No One Likes to Be Second Choice*

Yair Antler

University of Essex

September 2016

Abstract

A decision maker is interested in appointing one individual from a group of candidates to a public position with an exogenous wage. He must decide in what order to approach them. Candidates who are more valuable to the decision maker are less likely to be available. The candidates' preferences have a social component: each candidate finds the position more attractive if he is highly ranked by the decision maker (relative to the other candidates). However, the decision maker's preferences are his private information. As a result, candidates infer the decision maker's evaluation of them based on the number of candidates who have previously turned him down. The main result is that when the number of candidates is sufficiently large, all of the candidates reject the decision maker's offers, including those whom he most values. Moreover, the decision maker's payoff is not monotonic in the number of candidates.

*This paper is based on the second chapter of my Ph.D. dissertation. I acknowledge financial support from ERC Grant no. 230251. I am grateful to Rani Spiegler for his valuable guidance. I thank Benjamin Bachi, Kfir Eliaz, Alex Frug, Tobias Gamp, Jacob Glazer, Ariel Rubinstein, Heidi Thysen, and Asher Wolinsky for helpful comments. *Correspondence:* Yair Antler, University of Essex. *E-mail:* yair.an@gmail.com.

1 Introduction

There is a well-known scene in the movie "A Beautiful Mind" in which John Nash and his fellow students discuss how to approach a group of women in a bar: "If we all go for the blonde and block each other, not a single one of us is going to get her. So then we go for her friends, but they will all give us the cold shoulder because no one likes to be second choice. But what if none of us goes for the blonde? We won't get in each other's way and we won't insult the other girls. It is the only way to win."

We analyze a similar problem in which a single decision maker (hereafter, DM) wishes to appoint one member of a group of candidates to a public position with an exogenous wage (say, the chairman of a public committee). The innovation of the model lies in the fact that candidates' preferences include a social component: each candidate finds the position more lucrative if he is one of the DM's *k-most-valued* candidates. In contrast to the implicit assumption in "A Beautiful Mind," we assume that the candidates understand the DM's strategy such that John Nash's strategy may not be consistent with Nash equilibrium in our model.

In the model, the DM decides on the order in which to approach the candidates. The candidates observe the timing of the offers (each candidate knows how many candidates have already received offers) but they do not know the DM's preferences. Each of the candidates may or may not be available and this is his private information. The DM's value in appointing any particular candidate and the candidate's availability are negatively correlated (better candidates are less likely to be available). In a conventional setup (in which candidates do not have social preferences), the best strategy for the DM is to approach the candidates in order of value (i.e., the most-valued candidate first, the second-most-valued candidate second, and so on) regardless of their availability. However, in the case where candidates might reject his offers if they believe they have a low ranking, this strategy may not be consistent with a Nash equilibrium.

To illustrate, we use an example in the spirit of the discussion in "A Beautiful Mind." Assume that each of the women rejects John if she has a boyfriend or if she believes that she is not his first choice. Suppose that the women believe that John will approach his first choice first, and only if she rejects him, he will approach the other women. In that case, John can effectively ask only one of the women out since, if he is rejected by her, all of the other women will give him the cold shoulder as well. Therefore, when he decides on the first woman to approach, John takes into account the women's availability. If he believes that the woman whom he finds most attractive is unlikely to be available, he might give up on the idea of approaching his first choice in favor of approaching one of the other women. Note that in this example, the women's social preferences create an endogenous cost of delay for John. If he chooses to pursue his top choice, then he loses the opportunity to succeed with one of the other women.

In this model, it may be that all of the candidates reject the DM's offers in a Nash equilibrium even if they are available. We present a tight sufficient condition (on the number of candidates) which ensures the existence of such a Nash equilibrium. Under the assumption that the product of a candidate's availability and his value is decreasing in value, if the number of candidates is large, all of the DM's offers are rejected in each symmetric Nash equilibrium. Thus, increasing the number of candidates may worsen the DM's highest equilibrium payoff.

The model can be applied to numerous economic situations in which prestige plays a significant role. For example, an expert may be willing to serve as the chair of a public committee if he is the president's top choice, but may refuse to do so if he knows that he is a fallback option. In a different context, a student who is put on a waiting list may infer that he is ranked relatively low by a university and may prefer to attend a university that wants him more even if he is eventually admitted to the first one.

The paper proceeds as follows. Section 2 presents the model. Section 3 presents an illustrative example and Section 4 presents the general analysis. Section 5 compares the cases of private and public offers. Section 6 concludes and discusses the model in the context of the literature.

2 The Model

Let $I = \{1, ..., n\}$ be a set of candidates and suppose that a DM wishes to appoint one and only one of these candidates to a position. Let v_i denote the DM's value from appointing candidate $i \in I$ to the position. We assume that $v_1, ..., v_n$ are n random variables drawn independently from a density $f > 0$ on $[v_l, v_h]$, $v_h > v_l$. The values' realizations are the DM's private information while f is commonly known. For each candidate $i \in I$, $x_i = |\{j \in I / \{i\} | v_j > v_i\}|$ is the number of candidates who have a higher value than i. Let $k \in \{1, ..., n-1\}$. The utility that candidate $i \in I$ derives from accepting the DM's offer is 1 if $x_i < k$ and $1 - c < 0$ otherwise.[1] The utility derived by each $i \in I$ from rejecting the DM's offer is 0. Define $\phi := \frac{c-1}{c}$. Observe that a candidate who believes that he is one of the k-most-valued candidates with probability ϕ is indifferent between accepting and rejecting the DM's offer.

[1] One can interpret c as the intensity of the social preferences component.

Let $s_i \in \{0,1\}$ denote candidate i's availability, where $s_i = 1$ ($s_i = 0$) means that i is available (unavailable). Define $p(v_i) := Pr(s_i = 1|v_i)$. We assume that $p : [v_l, v_h] \to [0,1]$ is continuous and weakly decreasing in v. That is, there is a negative correlation between each candidate's value to the DM and the probability that the candidate is likely to be available. Each candidate's availability is his private information while p is commonly known. One can interpret s_i as agent i's outside option, such that unavailable (available) candidates are simply candidates with a good (bad) outside option.[2]

Denote the set of rounds by $N = \{1, ..., n\}$. In each round $t \in N$, the DM makes a binding offer to one candidate. If that candidate accepts, then he fills the position and the game ends. If he rejects the offer, then at round $t+1$ the DM makes an offer to a candidate who has not rejected an offer previously. If by the end of round n the DM's offers have all been rejected, the game ends and his payoff is 0. Denote the set of permutations of N by Π_N. A pure strategy for the DM is a permutation of N for each realization $(v_1, ..., v_n)$. In other words, a pure strategy $a : [v_l, v_h]^n \to \Pi_N$ maps realizations of candidates' values into permutations of the n rounds. A mixed strategy $\alpha : [v_l, v_h]^n \to \Delta(\Pi_N)$ maps realizations of candidates' values into probability distributions over permutations of N. For each realization $(v_1, ..., v_n)$, let $\alpha(\pi|v_1, ..., v_n)$ denote the probability that the mixed strategy α maps the realization $(v_1, ..., v_n)$ to the permutation π.

We assume that a candidate who is unavailable rejects the DM's offer. For each available candidate $i \in I$, a behavioral strategy $\sigma_i : N \to [0,1]$ is the probability of accepting an offer conditional on the round in which it is made. We say that the DM's strategy a treats the candidates in a *symmetric* manner if it is not conditioned on the candidates' labels. Formally, a treats the candidates in a symmetric manner if whenever $v_i \neq v_j$ for each $v_i, v_j \in (v_1, ..., v_n)$, and $(v_{\pi_1}, ..., v_{\pi_n})$ is a permutation of $(v_1, ..., v_n)$ such that $\pi_i = j$, then a maps candidate j to round h under $(v_1, ..., v_n)$ if and only if it maps candidate i to round h under $(v_{\pi_1}, ..., v_{\pi_n})$. Mixed strategies that treat the candidates in a symmetric manner are defined analogously.

