Portfolio Liquidity Risk Management with Expected Shortfall Constraints

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Abstract

In this thesis we quantify the potential cost of liquidity constraints on a long equity portfolio using the liquidity risk framework of Acerbi and Scandolo (2008). The model modifies the classical mark-to-market valuation model, and incorporates the impact of liquidity policies of portfolios on the liquidity adjustment valuation (LVA). Also, we suggest a quantitative indicator that scores market liquidity ranging from 0 to 1 (perfect liquidity) for a portfolio with possible liquidity constraints.

The thesis consists of three major studies. In the first one, we compute LVA given the cash, minimum weight and portfolio expected shortfall (ES) liquidity policies on a long equity portfolio. Several numerical examples in the results demonstrate the importance associated the incorporation of the liquidity policy in the liquidity risk valuation.

In the second study, we quantify the execution costs and the revenue risk when implementing trading strategies over multiple periods by employing the transaction costs measure of Garleanu and Pedersen (2013). The portfolio liquidity costs estimated from the model of Garleanu and Pedersen (2013) are compared with the costs estimated from the liquidity risk measure of Finger (2011).

In the third study, we estimate the liquidity-adjusted portfolio ES for a long equity portfolio with the liquidity constraints. Portfolio pure market P&L scenarios are based on initial positions, and the liquidity adjustments are based on positions sold, which depend on the specified liquidity constraints. Portfolio pure market P&L scenarios and state-dependent liquidity adjustments are integrated to obtain liquidity-adjusted P&L scenarios. Then, we apply the liquidity score method (Meucci, 2012) on the liquidity-plus-market P&L distribution to quantify the market liquidity for the portfolio.

The results show the importance of pricing liquidity risk with liquidity constraints. The liq-

uidity costs can vary greatly on different liquidity policies, portfolio MtM values, market situation and time to liquidation.

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Abbreviations and Notation

Abbreviations:

ADV Average daily volume

CPRM Coherent portfolio risk measure

CRM Coherent risk measure

ES Expected shortfall

GPD Generalised Pareto Distribution

LMtM Liquidation mark-to-market

LS Liquidity score

LVA Liquidity valuation adjustment

LVaR Liquidity-adjusted Value at Risk

MLE Maximum likelihood estimation

MSDCs Marginal supply and demand curves

P&L Profit and loss

UMtM Uppermost mark-to-market

Notation:

$\bar{J}$ Realized price impact

$\bar{p}$	A probability for each portfolio market P&L scenario
$\bar{r}_k^b$	A parameter to define the slope of a demand curve
β	Scale parameter of GPD
Λ	Kyle's lambda
$\mathbb{R}^+$	Set of positive real numbers
$\mathscr{L}$	Liquidity policy
$\overline{\Pi}$	Portfolio pure market-risk P&L scenarios
$\overline{\pi}$	Portfolio pure market risk P&L
Π	Portfolio liquidity-plus-market-risk P&L scenarios
π	Mark-to-market P&L of an asset
ρ	Risk measure
σ	Daily volatility
τ	Trading time step
θ	Toal number of shares in issue
$\tilde{S}$	Average realised price
ξ	Shape parameter of GPD
m^+	Best bid price
m^-	Best ask price
p_0	Cash component of the portfolio
a	Residual portfolio
C	Liquidity costs

D	Value when assets are immediately liquidated at the current MSDCs
d	Cash amount
h	Number of positions remaining to trade
j	Index for a scenario
k	Index for an asset
L	Cost/risk leverage parameter
n	Index for trading period
p	Initial positions of an asset
q	Cash value of positions liquidated
T	Trade duration in days
U	Uppermost mark-to-market value
V	Average daily volume
w	Asset weight
x	Cash value of positions remaining to trade
z	Number of positions liquidated

Chapter 1

Introduction

This thesis provides applications for quantifying the liquidity costs for a long equity portfolio, considering the transaction costs and the revenue risk in implementation of trading strategies, under liquidity policies an investor might be constrained in practice. We make use of the liquidity risk measure of Acerbi and Scandolo (2008) and Garleanu and Pedersen (2013) to quantify the liquidity risk depending on market depth and a portfolio owner. In particular, the study adopts the power-law marginal supply and demand curve (MSDC) to model non-unique asset prices. Liquidity policies are intertwined with the MSDC to incorporate the specific financial situation of an investor when measuring the price impact. Also, the thesis employs the optimal execution paradigm of Almgren and Chriss (2000) that includes the transaction costs and the revenue risk terms incurred when implementing the trading strategies over multiple trading periods (the total number of positions sold is varied on the specified liquidity policies). The study also suggests the liquidity score method that measures the portfolio market liquidity risk which can help investors to construct and maintain a sufficiently liquid portfolio (given the liquidity policies) (Meucci, 2012).

This chapter is organized as follows. Section 1.1 highlights the potential benefits of the liquidity risk model, and motivates the applications of them. Section 1.2 describes the main contributions of this thesis to the literature on market liquidity risk estimation, and liquidity-adjusted risk measurement. Section 1.3 outlines the general structure of the thesis.

1.1 Background and Motivation

The systemic contagious collapse caused by the US sub-prime mortgage market in the summer of 2007 demonstrated the importance and need for a robust liquidity risk management framework.

Authors such as van den End and Tabbae (2012) noted that the crisis developed through three stages: market and funding liquidity risk, counterparty risk finally resulting in liquidity stress on banks (commercial and investment). After the beginning of the crisis, the large discount in the asset values required financial institutions to post higher collateral, and they were reluctant to approve lending to each other. This caused a large increase in funding spreads in 2007-2008. Accordingly, there was an increased counterparty risk because of the possibility of bank failures; thus, the liquidity crisis in the financial market followed. The deterioration of market liquidity can destabilize the entire banking system. The pricing of liquidity risk is therefore a key focus for both practitioners and academics (Acerbi and Scandolo, 2008; Amihud et al., 2013; Wagner, 2007; Weber et al., 2012). This sequence of events led to the introduction of more robust liquidity measures under Basel III and in particular the liquidity coverage ratio[1].

The failure in liquidity risk management, as Brunnermeier and Pedersen (2009) have reported, was an important factor for the liquidity crisis. In order to understand rapid development of liquidity risk in the whole financial system, we classify liquidity into two categories namely, funding liquidity and market liquidity. Funding liquidity refers to the ability to raise money for investments, for example, through collateralized borrowing against their assets. If the assets are traded at fire-sale prices in stressed market situation, then both margins and haircuts on their assets would be increased. Funding liquidity worsens as market liquidity worsens.

Market liquidity is defined as the ease of trading. Institutions can raise cash easily by selling their assets when market liquidity is high. The cost of trading, which is the difference between the average realized price and the market prices, is small. According to Kyle (1985), market liquidity has three main dimensions namely bid-ask spread, market depth (the size of quotes that can be traded without any price movement), and market resilience (the speed the asset price to converges to its fundamental value).

Market liquidity dried up quickly during the financial crisis starting in 2007. Institutions had difficulty to unload positions, and were forced to make fire sales for increased margin requirements. They relied on short-term wholesale funding from other institutions to fund trading activities, and held too few liquid assets to survive in periods of unexpected funding outflows (Cifuentest et al., 2005).

In response to this liquidity crisis, the Basel Committee on Banking Supervision (BCBS) published global standards for liquidity risk compliance: the Liquidity Coverage Ratio (LCR) and the

[1] Basel III: The Liquidity Coverage Ratio and liquidity risk monitoring tools (available at http://www.bis.org/publ/bcbs238.pdf).

Net Stable Funding Ratio (NSFR).[2] LCR requires banks to hold high quality liquid assets (HQLA) to survive in stress environments. The amount of HQLA should be at least equal to the net cash outflow lasting for 30 days. NSFR is the ratio of the available stable funding (ASF) with respect to required stable funding (RSF). It requires banks to fund trading activities with long-term stable sources, which include term deposits, capital instruments and liabilities with their residual maturities greater than one year.

Several regulatory institutions propose principals and applications for managing liquidity risk related to operations of funds. International Organisation of Securities Commissions (IOSCO) (2013) outlines principles on liquidity management tools for collective investment schemes (CIS). As the report points out, net asset value (the value of the portfolio securities and other assets, less liabilities) (NAV) should be computed according to the principle that all fund holders are treated fairly. Open-ended funds in practice face redemption requests from investors on a daily basis. Good operation of the funds is to meet redemption obligation continuously on any business day, but eliminate any incentives of early redemptions. When the fund experiences large investor outflows, the NAV of existing shareholder may be adversely affected by the trading costs of transacting shareholders. Remaining investors should not bear the trading costs incurred by early redeemers; otherwise, there would be incentives for the redeeming investors. It may increases the risk of a run on the fund. Also, the liquidity risk of the portfolio needs to be unchanged following the sales (or the purchases), so the fund maintains the ability to meet redemption requests for remaining shareholders. U.S. Securities and Exchange Commission (SEC) (2015) proposes a liquidity management tool called swing pricing for open-ended mutual funds and exchange-traded funds (ETFs). The key objective of swing pricing is to protect existing investors from the transaction costs associated to redemptions (or subscriptions) of other shareholders. Swing pricing allows (but not requires) investment management companies to adjust a fund's NAV per share when there are net redemption (or net subscription) requests. There are two situations that trigger swing pricing: full and partial swinging. Full swinging adjusts the fund's NAV per share whenever net outflows (or net inflows) occur. However, it increases volatility in the NAV. Partial swinging, on the other hand, is triggered when the net outflows (or net inflows) from investors has exceeded a pre-defined swing threshold (as a percentage of the fund's NAV). Swing threshold requires approval by the fund's board, and must be reviewed at least annually. When the application is triggered, the redeemer receives an adjusted share price, which is decreased for sell and increased for buy orders,

[2]Details available at http://www.bis.org/publ/bcbs271.pdf

by the liquidity costs per share (also referred to as swing factor) from the sale (or the purchase) of portfolio assets. It compensates the remaining shareholders for the NAV dilution following the sales.

In this study, we quantify the liquidity risk for a long equity portfolio using the liquidity risk framework of Acerbi and Scandolo (2008). The liquidity adjustment on the asset value depends on the quantities traded and the profile of the owner of the portfolio. The marginal supply-demand curve (MSDC) measures the price impact on the trading volume. Moreover, liquidity constraint models limit access to financing, which may force the portfolio owner to liquidate a fraction of one's assets. The state of the market and the limited access to financing the portfolio owner has are intertwined to describe the portfolio liquidity risk.

1.2 Contribution

In this section, I describe the main contributions of this thesis to the studies on market liquidity risk and liquidity-adjusted risk measurement.

In Chapter 3, we quantify the potential cost of liquidity constraints on a long equity portfolio and compute the value given the liquidity policy. Three typical liquidity policies, namely cash, minimum weight and portfolio expected shortfall (ES) liquidity policies on a long equity portfolio are combined and used as constraints on the performance of the portfolio. We derive a power-law MSDC from the results of Almgren et al. (2005), correcting a formula presented by Finger (2011). We quantify the market liquidity risk in the presence of liquidity constraints using the liquidity risk framework of Acerbi and Scandolo (2008) and the power-law MSDC.

In Chapter 4, we apply the cash and portfolio ES limit constraints in the transaction costs measure of Garleanu and Pedersen (2013), and compare the liquidity costs for a long equity portfolio estimated from the model of Acerbi and Scandolo (2008) and Garleanu and Pedersen (2013). We use the Power-Law marginal supply and demand curves (MSDCs) to calibrate the parameter of market depth measure in the model of Garleanu and Pedersen (2013). We choose this approach to adjust a price impact per share on the volatility of the asset (it is fixed as 0.1% of the asset price in the model of Garleanu and Pedersen (2013)).

In Chapter 5, we measure the liquidity-adjusted portfolio ES for a long equity portfolio with 25% cash and portfolio ES limit constraints. We choose 25% redemption rates based on historical data for mutual funds (equity portfolios) in 2016 Investment Company Fact Book from Investment Company Institute (ICI) (2016). The report shows that the annual broad redemption rates, which include the sum of regular redemptions and exchange redemptions for the year, for equity funds varied between 24.7 and 25.3 percent of the fund's net assets from 2013 to 2015. Also, we quantify the portfolio market liquidity risk using the liquidity score (Meucci, 2012) which varies from 0 to 1 (perfect liquidity). The optimal execution paradigm of Almgren and Chriss (2000) was employed when finding the optimal execution strategy given a cost/risk leverage parameter. The cost/risk leverage parameter is adjusted according to the market risk scenarios in order to estimate the state-dependent liquidity adjustment parameters. Moreover, we stress the model with the potential liquidity constraints. The liquidity score was measured on the portfolio with 25% cash (e.g. the amount of client redemption, as a proportion of the portfolio value) and portfolio risk limit constraints (e.g. portfolio ES limit on the residual portfolio).

To the best of my knowledge, this is the first work that employs the portfolio valuation framework of Acerbi and Scandolo (2008) in computing liquidity score (Meucci, 2012) by estimating liquidity-adjusted P&L distribution incorporated cash and the portfolio ES constraints. In a study of Meucci (2012), the portfolio liquidated is given by an exogenous weight parameter (e.g. 20% of initial positions for each asset). In the model of Acerbi and Scandolo (2008), on the other hand, the portfolio liquidated is obtained by solving the constrained optimisation problem. It details liquidation process of liquid and illiquid assets depending on fund's liquidity policies (e.g. given market risk limit). The portfolio ES constraint enables a fund manager to estimate liquidity costs for a portfolio such that the liquidity characteristics of the fund unchanged following redemptions. The fund manager can impose a maximum portfolio ES value on the residual portfolio, forcing to sell illiquid assets to satisfy the given risk limit, so the existing investors are not left with illiquid assets. In order to remove first mover advantage, early redeemers should bear the liquidity costs (by receiving an adjusted fund's NAV per share), which protects remaining investors from NAV dilution.

1.3 Overview and Structure

This thesis introduces applications for liquidity adjustment valuation for a long equity portfolio when asset values are impacted by a deterioration in the market liquidity (market depth). The method modifies the classical mark-to-market valuation model, so it assumes that markets may not be perfectly liquid and hence assets have non-unique prices. The thesis also shows the key role that the liquidity policies of portfolios can have on the liquidity adjustment valuation. The portfolios with the identical positions but differing liquidity policies have different values. The thesis is structured as follows.

Chapter 2 provides a brief introduction to the coherent portfolio risk measure (CPRM) which is compatible with the liquidity risk, and describes the marginal supply-demand curve (MSDC). A short literature review on the liquidity risk framework of Acerbi and Scandolo (2008) is included. This chapter also defines the liquidity valuation adjustment (LVA) method, and derives expected shortfall (ES) that are applied throughout the thesis.

Chapter 3 quantifies the liquidity valuation adjustment for liquidity constraints on a long equity portfolio building on the liquidity risk framework of Acerbi and Scandolo (2008). A power-law MSDC from the results of Almgren et al. (2005) is derived. This chapter describes and analyses the relationship between the portfolio value and the liquidity policy (constraints that can potentially be called upon, and that are imposed externally on the portfolio owner). We employ a power-law Marginal Supply-Demand Curve (MSDC) to model market depth, together with various types of liquidity policies. We show the LVA changes when the portfolio MtM value becomes larger, and when the liquidity policy becomes stricter with the small values of portfolio ES limit and the trade duration.

Chapter 4 compares the liquidity costs from the model of Acerbi and Finger (2010) and Garleanu and Pedersen (2013) for a long equity portfolio with cash and the portfolio Expected Shortfall (ES) limit constraints when positions are sold according to the optimal trading trajectory in multiple periods. We employ the liquidity-adjusted Value-at-Risk (LVaR) model of Almgren and Chriss (2000) to find the optimal trading strategy that minimize the transaction costs and the revenue risk over the specified trading interval. We apply Power-Law Marginal Supply-Demand Curves (MSDCs) to calibrate the parameter of the market depth measure of Garleanu and Pedersen (2013).

Chapter 5 quantifies the market liquidity risk for a long equity portfolio using the liquidity score (Meucci, 2012), which varies from 0 to 1 (perfect liquidity). This chapter introduces a quantitative indicator to measure the level of market liquidity over time to construct (and maintain) a portfolio which is liquid enough (to mitigate significant liquidation costs from fire-sale in a stressed market situation). The execution costs and the volatility risk terms are integrated with the market risk to measure the liquidity-adjusted portfolio risk. We introduce the cost/risk leverage parameter that depends on the market risk scenarios, which is used to obtain the state-dependent liquidity adjustment parameters. The portfolio is constrained with cash and the portfolio Expected Shortfall

(ES) limit. The model quantifies the liquidity-adjusted portfolio ES and the liquidity score for a portfolio with potential liquidity restrictions. The liquidity scores on different portfolio MtM values and liquidity policies are compared, and the liquidity adjustment parameters according to liquidation time varying from 1 to 10 days are shown.

Chapter 6 summarizes the main findings of this thesis and concludes. It also discusses ideas for future research.

Chapter 2

Liquidity risk framework

In this chapter, we describe the marginal supply and demand curves (MSDCs) that capture the non-uniqueness of asset prices. The concept of the marginal supply and demand curves explained in this chapter is used to develop a generic function form MSDC in Chapter 3, and it is applied from Chapter 3 to 5 to measure the price impact, and to estimate the parameter of a market depth measure. A literature review on the liquidity risk framework of Acerbi and Scandolo (2008) and the definition of the liquidity valuation adjustment are also included. Moreover, the coherent portfolio risk measure (CPRM) which is compatible with the liquidity risk is introduced.

This chapter is organized as follows. Section 2.1 provides a general introduction to the characteristics of asset prices in illiquid market, and exemplifies the portfolio valuation problem that depends on the profile of its owner. Section 2.4 looks at the coherent risk measure (CRM) and the coherent portfolio risk measure (CPRM). Section 2.2 describes the feature of the MSDCs which models the non-uniqueness of the asset prices. Section 2.3 explains the liquidity risk measure of Acerbi and Scandolo (2008), and defines the liquidity valuation adjustment. Section 2.5 derives expected shortfall (ES) that will be used as a risk constraint throughout the thesis. Finally, Section 2.6 summarizes the chapter.

2.1 Introduction

A unique value of a portfolio value, as Weber et al. (2012) and Cetin et al. (2004) have pointed out, is an artificial quantity. Asset value may not be unique in practice because of a limited trading volume available in the market. As the trade size increases, the price executed for each position is gradually lower than the best bid price for sell trade, and increasingly higher than the best ask price for buy trade. For example, two portfolios have the same marked-to-market (MtM) value of the portfolio, but one is concentrated on one specific illiquid asset, and has a large number of positions. Suppose that a fragment of the portfolio should be liquidated for client redemption. As the trade size increases, the concentrated portfolio would suffer from a large price discount in the liquidation process, so the portfolio value can be significantly lower than the diversified one.

The profile of the owner of a portfolio is an important factor which affects the portfolio value according to Weber et al. (2012) and Acerbi and Scandolo (2008). The work of Acerbi (2008) shows the Alan and Ben example that the two traders have different portfolio values because of different liquidity policy. Liquidity policy refers to constraints which restrict a portfolio in practice. Alan is an investor that has no cash constraint (i.e. no payment obligations), so the Alan's portfolio can be valued by MtM practice (marking all the positions to the best bid and the ask prices). Ben, on the other hand, has periodic payments (e.g. 25% of the portfolio MtM value) to his debts, so this cash constraint has an impact on the portfolio over the investment period. The portfolio value should be quantified assuming that it can generate cash which is worth 25% of the portfolio MtM value.

One of the interesting issues in recent literature is quantifying a portfolio market liquidity risk. The market liquidity risk in this thesis refers to the risk caused by trading assets in the market where they cannot be sold immediately at their best price. In practice transaction availability is limited, so an investor may observe a gradual price decrease when liquidating a large number of positions. Marginal supply and demand curves (MSDCs) can be used to model this price impact. The MSDCs describe the amount of market liquidity in the form of slope. Market liquidity risk can be measured as the difference between the MtM value and the value observed from the MSDCs (i.e. the cash amount obtained when liquidating positions at prices according to the MSDCs). In practice, a risk management department may force a portfolio to reserve a minimum amount of capital for possible client redemption (e.g. 25% of portfolio MtM value). An investor has to sell assets if the cash component of the portfolio is not sufficient to satisfy the liquidity policy. In the liquidation process, if relatively liquid and low volatility assets are sold in order to avoid the liquidity costs, then the residual portfolio would have relatively illiquid and high volatility securities. Both the market risk as well as illiquidity on the residual portfolio would increase. A minimum liquidation weight can be constrained on the liquid assets; also, the market risk limit can be imposed on the residual portfolio to alleviate this issue. My aim in this thesis is to quantify the portfolio liquidity costs with potential liquidity constraints.

2.2 The marginal supply and demand curve (MSDC)

In this section, we show the characteristics of the MSDC, and illustrate asset prices for a given trade size obtained from the MSDC. An asset value may not be unique in practice due to limited trading volume availability. As the size of a trade increases, it is likely that the price executed for each position is lower than the best bid prices and higher than the best ask prices. We follow the convention that sold units are positive and bought units are negative. In order to model the market depth, which we denote by $m(z)$, is defined as the final price obtained when z units of the asset are sold or $|z|$[1] units are bought. To illustrate this idea we use the empirical MSDC for GlaxoSmithKline at a particular point in time on the morning of 1st March 2007 is shown in Figure 2.1. Note by convention the supply curve has negative trading volume, whilst the demand curve has positive trading volume. It is clear that the MSDC for buying 30,000 shares is $m(-30,000) =$ £14.23, and the effective price achieved for buying 30,000 shares is obtained from the MSDC as £14.22. In contrast, to sell 60,000 shares the MSDC is $m(60,000) =$ £14.16 and the effective selling price is £14.18. Empirical MSDCs have a step-like form as illustrated in Figure 2.1. In order to develop this framework a continuous approximation is convenient.

Formally the MSDC is a map that associates an asset with its available prices for a given trade size, denoted by $m : R \setminus \{0\} \to R$. The two properties of MSDC are

$$m(z_1) \geq m(z_2) \qquad if \quad z_1 < z_2 \tag{2.1}$$

$$m \ is \ cadlag \ for \ z_k < 0 \ and \ ladcag \ for \ z_k > 0. \tag{2.2}$$

Property (2.1) is a no-arbitrage requirement, and property (2.2) implies that ask prices are right continuous with left limits, and bid prices are left continuous with right limits. The best ask (offer) price is denoted as m^-, and the best bid price is represented as m^+, with the bid-ask spread

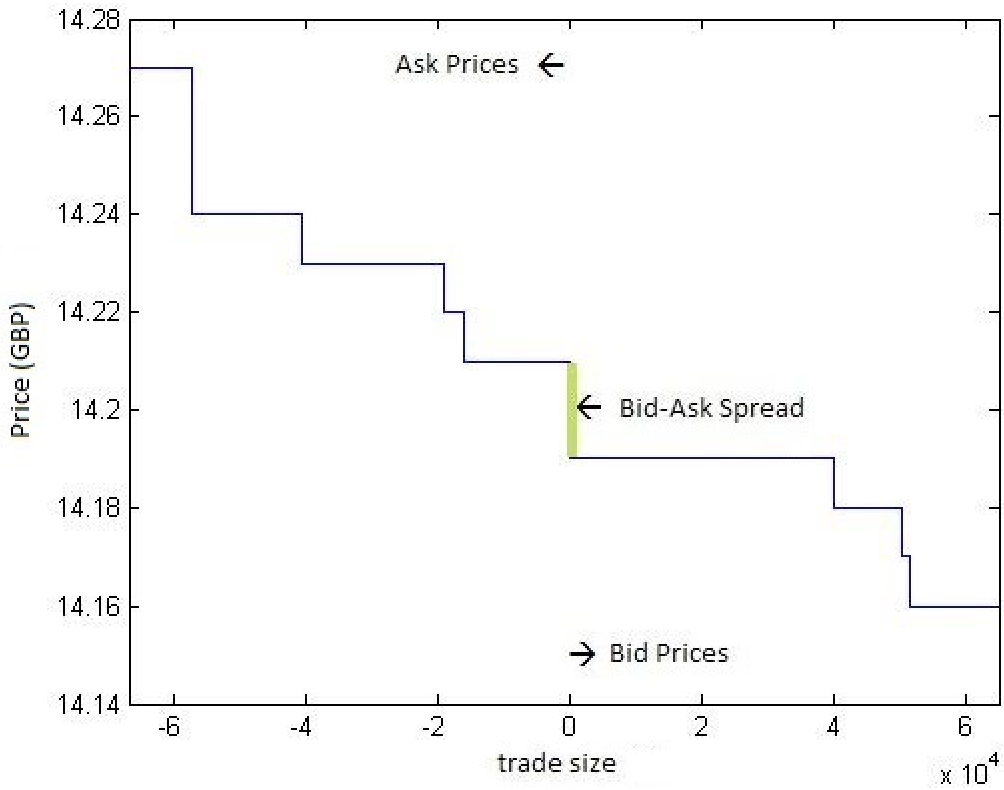
$$\delta m = m^- - m^+. \tag{2.3}$$

For sell transactions ($z_k > 0$), the investor receives $\int_0^{z_k} m_k(s)ds$, while for buy transactions ($z_k < 0$), and pays $\int_{-|z_k|}^{0} m_k(s)ds$. The average realized price depends on the shape of the order book. The MSDC models the supply and demand structures that affect the strength of the price impact. MSDC

[1]Positions for both sell and buy trades should stay positive in order to make the sign of the realized price impact in Power-law MSDC (derived in Chapter 3) correct.

fits the purpose of the thesis because we measure the market liquidity given the liquidity policy, which forces asset liquidation (e.g. asset fire sales due to limited access to financing).

Figure 2.1: Empirical MSDC



This figure plots the empirical MSDC of GlaxoSmithKline on 1st March, 2007 between 10:46am and 10:48am obtained from the London Stock Exchange database.

It is important for practical applications of the model to determine a generic functional form MSDC which depends on readily available market data. In a recent study by Tian et al. (2013), the MSDC is approximated by exponentials, however, this requires high frequency price information in the order book to model a ladder MSDC. In order to determine a practical MSDC lacking this price information, a power-law MSDC is developed in Chapter 3 derived from the seminal work of Almgren et al. (2005) who proposed that impact is a $\frac{3}{5}$ power law of block size.

2.3 Portfolio value given a Liquidity policy

In this section, we determine the value of a long only mutual fund given various liquidity policies. Suppose a significantly large volume needs to be sold because the portfolio has to convert the assets into cash immediately as a result of adverse redemption. If there is limited trading volume available in the market, this may result in significant liquidity costs. On the other hand, if there are sufficiently large quotes available at each price interval, then the transaction can be executed at once without causing significant liquidity costs (Acerbi and Scandolo, 2008).

The fund value under different market situations can be obtained by buying or selling assets at prices obtained from the MSDC. The monetary amount resulting from trading the asset k, $P_k(z_k)$, and the average price obtained $\tilde{S}_k(z_k)$ are given by

$$P_k(z_k) = \int_0^{z_k} m_k(s)ds \tag{2.4}$$

$$\tilde{S}_k(z_k) = \frac{1}{z_k}\int_0^{z_k} m_k(s)ds \tag{2.5}$$

where z_k is the size of a trade, and m_k is the asset price for the sth position obtained from the MSDC, $z_k > 0$ for sell and $z_k < 0$ for buy transactions.

In this thesis we focus on long only funds, however we should say something regarding short positions. In order to short shares, an investor enters into a securities loan transaction (D Avolio, 2002). When shorting shares, collateral is often posted to the lender. The amount is typically 105% of the market value of the stock loan, and is mark-to-market daily. The collateral posted can be either in the form of cash or US Treasury bonds. The collateral is used to buy back the shares if the borrower fails to return them to the lender when the loan is recalled which can be within 48 hours. Therefore, short positions are accompanied by certain contractual obligation for the stock borrower, so the positions are themselves subject to supply and demand. The modeling of these contracts is left for future work. However, if the stock borrowed is guaranteed for a period (typically 3 months), then it may be justifiable to incorporate short positions within the framework developed below.

