Accepted Manuscript

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PII: S0165-1889(17)30138-0
DOI: 10.1016/j.jedc.2017.06.003
Reference: DYNCON 3445

To appear in: *Journal of Economic Dynamics & Control*

Received date: 16 November 2016
Revised date: 12 June 2017
Accepted date: 15 June 2017

Please cite this article as: Paolo Pin, Elke Weidenholzer, Simon Weidenholzer, Constrained Mobility and the Evolution of Efficient Outcomes, *Journal of Economic Dynamics & Control* (2017), doi: 10.1016/j.jedc.2017.06.003

Constrained Mobility and the Evolution of Efficient Outcomes

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June 2017

Abstract

We study an evolutionary model akin to the one studied in Anwar (2002) where a set of agents use myopic best response learning to i) determine their action in a 2×2 coordination game and ii) to choose on which of multiple islands to interact. We focus on the case where the number of agents maximally allowed on each islands is constrained. We extend Anwar's original analysis by considering the case when there may be more than two islands. We find that if the constraints are such that one island may be empty, universal coordination on the payoff dominant action is possible in the long run. If the constraints are such that all islands will be full, then for relatively mild constraints, and apart from special cases, the coexistence of conventions will occur, with one island coordinating on the risk dominant action and all remaining islands coordinating on the payoff dominant action. For relatively stringent constraints all agents will play the risk dominant action.

Keywords: Mobility, Local Interactions, Learning, Coordination Games.

JEL Classification Numbers: C72, D83.

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1 Introduction

There are many circumstances where individuals can benefit from coordinating on the same action, as e.g. a common technology standard (e.g. IOS vs. Android OS) or norm (metric vs. imperial system of measurement). These situations give rise to coordination games with multiple strict Nash equilibria. A large literature has addressed the question of which equilibria emerge in the long run when agents adopt their behavior using simple rules of thumb, such as imitation or (myopic) best response learning (see e.g. Weidenholzer (2010) for a survey). The main message that arises from this literature is that when the interaction structure is fixed, players reach profiles where everybody chooses the same action. Moreover, when agents use myopic best response learning risk dominant conventions will emerge, that is the population will end up using strategies that do well against mixed profiles, but do not necessarily carry a high payoff when everybody adopts them (see e.g. Ellison (1993, 2000) and Young (1993)).[1,2]

In such a setting it is a natural question to ask what happens if agents are not organized in a fixed interaction structure, but may influence the set of their interaction partners. One way of choosing one's interaction partners is presented by models of network formation, see e.g. Jackson & Watts (2002), Goyal & Vega-Redondo (2005), or Staudigl & Weidenholzer (2014). An alternative branch of the literature has considered settings (see e.g. Oechssler (1997), Dieckmann (1999), Anwar (2002), Ely (2002) and Blume & Temzelides (2003)) where agents may identify their preferred interaction partners by deciding on which of multiple islands to play the game on. The interaction structure on islands is fully connected, meaning that everybody interacts with everybody else on their island, and there are no interactions across islands. Interactions on islands are thus characterized by extreme clustering. This form of endogenously formed interactions corresponds to choosing circles of friends or cliques rather

[1]Under imitation learning which convention will be selected depends on the information and interaction structures and the properties of the imitation rule, see e.g. Kandori et al. (1993) Robson & Vega-Redondo (1996) or Alós-Ferrer & Weidenholzer (2008, 2014).

[2]See also Dawid (1997) for a learning rule, using even less information than imitation learning, which converges to the risk dominant convention in the presence of mistakes.

than individually picking interaction partners.

We contribute to this literature by studying the implications of constraints limiting the number of players that may reside at each location. Such constraints could for instance be the result of natural limitations such as the space available on each island. In particular, we present a model with islands, similar to the one of Anwar (2002), where each agent interact with all the other agents in the same island, and agents use myopic best response learning to i) determine their action in a coordination game with a payoff dominant- and a risk dominant- equilibrium and to ii) choose on which of multiple islands to play the game on. Under myopic best response learning players choose an action and island that gives the highest payoff against the previous distribution of strategies of their opponents. This captures the idea that they are not able to form a forecast on their opponents' future behavior and, thus, react to the current distribution of play.[3] We believe this assumption to be particulary apt for consumers' choices of a technology standard or norm which most probably rely to a large extent on the current distribution in the population rather than on the consumers' beliefs on future distributions of play. In addition, we assume that when an agent is indifferent between various actions and/or islands, she randomizes between the set of all profiles giving the highest payoffs. Further, whenever a desired location is at capacity, moving there is not possible, and whenever the set of agents wanting to move to an island exceeds its capacity only a random subset of agents is allowed to move.

We extend Anwar's (2002) model by allowing the set of available locations to be larger than two. By considering this more general and realistic setting, we are able to provide a comprehensive picture of the role of (restricted) mobility on long run behaviour.

As a possible application of the setting we have in mind, consider the following example which borrows heavily from Shi (2015). There is a number of industrial parks for a population of firms developing software. The number of firms that may reside at each park is constrained by space. Firms receive payoff from engaging in projects with the other firms

[3] We remark that the main results of our model would also hold under the imitate the best max rule (see e.g. Robson & Vega-Redondo 1996) where players imitate the actions of the most successful agents in the population. See footnote 13 for details.

at their industrial park but do not benefit from firms at other parks. There are two different development platforms. The first is more efficient in the sense that it gives the highest payoff when used against itself. The second is more compatible in the sense that it gives a lower payoff when used against itself but gives a relatively high payoff when used against the former. Thus, the first platform is payoff dominant and the second is risk dominant. In this setting we provide a characterization of the set of long run outcomes depending on the properties of the platforms, the size of the population and the size of each software park.

We find that the best response dynamic will converge to states with the following properties: i) On each island only the same action is played. This stems from the fact that the underlying game is a coordination game. ii) The population is concentrated on the fewest possible islands. The reason behind this observation is that agents who are indifferent among islands (where the same action is played) are assumed to choose all such islands with positive probability and will, thus, move across islands. Hence, the process will at some point reach a state where certain islands are empty. Note that whether there are empty islands or not will depend on the capacity constraint. iii) Islands where the payoff dominant action is played will be at capacity. For, if there are payoff dominant islands with spare capacity, agents on islands where the risk dominant action is played will change their action and move to them.

We are interested in which state will emerge in the long run when the agent's choices are perturbed by occasional mistakes à la Kandori et al. (1993) and Young (1993). In a nutshell, states that are most robust to mistakes will emerge as *long run equilibria, LRE.*

Anwar (2002) has shown that with two islands either all agents will choose the risk dominant action in the long run or the coexistence of conventions will occur, where agents on one island will choose the payoff dominant action and agents on the other island will choose the risk dominant action.[4] In the case where there may be more than two islands, we find that if the constraints are such that one island may be empty, universal coordination on the payoff dominant action is possible in the long run. If the constraints on capacity are such that all islands will be occupied, our results generalize those in Anwar (2002) to

[4] See Goyal & Janssen (1997) and Alós-Ferrer & Weidenholzer (2007) for models where the co-existence of conventions may be observed on the circle.

more than two islands. In particular, if the constraints are relatively mild (within the range of relevant constraints) such that the smallest size of a populated island is relatively small compared to the largest size of an island (implied by capacity), apart from special cases, the coexistence of conventions will occur, with one island coordinating on the risk dominant action and all remaining islands coordinating on the payoff dominant action. For relatively stringent constraints on capacity all agents will play the risk dominant action.

