A SIGNALLING-BASED THEORY OF CONTRACTUAL COMMITMENT TO RELATIONSHIPS

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ABSTRACT. In this paper I present signalling as an explanation for how and why parties commit to relationships when they initially contract about the terms of those relationships. Two forms of contractual commitment to a relationship are considered: a promise to trade in the future (contracted quantity); and a promise not to trade with anyone else (contracted exclusivity). A party is said to commit more to a relationship if it commits initially to trade a higher quantity and/or to a higher level of exclusivity. I characterize equilibrium contracts and therefore commitment. Both the ability to signal information through an exclusivity commitment and whether the informed party commits more to the relationship when the relationship is more likely to succeed depend on the source of the asymmetry of information.

Keywords: Contractual commitment, signalling, informed principal.

JEL Classification: D2, D8, L14, J41.

1. Introduction

Parties’ initial commitment to relationships may vary considerably from one case to another. For example, in the academic world, while universities often hire scholars offering them a tenured position, sometimes universities also hire scholars under a non-tenure contract that may be extended later on. Similarly, in the case of vertical relationships, while manufacturers often sell their products to consumers through many retailers, it is also common for manufacturers to concede exclusivity to one retailer. Such exclusivity contracts often take the specific form of exclusive territories. Finally, also in the case of vertical relationships, producers frequently write contracts with a given supplier pre-ordering almost all of their needs of a specific input they will use in the future, while in some other cases they follow a more conservative approach by initially agreeing on more modest supply contracts that may be revised upward later on.

A few explanations for how parties initially commit (i.e. at the ex-ante contracting stage) to a relationship have been put forward in the literature. For example, the hold up literature

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highlights that a buyer may commit to trade a high quantity (or exclusively) with a supplier
to provide the supplier with incentives to invest in relationship-specific investment (e.g., Edlin
and Reichelstein, 1996; De Meza and Selvaggi, 2007). Similarly, in situations where agents
are not risk neutral, parties may commit ex-ante to trade a high (or a low) quantity so
as to achieve efficient risk allocation (e.g., Chung, 1991). In this paper, I present another
explanation for how parties initially commit to a relationship: signalling information about
the value of the relationship to the other party. The theory developed in the paper offers
clear predictions on how parties commit ex-ante to relationships. In particular, it highlights
which forms of commitment parties may use to signal information depending on the type of
private information they have, as well as circumstances under which they will commit more
(or less) to a relationship.

Instead of analyzing one of the specific examples mentioned above, I consider a more styl-
ized model of contracting under asymmetric information. Specifically, I consider a situation
where two parties, a principal (e.g., a buyer) and an agent (e.g., a supplier), contract on the
terms of a future transaction, knowing that the principal may later wish to trade with an
external party instead. At the contracting stage, the value of trade between the principal and
the agent is still uncertain. So is the value of trade between the principal and the external
party. However, the principal is better informed than the agent about these values. For
example, a firm may know better its own valuation of a good or service than the supplier
of that good or service; or the firm may have a better idea on the value of trading with an
alternative supplier in the future. At a later stage, before trade occurs, the values of trade are
realized and observed by the principal and the agent. At that moment they may renegotiate
the initial contract if it prescribes an inefficient level of trade. Despite renegotiation, the
initial contract is important as it determines the default positions of the principal and agent
during renegotiation and, consequently, the distribution of surplus.

The paper focuses on situations where agreeing initially with the agent on a contract, as
opposed to simply waiting to trade later with the external party, involves a basic trade-off for
the principal: it increases the (expected) total surplus from trade, but it also implies a loss
of some of the principal’s full bargaining power to the agent in future negotiations. When
two parties agree on a future transaction, they often begin interacting so as to prepare for it.
For example, after agreeing on a transaction, manufacturers and their suppliers frequently
work together on the potential customization of the good, planning delivery conditions, or
on eventual adjustments in their production processes. Because of such preparation, their
transaction may create more value than a transaction with an external party (e.g., a trans-
action on the spot market). This is captured in the paper by assuming that if a contract is
signed, the principal can (potentially) trade with the agent or with the external party, with
the value of trade between the principal and agent being possibly higher than that between
the principal and the external party; and that if a contract is not signed the principal can
only trade with the external party. But another implication of agreeing on a contract and
of such pre-trade interactions is that they often allow some learning about the other party
in the relationship, which may shift parties’ bargaining power in future negotiations. For example, a supplier may learn how important timely delivery of the good or service is to the manufacturer, or may even learn about negotiation techniques usually employed by the manufacturer.¹ The increase in the agent’s bargaining power means that the agent appropriates some of the renegotiation surplus when renegotiating a contract that turns out to be inefficient ex-post. This generates a problem of rent extraction for the principal when she negotiates the initial contract with the agent. A problem that becomes intricate because she has private information on the value of the relationship.

I analyze how the principal designs the initial contract, in particular, how she commits ex-ante to the relationship with the agent so as to transmit information to the agent about the value of their relationship and appropriate more of the rents created by the relationship. The principal can commit to the relationship in two ways: by promising ex-ante to trade a given quantity with the agent (contracted quantity) and/or by promising not to trade with the external party in the future (contracted exclusivity). The way in which the principal commits to the relationship depends on the type of private information she has. Three different types of private information are considered in the paper.

Suppose first the principal’s private information is about the principal’s value of trading with the agent. When the principal is a buyer and the agent is a supplier, for example, this corresponds to the situation where the buyer is better informed than the supplier about her valuation of the supplier’s input. I show that the principal commits initially to trade a higher quantity with the agent when she expects a higher value of trading with the agent. By doing so, the principal signals that the relationship is likely to create a high value, leading the agent to accept a contract that is more favorable to the principal. In fact, by choosing contracted quantity appropriately, the principal appropriates all the surplus generated by the relationship. A high contracted quantity credibly signals a high valuation of trade because it is more “costly” to a principal whose valuation of trade with the agent is low to commit ex-ante to trade a higher quantity with the agent than it is to a principal whose valuation of trade with the agent is high. This holds even if the contract can be renegotiated. Everything else equal in the contract, a higher contracted quantity leaves the principal in a weaker bargaining position (which is determined by the initial contract) in the event of a renegotiation when her value of trade with the agent is low than when it is high. In this case, the principal commits more to the relationship (i.e., commits to trade a higher quantity with the agent) when their relationship creates more value.

Suppose now the principal’s private information is about the agent’s value of trading with the principal. As an example of such a situation, consider a buyer (the principal) contracting

¹The idea that parties’ bargaining power may evolve during a relationship is not new. Williamson (1985) refers to the change in bargaining positions that may occur when parties make relationship-specific investments as the “Fundamental Transformation”. Because of this transformation, even the initial winner of a bidding competition may thereafter enjoy an advantage over rival suppliers. While investments by parties are not explicitly considered and modeled in this paper, they could eventually be another explanation for the change in the agent’s bargaining power.
with a new supplier (the agent) about the delivery of a specific input that the buyer needs. The buyer may have better information than the new supplier about the production cost of the input. This may be because of the buyer’s past experience with other suppliers of that (or similar) inputs, or because the buyer used to produce the input and is now outsourcing its production. In this case, the principal commits ex-ante to trade a lower quantity when she expects the agent’s value of trade with her to be higher. This is because a commitment to trade a high quantity with the agent is more costly to the principal when the agent’s value of trade is high than when it is low. Even when the initial contract is renegotiated, everything else equal in the contract, a higher contracted quantity leaves the agent in a better position during renegotiation when his value of trade with the principal is higher. Interestingly, in contrast with the previous case, the principal commits less to the relationship (i.e., commits to trade a lower quantity) when their relationship is expected to create more value.

These two cases constitute what is referred in the paper as the case of private internal information (as they concern private information about valuations of trade that are “internal” to the relationship). In both, a commitment by the principal to trade exclusively with the agent is totally ineffective as an instrument to signal information. This is because contracted exclusivity affects only the principal’s ability to trade with the external party in the future. Thus, when the principal has no private information on the value of trade with the external party, a commitment to trade exclusively with the agent cannot convey any of her private information. In fact, contractibility of exclusivity has no effect on the set of equilibrium payoffs of the principal and the agent. This is not the case, however, when the principal has private information about the value of trading with external parties.

When the principal’s private information is about the principal’s value of trading with the external party, a commitment by the principal to trade exclusively with the agent, or to trade a high quantity with the agent, or both, constitutes a credible signal of a low expected value of trade with the external party. This is because it is more costly to the principal to give away the possibility of trading with the external party (either directly through an exclusivity contract or through a contract where the principal allocates a large share of her “trade capacity” to trade with the agent) when she expects to have good external parties to trade with in the future than when she expects the opposite. Again, this holds even if the initial contract needs to be renegotiated. In equilibrium, the principal commits more to the relationship (through a higher contracted quantity, a higher exclusivity level, or both) when she expects a lower value of trade with the external party. Thus, as in the case where the principal has private information about her value of trading with the agent, the principal commits more to the relationship when the incremental value of the relationship is higher.

The asymmetry of information between the principal and the agent plays a crucial role in these results. If the principal and the agent have the same information about the trade valuations when they initially contract about the transaction, the initial commitment of the principal to the relationship is fully indeterminate in equilibrium. Any combination of contracted quantity and contracted exclusivity can arise in equilibrium. Regardless of
contracted quantity and exclusivity and regardless of parties’ common information about trade valuations, the principal can always extract all the expected surplus from trade through an up-front transfer. This is not possible when the principal has private information on the value of the relationship. In this case, the maximum transfer the agent is willing to accept depends on his beliefs about the value of relationship, which may depend on the type of contract offered by the principal.

The results in the paper are important because they offer clear predictions on how parties commit ex-ante to relationships. They are also important because they may have implications for competition policy. Exclusive contracts have received much attention from competition authorities because of the concern that they may be used to reduce entry and competition in the market, both of which may reduce efficiency and total welfare. The economics literature has shown that there are indeed circumstances in which this can happen, validating such concerns. For example, Aghion and Bolton (1987) show that an incumbent seller and a buyer may write a contract specifying a damage fee that the buyer has to pay the seller if she later trades with an entrant (a form of exclusive contract) so as to extract surplus from the entrant. Such contracts may deter entry of a more efficient entrant than the incumbent. Rasmusen et al. (1991) and Segal and Whinston (2000a) show that when there are economies of scale, an incumbent can profitably deter entry by writing exclusive contracts with (only) some customers. By monopolizing part of the customers through exclusivity, the incumbent actually monopolizes the entire market through the exclusion of competitors. Exclusivity contracts can also be used to reduce competition directly. For example, a manufacturer may concede exclusivity in the distribution of its products to a retailer so as to reduce competition in the retail market (and achieve higher profits). Exclusivity here serves as a commitment device. Without exclusivity, once the retailer agrees on a contract, the manufacturer may have an incentive to start selling through other retailers, which in equilibrium leads to more competition and lower profits (e.g., Hart and Tirole, 1990; O’Brien and Shaffer, 1992; and McAfee and Schwartz, 1994). And exclusivity contracts between manufacturers and retailers (or distributors) may also be used to reduce competition between manufacturers (e.g., Rey and Stiglitz, 1995; Piccolo and Reisinger, 2011).

