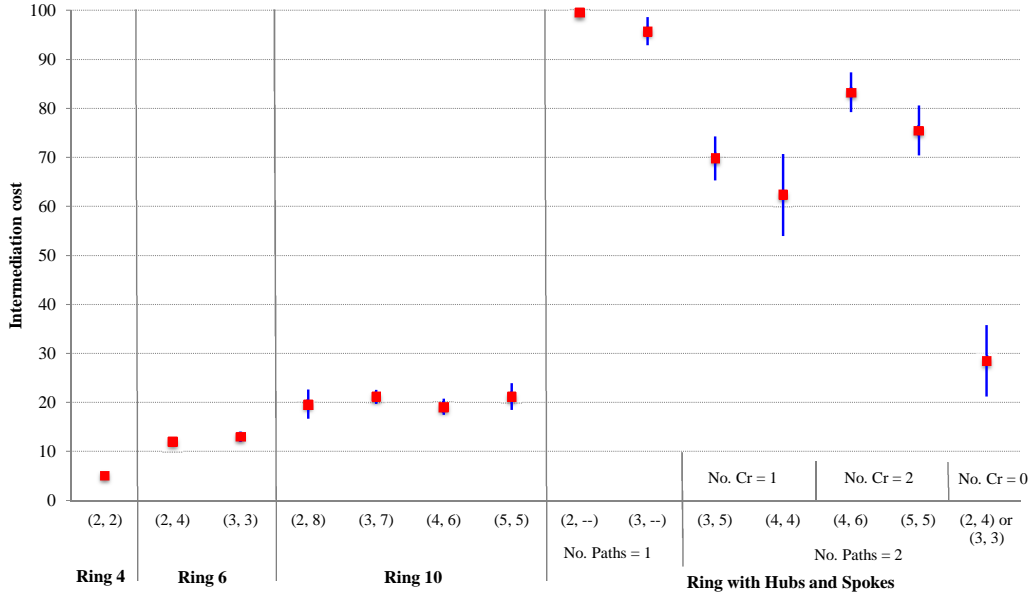


Figure 2: Costs of intermediation

reported in the experimental literature. This suggests that traders located at critical nodes in a network interpret their location as a form of ‘earned endowment,’ in the sense of Cherry, Frykblom and Shogren (2002). This may give rise to a sense of entitlement that is distinct from the standard dictator game.<sup>13</sup>

When  $\mathcal{S}$  and  $\mathcal{D}$  are connecting via one single path with two intermediaries, the game played by the two intermediaries is analogous to a symmetric Nash demand game. We find that intermediaries extract, in total, around

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<sup>13</sup>We also note that in our design, in some situations, both  $\mathcal{S}$  and  $\mathcal{D}$  are computer-generated agents, while in others, one of them is a human subject. We find no behavioral difference across these cases. This leads us to believe that the human subject vs. computer issue does not play a major role in explaining the behavior of the subjects in our experiment.

96 percent of the surplus and that they share it roughly equally.<sup>14</sup> These findings are consistent with those in the experimental literature of Nash bargaining (e.g., Roth and Murnighan (1982) and Fischer et al. (2006)).

Finally, when there are two competing paths and critical traders, the intermediation cost ranges between 62 percent and 83 percent. In the case without critical intermediaries, this cost falls sharply to around 28 percent, which is comparable to the low-cost outcome found in Rings. We summarize this discussion in our second finding.

**Finding 2:** *The presence of critical traders is both necessary and sufficient for large surplus extraction by intermediaries. In Rings with four, six, and ten traders, intermediation costs are small (ranging from 5 percent to 20 percent). In the Ring with Hubs and Spokes, with critical traders, intermediation costs are large (ranging from 60 percent to over 95 percent).*

We now turn to the issue of how surplus is divided between critical and non-critical intermediaries. Table 3 presents the average fraction of intermediation costs charged by critical traders, conditional on exchange (here, data are grouped into the blocks of 20 rounds, due to small samples). The number within parentheses is the number of group observations. Looking at the last 20 rounds, we observe that 67 percent to 80 percent of intermediation costs go to critical trader(s). In all the cases, regardless of whether an exchange takes place along the shorter or longer path, the number of

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<sup>14</sup>See Table 3 in online Appendix A.

Network	(#Cr,#Paths, d(q),d(q'))	Rounds		
		1 ~ 20	21 ~ 41	41 ~ 60
Ring with Hubs and Spokes	(1, 2, 3, 5)	0.56	0.68	0.72
		(20)	(26)	(25)
	(1, 2, 4, 4)	0.48	0.56	0.67
		(16)	(13)	(10)
	(2, 2, 4, 6)	0.73	0.77	0.80
		(16)	(19)	(24)
(2, 2, 5, 5)	0.65	0.67	0.74	
	(8)	(8)	(11)	

Notes. The number in a cell is the average fraction of costs charged by critical traders. The number of observations is reported in parentheses. #Cr denotes the number of critical intermediaries, #Paths denotes the number of paths connecting buyer and seller, d(q) denotes the length of path q between buyer and seller.

Table 3: Surplus division among intermediaries

non-critical traders is at least as large as the number of critical traders. To summarize:

**Finding 3:** *In the Ring with Hubs and Spokes, critical intermediaries set higher prices and earn a much higher share of surplus than non-critical intermediaries.*

We have established that network structure – reflected in the criticality of nodes – has powerful effects on intermediation costs and the division of surplus. To gain a deeper understanding of the mechanisms of competition and market power, we now examine the pricing behavior of traders directly.

We focus on the last 20 rounds and Figure 3 depicts average prices.<sup>15</sup> In the Ring with six and ten traders, there is tight competition between paths. Intermediaries on a longer path choose, on average, prices somewhere

<sup>15</sup>In Online Appendix A, Table 4 reports average prices charged across rounds by intermediaries in Rings and Ring with Hubs and Spokes, respectively.

