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Abstract

This paper uses contingent claims analysis to answer two questions: (i) why are some subsidy markets apparently slow in attracting an optimal subsidy when others are not, and (ii) what can be done about it? The lack of activity in the green investment subsidy markets has been a concern as it appears optimal that countries should offer such support from a welfare point of view but progress has nonetheless been stalling, which motivates this paper. We show that free riding (which is likely to affect the green subsidy market) cools down the subsidy market with harmful welfare effects, and preemption (which is likely to affect the more active FDI subsidy market) overheats the subsidy market with similarly harmful effects. The theory dictates a taxation scheme that offsets these effects to restore the welfare to its maximum point.

JEL numbers: D92, E62, G31, H21.

Keywords: First-mover advantages, Free riding, Investment subsidy, Preemption risk, Subsidy tax.

1 Introduction

Contingent claims analysis has been one of the success stories in financial economics, and this paper uses this technology to answer questions related to policy on promoting green technology. Investment subsidy can correct the problem that firms fail to recognise the positive externality of their investments, but the subsidy market itself may fail. Two questions related to this problem are addressed. First, we ask what factors explain the activity in the subsidy markets, and second, what role can policy play to enhance the welfare from these markets. The contingent claims framework has the advantage that expressions for the option value of deferring subsidy decisions can be obtained, which can be used to evaluate the effects of policy in this area.

The need for policy is particularly pressing in the area of preventing climate change. The welfare benefits of combatting climate change are well documented but the positive externality associated with green investments has so far failed to attract the kind of investment subsidy we see in other areas such as foreign direct investment (FDI).[1] The policy on climate change has largely been a failure, and although hopes are that the recent 2015 Paris agreements can signify a turnaround, concerns still linger about commitment and enforcement mechanisms. An understanding of how the investment subsidy market works and how policy affects this market is, therefore, of interest.

The optimal timing of investment subsidy is a key theme in this paper. A dollar spent too soon

[1] Thomas (2007) estimates that within the European Union in 2005 €8.4bn were distributed in regional aid, a figure that probably underestimates the actual number, and he cites estimates of between $40-50bn for the US for 2002. An estimate by the World Trade Organization is a total of $250bn in 2003 by 21 developed countries. Dutz and Sharma (2012) survey the green investment subsidy market and find that subsidy is largely non-existent globally, and essentially confined to developed economies such as the US and the European Union.

or too late in an industry cannot compete with a dollar spent at the optimal time.[2] Timing and welfare are however not necessarily related. Just as a farmer may get the highest yield from the latest harvest, the highest welfare may come from investment subsidy markets with low activity levels. The fact that the green investment subsidy market has been slow to attract activity is, therefore, not in itself a reason for policy intervention.

A factor distorting the timing of subsidy is first-mover advantages. A subsidy offered by one country may, for instance, have negative impact on the welfare of other countries considering the same type of subsidy which creates a first mover advantage in the subsidy market. Rare R&D investments are good examples. A country that attracts a rare R&D investment through subsidy captures most of the welfare gains while the losing countries not only receive very little in welfare effects but may also have to wait for a long time for a similar opportunity to appear. In this case the best response is to become more aggressive in the subsidy market and as a consequence the subsidy market is heated up. Welfare is therefore sacrificed in order to ensure the country wins the race to attract the investment. A second factor is free-riding effects in the market for subsidy. Here, the welfare that results from a subsidised investment is captured by other, non-subsidising, countries. For instance, if a country is offering a subsidy for green investments to combat climate change the welfare effects will be shared by all countries, including the non-subsidising ones. The best response is to become less aggressive in the subsidy market and the subsidy market is cooled down. Welfare is also here sacrificed because of the free riding effect.

[2] This argument assumes the absence of frictions. In frictional economies it may be optimal to sacrifice timing efficiency in order to preserve budgets for future subsidy.

We show that the welfare distortions caused by heating and cooling of a subsidy market can be corrected by a surprisingly simple policy intervention. This intervention takes the form of a tax, or transfer payments, linked to a country's actual subsidy payments. The tax is negative in the heated subsidy markets so that the non-subsidising countries receive payments calibrated to the subsidy payments of the subsidising countries. The tax is positive in the cooled subsidy markets so that the non-subsidising countries make payments calibrated to the subsidy payments of the subsidising countries. This scheme makes it more attractive not to pay subsidy in the hot subsidy markets and less attractive not to offer subsidy in the cold subsidy markets. Moreover, the scheme is incentive compatible in the sense that both the subsidising countries (who pay or receive tax payments) and the non-subsidising ones (who do not) have the same welfare and are therefore better off with the taxation scheme than without. Finally, the tax may be self-financing if the positive tax payments collected from the cold subsidy markets exactly offset the negative tax payments in the hot subsidy markets. We discuss implementation issues of such a scheme in the main body of the paper. The current approach to policy on climate change, based on agreements on targets, has had limited success. There is however no reason to think that targets remove the free riding problem in the investment subsidy market, and therefore the incentive for individual countries is to undersupply subsidy to green investments. The taxation scheme outlined in this paper will in contrast remove the free-riding problem and therefore the decision to subsidise green investments can be delegated to individual countries.

The related literature consists of several strands. There is a literature that discusses aspects of subsidy design in dynamic models (see, for example, Pennings (2000, 2005), Yu, Chang and Fan

(2007), and Asano (2010)). Our model extends this literature. There is also a strand of literature that discusses the welfare effects of FDI subsidy, surveyed in Besley and Seabright (1999). Related contributions are Black and Hoyt (1989), Albornoz et al (2009), Chor (2009), and Fumagalli (2003). The essential divide between their work and ours is that we use a dynamic model, allowing us to study timing effects. There is a growing literature on policy to encourage investments in green technology, but this literature is still relatively thin (see Dutz and Sharma (2012) for an overview). Agliardi and Sereno (2012) build a model of the optimal switch from a non-renewable source of energy to a renewable one. They do not, however, analyse distortions to the timing of subsidy.

The paper proceeds as follows. In Section 2 we present the framework, including the earnings process that leads to commercial value for the firm, and the welfare effects of investment subsidy. Also, we set out the impact of preemption risk and free riding effects in this framework. In Section 3 we present the main theoretical findings, including the optimal form of investment subsidy, the optimal timing of subsidy, and the effects of preemption risk and free riding on the timing of subsidy. In Section 4 we discuss the results of the model and derive its policy implications, and Section 5 concludes the paper.

2 Framework

In this section we set out the model. The basic framework where the firm makes an investment under uncertainty is described in the first subsection. A country makes a subsidy decision under uncertainty, where the subsidy must be calibrated such that it solves the investment decision for the firm that receives it. The country therefore decides the timing of the subsidy. The second and third subsections describe these problems. Finally, the fourth subsection describes first-mover and free-riding effects that influence both decision problems.

2.1 Investment and Earnings

We outline a standard real options framework stripped down to its simplest form, where investments are equity financed. We suppose an investment I (net of all tax implications) at time s yields an earnings flow y_t, $s \leq t < \infty$. The earnings are taxed at a corporate rate τ.[3] The earnings flow y_t follows a geometric Brownian motion with risk neutral drift μ and diffusion σ, and we assume the firm can observe the earnings flow free of cost so as to make the investment at a time to maximize the net present value. The trigger value y^* is the solution to this problem, where investment is made at the optimal stopping time defined as the event that the process $y_t = y^*$ for the first time. The instantaneous risk free rate is r.

The value of a claim on the earnings flow y_t by paying the investment cost I, at the investment trigger point y^*, can be written as

$$V(y_t|t < \tau_1) = \mathbb{E}\left[e^{-r(\tau_1 - t)}\left(\int_{\tau_1}^{\infty} e^{-r(s-\tau_1)} y_s(1-\tau)ds - I\right)\right], \tag{1.a}$$

where τ_1 is the stopping time for the event that $y_t = y^*$ for the first time. The value of a claim on

[3]Strictly speaking, taxes are not essential to the story we tell in this paper. However, tax relief is a common ingredient of investment subsidy packages and extensively modelled in the related literature. We show in the next section that it does not matter whether tax relief is part of the subsidy package as long as other benefits make up for it but we include taxation to be consistent with the related literature.

the earnings flow y_t at the point the investment is just made is similarly

$$V(y_t|t \geq \tau_1) = \mathbb{E}\left(\int_t^{\infty} e^{-r(s-t)} y_s(1-\tau)ds\right) \tag{1.b}$$

Dixit (1993) shows that

$$\mathbb{L}(V(y_t|t < \tau_1)) = 0, \qquad \mathbb{L}(V(y_t|t \geq \tau_1)) + y_t(1-\tau) = 0, \qquad \mathbb{L} = \frac{1}{2}\sigma^2 y_t^2 \frac{d^2}{dy_t^2} + \mu y_t \frac{d}{dy_t} - r\mathbb{I}, \tag{2}$$

where $\mathbb{L}$ is the infinitesimal operator associated with the Brownian motion governing the earnings process y_t and $\mathbb{I}$ is the identity operator. The solution to the first equation is $V(y_t|t < \tau_1) = A_0 y_t^{\lambda_1} + B_0 y_t^{\lambda_2}$ where A_0, B_0 are arbitrary constants, and λ_1, λ_2 are the positive and negative root, respectively, of the characteristic equation $\frac{1}{2}\sigma^2\lambda(\lambda-1) + \mu\lambda - r = 0$. The roots are $\lambda_1 = \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right) + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}$ and $\lambda_2 = \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right) - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}$. Note that $\lambda_2 < 0 < 1 < \lambda_1$ for $r > \mu$. The solution to the second equation is $V(y_t|t \geq \tau_1) = A_1 y_t^{\lambda_1} + B_1 y_t^{\lambda_2} + \frac{y_t(1-\tau)}{r-\mu}$, where again A_1 and B_1 are arbitrary constants. Since the solution must satisfy the homogeneity constraint $V(ky_t|t \geq \tau_1) = kV(y_t|t \geq \tau_1)$ the constants A_1 and B_1 must be zero. We impose boundary conditions and smooth pasting conditions to identify the free parameters A_0 and B_0 in the expression for the value of the investment opportunity. Using these conditions we find the optimal investment trigger point y^* which is the solution of the problem of finding a smooth fit between the value of the investment opportunity $V(y_t|t < \tau_1)$ and the value of the investment itself $V(y_t|t = \tau_1) - I$. The investment trigger is given by $y^* = \frac{I}{1-\tau}(r-\mu)\frac{\lambda_1}{\lambda_1 - 1}$ (see for example

Dixit (1993)).

2.2 Investment Subsidy

An unsubsidised investment will be made in a country at the investment trigger y^*. Suppose a host country is willing to offer a subsidy package, denoted K, which if accepted leads to the firm making the investment at an earlier time than otherwise would happen. We assume the value of the subsidy package can be made dependent on the earnings level y_t, $K(y_t)$. Let τ_2 denote the stopping time that the subsidy is accepted and the investment is made. The value of the investment opportunity to a firm that accepts the subsidy is

$$V(y_t|t < \tau_2, \text{Subsidy}) = \mathbb{E}\left[e^{-r(\tau_2 - t)}\left(\int_{\tau_2}^{\infty} e^{-r(s-\tau_2)} y_s(1-\tau)ds + (K(y_{\tau_2}) - I)\right)\right]. \qquad (3)$$

Note that in this expression we capture all value effects of the investment subsidy into the value function K. To make the acceptance decision of the firm rational, we need to impose the constraint that the value of the subsidised investment opportunity is at least as large as the value of the unsubsidised investment opportunity, i.e. $V(y_t|t < \tau_2, \text{Subsidy}) \geq V(y_t|t < \tau_1)$. It is not credible that the firm would accept an investment subsidy which allowed the country offering the subsidy to extract rent from the firm. On the other hand, we expect that the subsidy is no greater than it needs to be, so the constraint above is likely to be binding, i.e. the subsidy is the smallest subsidy that equals the value of the unsubsidised investment opportunity.