Let us provide some intuition for Anwar's (2002) and our findings. First, consider the case where, as in Anwar (2002), the constraints and the population are such that it requires all of the islands to shelter the entire population. If the payoff dominant action is played on some (but not all) of the islands, then all agents want to move to these islands, up until the point where they are at full capacity. On the contrary, the population on the islands where the risk dominant action is played will be relatively small. Thus, while islands where the payoff dominant action is played take a small fraction of a large population to make a mistake in order for the island to switch, islands where the risk dominant action is played will take a large fraction of a small population to switch. Hence, whenever constraints on capacity are relatively mild, so that the minimum size of a populated island is small compared to the maximum size of an island, we will observe all but one island coordinating on the payoff dominant action. If the constraints are relatively strict, so that the minimum size of a populated island is relatively large, universal coordination on the risk dominant action will obtain. With more than two islands we may have a third force at play, as the constraints can be such that some islands are empty. If now an agent makes a mistake, moves to an empty island and starts playing the payoff dominant action, other agents will follow up to the point where the island is fully occupied. Thus, with one mistake we can increase the number of payoff dominant islands. This implies that whenever the population size and the constraints are such that there may be empty islands, the payoff dominant convention will be a long run equilibrium.

Our results, thus, suggest that constrained mobility does not necessarily have to go hand in hand with inefficient outcomes. What matters more is the questions of whether the constraints are such that there can be empty islands from which payoff dominant action may spread or whether there are no such islands. Thus, the number of islands is a policy variable (keeping total capacity fixed), it would be desirable from a welfare point of view to have a lot of relatively small islands. In the context of the software firm example from above a designer should aim for a setting where at least one industrial park may be potentially empty. This empty park will then provide firms with a springboard from which the efficient programming language may be introduced.[5]

Related Literature

Our results provide a bridge between the unconstrained mobility models of Oechssler (1997) and Ely (2002), where payoff dominant profiles are selected, and the models of restricted mobility of Anwar (2002) and others (see below). Whenever the constraints are such that there may be empty islands, our results are in line with those of the unrestricted mobility literature. If there are, however, no such islands, our results may be seen as a generalization of those by Anwar (2002) to the case of more than two islands.

In addition to Anwar (2002), Dieckmann (1999) and Blume & Temzelides (2003) present models where agents may reside on one of multiple islands and there are restrictions limiting movement across locations. Dieckmann (1999) studies imitative behavior and finds that imperfect observability of play and frictions in movement can not prevent efficient conventions from arising. Her analysis of capacity constraints is in line with Anwar in showing that payoff dominant conventions are never observed when there are only two islands. Blume & Temzelides (2003) differs by modelling restricted mobility through mobile and immobile agents, rather than restrictions in capacity. If there are few islands, universal coordination on risk dominant conventions is observed. If the number of islands is large, the payoff dominant action is played on one island and all mobile agents will reside at that island. All other islands will be inhabited by immobile agents playing the risk dominant action. Shi (2015) studies a game between two rational policy makers who set constraints on the number of

[5]In fact, from the point of view of the designer, there might be a trade off. On one hand, an empty park means a waste of resources; on the other hand, an empty park can lead all firms to coordinate on the efficient programming language or platform.

agents maximally allowed on their respective island and whose payoff depends on the resulting long run behavior of players on their island. The characterization of the game between policy makers encompasses a complete analysis of long run behaviour under asymmetric capacity constraints. Abstracting from special cases, the analysis of asymmetric contraints in Shi (2015) confirms the results of Anwar. If capacity constraints are stringent, universal coordination on risk dominate conventions will obtain and if both capacity constraints are large, the coexistence of convention arises with the payoff dominant action being played on the island with the larger capacity constraint.[6]

The remainder of this paper is organized in the following way. Section 2 presents the model. Section 3 presents and discusses our main results. In section 4 we discuss the implications of immobile agents and present results for the case when constraints are heterogenous across islands. Section 5 concludes.

2 The model

We consider a population of kN agents who reside on $k \geq 2$ different islands. Each of these islands can only shelter M agents. We assume $N < M < kN$, so that one location may not shelter the entire population and there is enough total capacity to shelter the entire population. Using the notation of Anwar (2002), the maximal number of agents on an island is $M = dN$ with $1 < d < k$.[7]

Agents only interact with other agents on their island and have to use the same action in all of their interactions. The payoff of an agent is given by the average payoff of playing

[6]See Section 4.2 for a discussion of heterogeneous capacity constraints when there are more than two islands.

[7]In fact, Anwar (2002) considers two forms of restriction on mobility. First, there are constraints on the maximally allowed number of agents on each island. Second, a certain fraction of the population of each island (the so-called patriots) may never change location. Section 4.1 provides a discussion of patriots in the case where $k \geq 2$. While the two forms of restricted mobility are equivalent when $k = 2$ (see footnote 14) they will have different implications when $k \geq 3$.

the following coordination game against all other agents on her island.

	s_1	s_2
s_1	A, A	B, C
s_2	C, B	D, D

We assume $A > C$ and $D > B$, so that (s_1, s_1) and (s_2, s_2) are strict Nash equilibria. We further assume $A > D$, so that the equilibrium (s_1, s_1) is payoff dominant, and $C+D > A+B$, so that (s_2, s_2) is risk dominant, i.e. s_2 is the unique best response against an agent playing both strategies with probability $\frac{1}{2}$. We denote the critical mass on action s_1 in the mixed strategy Nash equilibrium by

$$q^* = \frac{D - B}{A - C + D - B}.$$

Note that by risk dominance $q^* > \frac{1}{2}$. In addition, we focus on coordination games where $C \geq D$.[8] We further assume that an agent who is alone on an island earns a payoff of $\bar{u} < B$, implying that agents prefer playing the game over being alone on an island.

Time is discrete $t = 1, 2, \ldots$. Each period each agent chooses an action that gives the highest average payoff against the profile of actions played on her island in the previous period. In addition, with positive probability each agent may choose an island where to reside. When such an opportunity arises an agent chooses an action and a location that maximize the payoff given the overall distribution of actions across all islands in the previous period. We assume random tie breaking, i.e. in case of multiple best responses an agent randomizes among all of them. If an island is at capacity M, an agent is not allowed to move to this island and stays at her original island. If the number of agents intending to move to an island exceeds available capacity, then only a random subset (equal to available capacity) is allowed to move. Thus, as in Anwar (2002), we consider a myopic best response process without inertia in the action choice but with inertia in the choice of islands. With

[8]It has been observed by Shi (2013) that if $C < D$, the transitions among the various absorbing sets can differ from the one found in Anwar (2002). For ease of exposition we have decided to focus on the case where this does not occur. The qualitative results of the present model will stay the same, though, if $C < D$. See also footnote 11.

probability ϵ, independent across agents and time, an agent ignores the prescription of the adjustment process and chooses a location and an action at random.

