Essex Finance Centre
Working Paper Series

Working Paper No15: 01-2017

"Wavelet-based option pricing: An empirical study"

"Xiaoquan Liu and Liya Shen"

Essex Business School, University of Essex, Wivenhoe Park, Colchester, CO4 3SQ
Web site: http://www.essex.ac.uk/ebs/

Wavelet-based option pricing: An empirical study

Xiaoquan Liu* Liya Shen†

December, 2016

*Nottingham University Business School, University of Nottingham Ningbo China, 199 Taikang East Road, Ningbo, 315100, China. Email: Xiaoquan.Liu@Nottingham.edu.cn. Financial support from the INQUIRE UK Grant 2007-06 is gratefully acknowledged.

†Corresponding author. Essex Business School, University of Essex, Colchester CO4 3SQ, UK. Email: lshenb@essex.ac.uk. Phone: +44 1206 874679.

Abstract

In this paper, we adopt a wavelet-based option valuation model and empirically compare the pricing and forecasting performance of this model with that of the stochastic volatility model with jumps and the spline method. Both the in-sample valuation and out-of-sample forecasting accuracy are examined using daily index options in the UK, Germany, and Hong Kong from January 2009 to December 2012. Our results show that the wavelet-based model compares favorably with the other two models and that it provides an excellent alternative for valuing option prices. Its superior performance comes from the powerful ability of the wavelet method in approximating the risk-neutral moment-generating functions.

Keywords: Pricing; Option Pricing; Wavelet Method; Stochastic Volatility; Jump Risk

JEL codes: G12, G13

1 Introduction

Since the seminal work of Black and Scholes (1973), huge progress has taken place in the theoretical and empirical option valuation literature that has greatly improved our understanding of the options market as a place for trading information and gauging investor expectation. A large number of parametric and nonparametric methods have been proposed to relax one or more restrictions of the original Black-Scholes model.

One avenue for extending the Black-Scholes model is to develop nonparametric models that are better at capturing the volatility smile and the literature has seen innovative methods in this area. Wavelets are well-known for their remarkable ability in numerical approximation and the wavelet-based option pricing model developed in Ma (2011) takes advantage of this and approximates the implied risk-neutral moment-generating functions (MGF) using wavelets. It offers a novel approach in the nonparametric option pricing literature. Unlike many other nonparametric option valuation models that require a large collection of data, the wavelet-based option pricing model is computationally efficient and requires only a reasonable amount of different strikes. Using numerical experiments, Haven et al. (2009) demonstrate that this model can price and forecast options with great precision.

In this paper, we contribute to the literature by taking this further to empirically compare the valuation and forecasting performance of the wavelet-based model with two other well-established models, namely the parametric stochastic volatility model with jumps (SVJ) and the nonparametric spline method. We focus on the key research questions of whether the excellent performance of this model in simulation still remains in the acid test with market data, and how its empirical performance compares with that of widely-accepted models in the literature.

To the best of our knowledge, this is the first research that subjects the wavelet-based option pricing model to market data. We use daily index options written on the FTSE-100 index and the DAX-30 index, the major financial indices in Europe, and the Heng Seng index in Hong Kong, the financial hub of Asia, from January 2, 2009, to December 28, 2012. Our main empirical findings can be summarized as follows. In the in-sample test across the three markets, the wavelet-based model produces smaller pricing errors than the SVJ and the spline method for the medium- and long-term options. For the short-term options, the performance of the three models are similar and the spline method produces slightly smaller average pricing errors than the other two models. When it comes to the out-of-sample forecasts, the wavelet-based model outperforms the other two methods by big margin for all three maturities across all markets. Our strong empirical evidence substantiates the wavelet-based option pricing model as an excellent alternative in the option valuation literature.

The rest of the paper is organized as follows. In Section 2, we review the relevant literature that motivates our study. Section 3 introduces the wavelet-based option pricing model, the SVJ model, and the spline method. In Section 4, we describe data and analyze empirical results. Finally, Section 5 concludes.

2 Literature review

In this section, we review two strands of the literature to which our paper makes a contribution, namely the literature of option pricing and that of the wavelet method.

In the parametric option pricing literature, researchers have identified a number of priced factors essential in capturing the volatility smile, which has become a stylized fact since the market crash of 1987. For example, volatility is shown to relate negatively to the underlying asset returns and that delta-hedged portfolios of options and the underlying stocks produce statistically significant negative returns (see, for instance, Bakshi and Kapadia (2003), Coval and Shumway (2001), Heston (1993), and Wong and Lo (2009)). Moreover, the random and unexpected jumps are also found to command significant risk premium in the options market (Bates (1996, 2000), Cai and Kou (2011), and Pan (2002)). Another priced factor worth noting is the demand pressure in the market, which affects option prices in incomplete markets (Gârleanu et al. (2009)).

Bakshi et al. (1997) propose a closed-form parametric option pricing model that simultaneously admits the stochastic volatility risk, the jump risk, and the stochastic interest rate risk. One or more risks can be singled out by setting the parameters of the remaining risk factors to zero so that the importance of each risk factor can be closely investigated. Based on the pricing, forecasting, and hedging performance of nested models, they show that the stochastic volatility and jumps are of first-order importance when it comes to accommodating the volatility smile observed in the market.

Parallel to the intensive interest in the parametric option pricing literature, a large number of nonparametric models have also been proposed. Although the nonparametric models lack the economic interpretation that the parameters contain in the parametric family of models, they are often more flexible as they impose no prior assumption on the underlying asset process.

These include the flexible distribution method (Rubinstein (1994)), the cubic spline method (Shimko (1994)), which is further developed by Bliss and Panigirtzoglou (2002), the kernel estimation method (Aït-Sahalia and Lo (1998), Aït-Sahalia and Duarte (2003) and Birke and F. (2009)), the neural network method (Hutchinson et al. (1994), Garcia and Gençay (2000) and Andreou et al. (2008)), and the ϵ-arbitrage replicating portfolio method (Bandi and Bertsimas (2014)).

The cubic spline method seeks to relax the constant volatility assumption in the Black-Scholes paradigm. By allowing the implied volatilities to be a nonlinear function of option moneyness, Shimko (1994) proposes the method that successfully captures the volatility smile and the heavily left-skewed risk-neutral densities embedded in option prices. Modifying the methodology, Bliss and Panigirtzoglou (2002) argue that it is more accurate and computationally efficient to interpolate the implied volatilities in the volatility-option delta space, rather than in the volatility-moneyness space. They fit a piecewise cubic polynomials between option deltas and let the function be linear outside the deltas. In this way, the implied volatilities are no longer constant.

A notable shortcoming of the cubic spline method is that it does not ensure non-negativity of the risk-neutral probability density function (PDF). This problem is addressed in Monteiro et al. (2008), where non-negativity is guaranteed by replacing the quadratic programming (QP) approach with the semi-definite programming (SDP). However, it is also noted in Monteiro et al. (2008) that the QP approach is generally sufficient to recover an appropriate risk-neutral PDF both with simulated and market data. Hence we adopt the QP method for the spline method in this paper.

