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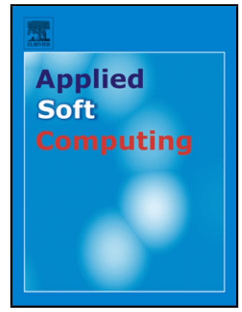

Title: Hybrid Non-dominated Sorting Genetic Algorithm with Adaptive Operators Selection

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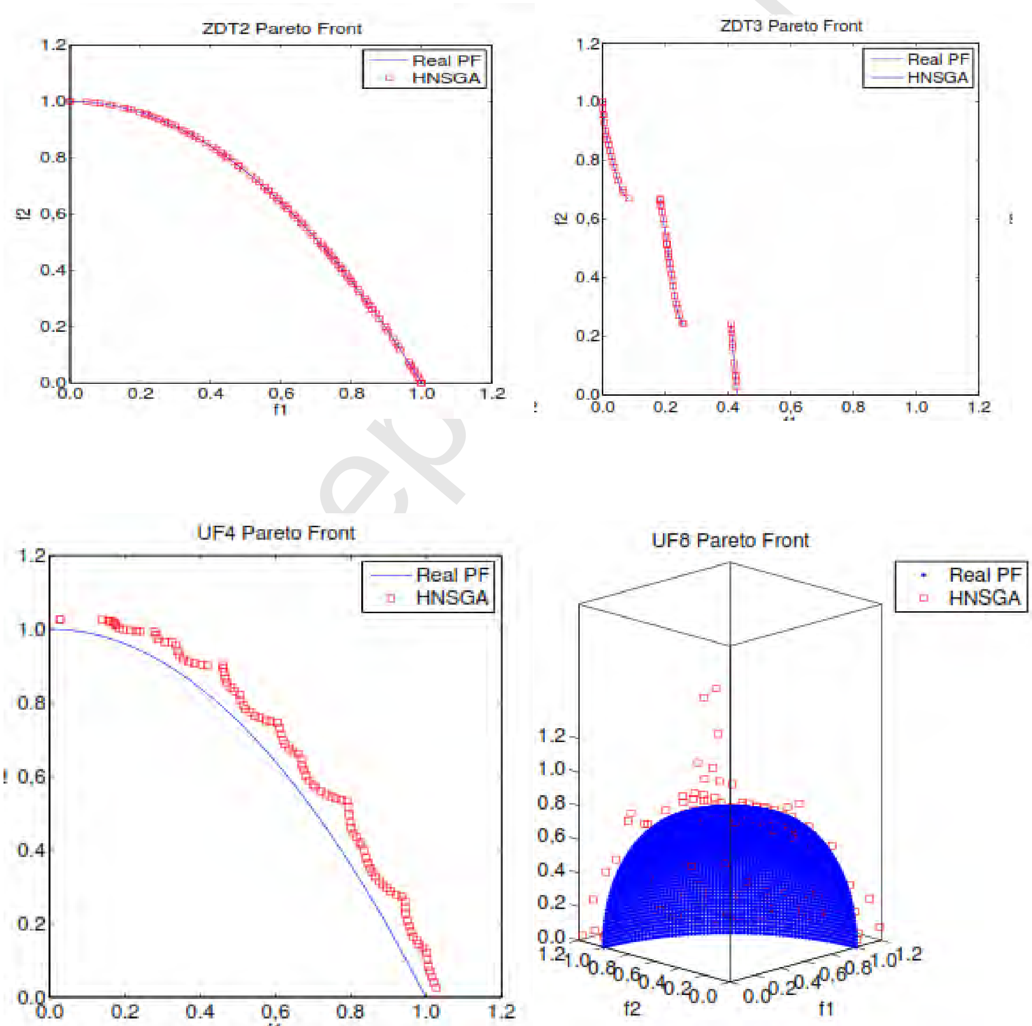
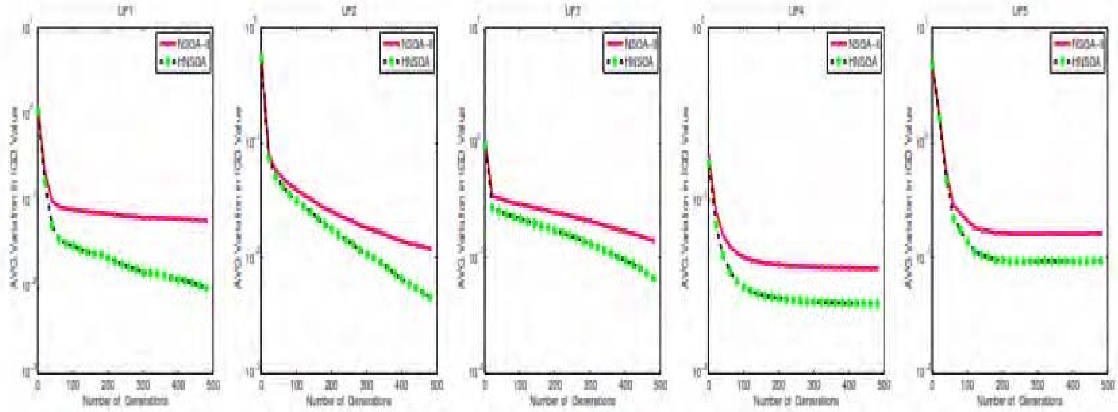
Highlights for HNSGA

- ***A novel hybrid non-dominated sorting genetic algorithm (HNSGA) for multi-objective optimization with continuous variables is developed.***
- ***HNSGA includes adaptive operator selection to allocate resources to multiple search operators based on their individual performance at the subpopulation level.***
- ***HNSGA is tested in classical benchmark problems taken from the ZDT and CEC'09 suites.***
- ***Inverted generational distance (IGD), relative hypervolume (RHV), Gamma and Delta functions are used as performance indicators.***
- ***The new algorithm is very competitive with other state-of-the-art optimizers such as AMALGAM, NSGA-II, MOEA/D, Hybrid AMGA, OMOEA, PA-DDS etc.***

Graphical Abstracts of the HNSGA Based on Adaptive Operator Selection Strategy

The main goal of this paper is to investigate the effect of the multiple search operators with adaptive selection strategy and to develop hybrid version of non-dominated sorting genetic algorithm (HNSGA) for solving recently developed complicated multi-objective optimization test suit for multi-objective evolutionary algorithms (MOEAs) competition in the special session of the congress on evolutionary computing held at Norway in 2009 (CEC'09). The Inverted generational distance (IGD) has been used performance indicator to establish valuable comparison between the suggested algorithm and NSGA-II as shown in the figure below. A set of Pareto optimal solutions with smaller is the IGD values confirm that approximated Pareto front (PF) will cover whole part of true PF in term of proximity and diversity.





The average IGD-metric values evolution obtained by HNSGA versus NSGA-II for UF1-UF5 within allowable resources of 300,000 Function Evaluations.

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Hybrid Non-dominated Sorting Genetic Algorithm with Adaptive Operators Selection

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Abstract

Multiobjective optimization entails minimizing or maximizing multiple objective functions subject to a set of constraints. Many real world applications can be formulated as multi-objective optimization problems (MOPs), which often involve multiple conflicting objectives to be optimized simultaneously. Recently, a number of multi-objective evolutionary algorithms (MOEAs) were developed suggested for these MOPs as they do not require problem specific information. They find a set of non-dominated solutions in a single run. The evolutionary process on which they are based, typically relies on a single genetic operator. Here, we suggest an algorithm which uses a basket of search operators. This is because it is never easy to choose the most suitable operator for a given problem. The novel hybrid non-dominated sorting genetic algorithm (HNSGA) introduced here in this paper and tested on the ZDT (Zitzler-Deb-Thiele) and CEC'09 (2009 IEEE Conference on Evolutionary Computations) benchmark problems specifically formulated for MOEAs. Numerical results prove that the proposed algorithm is competitive with state-of-the-art MOEAs.

Keywords: Multiobjective Optimization, Evolutionary Computation, Multiobjective Evolutionary Algorithms (MOEAs), Pareto Optimality, Adaptive Operator Selection.

1. Introduction

Multi-objective optimization deals with problems involving two or more conflicting objectives. In general, optimization problems can be combinatorial, continuous or both. The traveling salesman problem (TSP) [42] and minimum spanning tree (MST), for instance, are two well-known combinatorial problems. Combinatorial optimization has various applications in air traffic routing, design telephonic networks, electrical, hydraulic, TV cables and computer systems, road to deliver packages etc. Continuous optimization is widely utilized in mechanical design problems [24, 52]. This study is concerned with the minimization of multiple objectives within optimization problems (MOPs) involving discrete and/or continuous variables. The general formulation of a MOP is vas follows.

$$\begin{aligned} &\text{minimize } F(x) = (f_1(x), \ldots, f_m(x))^T \\ &\text{subject to } x \in \Omega \end{aligned} \tag{1}$$

where Ω is the decision space, $x = (x_1, x_2, \ldots, x_n)^T$ is a decision vector and x_i, $i = 1, \ldots, n$ are decision variables, $F(x) : \Omega \to R^m$ includes m real valued objective functions in the objective space R^m. If Ω is a closed and connected region in R^n and all objective functions involve only continuous variables then problem (1) is a continuous MOP.

In real-world multi-objective optimization problems, the objective functions are usually in conflict or mostly incommensurable. Consequently, there is not a unique solution that minimizes all the objective functions at the same time. The problem must be solved in terms of Pareto optimality.

A solution $u = (u_1, u_2, \ldots, u_n) \in \Omega$ is said to be Pareto optimal if there does not exist another solution $v = (v_1, v_2, \ldots, v_n) \in \Omega$ such that $f_j(u) \leq f_j(v)$ for all $j = 1, \ldots, m$ and $f_k(u) < f_k(v)$ for at least index k. An objective vector is Pareto optimal if the corresponding decision vector is Pareto optimal. All Pareto optimal solutions in the decision space form a Pareto Set (PS) and their image in the objective space forms a Pareto Front (PF) [37, 9, 12].

