The Evolution of Civil War Severity, 1816 to 2005[•]

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Abstract

Previous analyses of civil war trends tend to be informal and consider only post 1945 data. We examine data on civil wars over the period 1816 to 2005, using new methods for evolutionary growth processes. We find a number of new patterns and trends in civil war that have received little attention in previous research, including a structural break in frequency of conflict with decolonialization, as well as evidence of periodicity in civil conflict. We develop new measures of civil war intensity and impact, and find that conflicts have been generally more severe in the 20th than in the 19th century. We also find that the frequency-severity distribution of civil war does not appear to follow a power-law distribution, unlike data on many other types of conflict. Although structural trends suggest an increase in future civil wars, we discuss possible limiting factors that might prevent this in light of the recent observed decline in civil wars after the Cold War.

Introduction

Questions about trends in the frequency and severity of conflict, such as whether conflicts are becoming more common or more severe, have always been central to conflict research (see, e.g., Levy 1983; Richardson 1941; Wright 1965) . Such questions are best answered by systematic analyses of data. Whereas many scholars made very pessimistic forecasts about how the end of the Cold War would usher in a new era of increased instability and conflict (see, e.g., Mueller 1994 for a review), many subsequent empirical analyses actually show a recent decline in conflict, following an increase in the frequency of civil war in the immediate aftermath of the Cold War (Cederman et al. 2016; Goldstein 2002, 2011; Gurr 2002; Mack 2007; Pinker 2011). Although there are many analyses of trends in civil war and the potential factors that may account for these, most such studies examine only data after 1945 and typically rely on informal analysis of trends. In this article, we consider trends and patterns in civil wars over a much longer period – 1816 to 2005, using updated data from Gleditsch (2004) – and assess the time evolution of ongoing conflicts by year using a new multi-logistic methodology for analyzing evolutionary growth processes proposed by Miranda (2010) and Miranda and Lima (2010). Our analyses suggest a number of patterns and trends that so far have received little attention in conflict research. We find a clear structural break in the frequency of civil war before and after the period of decolonialization in the early 1960s. Moreover, there is some evidence of periodicity in civil conflict. We propose two new measures of civil war severity, which we call *conflict intensity*, based on the rate of casualties of a conflict over time, and *conflict impact*, based on the share of population killed in civil wars. Our analysis indicates that civil wars in the 20th century have been generally more severe than in the 19th century. Unlike many other types of conflicts, we find that the frequency-severity distribution of civil war does not appear to follow a power-law distribution, suggesting instead a mixture of qualitatively different classes of conflicts. Although structural trends indicate that we should see an increase in future civil wars, we discuss some possible limiting factors that might prevent this in light of the recent observed decline in civil wars after the Cold War.

Descriptive Analysis of Trends in Civil War

We start by inspecting the yearly distribution of the number of ongoing civil wars from 1816 to 2005, using version 1.52 of Gleditsch's (2004) Expanded War Data. Figure 1 displays the count of number of ongoing civil wars up to 2005, with two smoothed versions of the data, based on five year and ten year moving averages of the original data. We note that the main characteristics of the original data are preserved in both smoothed figures.

[Figure 1 about here]

Figure 2 displays the corresponding accumulated number of conflicts. A preliminary analysis of figures 1 and 2 suggests the existence of two distinct regimes: a first regime between 1816 and the beginning of the 1960 decade, with an average number of 4.79 conflicts per year, and a second regime from 1960 to 2005, with a much higher average number of 16.42 conflicts per year. Figure 2 shows a marked change of inflection in the accumulated number of conflicts by 1960, sharply separating the two regimes. The two regimes will be referred to as ranges 1 and 2, respectively. The civil war frequency in range 2 is roughly three times that of range 1. We note that the inflection point in this figure seems to correspond to the period of decolonization, and the emergence of new independent states that for various reasons may be more prone to conflict (see Sambanis 2002).

Another prominent feature in Figure 1 is that the yearly number of ongoing conflicts evolves as a sequence of somewhat regular bursts of conflicts. In fact, close inspection of Figure 1 indicates that the yearly number of ongoing conflicts peaks roughly around 1835, 1875, 1900, 1930, 1955, 1975, and 1995, or with an average of 26 years between peaks. In the next section, we proceed from a descriptive analysis to a more formal assessment of the time evolution of civil war, using a new multi-logistic methodology for analyzing evolutionary growth processes.

[Figure 2 about here]

Multi-logistic Modeling of Civil War Trends as an Evolutionary Growth Process

In this section we consider the event time series of ongoing civil war as an evolutionary process to examine the existence of trends, using a new multi-logistic methodology for analyzing evolutionary growth processes, recently proposed by Miranda and Lima (2010) and further detailed by Miranda (2010). Logistic modeling has traditionally been used to analyze growth processes in natural systems, such as population growth. Miranda and Lima (2010) and Miranda (2010) show how this methodology can be extended to consider trends in growth processes involving complex mixes of social, economic, and technological processes such as innovations, and also present evidence that considering such features can improve forecasts. In the case of civil war, this analysis will allow us to examine more formally whether there are regular trends in the frequency of conflict as well as notable changes over time.

Before turning to the application to civil war data we first outline the general concepts of logistic modeling and the main aspects of the methodology since these are unlikely to be familiar to social scientists. In general, a typical evolutionary growth process described by some cumulative indicator evolves over time following an S-shaped or sigmoid curve, as shown schematically in Figure 3. After reaching a maximum growth rate, somewhere along its evolutionary trajectory, it eventually saturates. The dynamics of such a process is determined by the way that it grows in its early stages and later declines, that is, it follows from the specific relationship between its amplitude and its growth rate, or growth frequency, at a given time.

[Figure 3 about here]

One of the most widely used functions to describe the evolution of S-shaped processes is the so-called Verhulst logistic function, which can be given as

$$f(t) = K \frac{e^{(t-t_c)/\tau}}{1 + e^{(t-t_c)/\tau}} \quad . \tag{1}$$

Here f (t) denotes the time-dependent variable describing the time evolution of the system, K, normally referred to as the system carrying capacity, denotes the excursion towards saturation, t_c is the inflection point of maximum rate of change of f and τ^{-1} is the growth rate parameter. The time derivative of Eq (1), namely,

$$g(t) = \frac{K}{\tau} \frac{e^{(t-t_c)/\tau}}{[1 + e^{(t-t_c)/\tau}]^2}, \tag{2}$$

gives the *logistic pulse*, depicted in Fig. 2 by the dotted curve, which describes the rate at which the logistic process evolves. The time interval around the center point in which the logistic pulse is within half of its maximum height is given by $t_c \pm \Delta t / 2$, where Δt is related to the parameter τ by $\Delta t = 3.526\ \tau$. That is, Δt is a measure of the logistic pulse time duration.

The logistic function was originally proposed by Verhulst (1845) to model human population dynamics, but remained practically forgotten for almost seventy years. In the 1920s Pearl and Reed (1920) rediscovered the Verhulst logistic model, and it soon attracted the attention of several researchers who applied it to several problems other than human population dynamics, ranging from population ecology to social behavior, from the spread of epidemics to the modeling economic processes, as well as to technological forecasting and substitution.

Many of these studies consider systems for which the use of a single logistic model suffices, namely systems where it is reasonable to assume a constant carrying capacity. However, this is not realistic in many natural and manmade processes. Bacterial growth, for instance, provides a good example of a multi-step growth process where the carrying capacity is subject to change. In many biological systems, carrying capacity may change as species can expand or shrink their niches. In manmade or human controlled systems such as energy production, transportation, communications, the carrying capacity at a given period is often limited by the available technology. Advances in scientific knowledge and technology introduce new products and social processes that ultimately alter the dynamics of a specific sector. For instance, coal has gradually come to replace wood as a primary energy source, and has in turn been partially replaced by oil and gas. At each stage of the technological substitution process, the carrying capacity of an energy supply system changes with the emergence of new primary energy sources.

Miranda and Lima (2010) and Miranda (2010) discuss a new methodology to improve the multi-logistic analysis of evolutionary time series. Their main concern is how to deal with extended series, especially series corresponding to a sequence of manmade events involving a complex mix of social, economical, technological and political factors. The methodology has been used to evaluate possible periodic structures that can be attributed to interaction of such components as well as whether short-term forecasts can be improved by taking into account such features.

Given the complexity of factors that may affect the evolution of a time series it is unlikely that the relative influence of components will be homogeneous over time. Each feature is likely to have its own characteristic evolution time, and some components may dominate over others during certain periods. Hence, we would rarely expect an entire process to be represented by a smooth single evolutionary logistic curve. As such, the analysis of extended time series calls for a multi-logistic description. The multi-logistic description of a given time series uses a series of either logistic functions, when dealing with a cumulative indicator, such as the stock of automobiles in a given country, or the derivatives of the logistic functions, the so-called logistic pulses, in the case of event frequency time series such as the number of conflicts per year.

The data fitting procedure is another important aspect of the methodology. If a model provides a good approximation to the data, the estimated parameters of the model function can be considered an optimal summary of the time evolution of the series of interest. However, the results can also provide information beyond the characteristic components of the series, for instance through the structure of the residuals of the fitted model. In general, the residuals will consist of random fluctuations or noise, reflecting unrelated components. In some systems, however, the residuals series display structured oscillatory behavior. This may reflect important relationships among components where the correlations can reveal important intrinsic properties of the system represented by the original time series. The residuals or deviations correspond to the influences of the events or processes that decisively drive the initiatives represented by the data set. If the pattern of the residuals for example can be described by a truncated harmonic sine series, its fundamental frequency will directly reflect actual system parameters, very much in the same sense as the fundamental frequency of a stretched string (like that of a piano) is entirely determined by the string’s length, the material density, and the tension at which it was originally stretched.

