How to Increase Energy Efficiency in Cognitive Radio Networks

Mohammad Robat Mili, Leila Musavian, Khairi Ashour Hamdi and Farokh Marvasti

***Abstract*—In this paper, we investigate the achievable energy efficiency of cognitive radio networks where two main modes are of interest namely; spectrum sharing (known as underlay paradigm) and spectrum sensing (or interweave paradigm). In order to improve the energy efficiency, we formulate a new multi-objective optimization problem that jointly maximizes the ergodic capacity and minimizes the average transmission power of the secondary user network while limiting the average interference power imposed on the primary user receiver. The multiobjective optimization will be solved by first transferring it into a single objective problem (SOP), namely, a power minimization problem, by using ε-constraint method. The formulated SOP will be solved using two different methods. Specifically, the minimum power allocation at the secondary transmitter in a spectrum sharing fading environment are obtained using iterative search based solution and augmented Lagrangian approach for single and multiple secondary links, respectively. The significance of having extra side information and also imperfect side information of cross channels at the secondary transmitter are investigated. The minimum power allocations under perfect and imperfect sensing schemes in interweave cognitive radio networks are also found. Our numerical results provide guidelines for the design of future cognitive radio networks.**

***Index Terms*—Energy Efficiency, Spectrum Sharing, Spectrum Sensing, Multiobjective Optimization, Power Allocation.**

I. Introduction

In today's wireless systems, there is an increased demand for the radio spectrum access due to many new wireless networks such as wireless sensor networks, wireless local area networks, Bluetooth, and so on. The frequency allocation chart of the Federal Communications Commission (FCC) shows that a severe under-utilization of the licensed spectrum has been observed. Cognitive radio networks have been proposed as an efficient method to address the problem by opportunistically accessing the spectrum across different networks of users [1]-[4]. A cognitive radio network consists of primary users (PU) and secondary users (SU). As described in [5], a PU has the legacy priority access to the spectrum while an SU uses the spectrum when the performance of the PUs is not harmfully affected by the SU network operation. The utilization of the spectrum in traditional wireless networks is improved by cognitive radio technology such that it increases the number of application and services in wireless systems. A cognitive radio network recognizes its communication environment and changes the parameters of its communication scheme to increase the quality of service (QoS) of SUs [6].

Cognitive radio technologies can be divided into two main modes, namely; spectrum sensing and spectrum sharing. In the latter, the SU is required to detect the spectrum opportunities and then transmit while the PU is absent [7]. In the former, SUs employ spectrum sharing technique while avoiding considerable interference to the primary receivers. In such systems, a medium access control layer protocol with ability to fairly allocate the spectrum between secondary users is required [8].

In underlay cognitive radio networks, several approaches have been considered in the literature to protect the primary user performance from the operation of the co-existing secondary user networks. These include limiting the average or peak values of the interference power imposed on the primary receiver as a result of the secondary user network transmission, or protecting the primary user rate from falling below a certain threshold at all times (or for a certain percentage of the time). The idea of limiting the imposed interference power was originally motivated by the concept of the interference power temperature [5] introduced by the FCC in 2003.While considering the approach for limiting the capacity-fall of the primary user may seem more protective for the primary user performance, but there will be a challenge of not having the information of the primary user channel (that is the channel between the primary transmitter and the primary receiver) at the secondary user network. In such cases, by considering a certain capacity-outage allowance in the primary user network, the constraint on the primary user capacity-fall can be formulated as an interference power constraint, [9], [10]. Henceforth, limiting the interference induced from secondary transmitter on the primary receiver, given that the interference threshold is chosen according to the capacity-outage constraint, could lead to protecting the capacity of primary user not to fall below a certain threshold with a close-to-one probability.

In recent years, the concept of energy efficiency has been much discussed in several wireless systems, e.g., in wireless sensor networks. In such networks, sensor nodes are typically powered by small limited-life batteries which are very expensive and difficult to be replaced or recharged. Therefore, wireless nodes must be operated without battery replacement for many years. On the other hand, recently there has been a focus on improving the energy efficiency in general wireless communications due to environmental concerns. It is known that the Information Communication Technologies (ICT) already represents about 2% of the total world CO_2 gas

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emission. Also, it is expected to increase from 0.53 billion ton (Gt) CO_2 in 2002 to 1.43 Gt in 2020. Mobile communication consume 15-20 percent of entire ICT energy foot print [11], [12].

As green radio (GR) becomes an inevitable trend, energy efficiency is becoming increasingly important for wireless and especially mobile systems due to the slow advances in battery technology. Energy efficiency is investigated in many recent publication. For example, the authors in [13]-[16] maximized the energy efficiency for different emphases and application scenarios. Spectrum access strategies in cognitive radio, aiming at optimizing the average energy efficiency were addressed in [13]. For better mathematical tractability, continuous and convex relaxation to modify the problems has been utilized in [13]. In [14], the maximization of energy efficiency subject to different constraints is formulated and also the fundamental trade-off between energy efficiency and spectral efficiency in downlink communications is addressed. In [16], energy efficient power adaptation in sensing-based spectrum sharing cognitive radio networks has been investigated by focusing on the maximization of the energy efficiency subject to peak/average transmission power and average interference constraints. In addition, the authors in [17] discussed the challenges of designing energy-efficient cognitive radio networks by focusing on the fundamental trade-offs which are identified as QoS, fairness, primary user interference, network architecture, and security. Energy efficient non-cooperative cognitive radio networks from the spectrum sensing and sharing perspectives are overviewed in [18].

In contrast to these works, in this paper, we minimize the power which brings energy efficiency enhancement to the cognitive radio network. In high transmission powers, the energy efficiency can be improved by decreasing the transmission power in wireless systems. Power minimization in cognitive radios has been studied in [19] where a group of cognitive radios access the resources of a primary systems. The authors in [19] developed an optimum resource allocation strategy, using cooperative game theory which guarantees the QoS in primary network and allocates an achievable rate for secondary networks. Our contribution ties to the power control schemes in cognitive radio networks by using the optimization problem. The maximization of ergodic capacity of a secondary link in dynamic spectrum sharing approach while ignoring the interference from primary transmitter to secondary receiver has been investigated in [20]-[22].

Energy efficiency as a measurement for the performance of wireless communication systems is considered as the main aim of this paper. Hence, the major contributions in this paper are as follows:

- In order to increase the energy efficiency in cognitive radio networks, this paper first provides a multiobjective optimization problem (MOP) that jointly maximizes the ergodic capacity and minimizes the average transmission power. In order to solve the MOP, we convert it into a single objective optimization (SOP) by using the ε-constraint method. Specifically, the MOP is converted into a SU power minimization problem with joint constraint on the ergodic capacity of the secondary link and the interference power on the primary receiver. An iterative algorithm based on the sub-gradient method is proposed to obtain the minimum transmit power in such optimization problem.
- We then extend the model to a general model where multiple secondary links share the spectrum with the existing multiple primary links. We evaluate the minimum transmission power while considering the impact of other secondary links and primary transmitters. Due to nonconvexity of optimization problem in the general model, augmented Lagrange method for the inequality constraints will be offered.
- Further, we investigate the significance of providing extra channel side information (CSI) at the secondary transmitters under spectrum sharing for single and multiple links. Since, having the extra CSIs at the secondary transmitter can be very difficult, we reduce these extra CSIs and derive new expressions for corresponding transmission power at the secondary transmitter. The imperfect side information of cross channels for single secondary link is also investigated.
- We finally provide a power minimization allocation approach in spectrum sensing cognitive radio systems with perfect and imperfect sensing.

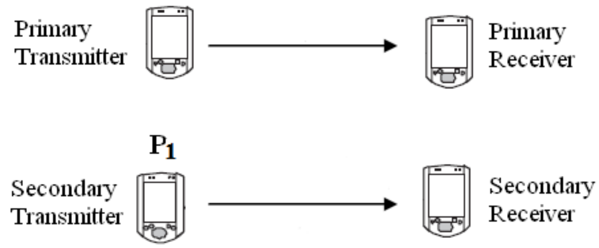

Fig. 1. Cognitive radio with one primary and one secondary link

The rest of this paper is organized as follows. Section II describes the system model of a cognitive radio. Sections III and IV derive expressions for minimum transmission power under spectrum sharing and sensing, respectively. In these two sections, new closed form expressions for evaluating the minimum power under single secondary link are derived. Numerical results are presented in section V and finally next section concludes the paper.

II. System Model

In this paper, we consider a cognitive radio with the primary and secondary links as shown in Fig. 1. A flat fading channel with perfect channel side information (CSI) between the secondary transmitter and the secondary receiver with instantaneous channel power gain g_1 and the additive white Gaussian noise (AWGN) n_1 has been assumed. Furthermore, the additive noise n_1 is an independent random variable with the distribution $CN(0; N_0)$ (circularly symmetric complex Gaussian variable with mean 0 and variance N_0). We consider ρ as the constant power for the primary transmitter and also the instantaneous power at the secondary transmitter can be written as P_1.

III. Minimum Power Under Spectrum Sharing

Under spectrum sharing, we assume that two links, primary and secondary links, use the same frequency band, therefore, the secondary transmitter imposes interference on the primary receiver, and also the primary transmitter affects the secondary receiver. It is assumed that the interference channel with full CSI between the primary transmitter and the secondary receiver is a flat fading channel characterized by instantaneous fading channel power gain h_1. The instantaneous received signal-to-interference-plus-noise ratio (SINR) at the secondary receiver becomes

$$\text{SINR} = \frac{P_1 g_1}{N_0 + \rho h_1} \tag{1}$$

where ρh_1 represents the interference caused as a result of the PU transmission, measured at the secondary receiver.

