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Scott Condie, Jayant Ganguli

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The pricing effects of ambiguous private information*

Scott Condie[†] Jayant Ganguli[‡]

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Abstract

When private information is observed by ambiguity averse investors, asset prices may be informationally inefficient in rational expectations equilibrium. This inefficiency implies lower asset prices as uninformed investors require a premium to hold assets and higher return volatility relative to informationally efficient benchmarks. Moreover, asset returns are negatively skewed and may be leptokurtic. Inefficiency also leads to amplification in price of small changes in news, relative to informationally efficient benchmarks. Public information affects the nature of unrevealed private information and the informational inefficiency of prices. Asset prices may be lower (higher) with good (bad) public information.

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[†]Department of Economics, Brigham Young University, Provo, UT 84602, USA. Telephone: +1 801 422 5306, Fax: +1 801 422 0194, Email: `ssc@byu.edu`

[‡]Department of Economics, University of Essex, Colchester, Essex CO4 3SQ, UK. Telephone: +44 (0) 1206 872723, Fax: +44 (0) 1206 872724, Email: `jayantvivek@gmail.com`

1 Introduction

Along with their role in rationing assets, market prices aggregate and convey information. In many asset pricing models prices always react to and reveal information.[1] However, growing empirical research indicates that prices react to news in differing ways depending on the state of the economy, which may affect information transmission.[2] This paper investigates the ability of market prices to transmit private information when it is observed by ambiguity averse investors and shows that the reaction of market prices to news can be very different than in traditional asset market models.

When ambiguity averse investors observe ambiguous private information in an otherwise frictionless market, a range of this information will not be revealed by asset prices in REE (rational expectations equilibrium) in the framework we study. This is in contrast to the case of private information observed by ambiguity-neutral investors.

This informational inefficiency of prices leads to several interesting phenomena. First, asset prices incorporate a premium due to the unrevealed information and are thus lower than they would otherwise be. This premium increases with fundamental risk and return volatility is higher relative to informationally efficient benchmarks, where there is no asymmetry of information or there is no ambiguity in information. Asset returns exhibit negative skewness and may also exhibit excess kurtosis. Moreover, informational inefficiency can amplify price reaction to small changes in news and implies price volatility changes with informational efficiency of price. Finally, public information affects the nature of the unrevealed information and the informational efficiency of prices. This leads to the seemingly anomalous result that an asset's price is lower (respectively, higher) when public information conveys good (respectively, bad) news.

These results stem from two facts. The first is that ambiguity averse informed traders who receive ambiguous private information about an asset will trade off their asset holdings unless they are compensated by an ambiguity premium. Moreover, they will do so at the same price for a range of information, a property we term portfolio inertia in information. Uninformed traders who take positive positions in the asset will then require a market risk premium which compensates them for fundamental risk and for the reduction in asset holders in addition to a premium for the unrevealed information. Non-revelation of information in REE arises when the premium required by the uninformed traders due to the reduction in

[1] For instance, Grossman (1981), Radner (1979) *inter alia*.

[2] See for example, Andersen, Bollerslev, Diebold, and Vega (2007), Faust, Rogers, Wang, and Wright (2007), and others.

asset holders is lower in aggregate than the ambiguity premium required by informed traders. In the informationally inefficient REE, the unrevealed information premium is higher for riskier assets and the informational inefficiency in price leads to returns that are negatively skewed and, for some parameter values, leptokurtic.

The second fact is that the price of an asset changes discontinuously relative to news as the informational efficiency of price changes. The non-revelation of some information implies that uninformed traders' beliefs are based on a set of possible signal values as opposed to being based on exact information. This implies that beliefs will differ discontinuously. Since these uninformed traders' beliefs drive asset prices in equilibrium, prices are discontinuous relative to news. This discontinuity implies price amplification of news changes. Since public information affects the range of unrevealed information, it affects the beliefs of informed and uninformed investors. This may lead to asset prices being lower despite good public news because public information affects what private information is revealed.

To establish the above results, we extend the standard CARA-normal REE model where market prices aggregate and communicate information (see Grossman (1976) or Radner (1979) among others). In the main model that we analyze ambiguity averse informed traders receive ambiguous private information. That is, their beliefs are represented by a set of probability distributions over the underlying fundamentals rather than a single distribution. These traders are ambiguity averse in the sense of the Gilboa and Schmeidler (1989) multiple priors (MEU) representation.[3] Uninformed traders can be ambiguity-neutral or averse.

The key property of portfolio inertia in information is a consequence of the non-smooth MEU representation. It is distinct from the portfolio inertia in prices property identified by Dow and da Costa Werlang (1992b), but related since both follow from non-smoothness of the representation. Incorporating this non-smooth decision-making model has provided a number of insights in studying financial markets (Epstein and Schneider 2010).[4] Smooth preference representations such as Klibanoff, Marinacci, and Mukerji (2005), Maccheroni, Marinacci, and Rustichini (2006), and Hansen and Sargent (2007) do not yield inertia in information and so will not generate the informational inefficiency we study here. Experi-

[3]This representation captures the degree of confidence decision-makers have in probabilistic assessments based on the quality of information, unlike the Savage (1954) decision-making model (Gilboa and Marinacci (2012)). See also Ellsberg (1961), Keynes (1921) and Knight (1921).

[4]Much of these are developed with representative investor or homogeneous information frameworks. Unlike in a representative investor framework, equilibrium asset prices under partial revelation here are not driven by the worst-case assessment of ambiguity averse investors. Moreover, Chapman and Polkovnichenko (2009) show that ignoring underlying heterogeneity can significantly change estimates for the equity premium and risk free rate.

mental evidence in Ahn, Choi, Kariv, and Gale (2011), Asparouhova, Bossaerts, Eguia, and Zame (2012), and Bossaerts, Ghirardato, Guarneschelli, and Zame (2010) provides persuasive support of non-smooth models of ambiguity aversion in financial markets.

Information non-revelation under ambiguity differs from informational inefficiency due to noise-, endowment-, or taste-shock mechanisms. These models introduce additional exogenous randomness in price to impede information revelation.[5] Our analysis suggests that the ambiguity-based and noise-based mechanisms provide differing, but complementary means of studying financial markets.

This paper fits into a growing literature studying informational efficiency and ambiguity averse traders including Tallon (1998), Caskey (2008), Ozsoylev and Werner (2011), and Mele and Sangiorgi (2015). Each of these papers use the noise trader mechanism for informational inefficiency. On the other hand, Condie and Ganguli (2011a), Easley, O'Hara, and Yang (2011), and Yu (2014) do not include noise traders in their market model.[6] Condie and Ganguli (2011a) demonstrates that the informational inefficiency studied here has the desirable property of being robust in the context of general financial market economies with finitely many states and signal values, similar to those studied in Radner (1979).

The paper proceeds as follows. We first develop the financial market model in Section 2. Section 3 describes the conditions for and nature of non revelation of information. Section 4 elaborates the pricing implications of partial revelation and Section 5 examines the effects of public information. Section 6 discusses the model in the context of noise-based partial revelation, other sources of inertia in information, and strategic trading behavior. Section 7 concludes. All proofs for the results in the main text are in Appendix A. Supplementary appendix B contains extensions of the baseline model and shows that similar qualitative results hold. These extensions include (i) non-tradable labour income as a source of ambiguous information, (ii) ambiguity-averse uninformed traders, (iii) all traders observing private information, and (iv) ambiguity averse informed investors with ambiguous priors who receive unambiguous private information.

[5] See Dow and Gorton (2008) for a recent discussion of these.

[6] de Castro, Pesce, and Yannelis (2010) introduce and prove existence, incentive compatibility, and Pareto efficiency of a separate equilibrium concept they call 'maximin rational expectations equilibrium'.

2 A model of ambiguous private information

The model is populated by two types of investors, denoted by $n \in \{I, U\}$. I-investors receive a private signal and are referred to as informed investors whereas U-investors don't receive any private information and are referred to as uninformed. The mass of I-investors is $0 < x_0^I < 1$ and that of U-investors is $0 < x_0^U < 1$, with $x_0^I + x_0^U = 1$. All investors live for 3 periods and trade assets in the market. Time is indexed by $t = 0, 1, 2$. Investors observe information and trade at $t = 1$. All uncertainty is resolved and consumption occurs at $t = 2$.

Two assets are traded in the market. The first asset is a risk-free bond whose payoff is denoted $v_f = 1$.[7] This asset is in perfectly elastic supply and we normalize its gross rate of return to one. The second asset, called the stock, has an uncertain terminal value denoted by v. It is assumed to be in unit net supply. At time 0, type n-investors are endowed in aggregate with $x_0^n > 0$ of the uncertain asset and 0 units of the bond. Trade occurs in period 1 with the resolution of uncertainty occurring in period 2.

We assume that the stock payoff v is normally distributed with mean μ_0 and precision ρ_0. In period 0, all investors have identical information about the stock. However, the two types of traders differ in their receipt and perception of information in period 1. At $t = 1$, I-investors receive a private signal

$$s = v + \epsilon \tag{1}$$

that conveys information about v, where ϵ is a stochastic error term. The signal is interpreted differently by the informed I-investors and the uninformed U-investors, if the latter observe it. This differential interpretation is related to the signal error term ϵ.

Both types of investors agree that the signal error ϵ is distributed normally with precision ρ_ϵ but have differing assessments of the mean μ_ϵ of the error term. I-investors believe the information may be biased but are unsure about the direction of this bias. I-investors' lack of knowledge about the signal bias is modeled as *ambiguity in the signal* in the sense that they know only that $\mu_\epsilon \in [-\delta, \delta]$ where $\delta > 0$. The size of this interval captures the I-investors degree of confidence in the information. Moreover, I-investors are averse to this perceived ambiguity. In this structure, I-investors use a set of likelihoods, indexed by $\mu_\epsilon^I \in [-\delta, \delta]$, in updating their beliefs, which we discuss formally in section 2.1.[8]

I-investors may doubt the unbiasedness of a signal because of concerns about the signal

[7] It would perhaps be more appropriate to use the term 'uncertainty-free' to describe this asset in our setting, but we stay with the usual terminology.

[8] Yu (2014) considers a related information structure with multiple likelihoods but where signals are drawn from a finite set and their relation to the underlying fundamental is not explicitly modeled.

source, because the information is intangible in the sense of Daniel and Titman (2006), or because the relationship between the signal and the stock is ambiguous, for example, receiving ambiguous private information about a non-traded asset like labor income, whose payoff is correlated with that of the stock (see Section B.1), among other possibilities. See also the discussions in Epstein and Schneider (2008) and Illeditsch (2011).[9]

On the other hand, U-investors do not perceive ambiguity in the signal and believe it is unbiased, i.e. their assessment of the mean $\mu_\epsilon^U = 0$. This assumption is for tractability and is relaxed in Section B.3 to allow U investors to be ambiguity averse at the cost of some notational simplicity, but without much additional insight into the nature of partial revelation. The key requirement is that U investors perceive less ambiguity in the signal, when they observe it through price, than I investors. We show in Section B.4 that similar results on partial revelation also hold in an alternative setting where I investors have prior beliefs represented by a set of distributions and consider the signal is unambiguous in the sense that they consider that $\mu_\epsilon = 0$ like U investors.[10]

These structures imply that the informational inefficiency found in this paper derives from the ambiguity-aversion of the private information recipients and not of the uninformed investors. That is, it is not the uninformed investors' inability to interpret information which drives informational inefficiency. We do not claim that such heterogeneity in ambiguity aversion or perception of ambiguity in information are pervasive. However, we think it is reasonable that such differences exist, especially since our results demonstrate that partial revelation can arise when a small fraction of investors perceive their own information to be ambiguous.[11] In this respect, our results illustrate how ambiguity can affect market efficiency and market aggregates even if it is not embodied by a large presence in the market, unlike for example, models with a representative ambiguity averse investor.

This model can be extended to allow for U investors to receive private signals as well, as

[9]These papers model ambiguity through an interval of signal variances. We do not explore this additional interesting avenue for ambiguity in information here.

[10]U investors, or some subset of U investors, may be considered as competitive risk-averse market makers, along the lines discussed for example in Vives (2008) (Chapters 4 and 8).

[11]One way to think about why U investors may consider the information to be unambiguous or less ambiguous than I investors is along the lines of the discussion in Gilboa, Postlewaite, and Schmeidler (2012) and Gilboa and Marinacci (2012), which point out that the Savage (1954) SEU representation does not allow a lack of ignorance or confidence to be captured. In this very specific sense, I and U investors in the baseline model could be considered to be representing investors who have differential attitudes toward ignorance. I investors are sensitive (averse) to this, while U investors are not. We do *not* consider this to be the same phenomena as the notion of overconfidence developed in, for example Daniel, Hirshleifer, and Subrahmanyam (1998), although we note that there is no contradiction between the two ideas. For example, the true signal bias could be below any fixed bias that U investors believe is present in the information.

we show in Section B.2. If these are unambiguous or if U investors are ambiguity-neutral, then such signals will be revealed in equilibrium and the qualitative results on non-revelation of ambiguous private information would hold similarly.[12]

2.1 Decision making

Investors' von Neumann-Morgenstern utility u is in the constant absolute risk aversion (CARA) class with common CARA coefficient γ, i.e.

$$u(w) = -\exp\left(-\gamma w\right), \tag{2}$$

where terminal wealth $w(\theta) = \theta v + m$ with stockholding θ and bondholding m. Since initial wealth of each trader is $w_0 = p$, the period 0 budget constraint $p = \theta p + m$ implies $w = p + \theta(v - p)$.

Ambiguous information is processed and incorporated using the updating rule developed in Epstein and Schneider (2007) and Epstein and Schneider (2008). This rule reduces to Bayes' rule when the information is unambiguous. The following result characterizes these updated beliefs for I-investors.[13]

Lemma 1. *The updated beliefs of an I-investor about v after observing signal s are represented by the set of normal distributions with precision $\rho^I|s = \rho_0 + \rho_\epsilon$ and means*

$$[\underline{\mu}^I|s, \overline{\mu}^I|s] = \left[\frac{\rho_0\mu_0 + \rho_\epsilon(s-\delta)}{\rho_0 + \rho_\epsilon}, \frac{\rho_0\mu_0 + \rho_\epsilon(s+\delta)}{\rho_0 + \rho_\epsilon}\right]. \tag{3}$$

The updated beliefs of U-investors depend on their inference of information from price and will be derived as part of the equilibrium below.

Ambiguity averse investors make decisions using the multiple prior max-min expected utility (MEU) criterion, which was axiomatized by Gilboa and Schmeidler (1989).[14] Denoting by F^n the set of distributions representing n investor beliefs given information as in Lemma

[12] If both I and U investors observe private information which they perceive to be ambiguous, then the analysis we carry out in Section B.2 and Section B.3 is suggestive that if information is considered more ambiguous by those who observe than those who do not, it may not be revealed through price.