Modeling complex evolutionary systems requires both finding a good model function fitting the data, describing the trend exhibited by the events themselves, as well as a thorough analysis of the resulting residuals. In the multi-logistic approach we model the trend of a given series of events either by a multi-logistic function or by multi-logistic pulses, whose model functions are written either as

$$F(t) = \sum_i A_i \frac{\exp((t - t_{ci})/\tau_i)}{1 + \exp((t - t_{ci})/\tau_i)}, \tag{3}$$

in the case of a cumulative series of events, or,

$$F(t) = \sum_i A_i \frac{\exp((t - t_{ci})/\tau_i)}{[1 + \exp((t - t_{ci})/\tau_i)]^2}, \tag{4}$$

in the case of events per unit time series. In eqs (3) and (4), the parameter A_i denotes the excursion towards saturation during the *i-th* growth sequence, in the case of cumulative events, or the yearly amplitude in the case of a time series of events frequency. The parameter t_{ci} is the inflection point of maximum rate of change of f in the sequence i and τ_i^{-1} is its growth rate parameter.

The parameter τ in Eqs. (3) and (4) is usually associated with the time it takes for the process to reach saturation. This time, also referred to as the process duration, is defined as the time required for the system to complete 80% of its excursion, corresponding to the system evolving from 10% to 90% of the total excursion. It turns out that, for all practical purposes, this lapse of time is approximately equal to 4.39 times the value of τ.

For the residuals reconstruction we rely on the straightforward use of a truncated sinusoidal Fourier series, although other signal reconstruction techniques such as wavelets also could be used. Accordingly, the data-to-model residuals were reconstructed using a truncated sine series of the type

$$R = \sum_{n=1}^{N} A_n \cdot \sin(2\pi\, n\, f_o\, t + \phi_n) \tag{5}$$

Here, A_n and ϕ_n represent the amplitude and the phase of the n-th harmonic and f_0 stands for the frequency of the fundamental mode.

Civil War Trend Analysis

After reviewing key aspects of the methodology, we now return to the specific application of this approach to civil wars. We analyze the time evolution of civil wars in each regime using a ten year moving average smoothed time series. The results for regime 1 are shown in Figure 4, where the crosses represent the smoothed empirical data, and the solid line the results of the logistic pulse modeling as given by Eq (4). The number of ongoing conflicts in this period is best modeled by a five logistic pulses model, with amplitude A_i's, center point t_i's, and time constant τ_i's as given in Table 1. We find conflict peaks around 1836, 1873, 1902, 1928, and 1958. These peaks generally seem to reflect periods of upheaval in the number of independent states and disputes over specific governments. For example, the first peak reflects a number of civil wars over control of the government in Europe and Latin America, as well as various secessionist conflicts at the fringes of the Russian and Ottoman empires, while the last peak reflects a number of independence movements in colonies as well as Marxist insurgencies.

[Figure 4 about here]

[Table 1 about here]

For the second regime, we find that a two logistic pulses model as shown in Fig 5 provides the best fit, with corresponding fitting parameter values listed in Table 2. The first peak is around 1972, with a broad time span, which may reflect both civil wars sustained by the Cold War as well as various separatist conflicts, whereas the 1993 peak to a large extent seems to be related to the dissolution of the USSR and Yugoslavia.

[Figure 5]

[Table 2 about here]

Figure 6 summarizes the overall picture emerging from this analysis. The horizontal dotted lines represent the average number of conflicts per year in ranges 1 and 2, respectively, while the solid line gives the number of conflicts obtained from the logistic modeling. From the peak positions listed in Tables 1 and 2, we find an average peak-to-peak time span of about 26.3 years, indicating that the world has experienced a peak in the number of conflicts every, say, 26 years.

[Figure 6]

This pattern and observed peaks lead almost inevitably to a forecasting exercise, and this in turn suggests another peak in the number of civil wars by around 2019 given the observed 1993 peak. We stress, however, that such a forecast is only valid if there are no structural breaks in the series or other factors limiting conflict do not apply. We will return to this issue later.

Residuals Analyses of Civil Wars Trend

We now investigate whether we can extract additional information from the data by an analysis of the residuals. The residuals are given by the difference between the data points and the model fitted values, so that a positive residual reflect observed data values higher than the model predictions and vice-versa. As we discussed previously, non-randomly distributed residuals may reflect interesting intrinsic properties of a system. The residuals have been fitted by a truncated sum of sine harmonics, as given in Eq. (5). Table 3 summarizes the results for the fundamental and dominant modes as obtained from the fitting of the residuals. From the fundamental mode results, we find a time period of 27.8 and of 30.8 years for the periodic behavior of the residuals in ranges 1 and 2, respectively. Figures 7 and 8 display the corresponding residuals reconstruction for ranges 1 and 2, respectively, while Figure 9 displays the amplitude of the fundamental and dominant modes for both ranges.

[Figure 7 about here]

[Figure 8 about here]

[Figure 9 about here]

These results suggest that the residuals for annual civil war incidence are not random and that there exists some underlying oscillatory structure. As shown in Table 3, the residuals for range 1, besides the fundamental period of 27.8 years, display dominant harmonics corresponding to periods of 13.9, 7.0, and 9.3 years. The corresponding results for range 2, also shown in Table 3, indicate that the dominant period (namely. 7.7 years) is roughly half that of range 1 (13.9 yrs). Coincidently, the dominant period of about 13.9 years we have found for the residuals in range 1 is similar to the 13 years periodicity of worldwide terrorist attacks reported by Clauset et al. (2007) based upon an autocorrelation analysis of such events between 1968 and 2006.

[Table 3 about here]

We perform an autocorrelation analysis to further examine whether these residuals represent a sequence of random or auto-correlated events (see Box and Jenkins 1970). The plot of the autocorrelation coefficients of the range 1 residuals as a function of varying time lags is shown in Figure 10. A similar autocorrelation behavior is also found for the residuals of the logistic modeling of range 2. If random, such autocorrelations should be near zero for any and all time-lag separations. If one or more of the autocorrelations are significantly different from zero the series may be non-random. The plot in Figure 10 shows a sequence of positive and negative spikes. Such a pattern is typical for the autocorrelation plot of a sequence of events exhibiting a strong degree of periodicity.

[Figure 10 about here]

Based on this evidence of periodicity in the residual data, we use the residuals for an exercise aimed at forecasting the next peaks in the number of conflicts. As we emphasized previously, the trend analysis is related to the actual occurrence of the events represented by the empirical data while the residual or deviations from the trend are likely to reflect a complex blend of political, economical, social and technological components. By extrapolating the fitted curve up to 2035, a sequence of peaks is obtained at about 2006, 2010, 2014, followed by the ones marked in Figure 8 around, 2017, 2023, and 2031.

We note, however, that we find very large negative residuals in the last years in the data, indicating that the model predictions are higher than the observed number of conflicts. Although the most recent data from the Uppsala Conflict Data Program suggest an increase in ongoing civil wars from 2013 to 2014 (see Petterson and

Wallensteen 2015: 537), the total number of conflict still remains below the expected ot Cold War levels. This deviation between predicted and observed values raises question about whether the end of Cold War may represent a new structural shift in the series, and whether the frequency of civil war may have fallen to a permanently lower level following the initial spike around 1993. Although we cannot examine whether there may be such structural shifts without additional actual data, our model based predictions provides a useful basis for a structural forecast of the expected frequency of conflict based on past trends to which the observed record can be compared. If we find that the number of conflicts in the post-Cold War remains lower than the expected level, we could then consider propositions on what may have decreased the actual number of conflict relative to the counterfactual structural prediction, including features such as political reform and democratization, or advances in conflict management or more effective international organizations (see Cederman et al. 2016; Gurr 2000; Golstein 2011; Mack 2007; Pinker 2011).

Civil War Severity

So far we have been discussing the time evolution of the number of civil wars rather than the severity of these conflicts. The analysis of conflict severity is a multifaceted problem that can be analyzed from different viewpoints, but conflict "intensity" and "impact" are clearly among the most central components. The concept of war intensity was introduced in the 1940s through Richardson's (1935, 1941, 1960) pioneering work, which proposed the number of casualties as a measure of war intensity. However, a plausible operationalization of the intensity of a given phenomenon depends not only on the total number of events but also on its rate. In other words, intensity is usually related to the number of events per time units or exposed area. For instance, in the case of solar radiation incident on Earth, the intensity parameter might be taken as the amount of solar radiation power per square meter. An alternative way to look at civil war severity in terms of intensity may be given by the number of casualties per day.

An analysis of war severity should go beyond describing trends as measured by the simple intensity parameter. Among the many features that may affect conflict severity, population size is likely to exert a particularly strong impact on the magnitude of a conflict, and the severity of a conflict relative to population is also an important aspect of a conflict's relative impact (see Cirillo and Taleb 2015; Pinker 2011). Accordingly, we propose the relative number of casualties, or the number of casualties per hundred thousand inhabitants, as a measure of conflict impact.

In this section we present an analysis of civil wars severity based the daily death toll, as a measure of civil war intensity, as well as the death toll per hundred thousand inhabitants as a measure of their impacts. We have information on duration and death for 291 out of the 373 conflicts in the Gleditsch (2004) data. For each conflict, we compute a conflict intensity indicator based on the average daily casualty indicator by dividing the total death toll by the total duration. We derive a conflict impact indicator by dividing the total civil war death toll by the population of the country at the beginning of the conflict, using population data from Maddison (2003).

In Table 4 we present a summary of the main characteristics of the severity indicators for the major civil wars. In the case of the civil war intensity parameter we considered as major conflicts those where the intensity parameter exceeds 100 fatalities per day. From the original 291 events only 43 fall in this category. Regarding the impact parameter, we consider conflicts as major if the total death toll during the conflict is equal or greater than 1% of the country's population at the beginning of the conflict. Table 5 lists the ten most severe conflicts based on each indicator.

