An Investigation Into The Relationship Between Wages, Mismatch, On-the-Job Search and Education

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Contents

Dedicated to my mother, Durre Maknoon Syed, who passed away a few months before the submission of my thesis. I hope you are proud and smiling from up in the heaven.

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Summary

This thesis contributes to the existing literature by studying the link between educational choices, skill mismatch and wages in a labour market with search frictions with on-the-job search.

In the first paper, I used empirical techniques to look at the link between skill mismatch and wages. I found that over-education and mismatch is part of a career mobility or job-to-job transition in the labour market. Workers accept jobs for which they are overqualified and search on-the-job to move to jobs that are more matched to their educational level. In the process they accept a wage cut which is temporary until they are able to find a job better suited to their level of education.

In the second paper, I used search and matching framework to study the link between on-the-job search and wages in an economy where high and low ability workers compete for jobs. On-the-job search is a way in which workers reduce the extent of mismatch and firms react to this. However, this interaction implies that when more workers try to relocate the friction in the market reduces the efficiency of resource allocation (by increasing mismatch) and it also creates more wage inequality between the different types of workers.

Finally in the third paper, I looked at the link between educational choices, and skill mismatch in a labour market with search frictions. I found that fewer search frictions lead to higher inequality in wages. If the cost of education is low enough, more individuals choose to acquire education and get trained. As a consequence mismatch increases.

Part I

Over-Education, Mismatch & Wage Penalty

1 Abstract

This paper adds to existing literature on mismatch, over-education, and wage penalty by utilising Panel Data to investigate the permanence of over-education wage penalties, while controlling for unobserved heterogeneity. The fixed effects estimates show that unobserved heterogeneity cannot account for all the difference in wage penalty. However, for both non-graduates and graduates the wage penalty for over-education is temporary. In this sense over-education is part of a career mobility or job-to-job transition in the labour market. Workers accept jobs for which they are overqualified and search on-the-job to move to jobs that are more matched to their educational level.

2 Introduction

Over the past decades in all Western countries there has been an increase in the educational level of the population. In the OECD countries about 38% of population aged 55-64 years had at least an upper secondary education in 1992. About 65% of the population aged 25-34 years had at least an upper secondary education. This translates into a 70% increase in the share of population with at least an upper secondary education in less than 30 years (Groot and Maassen van den Brink, 1996). With a rapid increase in the education level of workers there has been a higher than average increase in jobs requiring highly educated workers. For many jobs skill upgrade has been necessary to perform effectively. Despite this, the increase in the demand for higher-educated labour has not kept pace with the increase in the supply of skilled workers (Manacorda and Petrongola, 1998). Mismatch arises if the growth in the supply of higher-educated workers is more than the growth in demand. This particular mismatch between the workers and jobs is over-education. Workers are overeducated if their skills as approximated by their level of education is higher than the skills required to do the job they hold. Thus, the allocation of skills over jobs is not optimal.

Mismatch can be of two types, vertical mismatch and horizontal mismatch. Vertical mismatch occurs when an individual accepts a job for which he/she is either over-qualified or under-qualified. Horizontal mismatch on the other hand occurs if a person specialising in one area of study accepts a job in another sector. In this example if an economist whose highest level of education is a PhD takes up a job as a historian then he/she will be considered mismatched. The job of a historian may require a PhD as well but it would be a PhD in a relevant sector.

Many studies have shown that a large proportion of employees are mismatched or overqualified for the jobs they do (Borghans and de Grip, 2000; Hartog, 2000). This is achieved by deconstructing the effect of education within a wage equation into a part that the job requires (usually approximated through the respondent's self-assessment of years of education required to do the job effectively) and if the respondent has a higher or lower level of education than that required (Brynin, Lichtwardt and Longhi, 2006).

Over-education can be seen as a compensation for lacking other human capital endowments, such as ability, on-the-job training or experience (Brynin, Lichtwardt and Longhi, 2006). According to Groot (1993, 1996) and Sicherman (1991) over-educated workers tend to have less experience and on-the-job training compared to well-matched workers. Individuals having gone through a career break — for instance women with children — have a higher likelihood of being in jobs for which they are over-educated (Groot and Maassen van den Brink, 1996). On the other hand, over-education can be a part of a career mobility or job-to-job transition in the labour market. Workers may accept jobs for which they are overqualified and search on-the-job to move to jobs that are more matched to their educational level. After controlling for experience, younger workers have a higher probability of being over-educated than older workers (Groot, 1996; Groot and Maassen van den Brink, 1996). Also, over-educated workers change jobs more frequently, Sicherman (1991). This further implies that over-education is merely a stage of acclimatization in the early years of a career.

People who are over-educated for a particular job tend to be paid better than someone with the same job, but worse than someone with the same education, but in a job which is considered adequate for their level of education (Duncan and Hoffman, 1981; Hartog and Oosterbeek, 1988; Sicherman, 1991; Hersch, 1991; Cohn and Khan, 1995; Van Smoorenburg and Van der Velden, 2000). Furthermore, people who are undereducated tend to earn more than someone of the same education in a job that is adequate for them, but less than someone who is in the same job that is adequate for them. This further suggests that being in a job in which the employee is classified as over-educated could be viewed as a "transient state", where workers are attempting to gain additional information on labour market opportunities and adjust their present position through additional job search (Hartog, 2000). Most of the evidence in literature on over-education uses cross-sectional data. Following Lindley and McIntosh (2010) this paper uses panel data from the UK to examine three important aspects of over-education:

1. To what extent is the wage penalty to over-education attributable to unobserved characteristics of the individuals?

2. Is the wage penalty for over-education fixed for all individuals or does it change by whether the individuals escape over-education to move into matched jobs?

3. Is the wage penalty permanent for individuals who were able to escape over-education and move into matched jobs compared to those who were always in a matched job?

The main thought behind these questions is to analyse if there are differences in unobserved characteristics between workers that are mismatched and those that are not, and those who escape mismatch and those who do not. Given the increase in participation of individuals in higher education over the recent decades, the impact of mismatch on wages is an important issue. The next section of the paper examines the methodology used in the estimation of the wage equations followed by a section on the data set to be used. The next sections contain analysis of the three questions mentioned above followed by an analysis of horizontal mismatch. This is followed by some concluding remarks.

3 Methodology

In the analysis, education level or qualification is measured in terms of the highest qualification achieved, on a scale of 1 (less than O'levels) to 6 (higher education) as shown in table 1.

Education Level	Frequency	Percentage	Cumulative
Less O'levels (1)	39,372	24.04	24.04
O'levels (2)	32,077	19.58	43.62
A'levels (3)	24,165	14.75	58.37
Vocational (4)	44,133	26.94	85.32
First Degree (5)	19,284	11.77	97.09
Higher Degree (6)	4,763	2.91	100.00
Total	163,794	100.00	

Table1: Education Level

Worker i is defined as over-educated if his/her actual highest qualification exceeds the required qualification at time t. To measure over qualification, I follow Mendes de Oliveira et al. (2000), Bauer (2002) and Battu and Sloane (2004). I measure required qualification as the modal highest qualification in each occupation. This method is a variation of the method proposed by Verdugo and Verdugo (1989) where they measure the required education as the mean level of education amongst workers in the individual's occupation. An individual is over-educated if their actual qualification is one standard deviation above the mean or more. According to Mendes de Oliveira et al. (2000), the modal method is preferred because it is independent of the arbitrary use of the standard deviation and also since this method is less affected by outliers.

However, both methods use actual qualification to measure required qualification rather than the job requirement. In case the general qualification level of population rises, the average qualification of workers hired in all populations increases.

The mean and the mode after the tipping point in the most frequent qualification is reached also increase. However, this is not a serious problem over the period considered because even if there is no change in the job requirements, an increase in qualifications increases the required education so there is an under estimation of over-education.

3.1 Earnings

To analyse the affect of education on earnings, I use a variation of over-required and under-required (ORU) specification used by Hartog (1997) and Lindley and McIntosh (2010). The model includes dummy variables for each level of required education (RQ^m_{it}) for m=2. . . 6 with m=1 being the base level. Over-education is measured using 5 dummy variables (D^k_{it}) indicating the distance between actual and required education when this value is positive. This specification allows the returns to required-education and over-education to increase non-linearly across levels. Variables are also included for levels of under-education, $SU^1_{it}, SU^2_{it}, SU^3_{it}$. Again, the three variables indicate the distance between actual and required education. Note, the reason only three variables for under-education are included is that none of the individuals in my sample have an under-education of level 4 or 5.

$$Y_{it} = \beta X_{it} + \gamma_2 RQ^2_{it} + ... + \gamma_6 RQ^6_{it} + \delta_1 D^1_{it} + ... + \delta_5 D^5_{it}$$

$$\mu_1 SU^1_{it} + \mu_2 SU^2_{it} + \mu_3 SU^3_{it} + \varepsilon_{it} \tag{1}$$

Y_{it} is the net pay per month, X_{it} contains the relevant socio-economic and job characteristics for worker i at time t that explain earnings.

I estimate the parameters in equation (1) using fixed effect estimator to allow for the possibility of unobserved heterogeneity biasing the parameters through correlation between education and the error term ε_{it}. An individual should earn the same (determined by ability) regardless of the job after controlling for heterogeneity if unobserved heterogeneity is the only reason individuals work for jobs for which they are over-qualified. In such a case the returns to an incremental level of over-education,$\delta_k - \delta_{k-1}$, should be the same as the returns to an incremental level of required education, $\gamma_m - \gamma_{m-1}$.

The next step is to estimate whether the variation in estimated returns to over-education, change according to whether or not an individual moves to a well-matched job. This is done by interacting a variable that gives the amount of over-education experienced by an individual i at a time t, OL_{it}, with an indicator of whether the individual is still over-educated in the next period, OE_{it+1}. In order to reduce the number of interaction terms, over-education is constrained to be linear. This is equation 3. For comparison a similar equation but without the interaction terms was also estimated. This is equation 2.

$$Y_{it} = \beta X_{it} + \gamma_1 RQ_{it}^2 + ... + \gamma_6 RQ_{it}^6 + \alpha OL_{it} + \mu_1 SU_{it}^1 + \mu_2 SU_{it}^2 + \mu_3 SU_{it}^3 + \varepsilon_{it} \quad (2)$$

$$Y_{it} = \beta X_{it} + \gamma_1 RQ_{it}^2 + ... + \gamma_6 RQ_{it}^6 + \varphi OL_{it} + \pi(OL_{it} * OE_{it+1})$$

$$+\mu_1 SU_{it}^1 + \mu_2 SU_{it}^2 + \mu_3 SU_{it}^3 + \varepsilon_{it} \quad (3)$$

The over-education variable, OE_{it+1}, was interacted with over-education rather than required education since the former contains returns to individual characteristics and the later returns to job level characteristics. The interaction term was included to estimate whether earnings vary by unobserved individual characteristics, as proxied by future job matching, so interaction with over-education is more suitable.

Finally, I investigate whether over-education penalty is related to job specific characteristics or individual characteristics of workers in over-educated jobs, by estimating whether the wage penalty remains after individuals move to a well-matched job, compared to individuals who were already well-matched.

To do this, wage equations are estimated on samples of workers in matched jobs at $t = 1996, 2001, 2006$ respectively, with a dummy variable for whether individual i was over-educated at $t = 1991$, $OE_{it=1991}$. This is equation 4.

$$Y_{it} = \beta X_{it} + \gamma_1 RQ_{it}^2 + ... + \gamma_6 RQ_{it}^6 + \lambda OL_{it=1991} + \mu_1 SU_{it}^1 + \mu_2 SU_{it}^2 + \mu_3 SU_{it}^3 + \varepsilon_{it} \quad (4)$$

Hence, λ measures the earnings differential in 1996, 2001 and 2006, between workers who were always in a well-matched job and workers who were over-educated in 1991.

Finally, I will estimate equation 4 again including an interaction term between the variable OE_{it} and a variable indicating that an individual's highest qualification is a degree or above (HQ_{it}^5). This allows me to determine whether the effect of a history of over-education on future matched wages is the same for graduates and non-graduates. This is equation 5.

$$Y_{it} = \beta X_{it} + \gamma_1 RQ_{it}^2 + ... + \gamma_6 RQ_{it}^6 + \vartheta OE_{it=1991} + \sigma(HQ_{it}^5 * OE_{it=1991})$$

$$+ \mu_1 SU_{it}^1 + \mu_2 SU_{it}^2 + \mu_3 SU_{it}^3 + \varepsilon_{it} \quad (5)$$

Here,σ measures the difference in the penalty to previous (1991) over-education between graduates and non-graduates.

4 Data and Descriptive Information

The British Household Panel Survey (BHPS) is a longitudinal survey of households in Great Britain. The first wave of data was collected in 1991 with the survey then repeated each year. I use data from the first 18 waves (1991-2008). Note to check for robustness I also carried out the analysis using the Skills and Employment Survey Series Dataset and the results were broadly similar. However, those results are not included in the paper because they are less reliable and the longitudinal version of the survey has missing years.

In the BHPS dataset, in the first wave a nationally representative sample of 10,000 individuals, in 5,000 households, were interviewed from 250 areas of Great Britain. In subsequent years these same individuals were re-interviewed, as were any new members of their household, plus members of the new households of any individual who left their original household, as well as new households to replace any households that left the survey. Information was collected at both individual and household level, and includes individual questions on human capital and qualifications, as well as socio-economic characteristics such as income, employment status and region of residence, and job characteristics such as promotion prospects and firm tenure.

The BHPS data has been used to create a panel of 173,332 working age adults (from ages 18-60 years), with 81,303 male and 92,029 female observations. Table 2 provides summary statistics of net monthly earnings as well as all the explanatory variables used throughout the analysis.

The BHPS data set is used to calculate required qualification (RQ_j) using the mode level of HQ_i (highest qualification) by the occupation category j (taken from the variable "jbgold") averaged across 1991-2008. The data show that around 38.4% of the individuals are over-educated. To see if over-education is temporary, table 3 looks at transitions out of over-education for each occupational category for a balanced panel between 1991 and 2008. The table shows that of the 38.4% over-educated individuals around 14.8% of people remain over-educated in the subsequent wave.

Variable	Mean	Standard Deviation
$Log(wage)$	6.77	0.70
Age	40.00	10.14
Age^2	1703.29	826.10
$FirmSize$	5.06	2.45
$HoursWorked$	33.49	11.79
$FullTime$	0.78	0.42
$UnemploymentHistory$	0.59	4.07
$SpouseEmployed$	0.85	0.35

Table 2

Job categories	Over-Educated	ΔOver-Educated
Service Class Higher	2,174	377
Service Class Lower	9,626	1,100
Routine Non-Manual	57,075	8,637
Personal service	72,544	10,427
Small Proprietor w Employees	3,675	565
Small Proprietor w/o Employees	3,872	593
Farmers, Smallholders	53,994	8,304
Foreman, Technicians	3,877	583
Skilled Manual	55,312	8,540
Semi, Unskilled Manual	74,716	10,733
Agricultural	3,590	563
Percentage	38.4%	14.8%

Table 3

Figure 1 shows the proportion of workers in employment in the UK who are over-qualified or under-qualified. The figure shows that post 2008 the proportion of over-qualified has increased.

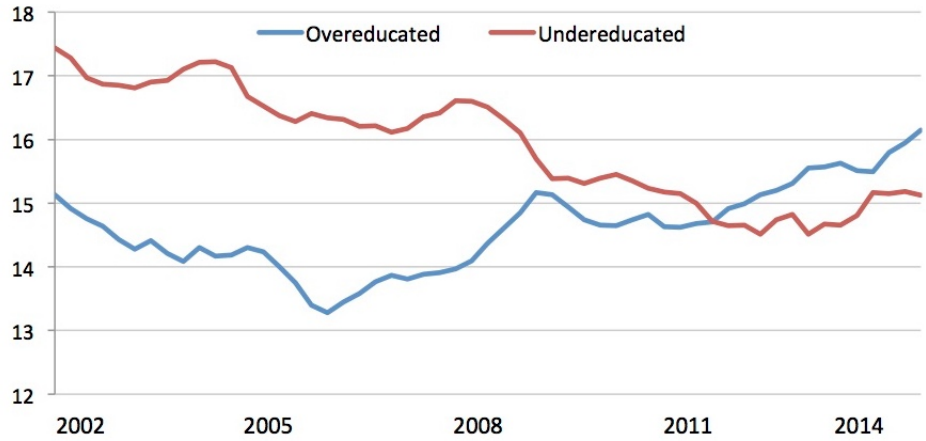

Figure 1: Over and Under Educated as a Proportion of those in Employment

4.1 Job Flows

Job flows help in understanding the flow of workers between different jobs. Figure 2 shows the job move rate. This is defined as the proportion of workers who change jobs between two periods. The figure aslso shows the reasons for the job moves.

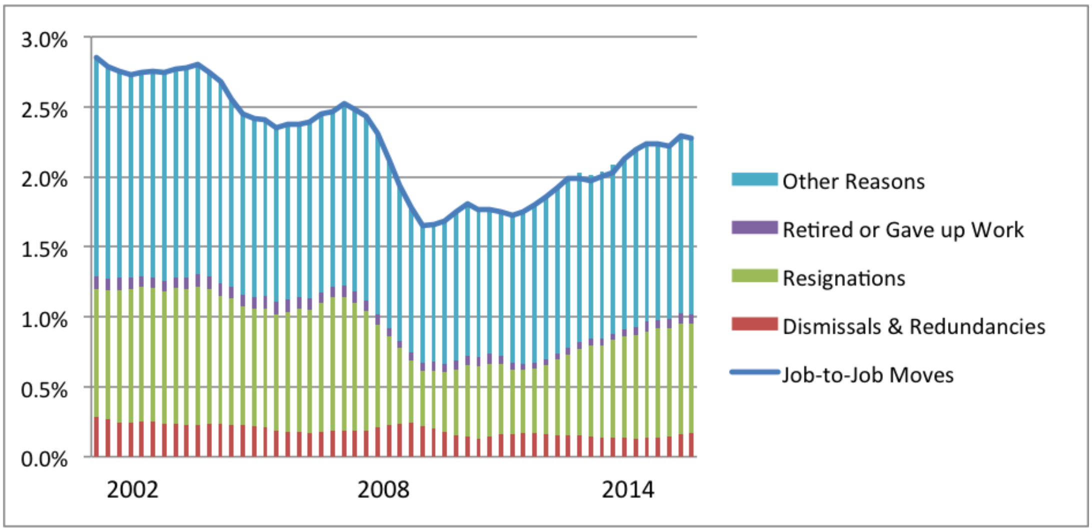

Figure 2: Contributions to Job Moves by Reason for Leaving Last Job (%)

The job move rate is the proportion of workers who move between one employer and another each period. “Other reasons” for move include workers whose temporary job finished, workers who gave up work for health reasons, for family or personal reasons, for education or for some other reason.

During the economic downturn (2008-2009), the rate of job to job moves fell by more than a third. During 2009 since the redundancy rate increased, the number of people resigning from their jobs fell markedly, reflecting the unwillingness of workers to risk moving from their current positions in a climate of elevated economic uncertainty. From 2011 onwards the rate of job-to-job transitions rose steadily.

While figure 2 shows the rate of job-to-job transitions, figure 3 shows the nature of the jobs towards which individuals are moving to. Figure 3 shows the contributions of job-to-job transitions both within the same industry and between different industries. Among the former are workers who arrange to move to a new post within the same industry – perhaps taking with them the specific skills and knowledge that they have acquired in their current position. Among the latter, are a group who are moving between industries – either seeking to change direction, or to deploy their skills in a new way.

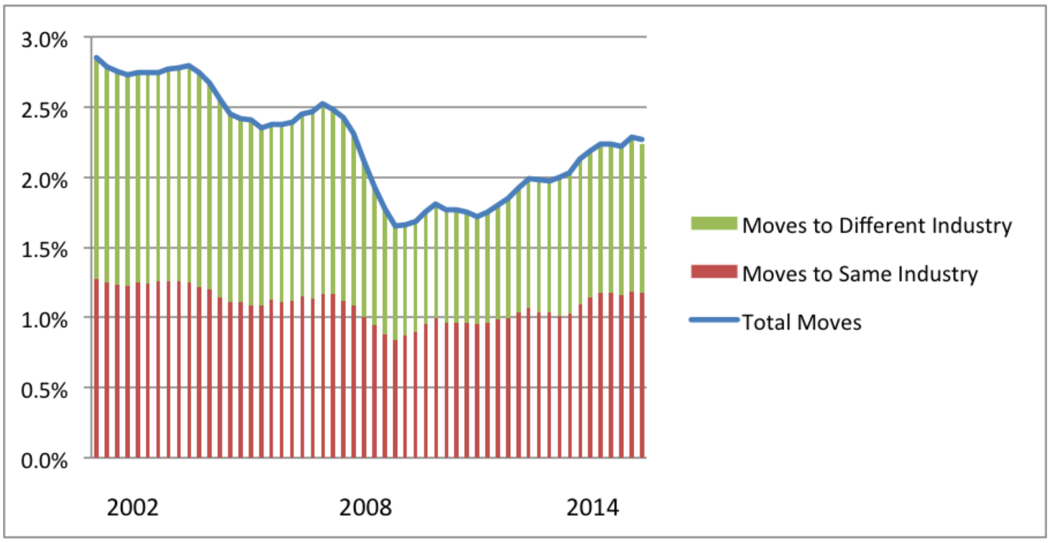

Figure 3: Moves within and Between Industries (as a %age of total employees)

Figure 3 shows that while both types of move were affected by the economic downturn, moves between industries fell more than moves within industries. : This could either be due to workers willing to take fewer risks, or because employers sought to benefit from the training and development of other similar firms, rather than investing in new staff.

Figure 4 shows the transitions of two different types of workers from and into the two occupational groups.

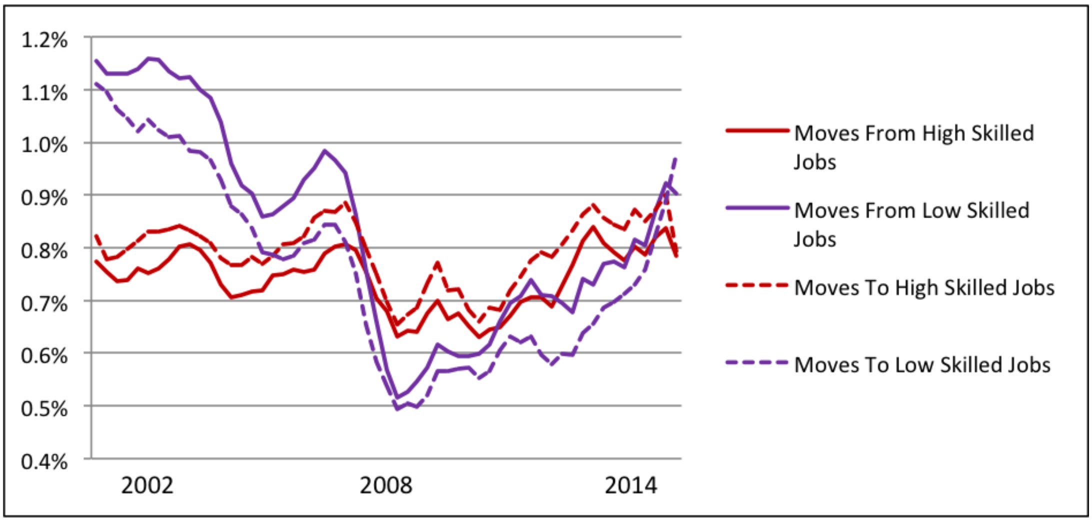

Figure 4: Transitions to and from High and Low- Skilled Occuptaions (as % age of total employees each period)

Higher turnover among the less-educated workers in the low-skill occupations has a greater contribution to in- and out-flows compared with high-skill occupations since they have a higher transition rate throughout the period. This high rate of turnover fell sharply during the downturn, reducing job mobility among lower occupational groups relative to higher-occupational groups. Low-skill occupations comprise elementary occupations, sales & customer services operators and process, plant & machine operatives, while high-skill occupations comprise managers, directors & senior officials, professional occupations and associate professional & technical occupations.

4.2 Panel Data Analysis of Earnings Equations

I will now use the BHPS data to estimate the wage equations to see if the wage penalties to over-education exist using panel data. I will use fixed effects estimation to see how much of the wage penalty is attributed to unobserved heterogeneity. Table 4 shows the results of the regression equation 1.

Table 4 shows key educational returns for the ORU earnings equations estimated using fixed effects and OLS. The first and third columns show the fixed effects estimates for men and women in the ORU model respectively. The second and fourth columns show the standard OLS wage equations using the ORU specification for men and women respectively. Comparing the OLS estimates like Lindley and McIntosh (2010) the incremental return to a level of required education is always greater than that to all over-education levels. Not only that but there is a direct negative impact on wages due to over-education. The largest increment for required education is a move from level 4 to level 5 followed by a move from level 3 to level 4. Returns to required education are higher for men than for women in general. However, the penalty for over-education is also higher for men than for women.

Once the individual unobserved heterogeneity is controlled one would expect an individual to earn the same regardless of the job if over-education is merely an indicator of unobserved heterogeneity and thus the penalties would be close to zero. This, however, is not the case as shown by columns 1 and 3. Controlling for fixed effects, men and women earn different amounts depending on the level of job they do which implies that over-education wage penalty is not a penalty attributed to low ability. Like Verdugo and Verdugo (1989), in my model the return to under-education is positive and significant. This is probably due to how under-education is defined. A person is under-educated if they have less education compared to their colleagues in a similar job. Hence, the wage for under-education is more job specific than individual specific.

Turning explicitly to the wage penalties associated with over-education one sees that the higher the level of mismatch, the higher the wage penalty. Looking at the OLS estimates for women and men the wage penalty for over-education is 0.143 log points for women and 0.115 log points for men if the level of over-education is 1.