A more recent addition to this growing literature is Ma (2011). The paper develops a non-parametric option pricing model that focuses on approximating the implied risk-neutral MGF of the underlying asset returns using wavelets. The risk-neutral MGF has a number of advantages compared with the implied risk-neutral PDF although there is a one-to-one relationship between them. For example, the MGF is more tractable when jumps are present in the underlying price process; the MGF obtained from options is a continuous function; all the statistical moments of the underlying asset distributions can easily be obtained from the MGF; and out-of-sample options with different maturity dates can be directly estimated using the risk-neutral MGF.[1]

Ma (2011) also represents another effort in applying the wavelet methods, already a widely

[1] See Haven et al. (2009) for detailed properties of the risk-neutral MGF.

used tool in science and engineering, in the area of economics and finance.[2] As pointed out in Haven et al. (2009) and Haven et al. (2012), there are mainly three types of application of wavelet methods in finance and economics. First of all, wavelets are used for multi-scaling analysis. For example, Ramsey and Lampart (1998a,b) use the wavelet method to analyze the relationship between economic variables at different scales and suggest that the relationship changes over different time horizons. Gençay et al. (2001a,b, 2003, 2005) employ the wavelet multi-scaling approach to examine intra-day seasonalities, foreign exchange volatilities, and systematic risk. Weron (2009) implement the wavelet method to de-seasonalize electricity prices. More examples can be found in Zapart (2002), Connor and Rossiter (2005), Kim and In (2005), Mitra (2006), In and Kim (2006), Fernandez (2006), Lien and Shrestha (2007), Gallegati and Gallegati (2007), and Nikkinen et al. (2011).

Secondly, wavelets are used to de-noise raw data. Capobianco (1999, 2001) show that wavelets as a pre-processing de-noising tool are useful for improving volatility analysis. The superior de-noising ability of the wavelet is also recognized in Haven et al. (2012) which apply the wavelet method to de-noise option prices before estimating the implied risk-neutral PDF from the option prices. Their findings show that the wavelet de-noising process significantly improves the density estimation quality and the forecasting abilities of the estimated densities. Sun and Meinl (2012) substantiates the superior performance of wavelet-based local linear scaling approximation algorithm in denoising high-frequency financial data. Asgharian (2011) de-noise frequency variations in the first principal component of a business cycle with wavelets. Other research in this stream includes, among others, Averkamp and Houdré (2003) and Lada and Wilson (2006).

Finally, wavelets are utilized to estimate unknown parameters of a model. For example, Jensen (1999, 2000) and Ko and Vannucci (2006) adopt wavelets for calibrating parameters of long memory processes. Genon-Catalot and Laredo (1992) apply wavelets in estimating a diffusion function non-parametrically. Matache et al. (2005) adopt wavelet to price American-style options driven by Lévy processes. Manchaldore et al. (2010) implement the wavelet method to obtain intra-day volume. Ortiz-Gracia and Oosterlee (2016) use Shannon wavelet to price European options. Additional references include Bayraktar et al. (2004), Hong and Kao (2004), Dong and He (2007), Esteban-Bravo and Vidal-Sanz (2007), and Haven et al. (2009).

[2] For excellent reference for applications of the wavelet method in finance and economics, see Gençay et al. (2002). See also Percival and Walden (2000) for applications of the wavelet method in the time series analysis.

3 Model specifications

In this section, we outline the wavelet-based option pricing model of Ma (2011), the SVJ of Bakshi et al. (1997), and the spline method of Bliss and Panigirtzoglou (2002) in detail.

The wavelet-based option pricing model

The wavelet-based model by Ma (2011) is the latest theoretical contribution to the option pricing literature. Its motivation and relation to the option-implied risk-neutral MGF and the option-implied risk-neutral PDF are discussed in Haven et al. (2009) and Ma (2011).

With fairly general assumptions including i.i.d. distribution for asset returns, the wavelet-based option pricing model can be expressed as follows,

$$C_t(S_t, X, T) = Xe^{-r(T-t)}\mathcal{L}^{-1}\left(\frac{\Theta^{T-t}(s)}{s(s+1)}\right)\left(\ln\frac{X}{S_t}\right), \tag{1}$$

where $\mathcal{L}^{-1}$ denotes the bilateral inverse Laplace transform[3], C_t is the time-t price for a European call option written on asset whose price is S_t with strike price X and a future maturity date T. Interest rate r is assumed to be constant.

The core of this pricing model is $\frac{\Theta^{T-t}(s)}{s(s+1)}$, where s is a complex value whose real part is defined to be less than -1. The MGF $\Theta^{T-t}(s)$ of the logarithmic returns $\ln\frac{S_T}{S_t}$ captures the underlying asset dynamics and investors expectation embedded in option prices and needs to be approximated with wavelets.

To approximate the implied MGF with the wavelet method, a particular wavelet need to be chosen from a huge family of wavelet functions. Probably a large number of wavelet functions are able to approximate the MGF with reasonable accuracy as wavelets are well known for their ability in function approximation. The wavelet literature seems to agree that there is no best wavelet for a particular application. Therefore, we follow Mallat (1999) and try to choose a wavelet that can achieve a reasonable level of accuracy with minimum number of wavelet terms. The Franklin hat function performs well on this criterion. In addition, it has the properties of being symmetric, smooth, and piecewise continuous, and it closely emulates the probability density function of asset returns.

The risk-neutral MGF $\Theta(s)$ of the return per unit of time is therefore estimated using the

[3] See Appendix for the definition and properties of the bilateral inverse Laplace transform

Franklin hat function $h(t)$, which is defined as follows:

$$h(t) = \begin{cases} (1-|t|) & \text{if } -1 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}. \tag{2}$$

The Laplace transform of $h(t)$ is denoted as $m_h(s)$:

$$m_h(s) = (\frac{e^{s/2} - e^{-s/2}}{s})^2. \tag{3}$$

A set of generalized functions can be generated from the Franklin hat function $h(t)$:

$$h_{l,k}(t) = 2^{\frac{l}{2}} h(2^l t - k),\ \ l, k = 0, \pm 1, \pm 2, ... \tag{4}$$

The scaling parameter l determines the degree of dilation or contraction and the shifting parameter k controls the horizontal location of the function. Perform Laplace transform on $h_{l,k}(t)$, we obtain $m_{l,k}(s)$ as follows:

$$m_{l,k}(s) = 2^{-\frac{l}{2}} e^{-\frac{ks}{2^l}} m_h(\frac{s}{2^l}), l, k = 0, \pm 1, \pm 2, ... \tag{5}$$

The risk-neutral MGF of the return per unit of time $\Theta(s)$ can be expanded using the Laplace transform of the set of the generalized Franklin function as follows:

$$\Theta(s) = \sum_{l=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} a_{lk} m_{l,k}(s), \tag{6}$$

where $a_{l,k}$ is a set of unknown coefficients and needs to be estimated by minimizing the sum of squared error between the true option prices and the estimated option prices. We follow the procedure in Haven et al. (2009) and estimate the unknown coefficients as follows.

1. Truncate the coefficients a_{lk} setting $a_{lk} = 0$ for all $|l| > L$ and $|k| > K$, where L and K are positive integers.[4] Let $\theta_{L,K} \equiv \{a_{lk}\}_{l=L, |k| \leq K}$.

2. Given a collection of market data set for options at time-t,

$$\{S_t, X_i, C_{t,i}, T, r\}, \tag{7}$$

where i=1,2,..., N, we estimate the unknown coefficients $\theta_{L,K}$ by minimizing the sum of squared errors between the market option prices $C_{t,i}$ and the estimated prices $C^w_{t,i}$:

$$\min_{\theta_{L,K}} \Sigma_i (C_{t,i} - C^w_{t,i}(\theta_{L,K}, S_t, X_i, T, t, r))^2. \tag{8}$$

[4] According to Haven et al. (2009), K is chosen so that it equals the smallest integer greater than $0.7 * 2^l + 1$. This is because the log return typically lies in the range $[-0.7, 0.7]$. The value of K can be easily adapted according to specific situation.