In the rest of this section, the average minimum power for single and multiple secondary links over fading channels to maximize the energy efficiency of the SU links will be obtained.

A. Minimum Power for Single Secondary Link

Energy efficiency (EE) as a measurement for the performance of wireless communication systems over fading channels can be defined by

$$\text{Energy Efficiency} = \frac{\mathbb{E}\left[\ln\left(1+\text{SINR}\right)\right]}{\mathbb{E}[P_1] + p_{\text{c}}}, \tag{2}$$

where p_{c} is a constant value of the circuit power consumption and $\mathbb{E}\left[\cdot\right]$ is the expectation operator. To increase the energy efficiency given in (2), we formulate a MOP that jointly maximizes the ergodic capacity of the secondary link (the numerator of (2)) and minimizes the average power of secondary transmitter (the denominator of (2)) while satisfying a target on the average received power constraint at the primary receiver, as following

$$\min_{P_1} \quad \mathbb{E}[P_1\left(g_1, h_1, f_1\right)] \tag{3a}$$

$$\max_{P_1} \quad \mathbb{E}\left[\ln\left(1+\text{SINR}\right)\right] \tag{3b}$$

$$\text{s.t.} \quad \mathbb{E}[P_1\left(g_1, h_1, f_1\right) f_1] \leq Q_{\text{average}} \tag{3c}$$

where Q_{average} is the maximum average interference on the primary receiver and f_1 is the instantaneous channel power gain between the secondary transmitter and the primary receiver.

The ϵ-constraint method is a well-known technique to solve MOPs. In this approach, one of the objectives is minimized while the others are used as constraints bound by some allowable levels ϵ [23]. Hence, the above MOP can be changed into an SOP according to

$$\min_{P_1} \quad \mathbb{E}[P_1\left(g_1, h_1, f_1\right)] \tag{4a}$$

$$\text{s.t.} \quad \mathbb{E}\left[\ln\left(1+\text{SINR}\right)\right] \geq R_{\text{average}} \tag{4b}$$

$$\text{s.t.} \quad \mathbb{E}[P_1\left(g_1, h_1, f_1\right) f_1] \leq Q_{\text{average}} \tag{4c}$$

The lower limit of the capacity constraint in (4), referred to by R_{average}, is limited in between two extreme values. R_{average} is limited to a maximum value, referred to in the revised manuscript by R_{max}, which is a function of the maximum transmit power of the secondary user. R_{average} from the lower side is to a minimum value, referred to by R_{min} in the revised manuscript, which reflects a minimum rate requirement of the secondary user as a quality-of-service (QoS) metric. The chosen value for R_{average} affects the multiple objective optimization optimal point. By swiping the R_{average} from its minimum to its maximum values, the Pareto Optimal Points of the MOP can be obtained. It is worth noting that higher values of R_{average} gives more preference to maximizing the rate of the secondary user link, whereas lower values of R_{average} gives more preference to minimizing the power of the secondary user, in the multiple objective optimization problem considered in (3).

We note that if the average power is used as a constraint bound, the obtained single optimization problem will be equivalent to maximizing the ergodic capacity of the SU link and, henceforth, the existing water-filling power allocation approach [20] gives the solution. Later, in the numerical results section, we will compare the energy efficiency achieved by the average power minimization problem (4) and the ergodic capacity maximization problem.

The above optimization problem is equivalent to solving the following Lagrangian approach [24]

$$\begin{aligned} L(P_1, \lambda) &= \mathbb{E}[P_1\left(g_1, h_1, f_1\right)] \\ &\quad - \lambda_s \left(\mathbb{E}\left[\ln\left(1 + \frac{P_1\left(g_1, h_1, f_1\right) g_1}{N_0 + \rho h_1}\right)\right] - R_{\text{average}}\right) \\ &\quad + \lambda_p \left(\mathbb{E}[P_1\left(g_1, h_1, f_1\right) f_1] - Q_{\text{average}}\right), \end{aligned} \tag{5}$$

where λ_s and λ_p are the nonnegative dual variables corresponding to the constraints (4b) and (4c). Taking the derivative of Lagrangian in (5) with respect to P_1 and equalling it to zero gives

$$\frac{\partial L(P_1, \lambda)}{\partial P_1} = \mathbb{E}\left[1 - \frac{\lambda_s g_1}{N_0 + \rho h_1 + P_1 g_1} + \lambda_p f_1\right] = 0, \tag{6}$$

which results into

$$P_1\left(g_1, h_1, f_1\right) = \frac{\lambda_s}{1 + \lambda_p f_1} - \frac{\left(N_0 + \rho h_1\right)}{g_1}, \tag{7}$$

where P_1 is a function of direct channel gains g_1 and f_1, and also cross channel gain h_1. In (7), by considering the non-negativity of the power, i.e., constraint $P_1 \geq 0$, we get

$$g_1 \geq \frac{\left(N_0 + \rho h_1\right)}{\lambda_s}\left(1 + \lambda_p f_1\right). \tag{8}$$

We note that when (8) hold, the optimum power allocation strategy follows (7). Otherwise, the transmission power should be limited to zero. In order to account for the minimum transmission power, the following iteration search based on the sub-gradient method can be implemented while substituting the obtained power (7) in constraints (4b) and (4c) as equation (9) and (10) (shown at the beginning of next page) where $[.]^+$ denotes the projection onto the nonnegative area, α is a positive gradient search stepsize, and λ_s^n and λ_p^n are the

$$\lambda_s^{n+1} = \left[\lambda_s^n - \alpha\left(\mathbb{E}\left[\ln\left(\frac{g_1}{N_0+\rho h_1}\frac{\lambda_s^n}{1+\lambda_p^n f_1}\right)\right] - R_{\text{average}}\right)\right]^+ \tag{9}$$

$$\lambda_p^{n+1} = \left[\lambda_p^n - \alpha\left(\mathbb{E}\left[\left(\frac{\lambda_s^n}{1+\lambda_p^n f_1} - \frac{(N_0+\rho h_1)}{g_1}\right) f_1\right] - Q_{\text{average}}\right)\right]^+ \tag{10}$$

$$\begin{aligned}
\mathbb{E}[P_1(g_1,h_1,f_1)] &= \mathbb{E}\left[\frac{\lambda_s^{n+1}}{1+\lambda_p^{n+1}f_1} - \frac{(N_0+\rho h_1)}{g_1}\right] \\
&= \int_0^\infty \int_0^\infty \int_{\frac{(N_0+\rho h_1)}{\lambda_s^{n+1}}\left(1+\lambda_p^{n+1}f_1\right)}^\infty \left(\frac{\lambda_s^{n+1}}{1+\lambda_p^{n+1}f_1} - \frac{N_0+\rho h_1}{g_1}\right) \times e^{-f_1-h_1-g_1} dg_1 dh_1 df_1 \\
&= (N_0+\rho)\,\mathrm{Ei}\left(-\frac{N_0}{\lambda_s^{n+1}}\right) - e^{-\frac{N_0}{\rho}}\rho\,\mathrm{Ei}\left(-\frac{N_0\left(\rho+\lambda_s^{n+1}\right)}{\rho\lambda_s^{n+1}}\right) - \left(N_0+\rho+\frac{\lambda_s^{n+1}}{\lambda_p^{n+1}}\right)e^{\frac{1}{\lambda_p^{n+1}}} \\
&\quad \times \mathrm{Ei}\left(-\frac{N_0\lambda_p^{n+1}+\lambda_s^{n+1}}{\lambda_p^{n+1}\lambda_s^{n+1}}\right) + e^{\frac{N_0\lambda_p^{n+1}+\lambda_s^{n+1}}{\rho\lambda_p^{n+1}}}\rho\,\mathrm{Ei}\left(-\frac{\left(\rho+\lambda_s^{n+1}\right)\left(N_0\lambda_p^{n+1}+\lambda_s^{n+1}\right)}{\rho\lambda_s^{n+1}\lambda_p^{n+1}}\right)
\end{aligned} \tag{11}$$

value of λ_s and λ_p at stage n, respectively. In theory, when the stepsize α is small enough this approach converges to a definite number [25]. Then, we can get the minimum average power of secondary transmitter under Rayleigh fading by (7) and using [26, eqs. (3.351.5), (3.352.4), (6.455.1), (9.121.6), (5.231.1), (9.131.1) and (4.337.1)] as equation (11) (shown at the next page) where $\mathrm{Ei}(.)$ is the exponential integral function defined as $\mathrm{Ei}(x) = \int_{-\infty}^{x}\frac{e^t}{t}dt$. The steps for the power allocation algorithm is shown in Table 1, in which λ_s^{n+1} and λ_p^{n+1} are updated to minimizing the power.