	Male		Female	
	FE	OLS	FE	OLS
	$LogWage$	$LogWage$	$LogWage$	$LogWage$
$RequiredEd^2$	0.0445**	0.240***	0.0860***	0.243***
	(2.62)	(25.86)	(5.51)	(25.40)
$RequiredEd^3$	0.145***	0.512***	0.112***	0.370***
	(7.80)	(47.47)	(7.23)	(38.78)
$RequiredEd^4$	0.229***	0.692***	0.137***	0.518***
	(14.81)	(84.73)	(10.58)	(64.24)
$RequiredEd^5$	0.372***	1.012***	0.235***	0.745***
	(14.77)	(101.74)	(9.42)	(79.81)
$RequiredEd^6$	0.524***	1.176***	0.381***	0.924***
	(13.62)	(68.86)	(12.45)	(66.86)
$OverEd^1$	-0.0341***	-0.115***	-0.0456***	-0.143***
	(-3.88)	(-13.12)	(-6.55)	(-17.90)
$OverEd^2$	-0.0994***	-0.222***	-0.0886***	-0.227***
	(-11.34)	(-25.11)	(-12.42)	(-27.89)
$OverEd^3$	-0.175***	-0.422***	-0.138***	-0.338***
	(-17.56)	(-45.87)	(-15.67)	(-36.03)
$OverEd^4$	-0.466***	-0.688***	-0.258***	-0.557***
	(-16.71)	(-24.49)	(-10.89)	(-19.32)
$OverEd^5$	-0.437***	-0.964***	-0.384***	-0.653***
	(-5.11)	(-9.61)	(-5.05)	(-5.71)
$UnderEd^1$	0.0620***	0.0754***	0.0263**	0.0588***
	(5.92)	(7.41)	(3.11)	(6.56)
$UnderEd^2$	0.106***	0.246***	0.0433***	0.103***
	(10.30)	(25.29)	(4.39)	(9.69)
$UnderEd^3$	0.116***	0.239***	0.0847***	0.217***
	(8.68)	(17.90)	(7.88)	(19.61)
$Constant$	3.349***	4.402***	3.516***	4.815***
	(73.04)	(126.03)	(84.93)	(132.90)
N	36392	36392	33801	33801

t statistics in parentheses

* $p<0.05$, ** $p<0.01$, *** $p<0.001$

Table 4

Over-education of level 1 happens for instance if the required education for a job is first degree and the individual has a highest qualification of a higher degree. When the level of over-education increases to 2 the wage penalty associated with it increases to 0.227 log points for women and 0.22 log points for men. This continues increasing non-linearly till an over-education level of 5 where the wage penalty associated with it is 0.653 log points for women and 0.964 log points for men.

Fixed effects estimates show a similar pattern. Whereas if unobserved heterogeneity was the sole cause of wage penalty controlling for this would imply that the wage penalty associated with over-education is now zero. Columns 1 and 3 of table 4 show that although, as expected, the penalties for over-education are lower for the fixed effects estimation compared to OLS, the wage penalties do not disappear. Thus, even after controlling for unobserved heterogeneity the wage penalty continues to grow non-linearly with the extent of over-education. Unobserved heterogeneity is no doubt a very important factor in explaining the wage penalties to over-education but is not the only factor that explains the existence of over-education. This is consistent with Chevalier (2003) and Lindley and McIntosh (2010).

One important thing to note is that in the fixed effects specification, the effect of over-education on wages is due to the transitions between differently matched jobs. It is likely that such transitions are not random. Thus, the results might not be representative of all the over-educated workers. Thus, I explore the wage analysis for more evidence on over-education.

4.3 Returns to Over-Education by Future Matched Status

Table 5 shows the results for equations (2)-(5) estimated for workers observed in 1991, 1996, 2001 and 2006. To ensure a reasonable sample the data for men and women are pooled. Equation 2 is the ORU wage equation estimated previously but now estimated separately for the four years mentioned. The extent of over-education is now estimated as a single variable with levels. The wage penalty for over-education is similar for the cross-sections considered with the penalty slightly greater in 1996. Thus, all the cross-sections examined tell a similar story.

	1991	1996	2001	2006
Equation (2)				
Unbalanced Panel ($\hat{\alpha}$)	-0.167***	-0.177***	-0.162***	-0.165***
	(-7.16)	(-8.36)	(-11.29)	(-11.46)
Equation (3)	-0.0618***	-	-	-
Found a match in 1996 ($\hat{\varphi}$)	(-4.68)	-	-	-
Differentital b/w Matched & OE in 1996 ($\hat{\pi}$)	-0.233***	-	-	-
	(-24.92)	-	-	-
Found a match in 2001 ($\hat{\varphi}$)	0.141***	0.142***	-	-
	(12.70)	(12.80)	-	-
Differentital b/w Matched & OE in 2001 ($\hat{\pi}$)	-0.148***	-0.0463***	-	-
	(-19.73)	(-8.90)	-	-
Found a match in 2006 ($\hat{\varphi}$)	0.301***	0.302***	0.306***	-
	(24.37)	(24.44)	(24.75)	-
Differentital b/w Matched & OE in 2006 ($\hat{\pi}$)	-0.147***	-0.0445***	0.0144**	-
	(-19.57)	(-8.58)	(2.71)	-
Equation (4)				
Penalty to now matched, previously over-educated, workers, relative to those already matched in 1991 ($\hat{\lambda}$)	-	-0.0329	-0.0244	-0.0143
	-	(-1.23)	(-0.75)	(-0.32)
Equation (5)				
Penalty to now matched, previously over-educated, non-graduates, relative to those already matched in 1991 ($\hat{\vartheta}$)	-	-0.0273	-0.00541	0.0143
	-	(-0.90)	(-0.15)	(0.27)
Differential in penalty to previous over-education between non-graduates and graduates ($\hat{\sigma}$)	-	-0.0257	-0.0876	-0.104
	-	(-0.40)	(-1.11)	(-1.03)

Table5: Returns to a level of over-education by key cross sections

Equation 3 allows the estimated returns to over-education in any year to differ according to whether or not an individual gets matched afterwards. The results show that individuals who get matched in 1996 suffer a wage penalty of 0.23 log points for being over-educated in a job in 1991. Thus, with the job requirements held constant there will still be variation among wage penalty suffered by over-educated employees at a level. Those who find jobs five years later already earn a higher wage (or a smaller over-education wage penalty) during their period of over-education.

The subsequent rows in the 1991 column of table 5 show that the longer a worker has to wait to get matched, the smaller their advantage in terms of return to over-education in 1991 relative to those who never get matched since the coefficients of $\hat{\pi}$ are declining. Thus, those who find a well-match job sooner, earn higher wages whilst still over-educated.

4.4 Wage Penalties to Previous Over-Education Status

Equations (4) and (5) show the impact of previous over-education status on the wages of workers who are now matched. Equation 4 shows that matched workers who were previously over-educated in 1991 earn wages around 0.01-0.3 log points lower compared to those already matched in 1991. Therefore, wages are not only determined by job characteristics but also by unobserved ability of workers who fill them, in this case determined by prior over-education experience. Holding constant the job requirements, lower wages, apparently indicating lower unobserved ability, are obtained for individuals previously over-educated. Thus, over-education is partly a characteristic of unobserved ability. However, in this analysis the results are not significant implying that the wage penalties are not permanent.

The last two rows of table 5 show the negative impact of previous over-education on matched wages for non-graduates and graduates. Again the results show that the penalty for over-education is temporary. Not only this but the difference between the wage penalty for non-graduates and graduates being insignificant implies that over-education wage penalty is only temporary regardless of the level of education. Thus, over-education is not an indication of lower ability within a qualification category for both non-graduates and graduates. Short periods of over-education are a natural event with no great future consequences, as individuals learn on the job, before being offered responsibilities equal to their skills.

5 Horizontal Mismatch

So far the analysis on over-education has focused on the vertical mismatch between an individual's education level and the skill required in the job he/she occupies. Recently the literature on over-education and wage penalty has focused on the "task approach". The idea here is to classify jobs according to their core task requirements and then look at the set of skills necessary for these tasks. Autor and Handel (2013) argue that the task approach has the potential to offer a micro foundation for a link between the aggregate demand for skill in the labour market to the specific skill demands of a given job activity.

However, there are two main challenges facing the task approach. The first is conceptual. So far the task approach has been unable to make explicit links between tasks, which are characteristics of jobs, and human capital, which is a characteristic of workers. The second challenge is measurement. The data sets available for studying employment and earnings give rough measures of workers' human capital, such as education and experience, but almost no information on their job tasks. To get around this limitation, researchers usually impute task requirements to person-level observations.

The BHPS dataset does not have the required data to carry out this analysis so I turned to the Skills and Employment Surveys Series Dataset, 1986, 1992, 1997, 2001, 2006 and 2012. The combined dataset includes comparable variables from the six surveys of the series. Therefore, it includes variables that appear at least twice in the series. These are a series of nationally representative sample surveys of individuals in employment aged 20-65 years old. The numbers of respondents were: 4,047 in the 1986 survey; 3,855 in 1992; 2,467 in 1997; 4,470 in 2001; 7,787 in 2006; and 3,200 in 2012. For each survey, weights were computed to take into account the differential probabilities of sample selection, the over-sampling of certain areas and some small response rate variations between groups (defined by sex, age and occupation). All of the analyses that follow use these weights.

The Skills and Employment Surveys Series Dataset provides information on job characteristics only at the level of occupations, not workers. This makes analysis of within- occupation heterogeneity in task demands and its relationship to earnings infeasible. I present evidence below both that job tasks differ among workers within an occupation and that this variation is an important determinant of earnings.

The current paper provides an exploratory effort to confront both of the limitations above: a lack of conceptual structure for analysing the wage "returns" to tasks and a lack of data for analysing the person-level relationship between tasks, education, and wages. The second goal of my paper is to explore the value added of task measurement at the person level for analysing job content and wage determination.

In order to look at the extent to which within-occupation variation in self-reported job tasks captures differences in wages, I regress log wages on task scales, demographic variables, and occupation dummies. The task scale here is a self-reported variation in job tasks such as routine repetitive tasks. I have also included self-reported skill-mismatch and matching variables that predict if the skill learnt during schooling is relevant to the job-tasks and if the past job skills are relevant to the job tasks. If the self-reported variation in job tasks is a robust predictor of wages, then there is evidence that self-reported task variation is a useful predictor of job content even within occupations.

As a benchmark, table 6 shows the relationship between log wages, human capital, and demographic variables for the six cross-sections (years 1986, 1992, 1997, 2001, 2006, and 2012) that data is available for. Most of the variables have signs as expected. The race dummies are mostly insignificant which shows that race is not an important predictor of wage variation. The variable search is a dummy variable that determines if an individual is actively searching on-the-job. Given firms prefer workers to stick around the negative impact on earnings of on- the-job search is as expected.

	1986	1992	1997	2001	2006	2012
Log Hours	-	-	1.606***	1.771***	1.712***	1.694***
	-	-	(12.56)	(16.19)	(22.93)	(16.92)
University	-	-	-	0.924***	0.546***	0.578***
	-	-	-	(7.21)	(4.78)	(3.37)
Masters & PhD	-	-	-	1.122***	0.977***	0.478
	-	-	-	(4.66)	(3.68)	(1.29)
No Qualification	-	-	-	-0.679***	-1.299***	-0.956***
	-	-	-	(-5.11)	(-14.23)	(-5.97)
Work Experience	0.0188***	0.0151***	0.0447*	0.0578***	0.0376***	0.0599***
	(6.88)	(5.37)	(2.09)	(3.36)	(3.92)	(4.73)
Work Experience Sq	-0.000391***	-0.000340***	-0.000929*	-0.000999**	-0.000690***	-0.00120**
	(-6.22)	(-5.48)	(-2.00)	(-2.67)	(-3.47)	(-3.14)
Female	-0.333***	-0.248***	-0.267*	-0.194*	-0.323***	-0.0816
	(-20.20)	(-15.69)	(-2.23)	(-2.00)	(-5.26)	(-0.96)
Supervise	-	-	0.887***	0.657***	0.717***	0.642***
	-	-	(7.75)	(6.96)	(11.99)	(7.67)
Permanent	0.0666*	0.0194	0.549**	0.551**	0.433***	0.137
	(2.42)	(0.77)	(2.59)	(2.96)	(3.30)	(0.75)
Union	0.114***	0.129***	0.496***	0.131	0.551***	0.349***
	(6.95)	(7.90)	(4.41)	(1.42)	(9.28)	(4.05)
Black	-0.0133	0.0233	0.311	-0.462	-0.431	-0.815**
	(-0.17)	(0.58)	(0.72)	(-1.31)	(-1.89)	(-2.80)
Asian	-0.0402	-0.0239	1.055**	-0.231	-0.261	-0.199
	(-0.28)	(-0.40)	(2.79)	(-0.82)	(-1.34)	(-0.97)
Trained	0.230***	0.293***	0.919***	0.537***	0.539***	0.438***
	(14.23)	(18.04)	(8.24)	(5.90)	(9.14)	(5.30)
Effort	-	0.0163	0.0229	0.0747	0.0448	-0.0274
	-	(1.55)	(0.30)	(1.12)	(1.05)	(-0.44)
London	-	-	0.503**	0.650***	0.390**	0.463**
	-	-	(2.98)	(4.31)	(2.79)	(2.86)
Private	-	-	-	0.322	0.623***	0.274
	-	-	-	(1.70)	(4.78)	(1.58)
First	-	-	-	-	0.719***	0.698***
	-	-	-	-	(3.58)	(3.30)
Search	-0.0461**	-	-	-	-0.137*	-
	(-2.83)	-	-	-	(-2.47)	-
Constant	0.994***	1.433***	0.708	0.317	0.863**	1.826***
	(26.84)	(22.91)	(1.19)	(0.65)	(2.62)	(4.21)
N	2948	2654	2121	3883	5715	2176

t statistics in parentheses

* $p<0.05$, ** $p<0.01$, *** $p<0.001$

Table 6: OLS Regressions of Log Wages on Demographic Variables

Table 7 replaces the human capital and demographic controls with the task and skill-matching scales that predict significant wage differentials. In 2001, a mismatched person, defined as a person who does not utilise his/her skills accumulated during education, tends to earn 72% less than a person who utilises his/her skills acquired during education. Figures for 2006 and 2012 are 59% less and 68% less respectively.

	1986	1992	1997	2001	2006	2012
Mismatched	-	-	-	-0.719**	-0.592***	-0.676**
	-	-	-	(-3.11)	(-3.81)	(-2.60)
Repetitive Tasks	-	-	-0.565***	-0.437***	-0.636***	-0.609***
	-	-	(-4.19)	(-4.05)	(-8.72)	(-5.71)
Past Skill	-	0.122***	-	0.513***	0.491***	0.365***
	-	(15.61)	-	(11.10)	(15.66)	(7.54)
Constant	-	1.392***	8.789***	7.499***	7.397***	8.087***
	-	(56.73)	(75.79)	(42.47)	(61.87)	(43.18)
N	-	2726	2179	3974	6067	2287

t statistics in parentheses

* $p<0.05$, ** $p<0.01$, *** $p<0.001$

Table 7: OLS Regressions of Log Wages on Task Scales

In 1992, a person who utilises the skills acquired during past jobs tends to earn 12% more than a person who does not. Figures for 2001, 2006, and 2012 are 51% more, 49% more, and 37% more respectively. The wage penalty for repetitive tasks is 57% in 1997, 44% in 2001, 64% in 2006, and 61% in 2012.

Table 8 demonstrates that these three sets of variables; human capital and demographics, job task and skill matching, and occupation, capture distinct sources of wage variance. The repetitive task measure remains a significant predictor of wages conditional on either human capital and demographic measures or a full set of occupation dummies.

	1986	1992	1997	2001	2006	2012
Mismatched	-	-	-	-0.440*	-0.246	0.0645
	-	-	-	(-2.07)	(-1.79)	(0.28)
Repetitive Tasks	-	-	-0.286*	-0.0918	-0.287***	-0.254**
	-	-	(-2.36)	(-0.91)	(-4.36)	(-2.68)
Past Skills	-	0.0769***	-	0.161***	0.179***	0.148***
	-	(10.41)	-	(3.59)	(6.11)	(3.41)
Log Hours	-	-	1.611***	1.704***	1.615***	1.634***
	-	-	(12.83)	(15.68)	(21.74)	(16.19)
University	-	-	-	0.705***	0.301**	0.396*
	-	-	-	(5.31)	(2.58)	(2.22)
Masters & PhD	-	-	-	0.951***	0.846**	0.341
	-	-	-	(3.84)	(3.20)	(0.90)
No Qualification	-	-	-	-0.583***	-1.176***	-0.959***
	-	-	-	(-4.36)	(-12.99)	(-5.93)
Work Experience	0.0169***	0.0104***	0.0294	0.0407*	0.0296**	0.0567***
	(6.29)	(3.84)	(1.46)	(2.55)	(3.11)	(4.42)
Work Experience Sq	-0.000351***	-0.000249***	-0.000553	-0.000658	-0.000568**	-0.00115**
	(-5.67)	(-4.19)	(-1.30)	(-1.95)	(-2.89)	(-2.98)
Female	-0.298***	-0.211***	-0.142	-0.0918	-0.266***	-0.0347
	(-18.03)	(-13.71)	(-1.17)	(-0.94)	(-4.31)	(-0.40)
Supervise	-	-	0.646***	0.415***	0.470***	0.483***
	-	-	(5.28)	(4.15)	(7.46)	(5.42)
Permanent	0.0558*	0.00143	0.520*	0.490**	0.430***	0.136
	(2.06)	(0.06)	(2.46)	(2.63)	(3.30)	(0.75)
Union	0.139***	0.163***	0.613***	0.244**	0.672***	0.441***
	(8.53)	(10.29)	(5.33)	(2.60)	(11.20)	(4.98)
Black	-0.0246	-0.000843	0.346	-0.361	-0.314	-0.636*
	(-0.33)	(-0.02)	(0.80)	(-1.02)	(-1.39)	(-2.22)
Asian	-0.0424	-0.0284	1.183**	-0.290	-0.237	-0.184
	(-0.31)	(-0.49)	(3.11)	(-1.03)	(-1.23)	(-0.88)
Trained	0.215***	0.220***	0.880***	0.445***	0.452***	0.391***
	(13.43)	(13.60)	(7.87)	(4.82)	(7.64)	(4.67)
Effort	-	-0.0109	0.0118	0.0312	0.000362	-0.0682
	-	(-1.06)	(0.15)	(0.46)	(0.01)	(-1.07)
London	-	-	0.484**	0.664***	0.346*	0.380*
	-	-	(2.86)	(4.36)	(2.49)	(2.32)
Private	-	-	-	0.210	0.521***	0.118
	-	-	-	(1.09)	(4.00)	(0.67)
First	-	-	-	-	0.556**	0.494*
	-	-	-	-	(2.76)	(2.22)
Search	-0.0457**	-	-	-	-0.0868	-
	(-2.85)	-	-	-	(-1.56)	-
Constant	0.972***	1.365***	0.893	0.434	1.098**	1.901***
	(26.63)	(22.48)	(1.50)	(0.87)	(3.27)	(4.12)
N	2948	2626	2121	3891	5712	2172

t statistics in parentheses

* p<0.05, ** p<0.01, *** p<0.001

Table 8: OLS Regressions of Log Wages on Task Scales, Demographic Variables, and Occupation Dummies

Similarly the past skill variable remains a significant predictor of wages conditional on either human capital and demographic measures or a full set of occupation dummies. However, the mismatch variable only remains a significant predictor of wage in 2001 at 5% level of significance.

While statistical significance does not imply economic significance, the magnitude of the relationship between tasks and wages net of other variables is still significant and substantial. Within occupations, a person utilising past job skills predicts an 8% increase in wage earnings in 1992. Figures for 2001, 2006, and 2012 are 16% increase, 18% increase and 15% increase respectively. Similarly the figures for repetitive tasks in 1997, 2001, 2006 and 2012 are 29% decrease, 9% decrease, 29% decrease and 25% decrease respectively. Although the effects are diminished they remain large and significant. Among these three measures, the mismatch variable proves least robust, losing significance when human capital and demographic controls, and occupational controls are included. These results suggest that the self-reported job tasks show sizable differences in job activities among workers, both within and between occupations.

6 Matching Areas of Study to Occupations

As an added analysis on the effect of horizontal mismatch on wages I focus on the 2006 cross-section that has the most detailed breakdown of job type. I limit analysis on individuals with an undergraduate degree because that is the only point where data is available for the subjects studied. Unfortunately that limits the data to only 1498 individuals. Thus, an analysis on the effect of mismatch on wages cannot be done through a regression. There are a total of 197 different occupations and 11 different undergraduate areas of study, namely mathematics, computing, physical science and engineering, biological science, social science, English and cultural studies, art and design studies, business and management studies, humanities, law, and medicine. For each occupation I searched for the entry-level requirements on job sites (such as Prospects) and marked as mismatched all individuals who are in a particular occupation but have studied a subject area that is unrelated to that occupation. For example, if an individual has studied biological science but is working as a financial institution manager, then that individual is considered mismatched. Surprisingly, using this definition of mismatch the number of individuals who are mismatched for each subject category is very high. In particular of the 55 individuals who have studied maths in their undergraduate, 39 are mismatched (71%). The figures for the rest of the occupations are as follows, 32 out of 61 for computing (52%), 138 out of 169 for engineering (82%), 86 out of 108 for biological sciences (80%), 131 out of 162 (81%) for social sciences, 80 out of 86 for English and cultural studies 93%, 56 out of 75 for art and design studies (75%), 125 out of 185 for business and management studies (68%), 93 out of 96 for humanities (97%), 32 out of 56 for law (57%), and 7 out of 34 for medicine (21%).

In order to estimate the relationship between wage and each of the 11 mismatched categories I calculate the correlation matrix between the mismatched variable and log of wage. Table 9 shows the results. As the results show only three mismatch variables (mathematics, computing and social science) show a significant negative impact on wages at 5% level. However, this result should be taken with extreme caution since the data available is very limited.

	Lwage	Maths	Comp	Engineer	Bio	SocialS	English	Art	BM	Human	Law	Medicine
Lwage	1.0000											
Maths	-0.3484*	1.0000										
	(0.0190)											
Comp	-0.2985*		1.0000									
	(0.0316)											
Engineer	-0.0775			1.0000								
	(0.3663)											
Bio	-0.1061				1.0000							
	(0.3037)											
SocialS	-0.2407*					1.0000						
	(0.0046)											
English	0.0188						1.0000					
	(0.8769)											
Art	-0.1652							1.0000				
	(0.2372)											
BM	-0.0979								1.0000			
	(0.2447)											
Human	-0.0621									1.0000		
	(0.5890)											
Law	-0.1965										1.0000	
	(0.2580)											
Medicine	0.2038											1.0000
	(0.3179)											

p-value in parentheses

* $p<0.05$

Table 9: Correlation matrix of log of wage and the 11 mismatched variables

7 Conclusion

This paper analysed the hypothesis that over-education and mismatch have a negative impact on the wages of workers. Over-education is partly an indicator of low-ability among groups of workers who share similar qualifications but have unobserved heterogeneity amongst them in terms of ability. This comes from the fact that using the BHPS dataset, the workers who were over-educated in 1991 did not all receive the same wage penalty to over-education even after controlling for the required qualifications for their jobs. The over-education wage penalty is smaller amongst workers who would get matched in 5 years. This implies that some of the heterogeneity amongst the over-educated, especially those with the lowest ability remain over-educated longer. However, the over-education wage penalty grows smaller and smaller in subsequent years, and the results for wage penalty for workers who eventually get matched become statistically insignificant.

Like Lindley and McIntosh (2010), the results show heterogeneity in skill amongst the over- educated. More trained individuals with higher initial earnings are more likely to escape over-education. However, unlike Lindley and McIntosh (2010) there is no evidence of heterogeneity between workers who experience a spell of over-education compared to those who do not, after controlling for the qualifications obtained. This comes from the fact that the results of wage penalty for a matched worker who was previously over-educated in 1991 are insignificant as indicated by the high p-values and low t-values. This holds true for both non-graduates and graduates. Thus, temporary over-education is not an indicator of low-ability but merely a reflection of experience gained over the years to complement the qualification before moving to a matched job.

The results show the presence of unobserved ability between workers; amongst workers who were always in a matched job and amongst workers who were over-educated but later found a matched job. None of our workers remains permanently over-educated. Thus, controlling for heterogeneity explains the difference in wages amongst those currently matched and those over-educated as shown by the fixed effects estimates for women and men. A substantial portion of over-education wage penalty observed in the OLS estimates is removed after controlling for unobserved heterogeneity.

When using the Skills and Employment Surveys Series Dataset and looking at the horizintal mismatch between workers, it turns out that once the occupation variables are controlled for mismatch is not a significant predictor of wage penalty. However, at the task-level repetitive tasks and past experience in a job remain significant predictors of wage even after controlling for occupation.

A lot of work needs to be done on the horizontal mismatch and task approach to understand the relationship between mismatch and wages at the person level. More data needs to be collected at the individual level to get a more detailed picture of a workers human capital in terms of the area of study and the type of job-tasks involved. This would be the next step towards understanding the variation in wages of workers in the United Kingdom.

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Part II

Wage Determination with On-the-Job Search and Bargaining

9 Abstract

On-the-job search by workers is an important feature in labour markets. In this paper I have analysed how this phenomenon affects the structure of employment and wages in an economy where high and low ability workers compete for jobs. I find that when all workers search on-the-job the outcome is worse for the labour market position of untrained workers compared to when only mismatched workers search while employed. This is because it causes a move in the job distribution towards high-technology firms and decreases the overall stability of unskilled jobs. These results are consistent for when the motive of job-to-job transitions is the pursuit of a better match or when identical workers compete in same job types. On-the-job search is a way in which workers reduce the extent of mismatch and firms react to this. However, this interaction implies that when more workers try to relocate the friction in the market reduces the efficiency of resource allocation (by increasing mismatch) and it also creates more wage inequality between the different types of workers.

10 Introduction

Economies are characterised by a significant amount of skill mismatch. According to McGuinness (2006) using post 1980 data the skill mismatch range is reported as follows, 13-50 per cent for the United States, 31 per cent for Canada, 11-40.7 per cent for United Kingdom, 11.5-17.5 per cent for Germany, 17 per cent for Spain, 20 per cent for Ireland and 11.85-30.6 per cent for Netherlands. A possible explanation for the existence of mismatch is search frictions. However, workers move between jobs, which in principle suggests that some of this mismatch may not be permanent.