But arguments in favor of exclusive contracts have also been put forward. The most prominent has been that they may enhance efficiency by increasing investment incentives. For example, a manufacturer may concede an exclusive territory to a retailer so as to provide the retailer with the right incentives to invest in retailer services that would otherwise be eroded by intra-brand competition. Similarly, a retailer may sell only the products of one manufacturer to increase the manufacturers investment in activities (e.g., advertising) that attracts customers to the retailer’s shop (see, e.g., Segal and Whinston, 2000b, for an analysis of the effect of exclusive contracts on investments and welfare).\footnote{Other arguments in favor of exclusive contracts (and exclusive territories) include that they can prevent inefficient entry, and that by generating enough rents to a retailer they may improve incentives for maintaining a reputation.} Thus, there is a long and
unsettled debate on the pros and cons of exclusive contracts and on whether they should be allowed by courts or not. The present paper contributes to this debate by highlighting a new motive why firms may (wish to) sign exclusive contracts, which is not anti-competitive. Firms may want to sign exclusive contracts simply to signal a low outside option and obtain a more favorable deal. Thus, the results in this paper reinforce the idea that the application by courts of the rule of reason when deciding on the legality of exclusive contracts, as is currently done in the US and in the EU, constitutes perhaps a better approach than simply seeing exclusive contracts as negative and systematically prohibiting them.

Like this paper, other articles in the literature have considered situations where the party that designs the contract has private information and have analyzed how that information affects the contract’s terms. In a more applied strand of the literature, Aghion and Bolton (1987), Aghion and Hermalin (1990) and Spier (1992) are examples. Aghion and Bolton (1987) analyze how an incumbent seller may use stipulated damages (or contract duration) when contracting with a buyer to signal the probability of entry of another seller. The analysis in the present paper differs from that in Aghion and Bolton (1987) in two aspects. First, in the present paper, it is the informed party and contract designer (the principal) that has the possibility to trade with an external party in the future, not the uninformed party. Second, the present paper analyzes a setting with ex-post renegotiation. Aghion and Hermalin (1990) show that signalling through the terms of a contract may lead to welfare losses. The authors use this result to argue that imposing restrictions on private contracts may improve welfare. Spier (1992) shows that asymmetric information may lead to contractual incompleteness. Specifically, that contractual incompleteness may signal information in the presence of transaction costs. More recently, Bénabou and Tirole (2003) and Martimort and Sand-Zantman (2006) analyze the interaction between signalling information through the contract and using it to incentivize effort; and Vasconcelos (2014) analyses the interaction between contractual signalling and using the contract to provide incentives for relationship-specific investment in a situation of hold-up.

A more theoretical strand of the literature has analyzed the contracting problem faced by the informed principal in a general framework (e.g., Myerson, 1983; Maskin and Tirole, 1990, 1992; Beaudry and Poitevin, 1993; Mylovanov and Tröger, 2014; Balkenborg and Makris, 2015). Although the setting in the present paper is more applied, it is akin to that in Maskin and Tirole (1992), with the difference that parties renegotiate contractual outcomes that are inefficient ex-post. Contract renegotiation implies that the sum of the payoffs of the principal and agent is always identical to efficient total surplus and independent of the initial contract signed by the parties.

The paper is organized as follows. In Section 2, I describe the model. In Section 3, I derive expected payoffs given an initial contract and define probability of success of the relationship.
In Section 4, I characterize equilibrium contracts and commitment for each of the different types of private information. In Section 5, I discuss a natural extension of the baseline model. In Section 6, I present concluding remarks.

2. Model

Players and Sequence of Events. A principal (e.g., a buyer) and an agent (e.g., a supplier) contract on the terms of a future transaction. Both know that ex-post the principal has the possibility of instead trading with another (external) agent. Specifically, there are two stages. At the beginning of the first stage the principal and the agent meet and contract about a transaction that is supposed to occur in stage two. During stage one, which may also involve preparation for trade by the principal and the agent (that is not explicitly modelled here), trade valuations are uncertain and the principal is better informed than the agent about them. At the beginning of stage two, the values of trade between the principal and the agent and between the principal and the external party become known. After learning these values, the principal and the agent may renegotiate the initial contract. They may decide to trade a quantity higher than that specified in the initial contract. They may also decide not to trade with each other, in which case the principal trades with the external party. All transactions occur at the end of period two. The sequence of events is illustrated in Figure 1. I next specify in detail the payoffs from trade, the principal’s superior information on trade valuations, the contracts that the principal and agent can write, and how they renegotiate the initial contract.

Payoffs From Trade. The principal values trade with the agent (per unit) as \( v_P \), the agent values trade with the principal (per unit) as \( v_A \), and the payoffs of the principal and agent are quasi-linear in money. Thus, if the principal and agent trade a quantity \( q \) they obtain, in addition to any transfers involved in the transaction, \( qv_P \) and \( qv_A \), respectively. For future convenience, the total value of trade between the principal and the agent is denoted

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v := v_P + v_A.\]

The principal values trade with the external agent (per unit) as \( v_E \). To
simplify notation, the value of trade to the external party when trading with the principal is normalized to zero. Thus, $v_E$ also denotes the total value of trade between the principal and the external party. This value of trade is assumed to be always non-negative.

(The Principal’s Superior) Information. In stage one (ex-ante), when the principal and the agent negotiate the contract, $v_P$, $v_A$ and $v_E$ are stochastic and their probability distributions (may) depend on the state of world $\theta \in \{\theta_L, \theta_H\}$. The state $\theta$ is known to the principal but not to the agent. The agent knows only that $\theta = \theta_i$ with probability $p_i$, $i = L, H$. Throughout the c.d.f. of the trade valuation $v_j$ given state $\theta$ is denoted by $F_{v_j}(. \mid \theta)$ for all $j \in \{P, A, E\}$ and $\theta \in \{\theta_L, \theta_H\}$. For convenience of exposition, the precise way in which the state of the world affects the probability distribution of the trade valuations will be specified later. The asymmetry of information between the principal and the agent vanishes at the beginning of stage two when both observe the realization of trade valuations $v_P$, $v_A$ and $v_E$.

Contracts. At the beginning of period one, the principal offers a contract to the agent. The agent either accepts the offer or rejects it. The agent’s decision depends on the contract’s terms and on his beliefs about the state $\theta$ following the observation of the contract offered. It is assumed that the agent accepts the offer whenever indifferent between accepting it and rejecting it.\[6\] If the agent accepts the offer, the principal and agent can then prepare for trade in stage two. If the agent rejects the offer, the principal and agent obtain their reservation payoffs, which are, respectively, the expected value of dealing later with the external party and zero. In the spirit of Maskin and Tirole (1992), we could allow the (informed) principal to offer the agent a menu of contracts (as opposed to a single contract). Doing so, however, would not affect the results obtained in the paper. Indeed, the set of equilibrium payoffs obtained is the same regardless of whether the principal can offer menus of contracts or just single contracts. Moreover, the characterization of the equilibrium levels of commitment (quantity and exclusivity level) obtained in the paper remain the same if the principal can offer menus of contracts.

A contract can specify an up-front transfer $t \in \mathbb{R}$ from the agent to the principal, a quantity $q \in Q \subseteq [0,1]$, and a level of exclusivity $e \in E \subseteq [0,1]$. A negative transfer $t$ corresponds to a transfer from the principal to the agent. Quantity and exclusivity are modeled as probabilities. Quantity $q$ denotes the probability that the principal and the agent must trade. The exclusivity variable $e$ denotes the probability that the agreement is exclusive, i.e., that the principal cannot trade with the external party in stage two. Hence, a contract is an object of the form $c := (t, q, e) \in C$, where $C = \mathbb{R} \times Q \times E$. The expected payoffs of the principal and the agent for any given contract $c$ are derived in the next section. Observe that the quantity and exclusivity variables can be interpreted as proportions of trade capacity. Under

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this interpretation, quantity $q$ represents the proportion of the trade capacity of the principal that is contractually allocated to the agent, and exclusivity $e$ represents the proportion of the remaining $(1-q)$ of the trade capacity of the principal that cannot be traded with an external party. Also, the assumption that $e$ is a proportion is not crucial. As it will become clear from the analysis, the results in the paper hold if contracts can only prescribe full exclusivity ($e = 1$) or full non-exclusivity ($e = 0$).

**Contract Renegotiation.** Upon learning the valuations of trade at beginning of stage two, the principal and the agent renegotiate trade to the efficient level whenever the initial contract prescribes an inefficient level of trade. Renegotiation is modeled as in Che and Hausch (1999) and Segal and Whinston (2000b, 2002), where each party obtains an exogenously determined proportion of the gains from renegotiation. Specifically, I assume that the principal and agent receive, respectively, fixed (bargaining) shares $\lambda_P$ and $\lambda_A$, where $\lambda_P + \lambda_A = 1$ and $\lambda_i \in (0,1)$, $i = A, P$, of the renegotiation surplus over the disagreement point determined by the original contract.\(^7\) Despite renegotiation, the original contract still matters because it affects the distribution of ex-post surplus, which in turn is important for surplus extraction by the principal. Finally, it is assumed that the external party with whom the principal can alternatively trade in stage two receives no surplus. This is consistent, for instance, with a case of competition amongst many external parties who are willing to deal with the principal in case she does not trade with the agent.

The equilibrium concept used is the Perfect Bayesian Equilibrium (PBE).

I conclude the description of the model with two observations. First, the fact that $\lambda_A > 0$ means that the agent has some bargaining power at the contract renegotiation stage. Since at the initial contracting stage the principal makes a take it or leave it offer to the agent, this means that the agent gains (at least) some bargaining power when a contract is signed and a relationship with the principal is initiated. There are several reasons why this may happen in reality. As mentioned in the Introduction, as part of preparing for the transaction, the agent may obtain information about the principal that puts him in a better position to negotiate with the principal. Parties may also make investments that may affect their bargaining positions. Or, the agent’s bargaining power may increase simply because an eventual renegotiation of the initial contract occurs closer to the date of the transaction, which may leave the principal more impatient to reach a deal.\(^8\)

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\(^7\)It is possible to specify an underlying bargaining game that corresponds to a constant bargaining share. Consider, for example, a generalized Nash bargaining game or a Rubinstein bargaining game with different discount factors. Some articles have considered contracts incorporating schemes that *ex-ante* manipulate parties’ future bargaining power (e.g., Chung, 1991; and Aghion et al., 1994). Such schemes are not considered here. Their implementation may be quite elaborate. Moreover, they may fail if parties always renegotiate inefficient outcomes as is the case here or face financial constraints. For a more thorough discussion of this issue, see Che and Hausch (1999).