We restrict our attention to Nash equilibria in which the DM's strategy treats the candidates in a symmetric manner and the candidates use symmetric strategies. We refer to this solution concept as a symmetric Nash equilibrium (SNE). Since the candidates' strategies are symmetric we omit the subscript from the description of their

[2] All of the results presented in the paper continue to hold if we assume that candidate i derives utility of $1 - s_i$ from rejecting the DM's offer and that the utility derived by candidate i from accepting the DM's offer is 0.5 ($0.5 - c < 0$) if he is (not) one of the k-most-valued candidates.

strategies and use σ^j to denote the probability that an available candidate who receives an offer in the jth round accepts it. Restricting our attention to equilibria in which the DM's strategy treats the candidates in a symmetric manner enables us to obtain full support and thereby avoid making assumptions about the candidates' beliefs off the equilibrium path. To see this, observe that the candidates' values are i.i.d. so there is a strictly positive probability for each permutation of the DM's offers.

At this point, we find it useful to discuss the model's key assumptions. First, we assume that the candidates find the position to be attractive only if they are highly ranked by the DM. We find it plausible that in situations in which the prestige of a position has a significant effect on its attractiveness, the value that is derived by a candidate from filling it is affected by his relative ranking among the candidates. In particular, it is plausible that this value depends on whether a candidate is the top choice for the position or a fallback option. Examples for such situations are chairing a public committee and holding an honorary/volunteer position. The aforementioned assumption seems plausible in the context of romantic relationships where egos play a significant role. In that context, it is supported by evidence from the social psychology literature (see, e.g., Curtis and Kim, 1986, and Condon and Crano, 1988). The second key assumption is that $p(v)$ is weakly decreasing. Namely, the more the candidate is valued, the less likely he is to be available. This assumption can be justified by thinking of the value derived by the DM as containing a component that is common both to the position that he wishes to fill and to other positions that candidates may hold (candidates who hold such positions are unavailable).

Before proceeding to the analysis, we present a simple result that will serve us as a benchmark. It establishes that in the absence of the social preferences component ($c = 0$) in the unique SNE, the DM approaches the candidates in the order of their values (i.e., he approaches his most-valued candidate first, his second-most-valued candidate second, and so on).

Claim 1 *Suppose that $c = 0$. Then, in the unique SNE, the DM approaches the candidates in the order of their value.*

Proof. Since $c = 0$, each available candidate accepts an offer regardless of the round in which it is made. Therefore, approaching the candidates in the order of their value first-order stochastically dominates any other strategy. ■

This result follows directly from the fact that there is no cost for delay because available candidates always accept the DM's offers. That is, there is no cost in making offers to candidates who are less likely to be available. However, once candidates have social preferences, their likelihood of being available plays an important role in the DM's strategic considerations. For example, consider an extreme case in which candidates who do not receive an offer in the first round believe that they were ranked low, and therefore reject the DM's subsequent offers. In this case, the DM has one effective offer to make (in the first round). In this case, he will make the first offer to the candidate who maximizes $vp(v)$. This candidate need not be the most-valued one.

3 An Illustrative Example

In order to demonstrate the trade-offs that emerge in the model we provide the following example. We assume that $n = 2$, $k = 1$, $p(v) = 1 - v$, and that the candidates' values are drawn from the uniform distribution F on $[0.5, 1]$. First, we show that the strategy of always making the first offer to the highly valued candidate is not a part of a SNE.

Claim 2 *The strategy of making the first offer to the highly valued candidate is not a part of a SNE.*

Proof. Assume by negation that there exists a SNE in which the DM always makes the first offer to the candidate whom he values most. Then, the candidate who receives the second (first) offer infers that he is the candidate who is least (most) valued by the DM. It follows that $\sigma^1 = 1$ and $\sigma^2 = 0$ in this SNE. Since there is only one round in which the DM's offers are accepted, he prefers to make the first offer to the candidate for whom $vp(v)$ is maximized. Since $v(1 - v)$ is decreasing in $[0.5, 1]$, we get a contradiction. This is because the DM's best response is to make the first offer to the candidate whom he values least. ■

The next corollary follows from Claim 2.

Corollary 1 *There exists no SNE in which $\sigma^i = 0$ and $\sigma^{j \neq i} > 0$.*

Proof. When the DM has only one effective round in which he can recruit a candidate (i.e., there is only one round in which his offers are accepted with a strictly positive probability), he prefers to make an offer in that round to the candidate for whom $vp(v)$ is maximized. Since $vp(v)$ is strictly decreasing in v, the DM makes that offer to the candidate whom he values less. ■

The next claim shows that it cannot be the case that in a SNE $\sigma^2 \geq \sigma^1 > 0$. Namely, if the DM's offers are accepted (with strictly positive probability) in both rounds, it must be that, conditional on the recipient being available, the DM's offer is more likely to be accepted in the first round.

Claim 3 *There exists no SNE such that $\sigma^2 \geq \sigma^1 > 0$.*

Proof. Suppose that $\sigma^2 \geq \sigma^1 > 0$ is a part of a SNE. Consider an arbitrary realization (v_i, v_j) such that $v_i > v_j$. The DM prefers to make the first offer to candidate i if and only if

$$\left(\sigma^1 - \sigma^2\right)\left(v_i p\left(v_i\right) - v_j p\left(v_j\right)\right) + \sigma^1 \sigma^2 p\left(v_i\right) p\left(v_j\right)\left(v_i - v_j\right) > 0 \tag{1}$$

Since $v\left(1 - v\right)$ is decreasing in v, $\left(\sigma^1 - \sigma^2\right)\left(v_i p\left(v_i\right) - v_j p\left(v_j\right)\right) \geq 0$. It follows that inequality (1) holds for each realization (v_i, v_j) such that $v_i > v_j$. Therefore, the DM always makes the first offer to his preferred candidate. It follows that $\sigma^2 > 0$ cannot be a part of a SNE. ■

Claim 3 shows that conditional on the recipient being available, the DM's offer is more likely to be accepted in the first round. The next claim demonstrates that there may exist SNEs in which the DM's offers are rejected in both rounds, regardless of the candidates' availability.

Claim 4 *Let $\phi > \frac{1}{3}$. There exists a SNE in which the DM randomizes uniformly between permutations and the candidates reject both of his offers (i.e., $\sigma^1 = \sigma^2 = 0$).*

Proof. Given the candidates' strategies, the DM is indifferent between both permutations. Therefore, it is left to check the candidates' strategies. Consider an arbitrary available candidate i ($s_i = 1$). His interim belief about v_i is

$$f\left(v_i | s_i = 1\right) = \frac{2\left(1 - v\right)}{\int_{0.5}^{1} 2\left(1 - v\right) dv} = 8\left(1 - v\right) \tag{2}$$

It follows that

$$Pr\left(v_i > v_{j \neq i} | s_i = 1\right) = \int_{0.5}^{1} 8\left(1 - v\right) 2\left(v - 0.5\right) dv = \frac{1}{3} < \phi \tag{3}$$

Since $\sigma^1 = 0$ and the DM mixes uniformly between the two permutations, candidate i does not receive any additional information. It follows that rejecting the DM's offers is a best response in both rounds. ■

In the SNE that is described in Claim 4, the DM's strategy does not convey any information to the candidates. Since $\sigma^1 = 0$, the candidate who is approached second does not receive any information from the fact that the first offer was rejected. The candidates reject the DM's offers since each available candidate's interim belief is that he is top ranked with probability $\frac{1}{3} < \phi$. The next claim establishes that if ϕ is high, then all of the DM's offers are rejected in each SNE.