The value of a portfolio, given a particular liquidation policy, is bounded between two extreme cases. The upper bound is simply the portfolio value marked at the best bid and offer prices; this is denoted as the 'uppermost mark-to-market' (UMtM) value of the portfolio. This is formally define by the operator U:

$$U(\mathbf{p}) = p_0 + \sum_{k=1}^{K} \left(m_k^+ p_k \theta(p_k) + m_k^- p_k \theta(-p_k) \right) \tag{2.6}$$

where p_k is the initial positions in the kth asset (p_0 the cash component of the portfolio), $\mathbf{p} = (p_1, p_2, \ldots, p_K) \in \mathbb{R}^{+K}$ is the vector of the initial K non-cash positions, and $\theta(\cdot)$ is the Heaviside step function. The U operator values the assets at either the best bid or ask price for all positions. This is the case when the trade is filled at once, and the liquidation does not cause any price depreciation by its own selling activity. Another situation where this mark-to-market policy is applicable is for portfolios that can be held for unlimited amount of time because the portfolio is not restricted by any payment obligation, so the asset price is assumed to adopt its theoretical value. In contrast, the lower bound is the value of the portfolio if it is fully and immediately liquidated at the current MSDCs. This value is formally defined by the operator D:

$$D(\mathbf{p}) = p_0 + \sum_{k=1}^{K} \int_0^{p_k} m_k(s) ds \tag{2.7}$$

This is an extreme example which determines the 'fire sell' value of the portfolio and is denoted as the 'liquidation mark-to-market' value of the portfolio (LMtM). The liquidation cost of a portfolio is given by

$$C(\mathbf{p}) = U(\mathbf{p}) - D(\mathbf{p}) > 0 \tag{2.8}$$

The incorporation of the liquidity policy, will result in an *actual* mark-to-market portfolio value that lies between the UMtM and LMtM bounds. The simplest liquidity policy is that of being able to have sufficient cash ($= d$) to meet possible obligations (e.g. adverse redemption). This liquidity policy is defined by

$$\mathscr{L}(d) = \{(p_0, \mathbf{p}) \in P | p_0 \geq d\}, \; d \in \mathbb{R}^+ \tag{2.9}$$

where P is the space of all portfolios. If the cash amount p_0 is not enough to satisfy the cash constraint, then an additional amount of cash needs to be raised by selling a portion of the assets from $\mathbf{p}$. In all that follows we set $p_0 = 0$. The value of the portfolio $V^{\mathscr{L}}(\mathbf{p})$ that complies with a general liquidity policy $\mathscr{L}$ is defined by (Acerbi and Scandolo, 2008)

$$V^{\mathscr{L}}(\mathbf{p}) = sup\{U(\mathbf{b})|\mathbf{b} \in Att(\mathbf{p}) \cap \mathscr{L}\}. \tag{2.10}$$

The value of the portfolio is the supremum of the UMtM of all portfolios, $\mathbf{b}$, attainable from the original portfolio $\mathbf{p}$, is defined as

$$\mathbf{b} = \mathbf{p} - \mathbf{z} + D(\mathbf{z}) \tag{2.11}$$

which satisfy the liquidity constraint, where $D(\mathbf{z})$ is given in (2.7) (i.e. the cash amount resulting from selling the portfolio $\mathbf{z}$). The portfolio $\mathbf{z}$ is obtained by minimizing the liquidity cost function

$$C(\mathbf{z}) = U(\mathbf{z}) - D(\mathbf{z}) = U(\mathbf{p}) - U(\mathbf{b}) \tag{2.12}$$

Different forms of liquidity policies such as a selling restriction or some given limit on the residual market risk can also be imposed on the portfolio. These liquidity policies may influence the proportion of illiquid and high volatility assets that the portfolio is allowed to hold. In addition to the portfolios $\mathbf{p}$ and $\mathbf{b}$, we define $\mathbf{a}$ as the residual portfolio and $\mathbf{z}$ as the portfolio liquidated, which gives

$$\mathbf{a} = \mathbf{p} - \mathbf{z} = \mathbf{b} - D(\mathbf{z}). \tag{2.13}$$

Finally we define the 'liquidity valuation adjustment' (LVA) as the difference in value between the widely employed UMtM (marking the portfolio at best bid and ask) and the actual value of the portfolio under the given liquidity policy, that is, LVA is defined as,

$$LVA^{\mathscr{L}}(\mathbf{p}) = U(\mathbf{p}) - V^{\mathscr{L}}(\mathbf{p}) \geqslant 0. \tag{2.14}$$

2.3.1 Exponential MSDC

In this section, we describe the Lagrangian derivation that computes a portfolio value with a cash constraint using exponential MSDC as shown from the work of Acerbi (2008). The definition of exponential marginal supply and demand curve is given by

$$m_k(z_k) = \begin{cases} m_k^+ e^{-\bar{r}_k^b z_k}, & z > 0 \\ m_k^- e^{-\bar{r}_k^a z_k}, & z < 0 \end{cases} \tag{2.15}$$

where z_k is given in (2.4), m_k^+ and m_k^- are given in (2.6), and $\bar{r}_k^b$ and $\bar{r}_k^a$ are the parameters defining the slope of demand and supply curves for the kth asset. The greater number of positions is sold, the lower bid prices are received. Similarly, the larger number of positions is bought, the higher ask prices are offered. The parameters of the exponential MSDC, which are $\bar{r}_k^b$ and $\bar{r}_k^a$, model this price impact.

When computing the portfolio value, the Lagrangian derivation employs the convention from Table 2.1 on the signs of model parameters: the trading volume z_k and the cash constraint d.

Table 2.1: Convention used in the Lagrangian derivation

	Selling	Buying
Trading volume	$z_k > 0$	$z_k < 0$
Cash constraint	$d > 0$ and $D(z_k) = d > 0$	$d > 0$ and $D(z_k) = -d < 0$

This table reports the convention used in the Lagrangian derivation.

The number of positions sold z_k is positive when assets are sold. In contrast, when positions are bought, the value z_k is negative. Also, the liquidation of positions increase cash amount, so the value of the operator D is positive; when positions are bought, the cash amount is decreased, so the value of the operator D is negative.

2.3.2 Lagrangian with Exponential MSDC for selling assets

The Lagrangian has two parts: an objective function and a constraint term multiplied by a Lagrange multiplier λ, and is written as

$$\Im(\mathbf{z},\lambda) = -\left(U(\mathbf{z}) - D(\mathbf{z})\right) + \lambda\left(D(\mathbf{z}) - d\right). \tag{2.16}$$

The symbol D in (2.16) denotes the value from selling z_k positions for K assets on prices obtained from the exponential demand curve, that is

$$\begin{aligned} D(\mathbf{z}) &= \sum_{k=1}^{K} \int\limits_{0}^{z_k} m_k(s)ds \\ &= \sum_{k=1}^{K} \int_{0}^{z_k} m_k^+ e^{-\bar{r}_k^b s} ds. \end{aligned} \tag{2.17}$$

The bid prices on the trading volume z_k for each asset are obtained from the exponential demand curve which has the slope described by the parameter $\bar{r}_k^b$. The symbol U in (2.18) is the uppermost market-to-market (UMtM) value of the portfolio, which is given by

$$U(\mathbf{z}) = \sum_{k=1}^{K} m_k^+ z_k. \tag{2.18}$$

The objective function $-(U(\mathbf{z}) - D(\mathbf{z}))$ in (2.16) indicates that an investor aims to minimise the liquidity costs from choosing the best selling volume z_k for each k. The constraint term constraint term $D(\mathbf{z}) - d$ represents that the portfolio has a cash constraint d; so, the cash amount d should be realised as a result of liquidation. By substituting equations (2.18) and (2.17) into (2.16), it gives

$$\Im(\mathbf{z},\lambda) = -\left(\sum_{k=1}^{K} m_k^+ z_k - \sum_{k=1}^{K} \int_{0}^{z_k} m_k^+ e^{-\bar{r}_k^b s} ds\right) + \lambda\left(\sum_{k=1}^{K} \int_{0}^{z_k} m_k^+ e^{-\bar{r}_k^b s} ds - d\right) \tag{2.19}$$

The extreme point that minimises the difference of U and D can be found by differentiating the Lagrangian (2.19) on z_k. The first derivative with respect to z_k in (2.19) gives

$$\frac{\partial \Im}{\partial z_k} = 0 = m_k^+ \left(e^{-\bar{r}_k^b z_k} - 1\right) + \lambda\left(m_k^+ e^{-\bar{r}_k^b z_k}\right) \tag{2.20}$$

and

$$m_k^+\left(e^{-\bar{r}_k^b z_k}-1\right)+\lambda\left(m_k^+ e^{-\bar{r}_k^b z_k}\right)=0$$
$$(1+\lambda)\,m_k^+ e^{-\bar{r}_k^b z_k}=m_k^+$$
$$log\left\{(1+\lambda)\,m_k^+ e^{-\bar{r}_k^b z_k}\right\}=log\left(m_k^+\right).$$

Rearranging the equation (2.20) on the optimal volume to sell for each asset z_k^* yields

$$z_k^*=\frac{log(1+\lambda)}{\bar{r}_k^b}. \tag{2.21}$$

The first derivative of the Lagrangian (2.19) with respect to λ is written as

$$\frac{\partial\Im}{\partial\lambda}=0=\sum_{k=1}^K\left\{\frac{m_k^+}{\bar{r}_k^b}\left[1-e^{-\bar{r}_k^b z_k}\right]\right\}-d. \tag{2.22}$$

By substituting an equation (2.21) into (2.22), it gives

$$\sum_{k=1}^K\left\{\frac{m_k^+}{\bar{r}_k^b}\left[1-\frac{1}{1+\lambda}\right]\right\}=d. \tag{2.23}$$

Rearranging the equation (2.23) on λ yields

$$\lambda=\frac{d}{\sum_{k=1}^K\frac{m_k^+}{\bar{r}_k^b}-d}. \tag{2.24}$$

By substituting the value of λ in (2.24) into (2.21), and the optimal volume to sell for each asset z_k^* (2.21) into (2.10) yields the portfolio value with a cash constraint (Acerbi, 2008), that is

$$
\begin{aligned}
V^{\mathscr{L}}(\mathbf{p}) &= U\left(\mathbf{p}-\mathbf{z}+D(\mathbf{z})\right) \\
&= U(\mathbf{p}) - \sum_{k=1}^{K} m_k^+ z_k^* + d \\
&= d + U(\mathbf{p}) - \sum_{k=1}^{K} m_k^+ \frac{log(1+\lambda)}{\bar{r}_k^b} \\
&= d + U(\mathbf{p}) - log\left(\frac{\sum_{k=1}^{K}\frac{m_k^+}{\bar{r}_k^b}}{\sum_{k=1}^{K}\frac{m_k^+}{\bar{r}_k^b} - d}\right)\sum_{k=1}^{K}\frac{m_k^+}{\bar{r}_k^b}.
\end{aligned}
$$

2.4 Risk measure with liquidity risk

In this section the properties of coherent risk measure (CRM) and coherent portfolio risk measures (CPRM) are described (Acerbi and Scandolo, 2008; Follmer and Schied, 2010). Risk measure ρ quantifies the uncertainty in the future value of the portfolio. From the point of financial regulators, a number given by $\rho(A)$ represents a capital requirement that should be raised in order to make the portfolio A acceptable.

Coherent risk measure shows basic principles that sensible risk measures should satisfy. It satisfies the following axioms. The variable A in CRM denotes portfolio payoff profile (i.e. sum of P&L random variables).The symbol λ is the scaling constant, and d is the risk-free cash portfolio.

- (M) Monotonicity: If $A_1 \geq A_2$, then $\rho(A_1) \leq \rho(A_2)$.
- (TC) Translational covariance: If $d \in \mathbb{R}$, then $\rho(A+d) = \rho(A) - d$.
- (PH) Positive homogeneity: If $\lambda \geq 0$, then $\rho(\lambda A) = \lambda \rho(A)$.
- (S) Subadditivity: $\rho(A_1+A_2) \leq \rho(A_1) + \rho(A_2)$.

Monotonicity (M) means that if the payoff profile of the portfolio A_2 is worse than the profile of the portfolio A_1, then the greater risk is associated with portfolio A_2 than the portfolio A_1. The larger capital should be required for the portfolio A_2 than the portfolio A_1. Translational covariance (TC) property implies that if the cash amount d is added to the portfolio, then the risk of the portfolio should be decreased by the same amount. The risk reduced by adding one unit of capital is $\rho(1) = -1$. If d units of cash is added to the portfolio, then the reduction in capital requirement is $\rho(A+m) = \rho(A) + \rho(m) = \rho(A) - m$. Positive homogeneity (PH) indicates that the risk of the portfolio having an increased amount of the assets λA is equivalent to the increased risk λ of the initial portfolio A. Subadditivity (S) refers to a situation that the risk of a diversified portfolio $(A_1 + A_2)$ should be lower than the sum of the risks of the two individual portfolios A_1 and A_2.

Translational covariance (TC) property of the coherent risk measure, as Acerbi and Scandolo (2008) point out, may not be compatible with the liquidity risk. For instance, in illiquid market, the portfolio risk decreased may be larger than the amount of cash added. The value of cash can be larger than its actual amount when funding liquidity is very tight in liquidity crisis. If the cash amount d is added to the portfolio, so the cash constraint is decreased by d, then the risk reduced can be greater than the amount d. This is because a portfolio can meet the collateral requirement with the cash added instead of selling assets to generate cash, thus it can avoid the potential liquidity costs. Another violation in the coherent risk measure is (PH) and (S) axioms. The portfolio risk increase may not be the same as the amount of illiquid assets added. For example, if illiquid assets are doubled in the portfolio, then the risk of the portfolio with the doubled amount of the illiquid assets would be higher than the doubled risk of the portfolio with initial amount of the illiquid assets.

Coherent portfolio risk measure, on the other hand, is compatible with liquidity risk. According to Acerbi and Scandolo (2008), coherent portfolio risk measure, $\rho^{\mathscr{L}}$, is defined as

$$\rho^{\mathscr{L}}(\mathbf{p}) = \rho\left(V^{\mathscr{L}}(\mathbf{p})\right) \tag{2.25}$$

where $V^{\mathscr{L}}(\mathbf{p})$ is given in (2.10). Unlike CRM, CPRM employs portfolio value function associating a liquidity policy $\mathscr{L}$ chosen on the vector space of portfolios $\mathbf{p}$. Coherent portfolio risk measure satisfies the following properties.

- (M) Monotonicity: If $V^{\mathscr{L}}(\mathbf{p_1}) \geq V^{\mathscr{L}}(\mathbf{p_2})$, then $\rho^{\mathscr{L}}(\mathbf{p_1}) \leq \rho^{\mathscr{L}}(\mathbf{p_2})$.
- (TS) Translational subvariance: If $d \in \mathbb{R}$, then $\rho^{\mathscr{L}}(\mathbf{p}+d) \leq \rho^{\mathscr{L}}(\mathbf{p}) - d$.
- (C) Convexity: $\rho^{\mathscr{L}}(\theta\mathbf{p_1} + (1-\theta)\mathbf{p_2}) \leq \theta\rho^{\mathscr{L}}(\mathbf{p_1}) + (1-\theta)\rho^{\mathscr{L}}(\mathbf{p_2})$, for $0 \leq \theta \leq 1$.

Monotonicity (M) still holds as in CRM. For given the liquidity policy, if the portfolio value on positions $\mathbf{p_1}$ is higher than the value on positions $\mathbf{p_2}$, then the risk of portfolio $\mathbf{p_1}$ is lower than the portfolio $\mathbf{p_2}$, so the capital requirement given by $\rho^{\mathscr{L}}$ on the portfolio $\mathbf{p_1}$ would be less than the amount on the portfolio $\mathbf{p_2}$. Translational subvariance (TS) means that the risk of a portfolio with an additional cash amount d is lower than the portfolio risk decreased by the amount of cash d. One additional unit of cash can be worth more than its nominal value in illiquid market, so the risk decreases more than proportionally when the fixed quantity d is added to the portfolio value. Convexity property (C) shows the "granularity effect" of the portfolio. If two illiquid portfolios, which are both under the liquidation operator, are combined with the weight θ, then the risk becomes lower than the weighted sum of the risk of the two individual illiquid portfolios. This is because relatively liquid assets can be chosen to sell from the portfolio with a larger set of assets on the given liquidity policies, so it incurs the lower liquidity costs. That is, there is a diversification benefit in the portfolio liquidity risk.

2.5 Expected shortfall derivation

In this section, we show expected shortfall (ES) derivation. The ES is the average size of the loss given VaR has been exceeded. The ES for the confidence level c is written as

$$ES_c = \frac{1}{1-c}\int_c^1 VaR_{c'}dc'. \tag{2.26}$$

The excess losses over the threshold (e.g. the 95th percentile of P&L distribution) y is defined as

$$y_{(i)} = r_{(i)} - u, \qquad i = 1,...,N_u \tag{2.27}$$

where r is the returns (i.e. losses become positive, and profits become negative), u is the threshold level, and N_u is the number of losses greater than the threshold. The conditional probability distribution for the excess losses given that those losses are greater than the threshold is given by

$$
\begin{aligned}
F_u(y) &= Pr(r-u<y|r>u) \\
&= \frac{Pr(r-u<y \quad and \quad r>u)}{Pr(r>u)} \\
&= \frac{Pr(u<r<y+u)}{Pr(r>u)}.
\end{aligned} \tag{2.28}
$$

The probability that the return r falls between the threshold u and the excess losses added on the threshold is written as

$$Pr(u<r<y+u) = F(y+u) - F(u) \tag{2.29}$$

, and the probability that the return r is greater than the threshold level u is given by

$$Pr(r>u) = 1 - F(u). \tag{2.30}$$

The distribution function in (2.28) can also be written as

$$F_u(y) = \frac{F(y+u) - F(u)}{1 - F(u)}. \tag{2.31}$$

The standard cumulative distribution function (CDF) of a Generalised Pareto Distribution (GPD) is defined as ($y>0$)

$$G_{\xi,\beta}(y) = 1 - \left(1 + \frac{\xi y}{\beta}\right)^{-\frac{1}{\xi}}, \quad \xi \neq 0 \tag{2.32}$$

where ξ is the shape parameter, and β is the scale parameter. The shape parameter refers to the tail-heaviness of the distribution, and the scale parameter refers to the dispersion of the extreme data. According to the Pickands-Balkema-de Haan Theorem (Balkema and de Haan, 1974; Pickands, 1975), the distribution of excess losses over the threshold tend to follow a GPD, that is

$$F_u(y) \approx G_{\xi,\beta}(y). \tag{2.33}$$

Then the distribution function in (2.31) can be written as

$$F(r) = [1 - F(u)]\, G_{\xi,\beta}(r-u) + F(u). \tag{2.34}$$

The probability that the return r is less than the threshold u is given by

$$\begin{aligned} F(u) &= Pr(r < u) \\ &= \frac{N - N_u}{N} \end{aligned} \tag{2.35}$$

where N is the total number of returns, $N_u$is given in (2.27).

By substituting equations (2.32) and (2.35) into (2.34), it gives

$$\begin{aligned} F(r) &= \left[1 - \left(1 - \frac{N_u}{N}\right)\right]\left[1 - \left(1 + \frac{\xi\,(r-u)}{\beta}\right)^{-\frac{1}{\xi}}\right] + \left(1 - \frac{N_u}{N}\right) \\ &= 1 - \frac{N_u}{N}\left(1 + \frac{\xi\,(r-u)}{\beta}\right)^{-\frac{1}{\xi}}. \end{aligned} \tag{2.36}$$

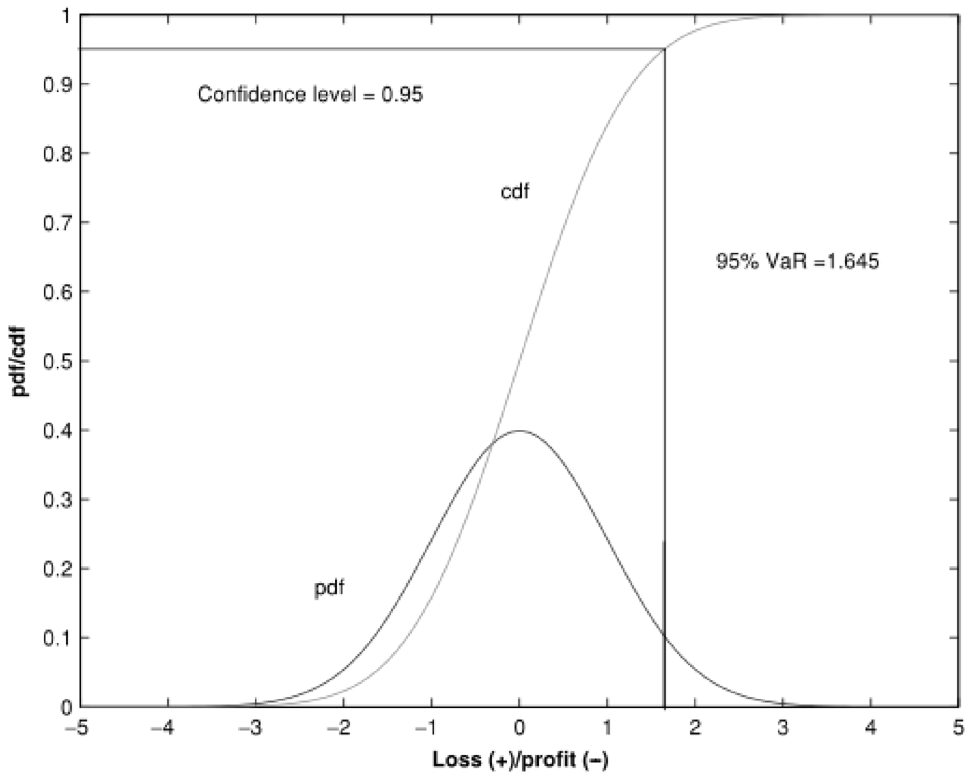


Figure 2.2: 95% VaR on profits/losses from standard normal PDF/CDF (source: Dowd, 2002)

From Figure 2.2, VaR at confidence level c satisfies

$$F(VaR_c) = c. \tag{2.37}$$

, so an equation (2.36) can be written as

$$c = 1 - \frac{N_u}{N}\left(1 + \frac{\xi\,(VaR_c - u)}{\beta}\right)^{-\frac{1}{\xi}}. \tag{2.38}$$

Rearranging (2.38) on VaR_c yields

$$VaR_c = \left(u - \frac{\beta}{\xi}\right) + \frac{\beta}{\xi}\left[\left(\frac{N}{N_u}(1-c)\right)^{-\xi}\right]. \tag{2.39}$$

The integral of $VaR_{c'}$ over all possible values between the confidence level c and the maximum confidence level 1 from (2.26) is written as

$$\int_c^1 VaR_{c'}dc' = \left(u - \frac{\beta}{\xi}\right)\int_c^1 dc' + \frac{\beta}{\xi}\left(\frac{N}{N_u}\right)^{-\xi}\int_c^1 \left[\frac{1}{(1-c')^{\xi}}\right]dc'. \tag{2.40}$$

Let $l = 1 - c'$, then $\frac{dl}{dc'} = -1$, so

$$dl = -dc'. \tag{2.41}$$

The integral in the first term in (2.40) yields

$$\int_c^1 dc' = 1 - c. \tag{2.42}$$

Let the integral in the second term in (2.40) as $I = \int_c^1 \left[\frac{1}{(1-c')^{\xi}}\right]dc'$. Using the result in (2.41), it gives

$$\begin{aligned} I &= \int_0^{1-c} l^{-\xi} dl \\ &= \left[\frac{1}{1-\xi} l^{1-\xi}\right]_0^{1-c} \\ &= \frac{(1-c)^{1-\xi}}{1-\xi}. \end{aligned} \tag{2.43}$$

By substituting (2.42) and (2.43) into (2.40), it can be written as

$$\int_c^1 VaR_{c'}dc' = \left(u - \frac{\beta}{\xi}\right)(1-c) + \frac{\beta}{\xi}\left(\frac{N}{N_u}\right)^{-\xi}\left(\frac{(1-c)^{1-\xi}}{1-\xi}\right). \tag{2.44}$$

By substituting (2.40) into (2.26), it gives

$$
\begin{aligned}
ES_c &= \frac{1}{1-c}\int_c^1 VaR_{c'}dc' \\
&= \frac{1}{1-c}\left[\left(u-\frac{\beta}{\xi}\right)(1-c)+\frac{\beta}{\xi}\left(\frac{N}{N_u}\right)^{-\xi}\left(\frac{(1-c)^{1-\xi}}{1-\xi}\right)\right] \\
&= u-\frac{\beta}{\xi}+\frac{\beta}{\xi}\left(\frac{N}{N_u}\right)^{-\xi}\frac{(1-c)^{-\xi}}{1-\xi}.
\end{aligned}
\tag{2.45}
$$

An equation in (2.39) can be written as

$$
\frac{VaR_c}{1-\xi} = \frac{\beta}{\xi}\left(\frac{N}{N_u}\right)^{-\xi}\frac{(1-c)^{-\xi}}{1-\xi}+\frac{u}{1-\xi}-\frac{\beta}{\xi(1-\xi)}.
\tag{2.46}
$$

Using the result in (2.46), an equation (2.45) becomes

$$
\begin{aligned}
ES_c &= u-\frac{\beta}{\xi}+\frac{\beta}{\xi}\left(\frac{N}{N_u}\right)^{-\xi}\frac{(1-c)^{-\xi}}{1-\xi} \\
&= u-\frac{\beta}{\xi}+\frac{\beta}{\xi}\left(\frac{N}{N_u}\right)^{-\xi}\frac{(1-c)^{-\xi}}{1-\xi}+\frac{u}{1-\xi}-\frac{\beta}{\xi(1-\xi)}-\frac{u}{1-\xi}+\frac{\beta}{\xi(1-\xi)} \\
&= u-\frac{\beta}{\xi}+\frac{VaR_c}{1-\xi}-\frac{u}{1-\xi}+\frac{\beta}{\xi(1-\xi)} \\
&= \frac{VaR_c}{1-\xi}+\frac{\beta-u\xi}{1-\xi}.
\end{aligned}
\tag{2.47}
$$

2.6 Summary

In this chapter, we introduce the liquidity risk framework of Acerbi and Scandolo (2008). In particular, the chapter focuses on the liquidity valuation adjustment (LVA) method that measures the liquidity risk for a long equity portfolio with potential liquidity constraints.

The LVA method is able to incorporate liquidity restrictions that can be imposed externally on the portfolio owner. The value of an asset in the method is obtained from marginal supply and demand curves (MSDCs) for the given trading volume, which depends on the liquidity restrictions. The amount of market liquidity in the MSDCs is described in the form of slope. The liquidity mark-to-market value is estimated by selling positions on the prices obtained from the MSDCs.

In the next chapter, we make use of the LVA in our numerical examples to demonstrate the importance associated with the proper incorporation of the liquidity policy in order to have a consistent liquidity risk valuation. This is a novel application of liquidity risk measure in finance. The classical asset pricing theory follows mark-to-market valuation model, and ignores the role of liquidity policies in the portfolio valuation.

Chapter 3

Portfolio valuation under liquidity and expected shortfall constraints

In this chapter, we quantify the potential cost of liquidity constraints on a long equity portfolio using the liquidity risk framework of Acerbi and Scandolo (2008) and a power-law MSDC derived in this thesis from the results of Almgren et al. (2005). We focus on the pricing of liquidity risk for buy side mutual funds. This not only refers to the risk associated with trading assets in a market where they cannot be sold immediately at their best bid or offer prices, but also accounting for different liquidity policies applied externally on the fund; the two are intertwined (Acerbi and Scandolo, 2008; Acerbi and Finger, 2010). We look at the relationship between the portfolio value and constraints the portfolio may be subject to in practice. The three typical liquidity policies, namely cash, minimum weight and portfolio expected shortfall (ES), are constrained in the portfolio liquidity costs estimation. The results suggest that liquidity policies has an impact on the portfolio in varying degree because the liquidation process of illiquid and high volatility assets changes according to the liquidity restrictions.

The chapter is structured as follows. In Section 3.1, we motivate the pricing of liquidity risk when assets have non-unique prices. In Section 3.2, we derive the power-law MSDC from the results of Almgren et al. (2005), and describe the features of the model. In Section 3.3, we show the Lagrangian derivation with a cash constraint. In Section 3.5, we compute the portfolio liquidity risk subject to various liquidity constraints. Finally, Section 3.6 concludes.