[13] Investors make decisions only once after receiving information, so issues of dynamic inconsistency do not arise, but inter-temporal decision making would be dynamically consistent with this updating rule and our assumptions.

[14] The ambiguity aversion of investors in this representation can be formalized using the analysis of Gajdos, Hayashi, Tallon, and Vergnaud (2008).

1, the utility from a portfolio with stock demand θ is

$$\mathbf{U}^n(\theta) = \min_{F\in F^n} \mathbb{E}_F[u(w(\theta))] = \min_{F\in F^n} \mathbb{E}_F\left(-\exp\left(-\gamma w(\theta)\right)\right), \tag{4}$$

This includes the case of Savage (1954) expected utility U-investors who do not perceive any ambiguity when F^U is a single probability distribution.

Utility $\mathbf{U}^I$ is everywhere differentiable except when the terminal wealth from portfolio holdings is not uncertain, i.e. when the investor trades away his holdings of the stock and holds only the risk-free asset. This non-differentiability is key for the partial revelation equilibria.[15]

2.2 Market prices and rational expectations equilibria

Trade in the assets occurs in period 1. A price function $\mathbf{p}$ maps signal values s to asset prices, i.e. $\mathbf{p}(s) = (p(s), p_f(s))$, where p denotes the stock price and p_f the bond price. Since we have normalized the price of the bond to 1, we study the price function $\mathbf{p}(s) = (p(s), 1)$ and abusing notation use $p(s)$ to denote the function hereafter. Information is revealed through prices when the function $p(s)$ is invertible. When this occurs for all signals, U-investors correctly infer each signal by observing the market price and the price function is said to be *fully-revealing*.

When the function is not invertible, the market prices will not reveal all information and the function is said to be *partially revealing*. When prices are partially revealing, multiple signal values may be consistent with the observed market price $\bar{p}$ and U-investors know only that some signal from the set $p^{-1}(\bar{p})$ was observed by I-investors.

The market clearing condition for the stock is

$$x_0^I\theta^I + x_0^U\theta^U = 1 \tag{5}$$

The rational expectations equilibrium (REE) concept requires that individuals behave optimally given the information that they have and that they make use of all available information.

[15]Though we will not explore this further, other portfolio positions where utility is non-differentiable could be used for studying the kind of partial revelation we present here. For example, Epstein and Schneider (2010) (section 3.1.2) suggest a formulation which may yield non-differentiability at a non-zero portfolio position.

Definition 1. A *rational expectations equilibrium* is a set of portfolios $\{\theta^I(s), \theta^U(s)\}_s$ and a price function $p(\cdot)$, which specifies stock price $p(s)$ for each signal s, such that the following hold almost surely.

1. Each I-investor has information s and chooses a stock demand $\theta^I(s)$ measurable with respect to s, that satisfies
$$\theta^I(s) \in \arg\max \mathbf{U}^I(\theta|s) \tag{6}$$
subject to the trader's budget constraint.

2. Each U-investor has information $p^{-1}(\bar{p})$ at stock price $\bar{p}$ and chooses a stock demand $\theta^U(s)$ measurable with respect to p, subject to the budget constraint, that satisfies
$$\theta^U(s) \in \arg\max \mathbf{U}^U(\theta|p^{-1}(\cdot)) \tag{7}$$

3. The market clearing condition (5) holds.

Given this definition, an REE is said to be *fully revealing* when the equilibrium price function is fully revealing and it is said to be *partially revealing* otherwise. In the above definition, we specify I-investors' information as the private signal s since the price does not convey any additional information to them.

2.3 Informed investor demand and inertia

I-investor demand is given in the following result, which also characterizes the *ambiguity premium* required by these investors to hold a non-zero position in the stock.

Proposition 1. *Suppose the stock price is p. The optimal portfolio of I-investors who observes signal s is given by*

$$\theta^I(s,p) = \begin{cases} \gamma^{-1}\rho^I|s\left(\underline{\mu}^I|s - p\right) & s > (\mu_0+\delta) - \frac{\rho_0+\rho_\epsilon}{\rho_\epsilon}(\mu_0 - p) \\ 0 & (\mu_0-\delta) - \frac{\rho_0+\rho_\epsilon}{\rho_\epsilon}(\mu_0 - p) \le s \le (\mu_0+\delta) - \frac{\rho_0+\rho_\epsilon}{\rho_\epsilon}(\mu_0 - p) \\ \gamma^{-1}\rho^I|s\left(\overline{\mu}^I|s - p\right) & s < (\mu_0-\delta) - \frac{\rho_0+\rho_\epsilon}{\rho_\epsilon}(\mu_0 - p) \end{cases} \tag{8}$$

I-investors require an ambiguity premium of $\delta\rho_\epsilon$ to be long or short in the stock.

I-investors require an ambiguity premium whenever they do not trade away their stock holding to a zero position. This premium is in addition to the usual risk premium required by risk-averse investors. I-investors require a reduction (respectively, an increase) of $\delta\rho_\epsilon$ in the stock price when they are long (respectively, short) in the stock given their effective belief $\underline{\mu}^I|s$ (respectively, $\overline{\mu}^I|s$). Whenever the price does not incorporate this ambiguity premium, they trade away their stock holding to a zero position.

In the above expression, note that the case of $(\mu_0 - \delta) - \frac{\rho_0+\rho_\epsilon}{\rho_\epsilon}(\mu_0 - p) \leq s \leq (\mu_0 + \delta) - \frac{\rho_0+\rho_\epsilon}{\rho_\epsilon}(\mu_0 - p)$ corresponds to a situation where I-investors trade from their non-zero initial stock position to a zero position in the stock. Thus, this demand does not represent a no-trade position since aggregate trade is then $x_0^I > 0$.

I-investors' demand also exhibits two interesting and complementary phenomena. The first is that for any given signal value s, there exists a range of prices for which it is optimal for I-investors to trade away their stock holdings to a zero position ($\theta^I = 0$). This corresponds to portfolio inertia in prices at the risk-free portfolio first noted by Dow and da Costa Werlang (1992b).[16]

The second fact is that for a given price p, I-investors will find it optimal to trade to a zero position under distinct signals s, s' when

$$s, s' \in \left[(\mu_0 - \delta) - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon}(\mu_0 - p), (\mu_0 + \delta) - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon}(\mu_0 - p)\right]. \tag{9}$$

That is, at $\theta^I(s) = \theta^I(s') = 0$, there is *portfolio inertia with respect to information* (Condie and Ganguli (2011a)).[17] The range of signals for which I-investors exhibit portfolio inertia in information at a given price p is characterized below as a corollary of Lemma 1.

Corollary 1. *I-investors trade away their stockholding at a given price p for an interval of signal values with length 2δ and upper bound b, where*

$$b = (\mu_0 + \delta) - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon}(\mu_0 - p). \tag{10}$$

The mid-point of this interval is given by

$$\mu_0 - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon}(\mu_0 - p) \tag{11}$$

We show below that this portfolio inertia in information leads to the existence of partially

[16] Given signal s, portfolio inertia in price would arise for all prices p, p' that satisfy $p, p' \in [\underline{\mu}^I|s, \overline{\mu}^I|s]$.

[17] As evident from the above discussion and as noted in Condie and Ganguli (2011a) the property of inertia in information and the property of inertia in price are distinct but related since both obtain from the non-differentiability of the MEU criterion.

revealing REE.[18] Whether or not the price incorporates the ambiguity premium of $\delta\rho_\epsilon$ plays an important role in informational inefficiency since it determines whether the inertia position is optimal. Finally, note also that this inertia does not appear in smooth models of preferences and so these models will not display the partial revelation property we study here.

3 Equilibrium partial revelation

3.1 The necessity of inertia for partial revelation

Non-revelation of signals s and s' requires that $p(s) = p(s')$. If I-investors find it optimal to not trade away their stock holdings then the equilibrium price will be monotone in the signal and hence revealing, as the next result shows.

Proposition 2. *If markets clear at signal value s with $\theta^I(s) \neq 0$, then the market clearing stock price reveals signal s in rational expectations equilibrium.*

Thus, the existence of partial revelation requires that for a given price there is a range of signals for which I-investors wish to trade to a zero position in the stock. Moreover, we note that for $\theta^I(s) \neq 0$, the market clearing price at s must include an *ambiguity premium* over and above the usual *market risk premium*.

Corollary 2. *If markets clear with $\theta^I(s) \neq 0$, the market clearing price $p(s)$ includes the usual market risk premium $\frac{\gamma}{\rho_0+\rho\epsilon}$ and an additional ambiguity premium $x_0^I \frac{\delta\rho_\epsilon}{\rho_0+\rho_\epsilon}$.*

3.2 Uninformed investor demand

The above requirements of optimality and market-clearing with $\theta^I = 0$ for partial revelation are related to and complicated by the fact that U-investors infer information from the prevailing price. This inference potentially leads to changes in the beliefs of U-investors which leads to changes in market prices. Thus, equilibrium prices and the beliefs of U-investors must be solved for simultaneously.

The solution to this problem is a set of signals that are not revealed in REE and beliefs for U-investors that are consistent with the knowledge that a signal in the set of unrevealed signals has been received. Given Corollary 1 and Proposition 2, we demonstrate the existence

[18] Condie and Ganguli (2011a) first noted this property and used it in the context of general financial market economies to establish robust existence of partially revealing REE when payoff states and signals can take only finitely many values, unlike the case here with normally distributed payoff and signal structures.

of partially revealing REE by conjecturing and verifying the existence of an interval $[b-2\delta, b]$ of signals that will not be revealed, while signals outside the interval $[b-2\delta, b]$ are revealed.

The first step is to characterize U-investor demand when signals in $[b-2\delta, b]$ are not revealed and those outside of $[b-2\delta, b]$ are revealed for a given price p.

Proposition 3. *If $s \in [b-2\delta, b]$ are not revealed and $s \notin [b-2\delta, b]$ are revealed at stock price p, the optimal portfolio for U-investors under updated beliefs about v at stock price p is given by*

$$\theta^U(s) = \begin{cases} \gamma^{-1}(\rho_0+\rho_\epsilon)(\mu_0 + \frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}(s-\mu_0) - p) & \text{if } s \notin [b-2\delta, b] \\ \gamma^{-1}\rho_0\left(\mu_0 + \frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}\Delta\left(b-2\delta+\frac{\gamma\theta^U(s)}{\rho_0}, b+\frac{\gamma\theta^U(s)}{\rho_0}\right) - p\right) & \text{if } s \in [b-2\delta, b], \end{cases} \tag{12}$$

where

$$\Delta\left(b-2\delta+\frac{\gamma\theta^U(s)}{\rho_0}, b+\frac{\gamma\theta^U(s)}{\rho_0}\right) = \sqrt{\frac{\rho_0+\rho_\epsilon}{\rho_0\rho_\epsilon}}\left[\frac{\phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}\left(b-2\delta-\mu_0+\frac{\gamma\theta^U(s)}{\rho_0}\right)\right)-\phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}\left(b-\mu_0+\frac{\gamma\theta^U(s)}{\rho_0}\right)\right)}{\Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}\left(b-\mu_0+\frac{\gamma\theta^U(s)}{\rho_0}\right)\right)-\Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}\left(b-2\delta-\mu_0+\frac{\gamma\theta^U(s)}{\rho_0}\right)\right)}\right] \tag{13}$$

Note that for $s \in [b-2\delta, b]$, U-investor demand $\theta^U(s)$ is constant in s and defined implicitly, while for $s \notin [b-2\delta, b]$, $\theta^U(s)$ is monotone in s and a closed form expression is available.

3.3 Partially revealing REE price function

Using Proposition 3 and the market clearing condition (5), the next result (Proposition 4) characterizes the unique partially revealing REE price function, trade volume, and the mass of unrevealed signals. Conditions for existence of the partially revealing equilibrium are given in Proposition 5 below.

Proposition 4. *Suppose I investors observe private signal s.*

1. *The market clearing price $p_{PR}(\cdot)$ when $\theta^I(\cdot) = 0$ satisfies*

$$p_{PR}(s) = \mu_0 + \frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}(\psi(s)-\mu_0) - \frac{\gamma}{x_0^U}\frac{1}{\rho_0+\rho_\epsilon} \tag{14}$$

where

$$\psi(s) = \begin{cases} s & \text{if } s \notin [b - 2\delta, b] \\ \mu_0 + \Delta\left(b + \frac{\gamma}{x_0^U \rho_0} - 2\delta, b + \frac{\gamma}{x_0^U \rho_0}\right) - \frac{\gamma}{x_0^U \rho_0} & \text{if } s \in [b - 2\delta, b] \end{cases} \tag{15}$$

This function is non-linear in s and discontinuous at $b - 2\delta$ and b.

2. *The length of the interval of signals $[b - 2\delta, b]$ where $p_{PR}(\cdot)$ does not reveal the signal is 2δ.*

3. *Trade volume is x_0^I for all s.*

Figure 1(a) illustrates the price function. It follows from the characterization of the partially revealing price function in (14)-(15) that price volatility is higher when information is revealed than when it is not, since changes in price are the mechanism through which information is transmitted.[19]

Using the relation between b and price in (10) (Corollary 1) and the expression for the price when $s \in [b-2\delta, b]$ in (14) yields that the existence of an interval $[b-2\delta, b]$ of unrevealed signals and hence the existence of partially revealing REE follows from the existence of a solution b to the following equation.

$$b + \frac{\gamma}{x_0^U \rho_0} - \mu_0 - \Delta\left(b + \frac{\gamma}{x_0^U \rho_0} - 2\delta, b + \frac{\gamma}{x_0^U \rho_0}\right) = \delta - \frac{\gamma}{x_0^U \rho_\epsilon} \tag{16}$$

The next result establishes that a solution to (16) always exists and is unique when the additional ambiguity premium required by I-investors (Corollary 2) above the usual market risk premium is larger than the premium that would be required by U-investors to hold all of the stock (as discussed in the following section). There is no closed form analytical solution for b generally, though one can be found numerically.

Proposition 5. *Markets clear with $\theta^I(s) = 0$ for all s and an interval $[b-2\delta, b]$ of unrevealed signals, with b unique, exists if and only if $\gamma \in [0, x_0^U \delta \rho_\epsilon]$.*

The partially revealing equilibrium price reveals s when $s \notin [b - 2\delta, b]$ and obscures s if $s \in [b - 2\delta, b]$ as conjectured and trading volume is x_0^I for all s. As discussed in Section 3.1, $\theta^I(s) = 0$ at market-clearing if price does not incorporate the ambiguity premium. On the

[19] Differing volatility across regimes is suggestive of time-varying volatility in dynamic models, see Andersen, Bollerslev, and Diebold (2009) for an introduction to the extensive literature on time-varying volatility.

other hand, for U-investors to to hold all the stock, the price must incorporate a premium related to x_0^U, as discussed below. The partially revealing REE exists when this premium is lower than the ambiguity premium I-investors require to hold the stock. This is true for all s when $\frac{\gamma}{x_0^U} \leq \delta\rho_\epsilon$.