Claim 5 *There exists a $\phi^{\star}$ such that for each $\phi > \phi^{\star}$, in each SNE $\sigma^1 = \sigma^2 = 0$.*

Proof. Consider a SNE such that $\sigma^1, \sigma^2 > 0$, i.e., a SNE in which the DM's offers are accepted with strictly positive probability in both rounds. In such a SNE, each candidate must believe that he is the most valued one with probability greater than ϕ. Let us write these probabilities explicitly. Given that he is available, the candidate who receives the first offer believes he is the most valued one with probability

$$\frac{\int_{0.5}^{1}\int_{0.5}^{1}\sum_{\pi}\alpha\left(\pi|v_1,v_2\right)\mathbb{1}\left(\pi,v_1,v_2\right)4\left(1-v_{\pi_1(v_1,v_2)}\right)dv_1dv_2}{\int_{0.5}^{1}\int_{0.5}^{1}\sum_{\pi}\alpha\left(\pi|v_1,v_2\right)4\left(1-v_{\pi_1(v_1,v_2)}\right)dv_1dv_2} \tag{4}$$

where $\mathbb{1}\left(\pi,v_1,v_2\right)$ is an indicator that equals 1 if and only if $v_{\pi_1(v_1,v_2)} > v_{\pi_2(v_1,v_2)}$. That is, $\mathbb{1}\left(\pi,v_1,v_2\right) = 1$ if and only if the value of the candidate who receives the first offer is higher than the value of the candidate who receives the second offer. Given that he is available, the candidate who receives the second offer believes he is the most valued one with probability

$$\frac{\int_{0.5}^{1}\int_{0.5}^{1}\sum_{\pi}\alpha\left(\pi|v_1,v_2\right)\left(1-\mathbb{1}\left(\pi,v_1,v_2\right)\right)4\left(1-v_{\pi_2(v_1,v_2)}\right)\left(1-\sigma^1\left(1-v_{\pi_1(v_1,v_2)}\right)\right)dv_1dv_2}{\int_{0.5}^{1}\int_{0.5}^{1}\sum_{\pi}\alpha\left(\pi|v_1,v_2\right)4\left(1-v_{\pi_2(v_1,v_2)}\right)\left(1-\sigma^1\left(1-v_{\pi_1(v_1,v_2)}\right)\right)dv_1dv_2} \tag{5}$$

Since $\mathbb{1}\left(\pi,v_1,v_2\right)$ appears in the numerator of expression (4) and $1-\mathbb{1}\left(\pi,v_1,v_2\right)$ appears in the numerator of expression (5) there must be a $\phi^{\star}$ such that for each $\phi > \phi^{\star}$, either the candidate who receives the first offer believes that he is the most valued one with probability lower than ϕ or the candidate who receives the second offer believes that he is the most valued one with probability lower than ϕ. By Corollary 1, there exists no SNE in which $\sigma^i = 0$ and $\sigma^{j\neq i} > 0$. It follows that for a sufficiently large ϕ, $\sigma^1 = \sigma^2 = 0$ in each SNE. ■

In order for accepting an offer to be a best response for a particular candidate, he must believe that he is likely to be the most-valued candidate. Since $\mathbb{1}\left(\pi,v_1,v_2\right)$ appears in the numerator of expression (4) and $1-\mathbb{1}\left(\pi,v_1,v_2\right)$ appears in the numerator of expression (5), any positive signal about the ranking of a candidate who receives an offer in round t necessarily sends a negative signal about the ranking of the candidate who receives an offer in round $\tau \neq t$. In this example, one can choose $\phi^{\star} = 0.333$.

At this point, it is worth mentioning that there exist other (non-symmetric) Nash equilibria in this example. For example, suppose that $\phi = \frac{1}{3}$. Consider a profile of strategies in which the DM makes an offer to candidate 1 (2) first (second), candidate 1 rejects offers in both rounds, while candidate 2 accepts an offer only in the second round. Assume that the candidates' beliefs off the equilibrium path are such that candidate 1 (2) believes that he is the DM's second choice with probability 1 if he gets an unexpected offer in round 2 (1). Any deviation by the DM induces a payoff of 0 since the candidates' beliefs off the equilibrium path are such that they reject all of his offers. Under this profile of strategies, no information is conveyed to the candidates, and, therefore, they are indifferent between accepting and rejecting an offer. However, our anonymity assumption excludes equilibria of this kind.

4 Analysis

In this section, we present the main results of the paper, which focus on SNEs in which all of the DM's offers are rejected. The first proposition provides a tight sufficient condition for the existence of such a SNE.

Proposition 1 *Suppose that $\frac{k}{n} \leq \phi$. A profile of strategies in which $\sigma^1 = ... = \sigma^n = 0$ and the DM mixes uniformly among all permutations is a SNE.*

Proof. Let $\sigma^1 = ... = \sigma^n = 0$. Clearly, each strategy is a best response for the DM. Since $p(v)$ is weakly decreasing in v, each available candidate's interim belief that he is one of the DM's k-most-valued candidates is lower than $\frac{k}{n}$. Since $\sigma^1 = ... = \sigma^n = 0$ and the DM mixes uniformly among permutations, the candidates do not receive any additional information. Since, by assumption, $\frac{k}{n} \leq \phi$, $\sigma^1 = ... = \sigma^n = 0$ are best responses. ■

The condition in Proposition 1 implies that the position is not attractive for available candidates who do not receive any information about their relative ranking. Note that although $\frac{k}{n} \leq \phi$ is only a sufficient condition for the existence of such a SNE, it is tight in the sense that if $\frac{k}{n} > \phi$, then one can find a distribution of candidates' values and a function $p(v)$ such that the DM's offers are accepted with strictly positive probability in each SNE.

The next proposition is the paper's main result. We show that for a sufficiently large number of candidates, the DM's payoff in every SNE is 0. That is, the DM cannot recruit any candidate in a SNE. Moreover, this proposition implies that in contrast to a setup with no social preferences, increasing the number of candidates may decrease the DM's highest equilibrium payoff.

Proposition 2 *Let $v^\star \in (v_l, v_h)$ and suppose that $vp(v)$ is strictly decreasing in v for $v > v^\star$. There exists a number $n^\star$ such that for each $n > n^\star$, in every SNE, all of the DM's offers are rejected (i.e., $\sigma^1 = ... = \sigma^n = 0$).*

The proof of Proposition 2 is relegated to the Appendix. We prove Proposition 2 by contradicting the assumption that there exists a SNE in which the DM's offers are accepted with strictly positive probability. The proof consists of two steps. In the first step, we show that the number of rounds in which offers are accepted with positive probability cannot exceed some number $T > 0$, regardless of the number of candidates n. Since the share of rounds in which the DM's offers are accepted goes to 0 as n approaches infinity, for large values of n the DM has to decide how to allocate the many candidates whom he values more than $v^\star$ to the smaller number of rounds in which his offers might be accepted.

Let us try to get some intuition as to why the number of rounds in which the DM's offers might be accepted cannot exceed some number T. Consider candidate i who receives an offer in round t. Candidate i has three sources of information: x_i, the fact that $t-1$ offers were rejected, and the DM's strategy. The first two sources of information are private. The third source of information is commonly known in a SNE. To get a clear intuition, let us assume for a moment that there is no private information and denote the probability that the DM approaches one of the k-most-valued candidates in round t by q_t. Since $\sum_{t \in N} q_t = k$, at most $\lfloor \frac{k}{\phi} \rfloor$ candidates can believe that they are one of the k-most-valued candidates with probability greater than ϕ. Roughly speaking, a positive signal about the ranking of a candidate who receives an offer in some round is a negative signal regarding the ranking of candidates who receive offers in other rounds.

