Energy Efficiency Optimization with Energy Harvesting using Harvest-Use Approach

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***Abstract*—Energy harvesting is emerging as a promising approach to improve the energy efficiency (EE) and to extend the life of wireless networks. This paper focuses on energy-efficient transmission power allocation techniques for a point-to-point communication channel, equipped with a fixed-power battery as well as a harvest-use battery. Using the fact that the harvested energy does not consume from the fixed battery, EE is formulated as the ratio of Shannon limit (as a function of the sum of the power consumed from the fixed battery and the harvest-use battery) to the sum of the circuit power and power consumed from the fixed battery. For the considered energy harvest-use technique, a time switching approach is used that in each frame, it harvests energy for a period of time and transmits data for the rest of the frame time. Using the fact that the formulated EE is a quasi-concave function in transmission power, we use fractional programming to obtain the optimal power level, $\overline{P}_{\mathrm{u}}$, and in-turn, the maximum achievable EE. Analytical derivations show that the maximum achievable EE monotonically increases with harvested power, whereas, $\overline{P}_{\mathrm{u}}$ monotonically decreases with it. Simulation results show the effects of harvested energy, fixed-battery power limit, and time switching rate on the maximum achievable EE.**

***Index Terms*—Energy harvesting, energy efficiency, convex optimization, fractional programming.**

I. INTRODUCTION

Green wireless communication technologies have attracted significant attention in the last few years as to provide possible solutions to energy limitation problems in information and communication technologies (ICT) [1]. Specifically, due to the increase in the number of mobile users, demand for higher data rates, and the base station (BS) is required to increase its transmission power, which in turn, results in higher greenhouse gases, pollution, and also higher costs [2]. Furthermore, the battery technology has not progressed with the same pace, which has resulted in increasing gap between increasing demand for power and the battery advancements [3]. These facts have motivated research in investigating new ways to get energy from re-usable sources.

Energy efficiency (EE), which is defined as the data transferred per unit energy consumed, with unit of b/J/Hz, has recently received a great deal of interest [4]. Improving EE or maximizing throughput, has been investigated widely in literature, e.g., [5], [6], which show that increasing EE in many cases results in decreasing rate. In this trend, recently, energy harvesting has emerged as a new technology to improve EE, while maintaining the spectral efficiency (SE) [7].

Energy harvesting has indeed the capability of reducing the carbon consumption of high data rate wireless systems by reusing the energy which is available at the surrounding environment [8]. In addition, energy harvesting techniques are used to increase the life of battery limited devices, e.g, sensor nodes that are implemented in remote access areas [9]. Energy harvesting is, infact, a low cost source of energy, which can provide self-sustainability to sensor networks [10], and hence, it can be regarded as a free of cost energy resource. Energy can be harvested from sources like solar, wind, vibrations and thermo-electric [11]. Another source of energy which can be harvested in communication devices is the radio frequency (RF) [12], [13]. We note that when RF harvested energy is used, the harvested energy level will be a random parameter that depends on the channel fading coefficients [14]. The harvested energy can be used to run devices with energy constrained sources [15].

From architectural point of view, there exist two main categories of energy harvesting techniques [15]:(a) harvest-store-use (HSU), where harvested energy can be accumulated for future use, and (b) harvest-use (HU). Specifically, in HU [16], energy cannot be stored and must be used immediately when it becomes available to the transmitter [17]. This technique is suitable for applications where nodes exchange short messages, e.g., sensor monitoring networks [18]. Even though the literature related to this technique is very limited, it is considered as a suitable approach for systems with limited battery storage [19]. Since in HU technique, the device cannot simultaneously transmit information data and harvest energy, a time switching technique that assures the source to either harvest energy or transmit information data is proposed in [20]. Time switching is necessary as the information and energy receivers in practice operate with different power levels [21].

In this paper, we propose an energy-efficient power allocation technique in a point-to-point Rayleigh flat-fading channel that is equipped with an energy harvesting battery source in addition to its fixed battery. HU technique is used with time switching approach.[1] In the considered system, the Shannon limit is a function of the total transmission power, which is the sum of power consumed from the fixed battery and the HU battery. We formulate EE as a ratio of Shannon limit to the sum of the circuit power and the power consumed from the fixed-battery. We first look at an EE-maximization problem of a system without a fixed-battery power limit. By converting the EE quasi-concave maximization problem into a concave optimization problem, we obtain the optimal power allocation strategy to find the input power, $\overline{P}_{\mathrm{u}}$, that

[1] As storing energy for future use should always be beneficial, HSU is being considered for an ongoing future work. The work presented in this paper paves the way for considering more complicated EH techniques in future.