\(^8\)This will be particularly the case if completing the transaction on time is important for the principal. For example, it might be important for a manufacturer to close a transaction with a supplier by a certain date so that the manufacturer can honour commitments with clients.
Second, while the model focuses on the contracting problem between a principal and a given agent, it is consistent with the following situation. There are initially many agents competing to trade in future (and therefore to sign a contract) with the principal; if the principal initiates a relationship with one agent by agreeing on a contract with him, trade between them generates $v_P + v_A$, while trade between the principal and each of the other (standard) agents generates $v_E$; and if the principal fails to sign a contract initially with one agent, then trade with any of the agents generates again $v_E$. In this context, agreeing on a contract with one agent creates the possibility of developing a transaction that may generate more surplus than a standard spot market transaction.

3. EXPECTED PAYOFFS AND PROBABILITY OF SUCCESS OF THE RELATIONSHIP

We are interested in characterizing the contract agreed by the principal and the agent at the beginning of stage one. The decisions of the principal and the agent regarding that contract depend on their expected payoffs given different contract choices. It is therefore convenient to derive those payoffs before proceeding to the analysis of equilibrium outcomes. The expected payoffs of the principal and agent at the beginning of stage one take into account the uncertainty on the trade valuations as well as the outcome of an eventual renegotiation. Thus, to derive them, one must first characterize the parties’ post-renegotiation payoffs.

At the renegotiation stage, the principal and agent receive their bargaining shares of the renegotiation surplus in addition to their disagreement payoffs. The disagreement payoffs of the principal and agent are the payoffs in the event they do not reach a renegotiation agreement, in which case the initial contract is executed. Given a contract $c$ and trade valuations $v_A, v_P$ and $v_E$, the disagreement payoff of the agent is $qv_A - t$ and that of the principal is $qv_P + (1 - q)(1 - e)v_E + t$. The renegotiation surplus is the difference between the efficient total surplus, which is $\max\{v, v_E\}$, and the sum of the disagreement payoffs.

From the above, it follows that given a contract $c$ and trade valuations $\varphi := (v_A, v_P, v_E)$, the agent’s post renegotiation payoff is given by

$$u_A(c; \varphi) = (qv_A - t) + \lambda_A \max\{v, v_E\} - (qv_A - t) - (qv_P + (1 - q)(1 - e)v_E + t)$$

(1)

$$= \lambda_A \max\{v, v_E\} + (1 - \lambda_A)(qv_A - t) - \lambda_A[qv_P + (1 - q)(1 - e)v_E + t].$$

Similarly, the principal’s post renegotiation payoff can be written as

$$u_P(c; \varphi) = \lambda_P \max\{v, v_E\} + (1 - \lambda_P)[qv_P + (1 - q)(1 - e)v_E + t] - \lambda_P(qv_A - t).$$

(2)

Thus, the post renegotiation payoffs of the principal and the agent depend positively on the total efficient surplus (the first term in (1) and in (2)); positively on the own disagreement payoff (the second term in (1) and in (2)); and negatively on the disagreement payoffs of the other party (the third term in (1) and in (2)). By affecting parties’ disagreement payoffs at the renegotiation stage, the initial contract affects the distribution of rents between the principal and the agent.
The ex-ante expected payoffs of the principal and the agent if they agree on a contract correspond to the expected value in stage one of their post-renegotiation payoffs. Since the principal knows $\theta$ and the agent does not, the expected payoff of the principal depends on state $\theta$, while the expected payoff of the agent depends on his beliefs about $\theta$. These beliefs may or may not coincide with the agent’s initial beliefs. The agent may revise his beliefs about $\theta$ after observing the contract offered by the principal. Thus, given contract $c$, when the agent believes that $\theta = \theta_H$ with probability $\hat{p}_H$, his expected payoff is given by

$$U_A(c; \hat{p}_H) = (1 - \hat{p}_H)E[u_A(c; \varphi) \mid \theta_L] + \hat{p}_H E[u_A(c; \varphi) \mid \theta_H].$$

The principal’s expected payoff given contract $c$ and state $\theta$ is

$$U_P(c; \theta) = E[u_P(c; \varphi) \mid \theta].$$

With a slight abuse of notation, $U_A(c; 0)$ and $U_A(c; 1)$ will be frequently denoted by $U_A(c; \theta_L)$ and $U_A(c; \theta_H)$, respectively. Using this notation, we can write $U_A(c; \hat{p}_H) = (1 - \hat{p}_H)U_A(c; \theta_L) + \hat{p}_H U_A(c; \theta_H)$.

Because the principal and the agent renegotiate the initial contract whenever it prescribes an inefficient level of trade, $u_A(c; \varphi) + u_P(c; \varphi) = s(\varphi)$, for all $c$ and $\varphi$, where $s(\varphi) := \max\{v, v_E\}$ denotes the efficient total surplus (hereinafter total surplus). Letting $S(\theta) := E[s(\varphi) \mid \theta]$ denote the expected total surplus given state $\theta$, this implies that for all $c \in C$ and $\theta \in \{\theta_L, \theta_H\}$,

$$U_A(c; \theta) + U_P(c; \theta) = S(\theta).$$

This property of the expected payoffs will be important in the analysis that follows.

A relationship between the principal and the agent is said to be successful if at the end of stage two the principal and the agent trade with each other. Because renegotiation always leads to an efficient outcome, they do so if and only if $v \geq v_E$. Hence, from an ex-ante perspective, the probability of success of their relationship is $P(\theta) := P_r[v \geq v_E \mid \theta]$. Throughout, I focus on the case in which $P(\theta)$ is strictly positive for all $\theta \in \{\theta_L, \theta_H\}$.

4. **Ex-ante commitment and surplus extraction**

We can now characterize the contract agreed by the principal and the agent in equilibrium and, consequently, equilibrium ex-ante commitment by the principal to the relationship. Since renegotiation implies that the ex-post trade decisions are always efficient, the main goal of the principal when designing the contract is to appropriate as much of the surplus generated by their relationship as possible. A problem of rent appropriation by the principal emerges here because the agent’s decision of whether or not to accept a given contract if offered by the principal may depend on the agent’s beliefs about the state $\theta$. Thus, the contract offered by the principal (and more specifically its design) is also important because it may convey
information about $\theta$ to the agent. In equilibrium, the principal and the agent always agree on a contract.\footnote{Observe, for example, that a contract specifying a sufficiently low transfer, quantity, and exclusivity level is always accepted by the agent and gives the principal a higher payoff than her outside option. This is essentially because, regardless of state $\theta$, initiating a relationship creates more joint value to the principal and agent than taking their outside options.} Therefore, in what follows, the focus is on the type of contracts they sign.

As a benchmark, consider the case where only the transfer $t$ is contractible. In this case, no commitment by the principal to the relationship is possible. Only pooling equilibria where the principal proposes as initial contract the same transfer regardless of the state $\theta$ exist. In equilibrium the agent learns nothing about state $\theta$ from the principal’s contract offer. This implies that the highest transfer that the agent is willing to accept in equilibrium is such that his expected payoff $U_A(c; p_H)$ is zero.\footnote{There is a continuum of equilibria. Any transfer $t \in [\min\{U_A(c; \theta_L), U_A(c; \theta_H)\}, U_A(c; p_H)]$ can be proposed by the principal (and accepted by the agent) in equilibrium.} In one of the states of the world the agent is left with positive surplus, which the principal would like appropriate.

I next return to the case where the contract can specify a quantity and an exclusivity levels, meaning that commitment by the principal to the relationship is possible. I first analyze the case where the principal’s private information is either about the trade valuation $v_P$ or about the trade valuation $v_A$ (private internal information) and then the case where the principal’s private information is about the “external” valuation $v_E$ (private external information).

### 4.1. Private internal information

Suppose the principal’s private information is either about $v_P$ or about $v_A$. Whether the principal has private information on $v_P$ or on $v_A$ is relevant in terms of the analysis and results. Therefore, the two cases are analyzed separately. They are defined in the following way: (1) Private information about $v_P$: $F_{v_P}(\cdot | \theta_H)$ strictly first order stochastically dominates $F_{v_P}(\cdot | \theta_L)$, and the distribution of $v_A$ does not depend on state $\theta$, i.e., $F_{v_A}(\cdot | \theta_L) = F_{v_A}(\cdot | \theta_H)$; (2) Private information about $v_A$: $F_{v_A}(\cdot | \theta_H)$ strictly first order stochastically dominates $F_{v_A}(\cdot | \theta_L)$, and the distribution of $v_P$ does not depend on state $\theta$, i.e., $F_{v_P}(\cdot | \theta_L) = F_{v_P}(\cdot | \theta_H)$.\footnote{A distribution $F(x)$ strictly first order stochastically dominates a distribution $G(x)$ if $F(x) \prec G(x)$ for all $x$ such that $G(x) \neq 0$ and $F(x) \neq 1$.} In both cases the distribution of $v_E$ is independent of $\theta$, i.e., $F_{v_E}(\cdot | \theta_L) = F_{v_E}(\cdot | \theta_H)$.

Observe that in both cases the principal expects the relationship with the agent to create more value in state $\theta_H$ than in state $\theta_L$. In the case of private information about $v_P$, this is because the principal expects her value of trading with the agent to be higher in state $\theta_H$ than in state $\theta_L$. In the case of private information about $v_A$, this is because the principal expects the agent’s value of trading with her to be higher in state $\theta_H$ than in state $\theta_L$. The above specifications imply that $P(\theta_L) \leq P(\theta_H)$. That is, the probability of success of the relationship between the principal and the agent is higher in state $\theta_H$ than in state $\theta_L$.

I begin the analysis of private internal information with an observation that follows from the fact that $v_E$ is independent of the state $\theta$. 


Proposition 1. Suppose that the principal’s private information is only about the internal values of trade \( v_P \) or \( v_A \), i.e. \( F_{\theta_E}(\cdot \mid \theta_L) = F_{\theta_E}(\cdot \mid \theta_H) \). Then, exclusivity has no effect on the set of equilibrium payoffs: for any equilibrium when the contract space is \( C = R \times Q \times E \), there exists a payoff equivalent equilibrium when the contract space is \( C' = R \times Q \times \{0\} \). Furthermore, exclusivity is fully indeterminate in equilibrium: contracts specifying any exclusivity level \( e \in [0,1] \) can be chosen by the principal in equilibrium and in any state \( \theta \).