2.3 Timing of Subsidy

The host country receives welfare benefits from attracting the investment earlier than it otherwise would be. The benefit takes the form of a constant flow w for the duration between the timing of the subsidised investment to the time the investment would have been made in any case. Whereas the earnings flow y can be thought of as the commercial profits following the investment, the welfare flow w can be thought of as the wider social externalities following the investment. For instance, the establishment of a manufacturing plant in an area with high unemployment provides commercial benefits for the firm that makes the investment as well as social benefits in terms of lowering unemployment, maintaining skill levels of the workforce, and boosts to local businesses. We assume y is a stochastic process but that w is constant. This is done primarily because it is analytically difficult to model two stochastic processes in the same model. However, it is natural to assume that the social benefits that arise from an investment are less risky than the commercial benefits from the same investment.

The stopping time for the event that the unsubsidised investment takes place is the event that $y_t = y^*$ for the first time. If the country seeks to attract investment early, say at the stopping time for the event that $y_t = y^{**} < y^*$ for the first time, the country receives a constant welfare benefit flow w from the latter stopping time to the former stopping time, i.e. in the time it takes for the earnings flow to go from y^{**} to the first instance the earnings flow hits y^*. The cost of inducing early investment is $K(y^{**})$, and the country will decide the optimal timing of its subsidy package to a time that maximizes the welfare benefit of receiving the welfare flow w against the cost of inducing investment. If the country waits until $y_t = y^*$ the net welfare effect is zero because the value of the benefit flow w is zero, and because the cost of the subsidy is also here zero. If w is sufficiently large it is always optimal to offer subsidy prior to this point in time. We can write the welfare benefit of the subsidy as

$$W(y_t|t < \tau_2) = \mathbb{E}\left[e^{-r(\tau_2 - t)}\left(\int_{\tau_2}^{\tau_1} e^{-r(s-\tau_2)} w ds - K(y^{**})\right)\right], \tag{4.a}$$

where τ_2 is the stopping time for the event that $y_t = y^{**}$ for the first time, and τ_1 is as before the stopping time for the event that $y_t = y^*$ for the first time. After a subsidy is offered and accepted at $y_t = y^{**}$, the welfare benefits take the value

$$W(y_t|t \geq \tau_2) = \mathbb{E}\left(\int_t^{\tau_1} e^{-r(s-t)} w ds\right). \tag{4.b}$$

These expressions satisfy the conditions

$$\mathbb{L}(W(y_t|t < \tau_2)) = 0 \qquad \mathbb{L}(W(y_t|t \geq \tau_2)) + w = 0, \tag{5}$$

where $\mathbb{L}$ is defined as above. The optimal timing of subsidy is the trigger point y^{**} which maximizes the welfare of the subsidy, $W(y_t|t < \tau_2)$. Therefore, y^{**} is the solution to the problem $\max_{y^{**}} W(y_t|t < \tau_2)$ subject to τ_2 being the stopping time for the event that $y_t = y^{**}$ for the first time.

2.4 Preemption Risk and Free-Riding

Finally, we investigate the impact of preemption risk in the market for investments, preemption risk in the market for subsidy, and free-riding in the market for subsidy. Consider the market for investments first. As long as no firm has made an investment, the competition is still alive and all firms know that it is still unclear which is the winner. At some point one firm makes the investment, and then the winning firm becomes known and the losing firms exit the market for the investment. Let τ_C be the stopping time for the event that the winning firm becomes known. The value of the investment opportunity, conditional on it belonging to the winning firm, is

$$V(y_t|t < \tau_C, \text{Win}) = \mathbb{E}\left[e^{-r(\tau_C - t)}\left(\int_{\tau_C}^{\infty} e^{-r(s-\tau_C)} y_s ds + (K(y_{\tau_C}) - I)\right)\right], \tag{6.a}$$

and the value, conditional on it belonging to the losing firm, is

$$V(y_t|t < \tau_C, \text{Lose}) = 0, \tag{6.b}$$

where the right hand side is zero by the fact that the winner makes the investment destroys the value of the investment for the loser. An example of this can be the investment in the manufacturing of a product whose demand can be met by one firm, or the investment in the development of a drug (which can be patented) for the cure of a certain decease. Once a firm successfully makes the investment in such areas the investment opportunities for the losing firms disappear. Preemption risk of this kind is studied in Lambrecht and Perraudin (2001).

When there is preemption risk in the market for investment subsidy, the winning country receives the welfare benefit but the loser will receive nothing. Again, let τ_C denote the stopping time the winning and the losing country become known and the subsidy is offered. We find that the welfare, conditional on it belonging to the winning country, is

$$W(y_t|t < \tau_C, \text{Win}) = \mathbb{E}\left[e^{-r(\tau_C - t)}\left(\int_{\tau_C}^{\tau_1} e^{-r(s-\tau_C)} w ds - K(y^{**})\right)\right], \tag{7.a}$$

and the welfare, conditional on it belonging to the losing country, is

$$W(y_t|t < \tau_C, \text{Lose}) = 0. \tag{7.b}$$

The incremental welfare of the option to attract business by giving investment subsidy will, therefore, vanish completely unless there is some probability that the country can win the preemption game.

Note that the form of competition implied by preemption in the market for investments will have a rent dissipation effect in the sense that when the probability of winning the preemption game is low the firm makes investments that have relatively low financial value since they are made earlier than otherwise would be the case. There is no effect on the product prices or the earnings flow to the winning firm. This assumption can be made because there will only be one winning firm in the industry after the investment is made, so there are no competitive pressures on the product prices ex post, but only rent dissipation ex ante. The case where competition has an impact on product prices is one we leave for separate treatment.

We also study the effect of free-riding in the market for subsidy. The welfare, conditional on it belonging to the winning country, is given by (7.a) above as before, but the welfare, conditional on belonging to the losing country, is

$$W(y_t|t < \tau_C, \text{Lose}) = a\mathbb{E}\left[e^{-r(\tau_C - t)}\left(\int_{\tau_C}^{\tau_1} e^{-r(s-\tau_C)} w ds\right)\right], \tag{7.c}$$

where $0 < a \leq 1$ is the fraction of the winning country's welfare that is captured by the loser. The loser will however not have to pay the subsidy so a free-riding effect arises.

3 Theoretical Results

In this section we study optimal subsidy in the framework described above. We first look at the problem of optimally designing a subsidy package, then at the problem of optimal timing of the delivery of the subsidy package, and finally at the problem of identifying the factors that influence the optimal timing.

3.1 Optimal Subsidy Packages

The first problem addresses optimal subsidy package design (studied in, for example, Pennings (2000, 2005) and Yu, Chang and Fan (2007)). Our analysis show key differences to this literature, and we will spend some time highlighting these. Throughout, consider a subsidy $K(y_t)$ offered when $y_t = y^{**}$. The trigger point y^{**} is the subsidised investment trigger, and let y^* as before denote the unsubsidised investment trigger. First, we map the distance between y^{**} and y^* to the subsidy package $K(y^{**})$ and its derivative $K'(y^{**})$, which can be interpreted as the minimum subsidy needed to move the investment point from y^* to y^{**}. We find the following result.

Lemma 1: *The firm is indifferent between making a subsidised investment at y^{**} and making an unsubsidised investment at $y^* \geq y^{**}$ when the following relationship holds,*

$$y^{**} = \left(y^* - K(y^{**})\frac{r-\mu}{1-\tau}\frac{\lambda_1}{\lambda_1 - 1}\right)\left(1 - \frac{r-\mu}{(1-\tau)(\lambda_1-1)}K'(y^{**})\right)^{-1}, \tag{8}$$

where $y^* = \frac{I}{1-\tau}(r-\mu)\frac{\lambda_1}{\lambda_1-1}$ is the unsubsidised investment trigger. For $y > y^*$ a subsidy makes no impact on the investment decision.

We notice that the subsidy compensates for early investment in two ways. The net investment cost with a subsidised investment is $I - K(y^{**})$, which can be expressed as a constant $\frac{1-\tau}{r-\mu}\frac{\lambda_1-1}{\lambda_1}$ times the term in the first bracket in (8). The lowering of the net investment cost is the first effect. There is, however, a second effect measured by the term in the second bracket of (8) since the marginal value of the subsidy matters too. If the marginal value is zero, for instance if K is a lump sum unaffected by changes to y_t, then $K'(y^{**}) = 0$ and the second bracket equals 1. The investment trigger y^{**} can be lowered further by making $K'(y^{**})$ negative. This can be understood in the context of incentivising the firm to invest earlier by offering more subsidy for lower earnings levels. Speeding up investment will in this case happen because the firm will avoid reducing the subsidy by delaying acceptance. This explains Pennings (2000) suggestion that taxing FDI investments to finance an investment support package which is self-financing at the point y^* leads to investment at $y^{**} < y^*$. If we denote the investment support package as a lump sum $\Delta > 0$ and the extra tax on the earnings of the investment as the rate $\delta > 0$, then the value of the subsidy at y_t is $K(y_t) = \Delta - \frac{\delta y_t}{r-\mu}$, and if we calibrate the package such that $\Delta - \frac{\delta y^*}{r-\mu} = 0$ it becomes self-financing at the unsubsidised investment trigger point y^*. Since the derivative is $K'(y_t) = -\frac{\delta}{r-\mu}$ is negative, however, the firm will speed up the investment such that $y^{**} < y^*$. Therefore, the country receives extra tax revenues that more than exceeds the value of the investment support, an apparent win-win situation. This argument ignores the outside options for the firm. The constraint that $V(y_t|t < \tau_2, \text{Subsidy}) \geq V(y_t|t < \tau_1)$ implies that $K(y^*) = K'(y^*) = 0$ since it is not optimal for the firm to be charged a negative subsidy for $y_t > y^*$ where the investment is made anyway, and therefore it is not credible for the country to threaten $K'(y^*) < 0$. Including this constraint determines optimal subsidy design and we find the following result.

Proposition 1: *The optimal investment subsidy $K(y^{**})$ which leads to investment at time y^{**} satisfies the ordinary differential equation*

$$\frac{d}{dy_t}K(y_t) - \frac{\lambda_1}{y_t}K(y_t) = \lambda_1 I\left(\frac{1}{y^*} - \frac{1}{y_t}\right). \tag{9.a}$$

The solution to this problem, taking into account the boundary condition $K(y^*) = \frac{d}{dy}K(y^*) = 0$, is *given by*

$$K(y^{**}) = \frac{I}{\lambda_1 - 1}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1} + I\left(1 - \frac{y^{**}}{y^*}\frac{\lambda_1}{\lambda_1 - 1}\right), \tag{9.b}$$

The ODE in (9.a) is obtained from Lemma 1. Imposing the boundary condition $K(y^*) = K'(y^*) = 0$ we find (9.b) which is the optimal subsidy. The sum $K(y^{**}) + \left(\frac{y^{**}(1-\tau)}{r-\mu} - I\right)$, which is the sum of the investment and the net present value of the investment, is exactly equal to the value of the unsubsidised investment opportunity at y^{**}. The optimal subsidy will however never give the firm strictly more than the value of the unsubsidised investment opportunity. This constraint is always binding with the optimal subsidy.

Figure 1 shows the relationship between the optimal subsidy $K(y^{**})$, the net present value of the investment $NPV(y^{**}) = \frac{(1-\tau)y^{**}}{r-\mu} - I$, and the option value of the unsubsidised investment opportunity evaluated at y^{**}, $V(y^{**})$. The relationship $V(y^{**}) = K(y^{**}) + NPV(y^{**})$ always holds.