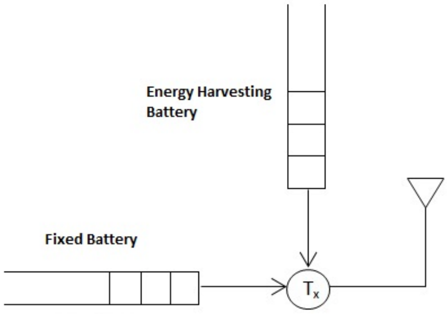

Fig. 1: System model.

maximizes EE. We analytically prove that EE increases with harvesting power, whereas, $\overline{P}_{\mathrm{u}}$ decreases with it. We further prove that in a system with fixed-battery power limit of P_{max}, the optimum operational power should be set at $\min(\overline{P}_{\mathrm{u}}, P_{\mathrm{max}})$. The effect of harvested energy along with time switching rate is analytically investigated in the paper. Closed-form expressions for maximum EE are derived.

II. System Model

We consider a point-to-point communication over a wireless fading channel with a total bandwidth of B. The channel is block fading where, the channel gain remains invariant in each fading-block and is independent from one fading-block to another. The channel state information (CSI) is estimated at the receiver and is assumed to be fed back to transmitter through an error-free feedback channel.

The transmitter is equipped with a fixed battery and an additional energy-harvesting battery, as shown in Fig. 1. HU technique, which implies that harvested energy is utilized by the transmitter whenever it becomes available, is considered in this paper. The proposed model can replenish energy from additional energy sources, e.g, radio frequency. At transmitter, the device either harvests energy or transmits data. Therefore, time switching approach is used as shown in Fig. 2. For each frame transmission, $0 \leq \tau \leq 1$ is the fraction of time in which energy is harvested, and $1-\tau$ is the remaining time for information transmission. In this case, the information rate R in units of b/s/Hz is given by

$$R = (1-\tau)\mathbb{E}\left(\log_2 \left(1 + (P_{\mathrm{t}}(t) + k_{\mathrm{eh}}\tau)\frac{\gamma}{K_{\mathcal{L}}}\right)\right),$$

where $P_{\mathrm{t}}(t)$ is the instantaneous transmission power at time t, γ is the channel power gain, $k_{\mathrm{eh}}\tau$ is the harvested energy, $\mathbb{E}(.)$ indicates the expectation operator, $K_{\mathcal{L}} = P_{\mathrm{L}}\sigma_{\mathrm{n}}^2 B$, with P_{L} indicating the path loss, and σ_{n}^2 representing the additive white gaussian noise (AWGN) variance.

EE, which is defined as the number of bits per unit power consumed from the fixed battery, is formulated as ratio of information rate, R, to the sum of total transmission power $P_{\mathrm{t}}(t)$, and circuit power, P_{c}, given by $P_{\mathrm{total}}(t) = P_{\mathrm{c}} + \dfrac{1}{\varepsilon}\mathbb{E}\Big(P_{\mathrm{t}}(t)\Big)$, where ε is the power amplifier efficiency with the range of $0 \leq \varepsilon \leq 1$. Therefore, the maximum achievable EE, defined by η, can be expressed as

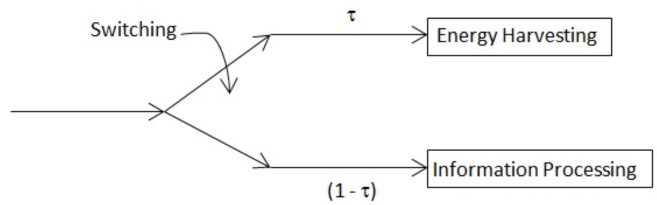

Fig. 2: Time switching scheme for energy harvesting and information processing.

$$\eta = \max_{P_{\mathrm{t}}(t)\geq 0} \frac{\mathbb{E}(R)}{P_{\mathrm{c}} + \frac{1}{\varepsilon}\mathbb{E}(P_{\mathrm{t}}(t))}. \tag{1}$$

III. Energy-Efficient Power Allocation

Given that the rate and energy consumption are determined by the transmit power of a system, EE can be optimized by adaptively allocating power based on channel condition and the system requirements. The instantaneous transmit power $P_{\mathrm{t}}(t)$ is, hence, replaced by $P_{\mathrm{t}}(\gamma)$, which shows that the transmission power is a function of channel power gain γ. The channel power gain is assumed to be a Rayleigh fading with unit variance and the probability density function (PDF) given as $f_\gamma = \mathrm{e}^{-\gamma}$.

A. *Energy-Efficient Power allocation without Input Power Constraint*

In this section, we consider an EE-maximization problem when no constraint is imposed on the total power of the fixed battery. The results of this section will pave the way for power-constrained EE-maximization problem considered in the next section. We start by formulation the EE-maximization problem, according to

$$\eta = \max_{P_{\mathrm{t}}(\gamma)\geq 0} \frac{(1-\tau)\mathbb{E}_\gamma\left(\log_2 \left(1 + (P_{\mathrm{t}}(\gamma) + k_{\mathrm{eh}}\tau)\frac{\gamma}{K_{\mathcal{L}}}\right)\right)}{P_{\mathrm{c}} + \frac{1}{\varepsilon}\mathbb{E}_\gamma(P_{\mathrm{t}}(\gamma))},$$