3. Increase L by 1 at a time and repeat the above steps until a satisfactory result is achieved.

The above optimization process yields an estimate of the risk-neutral MGF, which is expressed as a series of the Laplace transform of the set of the generalized Franklin functions:

$$\hat{\Theta}(s) = \sum_{|l|=L} \sum_{|k| \leq K} \hat{a}_{lk} m_{lk}(s). \tag{9}$$

The stochastic volatility model with jumps (SVJ)

The volatility and jump risks have long been shown in the literature to be priced factors in the options market and should be included in option pricing models (Coval and Shumway (2001), Bates (1996), Huang and Wu (2004), Pan (2002), Santa-Clara and Yan (2010)). Bakshi et al. (1997) propose a parametric model that incorporates a mean-reverting stochastic volatility component that correlates with the underlying stock and a jump process that follows the Poisson distribution.

Assuming constant interest rate, the closed-form formula for European call options is as follows,

$$c(t, T) = S_t \Pi_1(t, T, S_t, V_t) - X \exp(-r(T-t)) \Pi_2(t, T, S_t, V_t), \tag{10}$$

where the risk-neutral probabilities Π_1 and Π_2

$$\Pi_j(t, T, S_t, V_t) = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} Re \left[\frac{\exp(-i\phi \ln(X) f_j(t, T, S_t, V_t; \phi)}{i\phi} \right] d\phi, j = 1, 2$$

are obtained by inverting the characteristic functions f_1 and f_2 whose exact specifications are given in the appendix of the paper by Bakshi et al. (1997) .

There are a number of parameters in the SVJ model. The jump process is described with the mean jump size μ_J, the standard deviation of jump size σ_J, and the jump frequency λ. The mean-reverting stochastic volatility process V_t are parameterized by the speed of adjustment κ_v, the long-term mean of the volatility θ_v/κ_v, and the variation coefficient of the diffusion volatility σ_v. The volatility process and the underlying asset dynamics are correlated with coefficient ρ. For index options and most equity options, ρ is negative corresponding to the negative skewness found in the risk-neutral distributions.

The spline method

The spline method is first proposed in Bliss and Panigirtzoglou (2002) and applied in Bliss and Panigirtzoglou (2004) and Liu et al. (2007). It is a very flexible nonparametric method that estimates risk-neutral densities from option prices, which can be used to price options and other derivatives written on the same underlying asset.

To apply this method, we first obtain market implied volatilities σ_i, where $i = 1, 2, ...N$, for all the N observed options. The implied volatilities are to be fitted by spline function of option deltas Δ_i[5] and a parameter vector θ. The spline function $\sigma(\Delta|\theta)$ is composed of linear pieces and cubic polynomials defined over intervals between N observations $\Delta_1 < \Delta_2 < \ldots < \Delta_N$. The cubic is defined over intervals between point Δ_i to Δ_{i+1} while the function is linear for $\Delta \leq \Delta_1$ and $\Delta \geq \Delta_N$.

The spline function is constrained by the requirement that its first two derivatives are continuous functions. It has N free parameters and there is a unique spline with the required property that passes through the points (Δ_i, σ_i). The parameter vector of the risk-neutral densities θ is obtained by minimizing a function that combines the accuracy and the smoothness of the fitted spline function. For N market implied volatilities σ_i and a set of weights w_i, we minimize the following function,

$$\eta \sum_{i=1}^{N} w_i(\hat{\sigma}_i(\Delta|\theta) - \sigma_i)^2 + (1-\eta) \int_{\Delta_1}^{\Delta_N} \sigma^{''}(\Delta|\theta)^2 d\Delta, \tag{11}$$

where w_i is the weight[6]; $\hat{\sigma}_i(\Delta|\theta)$ is the fitted volatility based on the estimated parameter θ; and $\sigma^{''}(\Delta|\theta)^2$ is the squared curvature of the regression function $\sigma(\Delta|\theta)$. The parameter η is between zero and one, and it controls the trade-off between accuracy and smoothness of the fitted probability density function. The closer the η is to 1, the more accurate and jagged the density is. Conversely, the closer the η is to 0, the smoother the density is. A straightforward solution to the above optimization problem is provided by Lange (1998).

A shortcoming of the cubic spline method is that it does not ensure non-negativity of the risk-neutral PDF. This issue is addressed in Monteiro et al. (2008), where non-negativity is ensured by replacing the Quadratic Programming (QP) approach with semidefinite programming (SDP). However, it is also noted in Monteiro et al. (2008) that the simpler QP approach is generally sufficient to recover an appropriate risk-neutral PDF with both simulated and market

[5] The Greek letter Δ stands for the partial derivative of the option price with respect to the underlying asset price. It measures the sensitivity of the option price to the underlying asset price.

[6] Following Bliss and Panigirtzoglou (2002, 2004), we use option vegas as the weights in equation (11).

data. Hence we adopt the QP method for the spline method in this paper.

4 Data and empirical analysis

In this paper, we use daily FTSE-100 index options (ESX) in the UK, the DAX-30 options (DAX) in the German market, and Hang Seng index standard options (HSI) in Hong Kong for our empirical investigation. The sample period is from January 2, 2009 to December 28, 2012. Option prices are calculated as the average of end-of-day bid price and ask price.[7] Interest rates and dividend yield are obtained from the Datastream. We take the LIBOR rate as the interest rate for the UK market, the Euro LIBOR rate for the German market, and the Hong Kong interbank rate for the Hong Kong market. Interest rates that we use have five different maturities from one month to 12 months. They are matched with the options data based on their maturities. We apply conventional exclusion rules to clean the raw options data. The rules include the following:

- Out-of-money calls and puts, which are more frequently traded than at-the-money or in-the-money options are used;
- Options with prices below unity are removed to avoid microstructure issues;
- Options with less than 14 days to maturity or more than 365 days to maturity are excluded;
- Options with less than 10 trading volume are excluded;
- Options with less than 9 different strike prices are removed as we need sufficient strikes for parameter estimation.

Options traded on the same day with the same expiry date are put into the same group. After applying these rules, we have 2465 groups of call options and 4133 groups of put options for ESX over 988 business days. For DAX options, we have 4401 groups of calls and 4563 groups of puts over 1004 business days. For the Heng Seng options, the corresponding numbers are 1998 for calls and 2298 for puts, respectively, over 976 business days. The number of strikes in each group varies across trading days and across markets. On average, the DAX has the largest number of strikes per day among the three markets.

We separate the data into call and put options to remove microstructure errors as the microstructure issues are likely to affect the estimation and forecast precision. Put prices are converted to call prices using the put-call parity. We take the bid-ask spread into account by

[7] The data are obtained from www.ivolatility.

Table 1. Summary statistics of call options

This table provides summary statistics of call options written on the UK FTSE-100 index (ESX), the German DAX-30 index (DAX) and the Hong Kong HangSeng index (HSI). The sample period is from January 2, 2009, to December 28, 2012. The proportion of options for each moneyness category is reported in parentheses.