Occupational mobility and reallocation leads to temporary mismatch in an economy because the labour market is characterised by frictions. There is evidence in literature for on-the-job search. Fallick and Fleischman (2001) use data on US labour market to show that job-to-job transitions account for 50% of separations among college workers. Pissarides (1994) shows that job-to-job flows account for at least 40% of all separations in the UK in 1980s, while in Germany Bachman (2006) estimates that job-to-job flows accounted for about 35% of monthly separations during 1980-2000.

From time to time workers reallocate; sometimes within the same occupation and sometimes they switch occupations. Hall (2006) uses data from the US labour market and estimates the job finding rate to be around 40% per month while Brainard and Perry (2002) estimate the US quarterly job finding rate to be around 70% for the first year of search.

Reallocation occurs not only through unemployment but also through employment. Taylor and Longhi (2011) show that for both employed and unemployed job seekers the probability of finding a new job in the same occupation as the previous job is relatively low (around 30%), while more than one half experience a major occupational change. Moscarini and Thomsson (2007) suggest that employer-to-employer transitions are around 2.7% for the US from 1994-2004.

Kambouro and Manovski (2008) suggest that in the United States among male workers over the 1968-1997 period the average level of occupational mobility at a one-digit level is around 13% and the industry mobility is around 10%. At a one-digit-level occupational mobility has increased from 10% to 15%, while industry mobility has increased from 7% to 12%. Occupational and industry switches are fairly permanent: around 30% of the workers switching occupations (industries) return to their one-digit occupation (industry) within a four-year period after the switch.

The trends in mobility of unemployed workers differ substantially from employed workers. Taylor and Longhi (2011) used data from the Labour Force Survey to show that employed and unemployed job seekers in Great Britain originate from different occupations and find jobs in different occupations. They found substantial differences in occupational mobility between job seekers: employed job seekers are most likely to move to occupations paying higher average wages relative to their previous occupation, while unemployed job seekers are most likely to move to lower paying occupations. This suggests that for unemployed people a change in occupation is likely to have a negative impact on future wage growth while for employed people an occupational change is more often associated with better prospects for wage growth.

This model speaks about workers of different skills that face jobs with different skill requirements. Random search generates a temporary skill mismatch since highly qualified workers can be employed in jobs with low skill requirements. On-the-job search allows the workers to obtain a better employment. This is a model in which a brain surgeon can be a taxi driver but not the other way around. It is in that sense that the model can be linked to occupational mismatch.

Wage inequality and occupational mobility are interrelated phenomena. Using data from the US, Kambouro and Manovski (2008) show changes in wage inequality in the U.S. from the early 1970s to the mid 1990s. They show that inequality of hourly wages has increased over the period – the variance of logs has increased from 0.225 to 0.354, or 57%, while the Gini coefficient has increased from 0.258 to 0.346, or 34%. Most of the increase in wage inequality was due to rising inequality within narrowly defined age-education subgroups. The increase in wage inequality reflects increased dispersion throughout the entire wage distribution. Individual earnings became substantially more volatile. Kambouro and Manovski (2008) show that between-occupation inequality is much higher than within occupation inequality. The Within-Group measure of wages, displays an increase in inequality between occupations from 0.067 to 0.141 and an increase in the inequality within occupations from 0.109 to 0.140.

While workers search for jobs during unemployment as well as during employment, frictions in the market imply that there will always be some level of unemployment in the economy. Since the labour market is not frictionless it also means that the economy will always be characterised by some level of skill mismatch. This mismatch is reduced with a higher job turnover. However, this skill mismatch is never reduced to zero unless a specific policy is introduced that bars workers from taking up jobs not suited to their level of skill. As long as highly educated workers find it profitable to accept jobs requiring a low skill, there will be a positive amount of mismatch in a market with frictions.

This paper explores the concept of mismatch between workers and jobs and how it affects wages in a market. Wages are determined through Cahuc, Postal-Vinay, Robin (2006), through bargaining between employers and workers under complete information and Bertrand Competition between firms. Wages are determined through a game that generates a result like the generalised Nash-bargaining solution. Workers search on-the-job to improve their employment prospects. The paper shows that wage inequalities are exacerbated by on-the-job search, which is consistent with literature. The first section of the paper gives a general literature on the topic. This is followed by an explanation of the model. To begin with only mismatched workers search on-the-job which is an assumption that is later relaxed.

This paper contributes to the existing literature on wage inequality with on-the-job search by exploring the wage determination through a sequential bargaining game. It also allows for all worker types to search for jobs not only when they are unemployed but also when they are employed. As discussed, workers reallocate not only through unemployment but also through employment so this logical extension further adds realism to the model.

On-the-job search is a way in which workers reduce the extent of mismatch and firms react to this. However, this interaction implies that when more workers try to relocate the friction in the market reduces the efficiency of resource allocation (by increasing mismatch) and it also creates more wage inequality between the different types of workers.

11 Literature Review

Although, much has been said about the existence of skill mismatch and job-to-job transitions, there is insufficient literature on mismatch and on-the-job search coupled together with bargaining for wages between firms and workers and among firms. It is important to look at the effect of bargaining between searching workers and firms since evidence shows that workers search on-the-job for prospects of better employment. In this paper I will study the effect of skill mismatch on wages by developing a search-based model allowing for on-the-job search and wage bargaining. Like Albrecht and Vroman (2002) I consider an economy with heterogeneous workers and firms. Workers are of two types, highly educated ones that are considered skilled labourers and less educated ones that are considered unskilled labourers. This distribution of skill is exogenous in my model. Firms post vacancies that require either a high or a low skill level. The distribution of vacancies is endogenously determined in the model. Technology is such that while highly educated workers are able to occupy both types of vacancies, the less educated workers are only able to occupy low skilled vacancies. Thus, while a surgeon can operate on a patient and perform janitorial duties, a janitor can only perform janitorial duties and cannot operate on patients. By definition the surgeon is a highly educated worker while the janitor is a less educated worker.

Although highly educated workers can occupy both types of vacancies, they do not produce more than less educated workers in a low skilled job. While highly educated workers can occupy low skilled vacancies the match produced is less than ideal since such workers are more likely to break the match and move to a well-matched job. In such a framework the worker's outside option determines his/her wage. Since actual gains from trade accrue to the worker and not the firm, mismatched workers compensate their employers by accepting a wage reduction.

Consistent with Shi (2002), my model has a similar restriction that comes from the literature on skill heterogeneity that assumes that most jobs require a minimum skill requirement. Other authors that use a similar approach to skill heterogeneity include Vroman (1987), Mortensen and Pissarides (1999) and Dolado, Jansen and Jimeno (2008).

Like Bonilla and Burdett (2010) this model allows for on-the-job search but instead of using a lottery system to determine wages, it uses sequential bargaining. It is similar to that of Dolado, Jansen and Jimeno (2008) in that it deals with heterogeneous agents but allows for on-the-job search. However, Dolado, Jansen and Jimeno (2008) use a linear splitting the surplus rule for wage determination compared to the sequential bargaining rule used here. Due to labour market frictions, there is unemployment in equilibrium. The matching function is adopted from Diamond (1982) and Mortensen (1982). Unlike Shi (2002) search is undirected such that highly educated workers encounter low skilled vacancies with a probability per unit time of the fraction of vacancies that require low-skill levels. Similarly, less educated workers encounter high-skilled vacancies (and thus are unable to form a match) with a probability proportional to the fraction of vacancies that require high-skill levels. Thus, the undirected search process is used to capture the idea that given the overall labour market conditions less educated workers are better off, the greater the proportion of low-skilled vacancies available and vice versa for highly educated workers. Similarly, firms advertising low-skilled vacancies are better off, the greater the fraction of less educated workers in the economy. Equilibrium in the model is determined using free-entry conditions that state that the value of maintaining vacancies is zero and the steady state conditions that state that the flow of workers into and out of unemployment and employment are equal.

Several papers published are relevant to my model. The model introduced by Albrecht and Vroman (2002) is one. While their model deals with unemployed workers searching, my model allows both the unemployed and the employed workers to search on-the-job. In this respect, my model is closest to that of Dolado, Jansen and Jimeno (2008) who look at on-the-job search in a matching model with heterogeneous agents. However, my model also allows all types of workers to search on-the-job, where as Dolado, Jansen and Jimeno (2008) only allow mismatched workers to search while employed. Papers by McKenna (1996) and Gautier (1999) are also related since they use the same definition of skill differential as this model. However, while in this model, mismatched workers have the same productivity in low-skilled jobs as less educated workers, Gautier (1999) postulates that mismatched workers may or may not be more productive than less educated workers in low-skilled jobs. Thus, highly educated workers might be worse at simple repetitive jobs or they may be better at simple tasks, for instance waiters who know several languages. Like McKenna (1996) and Gautier (1999) my model does not allow less educated workers to match with complex jobs. Like Albrecht and Vroman (2002), the contact rate in my model is endogenous, which is different from Acemoglu (1999) who works with undirected search with heterogeneous agents and a constant exogenous contact rate. Bonilla and Burdett (2010) also work with a constant exogenous contact rate. However, their model is based on homogenous agents. Other related literatures are Mortensen and Pissarides (1999) and Shi (2002), both working with heterogenous agents, however, the search process in their models is a directed one rather than an undirected one. Thus, the equilibrium that they achieve is that of ex-post segmentation with perfect matching.

While the above models allow for search when workers are unemployed, the current model allows for on-the-job search by both unemployed and employed workers. Related to on-the-job search is a vast literature that assumes that only workers search while employed or unemployed, while firms without vacancies do not search. However, papers by Kiyotaki and Lagos (2007) and Burdett, Imai, and Wright (2003) develop more general models where both parties in a bilateral match may choose to search. In this model, initially only mismatched workers may search on-the-job. This is relaxed later. Search is assumed to be costless such that all mismatched workers necessarily search on-the-job. This is similar to Dolado, Jansen and Jimeno (2008).

This model follows Cahuc, Postel-Vinay and Robin (2006) in assuming that in wage determination, firms consider the worker's outside option. Thus, the incumbent firm and the newly contacted firm bargain for the service of highly educated employees. However, while Pissarides (1994) and Shimer (2006) have models in which firms cannot commit to such a wage offer, this model assumes that all wages are binding. This paper deals with all the above issues. After stating the value functions, the model first assumes that only mismatched workers search on-the-job. I compare the model for the case when firms have all the bargaining power to the case where both firms and workers have equal bargaining power. On-the-job search is considered costless and mismatched workers always search on-the-job. Later I look at the case when all workers are searching on-the-job.

12 Only Mismatched Workers Search On-the-Job

12.1 Basic Framework

I will develop a steady state matching model of unemployment. There are two sectors producing a single good. There is a continuum of workers with mass equal to one. Time is continuous. Agents are risk neutral. Agents discount future at the rate $r > 0$. Workers and firms are heterogeneous in skill and technology respectively. At each instant $\mu > 0$ workers die.

There are two types of workers, untrained $e = 0$ and trained $e = 1$, that is, $e \epsilon [\, 0,1]$. The fraction of untrained workers is exogenously determined and is given by γ_0 .

A worker can be unemployed or employed. Let λ be the effort with which employed workers search on-the-job. An unemployed worker receives a flow pay-off b from leisure, where $0 < b < 1$.

Each employer employs at most one worker. There are two types of jobs available in the labour market, low-technology (L) and high-technology (H) : $j \epsilon \{L, H\}$. While any type of worker can fill a low-technology job, only trained workers are productive in high-technology jobs. In low-technology firm, output y^L is independent of the worker's training e. In high-technology firms, output is y^H if $e = 1$, and is zero otherwise.

There is free entry of firms into each technology sector, where free entry implies new firms in each sector make zero expected profit. The firms pay a cost c for posting any vacancy.

There is job destruction which occurs at a rate of $\delta > 0$. There is also turnover through on-the-job search, which I now describe in detail.

12.2 Matching

Let U_0 be the measure of unemployed workers who are untrained and U_1 the measure of unemployed workers who are trained. Let E_0 be the measure of employed workers who are untrained and E_1 the measure of employed workers who are trained. Let E_{1L} denote the measure of mismatched workers described as trained workers who are employed in a low-technology firm.

Unemployed workers and vacancies are assumed to meet each other according to a random matching technology, where $M = M(K, V)$ denotes the flow number of contacts.

$K = U_0 + U_1 + \lambda E_{1L}$, is the aggregate job search effort because only unemployed and mismatched workers search for jobs. $V = V_L + V_H$, is the total vacancies. Assuming constant returns, define labour market tightness $\theta = V/K$ and $m(\theta) = \frac{M(K,V)}{K} = M(1, \theta)$ which describes the arrival rate of job offers per unit effort.

I assume that $m'(\theta) > 0$ and that $lim_{\theta\to\infty} m'(\theta) = 0$.

Unemployed workers sample job offers randomly at Poisson rate $m(\theta)$. Trained workers employed in low-technology firms may also search for a better job and the arrival rate of offers to on-the-job searchers is $\lambda m(\theta)$. λ is exogenous.

Similarly, vacancies meet unemployed workers at rate $\frac{m(\theta)}{\theta}$. I assume this rate is decreasing in θ and that $lim_{\theta\to 0}[m(\theta)/\theta] = \infty$.

13 Wage Determination

Wages are determined using the Cahuc, Postal-Vinay, Robin (2006) bargaining framework. Given contact and a gain to trade exists, the wage contract is negotiated following a set of rules that I now explain.

Wages are bargained over by workers and employers in a complete information context. Wage contracts stipulate a fixed wage that can be renegotiated by mutual agreement only: renegotiations thus occur only if one party can credibly threaten the other to leave the match if the latter refuses to renegotiate. There are no renegotiation costs.

When between-employer competition for labour services is not perfect, firm-worker matches are associated with a positive rent, defined as the expected value of future match output flows net of the worker's and firm's outside options.

When an unemployed worker meets a firm, the wage is determined as the worker's outside option plus a share β of the match surplus. This game delivers the generalized Nash-bargaining solution, where the worker receives a constant share β of the match rent. This latter parameter β is referred to as the worker's bargaining power.

When an employed worker contacts an outside firm, the situation becomes more favourable to the worker because she/he can now force the incumbent and poaching employers to compete. Competition between the two employers over the worker's services can be seen as an auction where the bidder with the higher valuation wins and pays the second price. I first assume that only trained workers employed in low-technology firms search on-the-job. This assumption will be relaxed later.

13.1 Equilibrium Wage Bargaining

13.1.1 Untrained workers $e = 0$

While unemployed, let V_e^u denote the expected lifetime value of a worker of type e being unemployed using an optimal strategy. If employed at type $j \epsilon \{L, H\}$ firm on wage w, let $V_e^j(w)$ denote the expected lifetime payoff.

Since, the flow payoff during unemployment $b > 0$, an untrained worker has no gains to trade with a high-technology firm. This is because these workers receive $b > 0$ during unemployment while the maximum they can receive in the high-technology firm is their productivity which is 0.

When the worker is paid her/his marginal productivity, the employer makes zero marginal profit on this worker, who therefore receives the entire match value $V_0^L\left(y^L\right)$. Further assuming that a vacant job has zero value to the employer, the difference between the match value $V_0^L\left(y^L\right)$ and the unemployment value defines the match surplus: $\left[V_0^L\left(y^L\right) - V_0^u\right]$.

When an unemployed worker is matched with a low-technology firm, she/he obtains her/his reservation utility, V_0^u , plus a share β of the maximum match surplus that she/he can get with a wage equal to the marginal productivity y^L in the low-technology firm. Let $\underline{w}$ be the equilibrium negotiated wage given a contact between an untrained, unemployed worker and a low technology firm. Equilibrium bargaining implies $\underline{w}$ solves:

$$V_0^L(\underline{w}) = V_0^u + \beta\left[V_0^L\left(y^L\right) - V_0^u\right] \tag{6}$$

13.1.2 Bargaining with trained workers

Suppose a trained unemployed worker contacts a low-technology firm. The negotiated wage, denoted w^{uL}, solves:

$$V_1^L(w^{uL}) = V_1^u + \beta\left[V_1^L\left(y^L\right) - V_1^u\right] \tag{7}$$

When a trained, unemployed worker is matched with a low-technology firm, she/he obtains her/his reservation utility, V_1^u , plus a share β of the maximum match surplus that she/he can get with a wage equal to the marginal productivity y^L in the low-technology firm. The worker only accepts the wage offer if

$V_1^L\left(y^L\right) > V_1^u$, otherwise she/he prefers to stay unemployed.

When an unemployed worker is matched with a high-technology firm, she/he obtains her/his reservation utility, V_1^u , plus a share β of the maximum match surplus that she/he can get with a wage equal to the marginal productivity y^H in the high-technology firm.

Let w^{uH} be the equilibrium negotiated wage given a contact between a trained, unemployed worker and a high technology firm. Equilibrium bargaining implies:

$$V_1^H(w^{uH}) = V_1^u + \beta\left[V_1^H\left(y^H\right) - V_1^u\right] \tag{8}$$

When a trained worker employed in a low-technology firm receives an outside job offer from another low-technology firm, a three-player bargaining process is started between the worker, her/his initial employer, and the employer who made the outside offer. No employer will pay more than match productivity. The auction forces a low-technology firm to place a bid equal to marginal productivity, y^L, of the worker in that job. The worker stays in her/his current job but her/his wage is raised to the marginal productivity in the low-technology firm.

Let w^{LL} be the equilibrium negotiated wage given a contact between a trained, worker employed in a low-technology firm meeting another low-technology firm with a Bertrand Competition. Equilibrium bargaining implies:

$$w^{LL} = y^L \tag{9}$$

When a type-$e = 1$ employee in a $j = L$ firm receives an outside offer from a $j = H$ firm, the worker moves to the $j = H$ firm, where she/he gets wage w^{LH}:

$$w^{LH} = y^L \tag{10}$$

This section determines $W = \left[\underline{w}, w^{uL}, w^{uH}, w^{LH}, w^{LL}\right]$ as a function of V_e^u and $V_e^j\left(w\right)$. The next step is to determine V_e^u and $V_e^j\left(w\right)$ consistent with $W\left(e\right)$.

13.2 Worker's Problem

13.3 Untrained Worker $e = 0$

Let $\zeta = \frac{V_L}{V_L + V_H}$ denote the fraction of vacancies which are offered by low-technology firms. As untrained workers have no gains to trade with high-technology firms, their effective arrival rate of job offers is $\zeta m(\theta)$ during unemployment.

The Bellman equation describing the value of a type e worker with $e = 0$ when unemployed is:

$$(r + \mu)V_0^u = b + m(\theta)\,\zeta\left[V_0^L(\underline{w}) - V_0^u\right] \tag{11}$$

The value of unemployment for an untrained worker incorporates the assumption that an untrained worker is not productive in a high-technology job. The gains from trade from the job are $\left[V_0^L(\underline{w}) - V_0^u\right]$. The flow payoff from unemployment is b.

The Bellman equation describing the value of a type e worker with $e = 0$ hired from unemployment is:

$$(r + \mu)V_0^L(\underline{w}) = \underline{w} + \delta\left[V_0^u - V_0^L(\underline{w})\right] \tag{12}$$

The flow value of an untrained worker hired from unemployment equals the sum of the flow return, $\underline{w}$, plus the expected instantaneous capital loss, $\delta\left[V_0^u - V_0^L(\underline{w})\right]$ from the job breaking.

Note the 2 equations (11-12) which determine V_0^u and $V_0^L(\underline{w})$. As equation (6) determined $\underline{w}(a)$ there are 3 equations for 3 unknowns.

Lemma 1

For an $e = 0$ worker, wage $\underline{w}$ and corresponding value functions V_0^u and $V_0^L(\underline{w})$ a solution exists and is bound for all $\zeta \in [0, 1]$ and $\theta \geq 0$. Equilibrium wage bargaining implies:

$$\underline{w} = \beta y^L + (1-\beta)\, b \tag{13}$$

$$V_0^u = \frac{b + m(\theta)\zeta\beta\left[\frac{y^L - b}{r+\mu+\delta+m(\theta)\zeta}\right]}{r+\mu} \tag{14}$$

$$V_0^L(\underline{w}) = \frac{b + m(\theta)\zeta\beta\left[\frac{y^L - b}{r+\mu+\delta+m(\theta)\zeta}\right]}{r+\mu} + \beta\left[\frac{y^L - b}{r+\mu+\delta+m(\theta)\zeta}\right] \tag{15}$$

13.4 Trained Worker $e = 1$

The Bellman equation describing the value of a worker with $e = 1$ when unemployed is:

$$(r+\mu)V_1^u = b + m(\theta)\left\{\zeta[V_1^L(w^{uL}) - V_1^u] + (1-\zeta)\left[V_1^H\left(w^{uH}\right) - V_1^u\right]\right\} \tag{16}$$

The value of unemployment for a trained worker incorporates the assumption that these workers might be productive in both low and high technology firms. An unemployed trained worker meets a low-technology firm at an arrival rate of $m(\theta)\zeta$ and a high-technology firm at an arrival rate of $m(\theta)(1-\zeta)$. The gains from trade with a high-technology firm are $[V_1^H(w^{uH}) - V_1^u]$. Assuming surplus exists the gains from trade with a low-technology firm are $[V_1^L(w^{uL}) - V_1^u]$. The flow payoff from unemployment is b.

The Bellman equation describing the value of a worker with $e = 1$ employed in a $j = L$ firm hired from unemployment is:

$$(r+\mu)V_1^L\left(w^{uL}\right) = w^{uL} + \delta\left[V_1^u - V_1^L\left(w^{uL}\right)\right]$$

$$+\lambda m(\theta)(1-\zeta)\left[V_1^H\left(w^{LH}\right) - V_1^L\left(w^{uL}\right)\right]$$

$$+\lambda m(\theta)\zeta\left[V_1^L\left(y^L\right) - V_1^L\left(w^{uL}\right)\right] \tag{17}$$

The flow value of employment of a trained worker employed in low-technology firm hired from unemployment equals the sum of the flow return, w^{uL}, plus the expected instantaneous capital loss, $\delta\left[V_1^u - V_1^L\left(w^{uL}\right)\right]$ from the job breaking up and the capital gains from on-the-job search. With an arrival rate of $\lambda m(\theta)(1-\zeta)$, the worker meets a high-technology firm and gains $[V_1^H\left(w^{LH}\right) - V_1^L\left(w^{uL}\right)]$. With an arrival rate of $\lambda m(\theta)\zeta$ the worker meets a low-technology firm. Assuming moving is costly, the worker stays with the current employer but is able to bargain her/his wage to $w^{LL} = y^L$, where $y^L > w^{uL}$. The gain to the worker in this case is $\left[V_1^L\left(y^L\right) - V_1^L\left(w^{uL}\right)\right]$.

The Bellman equation describing the value of a worker with $e = 1$ employed in a $j = H$ firm hired from unemployment is:

$$(r+\mu)V_1^H\left(w^{uH}\right) = w^{uH} + \delta\left[V_1^u - V_1^H\left(w^{uH}\right)\right] \tag{18}$$

The flow value of employment of a trained worker employed in a high-technology firm hired from unemployment equals the sum of the flow return, w^{uH}, plus the expected instantaneous capital loss, $\delta\left[V_1^u - V_1^H\left(w^{uH}\right)\right]$ from the job breaking up.

Note the 3 equations (16-18) which determine V_1^u and $V_1^j(w^{ij})$. As equations (7-8) and (10) determined w^{uL}, w^{uH} and w^{LH} there are 6 equations for 6 unknowns.

Lemma 2

For an $e = 1$ worker with wage w^{ij} and corresponding value functions V_1^u and $V_1^j(w^{ij})$ for $i = u, L, H$ and $j = L, H$ a solution exists and is bound for all $\zeta \in [0,1]$ and $\theta \geq 0$.

Proof

Equilibrium wage bargaining implies:

$$V_1^H(y^H) = \frac{y^H + \delta V_1^u}{r + \mu + \delta} \in \left[b, y^H\right] \tag{19}$$

$$V_1^L(y^L) = \frac{y^L + \delta V_1^u + \lambda m\left(\theta\right)\left(1 - \zeta\right) V_1^H\left(w^{LH}\right)}{r + \mu + \delta + \lambda m\left(\theta\right)\left(1 - \zeta\right)} \in \left[b, y^H\right] \tag{20}$$

$$V_1^H(w^{LH}) = \left(1 - \beta\right) V_1^u + \beta V_1^L(y^L) \in \left[b, y^H\right] \tag{21}$$

$$V_1^H(w^{uH}) = \left(1 - \beta\right) V_1^u + \beta V_1^H(y^H) \in \left[b, y^H\right] \tag{22}$$

$$V_1^L(w^{uL}) = \left(1 - \beta\right) V_1^u + \beta V_1^L(y^L) \in \left[b, y^H\right] \tag{23}$$

$$V_1^u = \frac{b + \beta m(\theta)\left[\zeta V_1^L(y^L) + (1 - \zeta) V_1^H(y^H)\right]}{r + \mu + m\left(\theta\right)\beta} \in \left[b, y^H\right] \tag{24}$$

Each value function is continuous. Further, each value function is a function of another value function multiplied by a discount factor < 1. Also, each value function is bounded between $\left[b, y^H\right]$. By Brouwer fixed-point theorem, there exists at least one continuously differentiable solution on $\left[b, y^H\right]$.

14 Firm's Problem

Let $J_e^j(w)$ denote the expected lifetime value of a firm of type $j \epsilon \{L, H\}$ employing a worker with training $e \in (0, 1)$, paying wage w. There is free entry of firms into each technology sector. Free entry implies new firms in each sector make zero expected profit. The value to the firm of a vacancy is zero.

14.1 Low-Technology Firm

Consider a low-technology firm holding a vacancy. Trained workers might find it worthwhile to accept these jobs.