where $\mathbb{E}_\gamma(.)$ is the expectation operator as a function of channel power gain γ. Note that, the maximum achievable EE with additional harvesting energy is different from the traditional EE, as in numerator, the harvested energy $k_{\mathrm{eh}}\tau$ is added to total transmission power $P_{\mathrm{t}}(\gamma)$. The addition of an extra harvested battery will help the transmitter to immediately use energy, when it is available.
The EE-maximization problem can further be normalized with $K_{\mathcal{L}}$, yielding

$$\eta = \max_{P_{\mathrm{r}}(\gamma)\geq 0} \frac{(1-\tau)\mathbb{E}_\gamma\left(\log_2 \left(1 + (P_{\mathrm{r}}(\gamma) + k_{\mathrm{ehr}}\tau)\gamma\right)\right)}{K_{\mathcal{L}}\left(P_{\mathrm{cr}} + \frac{1}{\varepsilon}\mathbb{E}_\gamma(P_{\mathrm{r}}(\gamma))\right)}. \tag{2}$$

Here, the signal-to-noise, circuit-to-noise and harvest-to-noise power ratios are represented by $P_{\rm r}(\gamma) = P_{\rm t}(\gamma)/K_{\mathcal{L}}$, $P_{\rm cr} = P_{\rm c}/K_{\mathcal{L}}$ and $k_{\rm ehr} = k_{\rm eh}/K_{\mathcal{L}}$. The maximization problem (2) involves maximization of a ratio of two functions of $P_{\rm r}(\gamma)$, and is not concave [22]. However, the denominator of (2) is affine and numerator is concave in transmission power. The concavity proof of R can be obtained using [23]. The EE-maximization objective function is, hence, a strictly quasi-concave function in $P_{\rm r}(\gamma)$ with a unique global maximum. A general methodology is used for transformation of quasi-concave function into a concave optimization problem through fractional programming [22]. Using variable transformation with inverse power dissipation parameter for $t = P_{\rm cr} + \frac{1}{\varepsilon}\mathbb{E}_\gamma(P_{\rm r}(\gamma))$, the EE-maximization problem is converted into

$$\eta = \max \qquad t^{-1}\Bigg((1-\tau)\mathbb{E}_\gamma\Bigg(\log_2\big(1+(P_{\rm r}(\gamma)+k_{\rm ehr}\tau)\gamma\big)\Bigg)\Bigg) \tag{3}$$

subject to:

$$t^{-1}\Bigg(P_{\rm cr}+\frac{1}{\varepsilon}\mathbb{E}_\gamma(P_{\rm r}(\gamma))\Bigg)=1 \tag{4}$$

$$P_{\rm r}(\gamma)\geq 0. \tag{5}$$

The objective function in (3) is concave in $P_{\rm r}(\gamma)$, continuously differentiable, and equality constraint is affine. Therefore, the Karush-Kuhn-Tucker (KKT) conditions are both sufficient and necessary for optimal solution. Considering the Lagrangian multiplier, λ, the Lagrangian function is formulated as

$$L(P_{\rm r}(\gamma),t) = t^{-1}\Bigg((1-\tau)\mathbb{E}_\gamma\Bigg(\log_2\left(1+(P_{\rm r}(\gamma)+k_{\rm ehr}\tau)\gamma\right)\Bigg)\Bigg) \\ -\lambda\Bigg(\Bigg(t^{-1}(P_{\rm cr}+\frac{1}{\varepsilon}\mathbb{E}_\gamma\left(P_{\rm r}(\gamma)\right)-1\Bigg)\Bigg). \tag{6}$$

The stationary conditions are, hence,

$$\frac{\partial L(P_{\rm r}(\gamma),t)}{\partial P_{\rm r}(\gamma)}=0 \implies \tag{7}$$

$$\frac{(1-\tau)(\gamma)}{1+\left(P_{\rm r}(\gamma)+k_{\rm ehr}\tau\right)\gamma}-\frac{\lambda}{\varepsilon}=0, \tag{8}$$

and

$$\frac{\partial L(P_{\rm r}(\gamma),t)}{\partial t}=0 \implies \tag{9}$$

$$(1-\tau)\mathbb{E}_\gamma\Bigg(\log_2\left(1+(P_{\rm r}(\gamma)+k_{\rm ehr}\tau)\gamma\right)\Bigg) \\ -\lambda\left(P_{\rm cr}+\frac{1}{\varepsilon}\mathbb{E}_\gamma(P_{\rm r}(\gamma))\right)=0. \tag{10}$$