In the last few year, several multi-objective evolutionary algorithms (MOEAs) were developed and successfully applied to various real-world optimization tasks [31, 36, 8, 6, 7, 27, 57, 30, 25]. Classical MOEAs can generally be categorized into three main paradigms such as Pareto dominance based MOEAs [13, 59, 58, 39, 17], indicator based evolutionary algorithms (IBEAs) [62, 63, 5, 3, 4, 14, 51] and decomposition based MOEAs [54, 23, 55, 32, 34, 29, 21]. Among decomposition methods, (MOEA/D) [54] is recently newly developed paradigm that transforms the given MOP into a number of different single objective problems (SOPS) and then applies generic EA to simultaneously optimize all these SOPs in single simulation runs to get optimal set solutions. MOEA/D has several enhanced variants (e.g.[23, 55, 32, 34, 29, 21]). Decomposition and Pareto dominance approaches are the best choice for the adaptation of evolutionary operators and control parameters. IBEAs and decomposition based EAs do not use Pareto ranking directly as in Pareto dominance based MOEAs. All the above categorized MOEAs have two main goals: convergence towards the true Pareto front and maintaining a diverse set of solutions. They are population based stochastic techniques and approximate a set of optimal solutions in a single simulation run for the problem at hand. MOEAs maintain diversity within this set of solutions using different measures such as the fitness sharing technique, the niching approach, the Kernel approach, the nearest neighbor approach, the histogram technique, the crowding/clustering estimation technique, the relaxed form of dominance and restricted mating and many others.

A fast non-dominated sorting genetic algorithm II (NSGA-II) [13], SPEA2 [58], Pareto archive evolution strategy (PAES) [22], multi-objective genetic algorithm (MOGA) [15], and niched Pareto genetic algorithm (NPGA) [17] are well known Pareto dominance based MOEAs. Among them, NSGA-II [13] is an improved version of the non-dominated sorting genetic algorithm (NSGA)[20]. It generates offspring with crossover and mutation and selects the next generation according to non-dominated sorting and crowding distance comparison. SPEA2 [58] is an improved version of strength Pareto evolutionary algorithm (SPEA) [60]. It incorporates a fine-grained fitness assignment strategy, a density estimation technique, and an enhanced archive truncation method in contrast to SPEA [60]. Furthermore, it is equipped with the k-Nearest Neighbor (kNN) mechanism and a specialized ranking system to sort the members of the population, and select the next generation of population by combining the current population and offspring population created via crossover and mutation. Both SPEA2 [58] and NSGA-II [13] showed excellent performance in solving various real-world, scientific and engineering problems.

Memetic algorithms (MAs) are a growing area of research motivated by the meme notion introduced by Dawkins [38]. MAs are hybrid algorithms which combine local search optimizers and genetic algorithms. The first multi-objective MA was developed by Ishibuchi and Murata [18] and then improved by Jaszkiewicz [1, 19]. These algorithms basically reformulate the given MOP into the simultaneous optimization of all weighted Tchebycheff functions or all weighted sum functions. The genetically adaptive multi-objective optimization algorithm (AMALGAM) [49] blends multiple search operators to evolve new populations of solutions. The probability of the used different operators are updated based on their particular current performances.

1.1. Motivation and Contributions

This paper presents a novel hybrid non-dominated sorting genetic algorithm (HNSGA) with an adaptive operator selection, inspired by evolutionary computing (EC) [49, 50, 29, 28, 32, 34, 35, 33]. The algorithm, developed starting from NGSA-II [13], and uses multiple search operators such as simulated binary crossover (SBX) [11], differential evolution (DE) [40], center of mass crossover (CMX) [46] and simplex crossover (SPX) [47] to evolve population evolution with a self-adaptive procedure.In particular, HNSGA divides candidate designs in subpopulations according to number of operators, allocates different resources to each subpopulation in terms of selecting different search operators for each subpopulation and updates size of each subpopulation. Using multiple search operators allows to increase the probability of selecting the most suitable operator for the problem at hand in each generation. Picking one operator at random or just because a search paradigm has always used is not really enough. The multioperator approach reduces the probability of selecting the wrong operator in the first place and increase our chances of using a more suitable operator for our problem, possibly the most suitable. The rate of use of an operator (i.e. the resources allocated in the optimization process for that operator) must depend on its performance. The best operators should work more in the optimization process. Should an operator work well in a generation it would be used in the next generation as well. This mechanism will be clarified in the pseudo-code of the proposed algorithm.

The main contributions of this paper are as follows.

- The suggested Hybrid NSGA employ multiple search operators based on adaptive procedure and its algorithmic behavior is tested on classical benchmark problems such as the ZDT [61]and CEC09 problems [56].
- Optimization results are compared with those of state-of-the-art MOEAs such as NSGA-II [13], AMALGAM [50], MOEA/D [55], hybrid archive-based micro genetic algorithm (AMGA) [45], orthogonal multi-objective evolutionary algorithm (OMOEA) with lower-dimensional crossover [16], Pareto archived dynamically dimensioned search (PA-DDS) with hypervolume based selection for multi-objective optimization [2], and differential evolution with self-adaptation and local search for constrained multiobjective optimization algorithm (DECMOSA-SQP) [53].
- The inverted generational distance (IGD) [56], relative hypervolme (RHV) [48, 49], gamma Υ [13] and delta Δ [13] are used as performance indicators. In particular, IGD metric gives information on both convergence and spread of optimized solutions.
- It is found that HNSGA outperforms the above mentioned competitors as it always finds approximate Pareto fronts (PF) closer to the true PF for most test problems.

The rest of the article is organized as follows. Section 2 outlines the new algorithm. Section 3 describes test problems and performance metrics. Section 4 presents and discusses optimization results. Section 5 summarizes the main findings of this study and outlines directions of future research.

2. Hybrid Non-dominated Sorting Genetic Algorithm with Adaptive Operators Selection

The pseudo-code of the proposed algorithm (HNSGA) is outlined in the Algorithm 1. HNSGA is an improved version of NSGA-II [13]. Similar to NSGA-II [13], the present algorithm randomly generates a population set P_t of size N, uniformly distributed over the search space of the problem at hand.The t subscript denotes the number of the current generation.

HNSGA initially divides the population P_t into q sub-populations $P_t(k)$ where q is the number of operators selected for the search process. For example, if population size is $N = 100$ and there are 4 operators, it holds $P_t = [P_1, P_2, P_3, P_4] = [25, 25, 25, 25]$. The initial assignment of sub-populations to operators is not based on fitness evaluation. The $q = 4$ search operators selected in this study are differential evolution (DE), simplex crossover (SPX), simulated binary crossover (SBX) and center of mass crossover (CMX), respectively, for sub-populations 1, 2, 3 and 4.Each operator perturbs the designs included in its corresponding subset. For each sub-population $P_t(k)$,an offspring population $Q_t(k)$ is thus generated.The above mentioned tasks are completed in steps 4 through 26 of Algorithm 1.Offsprings are merged in the population Q_t including N elements.After first generation (t=1), sizes of the P_k sub-population sets are updated based on individual performances of the q operator. NHSGA combines parent and offspring populations into the population $R_t = P_t \cup Q_t$ of size $2N$.

In step 31, a fast non-dominated sorting procedure of NSGA-II [13] is applied to population R and best N solutions are extracted and stored in the new population $\grave{P}_t$. Each operator gets resources in terms of sub-population $P_t(k)$ size based on its contribution to the new population as outlined in Algorithm 2. Credit assignment procedures adopted in HNSGA assign rewards to operators based on their offspring solutions that can survive to the next generation. HNSGA counts the solution members of Q_t retained in the new population $\grave{P}_t$ and allocates $1s$ more of CPU time to the operator that generated a trial solution able to replace a parent solution.Conversely, if the trial solution generated by an operator did not improve current population, that operator does not get any reward and its allocated CPU time is increased by $0s$ (i.e. it remains unchanged). The counting of solutions retained in the new population $\grave{P}_t$ or each operator is done in steps $33-34$ of the Algorithm 2.For example, $P_t = [30, 23, 21, 26]$ indicates that 30 elements have been extracted from $P_1 \cup Q_1$, 23 elements from $P_2 \cup Q_2$, 21 elements from $P_3 \cup Q_3$ and 26 elements from $P_4 \cup Q_4$. Therefore, the first crossover operator got a larger population than other search operators. Finally, HNSGA provides a set of non-dominated solutions as termination criteria are satisfied.

3. Test Problems and Performance Metrics

Since real-world problems usually include many objective functions, different test suites for MOPs have been developed by evolutionary computation experts [10, 43]. In the present study, we have selected the ZDT [61] and CEC'09 [56] test problems. Their main features of ZDT problems are summarized in the Table 1 and CEC'09 test instances's characteristics are explained in the Table 2.

Table 1. Characteristics of the Zitzler-Deb-Thiele's (ZDT) Benchmark Functions

Name	Functions	Side Constraints	Characteristics of PF
ZDT1	2	$[0, 1]^n$	convex PF
ZDT2	2	$[0, 1]^n$	nonconvex PF
ZDT3	2	$[0, 1]^n$	discontinuous PF
ZDT4	2	$[0, 1] \times [-5, 5]^{n-1}$	many local Pareto fronts
ZDT6	2	$[0, 1]^n$	local density solutions near Pareto front/nonuniformly spaced, nonconvex

Table 2. Details of CEC'09 benchmark functions

CEC'09	Functions	Side Constraints	Characteristics of PF
UF1	2	$[0, 1] \times [-1, 1]^{n-1}$	Concave
UF2	2	$[0, 1] \times [-1, 1]^{n-1}$	Concave
UF3	2	$[0, 1]^n$	Concave
UF4	2	$[0, 1] \times [-2, 2]^{n-1}$	Convex
UF5	2	$[0, 1] \times [-1, 1]^{n-1}$	21 point front
UF6	2	$[0, 1] \times [-1, 1]^{n-1}$	One isolated point and two disconnected parts
UF7	2	$[0, 1] \times [-1, 1]^{n-1}$	Continuous straight line
UF8	3	$[0, 1]^2 \times [-2, 2]^{n-2}$	Parabolic
UF9	3	$[0, 1]^2 \times [-2, 2]^{n-2}$	Planar
UF10	3	$[0, 1]^2 \times [-2, 2]^{n-2}$	Parabolic

MOEAs usually involve a number of internal parameters whose setting may greatly affect the computational efficiency of the optimizer. In this study, experiment were carried out using the following values of the internal parameters to solve the ZDT problems [61] and CEC'09 test instances [56].