We denote by n_i the number of players on island i and by n_i^1 the number of s_1 players on island i. The number of s_2 players on location i is, thus, given by $n_i^2 = n_i - n_i^1$. In the following we refer to a populated island i where all players play the payoff dominant action, $n_i^1 = n_i > 0$, as payoff dominant island and to a populated island i where all players play the risk dominant action, $n_i^2 = n_i > 0$, as risk dominant island. Using vector notation, so that $\vec{n}, \vec{n}^1, \vec{n}^2 \in \mathbb{R}_+^k$, we denote a state of this system by a tuple of vectors $(\vec{n}, \vec{n}^1)$. Note that $\vec{n}^2 = \vec{n} - \vec{n}^1$.

The state space of our model can, hence, be characterized as

$$S = \left\{ (\vec{n}, \vec{n}^1) \,\middle|\, n_i \in \{0, \ldots, M\}, n_i^1 \in \{0, \ldots, n_i\} \right\}.$$

We denote by $\underline{k} = \lceil \frac{kN}{M} \rceil$ the smallest number of locations required to shelter the entire population. Finally, we denote by m the size of the location when the other $\underline{k} - 1$ locations are at capacity,

$$m = kN - M(\underline{k} - 1).$$

2.1 Review of techniques

The process without mistakes ($\epsilon = 0$) is called *unperturbed* process. Ω denotes the set of absorbing sets of this process and $\omega \in \Omega$ denotes one such absorbing set. The process with mistakes ($\epsilon > 0$) is referred to as *perturbed process.* Any two states can be reached from each other under the perturbed process. Hence, there is only absorbing set which process corresponds to the entire state space, implying that the process is ergodic. The unique invariant distribution of this process is denoted by $\mu(\epsilon)$. We are interested in the *limit invariant distribution* (as the error rate goes to zero), $\mu^* = \lim_{\varepsilon \to 0} \mu(\varepsilon)$. This distribution exists (see Foster & Young (1990), Young (1993), or Ellison (2000)) and it is an invariant distribution of the unperturbed process. It provides a stable prediction for the unperturbed process. If ϵ is small enough the play in the long run corresponds to the distribution of play described by μ^*. States in the support of μ^*, are referred to as stochastically stable states or *Long Run Equilibria (LRE)*. We denote the set of LRE by $\mathcal{S} = \{\omega \in \Omega \mid \mu^*(\omega) > 0\}$. Let us provide an overview of the Freidlin & Wentzell (1988) algorithm to identify the set of LRE.[9] Consider two absorbing sets of states ω and ω' and let $\tau(\omega, \omega') > 0$ be the *transition cost*, i.e. the minimal number of mistakes under the perturbed process for moving from ω to ω'. An ω-tree corresponds to a directed tree where the nodes of the tree are given all absorbing sets, and the tree is directed into the root ω. The cost of a tree is calculated as the sum of the costs of transition on each edge. Freidlin & Wentzell (1988) have shown that a set ω is a LRE (or stochastically stable) if and only if it is the root of a minimum cost tree.

3 Results

We first characterize the absorbing sets of our process. In a nutshell, these will be made up of states where i) the population is concentrated on the fewest islands possible, ii) on each island all players play the same strategy, and iii) all islands where the payoff dominant action is played will be at capacity (unless the entire population chooses the payoff dominant action). In the following we refer to islands where all players play the risk dominant action as risk dominant islands and to islands where all players play the payoff dominant as payoff dominant islands. Note there are various ways in which the populated islands can be distributed among all islands and there are numerous ways in which payoff- and risk- dominant islands can be distributed among the occupied islands. Thus, formally characterizing the absorbing sets is a bit more cumbersome and requires more notation.

We use $C(a, \vec{x})$ to indicate the occurrences of element a in vector $\vec{x}$.[10] We start with the following observation.

Lemma **1.** *Any absorbing set $\omega \in \Omega$ is such that for all states $(\vec{n}, \vec{n}_1) \in \omega$,*

[9] See Samuelson (1997) for a textbook exposition. Ellison (2000) provides an alternative way of identifying LRE. We work with the original formulation which also allows for a characterization in case of multiple LRE.

[10] As an example, if $\vec{x} = (1, 2, 3, 1)$, we have that $C(1, \vec{x}) = 2$, $C(3, \vec{x}) = 1$ and $C(5, \vec{x}) = 0$. This operator is called *count* operator in the *Z notation*, used in computer science (see e.g. Spivey & Abrial (1992)).

i) $n_i^1(n_i - n_i^1) = 0$ *for all* $i = 1, \ldots, k$

ii) $C(0, \vec{n}) = k - \underline{k}$

Proof. The first part follows from the observation that on each island all players have to adopt the same action. Thus, if for some island $n_i^1 > 0$ it has to be the case that $n_i^2 = 0$ and if $n_i^2 > 0$, $n_i^1 = 0$ has to hold. To see the second part, observe that agents who are indifferent between various islands will move to each of these islands with positive probability. Thus, with positive probability the process converges to states where the population is concentrated on the fewest islands possible, $\underline{k}$. Further, no agent at a populated islands has an incentive to move to an empty island. Thus, at any absorbing set $k - \underline{k}$ islands will have to be empty. $\square$

Note that each absorbing set ω may contain multiple states which the process visits with positive probability. In order to characterize theses states, we introduce a distance relationship between them. Consider two islands, x and y, we define

$$\delta(x, y) = \begin{cases} 0 \quad if \quad n_x = 0 & \& \ n_y = 0 \\ 0 \quad if \quad n_x^1 = n_x > 0 & \& \ n_y^1 = n_y > 0 \\ 0 \quad if \quad n_x - n_x^1 = n_x > 0 & \& \ n_y - n_y^1 = n_y > 0 \\ 1 \qquad otherwise \end{cases} .$$

In words, $\delta(x, y) = 0$ if both islands are empty or the same action is played on them. We then define the distance between two states $(\vec{n}, \vec{n}^1)$ and $(\vec{n}', \vec{n}'^1)$ as

$$d\left(\left(\vec{n}, \vec{n}^1\right), \left(\vec{n}', \vec{n}'^1\right)\right) = \sum_{i=1}^{k} \delta(n_i, n_i').$$

We now have:

Lemma **2.** *For any two states* $(\vec{n}, \vec{n}^1)$ *and* $(\vec{n}', \vec{n}'^1)$ *contained in a non-singleton absorbing set* ω *we have* $d\left(\left(\vec{n}, \vec{n}^1\right), \left(\vec{n}', \vec{n}'^1\right)\right) = 0$.

Proof. In any absorbing set the process will only move among states where the same islands are populated and on the populated islands the same action is chosen, implying

$d\left(\left(\vec{n},\vec{n}^{1}\right),\left(\vec{n}',\vec{n}'^{1}\right)\right)=0.$ □

Let now Ω_ℓ denote the set of absorbing sets where on ℓ islands the payoff dominant action is played, i.e.

$$\Omega_\ell = \{\omega \in \Omega | C\left(0,\vec{n}\right) = k - \underline{k} \,\&\, C\left(0,\vec{n}^1\right) = k - \underline{k} + \ell \,\forall \left(\vec{n},\vec{n}^1\right) \in \omega\}.$$

An element of Ω_ℓ is denoted by ω_ℓ. We refer to the set of absorbing sets Ω_0 and $\Omega_{\underline{k}}$ as the risk dominant- and the payoff dominant- convention, respectively. All other sets of absorbing sets feature the coexistence of conventions and are referred to as mixed sets. The following lemma characterizes the size of the population on each occupied island in an absorbing set.