Now, the EE-maximization power allocation strategy can be derived using (7), according to,

$$P_{\rm r}(\gamma)=\left[\frac{\varepsilon(1-\tau)}{\lambda}-\frac{1}{\gamma}-k_{\rm eh_r}\tau\right]^+, \tag{11}$$

where $[x]^+$ return the $\max(0,x)$. We note that the power allocation (11) is different from traditional water-filling approach in a sense that (11) is scaled and shifted version of traditional water-filling power allocation. This is due to the presence of an additional harvested energy, $k_{\rm eh}\tau$, and time switching parameter, τ. The expectation in (10) can be solved to carry out a closed-form expression. In order to obtain λ, we insert (11) into (10) and solve the expectation operators, yielding

$$(1-\tau)\log_2[((1-\tau)\beta)]{\rm e}^{-\beta}+(1-\tau){\rm Ei}(\beta)- \\ \left[(1-\tau){\rm e}^{-\beta}\log_2\left(\frac{\beta(1-\tau)}{1+\beta\tau k_{\rm ehr}}\right)\right]-\frac{\beta\varepsilon(1-\tau)P_{\rm cr}}{(1+\beta\tau k_{\rm ehr})} \\ -\frac{(1-\tau){\rm e}^{-\beta}}{1+\beta\tau k_{\rm ehr}}+\frac{\beta(1-\tau){\rm Ei}(\beta)}{1+\beta\tau k_{\rm ehr}}=0. \tag{12}$$

where $\beta = \frac{\lambda}{\varepsilon(1-\tau)-\tau k_{\rm eh_r}}$, and ${\rm Ei}(\beta)$ indicates the exponential integral which is defined by ${\rm Ei}(\beta)=\int_\beta^\infty \frac{{\rm e}^{-t}}{t}dt$ [23]. Let us assume β^* is the optimal β, that solves (12). The average input power at this point $\overline{P}_{\rm u}$ can, hence, be found as $\overline{P}_{\rm u} = \mathbb{E}_\gamma(P_{\rm t}(\gamma))\Big|_{\beta=\beta^*}$, wherein $P_{\rm t}(\gamma) = \frac{P_{\rm r}(\gamma)}{K_{\mathcal{L}}}$, for which $P_{\rm r}(\gamma)$ is given in (11). Using $\overline{P}_{\rm u}$, one can show that the unconstrained EE-maximization problem can be simplified into a SE-maximization problem, subject to an input power constraint with the constraint power level set at $\overline{P}_{\rm u}$.

B. *Effect of $k_{ehr}\tau$ on $\overline{P}_u$*

The EE-optimal power allocation strategy (11) depends on β^*, which itself depends on the harvested power $k_{\rm ehr}$, and the time switching parameter τ. The effect of $k_{\rm ehr}\tau$ on β^*, and in-turn, on the optimum power allocation, however, cannot be analysed clearly from (11) and (12). Here, we want to analyse the effect of $k_{\rm ehr}\tau$ on the maximum achievable EE and inturn on $\overline{P}_{\rm u}$. We start by analyzing the effect of $k_{\rm ehr}\tau$ over $\overline{P}_{\rm u}$.

Lemma 1: The optimum input power $\overline{P}_{\rm u}$ monotonically decreases with $k_{\rm ehr}\tau$.

Proof: The proof is provided in Appendix A.

Lemma 1 implies that the average power associated to the maximum achievable power-unconstrained EE is a decreasing function of $k_{\rm ehr}\tau$.

C. *Energy-Efficient Power Allocation under Input Power Constraint*

In this section, the EE-maximization problem under an average transmit power constraint with a limit set at $P_{\rm max}$ is considered. In this case, the EE-maximization problem is formulated, according to

$$\eta = \max_{P_{\rm r}(\gamma)\geq 0} \frac{(1-\tau)\mathbb{E}_\gamma\Bigg(\log_2\left(1+(P_{\rm r}(\gamma)+k_{\rm ehr}\tau)\gamma\right)\Bigg)}{K_{\mathcal{L}}\left(P_{\rm cr}+\frac{1}{\varepsilon}\mathbb{E}_\gamma(P_{\rm r}(\gamma))\right)} \tag{13}$$

subject to:

$$\mathbb{E}_\gamma(P_{\rm r}(\gamma))\leq\frac{P_{\rm max}}{K_{\mathcal{L}}}, \tag{14}$$

Using the results of the last section, the power-unconstrained EE-maximization problem simplifies into a power-constrained

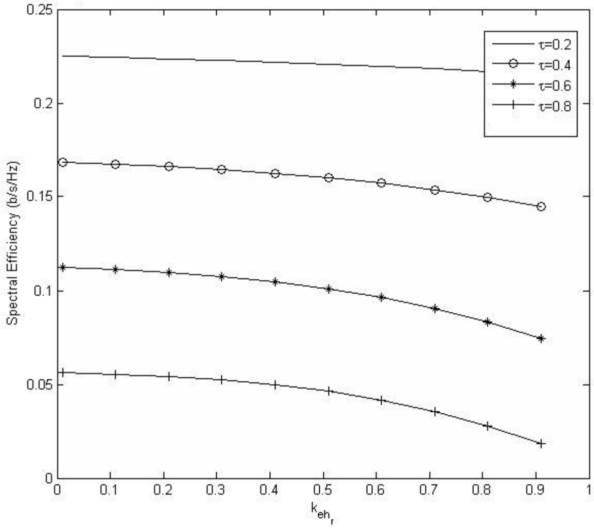

Fig. 3: SE versus harvesting power $k_{\mathrm{eh_r}}$ for various values of τ for unconstrained EE-maximization case.