We can look at two special cases. First, suppose the investment support is offered as the only ingredient in the subsidy package such that we can write $K(y_t) = \Delta(y_t)$ where $\Delta(y_t)$ is a lump sum offered to the firm at the time of investment which we allow to vary with y_t, so that $K'(y_t) = \Delta'(y_t)$. Similarly, suppose that the subsidy consists of tax relief only, $\delta(y_t)$, where $\delta(y_t)$ is the reduction in the tax rate of the firm's earnings flow the firm is entitled to over the lifetime of the investment. The effect of the tax relief is lost tax revenues $\frac{\delta(y_t)y_t}{r-\mu}$ over the life time of the investment, however there is also an increase in tax revenue between y_t and y^* where the country is paid a tax which otherwise would not materialise. The value of this tax revenue is $\frac{\tau y_t}{r-\mu} - \frac{\tau y^*}{r-\mu}\left(\frac{y_t}{y^*}\right)^{\lambda_1}$. So in total, therefore, $K(y_t) = \frac{\delta(y_t)y_t}{r-\mu} - \left(\frac{\tau y_t}{r-\mu} - \frac{\tau y^*}{r-\mu}\left(\frac{y_t}{y^*}\right)^{\lambda_1}\right)$ and, by straight

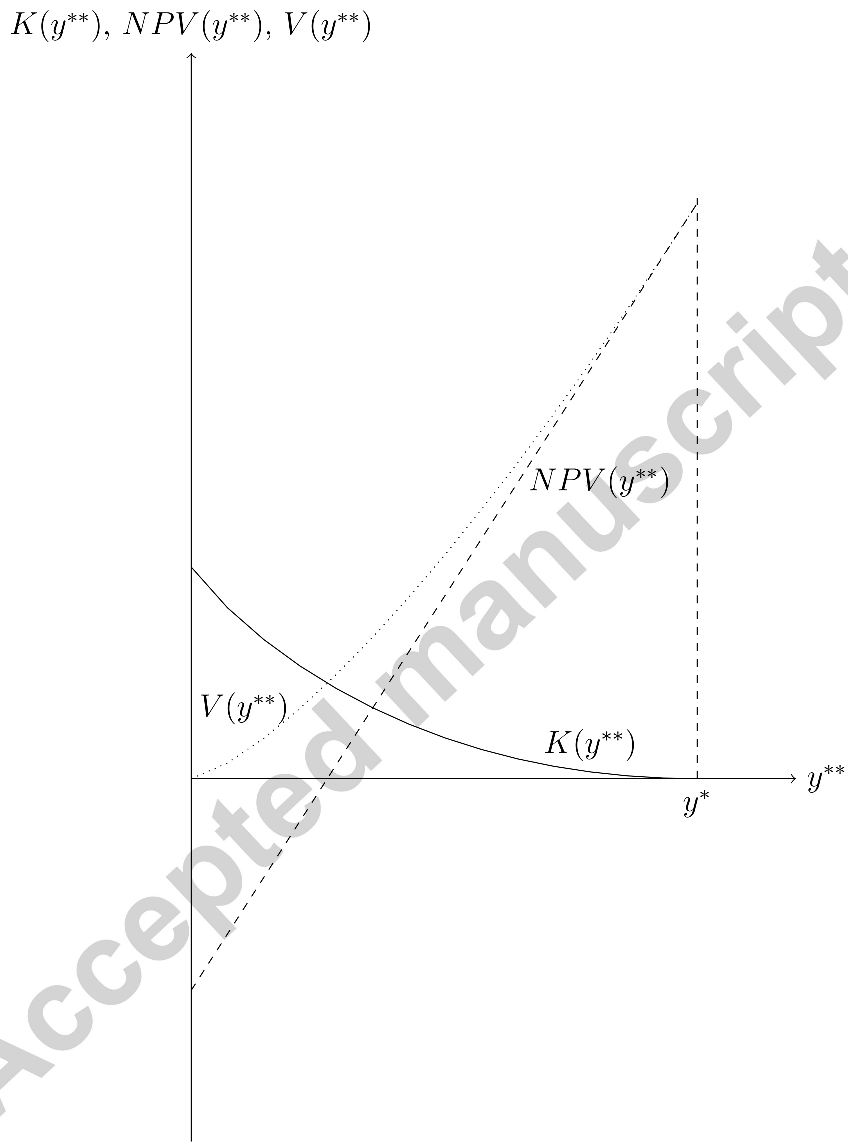

Figure 1: The figure shows the relationship between the optimal subsidy $K(y^{**})$ (solid lines), the unsubsidised net present value of the investment $NPV(y^{**})$ (dashed lines), and the unsubsidised option value of the investment opportunity $V(y^{**})$ (dotted lines).

differentiation, $K'(y_t) = \frac{\delta'(y_t)y_t}{r-\mu} + \frac{\delta(y_t)}{r-\mu} - \frac{\tau}{r-\mu}\left(1-\lambda_1\left(\frac{y_t}{y^*}\right)^{\lambda_1-1}\right)$. We can substitute the expressions of $K(y_t)$ into (9.b) to obtain the optimal investment support Δ and tax relief δ. We find the following result.

Corollary 1: *The optimal investment support for investment at y^{**} is given by*

$$\Delta(y^{**}) = \frac{I}{\lambda_1 - 1}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1} + I\left(1 - \frac{y^{**}}{y^*}\frac{\lambda_1}{\lambda_1 - 1}\right), \tag{10.a}$$

and the optimal tax relief for investment at y^{**} is given by

$$\delta(y^{**}) = \tau\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1-1}\right) + \frac{1-\tau}{\lambda_1}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1-1} - (1-\tau)\left(1 - \frac{y^*}{y^{**}}\frac{\lambda_1 - 1}{\lambda_1}\right). \tag{10.b}$$

These results are immediate from Proposition 1 and we see that both Δ and δ (as well as their derivatives) go towards zero if y^{**} goes towards y^*. The key is that the restrictions on K satisfy Proposition 1, not whether the subsidy consists of direct investment support or tax relief. This result contrasts existing results in the literature such as Pennings (2000) and Yu et al (2007), but these papers do not impose the boundary condition that $K(y^*) = K'(y^*) = 0$. In Figure 2 we illustrate the investment choices set out in Pennings (2000). Pennings suggests that the host country should impose an extra tax on FDI firms that will fund a package of direct investment support, but we see that the subsidy, nor its derivative, is zero at y^*. In Figure 3 we illustrate the investment choices in Yu *et al* (2007). The host country seeks to reduce the investment trigger

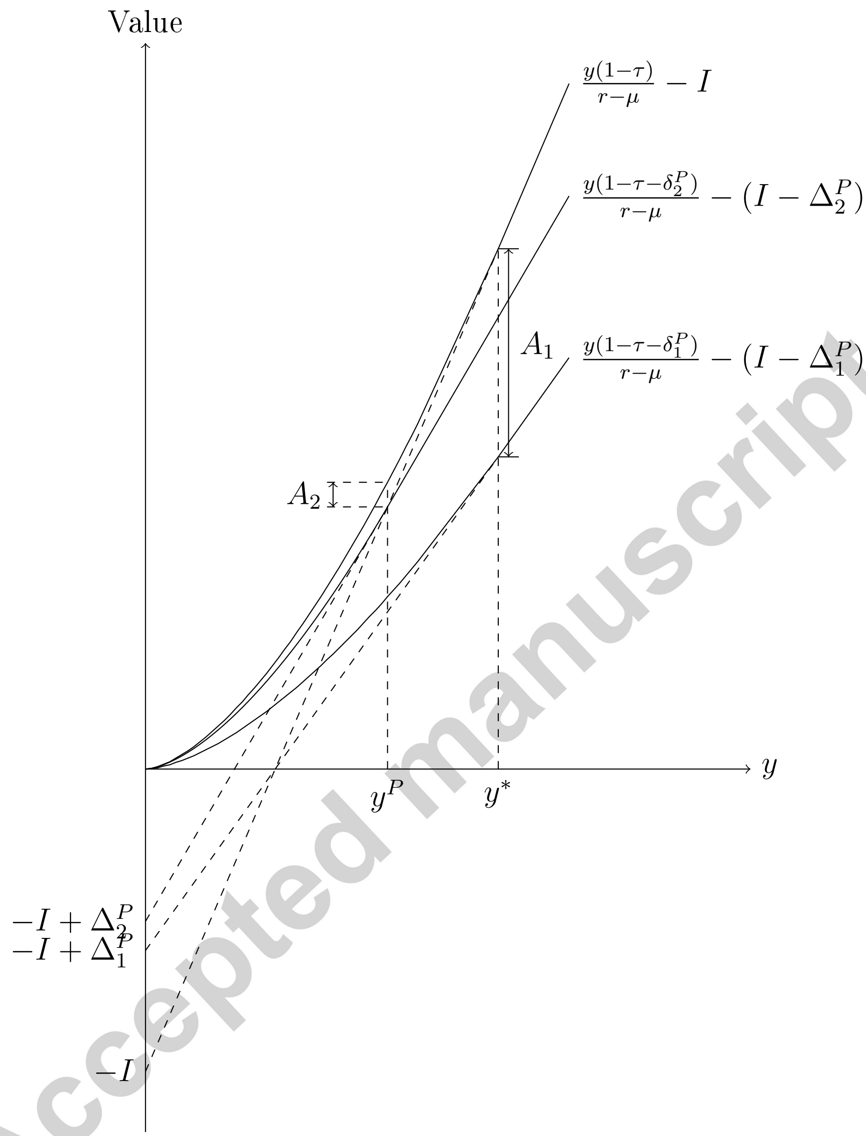

Figure 2: The straight dashed line marked $\frac{y(1-\tau)}{r-\mu} - I$ is the net present value of the unsubsidised investment, and the investment trigger y^* is the optimal timing of the investment without subsidy. Pennings (2000) demonstrates that the host country can extract rent from the firm by offering a direct investment support (denoted Δ_1^P) at the same time as increasing the tax rate (denoted δ_1^P) without changing the optimal investment trigger point. The magnitude of the rent extraction is A_1. Also, Pennings (2000) demonstrates that the host country can extract rent by offering a contract that is self-financing but will change the investment trigger point (denoted y^P). The subsidy is denoted Δ_2^P and δ_2^P, and the magnitude of the rent extraction is A_2.

such that the investment decision is speeded up, and the subsidy is either in the form of a direct investment support package worth Δ^Y, or a reduction of the tax rate equal to δ^Y. Again, we find that neither the subsidy, nor its derivative, is zero at y^*.

3.2 Optimal Timing

In this section we seek to derive the optimal timing of the optimal subsidy. The host country receives a welfare benefit w from the time of the investment, which is the stopping time for the investment trigger point y^{**} at which investment takes place at a cost $K(y^{**})$, to the time the investment would have taken place anyway, y^*. We find the following result.

Proposition 2: *Given the investment cost I and the unsubsidised investment trigger point y^* the optimal timing of the subsidy package $K(y^{**})$ is given by y^{**} such that*

$$y^{**} = y^* \max\left(0, 1 - \frac{w}{rI}\right). \tag{11}$$

Everything else being equal, y^{**} is closer to y^* the greater the investment cost I and the lower the welfare flow w. The welfare at the time the subsidy is offered is,

$$W(y^{**}) = \frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**}). \tag{12}$$

The optimal timing of the investment subsidy can be found by evaluating the point at which the