Lemma **3.** *Consider an absorbing set ω_ℓ. Then it holds:*

i) If $0 \le \ell \le \underline{k} - 2$, then for all states $\left(\vec{n},\vec{n}^1\right) \in \omega_\ell$, $n_i^1 = M$ for all islands i with $n_i^1 > 0$ and $n_j - n_j^1 \in \{m, m+1, \ldots, M\}$ for all islands j with $n_j - n_j^1 > 0$.

ii) If $\ell = \underline{k} - 1$, then for all states $\left(\vec{n},\vec{n}^1\right) \in \omega_\ell$, $n_i^1 = M$ for all islands i with $n_i^1 > 0$ and $n_j - n_j^1 = m$ for the island j with $n_j - n_j^1 > 0$.

iii) If $\ell = \underline{k}$, then for all states $\left(\vec{n},\vec{n}^1\right) \in \omega_{\underline{k}}$, $n_i^1 \in \{m, m+1, \ldots, M\}$ for islands i with $n_i^1 > 0$.

Proof. Let us start with ii) where the risk dominant action is played on one island. As agents who are given the opportunity to switch islands will move to islands where s_1 is played, the $\underline{k} - 1$ islands where s_1 is played will be full. On the only islands where s_2 is played m agents will be stuck using the risk dominant action. To see i), note that if s_2 is played on more than one island, again islands where s_1 is played (provided that there are any) will have to be full. The population on the islands where s_2 is played will be indifferent on which of the s_2 islands to reside and, thus, will move among those islands. Hence, the population on the s_2 islands will fluctuate between m and M. In case iii) s_1 is played on all islands and, thus, agents are indifferent among which of them to reside. □

Thus, in mixed states islands where the payoff dominant action is played will be at capacity. If there is only one island where the risk dominant action is played, its population is m. If there is more than one island where the risk dominant action is played the population on these islands will fluctuate between m and M. Similarly, if the payoff dominant action is played on all islands the population on each of these islands will also fluctuate between m and M.

Having characterized the absorbing sets of our process, we now move on to determine transition costs among them. To this end, we say that two absorbing sets ω and ω' are *adjacent* if every state $(\vec{n}, \vec{n}^1) \in \omega$ is at distance one to every state $(\vec{n}', \vec{n}'^1) \in \omega'$, i.e. the same islands are occupied and on all but one of these islands the same action is played. Furthermore, if an absorbing set ω_ℓ (with $1 \leq \ell \leq \underline{k} - 1$) is adjacent to another absorbing set ω_m, it follows that either $m = \ell - 1$ or $m = \ell + 1$.

The following series of lemmata characterizes transition costs among absorbing sets. The first two of those focus on the case when there are empty islands, which means that $\underline{k} < k$.

Lemma **4.** *If $\underline{k} < k$, then for every absorbing set ω_ℓ with $0 \leq \ell \leq \underline{k} - 1$ there exists an absorbing set $\omega_{\ell+1}$ with $\tau(\omega_\ell, \omega_{\ell+1}) = 1$.*

Proof. Assume an agent who resides on an island where everybody chooses the risk dominant action makes a mistake, moves to an empty island, and starts playing the payoff dominant action. All agents given revision opportunity will follow the mutant up to the point where the island is full. □

Lemma **5.** *If $\underline{k} < k$, then all absorbing sets $\omega_\ell \in \Omega_\ell$ can be reached from one another via a chain of single mutations.*

Proof. Again, assume one agent mutates, moves to an empty island but, this time, keeps his current action. With positive probability, agents from other islands where that strategy is played will follow and we reach a new state where the initially empty island is occupied and some initially occupied island is empty. Iterating this argument, we can move among all sets in Ω_ℓ via a chain of single mutations. □

The following lemma characterizes transitions to sets of states with more risk dominant islands.[11]

LEMMA **6.**

i) For any two adjacent absorbing sets $\omega_{\underline{k}}$ *and* $\omega_{\underline{k}-1}$*, the transition cost is* $\tau(\omega_{\underline{k}}, \omega_{\underline{k}-1}) = \lceil m(1-q^*) \rceil$*;*

ii) For any two adjacent absorbing sets ω_ℓ *and* $\omega_{\ell-1}$*, with* $0 \leq \ell \leq \underline{k}-1$*, the transition cost is* $\tau(\omega_\ell, \omega_{\ell-1}) = \lceil M(1-q^*) \rceil$*.*

Proof. Consider part i). For any absorbing set $\omega_{\underline{k}}$ the process moves among states where the population on each island fluctuates between m and M. Consider a state where the number of s_1 players on one island is minimal at m. If $\lceil m(1-q^*) \rceil$ agents mutate to s_2 all remaining s_1 players will find it optimal to switch to s_2 and we reach an adjacent state in $\Omega_{\underline{k}-1}$.

Part ii) follows from the fact that in all absorbing sets ω_ℓ with $1 \leq \ell \leq \underline{k}-1$ the s_1 islands are full. Thus, it takes $\lceil M(1-q^*) \rceil$ mutations to reach an adjacent state in $\Omega_{\ell-1}$. □

The next lemma analyzes transitions where the number of payoff dominant islands increases in the scenario where all islands are populated.

LEMMA **7.** *If* $\underline{k} = k$*, for any two adjacent absorbing sets* ω_ℓ *and* $\omega_{\ell+1}$*, with* $0 \leq \ell \leq \underline{k}-1$*, the transition cost is given by* $\tau(\omega_\ell, \omega_{\ell+1}) = \lceil mq^* \rceil$*.*

Proof. For any absorbing set ω_ℓ, with $0 \leq \ell \leq \underline{k}-1$, the process moves among states where the population on the risk dominant islands is either m or fluctuates between m and M. Consider a state such that the number of s_2 players on a risk dominant island is minimal at m. If $\lceil mq^* \rceil$ players on this island mutate to s_1 all remaining s_2 players will find it optimal to switch to s_1 and we have reached an adjacent state with one more payoff dominant island. □

[11] Shi (2013) has shown that if $D > C$ transitions that increase the number of islands on which s_2 is played may occur through an alternative cheaper route. In a first step, s_1 players have to mutate to s_2 such that all players allowed will move to an s_2 island. Once this is achieved, $\lceil m(1-q^*) \rceil$ of the remaining players still have to switch. While this may change the number of mutation required, it is, however, still true that a certain fraction of players on an island has to mutate. While the derivation of the number of mistakes required for a transition may be different when $D > C$, the qualitative nature of the results in Anwar (2002) remains.