- $N = 100$: population size for 2-objective test instances.
- $F = 0.5$: scaling factor of the DE;

Algorithm 1 Hybrid Non-dominated Sorting Genetic Algorithm with Adaptive Operators Selection

[**Input:**] N: Population size, P_m: Probability of mutation, Max_{Gen}: Maximum number of generations or Termination criterion, n: number of decision variables) and $q = 4$ number of search operators.
[**Output:**] Pareto Set $(PS) = \{x^1, \ldots, x^N\}$ and Pareto Front $(PF) = \{F(x^1), \ldots, F(x^N)\}$;
$P_t \leftarrow$ **Uniform-Random**(N, n) ▷ Generate population set P_t uniformly and randomly.
Evaluate-Fitness(P_t) ▷ Evaluate the fitness values of P_t solutions.
$P_t(k) \leftarrow \{N \times \frac{1}{q}, k = 1, 2, \ldots, q\}$ ▷ Select randomly equal number of solutions for each search operator at $t = 1$.
while $Termination\ Condition\ is\ not\ Satisfied$ **do**
  for $i \leftarrow 1 : N$ **do**
    if $i \in P1$ **then**
      $x^j, x^k, x^i \leftarrow$ **Random-Selection**$(i, P1)$ such that $x^i \neq x^j \neq x^k$.
      $Q1' \leftarrow$ $\mathbf{XOR_1}(x^i, x^j, x^k)$ ▷ A Crossover XOR_1 can be DE.
      $Q1 \leftarrow$ **Polynomial-Mutation**$(Q1', Params)$ together with **Repair-Strategy**$(Q1')$.
    else
      if $i \in P2$ **then**
        $x^j, x^k, x^i \leftarrow$ **Random-Selection**$(i, P2)$ such that $x^i \neq x^j \neq x^k$.
        $Q2' \leftarrow$ $\mathbf{XOR_2}(x^i, x^j, x^k)$ ▷ XOR_2 can be SPX.
        $Q2 \leftarrow$ **Polynomial-Mutation**$(Q2', Params)$ together with **Repair-Strategy**$(Q2')$.
      else
        if $i \in P3$ **then**
          $x^j, x^k, x^i \leftarrow$ **Random-Selection**(i, P_3) such that $x^i \neq x^j \neq x^k$.
          $Q3' \leftarrow$ $\mathbf{XOR_3}(x^i, x^j, x^k)$ ▷ XOR_3 can be SBX.
          $Q3 \leftarrow$ **Polynomial-Mutation**$(Q3', Params)$ together with **Repair-Strategy**$(Q3')$.
        else
          if $i \in P4$ **then**
            $x^j, x^k, x^i \leftarrow$ **Random-Selection**$(i, P4)$ such that $x^i \neq x^j \neq x^k$.
            $Q4' \leftarrow$ $\mathbf{XOR_4}(x^i, x^j, x^k)$ ▷ XOR_4 can be CMX.
            $Q4 \leftarrow$ **Polynomial-Mutation**$(Q4', Params) \leftarrow$ **Repair-Strategy**$(Q4')$.
          end if
        end if
      end if
    end if
    $Q_t \leftarrow$ **Combine-Offspring**$\{Q1 \cup Q2 \cup Q3 \cup Q4\}$ ▷ Combine sub-offspring population sets.
    Evaluate-Fitness(Q_t) ▷ Evaluate offspring population Q.
  end for
  $R_t \leftarrow$ **Combine-Parent-Offspring**$(P_t \cup Q_t)$ ▷ Combine parent and offspring populations.
  Ranking+Crowding(R_t) ▷ Find ranks and measure crowding distance of R_t population.
  $\grave{P}_t \leftarrow$ **Select-Best-Individuals**(R_t) ▷ Select N best Individuals from R_t population.
  $\{I_i | i = 1 : q\} \leftarrow$ **Count-Indices-XORs**$(\grave{P}_t)$ ▷ Count the individuals of each crossover to enter into new population P.
  Update$P_t(k)$ ▷ For explanation go to Algorithm 2.
  $t = t + 1$;
end while

Algorithm 2 Adaptive Operators Selection Strategy

1: In steps 32 and 33 of Algorithm 1, we count the number of solutions of each operator that are retained in the new population $\grave{P}$ in each generation of the algorithm.
2: Each successful solution generated by some operator leads to a reward of $1s$ for that operator, while unsuccessful solutions to a reward of $0s$ for the operators that generated them. Here, s stands for $seconds$.
3: **for** $k \leftarrow 1 : q$ **do**
4: $\delta(k) \leftarrow$ **Count-Successful-Solutions**$(\grave{P}, q)$ ▷ Count the solutions of each XOR_k crossover that belong to $\grave{P}$.
5: $\zeta(k) \leftarrow \frac{\delta(k)}{\sum_{k=1}^{q} \delta(k)}$;
6: $P_{t+1}(k) \leftarrow \alpha \times P_t(k) + (1-\alpha)\frac{\zeta_k}{\sum_{k=1}^{4} \zeta_k} \times N$▷; where α is a user defined parameter; here $\alpha = 0.2$.
7: Update the resources allocation set of crossovers denoted by XOR
8: $P_t(k) \leftarrow P_{t+1}(k), k = 1, \ldots, q$
9: **end for**

- $CR = 0.5$: probability of crossover;
- $F_{eval} = 25000$: maximum number of function evaluations;
- $\eta_c = \eta_m = 20$ distribution indices for the SBX and polynomial mutation, where η_c measures the distance of child solution from their parent solution and η_m defines the polynomial probability distribution.
- $p_c = 0.7$ and $p_m = \frac{1}{n}$ are probability of crossover and mutation, respectively.

✠ Values of internal parameters for CEC'09 problems were instead set as follows.

- $N = 600$: population size for 2-objective test problems;
- $N = 1000$: for 3-objective test problems;
- $F = 0.5$: scaling factor of the DE;
- $CR = 1$: crossover probability for DE;
- $F_{eval} = 300,000$: maximum number of function evaluations;

3.1. Performance Metric

The quality of the final set of non-dominated solutions must be assessed in terms of convergence and diversity. The former depicts the closeness of the final set of non-dominated solutions to the true Pareto front (PF), while the latter aims to reach a uniform distribution of the final set of solutions over the true PF. Many performance metrics for measuring the quality of obtained solutions are suggested in the literature [41, 48, 12, 64]. Indicators including relative hypervolume indicator (RHV) [48, 49], gamma (Υ) [13] and delta (Δ) [13] were used as indicators to quantitatively assess performance of the used MOEAs in this paper.

3.2. Inverted Generational Distance (IGD)

In this study, the inverted generational distance (IGD) [56] was utilized to evaluate the performance of the proposed algorithm. IGD measures both convergence and diversity of the approximate Pareto Front (APF) over the true PF.

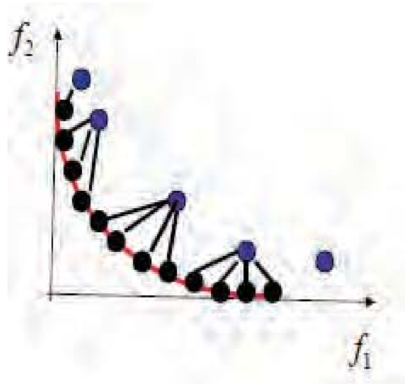

Figure 1. Explanation of the inverted generational distance performance indicator.

Let P^* be a generated set of uniformly distributed points along the true PF (the black points) as shown in Figure 1. The average distance from P^* to the approximated set A (the green points) is defined as [56]:

$$D(A, P) = \frac{\sum_{v \in P^*} d(v, A)}{|P^*|}$$

where $d(v, A)$ is the minimum Euclidean distance between v and the points in A. If P^* is large enough to represent the PF very well, $D(A, P)$ can measure both the diversity and convergence of A: the smaller the value of the IGD metric is, the better will be the obtained solution set. Here, we selected $P^* = 500$ uniformly distributed solutions over the true PF for two-objective problems and $P^* = 1000$ individuals for problems with three objective functions.

3.3. Relative Hypervolume Indicator (RHV)

The relative hypervolume (RHV) indicator can mathematically be expressed as

$$RHV(A) = \frac{HV(P^*) - HV(A)}{HV(P^*)}$$

where $(HV(.))$ denotes the hypervolume of approximated sets A and P^*, calculated as follows [48, 49]:

$$HV(A) = \text{volume} \cup_{i=1}^{|A|} z_i$$

where $i \in A$ and z_i is the i^{th} hypercube constructed with respect to reference point W and the solution i as the diagonal corners of the hypercube.The approximated set of solutions will tend to the true Pareto-optimal set as the value of RHV tends to 0.

3.4. The Gamma (Υ) Indicator

In order to use Υ metric [13], $P^* = 500$ uniformly spaced solutions were generated on the true Pareto-optimal front of the problem at hand. For each solution included in the approximated set A, the minimum Euclidean distance from the generated set P^* solution is computed. The average of these distances is defined as the is defined as gamma Υ metric values. Hence, the approximated set will tend to the true Pareto-optimal set as the Υ-metric tends to 0. Furthermore, this indicator measures the quality of convergence to a known set of Pareto-optimal solutions. The smaller the value of Υ is, the better distribution and diversity of obtained non-dominated solutions will be. However, since this metric sometimes cannot give information on the spread of obtained solutions, the Δ-metric was utilized in this study.

3.5. The Delta (Δ) Indicator

The Δ metric function is calculated as, [13]:

$$\Delta = \frac{d_f + d_I + \sum_{i=1}^{N-1} d_i - \overline{d}}{d_f + d_I + (N-1)\overline{d}},$$

where d_f and d_I are the Euclidean distances of the extreme solutions and the boundary solutions belonging to the approximated set of optimal solutions; $\overline{d}$ denotes the average of all Euclidean distances d_i between consecutive solutions in the final approximated set of optimal solutions provided by a particular algorithm. Very small values of Δ result in better distribution and diversity of the approximated solution set.

4. Results and Discussion

The optimization algorithms compared in this study were coded in MATLAB and 30 independent optimization runs were performed for each test problem (with the parameter settings as specified in Section 3,consistently with the values indicated in literature.) on a 2.4 GHz Core 2 Quad processor with 4 GB of RAM memory working on Windows XP Professional Operating System.