Condie and Ganguli (2011a) showed that partially revealing REE exist when ambiguity averse investors observe private information. However, the characterization and existence of partial revelation in the present setting are not corollaries of the Condie and Ganguli (2011a) results since those results are established for a setting with finitely many states and signal values unlike the present setting. Moreover, Condie and Ganguli (2011a) do not explicitly model the link between information and updated beliefs, unlike the present setting and so do not deal with the inference problem for U investors and the consequent existence problem as discussed in Proposition 3 and Proposition 5.[20]

4 Pricing implications of partial revelation

In this section, we establish the effects of partial revelation of ambiguous private information. In order to demarcate the effects due to partial revelation, we provide comparisons with two benchmark economies, which we describe first. Both of these benchmarks are informationally efficient, i.e. the REE price function is fully revealing in each.

4.1 Benchmark economies

The first benchmark economy, which we term the *full information economy*, is the economy where I investors and U investors are as in the current economy, but there is no asymmetry of information. This benchmark full information economy is what would obtain in a fully revealing REE, where price reveals the signal to U investors and I investors trade away their asset holdings to U investors. We establish that this fully revealing equilibrium exists when $\gamma \in [0, \delta x_0^U \rho_\epsilon]$ in the proof of Proposition 11 below.

[20] The existence results are established in different parameter spaces and so need to be stated and established independently. The parameter space in Condie and Ganguli (2011a) is the space of belief representations taking the endowment distributions and risk preferences as primitive while the result in Proposition 5 takes the belief representation (normal distributions) as primitive and establishes existence in the space of risk preference (γ) and endowment distributions (x_0^U) given ambiguity δ and signal error precision ρ_ϵ. The conceptual link between the two existence results is that finding the equilibrium beliefs of the traders is key to finding the equilibrium price function and vice versa.

In the fully revealing case, the price function, denoted $p_{FR}(\cdot)$, is given by

$$p_{FR}(s) = \mu_0 + \frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon}(s - \mu_0) - \frac{\gamma}{x_0^U}\frac{1}{\rho_0 + \rho_\epsilon}. \tag{17}$$

The full information benchmark and the partially revealing equilibrium therefore only differ in the fact that in the latter, information is not revealed to U investors.

The second benchmark economy, which we term the *no ambiguity economy*, is characterized by I investors who do not perceive any ambiguity in the signal ($\delta = 0$) and like U investors consider the signal to be unbiased. That is, this economy is populated only by (Savage 1954) SEU investors. In this economy, the REE price is always fully revealing, so there is no asymmetry of information as in the first benchmark. However, in this economy, the I investors do not trade away their stock holding in equilibrium. Full revelation in this no-ambiguity economy essentially follows from the results of Grossman (1976) and Radner (1979), but we state and prove the result for completeness.

Proposition 6. *Suppose $\delta = 0$. Then there exists a fully revealing equilibrium, which is the unique rational expectations equilibrium.*

The equilibrium price function in the no-ambiguity economy is denoted $p_{NA}(\cdot)$ and given by

$$p_{NA}(s) = \mu_0 + \frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon}(s - \mu_0) - \frac{\gamma}{\rho_0 + \rho_\epsilon}. \tag{18}$$

The only difference in prices between the full information and the no ambiguity benchmarks is the term $\frac{1}{x_0^U}$ found in the risk premium. This term captures the premium due to the reduction in stockholders from I investors and U investors in the no ambiguity economy to only U investors in the full information economy. This reduction is due to the ambiguity aversion of the I investors and has been noted previously in the literature, for example in Easley and O'Hara (2009) and Cao, Wang, and Zhang (2005).

In the full information economy, U investors require a premium of $\gamma(x_0^U(\rho_0 + \rho_\epsilon))^{-1}$ to hold all the stock and we refer to this as the *reduced stockholders market risk premium.* It comprises two portions. One is determined by x_0^U and measures the premium due to the reduction in the mass of stockholders as noted above. If x_0^U is small, U-investors are required to purchase a large fraction of the total asset stock from I-investors when the latter wish to trade to a zero position. This purchase involves an increasingly risky portfolio and U-investors require an increasing amount of compensation to take on this additional risk. However, if x_0^U is large, then U-investors own most of the market and taking on the remainder of the assets does not greatly increase the compensation they require. The other portion of the premium is determined by $\gamma(\rho_0 + \rho_\epsilon)^{-1}$ which measures the risk premium due to the risk faced by U-investors with risk aversion γ and conditional stock payoff volatility $(\rho_0 + \rho_\epsilon)^{-1}$.

In short, comparing the full information and no ambiguity benchmarks provides the effect of ambiguity aversion in the absence of information asymmetry. However, when there is information asymmetry, as in the partially revealing equilibrium, there are additional effects which we describe next.

4.2 Premia due to unrevealed information

The partially-revealing price function p_{PR} in (14) differs from the prices in the two benchmarks. When information is not revealed, i.e. U-investors know only that $s \in [b - 2\delta, b]$, the price p_{PR} includes a premium

$$-\frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon}(\psi(s) - \mu_0) = -\frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon}\left(\Delta\left(b - 2\delta + \frac{\gamma}{x_0^U \rho_0}, b + \frac{\gamma}{x_0^U \rho_0}\right) - \frac{\gamma}{x_0^U \rho_0}\right). \tag{19}$$

We refer to this as the *unrevealed information premium* (UIP). This premium is novel to this paper and is the reduction in price required by U-investors when they know there is information in the market that they haven't observed (see Proposition 7 below). For $s \in [b - 2\delta, b]$, the premium is measured by

$$-\left(\Delta\left(b - 2\delta + \frac{\gamma}{x_0^U \rho_0}, b + \frac{\gamma}{x_0^U \rho_0}\right) - \frac{\gamma}{x_0^U \rho_0}\right). \tag{20}$$

The next result characterizes the UIP and its relation to the market risk premium and describes how U-investors perceive unrevealed information.

Proposition 7. *The following hold in the partially revealing equilibrium.*

1. *The unrevealed information premium is positive and increasing in γ whenever γ is positive.*

2. *If information is not revealed then the unrevealed information premium exceeds that measured by the expected value of unrevealed information,*

$$-\left(\Delta\left(b - 2\delta + \frac{\gamma}{x_0^U \rho_0}, b + \frac{\gamma}{x_0^U \rho_0}\right) - \frac{\gamma}{x_0^U \rho_0}\right) \geq -(\mathbb{E}[s|s \in [b - 2\delta, b]] - \mu_0) \tag{21}$$

and U-investors consider unrevealed information to be, on average, bad news, i.e. $\mathbb{E}[s|s \in [b-2\delta, b]] \leq \mu_0$.

Unsurprisingly, as U investors' risk aversion increases the premium that they require for unrevealed information increases. Comparing the unrevealed information premium to a benchmark given by the average value of unrevealed information clarifies the nature of unrevealed news. When information is not revealed, the risk averse U investors require a premium which exceeds that given by the average value of unrevealed information. So, the price that I investors receive is lowered by more than the average value of unrevealed information. For I investors to sell their stockholding at a price with this discount, given the price function, U investors infer that the news that they observe but is not revealed by price must be bad news on average. That is, the asymmetry that unrevealed information is considered bad news on average arises from interaction of the risk aversion of the U investors and the feature that I investors accept a constant price to sell their stockholding for a range of information, i.e. exhibit portfolio inertia in information.[21]

Figure 1(b) illustrates the features of equilibrium given in Proposition 7. The set $[b-2\delta, b]$ of unrevealed signal values moves to the left and the price in the partial-revelation region declines as the risk aversion increases due to the increase in the UIP. When the risk aversion is zero or moderate, both moderately good news ($s > \mu_0$) and bad news ($s < \mu_0$) are not revealed. Similar results apply if we consider instead the premium $\frac{\gamma}{x_0^U \rho_0}$ as the varying parameter.

Proposition 5 and Proposition 7 highlight the role that the relative market share of I-investors plays in the revelation of information. From these two results it follows that if the market share of those who have received the private signal is large enough, non-revelation of information will not occur. That is, if enough investors in the market know the information, it will be revealed. As the market share of those who are privately informed increases, so does the market risk premium. U-investors are required to hold an increasingly large portion of the market and must be compensated to do so. As this market risk premium increases, the willingness of I-investors to hold the stock increases which in turn leads to their information being revealed.

[21] The unrevealed information premium is the equilibrium risk premium required by the CARA utility U investors when facing a non-normal distribution for v. The difference between the conditional average of unrevealed signals and the prior mean is a natural benchmark to compare this premium with and it is not a priori obvious whether the premium, which potentially depends on third or higher order moments of the non-normal distribution, would be lower or higher. Our result shows that the premium is higher than the benchmark.

Finally, we note that Proposition 4.2 implies that the informativeness of prices depends fundamentally on the amount of ambiguity in the signal. As ambiguity, measured by δ, increases so does the set of signals that don't get revealed in equilibrium. Market environments that are characterized by large amounts of ambiguous information will also tend to have prices that are less informative.

4.3 Jumps and amplification

The partially revealing price function is suggestive of *crashes, jumps, and amplification* of small changes in news (s). Since s is the private information of I investors, these effects are illustrated in terms of prices, which are publicly observable.[22] To illustrate this we compare the partially revealing price with the full information benchmark price. Comparing with the no-ambiguity benchmark price would give similar results.

In the benchmark full information economy, price p_{FR} is linear and strictly increasing in s with no points of discontinuity. The discontinuity in p_{PR} is suggestive of jumps and crashes relative to the full information benchmark and provides an amplification mechanism, wherein small differences in news s lead to relatively large changes in price p_{PR}. This amplification can be quantified in terms of the difference in volatility of partially revealing and fully revealing price in neighborhoods of the points of discontinuity.

Near the upper discontinuity point b, if the news is just slightly better, then it will be revealed through a discontinuous increase in price. This is true for all unrevealed signals that are better news than the average of the unrevealed signals. On the other hand, this price jump will be negative if the revealed signal is worse than the average unrevealed signal, and hence is below $b - 2\delta$. Similarly, a change from revelation to non revelation of signals is accompanied by a discontinuous change in price as U-investors' information changes from an exact signal value to a set of possible signal values. Figure 2 depicts this amplification in terms of the relative volatility of prices for signals close to b.

The comparative statics for price volatility and discontinuous price changes from these results are suggestive of interesting properties that may arise in an intertemporal setting, where information is recieved every period but informational efficiency of price change across periods. For example, a change from periods of information non-revelation to those with revelation may coincide with a intial large change in price relative to the change in news, an increase in price volatility, and expectation of continuing higher volatility. On the other

[22] The amplification would arise if we compared p_{PR} and s directly as well, but we do not do so here since s is in principle not publicly observable.

hand, with a change in the other direction, a relatively large initial price change, lower volatility, and expectation of lower future volatility may arise.[23]

These mechanisms and results are different from those in other papers. For example, Illeditsch (2011) shows that in a market with a representative ambiguity and risk averse investor holding the stock, there is an interval of market-clearing prices when a public signal confirms the prior mean. The multiplicity of prices arises due to the ambiguity averse investor's worst-case assessment (as measured by the variance of the signal) changing as the signal changes. This multiplicity of prices can act as an amplification mechanism for small changes in news due to the fundamental indeterminacy of prices.

This can be quantified by the higher volatility of price relative to the volatility of news. In contrast, our amplification result does not rely on ambiguity averse investors holding the stock, indeed the marginal investors are ambiguity-neutral. The worst-case assessment of the ambiguity averse I investors does not change around the points of discontinuity.[24] The result is due to the change in the inference of the U investors from a point to a set or vice versa at the points of discontinuity. Moreover, our result does not rely on equilibrium indeterminacy, since the partially revealing price function is discontinuous but unique and so the limit price of a sequence of news signal converging to b (or $b - 2\delta$) is distinct from the limit of the corresponding price sequence. Moreover, the result in this paper holds with risk neutrality as well.

Mele and Sangiorgi (2015) find multiplicity of equilibria in the information market when ambiguity averse investors acquire costly information and there is noise-based partial revelation in the financial market. Multiplicity of information market equilibria leads to price swings in the financial market due to switching between information market equilibria with changes in the level of uncertainty. In contrast, our result on price swings does not rely on multiple information market equilibria and arises due to discontinuity in the unique partially revealing equilibrium.

4.4 Implications for return moments.

The partially revealing equilibrium also has implications for the distribution of returns that are consistent with stylized facts of returns for the aggregate market index and in the cross section in the US. An extensive literature exists regarding the moments of excess returns

[23] We emphasize that this is just an illustration of possible richer implications in an intertemporal setting. A formal development of such a setting is left for future research.

[24] The worst case assessment is always given by $-\delta$ around b.

for the aggregate market and cross section.[25] This literature documents that the equity premium and return volatility are higher than is implied by many existing estimates of investor risk aversion and models of asset prices. Returns for the aggregate market and in the cross-section are negatively skewed and leptokurtic. Campbell, Lo, and Mackinlay (1995) documents evidence for the aggregate stock market on all four moments using daily and monthly returns data.[26] For the cross section, Conrad, Dittmar, and Ghysels (2013) document evidence on skewness and kurtosis using daily and monthly returns data.[27]

Since the model has just a single risky asset it is natural to interpret the results in terms of the aggregate market returns. However, it also is natural to interpret the private information setting in the context of the cross section rather than the aggregate market. Nothing in our analysis precludes interpreting the results in relation to the cross section. We do not provide a multi-asset version of the model, but such an analysis would be feasible and a careful study of the covariance structure of assets in the cross section together with the partial revelation of ambiguous private information for some or all of the cross-section would likely yield richer implications than those we document here.[28]

The static CARA-Normal framework used in this paper is not directly comparable to the empirical evidence in the literature, so we compare with the two benchmark models within the CARA-Normal framework to illustrate the marginal impact of non-revelation of ambiguous private information to an otherwise standard CARA-Normal model.[29]

The results in this section indicate that partial revelation due to the presence of ambiguous private information leads to a higher equity premium and higher return volatility than the benchmark economies. More interestingly, under partial revelation, returns are negatively skewed and leptokurtic for a range of parameter values. In contrast, in the benchmark

[25] Given our normalization of the risk-free rate, excess return coincide with stock return in our setting.

[26] See, for example, Table 1.1 of Campbell, Lo, and Mackinlay (1995). More recently, Albuquerque (2012) documents negative skewness for the the aggregate market using daily returns data.

[27] Harvey and Siddique (2000) document evidence on skewness in the cross section using monthly data while Dittmar (2002) documents evidence on kurtosis for the cross section of returns for annual data. Harvey and Siddique (2000) are primarily concerned with conditional skewness in the intertemporal asset pricing context, but document unconditional skewness as well.

[28] A natural question is whether asset specific ambiguous private information for individual assets could be diversified away if there are many assets and the equilibrium price function approaches a linear price function. However, the work of Epstein and Schneider (2008) (section B.3) and Epstein and Schneider (2007) (Theorem 1 and related discussion) suggests that the effects ambiguous information will not be diversified away. Moreover, a private information setting such as in this paper is likely to provide much richer implications. We leave this for future research.