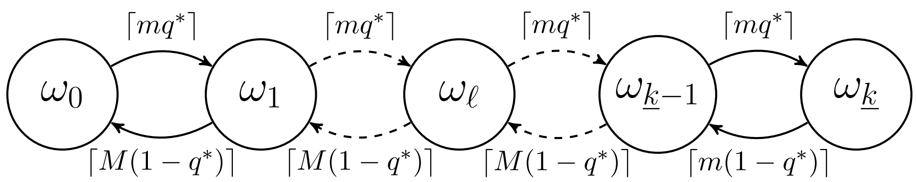

Figure 1: Transition costs for $\underline{k} = k$

We summarize the transition costs in the case where $\underline{k} = k$ in Figure 1.

The final lemma shows that when finding minimum cost trees for the various absorbing sets, it is sufficient to restrict attention to sets of absorbing sets Ω_ℓ. To this end, we define a *reduced* ω_ℓ-tree as a tree directed into the root ω_ℓ with the set of nodes being comprised by one absorbing set ω_m for each of the sets of absorbing sets Ω_m with $m \neq \ell$.[12]

Lemma 8. *If for an absorbing set ω_ℓ there exists a reduced minimum cost ω_ℓ-tree, then for each $\omega'_\ell \in \Omega_\ell$ there also exists a ω'_ℓ-minimum cost tree.*

Proof. If $k > \underline{k}$ lemma 5 implies that all absorbing sets $\omega_\ell \in \Omega_\ell$ can be reached from one another via a chain of single mutations and the claim follows. Next, consider $k = \underline{k}$. Since there are no empty islands, Ω_0 and Ω_ℓ each contain one unique absorbing set. Consider an absorbing set ω_ℓ. We can now construct a branch from $\omega_{\underline{k}}$ to an adjacent absorbing set $\omega_{\underline{k}-1}$, and so forth, finally connecting an adjacent absorbing set $\omega_{\ell+1}$ into ω_ℓ. In the same way, we can construct a path connecting ω_0 into ω_ℓ. As the cheapest way to escape every absorbing set is to move to an adjacent set and since in this construction only one island is changed at a time, the constructed reduced ω_ℓ-tree will have a cost no larger than the cost of any alternative reduced ω_ℓ- tree. Further, note that in the same fashion we can construct a reduced ω'_ℓ-tree of the same cost for each $\omega'_\ell \in \Omega_\ell$.

We now show that if there exists a reduced minimum cost ω_ℓ-tree there also exists a minimum cost ω_ℓ-tree. To this end, we will connect all remaining absorbing sets to the reduced ω_ℓ-tree. Again, note that the cheapest way to leave an absorbing set is by only

[12] While the original cost trees in the sense of Kandori et al. (1993) and Young (1993) are defined over all absorbing sets, reduced cost trees are only defined over a subset of absorbing sets compromising one set for each class of absorbing sets. This lemma generalizes lemma 1 in Anwar to the case of more than two islands.

changing the population on one island (i.e. moving to an adjacent set). By lemmata 6 and 7, for all absorbing sets, different from ω_0 and $\omega_{\underline{k}}$, it is either cheaper to increase or decrease the number of s_1 islands. Thus, if it is cheaper to increase (decrease) the number of s_1 islands for some ω_m, with $1 \leq m \leq \underline{k} - 1$, then it is cheaper to increase (decrease) the number of s_1 islands for all absorbing sets ω_g, with $1 \leq g \leq \underline{k} - 1$. Thus, we can link all remaining absorbing sets (possibly through a sequence of other sets) to the already existing part of the tree, by simply adding branches to each absorbing set ω_g that go either to an adjacent set in $\Omega_g + 1$ or in $\Omega_g - 1$ (depending on which direction is cheaper). By moving only in the least costly direction the total cost of the added part is minimal. Since also the first part is of minimum cost, the resulting tree is a minimum cost tree rooted into ω_ℓ. □

We can now state our main result.

PROPOSITION **1.**

a) If $\underline{k} < k$ *and*

i) if $\lceil m(1-q^*)\rceil > 1$, *then* $\mathcal{S} = \Omega_{\underline{k}}$

ii) if $\lceil M(1-q^*)\rceil > \lceil m(1-q^*)\rceil = 1$, *then* $\mathcal{S} = \Omega_{\underline{k}} \cup \Omega_{\underline{k}-1}$

iii) if $\lceil M(1-q^*)\rceil = \lceil m(1-q^*)\rceil = 1$, *then* $\mathcal{S} = \bigcup_{\ell=0}^{\underline{k}} \Omega_\ell$.

b) If $\underline{k} = k$ *and*

i) if $\lceil mq^*\rceil < \lceil m(1-q)^*\rceil < \lceil M(1-q^*)\rceil$, *then* $\mathcal{S} = \Omega_{\underline{k}-1}$

ii) $\lceil mq^*\rceil = \lceil m(1-q)^*\rceil < \lceil M(1-q)^*\rceil$, *then* $\mathcal{S} = \Omega_{\underline{k}} \cup \Omega_{\underline{k}-1}$

iii) if $\lceil mq^*\rceil > \lceil M(1-q^*)\rceil$, *then* $\mathcal{S} = \Omega_0$

iv) if $\lceil mq^*\rceil = \lceil M(1-q^*)\rceil > \lceil m(1-q^*)\rceil$, *then* $\mathcal{S} = \bigcup_{\ell=0}^{\underline{k}-1} \Omega_\ell$

v) if $\lceil mq^*\rceil = \lceil M(1-q^*)\rceil = \lceil m(1-q^*)\rceil$, *then* $\mathcal{S} = \bigcup_{\ell=0}^{\underline{k}} \Omega_\ell$.

Proof. By lemma 8 we only have to consider reduced minimum cost trees. Note that such a reduced minimum cost tree will necessarily only involve transitions among adjacent absorbing sets as any transition involving two or more islands at the same time is more costly. Let us first consider ω_0 trees. By lemma 6 the cost of every reduced minimum cost ω_0-tree is $(k-1)\lceil M(1-q^*)\rceil + \lceil m(1-q^*)\rceil$. Now consider the case where $\underline{k} < k$. By lemma 4 the cost of every reduced minimum cost $\omega_{\underline{k}}$-tree is $\underline{k}$. Combining lemma 4 and lemma 6 reveals that the cost of every reduced minimum cost ω_ℓ-tree (with $1 \leq \ell \leq k-1$) is $\ell + (\underline{k} - \ell - 1)\lceil M(1-q^*)\rceil + \lceil m(1-q^*)\rceil$. Pointing out that $\lceil M(1-q^*)\rceil \geq \lceil m(1-q^*)\rceil \geq 1$ establishes the claim in part a).