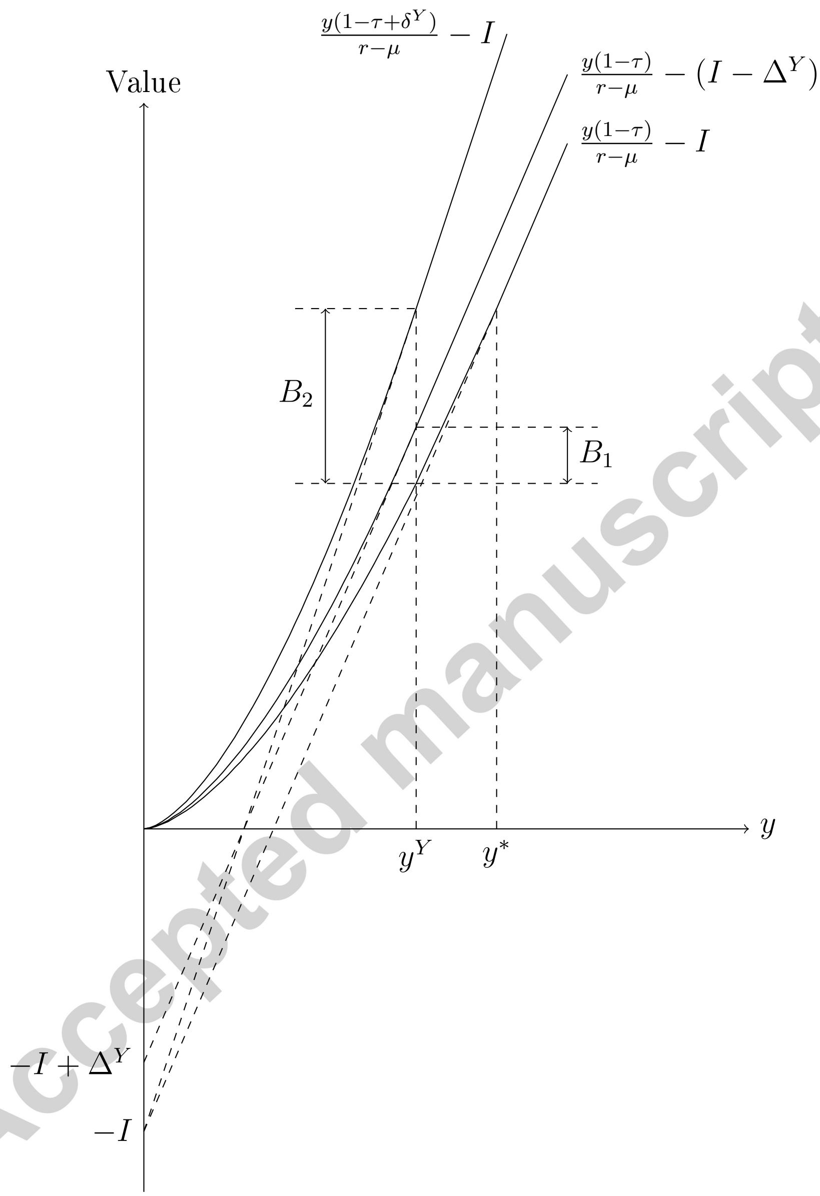

Figure 3: The straight dashed line marked $\frac{y(1-\tau)}{r-\mu} - I$ is the net present value of the unsubsidised investment, and the investment trigger y^* is the optimal timing of the investment without subsidy. Yu *et al* (2007) show that it is cheaper to offer a direct investment subsidy (denoted Δ^Y) than a tax relief (denoted δ^Y) if the host country wants to reduce the investment trigger to a point $y^Y < y^*$. Both lead to rent extraction by the firm denoted B_1 and B_2, respectively.

welfare of the option to offer subsidy achieves a smooth transition to the welfare of the investment. Additional boundary conditions are found by evaluating the welfare of the subsidy option as the earnings flow is sufficiently close to zero and here there are two possibilities: either the subsidy trigger is zero and no smooth transition to the welfare of the investment is found, or there is no likelihood that the subsidy trigger is reached. At the time the subsidy is made the welfare is the expected value of the welfare flow w, the first term in (12), minus the value of the subsidy package itself, the second term in (12).

A starting point for discussing optimal timing is the ratio $\frac{w}{rI}$. The numerator is the welfare flow that arises from the investment and the denominator is the capital flow that is necessary to justify the investment cost I. If this ratio is greater than one the welfare flow w dominates the capital flow rI so investment can be justified on welfare grounds alone even if the earnings flow is zero. In this case it is optimal to subsidise investment immediately. When the ratio is less than one $y^{**} > 0$ but always lower than the unsubsidised trigger point y^{*}. Therefore, as long as there is a positive incremental welfare benefit, it will always be optimal to offer a subsidy package which speeds up the investment point. The optimal subsidy is offered sooner the greater the welfare flow w and later the greater the capital flow rI, everything else being equal. Figure 4 illustrates the optimal stopping time for the event that the country offers the optimal subsidy to induce private investment. The figure assumes that $w < rI$ which implies that $y^{**} > 0$.

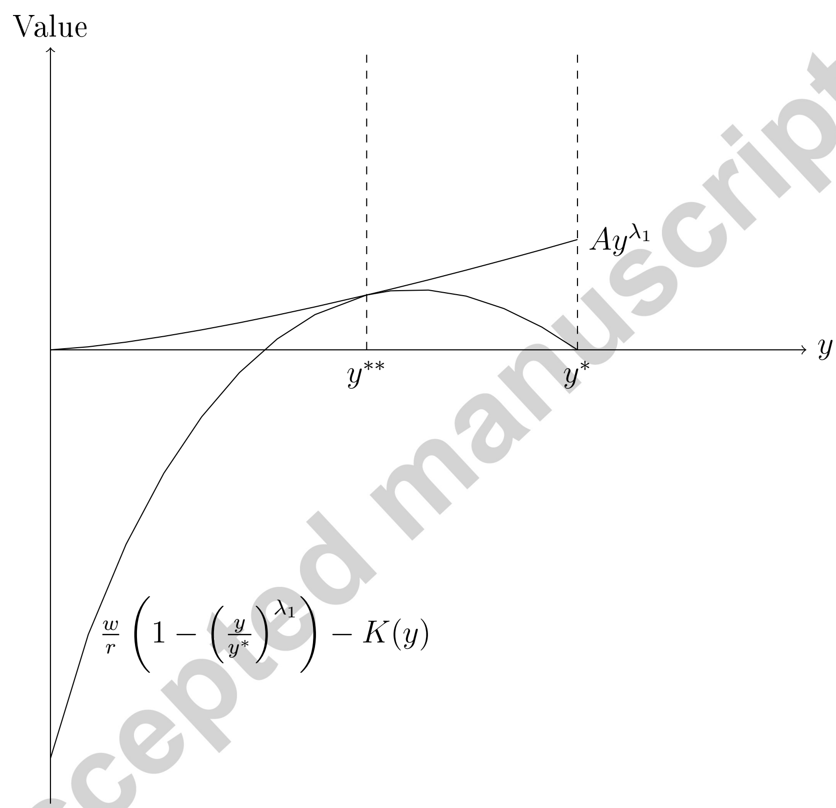

Figure 4: Optimal timing of investment subsidy. The investment trigger y^* is the unsubsidised investment trigger, and the investment trigger y^{**} is the optimal subsidised investment trigger.

3.3 Preemption Risk

Preemption risk in the market for investments arises when the event that one firm invests generates a negative externality on the value of other firms' investment opportunities. Preemption risk in the market for subsidy arises similarly when the event that one country subsidises investment generates a negative externality on the value of other countries' subsidy opportunities. For instance, if the investment being subsidised is a unique investment, then the winning firm obtains all the benefits from subsidising this investment whereas the losing firms lose the ability to subsidise similar investment. An example of a unique investment is an investment in a large production plant capturing economies of scale. Once the investment is made a second investment is unlikely to follow. The first problem is studied in Lambrecht and Perraudin (2001) and we can use their results directly. Define $F(y)$ as the probability that the nearest competing firm makes an investment at $y_t \leq y$, $F(y) = \mathbb{P}(y_C^* \leq y^*)$, and define the hazard rate $h_F(y) = \frac{f(y)}{1-F(y)}$ ($f(y)$ is the density function $f(y) = \frac{d}{dy}F(y)$). The hazard rate can be interpreted such that $h_F(y)dy$ is the probability that a firm makes the investment in the increment $[y, y+dy]$, conditional on the event that no investment has been made up to the point that $y_t = y$ for the first time. Lambrecht and Perraudin (2001) show that the unsubsidised investment trigger is $y^* = \frac{I}{1-\tau}(r-\mu)\frac{\lambda_1 - y^* h_F(y^*)}{\lambda_1 - 1 + y^* h_F(y^*)}$. When the hazard rate goes to zero the unsubsidised investment trigger converges to the normal trigger $\frac{I}{1-\tau}(r-\mu)\frac{\lambda_1}{\lambda_1 - 1}$.

With preemption risk the investment trigger is lowered. Note however that Proposition 2 still applies and the optimal subsidy will still be equal to $K(y^{**}) = \frac{I}{\lambda_1 - 1}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1} + I\left(1 - \frac{y^{**}}{y^*}\frac{\lambda_1}{\lambda_1 - 1}\right)$, only with a lower unsubsidised investment trigger y^*. We find the following result.

Proposition 3: *Preemption risk in the market for investment lowers the unsubsidised investment trigger point* $y^* = \frac{I}{1-\tau}(r-\mu)\frac{\lambda_1 - y^* h_F(y^*)}{\lambda_1 - 1 + y^* h_F(y^*)}$ *and the optimal timing of subsidy is*

$$y^{**} = y^* \max\left(0, 1 - \frac{w}{rI}\right), \tag{13}$$

as before. The welfare at the investment trigger point is $W(y^{**}) = \frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**})$, *independent of the changes to* y^* *and therefore at the same level as before at the point of subsidy.*

The subsidy trigger y^{**} enters the first term of $W(y^{**})$ through the ratio $\frac{y^{**}}{y^*}$. However, studying the expression of $K(y^{**})$ from Proposition 1, the subsidy trigger y^{**} also enters the second term through the ratio $\frac{y^{**}}{y^*}$ only (see (9.b)). Proposition 3 states that a given reduction of the investment trigger y^* from preemption risk in the market for investment feeds through to the same lowering of the subsidy trigger y^{**}. The ratio $\frac{y^{**}}{y^*}$ remains, therefore, constant and the welfare is the same at the point of subsidy. The timing of subsidy changes, however, as a consequence of the preemption risk effects. The case that $\frac{w}{rI} \geq 1$ is in this context not very interesting since here the subsidy trigger is always minimal at $y^{**} = 0$. Therefore, we focus the discussion on the case that $\frac{w}{rI} < 1$. Figure 5 illustrates this result.

Although the subsidy trigger y^{**} is lowered with preemption risk the ratio $\frac{y^{**}}{y^*}$ remains constant. This implies that the welfare benefit and the investment subsidy, which both depend on this ratio, are at the same level as they would be without preemption risk, at the time of optimal subsidy.

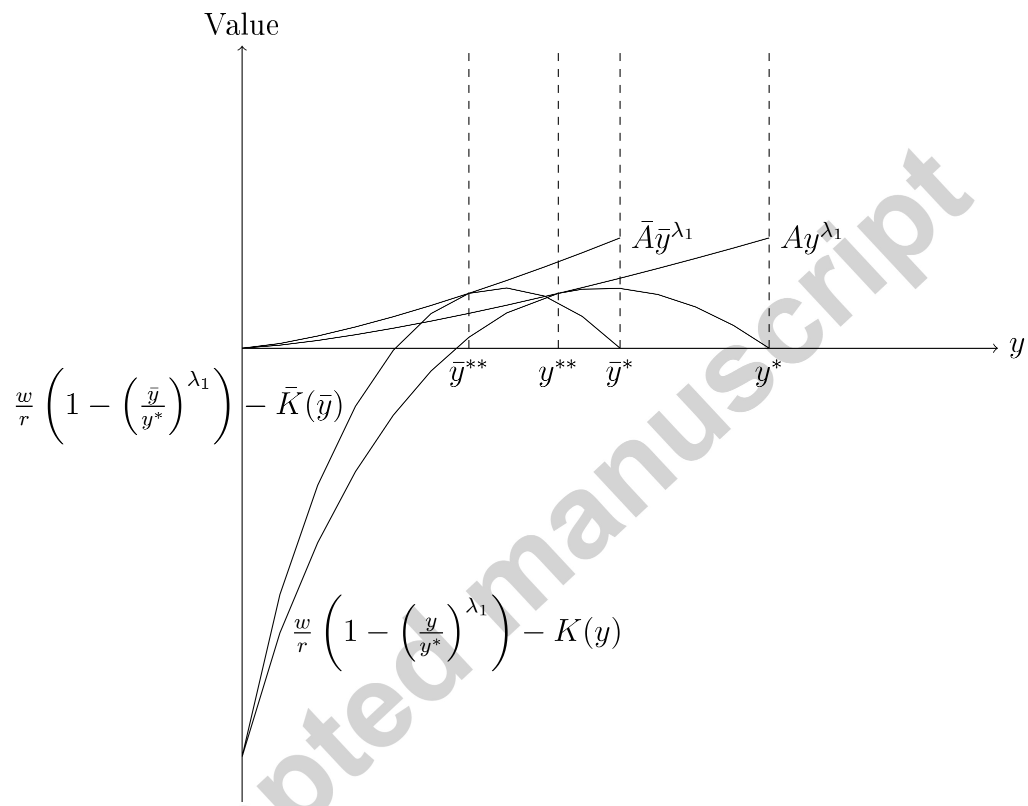

Figure 5: The effect of preemption risk between firms on the optimal timing of investment. We use y^* as the unsubsidised investment trigger, y^{**} as the subsidy trigger, Ay^{λ_1} as the welfare of the option to offer subsidy, and $\frac{w}{r}\left(1-\left(\frac{y}{y^*}\right)^{\lambda_1}\right)-K(y)$ as the welfare of the investment subsidy without preemption risk (corresponding to Figure 3). We use correspondingly $\bar{y}^*$, $\bar{y}^{**}$, $\bar{A}\bar{y}^{\lambda_1}$, and $\frac{w}{r}\left(1-\left(\frac{\bar{y}}{y^*}\right)^{\lambda_1}\right)-\bar{K}(\bar{y})$, respectively, with preemption risk.