	ESX	DAX	HSI
total number of options	29486	65867	30420
total number of trading days	988	1004	976
Panel A. Short-term options ($\leq$ 90 days)			
Avg. No. of options per day	12	15	16
range of moneyness	[6.84e-06,0.40]	[4.17e-06,0.65]	[1.33e-06,0.56]
0<moneyness≤0.1	18757(82.03%)	26539(75.75%)	18283(64.81%)
0.1<moneyness≤0.2	3878(16.96%)	7902(22.55%)	8986(31.85%)
0.2<moneyness≤0.3	228(1.00%)	564(1.61%)	838(2.97%)
moneyness>0.3	2(0.00%)	31(0.00%)	104(0.37%)
Panel B. Medium-term options (90 to 180 days)			
Avg. No. of options per day	11	18	12
range of moneyness	[1.86e-05, 0.38]	[4.17e-06, 0.47]	[8.48e-05,0.56]
0<moneyness≤0.1	2567(57.61%)	8768(55.39%)	992(48.02%)
0.1<moneyness≤0.2	1496(33.57%)	5450(34.43%)	761(36.83%)
0.2<moneyness≤0.3	356(7.99%)	1426(9.01%)	218(10.55%)
moneyness>0.3	37(0.83%)	187(1.18%)	95(4.6%)
Panel C. Long-term options (>180 days)			
Avg. No. of options per day	10	14	10
range of moneyness	[3.06e-04, 0.44]	[4.17e-06,0.65]	[0.00,0.49]
0<moneyness≤0.1	955(44.11%)	6526(43.51%)	45(31.47%)
0.1<moneyness≤0.2	764(35.29%)	4888(32.59%)	45(31.47%)
0.2<moneyness≤0.3	385(17.78%)	2485(16.57%)	34(23.78%)
moneyness>0.3	61(2.82%)	1101(7.34%)	19(13.29%)

using the following put-call parity relationship: $P_b = C_a + Ke^{-rT} - Se^{-yT}$, where y stands for the dividend yield, which is obtained from the Datastream. Index options are adjusted to reflect dividend payment by discounting the index level by annual dividend yields. We report the empirical results based on call options only due to space limit.[8]

[8] The results from put options are qualitatively the same and available upon request.

Descriptive statistics for the call options are summarized in Table 1. Following Bates (1996), we divide the options into three categories. Short-term options have 90 days or less before expiry; medium-term options are between 90 to 180 days to maturity; and long-term options are between 180 and 364 days to expiry. In this way we avoid weighting the longer-term options more heavily than shorter-term options in the parameter estimation later to be carried out (Huang and Wu (2004)). The majority of short term call options are near-the-money options, with moneyness[9] in the range of 0 and 0.1. This is expected as when it is close to expiration, the chance of large price changes is small and near-the-money calls will be more frequently traded than those with the same expiration with deeper moneyness. As time to maturity increases, options become progressively more out-of-the-money.

For the wavelet method, the scaling parameter L and the shift parameter K are chosen by the optimisation programme so that a satisfactory estimation result can be obtained. The optimisation programme usually stops when K reaches 4 or 6. For the spline method, we test both cases when $\eta = 0.1$ and when $\eta = 0.9$. We find that the average estimation and forecasting errors are generally slightly smaller when $\eta = 0.9$. However, in some cases with extreme outliers, the estimation and forecasting errors are much higher when $\eta = 0.9$ resulting in extremely larger pricing errors. Hence the results we report are based on $\eta = 0.1$.

[9] Moneyness is defined as log(K/S).

Table 2. In-sample performance of the option pricing models

This table summarizes of the Mean Squared Errors (MSE) for the alternative option pricing models, the wavelet-based model (Wavelet), the stochastic volatility model with jumps (SVJ), and the spline method (Spline) for call options written on the UK FTSE-100 index (ESX), the German DAX-30 index (DAX) and the Hong Kong HangSeng index (HSI). We report the mean, standard deviation (std), minimum (min), maximum (max) and the median of the MSE. The sample period is from January 2, 2009, to December 28, 2012.

	ESX			DAX			HSI		
	Panel A. Short-term options								
	Wavelet	SVJ	Spline	Wavelet	SVJ	Spline	Wavelet	SVJ	Spline
Mean	0.73	0.70	0.62	2.86	2.34	85.98	92.29	91.54	54.32
Std	9.13	8.72	8.20	68.75	50.17	2706.73	351.44	326.91	490.15
Min	1.61e-04	1.05e-03	5.30e-04	7.61e-05	3.52e-04	3.98e-04	3.13e-03	0.06	2.11e-03
Max	334.65	304.7	310.29	3077.2	2201.9	97512	6558	5040	17850
Median	0.028	0.034	0.04	0.020	0.03	0.023	4.98	6.21	2.65
	Panel B. Medium-term options								
	Wavelet	SVJ	Spline	Wavelet	SVJ	Spline	Wavelet	SVJ	Spline
Mean	0.48	1.08	6.37	12.78	10.27	422.27	8397.5	7039.3	2073.5
Std	2.22	8.96	28.46	156.71	122.90	6265.85	1.08e+05	8.94e+04	1.31e+04
Min	1.21e-03	5.19e-03	8.13e-04	1.74e-04	3.41e-03	3.21e-04	0.28	0.53	0.07
Max	22.47	126.34	244.42	2423.63	1930.13	1.61e+05	1.43e+06	1.19e+06	1.48e+05
Median	0.05	0.08	0.06	0.05	0.12	0.08	22.84	33.88	11.72
	Panel C. Long-term options								
	Wavelet	SVJ	Spline	Wavelet	SVJ	Spline	Wavelet	SVJ	Spline
Mean	0.67	1.74	74.13	4.90	8.69	820.24	32.12	48.97	1320.2
Std	3.20	12.33	413.01	68.07	93.64	2576.3	54.63	67.81	2599.6
Min	6.89e-04	4.57e-03	6.06e-03	1.28e-05	0.01	1.16e-03	0.24	3.92	27.86
Max	27.61	173.41	4335.7	1.81e+03	2.16e+03	2.77e+04	194.86	255.16	1.01e+4
Median	0.07	0.15	0.93	0.07	0.21	31.53	8.12	14.69	570

We use the entire sample period for in-sample fitting. Table 2 summarizes the mean squared errors (MSE), a popular measure of pricing accuracy, of the in-sample performance of the three models for the FTSE-100 index options, the DAX-30 index options, and the Heng Seng index options. The mean, standard deviation, minimum, maximum, and median of the MSE are reported across different maturities for the models. For the ESX options, the wavelet-based method generally produces smaller estimations errors than the other two models, except for the short-term options where the spline method produces the smallest average MSE. For the DAX options, the wavelet-based method is the best-performing model for long-term options. For the short-term and medium-term options, the wavelet-based model and the SVJ method exhibit similar performance. But the spline method is undoubtedly the poorest for the DAX options. In the Heng Seng index options market, the spline method tend to beat the other two methods for the short-term and medium-term options. But the wavelet-based method is still the best for the long-term options. To summarize, the wavelet-based method consistently performs better than the other two methods for long-term options. But for short-term and medium-term options, there is no clear winner. It is worth noting that in most cases the spline method produces much larger in-sample average pricing errors than the other two methods. However, when looking at the individual pricing errors on a daily basis, we notice that that the spline method does sometimes perform better than the wavelet-based method. However, there are incidents of extremely large pricing errors due to outliers. For example, in Figure 1 we plot the implied volatilities of the FTSE-100 index options on 27th January, 2009. There is an outlier of 0.5327 in the volatilities implied with 25 days to maturity when all other implied volatilities are about 0.3538 and less. The pricing error for the spline method is 64.42, over 100 times larger than the MSE the other two models. As the spline method fits the implied volatilities directly, it is difficult to handle the outliers leading to large valuation errors on average, and it is the method that fares poorest when outliers are present in the data.