[29] The results from the static framework in this paper are only suggestive. A more comprehensive quantitative exercise is beyond the scope of this paper.

economies, returns are not skewed and do not exhibit excess kurtosis. We illustrate the effects of partial revelation in comparison to the informationally efficient benchmarks for all four moments below.

Our results below also show that ambiguity itself, without asymmetry of information, only affects the first moment of the return distribution. That is, the full information and no-ambiguity benchmarks only differ in terms of the equity premium. Ambiguity itself, as captured in the full information economy benchmark has no effect on the volatility, skewness, and kurtosis of returns. It is the interplay of private information and ambiguity as captured in the partial information economy which has an effect on the higher order moments of returns.

4.4.1 The equity premium

The equity premium in the partially revealing equilibrium exceeds the premium in the full information and no-ambiguity benchmarks.

Proposition 8. *For all* $\gamma \in (0, \delta x_0^U \rho_\epsilon)$, $\mathbb{E}[v - p_{PR}] > \mathbb{E}[v - p_{FR}] > \mathbb{E}[v - p_{NA}]$.

Figure 3 demonstrates the impact on the equity premium from partial revelation relative to the two benchmarks discussed previously for different levels of γ and δ. First, note that the equity premium is always higher in the full information economy (dashed curves) than in the no-ambiguity economy (dotted curves) for all γ and δ. This difference is due to the reduced stockholder market risk premium. Ambiguity averse investors trade away their stockholding when the price does not include an ambiguity premium (Proposition 1), while ambiguity-neutral investors hold the stock when compensated through the market risk premium.

However, as can be seen in both graphs, there is an additional premium in the partial information economy (solid curves) relative to the full information economy. This is the UIP which compensates U-investors for holding the stock in the partially revealing equilibrium and it increases in δ, since the amount of information revealed decreases (Proposition 4.2). On the other hand, the UIP is non-monotonic in γ. There are two opposing effects on the UIP as γ changes and these underlie the non-monotonicity.

Since b is decreasing in γ (Proposition 7 and Figure 1(b)) the set of unrevealed signals shifts to the left as γ increases. This shift implies that conditional on non-revelation, the unrevealed information is worse ($\mu_0 - (b - 2\delta)$ is higher). On the other hand, as b decreases, the probability that I investors receive a signal in the set of unrevealed signals decreases.

As γ increases from 0, initially the U investors risk aversion is low enough that the UIP is positive but not very high relative to the full information and no-ambiguity benchmarks. However, for higher values of γ the worsening nature of unrevealed information implies that the first effect dominates and the UIP increases. Eventually, with high enough γ, the second effect dominates and the UIP decreases with increasing γ.

4.4.2 Return volatility

Figure 4 provides the unconditional variance of asset returns $(v - p)$ as a function of γ and δ. The volatility of returns is higher in the partial information economy relative to both benchmark economies and is increasing in δ and decreasing in γ. Return volatility increases with δ because the size of the unrevealed information region increases. Since asset return variance is composed of both the conditional variance of asset returns given the signal, and the marginal variance of the signal, these two sources of variance lead to greater asset return variance when ambiguity is large.

Return volatility decreases in γ because b decreases, which implies a decreasing probability that the I investors observe information that is not revealed. This in turn implies a decrease in the return volatility.

Yu (2014) uses a related framework to study a dynamic economy with collateral constraints and asymmetric ambiguous information about the growth rate of dividends. Yu (2014) assumes that ambiguity averse investors can observe a private signal with two values: one which indicates high dividend growth and the other low growth, unlike the normally distributed signals observed here. He shows that when price does not distinguish whether a high growth signal or low growth signal was received, the conditional volatility of future returns and conditional risk premia are higher than if the price reveals these, unlike the unconditional moments we study.[30]

4.4.3 Return skewness

The non-linearity in $p_{PR}(\cdot)$ leads to negatively skewed returns when traders are risk averse $(\gamma > 0)$, as shown in Figure 5. There is no skewness in the returns of either of the two

[30]Price volatility can exceed fundamental volatility as measured by the stock payoff in the presence of ambiguity aversion. Ozsoylev and Werner (2011) show that in the presence of ambiguity averse uninformed investors if the volatility of noise trading is high enough, then price volatility can exceed stock payoff volatility. Dow and da Costa Werlang (1992a) provides an example of excess volatility due to a violation of the standard Bayesian variance decomposition formula under ambiguity, which is not required here. Mandler (2012) shows excess volatility relative to a stochastic technology parameter in a sequential production economy.

benchmark economies. In the partially revealing REE, on the other hand, negative skewness in returns arises. In this REE, there is positive probability mass at the non-revealing price value, corresponding to the probability mass of the range of unrevealed bad news. Negative skewness arises since this price value is lower than the price value that would be implied by the average value of unrevealed information, as noted in (21).

Returns in the partial information economy are more negatively skewed as δ increases, as shown in the right panel of Figure 5. A higher δ implies a larger range of unrevealed information, i.e. a larger range where price is constant, which leads to greater skewness in returns.

The effect of increasing γ on skewness is non-monotonic as a result of the interplay of the two effects noted in the discussion of volatility – worsening news and decreasing probability. When $\gamma = 0$, since the non-revelation region is symmetric around the mean μ_0, non-revelation does not introduce skewness into the distribution of $v-p$. As γ increases, b decreases, moving the region to the left. Initially, the effect from worsening news dominates and the skewness becomes more negative. However, as γ becomes large enough the lower probability effect dominates and returns become less negatively skewed.

Epstein and Schneider (2008) show that with a representative ambiguity averse investor holding stocks, negative skewness can arise in returns.[31] In contrast, in our model skewness arises even when the marginal stockholder is ambiguity neutral. Yuan (2005) shows that when borrowing constraints are present in a noise-based CARA-normal partial revelation setting, negative skewness arises due to non-linearity of the partially revealing price function. In recent work, Palvolgyi and Venter (2015) show that negative skewness can arise conditional on low prices in a noise-based CARA-Normal partial revelation setting, when considering partially revealing equilibria where the price function is discontinuous. Our result documents unconditional negative skewness in returns in line with the evidence in the literature noted previously for the aggregate stock returns and for returns in the cross section.

4.4.4 Return kurtosis

As illustrated in Figure 6, returns may be leptokurtic in the partial information economy relative to the two benchmarks, which exhibit no kurtosis. The mechanism which leads to excess kurtosis is essentially similar to that for negative skewness, i.e. positive probability of the non-revealing price value and this value being lower than the average of the unre-

[31] More precisely, in the inter-temporal setting of Epstein and Schneider (2008), negative skewness arises when the investor's inter-temporal discount rate is low, such as for high frequency data.

vealed information. The effect of increasing γ on excess kurtosis is non-monotone because of the effects of worsening unrevealed information and decreasing probability of unrevealed information as in the case of the other moments.

However, unlike the other moments, excess kurtosis is not monotone in δ. As δ increases b increases and $b - 2\delta$ decreases. This implies an increase in the probability of the interval at the expense of probability of realizations to the left and the right of the range. When the interval is relatively small, the reduction in probability of realizations above b imply relatively large increases in the probability of the tails of the distribution of return. This implies that the kurtosis of returns exceeds that of the normal distribution. However, as the interval gets wider, especially as b increases, probability mass from relatively extreme positive realizations is transferred toward the center. Eventually, this consolidation effect dominates and kurtosis falls below that of the normal distribution.

5 Public information

Public information affects both I- and U-investors by reducing the disparity in their beliefs, which affects prices and informational efficiency. To model this, suppose all investors observe *a public signal*

$$\zeta = v + \epsilon_\zeta, \tag{22}$$

where ϵ_ζ is normally distributed with mean 0 and precision ρ_ζ. Since public information is observed by all investors we assume for simplicity that it is unambiguous.

Let $\hat{\rho}_0 = \rho_0 + \rho_\zeta$ and $\hat{\mu}_0(\zeta) = (\rho_0 \mu_0 + \rho_\zeta \zeta)/\hat{\rho}_0$. I-investors' updated beliefs about v with both the public and private information are characterized as follows.

Lemma 2. *I-investors' updated beliefs about v with the public signal ζ and private signal s are represented by the set of distributions with precision $\hat{\rho} + \rho_\epsilon$ and means*

$$\left[\underline{\mu}^I|(s,\zeta), \overline{\mu}^I|(s,\zeta)\right] = \left[\frac{\hat{\rho}_0\hat{\mu}_0(\zeta) + \rho_\epsilon (s - \delta)}{\hat{\rho}_0 + \rho_\epsilon}, \frac{\hat{\rho}_0\hat{\mu}_0(\zeta) + \rho_\epsilon (s + \delta)}{\hat{\rho}_0 + \rho_\epsilon}\right]. \tag{23}$$

Reasoning similar to that for Proposition 4 yields that with public information, a range of unrevealed private information will exist when the market risk premium doesn't exceed the ambiguity premium. The range will have length 2δ as before but the bounds will be distinct depending on the public signal. Denote the range of unrevealed private information by $[b_\zeta - 2\delta, b_\zeta]$.

Proposition 9. *Suppose public signal ζ is observed by the investors.*

1. *An interval of unrevealed private signals $[b_\zeta - 2\delta, b_\zeta]$ exists if and only if $\gamma \in [0, x_0^U \delta \rho_\epsilon]$, where b_ζ is the unique solution to*

$$b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} - \hat{\mu}_0(\zeta) - \Delta_\zeta \left(b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} - 2\delta, b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} \right) = \delta - \frac{\gamma}{x_0^U \rho_\epsilon} \tag{24}$$

2. *The partially revealing price function with public information $p_{PR}(\cdot, \zeta)$ is given by*

$$p_{PR}(s, \zeta) = \hat{\mu}_0(\zeta) + \frac{\rho_\epsilon}{\hat{\rho}_0 + \rho_\epsilon}(\psi(s, \zeta) - \hat{\mu}_0(\zeta)) - \frac{\gamma}{x_0^U} \frac{1}{\hat{\rho}_0 + \rho_\epsilon} \tag{25}$$

where

$$\psi(s, \zeta) = \begin{cases} s & \text{if } s \notin [b_\zeta - 2\delta, b_\zeta] \\ \hat{\mu}_0(\zeta) + \Delta_\zeta \left(b_\zeta - 2\delta + \frac{\gamma}{x_0^U \hat{\rho}_0}, b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} \right) - \frac{\gamma}{x_0^U \hat{\rho}_0} & \text{if } s \in [b_\zeta - 2\delta, b_\zeta] \end{cases} \tag{26}$$

and

$$\begin{aligned} &\Delta_\zeta \left(b_\zeta - 2\delta + \frac{\gamma}{x_0^U \hat{\rho}_0}, b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} \right) \\ = &\sqrt{\frac{\hat{\rho}_0 \rho_\epsilon}{\hat{\rho}_0 + \rho_\epsilon}} \left[\frac{\phi\left(\sqrt{\frac{\hat{\rho}_0 \rho_\epsilon}{\hat{\rho}_0 + \rho_\epsilon}} \left(b_\zeta - 2\delta - \hat{\mu}_0(\zeta) + \frac{\gamma}{x_0^U \hat{\rho}_0} \right) \right) - \phi\left(\sqrt{\frac{\hat{\rho}_0 \rho_\epsilon}{\hat{\rho}_0 + \rho_\epsilon}} \left(b_\zeta - \hat{\mu}_0(\zeta) + \frac{\gamma}{x_0^U \hat{\rho}_0} \right) \right)}{\Phi\left(\sqrt{\frac{\hat{\rho}_0 \rho_\epsilon}{\hat{\rho}_0 + \rho_\epsilon}} \left(b_\zeta - \hat{\mu}_0(\zeta) + \frac{\gamma}{x_0^U \hat{\rho}_0} \right) \right) - \Phi\left(\sqrt{\frac{\hat{\rho}_0 \rho_\epsilon}{\hat{\rho}_0 + \rho_\epsilon}} \left(b_\zeta - 2\delta - \hat{\mu}_0(\zeta) + \frac{\gamma}{x_0^U \hat{\rho}_0} \right) \right)} \right] \end{aligned} \tag{27}$$

The price function is non-linear in s and discontinuous at $b_\zeta - 2\delta$ and b_ζ.

Figure 7 illustrates the effect of public information which confirms the prior mean, $\zeta = \mu_0$. This effect is due to the increased precision $(\rho_0 + \rho_\zeta)$ under public information. The dotted lines depict the price function with no public signal, while the solid lines depict the price function for public signal $\zeta = \mu_0$. With the public signal, the set of unrevealed private signals comprises better private signal values relative to without the public signal, i.e. $b_\zeta > b$. The stock price with public information is higher for signal s when $b_\zeta - 2\delta \leq s \leq b$, but lower when $b - 2\delta \leq s < b_\zeta - 2\delta$ or $b < s \leq b_\zeta$.

5.1 Effects of public information: Unrevealed information and price non-monotonicity

Public information affects what private information is revealed and has implications for stock price different from those in standard settings such as the full information benchmark. First, public information affects the range of unrevealed private information. In particular, good (resp. bad) public information, i.e. a public signal realization $\bar{\zeta} > \mu_0$ (resp. $\underline{\zeta} < \mu_0$) implies that better (resp. worse) private information is not revealed. That is, the interval of unrevealed information lies to the right for good public information and to the left for bad public information.

Second, the non-linear nature of the partially revealing price implies that the price for a given private signal could be non-monotonic in small changes in the public information. That is, the price could be lower (respectively, higher) when the public signal is slightly higher (respectively, slightly lower). This is in contrast to the full information benchmark, where price for a private signal changes monotonically with changes in public information.

We formalize these effects of public information in Proposition 10, by comparing two public signal realizations. One public signal realization confirms the prior mean μ_0 while the other public signal realization is higher than the prior mean, $\bar{\zeta} > \mu_0$. An analogous result holds for the case where the public signal realization $\underline{\zeta}$ is lower than μ_0. Let b_{μ_0} (respectively, $b_{\bar{\zeta}}$) denote the upper bound of the non-revealed interval when the public signal realization is μ_0 (respectively, $\bar{\zeta}$).