Table 3 compares the minimum, median, mean, standard deviation and maximum IGD-metric values evaluated for the present HNSGA algorithm and other state-of-the-art MOEAs such as (a) HNSGA the suggested algorithm and the state-of-the-art MOEAs: (b) AMALGAM [49], (c) NSGA-II [13], (d) MOEA/D [54] in the ZDT test problems [10].It can be seen that HNSGA always found better approximated solutions set in terms of proximity to true PF and diversity. The same comparison is presented in the Table 4 with respect to the RHV-metric indicator while Table 5 evaluates the performance of HNSGA in terms of ϒ and Δ functions metrics. Again, the present algorithm was the most efficient optimizer overall.

Table 6 statistically compare the IGD-metric values produced by HNSGA in the CEC'09 problems with those relative to several state-of-the-art algorithms such as ALMALGAM [49], NSGA-II-SQP [44], NSGA-II [13], hybrid AMAGA [45], OMOEA [16], PA-DS [2], and DECMOSA-SQP [53]. In order to have a homogeneous basis of comparison, HNSGA optimizations were run setting values of parameters shared with referenced algorithms equal to those indicated in the literature. It can be seen from the tables that HNSGA always was very competitive with other optimizers outperforming other methods in most cases. Statistics expressed in terms of ϒ and Δ indicators (see Table 8) demonstrate that in the CEC'09 problems HNSGA could obtain a set of optimal solutions with better convergence properties spanning the solution sets over the entire Pareto-optimal region in most cases.

Figures 2 compares the best approximate Pareto fronts (PFs) of the five ZDT problems found by the HNSGA and NSGA-II [13] algorithms within 25000 function evaluations.It appears that the present algorithm can better reconstruct the true PF. The results obtained by HNSGA and NSGA-II [13] in the 30 independent simulations employing random seeds are compared in the Figures 3. It can be seen that all of the 30 PFs displayed for HNSGA entail a better distribution of solutions than in the case of NSGA-II [13].

Finally, Figure 4 compares the average variation of the IGD indicator for HNSGA and NSGA-II [13] confirming the superiority of the present algorithm that found the lowest values for this metric average variations in IGD-metric. The best approximate PFs found by HNSGA in the case of the CEC'09 test problems are shown in the Figures 5 while Figures 8 plots the corresponding results for NSGA-II [13]. In the present case, the best approximated PFs for the 2-objective function problems UF1-UF7 and the 3-objective function problems UF8-UF10) appear to be better in terms of diversity and proximity to true PF.

The PFs obtained in the 30 optimization runs of HNSGA are shown in the Figures 6 for problems UF1-UF4 and UF7- UF1-UF4, UF7-UF9. The corresponding plots for problems UF1-UF4 and UF7-UF10 solved with NSGA-II [13] are instead shown in Figure 7.

Figure 9 shows the evolution of the average IGD-metric with respect to the number of generations in problems UF1-UF7, UF9 and UF10. It can be seen that HNSGA always outperformed NSGA-II shows the average evolution in IGD-metric values versus number of generations spent by HNSGA and NSGA-II for dealing with UF1-UF7, UF9 and UF10. These figure demonstrate that HNSGA has tackled most test problems in much better average variation in the IGD-metric values as compared to NSGA-II.

Table 3. Statistical comparison of the IGD-metric values obtained by (a) HNSGA, (b) ALMALGAM [49], (c) NSGA-II [13] for ZDT problems.

CEC'09	Min	Median	Mean	StD	Max	*Algorithms*
ZDT1	0.003764	0.003795	0.003804	0.000031	0.003912	a
	0.004421	0.004623	0.004705	0.000237	0.005481	b
	0.0042193	0.004472	0.004369	0.000139	0.004258	c
	0.0040137	0.004196	0.004215	0.0001186	0.004557	d
ZDT2	0.003852	0.003897	0.003897	0.000032	0.003975	a
	0.004521	0.004893	0.004912	0.000269	0.005744	b
	0.0043213	0.004649	0.004656	0.000182	0.005011	c
	0.003837	0.003876	0.003886	0.0000426	0.004039	d
ZDT3	0.0050394	0.0052055	0.0052010	0.00697	0.005359	a
	0.004521	0.004893	0.004912	0.000269	0.005744	b
	0.005132	0.00546	0.00912	0.01388	0.0602182	c
	0.008484	0.009063	0.00915	0.000662	0.012523	d
ZDT4	0.003811	0.003907	0.003921	0.000060	0.004145	a
	0.004814	0.005297	0.005287	0.000171	0.005588	b
	0.004814	0.005297	0.005287	0.000171	0.005588	c
	0.011963	0.030562	0.042586	0.03342	0.157978	d
ZDT6	0.003398	0.003448	0.003453	0.000032	0.003531	a
	0.003821	0.004049	0.004055	0.000182	0.004732	b
	0.005606	0.007045	0.007003	0.0005878	0.0080474	c
	0.00856	0.015103	0.01479	0.004053	0.023509	d

Figure 10 visualize the contribution of each used crossover operator during the search process of the HNSGA to cope with tested MOPs. These figures demonstrate the adaptive searching behavior of the used crossover while producing an successful offspring solutions to the next generation of population evolution in HNSGA framework.

The above discussed results confirm that HNSGA could reach global convergence and reconstruct the complete Pareto optimal frontier for almost all test problems selected from CEC'09 [56] and ZDT test suites [61]. However, the objective functions of problems UF5 and UF6 are multi-modal near the global Pareto-optimal frontier even and slight perturbations of optimization variables may cause solutions to become dominated and trapped in their local basin of attraction. Similar to its competitors, HNSGA faced genetic drift as population follows good solutions found in the early stages of search process. This results in the clustering of solutions around these early discovered points. The very good performance of HNSGA is mainly due to the use of multiple operators with self-adaptive strategies. In fact, different operators may be suited for a larger variety of problems while the single operators utilized in the other algorithms (e.g. NSGA-II [61]) may not keep best performance during the whole optimization process. For this reason, multiple ensemble search operators should be utilized for more complicated real-world problems than those considered in this study.

Table 4. Statistical comparison of the RHV-metric values obtained for (a) HNSGA, (b) ALMALGAM [49], (c) NSGA-II [13] for ZDT problems.

ZDT	Min	Median	Mean	StD	Max	*Algorithms*
ZDT1	0.0049897	0.0053957	0.0054043	0.0002026	0.0057415	a
	0.0052387	0.0056880	0.0057441	0.0002893	0.0065897	b
	0.0053851	0.0058431	0.0058705	0.0002625	0.0064456	c
	0.0177029	0.020013	0.01991859	0.0004823	0.020497	d
ZDT2	0.0049736	0.0051240	0.0051327	0.00002176	0.0052514	a
	0.0050627	0.0055745	0.0056204	0.0002371	0.0061003	b
	0.0052781	0.0057286	0.0057248	0.0002348	0.0061626	c
	0.00505715	0.0051363	0.00514	0.00004208	0.0052718	d
ZDT3	0.0029581	0.0032372	0.00326405	0.0001879	0.0038705	a
	0.0032145	0.00354485	0.00356853	0.00020146	0.0039716	b
	0.0051326	0.0054674	0.0091267	0.0138825	0.0602182	c
	0.00477953	0.0048314	0.004828423	0.0001887	0.00483530	d
ZDT4	0.0068649	0.0074254	0.0074463	0.0003463	0.0076536	a
	0.0070600	0.0077258	0.0077495	0.0003516	0.00840719	b
	0.0069089	0.0092493	0.0079564	0.0048926	0.0091253	c
	0.0189880	0.0201214	0.020272	0.00060908	0.0218068	d
ZDT6	0.0040689	0.0041922	0.0050197	0.0004514	0.0015346	a
	0.0059667	0.0065878	0.0341751	0.1509787	0.8335517	b
	0.006189	0.0069388	0.0068906	0.0003541	0.0077600	c
	0.0048518	0.0050767	0.0052309	0.000459	0.0052243	d

Table 5. Statistical comparison of the Υ and Δ-values for the ZDT problems [61].

ZDT	Min	Median	Mean	StD	Max	Metrics
ZDT1	0.0315434	0.0372392	0.0279883	0.0025911	0.0102020	Υ
	0.2162211	0.30332014	0.3042615	0.01534164	0.3421585	Δ
ZDT2	0.0142695	0.0215163	0.0234601	0.0047628	0.0284613	Υ
	0.2264769	0.32186711	0.3231461	0.0231412	0.3603254	Δ
ZDT3	0.0432680	0.0553217	0.0568125	0.0045721	0.0600402	Υ
	0.449444	0.5045672	0.5046453	0.0202261	0.5131220	Δ
ZDT4	0.0250162	0.0300238	0.0355547	0.0109218	0.0703774	Υ
	0.3258961	0.3054030	0.3054483	0.0308418	0.4302315	Δ
ZDT6	0.01034511	0.01504932	0.0151906	0.0057525	0.0316515	Υ
	0.21454965	0.25196317	0.2538762	0.01027606	0.323209	Δ

Table 6. Statistical comparison of IGD-metric values obtained for (a) HNSGA, (b) ALMALGAM [49], (c) NSGA-II-SQP [44] (d) NSGA-II [13] on CEC'09 test instances.