In the second step of the proof, we build on the fact that in the last round t in which the DM's offers might be accepted, he will make an offer to the candidate for whom $vp(v)$ is maximized among the group of candidates who receive offers in rounds $\left\{t' | \sigma^{t'} = 0\right\} \cup \{t\}$. The assumption that $vp(v)$ is decreasing in v implies that if there are many candidates with values greater than $v^\star$ for the DM to choose from in round t, the DM will not make an offer to one of the k-most-valued candidates. This results in a contradiction to the existence of a SNE in which the DM's offers are accepted with strictly positive probability.

Proposition 2 establishes that increasing the number of candidates may decrease the DM's highest induced SNE payoff. Recall that in the absence of a social preferences component, increasing the number of candidates strictly increases the DM's payoff. This effect is generated because it is harder for the DM to signal his preferences to the candidates whom he values most as the number of candidates increases.

This result implies that in markets in which egos play a significant role, it may be beneficial to focus on a smaller group of relevant candidates. It also implies that in such cases, it might be better to make the offers privately. In the next section we study the case in which the offers are made privately.

5 Private Offers

The model assumes that candidates observe the offers made or at least their timing (i.e., each candidate knows in which round he received an offer). We now compare the model with an alternative regime in which the offers are made privately. In order to study the case of private offers, we need to redefine the candidates' strategies. Since the offers are private, a candidate can only observe whether he received an offer or not. Therefore, a strategy $\sigma_i \in [0,1]$ for candidate i is simply the probability that i accepts the DM's offer in the case where he is available and receives one. As before, we restrict our attention to SNEs and omit the subscript i.

The fact that a candidate receives an offer is a positive signal about his relative ranking. To see this, note that when offers are made privately, there is no cost in approaching an unavailable candidate. Therefore, in any SNE, it is a dominant strategy for the DM to approach the candidates in the order of their values. It follows that the probability of receiving an offer is higher for the higher-ranked candidates.

The natural question to ask is which of the two offer regimes, private or public, induces a greater payoff for the DM. It turns out that there is no clear-cut answer. An example of a case in which a confidential offers regime induces a greater payoff for the DM compared to a public offers regime is the example given in Section 3 with $\phi \leq \frac{1}{3}$.

Claim 6 *In the example given in Section 3, with $\phi \leq \frac{1}{3}$, the DM's highest SNE payoff under private offers is higher than his highest SNE payoff under public offers.*

Proof. To prove this claim, we will show that under confidential offers there exists a SNE in which $\sigma = 1$, and under private offers there exists no SNE in which $\sigma^1 = \sigma^2 = 1$. First, observe that an available candidate believes that he is the most-valued one with probability $\frac{1}{3}$. Under private offers, if $\sigma = 1$, the DM's best response is to approach the candidates according to their values (i.e., approach the most-valued candidate first). It follows that $\sigma = 1$ is a best response since

$$Pr\left(v_i > v_j | s_i = 1, i \text{ receives an offer}\right) > Pr\left(v_i > v_j | s_i = 1\right) = \frac{1}{3} \tag{6}$$

By Claim 3, under a public offers regime, there exists no SNE in which $\sigma^1 = \sigma^2 = 1 > 0$. ■

On the other hand, the next claim presents a case in which the highest SNE payoff under public offers is greater than the highest SNE payoff the DM attains under a private offers regime.

Claim 7 *Suppose $n = 2, k = 1, \phi = 0.75, p(v) = 1 - v$, and v is drawn from the uniform distribution on $[0, 0.5]$. The highest SNE payoff that the DM obtains under public offers is greater than the one he obtains under private offers.*

Proof. First, consider the case of public offers. We show that there exists a SNE in which the DM obtains a strictly positive payoff. Set $\sigma^1 = 1, \sigma^2 = 0$. Since $v(1-v)$ is increasing in v, the DM's best response is to approach the most-valued candidate first given any realization (v_1, v_2). It follows that $\sigma^1 = 1, \sigma^2 = 0$ are best responses.

Let us consider the case of private offers. Observe that $f(v_i|s_i = 1) = \frac{2(1-v)}{\int_0^{0.5} 2(1-v)dv} = \frac{8(1-v)}{3}$. It follows that i's interim belief of being the most-valued candidate is

$$Pr\left(v_i > v_j | s_i = 1\right) = \int_0^{0.5} f\left(v_i | s_i = 1\right) 2v dv = \int_0^{0.5} \frac{16v(1-v)}{3} = 0.444 \tag{7}$$

The probability of being the most-valued candidate given that i receives an offer is

$$\frac{0.444}{0.444 + \int_0^{0.5}\int_v^{0.5} \frac{16(1-v)x}{3} dx dv + (1-\sigma)\int_0^{0.5}\int_v^{0.5} \frac{16(1-v)(1-x)}{3} dx dv} \leq 0.711 < \phi \tag{8}$$

It follows that $\sigma > 0$ cannot be a best response. ■

The private offers regime induces a positive signal about the relative ranking of candidates who receive offers. Under the public offers regime, some of the candidates receive positive signals about their ranking when they are made an offer, while others view the offer they receive as a negative signal about their ranking. In the example, the public offers regime enables more flexibility in the intensity of the signal. In a SNE, being the first candidate to receive an offer is a stronger signal of being the DM's top choice than the signal a candidate could have received under the private offers regime.

In the next proposition we restrict our attention to the natural case of $k = 1$ and show that if the intensity of the social component c is low (high), then the private offers regime induces a higher (lower) payoff for the DM than the public offers regime.

Proposition 3 *Suppose $k = 1$. There exists a number $\phi^\star$ such that for each $\phi < \phi^\star$ ($\phi > \phi^\star$), the DM's highest SNE payoff under the public offers regime is weakly lower (higher) than his highest SNE payoff under the private offers regime.*

Proof. Consider the private offers regime and suppose that there exists a SNE such that $\sigma > 0$. Recall that the DM's unique best response is to approach the candidates according to the order of their values. Let $t \in \{1, ..., n\}$. Observe that

$$\begin{aligned} &Pr\left\{x_i = t-1, s_i = 1, i \textit{ receives an offer }\right\} = \\ &\int_{v_l}^{v_h} ... \int_{v_l}^{v_h} \left(1 - \sigma p\left(v_{\pi_1(v_1,...,v_n)}\right)\right) ... \left(1 - \sigma p\left(v_{\pi_{t-1}(v_1,...,v_n)}\right)\right) \\ &\qquad p\left(v_{\pi_t(v_1,...,v_n)}\right) f(v_1) ... f(v_n)\, dv_1 ... dv_n, \end{aligned} \tag{9}$$

where π is a permutation such that $v_{\pi_1(v_1,...,v_n)} \geq ... \geq v_{\pi_n(v_1,...,v_n)}$. Note that Expression (9) is decreasing in σ for $t \geq 2$. For $t = 1$, Expression (9) does not depend on σ. It follows that

$$Pr\left\{x_i = 0 | s_i = 1, i \textit{ receives an offer }\right\}$$

is increasing in σ. In words, when σ is higher, receiving an offer is a stronger signal of being the most-valued candidate. It follows that if there exists a SNE in which $\sigma > 0$, then there exists a SNE in which $\sigma = 1$.

Fix $\phi^\star$. If there exists a SNE in which $\sigma = 1$, then for $\phi < \phi^\star$ there exists a SNE in which $\sigma = 1$. If there exists no SNE in which $\sigma = 1$, then for $\phi > \phi^\star$, there exists no SNE in which $\sigma > 0$. A SNE in which $\sigma = 1$ and the DM approaches the candidates according to their values induces the highest possible payoff for the DM and a SNE in which $\sigma = 0$ induces the lowest possible payoff for the DM. ■

When the social component c is relatively low, the private offers regime induces a strictly higher SNE payoff for the DM, compared to his payoff under the public offers regime. This is because under the public offers regime, if the DM approaches the candidates according to their values, some of them infer that they are not among the k-most-valued candidates. Therefore, they reject his offers. Under the private offers regime, a candidate who receives an offer receives a positive signal on being one of the k-most-valued candidates. Thus, the DM can make all of the candidates believe that they are among the k-most-valued candidates with probability greater than ϕ.