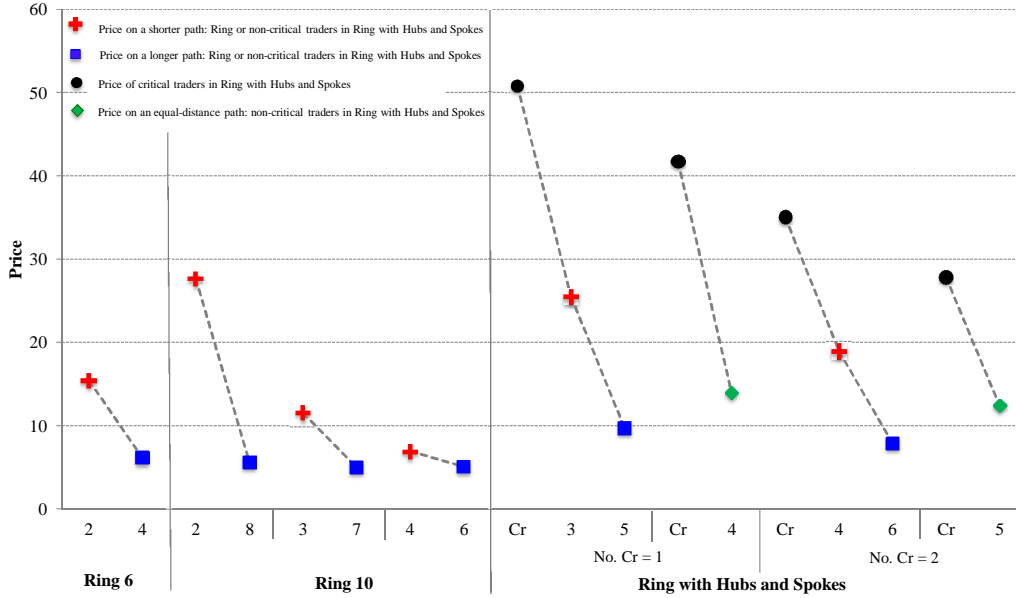


Figure 3: Price behavior across intermediary types

Network	$(d(q), d(q'))$	$ \text{cost1} - \text{cost2} $	Freq. on a shorter path
Ring 4	(2, 2)	3.99	--
Ring 6	(2, 4)	4.45	0.65
	(3, 3)	4.01	--
Ring 10	(2, 8)	15.20	0.64
	(3, 7)	5.30	0.68
	(4, 6)	6.82	0.68
	(5, 5)	5.01	--

Notes. We report the sample median of absolute differences of two competing paths, using the sample of last 20 rounds. The number in the last column is the frequency of trading on a shorter path.

Table 4: Short versus long paths



between five and ten, independently of the distances of the two paths across all ring networks. Responding strategically to this, intermediaries on a shorter path choose higher prices, which were proportionate to the difference in distance between the two paths. As a result, even when the two paths are very asymmetric, they have very similar intermediation costs and trade occurs frequently – roughly one third of the time – along the longer path! Table 4 provides data on these patterns.

In the Ring with Hubs and Spokes, the pricing of critical and non-critical intermediaries is very different. Critical intermediaries post much higher prices than non-critical intermediaries. The non-critical intermediaries post prices that are similar to intermediaries in Rings. For instance, when there is one critical intermediary and the two competing paths are of distance three and five, the critical intermediary charges, on average, a price close to 50; the only non-critical intermediary lying in the shorter path charges a price close to 24; and the three non-critical intermediaries in the longer path post a price around eight. Similar behavior is observed in the other cases. This demonstrates the strong impact of network criticality on pricing behavior and the division of surplus.

To further check the sharp differences in pricing behavior among different types of intermediaries presented in Figure 3, Table 5 presents the results of regressions of prices on dummies for critical and non-critical traders on a shorter path. Data are from the last 20 rounds, and we control for individual heterogeneity by including individual subject dummies. Robust

standard errors, clustered by individual subject, are reported in parentheses. In Ring networks, traders on a shorter path choose significantly higher prices than those on a longer path. In the Ring with Hubs and Spokes, critical intermediaries choose significantly higher prices than non-critical intermediaries. The price difference between non-critical intermediaries on short and long path is statistically significant.

Finally, while intermediation costs do take on extreme values, they depart significantly from the theoretical predictions. The observed departures from equilibrium pricing and surplus extraction are consistent with a model of noisy best response with risk aversion; a formal analysis is presented in online Appendix A.

Dependent variable: price	Ring 6			Ring 10			Ring with Hubs and Spokes				
	(2, 4)	(2, 8)	(3, 7)	(4, 6)	(3, 5)	(4, 4)	(4, 6)	(5, 5)	#Cr = 1	#Cr = 2	
(1) Non-critical & on a shorter path	9.478 (1.408)***	22.234 (6.597)***	6.316 (0.937)***	2.351 (0.453)***	18.571 (2.748)***		11.464 (1.565)***				
(2) Critical					40.173 (4.219)***	27.554 (5.749)***	27.923 (2.050)***	14.338 (2.191)***			
Constant	5.380 (0.166)***	2.250 (0.000)***	3.524 (0.375)***	4.454 (0.113)***	9.085 (1.944)***	13.333 (0.000)***	8.028 (0.745)***	5.662 (2.191)**			
$H_0: (1) = (2)$ or $H_0: (2) = 0$ (p -value)					0.000	0.000	0.000	0.000			
R-squared	0.313	0.464	0.336	0.330	0.728	0.771	0.776	0.840			
Number of obs.	376	112	224	200	134	50	156	66			

Notes: Each regression contains dummies for individual subjects. Robust standard errors, clustered by individual subject, are reported in parentheses. \*, \*\*, and \*\*\* represent 10%, 5%, and 1% significance level.

Table 5: Regression of pricing on network position

## 4 Uncertain demand: competition and market power

In our benchmark model, the number of intermediaries *per se* has no impact on the efficiency of trade because the value of surplus is perfectly known to all intermediaries. We now extend the benchmark model to allow for uncertain demand. We prove existence and provide a complete characterization of equilibrium in this model. As in the benchmark model, there typically exist multiple equilibria, with very different pricing, efficiency and division of surplus. However, the analysis also reveals important differences with the benchmark model: active intermediaries are predicted to all set the same price, and the number of active intermediaries has powerful effects on pricing and the efficiency of trade. Our experiments highlight the interplay between these theoretical predictions and the role of node criticality.

The next result proves existence and provides a complete characterization of equilibrium, for all networks. Let  $e(g, p)$  be the number of essential traders— i.e., the number of traders who lie on all paths  $q \in \mathcal{Q}^1$ . Define  $h(x) = f(x)/[1 - F(x)]$  to be the hazard rate.