The expected payoff of a low-technology firm holding a vacancy is:

$$c = \frac{m(\theta)}{\theta} \left\{ \left[\frac{U_0}{K} J_0^L (\underline{w}) \right] + \left[\frac{U_1}{K} J_1^L \left(w^{uL} \right) \right] \right\} \tag{25}$$

Conditional on a worker contact, $\frac{U_0}{K}$ is the probability the worker is untrained and unemployed and $\frac{U_1}{K}$ is the probability the worker is trained and unemployed. Vacancies meet unemployed workers at the rate $\frac{m(\theta)}{\theta}$. The capital gain from hiring an untrained worker is $J_0^L(\underline{w})$ and from hiring a trained worker is $J_1^L\left(w^{uL}\right)$.

Consider a low-technology firm hiring an untrained worker.

The expected payoff to a low-technology firm hiring an untrained worker from unemployment is:

$$J_0^L (\underline{w}) = \frac{y^L - \underline{w}}{r + \mu + \delta} \tag{26}$$

The worker-firm match is dissolved if the job is destroyed by a shock δ. $y^L - \underline{w}$ is the gain to the firm from hiring an untrained worker who produces y^L and is paid $\underline{w}$.

Consider a low-technology firm hiring a trained worker.

The expected payoff to a low-technology firm employing a trained worker hired from unemployment is:

$$J_1^L \left(w^{uL} \right) = \frac{y^L - w^{uL}}{r + \mu + \delta + \lambda_1 m(\theta)} \tag{27}$$

The worker-firm match is dissolved either if the worker changes her/his job, or if the job is destroyed by a shock. $y^L - w^{uL}$ is the gain to the firm from hiring a worker who produces y^L and is paid a wage w^{uL}. With a probability δ, the job is destroyed by a shock. With a probability $\lambda m(\theta)$ the worker meets another firm (low or high-technology).

14.2 High-Technology Firm

Consider a high-technology firm holding a vacancy. The expected payoff to a high-technology firm holding a vacancy is:

$$c = \frac{m(\theta)}{\theta}\left\{\left[\frac{U_1}{K}J_1^H\left(w^{uH}\right)\right] + \left[\frac{\lambda E_{1L}}{K}J_1^H\left(w^{LH}\right)\right]\right\} \tag{28}$$

The value to a high-technology firm holding a vacancy reflects the assumption that only trained workers are able to perform these jobs. Conditional on a worker contact, firms meet trained unemployed job seekers at a rate $\frac{m(\theta)}{\theta}\frac{U_1}{K}$ and trained workers employed in a low-technology firm at a rate $\frac{m(\theta)}{\theta}\frac{\lambda E_{1L}}{K}$. The capital gain from hiring a trained unemployed worker is $\left[J_1^H\left(w^{uH}\right)\right]$. Assuming surplus exists, the capital gain from hiring a trained worker employed in a low-technology firm is $\left[J_1^H\left(w^{LH}\right)\right]$.

Consider a high-technology firm hiring a trained worker. The expected payoff to a high-technology firm hiring a trained worker from unemployment is:

$$J_1^H\left(w^{uH}\right) = \frac{y^H - w^{uH}}{r + \mu + \delta} \tag{29}$$

$y^H - w^{uH}$ is the gain to the firm from hiring a trained worker who produces y^H and is paid w^{uH}. With a probability δ, the job is destroyed.

The expected payoff to a high-technology firm hiring a trained worker from employment in a low-technology firm is:

$$J_1^H\left(w^{LH}\right) = \frac{y^H - w^{LH}}{r + \mu + \delta} \tag{30}$$

The worker-firm match is dissolved if the job is destroyed by a shock. $y^H - w^{LH}$ is the gain to the firm from hiring a worker who produces y^H and is paid a wage w^{LH}. With a probability δ, the job is destroyed by a shock.

14.3 Firm Equilibrium with Free Entry

Equations (25) – (30) can be reduced to two equations given by:

$$c = \frac{m(\theta)}{\theta}\left\{\left[\frac{U_0}{K}\left[\frac{y^L - \underline{w}}{r+\mu+\delta}\right]\right] + \left[\frac{U_1}{K}\left[\frac{y^L - w^{uL}}{r+\mu+\delta+\lambda m(\theta)}\right]\right]\right\} \tag{31}$$

$$c = \frac{m(\theta)}{\theta}\left\{\left[\frac{U_1}{K}\left[\frac{y^H - w^{uH}}{r+\mu+\delta}\right]\right] + \left[\frac{\lambda E_{1L}}{K}\left[\frac{y^H - w^{LH}}{r+\mu+\delta}\right]\right]\right\} \tag{32}$$

14.4 Steady State Conditions

This model analyses the market in a steady state, where workers maximize expected discounted income while firms maximize expected discounted profit. The first steady state condition states that the outflow of untrained workers from unemployment equals their inflow back into unemployment.

$$U_0 m(\theta)\zeta = (\gamma_0 - U_0)\delta \tag{33}$$

The flow of untrained workers out of unemployment is then ζ (the fraction of low-technology vacancies) times U_0 (the mass of unemployed untrained workers) times $m(\theta)$ (the contact rate of workers). The flow into unemployment is then δ (the job destruction rate) times the mass of untrained employed workers. Since total population is normalized to 1, the share of untrained employed workers is then $\gamma_0 - U_0$ (proportion of untrained workers in the population minus those within them who are unemployed).

Solving the above for U_0 gives:

$$U_0 = \frac{\delta\gamma_0}{\delta + m(\theta)\zeta} \tag{34}$$

The second steady state condition states that the outflow of trained workers from unemployment equals their inflow back into unemployment.

$$U_1 m(\theta) = [(1-\gamma_0) - U_1]\delta \tag{35}$$

In equilibrium, the trained workers have a contact rate of $m(\theta)$ and the mass of trained unemployed workers is U_1. Thus, the outflow of trained unemployed workers is given by $m(\theta)U_1$. There are $1-\gamma_0$ trained workers of whom U_1 are unemployed. Subtracting the later from former times the job destruction rate gives us the inflow of trained workers into unemployment.

Solving for U_1 gives:

$$U_1 = \frac{\delta(1-\gamma_0)}{\delta + m(\theta)} \tag{36}$$

The third steady-state condition states that the outflow of trained workers employed in low-technology firm from unemployment equals their inflow back into unemployment.

$$[\delta + \lambda m(\theta)(1-\zeta)]E_{1L} = U_1 m(\theta)\zeta \tag{37}$$

The inflow of trained workers employed in a low-technology firm into employment is given by $U_1 m(\theta)\zeta$. There are U_1 trained unemployed people. With an arrival rate of $m(\theta)\zeta$, they meet a firm with a low-technology.

The outflow of trained workers employed in low-technology firms from employment happens when either the job is destroyed by a shock or there is worker turnover through on-the-job search. There are E_{1L} trained workers employed in low-technology firms. The probability of of meeting a high-technology job is $\lambda m(\theta)(1-\zeta)$, and the job is destroyed at an exogenous rate, δ.

Solving the above gives:

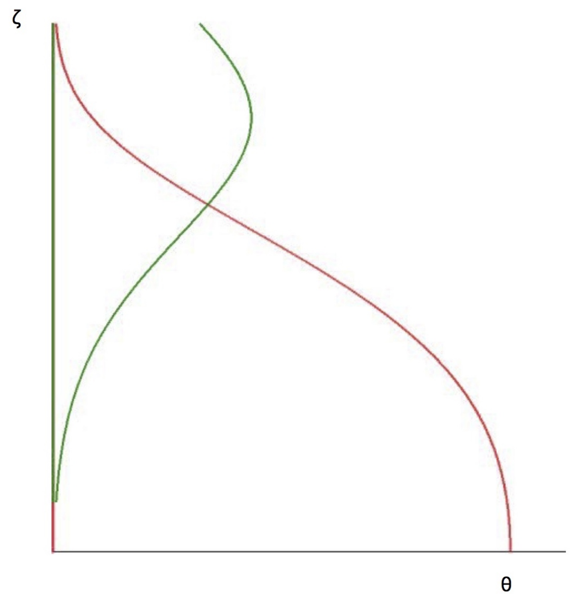
$$E_{1L} = \frac{m(\theta)\zeta\delta(1-\gamma_0)}{[\delta + m(\theta)][\delta + \lambda m(\theta)(1-\zeta)]} \tag{38}$$

14.5 Equilibrium

A steady-state equilibrium with on-the-job search consists of the wage $\underline{w}$ *given by (13) and a set of value functions for* V_1^u *and* $V_1^j(w^{ij})$ *that satisfy (19-24) plus a vector* $[\theta, \zeta, U_0, U_1, E_{1L}]$ *such that:*

1. All matches produce a non-negative surplus for the equilibrium values of $\{\theta, \zeta\}$.

2. The vector $[\theta, \zeta, U_0, U_1, E_{1L}]$ *solves the steady state conditions (34), (36) and (38) plus the free entry conditions (31) and (32).*



14.6 Numerical Solution

In this section some results of numerical simlutaions are illustrated. Following Petrongolo and Pissarides (2001) and Dolado, Jansen & Jimeno (2009), a standard Cobb-Douglas meeting function with a constant elasticity of 0.5 is assumed, i.e. $m(\theta) = A\sqrt{\theta}$, with A assumed to be equal to one. Time is measured in months. The parameter values are similar to Albrecht and Vroman (2002) and Dolado, Jansen & Jimeno (2009). Particularly, $r = 0.01$, $\mu = 0.05$, $\delta = 0.05$, $b = 0.1$, $\gamma_0 = 2/3$, $c = 0.5$, $\lambda = 0.7$, $y^H = 1.5$ and a normalised value $y^L = 1$. The baseline parameter values are chosen with three criteria in mind. First, the parameter values should be realistic. For example, Blanchard and Diamond, (1989) using labour market data from the US from 1968-1981 show that the returns to scale are close to one. Davis and Haltiwanger, (1990) using data from the U. S. manufacturing sector from 1972 to 1986 find that the quarterly job destruction rate is about 5.62%. Hall (2006) uses data from the US labour market and estimates the job finding rate to be around 40% per month while Brainerd and Perry (2002) estimate the US quarterly job finding rate to be around 70% for the first year of search. Second, the values of the endogenous variables that result from these parameter values must also be realistic. For example, the in the baseline model the parameters used result in an unemployment duration of 3 months. Finally, the baseline parameters must be such that likely variations show the different equilibrium and comparative statics possibilities. The first row shows the results when the bargaining power of worker β, is assumed to be equal to 0.5 and for comparison the second row shows the results when β is assumed to be 0.

	θ	ζ	U_0	U_1	E_0	E_1	E_{1L}	$\underline{w}$	w^{uL}	w^{uH}	w^{LH}	w^{LL}
$\beta = 0.5$	14.04	0.72	0.01	0.004	0.65	0.33	0.015	0.55	0.49	1.42	1.00	1.00
$\beta = 0$	66.4	0.79	0.005	0.002	0.66	0.33	0.010	0.10	-46.57	0.10	1.00	1.00

Table 1

For the chosen parameters a unique cross-skill matching equilibrium is obtained each time. The most striking feature of this model is that most of the vacancies are low technology. The reasons for this are that (i) most workers are low-skill ($\gamma_0 = 2/3$) and (ii) high technology jobs are only 50% more productive than low technology jobs. Since there are fewer high technology vacancies, the rate at which trained workers find jobs is only slightly above the corresponding rate for untrained workers. At the equilibrium value of θ, an untrained worker exits unemployment at a rate $\zeta m(\theta) = 2.69$ while a trained worker does so at a rate $m(\theta) = 3.75$. Unemployment in the model is low. The fraction of untrained unemployed workers among all the unemployed workers is 0.73 while that of trained unemployed workers is 0.27. This implies that trained workers, when they are employed, are typically working at high-technology firms.

The equilibrium value of $\theta = 14.04$ implies a steady-state measure of vacancies of $V = 0.38$. The average duration of unemployment is slightly more than 3 months (12 x$\frac{1}{\sqrt{14.0381}}) = 3.20$), while the average duration of a vacancy is close to 45 months (12 x $(\frac{14.0381}{\sqrt{14.0381}}) = 44.96$).

When employed in a low technology firm, untrained workers receive a wage that is a weighted average of their flow value of unemployment and the flow value of their productivity.

Trained workers have four different wages. Trained workers who are hired by a low-technology firm from unemployment receive a wage that is lower than the wage received by untrained workers hired by the same firm from unemployment. This is because these workers accept a wage cut since they have the option of raising their wage through on-the-job search. Trained workers who are hired by a high-technology firm from unemployment receive a wage that is a weighted average of the flow value of unemployment and the marginal productivity of the worker in a high-technology firm. There is one wage for a trained worker employed in a low-technology firm who indulges in on-the-job search. If this worker meets another low-technology firm, Bertrand competition between the two firms forces both firms to pay a wage equal to the marginal productivity of the worker in the low-technology firm and the worker stays in her/his current job. If the worker instead meets a high-technology firm, the worker moves jobs and is paid the same wage, that is, a wage equal to the marginal productivity of a trained worker in a low-technology firm (second price auction price).

When $\beta = 0$, the firms have all the bargaining power. The first difference to highlight when comparing the results when $\beta = 0$ compared to when $\beta = 0.5$ is that θ, the labour market tightness is much higher. The second striking difference is that the proportion of high-technology jobs, $(1 - \zeta)$, is 24.8% lower when firms have all the bargaining power (21.23%), compared to the case where the workers and firms have equal bargaining power (28.24%). This is because the low technology firms gain a lot of profit from a match with trained workers when $\beta = 0$. Consequently, with equal bargaining power untrained workers constitute 73.49% of the total unemployed while when firms have all the bargaining power untrained workers constitute 72.22% of the total unemployed.

The second difference worth stressing is that the share of mismatched workers among the trained employed workers is lower with $\beta = 0$ (3.11%) than with $\beta = 0.5$ (4.53%). This is because the arrival rate of job offers is higher when $\beta = 0$. When $\beta = 0$, at the equilibrium value of θ, an untrained worker exits unemployment at a rate $\zeta m(\theta) = 6.42$ while a trained worker does so at a rate $m(\theta) = 8.15$.

When $\beta = 0.5$ and a low-technology firm hires an untrained worker from unemployment, the wage it offers is equal to the weighted average of the worker's outside option, that is the flow value of unemployment, and the marginal productivity in the low-technology firm. However, when the firm has all the bargaining power, this worker receives a wage equal to the flow value of unemployment.

Similarly, when $\beta = 0$, and a trained worker is hired from unemployment by a high-technology firm, the worker receives a wage equal to the flow value of unemployment. However, if the trained worker is hired by a low-technology firm from unemployment, the worker accepts a large cut in wage below the flow value of unemployment. Accepting this wage cut is worthwhile because the worker has the option of on-the-job search to move to a high technology firm or to stay in the same job and receive a pay rise. Thus, the low technology firm extracts as much of the surplus it can while employing a trained worker from unemployment. When such a worker indulges in on-the-job search, she/he is able to raise her/his wage equal to her/his marginal productivity in the low-technology firm regardless of which type of firm it meets.

14.6.1 Comparing with DJJ

In order to compare the results of this model with that of Dolado, Jansen and Jimeno (2008), I now use the parameter specifications used by Dolado, et al. Time is in quarters and the parameters are given by: $r = 0.01$, $c = 0.5$, $\delta = 0.1$, $\mu = 0$, $b = 0.1$, $\gamma_0 = 0.75$, $\lambda = 1$, $\beta = 0.5$, $y^H = 1.5$ and a normalised value $y^L = 1$. The first row of the results show the results of my model with Dolado et al, parameters and the second row shows the results of the Dolado model taken from their paper.

	θ	ζ	U_0	U_1	E_0	E_1	E_{1L}	$\underline{w}$	w^{uL}	w^{uH}	w^{LH}	w^{LL}
DJJ parameters	15.08	0.79	0.02	0.006	0.73	0.24	0.021	0.55	0.12	1.42	1.00	1.00
DJJ Results	1.49	0.67	0.08	0.02	0.67	0.23	0.031	0.90	0.75	1.31	-	-

Table 2

For the chosen parameters a unique cross-skill matching equilibrium is obtained. The most striking result is that θ, the labour market tightness is much higher in this model compared to DJJ results and fewer high-technology vacancies are available in this model compared to DJJ. This is because firms offering high-technology vacancies gain less because they pay a large part of the output generated to the workers in form of wages under Bertrand Competition with other firms compared to a linear surplus splitting rule. With a sequential auction the firm is able to extract all the match rents ex-ante. With bargaining this is not so anymore. This model yields a higher share of untrained workers amongst all the unemployed workers (0.940) compared to DJJ (0.806). In order to determine wages, Dolado, Jansen and Jimeno (2008) impose the assumption of a linear surplus-splitting rule between the workers and firms. This reduces the arrival rate of jobs substantially when compared to wage determination through Bertrand Competition. Under Bertrand Competition workers indulge in on-the-job search in pursuit of a wage increase, which reduces the overall unemployment in the market and also reduces the mismatch.

At the equilibrium value of θ, an untrained worker exits unemployment at a rate $\zeta m(\theta) = 3.06$ (0.82 for DJJ) while trained workers do so at a rate $m(\theta) = 3.88$ (1.21 for DJJ). The second difference worth stressing is that the share of mismatched workers among the trained workers is lower under this model (8.49%) than the DJJ results (13.45%).

Under the linear-surplus splitting rule the wages determined through Nash bargaining are such that for all types of workers the wages are between their value of unemployment and the value of their productivity. However, with Bertrand Competition there are several wages a mismatched worker can get. One of these wages (paid to workers hired from unemployment) is lower than the wage received by untrained workers (when hired from unemployment). When hired from employment trained workers receive a wage equal to their marginal productivity in a low-technology firm. Trained workers hired by high-technology firm hired from unemployment receive a wage equal to the weighted average of their flow value of unemployment and their marginal productivity in the high-technology firm.

14.6.2 Comparative Statics

Table 3 shows the comparative static effect of changing variables one at a time. The first row is the same as the first row of table 1 replicated here for convenience.

	θ	ζ	U_0	U_1	E_0	E_1	E_{1L}	$\underline{w}$	w^{uL}	w^{uH}	w^{LH}	w^{LL}
baseline	14.04	0.72	0.012	0.0044	0.654	0.329	0.015	0.55	0.49	1.42	1.00	1.00
$b = 0.2$	12.26	0.70	0.013	0.0047	0.653	0.329	0.014	0.60	0.56	1.42	1.00	1.00
$c = 0.3$	38.07	0.72	0.007	0.0027	0.659	0.331	0.009	0.55	0.50	1.45	1.00	1.00
$\delta = 0.025$	22.97	0.71	0.004	0.0017	0.662	0.332	0.006	0.55	0.50	1.45	1.00	1.00
$\gamma_0 = 0.75$	15.15	0.78	0.012	0.0032	0.738	0.247	0.015	0.55	0.49	1.41	1.00	1.00
$y^H = 1.2$	5.60	0.86	0.016	0.0069	0.651	0.326	0.049	0.55	0.42	1.11	1.00	1.00

Table 3

Compared to the benchmark economy when the flow value of unemployment, b, is increased from 0.1 to 0.2, the labour market tightness is reduced by about 12.6%. The proportion of high-technology vacancies increases by 6.98%. The fraction of untrained unemployed amongst all the unemployed stays almost the same. The proportion of mismatched workers among the trained workers also remains almost the same. The duration of unemployment increases slightly to 3.43 months. This is because with an increase in the flow value of unemployment workers can afford to stay longer in unemployment. The gain from on-the-job search is lower compared to when $b = 0.1$.

When the cost of vacancy, c, is decreased from 0.5 to 0.3, the labour market tightness increases by 171%. The proportion of high-technology vacancies remains almost the same. The share of untrained unemployed among the unemployed stays almost the same. The proportion of mismatched workers among trained workers decreases by 37.9%. The duration of unemployment reduces to 2 months. Total vacancies increase to 0.64 and the duration of vacancy increases to 74 months. Lowering the cost of vacancy means that firms are able to hold vacancies for longer. It also favours the workers by reducing the duration of unemployment.

When the job destruction rate, δ, is decreased from 0.0.5 to 0.025, the labour market tightness increases by 62%. The proportion of high-technology vacancies remains almost the same. The share of untrained unemployed among the unemployed does not change. The proportion of mismatched workers among trained workers decreases by 60%. The duration of unemployment is reduced to 2.5 months while the duration of a vacancy increases to 57.5 months. With a lower job destruction rate the overall unemployment is reduced since fewer people enter unemployment due to an exogenous shock. Subsequently, the duration of unemployment is also lower.

When the share of untrained workers, γ_0, is increased from 2/3 to 0.75, the labour market tightness increases by 7.9%. The proportion of high-technology vacancies decreases by 23% and the fraction of untrained unemployed among the unemployed increases by 8.34%. The proportion of mismatched workers among the trained increased by 25.5%. With a smaller share of trained workers, the market starts offering more low-technology jobs. Hence, mismatch in the economy is also increased. This could be the difference between a more developed country and a less developed country with the later having more untrained workers.

Compared to the benchmark economy when the output gap between the untrained and trained workers is decreased from 0.5 to 0.2, the labour market tightness is decreased by 60%. The proportion of high-technology vacancies decreases by 50% and the fraction of untrained unemployed among the unemployed decreases by 4.29%. The proportion of mismatched workers among the trained workers increases by 232%. When trained workers are employed, more of them are working at low-technology firms. This is because the difference in productivity and hence the wages in a high-technology firm is not as substantial compared to in a low-technology firm. Since the gain from employing trained workers is reduced the number of high-technology vacancies also decline. Consequently, since high-technology jobs are rare, mismatch in the economy increases.

14.6.3 Wage Dispersion

To quantify the effect of these job-to-job transitions on wage dispersion, four useful statistics are reported. As a proxy for the degree of between group wage inequality, the ratio between the average wage of trained workers and the wage of untrained workers is calculated. Likewise, the within group wage inequality is measured by the ratio between the average wage of trained workers and their wage in a low technology job. Finally, to control for the relative size of the two groups, the total variance of the wage distribution, which is further decomposed into a permanent component due to between group wage differences and a transitory component due to within group wage differences is also computed. Table 4 shows the results obtained.

	Between Group Variation	Within Group Variation	Between Group Variance	Within Group Variance
baseline	1.78	1.31	0.09	0.03
$b = 0.2$	1.66	1.27	0.08	0.02
$c = 0.3$	1.80	1.31	0.10	0.03
$\delta = 0.025$	1.80	1.31	0.10	0.03
$\gamma_0 = 0.75$	1.77	1.31	0.09	0.03
$y^H = 1.2$	1.60	1.25	0.05	0.01

Table 4

Compared to the benchmark economy, when the flow value of unemployment is increased both the between-group and within group wage dispersion is reduced. A decrease in the cost of vacancy increases the between-group wage dispersion but the within-group wage dispersion remains the same. Thus, when a worker gains from an additional payment during unemployment the wage dispersion between the workers is reduced, however, when a firm receives the additional payment that reduces the cost of holding a vacancy, the wage dispersion is increased.

When the job destruction rate δ is reduced the between group wage dispersion is increased but the within group wage dispersion remains the same. Thus, a slower death rate of firms increases the wage dispersion among the workers. When the fraction of untrained workers among all the workers is increased the wage dispersion does not change significantly. This may be because the change in the fraction of untrained workers is not substantial.

Decreasing the output gap between the two types of workers substantially reduces both the between-group and within-group wage dispersion among the workers. When the two types of workers are almost equally productive, the average wages they receive are also similar. Hence, the wage dispersion among such workers is reduced significantly.

14.6.4 USA vs Europe

In this section the model is adapted to reflect the job markets in the US and in Europe. This is to compare markets with different levels of reallocations. In both markets job-to-job transitions explain between 40% and 50% of the separations. Also the unemployment rate is higher in Europe, however, wage inequality is higher in the US. Following Dolado, Jansen and Jimeno (2008), the parameter choice in the benchmark model is modified to reflect these facts. To capture the lower unemployment and higher wage inequality in the US, a higher matching efficiency is assumed for the US labour market than the European one. Thus, the meeting function is now $m(\theta) = A\sqrt{\theta}$, with $A = 1.25$ for US and $A = 1$ for Europe. In the current model, this change leads to lower unemployment rates for both types of workers and a higher in wage dispersion. However, this would lead to a fall in the share of mismatched workers, which would decline the job-to-job separations of highly educated workers. Hence, another parameter λ is changed such that the higher flexibility in the US market also means a higher value of λ there. Thus, for the US labour market $\lambda = 1$, whereas for the European labour market $\lambda = 0.5$. Table 5 shows the results of such simulations.

	θ	ζ	U_0	U_1	E_0	E_1	E_{1L}	$\underline{w}$	w^{uL}	w^{uH}	w^{LH}	w^{LL}
US	13.85	0.716	0.0099	0.003	0.657	0.3298	0.009	0.55	0.12	1.44	1.00	1.00
Europe	14.06	0.719	0.0121	0.004	0.654	0.3289	0.020	0.55	0.73	1.41	1.00	1.00

Table 5

The results show that in the first row there is a greater wage dispersion, which is synonymous to the case of the US, and the second row shows a higher unemployment rate and a lower matching efficiency synonymous to Europe.

15 Everyone Searches On-the-Job

In this section the assumption imposed earlier of only allowing mismatched workers to search is relaxed to give more realism to the model. As explained earlier, this is important because workers not only search for jobs while employed in pursuit of a better match but also indulge in on-the-job search to get a pay raise. In a market with frictions when more people search for jobs when employed, the mismatch is more and the wage inequality between the two types of workers is more.

15.1 Matching

Let U_0 be the measure of unemployed workers who are untrained and U_1 the measure of unemployed workers who are trained. Let E_0 be the measure of employed workers who are untrained and E_1 the measure of employed workers who are trained. Let E_{1L} denote the measure of trained workers who are employed in a low-technology firm.

Unemployed workers and vacancies are assumed to meet each other according to a random matching technology, where $M = M(K, V)$ denotes the flow number of contacts.

$K = \lambda_0(U_0+U_1)+\lambda_1(E_0+E_1)$, is the aggregate job search effort and $V = V_L+V_H$, is the total vacancies. Assuming constant returns, define labour market tightness $\theta = V/K$ and $m(\theta) = \frac{M(K,V)}{K} = M(1, \theta)$ which describes the arrival rate of job offers per unit effort.