However, since the subsidy is offered sooner the subsidy is worth more to the country as it has to wait for less time before harvesting the welfare benefits from the subsidy. The option to offer an investment subsidy is also therefore enhanced with preemption risk.

Next consider preemption risk in the market for subsidy, and we build the analysis again directly on Lambrecht and Perraudin (2001). Consider the probability distribution function G defined by

$$G(y) = \mathbb{P}(\text{Subsidy of nearest competing country is offered at } y_t \leq y). \tag{14}$$

Define further $\bar{y}_t = \max_{0 \leq x \leq t} y_t$ as the "all time high" of the earnings process y_s up to and including time t. If no subsidy has been offered at time t, we know that the probability that the subsidy is offered at earnings levels at or lower than $\bar{y}_t$ is zero. The conditional probability $G(y|\bar{y}_t)$, $y \geq \bar{y}_t$ is then defined as

$$G(y|\bar{y}_t) = \frac{G(y) - G(\bar{y}_t)}{1 - G(\bar{y}_t)}. \tag{15}$$

Let y^{**} be the trigger point for offering a subsidy, and τ_2 be the stopping time for the event that $y_t = y^{**}$ for the first time and τ_1 the stopping time for the event that the unsubsidised investment trigger $y_t = y^* > y^{**}$ is reached for the first time. The value of the option to offer a subsidy at the trigger point y^{**} is then

$$W(y_t|t \leq \tau_2, \bar{y}_t) = \left(1 - \frac{G(y_t) - G(\bar{y}_t)}{1 - G(\bar{y}_t)}\right) \mathbb{E}\left(\int_{\tau_2}^{\tau_1} e^{-r(s+\tau_2)} w ds - e^{-r\tau_2} K(y^{**})\right), \tag{16}$$

where the right hand side is the probability of having the winning subsidy at the trigger point y^{**} times the welfare of the winning subsidy. The event that a competing country has the winning subsidy leads to zero welfare. We find the following result.

Proposition 4: *The optimal timing y^{**} of an investment subsidy is given implicitly by the following equation.*

$$\frac{w}{r} + \frac{y^{**}}{\lambda_1} h_G(y^{**}) \left(\frac{w}{r} \left(1 - \left(\frac{y^{**}}{y^*} \right)^{\lambda_1} \right) - K(y^{**}) \right) = I \left(1 - \frac{y^{**}}{y^*} \right), \quad h_G(y^{**}) = \frac{g(y^{**})}{1 - G(y^{**})}. \quad (17)$$

When there is no preemption risk in the market for subsidy, the left hand side equals $\frac{w}{r}$ and the condition above implies Proposition 3. When the hazard rate $h_G(y^{**}) > 0$ is increasing in y^{**} the ratio $\frac{y^{**}}{y^*}$ is either zero or $\frac{y^{**}}{y^*} < 1 - \frac{w}{rI}$. The welfare at the time when the subsidy is offered is $W(y^{**}) = \frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**})$ and is affected by the preemption risk through the ratio $\frac{y^{**}}{y^*}$.

The effect of the preemption risk in the subsidy market is to speed up the timing of the subsidy, and the ratio $\frac{y^{**}}{y^*}$ is lowered relative to the level we would expect without preemption risk. This is implied by the fact that the second term on the left hand side is positive except in the special case where the welfare $W(y^{**}) = 0$. Thus the ratio $\frac{y^{**}}{y^*}$ is lowered and the welfare $W(y^{**})$ is lowered relative to what it would be without preemption risk. Preemption risk in the market for subsidy will therefore have a negative impact on welfare.

Proposition 2, which sets out the form of the optimal subsidy $K(y^{**})$, is robust to preemption risk in the subsidy market in the sense that the functional form of K remains intact. However, welfare is lowered because the ratio $\frac{y^{**}}{y^*}$ which enters the expression for K is lowered. With preemption risk in the market for investment the ratio $\frac{y^{**}}{y^*}$ will in fact not change.

3.4 Free-Riding

When a country can benefit not only from its own subsidised investments but also from investments subsidised by other countries a free-riding problem arises. An example is the subsidy of investments in green technology that lower emissions of greenhouse gasses. The cost of the subsidy is borne by the subsidising countries but the benefits are shared by both the subsidising and non-subsidising countries. The welfare of the winning country may be the same, but the losing country captures a fraction of this welfare at no cost. Recall that the parameter a defined in (7.c) in Section 2 defines the fraction of the welfare of the winning country captured by the losing country. The following result sets out the optimal timing of the subsidy with free-riding.

Proposition 5: *The optimal timing y^{**} of an investment subsidy is given implicitly by the following equation.*

$$\frac{w}{r} + \frac{y^{**}}{\lambda_1} h_G(y^{**})(1-a)\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - \frac{y^{**}}{\lambda_1} h_G(y^{**})K(y^{**}) = I\left(1 - \frac{y^{**}}{y^*}\right). \quad (18)$$

There exists a number $\bar{a} \in (0,1)$ *defined by*

$$\bar{a} = 1 - \frac{K(y^*(1 - w/rI))}{w/r(1 - (1 - w/rI)^{\lambda_1})} \tag{19}$$

For $a < \bar{a}$, $y^{**} < y^*(1 - w/rI)$, *for* $a > \bar{a}$, $y^{**} > y^*(1 - w/rI)$ *and for* $a = \bar{a}$, $y^{**} = y^*(1 - w/rI)$ *and the welfare is maximal (Proposition 3). The welfare at the time when the subsidy is offered is in either case given by* $W(y^{**}) = \frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**})$.

From (18) it is immediate that when the free riding effect is zero, i.e. when $a = 0$, y^{**} is determined only by preemption risk and corresponds therefore exactly to Proposition 4. What Proposition 5 tells us is that free-riding and preemption risk have offsetting effects and derives the exact cut-off point $\bar{a}$ where they neutralise each other. At this point the welfare is maximal, and the subsidy trigger point is determined by Proposition 3 (or Proposition 2 if the preemption risk in the market for investments is zero).

4 Discussion of Findings

Our model is designed to predict the timing and welfare of investment subsidy. In this section we derive the comparative statics of the model and discuss policy measures applied to the investment subsidy market.

4.1 Comparative Statics

When the subsidy trigger y^{**} is lowered we are more likely to observe an active subsidy market. This effect can arise in two ways. First, the unsubsidised investment trigger y^* is lowered and the ratio of the subsidy trigger to the unsubsidised investment trigger, $\frac{y^{**}}{y^*}$, remains constant. Second, the unsubsidised investment trigger y^* remains constant and the ratio $\frac{y^{**}}{y^*}$ is lowered. Either leads to a more active subsidy market, but this does not imply that the welfare generated by this market is enhanced. The welfare effect is measured more correctly by evaluating the value of the option to offer subsidy at a given point in time prior to the stopping time of the event that the subsidy is offered. Welfare cannot be evaluated consistently by looking at the welfare levels at the time the subsidy is offered as everything being equal the country would prefer to receive the welfare effect sooner rather than later. In this subsection we identify the factors that influence the value of the option to offer subsidy.

Lemma 2 (Value of Option to Subsidise): *Suppose $y_t < y^{**}$. Then the value of offering subsidy when the subsidy trigger y^{**} is reached is Ay^{λ_1} where*

$$A = I^{1-\lambda_1}\left(\frac{1-\tau}{r-\mu}\frac{\lambda_1 - 1}{\lambda_1}\right)^{\lambda_1}\left(\frac{1}{\lambda_1 - 1}\left(1 - \frac{w}{rI}\right)^{1-\lambda_1} - \left(\frac{1}{\lambda_1 - 1} + \frac{w}{rI}\right)\right) \tag{20}$$

We find the following result.

Proposition 6 (Comparative Statics without Preemption Risk): *For fixed t, the value* *of the option to offer subsidy at the subsidy trigger y^{**} is increasing in the welfare flow w and decreasing in the investment cost I.*

Proposition 6 demonstrates that with no preemption risk, the welfare and timing of subsidy are one-to-one so that the activity levels in the subsidy market is a proxy for the welfare generated by that market. Recall from Proposition 2 that the prime determinant of timing is the ratio of the welfare to the amortised investment cost, $\frac{w}{rI}$, with earlier subsidy the greater the ratio.[4] The value of the option to offer subsidy is also increasing in the ratio $\frac{w}{rI}$.

Next, consider preemption risk in the market for investments as described in subsection 3.3. Recall that the probability function $F(y)$ represents the probability that a competing firm makes the investment at $y_t \leq y$, preempting the firm in question from making the investment at y. Recall also that the hazard rate $h_F(y) = \frac{f(y)}{1-F(y)}$ is the likelihood that a competing firm makes the investment in the region $[y, y + dy]$, conditional on not having made the investment up to the point y. We study the impact on welfare arising from variation in the hazard rate h_F. We find the following result.

Proposition 7 (Comparative Statics with Preemption Risk in the Market for Investments): *For fixed t, the value of the option to offer subsidy at the subsidy trigger y^{**} is increasing in the level of preemption risk in the market for investments as measured by the hazard rate h_F.*

[4] Note that the ratio $\frac{w}{rI}$ is always less than 1 when there is a non-trivial timing issue at hand, as a ratio greater than 1 implies that the subsidy is offered for $y^{**} = 0$, i.e. the subsidy is offered at the first available opportunity.

We know already that preemption risk leads to a lowering of the unsubsidised investment trigger and that as a result the subsidy market becomes more active because the ratio $\frac{y^{**}}{y^{*}}$ remains constant (Proposition 3). Proposition 7 shows that the activity levels in the subsidy market are also a proxy for welfare. The intuition is that countries take advantage of the lowering of firm profits generated by the preemption risk in the investment market. This generates greater welfare from the subsidy market because the subsidy can be made sooner (recall from Proposition 3 that the welfare at the subsidy trigger is constant, but the timing is different for various levels of preemption risk).

So far, the results show a one-to-one relationship between the activity levels in the subsidy market and the welfare generated by this market. This will no longer be the case when we consider preemption risk and free-riding effects in the market for subsidy. Recall that $G(y)$ represents the probability that a subsidy is offered by a competing country at $y_t \leq y$, and that h_G is the hazard rate such that $h_G dy$ is the probability that a competing country offers a subsidy in the interval $[y, y + dy]$ conditional on the event that no subsidy has been offered when $y_t = y$ for the first time. We measure preemption risk in the subsidy market by the hazard rate h_G. Recall also that free-riding is measured by the parameter a which represents the fraction of the welfare captured by the winning country in the subsidy contest that can be consumed by the loser. Ideally we would work out the change in the value of the option to subsidise investment as a result of a change in the hazard rate h_G or the free-riding effect a, but analytical results are difficult to obtain. However, there is no ambiguity about which combination of preemption risk and free-riding effect is welfare optimal, as demonstrated in Proposition 5. Regardless of the hazard rate h_G, when the free-riding effect $a = \bar{a}$ (as defined in Proposition 5) the welfare generated by the subsidy market is optimal. When $a \neq \bar{a}$ both a and h_G matter but when $a = \bar{a}$ they do not.