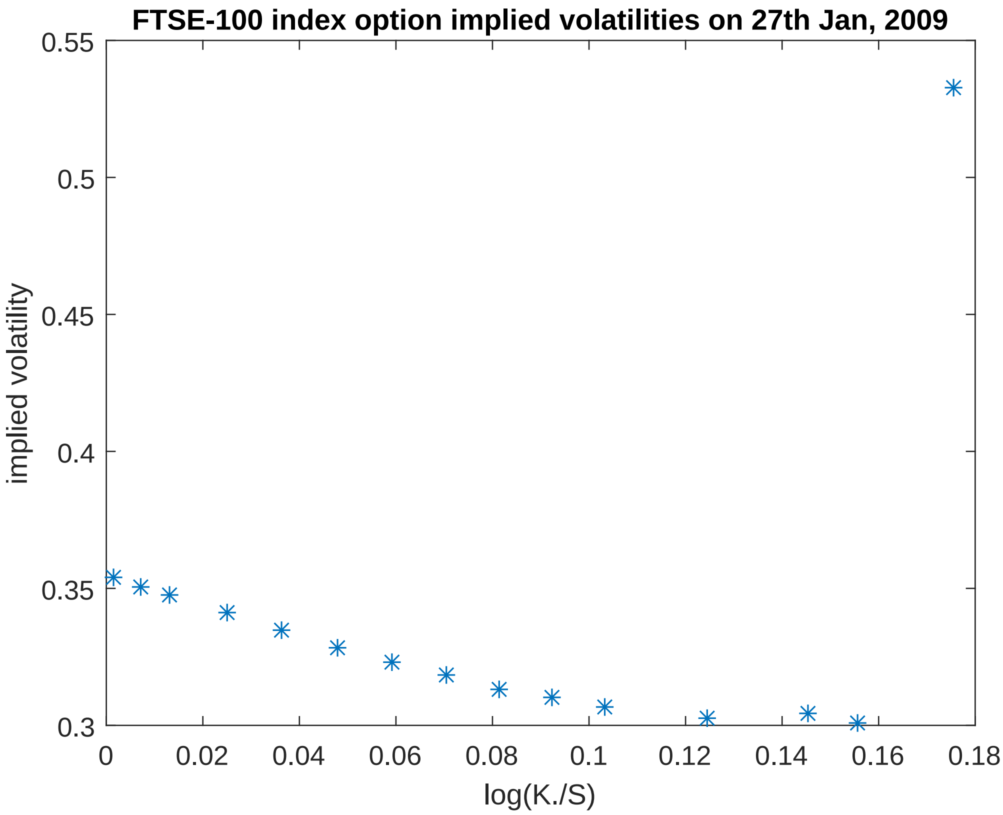
The wavelet-based and the SVJ models also produce larger pricing errors in this case due to the implied volatility outlier but the magnitude of the errors are much smaller than that of the spline method. On the day plotted in Figure 1, for example, the pricing error for the SVJ method is 34.94, about 50 times the average MSE. For the wavelet-based model the MSE is 6.82, about 9 times the average of the MSE. The wavelet-based model is the most robust estimation method among the three, particularly when dealing with noisy or polluted data. This is due to the fact that the wavelet method inherently has the ability of denoising, and it is able to de-noise option prices at the same time of estimating the risk-neutral MGF.

Table 3. Parameter estimates of the SVJ model

This table summarizes the in-sample parameter estimates of the SVJ model for call options written on the UK FTSE-100 index (ESX), the German DAX-30 index (DAX) and the Hong Kong HangSeng index (HSI). The sample period is from January 2, 2009, to December 28, 2012.

	λ	μ_J	σ_J	θ_v	κ_v	σ_v	V_t	ρ
Panel A. UK FTSE-100 index options								
Short-term options								
Mean	1.49	-0.08	0.08	0.26	5.72	0.66	0.04	-0.55
Std	1.41	0.08	0.09	0.23	3.12	0.27	0.04	0.26
Medium-term options								
Mean	1.84	-0.10	0.09	0.15	5.93	0.68	0.06	-0.60
Std	1.94	0.08	0.08	0.14	2.95	0.28	0.05	0.28
Long-term options								
Mean	1.34	-0.12	0.11	0.10	5.88	0.67	0.08	-0.61
Std	1.60	0.08	0.07	0.09	3.23	0.32	0.08	0.35
Panel B. German DAX-30 index options								
Short-term options								
Mean	1.85	-0.11	0.09	0.27	5.82	0.62	0.05	-0.54
Std	1.91	0.09	0.09	0.28	3.48	0.32	0.05	0.29
Medium-term options								
Mean	1.94	-0.13	0.10	0.12	5.66	0.60	0.07	-0.50
Std	2.49	0.11	0.09	0.13	3.42	0.38	0.07	0.44
Long-term options								
Mean	1.57	-0.19	0.13	0.07	4.74	0.55	0.10	-0.39
Std	2.20	0.14	0.11	0.09	3.81	0.41	0.09	0.63
Panel C. HongKong HSI index options								
Short-term options								
Mean	1.54	-0.09	0.10	0.26	5.52	0.61	0.04	-0.28
Std	1.68	0.10	0.10	0.27	3.46	0.29	0.04	0.46
Medium-term options								
Mean	1.79	-0.07	0.08	0.14	5.02	0.60	0.05	-0.18
Std	1.79	0.07	0.08	0.17	2.99	0.29	0.04	0.49
Long-term options								
Mean	1.79	-0.08	0.10	0.11	5.64	0.60	0.07	-0.22
Std	1.89	0.05	0.08	0.09	2.76	0.25	0.06	0.45

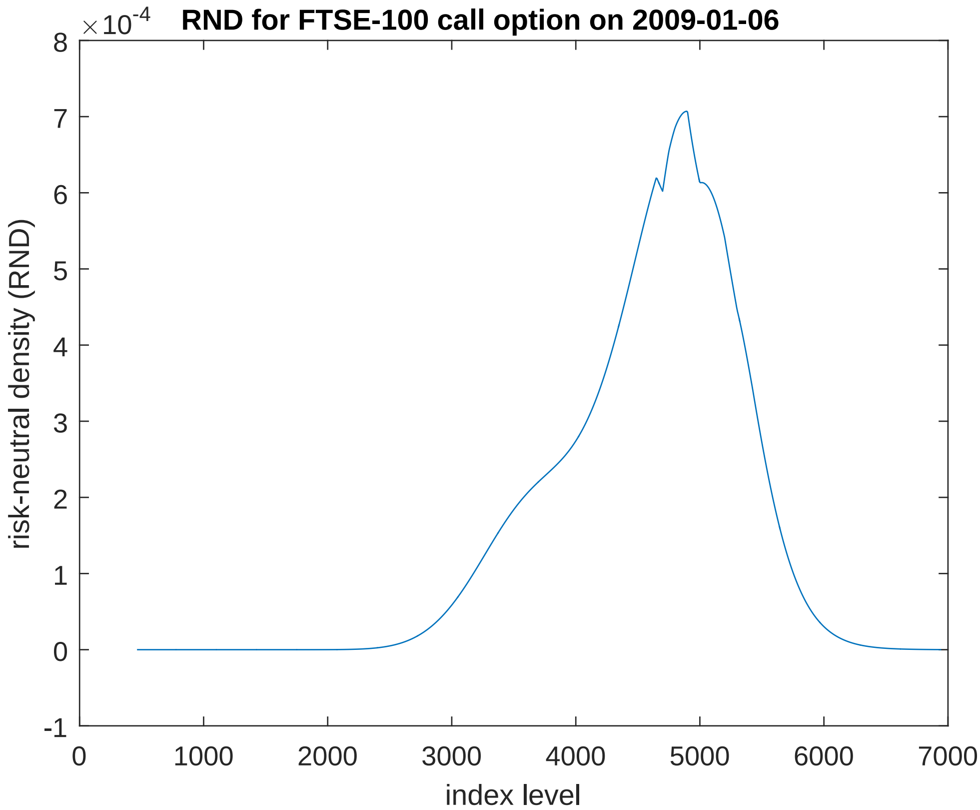
Figure 1. Option implied volatilities for the FTSE-100 index option on Jan 27, 2009



In Table 3 we report for the SVJ model the summary statistics of the parameters inferred from option prices. These parameters describe the underlying dynamics of the stochastic volatility and jump processes. The first three parameters relate to the jump process, including the implied jump frequency λ, mean jump size μ_J and jump volatility σ_J. For the FTSE-100 index, the implied jump size λ is between 1 and 2 across the three maturities, indicating that on average the FTSE-100 index tend to have jumps once or twice every year. The negative values of μ_J indicate that all the jumps are negative jumps. The jump size is generally larger over the long run than over the short run, and the jump volatility implied by long-term options also tends to be larger than short- and medium-term options. For example, the implied jump sizes is -0.12 for the long-term options and -0.08 for short-term options. The corresponding implied jump volatilities are 0.11 and 0.08, respectively. The highest jump frequency seems to be implied by medium-term options. The results for the DAX-30 index options and the Heng Seng index options tell a similar story.

