ЧЕЧЕНСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

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Таким образом для A_n имеем оценку

$$\|A_n u\|_{L_2(0;1)} 2\left(\frac{2}{3}\right)^{n-1} \frac{2}{3}\|u\|$$

Теперь из критерия голомофности следует, что ряд (1) для $|\varepsilon| < \frac{1}{2/3} = \frac{3}{2}$ образует голоморфное семейство типа (А).

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НЕКОММУНИКАТИВНОЕ РАСШИРЕНИЕ ОТОБРАЖЕНИЯ ЯНГА-БАКСТЕРА ТИПА АДЛЕРА ЯМИЛОВА

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A NONCOMMUTATIVE EXTENSION OF THE ADLER-YAMILOV YANG-BAXTER MAP

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В этой статье мы построим некоммутативное расширение отображения Янга-Бакстера типа Адлера-Ямилова, которое связано с нелинейным уравнением Шредингера. Кроме того, мы показываем, что это отображение частично интегрируемое.

Ключевые слова*: алгебры Грассмана, отображения Янга-Бакстера, уравнение Янга-Бакстера, интегрируемость по Лиувиллю.*

In this paper, we construct a noncommutative extension of the Adler-Yamilov Yang-Baxter map which is related to the nonlinear Schrödinger equation. Moreover, we show that this map is partially integrable.

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Key words: *Grassmann algebras, Yang-Baxter maps, Yang-Baxter equation, Liouville integrability.*

1 Introduction

The set theoretical solutions [9] of the YB equation have been of great interest for several researchers in the area of Mathematical Physics. The first examples of such solutions appeared in [18]. We refer to them using the shorter term "Yang-Baxter maps" which was proposed by Veselov in [21]. YB maps are related to several concepts of integrability as, for instance, the multidimensionally consistent equations [1, 2, 5, 15, 16]. Of particular interest are those Yang-Baxter maps which admit Lax representation [19]. They are connected with integrable mappings [21, 22] and they are also related to integrable partial differential equations via Darboux transformations [12].

Moreover, noncommutative extensions of integrable equations have been of great interest over the last decades [6, 7]. Darboux transformations for noncommutative-extended integrable equations were recently constructed; in the case of Grassman-extended nonlinear Schrödinger (NLS) equation in [11] and for the supersymmetric KdV equation in [23]. At the same time, the derivation of noncommutative versions of YB maps has gained its interest [8].

In this paper we construct a YB map associated to the noncommutative (Grassman) extension of the Darboux transformation in the case of the NLS equation which was recently appeared in [11]. The motivation comes from the study between Darboux transformations and Yang-Baxter maps which was studied in [12].

The paper is organised as follows: The next section deals with parametric Yang-Baxter maps, we give the definition of Liouville integrability, we present some basic properties of Grassmann algebras in order to make the text self-contained and we explain what is Lax representation of a Yang-Baxter map. In section 3, we present the Darboux transformation [11] of the noncommutative NLS equation. Section 4 is devoted in the derivation of a noncommutative ten-dimensional YB map and we also show that there exist suitable invariant leaves on which this ten-dimensional map can be restricted to a partially integrable eigh-dimensional YB map. The bosonic limit of the latter is the well-celebrated Adler-Yamilov map and we show that it is partially integrable. Finally, in chapter 5, we make some conclu-ding remarks.

2 Preliminaries

Let $Y_{ij} \in End(A \times A \times A)$, $i, j = 1,2,3$, $i \neq j$, where A is an algebraic variety in K^N, where K is any field of zero characteristic, such as $\mathbb{C}$ or $\mathbb{Q}$. Specifically, we define Y_{ij} by the following relations

$$Y^{12}(x, y, z) = (u(x, y), v(x, y), z), \tag{1a}$$

$$Y^{13}(x, y, z) = \big(u(x, z), y, v(x, z)\big), \tag{1b}$$

$$Y^{23}(x, y, z) = \big(x, u(y, z), v(y, z)\big), \tag{1c}$$

where $(x, y, z) \in A$.