**Theorem 2** *Assume that the hazard rate is increasing.*

A. **Existence:** *In every network, there exists an efficient or a partially efficient equilibrium.*

B. **Characterization:**

1. *An efficient equilibrium exists if, and only if, there are no critical traders.*
2.  *$p^*$  is a partially efficient equilibrium if, and only if, (a) there exists some essential trader— i.e.,  $e(g, p^*) \geq 1$ ; every essential trader charges a price*

$$\hat{p}^* = \frac{1}{h(e(g, p^*)\hat{p}^*)}; \quad (3)$$

*and every non-essential trader in the least-cost path sets a price equal to 0; and (b) if trader  $i$  belongs only to non-least-cost paths and he belongs to path  $q$ , then  $c_{-i}(q, p^*) \geq 1$ .*

3.  *$p^*$  is an inefficient equilibrium if, and only if, if trader  $i$  belongs to path  $q$  then  $c_{-i}(q, p^*) \geq 1$ .*

Theorem 2 brings out two important implications of pricing in networks under uncertain demand.<sup>16</sup> The first is that a lack of criticality is necessary and sufficient for the existence of an efficient equilibrium. So, whenever there are critical intermediaries, the equilibrium will involve some inefficiency (in the sense of lost trading opportunities). This is novel relative to Theorem 1. The second observation relates to equilibrium pricing by

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<sup>16</sup>All parts of the result, except for part [2], continue to hold if we relax the increasing hazard rate assumption. In part [2], we exploit the increasing hazard rate assumption for the sufficiency part of the proof only.

essential traders: they set a unique common price that solves condition (3). As  $c(p^*) \in (0, 1)$ , intermediaries always share surplus with  $\mathcal{S}/\mathcal{D}$ .<sup>17</sup>

We now show how pricing, efficiency and division of surplus, vary with the number of essential traders.

**Proposition 1** *Assume that the hazard rate is increasing. Suppose that  $p^*$  and  $p'$  are two partially efficient equilibria, with  $e(g, p^*) > e(g', p')$  essential traders, respectively. Then:*

1. *The price for essential traders under  $p^*$  is strictly lower than the price under  $p'$ .*
2. *The intermediation cost under  $p^*$  is strictly higher than under  $p'$  – i.e.,  $c(p^*) > c(p')$ . Hence,  $p^*$  is less efficient than  $p'$ .*
3. *The sum of intermediaries' payoffs and the sum of  $\mathcal{S}/\mathcal{D}$ 's payoffs are both lower under  $p^*$  than under  $p'$ .*

This proposition brings out another novel implication of pricing under uncertain demand: recall that, in the benchmark model, there is no systematic relation between the number of essential traders and intermediation

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<sup>17</sup>Observe that in case b.2 of Theorem 2, the cost of a least cost path must be interior. It then follows, using arguments along the lines of Theorem 1, that there cannot be multiple least cost paths. Otherwise, a trader on any one of these paths would have a strict incentive to slightly lower her price and discontinuously raise (expected) demand and thereby strictly raise profits.

costs (cf. Theorem 1). In contrast, under demand uncertainty, the more essential traders there are, the lower is the price that each charges, but the higher is the cost of intermediation. Hence, a greater number of essential traders leads to greater inefficiency. This follows from a classical problem of double-marginalization. Each of the essential traders faces a downward-sloping demand curve and has the incentive to mark up the intermediation price above its marginal cost. An increase in the number of essential traders reduces the mark-up charged by each intermediary, but the total intermediation cost must rise, because intermediaries do not fully internalize the benefit of lowering the mark-up.

## 4.1 Experimental design and procedures

We study the effects of uncertain demand on pricing, the division of surplus and efficiency of trade. In particular, we test the new theoretical predictions on equal pricing and on partially efficient equilibrium. With this in mind, in addition to rings of size four, six and ten and the Ring with Hubs and Spokes, we also consider Line networks with six and eight traders.<sup>18</sup> Figure 4 presents these networks.

Recall that in ring networks, there always exists an efficient equilibrium, but in rings with six and ten traders, there are also inefficient and partially efficient equilibria. In Lines and in Ring with Hubs and Spokes (with crit-

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<sup>18</sup>In the Line network with six and eight traders, the pair  $\mathcal{S}$  and  $\mathcal{D}$  are always the two end nodes and computer-generated agents.

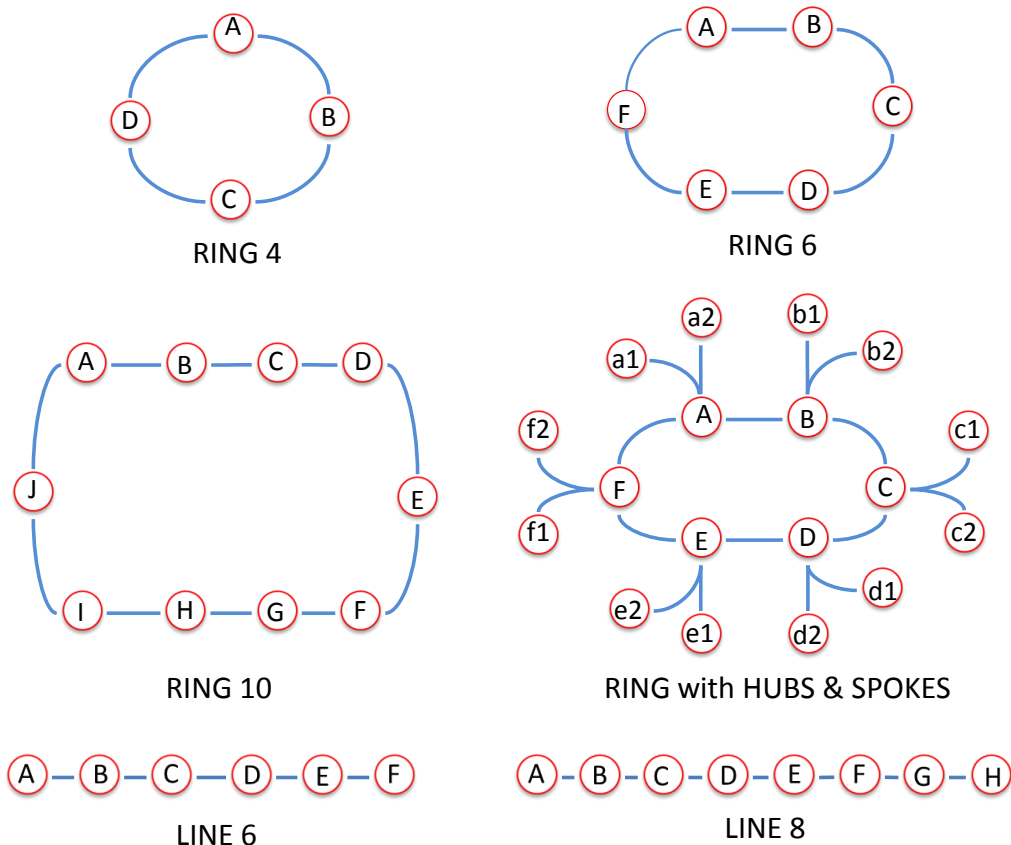


Figure 4: Networks in the uncertain demand case

ical intermediaries), an efficient equilibrium does not exist, but a partially efficient equilibrium does. The frequency of trade declines with the number of critical traders in this equilibrium. These observations motivate the following question.