Table. 1: Power allocation algorithm

Algorithm
1) Initialization $n=0$, α, λ_s^n and λ_p^n
2) While not converged do
3) update λ_s^{n+1} by (9)
4) update λ_p^{n+1} by (10)
5) calculate the minimum power by (11)
6) n=n+1
7) End While

1) The Effect of Reducing CSI h_1 at the Secondary Transmitter: Since having indirect channel gain h_1 at the secondary transmitter is a very difficult task to achieve and can cause major overhead on the system, we reduce the CSI h_1 from secondary transmitter, which in this case, P_1 becomes a function of g_1 and f_1. Hence, we modify the optimization problem (4) as following

$$\min_{P_1} \quad \mathbb{E}[P_1(g_1,f_1)] \tag{12a}$$

$$\text{s.t.} \quad \mathbb{E}\left[\ln\left(1+\frac{P_1(g_1,f_1)\,g_1}{N_0+\mathbb{E}[\rho h_1]}\right)\right] \geq R_{\text{average}} \tag{12b}$$

$$\text{s.t.} \quad \mathbb{E}[P_1(g_1,f_1)\,f_1] \leq Q_{\text{average}} \tag{12c}$$

Following the same procedure as found (7), we obtain the following power allocation of secondary transmitter as

$$P_1(g_1,f_1) = \frac{\mu_s}{1+\mu_p f_1} - \frac{N_0+\mathbb{E}[\rho h_1]}{g_1} \tag{13}$$

where μ_s and μ_p are the nonnegative dual variables. It is worthy to note that in (13) the transmission power is only a function of g_1 and f_1 and is independent of h_1 so only average value of interference from primary transmitter is available at the secondary transmitter. The similar iteration search based on the sub-gradient method is implemented as equation (14) and (15) (shown at the beginning of next page) where s is a positive gradient search stepsize, and μ_s^n and μ_p^n are the value of μ_s and μ_p at stage n, respectively. Hence, the minimum average power in this case under Rayleigh fading can be straightforwardly derived by using the identity [26, eqs. (3.351.5), (3.352.4), (6.455.1), (9.121.6) and (5.231.1)] as equation (16) (shown at the next page).

2) Imperfect CSI: Here, the perfect knowledge of the channel between secondary transmitter and the secondary receiver is assumed at secondary transmitter but only partial channel knowledge of the cross channel gains are available at the secondary transmitter.

With partial CSI of the cross channel gains at the secondary transmitter, we exploit the following model for the complex channel estimate of h and f at the secondary transmitter, i.e.

$$h = \sqrt{\sigma_{h_1}^2}\hat{h} + \sqrt{1-\sigma_{h_1}^2}\tilde{h} \tag{17}$$

$$f = \sqrt{\sigma_{f_1}^2}\hat{f} + \sqrt{1-\sigma_{f_1}^2}\tilde{f} \tag{18}$$

where $\tilde{h}$ and $\tilde{f}$ are the channel estimation errors which are complex Gaussian random variable with zero means and unit variances that are uncorrelated with $\hat{h}$ and $\hat{f}$ that denote the channel estimations, respectively. Also, σ_{h_1} and σ_{f_1} as correlation coefficients that are constant values between 0 and 1 give the average quality of channel estimate over all channel states

$$\mu_s^{n+1} = \left[\mu_s^n - s\left(\mathbb{E}\left[\ln\left(\frac{g_1}{N_0 + \mathbb{E}\left[\rho h_1\right]}\frac{\mu_s^n}{1+\mu_p^n f_1}\right)\right] - R_{\text{average}}\right)\right]^+ \tag{14}$$

$$\mu_p^{n+1} = \left[\mu_p^n - s\left(\mathbb{E}\left[\left(\frac{\mu_s^n}{1+\mu_p^n f_1} - \frac{N_0 + \mathbb{E}\left[\rho h_1\right]}{g_1}\right) f_1\right] - Q_{\text{average}}\right)\right]^+ \tag{15}$$

$$\begin{aligned}\mathbb{E}[P_1(g_1,h_1,f_1)] &= \mathbb{E}\left[\frac{\mu_s^{n+1}}{1+\mu_p^{n+1} f_1} - \frac{N_0+\mathbb{E}\left[\rho h_1\right]}{g_1}\right]\\ &= \int_0^\infty \int_{\frac{(N_0+\mathbb{E}[\rho h_1])}{\mu_s^{n+1}}\left(1+\mu_p^{n+1} f_1\right)}^\infty \left(\frac{\lambda_s^{n+1}}{1+\lambda_p^{n+1} f_1} - \frac{N_0+\mathbb{E}\left[\rho h_1\right]}{g_1}\right) e^{-f_1-g_1} dg_1 df_1\\ &= (N_0+\rho)\,\mathrm{Ei}\left(-\frac{N_0+\rho}{\lambda_s^{n+1}}\right) - e^{-\frac{1}{\lambda_p^{n+1}}}\left(N_0+\rho+\frac{\lambda_s^{n+1}}{\lambda_p^{n+1}}\right)\mathrm{Ei}\left(-\frac{N_0\lambda_p^{n+1}+\lambda_p^{n+1}\rho+\lambda_s^{n+1}}{\lambda_p^{n+1}\lambda_s^{n+1}}\right)\end{aligned} \tag{16}$$

of h and f, respectively. In addition, it is assumed that the secondary transmitter knows not only the imperfect channel information, $\hat{f}$ and $\hat{h}$, but also the correlation coefficient, σ_{h_1} and σ_{f_1}. Note that σ_{h_1} and σ_{f_1} which are function of the Doppler frequency and delay parameters can be used to access the impact of other factors on the CSI. This model which focusses on the effects of imperfect CSI is well established in the literature [22].

In this case, the channel power gains between primary transmitter and secondary receiver and also between secondary transmitter and primary receiver become

$$h_1 = \sigma_{h_1}^2 \hat{h}_1 + \left(1-\sigma_{h_1}^2\right)\tilde{h}_1 \tag{19}$$

$$f_1 = \sigma_{f_1}^2 \hat{f}_1 + \left(1-\sigma_{f_1}^2\right)\tilde{f}_1 \tag{20}$$

where $h_1 = |h|^2$, $\hat{h}_1 = \left|\hat{h}\right|^2$, $\tilde{h}_1 = \left|\tilde{h}\right|^2$, $f_1 = |f|^2$, $\hat{f}_1 = \left|\hat{f}\right|^2$, $\tilde{f}_1 = \left|\tilde{f}\right|^2$. In this case, the optimization problem can be updated as following

$$\min_{P_1} \quad \mathbb{E}\left[P_1(g_1,\hat{h}_1,\hat{f}_1)\right] \tag{21}$$

$$\text{s.t.} \quad \mathbb{E}\left[\ln\left(1+\frac{P_1(g_1,\hat{h}_1,\hat{f}_1)g_1}{N_0+\rho\left(\sigma_{h_1}^2\hat{h}_1+\left(1-\sigma_{h_1}^2\right)\tilde{h}_1\right)}\right)\right] \geq R_{\text{average}} \tag{22}$$

$$\text{s.t.} \quad \mathbb{E}\left[P_1(g_1,\hat{h}_1,\hat{f}_1)\left(\sigma_{f_1}^2\hat{f}_1+\left(1-\sigma_{f_1}^2\right)\tilde{f}_1\right)\right] \leq Q_{\text{average}}. \tag{23}$$

Under Rayleigh fading, the above equation can change into (24) subject to (25) and (26) (shown at the beginning of next page) where the expectation is with respect to the channel gains $g_1, \hat{h}_1$ and $\hat{f}_1$. Equation (24) subject to (25) and (26) can be equivalent to the equation (27) subject to (28) and (29) (shown at the next page) where $\mathbb{E}$ is respect to $g_1, \hat{h}_1, \tilde{h}_1$ and $\hat{f}_1$. In (27) subject to (28) and (29), we have changed the integration in (25) with respect to $\tilde{h}_1$ as an expectation with respect to a dummy random variable $\tilde{h}_1$, which is distributed with exponential distribution. Applying Lagrangian approach, $P^{'}(g_1,\hat{h}_1,\tilde{h}_1,\hat{f}_1)$ as a dummy power allocation is given by

$$P_1^{'}(g_1,\hat{h}_1,\tilde{h}_1,\hat{f}_1) = \frac{\nu_s}{1+\nu_p\hat{f}_1\left(2\sigma_{f_1}^2-1\right)} - \frac{N_0+\rho\left(\sigma_{h_1}^2\hat{h}_1+\left(1-\sigma_{h_1}^2\right)\tilde{h}_1\right)}{g_1} \tag{30}$$

where ν_s and ν_p are the nonnegative dual variables. Therefore, $P_1(g_1,\hat{g}_0,\hat{h}_1)$ can be obtained as

$$\begin{aligned}P_1(g_1,\hat{h}_1,\hat{f}_1) &= \int_0^\infty \left(\frac{\nu_s}{1+\nu_p\hat{f}_1\left(2\sigma_{f_1}^2-1\right)} - \frac{N_0+\rho\left(\sigma_{h_1}^2\hat{h}_1+\left(1-\sigma_{h_1}^2\right)\tilde{h}_1\right)}{g_1}\right) e^{-\tilde{h}_1} d\tilde{h}_1\\ &= \frac{\nu_s}{1+\nu_p\hat{f}_1\left(2\sigma_{f_1}^2-1\right)} - \frac{N_0+\rho\left(\sigma_{h_1}^2\hat{h}_1+1-\sigma_{h_1}^2\right)}{g_1}.\end{aligned} \tag{31}$$

Equation (31) is an expression for power allocation of the secondary transmitter when imperfect CSI of the cross channels are available at the secondary transmitter. Again, the similar iteration search based on the sub-gradient method should be implemented to obtain minimum transmit power where the minimum average power under Rayleigh fading becomes equation (32) (shown at the next page) where $\Gamma(0,.)$ is the incomplete Gamma function.