SE-maximization problem, with a power level set at $\overline{P}_{\mathrm{u}}$. Therefore, (13) and (14) can be simplified to

$$\max_{P_{\mathrm{r}}(\gamma)\geq 0}(1-\tau)\mathbb{E}_{\gamma}\Bigg(\log_2\Big(1+(P_{\mathrm{r}}(\gamma)+k_{\mathrm{eh_r}}\tau)\gamma\Big)\Bigg) \quad (15)$$

subject to:

$$\mathbb{E}_{\gamma}(P_{\mathrm{r}}(\gamma)) \leq \frac{P_{\max}}{K_{\mathcal{L}}}, \quad (16)$$

$$\mathbb{E}_{\gamma}(P_{\mathrm{r}}(\gamma)) \leq \frac{\overline{P}_{\mathrm{u}}}{K_{\mathcal{L}}}, \quad (17)$$

Since, in SE-maximization problem, the optimum input power level is set at the boundary, the optimum operational power for (15) should be set at $\min(\overline{\mathrm{P}}_{\mathrm{u}}, P_{\max})$.

IV. Numerical Results

In this section, we numerically evaluate EE (b/J/Hz) and SE (b/s/Hz) under fixed-battery power constraint, with respect to harvesting power $k_{\mathrm{eh_r}}$ and time switching parameter τ.

We start by plotting SE versus the harvesting power $k_{\mathrm{eh_r}}$ for various values of τ in a Rayleigh fading channel with $P_{\mathrm{cr}} = 2$dB in Fig. 3. The figure shows that SE decreases with increase in τ and $k_{\mathrm{eh_r}}$. This verifies the results in Appendix A and Lemma 1 which imply that, the maximum EE is achieved at lower values of $\overline{P}_{\mathrm{u}}$ when $k_{\mathrm{eh_r}}$ increases. Since SE is a monotonically increasing function of $\overline{P}_{\mathrm{u}}$, we conclude that less SE can be achieved at the EE-optimal point when $k_{\mathrm{eh_r}}$ increases.

Fig. 4 and Fig. 5 are the SE-maximization plots with a power constraint limit set at $P_{\max}$.[2] Fig. 4 includes the plots for SE versus time switching parameter τ, for various

[2]The plots presented in Fig. 4 and Fig. 5 are the results of an SE-maximization problem, rather than an EE-maximization problem, with HU approach.

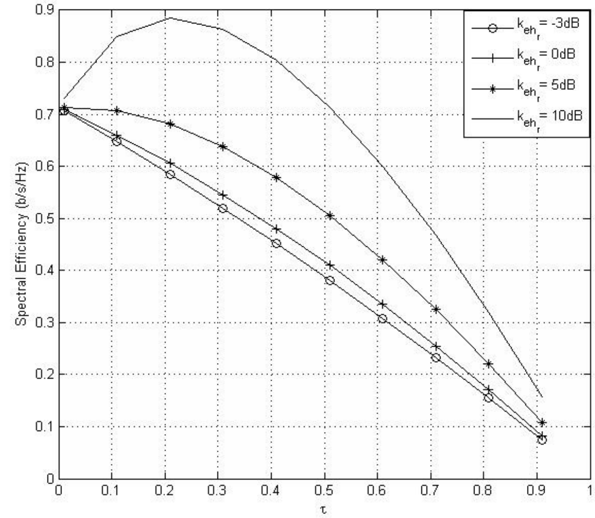

Fig. 4: SE versus τ for various energy harvesting values, $k_{\mathrm{eh_r}}$, with $P_{\max} = 0$dB.

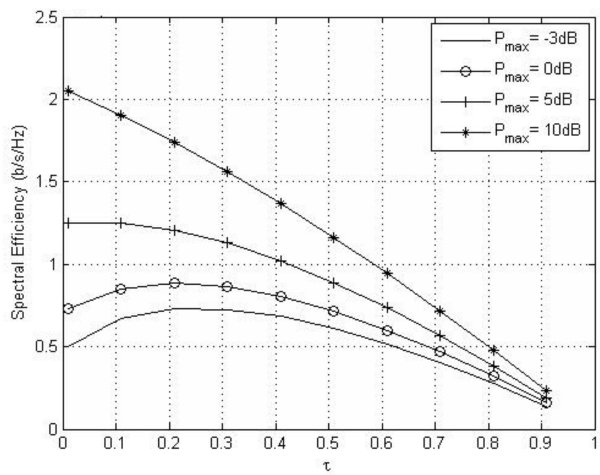

Fig. 5: SE versus τ for various values of $P_{\max}$ with $k_{\mathrm{eh_r}} = 10$dB.

harvesting power values with $P_{\max} = 0$dB. The figure shows that for the values of $k_{\mathrm{eh_r}} \leq 0$dB, SE monotonically decreases with τ. This happens since $k_{\mathrm{eh_r}}\tau$ is much smaller than $P_{\max}$, and the best use of time is to transmit information rather than to harvest energy. Similarly, with higher $k_{\mathrm{eh_r}}$, SE increases upto an optimal value, corresponding at a certain value for τ, and then decreases towards zero.