4.2 Policy Implications

The results in the previous section indicate that activity levels in the subsidy markets are one-to-one with welfare except when there is preemption risk and free-riding in the market for subsidy. In the latter case the activity levels in the subsidy market may be too high or too low, and the question we ask here is whether there exist policy measures to correct the activity levels towards the optimal levels. The optimal activity level is obtained for the free-riding parameter $a = \bar{a}$ (as defined in Proposition 5) so the search for policy measures can be narrowed down to those that correct for the free riding effect when $a \neq \bar{a}$. In fact, a surprisingly simple scheme will do this job. Consider that $a \neq \bar{a}$, and also consider the situation that a country has offered an investment subsidy $K(y)$ for $y_t = y$. Define a tax on all non-subsidising countries equal to $bK(y)$ so that every time one country pays a subsidy the other countries are taxed an amount $bK(y)$ which goes into or out from an international fund. We derive the following result. Denote by $W_S(y_t)$ the welfare of a subsidising country and $W_N(y_t)$ the welfare of a non-subsidising country.

Proposition 8 (Optimal Taxation): *Assume that* $\frac{w}{rI} < 1$. *Set the tax rate* b *such that*

$$b = \frac{a - \bar{a}}{1 - \bar{a}}. \tag{21}$$

Then $\frac{y^{**}}{y^*} = 1 - \frac{w}{rI}$ *and* $W_S(y_t) = W_N(y_t)$ *is maximal.*

The taxation scheme will therefore both remove the distortions to the optimal timing in the investment subsidy market and make the welfare of both the subsidising and the non-subsidising countries maximal. The intuition for Proposition 8 is straightforward. The incentive to defer the subsidy is brought about by the fact that the non-subsidising countries can obtain a times the welfare without incurring any of the cost of the subsidy. Therefore, if the non-subsidising countries obtain a times the welfare of subsidy but must pay b times the subsidy of the subsidising countries they are penalised for not offering subsidy. Offering subsidy is effectively a way of obtaining tax-free status. The tax is calibrated such that the countries are exactly indifferent between winning and losing the competition for subsidy. This eliminates the distortions to timing, and the welfare of both the subsidising and non-subsidising countries is maximal. Note that the tax rate b is positive if $a \geq \bar{a}$, which is likely to happen in cold subsidy markets, and it is negative if $a < \bar{a}$, which is likely to happen in hot subsidy markets.

The advantage of the taxation scheme outlined in Proposition 8 is that it removes the need for international agreements regulating directly the individual country's subsidy activity. The taxation scheme creates an alignment of the optimal subsidy activity for the individual country with the welfare maximising subsidy activity for that country. However, there are likely to be implementation problems, some of which we discuss here.

Uncertainty about the tax rate (the parameter b in Proposition 8): One may conceive that individual countries may lobby for a tax rate that is in their interests, but different from the optimal tax rate set out in Proposition 8. However, if the country's objective is to maximise the welfare of the subsidy the optimal scheme delivers this welfare level (regardless of whether the country pays taxes or engages in subsidy activity). Any other tax rate is likely to deliver suboptimal welfare levels. A country interested in maximising the welfare of its subsidy policy is, therefore, also likely to lobby for the correct tax rate.

Verifiability issues: The most important verifiability issue is that of misreporting of subsidy payments. A country could hide its subsidy activity, which is alleged to be a common feature in the FDI subsidy market currently (see Thomas (2007)). Or a country could try to classify a non-subsidy expense as an investment subsidy. Consider two countries, A and B, where country A reports subsidy activity $\hat{K}_A$ and country B reports subsidy activity $\hat{K}_B$. If the countries are active in two different subsidy markets, say country A is active in subsidy market 1 and country B in subsidy market 2, then country A should pay a tax of $b_2\hat{K}_B$ and country B should pay a tax of $b_1\hat{K}_A$. There is no incentive to report $\hat{K}_A$ or $\hat{K}_B$ incorrectly, since the effect of misreporting will only influence the other country's welfare through its tax payments. If both subsidies are made in the same subsidy market, say market 1, then country A pays a tax $b_1\hat{K}_B$ and country B pays a tax $b_1\hat{K}_A$. Therefore, there is no incentive to over- or under-report subsidy as misreporting only affects the other country's welfare. Verifiability is not likely to be an important concern for implementation of the taxation scheme. This conclusion would obviously change if the countries could collude. For instance, it may be in the interest of A that $\hat{K}_B$ was hidden as non-subsidy if b_1 is positive as this would lower the tax burden. If $\hat{K}_B$ was truly a non-subsidy expense for B, it would be in the interest of A that it is classified as a subsidy if b_1 is negative. Therefore, the taxation scheme is not robust to collusion.

Funding: A cold subsidy market generates positive tax payments from countries that participate in the scheme but are not active in the subsidy market. A hot subsidy market generates negative tax payments to countries that are not active in the subsidy market. Therefore, if the taxation scheme straddles both cold and hot subsidy markets the scheme could be funded without generating large positive or negative balances. There is however no guarantee that the scheme will not do so over time. If surplus cash need to be distributed or raised from the scheme's members it is necessary that this is done such as not to distort the member countries' future subsidy activity.

5 Conclusion

In this paper we study the optimal timing and welfare of investment subsidy and their determining factors. We find that preemption risk and free-riding in the subsidy market can create hot and cold subsidy markets, both of which are harmful to welfare and requires correction. We demonstrate that a policy scheme can eliminate this problem: simply transfer money (negative tax) for non-subsidy in the hot subsidy markets and charge a tax for non-subsidy in the cold subsidy markets. If the tax is calibrated properly it will make the activity in the market for subsidy robust against preemption risk and free-riding effects, and participation is therefore incentive compatible. Obtaining international agreement for such a scheme may be easier than obtaining international agreements about targets and quotas, which currently is the dominant policy measure aimed at making the subsidy market for green investments more active. The taxation scheme outlined in this paper has the benefit that it allows costless delegation of the decision to offering the welfare optimal subsidy to the individual country, whereas targets and quotas require a credible commitment device.

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Appendix: Proofs

Proof of Lemma 1: The firm regards the subsidy package as just another investment or borrowing opportunity, the net present value of which can be maximized by optimal timing. The investment opportunity (prior to investment) takes values that can be expressed as $Ay_t^{\lambda_1}$ for some constant A and the net present value of the investment (at the point of investment) including the value of subsidy package is

$$\left(\frac{y_t(1-\tau)}{r-\mu} - I\right) + K(y_t). \tag{A.1}$$

The firm decides on the optimal timing when the value of the investment opportunity equals the net present value of investment, and when there is a smooth fit at the investment trigger point. This yields two equations that determine the optimal investment trigger point y^{**} and the free constant A. The conditions are as follows,

$$\begin{aligned}
Ay^{**\lambda_1} &= \left(\frac{y^{**}(1-\tau)}{r-\mu} - I\right) + K(y^{**}), && \text{(Value matching VM)}\\
\lambda_1 A y^{**\lambda_1 - 1} &= \frac{1-\tau}{r-\mu} + \frac{d}{dy_t}K(y^{**}), && \text{(Smooth pasting SP)}\\
Ay^{**\lambda_1} &= \frac{1}{\lambda_1}\frac{y^{**}(1-\tau)}{r-\mu} + \frac{y^{**}}{\lambda_1}\frac{d}{dy_t}K(y^{**}), && \text{(Rearranging SP)}
\end{aligned}$$

$$\begin{aligned}
\left(\frac{y^{**}(1-\tau)}{r-\mu} - I\right) + K(y^{**}) &= \frac{1}{\lambda_1}\frac{y^{**}(1-\tau)}{r-\mu} + \frac{y^{**}}{\lambda_1}\frac{d}{dy_t}K(y^{**}), && \text{(Combining VM, SP)}\\
y^{**}\left(1 - \frac{1}{\lambda_1}\right) &= I\frac{I}{1-\tau}(r-\mu) - \frac{K(y^{**})(r-\mu)}{1-\tau} + \frac{r-\mu}{1-\tau}\frac{y^{**}}{\lambda_1}\frac{d}{dy_t}K(y^{**}), && \text{(Rearranging)}\\
y^{**}\left(\frac{\lambda_1 - 1}{\lambda_1} - \frac{r-\mu}{1-\tau}\frac{1}{\lambda_1}\frac{d}{dy}K(y^{**})\right) &= y^{*}\frac{\lambda_1 - 1}{\lambda_1} - \frac{K(y^{**})(r-\mu)}{1-\tau}, && \text{(Using definition of } y^{*}\text{)}\\
y^{**} &= \left(y^{*} - \frac{K(y^{**})(r-\mu)}{1-\tau}\frac{\lambda_1}{\lambda_1 - 1}\right)\left(1 - \frac{r-\mu}{1-\tau}\frac{1}{\lambda_1 - 1}\frac{d}{dy}K(y^{**})\right)^{-1}. && \text{(A.2)}
\end{aligned}$$

□

Proof of Proposition 1: Starting from the combination of the value matching and smooth pasting conditions outlined in the proof of Lemma 1, we find

$$\left(\frac{y^{**}(1-\tau)}{r-\mu} - I\right) + K(y^{**}) = \frac{1}{\lambda_1}\frac{y^{**}(1-\tau)}{r-\mu} + \frac{y^{**}}{\lambda_1}\frac{d}{dy_t}K(y^{**})$$

$$y^{**} - I\frac{r-\mu}{1-\tau} + \frac{r-\mu}{1-\tau}K(y^{**}) = \frac{1}{\lambda_1}y^{**} + \frac{y^{**}}{\lambda_1}\frac{r-\mu}{1-\tau}\frac{d}{dy}K(y^{**}) \qquad \text{(Rearranging)}$$

$$\frac{\lambda_1 - 1}{\lambda_1}y^{**} - I\frac{r-\mu}{1-\tau} = \frac{y^{**}}{\lambda_1}\frac{r-\mu}{1-\tau}\frac{d}{dy}K(y^{**}) - \frac{r-\mu}{1-\tau}K(y^{**}) \qquad \text{(Rearranging)}$$

$$I(y^{**} - y^{*}) = \frac{y^{**}y^{*}}{\lambda_1}\frac{d}{dy}K(y^{**}) - y^{*}K(y^{**}) \qquad \text{(Rearranging and using definition of } y^{*}\text{)}$$

$$\frac{d}{dy}K(y^{**}) - \frac{\lambda_1}{y^{**}}K(y^{*}) = \lambda_1 I\left(\frac{1}{y^{*}} - \frac{1}{y^{**}}\right) \tag{A.3}$$

and (A.3) is (9.a). (A.3) a linear first order ODE which implies that the solution can be written as

$$K(y) = Cy^{\lambda_1} + I(1 - \frac{\lambda_1}{\lambda_1 - 1}\frac{y^{**}}{y^{*}}), \tag{A.4}$$

where C is an integration constant. We use the constraint that $V(y_t|t < \tau_2, \text{Subsidy}) \geq V(y_t|t < \tau_1)$, which implies that at $y^{**} = y^{*}$ it must be true that $K(y^{*}) = \frac{d}{dy}K(y^{*}) = 0$, which yields

$$C = \frac{I}{\lambda_1 - 1}(y^{*})^{-\lambda_1}. \tag{A.5}$$

By substituting C from (A.5) back into (A.4) equation (9.b) follows. □

Proof of Proposition 2: Using arguments outlined in Section 2, we can write $W(y_t|t \geq \tau_2) = Ay_t^{\lambda_1} + By_t^{\lambda_2} + \frac{w}{r}$ for arbitrary constants A and B. We know that $\lim_{y_t \to 0} W(y_t|t \geq \tau_2) = \frac{w}{r}$ since there is no likelihood that τ_1 will be reached at this limit point, and this implies that $B = 0$. Also, we know that $\lim_{y_t \to y^{*}} W(y_t|t \geq \tau_2) = 0$