As far as the computation complexity of (32) is concerned, note that the double integral in (32) can be represented, to any desire degree of accuracy, in terms of the weights and abscissa of a Laguerre orthogonal polynomial [35, eq. (25.4.45)] as shown in (33) where β_n, α_n are, respectively, the sample points and weight factors of the Nth order Laguerre polynomial, tabulated in [35, Table 25.9] and R_N is the remainder. We observe that closed-form expressions is not obtainable for (33), and hence, we need to solve the equation numerically.

$$\min_{P_1} \quad \mathbb{E}\left[P_1(g_1,\hat{h}_1,\hat{f}_1)\right] \tag{24}$$

$$\text{s.t.} \quad \mathbb{E}\left[\int_0^\infty \frac{1}{\sigma^2_{h_1}} \ln\left(1+\frac{P_1(g_1,\hat{h}_1,\hat{f}_1)g_1}{N_0+\rho\left(\sigma^2_{h_1}\hat{h}_1+\left(1-\sigma^2_{h_1}\right)\tilde{h}_1\right)}\right) e^{-\frac{\tilde{h}_1}{\sigma^2_{h_1}}} d\tilde{h}_1\right] \geq R_{\text{average}} \tag{25}$$

$$s.t. \quad \sigma^2_{f_1}\mathbb{E}\left[P_1(g_1,\hat{h}_1,\hat{f}_1)\hat{f}_1\right]+\left(1-\sigma^2_{f_1}\right)\mathbb{E}\left[P_1(g_1,\hat{h}_1,\hat{f}_1)\hat{f}_1\right] \leq Q_{\text{average}} \tag{26}$$

$$\min_{P_1} \quad \mathbb{E}\left[P_1^{'}(g_1,\hat{h}_1,\tilde{h}_1,\hat{f}_1)\right] \tag{27}$$

$$\text{s.t.} \quad \mathbb{E}\left[\ln\left(1+\frac{P_1^{'}(g_1,\hat{h}_1,\tilde{h}_1,\hat{f}_1)g_1}{N_0+\rho\left(\sigma^2_{h_1}\hat{h}_1+\left(1-\sigma^2_{h_1}\right)\tilde{h}_1\right)}\right)\right] \geq R_{\text{average}} \tag{28}$$

$$s.t. \quad \sigma^2_{f_1}\mathbb{E}\left[P_1^{'}(g_1,\hat{h}_1,\tilde{h}_1,\hat{f}_1)\hat{f}_1\right]+\left(1-\sigma^2_{f_1}\right)\mathbb{E}\left[P_1^{'}(g_1,\hat{h}_1,\tilde{h}_1,\hat{f}_1)\right] \leq Q_{\text{average}} \tag{29}$$

$$\begin{aligned}
\mathbb{E}[P_1(g_1,\hat{h}_1,\hat{f}_1)] &= \mathbb{E}\left[\frac{\nu_s}{1+\nu_p\hat{f}_1\left(2\sigma^2_{f_1}-1\right)}-\frac{N_0+\rho\left(\sigma^2_{h_1}\hat{h}_1+\left(1-\sigma^2_{h_1}\right)\tilde{h}_1\right)}{g_1}\right] \\
&= \int_0^\infty\int_0^\infty\int_{\frac{\left(N_0+\rho\left(\sigma^2_{h_1}\hat{h}_1+1-\sigma^2_{h_1}\right)\right)\left(1+\nu_p\hat{f}_1\left(2\sigma^2_{f_1}-1\right)\right)}{\nu_s}}^\infty \left(\frac{\nu_s}{1+\nu_p\hat{f}_1\left(2\sigma^2_{f_1}-1\right)}-\frac{N_0+\rho\left(\sigma^2_{h_1}\hat{h}_1+\left(1-\sigma^2_{h_1}\right)\tilde{h}_1\right)}{g_1}\right) \\
&\times e^{-\hat{f}_1-\hat{h}_1-g_1}dg_1 d\hat{h}_1 d\hat{f}_1 = \int_0^\infty\int_0^\infty\left(\frac{\nu_s e^{-\frac{\left(N_0+\rho+\left(\hat{h}_1-1\right)\rho\sigma^2_{h_1}\right)\left(1+\hat{f}_1\left(2\sigma^2_{f_1}-1\right)\nu_p\right)}{\nu_s}}}{1+\hat{f}_1\left(2\sigma^2_{f_1}-1\right)\nu_p}-\left(N_0+\rho+\left(\hat{h}_1-1\right)\rho\sigma^2_{h_1}\right)\right. \\
&\left.\times\Gamma\left(0,\frac{\left(N_0+\rho+\left(\hat{h}_1-1\right)\rho\sigma^2_{h_1}\right)\left(1+\hat{f}_1\left(2\sigma^2_{f_1}-1\right)\nu_p\right)}{\nu_s}\right)\right)e^{-\hat{f}_1-\hat{h}_1}d\hat{h}_1 d\hat{f}_1
\end{aligned} \tag{32}$$

.

$$\begin{aligned}
\mathbb{E}[P(g_1,\hat{h}_1,\hat{f}_1)] &= \sum_{n=1}^N\sum_{m=1}^N \alpha_n\alpha_m\left(\frac{\nu_s e^{-\frac{\left(N_0+\rho+(\beta_n-1)\rho\sigma^2_{h_1}\right)\left(1+\beta_m\left(2\sigma^2_{f_1}-1\right)\nu_p\right)}{\nu_s}}}{1+\beta_m\left(2\sigma^2_{f_1}-1\right)\nu_p}-\left(N_0+\rho+(\beta_n-1)\rho\sigma^2_{h_1}\right)\right. \\
&\left.\times\Gamma\left(0,\frac{\left(N_0+\rho+(\beta_n-1)\rho\sigma^2_{h_1}\right)\left(1+\beta_m\left(2\sigma^2_{f_1}-1\right)\nu_p\right)}{\nu_s}\right)\right)+R_N
\end{aligned} \tag{33}$$

$$\min_{P_1,\ldots P_K} \qquad \sum_{k=1}^K \mathbb{E}[P_k] \tag{34}$$

$$\text{s.t.} \qquad \sum_{k=1}^K \mathbb{E}\left[\ln\left(1+\frac{P_k g_k}{N_0+\sum_{m=1}^M \rho_m h_{mk}+\sum_{j=1,j\neq k}^K P_j g_{jk}}\right)\right] \geq R_{\text{average}} \tag{35}$$

$$\text{s.t.} \qquad \sum_{k=1}^K \mathbb{E}[P_k f_{km}] \leq Q_{\text{average}} \qquad m=1,...,M \tag{36}$$

B. Minimum Power for Multiple Primary and Secondary Links

Here, we assume that multiple secondary links (K links) with multiple primary links (M Links) are present on the same spectrum while in the previous results, it was assumed that the licensed spectrum is being shared between one secondary link and one primary link. The optimization problem is modified as equation (34) subject to (35) and (36) (shown at the next page), where P_k is the transmission power of the kth secondary transmitter, g_k denotes the channel gain between secondary transmitter k and secondary receiver k, ρ_m is the transmission power of the mth primary transmitter, h_{mk} is the interference channel gain from the mth primary transmitter to the kth secondary receiver, g_{jk} denotes an interference channel gain for secondary receiver k from other secondary transmitters, f_{km} is the channel gain from the kth secondary transmitter to the mth primary receiver.

This optimization problem is a nonconvex problem due to having nonconvex constraint (35), which was recently shown to be NP-hard [27]. When a problem is not convex, duality gaps may occur that prevent ordinary Lagrangian duality from reaching the optimal solution. The authors in [28]-[31] augmented the Lagrangian function in order to eliminating duality gaps in nonconvex problems. The augmentation consists of a penalty-like quadratic term being introduced into the Lagrangian function to convexify the problem.

The approach, therefore, is to use augmented Lagrange method for inequality constraints which convert inequality constraints to equality constraints by using squared additional variables. Indeed, for sufficiently large penalty parameter, [28] shows that the augmented Lagrangian is locally convex.

We note that the augmented Lagrangian is different with standard Lagrangian in the presence of the squared terms. It is also different with the quadratic penalty function by the presence of the summation term involving lagrangian multipliers. In fact, the augmented Lagrangian method combines the Lagrangian function and the quadratic penalty function. In this method, the constraints are eliminated and added to the cost function as a *penalty term* that prescribes a high cost to infeasible points. A positive coefficient as a penalty parameter multiplied by the penalty terms determines the severity of the penalty, and therefore, the resulting unconstrained problem approximates the original constrained problem. When the introduced penalty parameter takes a higher value, the constraint violations will be penalized more severely, and therefore, the minimizer of the penalty function is forced closer to the feasible region for the constrained problem.

The augmented Lagrangian function can be expressed as equation (37) (shown at the beginning of next page) where μ_1 and μ_m are Lagrangian dual variables and σ is an adjustable penalty parameter. The equation (37) can be solved by an iterative algorithm to update μ_1, μ_m and σ until the convergence criteria is met. In this method, the following iterations for μ_1 and μ_m are implemented as equation (38) and (39) (shown at the next page) where $P_k^{(n)}, \mu_1^{(n)}$and $\mu_m^{(n)}$ are the values of P_k, μ_1 and μ_m at stage n, respectively. The steps for the power control algorithm are shown in Table 2, in which $P_k^{(n+1)}, \mu_1^{(n+1)}$and $\mu_m^{(n+1)}$ are updated to obtain the minimum transmission power.