[Table 4 about here]

Of the 43 major conflicts as given by the intensity criterion, 24 or 55.8% are conflicts with durations of less than 100 days. These intense short lived conflicts might be classified as heavily one-sided conflicts or perhaps even massacres by the government of much weaker opponents. By contrast, only 3 or 12% of the major conflicts as defined by the impact parameter fall in this category. Another interesting aspect that emerges from Table 4 is the involvement of European countries. With regards to the intensity criterion, the number of European countries involved in major events is more than four times greater than in the case of the impact criterion. We see this as reflecting how many high intensity conflicts pertain to colonial independence movements or involvements in conflicts in former colonies, where the demographic impact is much greater in the (ex-) colonies than in the metropoles. Another salient point in Table 4 is how civil wars in the 20th century tend to be much more severe than 19th century civil wars, irrespective of the indicator. This is also clear from Table 5, which shows that only two conflicts from the 19th century contribute to the top ten most severe conflicts (i.e., China versus Taipings 1860 for intensity and Mexicans vs. Yucatan Mayas 1847 in the case of the impact).

[Table 5 about here]

We now turn to consider the time evolution of the major civil wars as described by these indicators as well as the distribution of the civil wars as a function of their intensity and impact parameters. Figures 11 and 12 display the time evolution of the major civil wars as defined by their intensities (namely, those with intensities greater than 100 fatalities per day) and their impact factors (that is, total death toll during the conflict duration equal or greater than 1% of the country's population at the beginning of the conflict). Both figures clearly confirm that major civil wars during the 20th century have been more severe than civil wars in the 19th century.

[Figure 11 about here]

[Figure 12 about here]

The civil wars distributions as a function of the severity indicators were constructed by counting the corresponding number of conflicts, for each value indicator, as discussed in the Appendix. We have used two approaches. First, since the values of both indicators vary typically by four orders of magnitude, we assembled the distributions using logarithmic binning. Second, we worked directly with the corresponding cumulative distribution functions, commonly denoted by $P(X < x)$, and defined as the number of events having an indicator with values less than a given value x, divided by the total number of events. Its complementary distribution function, denoted as $P(X \geq x)$, is defined as the ratio of the number of events having the indicator with values greater or equal to x. This approach follows closely Clauset et al.'s (2007) analysis of the frequency of severe terrorist events. We provide a brief summary of the main characteristics of the distribution functions in the Appendix.

In Figure 13 we show the civil wars cumulative distribution function as a function of intensities. After successive trials in modeling this cumulative distribution using different functions, we found that a sum of four logistic functions provided the best fitting model. The solid line in this plot corresponds to the fitted predictions of this model, with the values of the fitting parameters given in Table 6. Figure 14, in turn, shows the complementary distribution function (open symbols) as a function of the war intensity indicator in logarithm scale, together with the corresponding model function as obtained from the above cumulative distribution function data fitting. Finally, Figure 15 presents the resulting probability density function obtained by taking the derivative with respect to the variable x of the cumulative distribution function. We note that the first three peaks, at x equal to 0.46, 1.46, and 2.66, corresponding to daily death tolls of the order of 1.6, 4.3, and 14.3, respectively, are quite visible in Figure 15, whereas the fourth peak at x = 3.96 (daily death toll of the order of 52.5) is not evident in this plot. The reason for this is that this is the weakest and widest out of the four peaks.

[Table 6 about here]

[Figure 13 about here]

[Figure 14 about here]

[Figure 15 about here]

The results presented in Figs. 13-15 contain a number of interesting aspects. First, contrary to what Clauset et al. (2007) find in the case of the severity of terrorism, the complementary cumulative distribution function shown in Figure 14 does not follow a so-called power law. That is, $P(X \geq x)$ does not scale as $x^{-\alpha}$ as seems to be the case for terrorism severity as discussed by Clauset et al. (2007), or as found in the case of interstate wars by Cederman (2003) and Richardson (1941). One interpretation of power laws is that there are no qualitative differences between small, medium and large events, and that a common underlying mechanism can account for the range of the distribution of events. We refer to Clauset et al. (2007) for further discussion. However, the results in Figure 15 indicate that the severity of civil wars, in terms of the daily death toll, seems to fall into distinct categories or classes. The strongest and clearest peak in the civil war intensity distribution corresponds to conflicts with an average daily death toll of 4.5 deaths per day, whereas the second dominant and broader peak, encompassing the majority of conflicts, refers to events with an average intensity of about 14 fatalities per day. This suggests that there may be different mechanisms generating the two classes of conflicts in terms of their intensity.

We have also applied similar methods to model the dependence of the civil wars distribution functions on the impact indicator. The results are shown graphically in Figures 16 and 17, and Table 6 lists the fitted parameter values. Figure 16 displays, on a bi-logarithmic scale, the complementary cumulative distribution function dependence on the civil wars impact indicators, while Figure 17 shows the corresponding probability density function. We now find that a three logistic function model provides the best fit.

[Figure 16 about here]

[Figure 17 about here]

The results shown in Figure 16, as well as in Figures 13 and 14, suggest that the proposed indicators provide a consistent picture of civil war severities. Figures 16 and

17 indicate that, in terms of their impact classification, civil wars do not exhibit a power law behavior, but rather fall into distinct classes according to the strength of their impact parameters. This is very well illustrated in Figure 17, where the two dominant peaks correspond to 5.2 and 42.7 fatalities per hundred thousand inhabitants, respectively. The distinct behavior of the civil wars severity distribution functions may be due to the different nature of the corresponding indicators. The impact parameter is by definition a relative quantity that can be compared across all conflicts, while the intensity parameter may reflect more specific dynamics of conflicts.

Discussion and Conclusions

We have presented a quantitative analysis of the time evolution of the civil wars between 1816 and 2005 and two new proposed measures of civil war severity. Our “coarse-grained” time evolution analysis, considering the number of conflicts per year, suggests seven phases over the time period considered. The analysis of the residuals derived from the yearly conflict distribution analysis offered a complementary “fine-grained” view of the evolution processes. Subtracting the overall trend off the civil wars yearly frequency time series, the residuals appear to exhibit a clear wavelike behavior. Furthermore, this wavelike behavior is not dominated by a single mode cycle, but rather by two or more periodic modes that add up to produce the observed wavy behavior of the time-series trend residuals, suggesting a complex mix of social, religious, economic, political and technological components involved in conflicts to varying degrees.

Table 7 consolidates the identified phases characterizing the trend of civil wars distribution summarized in Tables 1 and 2. Table 7 displays the central year of each phase, the average number of conflicts per year for each phase, the time lapse between the center year of consecutive phases (center year shift) and the difference between the average number of conflicts (conflict frequency shift) for the sequential phases.

[Table 7 about here]

Figure 18 displays the center year shifts and the war frequencies shifts between the sequential phases as listed in Table 7. The average time lag between these seven peaks is about 26.3 years. Nevertheless, there is some tendency for the phase separations to get closer, especially after 1960 (transitions 5 to 6 and 6 to 7). Combining this result with the information displayed in Figures 6, 11, and 12 reinforces the conclusion that the frequencies and severity of civil wars have increased over the 20th century, although it is possible that the post-Cold War decline on civil war may indicate that we have moved into a qualitatively new and more peaceful global context.

[Figure 18 about here]

It is natural to consider whether these phases in the frequency of civil war display any correspondence to other global trends or delineations of specific period. Maddison (1991), for example, identifies four phases in global capitalist development, 1870–1913, 1913–1950, 1950–1973, and 1973 onwards, using conventional macroeconomic indicators such as the rate of output growth, output per capita, capital stock, export volume, plus the cyclical variations in output, exports, level of unemployment and rate of price increase. We note that the Maddison’s first or “liberal” phase (1870–1913) follows closely after the 1873.17 second civil war phase peak, while the end of his “golden phase” (1950–1973), culminating in the 1973 oil crisis, coincides with the sixth civil war phase peak. Maddison emphasizes primarily economic policy stances such as the unemployment/price stability trade-off and the freedom of international trade of factor movements in distinguishing between the delineation of phases, and conflict and political trends that fuel conflict primarily enter the analysis as exogenous “shocks”. Our analyses suggest that the economic trends highlighted in Maddison’s phases as well as other features shaping global economic evolution, such as state formations in Europe around 1870 and global commodity prices in 1972-3, may also have an influence on civil war and the ideologies that may generate conflict. For example, economic change can promote the development of nationalist and separatist sentiments, and efforts to expand the reach of the state can generate tensions with groups that see themselves as distinct (see, e.g., Hechter 2001; Wimmer 2013). However, a more detailed analysis beyond the scope of this paper obviously will be necessary to examine the strength of the evidence for such relationships, but we see this as a very promising avenue for further research.

[Table 7 about here]