Thus, in the case of private internal information, not only the use of contracted exclusivity is irrelevant in terms of equilibrium payoffs but also the principal’s private information is immaterial in explaining contracted exclusivity levels. To understand the proposition, note that when \( v_E \) does not depend on \( \theta \), the expected payoff of the principal can be written as

\[
U_P(c; \theta) = \lambda_P S(\theta) + (1 - \lambda_P)\{qE[v_P \mid \theta] + (1 - q)(1 - e)E[v_E] + t\} - \lambda_P(qE[v_A \mid \theta] - t).
\]

In this expected payoff there is no interaction between the level of contracted exclusivity \( e \) and the state \( \theta \). Thus, the impact of changing \( e \) on the principal’s expected payoff is independent of the state \( \theta \). The same is true for transfer \( t \). This implies that for any given contract \( c \), it is possible to simultaneously change the exclusivity level and the value of the transfer, in this case increasing both or decreasing both, so as to obtain a new contract that confers the principal the same expected payoff as contract \( c \) in both states \( \theta_L \) and \( \theta_H \). More specifically, for any two contracts \( c = (t, q, e) \) and \( c' = (t', q, e') \) in which \( t' = t + (1 - \lambda_P)(1 - q)(e' - e)E[v_E] \),

\[
U_P(c; \theta) = U_P(c'; \theta) \quad \text{for all} \quad \theta \in \{\theta_L, \theta_H\}.
\]

Furthermore, since the sum of the expected payoffs of the principal and agent is always identical to the expected total surplus and the expected total surplus does not depend on the contract initially agreed by the principal and the agent (condition (3)), this also means that \( U_A(c; \theta) = U_A(c'; \theta) \) for all \( \theta \in \{\theta_L, \theta_H\} \). Thus, contracts \( c \) and \( c' \) are payoff equivalent to the principal and the agent regardless of the state \( \theta \). This implies that if there is an equilibrium in which the principal chooses contract \( c \) (with positive probability) in state \( \theta_L \) or in state \( \theta_H \), then there exists another equilibrium identical in every dimension to that one, except that contract \( c \) is replaced by contract \( c' \) in the principal’s choice of contract. This has two important implications. First, since exclusivity \( e' \) in contract \( c' \) can be anything (provided \( t' \) is adjusted properly), exclusivity is undetermined in equilibrium. Second, because \( e' \) can be zero, and equilibria in which contract \( c \) is replaced by contract \( c' \) are payoff equivalent, the set of equilibrium payoffs does not change if the principal and the agent are restricted to write non-exclusive contracts, i.e., contracts that specify \( e = 0 \). Since the proposition follows almost directly from this discussion, it is stated without further proof.

Another way of understanding Proposition 1 is to note that when information is only about internal values of trade, there is no cross effect between exclusivity and private information \( \theta \) in the expected payoffs of the principal and the agent. In other words, the expected utilities of the principal and the agent do not satisfy the strict single crossing property with respect to exclusivity. As a consequence, exclusivity cannot be used as a signalling device by the principal. This is somewhat analogous to a result in Segal and Whinston (2000b).

They study the effect of contracted exclusivity on the investment incentives of parties to a
relationship, and show that exclusivity has no effect on investments that affect only internal values. In Segal and Whinston (2000b), no cross effect between exclusivity and investments in the payoff functions implies no effect of exclusivity on investments. In this paper, no cross effect between exclusivity and private information \( \theta \) in the payoff functions implies that exclusivity has no effect on surplus extraction through information signalling.

I next analyze the contracts that are chosen in equilibrium by the principal and the agent as well as their expected payoffs. The cases of private information about \( v_P \) and private information about \( v_A \) are considered separately. Since exclusivity plays no role in neither case, in what follows I focus on the role of commitments to trade (i.e., contracted quantity) by restricting attention to non-exclusive contracts. Thus, in the remainder of this section, a contract \( c \) is a transfer-quantity pair \((t, q)\) and expected payoffs are written as \( U_A(t, q; \tilde{p}_H) \) and \( U_P(t, q; \theta) \).

4.1.1. Private information about \( v_P \). Consider first the case where the principal is better informed than the agent about her value of trade with the agent \( v_P \). Let \( q_i \) denote the contracted quantity chosen by the principal in state \( i, i = L, H \). The following proposition is proved in the Appendix.

**Proposition 2.** Suppose that the principal’s private information is about \( v_P \). Then, there exist both separating and pooling equilibria. In all the equilibria, the principal commits ex-ante to trade a (weakly) higher quantity with the agent in state \( \theta_H \) than in state \( \theta_L \). More specifically, there exists \( \tilde{q} \in (0, 1) \), the same across all equilibria, such that in every equilibrium \( q_L \leq \tilde{q} \leq q_H \). Furthermore, all the equilibria are payoff equivalent. In all of them the principal always appropriates all the surplus, i.e., the principal’s payoff in state \( \theta \) is \( S(\theta) \), for all \( \theta \in \{\theta_L, \theta_H\} \).

Two main reasons explain this proposition. First, it is less costly for the principal to commit to trade a high quantity with the agent in state \( \theta_H \) than in state \( \theta_L \). To see this, recall that the contract initially agreed by the principal and the agent crucially affects their disagreement payoffs during renegotiation and, therefore, the division of surplus. Since the disagreement payoff of the principal is \( qv_P + (1-q)v_E + t \), she is more willing to increase the quantity in the contract when she expects \( v_P \) to be high (i.e., in state \( \theta_H \)) than when she expects \( v_P \) to be low (i.e., in state \( \theta_L \)). This can also be seen by direct inspection of the expected payoff of the principal. When the principal’s private information is about \( v_P \), her expected payoff is given by

\[
U_P(c; \theta) = \lambda_P S(\theta) + (1 - \lambda_P)\{q\mathbb{E}[v_P \mid \theta] + (1-q)\mathbb{E}[v_E] + t\} - \lambda_P(q\mathbb{E}[v_A] - t).
\]

Contracted quantity \( q \) and the state \( \theta \) only interact in the second term, which is precisely the expected disagreement payoff of the principal. Since \( \mathbb{E}[v_P \mid \theta_H] \geq \mathbb{E}[v_P \mid \theta_L] \), the effect of increasing contracted quantity on the expected payoff of the principal is greater in state \( \theta_H \) than in state \( \theta_L \). Because of this, contracts \( c_L \) (chosen by the principal in state \( \theta_L \)) and \( c_H \) (chosen by the principal in state \( \theta_H \)) are incentive compatible for the principal, as required
in any equilibrium, only if $q_L \leq q_H$; and in separating equilibria the principal in state $\theta_H$ is able to signal to the agent a high expected value of trade by proposing a quantity strictly greater than that she proposes in state is $\theta_L$.

Second, the agent’s preferences (ranking) over the two states of the world depend on the quantity specified in the contract. To see this observe that the agent’s expected payoff given state $\theta$ and contract $c$ can be written as

$$U_A(c; \theta) = \lambda_A S(\theta) + (1 - \lambda_A)(q \mathbb{E}[v_A] - t) - \lambda_A \{q \mathbb{E}[v_P | \theta] + (1 - q) \mathbb{E}[v_E] + t\}. \quad (5)$$

The state $\theta$ affects this payoff in two ways. It affects the value of the total surplus, part of which the agent appropriates through renegotiation—the first term of $U_A(c; \theta)$. It also affects the disagreement payoff of the principal, which affects (negatively) the agent’s gains from renegotiation—the last term of $U_A(c; \theta)$. When contracted quantity is small (i.e., smaller than $\hat{q}$) the second effect is small and the agent prefers state $\theta_H$ to state $\theta_L$, as total surplus is larger in the former. In contrast, when contracted quantity is large (i.e., greater than $\hat{q}$), the second effect is large and dominant. The agent prefers state $\theta_L$ to state $\theta_H$, as in the latter the disagreement payoff of the principal is higher.

Because of these two reasons there exist contracts $c_L$ and $c_H$, the first specifying a small quantity and the second a large quantity, that: (i) are incentive compatible for the principal in the sense that she prefers contract $c_L$ to contract $c_H$ in state $\theta_L$ and contract $c_H$ to contract $c_L$ in state $\theta_H$; (ii) allow the principal to appropriate all the surplus in states $\theta_L$ and $\theta_H$, respectively; and (iii) are accepted by the agent regardless of the agent’s beliefs.

In fact there exists a continuum of equilibria. Consider the sets of contracts $C_H = \{c \in C : U_P(c; \theta_H) = S(\theta_H) \text{ and } q \geq \hat{q}\}$ and $C_L = \{c \in C : U_P(c; \theta_L) = S(\theta_L) \text{ and } q \leq \hat{q}\}$. For any pair of contracts $(c_L, c_H) \in C_L \times C_H$ there exist an equilibrium in which $c_L$ is the contract chosen in state $\theta_L$ and $c_H$ is the contract chosen in $\theta_H$. These are all the equilibria. Thus, there is a continuum of separating equilibria. In each, the principal commits to trade a higher quantity with the agent in state the $\theta_H$ than in state $\theta_L$ (i.e., $q_L < q_H$) and the principal’s contract offer fully reveals the true state $\theta$ to the agent. In these equilibria, the principal signals a high (low) expected valuation of trade $v_p$ to the agent by committing ex-ante to trade a high (low) quantity with him. This is in sharp contrast with the benchmark case discussed above where it is assumed that only transfers are contractible. In that case only pooling equilibria exist and the principal is unable to appropriate the entire surplus in both states of the world.\(^{12}\)

4.1.2. Private information about $v_A$. Consider now the case of private information about $v_A$. We can state the following proposition, which is proved in the Appendix.

\(^{12}\)There is one contract that is in both $C_L$ and $C_H$. It specifies quantity $\hat{q}$. As a consequence, there is one pooling equilibrium in which the principal offers the same contract to the agent regardless of the state $\theta$. The existence of this equilibrium is an artefact of considering a model with only two states, $\theta_L$ and $\theta_H$. Quantity $\hat{q}$ is the quantity for which the expected payoff of the agent in the two states intersect. In a model with more states, such a pooling equilibrium does not necessarily exist.
Proposition 3. Suppose that the principal’s private information is about \( v_A \). Then, there exist both separating and pooling equilibria and multiple equilibrium payoffs. In all these equilibria, the principal commits ex-ante to trade a (weakly) lower quantity with the agent in state \( \theta_H \) than in state \( \theta_L \), i.e., \( q_H \leq q_L \). Furthermore, contractibility of quantity expands the set of equilibrium payoffs of the principal in state \( \theta_L \) but not in state \( \theta_H \).

In contrast with the case of private information about \( v_P \), in the case of private information about \( v_A \) it is more costly for the principal to commit to trade a high quantity with the agent in state \( \theta_H \) than it is in state \( \theta_L \). The reason is the following. Contracted quantity affects the disagreement payoff of the agent in the event of a renegotiation, which is given by \( qv_A - t \). Since any improvement in the agent’s disagreement payoff reduces the principal’s ability to appropriate surplus during the renegotiation of the initial contract, the principal gains less by increasing contracted quantity when she expects a high \( v_A \) (i.e. in state \( \theta_H \)) than when she expects a low \( v_A \) (i.e., in state \( \theta_L \)). (As in the previous case, this could also be seen by direct inspection of the expected payoff of the principal.) Because of this, in any equilibrium \( q_H \leq q_L \), and in separating equilibria the principal in state \( \theta_H \) signals to the agent that his valuation of trade is likely to be high by proposing a quantity strictly smaller than that she proposes in state is \( \theta_L \). In contrast with the case of private information about \( v_P \), the principal commits ex-ante less to the relationship when its probability of success is higher.