3.1 Introduction

The work of Cetin et al. (2004) highlighted the fact that assets do not, in general, have unique values because of limited trading volume in the market. Suppose a fund manager has to liquidate (or purchase) a large volume of an illiquid asset, then the fund can suffer a sizable discount (increase) in its quoted bid (ask) price. This liquidity cost can be estimated using various means, in particular the Marginal Supply and Demand Curve (MSDC). The slope of the MSDC reflects the degree of liquidity in an asset price. The MSDC contains the possible prices that are gradually lower (higher) than the best bid (ask) price as the trade size increases. If the liquidity cost incurred is measured by trading the positions to the bid (ask) prices from the curve, then the liquidity risk can be estimated. In addition, Acerbi and Scandolo (2008) demonstrate that different liquidity policies result in different portfolio values for portfolios that contain the same assets. This is an important insight and they are then able to develop a general framework of liquidity risk, which in particular leads to a coherent measure of liquidity risk. Csoka and Herings (2014) recently developed this framework to the problem of risk allocation including liquidity risk. A key ingredient of this framework is the fund's liquidity policy which refers to constraints (possible external) which a portfolio is often subject to in practice. The liquidity policy is an important factor when measuring the liquidity risk because the estimated liquidity costs can vary greatly according to different liquidity policies. The liquidity policies the portfolio might face in practice include the following: reserving a certain amount of cash; requiring the ability to generate through liquidation a (maximum) amount of cash; restricting the liquidation of its liquid assets with a minimum weight; and constraining the residual portfolio via a given market risk limit.

Quantifying the fund liquidity risk under a particular liquidity policy is therefore an important consideration. The liquidity policy in effect constrains the available possible liquidation processes. If relatively liquid and low volatility assets were sold in order to avoid liquidity costs, then the residual portfolio could result in a concentration of illiquid and high volatility securities, which may result in higher (relative) market risks. In order to address this problem, the liquidation process needs to be changed. The liquidity risk, therefore, needs to be estimated by considering the liquidation process changed according to the liquidity policies.

We quantify the potential cost of liquidity constraints on a typical mutual fund and compute the value given the liquidity policy. My contribution is to formulate and combine cash, minimum weight and portfolio expected shortfall (ES) liquidity policies on a typical mutual fund (long equity portfolio), and to quantify the portfolio liquidity risk in the presence of liquidity constraints using the liquidity risk framework of Acerbi and Scandolo (2008) and a power-law MSDC derived in this chapter from the results of Almgren et al. (2005). We correct a formula presented by Finger (2011).

3.2 The power-law marginal supply and demand curve (MSDC)

In this section, we derive the Power-Law MSDC from the results of Almgren et al. (2005), and illustrate the asset prices obtained from the Power-Law MSDC, which encapsulates the non-uniqueness of asset prices. The power-law MSDC is a continuous approximation of the MSDC. It incorporates several readily available trading variables of an asset to describe the price changes for a given trade size. If the asset has a low volatility, then the curves tend to be flatter. In contrast, if the asset has a high volatility, then the curves are likely to be steeper. Moreover, if the asset has a low (high) average daily volume, then the curves will be steeper (flatter).

The power-law MSDC can reflect these characteristic of asset price changes because it describes the level of liquidity of the asset with the slope of curve using various trading parameters. The power-law MSDC has econometric parameters; the daily volatility and average daily volume to characterize the slope of the curve. Volatility, as Almgren et al. (2005) note, is empirically the most influential factor on the market impact cost. The flatter (steeper) the slope of the power-law demand curve, the more liquid (illiquid) the asset is. The asset has perfect liquidity if the curve is flat. As the size of trade is larger, an asset with the steeper MSDCs result in greater liquidity costs.

The power-law MSDC model is derived in the following way. The power-law MSDC is used to estimate the price impact for the given trade size z. The average realised price to trade z shares ($z > 0$ for a sell, $z < 0$ for a buy) is given by

$$\tilde{S}(z) = \frac{1}{z}\int_{0}^{z} m(s)ds. \tag{3.1}$$

This is the key ingredient required to make the connection to the seminal work of Almgren et al. (2005). Note that the sign convention employed by Almgren et al. (2005) where buy orders are positive and sell orders negative, is the opposite to the convention we employ in this chapter (we follow Acerbi and Scandolo (2008)). Differentiating (3.1) with respect to z results in

$$m(z) = z \cdot \tilde{S}'(z) + \tilde{S}(z). \tag{3.2}$$

The realized price impact $\bar{J}(z)$ for a sell trade is (Almgren et al., 2005)

$$\bar{J}(z) = \frac{m^+ - \tilde{S}(z)}{m^+}. \tag{3.3}$$

From equation (3.3), the average realized price $\tilde{S}(z)$ is

$$\tilde{S}(z) = m^+ - m^+ \bar{J}(z) \tag{3.4}$$

and the first derivative is given by

$$\tilde{S}'(z) = -m^+ \bar{J}'(z). \tag{3.5}$$

Substituting (3.4) and (3.5) into (3.2) yields

$$m(z) = m^+(1 - z\bar{J}'(z) - \bar{J}(z)). \tag{3.6}$$

The realized price impact (3.3) for both sell and buy trades is defined as (Almgren et al., 2005)

$$\bar{J}(z) = \frac{1}{2}\sigma\gamma\left(\frac{|z|}{V}\right)\left(\frac{\theta}{V}\right)^{\frac{1}{4}} + \sigma\eta\left(\frac{|z|}{VT}\right)^{\frac{3}{5}} + < noise > . \tag{3.7}$$

The $< noise >$ represents the error in explaining the cost due to the volatility of the price movements from pre-trade to post-trade. The noise term has a Gaussian distribution with mean 0, and standard deviation $\sigma\sqrt{T_{post}}$, where σ is the daily volatility, and T_{post} is the time one half hour after the last execution (Almgren et al., 2005); the error term averages to zero and hence is dropped from the analysis.

Differentiating the impact cost expression $\bar{J}(z)$ in (3.7) with respect to z gives (for $z > 0$)

$$\bar{J}'(z) = \frac{1}{2}\sigma\gamma\left(\frac{1}{V}\right)\left(\frac{\theta}{V}\right)^{\frac{1}{4}} + \frac{3}{5}\sigma\eta\left(\frac{1}{VT}\right)^{\frac{3}{5}} z^{-\frac{2}{5}}. \tag{3.8}$$

Substituting (3.7) and (3.8) into (3.6), we obtain the power-law MSDC as

$$m(z) = m^{+}\left(1 - \gamma\sigma\left(\frac{z}{V}\right)\left(\frac{\theta}{V}\right)^{\frac{1}{4}} - \frac{8}{5}\eta\sigma\left(\frac{z}{VT}\right)^{\frac{3}{5}}\right),\ z > 0. \tag{3.9}$$

The realized price impact for buy trade is

$$\bar{J}(z) = \frac{\tilde{S}(z) - m^{-}}{m^{-}}. \tag{3.10}$$

Rearranging (3.10) in terms of the average realized price $\tilde{S}(z)$ is

$$\tilde{S}(z) = m^{-} + m^{-}\bar{J}(z) \tag{3.11}$$

and differentiating (3.11) with respect to z , we get

$$\tilde{S}'(z) = m^{-}\bar{J}'(z). \tag{3.12}$$

By substituting (3.11) and (3.12) into (3.2), it reads

$$m(z) = m^{-}\left(1 + z\bar{J}'(z) + \bar{J}(z)\right). \tag{3.13}$$

Then, substituting (3.7) and (3.8) into (3.13), we obtain

$$m(z) = m^{-}\left(1 + \gamma\sigma\left(\frac{|z|}{V}\right)\left(\frac{\theta}{V}\right)^{\frac{1}{4}} + \frac{8}{5}\eta\sigma\left(\frac{|z|}{VT}\right)^{\frac{3}{5}}\right),\ z < 0. \tag{3.14}$$

The Power-Law MSDC of asset k is given by

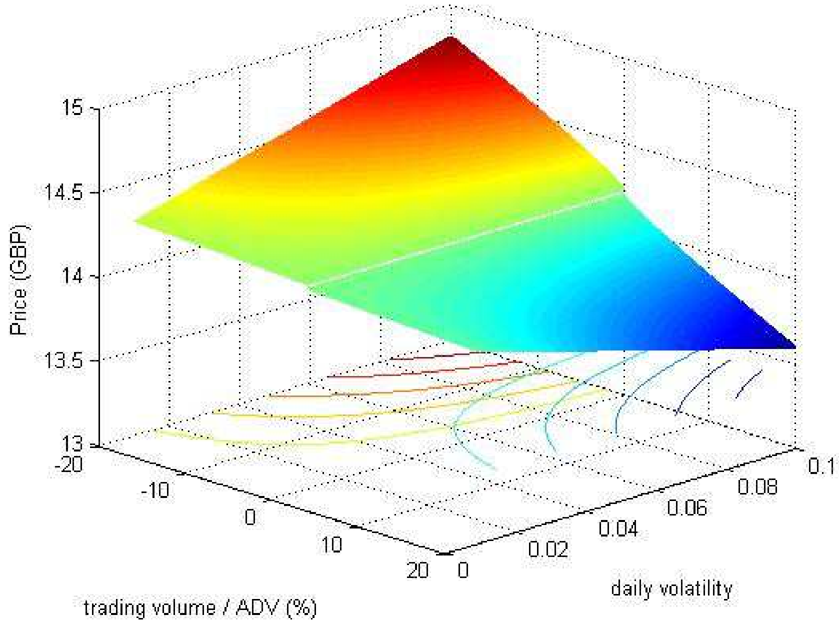
$$m_k(z) = \begin{cases} m_k^+ \left(1 - \gamma\sigma_k \frac{z}{V_k}\left(\frac{\theta_k}{V_k}\right)^{\frac{1}{4}} - \frac{8}{5}\eta\sigma_k\left(\frac{z}{V_k T}\right)^{\frac{3}{5}}\right) & , z > 0 \\ m_k^- \left(1 + \gamma\sigma_k \frac{|z|}{V_k}\left(\frac{\theta_k}{V_k}\right)^{\frac{1}{4}} + \frac{8}{5}\eta\sigma_k\left(\frac{|z|}{V_k T}\right)^{\frac{3}{5}}\right) & , z < 0 \end{cases} \tag{3.15}$$

where z is the number of shares traded, σ_k is the daily volatility, θ_k is the outstanding shares in issue, V_k is the average daily volume, T is the trade duration in days, and γ and η are the estimated market impact coefficients (See Almgren et al. (2005)). Equation (3.15) is a corrected version of that given by Finger (2011)[1]. It produces a demand curve when $z > 0$, and a supply curve when $z < 0$. In what follows we use the empirical values of the γ and η as 0.314 and 0.142 respectively determined by Almgren et al. (2005). The range of applicability for the model is discussed by Almgren (2008) and in particular, although the model is universally valid, it is poorer for larger trades. The applications we consider stress the model by varying the portfolio upper mark-to-market value (using best bid price) from £50M to £200M. Nevertheless, the model is straightforward to implement and as noted by Almgren (2008), despite its limitations (e.g. the order size up to 10% of average daily volume is considered in the model of Almgren (2008)), provides a useful approach to modeling liquidity costs.

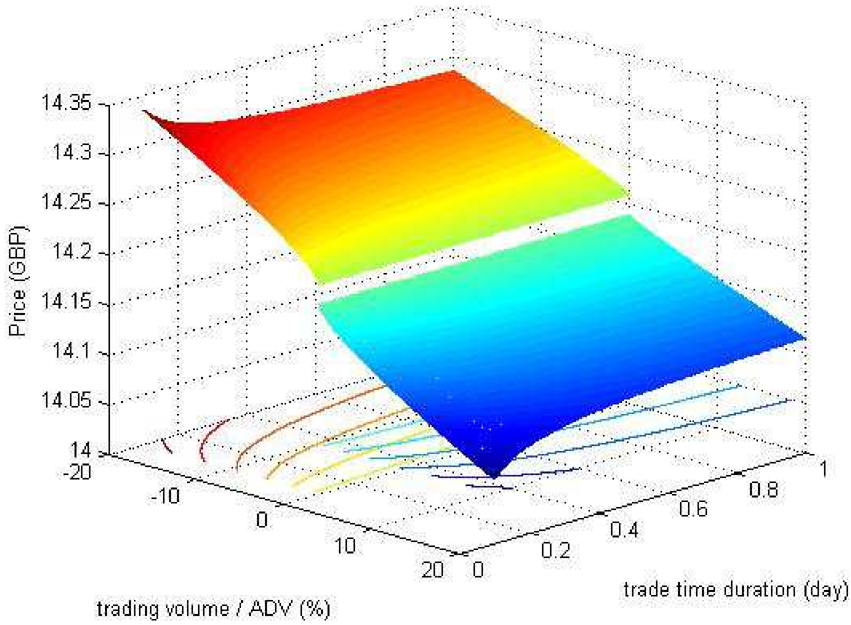
Figure 3.1, illustrates the behaviour of the MSDC when the trading volume (as a percentage of the average daily volume) varies from 0 to 20% and either daily volatility or trade time (in days) varies. It is noted that the slope of the power-law MSDC increases as the daily volatility rises, due to larger potential price deviations, and decreases as the daily volatility decreases. Moreover, as the trade time duration becomes shorter, the slope of the power-law MSDC becomes increasingly steeper. This is because when a large volume is traded in a short time, the price movement caused by the temporary market impact is greater. If a longer trade time is permitted to liquidate positions, then this temporary market impact can be mitigated, and the power-law MSDC flattens.

[1] In the formula by Finger (2011), the transient market impact term is incorrectly defined. The variable representing the number of stocks traded in the transient impact term should be removed as we have derived the Power-Law MSDC in equation (3.15).

Figure 3.1: Behaviour of Power-Law MSDC for different values of daily volatility and trade time



(a)



(b)

This figure plots the Power-law MSDC with daily volatility varying from 1% to 10 % and trade time duration (days) fixed as 0.5 (a), trade time duration (day) from 0.1 to 0.9 and daily volatility fixed as 1.4% (b). The trading volume as a proportion of the average daily volume varies from 0 to 20%. The bid and ask prices are set to 14.19 and 14.21. The average daily volume is 10.87 million, the total number of shares in issue is 4903 million are the trading variables for GlaxoSmithKline collected from Datastream as of 1st of February, 2013.

3.3 Lagrangian with Power-Law MSDC

In this section, we describe the Lagrangian (Chow, 1997) method used to solve the portfolio liquidity costs minimisation problem. We aim to minimise liquidity costs when selling a fragment of the portfolio. As a result of the liquidation, a certain cash amount that satisfies the cash requirement should be received. This leads us to a convex optimisation problem. From equation (2.19) in Chapter 2, we construct a Lagrangian, and this enables us to

$$\Im(\mathbf{z},\lambda) = O(\mathbf{z}) + \lambda\left(D(\mathbf{z}) - d\right) \tag{3.16}$$

We can specify for general equality and inequality constraints in the Lagrangian by having additional Lagrangian multipliers (Fletcher, 1987). For instance, we can impose the portfolio expected shortfall limit on the residual portfolio $\mathbf{a}$ at confidence level c by adding the term $\mu\left(ES_c(\mathbf{a}) - 0.035\right)$ in (3.16), this will be discussed in the following section. The objective function $O(\mathbf{z})$ allows the determination of the optimum portfolio $\mathbf{a}$. The term $D(\mathbf{z}) - d$ constrains the result of liquidating portfolio $\mathbf{z}$ to produce the cash amount d.

The Lagrangian (3.16) is now determined by explicit use of the power law MSDC (3.15)

$$\begin{aligned} D(\mathbf{z}) &= \sum_{k=1}^{K} \int_{0}^{z_k} m_k(s)ds \\ &= \sum_{k=1}^{K} \int_{0}^{z_k} m_k^{+}\left(1 - \gamma\sigma_k \frac{s}{V_k}\left(\frac{\theta_k}{V_k}\right)^{\frac{1}{4}} + \frac{8}{5}\eta\sigma_k\left(\frac{|s|}{V_kT}\right)^{\frac{3}{5}}\right)ds \\ &= \sum_{k=1}^{K} m_k^{+}\left\{z_k - \sigma_k\left(\gamma\left(\frac{\theta_k}{V_k}\right)^{\frac{1}{4}}\frac{1}{2V_k}z_k^2 + \eta\left(\frac{1}{V_kT}\right)^{\frac{3}{5}} z_k^{\frac{8}{5}}\right)\right\} \end{aligned} \tag{3.17}$$

and

$$U(\mathbf{z}) = \sum_{k=1}^{K} m_k^{+} z_k \tag{3.18}$$

where z_k is a number of stocks sold for the kth asset , By substituting equations (3.17) and (3.18) into equation (3.16), we obtain

$$\Im(\mathbf{z},\lambda) = \sum_{k=1}^{K} m_k^+ z_k - \sum_{k=1}^{K} m_k^+ \left\{ z_k - \sigma_k \left(\gamma \left(\frac{\theta_k}{V_k} \right)^{\frac{1}{4}} \frac{1}{2V_k} z_k^2 + \eta \left(\frac{1}{V_k T} \right)^{\frac{3}{5}} z_k^{\frac{8}{5}} \right) \right\} \\ + \lambda \left[\sum_{k=1}^{K} m_k^+ \left\{ z_k - \sigma_k \left(\gamma \left(\frac{\theta_k}{V_k} \right)^{\frac{1}{4}} \frac{1}{2V_k} z_k^2 + \eta \left(\frac{1}{V_k T} \right)^{\frac{3}{5}} z_k^{\frac{8}{5}} \right) \right\} - d \right]. \tag{3.19}$$

In this study, sequential quadratic programming (SQP), which is based on the Newton-Raphson method (Israel, 1966; Press et al., 2002), is used to find the optimum portfolio $\mathbf{z}$ and the Lagrangian multiplier λ. For details, see Nocedal and Wright (2006). Similar results are obtained for the additional liquidity policies discussed in the following section but these are not included for brevity.

Matlab 2012 Global Optimization Toolbox package that provides the SQP method is employed to solve the optimization problem. Matlab codes run on 64 bit Windows 7 operating system with Intel Core 2 Duo E8600 3.33 GHz processor. It appears that the SQP solves the optimisation problem more accurately than other non-linear programming methods as the smaller Lagrange multiplier values are produced. In order to avoid the issue of the local minima convergence, GlobalSearch method is combined with the SQP. The GlobalSearch method helps to find out the global optimum with the use of scatter-search algorithm (Glover et al., 2006; MathWorks, 2012).

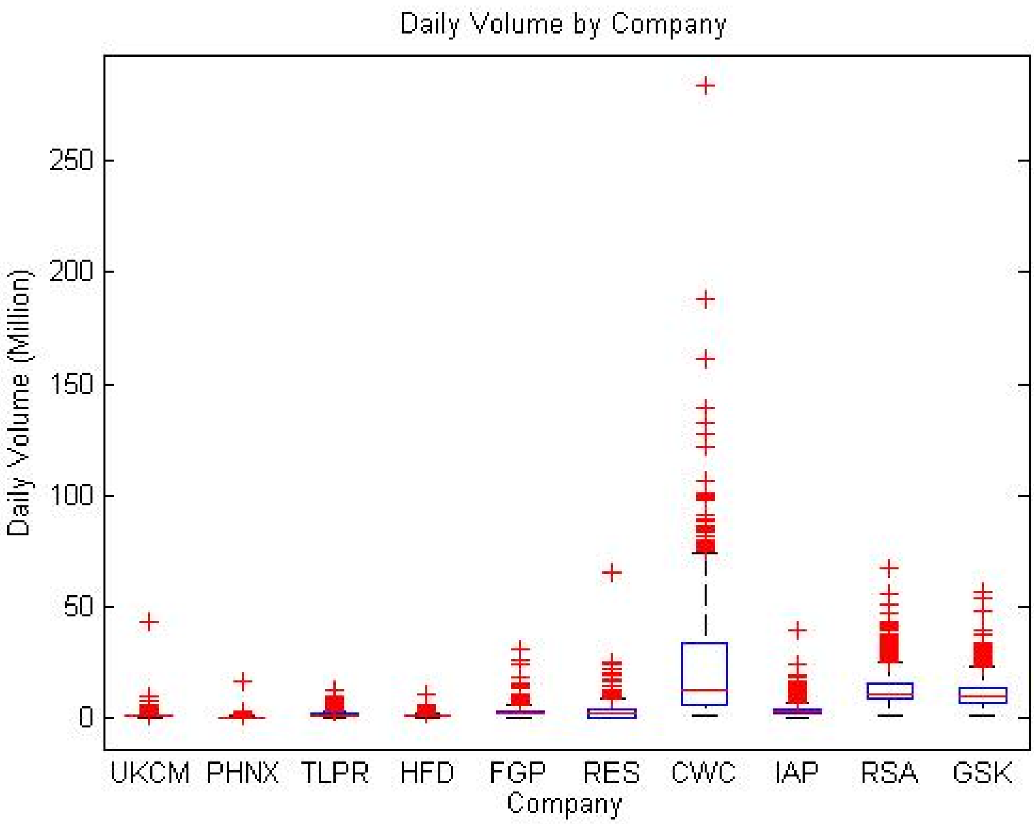
3.4 Data description

This section presents the data sets used in this study with their descriptive statistics. We obtain 20 equities data sets, which are combination of liquid and illiquid securities sampled at daily frequencies, in the FTSE 350. An equally weighted portfolio is constructed with the top twenty highest dividend yield stocks as of 1st of February, 2013: Cape (CIU), ICAP (IAP), Tullett Prebon (TLPR), FirstGroup (FGP), Cable & Wireless Communications (CWC), Halfords Group (HFD), Go-Ahead Group (GOG), RSA Insurance Group (RSA), Stobart Group (STOB), AMLIN (AML), BAE Systems (BA), Vodafone Group (VOD), UK Commercial Property Trust (UKCM), Merchants Trust (MRCH), Resolution (RES), Phoenix Group Holdings (PHNX), F&C Commercial Property Trust (FCPT), Scottish & Southern Energy (SSE), National Grid (NG), GlaxoSmithKline (GSK). End of day ask, bid, and turnover by daily volume of the securities for the period from 1st of February, 2008 to 1st of February, 2013 are collected from Thomson Reuters Datastream (TRD). This portfolio is used in the analysis in Chapter 4 and 5. The number of observations in the sample period are 1264 (1264 for ask, bid, and turnover by volume). Daily log returns are computed from mid prices.

We also compose the equally weighted portfolio for the analysis in Chapter 3, consisting of 10 stocks that provide relatively high dividend yield (on average 7.64%) selected from the portfolio above. A large number of stocks in a portfolio add the burden of a fund manager's responsibility, monitoring the performance of the shares. We choose 10 stocks because it achieves most of the risk reduction by increasing the number of stocks (Campbell et al., 2001). Also, the portfolio with 10 stocks complies with the portfolio diversification rule of undertakings for the collective investment in transferable securities (UCITS)[2]. The Article 52(2) of UCITS Directive (2009/65/EC) (amended by Directive 2014/91/EU) states that 5% or a maximum 10% of the portfolio can be invested in shares issued by the same body on condition that the amount invested does not exceed 40% of the value of its assets. We build two portfolios with different number of assets because we show that there is a diversification benefit in portfolio liquidity risk by increasing the number of stocks, which is described in Chapter 4.

[2]UCITS is an investment products regulation. Collective investment schemes (CIS) that comply with UCITS is allowed to operate freely throughout EU member states

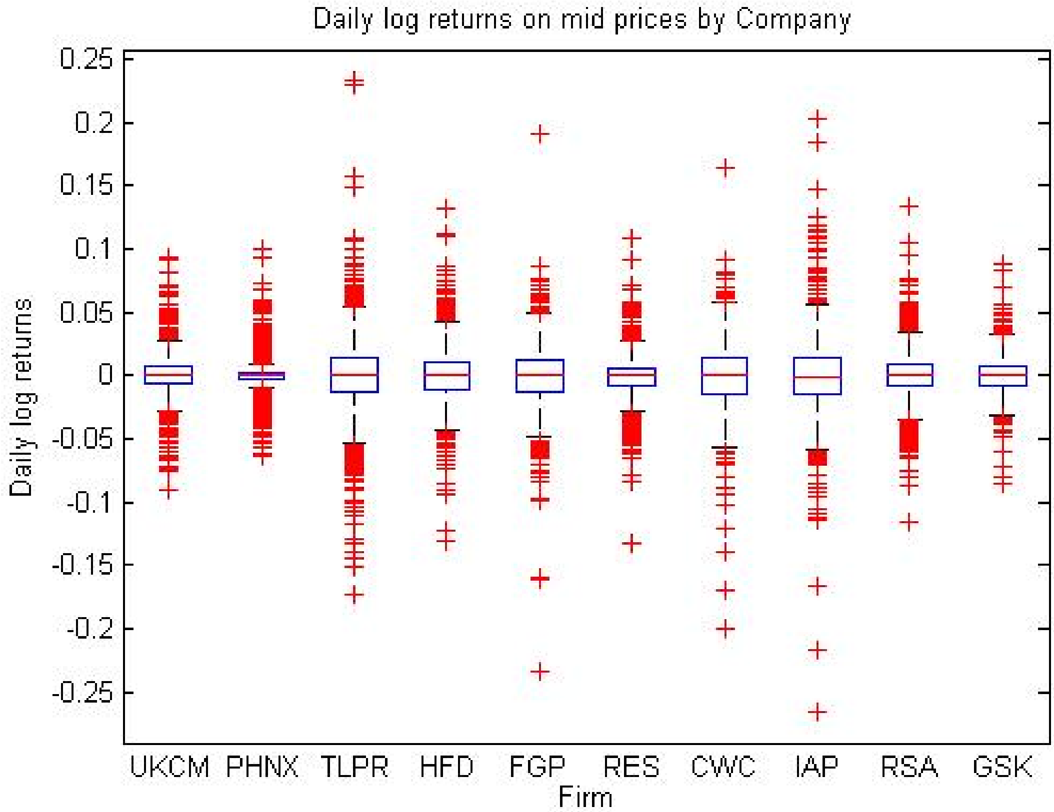
Figure 3.2: Daily volume for the sample of 10 firms



This figure shows the boxplot of daily volume (expressed in million) for the sample of 10 companies. The 10 relatively high dividend yield (on average 7.64%) stocks are selected from FTSE 350.

In order to divide securities into liquid and illiquid assets, it is useful to look at daily volume for the sample of 10 stocks. Figure 3.2 shows the median, quantiles, and outliers of the daily volume. Cable & Wireless Communications (CWC), RSA Insurance Group (RSA) and GlaxoSmithKline (GSK) are relatively liquid assets as their ADV are 22.21, 12.56 and 10.88 million, respectively, while UK Commercial Property Trust (UKCM) and Phoenix Group Holdings (PHNX) are relatively illiquid securities with their ADV as 0.47 and 0.1 million, respectively.

Figure 3.3: Daily log returns on mid prices in the sample of 10 firms



This figure shows the boxplots of daily log returns for the ten relatively high dividend yield stocks selected from FTSE 350.

Another importable variable in the market impact costs estimation is daily volatility (Almgren et al., 2005). The boxplot for daily log returns of mid prices are shown in Figure 3.3. Distribution of the returns for all assets has fat tails as the kurtosis is larger than 3. We compute the volatility on daily returns by giving equal importance to all 1263 data points. ICAP (IAP), FirstGroup (FGP) and Tullett Prebon (TLPR) are relatively high volatility securities in the data sets. From Table 3.1, both kurtosis and standard deviation for the assets are relatively larger than the values of other assets. GlaxoSmithKline (GSK), and Resolution (RES) are relatively low volatility assets. Standard deviations of their daily log return series are less than 2%.

Table 3.1: Descriptive statistics of daily log return series

Company	Std.Deviation (%)	Skewness	Kurtosis
UK Commercial Property Trust (UKCM)	1.77	-0.0240	4.72
Phoenix Group Holdings (PHNX)	1.68	0.6692	8.10
Tullett Prebon (TLPR)	3.14	0.3224	8.02
Halfords Group (HFD)	2.26	0.0420	4.48
FirstGroup (FGP)	2.50	-0.8098	11.55
Resolution (RES)	1.68	-0.1264	7.06
Cable & Wireless Communications (CWC)	2.41	-0.7986	9.59
ICAP (IAP)	3.16	-0.1861	9.71
RSA Insurance Group (RSA)	2.00	0.2693	5.11
GlaxoSmithKline (GSK)	1.45	0.0223	4.74

This table reports the descriptive statistics of daily log return series for the sample of ten relatively high dividend yield stocks selected from the FTSE 350. The statistics are computed over the period 1st of February, 2008 to 1st of February, 2013 (total 1263 data points). The mean values of the sample data sets are all close to 0.

3.5 Numerical results

In this section, we determine the value of a portfolio of equities under three liquidity policies. The portfolio is an equally weighted portfolio consisting of 10 stocks that provide relatively high dividend yield selected from the FTSE 350. The portfolio contains no cash, $p_0 = 0$. This choice provides a representative cross section of liquid and illiquid stocks, and is a common investment strategy for mutual funds. The UMtM value of the portfolio is GBP 200M (GBP 20M is allocated for each asset) although the UMtM value is varied in the analysis.