**Question 1A:** *In the presence of uncertain demand, how does the efficiency of trade vary with ring size and the presence of critical traders?*

Our theoretical analysis reveals that in equilibrium, all essential traders –critical and non-critical – must set the same price and that this price declines in the number of essential traders. This motivates our second



Treatment	Session				Total
	1	2	3	4	
Ring 4	16 / 240	24 / 360			40 / 600
Ring 6	18 / 180	18 / 180			36 / 360
Ring 10	30 / 180	30 / 180			60 / 360
Ring w. Hubs/Spokes	18 / 180	18 / 180	24 / 240	30 / 300	90 / 900
Line 6	16 / 240	20 / 300			36 / 540
Line 8	18 / 180	18 / 180			36 / 360

Table 6: Treatments with uncertain demand

question:

**Question 2A:** *In the presence of uncertain demand, how does pricing vary with network location and the number of critical traders?*

#### 4.1.1 Procedures

The experiment was run at the Experimental Laboratory of the University of Essex (ESSEXLab; <http://www.essex.ac.uk/essexlab/>) in May and October 2013. The subjects in the experiment were recruited from the ESSEXLab pool consisting of undergraduate and Master’s students across all disciplines at the University of Essex. The experimental procedures followed the one we described in Section 2.3. We note that in the experiment, the value of exchange  $v$  is randomly drawn to be an integer between 1 and 100 at the beginning of each round. Table 6 summarizes the experimental design and treatments. In each cell, we report number of subjects / number of group observations in a session.

#### 4.1.2 Findings

We start with an examination of efficiency of trade. Table 7 presents data on the frequency of trade across the different networks. We split the

data of Ring with Hubs and Spokes with respect to the number of paths. The cases in which there is only one path between  $\mathcal{S}$  and  $\mathcal{D}$  correspond to line networks with one or two critical intermediaries. In Table 7 and subsequent tables, we refer to these cases as Line 3 and Line 4, respectively. We refer to all other cases as belonging to Ring with Hubs and Spokes.

Our first observation is that, for fixed a network architecture, the distance between  $\mathcal{S}$  and  $\mathcal{D}$  has an impact on efficiency. In the Ring network with ten traders, the frequency of trade declines from 0.73 to 0.57 as we move from distance 2 to distance 5. In the Ring with Hubs and Spokes, the frequency falls from 0.60 to 0.45 as we move from distance 3 to distance 5. In line networks, the frequency of trade falls from 0.65 to 0.25 as we move from distance 2 to distance 6. Our second observation is on the effects of critical intermediaries. For a fixed distance, the frequency of trade in a ring network and in a line network differ considerably. The frequency of trade in Ring with Hubs and Spokes lies somewhere between that in rings and that in lines, for each fixed distance.

To draw out more clearly the effects of distance and the number of critical traders on efficiency, we compare efficiency between ring networks and line networks in Figure 5.<sup>19</sup> We calculate the frequency of trade in ring networks after pooling all the observations in rings with four, six and ten traders, where the length of the shortest path between  $\mathcal{S}$  and  $\mathcal{D}$  is the same

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<sup>19</sup>In online Appendix A, we report average intermediation costs (see table 6) and average prices for network location (see table 5) over time and across treatments.

Network	#Paths	minimum distance between buyer and seller						
		All ( $\geq 2$ )	2	3	4	5	6	7
Ring 4	2	0.89 (600)	0.89 (600)	--	--	--	--	--
Ring 6	2	0.73 (360)	0.74 (234)	0.69 (126)	--	--	--	--
Ring 10	2	0.64 (360)	0.73 (108)	0.62 (114)	0.60 (91)	0.57 (47)	--	--
Ring with Hubs and Spokes	2	0.51 (504)	--	0.60 (158)	0.47 (270)	0.45 (76)	--	--
Line 3	1	0.65 (227)	0.65 (227)	--	--	--	--	--
Line 4	1	0.53 (169)	--	0.53 (169)	--	--	--	--
Line 6	1	0.36 (540)	--	--	--	0.36 (540)	--	--
Line 8	1	0.25 (360)	--	--	--	--	--	0.25 (360)

Notes. The number of group observations is reported in parentheses. #Paths denotes the number of paths connecting buyer and seller. The samples of Line 3 and 4 are from sessions with Ring with Hubs and Spokes.

Table 7: Frequency of trade

(circles on the dotted line in Figure 5). The frequency of trade declines with distance. We also present the frequency of trade in line networks (squares on the solid line in Figure 5). We note that the frequency of trade is lower at every distance level and that the gradient remains significant all the way through. To summarize:

**Finding 1A:** *In the presence of uncertain demand, networks have large effects on efficiency. The frequency of trade falls with distance and falls even more sharply with the number of critical traders.*

We now turn to the pricing behavior of traders by focusing on the last 20 rounds of the experiment. We first present average prices of different types of intermediaries in the Ring networks and the Ring with Hubs and Spokes in Figure 6. In addition, we report in Table 8 the regression results of prices on dummies for critical and non-critical intermediaries, respectively,