Table. 2: Augmented Lagrange method

1) Choose tolerance $\epsilon = 10^{-3}$, initial value $n = 0$, initial penalty parameter $\sigma^{(n)} = 1$
starting points $P_k^{(n)} = 0, \mu_1^{(n)} = 0.1$ and $\mu_2^{(n)} = 0.15$
2) While
   a) Perform unconstrained optimization $L_\sigma(P_k, \mu_1, \mu_m)$ in (37) to get $P_k^{(n+1)}$
   b) Update $\mu_1^{(n+1)}$ by (38)
   c) Update $\mu_m^{(n+1)}$ by (39)
   d) $\sigma^{(n+1)} = 2\sigma^{(n)}$
   e) $n = n + 1$
3) If $|P_k^{(n+1)} - P_k^{(n)}| < \epsilon$ then stop.

The convergence rate of the augmented Lagrangian method depends heavily on the adjustable penalty parameter σ. In general, large σ results in fast convergence rate. However, large values of σ may introduce computational difficulty in minimizing the augmented Lagrangian. It is recommended in [28] to increase the adjustable penalty parameter gradually until it reaches a certain threshold value.

The quadratic penalty function is the simplest penalty function in which the penalty terms are the squares of the constraint violations. A suitable choice of the initial guess on $\sigma^{(0)}$ is necessary for the method to perform correctly. The initial guess should not be too large to the point that ill-conditioning results in the first unconstrained minimization. Our numerical analysis suggest that $\sigma^{(0)} \in [1, 5]$ works well in practice. In this case, the penalty parameter $\sigma^{(n)}$ is not increased too fast to the point that high ill-conditioning is forced upon the unconstrained minimization routine too early. On the other hand, $\sigma^{(n)}$ is not increased too slowly so that not to reduce the convergence rate. In fact, the subsequent values of $\sigma^{(n)}$ will be monotonically increased via the equation $\sigma^{(n+1)} = \upsilon\sigma^{(n)}$ where υ is a scalar with $\upsilon > 1$. Additional guidelines for choosing these parameters can be found in [28, Section 4.2].

In [30] and [31, Chapter 3], Bertsekas introduced an important result on the linear rate of convergence of the augmented Lagrangian method for nonconvex nonlinear optimization problems with inequality constraints. The rate of convergence tends to a constant value which is proportional to the ratio $1/\sigma$ where the penalty parameter σ is not less than a threshold $\bar{\sigma} > 0$ [29] and [28]. The significance of Bertsekas's result resides in the fact that theoretically, subject to numerical stability, a large σ can be selected to accelerate the convergence, which also demonstrates high practical performance of this method. The choice of a quadratic penalty function also has a substantial effect on the convergence rate. If a different penalty function is chosen, then the convergence rate can become sublinear or superlinear. Linear convergence rate for the augmented Lagrangian method with different conditions have been derived in literature. [1].

The complexity order in augmented Lagrange method proposed in Table. 2 to get minimum power in Steps 2b to 2e is

[1]For more on the augmented Lagrangian method for nonlinear programming, readers are referred to [32] and [33] and the survey paper [34]

$$L_\sigma(P_k,\mu_1,\mu_m)=\sum_{k=1}^{K}\mathbb{E}[P_k]$$
$$+\frac{1}{2\sigma}\left\{\left(\left(\max\left\{0,\mu_1+\sigma\left(\sum_{k=1}^{K}\mathbb{E}\left[\ln\left(1+\frac{P_kg_k}{N_0+\sum_{m=1}^{M}\rho_mh_{mk}+\sum_{j=1,j\neq k}^{K}P_jg_{jk}}\right)\right]-R_{\text{average}}\right)\right\}\right)^2\right.\right.$$
$$\left.\left.-\mu_1^2\right)+\sum_{m=1}^{M}\left(\left(\max\left\{0,\mu_m+\sigma\left(Q_{\text{average}}-\sum_{k=1}^{K}\mathbb{E}[P_kf_{km}]\right)\right\}\right)^2-\mu_m^2\right)\right\} \tag{37}$$

$$\mu_1^{(n+1)}=\max\left\{0,\mu_1^{(n)}+\sigma\left(\sum_{k=1}^{K}\mathbb{E}\left[\ln\left(1+\frac{P_kg_k}{N_0+\sum_{m=1}^{M}\rho_mh_{mk}+\sum_{j=1,j\neq k}^{K}P_jg_{jk}}\right)\right]-R_{\text{average}}\right)\right\} \tag{38}$$

$$\mu_m^{(n+1)}=\max\left\{0,\mu_m^{(n)}+\sigma\left(Q_{\text{average}}-\sum_{k=1}^{K}\mathbb{E}[P_kf_{km}]\right)\right\}\qquad m=1,...,M \tag{39}$$

$O(N)$ while in Step 2a the complexity order becomes $O(N^2)$, supposing that we use an iterative algorithm with a small step size [28] [29]. Therefore, the complexity of the augmented Lagrange method is $O(N^2)+O(N)+O(N)+O(N)+O(N)=O(N^2)$.

1) The Effect of Reducing Extra CSI at the Secondary Transmitter: Under multiple primary and secondary links, having extra CSI of interference channels, including from primary transmitters and also other secondary transmitters, at the secondary transmitter can be a strong assumption. So, we here present the case where only average values of extra CSI of interference channels are available at the secondary transmitter and compare with the pervious case where all CSIs are available at the secondary transmitter to study the significance of having extra CSI at the secondary transmitter.

In this case, the optimization problem can be simplified as equation (40) subject to (41) and (42) (shown at the next page). This optimization problem changes into a convex form. Following the same procedure as found (7), the power allocation of secondary transmitter can be expressed as

$$P_k\left(g_k,f_{k1},f_{k2},...,f_{kM}\right)=\frac{\eta_s}{1+\sum_{m=1}^{M}\eta_{pm}f_{km}}-\frac{N_0+\mathbb{E}\left[\sum_{m=1}^{M}\rho_mh_{mk}\right]+\mathbb{E}\left[\sum_{j=1,j\neq k}^{K}P_jg_{jk}\right]}{g_k} \tag{43}$$

where η_s and $(\eta_{p1},....,\eta_{pM})$ are the nonnegative dual variables. In this case, the transmission power P_k is only a function of g_k and $(f_{k1},f_{k2},...,f_{kM})$ and also is independent of extra CSI of interference channels.

We present the following iteration search based on the subgradient method as equation (44) and (45) (shown at the next page) where β is a positive gradient search stepsize, and η_s^n and η_{pm}^n are the value of η_s and η_{pm} at stage n, respectively.

IV. Minimum Power Under Spectrum Sensing

In spectrum sensing, when the primary user is active, the discrete received signal at the secondary user is expressed as

$$y(n)=hx(n)+u(n), \tag{46}$$

for $n=1,2,...,N$, where $x(n)$ is the signal sent by the primary transmitter, h represents the channel between the primary transmitter and the secondary transmitter, $u(n)$ denotes AWGN noise with zero mean and variance $E[|u(n)|^2]=\sigma_u^2$, and N is the number of samples. This case is referred to hypothesis H_1 as the presence of the primary user. When the primary user is inactive, the received signal at the secondary user can be given by

$$y(n)=u(n), \tag{47}$$

which is the output under hypothesis H_0 as the hypotheses of the absence of the primary user. We also assume that $N=f_s\tau$ where f_s is the sampling frequency and τ is the sensing time. Two probabilities are of interest in spectrum sensing known as probability of detection and probability of false alarm. Probability of detection (P_d) defines the probability of the algorithm correctly detecting the presence of primary signal under hypothesis H_1 whereas probability of false alarm (P_f) is referred to the probability of the algorithm falsely declaring the presence of primary signal under hypothesis H_0. Using energy detecting scheme and based on the probability density function (PDF) of the test static, P_d and P_f are expressed as

$$P_d(\epsilon,\tau)=Q\left(\left(\frac{\epsilon}{\sigma_u^2}-\gamma-1\right)\sqrt{\frac{\tau f_s}{2\gamma+1}}\right)$$
$$P_f(\epsilon,\tau)=Q\left(\left(\frac{\epsilon}{\sigma_u^2}-1\right)\sqrt{\tau f_s}\right)$$

respectively, where ϵ denotes the detection threshold, γ is the received signal-to-noise ratio (SNR) at the secondary transmitter, and $Q(\cdot)$ represents the complementary distribution function of the standard Gaussian [7]. Each frame under cognitive

$$\min_{P_1,\dots P_K} \quad \sum_{k=1}^{K} \mathbb{E}[P_k] \tag{40}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \mathbb{E}\left[\ln\left(1+\frac{P_k g_k}{N_0+\mathbb{E}\left[\sum_{m=1}^{M}\rho_m h_{mk}\right]+\mathbb{E}\left[\sum_{j=1,j\neq k}^{K} P_j g_{jk}\right]}\right)\right] \geq R_{\text{average}} \tag{41}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \mathbb{E}[P_k f_{km}] \leq Q_{\text{average}} \qquad m=1,...,M \tag{42}$$

$$\eta_s^{n+1} = \left[\eta_s^n - \beta\left(\sum_{k=1}^{K}\mathbb{E}\left[\ln\left(\frac{g_k}{N_0+\mathbb{E}\left[\sum_{m=1}^{M}\rho_m h_{mk}\right]+\mathbb{E}\left[\sum_{j=1,j\neq k}^{K} P_j g_{jk}\right]}\frac{\eta_s^n}{1+\eta_{pm}^n\sum_{m=1}^{M} f_{km}}\right)\right] - R_{\text{average}}\right)\right]^+ \tag{44}$$

$$\eta_{pm}^{n+1} = \left[\eta_{pm}^n - \beta\left(\sum_{k=1}^{K}\mathbb{E}\left[\left(\frac{\eta_s^n}{1+\sum_{m=1}^{M}\eta_{pm}^n f_{km}} - \frac{N_0+\mathbb{E}\left[\sum_{m=1}^{M}\rho_m h_{mk}\right]+\mathbb{E}\left[\sum_{j=1,j\neq k}^{K} P_j g_{jk}\right]}{g_k}\right) f_k\right] - Q_{\text{average}}\right)\right]^+ \quad m=1,...,M \tag{45}$$

---

radio network with periodic spectrum sensing includes one sensing slot and one data transmission slot. Denote τ as the sensing duration time, T as the frame duration time and $T-\tau$ as the data transmission slot duration. Secondary users can operate at the frequency band of primary network under two scenarios where in first scenario the primary user is not present and no false alarm is generated by the secondary user while in second, the primary user is active but it is not detected by the secondary user.