When calculating the portfolio value, the power-law MSDC is employed for each asset. Three liquidity policies are employed in this study are

1. Cash requirement
2. Cash requirement and minimum-weight restriction
3. Cash requirement and the portfolio expected shortfall (ES) limit constraints

The cash requirement constrains the portfolio to have the ability to generate cash due to possible adverse redemption. The cash requirement is set as 25% of the UMtM value of the portfolio. Simply applying a cash constraint, will as we shall see, leads to the optimisation procedure selling the most liquid assets. This then leads to a residual portfolio which may be illiquid and more volatile and hence have a higher proportional market risk. The minimum weight constraint, to some extent alleviates this issue by restricting the liquidation of the most liquid assets; the 70% minimum weight on the three most liquid assets constrains the portfolio to sell no more than 30% of their initial positions. A more appropriate constraint that addresses this is issue is to impose an appropriate market risk limit, specifically a limit on the expected shortfall (ES) of the residual portfolio. ES is the average loss over given the loss is greater than the Value at Risk at a given confidence level. We now apply these various constraints to value the equally weighted portfolio of the ten relatively high dividend yield (on average 7.64%) stocks in the FTSE 350.

3.5.1 Portfolio liquidity risk exposure given the cash and minimum-weight constraints

The liquidity policy for the cash constraint is given by

$$\mathscr{L}(\mathbf{a}) = \left\{ \mathbf{a} \in P \,\middle|\, \begin{array}{c} a_0 \geq 0.25U(\mathbf{p}) \\ a_k \geq 0, \quad \forall k = 1,\ldots,10 \end{array} \right\} \tag{3.20}$$

where a_0 is the cash component required . The condition $a_k \geq 0$ ensures that there are no short positions after liquidation. Under this straightforward liquidation policy the most liquid assets are liquidated in order to satisfy the cash constraints. liquidation policy. This is illustrated in 3.2 where the cash requirements is generating by liquidating all of the holding in GlaxoSmithKline, the holding with the lowest value for the ratio of position to average daily volume $(\tilde{p}_k = p_k/V_k)$, and then the those with successively larger values.

One of the concerns in a cash only constraint is the deteriorating liquidity of the residual portfolio. If most of the liquid assets have been liquidated subject to the cash liquidity policy, then the residual portfolio may end up holding the illiquid securities, often associated with high volatility. The weight restriction to the liquidation of relatively liquid assets can be made, so a certain proportion of the initial number of shares on the liquid assets remain; thus, it is likely that the illiquidity of the residual portfolio is partially alleviated. The most liquid assets are chosen as those which have the lowest ratio of position to average daily volume$\tilde{p}_k$.

Liquidity policy for a combined cash and minimum weight is given by

$$\mathscr{L}(\mathbf{a}) = \left\{ \mathbf{a} \in P \middle| \begin{array}{c} a_0 \geq 0.25U(\mathbf{p}) \\ a_k \geq 0.7 p_k, \quad \forall k = 1, ..., 3 \\ a_k \geq 0, \quad \forall k = 4, ..., 10 \end{array} \right\}. \tag{3.21}$$

.

An equally weighted long only portfolio, which allocates £10M for each asset, is constrained on the 25% cash and the minimum weight constraints for the three assets with the lowest $\tilde{p}_k$; no more than 30% of their initial positions can be liquidated. The LVA with 25% cash, and the liquidity policy (3.21) for the £200M equally weighted for a long only portfolio is shown in the Table 3.2. There is a reduction in the selling volume of GlaxoSmithKline and RSA Insurance Group shares because of the minimum weight restriction. A decrease in selling positions in those assets leads to an increased liquidation of other more illiquid assets with higher $\tilde{p}_k$. The portfolio subject to the minimum weight constraint increases the liquidity valuation adjustment (LVA) from 20.32 bp for the cash only constraint, to 53.89 bp for the cash constraint with a minimum liquidation.

3.5.2 Portfolio liquidity risk exposure with the cash, minimum-weight and ES constraints

In addition to the worsening liquidity, the increased relative market risk[3] of the residual portfolio is another issue of the cash-only constraint. When selling assets subject to a cash requirement, the relative market risk of the residual portfolio can be increased. Low volatility assets tend to be mainly sold with a cash only liquidity policy in order to avoid liquidity costs. As a result, the portfolio can be left with relatively high volatility assets. This increases the relative market risk of the residual portfolio. A possible market risk limit to impose could be value at risk (VaR); however, under the fundamental review of the trading book by the Basel Committee[4] a more appropriate measure going forward would be the expected shortfall (ES) which accounts for losses beyond VaR. It has the additional property of satisfying the axioms for a coherent measure of risk; VaR is not in general a coherent risk measure (Acerbi and Tasche, 2002). For these reasons we focus on ES, as opposed to VaR for setting the market risk constraint. The ES is defined (see e.g. Acerbi and Tasche (2002) and Dowd (2002)) as the average loss given that the loss is beyond VaR over a

[3] Specifically what we mean by relative market risk is the ratio of the monetary value of the residual portfolio ES, to the UMtM value of the residual portfolio.

[4] Details available at http://www.bis.org/publ/bcbs219.pdf

given time horizon (here we chose one day) and confidence level c (taken as 99%).

$$ES_c = E\left(Loss \mid Loss > VaR_c\right) \tag{3.22}$$

The methodology we employed to determine the ES of the fund is known as the 'peaks over threshold' method from extreme value theory (See McNeil et al. (2005)). This approach, via the Pickand-Balkema-de Haan theorem, models the tail of the distribution, beyond a specified threshold, as a Generalised Pareto Distribution (GPD). The GPD is characterised by a shape parameter (ξ) and scale parameter (β). These parameters are obtained from historical mid price data using standard maximum likelihood methods. The advantage of this methodology is its flexibility in parametrising the tail of the distribution, in particular capturing 'fat-tails'. After selling the estimated volume given less than 1 day trade duration, the ES of the residual portfolio should stay within the given limit. When computing portfolio returns in the ES estimation, we use mid prices as an approximation of asset prices. We compute log returns on the mid prices. We employ the one day historical VaR method on the portfolio log returns over five year horizon to define threshold u. The threshold level u is set to be the 99th percentile of the empirical distribution of daily log returns.

The extreme events are defined as exceedances above the threshold. The portfolio returns for K assets, as Tabner (2012), is defined by the formula

$$R_{k,t} = \ln\left(\frac{(m^+_{k,t} + m^-_{k,t})/2}{(m^+_{k,t-1} + m^-_{k,t-1})/2}\right), \quad t = 1, \ldots, T$$

and $R^{\mathbf{a}}_t$ the return of the residual portfolio **a** at time t is given by

$$R^{\mathbf{a}}_t = \sum_{k=1}^{10} w_k R_{k,t} \tag{3.23}$$

where $R_{k,t}$ is the log return at time t , and w_k is the weight of the kth asset .

The GPD can model the distribution of the excesses over a threshold. The excess losses are parametrically represented by fitting the data to the GPD (Castillo and Hadi, 1997). The cumulative distribution function (CDF) for the returns, $r > u$ is given by

$$F(r) = 1 - \frac{N_u}{N_T}\left(1 + \frac{\xi(r-u)}{\beta}\right)^{-\frac{1}{\xi}} \tag{3.24}$$

where u is a threshold level, N_T is the total number of returns, N_u is the number of threshold excesses, ξ is the shape parameter, and β is the scale parameter. The shape parameter represents how much fatter the tail is, and the scale parameter represents the bias in the dispersion of the extreme values. The two GPD parameters, ξ and β, are estimated via maximum likelihood estimation (MLE).

The VaR at confidence level c,VaR_c,satisfies $F(VaR_c) = c$; rearranging yields

$$VaR_c = u + \frac{\beta}{\xi}\left[\left(\frac{N_T}{N_u}(1-c)\right)^{-\xi} - 1\right]. \tag{3.25}$$

The ES for the confidence level c is given by

$$ES_c = \frac{1}{1-c}\int_c^1 VaR_y dy = \frac{VaR_c}{1-\xi} + \frac{\beta - \xi u}{1-\xi}. \tag{3.26}$$

The weight of each components in the residual portfolio is

$$w_k = \frac{(p_k - z_k)m_{k,T}^+}{\sum_{k=1}^{10}(p_k - z_k)m_{k,T}^+}, \quad k = 1, \ldots, 10 \tag{3.27}$$

where p_k is the initial position of the kth asset, and z_k is the selling volume resulting from the optimisation.

The liquidity policy applied here is given by

$$\mathscr{L}(\mathbf{a}) = \left\{ \mathbf{a} \in P \middle| \begin{array}{c} a_0 \geq 0.25U(\mathbf{p}) \\ a_k \geq 0, \quad \forall k = 1, \ldots, 10 \\ ES_{99\%} \leq 3.5\% \end{array} \right\} \tag{3.28}$$

$ES_{99\%}$ is the 99% portfolio ES value over one day horizon. Algorithm 3.1 outlines the liquidity

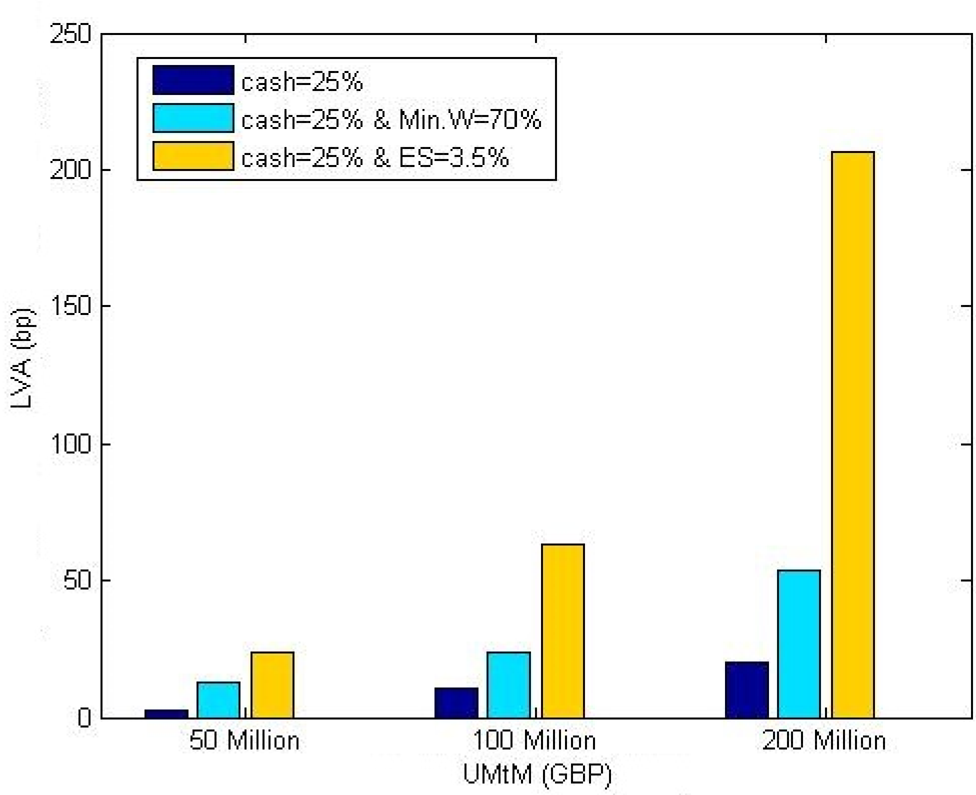
Algorithm 3.1 Algorithm for liquidity costs estimation with cash requirement and the portfolio ES limit constraints

- [Objective function]
$\mathbf{z} \equiv argmin_z\{U(\mathbf{z}) - L(\mathbf{z})\}$;
Calculate:
$U(\mathbf{z})$ [UMtM value of a portfolio in (3.18)]
$L(\mathbf{z})$ [Portfolio value with liquidity policies in (3.17)]
- [Cash requirement]
$L(\mathbf{z}) = 0.25U(\mathbf{p})$; [25% cash requirement]
- [Portfolio ES limit constraints]
do until $ES_c(\mathbf{w}) \leq \alpha\%$ [Constraining the market risk of the residual portfolio; we calculate **z** in order to satisfy the cash constraint, then compute the ES. The portfolio weight **w** is a function of **z**.]
  $a_k = p_k - z_k$ [**a** is the residual portfolio after selling a fragment of the portfolio **z**]
  $w_k = \frac{a_k m^+_{k,T}}{U(\mathbf{a})}$; [Given in (3.27)]
  $R^{\mathbf{a}}_t = \sum_{k=1}^{K} w_k R_{k,t}$; [Given in (3.23)]
  losses = sort($-R^{\mathbf{a}}_t$); [Losses are positive, profits negative]
  u = percentile(losses, 95%); [Threshold set at the 95th percentile of the empirical loss distribution]
  xtail = losses(losses > u); [Observations above the threshold]
  y = xtail - u; [Exceedances over the threshold]
  [ξ β] = maximum likelihood estimation (y); [Estimating the GPD parameters]
  Compute the portfolio VaR in (3.25). [The VaR confidence level c is 99%]
  Compute the portfolio ES in (3.26).
loop
return $U(\mathbf{z}) - L(\mathbf{z})$; [Estimating liquidity costs with the given cash and the portfolio ES constraints]

costs estimation pseudo-code for applying the liquidity policy 3.28.

The $ES_{99\%}$ of this portfolio over a one day horizon is 4.5% of the UMtM value (i.e. GBP 9m). Additionally, the residual portfolio is subject to $ES_{99\%} < 3.5\%$. It can be seen from Table 3.2 that with the ES constraint, the highest market risk assets, those with the largest daily volatility, are targeted in order to meet the requirement. For example the trading volume for the most volatile position, ICAP, has increased by 400% in comparison with the liquidation on the cash-only constraint. In contrast, the estimated liquidation volume for the lowest market risk asset, GlaxoSmithKline, has dropped by 94% from the estimated volume under the cash-only constraint. This is because, through the optimisation, a larger selling volume is estimated for the higher market risk assets than the lower market risk ones in order to lower the ES value of the residual portfolio. Therefore, this results in an increased LVA. The LVA for a liquidity policy that includes the cash and the portfolio ES constraints is GBP 4,135,345, which is 206.77 bp of its UMtM value. There is an approximate ten fold increase in the LVA by adding the 3.5% portfolio ES constraint.

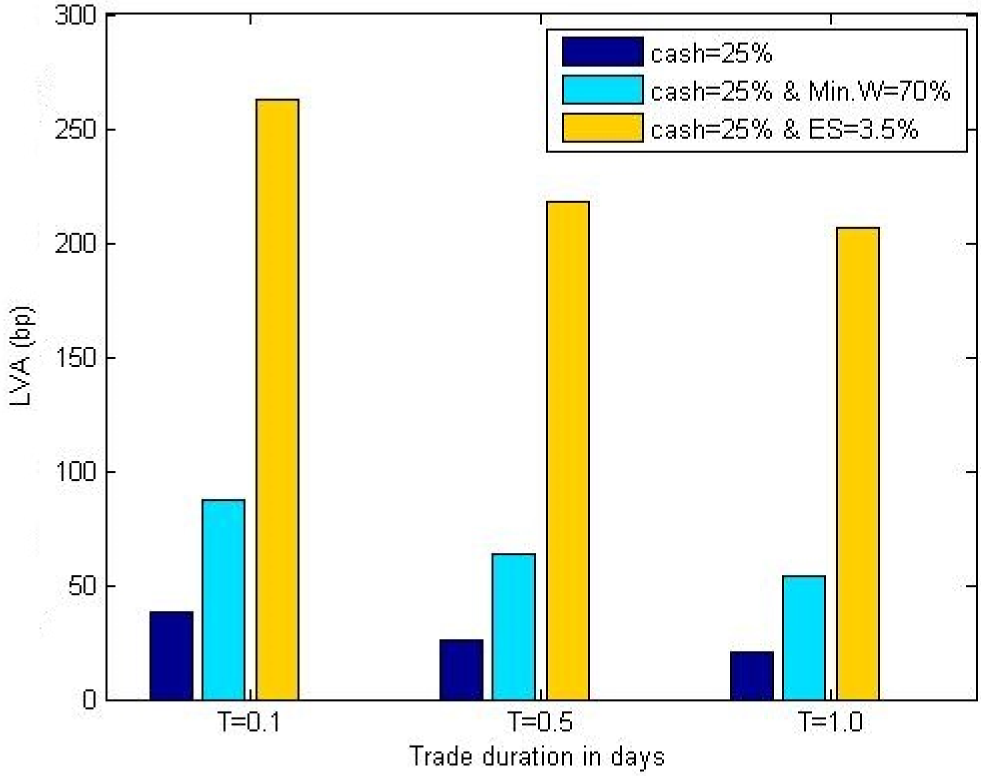
Figure 3.4: LVA with respect to the portfolio UMtM value



This figure shows the LVA with respect to the portfolio MtM values, which vary from £50 to £200M. The trade duration in days is set to 1. The portfolio is constrained by three different liquidity policies: 25% cash requirement, the liquidity policies (3.21), and (3.28).

There is a positive relationship between the size of the portfolio and liquidity costs. From Figure 3.4, as the UMtM value of an equally-weighted long portfolio becomes larger from £50M to £200M, the LVA with respect to the portfolio UMtM value increase from 23.49 bp to 206.77 bp for the 25% cash and the 3.5% portfolio ES constraints. This is because when the portfolio UMtM value becomes larger, the initial positions are larger, so it is likely that a larger volume is liquidated in comparison with its average daily volume. For instance, in the case of the £200M UMtM value of the portfolio, when it is subject to the ES 3.5% limit, the most volatile asset, ICAP, is completely liquidated. If the portfolio UMtM is set to be £50M, then 25% of the ICAP position is liquidated 25%.

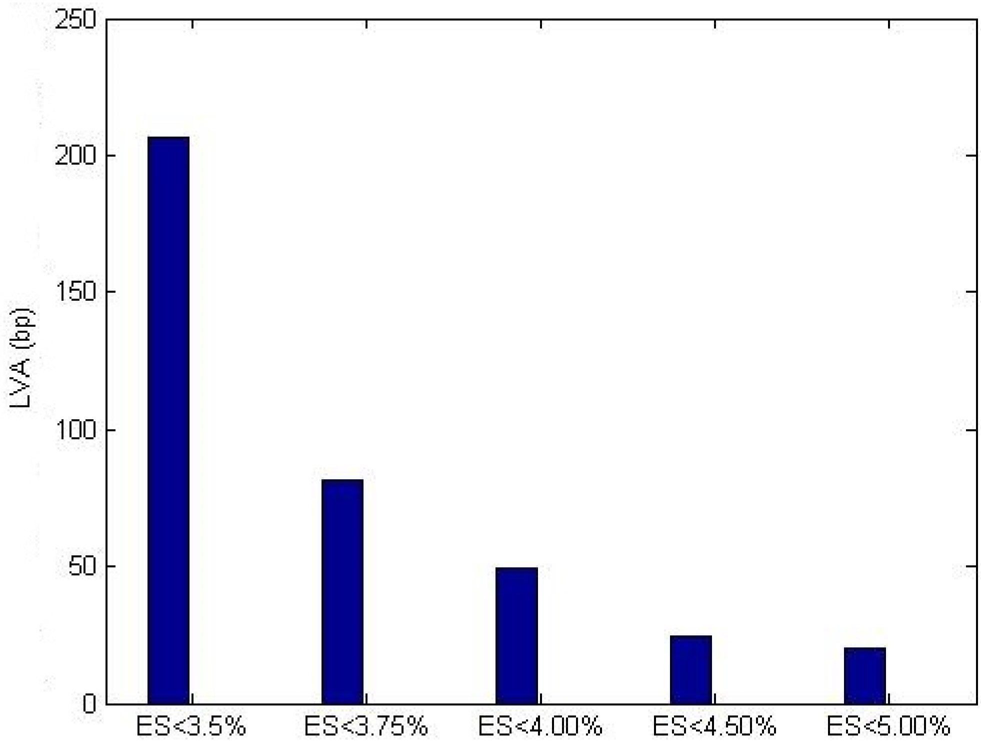
Figure 3.5: LVA with respect to the trade duration



This figures displays the LVA as a function of the portfolio UMtM values according to trade duration in days from 0.1 to 1. Portfolio UMtM value is set to £200M. The same liquidity policies given in Figure 3.4 are imposed on the portfolio.

Also, as can be seen from Figure 3.5, there is a negative relationship between the time scale of trade execution and the liquidity costs. For 25% cash and the portfolio ES 3.5% constraints, as the trade duration becomes longer from 0.1 to 1 days, the LVA with respect to the portfolio UMtM value decrease from 263.14 bp to 206.77 bp. The short execution time, as Almgren et al. (2005) emphasise, can cause a greater price movement by consuming liquidity available in the market during that short time. In contrast, the longer trade duration may cause a smaller price movement as this temporary market impact can be gradually mitigated. Given this evidence, the trade time has an impact on the LVA.

Figure 3.6: LVA with respect to the portfolio ES limits



This figure shows the LVA as a function of ES limits varying from 3.5% to 5%. Portfolio UMtM value is set to £200M, and the trade duration in days is set to 1. The portfolio is subject to the liquidity policy (3.28).

Strong evidence of a negative relationship between LVA and the portfolio risk limits was found when estimating the LVA according to different portfolio ES limits from 3.5% to 5%. As shown in Figure 3.6, there is a large increase in the LVA given more stringent ES constraints. The estimated LVA on the 25% cash and the portfolio $ES_{99\%} < 3.5\%$ constraint is 206.77 bp, while it is 20.32 bp on the 25% cash and the portfolio $ES_{99\%} < 5\%$, a less stringent constraint. The portfolio $ES_{99\%} < 3.5\%$ constraint increases the LVA more than double compared with the LVA on the portfolio with constraint $ES_{99\%} < 4.5\%$, or $ES_{99\%} < 5\%$. After a certain risk level, the the less stringent portfolio risk do not affect the LVA since the portfolio already satisfies the ES constraint, remember that the original portfolio has, $ES_{99\%} = 4\%$.

Table 3.2: Comparison of LVA estimation with liquidity policies

Liquidity policies	Liquidity valuation adjustment (LVA)	
cash (25%)	GBP 406,471	20.32 bp
cash (25%) and minimum weight (70%)	GBP 1,077,702	53.89 bp
cash (25%) and ES (3.5%)	GBP 4,135,345	206.77 bp

						$\widetilde{z}_i$ (%)		
Company	m^+ **(GBP)**	p	σ **(%)**	$\widetilde{\theta}$ (%)	$\widetilde{p}$ (%)	**C**	**C+MW**	**C+ES**
UK COMMERCIAL PROPERTY TRUST	0.68	29411765	1.77	2563	6296	1	1	0
PHOENIX GROUP HOLDINGS	6.37	3139717	1.68	1681	3024	5	0	1
TULLETT PREBON	2.41	8298755	3.14	227	866	8	11	89
HALFORDS GROUP	3.45	5797101	2.26	228	663	14	33	27
FIRST GROUP	1.98	10101010	2.50	200	419	17	42	38
RESOLUTION	2.62	7633588	1.68	607	327	21	63	0
CABLE & WIRELESS COMMUNICATIONS	0.41	48780488	2.41	114	220	6	20	2
ICAP	3.29	6079027	3.16	206	194	25	30	100
RSA INSURANCE GROUP	1.32	15151515	2.00	286	121	54	26	7
GLAXOSMITHKLINE	14.52	1377410	1.45	451	13	100	30	6

This table compares LVA estimation with the 25% cash (C); 25% cash and 70% minimum weight (MW); 25% cash and $ES_{99\%} \leq 3.5\%$ (ES) on the residual portfolio for the GBP 200M equally weighted for a long only portfolio. Trade duration is set to 1 day. The m^+ column is the asset bid price in GBP. The p column shows the initial positions. The σ column is the daily volatility in percentage. The $\widetilde{\theta}$ column represents the total number of shares in issue with respect to the average daily volume of the asset in percentage, i.e. $\frac{\theta_k}{V_k}$. The $\widetilde{p}$ column refers to the ratio of initial positions over the average daily volume in percentage, i.e. $\frac{p_k}{V_k}$. The $\widetilde{z}_k$ column represents the trading volume over the initial positions in percentage, i.e. $\frac{z_k}{p_k}$. The liquidity value adjustment (2.23) in Chapter 2 is given for each of the three liquidity policies in monetary terms and as a ratio of the UMtM.

3.6 Conclusions

We employ the portfolio liquidity valuation framework of Acerbi and Scandolo (2008) to determine the value of a long only mutual fund subject to various liquidity policies, namely; cash requirement, minimum weight and the portfolio ES. The methodology of Acerbi and Scandolo (2008) applied assumes a single period for transactions. A key component to the methodology is the MSDC for a particular asset, and we derive, using the seminal work of Almgren et al. (2005), functional forms for the curves that rely on readily available data. The methodology is applied to an equally weighted portfolio of the ten relatively high yielding (average 7.64%) stocks in the FTSE 350.

We demonstrate that these liquidity policies can have a significant effect on the value of the portfolio. This is highlighted by calculating the liquidity valuation adjustment (LVA) which is the difference between marking the portfolio to best bid and ask prices and the true valuation of the portfolio including the liquidity policy using Acerbi and Scandolo (2008). With the cash only constraint, mainly liquid and low volatility securities are sold to reduce liquidity costs; thus, the residual portfolio is left with relatively illiquid and high volatility securities. With the minimum weight constraint, larger positions of the liquid assets remain, so the illiquidity of the residual portfolio can be reduced. When the portfolio needs to satisfy cash and ES constraints, significant positions of high volatility assets potentially need to be liquidated in order to satisfy the portfolio ES limit. The liquidation of the high volatility assets can incur greater liquidity costs because the slope of the MSDCs on the assets increases with increased volatility. It is found that the application of an ES constraint can increases the LVA by a factor of ten.

Chapter 4

Portfolio liquidity adjustments over multiple execution periods

In this chapter, we compare the liquidity costs estimated from the model of Finger (2011) and Garleanu and Pedersen (2013) for a equity long portfolio with cash and the portfolio Expected Shortfall (ES) limit constraints (as defined in Chapter 3) when an investor unwinds positions according to the optimal trading trajectory in multiple periods.

We applied Power-Law Marginal Supply-Demand Curves (MSDCs) when estimating the measure of market depth parameter. We employed the portfolio liquidity-adjusted Value-at-Risk (LVaR) model of Almgren and Chriss (2000) that includes the transaction costs and the volatility risk terms. The investor aims to implement the best trading strategy that minimises the amount of the LVaR for a given confidence level.

The finding shows that the model of Garleanu and Pedersen (2013) tends to produce the greater liquidity costs than the costs from the liquidity measure of Finger (2011) as the size of trades increases because the former has a constant value in measuring market depth. The transaction cost unit contract is a linear function of the trading volume. On the other hand, the latter has a concave price impact function on the trading volume, so the transaction cost unit contract increases at decreasing rate.

The chapter is organized as follows. Section 4.1 motivates the estimation of the portfolio liquidity costs using the optimal execution paradigm of Almgren and Chriss (2000). In Section 4.2, we explain the transaction cost and the volatility risk estimation with multiple liquidation periods. In Section 4.3, we illustrate how to obtain the measure of market depth parameter. We use the Power-Law MSDC (as described in Chapter 3) to calibrate the market depth parameter. In Section

4.4 and 4.5, we compute and compare the portfolio liquidity costs with cash and the portfolio ES limit constraints estimated from the liquidity risk measure of Finger (2011) and Garleanu and Pedersen (2013). Finally, Section 4.6 concludes.

4.1 Introduction

The realized transaction price which an investor unwinds positions of shares may change due to market impact. Market impact is a phenomenon that when an investor executes large orders, its own operations results in adverse price movements on traded assets, so the investor is penalized by its own trading activity.

As unwinding positions in the portfolio increase, the depreciation of the realized transaction price can be greater. An investor, therefore, faces large transaction costs which results in a large implementation shortfall. The implementation shortfall is performance difference between a portfolio that trades positions at their initial market value and an implemented portfolio that executes shares at their effective price received. The revenue from the implemented portfolio would be less than the initial portfolio if each position is sold at the price lower than its theoretical price due to transaction costs (Perold, 1988; Guilbaud et al., 2013).

The development of a model that measures and manages this implementation shortfall is an important issue. The model can suggest an optimal execution strategy that minimise the liquidity costs. One simple strategy would be that a large order is split into many small children orders with different sizes. This time slicing on an order can reduce the transaction cost. However, it causes longer trade time to complete children orders. For high volatile assets, a slow trade may induce fundamental value change, so we need to liquidate the high volatile assets faster in order to reduce volatility risk.