Another important difference relative to the case of private information about \( v_P \) is that the principal is unable to extract all the surplus in both states of the world. In any equilibrium, the agent appropriates some of the surplus generated in state \( \theta_H \) (i.e., when his value of trade is expected to be high). This is because the agent always prefers state \( \theta_H \) to state \( \theta_L \) regardless of the contract agreed with the principal. Observe that in the case of private information about \( v_A \),

\[
U_A(c; \theta) = \lambda_A S(\theta) + (1 - \lambda_A)(qE[v_A | \theta] - t) - \lambda_A \{qE[v_P] + (1 - q)E[v_E] + t\},
\]

and since \( E[v_A | \theta_H] \geq E[v_A | \theta_L] \), the agent’s expected payoff satisfies \( U_A(c; \theta_H) \geq U_A(c; \theta_L) \) for all \( c \in C \). This implies that the payoff of the principal in state \( \theta_H \) must be at least \( S(\theta_L) \), as the agent accepts any contract \( c \) offered by the principal such that \( U_A(c; \theta_L) = 0 \) regardless of his beliefs about \( \theta \). However, in state \( \theta_H \), some surplus is left to the agent, even if the principal offers the agent a contract specifying a quantity of zero, the contracted quantity that more efficiently signals to the agent that the state is \( \theta_H \). While there exist multiple equilibria, they are not payoff equivalent. Indeed, the highest equilibrium payoff of the principal in state \( \theta_H \) is obtained when the principal proposes a contract specifying a quantity of zero and is the same as when quantity is not contractible.

4.2. Private External Information. Suppose now the principal’s private information when contracting with the agent is about the principal’s value of trading in the future with the external party \( v_E \). Specifically, suppose that: (i) the distribution \( F_{v_E}(. | \theta_L) \) strictly first order stochastically dominates the distribution \( F_{v_E}(. | \theta_H) \), and (ii) the distributions of internal values \( v_A \) and \( v_P \) do not depend on \( \theta \), i.e., \( F_{v_j}(. | \theta_L) = F_{v_j}(. | \theta_H) \) for \( j = A, P \).
Under these assumptions, the principal is more likely to have a good external party to trade with in state $\theta_L$ than in state $\theta_H$. This means that, in conformity with the cases of private information about $v_P$ and private information about $v_A$ studied above, state $\theta_H$ is associated with a higher probability of success of the relationship than state $\theta_L$, i.e., also in this case $P(\theta_L) \leq P(\theta_H)$.

Since exclusivity may play a role in the case of private information about the external value of trade $v_E$, I reconsider it in the analysis. Hence, a contract is a triple $c = (t, q, e) \in C$.

In what follows, let $(q_i, e_i)$ denote the contracted quantity-exclusivity pair chosen by the principal in state $\theta_i$, $i = L, H$. The main results of this section are presented in the following proposition, which is proved in the Appendix.

**Proposition 4.** Suppose that the principal’s private information is about $v_E$. Then, there exist both separating and pooling equilibria. In all the equilibria, the principal commits ex-ante (weakly) more to the relationship (either through a higher contracted quantity and/or through a higher level of exclusivity) in state $\theta_H$ than in state $\theta_L$. More specifically, there exists $\bar{x} \in (0, 1)$, the same across all the equilibria, such that $(1-q_H)(1-e_H) \leq \bar{x} \leq (1-q_L)(1-e_L)$. Furthermore, all the equilibria are payoff equivalent. In all of them the principal always appropriates all the surplus, i.e., the principal’s payoff in state $\theta$ is $S(\theta)$, for all $\theta \in \{\theta_L, \theta_H\}$.

The arguments behind this proposition resemble those underlying Proposition 2 regarding the case of private information on $v_P$. As in that case, two main reasons explain this proposition. First, a commitment to trade a high quantity or to deal exclusively (or almost exclusively) with the agent is less costly for the principal in state $\theta_H$ (when she expects a low value of trade with the external party) than in state $\theta_L$ (when she expects a high value of trade with the external party). The reason for this is simple. When both quantity and exclusivity are contractible, the disagreement payoff of the principal, which affects her position during renegotiation and ability to appropriate surplus, is given by $qv_P + (1-q)(1-e)v_E + t$. Thus, the higher are her expectations about $v_E$, the more she has to lose by offering a contract with a high quantity and/or exclusivity level. In other words, by agreeing in the initial contract to a high level of exclusivity or a high quantity, the principal reduces the possibility of trading later with the external party if the initial contract is enforced. From an ex-ante point of view, forgoing that possibility is more costly for the principal in state $\theta_L$ (when she expects a high $v_E$) than it is in state $\theta_H$ (when she expects a low $v_E$).

Second, the quantity and exclusivity level specified in the contract proposed by the principal affect the agent’s preferences (ranking) over the two states of the world. Following a reasoning similar to that used in the case of private information on $v_P$, we obtain that for contracts that specify a high quantity or a high exclusivity level (i.e., such that $(1-q_H)(1-e_H) \leq \bar{x}$) then $U_A(c; \theta_L) \geq U_A(c; \theta_H)$; and for contracts that specify a low quantity and a low exclusivity level (i.e., such that $\bar{x} \leq (1-q_L)(1-e_L)$), the opposite happens and $U_A(c; \theta_L) \leq U_A(c; \theta_H)$. Because of these two reasons, the principal always commits (weakly)
more to the relationship in state $\theta_H$ than in state $\theta_L$, she signals a low outside option in separating equilibria by committing strictly more (through a higher exclusivity level or quantity) to the relationship, and she always appropriates the entire surplus.

As in the case of private information about $v_P$, there exists a continuum of equilibria. Consider the following sets of contracts $C_H = \{c \in C : U_P(c; \theta_H) = S(\theta_H) \text{ and } (1-q)(1-e) \leq \hat{x}\}$ and $C_L = \{c \in C : U_P(c; \theta_L) = S(\theta_L) \text{ and } \hat{x} \leq (1-q)(1-e)\}$. For any pair of contracts $(c_L, c_H) \in C_L \times C_H$ there exists an equilibrium in which the principal offers contract $c_L$ in state $\theta_L$ and contract $c_H$ in state $\theta_H$. Thus, there is a continuum of separating equilibria where the principal’s contract offer fully reveals the true state $\theta$ to the agent. In these equilibria, the principal signals to the agent a low value of her outside option by committing ex-ante more to the relationship. As mentioned above, the principal does so by initially committing to trade more exclusively or a higher quantity (or both) with the agent.\(^{13}\)

Proposition 4 and the discussion above also apply if only quantity or only exclusivity is contractible. The characterization of the equilibria in the former case is given by setting $e = 0$ in the proposition and analysis, and in the latter case by setting $q = 0$. Thus, if quantity is contractible but exclusivity is not, the principal can still use quantity to signal information and appropriate surplus. Similarly, when quantity is not contractible (because for example it is not verifiable), the principal can use exclusivity to signal information so as to appropriate more surplus.

One way of interpreting Propositions 2-4 is that an observed higher ex-ante commitment to a relationship by two parties does not necessarily mean a higher probability of success of the relationship. As seen above, the principal commits more to the relationship when the probability of success of the relationship is higher if her private information is about her valuation of trade with the agent or about her valuation of trade with the external party. This is not the case, however, if the principal’s private information is about the agent’s value of trading with the principal. In fact, in this case the opposite happens.

5. Discussion

The analysis above considers separately the cases of private information about $v_P$, private information about $v_A$ and private information about $v_E$. A separate analysis of each case has the virtue of emphasizing how the level and form of commitment to the relationship by the informed principal is affected by the source of her private information. In reality, however, a party to a relationship may simultaneously have private information on more than one valuation of trade. I next briefly discuss how the analysis can be extended to accommodate such cases and how (qualitatively) the results obtained in the previous section change. Among other things, I argue that even when the state $\theta$ affects multiple valuations of trade, the principal’s incentive to commit to a higher (or lower) level of exclusivity depends

\(^{13}\)There exist contracts that are in both $C_L$ and $C_H$. They satisfy $(1-q)(1-e) = \hat{x}$. These are the contracts offered by the principal in pooling equilibria. Once again, the existence of this pooling equilibria is an artefact of considering a model with only two states, $\theta_L$ and $\theta_H$. 
only on how the state $\theta$ affects $v_E$; and, as before, the principal is more willing to commit to a higher level of exclusivity when she expects a low value of $v_E$. Regarding the principal’s incentives to choose a higher contracted quantity, they depend on how the state $\theta$ affects all valuations $v_A$, $v_P$ and $v_E$. Specifically, they increase when $v_P$ is expected to be higher relative to $v_A$ and $v_E$.

It is useful to begin with the case where the principal’s private information is only about the internal valuations $v_P$ and $v_A$. Accordingly, suppose the probability distributions of $v_P$ and $v_A$ depend both on the state $\theta$, but the distribution of $v_E$ does not. As before, only the principal observes $\theta$. The agent knows only that $\theta = \theta_1$ with probability $p_1$. To simplify notation, let $\Delta_{v_j} := \mathbb{E}[v_j | \theta_H] - \mathbb{E}[v_j | \theta_L]$ for $j = P, A$ and $\Delta_S := S(\theta_H) - S(\theta_L)$. Assume that $\Delta_S > 0$. Thus, as before, in the case of private internal information state $\theta_H$ is associated with a higher total surplus than state $\theta_L$. As a first observation, note that Proposition 1 also applies when the principal has private information on both $v_P$ and $v_A$. Therefore the contracted level of exclusivity cannot be used to signal such information and any exclusivity level may emerge in equilibrium. Regarding contracted quantity, whether in equilibrium it is greater in state $\theta_H$ than in state $\theta_L$ or vice versa critically depends on the relative impact of the state $\theta$ on the valuations $v_P$ and $v_A$. Specifically, following an analysis similar to that in Section 4.1, we obtain that the principal commits ex-ante to trade a higher quantity in state $\theta_H$ than in state $\theta_L$, i.e. that $q_H \geq q_L$ if

$$
(1 - \lambda_P)\Delta_{v_P} - \lambda_P \Delta_{v_A} > 0,
$$

and that $q_L \geq q_H$ otherwise. The intuition is similar to that when private information is only about $v_P$ or only about $v_A$. The key difference is that in this case the state $\theta$ affects simultaneously the disagreement payoffs of the principal and the agent – i.e., the payoffs if the initial contract is enforced. When condition (6) is satisfied, accounting for these two effects, the relative change in valuations and disagreement payoffs is such the principal gains more from increasing contracted quantity in state $\theta_H$ than in state $\theta_L$. In other words, it is less costly for the principal to commit to trade a higher quantity in state $\theta_H$ than in state $\theta_L$. This occurs when $\Delta_{v_P}$ is sufficiently large relative to $\Delta_{v_A}$. In contrast, when $\Delta_{v_A}$ is sufficiently large relative to $\Delta_{v_P}$, it is costlier for the principal to commit ex-ante to trade a higher quantity is state $\theta_H$ than in state $\theta_L$, and the principal commits to a lower quantity in state $\theta_H$ than in state $\theta_L$. The cases of private information on $v_P$ and of private information on $v_A$ studied in Section 4.1 are examples of each situation. In the former case, $\Delta_{v_P} > 0$ and $\Delta_{v_A} = 0$; in the latter, $\Delta_{v_P} = 0$ and $\Delta_{v_A} > 0$. Regarding surplus extraction, the principal will appropriate the entire surplus in both states if and only if $(1 - \lambda_P)\Delta_{v_P} - \lambda_P \Delta_{v_A} > (1 - \lambda_P)\Delta_S$. In such cases, the effect of the state $\theta$ on disagreement payoffs is large relative to its effect on total surplus that the agent’s ranking of the states depends on the contract. As in the case of private information on $v_P$ (which satisfies the above condition), there exist contracts $c_L$ (specifying a low quantity) and $c_H$ (specifying a high quantity) that are accepted by
the agent, are incentive compatible for the principal and allow her to appropriate the entire surplus in both states.