Fig. 5 shows the impact of maximum average transmit power limit, $P_{\max}$, on SE which is plotted versus the time switching parameter, τ, at $k_{\mathrm{eh_r}} = 10$dB. As the energy harvesting parameter τ increases, SE increases until it reaches its maximum value, after which it decreases towards zero. For $P_{\max} = 10$dB, SE is monotonically decreasing, whereas for the rest of the plotted figures with smaller values for $P_{\max}$, the curve is bell-shaped. The figure further shows that the maximum SE is achieved at lower values of τ for smaller $P_{\max}$.

Fig. 6 includes the plots for EE versus SE with $\tau = 0.8$,

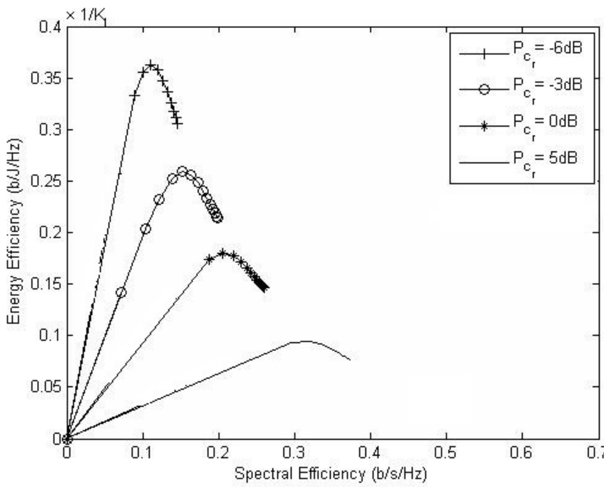

Fig. 6: EE versus SE for various values of circuit power P_c with $\tau = 0.8$ and $k_{\mathrm{ehr}} = 0$dB.

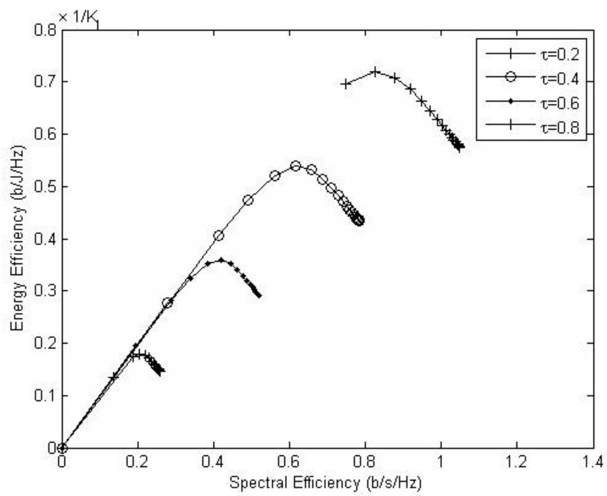

Fig. 7: EE versus SE for various values of time switching parameter τ with $k_{\mathrm{ehr}} = 0$dB and $P_{\mathrm{cr}} = 0$dB.

harvesting power $k_{\mathrm{ehr}} = 0$dB, and various values of P_{cr}. The results illustrate a higher EE against smaller SE at lower circuit powers. The optimal input power for each value for P_{cr} has been shifted to the left, i.e., the EE curve shrinks toward the zero y-axis. In this way, a higher EE is achieved at a lower average operational power.

Fig. 7 shows the plots for EE versus SE with $P_{\mathrm{cr}} = 0$dB, harvesting power $k_{\mathrm{ehr}} = 0$dB, and various values of τ. In this particular setting of k_{ehr} and P_{cr}, the figure shows that as the harvested energy portion of time (τ) is increased, both EE and SE increase. So, the choice of τ depends on the chosen k_{ehr} and P_{cr}. The EE bell shaped curve expands as τ increases resulting in achieving higher EE, as well as higher SE. Therefore, it is prominent for a system which is required to be both energy and spectral efficient.

V. Conclusion

We proposed an energy-efficient power allocation technique for a transmitter that is equipped with an energy harvesting technique to achieve maximum EE and throughput simultaneously. Harvest-Use technique is used for an additional source of energy which is used immediately as it becomes available. A time switching approach in which the transmitter switches over time between harvesting energy and transmitting data is defined. We show that the optimal EE is derived through a scaled water-filling power allocation scheme. A system with more harvested power is shown to achieve higher EE at a lower transmission power. The illustrative results show significant enhancement in maximum achievable EE as a result of an additional harvesting power. The selection for the time switching parameter is shown to depend on the harvesting power and the circuit power values, e.g., when the harvesting power is much smaller than P_{cr}, the best option is to transmit for the whole time. Whereas, as k_{ehr} increases, at a certain portion of time, we should set the time switching parameter so that we achieve the maximum EE. Harvest-Store-Use (HSU) technique, when the harvesting energy can be modelled as a random process, is considered for future work.