The volatility risk refers to an uncertainty about the final revenue because of price changes within the specified trading periods. For instance, for a high volatile security, the price traded in the current time step is more likely to be different from the price in the previous time step than in the case of a low volatile asset. A slow trade for high volatile securities would have greater uncertainty of the final revenue. If we trade the high volatile assets in one block, then it also results in large price impact. We need to implement the trade scheduling that balances between the revenue risk and the transaction costs for the liquidated portfolio by choosing the optimal trade speed for each execution.

Optimal trade scheduling refers to the optimal allocation of selling positions, which result in a large decrease in the volatility risk for a small increase in the transaction cost. In the optimal trade scheduling, as Guilbaud et al. (2013) indicate, there is a tradeoff between the volatility risk and the transaction costs. Suppose a trader needs to reserve cash which is worth 25% of the portfolio Mark-to-Market (MtM) value by selling a fragment of the portfolio in five days. Also, the investor should satisfy a market risk limit on the residual portfolio at the end of trading program.

Two extreme trading strategies are the following. An investor can sell positions at a constant rate in order to minimize the liquidity cost. The investor ignores the uncertainty of the total revenue. If one liquidates positions slowly, then the price executed may be impacted by fundamental value change within the specified trading interval. The investor, on the other hand, can sell the entire positions in the first time step in order to minimize the revenue risk, but the rapid trading can incur high transaction costs (Almgren and Chriss, 2000; Guilbaud et al., 2013). The trader wishes to construct an optimal trading trajectory that result in the minimum revenue risk as well as the minimum transaction cost over the specified time to liquidation.

The main goal of this chapter is to quantify the liquidity cost on a long equity portfolio for the cash and portfolio ES constraints and in the presence of the liquidity costs and the revenue risk tradeoff. The cash constraint is that an investor reserves a certain amount of cash by selling a fragment of the portfolio, and the portfolio Expected Shortfall (ES) constraint is to impose a market risk limit on the residual portfolio. The implemented portfolio is a fragment of the initial portfolio forced to sell to satisfy the given liquidity policies. We describe and analyse optimal trade scheduling that minimizes the trading costs and the volatility risk for the portfolio sold.

We measure the liquidity costs over multiple periods to liquidation. When finding out the best execution strategy that minimises the liquidity costs, in the model of Finger (2011), the decision on the selection of assets and the number of positions to sell is made once for given trade duration. This strategy depends only on information at the initial time. We use the transaction cost measure of Garleanu and Pedersen (2013) as the decision can be made multiple times over the specified trading periods. In each time period, the execution strategy can react to information as it is released. In other words, the choice of assets and the number of positions to liquidate shift optimally on mean and variance of transaction costs computed across multiple periods. Both models construct a constrained optimisation problem to find the optimal trading strategy, and compute the liquidity costs as a function of trade size for the securities chosen.

My contribution is to quantify the liquidity cost of the equity long portfolio using the transaction cost measure of Garleanu and Pedersen (2013) with cash and portfolio ES limit constraints. We apply the Power-Law marginal supply and demand curves (MSDCs) and the liquidity measure of Finger (2011) to calculate the liquidity cost per unit contract, which is used to obtain the measure of market depth parameter in the model of Garleanu and Pedersen (2013).

4.2 Transaction cost and revenue risk

This section outlines the features of execution costs and a defined risk function for a portfolio with K assets when securities are liquidated in multiple periods.

If positions for an asset k are liquidated in N periods, then the cash value of positions remaining to trade at the end of nth period is

$$x_{k,n} = x_{k,n-1} - q_{k,n},\ n = 1,\ldots,N \tag{4.1}$$

where $q_{k,n}$ is the cash value of positions liquidated at the nth period. On day 0, the original holdings to sell in cash value $x_{k,0}$ is allocated for an asset k for the given liquidity policy. The number of shares remaining to execute in cash value at the end of the following period $x_{k,1}$ is the value of shares traded $q_{k,1}$ deducted from the original value of share holdings $x_{k,0}$. The liquidation time T is specified in days e.g. one day ($T = 1$) or ten days ($T = 10$), The number of time periods to trade N is given by $\frac{T}{\tau}$, where τ is the time step. In this study, we vary the liquidation time from one day to ten days, and the trading time step parameter τ from 0.2 to 1. If the trading time step has a half day ($\tau = 0.5$), then it gives two time periods to trade per day ($N = 2$). The total cash value of positions liquidated is given by

$$x_{k,0} = m_k^+ h_{k,0} \tag{4.2}$$

where m_k^+ is the best bid price, and $h_{k,0}$ is the total number of positions traded for the kth asset. In practice, an actual realized average price in each time period would be different because of price impact. In this chapter, we assume that an asset price is fixed at each time period, and denote this by m_k ($= m_k^+$). The number of positions sold at time n is $z_{k,n}$, which gives $z_{k,n} = h_{k,n-1} - h_{k,n}$. For example, the cash value liquidated at time n is $z_{k,n} m_k = q_{k,n}$, and the cash value of positions remaining to trade at the end of time n is $h_{k,n}m_k = x_{k,n}$.

For a portfolio with K assets and N liquidation periods, $\sum_{k=1}^{K}\sum_{n=1}^{N} q_{k,n}$ is the cash constraint in the model of Acerbi and Scandolo (2008). If there are 20 assets in the portfolio, then the asset index k changes from 1 to 20. If we set the liquidation time T as one day, and the trading time step τ has a half day, then the index of liquidation periods n ranges from 1 to 2. We aggregate the total cash value of orders liquidated for a portfolio, and define this amount as a cash constraint.

For a portfolio with K securities, we employ the execution costs model of Garleanu and Pedersen (2013), that is

$$C = \tau^{-1}\sum_{n=1}^{N}\frac{1}{2}\mathbf{q_n}^{'}\Lambda\mathbf{q_n} \tag{4.3}$$

where $q_n = (q_{1,n}, q_{2,n}, ..., q_{K,n})^{'}$ is the $K \times 1$ vector of positions liquidated in cash value at the nth period, and Λ is the $K \times K$ matrix of Kyle's lambda with the off-diagonal elements set to be 0 (the main diagonal elements of Λ have the values of the Kyle's lambda for each k). The Kyle's lambda measures the market depth of a security, according to Kyle (1985). The smaller the value of the Kyle's lambda, the deeper the market is for the asset. The inverse of the value of it represents an order flow that results in a price movement (e.g. 0.1% of the asset price). Average price impact per share on the positions traded $\mathbf{z_n}$, as Almgren and Chriss (2000) and Garleanu and Pedersen (2013) have described, is obtained by $\frac{1}{2}\bar{\Lambda}\mathbf{z_n}$, where $\bar{\Lambda} = \Lambda m^2$ ($\bar{\Lambda}$ has units of (\$/share)/share). For the cash amount traded $\mathbf{q_n}$, the average price depression per unit currency is given by $\frac{1}{2}\Lambda\mathbf{q_n}$.

The inverse of the trading time step τ^{-1} in this term is an adjustment to the Kyle's lambda in order to make the execution costs independent of the trading frequency. In other words, a small τ does not affect the amount of the execution costs (Garleanu and Pedersen, 2013; Mastromatteo et al., 2014). According to Garleanu and Pedersen (2013), the execution costs is a fraction of the total amount of risk, and the costs is proportional to the covariance matrix, that is

$$\Lambda = \lambda\tilde{\Sigma} \tag{4.4}$$

where $\tilde{\Sigma}$ is the $K \times K$ covariance matrix with its off-diagonal elements set to be 0, and λ is the $K \times K$ matrix of the parameter of the market depth measure with its off-diagonal elements set to be 0. We explain how to estimate the values of λ in section 4.3.

The main diagonal of the covariance matrix $\tilde{\Sigma}$ is the variances for each asset, and the off-diagonal elements are set to be 0 because the execution costs for a portfolio is an aggregate amount of the execution costs from each asset (Garleanu and Pedersen, 2013; Meucci, 2012). It seems that a correlation between assets has not impact on the execution costs. It is unlikely that a price impact of one security is diversified by another asset which is negatively correlated in the portfolio. It appears that a correlation in the portfolio only has an impact on the volatility risk because when an investor holds a highly correlated portfolio, one would be more sensitive to the negative market P&L, so the investor would liquidate faster; thus, this can increase liquidity costs.

The volatility risk is the uncertainty about the final revenue due to price changes within specified trading interval. The larger the remaining value of positions to trade is, and the higher the volatility of an asset, the risk of total revenue becomes greater. For a portfolio with K assets, the risk term R can be written as,

$$R = \sqrt{\sum_{n=1}^{N} \tau \mathbf{h_n}' \Sigma \mathbf{h_n}} \tag{4.5}$$

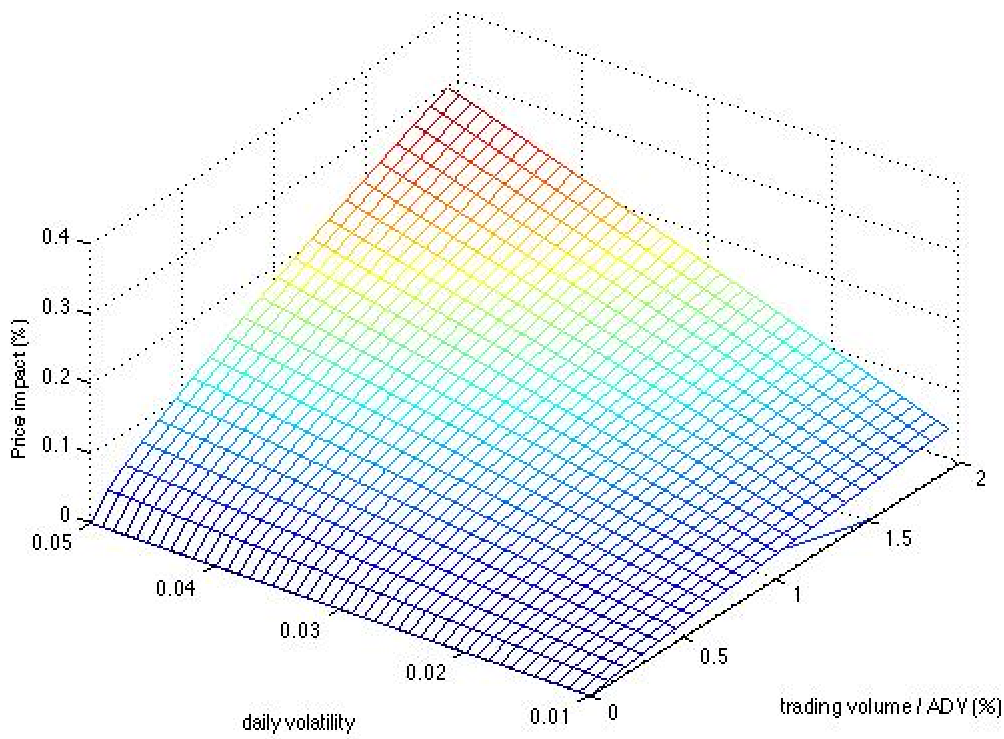
where $\mathbf{h_n} = (h_{1,n}, h_{2,n}, ..., h_{K,n})'$ is the $K \times 1$ vector of positions remaining to trade at the end of time n, and Σ is the $K \times K$ variance-covariance matrix (Almgren and Chriss, 2000). The volatility risk can be reduced by adding securities that are negatively correlated, so the speed of the trading can be slower (i.e. trading slow would, as Kharroubi and Pham (2010) state, decrease the liquidity costs, while increase the uncertainty of the total revenue).

4.3 The measure of market depth

This section illustrates how to estimate the market depth parameter λ_k of the Kyle's lambda for a security according to Garleanu and Pedersen (2013) and Power-Law MSDCs (Finger, 2011). Several studies (Garleanu and Pedersen, 2013; Engle et al., 2008; Breen et al., 2002) have stated that a price impact is 0.1% of the asset price when trading 1.59% of the daily volume. We use the Power-Law MSDCs to estimate a price impact per unit contract because the unit price impact would be greater for high volatility assets (i.e. a price impact per unit contract, which is the proportion of an asset price, for high volatility assets tends to be greater than 0.1%; conversely, the price impact would be less than 0.1% for low volatility assets) as shown in Figure 4.1. The

Power-Law MSDCs model includes volatility parameter measuring the market impact, so it can estimate a greater price impact per unit contract for high volatile assets. When calibrating the parameter of the market depth measure, we follow the method in Garleanu and Pedersen (2013), so the parameter is estimated once, and is used for all sizes of trades. The market depth parameter λ_k for the kth asset is estimated for 1.59% of the average daily volume. The proportion of the asset price as the price impact per unit contract is estimated from the Power-Law MSDCs model of Finger (2011).

Figure 4.1: Price impact estimated using the Power-Law demand curve of the GlaxoSmithKline, selected from FTSE350 on 1st of February, 2013



This figure shows the price impact of the GlaxoSmithKline estimated from the Power-Law demand curve on day volatility varying from 1% to 5 %. The trading volume as a proportion of the average daily volume varies from 0 to 2%. The price impact value represents the difference between the best bid price and the price obtained from the Power-Law demand curve with respect to the best bid price in percentage.

The liquidity costs per unit contract ($\$/share$) used in Garleanu and Pedersen (2013) is given by

$$\frac{1}{2}\lambda_k(m_k\sigma_k)^2 0.0159V_k = 0.001m_k \tag{4.6}$$

where m_k is the asset price, σ_k is the daily volatility, V_k is the average daily volume. The parameter λ_k is given by the expression

$$\lambda_k = \frac{0.1258}{m_k\sigma_k^2 V_k}.$$

We estimate the value of λ_k using the Power-Law MSDCs (Finger, 2011) for trading volume 1.59% of the average daily volume. The transaction cost on the order flow (z positions) from the Power-Law model is

$$\begin{aligned} C_k^A(z) &= U(z) - L(z) \\ &= m_k^+ z - \int_0^z m_k(u)du \\ &= m_k^+\sigma_k\left(\gamma\left(\frac{\theta_k}{V_k}\right)^{\frac{1}{4}}\frac{1}{2V_k}z^2 + \eta\left(\frac{1}{V_kT}\right)^{\frac{3}{5}}z^{\frac{8}{5}}\right) \end{aligned} \tag{4.7}$$

and

$$m_k(z) = \begin{cases} m_k^+\left(1-\gamma\sigma_k\frac{z}{V_k}\left(\frac{\theta_k}{V_k}\right)^{\frac{1}{4}} - \frac{8}{5}\eta\sigma_k\left(\frac{z}{V_kT}\right)^{\frac{3}{5}}\right) & , z>0 \\ m_k^-\left(1+\gamma\sigma_k\frac{|z|}{V_k}\left(\frac{\theta_k}{V_k}\right)^{\frac{1}{4}} + \frac{8}{5}\eta\sigma_k\left(\frac{|z|}{V_kT}\right)^{\frac{3}{5}}\right) & , z<0 \end{cases} \tag{4.8}$$

where m_k^+ is the best bid price, m_k^- is the best ask price, z is the number of shares traded, θ_k is the outstanding shares in issue, σ_k and V_k are given in (4.6), T is the liquidation time in days (as defined above), m_k^+z is the Mark-to-Market (MtM) value of the asset, $\int_0^z m_k(u)du$ is the asset value obtained from the Power-Law MSDCs, and γ and η are the market impact coefficients (Almgren et al., 2005). The Power-Law MSDCs measures the liquidity cost from the difference between the cash amount obtained from best bid prices (perfect liquidity) and the amount obtained from prices in Power-Law MSDCs. The transaction cost per unit-contract using the Power-Law MSDCs is given by the expression

$$\frac{1}{2}\lambda_k(m_k\sigma_k)^2 0.0159V_k = \frac{C_k^A}{0.0159V_k}. \tag{4.9}$$

where C_k^A is given in (4.7) for the order flow ($z = 0.0159V_k$ positions). The parameter λ_k in this study is given by

$$\begin{aligned}\lambda_k &= 2\frac{1}{(m_k\sigma_k)^2 0.0159V_k}\cdot\frac{C_k^A}{0.0159V_k} \\ &= \frac{7911.08C_k^A}{(m_k\sigma_k V_k)^2}.\end{aligned} \tag{4.10}$$

For instance, the measure of market depth parameter λ for the assets selected from FTSE 350 in Table 4.1 is estimated in the following way.

1. When the trading volume is 1.59% of the average daily volume (ADV), liquidity cost on average incurs 0.1% of the asset price per contract (Garleanu and Pedersen, 2013; Engle et al., 2008; Breen et al., 2002).

Table 4.1: Measure of market depth parameters for Cape

m (GBP)	σ (%)	V	λ	T	τ
2.21	5.15	711598	3.02x10^-5	1	1

This table reports the parameter used to measure the market depth for Cape. The parameter T is the trade duration in days (as defined in section 4.2), and τ is the trading time step in days, and the other parameters are given in (4.6).

2. The asset price is £2.21 and its ADV is 0.71M. When 1.59% of the ADV is traded, the transaction cost per contract results in 0.1% of the asset price, which is £2.21 $\times$ 0.1% = £0.002.

3. The number of positions traded is z, where $z = \frac{q}{m}$, q is the cash value traded, and m is the asset price. For the first asset, the number of positions sold z is set to 11325, which generates £25004.86, 1.59% of ADV in cash value.

4. The parameter λ is estimated by the following relationship:

$$0.001 \cdot m = \frac{1}{2}\lambda(m\sigma)^2 z \tag{4.11}$$

where σ is the daily volatility. For example, the value of λ is obtained by solving

$$0.001 \cdot 2.21 = \frac{1}{2}\lambda(2.21 \cdot 0.0515)^2 \frac{25004.86}{2.21}$$

As a result, the parameter λ is 3.02×10^{-5} , and this value is used to compute the liquidity costs for trading z positions. If the number of positions z is traded, then the right hand side of the equation (4.11) becomes

$$\frac{q}{m}\frac{1}{2}\lambda\,(m\sigma)^2\,\frac{q}{m} = q\frac{1}{2}\lambda\sigma^2 q. \tag{4.12}$$

Transaction costs for Cape in one liquidation period is estimated by equation (4.12). The transaction costs for selling 1.59% of the ADV on the first asset, which is 11325 positions and is worth £25004.86, is £25.03 with the one day trading horizon.

Instead of using 0.1% assumption as the proportion of an asset price for liquidity cost per unit contract, we can obtain the proportion from the model of Finger (2011). The transaction cost from the Power-Law model is given in (4.7). The liquidity cost per unit contract is then $\frac{C^A(z)}{z}$. The parameter λ is estimated as 3.24×10^{-5}. When 1.59% of the ADV of the first asset is traded, the liquidity cost using the Power-Law MSDCs is £26.85. This agrees with the assumption that the liquidity costs per unit contract on average is 0.1% of the asset price.

When trading 1.59% of the ADV in cash value, which is 353343 positions, for the second asset, the transaction cost from (4.3) produces £144.87. The transaction cost using the Power-Law MSDC for the second asset gives £69.81.

There is a difference between the transaction costs computed from the two models because in the transaction cost measure (4.3), it is assumed that the liquidity cost per unit contract is 0.1% of the asset price without regard to its ADV and daily volatility. In the liquidity cost measure of Finger (2011), an asset with greater ADV and low daily volatility incurs lower liquidity cost, so the liquidity cost per unit contract would be lower than 0.1% of the price for liquid assets.

Table 4.2: Measure of market depth parameters for Cable & Wireless Communications

m (GBP)	σ (%)	V	λ	T	τ
0.41	2.41	22212139	1.14x10^-5	1	1

This table reports the parameter used to measure the market depth for Cable & Wireless Communications (the asset is selected from FTSE 350). T is the trade duration in days (as defined in section 4.2), and τ is the trading time step in days, and the other parameters are given in (4.6).

The asset in Table 4.2 is more liquid asset than the asset in Table 4.1 because the ADV of it is greater than the ADV for Cape. Moreover, the daily volatility of this asset is lower than the volatility for Cape. The liquidity cost per unit contract obtained from the liquidity estimate of Finger (2011) is 0.048% of the asset price. The parameter of the market depth measure λ for selling 1.59% of the ADV is 1.14×10^{-5} by solving

$$0.00048 \cdot 0.41 = \frac{1}{2}\lambda(0.41 \cdot 0.0241)^2 \frac{144800}{0.41}. \tag{4.13}$$

Table 4.3: Measure of market depth parameters for a portfolio

m (GBP)	σ (%)	V	λ	T	τ
2.21	5.15	711598	3.24x10^-5	1	1
0.41	2.41	22212139	1.14x10^-5	1	1

This table reports the parameter used to measure the market depth for a portfolio with two assets: Cape and Cable & Wireless Communications. The parameters above are used in the transaction cost estimation.

For a portfolio with the two assets from Table 4.3 each of which sells the 1.59% of the ADV, which is worth £169804, and the covariance matrix $\tilde{\Sigma}$ (its off-diagonal elements set to be 0), and the matrix of Kyle's lambda $\Lambda = \lambda\tilde{\Sigma}$, the transaction cost (4.3) is obtained by

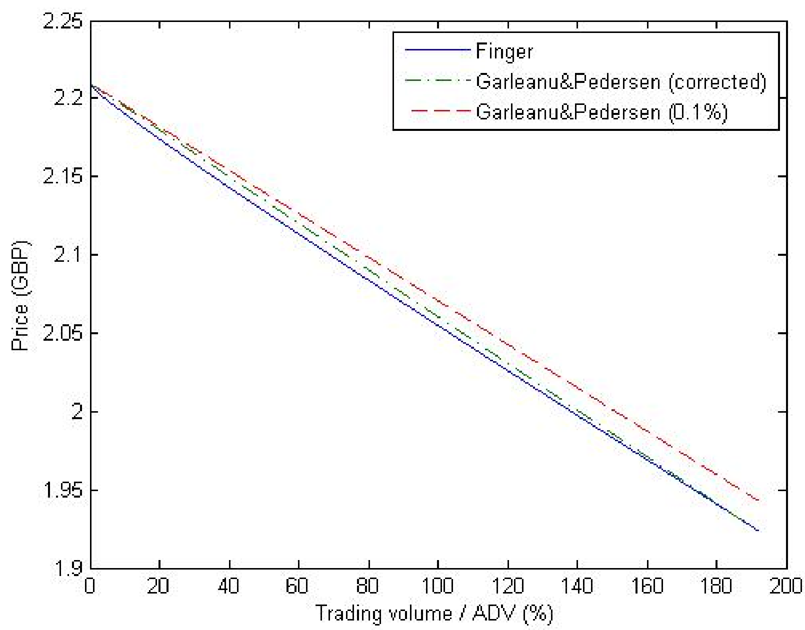
$$C = \frac{1}{2}\begin{bmatrix} 25004 & 144800 \end{bmatrix} \lambda \begin{bmatrix} 0.0515^2 & 0 \\ 0 & 0.0241^2 \end{bmatrix} \begin{bmatrix} 25004 \\ 144800 \end{bmatrix} \tag{4.14}$$

where

$$\lambda = \begin{bmatrix} 3.24 \times 10^{-5} & 0 \\ 0 & 1.14 \times 10^{-5} \end{bmatrix}.$$

When estimating a diagonal matrix of the parameter of the market depth measure λ, we use the liquidity cost per unit contract estimated from the liquidity measure of Finger (2011) (4.7) instead of 0.1% of the asset price. From Figure 4.2, the price impact from the Power-Law MSDCs and from the model of Garleanu and Pedersen (2013) is compared on the trading volume varying from 0 to 2 times of average daily volume for Cape (the asset is selected from the FTSE350 on 1st of February, 2013). The average realised asset prices from the model of Garleanu and Pedersen (2013) can be written as $m^{+} - \frac{C(z)}{z}$. The value $C(z)$ denotes the transaction costs when the number of positions z is liquidated. The price impact per unit contract is given by $\frac{C(z)}{z}$.

Figure 4.2: Price impact on the corresponding trading volume for Cape, selected from FTSE350 on 1st of February, 2013



This figure compares an average realised asset price for the corresponding trading volume for Cape (the asset is selected from FTSE350) from the Power-Law demand curve by Finger (2011) (solid line) and the transaction costs model of Garleanu and Pedersen (2013) (dotted lines). The average realized asset price from the transaction costs model of Garleanu and Pedersen (2013) are compared for the trading volume varying from 1 to 2 times of ADV when the proportion of an asset price for the liquidity costs per unit contract follows the model of Finger (2011) (green dotted line) and 0.1% of the asset price from Garleanu and Pedersen (2013) (red dotted line).

4.4 Transaction costs with a cash constraint

In this section, we construct an optimisation problem to find out the best execution schedule of an order, and quantify the liquidity costs of a long equity portfolio when a trader is subject to a cash constraint. A portfolio is constructed with equally-weighted 20 assets, which are the top 20 biggest annual yield companies selected from FTSE 350. The cash constraint means that an investor reserves a certain amount of cash by selling a fragment of the portfolio. We apply Value at Risk approach when choosing the optimal execution strategy that minimizes the liquidity costs and the volatility risk. The portfolio liquidity-adjusted Value-at-Risk (LVaR) is

$$LVaR = C + \alpha_c \sqrt{R} \tag{4.15}$$

where C is given by equation (4.3), $\alpha_c$is the VaR estimate, and R is given by equation (4.5). This liquidity-adjusted VaR is the c^{th} percentile possible transaction costs for a portfolio. The parameter α_c represents the VaR estimate. If the transaction cost changes are normally distributed, then the VaR estimate with 99% confidence level produces 2.33.

This VaR estimate, as Jaksa and Zapatero (2004) state, can be interpreted as an investor's level of the risk aversion. For higher value of the VaR estimate, the volatility risk becomes an important factor that constrain the trade. A trader should sell positions intensively at the beginning of the trading interval to avoid price change from the fundamental value. On the other hand, if the VaR estimate is 0, then the investor is indifferent to the volatility risk. Trading in equal sized packets is an optimal strategy for this type of investor because it results in the lowest transaction costs.

A rapid trading increases the execution costs, while a slow trading increases the variance of execution cost. The best tradeoff between the liquidity costs and the volatility is to minimise the value of portfolio liquidity-adjusted Value-at-Risk, that is

$$Min_{q_n} C + \alpha_c \sqrt{R} \tag{4.16}$$

The best trading trajectory for the cash value of shares traded $\mathbf{q_n}$ for each time period n and the cash value of shares remaining to trade $\mathbf{h_n}$ at the end of time n can be obtained by minimizing the value of the portfolio liquidity-adjusted VaR (Jaksa and Zapatero, 2004; Almgren and Chriss, 2000). The trader has various choices of $\mathbf{q_n}$ and $\mathbf{h_n}$ at each n for K assets when selling a given fraction of the portfolio value. The trader aims to select the trading strategy that obtains a large reduction in the transaction costs for smaller increase in the volatility risk.