Consider now the case where the principal has private information on both internal and external valuations of trade. In our setting such situation could be captured by assuming that not only the distributions of $v_P$ and $v_A$ depend on the state $\theta$ but also that of $v_E$. In what follows, let $\Delta_{v_E} := \mathbb{E}[v_E | \theta_H] - \mathbb{E}[v_E | \theta_L]$. In this case both contracted quantity and contracted exclusivity can be used by the principal to signal information. While feasible, a complete characterization of equilibrium outcomes is difficult to obtain. This is because equilibrium contracts depend on the relative effect (sign and magnitude) of the state $\theta$ on the different valuations, and many different subcases would need to be considered. (Of course, the cases considered in the previous section, can be seen as examples of such subcases.) As such, it is perhaps more instructive to briefly discuss how the state $\theta$ affects the principal’s motivations for choosing different quantity or exclusivity levels in a contract. This is done by looking at the requirement for two contracts $c_L$ and $c_H$ to be incentive compatible for the principal.\footnote{As before, this simply means that contracts $c_L$ and $c_H$ must satisfy $U_P(c_L, \theta_L) \geq U_P(c_H, \theta_L)$ and $U_P(c_H, \theta_H) \geq U_P(c_L, \theta_H)$. That is, the principal of type $\theta_L$ prefers contract $c_L$ to contract $c_H$, and the principal of type $\theta_H$ prefers contract $c_H$ to contract $c_L$.}

Consider first the case of exclusivity. Fix quantity across the two contracts, i.e. set $q_L = q_H = q < 1$, and let exclusivity vary. Incentive compatibility implies that

$$(1 - \lambda_p)(1 - q)\Delta_{v_E}(e_H - e_L) \leq 0.$$ 

We can make two observations. First, only the effect of $\theta$ on the external valuation is relevant. This is because a change in the exclusivity level affects the principal’s payoff only through its effect on the possibility of trading with the external party. Second, for contracts with the same quantity, incentive compatibility requires a higher exclusivity level when the value of the outside option is lower. The usual intuition applies here: it is costlier for a principal to commit to trade exclusively with the agent when the principal expects to have a good outside option in the future than when she expects a low outside option. In equilibrium, contracts $c_L$ (chosen by the principal in state $\theta_L$) and $c_H$ (chosen by the principal in state $\theta_H$) must be incentive compatible for the principal. Hence, for example in environments where quantity is not contractible (because it is not verifiable), this means that the principal will commit ex-ante to a higher exclusivity level when she expect a low outside option.

Consider now the case of quantity. Following the same procedure, fix the exclusivity level across the two contracts, i.e. set $e_L = e_H = e < 1$, and let quantity vary. Incentive compatibility implies that

$$(q_H - q_L)[(1 - \lambda_p)(\Delta_{v_P} - (1 - e)\Delta_{v_E}) - \lambda_p \Delta_{v_A}] \geq 0.$$ 

The effect of $\theta$ on all the trade valuations is now relevant. This is because a change in contracted quantity affects the disagreement payoff of the principal (which depends on $v_P$ and $v_E$) and the disagreement payoff of the agent (which depends on $v_A$), both of which affect
the principal’s expected payoff. When the term inside the square brackets is positive, the gains for the principal from increasing quantity are greater in state $\theta_H$ than in state $\theta_L$ and incentive compatibility requires $q_H \geq q_L$. If exclusivity clauses in contracts are not feasible (for example because they are not allowed by courts) and only quantity is contractible, this means that the principal will commit to trade a higher quantity in state $\theta_H$ whenever $\Delta v_P$ is large relative to $\Delta v_A$ and $\Delta v_E$.

In this discussion of the case where the principal has private information on both internal and external valuations of trade, we have kept one contractual variable fixed and analyzed changes in the other. When both quantity and exclusivity are contractible, it is of course the combination of both that matters. When considering any two contracts, the principal and agent will take into account the combined effect of the changes in quantity and exclusivity on their payoffs, which depends on how the state affects all valuations of trade.

6. Conclusion

This paper presents a theory of ex-ante commitment to relationships. The theory is based on a key element: the existence of asymmetry of information between parties when contracting about the terms of a future relationship. Another important element of the theory is the renegotiation of initially contracted terms when they turn out to be inefficient ex-post. In this context, I show how the level of contractual commitment to a relationship can be used by a better informed party to signal information about the value of the relationship.

While the theory offers clear predictions regarding which forms of commitment can be used to signal information and when parties will commit more (or less) to a relationship, the analysis shows that there are multiple equilibria (which is a typical feature of signalling games). In the cases of private information about $v_P$ and private information about $v_E$, different levels of contracted quantity and exclusivity may emerge in equilibrium, but all equilibria are payoff equivalent. In all of them the informed party appropriates the entire surplus despite the asymmetry of information. In the case of private information on $v_A$, both contracts and payoffs may differ across different equilibria.

The application of standard equilibrium refinements, such as the Intuitive Criterion and Universal Divinity, has no impact on the results in the cases of private information about $v_P$ and private information about $v_E$. In other words, all the equilibria identified in Section 4 for these cases survive the two refinements. The case of private information about $v_A$ is different. While the result regarding contracted quantity continues to hold (i.e., $q_H \leq q_L$ in equilibrium), the result regarding equilibrium payoffs changes. Specifically, equilibria in which the principal’s payoff exceeds the maximum equilibrium payoff that she can obtain when only transfers are contractible survive neither the Intuitive Criterion nor Universal Divinity. Thus, under these refinements, the set of equilibrium payoffs of the principal when quantity is contractible is identical to that when it is not. Hence, if we focus on equilibria that satisfy these refinements, there is an even greater difference between outcomes across the different sources of private information. While commitments to trade a certain quantity
and/or exclusively with the agent help the principal appropriate surplus when her private information is about $v_P$ or about $v_E$, they do not when her private information is about $v_A$.

The paper considers contracted quantity and contracted exclusivity as two forms of contractual commitment to a relationship. The way in which quantity and exclusivity are modelled enables broader interpretations. For example, contracted quantity may be interpreted as contract duration. Thus, this paper provides an information based theory of contract duration. For example, the results in the paper suggest that in trade relationships, parties with higher expected valuations from trade and with private information about those valuations will propose and agree on trade contracts with a longer duration. Similarly for parties that expect their value of trade with external parties or the value of trade to their partner in the relationship to be low. Aghion and Bolton (1987) also touch the issue of contract duration in a setting with asymmetric information. However, they do not consider the possibility of ex-post renegotiation.

Exclusivity, in turn, may be interpreted in terms of asset ownership. In this paper the effect of exclusivity in a relationship is that it constrains a party to dealing with other (external) parties. According to the property rights literature (e.g., Grossman and Hart, 1986; Hart and Moore, 1990), this is essentially the effect of forgoing ownership of an asset that is essential to trade with others. Thus, the results in the paper that characterize equilibrium exclusivity can also be interpreted as results that endogenize asset ownership. Specifically, they suggest that a party in a relationship may give up ownership of an asset that is essential to trade with others, to signal low outside options in the future.
Appendix

This appendix is organized as follows. I start by stating and proving two new lemmas, Lemma 1 and Lemma 2, which are used in the proofs of the propositions in the text. I then prove the propositions in the text, with the exception of Proposition 1 which, as mentioned in the text, is stated without further proof.

Lemma 1. Consider the case where the principal’s private information is about $v_P$ and let a contract be $c = (t, q)$. There exists $\tilde{q} \in (0, 1)$ such that for all $t \in \mathbb{R}$, $U_A(t, q; \theta_H) \geq U_A(t, q; \theta_L)$ if and only if $q \leq \tilde{q}$.

Proof. Taking expectation of (1) and rearranging terms, we obtain that when the principal’s private information is about $v_P$,

$$U_A(t, q; \theta) = (1 - \lambda_A)(q\mathbb{E}[v_A] - t) + \lambda_A\{S(\theta) - q\mathbb{E}[v_P | \theta] - (1 - q)\mathbb{E}[v_E] - t\}.$$

Thus, $U_A(t, q; \theta_H) \geq U_A(t, q; \theta_L)$ is equivalent to $S(\theta_H) - S(\theta_L) \geq q(\mathbb{E}[v_P | \theta_H] - \mathbb{E}[v_P | \theta_L]).$ Since by assumption $F_{v_P}(\cdot | \theta_H)$ strictly first order stochastically dominates $F_{v_P}(\cdot | \theta_L)$, $\mathbb{E}[v_P | \theta_H] > \mathbb{E}[v_P | \theta_L]$. Thus, $U_A(t, q; \theta_H) \geq U_A(t, q; \theta_L)$ if and only if $q \leq \hat{q} := \frac{S(\theta_H) - S(\theta_L)}{\mathbb{E}[v_P | \theta_H] - \mathbb{E}[v_P | \theta_L]}.$

It remains to show that $0 < \hat{q} < 1$. Since (i) $S(\theta) = \mathbb{E}[\max\{v_P + v_A, v_E\} | \theta]$, (ii) $\max\{v_P + v_A, v_E\}$ is an increasing function of $v_P$ and (iii) $F_{v_P}(\cdot | \theta_H)$ strictly first-order stochastically dominates $F_{v_P}(\cdot | \theta_L)$, then $S(\theta_H) > S(\theta_L)$. Hence, $\hat{q} > 0$. Furthermore, from the fact that (i) $S(\theta) - \mathbb{E}[v_P | \theta] = \mathbb{E}[\max\{v_P + v_A, v_E\} - v_P | \theta]$, (ii) $\max\{v_P + v_A, v_E\} - v_P$ is a decreasing function of $v_P$ and (iii) $F_{v_P}(\cdot | \theta_H)$ strictly first order stochastically dominates $F_{v_P}(\cdot | \theta_L)$, it follows that $S(\theta_H) - \mathbb{E}[v_P | \theta_H] > S(\theta_L) - \mathbb{E}[v_P | \theta_L]$, which is equivalent to $\hat{q} < 1.$

Lemma 2. Consider the case where the principal’s private information is about $v_E$ and let a contract be $c = (t, q, e)$. There exists $\tilde{x} \in (0, 1)$ such that for all $t \in \mathbb{R}$, $U_A(t, q, e; \theta_H) \geq U_A(t, q, e; \theta_L)$ if and only if $(1 - q)(1 - e) \geq \tilde{x}$.