CEC'09	Min	Median	Mean	StD	Max	*Algorithms*
UF1	0.013033	0.012027	0.011238	0.001417	0.020143	a
	0.029425	0.059633	0.057992	0.008557	0.070121	b
	0.009851	* * **	0.01153	0.0073	0.04734	c
	0.051996	0.106873	0.096076	0.024862	0.128739	d
UF2	0.003852	0.003897	0.003897	0.000032	0.003975	a
	0.011432	0.013029	0.013217	0.001367	0.016769	b
	0.006025	* * **	0.01237	0.009108	0.05455	c
	0.016012	0.019849	0.020050	0.001407	0.023589	d
UF3	0.010300	0.027521	0.028749	0.013865	0.066973	a
	0.091044	0.135348	0.136503	0.022927	0.199235	b
	0.03435	* * **	0.10603	0.06864	0.26207	c
	0.066353	0.098234	0.097065	0.017958	0.134235	d
UF4	0.040277	0.040458	0.041211	0.002399	0.059598	a
	0.040359	0.041061	0.041020	0.000332	0.041678	b
	0.04823	* * **	0.0584	0.005116	0.06975	c
	0.052199	0.054388	0.054551	0.001274	0.056679	d
UF5	0.259499	0.376031	0.379204	0.065761	0.509010	a
	0.166357	0.171420	0.171810	0.002873	0.178301	b
	0.29106	* * **	0.5657	0.1827	1.0498	c
	1.523087	1.671735	1.676288	0.099452	1.844279	d
UF6	0.077093	0.129060	0.150799	0.064798	0.281519	a
	0.068589	0.079046	0.078552	0.005998	0.089807	b
	0.08202	* * **	0.31032	0.19133	0.71745	c
	0.705834	0.762023	0.762271	0.028052	0.831784	d
UF7	0.007499	0.009677	0.009788	0.000970	0.012891	a
	0.014943	0.017678	0.017795	0.001254	0.020975	b
	0.007631	* * **	0.02132	0.01946	0.08801	c
	0.067270	0.114403	0.112305	0.012055	0.125719	d
UF8	0.090605	0.107091	0.08619	0.007010	0.123689	a
	0.103736	0.234141	0.230682	0.026012	0.261557	b
	0.06762	* * **	0.0863	0.01243	0.10911	c
	0.095436	0.108548	0.120433	0.030475	0.195112	d
UF9	0.073649	0.106394	0.1120152	0.087431	0.320933	a
	0.056616	0.067999	0.114652	0.085662	0.325894	b
	0.03873	* * **	0.0719	0.04504	0.19140	c
	0.088857	0.188603	0.160832	0.047975	0.218993	d
UF10	0.253304	0.307856	0.316548	0.020210	0.350921	a
	0.273304	0.327886	0.326948	0.020030	0.360955	b
	0.5339	* * **	0.84468	0.1626	1.1266	c
	0.473865	0.744428	0.781509	0.134987	1.043141	d

Table 7. Statistical comparison of IGD-metric values obtained for e) hybrid AMAGA [45], (f) Orthogonal MOEA (OMOEA) [16], (g) PA-DS with hypervolume based selection for multi-objective optimization [2], (h) DE with self-adaptation and local search for constrained multi-objective optimization (DECMOSA-SQP) [53] over 30 independent simulations on CEC'09 test instances [56].

CEC'09	Min	Max	Mean	StD	Algorithms
UF1	0.021023	0.059289	0.035886	0.010252	e
	0.078362	0.096748	0.085646	0.004070	f
	0.02909	0.10645	0.06234	0.02281	g
	0.055126	0.0880129	0.0770281	0.039379	h
UF2	0.011635	0.024160	0.016236	0.003167	e
	0.027570	0.034295	0.030572	0.001609	f
	0.00951	0.01909	0.01365	0.00232	g
	0.0173361	0.040226	0.0283427	0.0313182	h
UF3	0.037659	0.089363	0.069980	0.013954	e
	0.201978	0.353186	0.271415	0.037612	f
	0.08109	0.22473	0.12963	0.03291	g
	0.0305453	0.168162	0.0935006	0.197951	h
UF4	0.037688	0.044606	0.040621	0.001750	e
	0.044441	0.048181	0.046246	0.000966	f
	0.02927	0.03656	0.03229	0.00208	g
	0.0316247	0.035643	0.0339266	0.0053707	h
UF5	0.070599	0.134627	0.094057	0.0120555	e
	0.163349	0.178052	0.169201	0.003901	f
	0.13327	0.19261	0.21767	0.01718	g
	0.133012	0.237081	0.167139	0.0895087	h
UF6	0.045115	0.230019	0.129425	0.056588	e
	0.068193	0.079371	0.073381	0.002448	f
	0.06198	0.41434	0.22171	0.09903	g
	0.0579174	0.589904	0.126042	0.561753	h
UF7	0.013147	0.247734	0.057076	0.065309	e
	0.031179	0.038803	0.033548	0.001735	f
	0.13345	0.19234	0.21723	0.01709	g
	0.0198913	0.0427502	0.024163	0.0223494	h
UF8	0.139957	0.206937	0.171251	0.017224	e
	0.139163	0.201114	0.192005	0.012296	f
	0.08513	0.20854	0.13043	0.03932	g
	0.0989388	0.228895	0.215834	0.121475	h
UF9	0.112624	0.265932	0.188610	0.042137	e
	0.105055	0.341103	0.231795	0.064767	f
	0.02734	0.15901	0.04722	0.03041	g
	0.0772668	0.332909	0.14111	0.345356	h
UF10	0.201427	0.547349	0.324186	0.0957181	e
	0.439716	1.082671	0.627544	0.145954	f
	0.17627	0.74506	0.35129	0.20502	g
	0.238279	0.580852	0.369857	0.65322	h

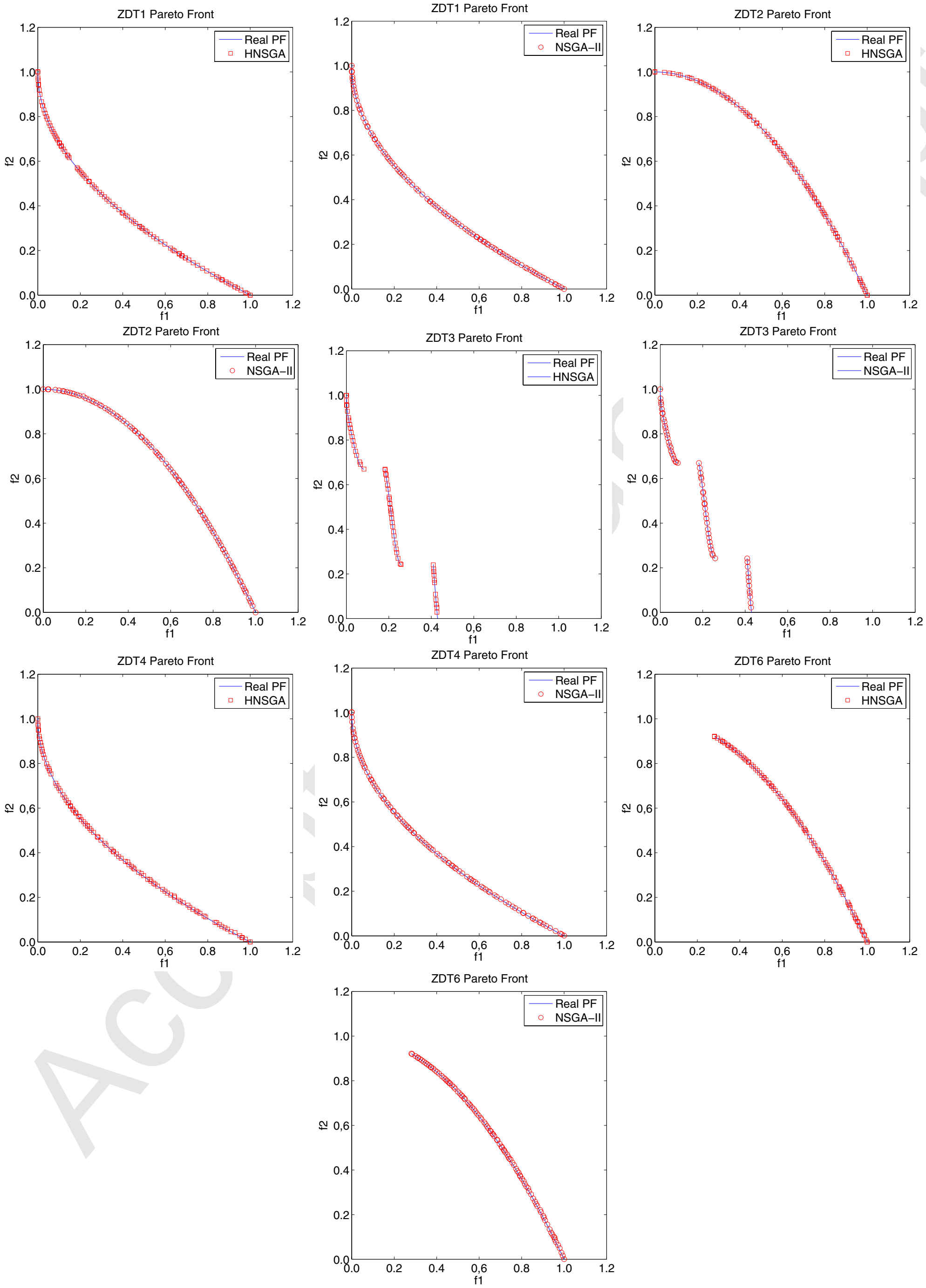

Figure 2. Comparison of approximate Pareto fronts obtained in the best optimization runs of HNSGA and NSGA-II on the ZDT test problems.