Now, let $Y \in End(A \times A)$ be a map $(x, y) \xrightarrow{Y} \big(u(x, y), v(x, y)\big)$, and $\tilde{Y} = \pi Y \pi$ where $\pi \in End(A \times A)$
is the permutation map $\pi(x, y) = (y, x)$. Map Y is called a *Yang-Baxter map* if it satisfies the following equation

$$Y^{12} \circ Y^{13} \circ Y^{23} = Y^{23} \circ Y^{13} \circ Y^{12}, \tag{2}$$

which is the so-called Yang-Baxter equation. Moreover, map Y is called *reversible* if the composition of $\tilde{Y}$ and Y is the identity map, namely

$$\tilde{Y} \circ Y = Id. \tag{3}$$

Furthermore, we use the term *parametric YB map* if two parameters $a, b \in K$are involved in the definition of the YB map, namely we have a map of the following form

$$Y_{a,b}: (x, y\,) \to \big(u(x, y; a, b), v(x, y; a, b)\big), \tag{4}$$

satisfying *the parametric YB equation*

$$Y^{12}_{a,b} \circ Y^{13}_{a,c} \circ Y^{23}_{b,c} = Y^{23}_{b,c} \circ Y^{13}_{a,c} \circ Y^{12}_{a,b}. \tag{5}$$

Now, following [10, 20] we define the *complete integrability* of YB maps which is essential for the construction of integrable lattices.

Definition 2.1
A $2N$-dimensional Yang-Baxter map,

$$Y: (x_1, \dots, x_{2N}) \mapsto (u_1, \dots, u_{2N}), \quad u_i = u_i(x_1, \dots, x_{2N}), \quad i = 1, \dots, 2N,$$

is said to be completely integrable or Liouville integrable if

1. *there is a Poisson matrix $J_{ij} = \{x_i, x_j\}$, of rank $2N$, which is invariant under the action of the YB map, namely J_{ij} and $\tilde{J}_{ij} = \{u_i, u_j\}$ have the same functional form of their respective arguments*;
2. *map Y has N functionally independent invariants, I_i, namely $I_i {}^\circ Y = I_i$, which are in involution with respect to the corresponding Poisson bracket*, i.e. $\{I_i, I_j\} = 0$, $i, j = 1, \dots, N$, $i \neq j$.

Whenever the number of Poisson-commuting invariants in the above definition is less than N, then the YB map is called *partially integrable*.

The Liouville integrability of a YB map is important for the construction of integrable lattices. In particular, for those YB maps which admit Lax representation, one could consider a family of integrable maps which preserve the spectrum of the correspond-ding monodromy matrix [21, 22]. The trace of the former provides us with invariants and one can claim integrability of the corresponding lattice, if the invariants are in involution with respect to a Poisson bracket.

2.1 Grassmann varieties

Here, we briefly present the basic properties of Grassmann algebras. For further details one could consult [4]. Let G be a $\mathbb{Z}_2$-graded algebra over $\mathbb{C}$ or, in general, a field $K$of character-ristic zero. Thus, G as a linear space is a direct sum $G = G0 \oplus G1$, such that $GiGj \subseteq Gi + j$. Those elements of G that belong either to $G0$ or to $G1$ are called *homogeneous*, the ones from $G0$ are called *even*, while those in $G1$ are called *odd*.

By definition, the parity $|a|$ of an even homogeneous element a is 0, and it is 1 for odd homogeneous elements. The parity of the product $|ab|$ of two homogeneous elements is a sum of their parities: $|ab| = |a| + |b|$. Grassmann commutativity means that $ba = (-1)^{|a||b|}ab$ for any homogeneous elements a and b. In particular, $\alpha_1^2 = 0$, for all $a1 \in G1$ and even elements commute with all the elements of G.

By *Grassmann algebraic variety* we mean the set of solutions of polynomial equations with even and odd variables.

Remark 2.1.1. In the rest of this paper we shall be using Latin letters for even elem-ents of Lax operators or entries of Darboux matrices, and Greek letters when referring to the odd ones.