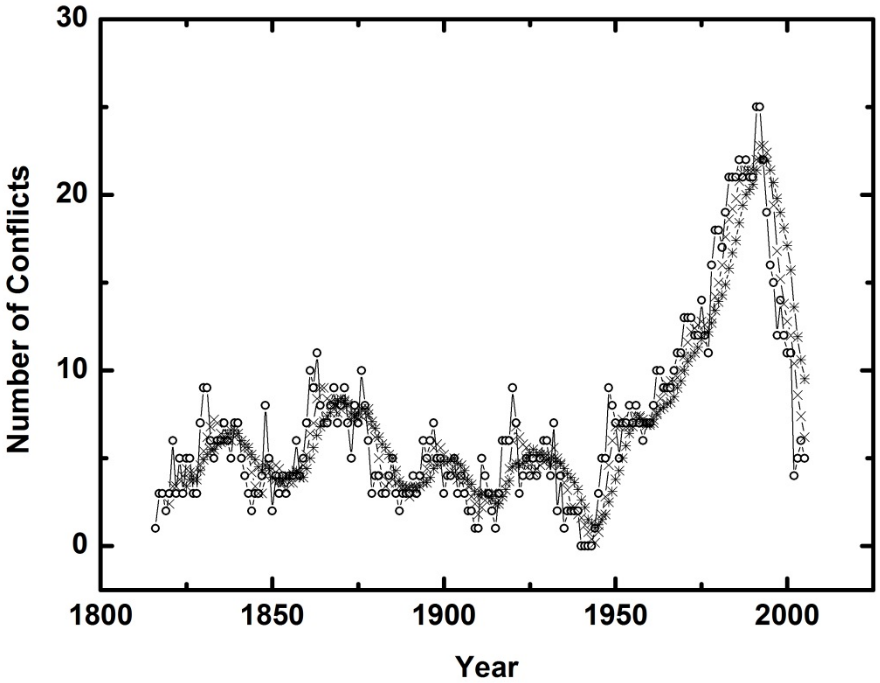
The fine-grained analysis of the residuals of the civil wars trend suggests an oscillatory structure with fundamental mode periods of 27.8 and 30.8 years in ranges 1 and 2 respectively, which is approximately of the order of a human generation (25 – 30 years). This lends some support to the conjecture that the rhythm of population dynamics can have an influence on social, political, and economic factors (see Turchin 2013, Turchin and Korotayev 2006; Turchin and Nefedov 2010). A number of scholars have identified cycles in economic activities, including Juglar (1856), Mitchell (1913, 1927), Kitchin (1923), Kondratiev (1922), Kuznets (1930), and Schumpeter (1939). The 26 years period for cyclical peaks in civil wars resembles the so-called Kuznets (1930) economic cycle period, related to investments in social infrastructure, which Kuznets found to have a characteristic period between 15 to 25 years. Modelski and Thompson (1996) suggest long political cycles of approximately 120 years, based on the leading global powers. Many researchers have tried to relate such cycles to trends in conflict, including early contributions by Dewey (1951), Wright (1965) and Richardson (1960), although most of these analyses have focused on interstate as opposed to civil wars (see Levy 1983; Goldstein 1988). Although it may seem tempting to interpret our findings in light of such cyclical theories, we stress that evolution of civil wars is likely to reflect more than simple economic trends, including the effectiveness and quality of governance, ethnic or religious exclusion, as well as technology and natural resources.

With regards to forecasting the future, the trend analysis and residuals reconstruction shown in Figure 8 predict an increase in civil war between 2020 and 2030. This forecast corresponds to a time span of about 110 to 120 years since the last change in the major powers, following the emergence of the US as the world's leading political and military power by the middle of the 1910s. This is consistent with Modelski and Thompson (1996) long cycle, suggesting that power transitions often are accompanied by severe conflicts. Likewise, our analysis of civil war intensity and the impact of civil wars indicate a clear tendency towards increasingly severity of the hostilities.

However, just as Richardson (1960) famously presented his analysis of arms races as a description of "what people will do if they do not stop to think", forecasts of the future based exclusively on past trends may be overly pessimistic. There are some signs that the number of civil wars have decreased since their peak immediately after the Cold War (Cederman et al. 2016; Goldstein 2002, 2011; Gurr 2000; Human Security Report 2005; Mack 2007; Pinker 2011). Although some have seen the relatively severe civil war in Syria as evidence that the trend towards a decline in conflict may be reversing (see Petterson and Wallensteen 2015), the evidence for a new upturn in the number of civil wars is modest and remains far below the peak at the end of the Cold War. There are a number of global trends including democratization, political reform, and increased economic growth in developing countries that may decrease incentives to use violence as well as possible improvements in conflict management efforts that may have helped decrease the escalation of major conflicts (see Cederman et al. 2016; Gurr 2002; Goldstein 2002, 2011; Gurr 2000; Mack 2007; Pinker 2011). Although the challenge of civil wars to humanity should not be understated, the projected increase in conflict frequency and severity is at least not inevitable.

Figure 1

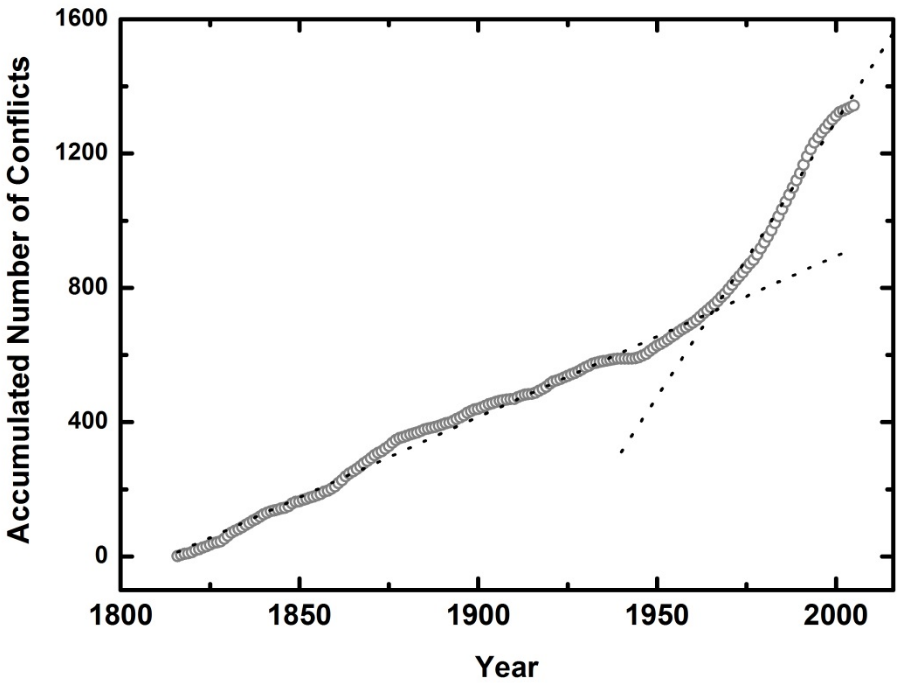
Civil Wars Time Evolution from 1816 to 2005



Note: Frequency of civil wars from 1816 to 2005 represented by the open circles, together with two smoothed versions given by the five (×) and ten (∗) years moving averages of the original data.

Figure 2

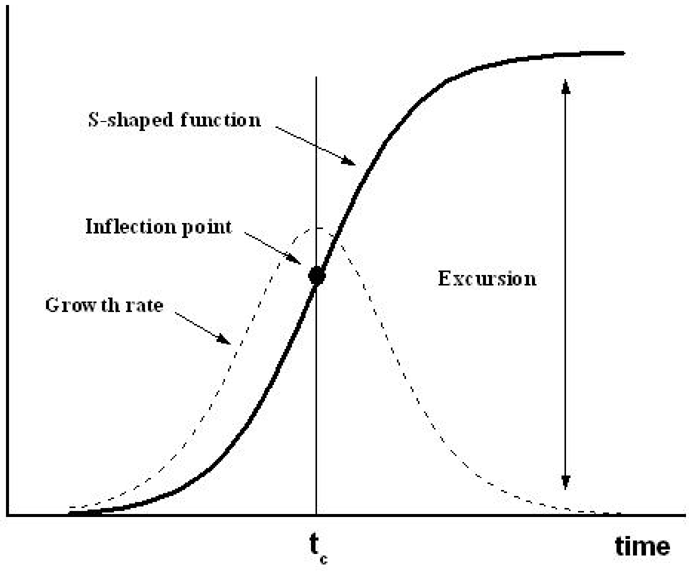
Time Evolution of the Accumulated Number of Conflicts from 1816 to 2005



Note: The evolution of the accumulated number of civil wars is represented by the open circles. We insert two linear regression lines for the periods 1816 to 1960 and 1960 to 2005 to emphasize the sharp transition between the two distinct regimes of the data trend.

Figure 3

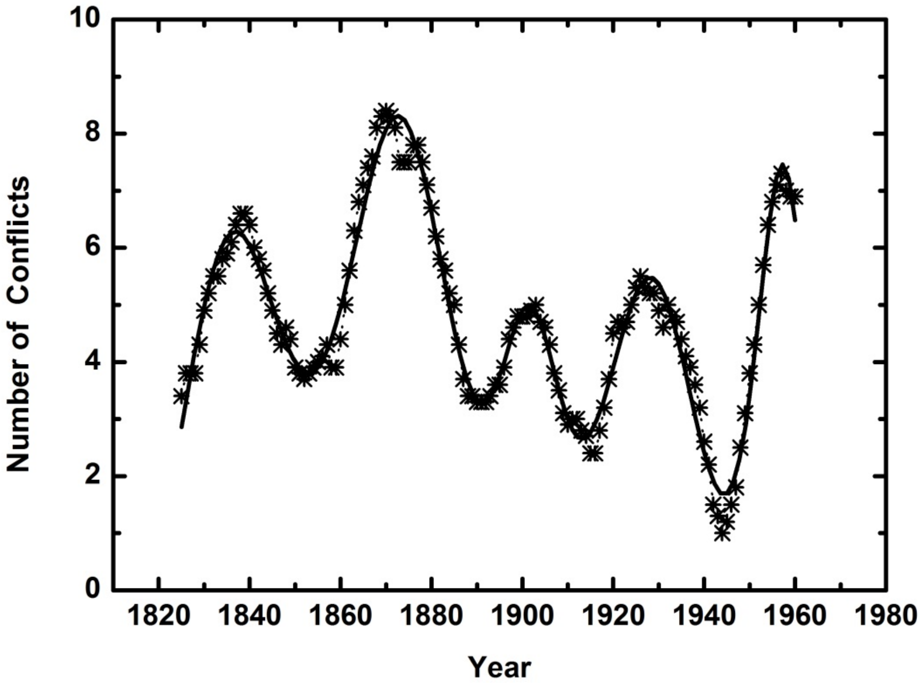
Characteristics of S-shaped Curves Describing Evolutionary Processes



Note: Schematic plot of a typical S-shaped curve, characterized by three parameters, namely, its excursion towards saturation, the point t_c at which the curve inflection occurs, as one goes from the initial growth stage to the final saturation, and a characteristic growth time parameter that indicates how sharp its growth is. The dashed curve is the time derivative of the S-shaped curve, representing its growth rate, and exhibits a characteristic "bell" shape.