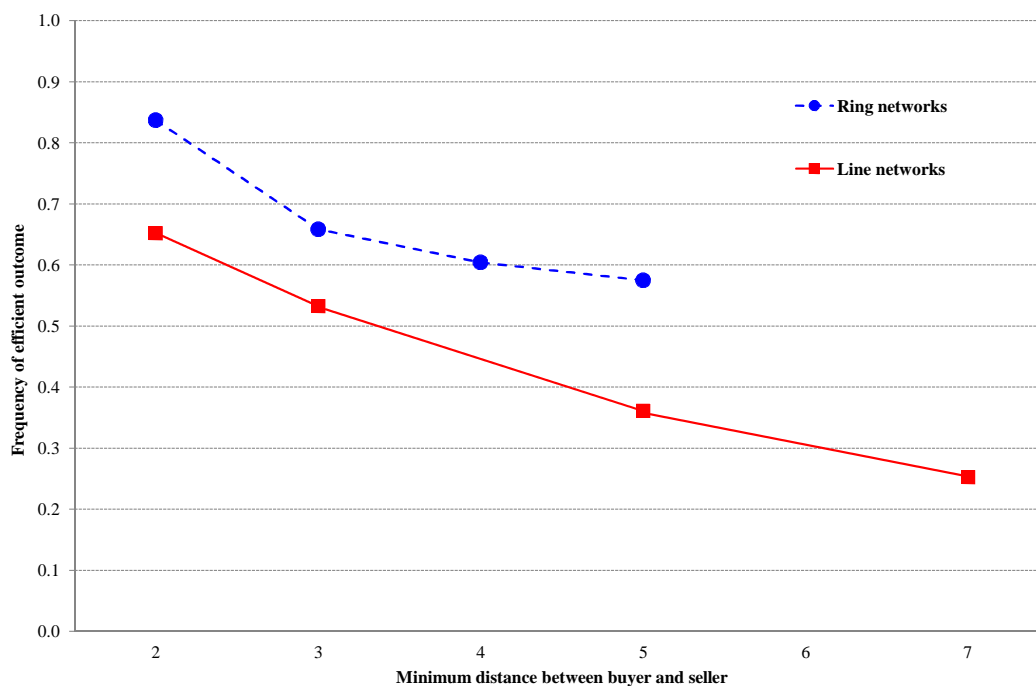


Figure 5: Efficiency and distance

on a shorter path. We control for individual heterogeneity by including dummies for individual subjects. As in our benchmark experiment, there is clear evidence that subjects responded strategically to the distances of the two paths. Intermediaries on a shorter path chose higher prices that appeared proportionate to the difference in distance between the two paths. In all the networks, this difference in prices chosen by those on a shorter path and on a longer path is statistically significant. As a consequence, trade often occurs along the longer path.

Our next finding pertains to pricing by critical versus non-critical traders in the Ring with Hubs and Spokes. We find that critical intermediaries choose prices that are similar to those of non-critical traders on a shorter

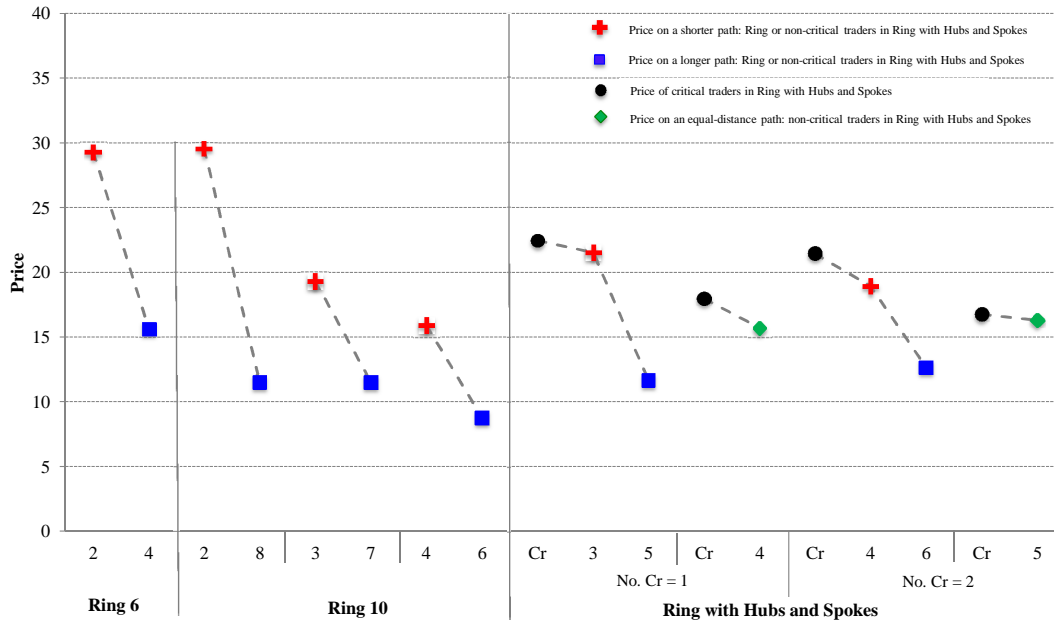


Figure 6: Pricing behavior in Rings and Ring with Hubs and Spokes

path or to all non-critical traders when the two paths are of equal distance. In all trading cases except for the case of  $(\#Cr, d(q), d(q')) = (2, 4, 6)$ , we cannot reject the null hypothesis either that prices chosen by critical and non-critical intermediaries on a shorter path are equal or that critical intermediaries choose the same price as non-critical intermediaries when the two paths are of equal distance. These findings are in line with the predictions of the theory.

Next, we examine the pricing behavior in Line networks. Theorem 2 (in a partially efficient equilibrium) predicts the declining patterns of prices with distance: 50 in Line 3; 33.3 in Line 4; 20 in Line 6; and 14.3 in Line 8. Figure 7 presents the sample average of prices with 95 percent confidence

	Ring 6	Ring 10			Ring with Hubs and Spokes			
	(2, 4)	(2, 8)	(3, 7)	(4, 6)	#Cr = 1		#Cr = 2	
Dependent variable: price	(2, 4)	(2, 8)	(3, 7)	(4, 6)	(3,5)	(4,4)	(4,6)	(5,5)
(1) Non-critical & on a shorter path	13.436 (2.722)***	19.086 (3.725)***	9.017 (1.865)***	6.977 (2.186)***	10.492 (1.896)***		4.511 (2.678)	
(2) Critical					11.318 (1.987)***	2.840 (2.520)	8.598 (1.520)***	-1.278 (3.089)
Constant	28.552 (0.961)***	7.750 (0.000)***	10.357 (0.678)***	1.907 (1.700)	6.647 (0.966)***	7.449 (0.360)***	7.536 (0.594)***	9.000 (0.000)***
$H_0: (1) = (2)$ or $H_0: (2) = 0$ (p-value)					0.641	0.269	0.117	0.682
R-squared	0.338	0.387	0.349	0.312	0.326	0.345	0.223	0.376
Number of obs.	312	280	312	200	280	145	378	144

Notes: Each regression contains dummies for individual subjects. Robust standard errors, clustered by individual subject, are reported in parentheses. \*, \*\*, and \*\*\* represent 10%, 5%, and 1% significance level.