Reasoning similar to that in the proof of Proposition 11 in Section 6.1, yields that with public information, for any public signal realization ζ, the full information economy price $p_{FR}(s, \zeta)$ is given by

$$p_{FR}(s, \zeta) = \hat{\mu}_0 + \frac{\rho_\epsilon}{\hat{\rho}_0 + \rho_\epsilon}(s - \hat{\mu}_0(\zeta)) - \frac{\gamma}{x_0^U(\hat{\rho}_0 + \rho_\epsilon)}. \tag{28}$$

Proposition 10. *Let $\bar{\zeta}, \mu_0$ be two realizations of the public signal, with $\bar{\zeta} > \mu_0$. The following hold.*

1. *$b_{\bar{\zeta}} > b_{\mu_0}$.*

2. *Suppose*

$$\mu_0 < \bar{\zeta} < \mu_0 + \frac{\hat{\rho}_0}{\rho_\zeta} \frac{\rho_\epsilon}{\hat{\rho}_0 + \rho_\epsilon} \left(\delta - \frac{\gamma}{x_0^U \rho_\epsilon} \right). \tag{29}$$

Then for all $s \in [b_{\mu_0}, b_{\bar{\zeta}}] \cup [b_{\mu_0} - 2\delta, b_{\bar{\zeta}} - 2\delta]$,

$$p_{PR}(s, \bar{\zeta}) < p_{PR}(s, \mu_0) \tag{30}$$

and

$$p_{FR}(s, \bar{\zeta}) > p_{FR}(s, \mu_0) \tag{31}$$

The condition (29) bounds the difference between the realizations of the public signal. The non-monotonicity under partial revelation arises for all small changes in public information which satisfy (29). For changes that violate the condition, the non-revealed region price given ζ, $p_{PR}(\cdot, \zeta)$, for all private signals is higher than the revealed region price given μ_0, $p_{PR}(\cdot, \mu_0)$.

To summarize, a small change in the range of unrevealed information implies price changes that might otherwise be considered anomalous. Typically, one would expect a higher stock price if $\zeta > \mu_0$ (good news) relative to $\zeta = \mu_0$ (neutral news. However, if the private information s is bad news and is revealed when $\zeta > \mu_0$ but not revealed when $\zeta = \mu_0$, then the price may be lower. The results of Andersen, Bollerslev, Diebold, and Vega (2007) for the US stock market are suggestive that this non-monotonicity result may be relevant. For example, they find that in relatively high growth periods, good public news can lead to decreases in asset prices (see pp.261-72).[32]

6 Model discussion and extensions

6.1 Noise-based partial revelation

The predominant approach to partial revelation introduces an exogenous source of stochastic variation in price such as noise traders, endowment shocks or taste shocks. We refer to this approach as the noise-based approach for brevity (see also Dow and Gorton (2008)). This added variation implies changes in price that are not due solely to changes in information, meaning price is not invertible as a function of private information and is therefore partially

[32] We do not claim that our static model's result explains the wide-ranging evidence documented in Andersen, Bollerslev, Diebold, and Vega (2007), which examines multiple financial markets and high frequency data in these inter-temporal markets. However, their findings for the US stock market summarized in pp. 261-12 and on pp. 11-12 (in relation to Tables 5A and 5B) of the corresponding working paper Andersen, Bollerslev, Diebold, and Vega (2005) are suggestive that the result of Proposition 10 may be worth exploring in an inter-temporal context.

revealing.

In the widely-studied Grossman and Stiglitz (1980) and related noise-based frameworks, the price function that is usually analyzed is linear due to the assumption of normal distributions, CARA utility, no wealth constraints, and unambiguous beliefs. Two exceptions are Marin and Olivier (2008) and Palvolgyi and Venter (2015). The first paper incorporates portfolio holding constraints into a CARA-normal REE model and establishes the existence of a continuum of non-linear and discontinuous REE price functions. The Marin and Olivier (2008) REE have a similar flavour to the unique REE of this paper in that (i) there is a discontinuity in the price function which coincides with the price function changing from a revealing increasing function to a constant non-revealing function and (ii) private information may be revealed even when informed traders' portfolio holdings are constant. Palvolgyi and Venter (2015) establish that REE with discontinuous price functions can exist as well in the noisy CARA-normal setting, without any trading or portfolio constraints, yielding multiplicity of noise-based partially revealing REE. Different distributional or utility assumptions (Mailath and Sandroni (2003), Barlevy and Veronesi (2003), Vanden (2008), Breon-Drish (2012)) or wealth constraints (Yuan (2005)) can also yield non-linear price functions, while exogenous portfolio insurance or hedging demand can yield a discontinuous price function (Gennotte and Leland (1990)).[33]

REE models with ambiguity averse traders and noise traders such as Ozsoylev and Werner (2011) and Mele and Sangiorgi (2015) and without noise traders such as Easley, O'Hara, and Yang (2011) feature continuous price functions. These papers differ significantly from the present paper. In addition to having noise-based partial revelation unlike this paper, in Ozsoylev and Werner (2011) ambiguity-averse traders do not receive any private signals nor do they perceive any information to be ambiguous. Mele and Sangiorgi (2015)also have noise-based partial revelation and private information in fact eliminates ex-ante ambiguity. Moreover, Mele and Sangiorgi (2015) are interested in time-inconsistent decision-making (Strotz (1955-1956)) by ambiguity averse investors, which has no role in our setting or results. In Easley, O'Hara, and Yang (2011), ambiguity-averse, uninformed 'simple' traders are ambiguous about the trading strategy of 'opaque' traders which yields a price function which is not fully informative for the 'simple' traders.

Under the noise-based approach, partial revelation typically does not have volatility implications beyond what noise adds in a manner qualitatively similar to how noise alters volatility in a symmetric information setting. Another distinguishing feature is that in

[33]Price is discontinuous at a point in the noise variable in Barlevy and Veronesi (2003).

common noise-based CARA-normal models information on trading volume makes partially revealing prices fully revealing (Blume, Easley, and O'Hara (1994) and Schneider (2009)). This is not true in the present framework since trade is constant at x_0^I for all signals.

The noise-based approach is also used to rule out the existence of fully revealing REE and provide a resolution to the Grossman and Stiglitz (1980) paradox of costly information acquisition.[34] Partial revelation under ambiguity in the competitive market setting of Sections 2-3 does not rule out full revelation REE; indeed one always exists as noted below.[35]

Proposition 11. *A fully revealing REE always exists with ambiguous information.*

Partial revelation under ambiguity involves a range of signal values not being revealed and yields a non-linear discontinuous price function and variation in price volatility as noted previously. Overall, the differences in partial revelation due to ambiguous information and noise-based partial revelation suggest that in principle, these approaches may provide differing testable implications and be useful in complementary ways for studying financial markets.

6.2 Implementability of REE and costly information acquisition

6.2.1 Implementability of REE

The REE obtained in the competitive market common-value and common-signal setting of Sections 3-4 are implementable as the equilibria of a trading game where traders submit demand schedules, in the tradition of Kyle (1989). We first establish implementability of the fully revealing REE in the *no ambiguity* benchmark economy (Section 4.1). It is well known that in a setting where the asset has common value and informed traders observe a common signal (s here) the fully revealing REE is implementable (see eg. Vives (2008) Section 4.2.2). However, we provide the analysis for completeness.

Proposition 12. *Suppose $\delta = 0$. There is a unique equilibrium with a linear price function* (18) *where I traders use the following symmetric demand schedule which is linear in s, p*

$$\hat{\theta}^I_{NA}(s,p) = \frac{\rho_0\mu_0}{\gamma} + \frac{\rho_\epsilon}{\gamma}s - \frac{\rho_0+\rho_\epsilon}{\gamma}p \tag{32}$$

[34] Exceptions to this include Rostek and Weretka (2012), Vives (2011), and Vives (2014) which analyze linear equilibria in a linear-quadratic / CARA-normal setting and do not rely on noise trading to obtain partial revelation.

[35] The analyzes of Radner (1979), Grossman (1981), and Condie and Ganguli (2011b) also suggest that a full revelation REE will exist.

and U traders use the symmetric constant demand schedule

$$\hat{\theta}_{NA}(p) = 1. \tag{33}$$

It is worth noting that the sensitivity of the demand schedule (32) to s is the mechanism through which s is revealed. The quantity demanded by each I traders and U traders at the market clearing revealing price is constant at 1 and 1 respectively across signal values, i.e. there is no trade.

We now establish implementability of the fully revealing REE price function (17) (Section 4.1) and the partially revealing REE price (14) (Section 3) in the economy with ambiguity averse I traders.

Proposition 13. *Suppose $\delta > 0$ and $\gamma \in [0, x_0^U \delta\rho_\epsilon]$. There exist two equilibria of the demand schedule game where I traders submit the following symmetric demand schedule which is piecewise linear in s, p*

$$\hat{\theta}^I(s,p) = \begin{cases} \frac{\rho_0\mu_0 - \delta\rho_\epsilon}{\gamma} + \frac{\rho_\epsilon}{\gamma}s - \frac{\rho_0+\rho_\epsilon}{\gamma}p & \text{if } \frac{\rho_0\mu_0+\rho_\epsilon(s-\delta)}{\rho_0+\rho_\epsilon} > p \\ 0 & \text{if } \frac{\rho_0\mu_0+\rho_\epsilon(s-\delta)}{\rho_0+\rho_\epsilon} \le p \le \frac{\rho_0\mu_0+\rho_\epsilon(s+\delta)}{\rho_0+\rho_\epsilon} \\ \frac{\rho_0\mu_0 + \delta\rho_\epsilon}{\gamma} + \frac{\rho_\epsilon}{\gamma}s - \frac{\rho_0+\rho_\epsilon}{\gamma}p & \text{if } \frac{\rho_0\mu_0+\rho_\epsilon(s+\delta)}{\rho_0+\rho_\epsilon} < p \end{cases} \tag{34}$$

and U traders use the symmetric constant demand schedule

$$\hat{\theta}(p) = \frac{1}{x_0^U}. \tag{35}$$

. *There is an equilibrium with a linear fully revealing price function* (17) *and there is an equilibrium with a non-linear and discontinuous partially revealing price function* (14)*.*

Note that in this game, the sensitivity of the ambiguity averse I traders' demand schedule (34) to s is the mechanism through which s is revealed. The quantity demanded by each I trader and U trader at the market clearing and revealing price is constant at 0 and $(x_0^U)^{-1}$ respectively, i.e. there is trade since I traders sell off their stockholding to U traders.

There is a multiplicity of market-clearing price functions due to the self-fulfilling nature of equilibrium. The partially revealing price arises when the Walrasian auctioneer sets a constant price across signal values to clear the market or equivalently, when U traders' conjecture that I traders' will trade at a constant price across a range of signal values. The fully revealing price arises when the Walrasian auctioneer sets market-clearing prices which are always sensitive to signal values or equivalently when the U traders conjecture that I traders will trade at a signal-sensitive price for all signal values.

Finally, it is worth emphasizing that the implementability of the equilibria with and without ambiguity averse traders is due to the fact that all I traders observe a common signal. Thus, price transmits information. In contrast, if the I traders observed diverse signals, such as in Grossman (1976) or Vives (2014) more recently, then the equilibria would not be implementable. With diverse signals, price would have to aggregate and transmit information. As noted, for example, in Vives (2014), if the asset in such a setting has common value across traders (like in this paper), then the fully revealing REE is not implementable even in an economy with ambiguity neutral I traders. The reason is that the traders' (linear) demand schedule is constant across signal values. This issue would arise in an economy with ambiguity averse I traders receiving diverse signals also. It is worth noting that the problem pertains to the revealing part of the REE price function and not to the non-revealing (constant) part.

On the other hand, Vives (2014) shows that if the informed traders' values from the stock are imperfectly correlated (private values) and traders receive diverse signals, then fully revealing REE can be implemented as the outcome of a demand schedule submission game.[36] In Condie and Ganguli (2016), we analyse a setting with one ambiguity averse informed trader and multiple uninformed ambiguity neutral strategic traders who compete in a demand schedule game along the lines of Kyle (1989) and Vives (2014). Following Vives (2014), we assume that the traders' values from the stock are not perfectly correlated. The presence of the ambiguity averse trader implies that the analysis is not a direct application of the work of Kyle (1989) or Vives (2014) for the linear case since the trading strategies and equilibrium price function may be non-linear, as suggested by the results of Proposition 13.

6.2.2 Costly information acquistion

This paper does not analyze costly information acquisition. Information used in financial markets may not always involve a direct cost. For example, information from a non-traded asset like labor income, whose payoff is correlated with, and hence informative about, the stock payoff will yield a similar information structure (see section B.1). However, costly information acquisition is a natural question to consider. In the common value and common

[36] Vives (2014) assumes risk-neutral traders who face linear marginal transaction costs, but the results of his analysis apply to the CARA-normal setting as well. Vives (2011) and Rostek and Weretka (2012) also have related analysis.

signal setting of this paper, I traders will not acquire costly information under the partially or fully revealing REE.

On the other hand, Vives (2014) provides a resolution to the Grossman and Stiglitz (1980) paradox of costly information acquisition and informational efficiency of price by providing conditions under which costly information is acquired when the equilibrium price function is privately revealing (see Proposition 2 and the related discussion on pp. 1214-1215).[37] [38]

The analysis of Vives (2014) suggests that costly information acquisition may occur in the setting of Condie and Ganguli (2016) and the analysis of this very interesting issue is a matter of future research. The analysis of Vives (2014) does not apply directly since the equilibrium price in Condie and Ganguli (2016) is not linear, unlike in Vives (2014).[39]

6.3 Inertial behavior and non-revelation of information

Portfolio inertia in information (section 2.3) is the key behavioral property underlying the non-revelation of information. Similar choice behavior may obtain due to sources different from that studied here and if so the framework and approach developed in this paper could be applied to study information revelation in those models.

For instance, an alternative preference-based source of inertia could be appropriately specified prospect-theoretic preferences as in Pasquariello (2012). The ambiguity averse representation and belief assumptions here and the specific preferences in Pasquariello (2012) both imply zero stockholding at a given price for a range of signals, though the overall demand for the latter is more complex than the one in (8). It would be interesting, but beyond the scope of the present paper to compare the general implications of these differing demand expressions.[40] One difference that suggests itself more generally is that non-smooth ambiguity averse preferences could allow for inertia at non-zero stockholding in addition, see for example, section 3.1.2 of Epstein and Schneider (2010) for an instance of non-differentiability at positive stockholding with multiple-prior mean-variance preferences. For prospect-theoretic preferences analyzed in Pasquariello (2012) inertia arises only at the reference point. This

[37] A privately revealing equilibrium is one where the price and the private information of a trader together comprise a sufficient statistic for the pooled information of all traders in the market.

[38] Bernardo and Judd (2000), Muendler (2007), and Krebs (2007) provide alternative analyses showing co-existence of informationally efficient prices and costly information acquisition in a competitive market setting.

[39] Moreover, the analysis of Mele and Sangiorgi (2015) indicates subtle issues involving time inconsistency may need to be dealt with when analysing costly information acquisition by ambiguity averse traders.

[40] As noted earlier, Pasquariello (2012) studies noise-based partial revelation and does not explore implications of the preference structure for information transmission by price.

is suggestive that the ambiguity preference structure may have richer implications, an interesting subject for future research.