In such scenario, the optimization problem for minimum power can be formulated as

$$\min_{P_1} \quad \mathbb{E}[P_1] \tag{48}$$

$$\text{s.t.} \quad \mathbb{E}\left[\frac{T-\tau}{T}\left(P(H_0)(1-P_f)\ln\left(1+\frac{P_1 g_1}{N_0}\right)+P(H_1)\times(1-P_d)\ln\left(1+\frac{P_1 g_1}{N_0+\rho h_1}\right)\right)\right] \geq R_{\text{avg\_sense}} \tag{49}$$

$$\text{s.t.} \quad P(H_1)(1-P_d)\,\mathbb{E}[P_1 f_1] \leq Q_{\text{avg\_sense}} \tag{50}$$

where $R_{\text{avg\_sense}}$ denotes the minimum capacity required in secondary network under spectrum sensing and $Q_{\text{avg\_sense}}$ refers to the maximum average interference on the primary user. The constraint (50) is added to the optimization problem in order to consider the simultaneous transmission of primary and secondary links. Secondary user causes interference to the primary user only when the primary user is active but it is not detected by the secondary user.

In what follows, we will obtain the minimum power under perfect and imperfect sensing.

A. *Perfect Sensing*

Under perfect sensing, we can assume that secondary transmitter perfectly detects the presence of the signal of the primary transmitter without false alarm, i.e., $P_d = 1$ and $P_f = 0$. So, the constraint in the above optimization problem can be modified as

$$\mathbb{E}\left[\frac{T-\tau}{T}\left(P(H_0)\ln\left(1+\frac{P_1 g_1}{N_0}\right)\right)\right] \geq R_{\text{avg\_sense}} \tag{51}$$

After applying the Lagrangian approach, we can find P_1 as

$$P_1 = \lambda P(H_0)\frac{T-\tau}{T} - \frac{N_0}{g_1} \tag{52}$$

The parameter λ that minimizes (48) can be obtained by substituting (52) in (49) as

$$R_{\text{avg\_sense}} = \mathbb{E}\left[P(H_0)\frac{T-\tau}{T}\ln\left(\frac{g_1}{N_0}\lambda P(H_0)\frac{T-\tau}{T}\right)\right] \tag{53}$$

which under Rayleigh fading, by using the definition of the Incomplete Gamma function, according to:

$$\int_a^\infty \ln\left(\frac{x}{a}\right)e^{-x}dx = \Gamma(0,a) \quad \text{if } a>0$$

and using [26, eq. (4.331.2) and eq. (8.359.1)], we get

$$R_{avg\_sense} = P(H_0)\frac{T-\tau}{T} \times \int_{\frac{N_0 T}{\lambda P(H_0)(T-\tau)}}^{\infty} \ln\left(\frac{\lambda P(H_0)(T-\tau)}{N_0 T}g_1\right)e^{-g_1}dg_1 \tag{54}$$

$$= P(H_0)\left(\frac{T-\tau}{T}\right)\Gamma\left(0,\frac{N_0 T}{\lambda P(H_0)(T-\tau)}\right) \tag{55}$$

We finally find the minimum power by using [26, eq. (3.351.5) and eq. (8.359.1)] as

$$\min \mathbb{E}[P_1] = \int_{\frac{N_0 T}{\lambda P(H_0)(T-\tau)}}^{\infty} \left(\lambda P\left(H_0\right) \frac{T-\tau}{T} - \frac{N_0}{g_1} \right) e^{-g_1} dg \tag{56}$$

$$= e^{-\frac{N_0 T}{\lambda P(H_0)(T-\tau)}} \lambda P\left(H_0\right) \frac{T-\tau}{T} - N_0 \Gamma \left(0, \frac{N_0 T}{\lambda P\left(H_0\right)\left(T-\tau\right)}\right) \tag{57}$$

B. Imperfect Sensing

Here, in the case of imperfect sensing as a more realistic model, where P_d is less than one and P_f is bigger than zero, we obtain the minimum transmission power.

Similarly, by employing the Lagrangian approach over the optimization problem (48) subject to (49) and (50), the power allocation of the secondary transmitter can be expressed as

$$P_1 = \frac{1}{2 g_1^2 A T} \left(-g_1 C - g_1 A \left(2 N_0 + h_1 \rho\right) T + \sqrt{g_1^2 \left(C^2 B^2 - 2 h_1 A C D \rho + \left(h_1 A \rho T\right)^2 \right)} \right) \tag{58}$$

where $A = f_1 \lambda_p \left(P_d - 1\right) P\left(H_1\right) - 1$, $B = P\left(H_1\right)\left(P_d - 1\right) + P\left(H_0\right)\left(P_f - 1\right)$, $C = g_1 \lambda_s T \left(t - T\right)$ and $D = \left(P_f - 1\right) P\left(H_0\right) + P\left(H_1\right)\left(1 - P_d\right)$. In a manner similar to the perfect sensing, we find the minimum transmission power under imperfect sensing.

C. Multiple Secondary Links

Here, we extend the model to multiple secondary links which in this case the optimization problem for the minimum power under spectrum sensing can be modified as equation (59) subject to (60) and (61). This optimization problem under perfect sensing ($P_d = 1$ and $P_f = 0$) gives the similar results as problem (48) subject to (49) and (50), while under imperfect sensing requires the augmented Lagrange method offered in the previous section.

V. Numerical Results

In this section, we present the numerical results to illustrate the energy efficiency, minimum power and achievable rate of the secondary network over Rayleigh fading channels under spectrum sharing and sensing schemes.

A. Spectrum Sharing

Fig. 2 shows the behavior of minimum transmission power of secondary transmitter over single secondary link scenario versus the number of iterations in power control algorithm proposed in Table 1 for $\rho = 5$dB. The convergence of the proposed algorithm can be observed in this figure. From this figure, we can further observe that the minimum power monotonically increases by increasing the number of iterations which itself depends on the chosen initial values for λ_s^0 and λ_p^0. Iteration should be applied until the difference between two consecutive steps becomes less than a small value.

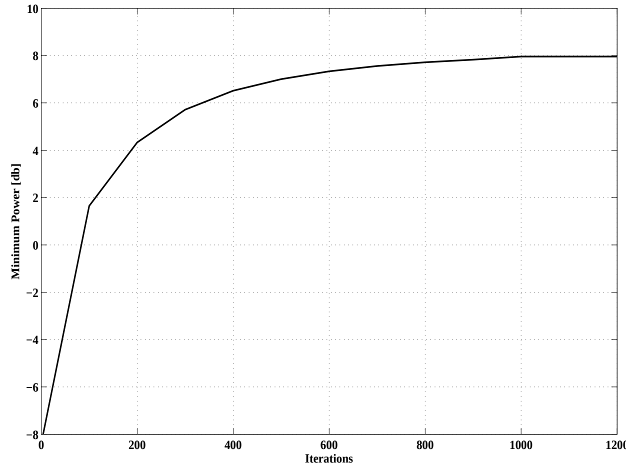

Fig. 2. The minimum power of the secondary transmitter versus the number of iterations for single secondary link scenario in power control algorithm proposed in Table 1.

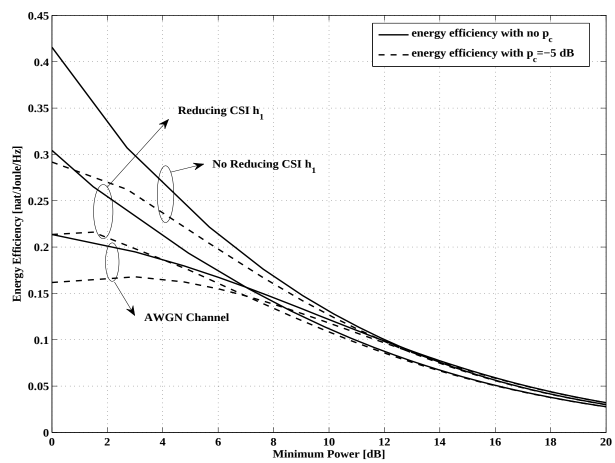

Fig. 3. Energy efficiency versus minimum transmit power for single secondary link.