since there is no likelihood that the welfare flow will continue, and this implies that $A = -\frac{w}{r}\left(\frac{1}{y^*}\right)^{\lambda_1}$. Using these, $W(y_t|t \geq \tau_2) = \frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right)$. If we look at the time prior to τ_2, we find that $W(y_t|t < \tau_2) = A' y_t^{\lambda_1} + B' y_t^{\lambda_2}$. B' must vanish because $\lim_{y_t \to 0} W(y_t|t < \tau_2) = 0$, so the value matching condition and the smooth pasting condition imply the following system, where y^{**} is the solution:

$$
\begin{aligned}
A' y^{**\lambda_1} &= \frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**}) && \text{(Value matching VM)}\\
&= \frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - \frac{I}{\lambda_1 - 1}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1} - I\left(1 - \frac{y^{**}}{y^*}\frac{\lambda_1}{\lambda_1 - 1}\right) && \text{(Using } K(y^{**})\text{)}\\
\lambda_1 A' y^{**\lambda_1 - 1} &= \frac{d}{dy}\left(\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**})\right) && \text{(Smooth pasting)}\\
&= -\lambda_1 \frac{w}{r}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1 - 1}\frac{1}{y^*} - \frac{\lambda_1}{\lambda_1 - 1} I \left(\frac{y^{**}}{y^*}\right)^{\lambda_1 - 1}\frac{1}{y^*} + \frac{\lambda_1}{\lambda_1 - 1}\frac{I}{y^*}, && \text{(Using } K(y^{**})\text{)}\\
A' y^{**\lambda_1} &= -\frac{w}{r}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1} - \frac{I}{\lambda_1 - 1}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1} + \frac{I}{\lambda_1 - 1}\frac{y^{**}}{y^*} && \text{(Rearranging SM)}\\
\frac{w}{r} - \frac{w}{r}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1} - I\frac{1}{\lambda_1 - 1}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1} - I + I\frac{\lambda_1}{\lambda_1 - 1}\frac{y^{**}}{y^*} & \\
&= -\frac{w}{r}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1} - I\frac{1}{\lambda_1 - 1}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1} + I\frac{1}{\lambda_1 - 1}\frac{y^{**}}{y^*} && \text{(Combining VM, SP)}\\
\frac{w}{rI} &= 1 - \frac{y^{**}}{y^*} && \text{(A.6)}
\end{aligned}
$$

We find that the right hand side is never greater than 1, so if the left hand side is greater than 1 it is optimal to offer the subsidy immediately, i.e. $y^{**} = 0$. If the left hand side is greater than 1, there is a unique solution. Consequently,

$$y^{**} = y^* \max\left(0, 1 - \frac{w}{rI}\right), \tag{A.7}$$

which yields (11). □

Proof of Proposition 3: The first part is obvious from (11): $\frac{dy^{**}}{dy^*} = \left(1 - \frac{w}{rI}\right) > 0$ for $\frac{w}{rI} < 1$. The second part can be evaluated by evaluating the derivative $\frac{dW(y^{**}(y^*))}{dy^*} = \frac{\partial W}{\partial y^{**}}\frac{\partial y^{**}}{\partial y^*} + \frac{\partial W}{\partial y^*}$. We find that $\frac{\partial y^{**}}{\partial y^*} = \left(1 - \frac{w}{rI}\right)$ and that

$$\frac{\partial W}{\partial y^{**}}\frac{\partial y^{**}}{\partial y^*} = -\lambda_1\left(\frac{w}{r} + \frac{I}{\lambda_1 - 1}\right)\frac{1 - \frac{w}{rI}}{y^*}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1 - 1} + \lambda_1\frac{I}{\lambda_1 - 1}\frac{1 - \frac{w}{rI}}{y^*} \tag{A.8}$$

$$\frac{\partial W}{\partial y^*} = \lambda_1\left(\frac{w}{r} + \frac{I}{\lambda_1 - 1}\right)\frac{y^{**}}{y^{*2}}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1 - 1} - \lambda_1\frac{I}{\lambda_1 - 1}\frac{y^{**}}{y^{*2}} \tag{A.9}$$

Adding (A.8) and (A.9), we find

$$\begin{aligned}\frac{\partial W}{\partial y^{**}}\frac{\partial y^{**}}{\partial y^*} + \frac{\partial W}{\partial y^*} &= -\lambda_1\left(\frac{w}{r} + \frac{I}{\lambda_1 - 1}\right)\frac{1 - \frac{w}{rI}}{y^*}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1 - 1} + \lambda_1\frac{I}{\lambda_1 - 1}\frac{1 - \frac{w}{rI}}{y^*} \\ &\quad + \lambda_1\left(\frac{w}{r} + \frac{I}{\lambda_1 - 1}\right)\frac{y^{**}}{y^{*2}}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1 - 1} - \lambda_1\frac{I}{\lambda_1 - 1}\frac{y^{**}}{y^{*2}} \qquad \text{((A.8) plus (A.9))} \\ &= \lambda_1\left(\frac{w}{r} + \frac{I}{\lambda_1 - 1}\right)\left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\left(\frac{1}{y^*} - \frac{1 - w/rI}{y^*}\right) \\ &\quad - \lambda_1\frac{I}{\lambda_1 - 1}\frac{y^{**}}{y^*}\left(\frac{1}{y^*} - \frac{1 - w/rI}{y^*}\right)\end{aligned} \tag{A.10}$$

Since $y^{**} = y^*(1 - w/rI)$, we find that $\frac{1}{y^*} = \frac{1 - w/rI}{y^{**}}$, so it follows that $\frac{dW(y^{**}(y^*))}{dy^*} = \frac{\partial W}{\partial y^{**}}\frac{\partial y^{**}}{\partial y^*} + \frac{\partial W}{\partial y^*} = 0$. □

Proof of Proposition 4: The value of the option to offer subsidy at time t, conditional on the all-time-high earnings level $\bar{y}_t$ can now be written as $Ay_t^{\lambda_1}$ for some constant A. At the optimal time of subsidy, this value smooth pastes into the welfare of the investment subsidy $(1 - G(y_t|\bar{y}_t))\left(\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**})\right)$. Adding the value matching condition, we find

$$Ay^{**\lambda_1} = (1 - G(y^{**}|\bar{y}_t))\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**}), \tag{A.11}$$

$$\lambda_1 Ay^{**(\lambda_1 - 1)} = -\frac{g(y^{**})}{1 - G(y^{**})}\left(\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**})\right) - \lambda_1\frac{w}{r}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1 - 1}\frac{1}{y^*} - \frac{d}{dy}K(y^{**}) \tag{A.12}$$

When the winning bid is known it must be the case that $\bar{y}_t = y^{**}$ and $G(y^{**}|\bar{y}_t) = 0$. Using this fact, and combining (A.10) and (A.11), we find

$$Ay^{**\lambda_1} = \frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**}) \qquad \text{(From (A.11))}$$

$$Ay^{**\lambda_1} = -\frac{y^{**}}{\lambda_1}\frac{g(y^{**})}{1-G(y^{**})}\left(\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**})\right) - \frac{w}{r}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1} - \frac{y^{**}}{\lambda_1}\frac{d}{dy}K(y^{**})$$

$$\text{((A.12) multiplied by } \tfrac{y^{**}}{\lambda_1}\text{)}$$

Since the left hand side is the same in both expressions the right hand side is also the same, which implies the following relationship:

$$\frac{w}{r} + \frac{y^{**}}{\lambda_1}\frac{g(y^{**})}{1-G(y^{**})}\left(\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**})\right) = K(y^{**}) - \frac{y^{**}}{\lambda_1}\frac{d}{dy}K(y^{**}) \qquad \text{(A.13)}$$

From Proposition 2 we know that $\frac{d}{dy}K(y^{**}) - \frac{\lambda_1}{y^{**}}K(y^{**}) = \lambda_1 I\left(\frac{1}{y^*} - \frac{1}{y^{**}}\right)$. Multiplying both sides of this expression by $\frac{y^{**}}{\lambda_1}$ and rearranging we find $\frac{y^{**}}{\lambda_1}\frac{d}{dy}K(y^{**}) - K(y^{**}) = I\left(1 - \frac{y^{**}}{y^*}\right)$. Since the preemption risk in the market for subsidy will not affect the subsidy itself at the time it is paid, the right hand side in (A.12) equals $I\left(1 - \frac{y^{**}}{y^*}\right)$. $\square$

Proof of Proposition 5: The value of the option to offer subsidy at time t, conditional on the all-time-high earnings level $\bar{y}_t$ can now be written as $Ay_t^{\lambda_1}$ for some constant A. At the optimal time of subsidy, this value smooth pastes into the welfare of the investment subsidy $(1 - G(y_t|\bar{y}_t))\left(\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**})\right) + aG(y_t|\bar{y}_t))\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right)$. We find, therefore, the following two conditions:

$$Ay^{**\lambda_1} = (1 - G(y^{**}|\bar{y}_t))\left(\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**})\right) + aG(y^{**}|\bar{y}_t))\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right), \qquad \text{(A.14)}$$

$$\begin{aligned}\lambda_1 Ay^{**(\lambda_1-1)} &= -\frac{g(y^{**})}{1-G(y^{**})}\left(\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**})\right) - \lambda_1\frac{w}{r}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1-1}\frac{1}{y^*} - \frac{d}{dy}K(y^{**}) \\ &\quad + a\frac{g(y^{**})}{1-G(y^{**})}\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - aG(y^{**}|\bar{y}_t)\lambda_1\frac{w}{r}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1-1}\frac{1}{y^*}, \qquad \text{(A.15)}\end{aligned}$$

When the winning bid is known the conditional probability $G(y^{**}|\bar{y}_t) = 0$, so taking this into account and combining (A.13) and (A.14), we find

$$Ay^{**\lambda_1} = \frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**}) \qquad \text{(From (A.14))}$$

$$Ay^{**\lambda_1} = -\frac{y^{**}}{\lambda_1}\frac{g(y^{**})}{1-G(y^{**})}(1-a)\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) + \frac{y^{**}}{\lambda_1}\frac{g(y^{**})}{1-G(y^{**})}K(y^{**}) - \frac{w}{r}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1}$$
$$- \frac{y^{**}}{\lambda_1}\frac{d}{dy}K(y^{**}) \qquad \left(\text{(A.15) multiplied by } \tfrac{y^{**}}{\lambda_1}\right)$$

Since the left hand side is the same in both expressions the right hand side is also the same, which implies the following relationship:

$$\frac{w}{r} + \frac{y^{**}}{\lambda_1}\frac{g(y^{**})}{1-G(y^{**})}(1-a)\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - \frac{y^{**}}{\lambda_1}\frac{g(y^{**})}{1-G(y^{**})}K(y^{**}) = K(y^{**}) - \frac{y^{**}}{\lambda_1}\frac{d}{dy}K(y^{**}) \qquad \text{(A.16)}$$

From Proposition 2 we know that $\frac{d}{dy}K(y^{**}) - \frac{\lambda_1}{y^{**}}K(y^{**}) = \lambda_1 I\left(\frac{1}{y^*} - \frac{1}{y^{**}}\right)$. Multiplying both sides of this expression by $\frac{y^{**}}{\lambda_1}$ we find $\frac{y^{**}}{\lambda_1}\frac{d}{dy}K(y^{**}) - K(y^{**}) = I\left(1 - \frac{y^{**}}{y^*}\right)$. Since the preemption risk in the market for subsidy will not affect the subsidy itself at the time it is paid, the right hand side in (A.12) equals $I\left(1 - \frac{y^{**}}{y^*}\right)$, yielding equation (18). If the preemption risk leads to a delay in the timing of subsidy beyond the point where the subsidy is optimally timed with no preemption risk at all, there must exist $\bar{a} \in (0,1)$ such that the left hand side in (A.12) equals $\frac{w}{r}$, and the value of y^{**} is then determined as in Proposition 3 by $y^{**} = y^*(1 - w/rI)$. Therefore, we are looking for a solution to the equation

$$(1-\bar{a})\frac{w}{r}\left(1 - \left(1 - \frac{w}{rI}\right)^{\lambda_1}\right) - K\left(y^*\left(1 - \frac{w}{rI}\right)\right) = 0 \qquad \text{(A.17)}$$