Finally, consider the case $\underline{k} = k$. By lemma 7 the cost of every reduced minimum cost $\omega_{\underline{k}}$-tree is $\underline{k}\lceil mq^*\rceil$. By lemmata 6 and 7 the cost of every reduced minimum cost ω_ℓ-tree (with $1 \leq \ell \leq k-1$) is $\ell\lceil mq^*\rceil + (\underline{k} - \ell - 1)\lceil M(1-q^*)\rceil + \lceil m(1-q^*)\rceil$. Noting that $\lceil mq^*\rceil \geq \lceil m(1-q^*)\rceil$ and $\lceil M(1-q^*)\rceil \geq \lceil m(1-q^*)\rceil$ and comparing the costs of the various reduced minimum cost trees establishes part b). $\square$

To interpret the result, recall that the smallest number of islands required to shelter the entire population $\underline{k}$ is given by $\lceil \frac{kN}{M} \rceil$. Thus, whenever the restrictions on mobility are weak and M is large enough so that some islands will be empty, only coordination on efficient outcomes will be observed (abstracting from the case where $\lceil M(1-q^*)\rceil = 1$ and/or $\lceil m(1-q^*)\rceil = 1$). If, however, there are no such islands ($\underline{k} = k$), then (apart from the special case $\lceil mq^*\rceil = \lceil m(1-q^*)\rceil$) either the co-existence of conventions or universal coordination on the risk dominant action will be observed. Note that in this case $m = M - k(M-N)$. Thus, m approaches M if the constraint becomes stricter (and M approaches N). If the constraint is relatively weak (M large relative to m), the co-existence of conventions may occur whereas if the constraint is relatively stringent (m large relative to M), universal coordination on the risk dominant convention will obtain. Thus, (abstracting from non-generic cases) as the constraint becomes stricter, the prediction switches from the payoff dominant convention, to the co-existence of conventions, and finally to the risk dominant convention.[13]

[13]A similar result can be obtained when players base their decisions on the imitate the best max rule. The only substantial difference is that it becomes slightly more difficult to populate previously empty islands as a lonely player on an island earns the lowest possible payoff and will never be imitated. Instead, two mutations are required to populate an island. Thus, the thresholds in Proposition 1 would change accordingly.

Let us provide some technical intuition for proposition 1. If the constraint is such that there may be empty islands, a single mistake is enough to move to states where the efficient action is played on more islands. This ensures universal coordination on the efficient convention in the long run. If the constraint is such that all islands will be occupied the picture is more complicated. If all agents choose the payoff dominant action, then the population on each island will fluctuate between m and M. Thus, with $\lceil m(1-q^*)\rceil$ mistakes we can traverse from these states to states where on one island the risk dominant action is chosen. Note that now all payoff dominant islands will have to be full, implying that further increasing the number of payoff dominant islands will take $\lceil M(1-q^*)\rceil$ mistakes. Thus, states where there is one island choosing the risk dominant action and the rest of the population chooses the payoff dominant action are more resilient to further increasing the number of risk dominant islands than states where everybody chooses the payoff dominant action. Finally, note that increasing the number of payoff dominant islands always takes $\lceil mq^*\rceil$ mistakes. Thus, whenever m is small relative to M and/or q^* is close to $\frac{1}{2}$ we observe the coexistence of conventions. Conversely, whenever q^* is large and m is large, we will observe everybody choosing the risk dominant action. Note that by the argument above, it also follows that whenever the coexistence of conventions emerges, only one island will choose the risk dominant action.

We end this section with a remark providing a simple necessary and sufficient condition when either part of Proposition 1, *a*) or *b*), holds.

Remark **1.**

i) If $k < \frac{M}{M-N}$*, then part a) of Proposition 1 holds.*

ii) If $k \geq \frac{M}{M-N}$*, then part b) of Proposition 1 holds.*

This statement can be easily verified by noting that $\underline{k} = \lceil \frac{kN}{M} \rceil$. Solving for $\underline{k} < k$ and $\underline{k} = k$ gives the required result. In addition, note that when we consider the case $k = 2$ as in Anwar (2002), the first condition reads $M < 2N$ meaning that no island can shelter the entire population, which was one of the initial assumptions of Anwar (2002) and this contribution. Thus, with only two islands only case a) can occur. Thus, profiles where everybody chooses the payoff dominant action can not occur in the long run (abstracting from the special case where $\lceil mq^* \rceil = \lceil m(1-q^*) \rceil$).

4 Extensions

In the following section we provide an informal discussion of two extensions of the model. On one hand, we provide insights on the role played by agents who are immobile and will never leave their native island. On the other hand, we discuss the consequences of heterogenous capacity constraints. While Anwar's original formulation with two islands encompasses patriots and Shi (2015) has analyzed the case of heterogenous capacity, the implications of these two extensions when there are more than two islands are not known. Thus, our discussion provides additional understanding of the mechanism at play and helps us explore the robustness of our results.

4.1 Patriots

First of all, let us consider (in the original spirit of Anwar (2002)) what happens if we include *patriot* agents in our model: i.e. agents who are immobile and cannot change islands. We do so assuming that in addition to our previously considered agents, there are $P \geq 2$ patriots on each island who cannot leave their home island. Thus, the overall population is now given by $k(N+P)$.[14] We assume that capacity on each island is adjusted so to make room for the additional (patriot) agents. That is capacity on each island is now given by $M' = M + P$ where M is the capacity in the model without patriots.

First, consider the case where in the original setup we have $\underline{k} = k$, i.e. even in the absence of patriots all islands would be populated. When we now add patriots to this setting (and

[14]When $k = 2$, as in Anwar (2002), the number of patriots (Np with $0 < p < 1$, in his notation) is in direct correspondence with capacity (Nc with $c > 1$, in his notation) of each island. Namely (see page 3 in Anwar (2002)): "The maximum number of agents who can reside at each location is therefore Nd where $d = \min\{c, 2-p\}$ and $1 < d < 2$".

adjust capacity accordingly), the prediction stays in line with the case where there are no patriots. The reason behind this insight is that the size of the smallest island now also adjusts to $m' = m + P$. Thus, when we determine the transition costs between the absorbing sets we now have to consider the new variables $m' = m + P$ and $M' = M + P$, instead of m and M. Thus, part b) of Proposition 1 ($\underline{k} = k$) can be easily adopted to the case where there are patriots with the only difference being that we now have to consider the variables m' and M'.

If instead $\underline{k} < k$, then the presence of patriots substantially changes the things. Most importantly, no island can be empty now. Thus, we do not only have to consider the set of absorbing sets from Ω_0 to $\Omega_{\underline{k}}$ as possible candidates for LRE, but also all sets of absorbing sets $\Omega_{\underline{k}+1}$ to Ω_k. Now every island has at least P inhabitants and in an absorbing set they will all play the same action. Note that in absorbing sets with less than $\underline{k}$ payoff dominant islands the island where the payoff dominant action is played will be at full capacity. If there are exactly $\underline{k}$ payoff dominant islands the population on these will fluctuate between $m' = M + P$ and $M' = M + P$. If there are, however, more than $\underline{k}$ payoff dominant islands, the population on these islands will fluctuate between P and $M' = M + P$. Note that the population of risk dominant islands will always be given by P, unless all islands choose the risk dominant island in which case it fluctuate between P and $M' = M + P$.

It is again possible to cut down the combinatorial problems associated with finding minimum cost trees by using an analogous result to the one of lemma 8. Hence, we can reduce the analysis of minimum cost trees to sets of absorbing sets characterized by the number of islands in which the payoff dominant action is played.