While the present paper shows non-revelation in a trading environment that is frictionless following any signal realization and with investors whose preferences are classical in the sense of being convex but non-smooth and broader than the Savage (1954) class, it may be possible for inertia to obtain in frictional trading environments with smooth preferences, eg. in the Savage (1954) class. These frictions include transactions costs (Vissing-Jorgensen (2002)) or portfolio constraints such as short sale constraints (Diamond and Verrecchia (1987)) or constraints from agency frictions (Almazan, Brown, Carlson, and Chapman (2004)) and may lead to no trade in equilibrium. A detailed analysis of the possibility of inertia, related non-revelation, and its implications with such frictions is an interesting subject for future research.[41]

7 Concluding remarks

In this paper, we show that partially revealing REE arise and affect market variables when private information is received by ambiguity averse investors who exhibit portfolio inertia with respect to information. This property of investor behaviour arises under the MEU decision-making criteria axiomatized in Gilboa and Schmeidler (1989).

Partial revelation of information leads to lower and more volatile stock prices than when information is symmetric or there are no ambiguity averse investors. Moreover, partial revelation leads to skewness and kurtosis in stock returns, which is consistent with stylized facts for the US. Small changes in information can lead to price swings and amplification due to changes in informational efficiency of price. Public information directly affects the informational efficiency of prices and in fact, stock prices can be lower (higher) when good (bad) public information is observed.

We have focused on a single type of informed investor and a single uncertain asset in order to highlight the information transmission role of prices. Future areas for research would allow for multiple types of informed investors who receive different information along the lines sketched out in the supplementary appendix (Section B), thus enabling the study of information aggregation and transmission as well as the study of multiple traded assets. In such models, non-revelation of information will require conditions similar to those in (16).

[41]See Yu (2014) for a brief comparison of the ambiguity aversion approach and an instance of transaction cost frictions.

A Appendix: Proofs of results

A.1 Proofs for Section 2

Proof of Lemma 1. In the updating rule developed in Epstein and Schneider (2007), if a decision maker has a prior and the set of likelihoods is $\{L(s|\cdot)\}_{L\in\mathcal{L}}$ for some index set $\mathcal{L}$, then the set of updated beliefs about event B is given by

$$\{Pr(B|s)\} = \left\{ \frac{Pr(B)L(s|B)}{\int L(s|B)dB} \middle| L \in \mathcal{L} \right\}. \tag{36}$$

The prior over v has mean μ_0 and precision ρ_0 and the set of likelihoods is indexed by $[-\delta, \delta]$. This updating rule implies that given μ_0 and $\mu_\epsilon \in [-\delta, \delta]$, v conditional on the signal s is normally distributed with mean $\mu|s$ and precision $\rho|s$ where

$$\mu|s = \frac{\rho_0\mu_0 + \rho_\epsilon (s - \mu_\epsilon)}{\rho_0 + \rho_\epsilon} \text{ and } \rho|s = \rho_0 + \rho_\epsilon. \tag{37}$$

Using these expressions yields the desired result. □

Proof of Proposition 1. Using our normalization of $p_f = 1$ and the expressions for the updated beliefs in (3), the optimal demand for I-investors is the solution to

$$\max_{\theta} \min_{\mu\in[\underline{\mu}^I|s,\overline{\mu}^I|s]} \theta(\mu - p) - \frac{\gamma\theta^2}{2\rho^I|s}. \tag{38}$$

The first order conditions for I-investors are

$$\begin{aligned} &0 = \underline{\mu}^I|s - p - \gamma\theta(\rho^I|s)^{-1} && \text{if } \theta > 0 \\ &0 \in [\underline{\mu}^I|s - p - \gamma\theta(\rho^I|s)^{-1}, \overline{\mu}^I|s - p - \gamma\theta(\rho^I|s)^{-1}] && \text{if } \theta = 0 \\ &0 = \overline{\mu}^I|s - p - \gamma\theta(\rho^I|s)^{-1} && \text{if } \theta < 0. \end{aligned} \tag{39}$$

which yields that

$$\theta^I(s,p) = \begin{cases} \gamma(\rho^I|s)(\underline{\mu}^I|s - p) & \text{if } \underline{\mu}^I|s - p > 0 \\ 0 & \text{if } \underline{\mu}^I|s - p \leq 0 \text{ and } \overline{\mu}^I|s - p \geq 0 \\ \gamma(\rho^I|s)(\overline{\mu}^I|s - p) & \text{if } \overline{\mu}^I|s - p < 0. \end{cases} \tag{40}$$

Using the expressions for $\underline{\mu}^I|s$ and $\overline{\mu}^I|s$ in Lemma 1 and simplifying yields the expression in

(8) for I-investor demand. □

Proof of Corollary 1. This follows from using demand expression (8) and the updated beliefs (3) for I-investors to see that the upper bound of the range of signals for which I-investors exhibit portfolio inertia in information is

$$\frac{(\rho_0+\rho_\epsilon)}{\rho_\epsilon}p - \frac{\rho_0}{\rho_\epsilon}\mu_0 + \delta \tag{41}$$

and the lower bound is

$$\frac{(\rho_0+\rho_\epsilon)}{\rho_\epsilon}p - \frac{\rho_0}{\rho_\epsilon}\mu_0 - \delta \tag{42}$$

which yields the desired results. □

A.2 Proofs for Section 3

Proof of Proposition 2. Suppose markets clear with $\theta^I(s) = \gamma^{-1}\rho^I|s\left(\mu^I|s - p\right) > 0$. Then the market clearing price satisfies

$$p = x_0^I\left(\frac{\rho_0\mu_0 + \rho_\epsilon}{\rho_0+\rho_\epsilon}(s-\delta)\right) + (x_0^U\theta^U - 1)\frac{\gamma}{\rho_0+\rho_\epsilon} \tag{43}$$

where θ^U denotes U-investor demand. If this price were to be non-revealing then θ^U must be a decreasing function of s, which cannot occur, either in partial or full revelation.

In full revelation, s is observed by U-investors and their updated beliefs are that v is normally distributed with precision $\rho_0 + \rho_\epsilon$ and mean

$$\frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0+\rho_\epsilon}. \tag{44}$$

Then, the optimal demand for U-investors is the solution to

$$\max_\theta \theta\left(\frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0+\rho_\epsilon} - p\right) - \frac{\gamma\theta^2}{2\rho_0+\rho_\epsilon} \tag{45}$$

which yields

$$\theta^U(s) = \gamma^{-1}(\rho_0+\rho_\epsilon)\left(\frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0+\rho_\epsilon} - p\right), \tag{46}$$

which is increasing in s.

On the other hand if $p(s') = p(s'')$ for $s' > s''$ (without loss of generality), i.e. under partial revelation, U-investor demand is constant in s, i.e. $\theta^U(s') = \theta^U(s'')$ due to measurability of $\theta^U(\cdot)$ in $p(\cdot)$ (measurability in $\mathbf{p}(\cdot)$ reduces to measurability in $p(\cdot)$ with our normalization of $p_f = 1$).

When the signal s is revealed by the price, given U-investors demand, the market clearing price satisfies

$$p = \frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} - \frac{\gamma}{\rho_0 + \rho_\epsilon} - x_0^I \frac{\delta\rho_\epsilon}{\rho_0 + \rho_\epsilon} \tag{47}$$

and again this price is consistent with revelation of s to U-investors since it is monotone in s. Hence with $\theta^I(s) > 0$, the market clearing price reveals the signal to U-investors. Similar arguments show that if $\theta^I(s) < 0$, then price reveals the signal s as well. □

Proof of Corollary 2. Suppose the market clears with $\theta^I(s) > 0$, then the market clearing price is given by (47) which includes the usual risk premium $\frac{\gamma}{\rho_0+\rho_\epsilon}$ required by all investors and an ambiguity premium $\frac{\delta\rho_\epsilon}{\rho_0+\rho_\epsilon}$ required by the x_0^I I-investors. A similar argument applies for $\theta^I(s) < 0$. □

Proof of Proposition 3. For $s \notin [b - 2\delta, b]$, the updated beliefs of U-investors are that v is normally distributed with precision $\rho_0 + \rho_\epsilon$ and mean

$$\frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon}. \tag{48}$$

Then, reasoning similar to that for the full revelation case in the proof of Proposition 2 shows that the optimal demand for U-investors is the solution to

$$\max_\theta \theta \left(\frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} - p \right) - \frac{\gamma\theta^2}{2\rho_0 + \rho_\epsilon} \tag{49}$$

which yields

$$\theta^U(s) = \gamma^{-1}(\rho_0 + \rho_\epsilon) \left(\frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} - p \right). \tag{50}$$

as desired.

For $s \in [b-2\delta, b]$, first note that U-investors prior beliefs are that s is normally distributed with mean μ_0 and variance $\rho_0^{-1} + \rho_\epsilon^{-1}$. Let $f(\cdot)$ and $F(\cdot)$ denote the density and distribution functions of s respectively, then conditional on the information that $s \in [b-2\delta, b]$, U-investors beliefs are that s follows a truncated normal distribution with density

$$f_{s|[b-2\delta,b]}(s)=\begin{cases}\frac{f(s)}{F(b)-F(b-2\delta)}, s\in[b-2\delta,b]\\ 0, s\notin[b-2\delta,b]\end{cases}\tag{51}$$

.

Let $f_{w,s}(\cdot)$ and $F_{w,s}(\cdot)$ denote the joint density and joint distribution functions of $(w(\theta),s)$ respectively. Then the density of $w(\theta)$ conditional on $s\in[b-2\delta,b]$ denoted $f_{w|[b-2\delta,b]}$ is

$$f_{w|[b-2\delta,b]}(w)=\frac{\int_{b-2\delta}^{b}f_{w,s}(w,s)ds}{F(b)-F(b-2\delta)}\tag{52}$$

.

U-investors choose θ to maximize

$$\mathbf{U}^U(\theta)=-\int_{-\infty}^{\infty}\exp\{-\gamma w(\theta)\}f_{w|[b-2\delta,b]}(w)dw\tag{53}$$

Using the fact that for the joint normal density $f_{w,s}(w,s)=f_{w|s}(w,s)f(s)$, rearranging the order of integrals, and using $\mathbb{E}[w|s]=\theta\left(\frac{\rho_0\mu_0+\rho_\epsilon s}{\rho_0+\rho_\epsilon}-p\right)+p$ and $\mathbb{V}[w|s]=\frac{\theta^2}{\rho_0+\rho_\epsilon}$ to denote the mean and variance of $w(\theta)$ conditional on s we have

$$\begin{aligned}\mathbf{U}^U(\theta)=&\ -\tfrac{1}{F(b)-F(b-2\delta)}\int_{b-2\delta}^{b}\left(\exp^{-\gamma\mathbb{E}(w|s)+\frac{1}{2}\gamma^2\mathbb{V}(w|s)}\right)f(s)ds\\ =&\ -\tfrac{\exp^{\frac{\gamma^2}{2}\mathbb{V}(w|s)}}{F(b)-F(b-2\delta)}\int_{b-2\delta}^{b}\left(\exp^{-\gamma\mathbb{E}(w|s)}\right)f(s)ds\\ =&\ -\exp^{\frac{\gamma^2}{2}\theta^2\frac{1}{\rho_0+\rho_\epsilon}+\gamma\theta\left(\frac{\rho_\epsilon\mu_0}{\rho_0+\rho_\epsilon}-(\mu_0-p)\right)-\gamma p}\int_{b-2\delta}^{b}\left(\exp^{-\gamma\theta\frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}s}\right)\tfrac{f(s)}{F(b)-F(b-2\delta)}ds\end{aligned}\tag{54}$$

Using the moment generating function of a truncated normal (see e.g. Johnson and Kotz (1970)) yields

$$\begin{aligned}=&\ -\exp^{\frac{\gamma^2}{2}\theta^2\frac{1}{\rho_0+\rho_\epsilon}+\gamma\theta\left(\frac{\rho_\epsilon\mu_0}{\rho_0+\rho_\epsilon}-(\mu_0-p)\right)-\gamma p}\exp^{-\frac{\gamma\theta\rho_\epsilon\mu_0}{\rho_0+\rho_\epsilon}+\frac{\gamma^2\theta^2}{2}\left(\frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}\right)^2\left(\frac{\rho_\epsilon+\rho_0}{\rho_0\rho_\epsilon}\right)}\times\\ &\left[\frac{\Phi\left((b-\mu_0)\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_\epsilon+\rho_0}}+\sqrt{\frac{\rho_\epsilon+\rho_0}{\rho_0\rho_\epsilon}}\frac{\gamma\theta\rho_\epsilon}{\rho_0+\rho_\epsilon}\right)-\Phi\left((b-2\delta-\mu_0)\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_\epsilon+\rho_0}}+\sqrt{\frac{\rho_\epsilon+\rho_0}{\rho_0\rho_\epsilon}}\frac{\gamma\theta\rho_\epsilon}{\rho_0+\rho_\epsilon}\right)}{\Phi\left((b-\mu_0)\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_\epsilon+\rho_0}}\right)-\Phi\left((b-2\delta-\mu_0)\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_\epsilon+\rho_0}}\right)}\right]\\ =&\ -\exp^{\frac{\gamma^2\theta^2}{2}\frac{1}{\rho_0}-\gamma\theta(\mu_0-p)-\gamma p}\times\\ &\left[\frac{\Phi\left((b-\mu_0)\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_\epsilon+\rho_0}}+\sqrt{\frac{\rho_\epsilon+\rho_0}{\rho_0\rho_\epsilon}}\frac{\gamma\theta\rho_\epsilon}{\rho_0+\rho_\epsilon}\right)-\Phi\left((b-2\delta-\mu_0)\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_\epsilon+\rho_0}}+\sqrt{\frac{\rho_\epsilon+\rho_0}{\rho_0\rho_\epsilon}}\frac{\gamma\theta\rho_\epsilon}{\rho_0+\rho_\epsilon}\right)}{\Phi\left((b-\mu_0)\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_\epsilon+\rho_0}}\right)-\Phi\left((b-2\delta-\mu_0)\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_\epsilon+\rho_0}}\right)}\right]\end{aligned}\tag{55}$$