Suppose that a trader wishes to devise an optimal unwinding strategy with 25% cash constraint. This can be written as the following optimisation problem:

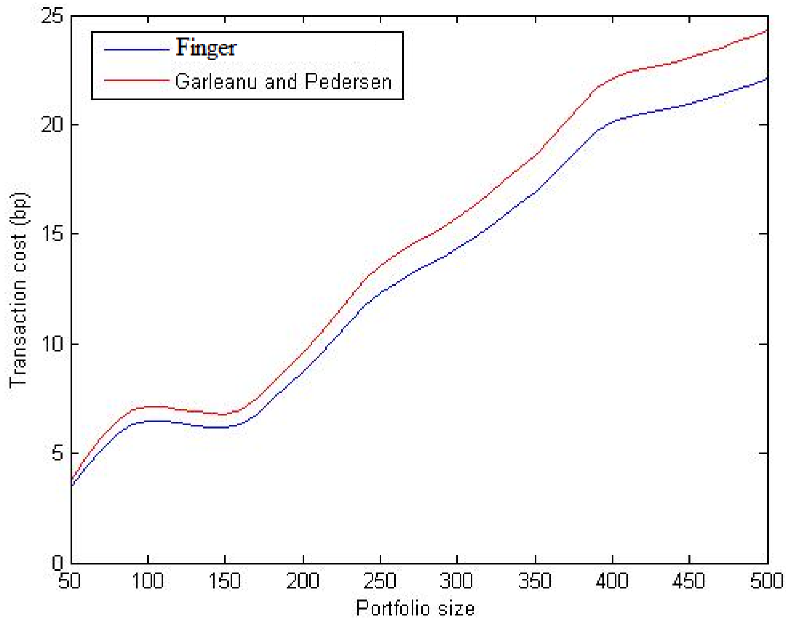
$$Min_{q_{k,n}} C + \alpha_{99\%}\sqrt{R} \tag{4.17}$$

$$s.t. \sum_{k=1}^{K}\sum_{n=1}^{N} q_{k,n} = 0.25 \sum_{i=1}^{K} p_k m_k \tag{4.18}$$

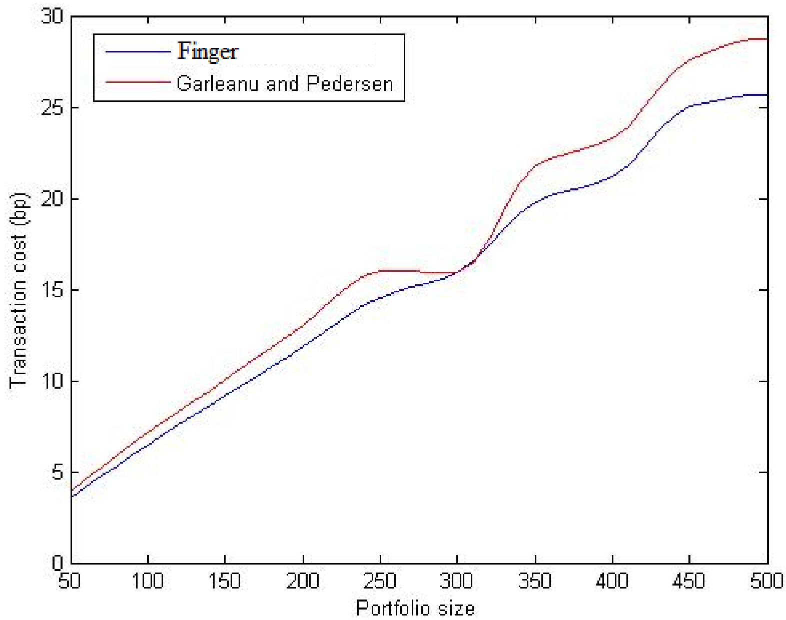
where C is given by equation (4.3), $\alpha_{99\%}$ is the normal VaR estimate with 99% confidence level, R is given by equation (4.5), p_k is the initial holdings for the nth asset, and m_k is given in (4.2).

Transaction costs for a portfolio with the equally-weighted two assets from Table 4.3 subject to 25% cash constraint obtained from the model of Finger (2011) and Garleanu and Pedersen (2013) are plotted in Figure 4.3. The portfolio sizes vary from £50M to £500M, and the liquidation time and the trading time step are set to 1 and 0.5, respectively.

Figure 4.3: Transaction costs estimated on different sizes of the portfolio



(a)



(b)

This figure shows the transaction costs (bp) comparison between the liquidity costs estimate of Finger (2011) and Garleanu and Pedersen (2013) for (a) a portfolio with the top 20 annual yield companies selected from FTSE 350 and for (b) a portfolio with the biggest (Cape) and smallest (GlaxoSmithKline) volatility assets selected from the (a) portfolio. Both portfolios are equally-weighted. The size of the portfolios varies from 50M to 500M.

From Figure 4.3, the liquidity cost decreases as the number of assets in a portfolio increases, which is a granularity effect shown in the liquidity risk framework of Acerbi and Scandolo (2008). The liquidity risk is more likely to be diversified by trading many small positions with the larger number of assets in a portfolio. The liquidity costs for the £500M portfolio with 20 assets subject to 25% cash constraint is smaller than the costs for the same size of the portfolio with two assets which is also subject to 25% cash constraint.

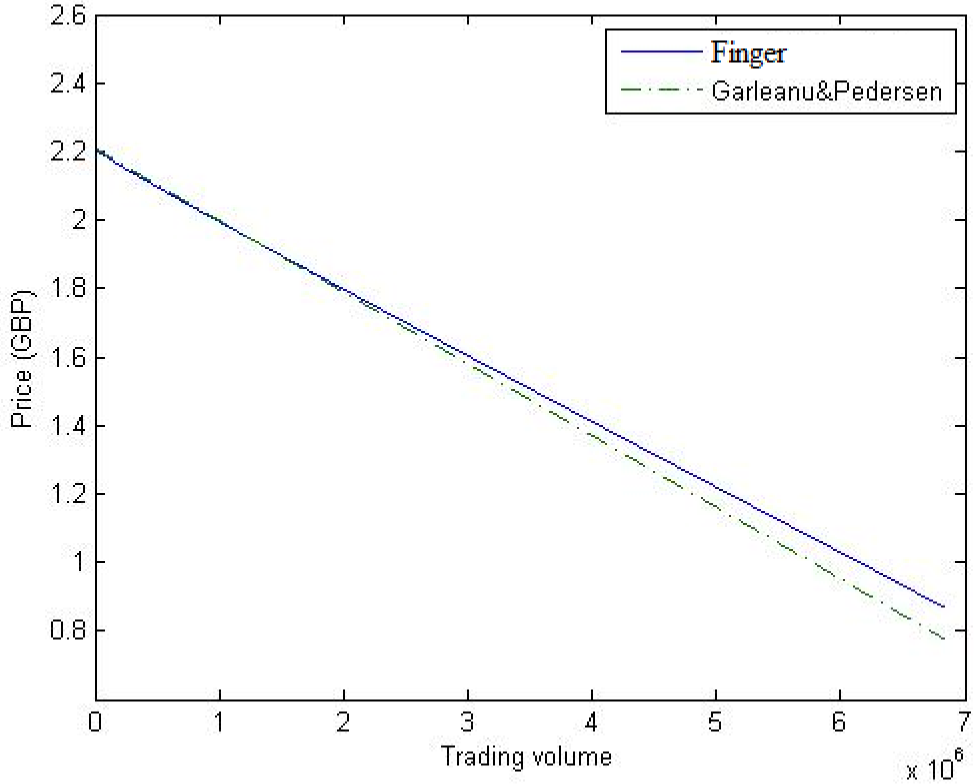
Table 4.4: Transaction costs estimation for different sizes of the portfolio with the 25% cash constraint

	(a) portfolio (20 assets)		(b) portfolio (2 assets)	
Portfolio size	**Finger**	**Garleanu & Pedersen**	**Finger**	**Garleanu & Pedersen**
50M	3.40	3.74	3.54	3.89
100M	6.50	7.15	6.49	7.14
150M	6.15	6.77	9.13	10.04
200M	8.75	9.63	11.86	13.04
250M	12.31	13.54	14.55	16
300M	14.35	15.78	15.92	15.92
350M	16.94	18.63	19.78	21.76
400M	20.11	22.12	21.16	23.27
450M	20.96	23.06	25.04	27.55
500M	22.10	24.31	25.70	28.77

This table reports the transaction costs with the 25% cash constraint estimated from the liquidity estimate of Finger (2011) and Garleanu and Pedersen (2013). The size of portfolios varies from £50M to £500M. The cost with respect to the initial portfolio value is expressed in basis points.

From the Table 4.4, the liquidity cost of Garleanu and Pedersen (2013) tends to estimate the larger execution costs for large trading volume than the cost from the liquidity measure of Finger (2011). This is because in the model of Garleanu and Pedersen (2013), the parameter for the measure of market depth λ is constant value without regard to the size of the trade (i.e. the value is not adjusted to produce a concave market impact). As can be seen from Figure 4.4, when the trading volume is about 2 times of the average daily volume, the slope of the two curves is similar to each other. As the trading volume increases, the slope of the curve from the model of Garleanu and Pedersen (2013) becomes larger than the Power-Law demand curve. This implies that the liquidity measure of Garleanu and Pedersen (2013) would produce a greater liquidity costs per unit contract than the cost from the model of Finger (2011) for the trading volume greater than 2 times of the ADV.

Figure 4.4: Average realized asset price for the corresponding trading volume for Cape, selected from FTSE 350 on 1st of February, 2013



The figure compares the average realized asset price $\widetilde{S}$ from the Power-Law demand curve by Finger (2011) in (4.8) and the model of Garleanu and Pedersen (2013) for the corresponding trading volume for Cape (the asset is selected from FTSE 350). The average realized asset price $\widetilde{S}$ from the model of Garleanu and Pedersen (2013) is given by $m^+ - \frac{C(z)}{z}$ for trading volume z. The symbol C is the transaction cost given in (4.3). Trading volume varies from 1 to 9 times of the ADV.

4.5 Transaction cost with cash and portfolio ES constraints

In this section, we measure the portfolio liquidity costs with the cash and the portfolio ES (McNeil, 1999) constraints. The portfolio ES constraint refers to the market risk limit imposed on the residual portfolio. The portfolio ES is calculated once after the last execution of the trade program.

The portfolio returns and the portfolio ES limit constraint are defined as

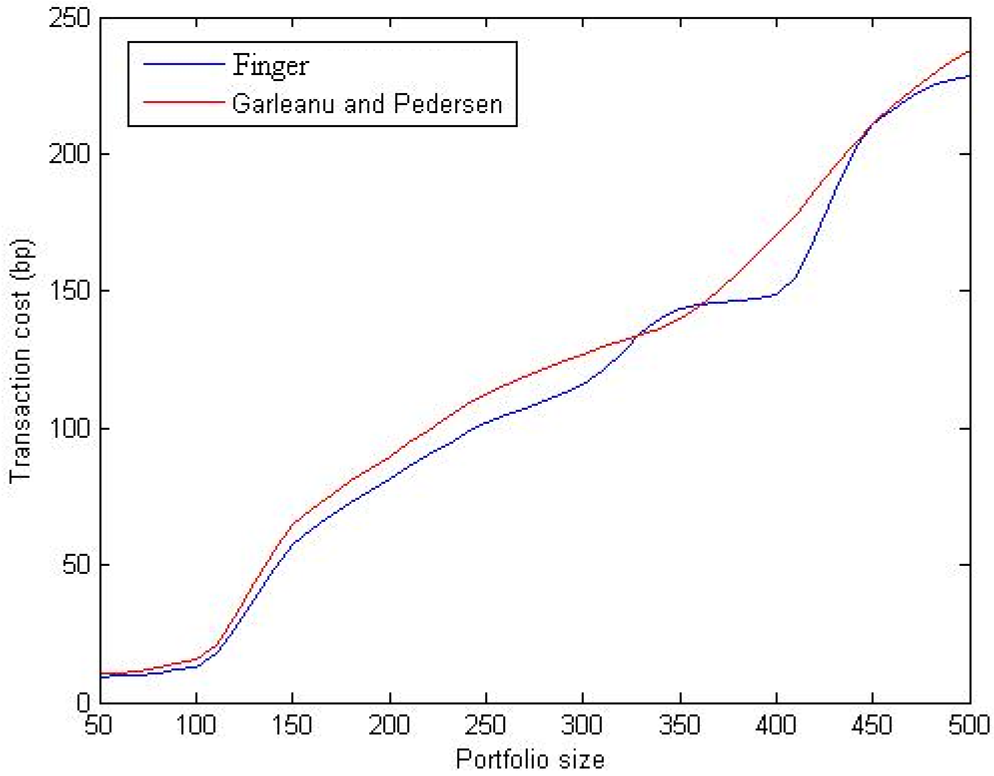
$$w_k = \frac{x_{k,N}}{\sum_{k=1}^{K} x_{k,N}},\ k = 1, ..., K \tag{4.19}$$

$$R_t^P = \sum_{k=1}^{K} w_k R_{k,t}, \tag{4.20}$$

$$ES_{99\%} \leq 4\%. \tag{4.21}$$

The transaction costs for different sizes of portfolios with 25% cash and 4% portfolio ES limit constraints are plotted in Figure 4.5. The 25% cash constraint indicates that the portfolio has the ability to generate 25% in cash, so it results in cash worth 25% of the portfolio MtM value after deducting the transaction costs C. The portfolio ES limit 4% is chosen as the optimisation procedure cannot find the solution below this level of risk. The portfolio sizes are ranged from £50M to £500M, and the trading horizon is one day. The costs with respect to the portfolio size is expressed in basis points.

Figure 4.5: Transaction costs estimated for portfolios with 25% cash and 4% portfolio ES limit constraints



This figure shows a comparison of the transaction costs (bp) for portfolios with 25% cash and 4% portfolio ES limit constraints from the liquidity measure of Finger (2011) and Garleanu and Pedersen (2013).

Table 4.5: Transaction costs estimation for different sizes of the portfolio with 25% cash and 4% portfolio ES limit constraints

Portfolio size	Finger	Garleanu and Pedersen
50M	9.44	10.38
100M	13.13	15.75
150M	57.22	64.34
200M	81.53	89.70
250M	101.63	112.32
300M	116.01	127.10
350M	143.69	139.89
400M	148.69	170.66
450M	211.20	210.86
500M	228.09	237.62

This table reports the transaction costs for different sizes of the portfolio with 25% cash and 4% portfolio ES limit constraints estimated from the model of Finger (2011) and Garleanu and Pedersen (2013). The portfolios sizes are ranged from £50M to £500M, and the liquidation time and the trading time step are set to 1 and 0.5, respectively. The costs with respect to the portfolio sizes are denoted in basis points.

The result both from the liquidity measure of Finger (2011) and Garleanu and Pedersen (2013) produces the greater liquidity costs as the portfolio size becomes larger, but the latter results in the larger liquidity costs as shown in Table 4.5. This is because in the model of Garleanu and Pedersen (2013), the parameter for the measure of market depth λ has a constant value for all the size of the trades.

In the case of £500M portfolio, when applying the portfolio ES 4% limit in addition to the 25% cash constraint, a larger amount of the higher volatility assets is sold in order to satisfy the portfolio ES limit, as shown in Table 4.6. The liquidation weight for the top highest volatility asset is 53%, and the rest 47% of cash is resulted from selling the other 15 assets.

Table 4.6: Transaction costs estimation for the £500M portfolio with 25% cash and 4% ES limit constraints

Firm	p	V	σ (%)	w (%)	z/V (%)
CAPE	11312217	711599	5.15	10.51	934.04
ICAP	7598784	3129990	3.16	9.44	128.17
TULLETT PREBON	10373444	958076	3.14	6.71	406.27
FIRSTGROUP	12626263	2412171	2.50	8.79	257.38
CABLE & WIRELESS COMMUNICATIONS	60975610	22212139	2.41	17.88	274.51
HALFORDS GROUP	7246377	873900	2.26	4.85	224.82
GO-AHEAD GROUP	1908397	143157	2.13	0.81	60.07
RSA INSURANCE GROUP	18939394	12561428	2.00	10.28	86.68
STOBART GROUP	25773196	366732	1.91	0.75	296.52
AMLIN	6410256	1713775	1.90	1.63	34.10
BAE SYSTEMS	7396450	11589550	1.86	5.74	20.49
VODAFONE GROUP	14450867	124822527	1.81	6.94	4.49
UK COMMERCIAL PROPERTY TRUST	36764706	467167	1.77	1.20	529.23
MERCHANTS TRUST	6024096	121861	1.75	0.06	15.59
RESOLUTION	9541985	2335041	1.68	0.39	8.83
PHOENIX GROUP HOLDINGS	3924647	103816	1.68	1.89	399.75
F&C COMMERCIAL PROPERTY TRUST	24271845	618225	1.59	0.96	211.32
SCOTTISH&SOUTHERN ENERGY	1750700	2449905	1.55	4.54	18.15
NATIONAL GRID	3591954	8263000	1.55	3.83	9.30
GLAXOSMITHKLINE	1721763	10876978	1.45	2.82	2.49

This table reports the liquidity costs estimation using the model of Garleanu and Pedersen (2013) for the £500M portfolio with 25% cash and 4% ES limit constraints. Liquidation time is set to be 1 day, and the trading time step is set to be 0.5. The column p represents the initial positions, and the column V refers to the average daily volume. The weight represents the proportion of each asset if the amount of cash liquidated is 100. The value $\frac{z}{V}$ shows the ratio of the total liquidation size for each asset with respect to the ADV in percentage.

4.6 Conclusions

Three main ideas have been presented in this chapter: market depth in the Power-Law MSDCs, mean-variance tradeoff in an optimal trade execution problem, and the liquidity costs estimation with cash and the portfolio ES constraints.

We began by looking at the characteristics of the transaction costs and the volatility risk function, and explained how to obtain the parameter of the market depth measure in the model of Garleanu and Pedersen (2013). The Power-Law MSDCs estimate post-trade price distortion given trading volume. We used the liquidity measure of Finger (2011) and the Power-Law MSDCs to obtain the transaction cost per unit-contract which depends on the volatility of assets.

Second, we described mean-variance tradeoff in an optimal order execution problem, and illustrated how to quantify the portfolio liquidity costs. We measured the liquidity risk of a portfolio that an investor unwinds positions according to the trading trajectory that minimises the value of LVaR for all the execution of the trade program. A constant rate of trade reduces the transaction costs, but increases the volatility risk because a slow trade may induce fundamental value change of an asset. Conversely, a rapid trade decreases the uncertainty of the total revenue over the trading period, but it can incur a large price impact from its own trade operation.

Third, we quantified and compared the portfolio liquidity costs computed from the model of Finger (2011) and Garleanu and Pedersen (2013) when the portfolio is constrained by the cash and the portfolio ES limit constraints. Liquidation of high volatile assets increased to satisfy given risk limit when the portfolio is constrained by the portfolio ES as well as cash. Both models showed increased liquidity costs as the portfolio sizes become larger; however, the model of Garleanu and Pedersen (2013) tends to estimate larger transaction costs than the costs from the model of Finger (2011) because the parameter of the Kyle's lambda is estimated only once and applied for all trade sizes.

It would be interesting if the parameter of the Kyle's lambda is computed according to different trade size. Also, instead of constant volatility, we can incorporate stochastic volatility for each time period because in practice volatility is varying randomly over time. Computing liquidity costs with time varying market state and implementing an optimal trajectory to unwind securities is a topic for further research.

Chapter 5

Portfolio liquidity management with liquidity score method

In this chapter, we quantify the market liquidity risk using the liquidity score (Meucci, 2012), which varies from 0 to 1 (perfect liquidity), for a long equity portfolio constrained with cash and the portfolio Expected Shortfall (ES) limit (as defined in Chapter 3).

Unlike Chapter 4, the execution costs and the volatility risk terms are integrated with the market risk in the market liquidity estimation model. We generate the scenarios of the liquidity adjustments that depend on the market situation using the cost/risk leverage parameter. Also, we stress the model with potential liquidity restrictions on a portfolio. Liquidity policies have an impact on the liquidity adjustments because the portfolio liquidated is obtained for the given liquidity policies. We find that the liquidity score drops significantly when the portfolio ES limit is imposed on the portfolio. The method can be a useful quantitative indicator to monitor market illiquidity changes over time with possible liquidity constraints for a long equity portfolio.

The reminder of the chapter is structured as follows. Section 5.1 motivates the application of the liquidity score in monitoring the portfolio market liquidity risk with various liquidity constraints. In section 5.2, we explain the state-dependent cost/risk leverage parameter applied in the optimisation problem with the execution costs and the volatility risk tradeoff (as defined in Chapter 4). In section 5.3, we describe the liquidity-adjusted P&L distribution, and define the liquidity score. In section 5.4, we measure the impact of liquidity policies on the liquidity-adjusted probability density function, and compute the liquidity score on various liquidity constraints. Finally, Section 5.5 concludes.

5.1 Introduction

Liquidity tends to dry up during the periods of financial stress. Investment management industry has suffered from a self-enforcing liquidity spiral in the 2007-8 financial crisis. When an asset price of a company falls, it requires a higher margin. The firm having funding problem reduces its positions to make funds available (e.g. quant-equity strategies), and causes a decline in market liquidity. This market illiquidity exacerbates funding problems (i.e. increased margin requirements), developing negative feedback loop that leads to the liquidity crisis (Amihud et al., 2013; Brunnermeier and Pedersen, 2009; Meucci, 2012).

As emphasized in Sarr and Lybek (2002), a quantitative indicator to monitor the changing level of market liquidity over time is needed to predict any future liquidity crisis. In this study, we integrate market risk with liquidity risk to obtain liquidity-adjusted P&L distribution, and compute a related liquidity score (LS). The liquidity score of a portfolio is the ratio of the expected shortfall (ES) on pure market P&L scenarios to the expected shortfall for liquidity-adjusted P&L scenarios (the liquidity costs is estimated only on the portfolio sold e.g. a 50M portfolio sold when a 200M portfolio is subject to 25% cash constraint). The expected shortfall is the average loss given that the loss is greater than the Value at Risk for a given confidence level (Acerbi and Tasche, 2002; McNeil et al., 2005). This liquidity risk indicator takes value from 0 to 1. When this ratio is close to one, liquidity risk is negligible. Conversely, when the ratio is close to 0, there is significant liquidity risk (Meucci, 2012).

The execution costs is the trading costs when an investor executes large orders in adverse price movements on the traded assets (Perold, 1988; Guilbaud et al., 2013). We use the transaction cost measure proposed by Garleanu and Pedersen (2013), and use Kyle's lambda (Kyle, 1985) which captures price impact given the order flow (Amihud, 2002).

The volatility risk is the uncertainty about trading revenue because of price changes during the course of trading. An investor is uncertain about the final revenue one will actually receive if a portfolio is not sold instantly, but liquidated over multiple time periods. This risk would be large if the orders are executed slowly within the specified trading interval, or the trade program lasts for long periods. A risk measure for portfolios of multiple assets, suggested by Almgren and Chriss (2000), is applied to quantify this revenue risk.

Unlike other studies on optimal execution (see e.g. Almgren and Chriss, 2000; Gatheral and Schied, 2011 and Brigo and Graziano, 2014), my work has 25% cash and portfolio ES constraints. The model can measure the liquidity adjustment given the amount of client redemption, which is the proportion of the portfolio value, as well as risk limit on the residual portfolio. We estimate liquidity adjustment given these liquidity policies on different liquidation time. Portfolio market P&L scenarios affect trading speed. The execution speed is incorporated in the liquidity adjustment parameter estimation.

In this study, the liquidity costs is estimated on the portfolio sold (e.g. a 50M portfolio when 200M portfolio is subject to 25% cash constraint). The liquidity adjusted portfolio value, as Weber et al. (2012) have point out, is given by the maximal mark-to-market value on the residual portfolio after accounting for losses on a fragment of the portfolio forced to liquidate. We assign the costs to illiquidity on the portfolio liquidated (e.g. a 50M portfolio), and the residual portfolio value is given by the maximal mark-to-market value.

The purpose of this study is to show changing liquidity score for the given liquidity policies. When computing the liquidity score, we sample portfolio pure market P&L and market-plus-liquidity P&L scenarios with the state-dependent liquidity adjustments. Also, we show that the portfolio market-plus-liquidity P&L distribution changes according to liquidity policies (i.e. if the portfolio is subject to 25% cash and 4% ES constraints, then it would sell greater number of positions in high volatility assets, so it would results in high liquidity costs. As a result, the portfolio market-plus-liquidity P&L would have more negative value).

When integrating liquidity risk with market risk, the portfolio liquidity adjustments are quantified and combined with the pure market P&L scenarios. Liquidity risk scenarios are generated by using stress factors. This is because the liquidity risk would be greater when the portfolio market P&L is very negative. Different values of the liquidity costs/revenue risk leverage parameter in the optimal trade execution problem are used to obtain the state dependent portfolio liquidity adjustments. Then they are integrated with the pure portfolio market P&L scenarios to obtain the liquidity-plus-market P&L distribution. Subsequently, portfolio expected shortfall values for a given confidence level for both pure market P&L and liquidity-adjusted P&L scenarios are measured, and then they are used to compute the liquidity score.

5.2 State-dependent liquidity adjustments

This section outlines our optimisation problem with the state-dependent liquidity adjustment parameters $[C, R]$ (originally defined in Chapter 2). According to Almgren and Chriss (2000) and Brigo and Graziano (2014), an optimal execution problem that minimises the execution costs and the volatility risk is given by the expression

$$Min_{\mathbf{q_n}} C + L \cdot R \tag{5.1}$$

where $\mathbf{q_n}$ is the vector of the positions liquidated in cash value at the nth period, C is the execution costs, R is the volatility risk function on positions remaining, and L is the leverage parameter between the cost and the risk. The portfolio liquidity adjustment parameters for K assets $[C, R]$ (as defined in Chapter 4) are given by

$$C = \tau^{-1} \sum_{n=1}^{N} \frac{1}{2} \mathbf{q_n}^{'} \Lambda \mathbf{q_n} \tag{5.2}$$

and

$$R = \sqrt{\sum_{n=1}^{N} \tau \mathbf{h_n}^{'} \Sigma \mathbf{h_n}} \tag{5.3}$$

where $q_n = (q_{1,n}, q_{2,n}, ..., q_{K,n})^{'}$ is the $K \times 1$ vector of positions liquidated in cash value at the nth period, and $\mathbf{h_n} = (h_{1,n}, h_{2,n}, ..., h_{K,n})^{'}$ is the $K \times 1$ vector of positions remaining to sell at the end of time n.

The cost/risk leverage parameter L is varied from 0 to 10^5 depending on pure market P&L scenarios in this study (Brigo and Graziano, 2014). The risk aversion parameter L can be thought of a Lagrange multiplier i.e. $\frac{dCost}{dRisk}$ in the constrained optimisation problem (Almgren, 2003). As the leverage parameter L becomes larger, the investor is more concerned about the uncertainty of liquidity adjustment (i.e. liquidate faster); conversely, if the parameter L is smaller, one is less risk averse (i.e. liquidate slower). If the parameter L is 0, then the investor ignores about the risk component (i.e. liquidate at an equal trading rate over the trading interval).

The optimal execution problem with the cost/risk leverage parameter becomes

$$Min_{\mathbf{q_n}} \left[\tau^{-1} \sum_{n=1}^{N} \frac{1}{2} \mathbf{q_n}^{'} \Lambda \mathbf{q_n} + L \cdot \left(\sqrt{\sum_{n=1}^{N} \tau \mathbf{h_n}^{'} \Sigma \mathbf{h_n}} \right) \right]. \tag{5.4}$$

When the market P&L is very negative, an investor tends to liquidate positions faster Meucci (2012). The cost/risk leverage parameter L from 5.1 can describe this by assigning a large value of L according to a stress factor. The stress factor applied for each portfolio market P&L scenario j are given by

$$\sigma_g = |\alpha_c|\, \sigma_{\overline{\Pi}} \tag{5.5}$$

$$g_{(j)} = -\frac{Min\left[\overline{\pi}_{(j)}, -\sigma_g\right]}{\sigma_g},\ j = 1,\ldots,J \tag{5.6}$$

where α_c is the value from the standard normal distribution (which is -1.64 at the 95% confidence level), $\overline{\Pi}$ is the distribution of the pure market P&L scenarios, $\overline{\pi}_{(j)}$ is the portfolio P&L scenarios for each j, $g_{(j)}$ is the stress factor which gives greater weight when the market P&L is very negative. The positions liquidated in cash value $\mathbf{q_n}$ for each period n is obtained by solving the optimisation problem

$$Min_{\mathbf{q_n}} C + L\left(g_{(j)}\right) \cdot R,\ j = 1,\ldots,J \tag{5.7}$$

and the state-dependent execution costs and volatility risk for each scenario j are given by

$$C_{(j)} = C\left(\mathbf{q_n}\right) \tag{5.8}$$

$$R_{(j)} = R\left(\mathbf{h_n}\right) \tag{5.9}$$

where C and $\mathbf{q_n}$, R and $\mathbf{h_n}$ are given in (5.2) and (5.3), respectively. In this study, we use a simple function that sets the cost/risk leverage parameter value as $L = 10^{1+g_{(j)}}$ to model the relationship between the portfolio market P&L scenario and the trading speed. For example, if a stress factor for the jth portfolio market P&L scenario is 4, then the cost/risk leverage parameter L is set to be 10^5. The large value of L conveys that an investor is more concerned about the uncertainty of liquidity adjustment, so one is trading fast. This results in greater liquidity costs, but smaller uncertainty of the liquidity costs.