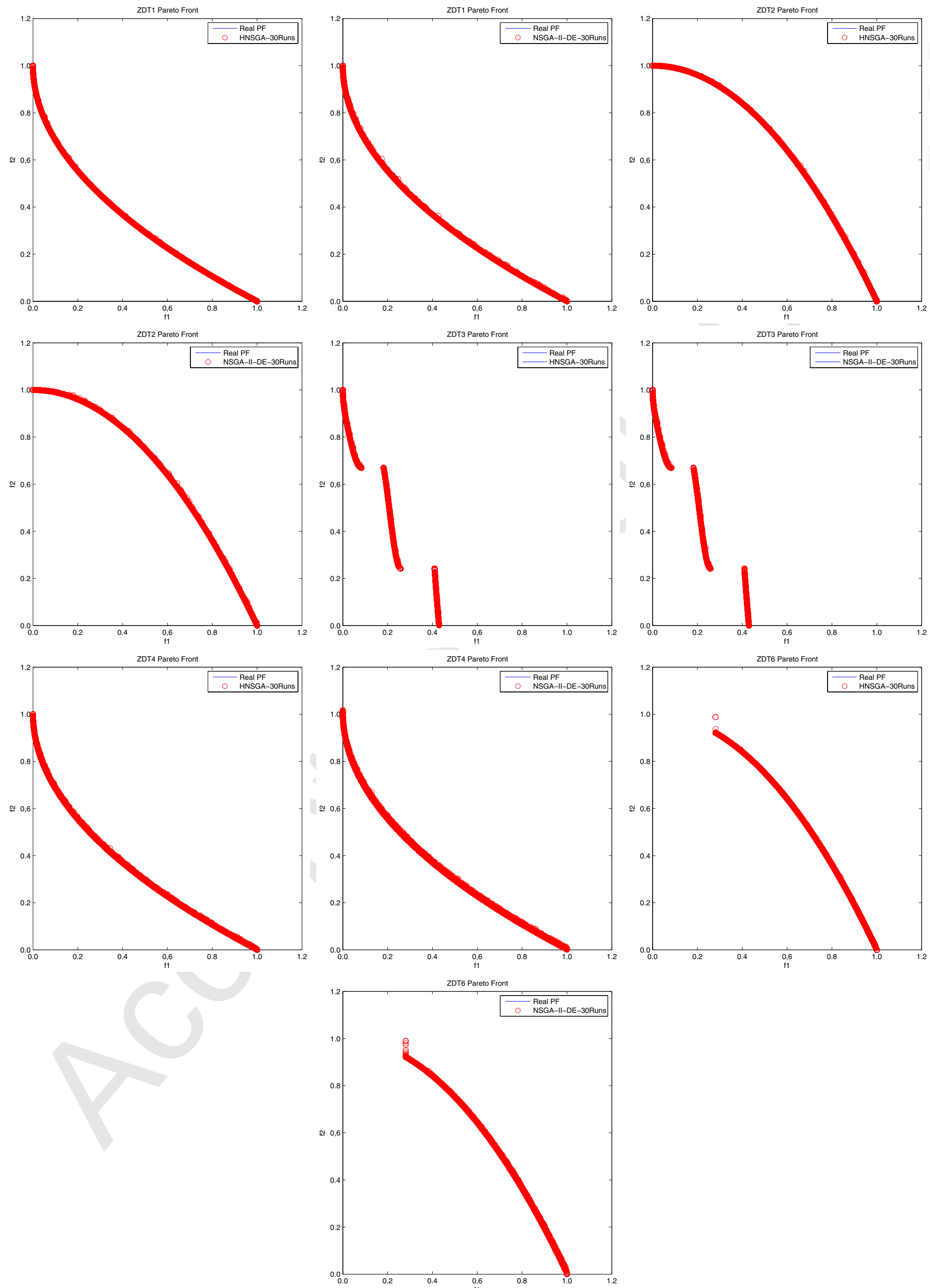

Figure 3. Comparison of approximate Pareto fronts obtained in all optimization runs of HNSGA and NSGA-II on the ZDT problems

Table 8. Statistical comparison of the relative hypervolume (RHV) and gamma (γ)-metric values obtained for HNSGA on CEC'09 test problems.

CEC'09	Min	Median	Mean	StD	Max	Metrics
	0.0101260	0.0109410	0.0108149	0.0682020	0.1020036	RHV
UF1	0.0108324	0.0237234	0.0239023	0.0120345	0.0210436	ϒ
	0.0002587	0.0101630	0.0109421	0.0162106	0.0979968	RHV
UF2	0.0021431	0.00320010	0.0032022	0.0004012	0.0030416	ϒ
	0.0013838	0.0021507	0.0021767	0.0003508	0.0030351	RHV
UF3	0.0103143	0.0123561	0.0124501	0.0020317	0.0134113	ϒ
	0.0013838	0.0021507	0.0021767	0.0003508	0.0030351	RHV
UF4	0.0100453	0.0210312	0.0213969	0.0001301	0.0310281	ϒ
	0.0307513	0.1208518	0.1314200	0.0401313	0.2087415	RHV
UF5	0.0361094	0.0317413	0.0375054	0.0212730	0.0624107	ϒ
	0.0020242	0.0011711	0.0110571	0.1013035	0.6670742	RHV
UF6	0.0010155	0.0120732	0.0123627	0.0020207	0.0303115	ϒ
	0.0003605	0.0009032	0.00921109	0.0102462	0.068491	RHV
UF7	0.0011005	0.0012051	0.0013203	0.0003216	0.0100234	ϒ
	0.0421403	0.0986121	0.0985931	0.0001995	0.0990095	RHV
UF8	0.0011005	0.0012051	0.0013203	0.0003216	0.0100234	ϒ
	0.0373526	0.0921153	0.0930047	0.0007384	0.0856440	RHV
UF9	0.0501054	0.5232083	0.5386664	0.0140512	0.0545365	ϒ
	0.0003605	0.0009032	0.00921109	0.0102462	0.068491	RHV
UF10	0.095102	0.0932820	0.0943213	0.0004272	0.0672061	ϒ

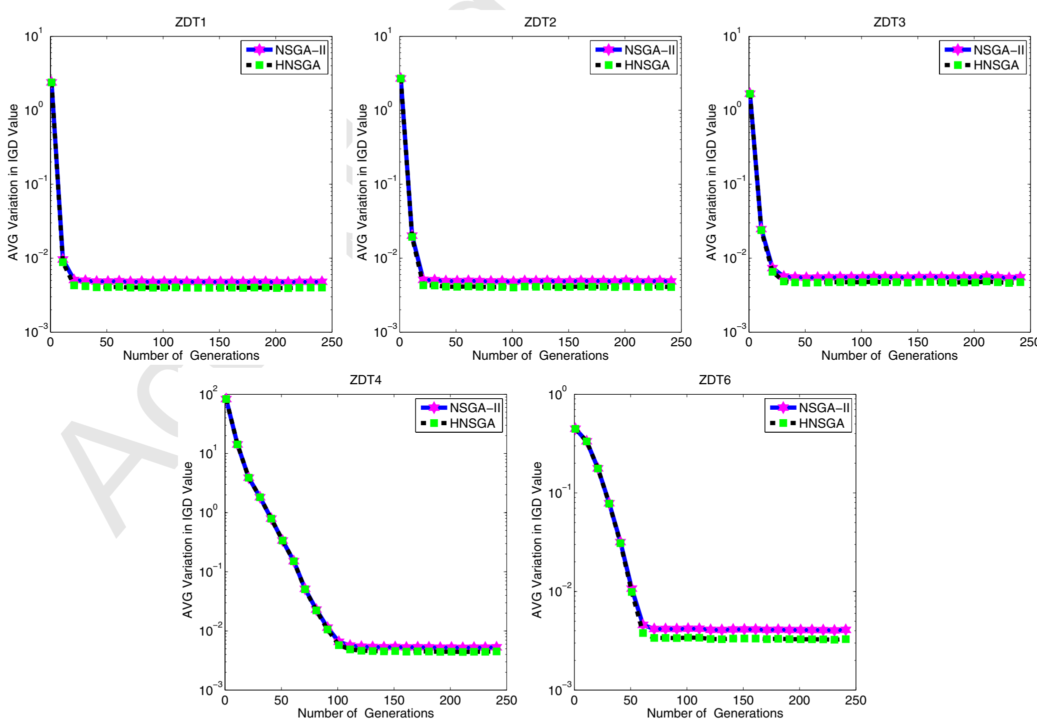

Figure 4. Comparison of IGD-metric average values in HNSGA and NSGA-II [13] on ZDT test problems.

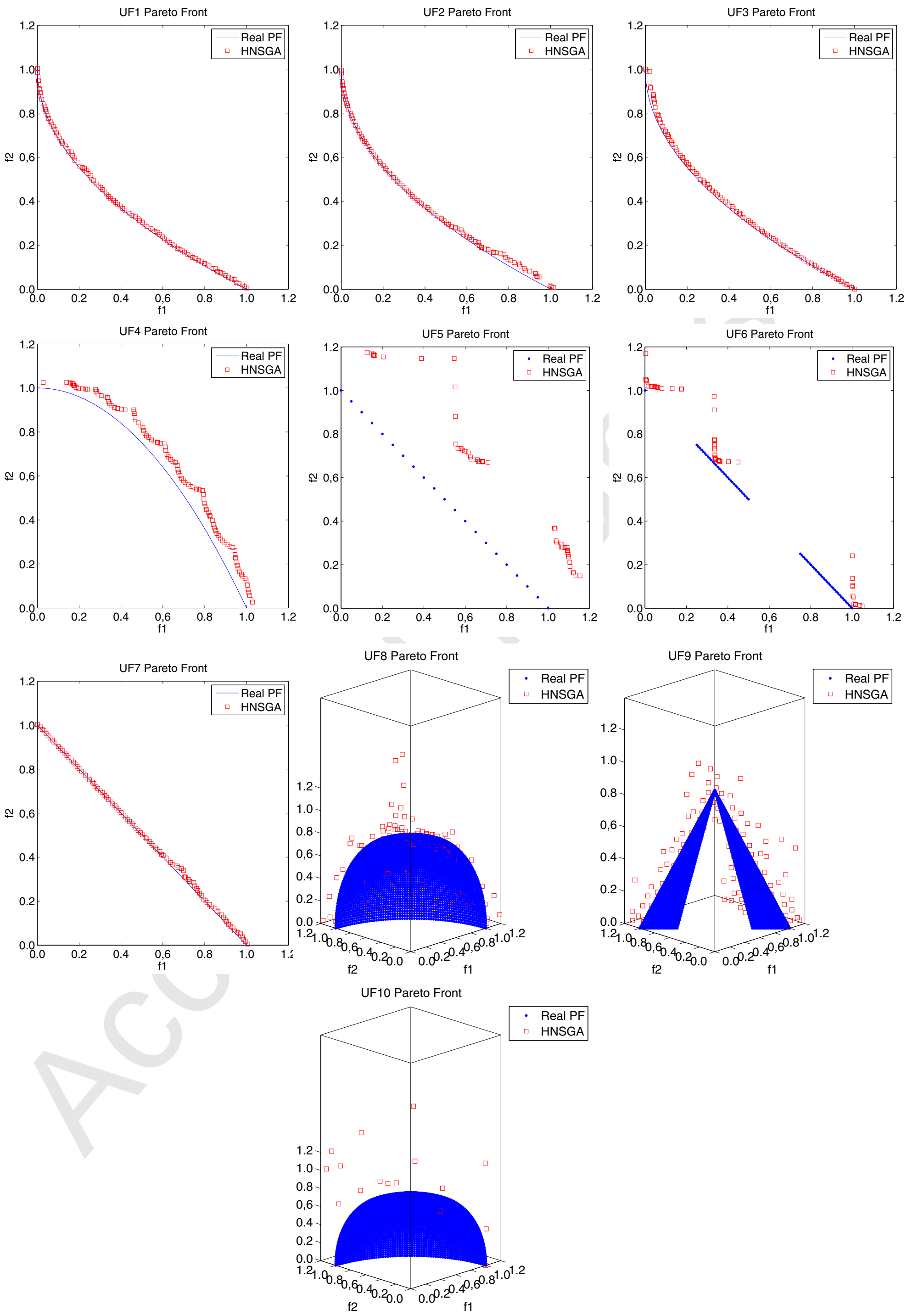

Figure 5. Comparison of approximate Pareto fronts obtained in the best optimization run of HNSGA for the CEC'09 test problems.