The first order condition for (53) with respect to θ yields

$$\begin{aligned} 0 = \; & \left(\gamma\theta\tfrac{1}{\rho_0} - (\mu_0 - p)\right) \times \\ & \left[\Phi\left((b-\mu_0)\sqrt{\tfrac{\rho_0\rho_\epsilon}{\rho_\epsilon+\rho_0}} + \sqrt{\tfrac{\rho_\epsilon+\rho_0}{\rho_0\rho_\epsilon}}\tfrac{\gamma\theta\rho_\epsilon}{\rho_0+\rho_\epsilon}\right) - \Phi\left((b-2\delta-\mu_0)\sqrt{\tfrac{\rho_0\rho_\epsilon}{\rho_\epsilon+\rho_0}} + \sqrt{\tfrac{\rho_\epsilon+\rho_0}{\rho_0\rho_\epsilon}}\tfrac{\gamma\theta\rho_\epsilon}{\rho_0+\rho_\epsilon}\right)\right] + \\ & \sqrt{\tfrac{\rho_0+\rho_\epsilon}{\rho_0\rho_\epsilon}}\tfrac{\rho_\epsilon}{\rho_0+\rho_\epsilon}\left[\phi\left((b-\mu_0)\sqrt{\tfrac{\rho_0\rho_\epsilon}{\rho_\epsilon+\rho_0}} + \sqrt{\tfrac{\rho_\epsilon+\rho_0}{\rho_0\rho_\epsilon}}\tfrac{\gamma\theta\rho_\epsilon}{\rho_0+\rho_\epsilon}\right) - \phi\left((b-2\delta-\mu_0)\sqrt{\tfrac{\rho_0\rho_\epsilon}{\rho_\epsilon+\rho_0}} + \sqrt{\tfrac{\rho_\epsilon+\rho_0}{\rho_0\rho_\epsilon}}\tfrac{\gamma\theta\rho_\epsilon}{\rho_0+\rho_\epsilon}\right)\right] \end{aligned} \tag{56}$$

which yields the following implicit expression for U-investor demand,

$$\theta = \; \gamma^{-1}\left(\rho_0(\mu_0 - p) + \tfrac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}\Delta_1(b - 2\delta + \tfrac{\gamma\theta}{\rho_0}, b + \tfrac{\gamma\theta}{\rho_0})\right) \tag{57}$$

where

$$\Delta(b-2\delta+\frac{\gamma\theta}{\rho_0}, b+\frac{\gamma\theta}{\rho_0}) = \sqrt{\frac{\rho_0+\rho_\epsilon}{\rho_0\rho_\epsilon}}\left[\frac{\phi(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}\left(b-2\delta-\mu_0+\frac{\gamma\theta}{\rho_0}\right)) - \phi(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}\left(b-\mu_0+\frac{\gamma\theta}{\rho_0}\right))}{\Phi(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}\left(b-\mu_0+\frac{\gamma\theta}{\rho_0}\right)) - \Phi(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}\left(b-2\delta-\mu_0+\frac{\gamma\theta}{\rho_0}\right))}\right] \tag{58}$$

□

Proof of Proposition 4. Market clearing with $\theta^I = 0$ requires $x_0^U\theta^U = 1$. For $s \notin [b-2\delta, b]$ and $s \in [b-2\delta, b]$, using $\theta^U = (x_0^U)^{-1}$ and the expression for U-investor demand in (12) in Proposition 3 provides the desired expression for $p_{PR}(s)$ in (14). Non-linearity of p_{PR} follows from the fact that the function is strictly increasing in s for $s \notin [b-2\delta, b]$ and constant for $s \in [b-2\delta, b]$. Discontinuity at b follows from

$$\lim_{s\downarrow b} p_{PR}(s) - \lim_{s\uparrow b} p_{PR}(s) = \frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}\left(b + \frac{\gamma}{x_0^U\rho_0} - \mu_0 - \Delta\left(b + \frac{\gamma}{x_0^U\rho_0} - 2\delta, b + \frac{\gamma}{x_0^U\rho_0}\right)\right) > 0 \tag{59}$$

where the strict inequality follows from the fact that by the properties of a truncated normal distribution (see eg. Johnson and Kotz (1970))

$$\mathbb{E}\left[s \,\middle|\, s \in \left[b + \frac{\gamma}{x_0^U\rho_0} - 2\delta, b + \frac{\gamma}{x_0^U\rho_0}\right]\right] \equiv \mu_0 + \Delta\left(b + \frac{\gamma}{x_0^U\rho_0} - 2\delta, b + \frac{\gamma}{x_0^U\rho_0}\right). \tag{60}$$

A similar inequality implies discontinuity at $b - 2\delta$. Trade volume is x_0^I since $\theta^I(s) = 0$ for all s. □

The next lemma collects several facts that are useful in proving subsequent results.

Lemma 3. *Signal s is normally distributed with mean μ_0 and variance $\sigma_s^2 \equiv \rho_0^{-1} + \rho_\epsilon^{-1}$. Let $f(\cdot)$ denote the density function of s. Then for scalars B_1, B_2,*

$$
\begin{aligned}
&1.\ \frac{\partial}{\partial B_2}\int_{B_2-B_1}^{B_2} sf(s)ds = B_2 f(B_2) - (B_2 - B_1)f(B_2 - B_1) \\
&2.\ \frac{\partial}{\partial B_2}\int_{B_2-B_1}^{B_2} f(s)ds = f(B_2) - f(B_2 - B_1) \\
&3.\ f(B_2 - B_1) = f(B_2)e^{-\frac{B_1(\frac{B_1}{2}+\mu_0)}{\sigma_s^2}} e^{\frac{B_1 B_2}{\sigma_s^2}} \\
&4.\ B_2 f(B_2) - (B_2 - B_1)f(B_2 - B_1) = \int_{B_2-B_1}^{B_2} f(s)ds - \int_{B_2-B_1}^{B_2} s\left(\frac{s-\mu_0}{\sigma_s^2}\right) f(s)ds \\
&5.\ f(B_2 - B_1) - f(B_2) = \frac{1}{\sigma_s^2}\int_{B_2-B_1}^{B_2} sf(s)ds - \frac{\mu_0}{\sigma_s^2}\int_{B_2-B_1}^{B_2} f(s)ds \\
&6.\ 0 < \frac{\partial}{\partial B_2}\mathbb{E}[s|B_2 - B_1 \leq s \leq B_2] < 1 \text{ for all } -\infty < B_2 < \infty
\end{aligned}
\tag{61}
$$

Proof. The first two results follow from Leibniz's rule and the third by rearranging terms in $f(B_2 - B_1)$. The fourth follows from integrating $\int_{B_2-B_1}^{B_2} f(s)ds$ by parts where $u = f(s)$ and $dv = ds$. The fifth follows from observing that

$$
f'(s) = -\left(\frac{s-\mu_0}{\sigma_s^2}\right) f(s) \tag{62}
$$

and integrating both sides of equation (62) over the region $[B_2 - B_1, B_1]$.

To show

$$
0 < \frac{\partial}{\partial B_2}\mathbb{E}[s|B_2 - B_1 \leq s \leq B_2] < 1 \tag{63}
$$

calculate

$$
\begin{aligned}
&\frac{\partial \mathbb{E}[s|B_2 - B_1 \leq s \leq B_2]}{\partial B_2} \\
=&\frac{\partial}{\partial B_2} \frac{\int_{B_2-B_1}^{B_2} sf(s)ds}{\int_{B_2-B_1}^{B_2} f(s)ds} \\
=&\frac{\left(\int_{B_2-B_1}^{B_2} f(s)ds\right)(B_2 f(B_2) - (B_2 - B_1) f(B_2 - B_1)) - \left(\int_{B_2-B_1}^{B_2} sf(s)ds\right)(f(B_2) - f(B_2 - B_1))}{\left(\int_{B_2-B_1}^{B_2} f(s)ds\right)^2} \\
=&\frac{\left(\int_{B_2-B_1}^{B_2} f(s)ds\right)(B_2 f(B_2) - (B_2 - B_1) f(B_2 - B_1))}{\left(\int_{B_2-B_1}^{B_2} f(s)ds\right)^2} \\
&+ \frac{\left(\int_{B_2-B_1}^{B_2} sf(s)ds\right)\left(\frac{1}{\sigma_s^2}\int_{B_2-B_1}^{B_2} sf(s)ds - \frac{\mu_0}{\sigma_s^2}\int_{B_2-B_1} B_2 f(s)ds\right)}{\left(\int_{B_2-B_1}^{B_2} f(s)ds\right)^2} \\
=&\frac{\left(\int_{B_2-B_1}^{B_2} f(s)ds\right)\left(\int_{B_2-B_1}^{B_2} f(s)ds - \int_{B_2-B_1}^{B_2} s\left(\frac{s-\mu_0}{\sigma_s^2}\right) f(s)ds\right)}{\left(\int_{B_2-B_1}^{B_2} f(s)ds\right)^2} \\
&+ \frac{\left(\int_{B_2-B_1}^{B_2} sf(s)ds\right)\left(\frac{1}{\sigma_s^2}\int_{B_2-B_1}^{B_2} sf(s)ds - \frac{\mu_0}{\sigma_s^2}\int_{B_2-B_1}^{B_2} f(s)ds\right)}{\left(\int_{B_2-B_1}^{B_2} f(s)ds\right)^2} \\
=&1 - \frac{1}{\sigma_s^2}\frac{\int_{B_2-B_1}^{B_2} s^2 f(s)ds}{\int_{B_2-B_1}^{B_2} f(s)ds} + \frac{\mu_0}{\sigma_s^2}\frac{\int_{B_2-B_1}^{B_2} sf(s)ds}{\int_{B_2-B_1}^{B_2} f(s)ds} \\
&- \frac{1}{\sigma_s^2}\left[\frac{\int_{B_2-B_1}^{B_2} sf(s)ds}{\int_{B_2-B_1}^{B_2} f(s)ds}\right]^2 - \frac{\mu_0}{\sigma_s^2}\frac{\int_{B_2-B_1}^{B_2} sf(s)ds}{\int_{B_2-B_1}^{B_2} f(s)ds} \\
=&1 - \frac{1}{\sigma_s^2}\mathbb{E}[s^2|B_2 - B_1 \leq s \leq B_2] + \frac{1}{\sigma_s^2}\mathbb{E}[s|B_2 - B_1 \leq s \leq B_2]^2 \\
=&1 - \frac{1}{\sigma_s^2}\mathbb{V}(s|B_2 - B_1 \leq s \leq B_2)
\end{aligned}
\tag{64}
$$

The third and fourth equalities follow from facts 5 and 4, respectively and the rest are simplifications. The result follows since

$$
0 < \mathbb{V}(s|B_2 - B_1 \leq s \leq B) < \sigma_s^2 \equiv \mathbb{V}(s). \tag{65}
$$

$\square$

Proof of Proposition 5. We characterize the existence of a solution to (16). We first prove that if a solution b exists then $\frac{\gamma}{x_0^U \rho_\epsilon} < \delta$. First, for any b, by the properties of a truncated normal distribution (see eg. Johnson and Kotz (1970)) note that

$$\mathbb{E}\left[s \,\middle|\, s \in \left[b + \frac{\gamma}{x_0^U \rho_0} - 2\delta, b + \frac{\gamma}{x_0^U \rho_0}\right]\right] \equiv \mu_0 + \Delta\left(b + \frac{\gamma}{x_0^U \rho_0} - 2\delta, b + \frac{\gamma}{x_0^U \rho_0}\right) \tag{66}$$

Define $h(\cdot)$ as

$$h(b) = b + \frac{\gamma}{x_0^U \rho_0} - \mu_0 - \Delta\left(b + \frac{\gamma}{x_0^U \rho_0} - 2\delta, b + \frac{\gamma}{x_0^U \rho_0}\right) \tag{67}$$

and rewrite equation (16) as

$$h(b) = \delta - \frac{\gamma}{x_0^U \rho_\epsilon}. \tag{68}$$

If $\frac{\gamma}{x_0^U \rho_\epsilon} > \delta$ then (16) requires $h(b) < 0 \Leftrightarrow b + \frac{\gamma}{x_0^U \rho_0} < \mathbb{E}\left[s \,\middle|\, b + \frac{\gamma}{x_0^U \rho_0} - 2\delta \leq s \leq b + \frac{\gamma}{x_0^U \rho_0}\right]$, which is a contradiction.

We prove sufficiency by showing that $h(\cdot)$ takes all values in $[0, \delta]$ as $b \to -\infty$. When $\gamma = 0$, ie. $\frac{\gamma}{x_0^U \rho_\epsilon} = 0$, $b_0 = \mu_0 + \frac{\gamma}{x_0^U \rho_0} + \delta = \mu_0 + \delta$ solves (16), ie. $h(b_0) = \delta$, since $\Delta(\mu_0 - \delta, \mu_0 + \delta) = 0$ given the symmetry of a normal density function around its mean.

For any $-\infty < b < \infty$, $h'(b)$ exists and from (63) $0 < h'(b) < 1$ with $B_2 = b + \frac{\gamma}{x_0^U \rho_0}$ and $B_1 = 2\delta$. Moreover, for $\gamma \in [0, \delta x_0^U \rho_\epsilon]$,

$$\lim_{b \to -\infty} b + \frac{\gamma}{x_0^U \rho_0} \leq \lim_{b \to -\infty} b + \frac{\delta \rho_\epsilon}{\rho_0} = -\infty. \tag{69}$$

Hence, it suffices to show that

$$\lim_{b \to -\infty} h(b) = 0. \tag{70}$$

Using L'Hopital's rule, 1-3 in Lemma 3 and rearranging terms,

$$
\begin{aligned}
&\lim_{b\to-\infty} \mathbb{E}\left[s \,\middle|\, s \in \left[b+\frac{\gamma}{x_0^U\rho_0}-2\delta, b+\frac{\gamma}{x_0^U\rho_0}\right]\right] \\
&= \lim_{b\to-\infty} \frac{\int_{b+\frac{\gamma}{x_0^U\rho_0}-2\delta}^{b+\frac{\gamma}{x_0^U\rho_0}} s f(s) ds}{\int_{b+\frac{\gamma}{x_0^U\rho_0}-2\delta}^{b+\frac{\gamma}{x_0^U\rho_0}} f(s) ds} \\
&= \lim_{b\to-\infty} \frac{\frac{\partial}{\partial b}\int_{b+\frac{\gamma}{x_0^U\rho_0}-2\delta}^{b+\frac{\gamma}{x_0^U\rho_0}} s f(s) ds}{\frac{\partial}{\partial b}\int_{b+\frac{\gamma}{x_0^U\rho_0}-2\delta}^{b+\frac{\gamma}{x_0^U\rho_0}} f(s) ds} \\
&= \lim_{b\to-\infty} \frac{\left(b+\frac{\gamma}{x_0^U\rho_0}\right) f\left(b+\frac{\gamma}{x_0^U\rho_0}\right) - \left(b+\frac{\gamma}{x_0^U\rho_0}-2\delta\right) f\left(b+\frac{\gamma}{x_0^U\rho_0}-2\delta\right)}{f\left(b+\frac{\gamma}{x_0^U\rho_0}\right) - f\left(b+\frac{\gamma}{x_0^U\rho_0}-2\delta\right)} \\
&= \lim_{b\to-\infty} \frac{\left(b+\frac{\gamma}{x_0^U\rho_0}\right)\left(f\left(b+\frac{\gamma}{x_0^U\rho_0}\right) - f\left(b+\frac{\gamma}{x_0^U\rho_0}-2\delta\right)\right) + 2\delta f\left(b+\frac{\gamma}{x_0^U\rho_0}-2\delta\right)}{f\left(b+\frac{\gamma}{x_0^U\rho_0}\right) - f\left(b+\frac{\gamma}{x_0^U\rho_0}-2\delta\right)} \qquad (71) \\
&= \lim_{b\to-\infty} b+\frac{\gamma}{x_0^U\rho_0} + 2\delta \frac{f\left(b+\frac{\gamma}{x_0^U\rho_0}-2\delta\right)}{f\left(b+\frac{\gamma}{x_0^U\rho_0}\right) - f\left(b+\frac{\gamma}{x_0^U\rho_0}-2\delta\right)} \\
&= \lim_{b\to-\infty} b+\frac{\gamma}{x_0^U\rho_0} + 2\delta \frac{f\left(b+\frac{\gamma}{x_0^U\rho_0}\right) e^{-\frac{2\delta(\delta+\mu_0)}{\sigma_s^2}} e^{\frac{2\delta\left(b+\frac{\gamma}{x_0^U\rho_0}\right)}{\sigma_s^2}}}{f\left(b+\frac{\gamma}{x_0^U\rho_0}\right)\left(1-e^{-\frac{2\delta(\delta+\mu_0)}{\sigma_s^2}} e^{\frac{2\delta\left(b+\frac{\gamma}{x_0^U\rho_0}\right)}{\sigma_s^2}}\right)} \\
&= \lim_{b\to-\infty} b+\frac{\gamma}{x_0^U\rho_0} + 2\delta \frac{e^{-\frac{2\delta(\delta+\mu_0)}{\sigma^2}} e^{\frac{2\delta\left(b+\frac{\gamma}{x_0^U\rho_0}\right)}{\sigma_s^2}}}{1-e^{-\frac{2\delta(\delta+\mu_0)}{\sigma_s^2}} e^{\frac{2\delta\left(b+\frac{\gamma}{x_0^U\rho_0}\right)}{\sigma_s^2}}},
\end{aligned}
$$