2.2 Lax representations of YB maps

Following Suris and Veselov in [19], we call a *Lax matrix* for a parametric YB map a square matrix, $L = L(x, \chi;\ a)$, depending on an even variable x, an odd variable χ, a parameter a and a spectral parameter λ, such that the Lax-equation

$$L(u, \xi;\ a)L(v, \eta;\ b) = L(y, \psi;\ b)L(x, \chi;\ a) \tag{6}$$

is satisfied whenever $(u, \xi, v, \eta) = Y_{\alpha,\beta}(x, \chi, y, \psi)$. Equation (10) is also called a *refactorisation problem*.

If the Lax-equation (6) has a unique solution, namely it is equivalent to a map

$$(u, \xi, v, \eta) = Y_{\alpha,\beta}(x, \chi, y, \psi), \tag{7}$$

then the Lax matrix L is said to be strong [14]. In this case (7) is a Yang-Baxter map and it is reversible [22].

3 Grassmann extensions of Darboux transformations

Let L be a Lax operator of the following AKNS form

$$L(p,q,\theta,\varphi;\lambda) = D_x + U(p,q,\theta,\varphi;\lambda), \tag{8}$$

where U is a matrix depending on two even potentials, $p = p(x)$ and $q = q(x)$, two odd potentials, $\theta = \theta(x)$ and $\varphi = \varphi(x)$, a spectral parameter λ and a variable x implicitly through the potentials.

By *Darboux transformation* we understand a map of the following form

$$L \to \tilde{L} = MLM^{-1}, \tag{9}$$

where $\tilde{L}$ is L updated with potentials $p_{10} = p_{10}(x)$, $q_{10} = q_{10}(x)$, $\theta_{10} = \theta_{10}(x)$and $\varphi_{10} = \varphi_{10}(x)$, namely $\tilde{L} = L(p_{10}, q_{10}, \theta_{10}, \varphi_{10};\, \lambda)$. Matrix M in (9) is called the *Darboux matrix*. Here, we shall be assuming that matrix M has the same λ-dependence with U. Moreover, we define the *rank* of a Darboux transformation to be the rank of the matrix which appears as coefficient of the highest power of the spectral parameter.

In this section we consider the Grassmann extension of the Darboux matrix correspo-nding to the NLS equation (see [11]).

3.1 Nonlinear Schrödinger equation

The Grassmann extension of the Darboux matrix for the NLS equation was constructed in [11]. In particular, they considered the following noncommutative extension of the NLS operator

$$L := D_x + U(p,q,\theta,\varphi;\lambda) = D_x + \lambda\, U_1 + U_0, \tag{10a}$$

where U^1 and U^0 are given by

$$U^1 = \mathrm{diag}(1,-1,0), \quad U^0 = \begin{pmatrix} 0 & 2p & \theta \\ 2q & 0 & \zeta \\ \varphi & \kappa & 0 \end{pmatrix}, \tag{10b}$$

where $p, q \in G_0$ and $\theta, \varphi, \kappa, \zeta \in G_1$.

It was shown that all the Darboux transformations of rank 1 associated to this operator are described by the following matrix

$$M(p,q,\theta,\varphi;\, c_1, c_2) = \begin{pmatrix} F+\lambda & p & \theta \\ q_{10} & c_1 & 0 \\ \varphi_{10} & 0 & c_2 \end{pmatrix}, \tag{11}$$

where c_1 and c_2 can be either 1 or 0. In the case where $c_1 = c_2 = 1$, the entries of $M(p,q,\theta,\varphi;\, 1,1)$ satisfy the following system of differential-difference equations

$$Fx = 2(pq - p_{10}q_{10}) + \theta\varphi - \theta_{10}\varphi_{10}, \tag{12a}$$

$$p_x = 2(Fp - p_{10}) + \theta\zeta, \tag{12b}$$

$$q_{10,x} = 2(q - q_{10}F) - \kappa_{10}\varphi_{10}, \tag{12c}$$

$$\theta_x = F\theta - \theta_{10} + pk, \tag{12d}$$

$$\varphi_{10,x} = \varphi - \varphi_{10}F - \zeta_{10}q_{10}, \tag{12e}$$

and the algebraic equations

$$\theta q_{10} = (S - 1)\kappa, \tag{13a}$$

$$\varphi_{10}p = (S - 1)\zeta. \tag{13b}$$

Moreover, system (12) admits the following first integral

$$\partial_x(F - pq_{10} - \varphi_{10}\theta) = 0, \tag{14}$$

which implies that $\partial_x(\mathrm{sdet}(M)) = 0$, since $\mathrm{sdet}(M) = \lambda + F - pq_{10} - \varphi_{10}\theta$.