Between the last two parameters, V_t is the diffusion process of the return variance when no jump occurs, and ρ is the correlation coefficient between the underlying asset price dynamics and the stochastic volatility process. For options written on the market-wide indices, ρ tends

Figure 2. The risk-neutral density for the FTSE-100 index options on January 6, 2009



to be negative reflecting the negative risk-neutral skewness found in empirical studies on the risk-neutral PDF inferred from index options (Liu et al. (2007) and Jackwerth (2000)). Our results are consistent with this. The correlation coefficient across the markets and maturities are all negative, indicating left-skewed distribution for the underlying index. See for example in Figure 2, where we plot the implied risk-neutral PDF estimated using the spline method for the FTSE-100 options traded on a randomly chosen day.

Out-of-sample forecasting is carried out for the entire sample period but the very first business day. For each business day, all options with a same maturity date are used to estimate model parameters, which are then used as inputs to forecast option prices for the following business day. The Out-of-sample forecasting performance is reported in Table 4. The main observation of this table is that the wavelet-based model consistently outperforms the SVJ and the spline method in the out-of-sample forecast. The SVJ method ranks the second and the spline method the third. For example, for the FTSE-100 index options, the average Mean Forecasting Squared Error MFSE for short-term options is 58.49, compared with 60.29 for the SVJ and 75.72 for the spline method. The difference between forecasting errors among the three models become larger with the time to maturity. These observations remain the same for the DAX-30 index options and the HSI index options. The only exception is the long-term options for the Heng Seng index whereby the SVJ model produces the smallest MFSE, followed by the wavelet-based model, and the spline model still has the largest forecasting error. For the rest of the results, the wavelet-based model consistently outperforms the SVJ and the spline model. The superior forecasting performance of the wavelet-based model is due to the inherent de-nosing and approximation ability of wavelets. Whilst estimating the implied risk-neutral MGF, wavelets automatically filter out the extremely noisy information and hence produce more accurate MGF estimation, which in turn provide cleaner information for forecasting.

We notice that some of the out-of-sample forecasting errors are large in magnitude but the corresponding percentage error is less so. This is consistent with the findings reported in Bakshi et al. (1997). For example, the average out-of-sample MSFE produced by the wavelet-based method for long-term DAX options is 210.89 but the corresponding percentage error is 6.84%. This suggests that a large proportion of the MSFE is contributed by options with higher prices. We also notice that both the in-sample valuation errors and the out-of-sample forecasting errors are larger for the Heng Seng index options than the other two types of index options. This could also due to the fact that the HSI index option prices are of a much larger magnitude than the DAX-30 index options and the FTSE-100 index options.

Table 4. Out-of-sample performance of the models for calls

This table summarizes of the out-of-sample Mean Forecasting Squared Errors (MFSE) for the alternative option pricing models, the wavelet-based model (Wavelet), the stochastic volatility model with jumps (SVJ), and the spline method (Spline) for call options written on the UK FTSE-100 index (ESX), the German DAX-30 index (DAX) and the Hong Kong HangSeng index (HSI). We report the mean, standard deviation (std), minimum (min), maximum (max) and the median of the MSE. The sample period is January 3, 2009 to December 28, 2012.

	ESX			DAX			HSI		
	Panel A. Short-term options								
	Wavelet	SVJ	Spline	Wavelet	SVJ	Spline	Wavelet	SVJ	Spline
mean	58.49	60.29	75.72	100.02	103.01	226.71	1240.9	1271.5	1516.9
std	132.16	135.83	170.09	232.08	236.36	2938.4	8473.8	8443.5	8950.2
min	0.01	0.03	0.02	0.03	0.02	0.03	0.86	0.65	0.95
max	1824.5	1903.1	3044.7	3794.4	4020.3	1.06e+05	2.10e+05	2.04e+05	2.14e+05
median	16.99	18.01	24.30	26.94	28.91	36.68	186.88	201.24	266.43
	Panel B. Medium-term options								
	Wavelet	SVJ	Spline	Wavelet	SVJ	Spline	Wavelet	SVJ	Spline
mean	92.65	99.11	123.69	155.21	171.83	602.81	1761.4	2645.8	3889.9
std	192.46	200.48	228.27	339.84	366.26	6044.4	6130.9	1.16e+04	1.50e+04
min	0.08	0.08	0.03	0.11	0.11	0.05	2.82	0.68	4.65
max	1686	1762.9	1890	4973.9	5101.6	1.51e+05	6.48e+04	1.30e+05	1.48e+05
median	28.96	31.06	38.05	44.44	46.93	63.91	391.07	490.21	582.83
	Panel C. Long-term options								
	Wavelet	SVJ	Spline	Wavelet	SVJ	Spline	Wavelet	SVJ	Spline
mean	113.17	135.95	191.53	210.89	238.19	1023.5	636.7	372	1168.3
std	180.45	225.8	327.4	517.43	565.89	2775.8	890.69	499.82	1535
min	0.10	0.17	0.12	0.04	0.11	0.03	11.68	16.98	37.45
max	1010.8	1616.4	1793.4	9158.3	9906.3	3.32e+05	2966.8	1627.3	4632.6
median	37.44	46.38	54.64	55.48	64.76	139.62	183.88	118.68	560.71

In addition to the pricing and forecasting errors, we have examined the calibration speed of the three models. The spline method is the fastest since there is a straightforward solution to the optimization problem provided by Lange (1998). The average calibration time for the spline method is 0.11 seconds. The wavelet-based and the SVJ models have similar number of unknown coefficients to estimate. The SVJ model has eight unknown parameters and the average time needed for optimisation is 18.04 seconds[10]. For the wavelet-based method, the number of unknown parameters depends on how many wavelets are adopted in the estimation. If the shift parameters are chosen to be $k = -4 : 4$ with unit increment, there are nine unknown wavelet coefficients to estimate, the wavelet-based model needs 9.47 seconds on average to produce the result. In most cases, $k = -6 : 6$ would suffice to produce a good estimation result. The calibration time increases to 11.47 seconds on average if the shift parameters are $k = -6 : 6$ with 13 unknown wavelet coefficients. Therefore, the wavelet-based model is computationally more efficient than the SVJ model. This perhaps is due to the fact that the SVJ model has two integrations to calculate for the two risk-neutral probabilities while the wavelet-based model needs one integration to calculate for the inverse Laplace transformation.