5.3 Liquidity-adjusted PDF/CDF and Liquidity score

In this section, we define the liquidity-adjusted P&L distribution, and describe the liquidity score. Suppose a portfolio liquidated $\mathbf{h}$ with K assets and the $K \times 1$ vector of the total number of positions sold $\mathbf{h} \equiv (\mathrm{h}_{1,0}, \ldots, \mathrm{h}_{\mathrm{K},0})^{'}$ for the given liquidity constraints. We denote π_k as the mark-to-market P&L for the kth security over one day investment horizon. The risk driver of the kth stock is its log price, that is

$$m_k = ln S_k, \tag{5.10}$$

and the mark-to-market P&L is given by

$$\pi_k = e^{m_k} - S_{k,0}. \tag{5.11}$$

According to Meucci (2012), the distribution of risk drivers (i.e. log prices of stocks with their respective probabilities) is

$$\mathbf{M} \sim \left[\mathbf{m}_{(j)}, \bar{p}_{(j)}\right]_{j=1,\ldots,J} \tag{5.12}$$

where j is a set of scenarios, $\bar{p}_{(j)}$ is a scalar value of the probability for the jth scenario (e.g. $\frac{1}{J}$), and $\mathbf{m}_{(j)} = \left(m_{1,j}, m_{2,j}, \ldots, m_{K,j}\right)^{'}$ is the $K \times 1$ vector of the risk drivers. A probability $\bar{p}_{(j)}$ is assigned to the jth scenario of the log prices $\mathbf{m}_{(j)}$. The risk drivers $\mathbf{m}_{(j)}$ are generated through Monte Carlo simulation. Random variables (i.e. J scenarios for K assets) are drawn from multivariate student-t distribution with a correlation matrix and the degrees of freedom parameter (set to be 7 in this study). Student-t distribution can deal with fat tails in empirical distribution of stock returns (Andreev and Kanto, 2005; Shaw, 2011). The smaller the degrees of freedom parameter, the fatter tails the distribution has. As the degrees of freedom parameter becomes larger, the distribution converges to a normal distribution. Historical simulations approach is used to generate the risk drivers. It is possible to stress-test the model by assigning different probabilities for the portfolio market P&L scenarios that should give a specified target level of average value of the P&Ls using, as Meucci (2010) indicates, relative entropy minimisation. For simplicity, an equal probability $\frac{1}{J}$ is assigned for each P&L scenario in this study. The portfolio P&L scenarios for each j are obtained by the following simple equation,

$$\overline{\pi}_{(j)} = \sum_{k=1}^{K} p_k \pi_k \left(m_{k,j}\right) \tag{5.13}$$

where p_k is the initial positions for the kth asset, π_k is the mark-to-market P&L, and $m_{k,j}$ is the risk driver (i.e. a log price of the stock simulated). We obtain the pure market P&Ls for each k as multiplying the initial positions for each asset (which is p_k) by their respective mark-to-market P&Ls for the jth scenario (which is $\pi_k \left(m_{k,j}\right)$). The probability for each portfolio P&L scenario remains the same as the probability for its risk driver, that is

$$\overline{\Pi} \sim \left[\overline{\pi}_{(j)}, \bar{p}_{(j)}\right]_{j=1,\ldots,J}. \tag{5.14}$$

where $\overline{\Pi}$ is the distribution of the portfolio pure market P&L scenarios with their respective probabilities (e.g. $\frac{1}{J}$).

The liquidity-plus-market-risk P&L (Π), as Meucci (2012) proposes, is the aggregation of the pure market-to-market component ($\overline{\Pi}$) and the liquidity adjustment ($\Delta\Pi$). The conditional PDF of the liquidity adjustment is given by

$$f_{\Delta\Pi|\mathbf{m}_{(j)}}(y) = \frac{1}{R_{(j)}} \varphi \left(\frac{y - C_{(j)}}{R_{(j)}}\right) \tag{5.15}$$

where $C_{(j)}$ and $R_{(j)}$ are given in (5.8) and (5.9), respectively, andφ is the standard normal distribution. The PDF of the total portfolio P&L marginalizing over the distribution of risk drivers can be written as

$$\begin{aligned} f_{\Pi}(y) &= \int f_{\Pi|\mathbf{m}}(y) f_{\mathbf{M}}(\mathbf{m}) d\mathbf{m} \\ &= \int f_{\overline{\Pi}+\Delta\Pi|\mathbf{m}}(y) f_{\mathbf{M}}(\mathbf{m}) d\mathbf{m} \\ &= \int f_{\Delta\Pi|\mathbf{m}} \left(y - \sum_{k=1}^{K} p_k \pi_k \left(m_{k,j}\right)\right) f_{\mathbf{M}}(\mathbf{m}) d\mathbf{m}. \end{aligned} \tag{5.16}$$

The PDF of Fully Flexible Probabilities risk drivers (Meucci, 2010) is defined as

$$f_{\mathbf{M}}(\mathbf{m}) = \sum_{j=1}^{J} \bar{p}_{(j)} \delta^{\mathbf{m}_{(j)}}(\mathbf{m}) \tag{5.17}$$

where $\delta^{\mathbf{m}_{(j)}}$ is the Dirac delta function at the origin $\mathbf{m}_{(j)}$.

By substituting (5.17) into (5.16), it gives

$$\begin{aligned} f_{\Pi}(y) &= \int f_{\Delta\Pi|\mathbf{m}} \left(y - \sum_{k=1}^{K} p_k \pi_k \left(m_{k,j} \right) \right) \sum_{j=1}^{J} \bar{p}_{(j)} \delta^{\mathbf{m}_{(j)}}(\mathbf{m}) d\mathbf{m} \\ &= \sum_{j=1}^{J} \bar{p}_{(j)} f_{\Delta\Pi|\mathbf{m}_{(\mathbf{j})}} \left(y - \overline{\pi}_{(j)} \right). \end{aligned} \tag{5.18}$$

Substituting (5.15) into (5.18) yields the liquidity-plus-market-risk P&L (Π) distribution of Meucci (2012), that is

$$f_{\Pi}(y) = \sum_{j=1}^{J} \frac{\bar{p}_{(j)}}{R_{(j)}} \varphi \left[\frac{y - \overline{\pi}_{(j)} - C_{(j)}}{R_{(j)}} \right] \tag{5.19}$$

where φ is the standard normal probability density, $\overline{\pi}_{(j)}$ is given in (5.13), $\bar{p}_{(j)}$ is given in (5.12), and $C_{(j)}$ and $R_{(j)}$ are given in (5.8) and (5.9), respectively. By integrating the liquidity-plus-market-risk P&L density, it gives the liquidity-plus-market-risk P&L cumulative distribution function (CDF),

$$F_{\Pi}(y) = \sum_{j=1}^{J} \bar{p}_{(j)} \Phi \left[\frac{y - \overline{\pi}_{(j)} - C_{(j)}}{R_{(j)}} \right] \tag{5.20}$$

where Φ is the standard normal CDF, and the other parameters are given in (5.19). From the liquidity-adjusted P&L CDF, liquidity-plus-market P&L scenarios can be generated using the inverse method,

$$u \sim U_{[0,1]} \tag{5.21}$$

$$\Pi \sim F_{\Pi}^{-1}(u) \tag{5.22}$$

where u follows a standard uniform distribution, $F_{\Pi}^{-1}(u)$ represents the inverse CDF of the liquidity-plus-market-risk P&L. A uniformly distributed random number u is used to sample the liquidity-plus-market-risk P&L (Π) numerically from the distribution (5.20). The liquidity score of a portfolio, as Meucci (2012) suggests, is the ratio of the expected shortfall (taken as 99%) on pure market P&L scenarios ($\overline{\Pi}$) to the expected shortfall for liquidity-adjusted P&L scenarios (Π),

$$LS_{99\%} = \frac{ES_{99\%}\{\overline{\Pi}\}}{ES_{99\%}\{\Pi\}}. \tag{5.23}$$

Algorithm 5.1 outlines the liquidity score estimation with portfolio ES values on the pure market P&L and the liquidity adjusted P&L scenarios. In this study, we obtain the total number of positions sold for each k for the given liquidity policies. In other words, for specified liquidity policies (e.g. 25% cash requirement, or 25% cash requirement and maximum portfolio ES), positions sold for the kth asset at the nth time period q_n is found for N periods by solving the optimal execution problem with the cost/risk leverage parameter set to be 0 (this gives the state-independent liquidity adjustments), The positions remaining to sell h_n may be deduced from the values of q_n. After that, a Monte Carlo simulation is used to generate pure market P&L scenarios ($\overline{\Pi}$) from multivariate student-t distribution. The state-dependent liquidity costs and the revenue risk are obtained from (5.8) and (5.9), respectively, after solving the optimisation problem in (5.7). We reduce our problem that the total number of shares sold for each asset, which is $h_{k,0}$, is the same for all values of L; the cost/risk leverage parameter does not affect the total number of positions to trade, but it does affect the speed of trading to produce the state-dependent liquidity adjustments (i.e. the liquidity adjustments for each j would be changed only because of the execution speed; the total number of shares sold for each asset is independent of the market state; the value $h_{k,0}$ for each k only depends on liquidity policies). A Monte Carlo simulation from standard uniform random variables and an inverse method are then applied to generate liquidity adjusted P&L scenarios (Π) from Liquidity adjusted CDF in (5.20). Finally, we compute the liquidity score from the expected shortfall relatives of the portfolio pure market P&L scenarios ($\overline{\Pi}$) and the liquidity-adjusted P&L

Algorithm 5.1 Algorithm to estimate a liquidity score with liquidity constraints: cash requirement and maximum portfolio ES limit

- [Objective function]
$q_n \equiv argmin_{q_n} \{C(q_n) + L \cdot R(x_n)\}$ [Optimal execution schedule in (5.4)]

- [Constraints]
$\sum_n q_n = 0.25pm^+$ [25% cash requirement]
$ES_c \leq \alpha\%$ on the portfolio weight $w_k = \frac{a_k m_k}{\sum_k a_k m_k}$ [Maximum portfolio ES $\alpha\%$ and holding positions $a_k = p_k - \sum_n z_{k,n}$]

- [State-independent liquidity adjustment parameters]
$C(q_n)$ [Liquidity adjustment given in (5.2)]
$R(x_n)$ [Uncertainty of the liquidity adjustment given in (5.3); cash value of positions remaining to sell at the end of nth period is given by $\mathbf{x_n} = \mathbf{x_{n-1}} - \mathbf{q_n}$]

- [Pure market P&L scenarios]
$\bar{\pi} = \pi \mathbf{p}$ [π is the $J \times K$ matrix of the portfolio P&L simulated in (5.13), and $\mathbf{p}$ is the $K \times 1$ vector of the initial positions; this results in a $J \times 1$ column vector of the portfolio pure market P&L scenarios.]
$\bar{\Pi} = \left[\bar{\pi}_{(j)}, \bar{p}_{(j)}\right]_{j=1,\ldots,J}$ [Portfolio P&L scenarios in (5.14); an equal probability $\left(\bar{p}_{(j)} = \frac{1}{J}\right)$ is applied for each scenario]

- [State-dependent liquidity adjustment parameters]
For $j = 1, \ldots, J$
 If $g_{(j)} > 1$[If the jth stress factor computed in (5.6) is greater than 1, then run the optimisation problem in (5.7)]
  $[C_{(j)}, R_{(j)}]$ [State-dependent liquidity adjustment parameters are estimated according to portfolio pure market P&L scenarios]
 End (If)
End (For)

- [Liquidity adjusted P&L scenarios]
$F_{\Pi}(y) = \sum_j \bar{p}_{(j)} \Phi \left[\frac{y - \bar{\pi}_{(j)} - C_{(j)}}{R_{(j)}}\right]$ [Liquidity adjusted CDF in (5.20)]
$\Pi \sim F_{\Pi}^{-1}(u)$ [Generate liquidity-plus-market P&L scenarios from the liquidity adjusted CDF]

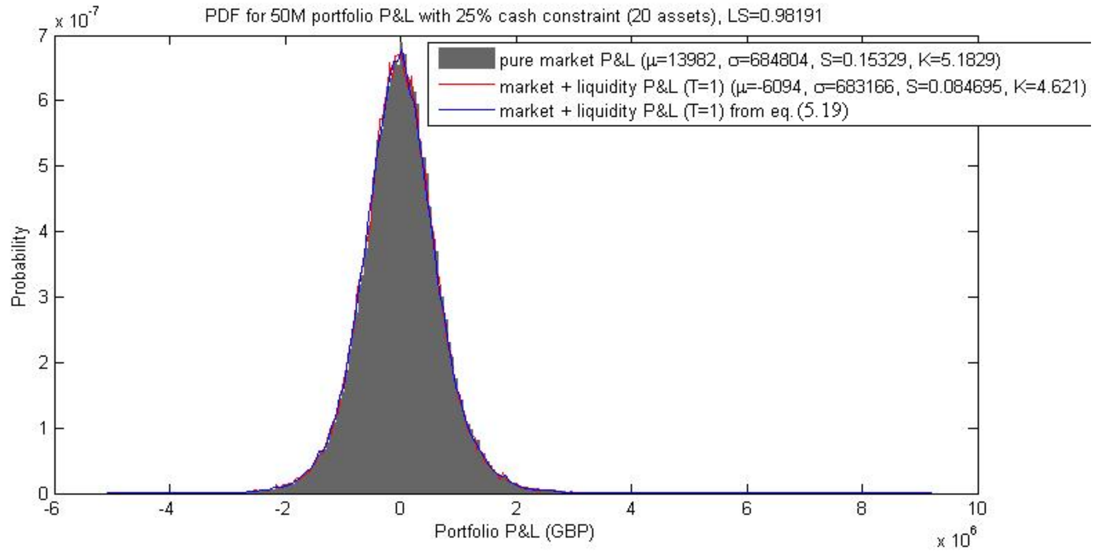
- [Liquidity score]
$LS_c = \frac{ES_c\{\bar{\Pi}\}}{ES_c\{\Pi\}}$. [Estimating liquidity score with the portfolio ES at confidence level c on the pure market P&L and the liquidity adjusted P&L scenarios]

scenarios (Π) over one day.

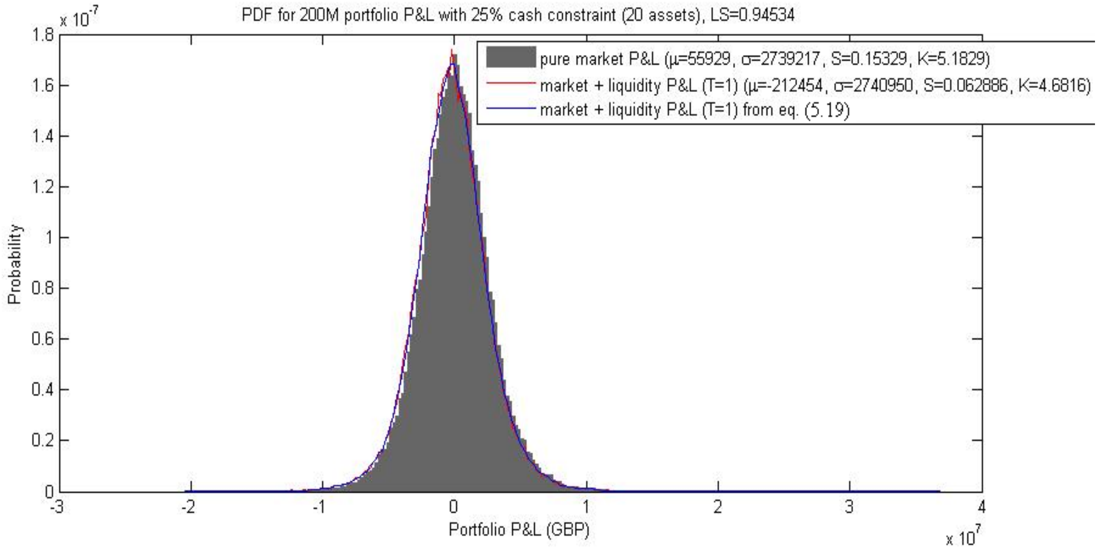
5.4 Result

In this section, we quantify the portfolio liquidity adjustment parameters on liquidity constraints, and show the impact of the liquidity policies on the liquidity-adjusted PDF and the liquidity score.

Figure 5.1: Market-plus-liquidity portfolio P&L distributions on different sizes of portfolio MtM value



(a)



(b)

This figures show the market-plus-liquidity portfolio P&L distributions for the portfolio MtM value (a) 50M and (b) 200M with 25% cash constraint. The liquidation time is 1 day, and the trading time step τ is set to 0.2.

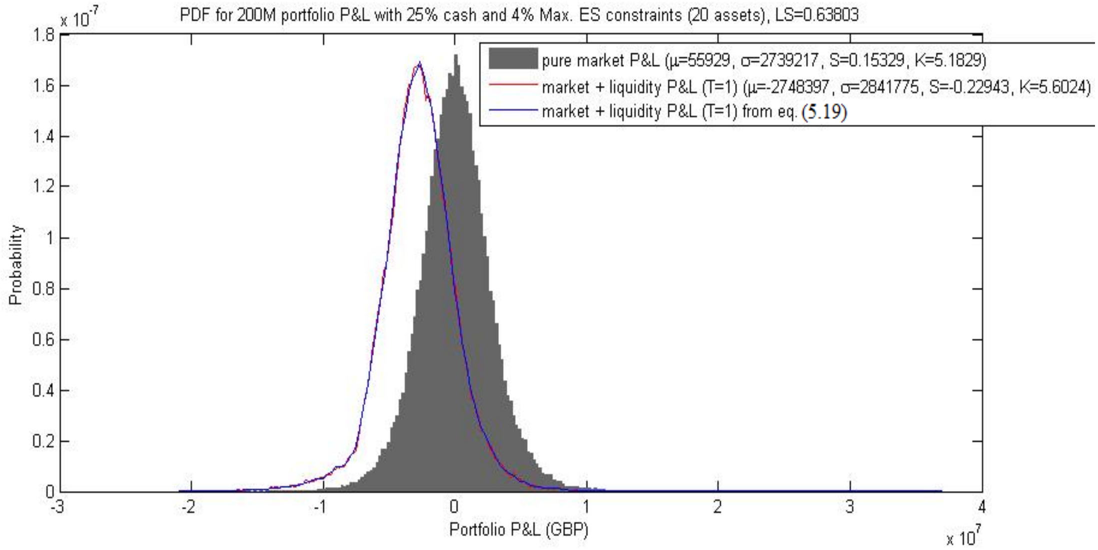
Table 5.1: Liquidity scores for the 50M and 200M portfolios both with 25% cash requirement

Portfolio size	C	R	$VaR_{99\%}\,(\Pi)$	$ES_{97.5\%}\,(\Pi)$	$ES_{99\%}\,(\bar{\Pi})$	$ES_{99\%}\,(\Pi)$	$LS_{99\%}$ (%)
50M	3	7	344	352	419	426	98
200M	13	6	358	366	419	444	94

This table reports the liquidity adjustment parameters $[C, R]$, expect shortfall values for pure market P&L ($\bar{\Pi}$) and market-plus-liquidity P&L (Π), and liquidity scores computed for the 50M and 200M portfolios both with 25% cash requirement. The liquidity adjustments and the expect shortfall values with respect to their portfolio MtM value are measured in basis points. The liquidity scores are expressed in percentage. $ES_{99\%}$ represents one day expected shortfall value at 99% confidence level.

We compare market-plus-liquidity portfolio P&L distributions on different sizes of portfolio MtM values from Figure 5.1. As indicated in Table 5.1, the 99% ES on the pure market P&L scenarios for both 50M and 200M portfolios produce the same values as 419 bp (which is positive homogeneity (PH) of the coherent risk measure). However, liquidity risk for the large portfolio tends to be greater; the 99% ES on the market plus liquidity P&L results in higher value (from 426 bp to 444 bp) for the 200M portfolio in comparison with the ES value for the 50M portfolio. For the portfolio MtM value 200M, the liquidity scores dropped 4% (from 98% to 94%). This is resulted from the liquidity costs increased from 3 bp to 13 bp as the size of the portfolio increases from 50M to 200M. A larger expected shortfall value is estimated on the total market-plus-liquidity P&L scenarios accordingly, widening of the two expected shortfall values.

Figure 5.2: Market-plus-liquidity portfolio P&L distribution under 25% cash and 4% maximum ES constraints



This figure shows the market-plus-liquidity portfolio P&L distribution for the portfolio MtM value 200M with 25% cash and 4% maximum ES constraints.

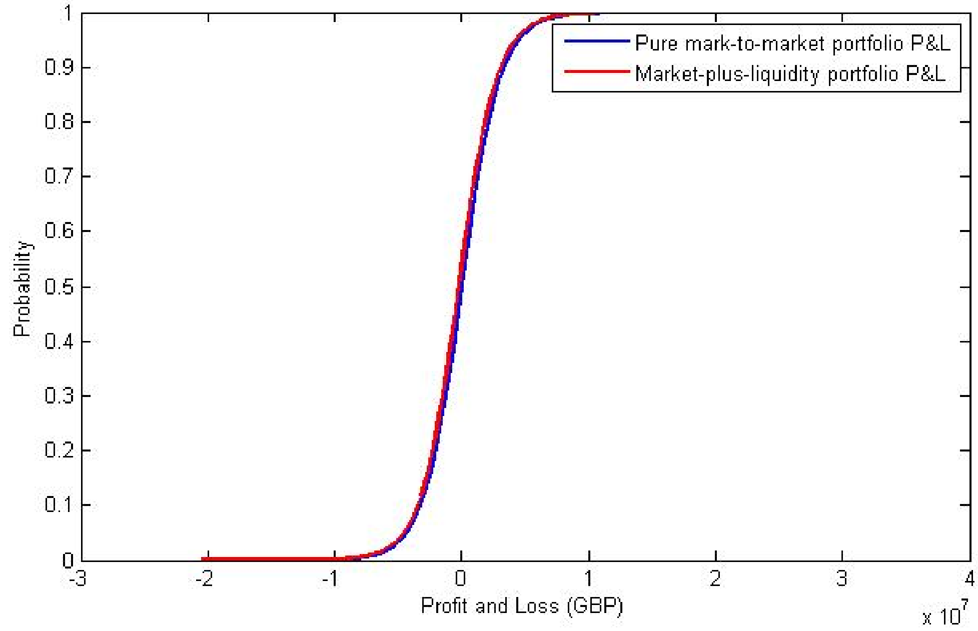
Table 5.2: Liquidity scores for the 200M portfolios with 25% cash and 4% maximum ES constraints

Constraint	C	R	$VaR_{99\%}\,(\Pi)$	$ES_{97.5\%}\,(\Pi)$	$ES_{99\%}\,(\bar{\Pi})$	$ES_{99\%}\,(\Pi)$	$LS_{99\%}$ (%)
Cash	13	6	358	366	419	444	94
Cash + ES	138	10	536	548	419	657	64

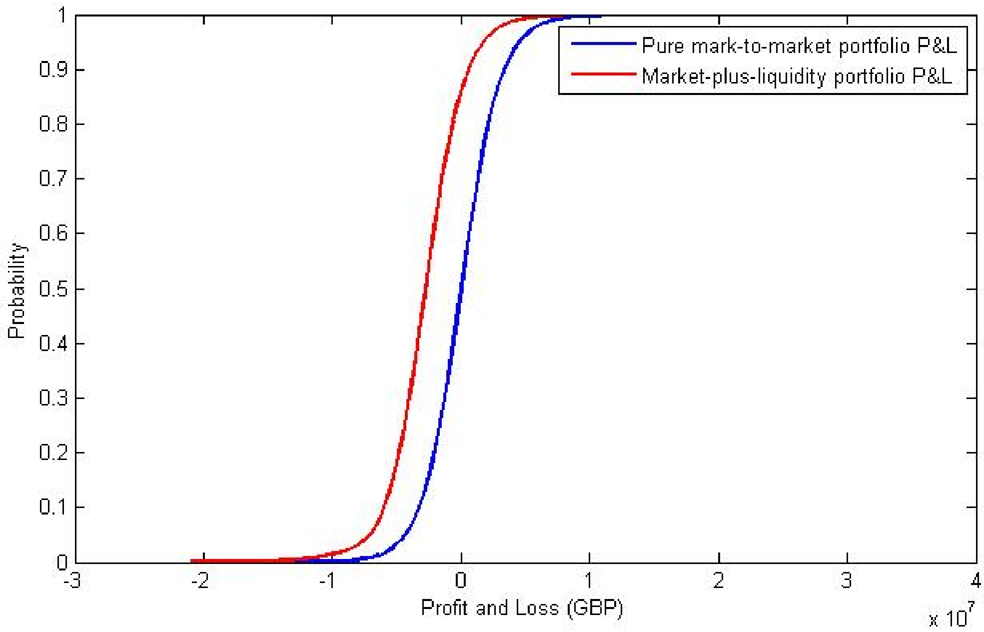
This table reports liquidity adjustment parameters $[C, R]$, expect shortfall values for pure market P&L ($\bar{\Pi}$) and market-plus-liquidity P&L (Π), and liquidity scores computed for the 200M portfolio with 25% cash, and 25% cash and 4% maximum ES constraints. The liquidity adjustments and the expect shortfall values with respect to their portfolio MtM value are measured in basis points. The liquidity scores are expressed in percentage. $ES_{99\%}$ denotes the one day expected shortfall value at 99% confidence level.

After adding the 4% maximum ES with 25% cash constraint, there is a dramatic decline in liquidity scores (from 94% to 64%) on the 200M portfolio. As can be seen from Table 5.2, the effect of the 4% maximum ES constraint is to increase the liquidity costs around 942%. This is because greater number of high volatility assets is sold in comparison with the liquidation on the cash-only constraint to reduce the portfolio ES as shown in Table 5.3. From Figure 5.2, the tail of the distribution became heavier when a larger number of high volatility assets is liquidated. The kurtosis of the pure market P&L and market plus liquidity P&L probability distributions are 5.18 and 5.60, respectively.

Figure 5.3: Cumulative distribution for the pure mark-to-market and the market-plus-liquidity portfolio P&Ls



(a)



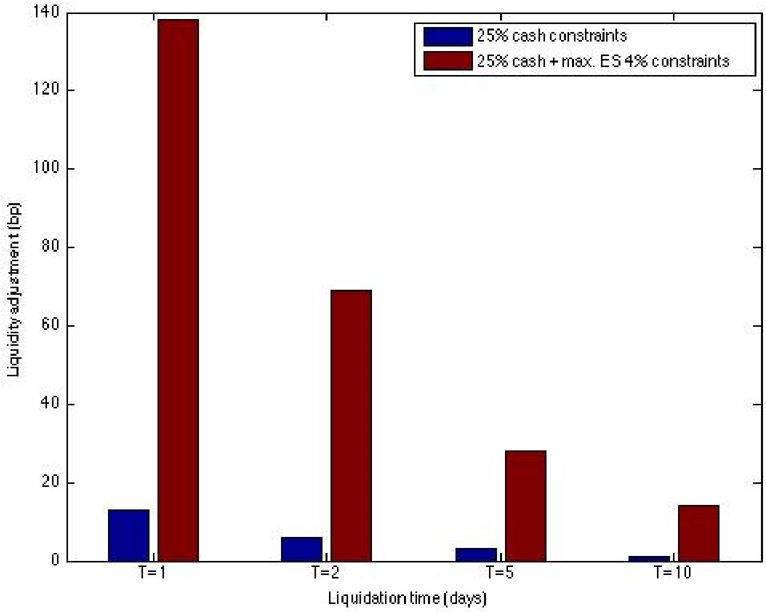
(b)

This figure shows the cumulative distributions for the pure mark-to-market and the market-plus-liquidity portfolio P&Ls on the 200M portfolio with 25% cash constraint (a) and 25% cash and 4% maximum ES constraints (b).

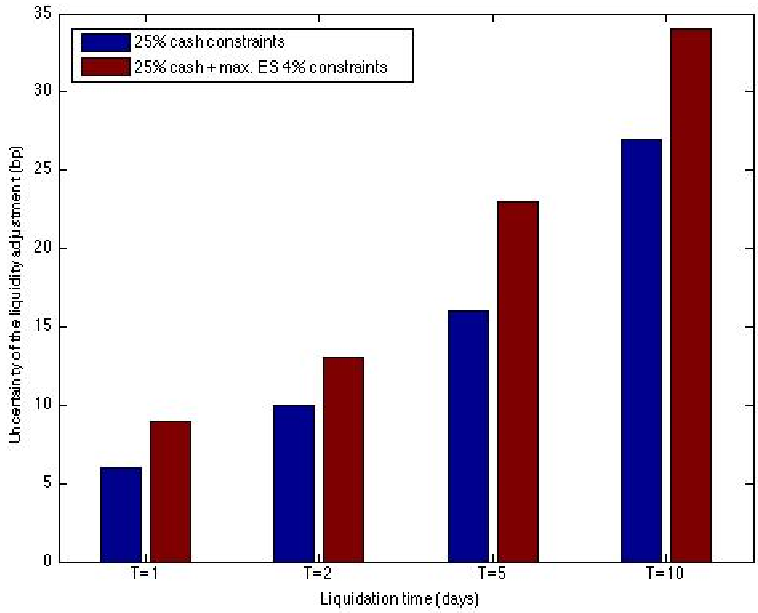
The cumulative distribution function for pure market P&L ($\bar{\Pi}$) and market-plus-liquidity P&L (Π) is shown in Figure 5.3. The effect of liquidity adjustment parameters $[C, R]$ on the pure market

P&L is to have a more negative P&L. With 25% cash and 4% maximum ES constraints, the market-plus-liquidity P&L becomes very negative because mainly high volatility assets are liquidated to satisfy the market risk limit on the residual portfolio.