Figure 4

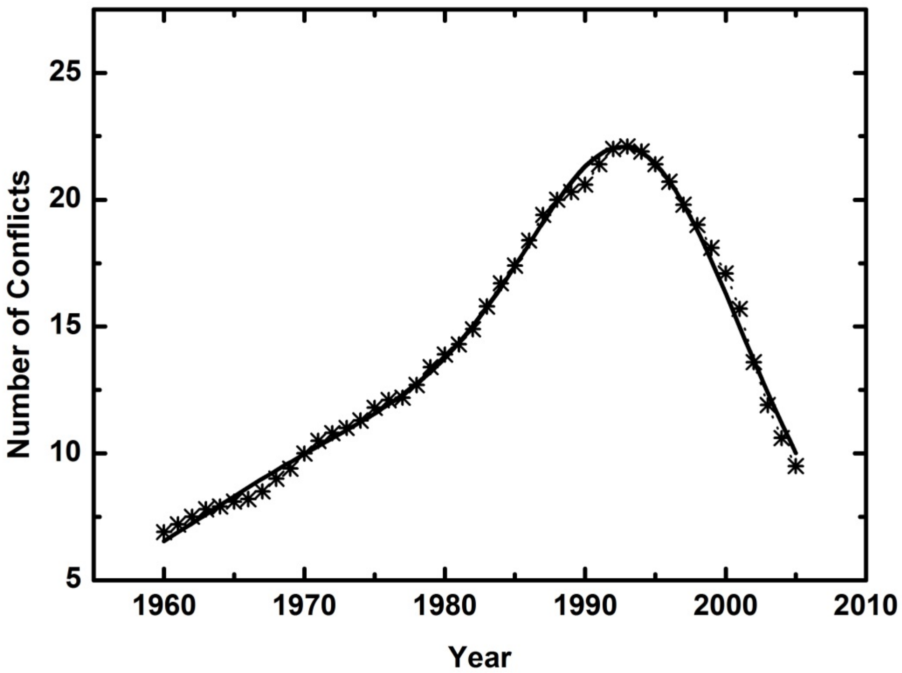
Time Evolution of the Number of Civil Wars between 1816 and 1960



Note: Time evolution of the number of civil wars between 1818 and 1960 (range 1). The solid line represents the results of logistic modeling as described in the text.

Figure 5

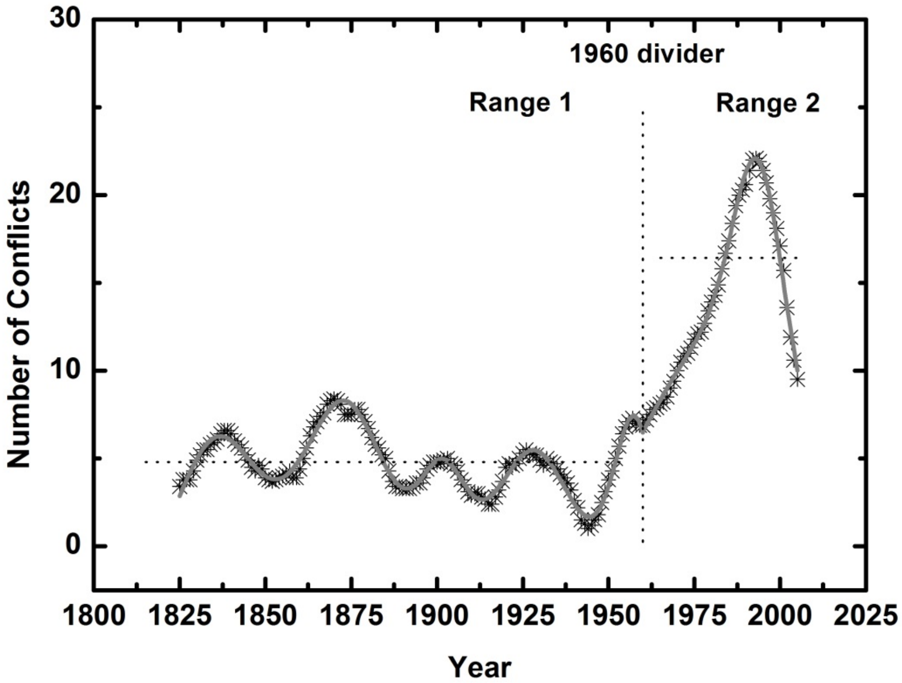
Evolution of the Number of Civil Wars between 1960 and 2005



Note: Evolution of the number of civil wars between 1960 and 2005 (range 2). The solid line represents the results of logistic modeling as described in the text.

Figure 6

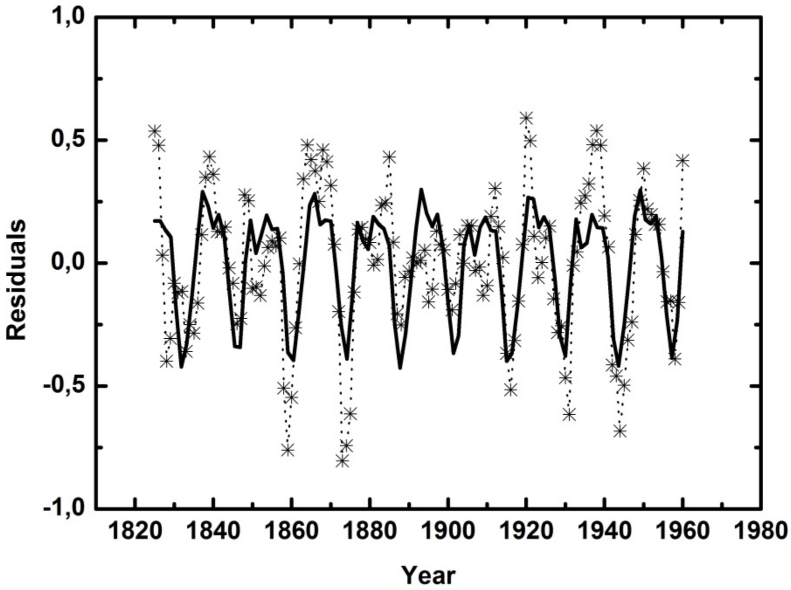
Consolidation of the Model Results for the Number of Conflicts, 1816 to 2005



Note: Combined view of the evolution of the number of civil wars in ranges 1 and 2. The horizontal dotted lines represent the average number of conflicts per year in each range.

Figure 7

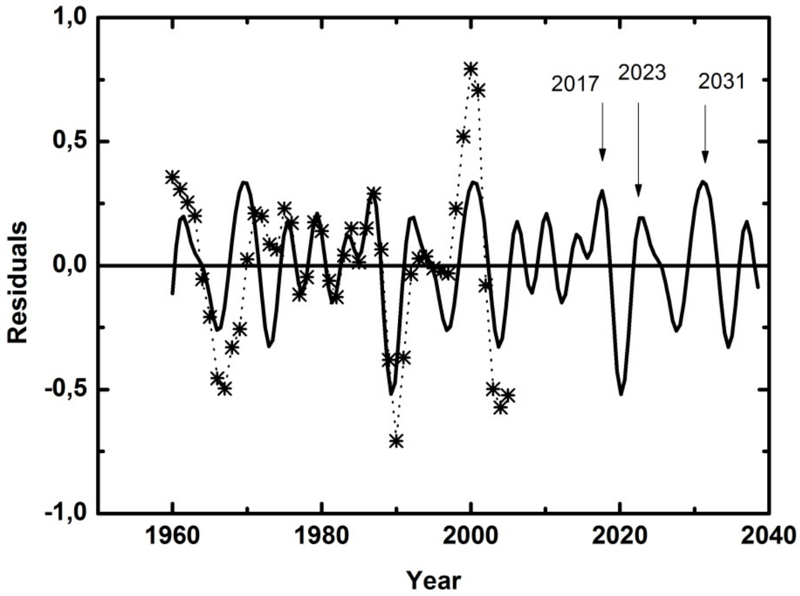
Reconstruction of the Residuals from the Modeling of the Number of Conflicts in Range 1



Note: Residuals from the five logistic pulses modeling of the time evolution of civil wars in range 1. The solid line represents the result of a truncated sine series reconstruction.

Figure 8

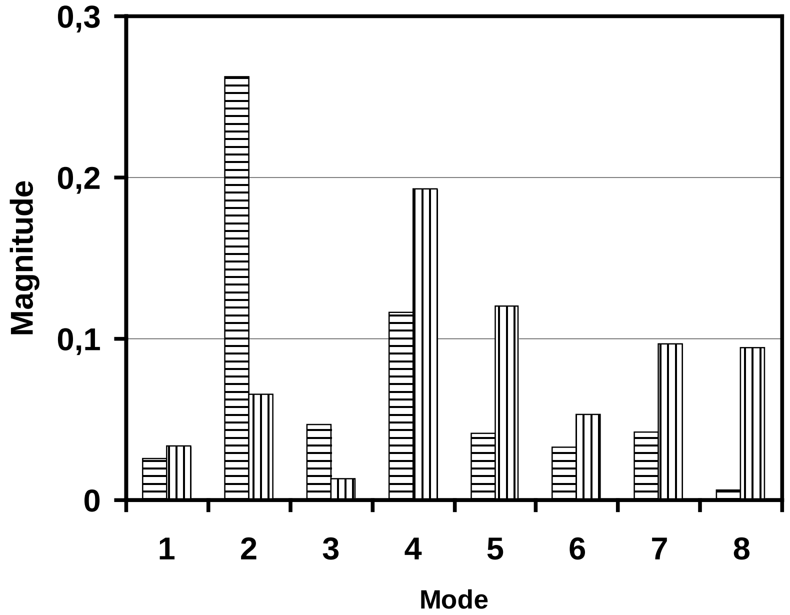
Reconstruction of the Residuals from the Modeling of the Number of Conflicts in Range 2



Note: Residuals from the two logistic pulses modeling of the time evolution of civil wars in range 2. The solid line represents the result of a truncated sine series reconstruction.