4 Derivation of a noncommutative Yang-Baxter map

Here, we are interested in the Grassmann extension of the Adler-Yamilov YB map, associated to the NLS equation and its integrability.

4.1 A ten-dimensional Yang-Baxter map

According to (11) we define the following matrix

$$\begin{pmatrix} X+\lambda & x_1 & \chi_1 \\ x_2 & 1 & 0 \\ \chi_2 & 0 & 1 \end{pmatrix}, \qquad x := (x_1, x_2, \chi_1, \chi_2, X), \tag{15}$$

Then, we substitute M to the Lax equation (6). The corresponding algebraic variety is a union of two ten-dimensional components. The first one is obvious from the refactorisation problem, and it corresponds to the permutation map

$$x \mapsto \boldsymbol{u} = \boldsymbol{y}, \;\; y \mapsto \boldsymbol{v} = \boldsymbol{x},$$

which is a trivial YB map. The second one can be represented as a ten-dimensional non-involutive Yang-Baxter map given by

$$x_1 \mapsto u_1 = y_1 - \frac{X - x_1x_2 - \chi_1\chi_2 - Y + y_1y_2 + \psi_1\psi_2}{1 + x_1y_2 + \chi_1\psi_2} x_1, \tag{16a}$$

$$x_2 \mapsto u_2 = y_2, \tag{16b}$$

$$\chi_1 \mapsto \xi_1 = \psi_1 - \frac{X - x_1x_2 - Y + y_1y_2 + \psi_1\psi_2}{1 + x_1y_2} \chi_1, \tag{16c}$$

$$\chi_2 \mapsto \xi_2 = \psi_2, \tag{16d}$$

$$X \mapsto U = \frac{X - x_1x_2 - \chi_1\chi_2 + (x_1y_2 + \chi_1\psi_2)Y + y_1y_2 + \psi_1\psi_2}{1 + x_1y_2 + \chi_1\psi_2}, \tag{16e}$$

$$y_1 \mapsto v_1 = x_1, \tag{16f}$$

$$y_2 \mapsto v_2 = x_2 + \frac{X - x_1x_2 - \chi_1\chi_2 - Y + y_1y_2 + \psi_1\psi_2}{1 + x_1y_2 + \chi_1\psi_2} y_2, \tag{16g}$$

$$\psi_1 \mapsto \eta_1 = \chi_1, \tag{16h}$$

$$\psi_2 \mapsto \eta_2 = \chi_2 + \frac{X - x_1x_2 - \chi_1\chi_2 - Y + y_1y_2}{1 + x_1y_2} \psi_2, \tag{16i}$$

$$Y \mapsto V = \frac{(x_1y_2 + \chi_1\psi_2)X + x_1x_2 + \chi_1\chi_2 + Y - y_1y_2 - \psi_1\psi_2}{1 + x_1y_2 + \chi_1\psi_2}. \tag{16j}$$

4.2 Restriction on invariant leaves: Noncommutative extension of the Adler-Yamilov map

In this section, we derive an eight-dimensional Yang-Baxter map from map (16), which is the Grassmann extension of the Adler-Yamilov map [3, 13, 17]. Our proof is motivated by the existence of the first integral (14) for system (12).

In particular, we have the following.