In terms of the transition costs, the presence of patriots changes the model along a number of dimensions. Most importantly, since there are no empty islands, we can no longer increase the number of payoff dominant islands by one mistake. When increasing the number of payoff dominant islands it now takes $\lceil Pq^* \rceil$ mistakes (since the population on risk dominant island is either P or may reach a state where it is P). When we want to increase the number of risk dominant islands, we now have to distinguish three cases: i) If the payoff

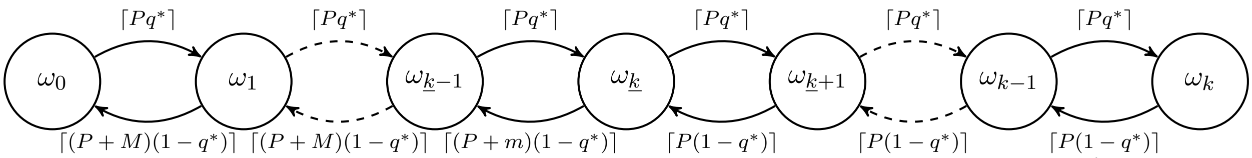

Figure 2: Transition costs with patriots.

dominant action is played on $\underline{k}+1$ to k islands (with at least one island where only patriots reside) this is possible with $\lceil P(1-q^*) \rceil$. ii) If the payoff dominant action is played on $\underline{k}$ islands, then the size of the smallest payoff dominant island is given by $m' = P + m$. It thus takes $\lceil (m+P)(1-q^*) \rceil$ mistakes. iii) If the payoff dominant action is played on less than $\underline{k}$ islands, all payoff dominant islands will be at full capacity $M' = M + P$, implying it will take $\lceil (M+P)(1-q^*) \rceil$ mistakes. We summarize this discussion in Figure 2.

We can now use these transition costs to determine the set of LRE. We summarize our findings in the remark below, for which a formal proof – despite being a straightforward generalization of the proof of Proposition 1 – goes beyond the scope of the current paper and is omitted. The mechanisms driving this remark can be easily deduced from Figure 2.[15]

Remark **2.** *If $\underline{k} < k$ and there are P patriots on each island, then:*

i) if $\lceil Pq^* \rceil > \lceil (P+M)(1-q^*) \rceil$, then $\mathcal{S} = \Omega_0$

ii) if $\lceil Pq^* \rceil = \lceil (P+M)(1-q^*) \rceil > \lceil (P+m)(1-q^*) \rceil$, then $\mathcal{S} = \bigcup_{\ell=0}^{\underline{k}-1} \Omega_\ell$

iii) if $\lceil Pq^* \rceil = \lceil (P+M)(1-q^*) \rceil = \lceil (P+m)(1-q^*) \rceil > \lceil P(1-q^*) \rceil$, then $\mathcal{S} = \bigcup_{\ell=0}^{\underline{k}} \Omega_\ell$

iv) if $\lceil Pq^* \rceil = \lceil P(1-q^*) \rceil = \lceil (P+M)(1-q^*) \rceil = \lceil (P+m)(1-q^*) \rceil$, then $\mathcal{S} = \bigcup_{\ell=0}^{k} \Omega_\ell$

v) if $\lceil (P+M)(1-q^*) \rceil > \lceil Pq^* \rceil > \lceil (P+m)(1-q^*) \rceil$, then $\mathcal{S} = \Omega_{\underline{k}-1}$

vi) if $\lceil (P+M)(1-q^*) \rceil > \lceil (P+m)(1-q^*) \rceil = \lceil Pq^* \rceil$ and $\lceil Pq^* \rceil > \lceil P(1-q^*) \rceil$, then $\mathcal{S} = \Omega_{\underline{k}-1} \cup \Omega_{\underline{k}}$

[15]Note that it is always the case that $\lceil Pq^* \rceil \geq \lceil P(1-q^*) \rceil$ and $\lceil (P+M)(1-q^*) \rceil \geq \lceil (P+m)(1-q^*) \rceil \geq \lceil P(1-q^*) \rceil$.

vii) if $\lceil (P+M)(1-q^*)\rceil > \lceil (P+m)(1-q^*)\rceil = \lceil P(1-q^*)\rceil = \lceil Pq^*\rceil$, then $\mathcal{S} = \bigcup_{\ell=\underline{k}-1}^{k} \Omega_\ell$

viii) if $\lceil Pq^*\rceil > \lceil P(1-q^*)\rceil$ and $\lceil (P+m)(1-q^*)\rceil > \lceil Pq^*\rceil$, then $\mathcal{S} = \Omega_{\underline{k}}$

ix) if $\lceil (P+m)(1-q^*)\rceil > \lceil Pq^*\rceil = \lceil P(1-q^*)\rceil$, then $\mathcal{S} = \bigcup_{\ell=\underline{k}}^{k} \Omega_\ell$.

In order to interpret this finding let us consider the limiting cases where the number of patriots is small ($P = 2$) and where it is large. In the former case we have $\lceil P(1-q^*)\rceil = 1$ and $\lceil Pq^*\rceil = 2$. If we consider islands large enough, so that $\lceil (2+M)(1-q^*)\rceil > 2$, then either the case *v)*, *vi)* or *viii)* will occur. It follows that only $\Omega_{\underline{k}}$ and $\Omega_{\underline{k}-1}$ may be subsets of the set of LRE. That is, we will observe the coexistence of conventions where either on $\underline{k}$ or on $\underline{k}-1$ islands the payoff dominant action will be played and these islands will be at capacity. If instead capacity M is relatively small as compared to the number of patriots, $\lceil (2+M)(1-q^*)\rceil \leq 2$,[16] then cases *i)*, *ii)* or *iii)* obtain. Thus Ω_0 is always a subset of the set of LRE, but there can be cases where other absorbing sets (including $\Omega_{\underline{k}}$) are also LRE.

In the other limiting case where P is large enough, we have $\lceil Pq\rceil > \lceil (P+M)(1-q^*)\rceil$. Thus only case *i)* can occur, implying that Ω_0 is unique LRE.

Abstracting from the special cases where the relevant transition costs are exactly the same, the following pattern emerges. If the number of immobile agents is relatively small, compared to capacity, we will observe profiles in the long run where almost all islands with mobile agents choose the payoff dominant action and the risk dominant action is only chosen on the smallest island containing mobile players and on all islands where only patriots live. Thus, provided there are not too many patriots a considerable share of the population will end up choosing the payoff dominant action. Interestingly there may be more than one island where the payoff dominant action is played. So, even in the presence of patriots mobile agents voting by their feet may be able to ensure high levels of efficiency. On the contrary, if patriots form a large part of the population, there is not enough flexibility for mobile agents to bring along efficient outcomes.

[16] It is possible that $\lceil (2+M)(1-q^*)\rceil = 2$ if $M \leq \frac{3q^*-2}{1-q^*}$, and this fraction can be arbitrarily large as $q^* \to 1$.

In this light our results for the patriots case may be seen as a generalization of those in Anwar (2002), albeit in a slightly more complex form.

4.2 Heterogeneous capacity

Let us now consider the implications of introducing heterogenous capacity constraints to our original model. Without loss of generality denote these constraints by $M_1 \leq M_2 \leq \cdots \leq M_k$. In this case, it is no longer possible to reduce the combinatorial complexity of the problem by considering sets of absorbing sets that are characterized by the number of islands on which the payoff dominant strategy is played. When computing the number of mistakes required for moving among islands with heterogenous capacity it matters which particular islands are occupied and which action is played on each of them.