Appendix A

We want to prove that $\overline{P}_{\mathrm{u}}$ decreases with increase in harvested energy $k_{\mathrm{ehr}}\tau$. We note that rate, R, is a concave function of transmission power $\mathbb{E}_\gamma(P_{\mathrm{r}}(\gamma))$, which is non-decreasing linear function of $P_{\mathrm{r}}(\gamma)$. Hence, EE is a quasi-concave function of $\mathbb{E}_\gamma(P_{\mathrm{r}}(\gamma))$, and its maximum can be achieved when $\eta' = 0$, with η' indicating the first derivative of η with respect to $\mathbb{E}_\gamma(P_{\mathrm{r}}(\gamma))$. This means that EE monotonically increases with $\mathbb{E}_\gamma(P_{\mathrm{r}}(\gamma))$ until it reaches its maximum and then it becomes a monotonically decreasing function of $\mathbb{E}_\gamma(P_{\mathrm{r}}(\gamma))$.

Now let us consider a system with fixed P_{cr} and energy harvested power value of $k_{\mathrm{ehr}_1}\tau$. We take the first derivative of η of the system with respect to $\mathbb{E}(P_{\mathrm{r}}(\gamma))$, yielding,

$$\eta'(\gamma)\Bigg|_{\substack{\mathbb{E}_\gamma(P_{\mathrm{r}}(\gamma))=\overline{P}^*_{\mathrm{u}_1}\\ k_{\mathrm{ehr}}=k_{\mathrm{ehr}_1}}} = \frac{R'(\gamma)(\overline{P}^*_{\mathrm{u}_1} + P_{\mathrm{cr}} - k_{\mathrm{ehr}_1}\tau) - R(\gamma)}{(\overline{P}^*_{\mathrm{u}_1} - k_{\mathrm{ehr}_1}\tau + P_{\mathrm{cr}})^2} = 0. \tag{18}$$

In more detail, (18) implies that for a system with $k_{\mathrm{ehr}}\tau$, η is maximized when $\mathbb{E}_\gamma(P_{\mathrm{r}}(\gamma)) = \overline{P}^*_{\mathrm{u}_1}$, and as a result, $\eta' = 0$ at $\mathbb{E}_\gamma(P_{\mathrm{r}}(\gamma)) = \overline{P}^*_{\mathrm{u}_1}$.
Now assume a system with higher energy harvesting power, i.e., $k_{\mathrm{ehr}_2}\tau = k_{\mathrm{ehr}_1}\tau + \Delta k_{\mathrm{ehr}}\tau$, when $\Delta k_{\mathrm{ehr}}\tau \geq 0$. In this system, the input power at which EE is maximized is achieved by $\overline{P}^*_{\mathrm{u}_2}$. Now update (18) with $k_{\mathrm{ehr}_2}\tau$ gives

$$\eta'(\gamma)\Bigg|_{\substack{\mathbb{E}_\gamma(P_{\mathrm{r}}(\gamma))=\overline{P}^*_{\mathrm{u}_1}\\ k_{\mathrm{ehr}_2}=k_{\mathrm{ehr}_1}+\Delta k_{\mathrm{ehr}}}} =$$

$$\frac{R'(\gamma)(\overline{P}^*_{\mathrm{u}_1} - (k_{\mathrm{ehr}_1}\tau + \Delta k_{\mathrm{eh}}\tau_{\mathrm{r}}) + P_{\mathrm{cr}}) - R(\gamma)}{(\overline{P}^*_{\mathrm{u}_1} - (k_{\mathrm{ehr}_1}\tau + \Delta k_{\mathrm{ehr}}\tau) + P_{\mathrm{cr}})^2}. \tag{19}$$

Using (18), we can further simplify (19), according to

$$\eta'(\gamma)\Bigg|_{\substack{\mathbb{E}_{\gamma}(P_{\mathrm{r}}(\gamma))=\overline{P}^{*}_{\mathrm{u}_1}\\ k_{\mathrm{ehr}_2}=k_{\mathrm{ehr}_1}+\Delta k_{\mathrm{ehr}}}} = \frac{-R'(\gamma)(\Delta k_{\mathrm{ehr}}\tau)}{(\overline{P}^{*}_{\mathrm{u}_1}-(k_{\mathrm{ehr}_1}\tau+\Delta k_{\mathrm{ehr}}\tau)+P_{\mathrm{cr}})^2}$$

$$\leq 0. \tag{20}$$

which shows $\eta'(\gamma)\Big|_{k_{\mathrm{ehr}}=k_{\mathrm{ehr}_2}}$ is decreasing at $\mathbb{E}_{\gamma}(P_{\mathrm{r}}(\gamma)) = \overline{P}_{\mathrm{u}_1}$, henceforth, η has already reached its maximum, which implies that $\overline{P}^{*}_{\mathrm{u}_2} \leq \overline{P}^{*}_{\mathrm{u}_1}$.