Proof. Taking expectation of (1) and rearranging terms we obtain that when the principal’s private information is about $v_E$,

$$U_A(t, q, e; \theta) = (1 - \lambda_A)(q\mathbb{E}[v_A] - t) + \lambda_A\{S(\theta) - q\mathbb{E}[v_P] - (1 - q)(1 - e)\mathbb{E}[v_E | \theta] - t\}.$$

It is then straightforward to obtain that $U_A(t, q, e; \theta_H) \geq U_A(t, q, e; \theta_L)$ is equivalent to $S(\theta_L) - S(\theta_H) \leq (1 - q)(1 - e)(\mathbb{E}[v_E | \theta_L] - \mathbb{E}[v_E | \theta_H])$. Since by assumption $F_{v_E}(\cdot | \theta_L)$ strictly first order stochastically dominates $F_{v_E}(\cdot | \theta_H)$, $\mathbb{E}[v_E | \theta_L] > \mathbb{E}[v_E | \theta_H]$. Thus, $U_A(t, q, e; \theta_H) \geq U_A(t, q, e; \theta_L)$ if and only if $(1 - q)(1 - e) \geq \hat{x} := \frac{S(\theta_L) - S(\theta_H)}{\mathbb{E}[v_E | \theta_L] - \mathbb{E}[v_E | \theta_H]}.$
It remains to show that $0 < \hat{x} < 1$. Since (i) $S(\theta) = \mathbb{E}[\max\{v_P + v_A, v_E\} | \theta]$, (ii) $\max\{v_P + v_A, v_E\}$ is an increasing function of $v_E$ and (iii) $F_{v_E}(\cdot | \theta_L)$ strictly first order stochastically dominates $F_{v_E}(\cdot | \theta_H)$, then $S(\theta_L) > S(\theta_H)$. Hence, $\hat{x} > 0$. Furthermore, since (i) $S(\theta) - \mathbb{E}[v_E | \theta] = \mathbb{E}[\max\{v_P + v_A, v_E\} - v_E | \theta]$, (ii) $\max\{v_P + v_A, v_E\} - v_E$ is a decreasing function of $v_E$ and (iii) $F_{v_E}(\cdot | \theta_L)$ strictly first order stochastically dominates $F_{v_E}(\cdot | \theta_H)$, then $S(\theta_L) - \mathbb{E}[v_E | \theta_L] < S(\theta_H) - \mathbb{E}[v_E | \theta_H]$, which is equivalent to $\hat{x} < 1$. ■

Proof of Proposition 2. The proof is given in three steps. In the first step I prove the result on the principal’s payoffs, in the second step I prove the result on the contracted quantities, and in the last step I prove the existence of equilibrium.

Step 1: In any equilibrium the principal’s payoff in state $\theta$ is $S(\theta)$ for all $\theta \in \{\theta_L, \theta_H\}$. Let $\hat{q} \in (0, 1)$ be the threshold quantity such that for all $t \in \mathbb{R}$, $U_A(t, q; \theta_H) \geq U_A(t, \hat{q}; \theta_L)$ if and only if $q \leq \hat{q}$. By Lemma 1 (in this Appendix) we know that $\hat{q}$ exists. Consider contracts $c_L = (t_L, q_L)$ and $c_H = (t_H, q_H)$ such that: $q_L \leq \hat{q} \leq q_H$, $t_L = U_A(0, q_L; \theta_L)$, and $t_H = U_A(0, q_H; \theta_H)$. Thus, $U_A(c_j; \theta_j) = 0$ for $j = L, H$ and

\[ U_A(c_L; \theta_H) \geq U_A(c_L; \theta_L) = 0 \]

and

\[ U_A(c_H; \theta_L) \geq U_A(c_H; \theta_H) = 0. \]

Conditions (7) and (8) imply, respectively, that $U_A(c_L; \hat{p}_H) \geq 0$ and $U_A(c_H; \hat{p}_H) \geq 0$ for all $\hat{p}_H \in [0, 1]$. Hence, regardless of the agent’s beliefs $\hat{p}_H$, he accepts contract $c_L$ as well as contract $c_H$ if they are offered by the principal. Since $c_L$ and $c_H$ satisfy $U_A(c_j; \theta_j) = 0$ for $j = L, H$, it follows by (3) that $U_P(c_j; \theta_j) = S(\theta_j)$ for $j = L, H$. Thus, in any equilibrium, the principal’s payoff in both states must be at least the expected total surplus. Clearly, the principal’s payoff in equilibrium cannot exceed $S(\theta_j)$ in any state $\theta_j$, as by (3) that would imply a negative expected payoff to the agent, in which case the agent would be better off rejecting the contracts offered by the principal.

Step 2: In any equilibrium $q_L \leq \hat{q} \leq q_H$. Consider an arbitrary equilibrium and let $c_L = (t_L, q_L)$ and $c_H = (t_H, q_H)$ denote contracts chosen with positive probability by the principal in states $\theta_L$ and $\theta_H$, respectively. Contracts $c_L$ and $c_H$ must satisfy

\[ U_P(c_H; \theta_H) \geq U_P(c_L; \theta_H) \]

and

\[ U_P(c_L; \theta_L) \geq U_P(c_H; \theta_L), \]

otherwise either in state $\theta_H$ or in state $\theta_L$ the principal would be better off deviating by mimicking the principal in the other state. Furthermore, by Step 1, contracts $c_L$ and $c_H$ must satisfy $U_P(c_j; \theta_j) = S(\theta_j)$ for $j = L, H$. This implies by (3) that $U_A(c_j; \theta_j) = 0$, which means that $t_L = U_A(0, q_L; \theta_L)$ and $t_H = U_A(0, q_H; \theta_H)$. From this and (3) again, it follows that we can write $U_P(c_H; \theta_H) = S(\theta_H) = U_P(0, q_L; \theta_H) + U_A(0, q_L; \theta_H), U_P(c_L; \theta_H) =$
Proof of Proposition 3. The proof is given in the following steps.

**Step 1: In any equilibrium** $q_H \leq q_L$. Consider an arbitrary equilibrium where $c_L = (t_L, q_L)$ and $c_H = (t_H, q_H)$ are contracts chosen with positive probability by the principal in states $\theta_L$ and $\theta_H$, respectively. Contracts $c_L$ and $c_H$ must be incentive compatible for the principal, i.e., they must satisfy $U_P(c_L; \theta_L) \geq U_P(c_H; \theta_L)$ and $U_P(c_H; \theta_H) \geq U_P(c_L; \theta_H)$. When the principal’s private information is about $v_A$, these conditions are equivalent to

$$
\begin{align*}
(11) & \quad t_H - t_L \leq (q_H - q_L) \{ \lambda_P \mathbb{E}[v_A | \theta_L] - (1 - \lambda_P)(\mathbb{E}[v_P] - \mathbb{E}[v_E]) \} \\
(12) & \quad t_H - t_L \geq (q_H - q_L) \{ \lambda_P \mathbb{E}[v_A | \theta_H] - (1 - \lambda_P)(\mathbb{E}[v_P] - \mathbb{E}[v_E]) \},
\end{align*}
$$

respectively. Conditions (11) and (12) hold simultaneously only if the right-hand side of (11) is greater than or equal to the right-hand side of (12). Since $\mathbb{E}[v_A | \theta_H] > \mathbb{E}[v_A | \theta_L]$, this is possible only if $q_H \leq q_L$. I next analyze equilibrium payoffs.

**Step 2: Characterization of the principal’s equilibrium payoffs.** I begin by deriving lower bounds for the principal’s equilibrium payoffs. When private information is about $v_A$,

$$
U_A(c; \theta) = \lambda_A S(\theta) + (1 - \lambda_A)(q \mathbb{E}[v_A | \theta] - t) - \lambda_A \{ q \mathbb{E}[v_P] + (1 - q) \mathbb{E}[v_E] + t \}.
$$

Since $S(\theta_H) > S(\theta_L)$ and $\mathbb{E}[v_A | \theta_H] > \mathbb{E}[v_A | \theta_L]$, then $U_A(c; \theta_H) > U_A(c; \theta_L)$ for all $c \in C$. This implies that the agent accepts any contract $c$ such that $U_A(c; \theta_L) \geq 0$ regardless of his beliefs about $\theta$. Hence, by (3), a lower bound for the payoff of the principal in state $\theta_L$ is $S(\theta_L)$. A lower bound for the payoff of the principal in state $\theta_H$ can be obtained by solving $\max_t U_P(c, \theta_M)$ subject to $U_A(c; \theta_L) \geq 0$. As $U_P$ and $U_A$ are quasi-linear in $t$, in any solution to this problem the constraint must hold with equality, and we can use it to eliminate $t$ from the problem. Next, observe that from (3) it follows that $\partial U_P(c; \theta_H)/\partial q = -\partial U_A(c; \theta_H)/\partial q$. Finally, since $\partial U_A(c; \theta_H)/\partial q > \partial U_A(c; \theta_H)/\partial q$, the solution to the problem involves $q = 0$, $t =$
let $U_\theta(0,0; \theta_L)$, and payoff to the principal $U_P(0,0; \theta_H) + U_A(0,0; \theta_L)$. To simply the exposition, let $	ilde{U}_P(\theta_L) := S(\theta_L)$ and $	ilde{U}_P(\theta_H) := U_P(0,0; \theta_H) + U_A(0,0; \theta_L)$. That is, $	ilde{U}_P(\theta_L)$ and $	ilde{U}_P(\theta_H)$ denote the lower bounds for the principal’s payoffs derived here. I next analyze the principal’s equilibrium payoffs. It is useful to consider separating and pooling equilibria separately.