6 Concluding Remarks

We presented a model in which the costs of pursuing one's top choice emerge endogenously as a result of the candidates' social preferences. The candidates are "insulted" when they learn that they are ranked relatively low. Therefore, some of the DM's offers will be rejected even if they are made to available candidates. Since the DM will inevitably "insult" some of the candidates, he prefers to insult those who are less likely to accept his offers in the first place.

The model captures considerations that play a significant role in many real-world situations. For example, a job-market candidate would prefer going to a department that likes him better. In the real world there are many considerations that come to mind, for example, wages which are kept fixed in the model. This describes situations in which the wage is highly regulated. Prominent examples are public positions (e.g., the chair of a central bank) and senior functionaries in international institutions (e.g., IMF). It is important to note that the outside options that are available to candidates for such prestigious positions are also of the same magnitude, thus, making this consideration relevant.

The model can be extended to situations in which the DM is interested in recruiting more than one candidate, such as in the case where he wants to recruit a committee of experts. Another extension of the model would be to include wages. Note that introducing wages into the model introduces another signaling device and a mechanism that enables the DM to compensate candidates who are not highly valued. In that case, the interesting question is whether a very small social preferences component would influence the equilibrium outcomes in a significant way.

Literature review

The model is related to the literature on interdependent preferences. Gul and Pesendorfer (2016) provide a framework for studying interdependent preferences with incomplete information. The social preferences in the present model can be viewed as a special case of their framework. Avery and Levin (2010) study early admissions at selective colleges using a model in which there are interdependencies between the preferences of the colleges and those of the students, such that a college will prefer students who are relatively enthusiastic about attending it. In their model, early admissions help students to signal their preferences for one college or another. In the context of interdependent preferences, we are unaware of any other models in which agents' preferences are inferred from the timing of their actions.

The social motives that drive the paper are broadly supported by experimental evidence from the social psychology literature in the context of romantic and work relations. Aronson and Worchel (1966), Curtis and Kim (1986), and Condon and Crano (1988) presented subjects with false evidence that other subjects have indicated that they like them. As a result, the liked subjects favored these other subjects and gave them more positive evaluations.

Antler (2015) studies the effect of social preferences on the design of mechanisms for two-sided matching. He extends the conventional two-sided matching problem by allowing the agents' preferences to depend on the endogenous *actions* of agents on the other side of the market and their interpretation. Specifically, when an agent's action expresses that he wishes to be matched with an agent on the other side of the market, this will affect the latter agent's preferences. In Antler (2015) the agents' endogenous actions do not reveal any new information to other agents.

References

[1] Antler, Y. (2015): "Two-sided Matching with Endogenous Preferences," *American Economic Journal: Microeconomics,* 7(3), 241–258.

[2] Aronson, E. and Worchel, P. (1966): "Similarity versus liking as determinants of interpersonal attractiveness," *Psychonomic Science,* 5(4), 157–158.

[3] Avery C. and Levin, J. (2010): "Early Admissions at Selective Colleges," *American Economic Review,* 100(5), 2125–2156.

[4] Condon J. and Crano, W. D. (1988): "Inferred Evaluation of the Relation between Attitude Similarity and Interpersonal Attraction," *Journal of Personality and Social Psychology,* 54(5), 789–797.

[5] Curtis, R. J. and Miller, K. (1986): "Believing Another Likes or Dislikes You: Behaviors Making the Beliefs Come True," *Journal of Personality and Social Psychology,* 51(2), 284–290.

[6] Gul, F. and Pesendorfer, W. (2016): "Interdependent Preference Models as a Theory of Intentions," *Journal of Economic Theory* (forthcoming).

7 Appendix: Proof of Proposition 2

Before we start with the proof, we need some additional notation. We denote a profile of candidates' strategies $(\sigma^1, ..., \sigma^n)$ by σ. For each realization $(v_1, ..., v_n)$ and profile of strategies (α, σ), denote by $V_t(\alpha, \sigma, v_1, ..., v_n)$ the continuation value of a DM who acts according to α (from round 1) and all of his offers in rounds $1, ..., t-1$ were rejected. Let $T(\sigma) := \{t \in N | \sigma^t > 0\}$ denote the set of rounds in which the DM's offers might be accepted in the profile σ. For each realization $(v_1, ..., v_n)$ and round $t \in N$, we denote the candidate who receives an offer in round t by $\pi_t(v_1, ..., v_n)$.

The candidate who receives an offer in round t is said to be active if he is available and the offers in rounds $1, ..., t-1$ were rejected. We denote the probability that the candidate who receives an offer in round t is active given the profile (α, σ) by $\lambda_t(\alpha, \sigma)$. We can write $\lambda_t(\alpha, \sigma)$ as

$$\begin{gathered}\int_{v_l}^{v_h} ... \int_{v_l}^{v_h} \sum_{\pi} \alpha(\pi | v_1, ..., v_n) \left(1 - \sigma^1 p\left(v_{\pi_1(v_1,...,v_n)}\right)\right) ... \\ \left(1 - \sigma^{t-1} p\left(v_{\pi_{t-1}(v_1,...,v_n)}\right)\right) p\left(v_{\pi_t(v_1,...,v_n)}\right) f(v_1) ... f(v_n) \, dv_1 ... dv_n\end{gathered} \tag{10}$$

Denote the probability that the candidate who receives an offer in round t is active and the candidate who receives an offer in round r is one of the k-most-valued candidates by $\lambda_{tr}(\alpha, \sigma)$. Observe that t and r need not be the same round. We can write $\lambda_{tr}(\alpha, \sigma)$ as

$$\begin{gathered}\int_{v_l}^{v_h} ... \int_{v_l}^{v_h} \sum_{\pi} \alpha(\pi | v_1, ..., v_n) \mathbb{1}_r(\pi, v_1, ..., v_n) \left(1 - \sigma^1 p\left(v_{\pi_1(v_1,...,v_n)}\right)\right) ... \\ \left(1 - \sigma^{t-1} p\left(v_{\pi_{t-1}(v_1,...,v_n)}\right)\right) p\left(v_{\pi_t(v_1,...,v_n)}\right) f(v_1) ... f(v_n) \, dv_1 ... dv_n,\end{gathered} \tag{11}$$

where $\mathbb{1}_r(\pi, v_1, ..., v_n)) \in \{0, 1\}$ is an indicator that equals 1 if and only if the candidate who receives an offer rth according to π is one of the k-most-valued candidates, that is, if $x_{\pi_r(v_1,...,v_n)} < k$. Observe that $\frac{\lambda_{tt}(\alpha,\sigma)}{\lambda_t(\alpha,\sigma)}$ is the probability that a candidate who is active in round t assigns to the event that he is one of the k-most-valued candidates.

Let $(\hat{\alpha}_n, \hat{\sigma}_n)_{n=1}^{\infty}$ be a sequence of SNEs such that $(\hat{\alpha}_n, \hat{\sigma}_n)$ is played in the game with n candidates. Our *goal* in the next four lemmata is to show that the number of rounds in which the DM's offers might be accepted is bounded. That is, there exists a number $T > 0$ such that for every $n \in \mathbb{N}$, $|T(\hat{\sigma}_n)| < T$. Let us assume by negation that this is not the case. For each $m \in \mathbb{N}$, define $n(m)$ to be the lowest n for which $|T(\hat{\sigma}_n)| \geq m$. Observe that $n(m) \geq m$. For every $m \in \mathbb{N}$, denote $(\tilde{\alpha}_m, \tilde{\sigma}_m) := \left(\hat{\alpha}_{n(m)}, \hat{\sigma}_{n(m)}\right)$. That is, $(\tilde{\alpha}_m, \tilde{\sigma}_m)_{m=1}^{\infty}$ is a sequence of SNEs such that $(\tilde{\alpha}_m, \tilde{\sigma}_m)$ is played in the game with $n(m)$ candidates and $|T(\tilde{\sigma}_m)| \geq m$.