I assume that $m'(\theta) > 0$ and that $lim_{\theta\to\infty} m'(\theta) = 0$.

Unemployed workers sample job offers randomly at Poisson rate $\lambda_0 m(\theta)$. Employees may also search for a better job while employed and the arrival rate of offers to on-the-job searchers is $\lambda_1 m(\theta)$. λ_0 and λ_1 are exogenous. I assume

$\lambda_0 > \lambda_1$.

Similarly, vacancies meet unemployed workers at rate $\frac{m(\theta)}{\theta}$. I assume this rate is decreasing in θ and that $lim_{\theta \to 0}[m(\theta)/\theta] = \infty$.

15.2 Equilibrium Wage Bargaining

15.2.1 Untrained workers $e = 0$

Since, the flow payoff during unemployment $b > 0$, an untrained worker has no gains to trade with a high-technology firm. This is because these workers receive $b > 0$ during unemployment while the maximum they can receive in the high-technology firm is their productivity which is 0.

When the worker is paid her/his marginal productivity, the employer makes zero marginal profit on this worker, who therefore receives the entire match value $V_0^L\left(y^L\right)$. Further assuming that a vacant job has zero value to the employer, the difference between the match value $V_0^L\left(y^L\right)$ and the unemployment value defines the match surplus: $\left[V_0^L\left(y^L\right) - V_0^u\right]$.

When an unemployed worker is matched with a low-technology firm, she/he obtains her/his reservation utility, V_0^u , plus a share β of the maximum match surplus that she/he can get with a wage equal to the marginal productivity y^L in the low-technology firm. Let $\underline{w}$ be the equilibrium negotiated wage given a contact between an untrained, unemployed worker and a low technology firm. Equilibrium bargaining implies $\underline{w}$ solves:

$$V_0^L(\underline{w}) = V_0^u + \beta\left[V_0^L\left(y^L\right) - V_0^u\right] \tag{39}$$

When an untrained employed worker receives an outside job offer, a three-player bargaining process is started between the worker, her/his initial employer, and the employer who made the outside offer. No employer will pay more than match productivity. The auction forces a low-technology firm to place a bid equal to marginal productivity, y^L, of the worker in that job.

Let $\bar{w}$ be the equilibrium negotiated wage given a contact between an untrained, employed worker and two low-technology firms. Equilibrium bargaining implies:

$$\bar{w} = y^L \tag{40}$$

15.2.2 Bargaining with trained workers

Again I start by considering an unemployed trained worker. Suppose the worker contacts a low-technology firm. The negotiated wage, denoted w^{uL}, solves:

$$V_1^L(w^{uL}) = V_1^u + \beta \left[V_1^L\left(y^L\right) - V_1^u\right] \tag{41}$$

When a trained, unemployed worker is matched with a low-technology firm, she/he obtains her/his reservation utility, V_1^u , plus a share β of the maximum match surplus that she/he can get with a wage equal to the marginal productivity y^L in the low-technology firm. Since, $\lambda_1 < \lambda_0$, the worker only accepts the wage offer if $V_1^L\left(y^L\right) > V_1^u$, otherwise she/he prefers to stay unemployed.

When an unemployed worker is matched with a high-technology firm, she/he obtains her/his reservation utility, V_1^u , plus a share β of the maximum match surplus that she/he can get with a wage equal to the marginal productivity y^H in the high-technology firm.

Let w^{uH} be the equilibrium negotiated wage given a contact between a trained, unemployed worker and a high technology firm. Equilibrium bargaining implies:

$$V_1^H(w^{uH}) = V_1^u + \beta \left[V_1^H\left(y^H\right) - V_1^u\right] \tag{42}$$

When a trained worker employed in a low-technology firm receives an outside job offer from another low-technology firm, a three-player bargaining process is started between the worker, her/his initial employer, and the employer who made the outside offer. No employer will pay more than match productivity. The auction forces a low-technology firm to place a bid equal to marginal productivity, y^L, of the worker in that job. The worker stays in her/his current job but her/his wage is raised to the marginal productivity in the low-technology firm.

Let w^{LL} be the equilibrium negotiated wage given a contact between a trained, worker employed in a low-technology firm meeting another low-technology firm with a Bertrand Competition. Equilibrium bargaining implies:

$$w^{LL} = y^L \tag{43}$$

When a trained employed worker employed in a high-technology firm receives an outside job offer from another high-technology firm, a three-player bargaining process is started between the worker, her/his initial employer, and the employer who made the outside offer. No employer will pay more than match productivity. The auction forces a high-technology firm to place a bid equal to marginal productivity, y^H, of the worker in that job. The worker stays in her/his current job but her/his wage is raised to the marginal productivity in the high-technology firm.

Let w^{HH} be the equilibrium negotiated wage given a contact between a trained worker employed in a high-technology firm who meets another high-technology firm. Equilibrium bargaining implies:

$$w^{HH} = y^H \tag{44}$$

Lemma 3

When a type-$e = 1$ employee in a $j = L$ firm receives an outside offer from a $j = H$ firm, the worker moves to the $j = H$ firm, where she/he gets wage w^{LH} that solves:

$$V_1^H(w^{LH}) = V_1^L(y^L) + \beta \left[V_1^H\left(y^H\right) - V_1^L(y^L)\right] \tag{45}$$

Proof

y^H is the highest wage in the market. It is always better to earn this wage than any other wage. Thus, $V_1^H(y^H) > V_1^L(y^L)$. The highest wage a low-technology firm can offer is y^L which the worker values at $V_1^L\left(y^L\right)$. The employee receives a positive surplus on top of this outside option from moving and hence prefers to move from a $j = L$ firm to a $j = H$ firm. Q.E.D.

Note that the wage w^{LH} obtained in the new firm can be smaller than the wage paid in the previous job, because the worker expects larger wage rises in firms with higher productivity. This option value effect implies that workers may be willing to take wage cuts just to move from a low- to a high-technology firm.

This section determines $W = \left[\underline{w}, \bar{w}, w^{uL}, w^{uH}, w^{LH}, w^{LL}, w^{HH}\right]$ as a function of V_e^u and $V_e^j(w)$. The next step is to determine V_e^u and $V_e^j(w)$ consistent with $W(e)$.

15.3 Worker's Problem

15.4 Untrained Worker $e = 0$

Let $\zeta = \frac{V_L}{V_L + V_H}$ denote the fraction of vacancies which are offered by low-technology firms. As untrained workers have no gains to trade with high-technology firms, their effective arrival rate of job offers is $\lambda_0 \zeta m(\theta)$ during unemployment and $\lambda_1 \zeta m(\theta)$ during employment.

The Bellman equation describing the value of a worker with $e = 0$ when unemployed is:

$$(r+\mu)V_0^u = b + \lambda_0 m(\theta)\zeta\left[V_0^L(\underline{w}) - V_0^u\right] \tag{46}$$

The value of unemployment for an untrained worker incorporates the assumption that an untrained worker is not productive in a high-technology job. The gains from trade from the job are $\left[V_0^L(\underline{w}) - V_0^u\right]$. The flow payoff from unemployment is b.

The Bellman equation describing the value of a worker with $e = 0$ hired from unemployment is:

$$(r+\mu)V_0^L(\underline{w}) = \underline{w} + \delta\left[V_0^u - V_0^L(\underline{w})\right]$$

$$+\lambda_1 m(\theta)\zeta\left[V_0^L(\bar{w}) - V_0^L(\underline{w})\right] \tag{47}$$

The flow value of an untrained worker hired from unemployment equals the sum of the flow return, $\underline{w}$, plus the expected instantaneous capital loss, $\delta\left[V_0^u - V_0^L(\underline{w})\right]$ from the job breaking and the capital gain from on-the-job search. An untrained worker meets a low-technology firm at the rate $\lambda_1 m(\theta)\zeta$. Both firms will offer a single wage $\bar{w} > \underline{w}$. Assuming that it is costly to move the worker stays with the current employer but is able to make capital gains of $\left[V_0^L(\bar{w}) - V_0^L(\underline{w})\right]$.

The Bellman equation describing the value of a worker with $e = 0$ hired from employment is:

$$(r+\mu)V_0^L(\bar{w}) = y^L + \delta\left[V_0^u - V_0^L(\bar{w})\right] \tag{48}$$

The flow value of an untrained worker hired from employment equals the sum of the flow return, $\bar{w} = y^L$, plus the expected instantaneous capital loss, $\delta\left[V_0^u - V_0^L(\bar{w})\right]$ from the job breaking. There are no capital gains from on-the-job search.

Note the 3 equations (46-48) which determine V_0^u, $V_0^L(y^L)$ and $V_0^L(\underline{w})$. As equation (39) determined $\underline{w}$ there are 4 equations for 4 unknowns.

Lemma 4

For an $e = 0$ worker, wage $\underline{w}, \bar{w}$ and corresponding value functions V_0^u and $V_0^j(w^{ij})$ for $i = uL, H$ and $j = L, H$ a solution exists and is bound for all $\zeta \in [0,1]$ and $\theta \geq 0$. Equilibrium wage bargaining implies:

$$V_0^u = \frac{1}{r+\mu}\left\{b\left[\frac{r+\mu+\delta}{r+\mu+\delta+\lambda_0 m(\theta)\zeta\beta}\right] + \left[\frac{\lambda_0 m(\theta)\zeta\beta y^L}{r+\mu+\delta+\lambda_0 m(\theta)\zeta\beta}\right]\right\}$$

$$\in [\frac{b}{r+\mu}, \frac{y^L}{r+\mu}] \tag{49}$$

$$V_0^L(y^L) = \frac{1}{r+\mu}\left\{b\left[\frac{\delta}{r+\mu+\delta+\lambda_0 m(\theta)\zeta\beta}\right] + \left[\frac{r+\mu+\lambda_0 m(\theta)\zeta\beta y^L}{r+\mu+\delta+\lambda_0 m(\theta)\zeta\beta}\right]\right\}$$

$$\in [\frac{b}{r+\mu}, \frac{y^L}{r+\mu}] \tag{50}$$

$$V_0^L(\underline{w}) = \frac{1}{r+\mu}\left\{b\left[\frac{(r+\mu)(1-\beta)+\delta}{r+\mu+\delta+\lambda_0 m(\theta)\zeta\beta}\right] + \left[\frac{[(r+\mu)\beta+\lambda_0 m(\theta)\zeta\beta]\, y^L}{r+\mu+\delta+\lambda_0 m(\theta)\zeta\beta}\right]\right\}$$

$$\in [\frac{b}{r+\mu}, \frac{y^L}{r+\mu}] \tag{51}$$

$$\bar{w} = y^L \tag{52}$$

$$\underline{w} = b - y^L(1-b)\left[\frac{\lambda_1 m(\theta)\zeta(1-\beta)}{r+\mu+\delta}\right] < b \tag{53}$$

Note the wage $\underline{w} < b$, the flow pay-off from leisure. Hence workers are willing to take wage cuts because they expect a larger wage rises through on-the-job search.

15.5 Trained Worker $e = 1$

The Bellman equation describing the value of a worker with $e = 1$ when unemployed is:

$$(r+\mu)V_1^u = b + \lambda_0 m(\theta)\{\zeta[V_1^L(w^{uL}) - V_1^u] + (1-\zeta)\left[V_1^H\left(w^{uH}\right) - V_1^u\right]\} \quad (54)$$

The value of unemployment for a trained worker incorporates the assumption that these workers are productive in both low and high technologies. An unemployed trained worker meets a low-technology firm at an arrival rate of $\lambda_0 m(\theta)\zeta$ and a high-technology firm at an arrival rate of $\lambda_0 m(\theta)(1-\zeta)$. The gains from trade with a high-technology firm are $[V_1^H(w^{uH}) - V_1^u]$. Assuming surplus exists the gains from trade with a low-technology firm are $[V_1^L(w^{uL}) - V_1^u]$. The flow payoff from unemployment is b.

The Bellman equation describing the value of a worker with $e = 1$ employed in a $j = L$ firm hired from unemployment is:

$$(r+\mu)V_1^L\left(w^{uL}\right) = w^{uL} + \delta\left[V_1^u - V_1^L\left(w^{uL}\right)\right]$$

$$+\lambda_1 m(\theta)(1-\zeta)\left[V_1^H\left(w^{LH}\right) - V_1^L\left(w^{uL}\right)\right]$$

$$+\lambda_1 m(\theta)\zeta\left[V_1^L\left(y^L\right) - V_1^L\left(w^{uL}\right)\right] \quad (55)$$

The flow value of employment of a trained worker employed in low-technology firm hired from unemployment equals the sum of the flow return, w^{uL}, plus the expected instantaneous capital loss, $\delta\left[V_1^u - V_1^L\left(w^{uL}\right)\right]$ from the job breaking up and the capital gains from on-the-job search. With an arrival rate of $\lambda_1 m(\theta)(1-\zeta)$, the worker meets a high-technology firm and gains $[V_1^H\left(w^{LH}\right) - V_1^L\left(w^{uL}\right)]$. With an arrival rate of $\lambda_1 m(\theta)\zeta$ the worker meets a low-technology firm. Assuming moving is costly, the worker stays with the current employer but is able to bargain her/his wage to $w^{LL} = y^L$, where $y^L > w^{uL}$. The gain to the worker in this case is $\left[V_1^L\left(y^L\right) - V_1^L\left(w^{uL}\right)\right]$.

The Bellman equation describing the value of a worker with $e = 1$ employed in a $j = L$ firm hired from employment is:

$$(r+\mu)V_1^L\left(w^{LL}\right) = y^L + \delta\left[V_1^u - V_1^L\left(w^{LL}\right)\right]$$

$$+\lambda_1 m(\theta)(1-\zeta)\left[V_1^H\left(w^{LH}\right) - V_1^L\left(w^{LL}\right)\right] \quad (56)$$

The flow value of employment of a trained worker employed in a low-technology firm hired from employment equals the sum of the flow return, $w^{LL} = y^{L}$, plus the expected instantaneous capital loss, $\delta \left[V_1^u - V_1^L \left(w^{LL}\right)\right]$ from the job breaking up and the capital gains from on-the-job search. With an arrival rate of $\lambda_1 m(\theta)(1-\zeta)$, the worker meets a high-technology firm and gains $\left[V_1^H \left(w^{LH}\right) - V_1^L \left(w^{LL}\right)\right]$. There are no gains from trade if this worker meets another low-technology firm.

The Bellman equation describing the value of a worker with $e = 1$ employed in a $j = H$ firm hired from unemployment is:

$$(r+\mu)V_1^H \left(w^{uH}\right) = w^{uH} + \delta \left[V_1^u - V_1^H \left(w^{uH}\right)\right]$$

$$+\lambda_1 m(\theta)\zeta \left[V_1^L \left(y^L\right) - V_1^H \left(w^{uH}\right)\right]$$

$$+\lambda_1 m(\theta)(1-\zeta) \left[V_1^H \left(w^{HH}\right) - V_1^H \left(w^{uH}\right)\right] \tag{57}$$

The flow value of employment of a trained worker employed in a high-technology firm hired from unemployment equals the sum of the flow return, w^{uH}, plus the expected instantaneous capital loss, $\delta \left[V_1^u - V_1^H \left(w^{uH}\right)\right]$ from the job breaking up and the capital gains from on-the-job search. With an arrival rate of $\lambda_1 m(\theta)\zeta$, the worker meets a low-technology firm. Assuming surplus exists the worker gains $\left[V_1^L \left(y^L\right) - V_1^H \left(w^{uH}\right)\right]$. The worker meets a high-technology firm at an arrival rate $\lambda_1 m(\theta)(1-\zeta)$. The worker's current employer and the new employer enter a bid for the worker's services. Assuming moving is costly, the worker stays with the current employer but is able to bargain her/his wage to $w^{HH} = y^H$, where $y^H > w^{uH}$. The capital gains are $\left[V_1^H \left(w^{HH}\right) - V_1^H \left(w^{uH}\right)\right]$.

The Bellman equation describing the value of a worker with $e = 1$ employed in a $j = H$ firm hired from employment is:

$$(r+\mu)V_1^H \left(w^{HH}\right) = w^{HH} + \delta \left[V_1^u - V_1^H \left(w^{HH}\right)\right] \tag{58}$$

The flow value of employment of a trained worker employed in a high-technology firm hired from employment equals the sum of the flow return, w^{HH}, plus the expected instantaneous capital loss, $\delta\left[V_1^u - V_1^H\left(w^{HH}\right)\right]$ from the job breaking. There are no capital gains from on-the-job search.

Note the 5 equations (54-58) which determine V_1^u and $V_1^j(w^{ij})$. As equations (41-42) and (45) determined w^{uL}, w^{uH} and w^{LH} there are 8 equations for 8 unknowns.

Lemma 5

For an $e = 1$ worker with wage w^{ij} and corresponding value functions V_1^u and $V_1^j(w^{ij})$ for $i = u, L, H$ and $j = L, H$ a solution exists and is bound for all $\zeta \in [0, 1]$ and $\theta \geq 0$.

Proof

Equilibrium wage bargaining implies:

$$V_1^H(y^H) = \frac{y^H + \delta V_1^u}{r + \mu + \delta} \in \left[b, y^H\right] \tag{59}$$

$$V_1^L(y^L) = \frac{y^L + \delta V_1^u + \lambda_1 m\left(\theta\right)\left(1 - \zeta\right) V_1^H\left(w^{LH}\right)}{r + \mu + \delta + \lambda_1 m\left(\theta\right)\left(1 - \zeta\right)} \in \left[b, y^H\right] \tag{60}$$

$$V_1^H(w^{LH}) = \left(1 - \beta\right) V_1^L\left(y^L\right) + \beta V_1^H(y^H) \in \left[b, y^H\right] \tag{61}$$

$$V_1^H(w^{uH}) = \left(1 - \beta\right) V_1^u + \beta V_1^H(y^H) \in \left[b, y^H\right] \tag{62}$$

$$V_1^L(w^{uL}) = \left(1 - \beta\right) V_1^u + \beta V_1^L(y^L) \in \left[b, y^H\right] \tag{63}$$

$$V_1^u = \frac{b + \beta\lambda_0 m(\theta)\left[\zeta V_1^L(y^L) + (1 - \zeta) V_1^H(y^H)\right]}{r + \mu + \lambda_0 m\left(\theta\right)\beta} \in \left[b, y^H\right] \tag{64}$$

Each value function is continuous. Further, each value function is a function of another value function multiplied by a discount factor < 1. Also, each value function is bounded between $\left[b, y^H\right]$. By Brouwer fixed-point theorem, there exists at least one continuously differentiable solution on $\left[b, y^H\right]$.

15.6 Firm's Problem With Free Entry

Let $J_e^j(w)$ denote the expected lifetime value of a firm of type $j\epsilon\{L, H\}$ employing a worker with training $e \in (0,1)$, paying wage w. There is free entry of firms into each technology sector. Free entry implies new firms in each sector make zero expected profit. The value to the firm of a vacancy is zero.

15.7 Low-Technology Firm

Consider a low-technology firm holding a vacancy. Trained workers find it worthwhile to accept these jobs.

The expected payoff of a low-technology firm holding a vacancy is:

$$c = \frac{m(\theta)}{\theta}\left\{\left[\frac{\lambda_0 U_0}{K}[J_0^L(\underline{w})]\right] + \frac{\lambda_0 U_1}{K}\left[J_1^L\left(w^{uL}\right)\right]\right\} \tag{65}$$

Conditional on a worker contact, $\frac{\lambda_0 U_0}{K}$ is the probability the worker is untrained and unemployed and $\frac{\lambda_0 U_1}{K}$ is the probability the worker is trained and unemployed. Vacancies meet unemployed workers at the rate $\frac{m(\theta)}{\theta}$. The capital gain from hiring an untrained worker is $J_0^L(\underline{w})$ and from hiring a trained worker is $J_1^L\left(w^{uL}\right)$.

Consider a low-technology firm hiring an untrained worker. There is no gain to the firm from hiring a worker from employment.

The expected payoff to a low-technology firm hiring an untrained worker from employment is:

$$J_0^L(\overline{w}) = 0 \tag{66}$$

The expected payoff to a low-technology firm hiring an untrained worker from unemployment is:

$$J_0^L(\underline{w}) = \frac{y^L - \underline{w}}{r + \mu + \delta + \lambda_1 m(\theta)\zeta} \tag{67}$$

The worker-firm match is dissolved if the job is destroyed by a shock δ, or if the worker changes her/his job. $\lambda_1 m(\theta)\zeta$ is the probability the worker meets a low-technology firm and changes her/his job. $y^L - \underline{w}$ is the gain to the firm from hiring an untrained worker who produces y^L and is paid $\underline{w}$.

Consider a low-technology firm hiring a trained worker. There is no gain to the firm from hiring a worker from employment.

The expected payoff to a low-technology firm employing a trained worker from employment is:

$$J_1^L\left(w^{LL}\right) = 0 \tag{68}$$

The expected payoff to a low-technology firm employing a trained worker hired from unemployment is:

$$J_1^L\left(w^{uL}\right) = \frac{y^L - w^{uL}}{r + \mu + \delta + \lambda_1 m(\theta)} \tag{69}$$

The worker-firm match is dissolved either if the worker changes her/his job, or if the job is destroyed by a shock. $y^L - w^{uL}$ is the gain to the firm from hiring a worker who produces y^L and is paid a wage w^{uL}. With a probability δ, the job is destroyed by a shock. With a probability $\lambda_1 m(\theta)$ the worker meets another firm (low or high-technology).

15.8 High-Technology Firm

Consider a high-technology firm holding a vacancy. The expected payoff to a high-technology firm holding a vacancy is:

$$c = \frac{m(\theta)}{\theta}\left\{\frac{\lambda_0 U_1}{K}\left[J_1^H\left(w^{uH}\right)\right] + \frac{\lambda_1 E_{1L}}{K}\left[J_1^H\left(w^{LH}\right)\right]\right\} \tag{70}$$

The value to a high-technology firm holding a vacancy reflects the assumption that only trained workers are able to perform these jobs. Conditional on a worker contact, firms meet trained unemployed job seekers at a rate $\frac{m(\theta)}{\theta}\frac{\lambda_0 U_1}{K}$ and trained workers employed in a low-technology firm at a rate $\frac{m(\theta)}{\theta}\frac{\lambda_1 E_{1L}}{K}$. The capital gain from hiring a trained unemployed worker is $\left[J_1^H\left(w^{uH}\right)\right]$. Assuming surplus exists, the capital gain from hiring a trained worker employed in a low-technology firm is $\left[J_1^H\left(w^{LH}\right)\right]$.

Consider a high-technology firm hiring a trained worker. There is no gain to the firm from hiring a worker from employment.

The expected payoff to a high-technology firm hiring a trained worker from employment in a high-technology firm is:

$$J_1^H\left(w^{HH}\right) = 0 \tag{71}$$

The expected payoff to a high-technology firm hiring a trained worker from unemployment is:

$$J_1^H\left(w^{uH}\right) = \frac{y^H - w^{uH}}{r + \mu + \delta + \lambda_1 m\left(\theta\right)\left(1 - \zeta\right)} \tag{72}$$

$y^H - w^{uH}$ is the gain to the firm from hiring a trained worker who produces y^H and is paid w^{uH}. With a probability δ, the job is destroyed. With a probability $\lambda_1 m\left(\theta\right)\left(1-\zeta\right)$, the worker meets a high-technology firm.

The expected payoff to a high-technology firm hiring a trained worker from employment in a low-technology firm is:

$$J_1^H\left(w^{LH}\right) = \frac{y^H - w^{LH}}{r + \mu + \delta + \lambda_1 m\left(\theta\right)} \tag{73}$$

The worker-firm match is dissolved either if the worker changes her/his job, or if the job is destroyed by a shock. $y^H - w^{LH}$ is the gain to the firm from hiring a worker who produces y^H and is paid a wage w^{LH}. With a probability δ, the job is destroyed by a shock. With a probability $\lambda_1 m(\theta)$ the worker meets another firm (low or high-technology).

15.9 Firm Equilibrium with Free Entry

Equations (65) – (73) can be reduced to two equations given by:

$$c = \frac{m(\theta)}{\theta}\left\{\frac{\lambda_0 U_0}{K}\left[\frac{y^L - \underline{w}}{r+\mu+\delta+\lambda_1 m(\theta)\zeta}\right] + \frac{\lambda_0 U_1}{K}\left[\frac{y^L - w^{uL}}{r+\mu+\delta+\lambda_1 m(\theta)}\right]\right\} \tag{74}$$

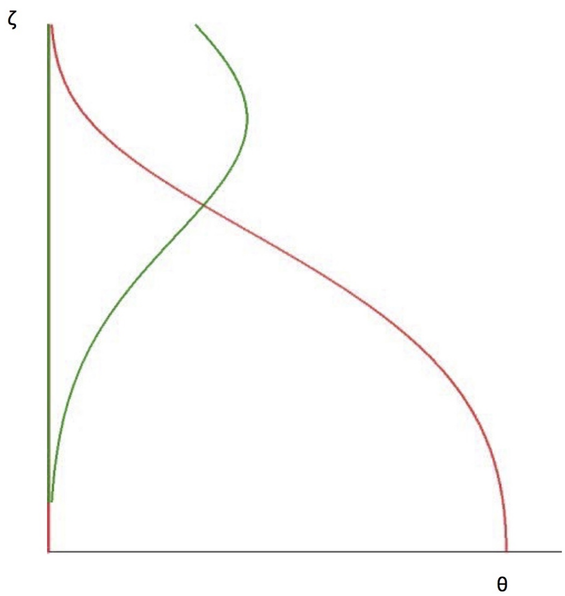
$$c = \frac{m(\theta)}{\theta}\left\{\frac{\lambda_0 U_1}{K}\left[\frac{y^H - w^{uH}}{r+\mu+\delta+\lambda_1 m(\theta)(1-\zeta)}\right] + \frac{\lambda_1 E_{1L}}{K}\left[\frac{y^H - w^{LH}}{r+\mu+\delta+\lambda_1 m(\theta)}\right]\right\} \tag{75}$$

15.10 Equilibrium

A steady-state equilibrium with on-the-job search consists of the wage $\underline{w}$ given by (53) and a set of value functions for V_1^u and $V_1^j(w^{ij})$ that satisfy (59-64) plus a vector $[\theta, \zeta, U_0, U_1, E_{1L}]$ such that:

1. All matches produce a non-negative surplus for the equilibrium values of $\{\theta, \zeta\}$.

2. The vector $[\theta, \zeta, U_0, U_1, E_{1L}]$ solves the steady state conditions (34), (36) and (38) plus the free entry conditions (74) and (75).