Table 8: Regressions of pricing on network position

interval across Line networks, along with the theoretically predicted price. As theory predicts, average prices fall with distance between  $\mathcal{S}$  and  $\mathcal{D}$ : 34 in Line 2; 24 in Line 3; 17 in Line 6; 13 in Line 8. However, average prices quantitatively depart from the predictions in a manner that subjects underprice relative to the equilibrium. The gap between empirical prices and equilibrium prices shrinks with distance. We shall return to these departures in the next section.

We finally turn to the empirical investigation of the theoretical prediction that critical traders across different positions set a common price. We focus on Line 6 and Line 8 networks for this analysis. Table 9 reports the regression results of prices on dummies for network positions, using the last 20 rounds of the data. The average prices in position  $A$  of Line 6 and Line 6 networks are, respectively, about 20 and 13. The coefficients of position dummies are not significantly far from zero, and we cannot reject the null hypothesis of the equivalence of prices between any two positions in each Line network at an usual significance level.

We summarize the pricing behavior in networks with demand uncer-

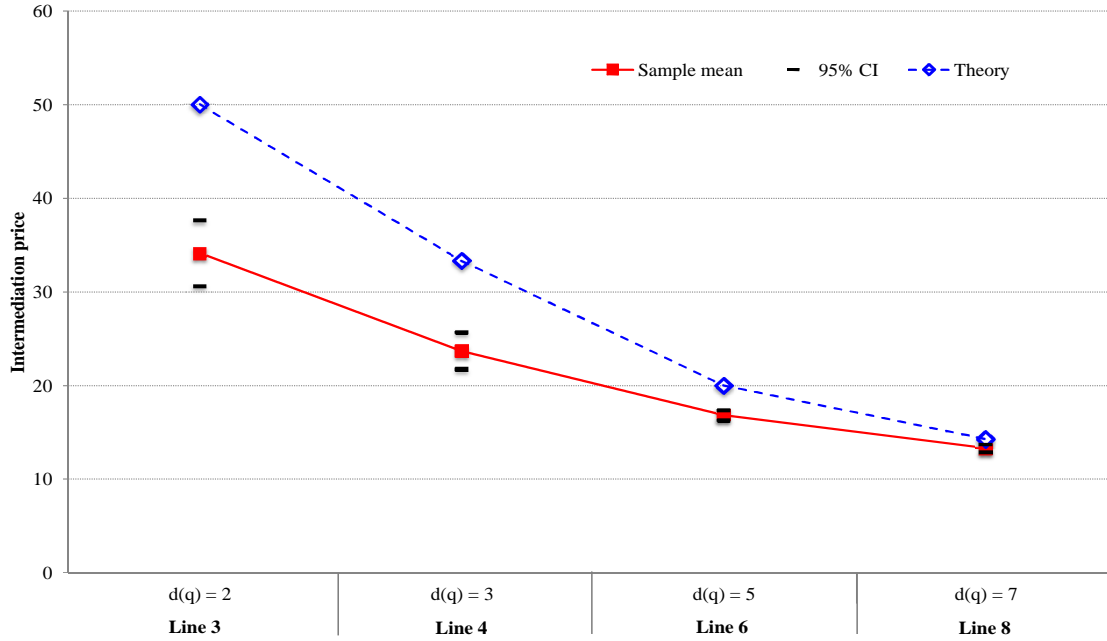


Figure 7: Pricing behavior in Line networks

Dependent variable: price	Line 6 ( $d(q) = 5$ )	Line 8 ( $d(q) = 7$ )
Constant	19.579 (0.348)***	12.753 (0.456)***
Position B	0.029 (0.606)	-1.030 (0.714)
Position C	-0.383 (0.588)	-0.588 (0.874)
Position D	0.362 (0.624)	-0.006 (0.682)
Position E		-0.205 (0.709)
Position F		-0.764 (0.806)
R-squared	0.200	0.190
Number of obs.	720	720

Notes: Each regression contains dummies for individual subjects. Robust standard errors, clustered by individual subject, are reported in parentheses. \*, \*\*, and \*\*\* represent 10%, 5%, and 1% significance level.

Table 9: Regressions of pricing in line network

tainty as follows.

**Finding 2A:** *(i) Subjects responded strategically to the distances of two paths. Critical traders and non-critical traders on a shorter path set similar prices, while non-critical traders on a longer path set much lower prices. (ii) Average prices in Line networks decline with distance, as theory predicts. However, average prices are lower than equilibrium prices; the gap between them shrinks with distance.*

## 5 Concluding remarks

We propose a general model of posted prices in networks. Our theoretical analysis provides a complete characterization of posted price equilibrium for arbitrary structures of intermediation. This is a first step towards understanding the functioning of intermediated networks. Our experiments complement our theoretical work and point to *node criticality* as an organizing principle for understanding pricing, efficiency and the division of surplus in networked markets.

Our model extends naturally to the case of an arbitrary number of source-destination pairs. The key assumption is that traders know the location of the source-destination in the network, and can discriminate based on this location. In some applications, traders set prices that apply uniformly to all intermediated trades, independently of the location of the origin and destination. An example of uniform prices are road tolls: two drivers who



use a bridge across a river will pay the same amount, regardless of where they started or where they end up. This motivates the study of pricing in a model in which the network origin and destination of trades are unknown.

In a companion paper, Choi, Galeotti and Goyal (2014), we study this setting. We suppose that all traders simultaneously post prices: the price that a trader sets applies to all potential trades that go through him. Once prices are set, an  $\mathcal{S}/\mathcal{D}$  pair is picked at random from the set of all traders. As before, a feasible least-cost path is selected. Given a profile of prices, a trader faces the following trade-off. A higher price raises the payoff if trade does take place, but it rules out long-distance trade between farther-away  $\mathcal{S}/\mathcal{D}$  pairs. The theory and experiments suggest that location uncertainty leads to breakdown of long distance trade and creates large losses in efficiency.