Proposition 4.2.1

1. The quantities $\Phi = X - x_1x_2 - \chi_1\chi_2$ and $\Psi = Y - y_1y_2 - \psi_1\psi_2$ are invariants (first integrals) of the map (16),
2. The ten-dimensional map (16) can be restricted to an eight-dimensional map $Y_{\alpha,\beta} \in \mathrm{End}(A_a \times A_b)$, where A_a, A_b are level sets of the first integrals Φ and Ψ, namely

$$A_a = \{(x_1, x_2, \chi_1, \chi_2, X) \in A^5; X = a + x_1x_2 + \chi_1\chi_2\}, \tag{17a}$$

$$A_b = \{(y_1, y_2, \psi_1, \psi_2, Y) \in A^5; Y = b + y_1y_2 + \psi_1\psi_2\}, \tag{17b}$$

3. The bosonic limit of map $Y_{a,b}$ is the Adler-Yamilov map.

Proof

1. It can be readily verified that (16) implies $U - u_1u_2 - \xi_1\xi_2 = X - x_1x_2 - \chi_1\chi_2$ and $V - v_1v_2 - \eta_1\eta_2 = Y - y_1y_2 - \psi_1\psi_2$. Thus, Φ and $\Psi$are invariants, i.e. first integrals of the map.
2. The existence of the restriction is obvious. Using the conditions $X = x_1x_2 + \chi_1\chi_2 + a$ a and $Y = y_1y_2 + \psi_1\psi_2 + b$, one can eliminate X and Y from (16). The resulting map, $\boldsymbol{x} \mapsto \boldsymbol{u}(\boldsymbol{x}, \boldsymbol{y})$, $\boldsymbol{y} \mapsto \boldsymbol{v}(\boldsymbol{x}, \boldsymbol{y})$, is given by

$$\boldsymbol{x} \mapsto \boldsymbol{u} = \left(y_1 + \frac{(b-a)(1 + x_1y_2 - \chi_1\psi_2)}{(1 + x_1y_2)^2} x_1, y_2, \psi_1 + \frac{b-a}{1 + x_1y_2} \chi_1, \psi_2\right), \tag{18a}$$

$$\boldsymbol{y} \mapsto \boldsymbol{v} = \left(x_1, x_2 + \frac{(a-b)(1 + x_1y_2 - \chi_1\psi_2)}{(1 + x_1y_2)^2} y_2, \chi_1, \chi_2 + \frac{a-b}{1 + x_1y_2} \psi_2\right). \tag{18b}$$

3. If one sets the odd variables of the above map equal to zero, namely $\chi_1 = \chi_2 = 0$ and $\psi_1 = \psi_2 = 0$, then the map (18) coincides with the Adler-Yamilov map. ∎

Now, one can use the condition $X = x_1x_2 + \chi_1\chi_2 + a$ to eliminate X from the Lax matrix (35), i.e.

$$M(\boldsymbol{x}; a, \lambda) = \begin{pmatrix} a + x_1x_2 + \chi_1\chi_2 + \lambda & x_1 & \chi_1 \\ x_2 & 1 & 0 \\ \chi_2 & 0 & 1 \end{pmatrix}, \qquad (19)$$

which corresponds to the Darboux matrix derived in [11]. Now, the Adler-Yamilov map's extension follows from the strong Lax representation

$$M(\boldsymbol{u}; a, \lambda)M(\boldsymbol{v}; b, \lambda) = M(\boldsymbol{y}; b, \lambda)M(\boldsymbol{x}; a, \lambda). \qquad (20)$$

Therefore, the extension of the Adler-Yamilov's map (18) is a reversible parametric YB map. Moreover, it is easy to verify that it is not involutive.

Proposition 4.2.2

The noncommutative extension of the Adler-Yamilov map is a partially integrable map.

Proof

From str($M(\boldsymbol{y}; b, \lambda)M(\boldsymbol{x}; a, \lambda)$) we obtain the following invariants for map (18)

$$T_1 = x_1x_2 + y_1y_2 + \chi_1\chi_2 + \psi_1\psi_2,$$

$$T_2 = (a + x_1x_2 + \chi_1\chi_2)(b + y_1y_2 + \psi_1\psi_2) + x_1y_2 + x_2y_1 + \chi_1\psi_2 - \psi_2\chi_1,$$

where we have omitted the additive constants. However, these invariants are linear combinations of the following integrals

$$I_1 = b(x_1x_2 + \chi_1\chi_2) + a(y_1y_2 + \psi_1\psi_2) + y_1y_2(x_1x_2 + \chi_1\chi_2) + x_1x_2\psi_1\psi_2 + x_2y_1 + x_1y_2 + \chi_1\psi_2 - \psi_2\chi_1, \qquad (41a)$$

$$I_2 = x_1x_2 + y_1y_2, \quad I_3 = \chi_1\chi_2 + \psi_1\psi_2, \quad I_4 = \chi_1\chi_2\psi_1\psi_2. \qquad (41b)$$