Overall, for the in-sample fit, the wavelet-based method consistently performs better than the other two methods for long-term options. For short- and medium-term options, there is no clear winner. In the out-of-sample forecasting exercise, the wavelet-based option pricing model outperforms the SVJ model and the spline method for all three options market and across all maturities. The wavelet-based model is also computationally more efficient than the SVJ model. This is the case despite a restrictive assumption of constant volatility and despite the fact that theoretically the SVJ model is designed to tackle stochastic volatility and jump risks and that the spline method focuses on fitting the implied volatility smile. This is evidence of the powerful approximation and de-noising ability of the wavelet methodology.

5 Conclusion

This paper empirically evaluates three option pricing models to compare their in-sample valuation and out-of-sample forecasting performance. The parametric SVJ model has the advantage that we can observe the estimated parameters and assess the economic intuition of the risk factors they represent. The spline method and the wavelet-based model, on the other hand, possess greater flexibility in capturing the underlying asset price dynamics and the return distributions.

[10] It is noted that Date and Islyaev (2015)) introduce a fast calibrating volatility model for option pricing based on higher moments. The calibration speed of the SVJ model could therefore be improved based on Date and Islyaev (2015)) method. In this paper, however, we focus on the original SVJ model.

The data we use in this paper include daily end-of-day bid and ask prices of the FTSE-100 index options, the DAX-30 index options, and the Hong Kong index options. The whole data sample is used for both in-sample and out-of-sample estimation. We show that in the in-sample valuation, the wavelet-based model performs as well as, if not better than, the SVJ and the spline model. For the out-of-sample forecasting exercise, the wavelet-based option pricing model significantly outperforms the SVJ model and the spline method. The wavelet method is also computationally more efficient than the SVJ model. This suggests that the wavelet method is effective in revealing the risk-neutral MGF and hence useful for option pricing and forecasting.

As suggested by Ma (1992, 2006, 2011), the statistical moments of the underlying asset distribution and the preference parameter of the utility function can be obtained from the risk-neutral MGF. Our future research is to reveal the option implied information on the risk preference and higher moments of the distribution using the wavelet-based option model. It would also be interesting to estimate the distribution function of the jump sizes of the underlying asset price with the wavelet method.

References

Aït-Sahalia, Y., Duarte, J., 2003. Nonparametric option pricing under shape restrictions. Journal of Econometrics 116, 9–47.

Aït-Sahalia, Y., Lo, A., 1998. Nonparametric estimation of state-price densities implicit in financial asset prices. Journal of Finance 52, 490–547.

Andreou, P. C., Charalambous, C., Martzoukos, S. H., 2008. Pricing and trading European options by combining artificial neural networks and parametric models with implied parameters. European Journal of Operational Research 185, 1415–1433.

Asgharian, H., 2011. A conditional asset-pricing model with the optimal orthogonal portfolio. Journal of Banking and Finance, 1027–1040.

Averkamp, R., Houdré, C., 2003. Wavelet thresholding for non-necessarily Gaussian noise: Idealism. The Annals of Statistics, 110–151.

Bakshi, G., Cao, C., Chen, Z., 1997. Empirical performance of alternative option pricing models. Journal of Finance 52, 2003–2049.

Bakshi, G., Kapadia, N., 2003. Delta-hedged gains and the negative market volatility risk premium. Review of Financial Studies 16, 527–566.

Bandi, C., Bertsimas, D., 2014. Robust option pricing. European Journal of Operational Research 239, 842–853.

Bates, D. S., 1996. Jumps and stochastic volatility: Exchange rate processes implicit in Deutsche mark options. Review of Financial Studies 9, 69–107.

Bates, D. S., 2000. Post-'87 crash fears in the S&P 500 futures option market. Journal of Econometrics 94, 181–238.

Bayraktar, E., Poor, V., Sircar, R. H., 2004. Estimating the fractal dimension of the S&P 500 index using wavelet analysist. International Journal of Theoretical and Applied Finance 5, 615–643.

Birke, M., F., P. K., 2009. Nonparametric option pricing with no-arbitrage constraints. Journal of Financial Econometrics 7, 53–76.

Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. Journal of Political Economy 81, 637–659.

Bliss, R. R., Panigirtzoglou, N., 2002. Testing the stability of implied probability density functions. Journal of Banking and Finance 26, 381–422.

Bliss, R. R., Panigirtzoglou, N., 2004. Recovering risk aversion from options. Journal of Finance 59, 407–446.

Bracewell, R., 1999. The Fourier Transform and Its Applications. McGraw-Hill, US.

Cai, N., Kou, S., 2011. Option pricing under a mixed-exponential jump diffusion model. Management Science 57, 2067 – 2081.

Capobianco, E., 1999. Statistical analysis of financial volatility by wavelet shrinkage. Methodology and Computing in Applied Probability 1, 423–443.

Capobianco, E., 2001. Wavelet transforms for the statistical analysis of returns generating stochastic processes. International Journal of Theoretical and Applied Finance 4, 511–534.

Connor, J., Rossiter, R., 2005. Wavelet transforms and commodity prices. Studies in Nonlinear Dynamics and Econometrics 9, 6.

Coval, J. D., Shumway, T., 2001. Expected option returns. Journal of Finance 56, 983–1010.

Date, P., Islyaev, S., 2015. A fast calibrating volatility model for option pricing. European Journal of Operational Research 243, 599–606.

Dong, M., He, D., 2007. Hidden semi-Markov models-based methodology for multi-sensor equipment health diagnosis and prognosis. European Journal of Operational Research 178, 858–878.

Esteban-Bravo, M., Vidal-Sanz, J. M., 2007. Computing continuous-time growth models with boundary conditions via wavelets. Journal of Economic Dynamics and Control 31, 3614–3643.

Fernandez, V. P., 2006. The CAPM and value at risk at different time-scales. International Review of Financial Analysis 15, 203–219.

Gallegati, M., Gallegati, M., 2007. Wavelet variance analysis of output in G-7 countries. Studies in Nonlinear Dynamics and Econometrics 11, Article 6.

Garcia, R., Gençay, R., 2000. Pricing and hedging derivative securities with neural networks and a homogeneity hint. Journal of Econometrics 94, 93–115.

Gârleanu, N., Pedersen, L. H., Poteshman, A. M., 2009. Demand-based option pricing. Review of Financial Studies 22, 4259–4299.

Gençay, R., Selçuk, F., Whitcher, B., 2001a. Differentiating intra-day seasonalities through wavelet multi-scaling. Physica 289, 543–556.

Gençay, R., Selçuk, F., Whitcher, B., 2001b. Scaling properties of foreign exchange volatility. Physica 289, 249–266.

Gençay, R., Selçuk, F., Whitcher, B., 2002. An introduction to wavelets and other filtering methods in finance and economics. Academic Press.

Gençay, R., Selçuk, F., Whitcher, B., 2003. Systematic risk and timescales. Quantitative Finance 3, 108–116.

Gençay, R., Selçuk, F., Whitcher, B., 2005. Multiscale systematic risk. Journal of International Money and Finance 24, 55–70.

Genon-Catalot, V., Laredo, C.and Picard, D., 1992. Nonparametric estimation of the diffusion coefficient by wavelet methods. Scandinavian Journal of Statistics 19, 317–335.

Haven, E., Liu, X., Ma, C., Shen, L., 2009. Revealing the implied risk-neutral MGF from options: The wavelet method. Journal of Economic Dynamics and Control 33, 692–709.

Haven, E., Liu, X., Shen, L., 2012. De-noising option prices with the wavelet method. European Journal of Operational Research 222, 104112.

Heston, S. L., 1993. A closed-form solution for options with stochastic volatility with applications to bond and currency options. Review of Financial Studies 6, 327–343.