We show that such value always exists for any $0 < \frac{w}{rI} < 1$ and we can then verify that the solution in (19) solves (A.17). We know that if a subsidy is offered the welfare effect of the subsidy, $\frac{w}{r}\left(1 - \left(1 - \frac{w}{rI}\right)^{\lambda_1}\right)$, is greater than the cost of the subsidy $K(y^{**})$, therefore, the equation in (A.17) must always have a solution $0 \leq \bar{a} \leq 1$. □

Proof of Lemma 2: The welfare of the option to offer subsidy can be written as $Ay_t^{\lambda_1}$ where the coefficient A is determined at the optimal point of subsidy $Ay^{**\lambda_1} = W(y^{**})$, such that $A = W(y^{**})\left(\frac{1}{y^{**}}\right)^{\lambda_1}$. The following is given in the text and in Proposition 2 and 3, and unless $y^{**} = 0$ we need to assume that $\frac{w}{rI} < 1$.

$$W(y^{**}) = \frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**}) \tag{A.18.a}$$

$$K(y^{**}) = \frac{I}{\lambda_1 - 1}\left(\frac{y^{**}}{y^*}\right)^{\lambda_1} + I\left(1 - \frac{y^{**}}{y^*}\frac{\lambda_1}{\lambda_1 - 1}\right) \tag{A.18.b}$$

$$y^{**} = y^*\left(1 - \frac{w}{rI}\right) \tag{A.18.c}$$

$$y^* = \frac{I}{1-\tau}(r-\mu)\frac{\lambda_1}{\lambda_1 - 1} \tag{A.18.c}$$

Solving for the constant A from above, we find the effect on welfare from changes in w and I by evaluating $\frac{d}{dw}Ay^{\lambda_1} = \lambda_1\frac{dA}{dw}$ and $dAy^{\lambda_1} = \lambda_1\frac{dA}{dI}$ and the sign is obviously determined by $\frac{dA}{dw}$ and $\frac{dA}{dI}$. The constant A is given by

$$\begin{aligned}
Ay^{**\lambda_1} &= \frac{w}{r}\left(1 - \left(1 - \frac{w}{rI}\right)^{\lambda_1}\right) - \frac{I}{\lambda_1 - 1}\left(1 - \frac{w}{rI}\right)^{\lambda_1} - I\left(1 - \left(1 - \frac{w}{rI}\right)\frac{\lambda_1}{\lambda_1 - 1}\right) && \text{(Value matching)}\\
&= \frac{I}{\lambda_1 - 1}\left(1 - \frac{w}{rI}\right) - \left(\frac{w}{r} + \frac{I}{\lambda_1 - 1}\right)\left(1 - \frac{w}{rI}\right)^{\lambda_1} && \text{(Rearranging)}\\
A &= \left(\frac{I}{\lambda_1 - 1}\left(1 - \frac{w}{rI}\right) - \left(\frac{w}{r} + \frac{I}{\lambda_1 - 1}\right)\left(1 - \frac{w}{rI}\right)^{\lambda_1}\right)\left(\frac{1}{y^{**}}\right)^{\lambda_1} && \text{(Dividing by } y^{**\lambda_1})\\
A &= \left(\frac{I}{\lambda_1 - 1}\left(1 - \frac{w}{rI}\right)^{1-\lambda_1} - \left(\frac{w}{r} + \frac{I}{\lambda_1 - 1}\right)\right)\left(\frac{I}{1-\tau}(r-\mu)\frac{\lambda_1}{\lambda_1 - 1}\right)^{-\lambda_1} && \text{(Using (A.18.c-d))}
\end{aligned}$$

The result follows directly. □

Proof of Proposition 6: First we evaluate $\frac{dA}{dw}$ and we show that the welfare is increasing in w by demonstrating that $\frac{dA}{dw} > 0$. Direct differentiation yields

$$\frac{dA}{dw} = \left(\frac{I}{1-\tau}(r-\mu)\frac{\lambda_1}{\lambda_1 - 1}\right)^{-\lambda_1}\frac{1}{r}\left(\left(1 - \frac{w}{rI}\right)^{-\lambda_1} - 1\right)$$

The sign of the first bracket is positive, and the sign of the second bracket is also positive since $\lambda_1 > 0$ and $0 < \frac{w}{rI} < 1$.

Next we evaluate $\frac{dA}{dI}$ and we show that the welfare is decreasing in I by demonstrating that $\frac{dA}{dI} < 0$. Direct differentiation yields

$$\begin{aligned}\frac{dA}{dI} &= (1-\lambda_1)I^{-\lambda_1}\left(\frac{1}{\lambda_1-1}\left(1-\frac{w}{rI}\right)^{-\lambda_1} - \left(\frac{1}{\lambda_1-1}+\frac{w}{rI}\right)\right)\left(\frac{1-\tau}{r-\mu}\frac{\lambda_1-1}{\lambda_1}\right)^{\lambda_1} \\ &+ I^{1-\lambda_1}\left(\frac{1-\lambda_1}{\lambda_1-1}\left(1-\frac{w}{rI}\right)^{-\lambda_1}\left(\frac{w}{rI^2}\right)+\frac{w}{rI^2}\right)\left(\frac{1-\tau}{r-\mu}\frac{\lambda_1-1}{\lambda_1}\right)^{\lambda_1} \\ &= \frac{1-\lambda_1}{y^{*\lambda_1}}\left(\frac{1}{\lambda_1-1}\left(1-\frac{w}{rI}\right)^{1-\lambda_1} - \left(\frac{1}{\lambda_1-1}+\frac{w}{rI}\right)\right) \\ &+ \frac{w/rI}{y^{*\lambda_1}}\left(1-\left(1-\frac{w}{rI}\right)^{-\lambda_1}\right)\left(\frac{1-\tau}{r-\mu}\frac{\lambda_1-1}{\lambda_1}\right)^{\lambda_1}\end{aligned} \tag{A.19}$$

The sign of the first term of (A.19) is determined by the sign of $1-\lambda_1$ which is negative. The sign of the second term is negative since $0 < \frac{w}{rI} < 1$ and $\lambda_1 > 0$. The result follows. □

Proof of Proposition 7: The expression for $Ay_t^{\lambda_1}$ follows from the same smooth pasting problem as in Proposition 6 so the only thing we need to be concerned with is the impact on y^{**} from a change in h_F. Differentiating $Ay_t^{\lambda_1}$ with respect to h_F, therefore, yields

$$\frac{dAy_t^{\lambda_1}}{dh_F} = -\frac{\lambda_1}{y^{**}}Ay_t^{\lambda_1}\frac{dy^{**}}{dh_F} = -\frac{\lambda_1}{y^{**}}Ay_t^{\lambda_1}\left(1-\frac{w}{rI}\right)\frac{dy^{*}}{dh_F} \tag{A.21}$$

Since $y^{**} = y^*\left(1-\frac{w}{rI}\right)$ we find $\frac{dy^{**}}{dh_F} = \left(1-\frac{w}{rI}\right)\frac{dy^*}{dh_F}$. Using the expression for y^* given in the text (this is derived in Lambrecht and Perraudin (2001)), we find that the optimality condition for y^* is given by

$$y^*(\lambda_1 - 1 + y^*h_F) - \frac{I}{1-\tau}(r-\mu)(\lambda_1 - y^*h_F) = 0 \tag{A.22}$$

Taking the total differential with respect to y^* and h_F on both sides of (A.22), dividing by dh_F and isolating the expression $\frac{dy^*}{dh_F}$ we find

$$\frac{dy^*}{dh_F} = -y^* \frac{y^* + \frac{I}{1-\tau}(r-\mu)}{\lambda_1 - 1 + 2y^* h_F + \frac{I}{1-\tau}(r-\mu)h_F} < 0 \tag{A.23}$$

Hence, $\frac{dAy_t^{\lambda_1}}{dh_F} > 0$. □

Proof of Proposition 8: The optimality conditions which leads to the welfare of the subsidy option $Ay_t^{\lambda_1}$ to smooth paste into the welfare of the subsidy at y^{**}, are given by

$$\begin{aligned} Ay^{**\lambda_1} &= (1 - G(y^{**}|\bar{y}_t)) \left(\frac{w}{r} \left(1 - \left(\frac{y^{**}}{y^*} \right)^{\lambda_1} \right) - K(y^{**}) \right) \\ &\quad + aG(y^{**}|\bar{y}_t) \frac{w}{r} \left(1 - \left(\frac{y^{**}}{y^*} \right)^{\lambda_1} \right) - bG(y^{**}|\bar{y}_t) K(y^{**}) \end{aligned} \tag{A.24.a}$$

$$\begin{aligned} \lambda_1 Ay^{**(\lambda_1 - 1)} &= -\frac{g(y^{**})}{1 - G(y^{**})} \left(\frac{w}{r} \left(1 - \left(\frac{y^{**}}{y^*} \right)^{\lambda_1} \right) - K(y^{**}) \right) - \frac{w}{r} \lambda_1 \left(\frac{y^{**}}{y^*} \right)^{\lambda_1 - 1} \frac{1}{y^*} - \frac{d}{dy} K(y^{**}) \\ &\quad + a \frac{g(y^{**})}{1 - G(y^{**})} \frac{w}{r} \left(1 - \left(\frac{y^{**}}{y^*} \right)^{\lambda_1} \right) - b \frac{g(y^{**})}{1 - G(y^{**})} K(y^{**}) \\ &\quad - aG(y^{**}|\bar{y}_t) \lambda_1 \frac{w}{r} \left(\frac{y^{**}}{y^*} \right)^{\lambda_1 - 1} \frac{1}{y^*} - bG(y^{**}|\bar{y}_t) \frac{d}{dy} K(y^{**}) \end{aligned} \tag{A.24.b}$$

Combining (A.24.a) and (A.24.b), and setting $G(y^{**}|\bar{y}_t) = 0$, we find the condition for the optimal stopping time y^{**}, which implies the condition:

$$\frac{w}{r} - K(y^{**}) + \frac{y^{**}}{\lambda_1} \frac{d}{dy} K(y^{**}) = -h_G \frac{y^{**}}{\lambda_1} \left((1-a) \frac{w}{r} \left(1 - \left(\frac{y^{**}}{y^*} \right)^{\lambda_1} \right) - (1-b) K(y^{**}) \right) \tag{A.25}$$

Substituting in $b = \frac{a - \bar{a}}{1 - \bar{a}}$ into (A.25), the right hand side vanishes. Recall from the proof of Proposition 2 that the optimal stopping time y^{**} satisfies

$$\frac{w}{r} - K(y^{**}) + \frac{y^{**}}{\lambda_1} \frac{d}{dy} K(y^{**}) = 0 \tag{A.26}$$

which is identical to (A.25) when $b = \frac{a-\bar{a}}{1-\bar{a}}$, hence the result follows from Proposition 2. It remains to show that $W_S(y^{**}) = W_N(y^{**})$. The welfare of a subsidising country at y^{**} is $W_S(y^{**})$ and the welfare of a non-subsidising country at y^{**} is $W_N(y^{**})$. We find that

$$W_S(y^{**}) = \frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - K(y^{**}) \tag{A.27.a}$$

$$W_N(y^{**}) = a\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - bK(y^{**}) \tag{A.27.b}$$

Assume a tax $b = \frac{a-\bar{a}}{1-\bar{a}}$. Recall from the definition of $\bar{a}$ that $(1-\bar{a})\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) = K(y^{**})$. Then we find that

$$\begin{aligned}
W_S(y^{**}) - W_N(y^{**}) &= (1-a)\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - (1-b)K(y^{**}) \\
&= (1-a)\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - \frac{1-a}{1-\bar{a}}K(y^{**}) && \text{(using definition of } b\text{)} \\
&= (1-a)\left(\frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right) - \frac{w}{r}\left(1 - \left(\frac{y^{**}}{y^*}\right)^{\lambda_1}\right)\right) && \text{(using expression for } (1-\bar{a})\text{)} \\
&= 0 && \text{(A.28)}
\end{aligned}$$

The result follows. □