Figure 9

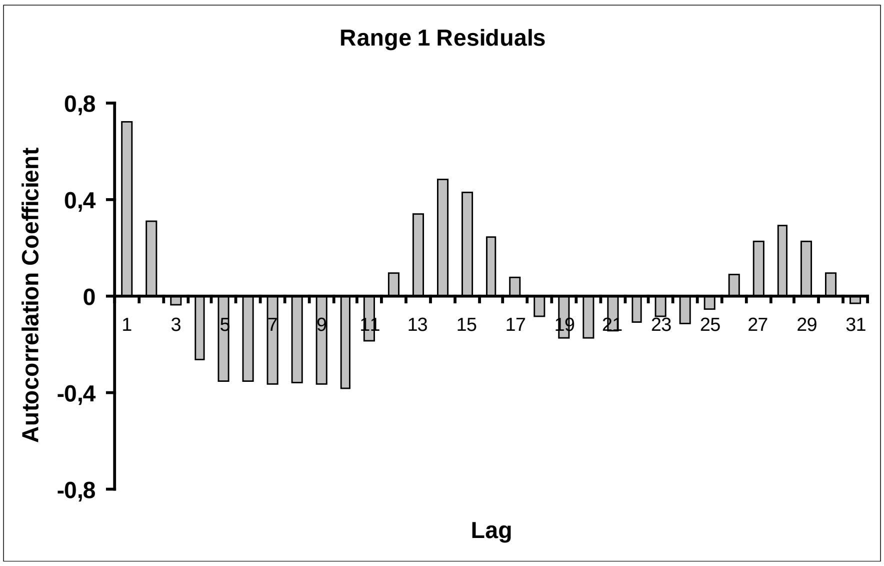
Magnitudes of the Harmonic Modes of the Residuals Reconstruction



Note: Magnitude distribution of the harmonic modes in the truncated sine series reconstruction of the residuals from the logistic modeling of the time evolution of the number of civil wars. The horizontal and vertical dashed columns refer to the residuals reconstruction in ranges 1 and 2, respectively.

Figure 10

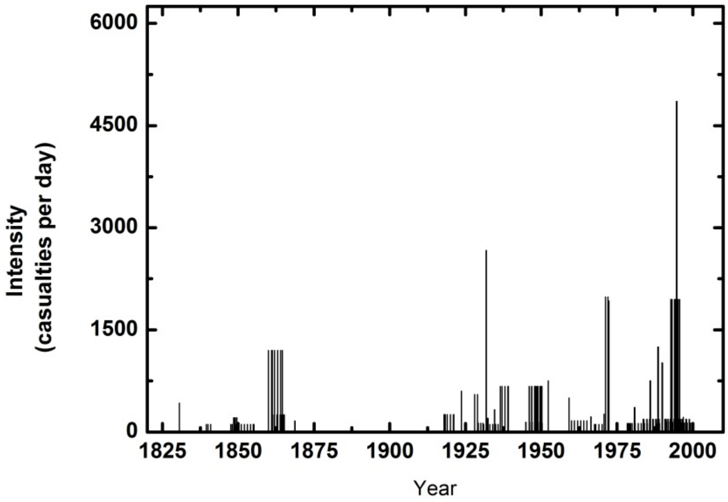
Autocorrelation Coefficients of the Residuals of Range 1 Modeling



Note: Autocorrelation coefficients for the residuals from the logistic modeling of the time evolution of the number of civil wars in range 1 (1818-1960).

Figure 11

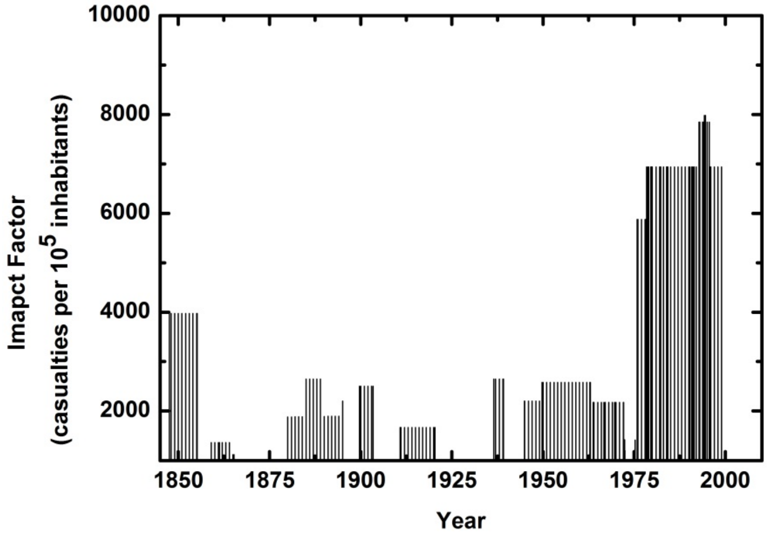
Time Evolution of Civil War Intensity



Note: Time distribution of the major civil wars intensities.

Figure 12

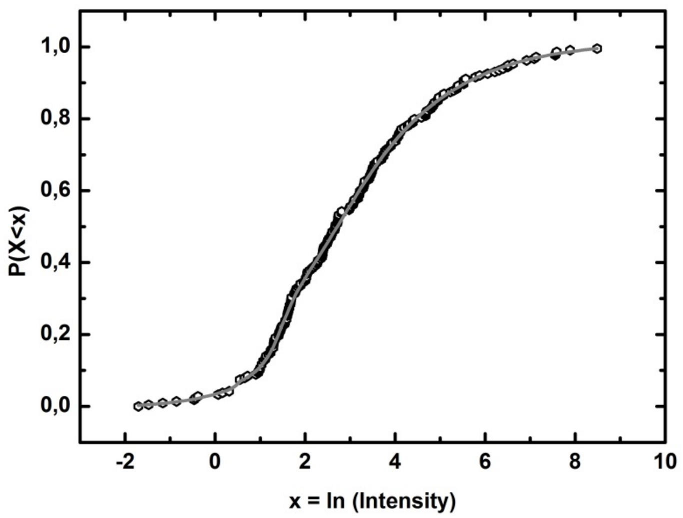
Time Evolution of Civil War Impact



Note: Time distribution of the major civil wars impact parameters.

Figure 13

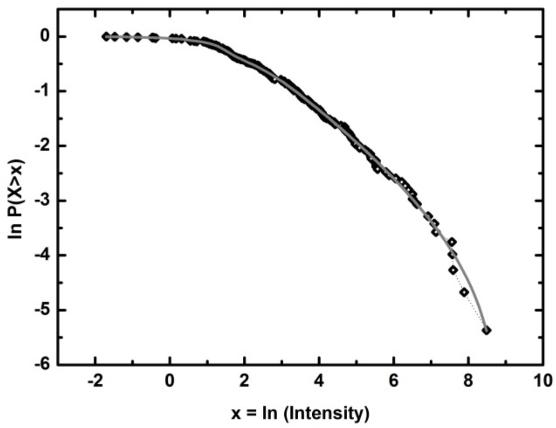
Civil War Cumulative Distribution as a Function of the Intensity



Note: Cumulative distribution of civil wars as a function of their intensities. The solid line represents the data fitting to the logistic model discussed in the text.

Figure 14

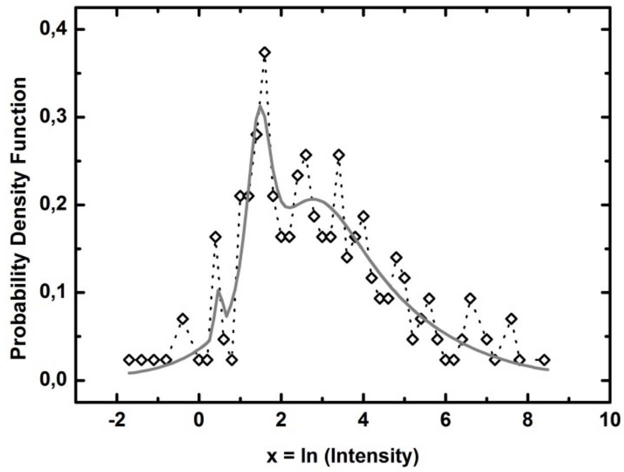
Civil War Complementary Cumulative Distribution as a Function of the Intensity



Note: Complementary cumulative distribution of civil wars as a function of their intensities. The solid line represents the theoretical model function as obtained from the cumulative distribution data fitting.

Figure 15

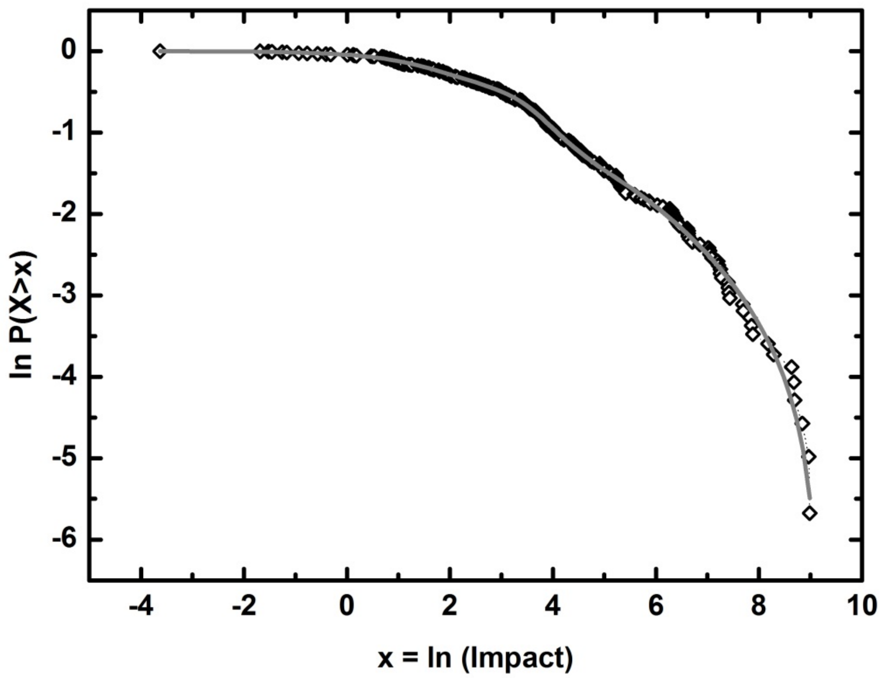
Civil War Probability Density as a Function of the Intensity



Note: Probability density function for the civil wars as a function of the logarithm of their intensities.