15.11 Numerical Solutions

In this section some results of numerical simulations are illustrated. As before a standard Cobb-Douglas meeting function with a constant elasticity of 0.5 is assumed, i.e. $m(\theta) = A\sqrt{\theta}$, with A assumed to be equal to one and time is measured in quarters. The parameters are kept the same as before for comparison. Particularly, $r = 0.01$, $\mu = 0.05$, $\delta = 0.05$, $c = 0.5$, $b = 0.1$, $\gamma_0 = 2/3$, $\lambda_1 = 0.5$, $\lambda_0 = 0.7$, $y^H = 1.5$ and a normalised value $y^L = 1$. Table 6 shows the results of the baseline model in equilibrium as well as the comparative static results from changing different parameters.

	θ	ζ	U_0	U_1	E_0	E_1	E_{1L}	$\underline{w}$	$\overline{w}$	w^{uL}	w^{uH}	w^{LH}	w^{LL}	w^{HH}
Baseline	0.98	0.64	0.067	0.022	0.60	0.31	0.044	-1.20	1.00	0.46	1.00	-4.87	1.00	1.50
$b = 0.2$	0.89	0.63	0.072	0.023	0.59	0.31	0.04	-0.88	1.00	0.53	1.04	-4.49	1.00	1.50
$c = 0.3$	1.66	0.62	0.055	0.017	0.61	0.32	0.03	-1.53	1.00	0.47	1.01	-6.83	1.00	1.50
$\delta = 0.025$	0.64	0.66	0.042	0.014	0.62	0.32	0.03	-1.30	1.00	0.46	1.03	-4.01	1.00	1.50
$\gamma_0 = 0.75$	0.98	0.71	0.069	0.017	0.68	0.23	0.04	-1.33	1.00	0.46	1.07	-5.33	1.00	1.50
$y^H = 1.2$	0.95	0.66	0.07	0.020	0.60	0.31	0.05	-1.23	1.00	0.43	0.65	-4.29	1.00	1.20

Table 6

For the chosen parameters a unique equilibrium is obtained each time. The first row of Table 6 shows the labour market results for the benchmark economy. Comparing the results to when only mismatched workers search in the market, when all workers search, θ, the labour market tightness is reduced by approximately 93% from 14.0381 to 0.9823. The proportion of high-technology vacancies is increased by 27.3% from 0.2824 to 0.3595. The total unemployment in the economy increases from 1.6% to 9% and the proportion of untrained unemployed workers among all the unemployed increases by 2.12%. The proportion of mismatched workers among all the trained employed workers increases by about 210%. Hence, compared to when only mismatched workers are searching on-the-job, when all workers indulge in on-the-job search in pursuit of better paying jobs, the total unemployment increases and the proportion of mismatch is increased substantially. The duration of unemployment is also increased substantially from about 3 months to about a year.

Seven wages are obtained when all workers indulge in on-the-job search compared to 5 when only mismatched workers search for jobs. Untrained workers have two wages. When employed from unemployment these workers receive a wage, which is less than the flow value of unemployment. Workers accept this wage cut because they have a possibility of a wage rise through indulging in on-the-job search. Untrained workers who are hired from employment receive a wage equal to their marginal productivity in a low-technology firm.

Trained workers have five possible wages. Those who are hired from unemployment receive a wage that is a weighted average of the flow value of unemployment and their marginal productivity in the job. Since trained workers are less productive in a low-technology firm compared to a high-technology firm, they receive a lower wage when employed in a low-technology firm from unemployment compared to a high-technology firm. In other words, when hired from unemployment, mismatched workers receive a lower wage compared to well-matched trained workers. If these mismatched workers meet another low-technology firm, they are able to obtain a pay rise through Bertrand competition between the firms. Such workers stay in their current jobs but obtain a wage equal to their marginal productivity in a low-technology firm. If however, the mismatched workers meet a high-technology firm, the workers move jobs but they must accept a pay cut when mismatched since they have the possibility of searching for better paying jobs while employed. Such workers obtain a wage lower than the flow value of unemployment. Such workers must accept a bigger wage cut compared to untrained workers hired from unemployment, since the wage gain from on-the-job search is higher for these workers compared to the untrained workers. Finally, if trained workers employed in a high-technology firm meet another high-technology firm, the workers obtain a pay rise through Bertrand competition between the employers. Such workers receive a wage equal to the marginal productivity of trained workers in a high-technology firm.

15.11.1 Comparative Statics

The second row of table 6 shows the equilibrium results, when the flow value of unemployment, b, is increased from 0.1 to 0.2. The results show that the labour market tightness is decreased by 9%. The proportion of high-technology vacancies increases by 4%. Total unemployment decreases by 6% and the duration of unemployment increases slightly from 12.1 months to 12.6 months. The proportion of mismatched workers among the trained employed workers declines by 1.7%. When only mismatched workers indulged in on-the-job search, the total unemployment increased and mismatch also increased. However, now because all worker types are searching the increase in flow value of unemployment is not enough to attract the workers to remain unemployed for longer.

The third row of table 6 shows the equilibrium results, when the cost of a vacancy, c, is reduced from 0.5 to 0.3. The results show that the labour market tightness increases by 69% and the proportion of high-technology vacancies increase by about 6%. Total unemployment decreases by 20% and the duration of unemployment decreased to about 9.3 months. The proportion of mismatched workers among the employed trained workers decreases by about 25%. Total vacancies increase by 68% and the duration of a vacancy increases by 30% from about 12 months to about 15 months. Thus, when firms face a lower cost of holding a vacancy, the total number of vacancies as well as their duration increases.

The fourth row of table 6 shows the equilibrium results, when δ is reduced from 0.05 to 0.025. The results show that the labour market tightness decreases by about 34.4% and the proportion of high-technology vacancies decrease by 5.2%. The total unemployment decreases by 37% but the duration of unemployment increases from about 12 months to 15 months. The proportion of mismatched workers among the employed trained workers is decreased by 27%.

When the share of less educated workers, γ_0 is increased from 2/3 to 3/4, the labour market tightness remains almost the same and the proportion of high-technology firms in the market decrease by 18.4%. Total unemployment decreases by 4% and the proportion of untrained unemployed workers among all the unemployed increased by 7.2%. The share of mismatched workers among the employed trained workers decreased by 28.7%. Thus, when the share of trained job seekers is decreased, fewer high-technology jobs are offered and mismatch in the economy is increased. This is in line with the previous results where only mismatched workers were searching on-the-job.

When the output gap $(y^H - y^L)$ is reduced from 0.5 to 0.2, the labour market tightness decreases by 3% and the proportion of high-technology firms in the market decrease by 6.5%. Total unemployment decreases by 1% and the proportion of untrained unemployed workers among all the unemployed decreased by 0.8%. The share of mismatched workers among the employed trained workers increased by 10.6%. Thus, when the productivity gap between untrained and trained workers is reduced, fewer high-technology jobs are offered since the gain to such firms from employing a trained worker have been reduced. As a consequence the share of mismatch in the economy increases.

15.11.2 Comparing to DJJ

The baseline results of this variation of the model are more similar to the DJJ results compared to the model that only allowed mismatched workers to search on-the-job. The labour market tightness in my model is 0.98 compared to 1.49 in the DJJ model. The proportion of workers who are unskilled is 64% in my model compared to 67% in the DJJ model. The proportion of unemployed low-skilled workers is about 7% in my model compared to 8% in the DJJ model. The proportion of skilled unemployed workers is almost similar in my model compared to DJJ (2%). The share of low-skilled employed workers is 60% in my model compared to 67% in the DJJ model. The share of high-skilled employed workers is 31% in my model compared to 23% in the DJJ model. The share of mismatched workers is about 4% compared to 3% in the DJJ model. The biggest difference in this variation of the model compared to the DJJ model however, is in the wages of workers. Whereas, in the DJJ model there are three possible wages, my model has 7 possible wages.

15.11.3 Welfare Cost of Mismatch

When a highly skilled worker accepts a low-skilled job, there is a potential loss in welfare to the economy because this worker can only be as productive as the productivity in a low-skilled job whereas if she/he were to be in a well-matched job their productivity would be higher ($y^H > y^L$). Thus, there is a potential loss to the economy of workers accepting a job to which they are not suited.

15.12 Wage Variation

As previously, to quantify the effect of these job-to-job transitions on wage dispersion, four useful statistics are reported. As a proxy for the degree of between-group wage inequality, the ratio between the average wage of high ability workers and the wage of low ability workers is calculated. Likewise, the within-ability wage inequality is measured by the ratio between the average wage of high-ability workers and their wage in unskilled jobs. Finally, to control for the relative size of the two groups, the total variance of the wage distribution, which is further decomposed into a permanent component due to between-group wage differences and a transitory component due to within-group wage differences is calculated. These statistics are reported in Table 7.

	Between Group Variation	Within Group Variation	Between Group Variance	Within Group Variance
Baseline	1.821	0.247	0.003	0.416
$b = 0.2$	1.364	0.111	0.011	0.361
$c = 0.3$	2.131	0.777	0.046	0.848
$\delta = 0.025$	0.023	0.005	0.011	0.269
$\gamma_0 = 0.75$	1.542	0.354	0.004	0.486
$y^H = 1.2$	1.786	0.283	0.004	0.419

Table 7

Table 7 shows that our benchmark model yields a considerably higher between-group wage variation when all workers indulge in on-the-job search, compared to when only mismatched workers search for jobs when employed. In contrast, there is a lower between-group wage variance when all workers indulge in on-the-job search, compared to when only mismatched workers search for jobs when employed. However, the within-group wage variation is much lower when all workers indulge in on-the-job search, compared to when only mismatched workers search for jobs when employed. Again in contrast, there is a higher within-group variance when all workers indulge in on-the-job search, compared to when only mismatched workers search for jobs when employed.

The wage dispersion is decreased when the flow value of unemployment is increased, but is substantially exacerbated when the cost of a vacancy is reduced. When the death rate of a firm δ, is reduced the wage dispersion is also reduced. When the proportion of untrained workers among the pool of workers is increased, the between-group wage variation is reduced but the between-group wage variance and within-group wage variation and variance are increased. When the output gap between a trained and an untrained worker is decreased, the between-group wage variation is reduced but the between-group wage variance and within-group wage variation and variance are increased.

On-the-job search is a way workers try to reduce mismatch and get a higher wage. In the process workers end up increasing the wage dispersion between the different types of workers. Trained workers however, are able to reduce the wage dispersion within their own type.

15.13 USA vs Europe

As previously in this section the model is adapted to reflect the job markets in the US and in Europe. As noted before, in both markets job-to-job transitions explain between 40% and 50% of the separations. Also the unemployment rate is higher in Europe, however, wage inequality is higher in the US. Thus, the parameter choice in the benchmark model is modified to reflect these facts. To capture the lower unemployment and higher wage inequality in the US, a higher matching efficiency is assumed for the US labour market than the European one. Thus, the meeting function is now $m(\theta) = A\sqrt{\theta}$, with $A = 1.25$ for US and $A = 1.20$ for Europe. In the current model, this change leads to lower unemployment rates for both types of workers and a higher in wage dispersion. However, this would lead to a fall in the share of mismatched workers, which would decline the job-to-job separations of highly educated workers. Hence, another parameter λ is changed such that the higher flexibility in the US market also means a higher value of λ there. Thus, for the US labour market $\lambda_0 = 1$ and $\lambda_1 = 0.75$, whereas for the European labour market $\lambda_0 = 0.90$ and $\lambda_1 = 0.70$. Table 8 shows the results of such simulations.

	θ	ζ	U_0	U_1	E_0	E_1	E_{1L}	$\underline{w}$	$\overline{w}$	w^{uL}	w^{uH}	w^{LH}	w^{LL}	w^{HH}
USA	0.57	0.55	0.07	0.0206	0.59	0.3127	0.0287	-1.81	1.00	6.58	1.00	-1.85	1.36	1.50
Europe	0.58	0.64	0.08	0.0227	0.58	0.3107	0.0305	-1.69	1.00	6.03	1.00	-1.80	1.35	1.50

Table 8

The results show that in the first row there is a greater wage dispersion, which is synonymous to the case of the US, and the second row shows a higher unemployment rate and a lower matching efficiency synonymous to Europe.

15.14 Existence of Mismatch

This paper has focused on an economy where highly-skilled workers find it optimal to accept low-skilled jobs. Is it too unrealistic to assume that some workers will find mismatched jobs attractive? The answer is no. Empirical evidence suggests that around 15.9% of workforce in the UK is highly-skilled but is occupying a low-skilled job.

This paper tries to explain some of the phenomena that exist in a labour market. Empirical evidence shows that mismatch exists and that mismatched workers earn less than their peers in who have a similar level of education to them but are occupying jobs that are more suited to them. Evidence also suggests that workers entering employment from unemployment earn less than workers who take up jobs from employment. On-the-job search and bargaining are important features of a labour market. By including them in the model my paper is more realistic that the DJJ paper.

16 Conclusion

On-the-job search by workers is an important feature in labour markets. In this paper I have analysed how this phenomenon affects the structure of employment and wages in an economy where trained and untrained workers compete for jobs. I find that when all workers search on-the-job the outcome is worse for the labour market position of untrained workers than when only-mismatched workers search on-the-job. This is because it causes a move in the job distribution towards high-technology jobs and decreases the overall stability of low-technology jobs. These results are consistent for when the motive of job-to-job transitions is the pursuit of a better match or when identical workers compete in same job types. When all worker types search on-the-job the job-to-job transitions are less frequent.

An interesting outcome of this research is that when all workers search on-the-job the proportion of high-technology vacancies is much higher compared to when only mismatched workers search during employment. Unemployment rate for both untrained and trained workers is higher when all worker types search on-the-job compared to when only mismatched workers search. Mismatch is higher in the market where all workers indulge in on-the-job search. Looking at the wage dispersion one observes that between group wage inequality is substantially exacerbated when all workers indulge in on-the-job search compared to when only mismatched workers search on the job. However, the within-group wage inequality is reduced when all workers search on-the-job compared to only the mismatched workers searching while employed.

On-the-job search is a way in which workers reduce the extent of mismatch and firms react to this. However, this interaction implies that when more workers try to relocate the friction in the market reduces the efficiency of resource allocation (by increasing mismatch) and it also creates more wage inequality between the different types of workers.

This paper contributes to the existing literature on wage inequality with on-the-job search by exploring the wage determination through a sequential bargaining game. It also allows for workers to search for jobs not only when they are unemployed but also when they are employed. It also allows not only the mismatched workers to search on-the-job but allows all worker types to search during employment and unemployment to improve their wage prospects.

An extension to this research would be to investigate the properties of job-to-job transitions. A utilitarian social planner might take into account effects of turnover on the rate of unemployment and employment for both untrained and trained workers to determine the best frequency of job-to-job transitions.

Another area of research could be to investigate the effects of productivity shocks and business cycle on the structure of employment and wages when on-the-job search affects the decisions of workers. Since, search is costly there might be pro-cyclical fluctuations in search intensities. Thus, for untrained workers competition with trained workers is stiffer during recessions. During a boom when more jobs are created there is a steady release of jobs by trained workers who move to better paying jobs thus, leaving the vacant jobs for untrained workers.

It would also be interesting to examine wage inequality, and job mobility empirically with a wide range of jobs. It will be interesting to see whether on-the-job search and mismatch can improve the empirical performance of the search and matching models.

Finally, it is worth noting that in the model developed, the share of low technology and high technology firms is fixed and determined by the free entry condition. This means that the firm side of the market cannot fully respond to the labour market by adjusting the types of job it offers. Thus, the model develpoed is not a complete general equilibrium analysis. The firm side analysis is only partially worked out.

17 References

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Part III

Self-selection in Education with Matching Frictions, on-the-job Search and Bargaining

18 Abstract

This paper contributes to existing literature by studying the link between educational choices, and (temporary) skill mismatch in a labour market with search frictions. The most important feature of this paper is the choice faced by a government in terms of policy. The paper shows that a policy of giving subsidy to the firms, which lowers their cost of vacancy is much more cost-effective compared to giving unemployment benefits that give more options to workers. Another important feature of the paper is the effect on average productivity in each education group of an increase in the number of people in education. An increase in the number of people in education means that the ablest among the uneducated becomes the least able among the educated. Hence, the mean ability in each education group decreases.

The results show that fewer people invest in education with a high cost of schooling and a low cost of firm vacancy. Total unemployment declines slightly with a declining cost of education. However, while fewer untrained workers are unemployed, the percentage of trained workers who are unemployed increases significantly and so does the proportion of trained workers accepting jobs offered by low technology firms. Net output also declines.

If the government target is to reduce the mismatch of workers and jobs, the best

option is not to decrease cost of education. At the same time in order to increase the net output in the economy the government can either give unemployment benefits or lower the cost of firm vacancy by giving subsidies to firms. This way the threshold level of education is high meaning fewer people choose to get trained and the average productivity in each education group is high. This model shows that a government subsidy to the firms, which reduces total unemployment and increases the net output in the economy, is far more effective than giving money to the unemployed workers in terms of benefits.

19 Introduction:

The decision to acquire education is made rationally by people based on several factors. According to the human capital theory illustrated by Becker (1962), the decision to invest in human capital rests on comparison with the costs and benefits of such an investment. Increase in future productivity can be attained only at a certain cost. These costs include the direct costs of schooling and indirect costs associated with foregone earnings while acquiring education. This paper's contribution is to study the link between educational choices, and (temporary) skill mismatch in a labour market with search frictions.

Burdett and Smith (2002), show that the investment decision of workers interacts positively with the decision of firms to offer jobs, such that an economy can get stuck in a bad trap, with high unemployment as well as a low investment in education. On the other hand Acemoglu (1996) shows that when there is frictional unemployment, there is an underinvestment in education since workers bear the cost of education and get paid a share less than their marginal productivity.

This model looks at the steady state equilibrium, where workers self-select themselves into education and firms are heterogeneous. Free entry into the market drives job creation. Search is random, as in Albrecht and Vroman (2002). Due to labour market frictions, there is unemployment in equilibrium. The productivity of workers is split in two parts; the first is the innate ability of the worker, while the second is the sector specific productivity related to education. Like Charlot and Decreuse (2005) acquiring education is assumed costly but this cost is assumed to be independent of the worker's ability. This means that given the fixed cost of investment into education, that is, the tuition fees, only those workers who are able enough, choose to acquire education and become qualified enough to work in high-productivity jobs.

The number of people receiving education increases only if less-able individuals are drawn into education (i.e. if the cost of education is low). This way the most able amongst the former uneducated workers become the least able among the now educated group. This means that the average productivity across the two groups declines.

Workers search for jobs from unemployment as well as during employment. On-the-job search allows the workers to obtain a better employment. Frictions in the market imply that there will always be some level of unemployment in the economy. Since the labour market is not frictionless it also means that the economy will always be characterised by some level of skill mismatch. Highly qualified workers may find it profitable to accept jobs requiring a low skill. Mismatch is reduced with a higher job turnover. Wages are determined through Cahuc, Postal-Vinay, Robin (2006), through a sequential bargaining between employers and workers under complete information.

Several studies have shown the importance of self-selection. Charlot and Decreuse (2005) build on Roy (1951) multi-sector model of occupational choices. They show that when frictions characterize markets, individuals fail to internalize the cost of their decision to acquire education and hence it causes inefficiency and over-education in the market. Cameron and Heckman (1998) argue that any analysis that ignores the effects of heterogeneity and self-selection on returns to education, present an overly optimistic view of policies that promote education. Andolfatto and Smith (2001) use search based frictions in a Roy model to study the dynamics of sectoral employment after a biased productivity shock. Saint Paul (1996) creates a model with fixed jobs where firms change their decisions to allocate vacancies between sectors according to the investment in education decision of workers.

This paper contributes to existing literature by studying the link between educational choices, and (temporary) skill mismatch in a labour market with search frictions. As shown in the paper, "Wage Determination with On-the-Job Search and Bargaining" (Syed 2015), more workers searching leads to higher inequality in wages. If the cost of education is low enough, more individuals choose to acquire education and get trained. As a consequence mismatch increases.

This model is related to several other matching models, for example Vroman (1987), Mortensen and Pissarides (1999), Dolado, Jansen and Jimeno (2008) and Bonilla and Burdett (2010). In these papers there are two sectors with heterogeneous workers and firms. There is no self-selection of workers into education. The skill level of workers is exogenously determined. Other models relevant to this paper are Burdett and Coles (1997) and Burdett and Wright (1998). These papers do not have a segmented market. The workers are assigned to a particular firm through setting reservation productivity. This means that a certain fixed number of workers meet a fixed number of firms in a market where both agents are heterogeneous.

20 Basic Framework

I will develop a steady state matching model of unemployment. There are two sectors producing a single good. There is a continuum of workers with mass equal to one. Time is continuous. Agents are risk neutral. Agents discount future at the rate $r > 0$. Workers and firms are heterogeneous in skill and technology respectively. At each instant $\mu > 0$ workers die.

Workers in each cohort are characterised by ability $a \in [0, 1]$ with distribution Φ and an associated density function $\phi > 0$ which is continuous. Workers choose education and become trained $e = 1$ or remain untrained $e = 0$, that is $e \in (0, 1)$.

A worker can be unemployed or employed. Let λ_0 be the effort with which unemployed workers search for jobs while λ_1 be the effort with which employed workers search on-the-job. λ_0, λ_1 are exogenous and assume that $\lambda_0 > \lambda_1$. An unemployed worker receives a flow pay-off ab from leisure, where $0 < b < 1$.

Each employer employs at most one worker. There are two types of jobs available in the labour market, low-technology (L) and high-technology (H) : $j \epsilon \{L, H\}$. While any type of worker can fill a low-technology job, only trained workers are productive in high-technology jobs. The output of a match depends on the worker's ability a and a worker's education choice $e \in (0, 1)$. In low-technology firm, output $y^L = a$ is independent of the worker's training e. In high-technology firms, output $y^H = \eta a$ if $e = 1$, where $\eta > 1$, and is zero otherwise.

There is free entry of firms into each technology sector, where free entry implies new firms in each sector make zero expected profit. The firms pay a cost c for posting any vacancy.

There is job destruction which occurs at a rate of $\delta > 0$. There is also turnover through on-the-job search, which I now describe in detail.

20.1 Matching

Let U_0 be the measure of unemployed workers who are untrained and U_1 the measure of unemployed workers who are trained. Let E_0 be the measure of employed workers who are untrained and E_1 the measure of employed workers who are trained. Let E_{1L} denote the measure of trained workers who are employed in a low-technology firm.

Unemployed workers and vacancies are assumed to meet each other according to a random matching technology, where $M = M(K, V)$ denotes the flow number of contacts.

$K = \lambda_0(U_0 + U_1) + \lambda_1(E_0 + E_1)$, is the aggregate job search effort and $V = V_L + V_H$, are the total vacancies. Assuming constant returns, define labour market tightness $\theta = V/K$ and $m(\theta) = \frac{M(K,V)}{K} = M(1, \theta)$ which describes the arrival rate of job offers per unit effort.

I assume that $m'(\theta) > 0$ and that $lim_{\theta\to\infty} m'(\theta) = 0$.

Unemployed workers sample job offers randomly at Poisson rate $\lambda_0 m(\theta)$. Employees may also search for a better job while employed and the arrival rate of offers to on-the-job searchers is $\lambda_1 m(\theta)$.

Similarly, vacancies meet unemployed workers at rate $\frac{m(\theta)}{\theta}$. I assume this rate is decreasing in θ and that $lim_{\theta\to 0}[m(\theta)/\theta] = \infty$.

21 Wage Determination

Wages are determined using the Cahuc, Postal-Vinay, Robin (2006) bargaining framework. Given contact and a gain to trade exists, the wage contract is negotiated following a set of rules that I now explain.

Wages are bargained over by workers and employers in a complete information context. Wage contracts stipulate a fixed wage that can be renegotiated by mutual agreement only: renegotiations thus occur only if one party can credibly threaten the other to leave the match if the latter refuses to renegotiate. There are no renegotiation costs.

When between-employer competition for labour services is not perfect, firm-worker matches are associated with a positive rent, defined as the expected value of future match output flows net of the worker's and firm's outside options.

When an unemployed worker meets a firm, the wage is determined as the worker's outside option plus a share β of the match surplus. This game delivers the generalized Nash-bargaining solution, where the worker receives a constant share β of the match rent. This latter parameter β is referred to as the worker's bargaining power.

When an employed worker contacts an outside firm, the situation becomes more favourable to the worker because she/he can now force the incumbent and poaching employers to compete. Competition between the two employers over the worker's services can be seen as an auction where the bidder with the higher valuation wins and pays the second price.

21.1 Equilibrium Wage Bargaining

21.1.1 Untrained workers $e = 0$

While unemployed, let $V_e^u(a)$ denote the expected lifetime value of a worker of type (a, e) being unemployed using an optimal strategy. If employed at type $j \epsilon \{L, H\}$ firm on wage w, let $V_e^j(a, w)$ denote the expected lifetime payoff.

Since, the flow payoff during unemployment $ab > 0$, an untrained worker has no gains to trade with a high-technology firm. This is because these workers receive $ab > 0$ during unemployment while the maximum they can receive in the high-technology firm is their productivity which is 0.

When the worker is paid her/his marginal productivity, the employer makes zero marginal profit on this worker, who therefore receives the entire match value $V_0^L(a,a)$. Further assuming that a vacant job has zero value to the employer, the difference between the match value $V_0^L(a,a)$ and the unemployment value defines the match surplus: $\left[V_0^L(a,a) - V_0^u(a)\right]$.