References

[1] E. Uysal-Biyikoglu, B. Prabhakar, and A. El Gamal, "Energy-efficient packet transmission over a wireless link," *IEEE/ACM Transactions on Networking (TON)*, vol. 10, no. 4, pp. 487–499, 2002.

[2] Z. Hasan, H. Boostanimehr, and V. K. Bhargava, "Green cellular networks: A survey, some research issues and challenges," *Communications Surveys & Tutorials, IEEE*, vol. 13, no. 4, pp. 524–540, 2011.

[3] X. Fafoutis, D. Vuckovic, A. Di Mauro, N. Dragoni, and J. Madsen, "Energy-harvesting wireless sensor networks," in *9th European Conference on Wireless Sensor Networks (EWSN 2012)*, 2012, pp. 84–85.

[4] G. Miao, N. Himayat, and G. Y. Li, "Energy-efficient link adaptation in frequency-selective channels," *Communications, IEEE Transactions on*, vol. 58, no. 2, pp. 545–554, 2010.

[5] R. S. Prabhu and B. Daneshrad, "An energy-efficient water-filling algorithm for OFDM systems," in *Communications (ICC), 2010 IEEE International Conference on*, 2010, pp. 1–5.

[6] J. Xu and R. Zhang, "Throughput optimal policies for energy harvesting wireless transmitters with non-ideal circuit power," *Selected Areas in Communications, IEEE Journal on*, vol. 32, no. 2, pp. 322–332, 2014.

[7] X. Zhou, R. Zhang, and C. K. Ho, "Wireless information and power transfer: architecture design and rate-energy tradeoff," in *Global Communications Conference (GLOBECOM), 2012 IEEE*, 2012, pp. 3982–3987.

[8] H.-H. Chen, Y. Qian, and S. Zhou, "Energy conservation and harvesting for green communications," *Communications, China*, vol. 11, no. 3, pp. i–ii, 2014.

[9] D. W. K. Ng, E. S. Lo, and R. Schober, "Wireless information and power transfer: Energy efficiency optimization in OFDMA systems," *CoRR*, vol. abs/1303.4006, 2013.

[10] L. Liu, R. Zhang, and K.-C. Chua, "Wireless information transfer with opportunistic energy harvesting," *Wireless Communications, IEEE Transactions on*, vol. 12, no. 1, pp. 288–300, 2013.

[11] J. Yang and S. Ulukus, "Transmission completion time minimization in an energy harvesting system," in *Information Sciences and Systems (CISS), 2010 44th Annual Conference on*. IEEE, 2010, pp. 1–6.

[12] T. Chen, Y. Yang, H. Zhang, H. Kim, and K. Horneman, "Network energy saving technologies for green wireless access networks," *Wireless Communications, IEEE*, vol. 18, no. 5, pp. 30–38, 2011.

[13] X. Lu, P. Wang, D. Niyato, D. I. Kim, and Z. Han, "Wireless networks with rf energy harvesting: A contemporary survey," *arXiv preprint arXiv:1406.6470*, 2014.

[14] Z. Popovic *et al.*, "Resistor emulation approach to low-power rf energy harvesting," *IEEE Trans. Power Electronics*, vol. 23, no. 3, 2008.

[15] R. Rajesh, V. Sharma, and P. Viswanath, "Information capacity of energy harvesting sensor nodes," in *Information Theory Proceedings (ISIT), 2011 IEEE International Symposium on*, 2011, pp. 2363–2367.

[16] I. Krikidis, G. Zheng, and B. Ottersten, "Harvest-use cooperative networks with half/full-duplex relaying," in *Wireless Communications and Networking Conference (WCNC), 2013 IEEE*, 2013, pp. 4256–4260.

[17] B. Medepally and N. B. Mehta, "Voluntary energy harvesting relays and selection in cooperative wireless networks," *Wireless Communications, IEEE Transactions on*, vol. 9, no. 11, pp. 3543–3553, 2010.

[18] N. S. Shenck and J. A. Paradiso, "Energy scavenging with shoe-mounted piezoelectrics," *IEEE Micro*, vol. 21, no. 3, pp. 30–42, 2001.

[19] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: Optimal policies," *Selected Areas in Communications, IEEE Journal on*, vol. 29, no. 8, pp. 1732–1743, 2011.

[20] Y. Gu and S. Aissa, "Interference aided energy harvesting in decode-and-forward relaying systems," in *Communications (ICC), 2014 IEEE International Conference on*, 2014, pp. 5378–5382.

[21] L. R. Varshney, "Transporting information and energy simultaneously," in *Information Theory, 2008. ISIT 2008. IEEE International Symposium on*. IEEE, 2008, pp. 1612–1616.

[22] S. Schaible, "Minimization of ratios," *Journal of Optimization Theory and Applications*, vol. 19, no. 2, pp. 347–352, 1976.

[23] S. Boyd and L. Vandenberghe, "Convex optimization. 2004," *Cambridge Univ. Pr*, 2004.