Figure 16

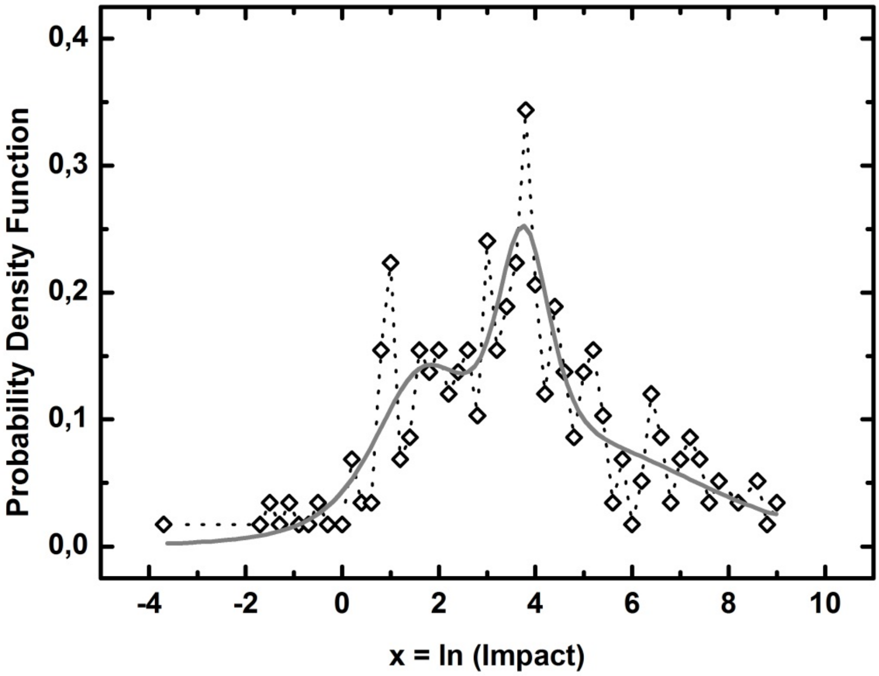
Civil War Complementary Cumulative Distribution as a Function of its Impact Indicator



Note: Complementary cumulative distribution of civil wars as a function of their impacts. The solid line represents the theoretical model function as obtained from the cumulative distribution data fitting.

Figure 17

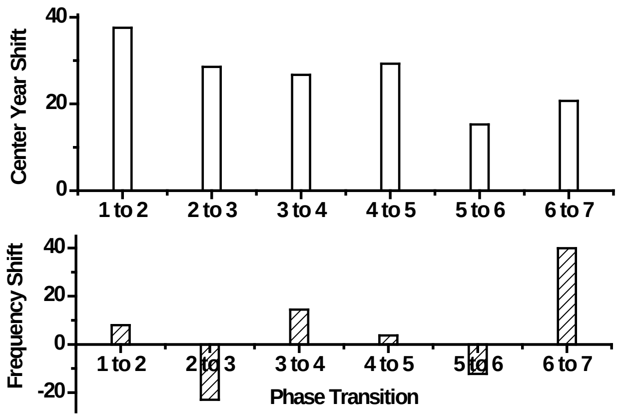
Civil War Probability Density as a Function of the Impact Indicator



Note: Probability density function for the civil wars as a function of the logarithm of their impacts.

Figure 18

Shifts of the Center Years and War Frequencies of the Different Phases of Civil Wars, 1816 to 2005



Note: Time interval between the center years and the war frequency shifts of the sequential civil wars phases as listed in Table 7.

Table 1

Summary of the Logistic Modeling Parameters Shown in Figure 4

Parameters	i = 1	i = 2	i = 3	i = 4	i = 5
A_i	40.51	48.54	25.53	38.97	43.65
t_i	**1835.61**	**1873.17**	**1901.71**	**1928.42**	**1957.71**
τ_i	8.83	8.73	4.99	8.23	8.23
Residual Sum of Squares: 11.801			Coefficient of determination, R^2: 0.964		

Note: Values found for the parameters of the five logistic pulses describing range 1 of the civil war conflicts.

Table 2

Summary of the Logistic Modeling Parameters Shown in Figure 5

Parameters	i = 1	i = 2
A_i	31.39	71.35
t_i	**1972.96**	**1993.64**
τ_i	**9.82**	5.84
Residual Sum of Squares: 4.50	Coefficient of determination, R^2: 0.995	

Note: Values found for the parameters of the two logistic pulses describing range 2 of the civil war conflicts.

Table 3

Summary of the Dominant Mode Characteristics of the Residuals Reconstruction

	Range 1			Range 2		
Fundamental period (yr)	27.8			30.8		
	Mode	Magnitude	Period (yr)	Mode	Magnitude	Period (yr)
Dominant modes	2	0.233	13.9	4	0.193	7.7
	4	0.080	7.0	5	0.0121	6.2
	3	0.063	9.3	7	0.097	4.4

Note: Dominant mode periods as obtained from the truncated sine series reconstruction of the data fitting of the residuals.

Table 4

Summary of the main Characteristics of the Severity Indicators of Major Civil Wars

Geographic Region of Participating Countries	Civil War Severity Indicators	
	Intensity greater than 100	Impact Factor greater than 1%
Europe	13	3
East Asia	9	1
Middle East	5	2
Africa	10	10
North America	1	1
Latin America	5	9
Number of conflicts in 19th century	8 (19%)	4 (15%)
Number of conflicts in 20th century	35 (81%)	22 (85%)
Total number of conflicts	43	26

Note: Main characteristics of the severity indicators for the major civil wars as defined in the text.

Table 5

Top Ten Conflicts as Determined by the Severity Indicators

ID	*Country*	*Starting Year*	*Intensity (deaths per day)*	*ID*	*Country*	*Starting Year*	**Impact (deaths per 10^5 inhabitants)**
2150	Rwanda	1994	4854.37	2150	Rwanda	1994	7982.12
1630	El Salvador	1932	2666.67	2120	Liberia	1992	7841.09
1870	Pakistan	1971	1984.13	1945	Afghanistan	1978	6938.59
2120	Liberia	1992	1948.05	2020	Sudan	1983	5976.74
1885	Burundi	1972	1923.08	1925	Angola	1975	5875.96
2050	Buruni	1988	1250.00	2105	Bosnia	1991	5619.24
1255	China	1860	1196.19	1170	Mexico	1847	3971.41
2060	Romania	1989	1014.00	2135	Burundi	1993	3550.51
1715	Bolivia	1952	750.00	1655	Spain	1936	2645.30
2045	DR Yemen	1986	750.00	1700	Colombia	1949	2578.43

Note: List of the top ten conflicts as determined by the civil war intensity and impact parameters as defined in the text.

Table 6

Summary of the Model Parameters for the Cumulative Distribution Functions

	Intensity indicator (death per day)				Impact indicator (deaths per 10^5 inhabitants)		
	i = 1	i = 2	i = 3	i = 4	i = 1	i = 2	i = 3
A_i	0.001	0.135	0.465	0.410	0.265	0.236	0.545
x_i	0.464	1.462	2.662	3.957	1.644	3.754	4.923
δ_i	0.012	0.175	0.791	1.458	0.635	0.348	1.704
R^2	0.9993				0.9995		

Note: Values of the fitting parameters of the logistic modeling for the civil wars cumulative distribution functions.

Table 7

Civil Wars Phase Characteristics

Phase	Center Year	Conflict Frequency (per year)	Phase Transition	Time Lapse between Center Years (yrs)	Difference in the Conflict Frequency (per year)
1	1835.61	40.51			
2	1873.17	48.54	1 → 2	37.56	8.03
3	1901.71	25.53	2 → 3	28.54	-23.01
4	1928.42	39.97	3 → 4	26.71	14.44
5	1957.71	43.65	4 → 5	29.29	3.68
6	1972.96	31.39	5 → 6	15.25	-12.26
7	1993.64	71.35	6 → 7	20.68	39.96

Note: Main characteristics of the seven identified phases of the trend of the Civil Wars between 1816 and 2005.

Table 8
Civil War Phases and World Economy Phases

Civil War Phases (present study)	World Economy Phases (Maddison, 1991)
1 – 1835,61	
2 – 1873,17 3 – 1901,71	1870–1913 (liberal phase; center year at 1891.5)
4 – 1928,42	1913–1950 (worldwide shock period; center year at 1931.5)
5 – 1957,71 6 – 1972,96	1950–1973 (golden phase; center year at 1961.5)
7 – 1993,64	1973 onwards

Note: Main characteristics of the civil wars phases between 1816 and 2005 as identified in the present work and the world economy phases investigated by Maddison (1991).