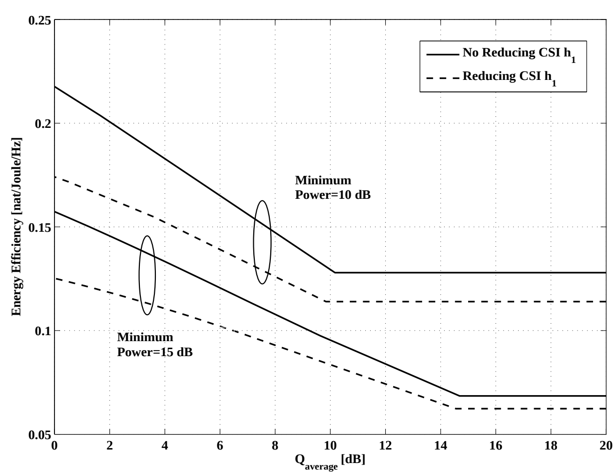

Fig. 4. Energy efficiency versus Q_{average} for single secondary link.

$$\min_{P_1,\ldots P_K} \quad \sum_{k=1}^{K} \mathbb{E}[P_k] \tag{59}$$

$$s.t. \quad \mathbb{E}\left[\frac{T-\tau}{T}\left(P\left(H_0\right)\left(1-P_f\right)\ln\left(1+\frac{P_1 g_1}{N_0}\right)+P\left(H_1\right)\right.\right.$$
$$\left.\left.\times\left(1-P_d\right)\sum_{k=1}^{K}\left[\ln\left(1+\frac{P_k g_k}{N_0+\mathbb{E}\left[\rho h_k\right]+\mathbb{E}\left[\sum_{j=1,j\neq k}^{K} P_j g_{jk}\right]}\right)\right]\right)\right] \geq R_{\text{avg\_sense}} \tag{60}$$

$$s.t. \quad P\left(H_1\right)\left(1-P_d\right)\sum_{k=1}^{K}\mathbb{E}[P_k f_k] \leq Q_{\text{avg\_sense}} \tag{61}$$

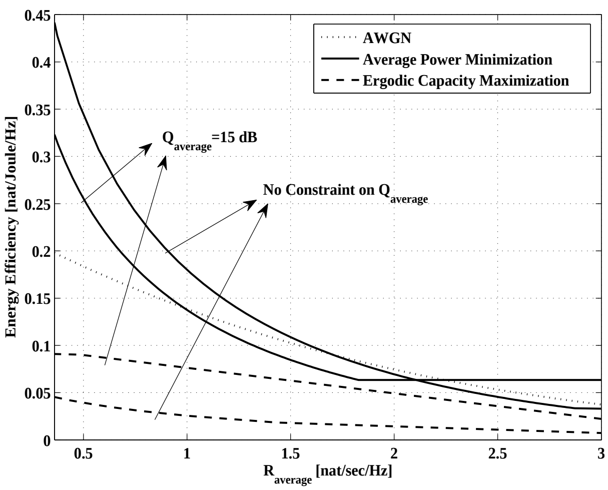

Fig. 5. Energy efficiency for average power minimization problem and the ergodic capacity maximization problem

Fig. 3 displays the energy efficiency against transmit power in single secondary link scenario when there is no constraint on Q_{average} with $\rho = 5$dB. For comparison, we also plot the energy efficiency results for AWGN channel. As we can see, the energy efficiency in all cases exponentially decreases with increasing the power, at high transmission powers. This figure includes the results for two different values of the circuit power with one case with $P_{\text{c}} = -5$dB and another case with no circuit power consideration, i.e., $P_{\text{c}} = 0$ watts. The figure shows that when $P_{\text{c}} = -5$dB, in AWGN channels, the EE first increases slowly and then decreases. When no circuit power is considered, the EE decreases monotonically with the minimum power. These results are expected from EE formula as the minimum power increases. The figure further reveals that the EE decreases rapidly at lower values of the minimum power, whereas the slope of the EE curve is slower at higher values of the minimum power. Also, the figure shows that the AWGN channel's EE performance is not necessarily higher than that of the Rayleigh fading channel. This is in contrast to the point-to-point channels for which fading causes performance degradation in wireless systems. This is due to the fact that in Rayleigh fading channels, the SU can benefit from the case when the interference channel is in deep fading, but, its direct channel is in good condition. This result is in line with achieving higher capacity in Rayleigh fading channel when compared to AWGN channels in spectrum sharing scenarios [36]. In addition to the above, we note that at each moment, with high probability, a user with the best joint channel condition is available which also can improve the performance of the secondary user network due to multiuser diversity gain.

Fig. 4 shows the effect of varying Q_{average} on the energy efficiency of cognitive network with different minimum transmit powers. This figure reveals that Q_{average} can limit the energy efficiency of cognitive network as a dominant constraint. We also include in Fig. 3 and Fig. 4 the energy efficiency results with a reduced side information on h_1 (where h_1 is not made available at the secondary transmitter). This is used to gain insight into the significance of having extra side information on the channel gain h_1 at the secondary transmitter. One important observation is that the energy efficiency when the power is a function of g_1, h_1 and f_1 is always higher than that when the power depends on g_1 and f_1. Examining Fig. 3, we can see that the difference between the energy efficiency with a reduced CSI h_1 and energy efficiency with no reduced CSI h_1 decreases as the transmit power increases. For instance, as the minimum power increases from 3dB to 18dB, the difference between the energy efficiencies decreases from 0.07 nat/Joule/Hz to nearly zero nat/Joule/Hz. Flooring in the case of 10dB and 15dB minimum powers, and in general, occurs because the minimum power becomes the dominant constraint in the MOP. In this case, the operational power does not increase further and the EE remains constant.

Fig. 5 shows the energy efficiency versus the ergodic capacity. This figure compares the energy efficiency obtained by the average power minimization problem (4) and the following ergodic capacity maximization problem

$$\max_{P_1} \quad \mathbb{E}\left[\ln\left(1+\frac{P_1 g_1}{N_0+\rho h_1}\right)\right] \tag{62a}$$
$$\text{s.t.} \quad \mathbb{E}[P_1\left(g_1,h_1,f_1\right)f_1] \leq Q_{\text{average}} \tag{62b}$$
$$\text{s.t.} \quad \mathbb{E}[P_1\left(g_1,h_1,f_1\right)] \leq P_{\text{average}} \tag{62c}$$

From Fig. 5, it can be seen that the difference between these two obtained results is considerable which indicates the effect of power minimization on the improving of energy efficiency. From this figure, we can observe that Q_{average} can prevent the

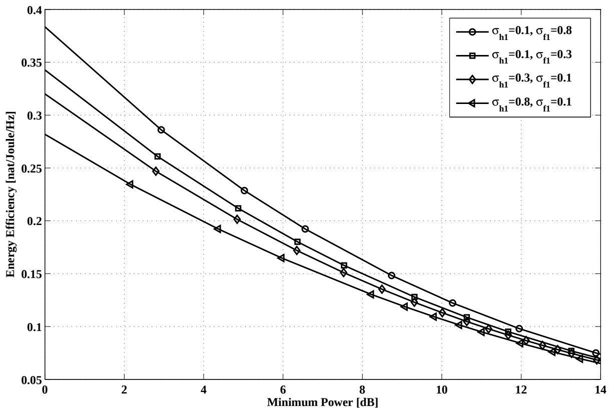

Fig. 6. The impact of varying σ_{h_1} and σ_{f_1} on the energy efficiency with $\rho = 5$dB and $Q_{\text{average}} = 12$dB

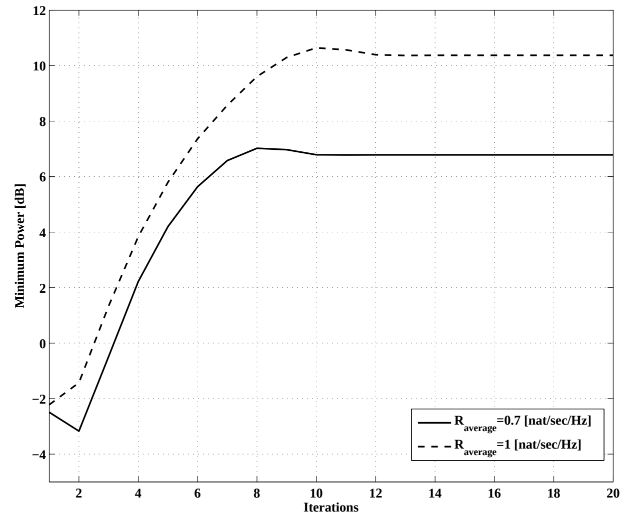

Fig. 7. The sum of minimum power of secondary transmitters versus the number of iterations for multiple secondary links in power control algorithm proposed in Table 2.

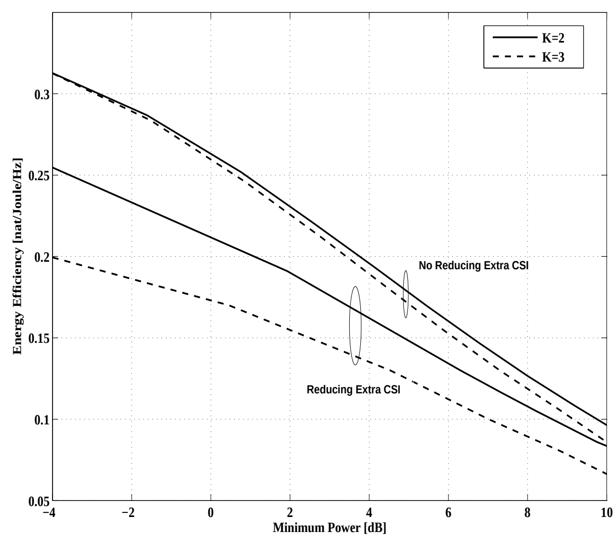

Fig. 8. Energy efficiency versus minimum power for multiple secondary links

energy efficiency reduction. This is because the constraint on Q_{average} limits the transmit power. This threshold, defines a maximum on the operational power that optimizes the power minimization problem. Henceforth, after a certain value of R_{average} flooring occurs and increasing the value of R_{average} beyond this point does not change the system performance. In other words, for lower values of R_{average}, the constraint on the R_{average} is the dominant constraint and limits the performance of the secondary user and increasing its maximum value reduces the system EE performance. On the other hand, at higher values of R_{average}, the constraint on the Q_{average} will be the dominant constraint and increasing R_{average} does not change the system performance, henceforth, flooring occurs.