We have assumed that all intermediaries have zero costs and that this is common knowledge. It would be natural to examine the case in which intermediaries have private information about their marginal costs. At a general level, residual uncertainty about marginal costs of upstream and downstream intermediaries leads to a trade-off similar to the one introduced by demand uncertainty or by uncertainty of the location of source and destination. That is, when an intermediary sets the price, he faces the trade-off between charging a high price and obtaining a high profit, but with low probability, or charging a low price, which leads to a low profit, but with high probability. In an early paper, Spulber (1995) studies a setting with

one intermediary per path; in a recent paper, Minarsch and Leister (2015) provide partial characterization of pricing equilibrium for special classes of networks. The analysis of pricing with asymmetric information in general networks remains an open problem.

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## Appendix

### Proof of Theorem 1:

**Existence:** If  $\mathcal{C} = \emptyset$ , set  $p_i^* = 0$  for all  $i \in \mathcal{N}$ . Note that no intermediary can earn positive profits by deviating and setting a positive price, because, since there are no critical traders, there is always an alternative zero cost path. If  $\mathcal{C} \neq \emptyset$ , then consider a price profile  $\mathbf{p}^*$  such that  $p_i^* = 0$  if  $i \notin \mathcal{C}$ , and for  $j \in \mathcal{C}$  set  $p_j^*$  so that  $\sum_{j \in \mathcal{C}} p_j^* = 1$ . It is easily checked that no critical or non-critical intermediary has a profitable deviation from this profile.

**Characterization:** We first show that  $c^*(p^*) \in (0, 1)$  cannot be sustained in equilibrium. We consider two cases.

**Case 1:** Suppose  $|\mathcal{Q}^*| = 1$ ; in this case a trader  $i$  on  $q \in \mathcal{Q}^*$  can raise his price slightly and strictly increase payoffs.

**Case 2:** Suppose  $|\mathcal{Q}^*| > 1$ ; consider a path  $q \in \mathcal{Q}^*$  and fix a trader  $i \in q$  with  $p_i > 0$ . Note that such a trader always exists, given that  $c(p^*) > 0$ . We have two possibilities:

**2a:** If intermediary  $i$  is essential, he can raise his price slightly and he will remain essential as all other prices remain as before and the sum of prices being less than 1. So there is a strictly profitable deviation.

**2b:** If  $i$  is not essential, given that  $|\mathcal{Q}^*| > 1$ , the probability that  $i$  is used in exchange is at most  $1/2$ . If trader  $i$  lowers his price slightly, he ensures that he is on the unique feasible least cost path. Thus the deviation strictly increases payoff.

Now we take up each of the remaining three possibilities with regard to intermediation costs and characterize the conditions for which they can be sustained in equilibrium.

**1.** Assume  $c(p^*) = 0$ . We first establish sufficiency. In equilibrium every trader makes payoff 0. Consider an increase in price by some intermediary  $i$ . As no intermediary is essential under  $p$ , there exists an alternative path between  $b$  and  $s$  at cost 0, and this path excludes trader  $i$ . So there is no profitable deviation, and  $p^*$  is an equilibrium.

We now establish necessity. Suppose there is a trader  $i$  who is essential under  $p^*$ . As  $c(p^*) = 0$ , essential trader  $i$  can raise his price slightly, still ensure that exchange takes place through him, and thereby he strictly raises his payoffs. So  $p^*$  is not an equilibrium.

**2.** Assume  $c(p^*) = 1$ . We first establish sufficiency. Consider intermediary  $j \in q$ , with  $q \in \mathcal{Q}^*$ . If  $p_j^* > 0$  then intermediary  $j$  is essential and so trade occurs with probability 1 via  $j$  and he earns  $p_j^*$ . If  $j$  raises his price then total costs of intermediation exceed 1 and no trade takes place, yielding a zero payoff to  $j$ . If  $j$  lowers his price, trade does occur with probability 1 via him, so he only succeeds in lowering his payoff below  $p_j^*$ . Next consider trader  $k \in q$  with  $q \in \mathcal{Q}^*$  such that  $p_k = 0$ . It is easily verified that  $k$  cannot increase his payoff by raising his price. Finally, consider  $l \notin q, \forall q \in \mathcal{Q}^*$ . This trader earns 0 in  $p^*$ . A deviation to a lower positive price leaves the trade probability via  $l$  at 0, as  $c_{-l}(q'^*) \geq 1$  for all  $q'$  such that  $l \in q'$ . We have shown that  $p^*$  is an equilibrium.

We now establish necessity. Suppose  $j \in q$ , with  $q \in \mathcal{Q}^*$ ,  $p_j^* > 0$  and  $j$  is not essential. So the probability that exchange occurs via trader  $j$  is at most  $1/2$ . Trader  $j$  can lower his price slightly and this will push the probability of trade via himself to 1, and thereby he strictly raises his payoff. Next consider  $k \notin q$  for all  $q \in \mathcal{Q}^*$  and suppose  $c_{-k}(q'^*) < 1$  for some  $q'$  such that  $k \in q'$ . Under  $p^*$ , the payoff to  $k$  is 0. But since  $c(p^*) = 1$ , there is a price  $p_k = 1 - c_{-k}(q'^*) - \epsilon$  such that, for small  $\epsilon > 0$ , the probability of trade via  $k$  is 1 and  $p_k > 0$ . This is therefore a profitable deviation.

**3.** Assume  $c(p^*) > 1$ . We first establish sufficiency. All traders earn 0 under profile  $p^*$ . It can be checked that no deviation to another price can generate positive payoffs given that  $c_{-j}(q, p^*) \geq 1$ , for all  $j$  and for all  $q \in \mathcal{Q}$ . A deviation to price 0 yields payoff 0. This proves sufficiency.

We now establish necessity. Suppose that  $c(p^*) > 1$  and that there is some  $j \in q$  such that  $c_{-j}(q, p^*) < 1$ . Then there is a price  $p_j = 1 - c_{-j}(p^*) - \epsilon$ , for some  $\epsilon > 0$  such that trade takes place via trader  $j$  with probability 1 and  $p_j > 0$ . This constitutes a profitable deviation.

■

### **Proof of Theorem 2:**

**Existence:** If there are no critical traders in  $g$ , then existence of efficient equilibrium follows the arguments developed in Theorem 1. If there are critical traders then set  $p_i = 0$  for every non-critical intermediary  $i$ , and for every critical intermediary set  $p^* = 1/h(\eta p^*)$ , where  $\eta$  is the number of crit-

ical players. The constructed profile satisfies part 2. Therefore there always exists a partially efficient equilibrium in the presence of critical traders.