When an unemployed worker is matched with a low-technology firm, she/he obtains her/his reservation utility, $V_0^u(a)$, plus a share β of the maximum match surplus that she/he can get with a wage equal to the marginal productivity a in the low-technology firm. Let $\underline{w}(a)$ be the equilibrium negotiated wage given a contact between an untrained, unemployed worker and a low technology firm. Equilibrium bargaining implies $\underline{w}(a)$ solves:

$$V_0^L(a, \underline{w}(a)) = V_0^u(a) + \beta\left[V_0^L(a,a) - V_0^u(a)\right] \tag{76}$$

When an untrained employed worker receives an outside job offer, a three-player bargaining process is started between the worker, her/his initial employer, and the employer who made the outside offer. No employer will pay more than match productivity. The auction forces a low-technology firm to place a bid equal to marginal productivity, a, of the worker in that job.

Let $\bar{w}(a)$ be the equilibrium negotiated wage given a contact between an untrained, employed worker and two low-technology firms. Equilibrium bargaining implies:

$$\bar{w}(a) = a \tag{77}$$

21.1.2 Bargaining with trained workers

Again I start by considering an unemployed trained worker with ability a. Suppose the worker contacts a low-technology firm. The negotiated wage, denoted $w^{uL}(a)$, solves:

$$V_1^L(a, w^{uL}(a)) = V_1^u(a) + \beta max \left\{ \left[V_1^L\left(a, a\right) - V_1^u(a) \right], 0 \right\} \tag{78}$$

When a trained, unemployed worker is matched with a low-technology firm, she/he obtains her/his reservation utility, $V_1^u(a)$, plus a share β of the maximum match surplus that she/he can get with a wage equal to the marginal productivity a in the low-technology firm. Since, $\lambda_1 < \lambda_0$, the worker only accepts the wage offer if $V_1^L\left(a, a\right) > V_1^u(a)$, otherwise she/he prefers to stay unemployed.

When an unemployed worker is matched with a high-technology firm, she/he obtains her/his reservation utility, $V_1^u(a)$, plus a share β of the maximum match surplus that she/he can get with a wage equal to the marginal productivity ηa in the high-technology firm.

Let $w^{uH}(a)$ be the equilibrium negotiated wage given a contact between a trained, unemployed worker and a high technology firm. Equilibrium bargaining implies:

$$V_1^H(a, w^{uH}(a)) = V_1^u(a) + \beta \left[V_1^H\left(a, \eta a\right) - V_1^u(a) \right] \tag{79}$$

When a trained worker employed in a low-technology firm receives an outside job offer from another low-technology firm, a three-player bargaining process is started between the worker, her/his initial employer, and the employer who made the outside offer. No employer will pay more than match productivity. The auction forces a low-technology firm to place a bid equal to marginal productivity, a, of the worker in that job. The worker stays in her/his current job but her/his wage is raised to the marginal productivity in the low-technology firm.

Let $w^{LL}(a)$ be the equilibrium negotiated wage given a contact between a trained, worker employed in a low-technology firm meeting another low-technology firm with a Bertrand Competition. Equilibrium bargaining implies:

$$w^{LL}(a) = a \tag{80}$$

When a trained employed worker employed in a high-technology firm receives an outside job offer from another high-technology firm, a three-player bargaining process is started between the worker, her/his initial employer, and the employer who made the outside offer. No employer will pay more than match productivity. The auction forces a high-technology firm to place a bid equal to marginal productivity, ηa, of the worker in that job. The worker stays in her/his current job but her/his wage is raised to the marginal productivity in the high-technology firm.

Let $w^{HH}(a)$ be the equilibrium negotiated wage given a contact between a trained worker employed in a high-technology firm who meets another high-technology firm. Equilibrium bargaining implies:

$$w^{HH}(a) = \eta a \tag{81}$$

Lemma 6:

When a type-e $= 1$ employee in a $j = L$ firm receives an outside offer from a $j = H$ firm, the worker moves to the $j = H$ firm, where she/he gets wage $w^{LH}(a)$ that solves:

$$V_1^H(a, w^{LH}(a)) = V_1^L(a, a) + \beta \left[V_1^H(a, \eta a) - V_1^L(a, a)\right] \tag{82}$$

Proof:

ηa is the highest wage in the market, it is always better to earn this wage than any other wage. Thus, $V_1^H(a, \eta a) > V_1^L(a, a)$. The highest wage a low-technology firm can offer is a which the worker values at $V_1^L(a, a)$. The employee receives a positive surplus on top of this outside option from moving and hence prefers to move from a $j = L$ firm to a $j = H$ firm. Q.E.D.

Note that the wage $w^{LH}(a)$ obtained in the new firm can be smaller than the wage paid in the previous job, because the worker expects larger wage rises in firms with higher productivity. This option value effect implies that workers may be willing to take wage cuts just to move from a low- to a high-technology firm.

This section determines

$W = \left[\underline{w}(a), \bar{w}(a), w^{uL}(a), w^{uH}(a), w^{LH}(a), w^{LL}(a), w^{HH}(a)\right]$ as a function of (a, e), $V_e^u(a)$ and $V_e^j(a, w)$. The next step is to determine $V_e^u(a)$ and $V_e^j(a, w)$ consistent with $W(a, e)$.

22 Worker's Problem

22.1 Untrained Worker $e = 0$

Let $\zeta = \frac{V_L}{V_L + V_H}$ denote the fraction of vacancies which are offered by low-technology firms. As untrained workers have no gains to trade with high-technology firms, their effective arrival rate of job offers is $\lambda_0 \zeta m(\theta)$ during unemployment and $\lambda_1 \zeta m(\theta)$ during employment.

The Bellman equation describing the value of a type (a, e) worker with $e = 0$ when unemployed is:

$$(r + \mu)V_0^u(a) = ab + \lambda_0 m(\theta)\zeta\left[V_0^L(a, \underline{w}(a)) - V_0^u(a)\right] \tag{83}$$

The value of unemployment for an untrained worker incorporates the assumption that an untrained worker is not productive in a high-technology job. The gains from trade from the job are $\left[V_0^L\left(a, \underline{w}(a)\right) - V_0^u\left(a\right)\right]$. The flow payoff from unemployment is ab.

The Bellman equation describing the value of a type (a, e) worker with $e = 0$ hired from unemployment is:

$$(r+\mu)V_0^L\left(a, \underline{w}\left(a\right)\right) = \underline{w}\left(a\right) + \delta\left[V_0^u\left(a\right) - V_0^L\left(a, \underline{w}\left(a\right)\right)\right]$$

$$+\lambda_1 m\left(\theta\right)\zeta\left[V_0^L\left(a, \bar{w}\left(a\right)\right) - V_0^L\left(a, \underline{w}\left(a\right)\right)\right] \tag{84}$$

The flow value of an untrained worker hired from unemployment equals the sum of the flow return, $\underline{w}\left(a\right)$, plus the expected instantaneous capital loss, $\delta\left[V_0^u\left(a\right) - V_0^L\left(a, \underline{w}\left(a\right)\right)\right]$ from the job breaking and the capital gain from on-the-job search. An untrained worker meets a low-technology firm at the rate $\lambda_1 m\left(\theta\right)\zeta$. Both firms will offer a single wage $\bar{w}\left(a\right) > \underline{w}\left(a\right)$. Assuming that it is costly to move the worker stays with the current employer but is able to make capital gains of $\left[V_0^L\left(a, \bar{w}\left(a\right)\right) - V_0^L\left(a, \underline{w}\left(a\right)\right)\right]$.

The Bellman equation describing the value of a type (a, e) worker with $e = 0$ hired from employment is:

$$(r+\mu)V_0^L\left(a, \bar{w}\left(a\right)\right) = \bar{w}\left(a\right) + \delta\left[V_0^u\left(a\right) - V_0^L\left(a, \bar{w}\left(a\right)\right)\right] \tag{85}$$

The flow value of an untrained worker hired from employment equals the sum of the flow return, $\bar{w}\left(a\right)$, plus the expected instantaneous capital loss, $\delta\left[V_0^u\left(a\right) - V_0^L\left(a, \bar{w}\left(a\right)\right)\right]$ from the job breaking. There are no capital gains from on-the-job search.

Note the 3 equations (83-85) which determine $V_0^u(a)$, $V_0^L(a, a)$ and $V_0^L(a, \underline{w}(a))$. As equation (76) determined $\underline{w}(a)$ there are 4 equations for 4 unknowns.

Lemma 7

For an $e = 0$ worker with ability a, wage $\underline{w}(a), \bar{w}(a)$ and corresponding value functions $V_0^u(a)$ and $V_0^j(a, w(a))$ for $j = L, H$ a solution exists and is bound for all $\zeta \in [0,1]$ and $\theta \geq 0$. Equilibrium wage bargaining implies:

$$V_0^u(a) = \frac{a}{r+\mu}\left\{b\left[\frac{r+\mu+\delta}{r+\mu+\delta+\lambda_0 m(\theta)\zeta\beta}\right]+\left[\frac{\lambda_0 m(\theta)\zeta\beta}{r+\mu+\delta+\lambda_0 m(\theta)\zeta\beta}\right]\right\}$$

$$\in [\frac{ab}{r+\mu}, \frac{a}{r+\mu}] \tag{86}$$

$$V_0^L(a,a) = \frac{a}{r+\mu}\left\{b\left[\frac{\delta}{r+\mu+\delta+\lambda_0 m(\theta)\zeta\beta}\right]+\left[\frac{r+\mu+\lambda_0 m(\theta)\zeta\beta}{r+\mu+\delta+\lambda_0 m(\theta)\zeta\beta}\right]\right\}$$

$$\in [\frac{ab}{r+\mu}, \frac{a}{r+\mu}] \tag{87}$$

$$V_0^L(a,\underline{w}(a)) = \frac{a}{r+\mu}\left\{b\left[\frac{(r+\mu)(1-\beta)+\delta}{r+\mu+\delta+\lambda_0 m(\theta)\zeta\beta}\right]+\left[\frac{[(r+\mu)\beta+\lambda_0 m(\theta)\zeta\beta]}{r+\mu+\delta+\lambda_0 m(\theta)\zeta\beta}\right]\right\}$$

$$\in [\frac{ab}{r+\mu}, \frac{a}{r+\mu}] \tag{88}$$

$$\bar{w}(a) = a \tag{89}$$

$$\underline{w}(a) = ab - a(1-b)\left[\frac{\lambda_1 m(\theta)\zeta(1-\beta)}{r+\mu+\delta}\right] < ab \tag{90}$$

Note the wage $\underline{w}(a) < ab$, the flow pay-off from leisure. Hence workers are willing to take wage cuts because they expect a larger wage rises through on-the-job search.

22.2 Trained Worker $e = 1$

The Bellman equation describing the value of a type (a, e) worker with $e = 1$ when unemployed is:

$$(r+\mu)V_1^u(a) = ab + \lambda_0 m(\theta)\,\zeta max\{[V_1^L(a, w^{uL}(a)) - V_1^u(a)], 0\}$$

$$+\lambda_0 m(\theta)(1-\zeta)\left[V_1^H\left(a, w^{uH}(a)\right) - V_1^u(a)\right] \tag{91}$$

The value of unemployment for a trained worker incorporates the assumption that these workers might be productive in both low and high technologies. An unemployed trained worker meets a low-technology firm at an arrival rate of $\lambda_0 m(\theta)\,\zeta$ and a high-technology firm at an arrival rate of $\lambda_0 m(\theta)(1-\zeta)$. The gains from trade with a high-technology firm are $[V_1^H(a, w^{uH}(a)) - V_1^u(a)]$. Assuming surplus exists the gains from trade with a low-technology firm are $[V_1^L(a, w^{uL}(a)) - V_1^u(a)]$ and zero otherwise. The flow payoff from unemployment is ab.

The Bellman equation describing the value of a type (a, e) worker with $e = 1$ employed in a $j = L$ firm hired from unemployment is:

$$(r+\mu)V_1^L\left(a, w^{uL}(a)\right) = w^{uL}(a) + \delta\left[V_1^u(a) - V_1^L\left(a, w^{uL}(a)\right)\right]$$

$$+\lambda_1 m(\theta)(1-\zeta)\left[V_1^H\left(a, w^{LH}(a)\right) - V_1^L\left(a, w^{uL}(a)\right)\right]$$

$$+\lambda_1 m(\theta)\,\zeta\left[V_1^L(a, a) - V_1^L\left(a, w^{uL}(a)\right)\right] \tag{92}$$

The flow value of employment of a trained worker employed in low-technology firm hired from unemployment equals the sum of the flow return, $w^{uL}(a)$, plus the expected instantaneous capital loss, $\delta\left[V_1^u(a) - V_1^L\left(a, w^{uL}(a)\right)\right]$ from the job breaking up and the capital gains from on-the-job search. With an arrival rate of $\lambda_1 m(\theta)(1-\zeta)$, the worker meets a high-technology firm and gains $[V_1^H\left(a, w^{LH}(a)\right) - V_1^L\left(a, w^{uL}(a)\right)]$. With an arrival rate of $\lambda_1 m(\theta)\,\zeta$ the worker meets a low-technology firm. Assuming moving is costly, the worker stays with the current employer but is able to bargain her/his wage to $w^{LL}(a) = a$, where $a > w^{uL}(a)$. The gain to the worker in this case is $\left[V_1^L(a,a) - V_1^L\left(a, w^{uL}(a)\right)\right]$.

The Bellman equation describing the value of a type (a,e) worker with $e = 1$ employed in a $j = L$ firm hired from employment is:

$$(r+\mu)V_1^L\left(a, w^{LL}(a)\right) = w^{LL}(a) + \delta\left[V_1^u(a) - V_1^L\left(a, w^{LL}(a)\right)\right]$$

$$+\lambda_1 m(\theta)(1-\zeta)\left[V_1^H\left(a, w^{LH}(a)\right) - V_1^L\left(a, w^{LL}(a)\right)\right] \tag{93}$$

The flow value of employment of a trained worker employed in a low-technology firm hired from employment equals the sum of the flow return, $w^{LL}(a) = a$, plus the expected instantaneous capital loss, $\delta\left[V_1^u(a) - V_1^L\left(a, w^{LL}(a)\right)\right]$ from the job breaking up and the capital gains from on-the-job search. With an arrival rate of $\lambda_1 m(\theta)(1-\zeta)$, the worker meets a high-technology firm and gains $\left[V_1^H\left(a, w^{LH}(a)\right) - V_1^L\left(a, w^{LL}(a)\right)\right]$. There are no gains from trade if this worker meets another low-technology firm.

The Bellman equation describing the value of a type (a,e) worker with $e = 1$ employed in a $j = H$ firm hired from unemployment is:

$$(r+\mu)V_1^H\left(a, w^{uH}(a)\right) = w^{uH}(a) + \delta\left[V_1^u(a) - V_1^H\left(a, w^{uH}(a)\right)\right]$$

$$+\lambda_1 m(\theta)\zeta max\left\{\left[V_1^L(a,a) - V_1^H\left(a, w^{uH}(a)\right)\right], 0\right\}$$

$$+\lambda_1 m(\theta)(1-\zeta)\left[V_1^H\left(a, w^{HH}(a)\right) - V_1^H\left(a, w^{uH}(a)\right)\right] \tag{94}$$

The flow value of employment of a trained worker employed in a high-technology firm hired from unemployment equals the sum of the flow return, $w^{uH}(a)$, plus the expected instantaneous capital loss, $\delta\left[V_1^u(a) - V_1^H\left(a, w^{uH}(a)\right)\right]$ from the job breaking up and the capital gains from on-the-job search. With an arrival rate of $\lambda_1 m(\theta)\zeta$, the worker meets a low-technology firm. Assuming surplus exists the worker gains $[V_1^L(a,a) - V_1^H\left(a, w^{uH}(a)\right)]$ and zero otherwise. The worker meets a high-technology firm at an arrival rate $\lambda_1 m(\theta)(1-\zeta)$. The worker's current employer and the new employer enter a bid for the worker's services. Assuming moving is costly, the worker stays with the current employer but is able to bargain her/his wage to $w^{HH}(a) = \eta a$, where $\eta a > w^{uH}(a)$. The capital gains are $\left[V_1^H\left(a, w^{HH}(a)\right) - V_1^H\left(a, w^{uH}(a)\right)\right]$.

The Bellman equation describing the value of a type (a, e) worker with $e = 1$ employed in a $j = H$ firm hired from employment is:

$$(r+\mu)V_1^H\left(a, w^{HH}(a)\right) = w^{HH}(a) + \delta\left[V_1^u(a) - V_1^H\left(a, w^{HH}(a)\right)\right] \qquad (95)$$

The flow value of employment of a trained worker employed in a high-technology firm hired from employment equals the sum of the flow return, $w^{HH}(a)$, plus the expected instantaneous capital loss, $\delta\left[V_1^u(a) - V_1^H\left(a, w^{HH}(a)\right)\right]$ from the job breaking. There are no capital gains from on-the-job search.

Note the 5 equations (91-95) which determine $V_1^u(a)$ and $V_1^j(a, w^{ij}(a))$. As equations (78-79) and (82) determined $w^{uL}(a)$, $w^{uH}(a)$ and $w^{LH}(a)$ there are 8 equations for 8 unknowns.

Lemma 8

For an $e = 1$ worker with ability a, wage $w^{ij}(a)$ and corresponding value functions $V_1^u(a)$ and $V_1^j(a, w^{ij}(a))$ for $i = u, L, H$ and $j = L, H$ a solution exists and is bound for all $\zeta \in [0, 1]$ and $\theta \geq 0$.

Proof

Equilibrium wage bargaining implies:

$$V_1^H(a, \eta a) = \frac{\eta a + \delta V_1^u(a)}{r + \mu + \delta} \in [ab, \eta a] \qquad (96)$$

$$V_1^L(a,a) = \frac{a + \delta V_1^u(a) + \lambda_1 m(\theta)(1-\zeta)\beta V_1^H(a,\eta a)}{r+\mu+\delta+\lambda_1 m(\theta)(1-\zeta)\beta} \in [ab, \eta a] \tag{97}$$

$$V_1^H(a, w^{LH}(a)) = (1-\beta) V_1^L(a,a) + \beta V_1^H(a,\eta a) \in [ab, \eta a] \tag{98}$$

$$V_1^H(a, w^{uH}(a)) = (1-\beta) V_1^u(a) + \beta V_1^H(a,\eta a) \in [ab, \eta a] \tag{99}$$

$$V_1^L(a, w^{uL}(a)) = (1-\beta) V_1^u(a) + \beta V_1^H(a,a) \in [ab, \eta a] \tag{100}$$

$$V_1^u(a) = \frac{ab + \beta\lambda_0 m(\theta)\left[\zeta V_1^L(a,a) + (1-\zeta)V_1^H(a,\eta a)\right]}{r+\mu+\lambda_0 m(\theta)\beta} \in [ab, \eta a] \tag{101}$$

Each value function is continuous. Further, each value function is a function of another value function multiplied by a discount factor < 1. Also, each value function is bounded between $[ab, \eta a]$. By Brouwer fixed-point theorem, there exists at least one continuously differentiable solution on $[ab, \eta a]$.

23 Optimal Education Choice

Solving for $V_0^u(a)$ and $V_1^u(a)$ gives:

$$V_0^u(a) = \frac{ab}{r+\mu} + \frac{a}{r+\mu}\left\{\left[\frac{\lambda_0 m(\theta)\zeta\beta(1-b)}{r+\mu+\delta+\lambda_0 m(\theta)\zeta\beta}\right]\right\} \tag{102}$$

and

$$V_1^u(a) = \frac{r+\mu+\delta}{r+\mu+\delta+\lambda_0 m(\theta)\beta}\frac{ab}{r+\mu}$$

$$+\frac{r+\mu+\delta}{r+\mu+\delta+\lambda_0 m(\theta)\beta}\frac{a}{r+\mu}\left[\frac{\lambda_0 m(\theta)\beta\left[(r+\mu+\delta)\zeta + \left[r+\mu+\delta+\lambda_1 m(\theta)\beta\right](1-\zeta)\eta\right]}{(r+\mu+\delta)\left[r+\mu+\delta+\lambda_1 m(\theta)(1-\zeta)\beta\right]}\right] \tag{103}$$

Lemma 9

$V_0^u(a)$ is an increasing function of θ and ζ, whereas $V_1^u(a)$ is an increasing function of θ but a decreasing function of ζ. Hence, there must be one point at which the two value functions $V_0^u(a)$ and $V_1^u(a)$ cross each other.

Proof

When $m(\theta) = 0$, $V_0^u(a) = \frac{ab}{r+\mu}$,

when $m(\theta) > 0$, $V_0^u(a) = \frac{ab}{r+\mu} + \frac{a}{r+\mu}\left\{\frac{\lambda_0 m(\theta)\zeta\beta(1-b)}{r+\mu+\delta+\lambda_0 m(\theta)\zeta\beta}\right\} > \frac{ab}{r+\mu}$.

When $m(\theta) = 0$, $V_1^u(a) = \frac{ab}{r+\mu}$,

when $m(\theta) > 0$,

$V_1^u(a) = \frac{r+\mu+\delta}{r+\mu+\delta+\lambda_0 m(\theta)\beta}\left\{\frac{ab}{r+\mu} + \frac{a}{r+\mu}\left[\frac{\lambda_0 m(\theta)\beta[(r+\mu+\delta)\zeta+[r+\mu+\delta+\lambda_1 m(\theta)\beta](1-\zeta)\eta]}{(r+\mu+\delta)[r+\mu+\delta+\lambda_1 m(\theta)(1-\zeta)\beta]}\right]\right\} > \frac{ab}{r+\mu}$.

When $\zeta = 0$, $V_0^u(a) = \frac{ab}{r+\mu}$,

when $\zeta = 1$, $V_0^u(a) = \frac{ab}{r+\mu} + \frac{a}{r+\mu}\left\{\frac{\lambda_0 m(\theta)\beta(1-b)}{r+\mu+\delta+\lambda_0 m(\theta)\beta}\right\} > \frac{ab}{r+\mu}$.

Finally, when $\zeta = 0$, $V_1^u(a) = \frac{r+\mu+\delta}{r+\mu+\delta+\lambda_0 m(\theta)\beta}\left\{\frac{ab}{r+\mu} + \frac{a}{r+\mu}\left[\frac{\lambda_0 m(\theta)\beta\eta}{r+\mu+\delta}\right]\right\}$,

and when $\zeta = 1$,

$V_1^u(a) = \frac{r+\mu+\delta}{r+\mu+\delta+\lambda_0 m(\theta)\beta}\left\{\frac{ab}{r+\mu} + \frac{a}{r+\mu}\left[\frac{\lambda_0 m(\theta)\beta}{r+\mu+\delta}\right]\right\} < \frac{r+\mu+\delta}{r+\mu+\delta+\lambda_0 m(\theta)\beta}\left\{\frac{ab}{r+\mu} + \frac{a}{r+\mu}\left[\frac{\lambda_0 m(\theta)\beta\eta}{r+\mu+\delta}\right]\right\}$

Since $V_0^u(a)$ and $V_1^u(a)$ are continuous and differentiable, there must be a point where the two functions cross each other.

23.1 Education Choice

Acquiring education is costly. Let $C > 0$ be the cost of schooling independent of ability. At birth, each individual compares the expected lifetime gain from getting educated and invests accordingly. An individual with ability a decides to get educated if and only if:

$$V_1^u(a) \geq V_0^u(a) + C \tag{104}$$

The RHS of the equation is the sum of the opportunity cost $V_0^u(a)$ plus the direct schooling cost C. The utility levels $V_0^u(a)$ and $V_1^u(a)$ depend on the fraction of vacancies which are offered by each firm type $[\zeta, (1-\zeta)]$ and the job-finding rate $m(\theta)$ in each market.

23.2 Self-selection Rule

The threshold individual, with innate ability a, is indifferent between education and no education. Let a^c be the critical cutoff ability such that $a \geq a^c \Rightarrow e = 1$ and $a < a^c \Rightarrow e = 0$. The cutoff ability $a^c \subset (0,1)$ is such that $V_1^u(a) = V_0^u(a) - C$. Since, $V_0^u(a)$ and $V_1^u(a)$ are linear functions of a, we can denote:

$$V_0^u(a) = a\phi_0^u$$

and

$$V_1^u(a) = a\phi_1^u$$

This implies:

$$a^c[\phi_1^u - \phi_0^u] = C$$

Therefore,

$$a^c = \frac{C}{\phi_1^u - \phi_0^u} \tag{105}$$

Lemma 10

When the fraction of low skilled vacancies ζ increases, $\phi_1^u - \phi_0^u$, decreases and hence, the cutoff ability a^c increases.

Proof

When $\zeta = 0$, $\phi_1^u - \phi_0^u = \frac{\lambda_0 m(\theta)\beta\{(r+\mu+\delta)(\eta-b)+\lambda_0 m(\theta)\beta\eta\}}{(r+\mu)(r+\mu+\delta)[r+\mu+\delta+\lambda_0 m(\theta)\beta]}$,

and when $\zeta = 1$, $\phi_1^u - \phi_0^u = \frac{(\lambda_0 m(\theta)\beta)^2}{(r+\mu)(r+\mu+\delta)[r+\mu+\delta+\lambda_0 m(\theta)\beta]} < \frac{\lambda_0 m(\theta)\beta\{(r+\mu+\delta)(\eta-b)+\lambda_0 m(\theta)\beta\eta\}}{(r+\mu)(r+\mu+\delta)[r+\mu+\delta+\lambda_0 m(\theta)\beta]}$.

That is, the more low-skilled vacancies in the market, fewer people decide to invest in education.

The effect of labour market tightness θ, on $\phi_1^u - \phi_0^u$ and hence, on a^c is unknown.

23.3 Composition Effect

The average abilities across the pools of unemployed in each sector are functions of the cut-off ability a^c such that:

$$\overline{a}_0 = E\left(a \mid a < a^c\right) = \int_0^{a^c} \frac{\phi\left(a\right)}{\Phi\left(a^c\right)} a da \tag{106}$$

$$\overline{a}_1 = E\left(a \mid a > a^c\right) = \int_{a^c}^{1} \frac{\phi\left(a\right)}{1 - \Phi\left(a^c\right)} a da \tag{107}$$

Thus, a shift in the selection threshold a^c involves a composition effect. The average productivity in each education group is increasing in a^c. Increase in a^c (i.e. the number of people in education goes down) means the least able among the educated becomes the ablest among the uneducated. The mean ability in each education group increases.