Under imperfect CSI, the behavior of the energy efficiency versus minimum transmit power with different values of σ_{f_1} and σ_{h_1} are shown in Fig. 6. In this figure, four different cases are studied, wherein the two first cases σ_{h_1} is fixed and σ_{f_1} takes two different values of $\sigma_{f_1} = 0.8$ and $\sigma_{f_1} = 0.3$. In the remaining cases, σ_{f_1} is fixed and σ_{h_1} is changed between $\sigma_{h_1} = 0.8$ and $\sigma_{h_1} = 0.3$. This figure indicates that by decreasing σ_{f_1} energy efficiency decreases whereas by varying σ_{h_1} from 0.3 to 0.8 energy efficiency decreases.

In Fig. 7, the behavior of total minimum transmission power versus the number of iterations in augmented Lagrange method proposed in Table 2 for $\rho = 5$dB is discussed. This figure indicates the convergence of the proposed algorithm.

Fig. 8 shows the energy efficiency versus transmit power under ergodic capacity and average interference power constraint for different values of K, $\rho = 5$dB and $Q_{\text{average}} = 15$dB. As we can see in this figure, for a constant transmit power, energy efficiency decreases when the number of secondary links (K) increases. In addition, the energy efficiency result with reduced extra side information at the secondary transmitter is also plotted in Fig. 8. This figure shows that the energy efficiency when the extra CSIs are available at the secondary transmitter always higher than that when extra CSIs at the secondary transmitter are reduced. The figure further shows that for a fixed K, the different between the achievable EE in two cases of full CSI and reduced CSI decreases as the minimum power increases. The plots for with full CSI and with reducing extra CSI have the same behavior when the value of K is constant.

B. Spectrum Sensing

Fig. 9 and Fig. 10 show the effect of τ and $P(H_0)$ on the energy efficiency under perfect sensing, respectively. For comparison, we also plot the energy efficiency results over AWGN channels in these figures. These two figures indicate that increasing τ reduces the energy efficiency while increasing $P(H_0)$ improves the energy efficiency. These results can be expected from equation (55). In addition, since ergodic capacity under fading channels is lower than that under AWGN channels and due to assuming constant transmit power in plotting Fig. 9, energy efficiency under fading channels becomes lower than that under AWGN channels.

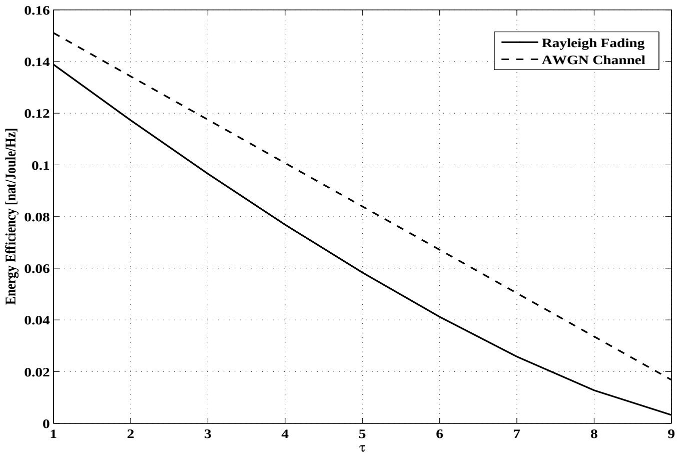

Fig. 9. Energy efficiency under perfect sensing versus τ for fixed transmit power and $P\left(H_0\right) = 0.7$.

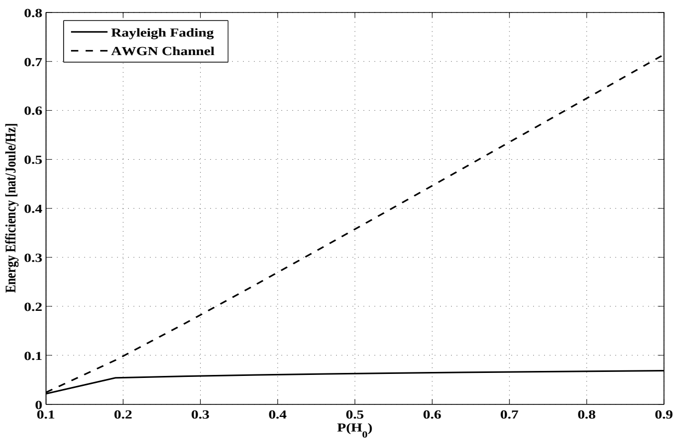

Fig. 10. Energy efficiency under perfect sensing versus $P\left(H_0\right)$ for fixed rate of secondary transmitter and $\tau = 2$.

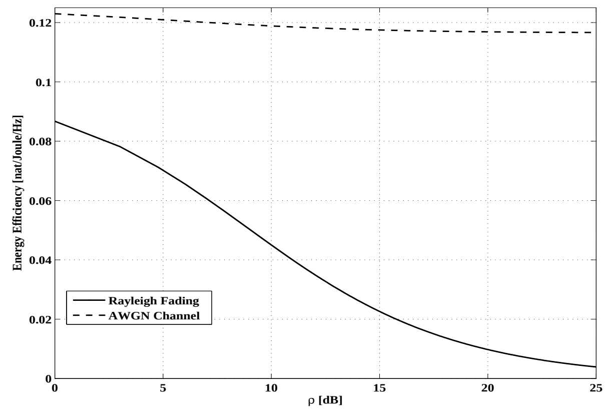

Fig. 11. Energy efficiency under imperfect sensing versus ρ for fixed transmit power.

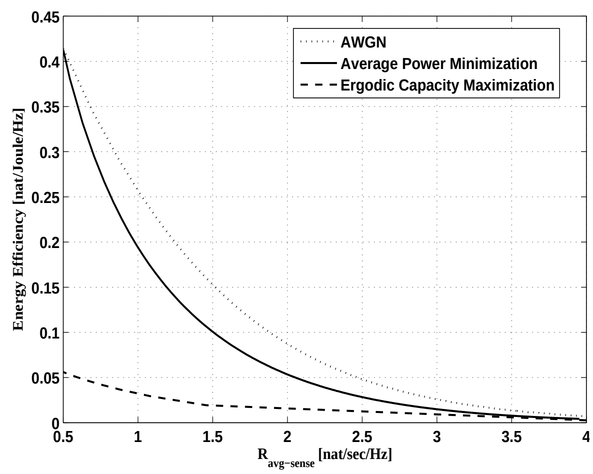

Fig. 12. Energy efficiency for average power minimization problem and the ergodic capacity maximization problem under perfect spectrum sensing

The behavior of ρ under imperfect sensing is shown in Fig. 11 for $P\left(H_0\right) = 0.6$, and fixed transmit power. From Fig. 11, it can be seen that the difference between energy efficiencies under AWGN channels and Rayleigh fading channels increases by increasing ρ.

Fig. 12 displays the energy efficiency against the ergodic capacity under perfect spectrum sensing with $P\left(H_0\right) = 0.7$ and $\tau = 1$. Similar to the result obtained for spectrum sharing, this figure also shows the significant impact of the power minimization on increasing the energy efficiency compared to ergodic capacity maximization. In this figure, for very low ergodic capacity, the transmit power of the proposed solution and the AWGN case are very low, giving energy efficiency close to each other. But for very high ergodic capacity, with increasing transmit power from a threshold point, ergodic capacities under proposed solution and AWGN case are constant, resulting in same energy efficiency. The ergodic capacity maximization case does not show improvement in energy efficiency with increasing ergodic capacity. This is because with increasing ergodic capacity, the transmit power should be simultaneously increased leading to decrease energy efficiency. Also, in energy efficiency equation, the transmit power in denominator directly affects the energy efficiency.

VI. Conclusion

This paper considered spectrum sharing and sensing systems in time varying channels motivated by the concept of cognitive radio networks. In this paper, we evaluate the energy efficiency by minimizing the transmission power at the secondary transmitter for both schemes of spectrum sharing and sensing over fading environments. Under spectrum sharing, the effects of extra CSI which can be available at the secondary transmitter are analyzed and closed-form expressions for corresponding transmission power are obtained. We also discussed the imperfect side information of cross channels when licensed spectrum is being shared between one secondary link and one primary link. In the case of spectrum sensing, the power allocation over perfect and imperfect sensing is minimized. Numerical results shows that minimizing power allocation at the secondary transmitter in the case of spectrum sharing improves the energy efficiency over Rayleigh fading channels even higher than that over AWGN channels, for some values of transmission power.

Numerical results also reveals that power minimization significantly increases the energy efficiency compared to ergodic capacity maximization.

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