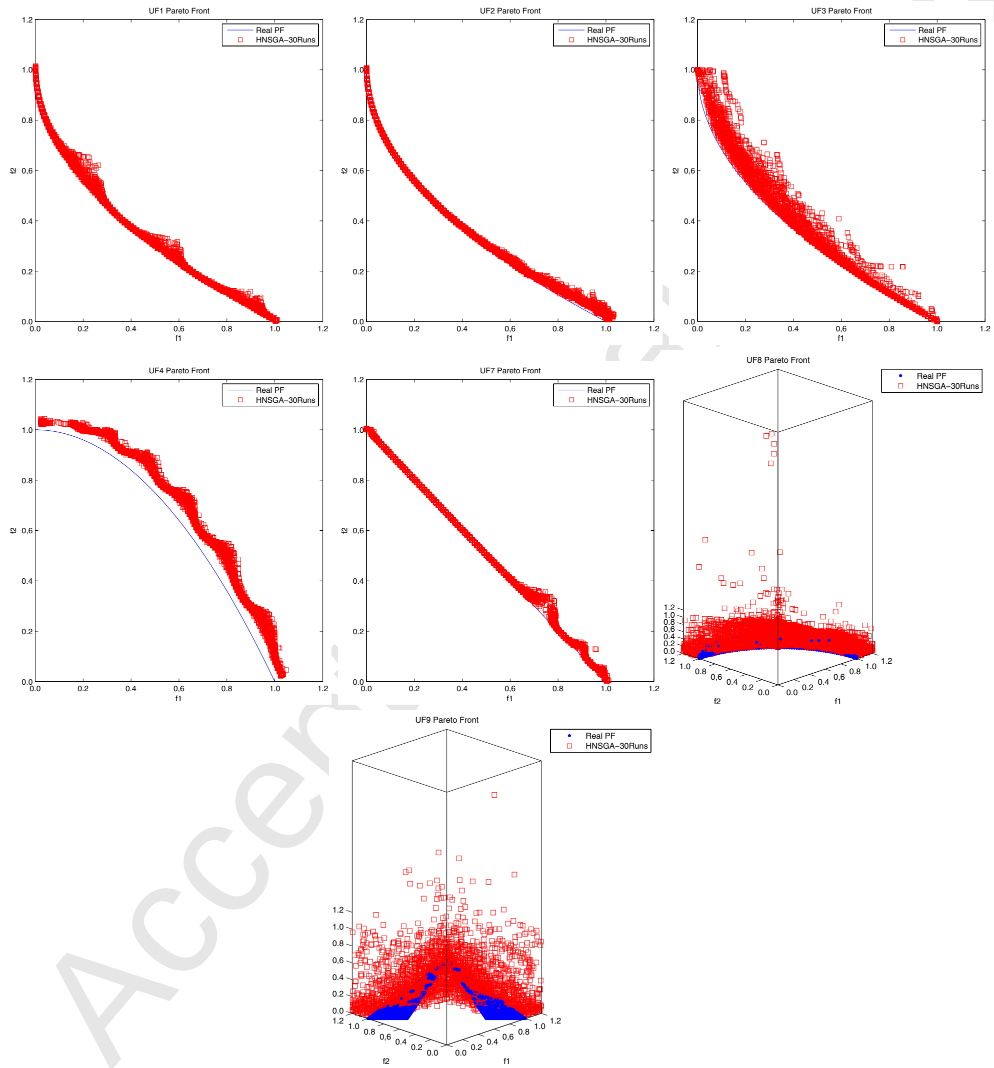

Figure 6. Comparison of approximate Pareto fronts obtained in all optimization runs of HNSGA on the CEC'09 test problems.

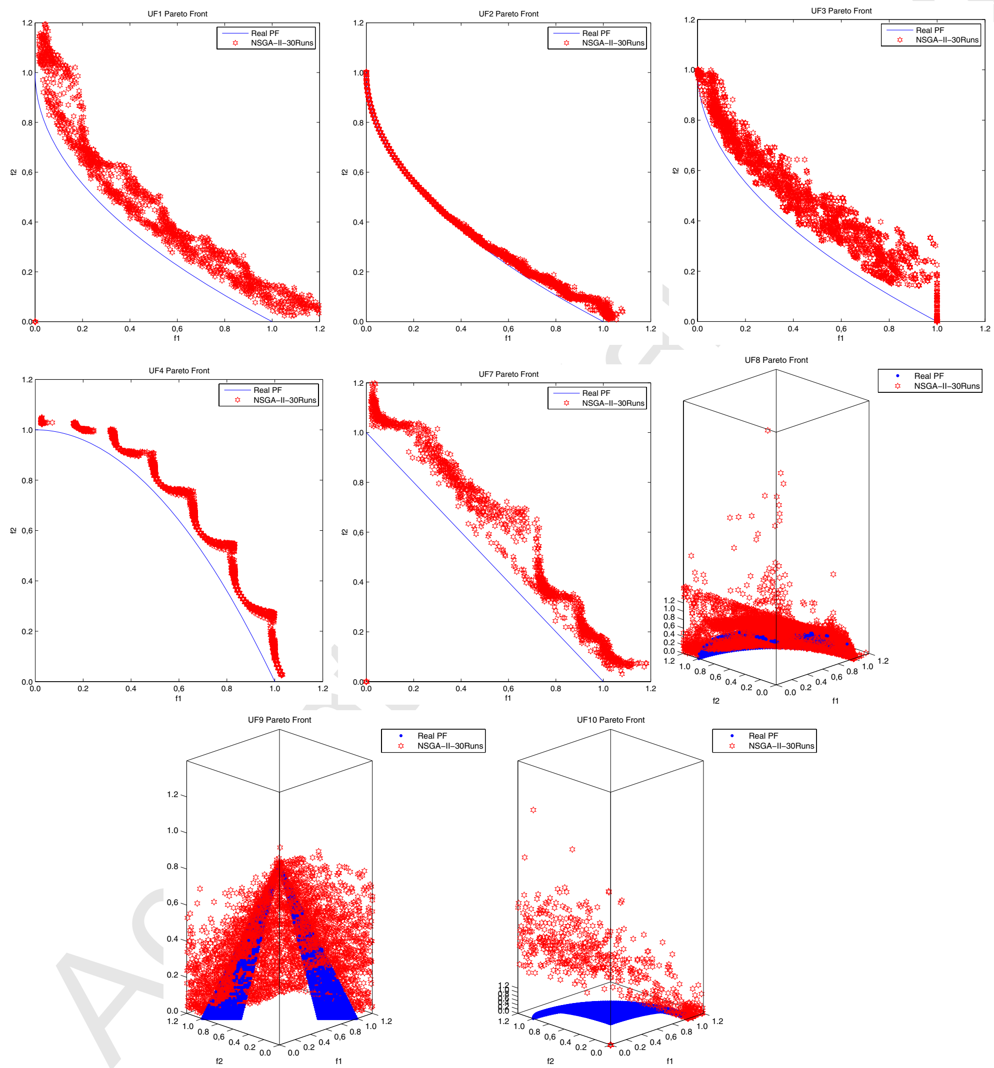

Figure 7. Comparison of approximate Pareto fronts obtained in all optimization runs of NSGA-II on the CEC'09 test problems.

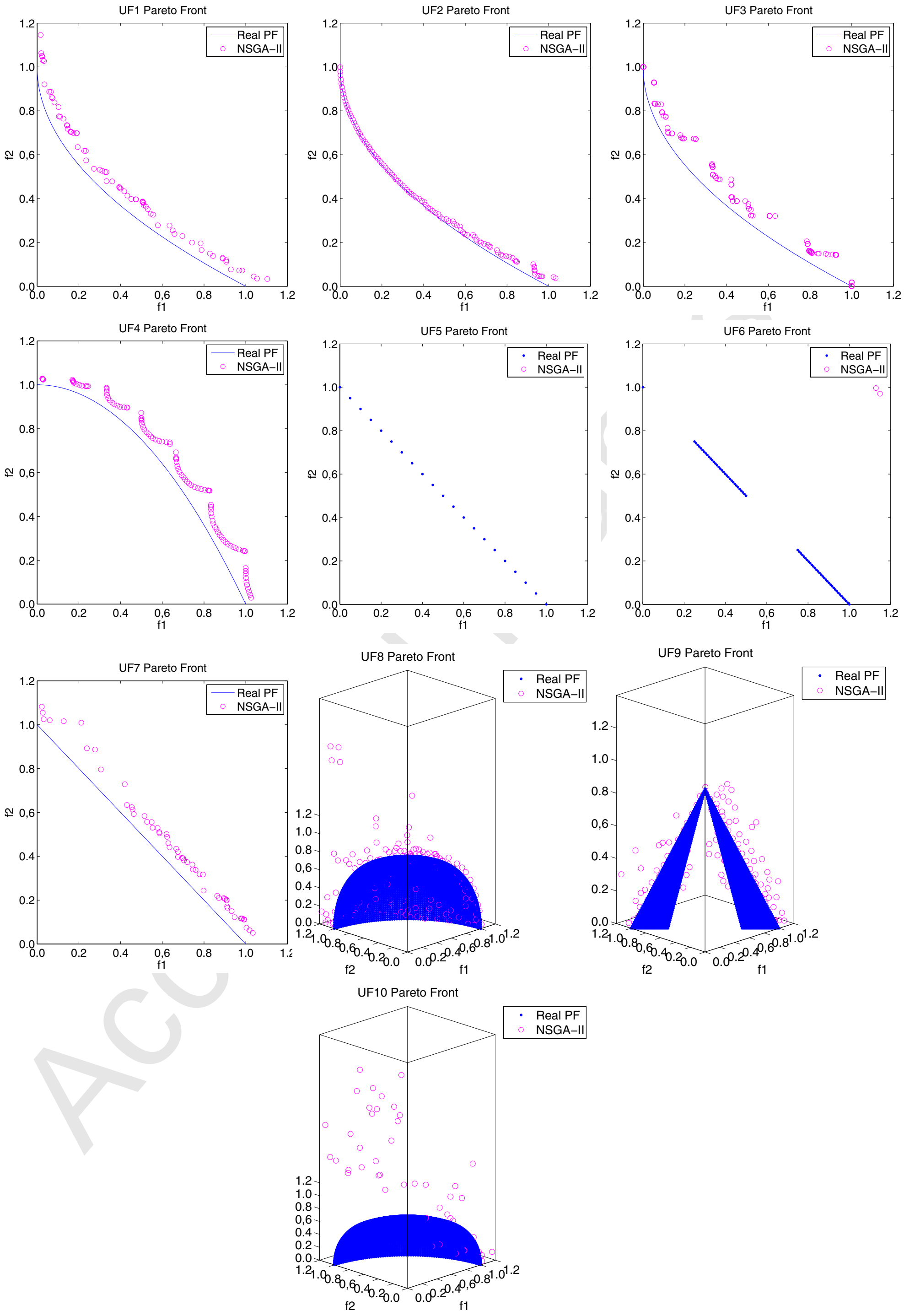

Figure 8. Comparison of approximate Pareto fronts obtained in the best optimization run of NSGA-II on the CEC'09 test problems