APPENDIX: Probability Density as a Function of the Severity Indicators

The civil wars distribution as a function of the severity indicators were constructed as by counting, for each value of the corresponding indicator, the number of conflicts. Since the values of both indicators vary typically four orders of magnitude, our distributions were assembled using a logarithmic binning for both indicators. In the Table below we present a summary of the data used in this study. Table 6 shows, for instance, that beginning with 1 conflict ln (Intensity) = -1.7 ± 0.1 we ended up with 1 war having log (Intensity) = 8.4 ± 0.1 (i.e., between roughly 6,310 and 10,000 battle deaths per day).

ln (Intensity)	Intensity Probability Density Function	ln (Impact)	Impact Probability Density Function a
-1.7 ± 0.1	0.023	-3.7 ± 0.1	0.017
-1.4 ± 0.1	0.023	-1.7 ± 0.1	0.017
-1.1 ± 0.1	0.023	-1.5 ± 0.1	0.034
-0.8 ± 0.1	0.023	-1.3 ± 0.1	0.017
-0.4 ± 0.1	0.070	-1.1 ± 0.1	0.034
0 ± 0.1	0.023	-0.9 ± 0.1	0.017
0.2 ± 0.1	0.023	-0.7 ± 0.1	0.017
0.4 ± 0.1	0.164	-0.5 ± 0.1	0.034
0.6 ± 0.1	0.047	-0.3 ± 0.1	0.017
0.8 ± 0.1	0.023	0 ± 0.1	0.017
1 ± 0.1	0.210	0.2 ± 0.1	0.069
1.2 ± 0.1	0.210	0.4 ± 0.1	0.034
1.4 ± 0.1	0.280	0.6 ± 0.1	0.034
1.6 ± 0.1	0.374	0.8 ± 0.1	0.155
1.8 ± 0.1	0.210	1 ± 0.1	0.223
2 ± 0.1	0.164	1.2 ± 0.1	0.069
2.2 ± 0.1	0.164	1.4 ± 0.1	0.086
2.4 ± 0.1	0.234	1.6 ± 0.1	0.155
2.6 ± 0.1	0.257	1.8 ± 0.1	0.137
2.8 ± 0.1	0.187	2 ± 0.1	0.155

3 ± 0.1	0.164	2.2 ± 0.1	0.120
3.2 ± 0.1	0.164	2.4 ± 0.1	0.137
3.4 ± 0.1	0.257	2.6 ± 0.1	0.155
3.6 ± 0.1	0.140	2.8 ± 0.1	0.103
3.8 ± 0.1	0.164	3 ± 0.1	0.241
4 ± 0.1	0.187	3.2 ± 0.1	0.155
4.2 ± 0.1	0.117	3.4 ± 0.1	0.189
4.4 ± 0.1	0.093	3.6 ± 0.1	0.223
4.6 ± 0.1	0.093	3.8 ± 0.1	0.344
4.8 ± 0.1	0.140	4 ± 0.1	0.206
5 ± 0.1	0.117	4.2 ± 0.1	0.120
5.2 ± 0.1	0.047	4.4 ± 0.1	0.189
5.4 ± 0.1	0.070	4.6 ± 0.1	0.137
5.6 ± 0.1	0.093	4.8 ± 0.1	0.086
5.8 ± 0.1	0.047	5 ± 0.1	0.137
6 ± 0.1	0.023	5.2 ± 0.1	0.155
6.2 ± 0.1	0.023	5.4 ± 0.1	0.103
6.4 ± 0.1	0.047	5.6 ± 0.1	0.034
6.6 ± 0.1	0.093	5.8 ± 0.1	0.069
7 ± 0.1	0.047	6 ± 0.1	0.017
7.2 ± 0.1	0.023	6.2 ± 0.1	0.052
7.6 ± 0.1	0.070	6.4 ± 0.1	0.120
7.8 ± 0.1	0.023	6.6 ± 0.1	0.086
8.4 ± 0.1	0.023	6.8 ± 0.1	0.034
		7 ± 0.1	0.069
		7.2 ± 0.1	0.086
		7.4 ± 0.1	0.069
		7.6 ± 0.1	0.034
		7.8 ± 0.1	0.052
		8.2 ± 0.1	0.034
		8.6 ± 0.1	0.052
		8.8 ± 0.1	0.017
		9 ± 0.1	0.034

References

Box, George and Gwilym Jenkins. 1970. *Time series analysis: Forecasting and control*. San Francisco, CA: Holden-Day.

Cederman, Lars-Erik. 2003. Modeling the size of wars: From billiard balls to sandpiles. *American Political Science Review* 97 (1): 135-150.

Cederman, Lars-Erik; Kristian Skrede Gleditsch and Julian Wucherpfennig. 2016. Predicting the Decline of Ethnic Civil War:\\ Was Gurr Right and For the Right Reasons? *Journal of Peace Research*, forthcoming.

Cirillo, Pasquale and Nasim Taleb. 2015. On the Statistical properties and Tail Risk of Violent Conflicts. *Physica A: Statistical Mechanics and its Applications*, forthcoming.

Clauset, Aaron, Michael Young, and Kristian Skrede Gleditsch. 2007. On the frequency of severe terrorist events. *Journal of Conflict Resolution* 51 (1): 58-87.

Dewey, Edward R. 1951. The 57-year cycle in international conflicts. *Cycles* 2: 2–4.

Gleditsch, Kristian S. 2007. *Expanded war data*, version 1.52. Available at http://privatewww.essex.ac.uk/~ksg/expwar.html.

Gleditsch, Kristian S. 2004. A revised list of wars between and within independent states, 1816-2002. *International Interactions* 30 (3)**:** 231-262.

Goldstein, Joshua. 1998. *Long cycles: War and prosperity in the modern age*. New Haven, CT: Yale University Press.

Goldstein, Joshua S. 2002. The worldwide lull in war. *Christian Science Monitor*, 14 May 2002, p. 9.

Goldstein, Joshua S. 2002. The worldwide lull in war. *Christian Science Monitor*, 14 May 2002, p. 9.

Goldstein, Joshua S. 2011. *Winning the War on War.* Hialeah, FI: Dutton/Penguin.

Gurr, Ted Robert. 2000. Ethnic warfare on the wane. *Foreign Affairs* 79 (3): 52–64.

Hechter, Michael. 2001. *Containing Nationalism.* Oxford: Oxford University Press.

Human Security Report. 2005. *Human security report 2005: War and peace in the 21st century.* Oxford: Oxford University Press.

Juglar, Clement. 1863. *Des crises commerciales et de leur retour periodique en France, en Angleterre et aux Etats Unis*. Strasbourg: Vve. Gerger-Levrault Press. Also available at http://gallica.bnf.fr/ark:/12148/bpt6k83714z.

Kitchin, Joseph. 1923. Cycles and trends in economic factors. *Review of Economics and Statistics* 5 (1): 10–16.

Kondratiev, Nikolai D. 1922. *The world economy and its conjunctures during and after the war* (in Russian); this paper was later expanded and published in German in 1926 as Langen Wellen der Konjunktur, *Archiv fur Sozialwissenschaften und Sozialpolitik* **56**: 573–609.

Kuznets, Simon S. 1930. *Secular movements in production and prices*. Boston, MA: Houghton Mifflin.

Levy, J.S. 1983. *War in the modern great power system. 1495-1975*. Lexington, KY: University Press of Kentucky.

Mack, Andrew. 2007. Global political violence: Explaining the post-Cold War decline. In *The state of Political Science in Western Europe*, edited by Hans-Dieter Klingemann. Leverkusen: Barbara Budrich.

Maddison, Angus. 1991. *Dynamic forces in capitalist development*. Oxford: Oxford University Press.

Maddison, Angus. 2003. *The world economy: Historical statistics*. Paris: OECD Publishing. The GDP and population data over the 1-2003 AD period for the different countries can be accessed at the website http://www.ggdc.net/maddison/ (last accessed Dec 2008).

Miranda, Luiz C.M. and Carlos A.S. Lima. 2010. A new methodology for the logistic analysis of evolutionary S-shaped processes: Application to historical time series and forecasting. *Technological Forecasting and Social Change* 77 (2): 175-192.

Miranda, Luiz C.M. 2010. A note on the periodicities in the logistic analyses of growth processes. *Technological Forecasting and Social Change* 77 (5): 823-830.

Mitchell, Wesley C. 1913. *Business cycles*. Berkeley, CA: University of California Press.

Mitchell, Wesley C. 1927. *Business cycles: The problem and its setting*. New York: National Bureau of Economic Research.

Modelski, George and William. R. Thompson. 1988. *Seapower in global politics, 1494–1993*. Seattle, WA: University of Washington Press.

Modelski, George and William R. Thompson. 1996. *Leading sectors and world powers: The Co-evolution of global economics and politics*. Columbia, SC: University of South Carolina Press.

Mueller, John. 1994. The catastrophe quota: Trouble after the Cold War. *Journal of Conflict Resolution* 38 (3): 355-375.

Pearl, Raymond and Lowell J. Reed. 1920. On the rate of growth of the population of the United States since 1790 and its mathematical representation. *Proceedings of the National Academy of Sciences USA* 6 (6): 275-288.

Pettersson, Therése, and Peter Wallensteen. 2015. Armed conflicts, 1946–2014. *Journal of Peace Research* 52 (4):536-50

Pinker, Steven. 2011. *The Better Angels of Our Nature: Why Violence has Declined.* New York: Viking.

Richardson, Lewis, F. 1935. Mathematical psychology of war. *Nature* 135 (3420): 830–831.

Richardson, Lewis. F. 1941. Frequency of occurrence of wars and other fatal quarrels. *Nature* 148 (3759): 598.

Richardson, Lewis F. 1960. *Statistics of deadly quarrels*. Pacific Grove, CA: Boxwood Press.

Sambanis, Nicholas. 2002. A review of recent advances and future directions in the literature on civil wars. *Defense and Peace Economics* 13 (2): 215-243.

Schumpeter, Joseph A. 1939. *Business cycles: A theoretical, historical, and statistical analysis of the capitalist process*. New York: McGraw-Hill.

Turchin, Peter. 2013. Modeling social pressures toward political instability. *Cliodynamics* 4: 241–280.

Turchin, Peter and Andrey Korotayev. 2006. Population dynamics and internal warfare: A reconsideration. *Social Science History* 5: 121–158.

Turchin, Peter and Sergey A. Nefedov. 2009. *Secular cycles*. Princeton, NJ: Princeton University Press.

Verhulst, Pierre F. 1845. Recherces mathematiques sur la loi d'accroissement de la population. *Nouvelle Memoires de la Academie Royale des Sciences et Belles-Lettres de Bruxelles* 18**:** 1-41.

Wimmer, Andreas. 2013. *Waves of War: Nationalism, State Formation, and Ethnic Exclusion in the Modern World*. Cambridge: Cambridge University Press.

Wright, Quincy. 1965. *A Study of War*. Chicago, IL: University of Chicago Press.