24 Firm's Problem With Free Entry

Let $J_e^j(a,w)$ denote the expected lifetime value of a firm of type $j\epsilon\{L,H\}$ employing a worker with training $e \in (0,1)$, paying wage w. There is free entry of firms into each technology sector. Free entry implies new firms in each sector make zero expected profit. The value to the firm of a vacancy is zero.

24.1 Low-Technology Firm

Consider a low-technology firm holding a vacancy. Trained workers might find it worthwhile to accept these jobs.

The expected payoff of a low-technology firm holding a vacancy is:

$$c = \frac{m(\theta)}{\theta}\left\{\left[\frac{\lambda_0 U_0}{K}\int_0^{a^c}\left[J_0^L\left(a,\underline{w}(a)\right)\right]\right]dG_u(a)\right\}$$

$$+\frac{m(\theta)}{\theta}\left\{\frac{\lambda_0 U_1}{K}\int_{a^c}^{1} max\left\{\left[J_1^L\left(a,w^{uL}(a)\right)\right],0\right\}dG_u(a)\right\} \tag{108}$$

Conditional on a worker contact, $\frac{\lambda_0 U_0}{K}$ is the probability the worker is untrained and unemployed and $\frac{\lambda_0 U_1}{K}$ is the probability the worker is trained and unemployed. Vacancies meet unemployed workers at the rate $\frac{m(\theta)}{\theta}$. The capital gain from hiring an untrained worker is $J_0^L\left(a,\underline{w}(a)\right)$ and from hiring a trained worker is $J_1^L\left(a,w^{uL}(a)\right)$.

Consider a low-technology firm hiring an untrained worker. There is no gain to the firm from hiring a worker from employment.

The expected payoff to a low-technology firm hiring an untrained worker from employment is:

$$J_0^L\left(a,\overline{w}(a)\right) = 0 \tag{109}$$

The expected payoff to a low-technology firm hiring an untrained worker from unemployment is:

$$J_0^L\left(a, \underline{w}\left(a\right)\right) = \frac{a - \underline{w}\left(a\right)}{r + \mu + \delta + \lambda_1 m\left(\theta\right)\zeta} \tag{110}$$

The worker-firm match is dissolved if the job is destroyed by a shock δ, or if the worker changes her/his job. $\lambda_1 m\left(\theta\right)\zeta$ is the probability the worker meets a low-technology firm and changes her/his job. $a - \underline{w}\left(a\right)$ is the gain to the firm from hiring an untrained worker who produces a and is paid $\underline{w}\left(a\right)$.

Consider a low-technology firm hiring a trained worker. There is no gain to the firm from hiring a worker from employment.

The expected payoff to a low-technology firm employing a trained worker from employment is:

$$J_1^L\left(a, w^{LL}\left(a\right)\right) = 0 \tag{111}$$

The expected payoff to a low-technology firm employing a trained worker hired from unemployment is:

$$J_1^L\left(a, w^{uL}\left(a\right)\right) = \frac{a - w^{uL}\left(a\right)}{r + \mu + \delta + \lambda_1 m\left(\theta\right)} \tag{112}$$

The worker-firm match is dissolved either if the worker changes her/his job, or if the job is destroyed by a shock. $a - w^{uL}\left(a\right)$ is the gain to the firm from hiring a worker who produces a and is paid a wage $w^{uL}\left(a\right)$. With a probability δ, the job is destroyed by a shock. With a probability $\lambda_1 m(\theta)$ the worker meets another firm (low or high-technology).

24.2 High-Technology Firm

Consider a high-technology firm holding a vacancy. The expected payoff to a high-technology firm holding a vacancy is:

$$c=\frac{m(\theta)}{\theta}\left\{\int_{a^c}^{1}\frac{\lambda_0 U_1}{K}\left[J_1^H\left(a,w^{uH}(a)\right)\right]dG_u(a)\right\}$$

$$+\frac{m(\theta)}{\theta}\left\{\int_{a^c}^{1}\frac{\lambda_1 E_{1L}}{K}max\left\{\left[J_1^H\left(a,w^{LH}(a)\right)\right],0\right\}dG_u(a)\right\} \tag{113}$$

The value to a high-technology firm holding a vacancy reflects the assumption that only trained workers are able to perform these jobs. Conditional on a worker contact, firms meet trained unemployed job seekers at a rate $\frac{m(\theta)}{\theta}\frac{\lambda_0 U_1}{K}$ and trained workers employed in a low-technology firm at a rate $\frac{m(\theta)}{\theta}\frac{\lambda_1 E_{1L}}{K}$. The capital gain from hiring a trained unemployed worker is $\left[J_1^H\left(a,w^{uH}(a)\right)\right]$. Assuming surplus exists, the capital gain from hiring a trained worker employed in a low-technology firm is $\left[J_1^H\left(a,w^{LH}(a)\right)\right]$, otherwise it is 0.

Consider a high-technology firm hiring a trained worker. There is no gain to the firm from hiring a worker from employment.

The expected payoff to a high-technology firm hiring a trained worker from employment in a high-technology firm is:

$$J_1^H\left(a,w^{HH}(a)\right)=0 \tag{114}$$

The expected payoff to a high-technology firm hiring a trained worker from unemployment is:

$$J_1^H\left(a,w^{uH}(a)\right)=\frac{\eta a-w^{uH}(a)}{r+\mu+\delta+\lambda_1 m(\theta)(1-\zeta)} \tag{115}$$

$\eta a-w^{uH}(a)$ is the gain to the firm from hiring a trained worker who produces ηa and is paid $w^{uH}(a)$. With a probability δ, the job is destroyed. With a probability $\lambda_1 m(\theta)(1-\zeta)$, the worker meets a high-technology firm.

The expected payoff to a high-technology firm hiring a trained worker from employment in a low-technology firm is:

$$J_1^H\left(a,w^{LH}(a)\right)=\frac{\eta a-w^{LH}(a)}{r+\mu+\delta+\lambda_1 m(\theta)} \tag{116}$$

The worker-firm match is dissolved either if the worker changes her/his job, or if the job is destroyed by a shock. $\eta a - w^{LH}(a)$ is the gain to the firm from hiring a worker who produces ηa and is paid a wage $w^{LH}(a)$. With a probability δ, the job is destroyed by a shock. With a probability $\lambda_1 m(\theta)$ the worker meets another firm (low or high-technology).

24.3 Firm Equilibrium with Free Entry

Equations (108) – (116) can be reduced to two equations given by:

$$c = \frac{m(\theta)}{\theta}\left\{\left[\int_0^{a^c} \frac{\lambda_0 U_0}{K}\left[\frac{a - \underline{w}(a)}{r+\mu+\delta+\lambda_1 m(\theta)\zeta}\right]\right] dG_u(a)\right\}$$

$$+\frac{m(\theta)}{\theta}\left\{\int_{a^c}^1 \frac{\lambda_0 U_1}{K} max\left\{\left[\frac{a - w^{uL}(a)}{r+\mu+\delta+\lambda_1 m(\theta)}\right], 0\right\} dG_u(a)\right\} \tag{117}$$

$$c = \frac{m(\theta)}{\theta}\left\{\int_{a^c}^1 \frac{\lambda_0 U_1}{K}\left[\frac{\eta a - w^{uH}(a)}{r+\mu+\delta+\lambda_1 m(\theta)(1-\zeta)}\right] dG_u(a)\right\} \tag{118}$$

$$+\frac{m(\theta)}{\theta}\left\{\int_{a^c}^1 \frac{\lambda_1 E_{1L}}{K} max\left\{\left[\frac{\eta a - w^{LH}(a)}{r+\mu+\delta+\lambda_1 m(\theta)}\right], 0\right\} dG_u(a)\right\}$$

25 Steady State Conditions

This model analyses the market in a steady state. Workers maximize expected discounted income. Firms maximize expected discounted profit. The steady state conditions are the following:

The first steady state condition states that the outflow of untrained workers from unemployment equals their inflow back into unemployment.

$$U_0 \lambda_0 m(\theta)\zeta = [\phi(a^c) - U_0]\delta \tag{119}$$

The flow of untrained workers out of unemployment is U_0 (the measure of unemployed untrained workers) multiplied by $\lambda_0 \zeta m(\theta)$ (the arrival rate of untrained unemployed workers).

The flow into unemployment is then δ (the job destruction rate) multiplied by the mass of employed untrained workers, $(\phi(a^c) - U_0)$, (proportion of untrained workers in the population minus those within them who are unemployed). Thus,

$$U_0 = \frac{\delta \phi(a^c)}{\delta + \lambda_0 m(\theta) \zeta} \tag{120}$$

The second steady state condition states that the outflow of trained workers from unemployment equals their inflow back into unemployment.

$$U_1 \lambda_0 m(\theta) = [(1 - \phi(a^c)) - U_1] \delta \tag{121}$$

$\lambda_0 m(\theta)$ is the arrival rate of trained workers. The outflow of trained workers is the arrival rate multiplied by the measure of trained unemployed workers, U_1.

There are $1 - \phi(a^c)$ trained workers of whom U_1 are unemployed. Subtracting the later from former multiplied by the job destruction rate, gives the inflow of trained workers into unemployment. Thus,

$$U_1 = \frac{\delta [1 - \phi(a^c)]}{\delta + \lambda_0 m(\theta)} \tag{122}$$

The third steady-state condition states that the outflow from unemployment of trained workers employed in low-technology firms equals their inflow back into unemployment.

$$U_1 \lambda_0 m(\theta) \zeta = [\delta + \lambda_1 m(\theta)(1 - \zeta)] E_{1L} \tag{123}$$

The outflow of trained workers employed in a low-technology firm from unemployment is given by $U_1 \lambda_0 m(\theta) \zeta$. There are U_1 trained unemployed people. With an arrival rate of $\lambda_0 m(\theta) \zeta$, they meet a firm with a low-technology.

The inflow of trained workers employed in low-technology firms into unemployment happens when either the job is destroyed by a shock or there is worker turnover through on-the-job search. There are E_{1L} trained workers employed in low-technology firms. The probability of of meeting a high-technology job is $\lambda_1 m(\theta)(1-\zeta)$, and the job is destroyed at an exogenous rate, δ. Thus,

$$E_{1L} = \frac{\lambda_0 m(\theta)\zeta\delta\left[1-\phi(a^c)\right]}{\left[\delta+\lambda_0 m(\theta)\right]\left[\delta+\lambda_1 m(\theta)(1-\zeta)\right]} \tag{124}$$

26 Equilibria

I will consider two types of equilibria; one in which trained workers do not find it optimal to accept jobs from low-technology firms and one in which they do. Consistent with Albrecht and Vroman (2002) the former is called an ex-post segmentation equilibrium and the later a cross-skill matching equilibrium. These two equilibria are mutually exclusive. In case of ex-post segmentation, highly qualified workers do not find it profitable to accept low-technology jobs, which means $\left[\frac{\eta a - w^{LH}(a)}{r+\mu+\delta+\lambda_1 m(\theta)}\right] \leq 0$ and $\left[\frac{a - w^{uL}(a)}{r+\mu+\delta+\lambda_1 m(\theta)}\right] \leq 0$. In a cross-skill matching equilibrium these two surpluses are positive. Empirical evidence suggests that around 31% of the UK workforce is mismatched (over and under-educated) while around 69% of the workforce is well-matched. Thus, even though an ex-post segmentation equilibrium is less relevant to my paper, it is still an important equilibrium since it explains the job market conditions of a majority of UK workforce.

26.1 Ex-Post Segmentation

This equilibrium can be reduced to a set of two equations given by:

$$c = \frac{m(\theta)}{\theta}\left\{\left[\int_0^{a^c}\frac{\lambda_0 U_0}{K}\left[\frac{a-\underline{w}(a)}{r+\mu+\delta+\lambda_1 m(\theta)\zeta}\right]\right] dG_u(a)\right\} \tag{125}$$

$$c = \frac{m(\theta)}{\theta}\int_{a^c}^1 \frac{\lambda_0 U_1}{K}\left[\frac{\eta a - w^{uH}(a)}{r+\mu+\delta+\lambda_1 m(\theta)(1-\zeta)}\right] dG_u(a) \tag{126}$$

where U_0 is given by (120), U_1 is given by (122), $\underline{w}(a)$ is given by (90) and $w^{uH}(a)$ is given by:

$$w^{uH}(a) = \frac{[r+\mu+\delta+\lambda_1 m(\theta)(1-\zeta)](1-\beta)ab}{[r+\mu+\delta+\lambda_0 m(\theta)(1-\zeta)\beta]}$$

$$+\frac{\{[r+\mu+\delta+\lambda_0 m(\theta)(1-\zeta)]\beta-\lambda_1 m(\theta)(1-\zeta)(1-\beta)\}\eta a}{[r+\mu+\delta+\lambda_0 m(\theta)(1-\zeta)\beta]} \qquad (127)$$

Lemma 11

Equation (125) *is a decreasing function of θ and ζ, whereas equation* (126) *is a decreasing function of θ and an increasing function of ζ. Hence, there must be one point at which the two functions cross each other.*

Proof

Since, $\frac{m(\theta)}{\theta}$ is a decreasing function of θ, both equation (50) and (51) are decreasing functions of θ.

For equation (125):

When $\zeta = 0$, $\left[\frac{a-\underline{w}(a)}{r+\mu+\delta+\lambda_1 m(\theta)\zeta}\right] = \frac{a-ab}{r+\mu+\delta}$.

When $\zeta = 1$, $\left[\frac{a-\underline{w}(a)}{r+\mu+\delta+\lambda_1 m(\theta)\zeta}\right] = \left[\frac{a-ab}{r+\mu+\delta}\right]\left[\frac{r+\mu+\delta+\lambda_1 m(\theta)(1-\beta)}{r+\mu+\delta+\lambda_1 m(\theta)}\right] < \left[\frac{a-ab}{r+\mu+\delta}\right]$.

Also, when ζ increases from 0 to 1, U_0 decreases from $\phi(a^c)$ to $\frac{\delta\phi(a^c)}{\delta+\lambda_0 m(\theta)}$.

Hence, equation (125) is a decreasing function of ζ.

For equation (126):

U_1 is independent of ζ.

However, when $\zeta = 0$, $\left[\frac{\eta a-w^{uH}(a)}{r+\mu+\delta+\lambda_1 m(\theta)(1-\zeta)}\right] = \frac{(1-\beta)(\eta a-ab)}{r+\mu+\delta+\lambda_0 m(\theta)\beta}$.

And when $\zeta = 1$, $\left[\frac{\eta a-w^{uH}(a)}{r+\mu+\delta+\lambda_1 m(\theta)(1-\zeta)}\right] = \frac{(1-\beta)(\eta a-ab)}{r+\mu+\delta} > \frac{(1-\beta)(\eta a-ab)}{r+\mu+\delta+\lambda_0 m(\theta)\beta}$.

Hence, equation (126) is an increasing function of ζ.

Since, both (125) and (126) are continuous, there must be one point at which the two functions cross each other.

26.2 Uniqueness

Re-write equation (125) as:

$z() = -c + \frac{m(\theta)}{\theta}\left[\int_0^{a^c} \frac{\lambda_0 U_0}{K}\left[\frac{a-\underline{w}(a)}{r+\mu+\delta+\lambda_1 m(\theta)\zeta}\right]\right] dG_u\left(a\right)$

and equation (126) as:

$x() = -c + \frac{m(\theta)}{\theta}\int_{a^c}^1 \frac{\lambda_0 U_1}{K}\left[\frac{\eta a-w^{uH}(a)}{r+\mu+\delta+\lambda_1 m(\theta)(1-\zeta)}\right] dG_u\left(a\right)$

Lemma 12

$z()$and $x()$ satisfy the weak axiom of revealed preference if for any vectors θ_0 and θ_1, where $\theta_0 < \theta_1$,

$z(\theta_0) \neq z(\theta_1)$ and $\theta_0 z(\theta_1) \leq 0$ implies $\theta_1 z(\theta_0) > 0$

$x(\theta_0) \neq x(\theta_1)$ and $\theta_0 x(\theta_1) \leq 0$ implies $\theta_1 x(\theta_0) > 0$

If $z()$ and $x()$ satisfy the weak axiom, then for any constant returns to scale technology, the set of equilibrium vectors is convex. If in addition the economy is regular, the equilibrium is unique.

Proof

Since $\theta_0 < \theta_1$, and $\frac{m(\theta)}{\theta}$ is a decreasing function of θ, then $z(\theta_0) \neq z(\theta_1)$ and $\theta_0 z(\theta_1) \leq 0$ implies $\theta_1 z(\theta_0) > 0$. Also $x(\theta_0) \neq x(\theta_1)$ and $\theta_0 x(\theta_1) \leq 0$ implies $\theta_1 x(\theta_0) > 0$.

26.3 Cross-Skill Matching

A cross-skill matching steady-state equilibrium with on-the-job search consists of the wage $\underline{w}(a)$ *given by (90) and a set of value functions for* $V_1^u(a)$ *and* $V_1^j(a,w)$ *that satisfy (96-101) plus a vector* $[\theta, \zeta, a^c, U_0, U_1, E_{1L}]$ *such that:*

1. All matches produce a non-negative surplus for the equilibrium values of $\{\theta, \zeta, a^c\}$.

2. The vector $[\theta, \zeta, a^c, U_0, U_1, E_{1L}]$ *solves the steady state conditions (120), (122) and (124) plus the free entry conditions (117) and (118).*

26.4 Numerical Solutions

I will solve the cross-skill matching equilibrium using numerical solutions. I will assume a standard Cobb-Douglas meeting function with a constant elasticity of 0.5, i.e $m(\theta) = B\sqrt{\theta}$, with B assumed to be equal to one. Time is measured in quarters. The parameters are, $r = 0.01$, $\mu = 0.15$, $\delta = 0.15$, $c = 0.25$, $b = 0.1$, $\lambda_1 = 0.5$, $\lambda_0 = 0.7$, $C = 0.5$ and $\eta = 1.5$. Below are the results obtained in equilibrium:

$$Equilibrium = \begin{cases} \theta = & 10.61 \\ \zeta = & 0.93 \\ a^c = & 0.98 \\ U_0 = & 0.04 \\ U_1 = & 0.001 \\ E_{1L} = & 0.01 \end{cases}$$

For the chosen parameters a unique equilibrium is obtained. The labour market tightness is around 10.61, which implies that unemployment duration is about

3.7 months and the duration of a vacancy of about 13 quarters. Low technology firms offer most of the vacancies in the market (93%). The cut-off ability below which workers do not acquire education is 0.98. Thus, since low-technology firms offer most of the jobs available, most of the workers choose to get no training. Of all the workers about 4% are unemployed. The share of untrained workers among the unemployed workers is 97.56%. Of all the trained employed workers about 41.96% are employed in a low-technology firm.

27 Policy

For the purpose of policy I am interested in focussing on the case where trained workers find it profitable to accept low-technology jobs.

Although obtaining the optimal policy is ideal, in this model it is very complicated. I will thus, look at comparative statics to understand the interaction between education and mismatch. I will compute the output produced net of all costs (cost of education for workers and cost of vacancy for a firm). This will enable me to obtain a policy that improves on the existing outcome. Table 1 shows the results when different policy variables are changed.

	θ	ζ	a^c	U_0	U_1	E_{1L}	Y	$Y-C$	Net Y
Baseline	10.61	0.93	0.98	0.04	0.001	0.01	0.4477	0.4474	0.197
$C = 0.25$	10.02	0.92	0.49	0.02	0.02	0.21	0.2969	0.2315	-0.018
$b = 0.2$	12.06	0.94	0.98	0.04	0.001	0.01	0.4513	0.4511	0.201
$c = 0.5$	10.36	0.21	0.15	0.03	0.04	0.01	0.6002	0.2356	-0.264

Table 1

The second row of table 1 shows the equilibrium results for the baseline model. When the cost of acquiring education, C, is decreased from 0.5 to 0.25, the labour market tightness is decreased by 5.53%. The proportion of low-technology vacancies decreases by 0.16%. The cut-off ability at which workers decide to acquire education decreases significantly (from 0.98 to 0.49). Total unemployment stays almost the same (unemployment for untrained workers decreases by 50% while that for trained workers increases by 2164%). The duration of unemployment increases from 3.7 months to 3.8 months. The proportion of mismatched workers among the trained employed workers increases substantially from 1% to 21% (an increase of 2106%). Total output produced decreases by 33.68%. This is interesting because although with a decrease in the cost of education more workers choose to get educated the total output produced in the economy declines. Looking more closely at the results one sees that in the baseline model most of the workers are untrained (about 98%). Thus, the output produced by trained workers has little contribution towards the total output produced. When the cost of education is decreased nearly half the workers choose to invest in education. However, since the gap between the productivity of the two types of workers is not very high, the total output produced by untrained employed workers in the baseline model exceeds the total output produced by trained workers when the cost of education is lowered simply because there are many more untrained workers in the baseline model than there are trained workers when the cost of education has declined.

Note that the share of low technology and high technology firms is fixed and determined by the free entry condition. Thus, when the cost of education is decreased and many more workers choose to get educated the firm side of the market does not respond by adjusting the types of job it offers. This is a limitation of the model, which can be addressed by endogenously determining the share of low and high technology firms. Due to this limitation, about 21% of the workers are now mismatched and these workers have the same technology specific productivity as the untrained workers. Mean ability in each education group decreases with a decrease in a^c. Since output is lower in this new equilibrium the net output is also lower compared to the baseline model. Thus, a decrease in the cost of education without the firm side of the market responding ends up having a very high social cost.

The third row of table 1 shows the equilibrium results, when the unemployment benefit independent of ability, b, is increased from 0.1 to 0.2. The results show that the labour market tightness increases by about 13.75%. The proportion of low-technology vacancies increases by 1%. The cut-off ability at which workers decide to acquire education remains the same. The total unemployment also remains the same but the duration of unemployment decreases from about 3.7 months to 3.5 months. The proportion of mismatched workers among the employed trained workers remains constant but the total output produced increases by 0.8%. Output net of education cost increases by 2%. Thus, a 100% increase in unemployment benefit increases net output by only 2%.

The fourth row of table 1 shows the equilibrium results, when the cost of a vacancy, c, is increased from 0.25 to 0.5. The results show that the labour market tightness decreases by 2.3% and the proportion of low technology vacancies decrease substantially from 93% to 21% (a decrease of about 77.5%). The threshold level of education decreases significantly from 0.98 to 0.15 (a decrease of 85%). Total unemployment increases by 37.2% and the duration of unemployment remains almost the same. The proportion of mismatched workers among the employed trained workers also remains almost the same. Total output produced in the economy increases by 33% but the output net of all costs decreases by 234%. Thus, an increase in the cost of vacancy shifts the market in favour of high-technology firms. As a result more workers choose to invest in education and with more trained individuals in the economy total output increases. However, this increase in output comes at a cost of an increase in the total costs. Thus, output net of all costs declines substantially.

As stated before, the average productivity in each education group is increasing in a^c. When a^c goes up (i.e. the number of people in education goes down), the least able among the educated becomes the ablest among the uneducated. The mean ability in each education group increases.

The government has three options; give a direct subsidy (or enforce a direct tax) to students for the cost of education, give unemployment benefits to unemployed workers, or give subsidy (or enforce a tax on) to the firms to lower (increase) the cost of posting a vacancy.

A government or social planner cares about the net output produced in the economy. Since a declining cost of education is associated with more workers choosing to get trained with the firm side of the economy not responding to this change effectively, the net output in the economy declines. Hence, it is in the interest of the government to keep the cost of education high so that only a small proportion of workers get trained. This ensures that only a small percentage of trained workers accept low-technology jobs. It is also in the interest of the government to give subsidies to firms to lower their cost of vacancy and increase the benefits to unemployed workers. However, in this example a 100% increase in the unemployment benefit increases the net output by only 2%. Thus, the government should offer subsidies to the firms to lower their cost of vacancy. In this example halving the cost of subsidy has increased the net output by about 175%. This policy not only increases the net output but also reduces the total unemployment in the economy.

28 Conclusion

This paper contributes to existing literature by studying the link between educational choices, and skill mismatch in a labour market with search frictions. As shown in the paper, "Wage Determination with On-the-Job Search and Bargaining" (Syed 2015), fewer search frictions lead to higher inequality in wages. If the cost of education is low enough, more individuals choose to acquire education and get trained. As a consequence mismatch increases.

The most important feature of this paper is the choice faced by a government in terms of policy. The paper shows that a policy of giving subsidy to the firms, which lowers their cost of vacancy is much more cost-effective compared to giving unemployment benefits that give more options to workers. Another important feature of the paper is the effect on average productivity in each education group of an increase in the number of people in education. An increase in the number of people in education means that the ablest among the uneducated becomes the least able among the educated. Hence, the mean ability in each education group decreases.

This paper shows that the cut-off ability threshold, a^c, rises with the cost of schooling, C, and a declining vacancy cost of the firms, c. This means that fewer people invest in education with a high cost of schooling and a low cost of firm vacancy. Total unemployment declines slightly with a declining cost of education. However, while fewer untrained workers are unemployed, the percentage of trained workers who are unemployed increases significantly and so does the proportion of trained workers accepting jobs offered by low technology firms. Net output also declines.

If the government target is to reduce the mismatch of workers, the best option is not to decrease cost of education. At the same time in order to increase the net output in the economy the government can either give unemployment benefits or lower the cost of firm vacancy by giving subsidies to firms. This way the threshold level of education is high meaning fewer people choose to get trained and the average productivity in each education group is high. The number of mismatched people is also lower.

In the model developed, the share of low technology and high technology firms is fixed and determined by the free entry condition. This means that the firm side of the market cannot fully respond to the labour market by adjusting the types of job it offers. Given the lag in the adjustment from the firm side of the market to labour market outcomes of the workers, this phenomenon is not uncommon. However, as a next step to add more realism, one should look into endogenously determining the share of low and high technology firms within this model. This would allow one to study in detail the response of the firms to the pool of untrained and trained workers in the market.

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