**Step 2.1 Principal’s payoffs in separating equilibria.** In any separating equilibrium the payoff of the principal in state $\theta_j$ is $\tilde{U}_P(\theta_j)$ for $j = L, H$. To see this, consider a separating equilibrium and let $c_j$ denote the contract offered by the principal in state $\theta_j$. The agent’s beliefs upon observing that contract $c_j$ is offered are that $\theta = \theta_L$ with probability one. Since the agent accepts the offer, then $U_A(c_L; \theta_L) \geq 0$, which by (3) implies that $U_P(c_L; \theta_L) \leq S(\theta_L) = \tilde{U}_P(\theta_L)$. Thus, $U_P(c_L; \theta_L) = S(\theta_L) = \tilde{U}_P(\theta_L)$. To obtain that $U_P(c_L; \theta_L) \leq \tilde{U}_P(\theta_L)$, observe that contracts $c_L$ and $c_H$ must be incentive compatible for the principal. Hence, $U_P(c_L, \theta_L) \geq U_P(c_H; \theta_L)$. Since $U_P(c_L, \theta_L) = S(\theta_L)$, it follows by (3) that $U_A(c_H; \theta_L) \geq 0$. Because $\tilde{U}_P(\theta_H)$ is the max, $U_P(c, \theta_H)$ subject to $U_A(c; \theta_L) \geq 0$, then $U_P(c_H; \theta_L) \leq \tilde{U}_P(\theta_H)$. Hence, $U_P(c_H; \theta_H) = \tilde{U}_P(\theta_H)$.

There exist a continuum of separating equilibria but they are all payoff equivalent. Specifically, for every $q \in (0,1]$, there exists a separating equilibrium in which $c_H = (0, t_H)$, $c_L = (q, t_L)$, where $t_H = U_A(0,0; \theta_L)$ and $t_L = U_A(0,q; \theta_L)$. They are supported by the off-the-equilibrium-path beliefs where for all $c \neq c_L$ and $c \neq c_H$, the agent believes that $\theta = \theta_L$ with probability one.

**Step 2.2 Principal’s payoffs in pooling equilibria.** There exists an equilibrium in which the principal offers in both states the contract $c = (t, q = 0)$ where $t = U_A(0,0; \theta_L)$, i.e. $t$ is such that $U_A(c; \theta_L) = 0$. This equilibrium is sustained by the off-the-equilibrium-path beliefs that if the principal offers any contract $c' \neq c$, then the agent believes that $\theta = \theta_L$ with probability one. In this equilibrium, the payoff of the principal of type $\theta_j$ is $\tilde{U}_P(\theta_j)$ for $j = L, H$. These are the lowest equilibrium payoffs of the principal in a pooling equilibrium.

The highest equilibrium payoff of the principal when the state is $\theta_j$ is given by

$$
\max_c U_P(c; \theta_j)
$$

$$
\text{s.t. } (i) \quad p_L U_A(c; \theta_L) + p_H U_A(c; \theta_H) \geq 0 \\
(ii) \quad U_P(c; \theta_i) \geq \tilde{U}_P(\theta_i) \text{ for } i = L, H.
$$

Observe that constraints $(i)$ and $(ii)$ characterize the set of pooling equilibria. For each contract $c$ that satisfies them, there exists a pooling equilibrium where the principal offers it and the agent accepts the offer. Once again, all these equilibria all supported by the off-the-equilibrium-path beliefs where, if the principal offers any contract $c' \neq c$, then the agent believes that $\theta = \theta_L$ with probability one.

To solve problem (14) first observe that in any solution, constraint $(i)$ must bind. Thus, I next analyze how the objective function evolves along constraint $(i)$. Replacing the transfer $t$ in the objective function by its value when constraint $(i)$ holds with equality, we obtain

$$
m_j(q) := U_P(0,q; \theta_j) + p_L U_A(0,q; \theta_L) + p_H U_A(0,q; \theta_H).
$$
From (3), it follows that $\partial U_A(c; \theta)/\partial q = -\partial U_P(c; \theta)/\partial q$. Thus,

$$\partial m_j(q)/\partial q = \partial U_P(0, q; \theta_j)/\partial q - [p_L \times \partial U_P(0, q; \theta_L)/\partial q + p_H \times \partial U_P(0, q; \theta_H)/\partial q].$$

Because

$$\partial U_P(0, q; \theta)/\partial q = (1 - \lambda_P)(\mathbb{E}[v_P] - \mathbb{E}[v_E]) - \lambda_P\mathbb{E}[v_A | \theta_H]$$

and $\mathbb{E}[v_A | \theta_H] > \mathbb{E}[v_A | \theta_L]$, we obtain that $\partial U_P(0, q; \theta_H)/\partial q < \partial U_P(0, q; \theta_L)/\partial q$. Thus, from direct inspection of (15), it follows that $\partial m_H(q)/\partial q < 0 < \partial m_L(q)/\partial q$.

Consider first the case of state $\theta_H$. Since $\partial m_H(q)/\partial q < 0$, a contract that solves problem (14) when ignoring constraints (ii) is $c^*_H = (0, t^*_H)$ where $t^*_H$ is such that constraint (i) binds. Since $c^*_H$ also satisfies constraints (ii) it is a solution to problem (14) when $j = H$. Thus the highest equilibrium payoff of the principal in a pooling equilibrium when the state is $\theta_H$ is $U_P(c^*_H; \theta_H) = U_P(0, 0; \theta_H) + t^*_H$, where $t^*_H = p_L U_A(0, 0; \theta_L) + p_H U_A(0, 0; \theta_H)$. This is the same as that when only transfers are contractible. Consider now the case of state $\theta_L$. Since, $\partial m_L(q)/\partial q > 0$, the objective function increases with $q$ along constraint (i). Moreover, constraints (ii) are slack when $q = 0$. Thus, denoting the solution by $c^*_L$, we obtain that $U_P(c^*_L; \theta_L) > U_P(0; \theta_L) + p_L U_A(0, 0; \theta_L) + p_H U_A(0, 0; \theta_H)$, which is the maximum equilibrium payoff of the principal when only transfers are contractible. These results together with those obtained in Step 2.1 imply that the ability to contract on quantity expands the set of the principal’s equilibrium payoffs in state $\theta_L$ but not in state $\theta_H$.

Finally, observe that to characterize the set of equilibrium payoffs there is no need to analyze semi-separating equilibria. This is because for any of such equilibria there is a payoff equivalent pooling equilibrium. Indeed, in any given semi-separating equilibrium at most one contract can be offered (with some probability) by both types of principal, as the principal’s indifference curves satisfy the single crossing property. Using (3) it is easy to obtain that there is a payoff equivalent pooling equilibrium where the principal offers that contract in both states (with probability one).

**Proof of Proposition 4.** This proof is similar to that of Proposition 2 and follows the same steps.

**Step 1:** In any equilibrium the principal’s payoff in state $\theta$ is $S(\theta)$ for all $\theta \in \{\theta_L, \theta_H\}$. Let $\hat{\alpha} \in (0, 1)$ be the threshold value such that for all $t \in \mathbb{R}$, $U_A(t, q, e; \theta_H) \geq U_A(t, q, e; \theta_L)$ if and only if $(1 - q)(1 - e) \geq \hat{\alpha}$. By Lemma 2 (in this Appendix) we know that $\hat{\alpha}$ exists. Consider contracts $c_L = (t_L, q_L, e_L)$ and $c_H = (t_H, q_H, e_H)$ such that: $(1 - q_H)(1 - e_H) \leq \hat{\alpha} \leq (1 - q_L)(1 - e_L)$, $t_L = U_A(0, q_L, e_L; \theta_L)$, and $t_H = U_A(0, q_H, e_H; \theta_H)$. By construction $U_A(c_j; \theta_j) = 0$ for $j = L, H$ and

$$U_A(c_L; \theta_H) \geq U_A(c_L; \theta_L) = 0$$

and

$$U_A(c_H; \theta_L) \geq U_A(c_H; \theta_H) = 0.$$
Conditions (16) and (17) imply, respectively, that \( U_A(c_L; \hat{p}_H) \geq 0 \) and that \( U_A(c_H; \hat{p}_H) \geq 0 \) for all \( \hat{p}_H \in [0, 1] \). Hence, regardless of the agent’s beliefs \( \hat{p}_H \), he accepts contract \( c_L \) as well as contract \( c_H \) if any of these contracts is offered by the principal. Since \( c_L \) and \( c_H \) satisfy \( U_A(c_j; \theta_j) = 0 \) for \( j = L, H \), it follows by (3) that \( U_P(c_j; \theta_j) = S(\theta_j) \) for \( j = L, H \). Thus, in any equilibrium, the principal’s payoff must be at least the expected total surplus in both states. Clearly, the principal’s payoff in equilibrium cannot exceed \( S(\theta_j) \) in any state \( \theta_j \), as by (3) that would imply a negative expected payoff to the agent, in which case the agent would prefer to reject the contract offered by principal.

**Step 2: In any equilibrium** \((1 - q_H)(1 - e_H) \leq \bar{x} \leq (1 - q_L)(1 - e_L)\). Consider an arbitrary equilibrium and let \( c_L = (t_L, q_L, e_L) \) and \( c_H = (t_H, q_H, e_H) \) denote contracts chosen with positive probability by the principal in states \( \theta_L \) and \( \theta_H \), respectively. Contracts \( c_L \) and \( c_H \) must satisfy

\[
U_P(c_H; \theta_H) \geq U_P(c_L; \theta_H) \tag{18}
\]

and

\[
U_P(c_L; \theta_L) \geq U_P(c_H; \theta_L), \tag{19}
\]

otherwise either in state \( \theta_H \) or in state \( \theta_L \) the principal would be better off deviating by mimicking the principal in the other state. Furthermore, by Step 1, contracts \( c_L \) and \( c_H \) must satisfy \( U_P(c_j; \theta_j) = S(\theta_j) \) for \( j = L, H \). This implies by (3) that \( U_A(c_j; \theta_j) = 0 \), which means that \( t_L = U_A(0, q_L, e_L; \theta_L) \) and \( t_H = U_A(0, q_H, e_H; \theta_H) \). From this and (3) again, it follows that we can write \( U_P(c_H; \theta_H) = S(\theta_H) = U_P(0, q_L, e_L; \theta_H) + U_A(0, q_L, e_L; \theta_H) \), \( U_P(c_L; \theta_H) = U_P(0, q_L, e_L; \theta_H) + U_A(0, q_L, e_L; \theta_L) \), \( U_P(c_L; \theta_H) = S(\theta_L) = U_P(0, q_H, e_H; \theta_L) + U_A(0, q_H, e_H; \theta_L) \), and \( U_P(c_H; \theta_L) = U_P(0, q_H, e_H; \theta_L) + U_A(0, q_H, e_H; \theta_H) \). Using this, we obtain that (18) is equivalent to \( U_A(0, q_L, e_L; \theta_H) \geq U_A(0, q_L, e_L; \theta_L) \) and (10) is equivalent to \( U_A(0, q_H, e_H; \theta_L) \geq U_A(0, q_H, e_H; \theta_H) \), which by Lemma 2 implies that \( \bar{x} \leq (1 - q_L)(1 - e_L) \) and \((1 - q_H)(1 - e_H) \leq \bar{x} \), respectively.

**Step 3: Existence of Equilibrium.** This part of the proof is totally analogous to Step 3 of the proof of Proposition 2. ■
REFERENCES