Figure 5.4: Liquidity adjustment parameters estimated on different liquidation time



(a)

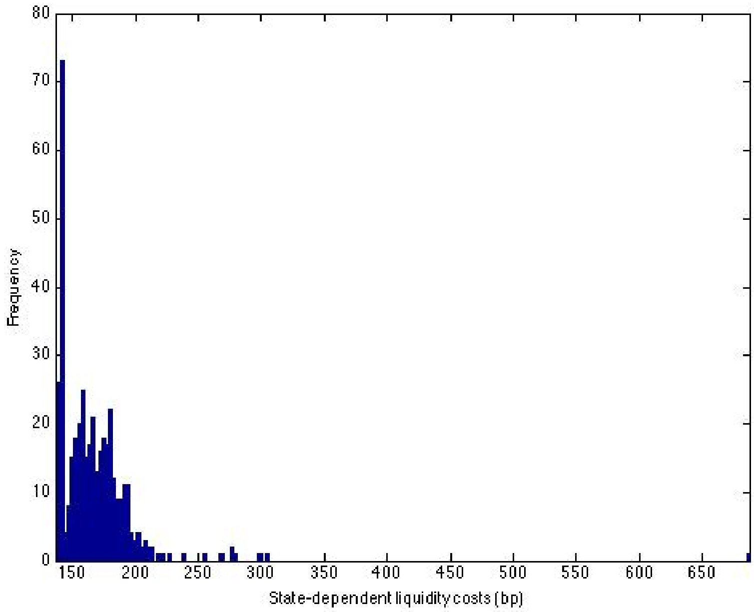


(b)

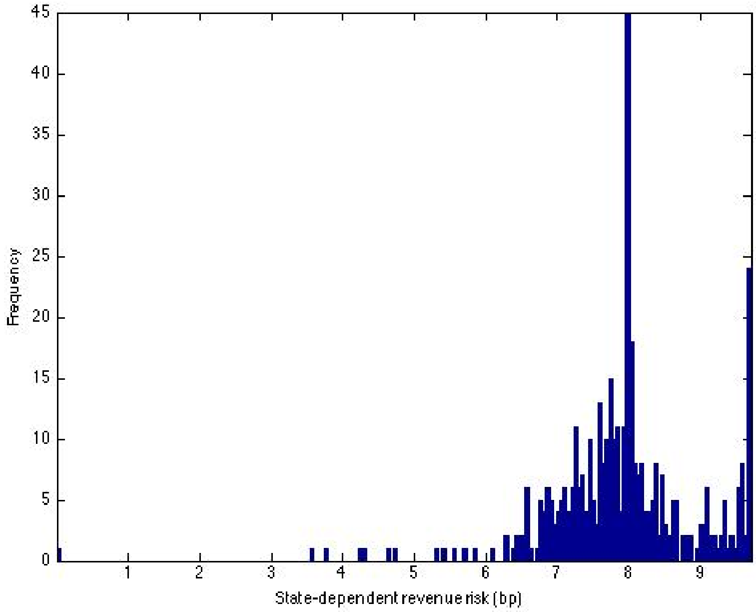
This figure displays liquidity adjustment (a) and uncertainty of the liquidity adjustment (b) with respect to the portfolio MtM value (200M) with 25% cash, and 25% cash and 4% maximum ES constraints according to liquidation time from 1 to 10 days. Trading time step τ is set to 0.2.

Liquidity adjustment with respect to the portfolio MtM value 200M on different liquidation time from 1 to 10 days is shown in Figure 5.4. As the liquidation time increases, the liquidity adjustment decreases because a large order can be split into many small children orders with longer liquidation time. This time slicing on an order can reduce the transaction cost. In contrast, longer liquidation time increases uncertainty of the liquidity adjustment. A slow trade would have greater uncertainty of liquidity costs because the price traded in the current time step is likely to be different from the price in the previous time step as liquidation time become longer.

Figure 5.5: State-dependent liquidity adjustment parameters



(a)

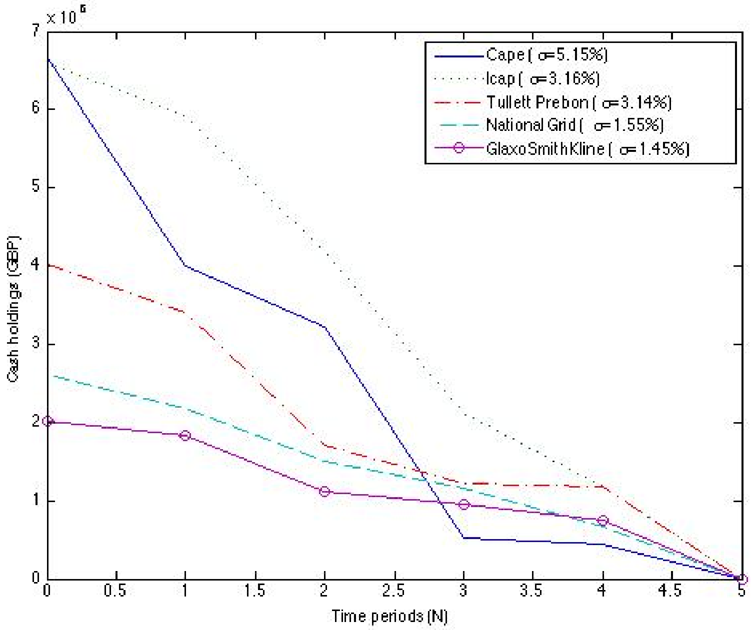


(b)

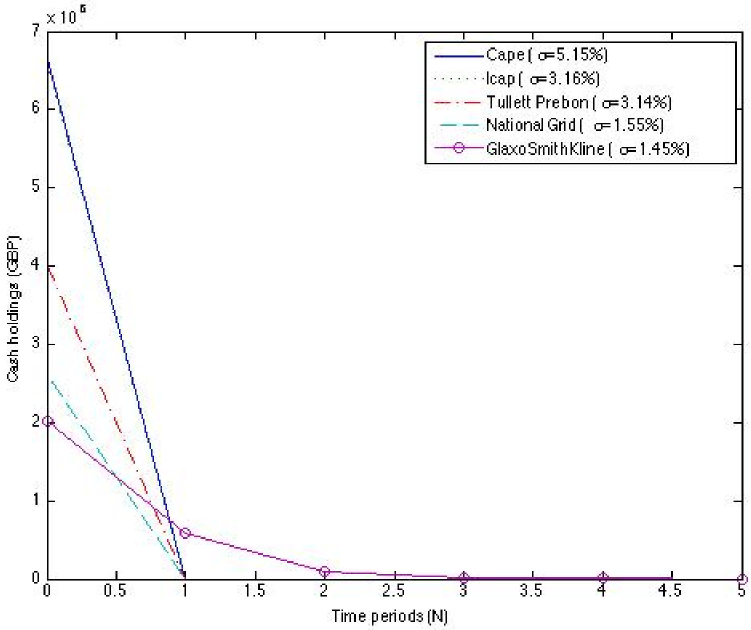
This figure plots the state-dependent liquidity costs (a) and revenue risk (b) with a stress factor greater than 1 (i.e. $g_{(j)} > 1$) for the portfolio MtM value (200M) with 25% cash and 4% maximum ES constraints. Liquidation time T is set to be 1 day, and trading time step τ is set to be 0.2. The liquidity costs with respect to the portfolio MtM value are measured in basis points. The number of portfolio market P&L scenarios that exceed 95% VaR (σ_g) value is 412. The total number of the portfolio market P&L scenarios J is set to be 10000.

An investor tends to liquidate positions faster as the portfolio market P&L becomes more negative; as a result, the liquidity costs would increase, while the revenue risk would decrease. A large value of the cost/risk leverage parameter L can model this. The cost/risk leverage parameter L depends on a stress factor $g_{(j)}$. The stress factor value is the relatives of a portfolio market P&L for the jth scenarios and the 95% VaR (σ_g) value on the portfolio market P&L distribution. The more negative the portfolio market P&L is, the larger the value of a stress factor $g_{(j)}$ is. The cost/risk leverage parameter applied in the state-dependent liquidity adjustments estimation for the jth scenario is given by $10^{1+g_{(j)}}$. For instance, if the portfolio market P&L is 5 times more negative than 95% VaR (σ_g) value on the distribution of the portfolio market P&L scenarios, then the stress factor $g_{(j)}$ is set to be 5, and the cost/risk leverage parameter L becomes 10^6. The liquidity costs and the revenue risk measured according to different cost/risk leverage parameter values for the 200M portfolio with 25% cash and 4% maximum ES constraints are shown in Figure 5.5. A larger value of the cost/risk leverage parameter L results in greater liquidity costs estimated and smaller revenue risk. State-independent liquidity costs is estimated as 138 bp, but state-dependent liquidity costs varies from 138 bp to 687 bp according to portfolio market P&L scenarios. Also, state-independent revenue risk is estimated as 10 bp, but state-dependent revenue risk ranges from 0 bp to 10 bp depending on the market P&L scenarios. The trading speed increases as the cost/risk leverage parameter L becomes larger. For the cost/risk leverage parameter L set to be 10^6, the liquidation for all the assets except the least volatile asset from Figure 5.6 is completed within 2 trading periods.

Figure 5.6: Trading speed on different cost/risk leverage parameter values



(a)



(b)

This figure shows the trading speed for the three most volatile, and the two least volatile assets in the 200M portfolio with 25% cash and 4% maximum ES constraints according to two different cost/risk leverage parameter values: (a) 10^3 and (b) 10^6. The liquidation time T is set to be 1 day, and the trading time step τ is set to be 0.2.

Table 5.3: Comparison of liquidity adjustment estimation with liquidity constraints

Liquidity policies	
cash (25%)	13 bp
cash (25%) and ES (4%)	138 bp

						$\sum_n \widetilde{z}_{k,n}$ **(%)**	
Company	m **(GBP)**	p	σ **(%)**	$\widetilde{\theta}$ (%)	$\widetilde{p}$ (%)	**Cash**	**Cash+ES**
CAPE	2.21	4524886	5.15	170	636	0	67
ICAP	3.29	3039513	3.16	206	97	7	66
TULLETT PREBON	2.41	4149377	3.14	227	433	0	40
FIRSTGROUP	1.98	5050505	2.50	200	209	0	47
CABLE & WIRELESS COMMUNICATIONS	0.41	24390243	2.41	114	110	33	25
HALFORDS GROUP	3.45	2898550	2.26	228	332	0	32
GO-AHEAD GROUP	13.1	763358	2.13	300	533	0	7
RSA INSURANCE GROUP	1.32	7575757	2.00	286	60	41	52
STOBART GROUP	0.97	10309278	1.91	947	2811	0	0
AMLIN	3.9	2564102	1.90	290	150	26	12
BAE SYSTEMS	3.38	2958579	1.86	281	26	60	37
VODAFONE GROUP	1.73	5780346	1.81	392	5	70	32
UK COMMERCIAL PROPERTY TRUST	0.68	14705882	1.77	2563	3148	0	1
MERCHANTS TRUST	4.15	2409638	1.75	3571	1977	0	0
RESOLUTION	2.62	3816793	1.68	607	163	39	0
PHOENIX GROUP HOLDINGS	6.37	1569858	1.68	1681	1512	0	0
F&C COMMERCIAL PROPERTY TRUST	1.03	9708737	1.59	1211	1570	0	2
SCOTTISH&SOUTHERN ENERGY	14.28	700280	1.55	391	29	70	33
NATIONAL GRID	6.96	1436781	1.55	444	17	75	26
GLAXOSMITHKLINE	14.52	688705	1.45	451	6	80	20

This table reports the comparison of liquidity adjustment estimation for the £200M equally weighted for a long only portfolio with the 25% cash (Cash); 25% cash and $ES_{99\%} \leq 4\%$ (ES) on the residual portfolio. Trade duration is set to 1. The m column is the asset price in GBP. The p column shows the initial positions. The σ column is the daily volatility in percentage. The $\sum_n \widetilde{z}_{k,n}$ column represents the trading volume over the initial positions in percentage, i.e. $\frac{\sum_n z_{k,n}}{p_k} 100$. The column $\widetilde{p}$ refers to the initial positions over the average daily volume in percentage, i.e. $\frac{p_k}{V_k} 100$. The column $\widetilde{\theta}$ represents the total number of shares in issue with respect to the average daily volume of the asset in percentage, i.e. $\frac{\theta_k}{V_k} 100$.

5.5 Conclusions

We measure liquidity score by Meucci (2012) for the portfolio in the presence of the liquidity policies: cash requirement and the portfolio expected shortfall limit constraints. An investor can monitor the market liquidity over time for the equity portfolio using the liquidity score, which is ranging from 0 to 1 (perfect liquidity). We compute liquidity score as the ratio of the expected shortfall (e.g. taken as 99%) on pure market P&L scenarios to the expected shortfall for liquidity-adjusted P&L scenarios. The liquidity-adjusted P&L scenarios are generated by combining the pure market P&L scenarios with the state-dependent liquidity adjustment parameters.

An investor aims to minimize the transaction costs and the revenue risk over the trade periods by selling assets according to optimal trade sizes and speeds. An investor liquidates positions faster when the portfolio market P&L is very negative. The execution costs and the revenue risk are quantified given the state-dependent cost/risk leverage parameter because the speed of the trading depends on the market situation (Almgren and Chriss, 2000; Brigo and Graziano, 2014; Gatheral and Schied, 2011).

We find the price impacts depend on liquidity policies (e.g. cash requirement and the portfolio ES constraints), different portfolio MtM values and time to liquidation. For the portfolio MtM value 200M, the liquidity score dropped 4% (from 98% to 94%) in comparison with the liquidity score for the 50M portfolio. The liquidity score decreases as the portfolio size becomes larger. Also, for the portfolio MtM value 200M, the liquidity score falls sharply (from 94% to 64%) when the portfolio has the 4% maximum ES limit as well as 25% cash constraint. The reason for the significant decrease in LS was an increase in the liquidation of high volatility assets in order to satisfy the risk limit. Longer liquidation time decreases the liquidity adjustment; however, it results in higher revenue risk because an asset price can change over the specified trading interval (Kharroubi and Pham, 2010).

The liquidity scores can be a useful quantitative indicator to measure the market liquidity given the liquidation time for an equity (long only) portfolio. It can be employed to construct (and maintain) a portfolio which is liquid enough (or help to avoid significant liquidation costs from a possible fire-sale) in stressed market situation.

Chapter 6

Conclusion

In this thesis, we have quantified the potential cost of liquidity constraints on a long equity portfolio using the liquidity risk measures of Finger (2011) and Garleanu and Pedersen (2013). The suggested applications are:

• first, the liquidity adjustment valuation under a particular liquidity policy;

• second, the estimation of the liquidity costs and the revenue risk using the optimal execution paradigm of Almgren and Chriss (2000);

• and third, the computation of the state-dependent liquidity adjustments (Meucci, 2012) and the liquidity-adjusted portfolio risk.

This chapter is organized as follows. Section 6.1 summarises the work presented in this thesis. Section 6.3 proposes possible future research directions.

6.1 Summary of the Work

This thesis has six chapters including the current one. Chapter 1 motivates applications for the pricing of liquidity risk.

Chapter 2 is the literature review of the liquidity risk framework of Acerbi and Scandolo (2008). We introduced the portfolio risk measure which incorporates the liquidity risk, and defined the marginal supply-demand curve and the liquidity adjustment valuation. Also, we showed expected shortfall (ES) derivation.

In Chapter 3, we employed the portfolio liquidity valuation framework of Acerbi and Scandolo (2008) to determine the value of a long only equity portfolio subject to three typical liquidity policies; a requirement to be able to generate an amount of cash (due to possible adverse redemption), a minimum weight requirement per asset, and a market risk constraint such as a maximum expected shortfall (ES). The methodology assumes a single period for transactions, so the trading strategy depends only on information known at the initial time. Power-law MSDC is applied to model the price of the asset impacted by deterioration of market liquidity (market depth). In particular, we demonstrated the effect of the liquidity policies on the value of the portfolio.

The findings in Chapter 3 show that the liquidity restrictions affect, to a varying degree, the liquidation process of illiquid and high volatility assets. We find that as the liquidity policy becomes stricter with the portfolio ES constraint, the liquidity costs increase significantly.

In Chapter 4, we compared the liquidity costs from the model of Finger (2011) and Garleanu and Pedersen (2013) for the equity long portfolio with cash and the portfolio Expected Shortfall (ES) limit constraints. We employed the portfolio LVaR method of Almgren and Chriss (2000) to describe a tradeoff between the transaction costs and the volatility risk incurred during the specified trading interval. We applied Power-Law Marginal Supply-Demand Curves (MSDCs) when estimating the market depth parameter of Garleanu and Pedersen (2013).

The results in Chapter 4 show that the model of Garleanu and Pedersen (2013) tends to produce the larger liquidity costs than the costs from the liquidity measure of Finger (2011) because the former computes the liquidity costs using the parameter of the market depth measure which has a constant value without regard to the size of the trade. The transaction costs per unit contract in the former is a linear function of the trading volume. The latter has a concave function on the volume traded when estimating the liquidity costs per unit contract.

In Chapter 5, we made use of the liquidity score method to estimate the amount of market liquidity in the equity (long only) portfolio. The liquidity adjustments, which include the execution costs and the volatility risk terms, are quantified for the given liquidity policies. We estimated the state-dependent liquidity adjustments varied according to market P&L scenarios. We combined the liquidity adjustments scenarios with pure market P&L scenarios to obtain liquidity-adjusted P&L distribution. We computed the liquidity-adjusted portfolio risk and the liquidity score for a portfolio constrained with cash and the portfolio Expected Shortfall (ES) limit.

The results show that the risk on the pure market P&L for both 50M and 200M portfolios produces the same value; however, as the size of the portfolio becomes larger, the risk on the market plus liquidity P&L produces greater value. This is because liquidity risk tends to be greater for larger portfolios, so the liquidity-adjusted portfolio risk has higher value. Also, when the risk limit is imposed on the residual portfolio the liquidity score dropped significantly as the liquidation of higher volatile securities increases to satisfy the given risk limit.

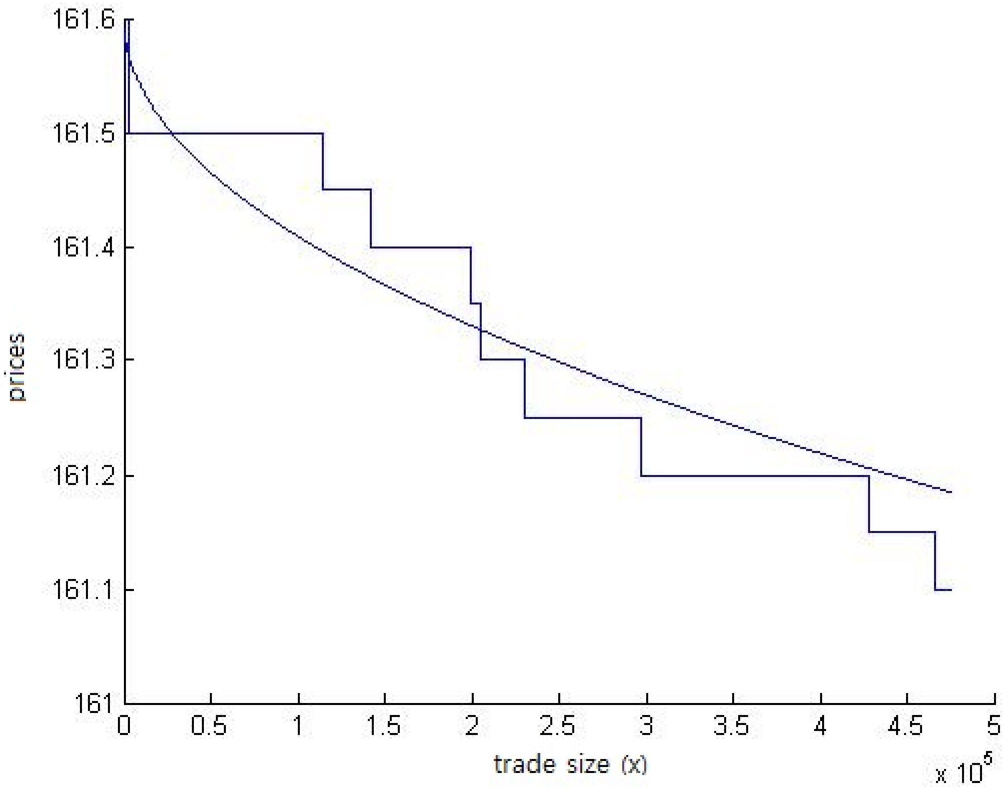
We have shown in this research that the liquidity scores can be a useful quantitative indicator to measure the market liquidity given the specific financial situation of an investor and liquidation time allowed for an equity (long only) portfolio. It can be employed to construct (and maintain) a portfolio which is liquid enough (or help to avoid significant liquidation costs from a possible fire-sale) in stressed market situation.

Also, fund managers can use the estimated liquidity costs as swing factor adjustment, which protects remaining investors not to bear the trading costs incurred by outgoing investors. This study suggests applications for fund managers to estimate liquidity costs, which can be used to remove an incentive of early redeemers. Fund managers should ensure the liquidity characteristics of the portfolio unchanged even after significant redemption request. In order to protect existing investors not to be left with illiquid assets, the applications in this study employ the maximum portfolio ES limit imposed on the residual portfolio. When the portfolio is subject to market risk limit of the remaining portfolio, illiquid assets in the initial portfolio are forced to sell to satisfy the given liquidity given risk limit. As a result, the liquidity profile of the portfolio can remain the same following redemptions. The liquidity costs incurred following this liquidation process, which can be used to adjust the fund's NAV, fall upon the outgoing investors. This protects the remaining investor in their fund's NAV not to be adversely affected by early redeemers' trading costs.

6.2 Limitation of this study

This section discusses limitations emerged during the research's progression. In Chapter 3 and 4, we used the Power-Law MSDC to model liquidity costs, but the actual liquidity cost is from the current order in the market. The estimated market impact coefficients γ and η in the Power-Law MSDC is determined by linear regression by Almgren et al. (2005), and the fixed values are applied in the application without regard to the market situation. In practice, these coefficients change randomly over time, so the values should be updated in each time period we compute liquidity costs to reflect the current market situation.

Figure 6.1: Exponential marginal demand curve versus empirical ladder marginal demand curve



This figure shows exponential marginal demand and empirical ladder marginal demand curve of Vodafone group on 2nd June, 2008 between 9:14am and 9:19am obtained from the LSE database.

The work of Tian et al. (2013) shows that the exponential and empirical ladder MSDC is used to measure liquidity costs using actual order data (by observing current bid prices quoted in the market and their corresponding trading volume). The bid part of order book records for 5 minutes for Vodafone group on 2nd June, 2008 is shown in Figure 6.1. The bid prices from the empirical ladder marginal demand curve can be approximated by exponential marginal demand curve, that is

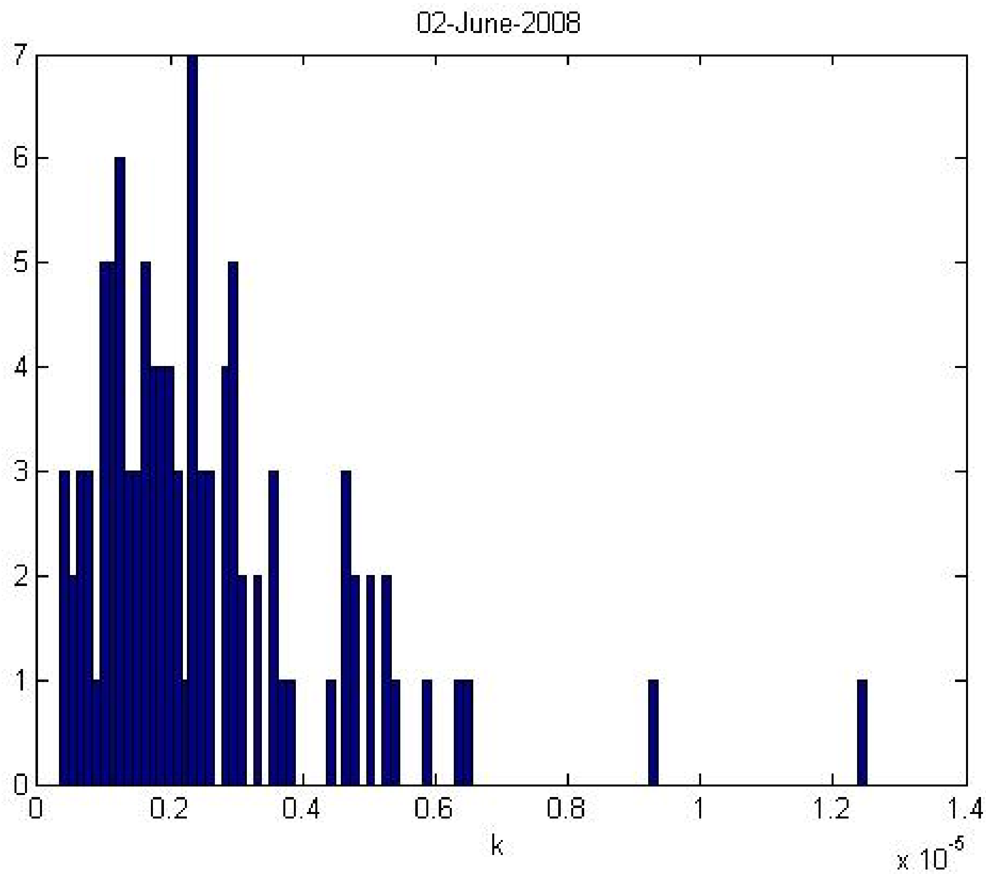
$$m(z) = m^+ e^{-r^b \sqrt{z}} \tag{6.1}$$

where m^+ is the best bid price, r^b is the liquidity risk factor, and z is the trade size. According to Tian et al. (2013), the liquidity risk factor r^b for large- and medium-cap shares is given by

$$\hat{r^b} = \frac{-\sum_{n=1}^{N} \sqrt{z_n} log\left(\frac{m(z_n)}{m^+}\right)}{\sum_{n=1}^{N} z_n} \tag{6.2}$$

where m^+ is given in (6.1), and z_n is the n^{th} discrete liquidation size. A histogram of the 5 minute frequency liquidity risk factors estimated on the ladder MSDC for Vodafone group on 2nd June, 2008 is shown in Figure .

Figure 6.2: Liquidity risk factors



This figure shows the histogram of liquidity risk factors each of which estimated on empirical ladder MSDC with order book records for 5 minutes for Vodafone group on 2nd June, 2008 from the LSE database.

The difference between the best bid price and the average realized bid price calculated using (6.1) and (6.2) is the liquidity costs per share estimated from the real data. Following this approach, it is possible to estimate liquidity costs that reflect the most recent data.

Another limitation of the research is that when sampling pure market P&L scenarios in Chapter 5, student-t distribution is used with degrees of freedom parameter set to be 7. The value is taken from historical data. However, we may observe more extreme losses than the negative returns simulated from the student-t distribution during periods of turmoil. The student-t distribution based on limited historical data may not capture such shocks in financial markets. It may be beneficial to use skewed student-t distribution to generate more extreme scenarios (with lower values for both degrees of freedom and non-centrality parameters than the values taken from historical data) that have not yet been experienced.

6.3 Future work

The work of this thesis can be further enhanced in the following way.

First, it would be useful to provide a detailed summary of the common practices in the portfolio management industry with regards to liquidity restrictions and portfolio liquidation procedures when computing the portfolio value according to liquidation policies.

Second, it would be interesting to study the common liquidity policies and restrictions applied in portfolio management firms, and analyse sensitivity of them on the theoretical portfolio value. In addition, the study can be extended by looking at how the liquidity restrictions affect the portfolio choice (i.e. the impact of regulation on portfolio choice) given the constraints that the portfolio value should remain the same.

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