Despite these difficulties it is possible to provide a simple result for the case in which every absorbing set has at least one empty island. In our setting, this corresponds to a scenario where all agents can be accommodated in all but the largest island, which gives rise to the condition $kN \leq \sum_{i=1}^{k-1} M_i$. This assumption is analogous to the assumption $\underline{k} < k$ in the original model. In this sense the following result is a generalization of part a) of Proposition 1.

PROPOSITION **2.** *If $kN \leq \sum_{i=1}^{k-1} M_i$, then $\mathcal{S}$ will always contain all absorbing sets where all agents play the payoff dominant action.*

Proof. Suppose there are L different absorbing sets. In every absorbing state at least one island is empty. This means that we can move from every absorbing set where at least one island is populated by risk dominant players, to an absorbing set where the empty island is populated by payoff dominant players in the following way: one agent from the original island moves to the empty island and switches to the payoff dominant action, by mistake. Consequently the previously empty island will be populated by payoff dominant players. In the same manner we can move among all absorbing sets where everybody chooses the payoff dominant action. Thus, all absorbing sets where the payoff dominant action is chosen by all players can be linked to all other absorbing sets via a chain of single mutations. Hence, for each absorbing set where the payoff dominant action is chosen by all players we can construct cost tree of $L-1$. Since no absorbing set can have a cost tree of cost smaller than $L-1$ the claim follows. □

This result does not tell us that there can be no LRE where the risk dominant action is played (on some or all islands), since it could be possible that in some absorbing sets an island is occupied by very few players. In this case if only one of them makes a mistake they could switch to the risk dominant action, thus possibly allowing us to construct minimum cost trees of the lowest possible cost $L-1$ for some mixed states or the absorbing set where everybody chooses the risk dominant action. This is analogous to the cases $\lceil m(1-q^*)\rceil = 1$ and/or $\lceil M(1-q^*)\rceil = 1$ in Proposition 1.

We remark that with heterogenous constraints it is a tedious problem to characterize the size of the population on the smallest island when all other islands are at capacity as this now depends on which islands are at capacity. This is also the main reason we believe it is difficult to provide general conditions (in the spirit of our main Proposition) for the case where there may be no empty islands.

Nonetheless we believe that the result above constitutes an important observation demonstrating that payoff dominant equilibria may arise in the long run for a wide range of parameters for more than three islands, thus supplementing the insights from Anwar (2002) and Shi (2015) where this was only possible for a very small parameter range (where the size of the population on the smallest island is very small).

5 Conclusion

We have extended the results from Anwar (2002), where agents had the possibility to move between two islands. We have relaxed constraints in a very natural way, increasing the number of available islands, and we have shown that when the constraints are less binding we have a long run stable equilibrium that was not considered in the original model: a configuration in which all agents play the payoff dominant strategy. The driving force behind this result is that whenever the restrictions are relatively weak there may be empty islands. This provides the payoff dominant action with a springboard from which it can play out its superiority. We can also reinterpret our results in the context of a model where agents choose circles of friends which are characterized by extreme clustering. In this context, universal coordination on the payoff dominant action in a society requires it to be possible that agents may completely abandon their current circle of friends and form new circles of friends. From this point of view, sufficient flexibility in creating new interaction structures leads to more efficient outcomes.

Acknowledgement: We are thankful to the Editor, Herbert Dawid, and two anonymous referees for their support and constructive comments. We would also like to thank Carlos Alós-Ferrer, Lukas Balafoutas, Maarten Janssen, Friederike Mengel, Wieland Müller, and Fei Shi for helpful discussions and useful suggestions. P. Pin acknowledges funding from the Italian Ministry of Education Progetti di Rilevante Interesse Nazionale (PRIN) grant 2015592CTH.

References

Alós-Ferrer, C. & Weidenholzer, S. (2007). Partial Bandwagon Effects and Local Interactions, *Games and Economic Behavior* **61**: 1–19.

Alós-Ferrer, C. & Weidenholzer, S. (2008). Contagion and efficiency, *Journal of Economic Theory* **143**(1): 251–274.

Alós-Ferrer, C. & Weidenholzer, S. (2014). Imitation and the role of information in overcoming coordination failures, *Games and Economic Behavior* **87**: 397–411.

Anwar, A. W. (2002). On the co-existence of conventions, *Journal of Economic Theory* **107**(1): 145–155.

Blume, A. & Temzelides, T. (2003). On the geography of conventions, *Economic Theory* **22**(4): 863–873.

Dawid, H. (1997). Learning of equilibria by a population with minimal information, *Journal of Economic Behavior & Organization* **32**(1): 1–18.

Dieckmann, T. (1999). The evolution of conventions with mobile players, *Journal of economic behavior & organization* **38**(1): 93–111.

Ellison, G. (1993). "Learning, Local Interaction, and Coordination", *Econometrica* **61**: 1047–1071.

Ellison, G. (2000). "Basins of Attraction, Long Run Stochastic Stability, and the Speed of Step-by-Step Evolution", *Review of Economic Studies* **67**: 17–45.

Ely, J. C. (2002). Local Conventions, *Advances in Theoretical Economics* **2**: 1–30.

Foster, D. & Young, P. (1990). Stochastic evolutionary game dynamics, *Theoretical population biology* **38**(2): 219–232.

Freidlin, M. & Wentzell, A. (1988). *Random Perturbations of Dynamical Systems, 2nd Ed.*, Springer Verlag, New York.

Goyal, S. & Janssen, M. C. (1997). Non-exclusive conventions and social coordination, *Journal of Economic Theory* **77**(1): 34–57.

Goyal, S. & Vega-Redondo, F. (2005). Network formation and social coordination, *Games and Economic Behavior* **50**: 178–207.

Jackson, M. O. & Watts, A. (2002). On the formation of interaction networks in social coordination games, *Games and Economic Behavior* **41**: 265–291.

Kandori, M., Mailath, G. J. & Rob, R. (1993). "Learning, Mutation and Long Run Equilibria in Games", *Econometrica* **61**: 29–56.

Oechssler, J. (1997). Decentralization and the coordination problem, *Journal of Economic Behavior & Organization* **32**(1): 119–135.

Robson, A. J. & Vega-Redondo, F. (1996). Efficient equilibrium selection in evolutionary games with random matching, *Journal of Economic Theory* **70**(1): 65–92.

Samuelson, L. (1997). *Evolutionary Games and Equilibrium Selection*, The MIT Press, Cambridge, Massachusetts.

Shi, F. (2013). Comment on "on the co-existence of conventions", *Journal of Economic Theory* **148**(1): 418–421.

Shi, F. (2015). Long-run technology choice with endogenous local capacity, *Economic Theory* **59**(2): 377–399.

Spivey, J. M. & Abrial, J. (1992). *The Z notation*, Prentice Hall Hemel Hempstead.

Staudigl, M. & Weidenholzer, S. (2014). Constrained interactions and social coordination, *Journal of Economic Theory* **152**: 41–63.

Weidenholzer, S. (2010). Coordination games and local interactions: a survey of the game theoretic literature, *Games* **1**(4): 551–585.

Young, H. P. (1993). "The Evolution of Conventions", *Econometrica* **61**: 57–84.