For each $m \in \mathbb{N}$ and $z = 1, ..., m$, define T_{zm} to be an arbitrary subset of $T(\tilde{\sigma}_m)$

that satisfies

(a) $|T_{zm}| = z$.

(b) If $t, t'' \in T_{zm}$, $t' \notin T_{zm}$, and $t > t' > t''$, then $t' \notin T(\tilde{\sigma}_m)$.

Lemma 1 will show that there is a number of rounds z such that for every $m \geq z$, there is a round $t \in T_{mz}$ for which $\tilde{\sigma}_m^t \geq \epsilon_z$, where $\epsilon_z > 0$ depends only on z.

Lemma 1 *There exists a number $z^\star$ such that for each $z \geq z^\star$ and $m \geq z$, there exists a number $\epsilon_z > 0$ (which is independent of m) such that for each T_{zm}, there exists a round $t \in T_{zm}$ such that $\tilde{\sigma}_m^t \geq \epsilon_z$.*

Proof. Consider $z \geq 2k$ and denote the median value of $\{\lambda_t(\tilde{\alpha}_m, \tilde{\sigma}_m) | t \in T_{zm}\}$ by $\lambda^\star_{zm}$. Define two disjoint sets of $\lfloor \frac{z}{2} \rfloor$ rounds, $T^\star_{zm} \subset T_{zm}$ and $T^{\star\star}_{zm} \subset T_{zm}$, such that for each $t \in T^\star_{zm}$, $\lambda_t(\tilde{\alpha}_m, \tilde{\sigma}_m) \leq \lambda^\star_{zm}$, and for each $t \in T^{\star\star}_{zm}$, $\lambda_t(\tilde{\alpha}_m, \tilde{\sigma}_m)) \geq \lambda^\star_{zm}$.

Consider $r \in T_{zm}$. Let us look at Expression (11) for $\lambda_{tr}(\tilde{\alpha}_m, \tilde{\sigma}_m)$. The *weight* of each realization $(v_1, ..., v_n)$ and permutation π such that $\tilde{\alpha}_m(\pi | v_1, ..., v_n) \mathbb{1}_r(\pi, v_1, ..., v_n) > 0$ is the probability that $t-1$ offers were rejected and that $\pi_t(v_1, ..., v_n)$ is available. It is given in (12).

$$\begin{aligned} w_t\left(\tilde{\sigma}_m, \pi, v_1, ..., v_{n(m)}\right) = &\left(1 - \tilde{\sigma}_m^1 p\left(v_{\pi_1\left(v_1, ..., v_{n(m)}\right)}\right)\right) ... \\ &\left(1 - \tilde{\sigma}_m^{t-1} p\left(v_{\pi_{t-1}\left(v_1, ..., v_{n(m)}\right)}\right)\right) p\left(v_{\pi_t\left(v_1, ..., v_{n(m)}\right)}\right) \end{aligned} \tag{12}$$

Since $p(v)$ is decreasing in v and $\mathbb{1}_r(\pi, v_1, ..., v_n) > 0$, there are at least $\lfloor \frac{z}{2} \rfloor - k$ candidates who receive offers in rounds $t \in T^\star_{zm}$ for whom $p\left(v_{\pi_t(v_1, ..., v_n)}\right) \geq p\left(v_{\pi_r(v_1, ..., v_n)}\right)$. Denote $\epsilon := max\{\tilde{\sigma}_m^t | t \in T_{zm}\}$. Since $p(v) \leq 1$, and $\tilde{\sigma}_m^i \leq \epsilon$ for each $i \in T_{zm}$,

$$w_t\left(\tilde{\sigma}_m, \pi, v_1, ..., v_{n(m)}\right) \geq w_r\left(\tilde{\sigma}_m, \pi, v_1, ..., v_{n(m)}\right)(1-\epsilon)^z$$

for each round t that is one of the $\lfloor \frac{z}{2} \rfloor - k$ rounds in $T^\star_{zm}$ in which $p\left(v_{\pi_t(v_1, ..., v_n)}\right) \geq p\left(v_{\pi_r(v_1, ..., v_n)}\right)$. Observe that in the last transition we used (b) and the fact that each round $t \notin T(\tilde{\sigma}_m)$ can be ignored since $\tilde{\sigma}_m^t = 0$. It follows that $\sum_{t \in T^\star_{zm}} \lambda_{tr}(\tilde{\alpha}_m, \tilde{\sigma}_m) \geq (1-\epsilon)^z \left(\lfloor \frac{z}{2} \rfloor - k\right) \lambda_{rr}(\tilde{\alpha}_m, \tilde{\sigma}_m)$ for each $r \in T_{zm}$.

Since $\frac{\lambda_{tr}(\tilde{\alpha}_m, \tilde{\sigma}_m)}{\lambda_t(\tilde{\alpha}_m, \tilde{\sigma}_m)}$ is the probability that a candidate who is active in the tth round assigns to the event that the candidate who receives an offer in the rth round is one of the k-most-valued candidates, it follows that $\sum_{r \in T_{zm}} \frac{\lambda_{tr}(\tilde{\alpha}_m, \tilde{\sigma}_m)}{\lambda_t(\tilde{\alpha}_m, \tilde{\sigma}_m)} \leq k$. Therefore,

$$
\begin{aligned}
kz \geq \sum_{t\in T_{zm}} \sum_{r\in T_{zm}} \frac{\lambda_{tr}\left(\tilde{\alpha}_m, \tilde{\sigma}_m\right)}{\lambda_{t}\left(\tilde{\alpha}_m, \tilde{\sigma}_m\right)} > \sum_{t\in T^{\star}_{zm}} \sum_{r\in T^{\star\star}_{zm}} \frac{\lambda_{tr}\left(\tilde{\alpha}_m, \tilde{\sigma}_m\right)}{\lambda_{t}\left(\tilde{\alpha}_m, \tilde{\sigma}_m\right)} \geq \\
\sum_{t\in T^{\star}_{zm}} \sum_{r\in T^{\star\star}_{zm}} \frac{\lambda_{tr}\left(\tilde{\alpha}_m, \tilde{\sigma}_m\right)}{\lambda^{\star}_{zm}} \geq (1-\epsilon)^z \sum_{t\in T^{\star\star}_{zm}} \frac{\left(\left\lfloor \frac{z}{2} \right\rfloor - k\right) \lambda_{tt}\left(\tilde{\alpha}_m, \tilde{\sigma}_m\right)}{\lambda^{\star}_{zm}} \geq \\
(1-\epsilon)^z \left(\left\lfloor \frac{z}{2} \right\rfloor - k\right) \left\lfloor \frac{z}{2} \right\rfloor \phi
\end{aligned}
$$

It follows that for a sufficiently large z and $m \geq z$, it must be that ϵ is greater than some $\epsilon_z > 0$ that is independent of m. ■

Lemma 1 shows that for large values of m, at least $\lfloor \frac{m}{z} \rfloor$ of the DM's offers are accepted (by available candidates) with probability greater than $\epsilon_z > 0$. In Lemmata 2 and 3, we consider large values of m. Roughly speaking, in these two lemmata we show that in the first rounds of $T\left(\tilde{\sigma}_m\right)$, the value of each candidate who receives an offer is close to v_h.

In Lemmata 2 and 3 we use the following notation. Fix arbitrary $\delta \in (0,1)$ and $\bar{\epsilon} \in \left(0, \frac{v_h - v_l}{2}\right)$. For every $m \in \mathbb{N}$, let $\left(\tilde{v}_1, ..., \tilde{v}_{n(m)}\right)$ be an arbitrary realization of candidates' values such that $|\left\{i | v_i \in \left(v_h - 2\bar{\epsilon}, v_h - \bar{\epsilon}\right)\right\}| \geq \delta m$. Observe that if $\delta < F\left(v_h - \bar{\epsilon}\right) - F\left(v_h - 2\bar{\epsilon}\right)$, then the probability of such a realization approaches 1 as m goes to infinity.