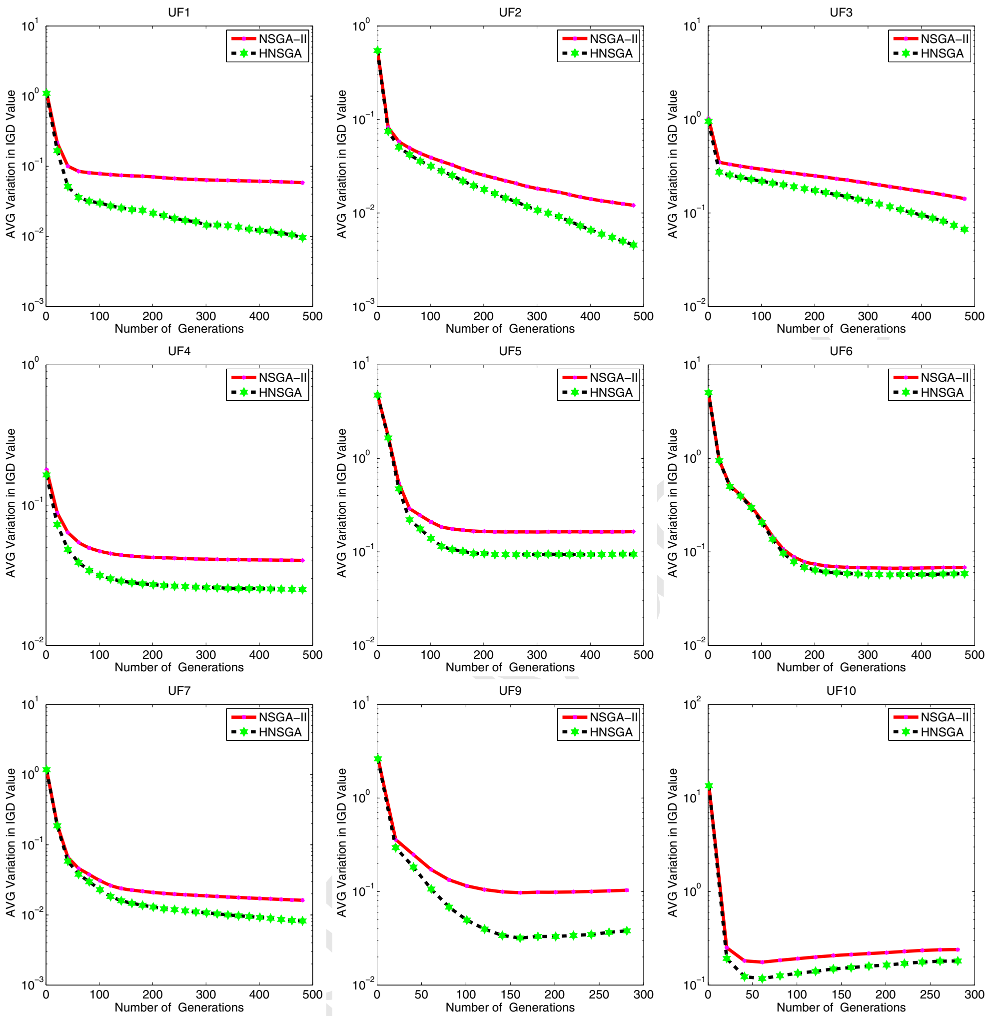

Figure 9. Comparison of IGD-metric average values for HNSGA and NSGA-II [13] on the CEC'09 test instances.

5. Conclusion and Future Work

Recently, a variety of multi-objective evolutionary algorithms (MOEAs) have been developed and tested on diverse test suites of MOPs including complicated real-world problems. Among these, the multi-objective evolutionary algorithm based on decomposition (MOEA/D) [23] is a paradigm that transforms the given MOP into a number of different single objective optimization problems (SOPS) and then applies a generic EA to simultaneously solve these SOPs in a single simulation run aiming at getting the optimal set of solutions. MOEA/D has several enhanced versions to be found in [26, 32, 29, 30, 27].

Pareto dominance based MOEAs do not rely on any decomposition strategy in their evolutionary process and solve MOP directly. Decomposition and Pareto dominance approaches are well suitable for the adaptation of evolutionary operators and tuning of control parameters. NSGA-II [13] is one of the most popular and efficient Pareto dominance based technique for dealing with diverse test suites of optimization and search problems.

This paper described a novel hybrid multiobjective evolutionary algorithm derived by combining NSGA-II, a state-of-the art Pareto dominance-based technique, with adaptive multiple operators selection strategy. The new algorithm,

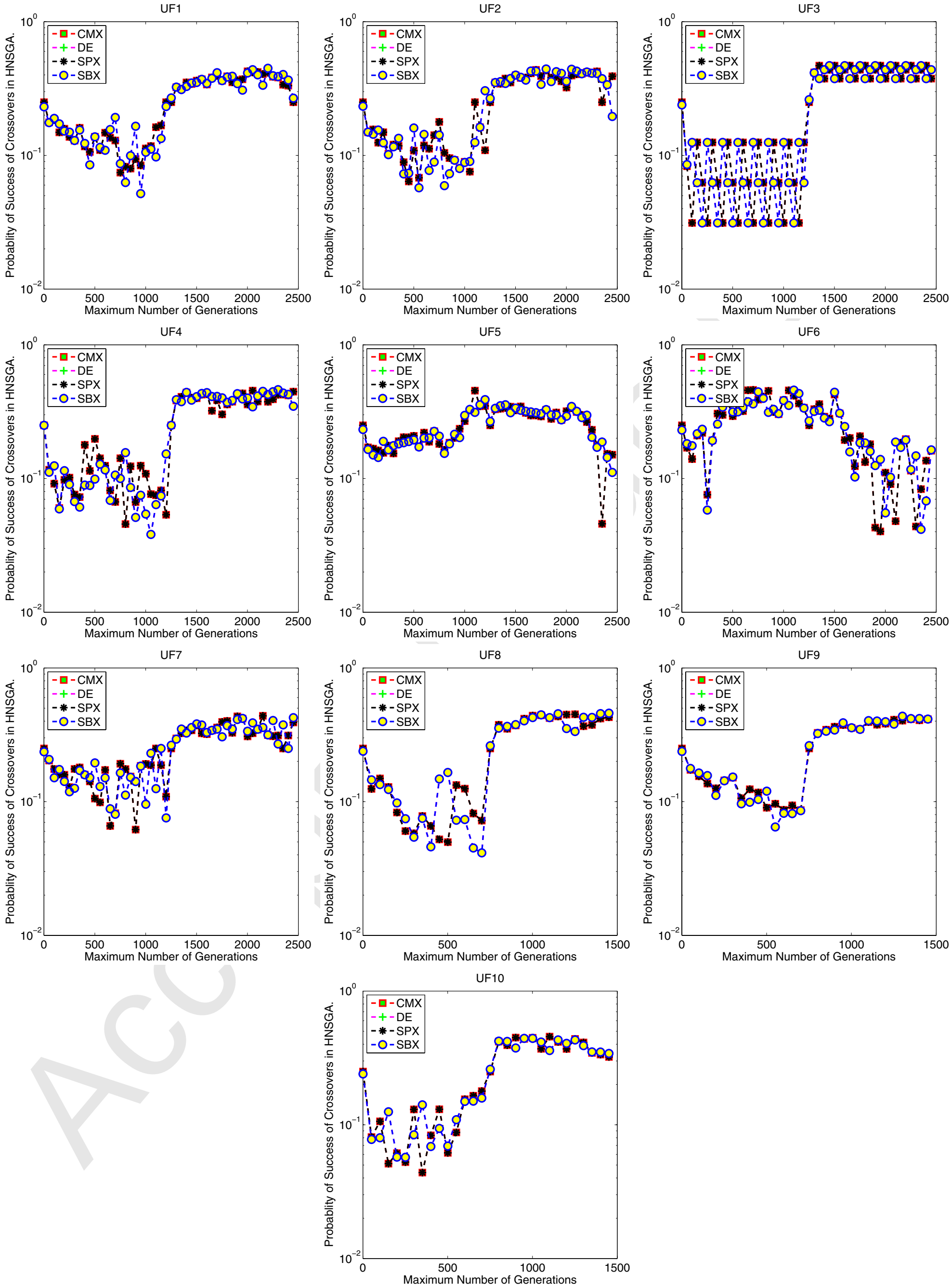

Figure 10. The proportion of crossover operators selected during the evolution process of HNGA in solving CEC'09 test problems

called HNSGA, was tested in two sets of benchmark problems (the commonly used ZDT problems [61] and the more difficult CEC'09 problems [61]) including 2 or 3 objective functions. It was found that the proposed approach outperforms other state-of-the-art evolutionary algorithms with respect to robustness and capability of reconstructing the true Pareto front. In the future, the suggested algorithm will be used for solving combinatorial optimization problems and more complicated real-world problems including multiple objectives and constraints. Furthermore, multiple ensemble local search operators will be employed together with search operators to examine their strength in memetic computation.

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