These are in involution with respect to the Poisson bracket $\{x_1, x_2\} = \{y_{1,}y_2\} = 1$, $\{\chi_1, \chi_2\}$ = $\{\psi_1, \psi_2\}$ = 1 and all the rest $\{x_i, x_j\} = \{y_{i,}y_j\} = \{x_{i,}y_j\} = 0$.

and the corresponding Poisson matrix is invariant under the YB map (18). However, I_3 and I_4 are not functionally independent, but $I_3^2 = 2I_4$, thus map (18) is partially integrable.

5 Conclusions

We showed that there is an explicit example of birational endomorphism of Grass-mann algebraic varieties which possesses the Yang-Baxter property. Specifically, we considered the case of the Grassmann extension of the Darboux transformation for the NLS equation. In this case a Darboux transformation appeared in [11]. Employing the associated Darboux matrix we derived a ten-dimensional map, which we restricted on invariant leaves to an eight-dimensional birational parametric YB map. The motivation for this restriction was the fact that the entries of the associated Darboux matrix satisfy a particular system of differential-difference equations which possesses a first integral. We showed that the eight-dimensional YB map is partially integrable and, at its bosonic limit, is equivalent to the famous Adler-Yamilov map.

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ОБ ОДНОМ КЛАССЕ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ С МНОГОТОЧЕЧНОЙ СИНГУЛЯРНОСТЬЮ И РАЗРЫВНЫМИ РЕШЕНИЯМИ

ABOUT ONE CLASS OF DIFFERENTIAL EQUATIONS WITH MULTIPOINT SINGULARLY AND DISCONTINUOUS SOLUTION

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В статье приводятся теоремы существования решения для системы ОПУ с разрывными правыми частями по независимой переменной и фазовым координатам, удовлетворяет многоточечным условиям в точках сингулярностей.

Ключевые слова: *точки сингулярности, разрывы по фазовым координатам, многоточечные краевые условия, теоремы существования решения.*

The article presents the existence theorem for the solutions of the income statement with discontinuous the right sides of the independent variable and phase coordinates satisfies multipoint conditions at singularities.

Key words: *singular point, breaks on phase coordinates multipoint boundary conditions, the existence theorem for solutions.*

I. Изучается квазилинейная система дифференциальных уравнений

$$y_i' = \Phi_i(x, y_i, y_2, ..., y_n)y_i + f_i(x, y_1, y_2, ..., y_n) \quad (i = 1, ..., n) \tag{1}$$

относительно точек области $D = Y \times R^n, Y = [a,b], R^n = \{|y_i| \le d_i, i = 1,...,n\}$. Здесь $d_i(i = 1,....,n)$ – известные числа, $Y = [a,b]$ заданный сегмент. Предполагается, что при $x = x_{i\nu_i} \in]a,b[$ $(\nu_i = 1,2,...,m_i, i = 1,...,n, m_i$ – натуральное число $i = 1,...,n)$, функции $\Phi_i(x,y) = \Phi_i(x, y_1, y_2, ..., y_n)$, $f_i(x.y) = f_i(x, y_1, y_2, ..., y_n)$, $y = (y_i)_{=1}^n$ имеют сингулярности, т.е. в области

$$D_i = Y_i \times R^n, Y_i = \bigcup_{k_i=0}^{m_i} Y_{ik_i}, Y_{i\nu_i'} =]x_{i\nu_i'}, x_{i\nu_i'+1}[(\nu_i' = 1,...,m_i - 1), Y_{i0} = [a, x_{i1}[, \quad Y_{im_i} =]x_{im_i}, b] \ (i = 1,...,n)$$

могут быть не ограничены суммируемыми функциями. Для таких систем во многих