Lemma 2 *Fix $t \in \mathbb{N}$. There exists a number $m^t \in \mathbb{N}$ such that for each $m \geq m^t$, $V_j\left(\tilde{\alpha}_m, \tilde{\sigma}_m, \tilde{v}_1, ..., \tilde{v}_{n(m)}\right) > v_h - 3\bar{\epsilon}$ for each $j \in \left\{t' \in T\left(\tilde{\sigma}_m\right) : |\left\{\bar{t} \in T\left(\tilde{\sigma}_m\right) : \bar{t} \leq t'\right\}| \leq t\right\}$.*

Proof. Denote the following strategy by α': in each round $t \in T\left(\tilde{\sigma}_m\right)$ make an offer to the lowest-valued candidate among those with value $v > v_h - 2\bar{\epsilon}$ who did not receive an offer previously. If such a candidate does not exist, make an offer to the highest-valued candidate who did not receive an offer previously. In each round $t \notin T\left(\tilde{\sigma}_m\right)$ make an offer to an arbitrary candidate whose value is $v \leq v_h - 2\bar{\epsilon}$. If such a candidate does not exist, make an offer to the highest-valued candidate who did not receive an offer previously.

For $j = 1, ..., |T\left(\tilde{\sigma}_m\right)|$, denote the following strategy by β_j : up to the jth round of $T\left(\tilde{\sigma}_m\right)$, the DM makes the offers in each round $t \in T\left(\tilde{\sigma}_m\right)$ according to $\tilde{\alpha}_m$. In each round $t \notin T\left(\tilde{\sigma}_m\right)$, he makes the offers according to the rule that defines α'. Starting from the jth round of $T\left(\tilde{\sigma}_m\right)$, the DM makes the offers according to the rule that defines α'. Let us denote the jth round of $T\left(\tilde{\sigma}_m\right)$ by $j^{\star}$. By the optimality of $\tilde{\alpha}_m$, if

$V_{j^\star}\left(\beta_j, \tilde{\sigma}_m, \tilde{v}_1, ..., \tilde{v}_{n(m)}\right) > v_h - 3\bar{\epsilon}$, then $V_{j^\star}\left(\tilde{\alpha}_m, \tilde{\sigma}_m, \tilde{v}_1, ..., \tilde{v}_{n(m)}\right) > v_h - 3\bar{\epsilon}$. It is left to show that for a sufficiently large m, $V_{j^\star}\left(\beta_j, \tilde{\sigma}_m, \tilde{v}_1, ..., \tilde{v}_{n(m)}\right) > v_h - 3\bar{\epsilon}$ for $j = 1, ..., t$.

By construction, according to β_j, in the rounds $\{t' \in T(\tilde{\sigma}_m) \,|t' \geq j^\star\}$, the DM makes at least $\lfloor \delta m \rfloor - j + 1$ offers to candidates with a value $v \in (v_h - 2\bar{\epsilon}, v_h - \bar{\epsilon})$. By Lemma 1, there is a number $z \in \mathbb{N}$ such that at least $\lfloor \frac{\lfloor \delta m \rfloor - j + 1}{z} \rfloor$ of these offers are accepted with a probability greater than $\epsilon_z p(v_h - \bar{\epsilon}) > 0$. If we chose a sufficiently large m, then $V_{j^\star}\left(\beta_j, \tilde{\sigma}_m, \tilde{v}_1, ..., \tilde{v}_{n(m)}\right) > v_h - 3\bar{\epsilon}$ for $j = 1, ..., t$. ■

In Lemma 2 we considered the first t rounds of $T(\tilde{\sigma}_m)$. We showed that for large values of m, the DM's continuation value in these rounds is arbitrarily close to v_h when he faces a realization $\left(\tilde{v}_1, ..., \tilde{v}_{n(m)}\right)$. In Lemma 3 we consider large values of m and show that in each round $j \in \{t' \in T(\tilde{\sigma}_m) : |\{\bar{t} \in T(\tilde{\sigma}_m) : \bar{t} \leq t'\}| \leq t\}$, if the DM does not offer the position to a candidate whose value is higher than $v_h - 4\bar{\epsilon}$ when he faces a realization $\left(\tilde{v}_1, ..., \tilde{v}_{n(m)}\right)$, then $\tilde{\sigma}_m^j < \epsilon$, where $\epsilon > 0$ is arbitrarily close to 0.

Lemma 3 *Fix $t \in \mathbb{N}, \epsilon > 0$. There exists a number $m^{t\epsilon}$ such that for each $m \geq m^{t\epsilon}$, if the DM faces a realization $\left(\tilde{v}_1, ..., \tilde{v}_{m(n)}\right)$ and makes an offer to a candidate with a value $v < v_h - 4\bar{\epsilon}$ in round $j \in \{t' \in T(\tilde{\sigma}_m) : |\{\bar{t} \in T(\tilde{\sigma}_m) : \bar{t} \leq t'\}| \leq t\}$, then $\tilde{\sigma}_m^j < \epsilon$.*

Proof. Consider candidate i who receives an offer in round j, which is one of the first t rounds of $T(\tilde{\sigma}_m)$, and suppose that $v_i < v_h - 4\bar{\epsilon}$. By definition,

$$V_j\left(\tilde{\alpha}_m, \tilde{\sigma}_m, \tilde{v}_1, ..., \tilde{v}_{n(m)}\right) = \tilde{\sigma}^j p(\tilde{v}_i)\, \tilde{v}_i + \left(1 - \tilde{\sigma}^j p(\tilde{v}_i)\right) V_{j+1}\left(\tilde{\alpha}_m, \tilde{\sigma}_m, \tilde{v}_1, ..., \tilde{v}_{n(m)}\right)$$

By Lemma 2, there exists a number m^j such that if $m > m^j$, then $V_j\left(\tilde{\alpha}_m, \tilde{\sigma}_m, \tilde{v}_1, ..., \tilde{v}_{n(m)}\right)$ $> v_h - 3\bar{\epsilon}$. Consider $m > m^j$. Since $v_i < v_h - 4\bar{\epsilon}$, it follows that $V_{j+1}\left(\tilde{\alpha}_m, \tilde{\sigma}_m, \tilde{v}_1, ..., \tilde{v}_{n(m)}\right)$ $> v_h - 3\bar{\epsilon}$.

Suppose that there exists a round r such that $\tilde{\sigma}_m^r = 0$. It must be that in round r the DM makes an offer to a candidate i' with a value $v_{i'} \leq v_i$. Otherwise, the DM could do better by switching between i and i', which is a contradiction to the optimality of $\tilde{\alpha}_m$. It follows that by round j the DM makes at most $t - 1$ offers to candidates with a value $v \in (v_h - 2\bar{\epsilon}, v_h - \bar{\epsilon})$. There are at least $\lfloor \delta m \rfloor - t + 1$ candidates with a value $v \in (v_h - 2\bar{\epsilon}, v_h - \bar{\epsilon})$ who did not receive an offer by round j. Denote the set of rounds in which they are scheduled to receive offers by Q. Let $\tilde{\sigma}_m^{min} := min\{\tilde{\sigma}_m^i : i \in Q\}$.