**Characterization:** The proof of Part 1 and Part 3 uses the arguments developed in the proof of Part 1 and Part 3 of Theorem 1, and are therefore omitted. We now prove Part 2.

First consider necessity. Suppose  $p^*$  is equilibrium and  $c(p^*) \in (0, 1)$ . Take an arbitrary least cost path  $q \in \mathcal{Q}^1$ . Observe that every player  $i$  who is not essential and who belongs to path  $q$  must set price 0. For otherwise, a positive price by player  $i$ ,  $p_i > 0$ , is dominated by a slightly lower price  $p'_i < p_i$ , that ensures the path  $q$  becomes the unique lowest cost path. This observation and the hypothesis that  $c(p^*) > 0$ , implies that there must exist essential players, i.e.,  $e(g, p^*) \geq 1$ , and that  $c(p^*) = \sum_{i \in \mathcal{E}(g, p^*)} p_i^*$ .

Second, the optimal price of an essential player  $i \in \mathcal{E}(g, p^*)$  solves

$$p_i^* = \arg \max p_i [1 - F(p_i + c_{-i}^*(p^*))]. \quad (\text{A.1})$$

It is easy to see that  $p_i^* \in (0, 1 - c_{-i}^*(p^*))$ ; the first order condition then says that for all  $i \in \mathcal{E}(g, p)$ ,

$$p_i^* = \frac{1 - F(c(p^*))}{f(c(p^*))}.$$

But this implies that  $\forall i, j \in \mathcal{E}(g, p^*)$ ,  $p_i^* = p_j^*$  and  $p_i^* \in \left(0, \frac{1}{e(g, p^*)}\right)$ . So

equilibrium price is given by

$$p_i^* = \frac{1 - F(e(g, p^*)p^*)}{f(e(g, p^*)p^*)}.$$

The existence of such a  $p^* \in \left(0, \frac{1}{e(g, p^*)}\right)$  follows from the assumption that  $f(\cdot)$  and  $F(\cdot)$  are both continuous functions and that  $f(0) > 0$ . Finally consider an intermediary  $i$  who does not belong to any path in  $\mathcal{Q}^1$  and suppose that  $c_{-i}(q'^*) < e(g, p^*)p^*$  for some path  $q'$  such that  $i \in q'$ . Then player  $i$  can charge a price  $p = e(g, p^*)p^* - c_{-i}(q'^*) - \epsilon > 0$  and now whenever trade occurs it will occur via path  $q'$ ; hence, this is a strictly profitable deviation for intermediary  $i$ . The proof that these conditions are sufficient is straightforward, given that the hazard rate is increasing. ■

**Proof of Proposition 1:** From Theorem 2 we know that in a partially efficient equilibrium every essential player sets price,  $p_i^*$ , such that:

$$p_i^* = \frac{1}{h(e(g, p^*)p_i^*)} \tag{A.2}$$

where  $e(g, p^*) \geq 1$ . The assumption of increasing hazard rate implies that there exists a unique  $p^*$  which solves  $p^* = 1/h(e(g, p^*)p_i^*)$ . We now prove the three parts in the proposition.

Part 1. Implicitly differentiating (A.2) and simplifying yields:

$$\frac{dp^*}{de(g, p^*)} = -\frac{h'(e(g, p^*)p_i^*)}{h^2(e(g, p^*)p_i^*) + h'(e(g, p^*)p_i^*)} < 0, \quad (\text{A.3})$$

where the inequality follows from the assumption of increasing hazard rate.

Part 2. Next, note that in a partially efficient equilibrium intermediation costs are  $e(g, p^*)p_i^*$  and therefore the probability that trade does not occur is  $F(e(g, p^*)p_i^*)$ . Again, implicit differentiation yields

$$\begin{aligned} \frac{dF(e(g, p^*)p_i^*)}{de(g, p^*)} &= f(e(g, p^*)p_i^*) \left[ p_i^* + e(g, p^*) \frac{dp_i^*}{de(g, p^*)} \right] \\ &= f(e(g, p^*)p_i^*) p_i^* \left[ 1 - \frac{h'(e(g, p^*)p_i^*)}{h^2(e(g, p^*)p_i^*) + h'(e(g, p^*)p_i^*)} \right] > 0 \end{aligned}$$

where the the second equality follows by substituting the expression for  $\frac{dp_i^*}{de(g, p^*)}$  from above, and the inequality follows from the assumption of increasing hazard rate.

Part 3. The expected payoff of an essential intermediary is  $p^*[1 - F(e(g, p^*)p^*)]$ ; since inessential intermediaries obtain a payoff of zero, the joint profits of intermediaries are

$$\sum_{i \in \mathcal{N}} \Pi_i(p^*) = e(g, p^*)p^*[1 - F(e(g, p^*)p^*)], \quad (\text{A.4})$$

and

$$\begin{aligned}
\frac{d \sum \Pi_i(p^*)}{de(g, p^*)p^*} &= [1 - F(e(g, p^*)p^*)] - e(g, p^*)p^* f(e(g, p^*)p^*) \\
&= [1 - F(e(g, p^*)p^*)](1 - e(g, p^*)) \leq 1, \tag{A.5}
\end{aligned}$$

where the second equality follows using equilibrium condition  $p^* = 1/h(e(g, p^*)p^*)$ , and the inequality follows because in a partially efficient equilibrium  $e(g, p^*) \geq 1$ .

1. Finally, the joint profit of  $\mathcal{S}$  and  $\mathcal{D}$  is

$$\begin{aligned}
\Pi_{\mathcal{S}}(p^*) + \Pi_{\mathcal{D}}(p^*) &= [1 - F(e(g, p^*)p_i^*)] [E[v|v \geq e(g, p^*)p_i^*] - e(g, p^*)p_i^*] \\
&= \int_{e(g, p^*)p_i^*}^1 x f(x) dx - e(g, p^*)p_i^* [1 - F(e(g, p^*)p_i^*)] \tag{A.6}
\end{aligned}$$

and therefore

$$\frac{d[\Pi_{\mathcal{S}}(p^*) + \Pi_{\mathcal{D}}(p^*)]}{de(g, p^*)p^*} = -[1 - F(e(g, p^*)p_i^*)] < 0. \tag{A.7}$$

■