Hong, Y., Kao, C., 2004. Wavelet based testing for serial correlation of unknown form in panel models. Econometrica 72, 1519–1563.

Huang, J., Wu, L., 2004. Specification analysis of option pricing models based on time-changed Levy processes. Journal of Finance 59, 1405–1439.

Hutchinson, J. M., Lo, A., Poggio, T., 1994. A nonparametric approach to the pricing and hedging of derivative securities via learning networks. Journal of Finance 49, 851–889.

In, F., Kim, S., 2006. The hedge ratio and the empirical relationship between the stock and futures markets: A new approach using wavelet analysis. Journal of Business 79, 799–820.

Jackwerth, J., 2000. Recovering risk aversion from option prices and realized returns. Review of Financial Studies 13, 433–451.

Jensen, M., 1999. Using wavelets to obtain a consistent ordinary least squares estimator of the long-memory parameter. Journal of Forecasting 18, 17–32.

Jensen, M., 2000. An alternative maximum likelihood estimator of long-memory processes using compactly supported wavelets. Journal of Economic Dynamics and Control 24, 361–387.

Kim, S., In, F., 2005. The relationship between stock returns and inflation: New evidence from wavelet analysis. Journal of Empirical Finance 12, 435–444.

Ko, K., Vannucci, M., 2006. Bayesian wavelet analysis of autoregressive fractionally integrated moving-average processes. Journal of Statistical Planning and Inference 10, 3415–3434.

Lada, E. K., Wilson, J. R., 2006. A wavelet-based spectral procedure for steady-state simulation analysis. European Journal of Operational Research 174, 1769–1801.

Lange, K., 1998. Numerical Analysis for Statisticians. Springer Verlag.

Lien, D., Shrestha, K., 2007. An empirical analysis of the relationship between hedge ratio and hedging horizon using wavelet analysis. Journal of Futures Markets 27, 127–150.

Liu, X., Shackleton, M. B., Taylor, S. J., Xu, X., 2007. Closed-form transformations from risk-neutral to real-world distributions. Journal of Banking and Finance 31, 1501–1520.

Ma, C., 1992. Two essays on equilibrium asset pricing and intertemporal recursive utility. Ph.D. thesis, Department of Economics,University of Toronto.

Ma, C., 2006. Intertemporal recursive utility and an equilibrium asset pricing model in the presence of Levy jumps. Journal of Mathematical Economics 42, 131–160.

Ma, C., 2011. Advanced Asset Pricing Theory. Imperial College Press.

Mallat, S., 1999. A Wavelet Tour of Signal Process. Academic Press.

Manchaldore, J., Palit, I., Soloviev, O., 2010. Wavelet decomposition for intra-day volume dynamics. Quantitative Finance 10, 917–930.

Matache, A.-M., Nitsche, P.-A., Schwab, C., 2005. Wavelet Galerkin pricing of American options on Levy driven assets. Quantitative Finance 5, 403–424.

Mitra, S., 2006. A wavelet filtering based analysis of macroeconomic indicators: The Indian evidence. Applied Mathematics and Computation 175, 1055–1079.

Monteiro, A., Tütüncü, R., Vicente, L., 2008. Recovering risk-neutral probability density functions from options prices using cubic splines and ensuring nonnegativity. European Journal of Operational Research 187, 525–542.

Nikkinen, J., pynnönen, S., Ranta, M., Vähämaa, S., 2011. Cross-dynamics of exchange rate expectations: A wavelet analysis. International Journal of Finance & Economics 16, 205–217.

Ortiz-Gracia, L., Oosterlee, C. W., 2016. A highly efficient Shannon wavelet inverse Fourier technique for pricing European options. SIAM Journal on Scientific Computing 38, B118–B143.

Pan, J., 2002. The jump-risk premia implicit in options: Evidence from an integrated time-series study. Journal of Financial Economics 63, 3–50.

Percival, D. B., Walden, A. T., 2000. Wavelet methods for time series analysis. Cambridge University Press.

Ramsey, J. B., Lampart, C., 1998a. The decomposition of economic relationship by time scale using wavelets: Expenditure and income. Studies in Nonlinear Dynamics and Econometrics 3, 23–42.

Ramsey, J. B., Lampart, C., 1998b. Decomposition of economic relationship by time scale using wavelets: Money and income. Macroeconomic Dynamics 3, 23–42.

Rubinstein, M., 1994. Implied binomial trees. Journal of Finance 49, 771–818.

Santa-Clara, P., Yan, S., 2010. Crashes, volatility, and the equity premium: Lesson from the S&P 500 options. Review of Economics and Statistics 92, 435–451.

Shimko, D., 1994. Bounds of probability. Risk 6, 33–37.

Sun, E. W., Meinl, T., 2012. A new wavelet-based denoising algorithm for high-frequency financial data mining. European Journal of Operational Research 217, 589–599.

Weron, R., 2009. Heavy tails and regime-switching in electricity prices. Mathematical Methods of Operations Research 69, 457–473.

Wong, H., Lo, Y., 2009. Option pricing with mean reversion and stochastic volatility. European Journal of Operational Research 197, 179–187.

Zapart, C., 2002. Stochastic volatility options pricing with wavelets and artificial neural networks. Quantitative Finance 2, 487–495.

Appendix: Bilateral Laplace transform

Following Haven et al. (2009), the bilateral Laplace transform is defined as follows. For a real-valued function $f(t)$ which is piecewise continuous on $[-\infty, \infty)$, its bilateral Laplace transformation is a complex valued function given by

$$\mathcal{L}\{f(t)\}(s) = F(s) = \int_{-\infty}^{\infty} f(t)e^{-st}dt; \tag{12}$$

where s is a complex value and $\mathcal{L}$ denotes the Laplace transform operator. The inverse Laplace transform, denoted by $\mathcal{L}^{-1}\{F(s)\}(t)$, can be written as:

$$\mathcal{L}^{-1}\{F(s)\}(t) = f(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} F(s)e^{st}ds; \tag{13}$$

where c is a specific real number.

Let $F(s)$ denote $\mathcal{L}\{f(x)\}(s)$ and $G(s)$ denote $\mathcal{L}\{g(x)\}(s)$, we have the properties of the Laplace transform summarized as following:

1. Linearity

$$\mathcal{L}\{af(x) + bg(x)\}(s) = aF(s) + bG(s); \tag{14}$$

$$\mathcal{L}^{-1}\{aF(s) + bG(s)\}(x) = af(x) + bg(x). \tag{15}$$

2. Frequency shifting

$$\mathcal{L}\{e^{-lx}f(x)\}(s) = F(s+l), \forall l \in R; \tag{16}$$

$$\mathcal{L}^{-1}\{F(s+l)\}(x) = e^{-lt}f(x), \forall l \in R. \tag{17}$$

3. Time shifting

$$\mathcal{L}\{f(x-x_0)\}(s) = e^{-x_0 s}F(s), \forall x_0 \in R; \tag{18}$$

$$\mathcal{L}^{-1}\{e^{-x_0 s}F(s)\}(x) = f(x-x_0), \forall x_0 \in R. \tag{19}$$

4. Convolution

$$\mathcal{L}\{f(x) * g(x)\} = F(s)G(s); \tag{20}$$

$$\mathcal{L}^{-1}\{F(s)G(s)\}(x) = f(x) * g(x). \tag{21}$$

where '*' indicates the convolution operator on f and g. This operator can be defined as (Bracewell (1999, page 25)),

$$f * g \equiv \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau = \int_{-\infty}^{\infty} g(\tau)f(t-\tau)d\tau. \tag{22}$$