Suppose that the game has reached round j (that is, the DM's offers in rounds $1, ..., j-1$ were rejected). The probability that the last (according to $\tilde{\alpha}_m$) of the candidates with a value $v \in (v_h - 2\bar{\epsilon}, v_h - \bar{\epsilon})$ will receive an offer must be lower than

$(1-\tilde{\sigma}_m^{min} p(v_h-\bar{\epsilon}))^{\lfloor \delta m \rfloor - t}$. For each $\epsilon' > 0$, there exists $m^{\epsilon'} \in \mathbb{N}$ such that for each $m > m^{\epsilon'}$, $\tilde{\sigma}_m^{min} < \epsilon'$ or $(1-\tilde{\sigma}_m^{min} p(v_h-\bar{\epsilon}))^{\lfloor \delta m \rfloor - t} < \epsilon'$. This implies that for a sufficiently large m, the DM could switch between i (who receives an offer in round j and has a value of $v_i < v_h - 4\bar{\epsilon}$) and a candidate with a value $v_{i'} > v_h - 2\bar{\epsilon}$ while making an arbitrarily small change in $V_{j+1}\left(\tilde{\alpha}_m, \tilde{\sigma}_m, \tilde{v}_1, ..., \tilde{v}_{n(m)}\right)$. Since $p(v_{i'}) \leq p(v_i)$ and $V_{j+1}\left(\tilde{\alpha}_m, \tilde{\sigma}_m, \tilde{v}_1, ..., \tilde{v}_{n(m)}\right) > v_h - 3\bar{\epsilon}$, if $\tilde{\sigma}^j \geq \epsilon$, we get a contradiction to the optimality of $\tilde{\alpha}_m$. ■

In Lemma 4 we show that the number of rounds in which the DM's offers might be accepted is bounded from above.

Lemma 4 *There exists a number $T > 0$ such that for every $n \in \mathbb{N}$, $|T(\hat{\sigma}_n)| < T$.*

Proof. Assume towards a contradiction that the claim is false. Then, the sequence $(\tilde{\alpha}_m, \tilde{\sigma}_m)_{m=1}^{\infty}$ can be defined as before. Fix an arbitrarily small $\bar{\epsilon} > 0$ and let $\delta(\bar{\epsilon}) := \frac{F(v_h-\bar{\epsilon})-F(v_h-2\bar{\epsilon})}{2}$. By Lemma 3, for each $t \in \mathbb{N}$ and $\epsilon > 0$, there exists a number $m^{t\epsilon}$ such that if $m > m^{t\epsilon}$ and $|\{i | v_i \in (v_h - 2\bar{\epsilon}, v_h - \bar{\epsilon})\}| \geq \delta(\bar{\epsilon}) m$, then in each round $j \in \{t' \in T(\tilde{\sigma}_m) : |\{\bar{t} \in T(\tilde{\sigma}_m) : \bar{t} \leq t'\}| \leq t\}$ the DM makes an offer to a candidate with a value $v > v_h - 4\bar{\epsilon}$ or $\tilde{\sigma}_m^j < \epsilon$.

Let us consider $\lambda_{jr}(\tilde{\alpha}_m, \tilde{\sigma}_m)$, where $j \in \{t' \in T(\tilde{\sigma}_m) : |\{\bar{t} \in T(\tilde{\sigma}_m) : \bar{t} \leq t'\}| \leq t\}$ and $r \in N$. Observe that the probability that the values' realization is such that $|\{i | v_i \in (v_h - 2\bar{\epsilon}, v_h - \bar{\epsilon})\}| \geq \delta(\bar{\epsilon}) m$ approaches 1 as m goes to infinity. Let us consider two such realizations: $\left(\tilde{v}_1, ..., \tilde{v}_{n(m)}\right)$, $\left(\tilde{v}'_1, ..., \tilde{v}'_{n(m)}\right)$, and two permutations: π and π', such that $\tilde{\alpha}_m(\pi | \tilde{v}_1, ..., \tilde{v}_n) > 0$ and $\tilde{\alpha}_m(\pi' | \tilde{v}'_1, ..., \tilde{v}'_n) > 0$. One can choose arbitrarily small $\epsilon, \bar{\epsilon} > 0$ such that for $m > m^{t\epsilon}$, the ratio between the weights $\frac{w_j\left(\tilde{\sigma}_m, \pi, \tilde{v}_1, ..., \tilde{v}_{n(m)}\right)}{w_j\left(\tilde{\sigma}_m, \pi', \tilde{v}'_1, ..., \tilde{v}'_{n(m)}\right)}$ given in (12) is arbitrarily close to 1. This follows from the fact that in each round $t < j$ such that $\tilde{\sigma}_m^t > \epsilon$, the DM approaches candidates with a value $v > v_h - 4\bar{\epsilon}$. Therefore, one can find $m^\star \in \mathbb{N}$ such that for each $m > m^\star$, $\frac{\lambda_{jr}(\tilde{\alpha}_m, \tilde{\sigma}_m)}{\lambda_j(\tilde{\alpha}_m, \tilde{\sigma}_m)}$ is arbitrarily close to

$$\frac{\int_{v_l}^{v_h} ... \int_{v_l}^{v_h} \sum_\pi \tilde{\alpha}_m\left(\pi | v_1, ..., v_{n(m)}\right) \mathbb{1}_r\left(\pi, v_1, ..., v_{n(m)}\right) f(v_1) ... f\left(v_{n(m)}\right) dv_1 ... dv_{n(m)}}{\int_{v_l}^{v_h} ... \int_{v_l}^{v_h} \sum_\pi \tilde{\alpha}_m\left(\pi | v_1, ..., v_{n(m)}\right) f(v_1) ... f\left(v_{n(m)}\right) dv_1 ... dv_{n(m)}} \quad (13)$$

It follows that for $i, i' \leq j$ and $r \in N$, $\frac{\lambda_{ir}(\tilde{\alpha}_m, \tilde{\sigma}_m)}{\lambda_i(\tilde{\alpha}_m, \tilde{\sigma}_m)}$ is arbitrarily close to $\frac{\lambda_{i'r}(\tilde{\alpha}_m, \tilde{\sigma}_m)}{\lambda_{i'}(\tilde{\alpha}_m, \tilde{\sigma}_m)}$. Recall that $\frac{\lambda_{ii}(\tilde{\alpha}_m, \tilde{\sigma}_m)}{\lambda_i(\tilde{\alpha}_m, \tilde{\sigma}_m)} \geq \phi$ for each $i \in T(\tilde{\sigma}_m)$. It follows that $\sum_{t' \in T(\tilde{\sigma}_m)} \frac{\lambda_{jt'}(\tilde{\alpha}_m, \tilde{\sigma}_m)}{\lambda_j(\tilde{\alpha}_m, \tilde{\sigma}_m)} > \phi(t-1)$. For large $t \in \mathbb{N}$ this is a contradiction since $\phi(t-1) > k$ and $\sum_{t' \in N} \frac{\lambda_{jt'}(\tilde{\alpha}_m, \tilde{\sigma}_m)}{\lambda_j(\tilde{\alpha}_m, \tilde{\sigma}_m)} = k$. ■

Proof of Proposition 2

By Lemma 4, there exists a number $T > 0$ such that for every $n \in \mathbb{N}$, $|T(\hat{\sigma}_n)| < T$. Let us choose a large $n^{\star}$ and consider a realization in which there are more than $T + k$ candidates with a value $v > v^{\star}$ (the probability of such a realization goes to 1 as n goes to infinity). Consider the last round $t \in T(\hat{\sigma}_n)$. There must be at least $k + 1$ candidates who are supposed to receive an offer in rounds $\{t\} \cup N/T(\hat{\sigma}_n)$ who each have a value greater than $v^{\star}$. The optimality of the DM's strategy requires that in round t he approach the candidate for whom $vp(v)$ is maximized (among those candidates and the other candidates who did not receive offers in rounds $\{t' \notin T(\hat{\sigma}_n) : t' < t\}$). By the assumption that $vp(v)$ is strictly decreasing for $v > v^{\star}$, there are at least k candidates who are more valued than the candidate who is approached in round t. Therefore, $\hat{\sigma}_n^t > 0$ cannot be a part of a SNE.