which yields

$$\begin{aligned}
\lim_{b\to-\infty} h(b) &= \lim_{b\to-\infty} b + \frac{\gamma}{x_0^U\rho_0} - \mathbb{E}\left[s \,\middle|\, s \in \left[b + \frac{\gamma}{x_0^U\rho_0} - 2\delta, b + \frac{\gamma}{x_0^U\rho_0}\right]\right] \\
&= \lim_{b\to-\infty} b + \frac{\gamma}{x_0^U\rho_0} - b - \frac{\gamma}{x_0^U\rho_0} - 2\delta \frac{e^{-\frac{2\delta(\delta+\mu_0)}{\sigma^2}} e^{\frac{2\delta\left(b+\frac{\gamma}{x_0^U\rho_0}\right)}{\sigma^2}}}{1 - e^{-\frac{2\delta(\delta+\mu_0)}{\sigma^2}} e^{\frac{2\delta\left(b+\frac{\gamma}{x_0^U\rho_0}\right)}{\sigma^2}}} \\
&= \lim_{b\to-\infty} -2\delta \frac{e^{-\frac{2\delta(\delta+\mu_0)}{\sigma^2}} e^{\frac{2\delta\left(b+\frac{\gamma}{x_0^U\rho_0}\right)}{\sigma^2}}}{1 - e^{-\frac{2\delta(\delta+\mu_0)}{\sigma^2}} e^{\frac{2\delta\left(b+\frac{\gamma}{x_0^U\rho_0}\right)}{\sigma^2}}} \\
&= 0.
\end{aligned} \tag{72}$$

Hence, a solution to (16) exists if and only if $\gamma \in [0, x_0^U\rho_\epsilon\delta]$. A consequence of the above is also that $-\Delta\left(b + \frac{\gamma}{x_0^U\rho_0} - 2\delta, b + \frac{\gamma}{x_0^U\rho_0}\right) > 0$ if $\gamma > 0$. The existence of a solution to (16) if and only if $\gamma \in [0, x_0^U\rho_\epsilon\delta]$ characterizes the existence of an interval $[b - 2\delta, b]$ of unrevealed signals. Finally, uniqueness of b follows from the fact that $h'(b) > 0$.

Market clearing with $\theta^I(s) = 0$ requires that the market clearing price $p(s) \in [\underline{\mu}^I|s, \overline{\mu}^I|s]$ given the I-investors demand expression (8). For $s \notin [b - 2\delta, b]$, using the expression for market clearing price $p_{PR}(s)$ in (14) and those for $\underline{\mu}^I|s$ and $\overline{\mu}^I|s$ from (3) shows that $p_{PR}(s) \in [\underline{\mu}^I|s, \overline{\mu}^I|s]$ if and only if $\gamma \in [0, x_0^U\rho_\epsilon\delta]$. For $s \in [b - 2\delta, b]$, since

$$b = \mu_0 + \Delta\left(b + \frac{\gamma}{x_0^U\rho_0} - 2\delta, b + \frac{\gamma}{x_0^U\rho_0}\right) - \frac{\gamma}{x_0^U}\left(\frac{1}{\rho_0} + \frac{1}{\rho_\epsilon}\right) + \delta \tag{73}$$

it follows that $p_{PR}(s) \in [\underline{\mu}^I|s, \overline{\mu}^I|s]$. On other hand if $\gamma > x_0^U\rho_\epsilon\delta$, there exists $s \in [b - 2\delta, b]$ such that $p_{PR}(s) < \underline{\mu}^I|s$ which contradicts market clearing with $\theta^I(s) = 0$. Hence, markets clear with $\theta^I(s) = 0$ for all s if and only if $\gamma \in [0, x_0^U\rho_\epsilon\delta]$. □

A.3 Proofs for section 4

Proof of Proposition 6. Suppose $\delta = 0$. Then, the updated belief of I investors about v is given by the normal distribution with mean $\frac{\rho_0\mu_0+\rho_\epsilon s}{\rho_0+\rho_\epsilon}$ and precision $\rho_0 + \rho_\epsilon$. The optimal portfolio $\theta^I_{NA}(s,p)$ of I investors is then given by

$$\theta^I_{NA}(s,p) = \gamma^{-1}(\rho_0+\rho_\epsilon)\left(\frac{\rho_0\mu_0+\rho_\epsilon s}{\rho_0+\rho_\epsilon} - p\right). \tag{74}$$

Clearly $\theta^I(s,p) = 0$ if and only if $s = \mu_0$. So, reasoning similarly to the proof of Proposition 2 (or to for example, the analysis in Grossman (1976)) shows that market clearing price will always reveal s to U investors. So, the U investors optimal portfolio given their updated beliefs about v is given by

$$\theta^U_{NA}(s,p) = \gamma^{-1}(\rho_0+\rho_\epsilon)\left(\frac{\rho_0\mu_0+\rho_\epsilon s}{\rho_0+\rho_\epsilon} - p\right) \tag{75}$$

and the unique market clearing price for each s is given by $p_{NA}(s)$ as in (18). This price function is clearly fully revealing, which provides the desired result. □

Proof of Proposition 7. The proof to Proposition 5 establishes that $b = \mu_0 + \delta$ solves (16) when $\gamma = 0$ and that in this case $\Delta(b - 2\delta, b) = 0$ due to the symmetry of the normal pdf around μ_0. To show that the premium is positive and increasing in γ whenever $\gamma > 0$, we show that

$$\frac{d}{d\gamma}\Delta\left(b+\frac{\gamma}{x^U_0\rho_0} - 2\delta, b+\frac{\gamma}{x^U_0\rho_0}\right) < 0 \tag{76}$$

Using the sixth result in Lemma 3 with $B_2 = b + \frac{\gamma}{x^U_0\rho_0}$ and $B_1 = 2\delta$ and (66) yields for any b

$$0 < \frac{\partial}{\partial b}\Delta\left(b+\frac{\gamma}{x^U_0\rho_0} - 2\delta, b+\frac{\gamma}{x^U_0\rho_0}\right) < 1 \tag{77}$$

and any γ,

$$0 < \frac{\partial}{\partial \gamma}\Delta\left(b+\frac{\gamma}{x^U_0\rho_0} - 2\delta, b+\frac{\gamma}{x^U_0\rho_0}\right) < \frac{1}{x^U_0\rho_0} \tag{78}$$

Applying the implicit function theorem to equation (16) then yields

$$\tfrac{d}{d\gamma}\Delta\left(b+\tfrac{\gamma}{x^U_0\rho_0} - 2\delta, b+\tfrac{\gamma}{x^U_0\rho_0}\right) = -\frac{\frac{1}{x^U_0\rho_0}+\frac{1}{x^U_0\rho_\epsilon}-\frac{\partial}{\partial\gamma}\Delta\left(b+\frac{\gamma}{x^U_0\rho_0}-2\delta,b+\frac{\gamma}{x^U_0\rho_0}\right)}{\left(\left(\frac{\partial}{\partial b}\Delta\left(b+\frac{\gamma}{x^U_0\rho_0}-2\delta,b+\frac{\gamma}{x^U_0\rho_0}\right)\right)^{-1}-1\right)} < 0 \tag{79}$$

We show that $\mathrm{E}[s|s\in[b-2\delta,b]] - \mathrm{E}[s|s\in[b+\frac{\gamma}{x^U_0\rho_0}-2\delta, b+\frac{\gamma}{x^U_0\rho_0}]] + \frac{\gamma}{x^U_0\rho_0} > 0$, which establishes the desired result given (66) . The following notation will be useful to show this fact: for any $\gamma \in (0, \delta x^U_0\rho_\epsilon)$, let $b(\gamma)$ denote the solution to the existence equation (83). Let

$\gamma \in (0, \delta x_0^U \rho_\epsilon)$. It follows that

$$\begin{array}{rl} & \mathrm{E}[s|s \in [b(\gamma) + \frac{\gamma}{x_0^U \rho_0} - 2\delta, b(\gamma) + \frac{\gamma}{x_0^U \hat{\rho}_0}]] - \mathrm{E}[s|s \in [b(\gamma) - 2\delta, b(\gamma)]] \\ = & \int_0^{\frac{\gamma}{x_0^U \rho_0}} \frac{\partial}{\partial k} \mathrm{E}[s|s \in [b(\gamma) + k - 2\delta, b(\gamma) + k]] dk \\ < & \int_0^{\frac{\gamma}{x_0^U \rho_0}} dk \\ = & \frac{\gamma}{x_0^U \rho_0}, \end{array} \tag{80}$$

where the first equality is due to the Fundamental Theorem of Calculus and we use Lemma 3.6 to obtain the strict inequality. This yields the desired result.

Finally, for any b, $\mathbb{E}\left[s \left| b - 2\delta \le s \le b\right.\right] < \mathbb{E}\left[s \left| b + \frac{\gamma}{x_0^U \rho_0} - 2\delta \le s \le b + \frac{\gamma}{x_0^U \rho_0}\right.\right]$ if $\gamma > 0$, which establishes that $\mathbb{E}\left[s \left| b - 2\delta \le s \le b\right.\right] < \mu_0$ given (66) since $\Delta\left(b + \frac{\gamma}{x_0^U \rho_0} - 2\delta, b + \frac{\gamma}{x_0^U \rho_0}\right) < 0$ when $\gamma > 0$. $\square$

Proof of Proposition 8. We show that $\mathbb{E}[v - p_{PR}] > \mathbb{E}[v - p_{FR}]$. Using (14) and (17), and $f(\cdot)$ and $F(\cdot)$ to denote the pdf and cdf respectively of s, we get

$$\begin{array}{rl} & \mathbb{E}[v - p_{PR}] - \mathbb{E}[v - p_{FR}] \\ = & \int_{b-2\delta}^{b} \left(s - \mu_0 - \Delta\left(b + \frac{\gamma}{x_0^U \rho_0} - 2\delta, b + \frac{\gamma}{x_0^U \rho_0}\right) + \frac{\gamma}{x_0^U \rho_0}\right) f(s) ds \\ = & [F(b) - F(b - 2\delta)] \left(\mathrm{E}[s|s \in [b - 2\delta, b]] - \mathrm{E}[s|s \in [b + \frac{\gamma}{x_0^U \rho_0} - 2\delta, b + \frac{\gamma}{x_0^U \rho_0}]] + \frac{\gamma}{x_0^U \rho_0}\right). \end{array} \tag{81}$$

It follows from the proof of Proposition 7 that $\mathrm{E}[s|s \in [b - 2\delta, b]] - \mathrm{E}[s|s \in [b + \frac{\gamma}{x_0^U \rho_0} - 2\delta, b + \frac{\gamma}{x_0^U \rho_0}]] + \frac{\gamma}{x_0^U \rho_0} > 0$. This yields the desired result.

Finally, using the fact that $p_{NA}(s) > p_{FR}(s)$ for all s yields $\mathbb{E}[v - p_{PR}] > \mathbb{E}[v - p_{FR}] > \mathbb{E}[v - p_{NA}]$ as desired. $\square$

A.4 Proofs for Section 5

Proof of Lemma 2. The proof is similar to that of Lemma 1. $\square$

Proof of Proposition 9 . Proceeding along the same lines as in section 3 shows that U-investor demand is given by

$$\theta^U(s) = \begin{cases} \gamma^{-1}(\hat{\rho}_0 + \rho_\epsilon)(\hat{\mu}_0(\zeta) + \frac{\rho_\epsilon}{\hat{\rho}_0 + \rho_\epsilon}(s - \hat{\mu}_0(\zeta)) - p) & \text{if } s \notin [b_\zeta - 2\delta, b_\zeta] \\ \gamma^{-1}\hat{\rho}_0\left(\hat{\mu}_0(\zeta) + \frac{\rho_\epsilon}{\hat{\rho}_0 + \rho_\epsilon}\Delta\left(b_\zeta - 2\delta + \frac{\gamma\theta^U(s)}{\hat{\rho}_0}, b_\zeta + \frac{\gamma\theta^U(s)}{\hat{\rho}_0}\right) - p\right) & \text{if } s \in [b_\zeta - 2\delta, b_\zeta], \end{cases} \tag{82}$$

the length of the interval of unrevealed signals is 2δ, and the existence of the interval follows from the existence of a solution to (24). The proof then follows reasoning similar to that for Proposition 5. $\square$

Proof of Proposition 10. We first show that $\frac{d}{d\zeta}b_\zeta = \frac{\rho_\zeta}{\hat{\rho}_0} > 0$ for any ζ. For any ζ, the equilibrium existence condition is

$$b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} - \hat{\mu}_0(\zeta) - \Delta_\zeta \left(b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} - 2\delta, b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} \right) = \delta - \frac{\gamma}{x_0^U \rho_\epsilon}. \tag{83}$$

From this it follows that $\frac{d}{d\zeta}b_\zeta = \frac{d}{d\hat{\mu}_0}b_\zeta \frac{d}{d\zeta}\hat{\mu}_0(\zeta)$.

Let $f(\cdot|\zeta)$ denote the distribution of the private signal given public signal ζ. Note that

$$\frac{\partial}{\partial \hat{\mu}_0(\zeta)} f(s|\zeta) = f(s|\zeta) \left(\frac{s - \hat{\mu}_0(\zeta)}{\sigma_s^2} \right) \tag{84}$$

where $\sigma_s^2 = \hat{\rho}_0^{-1} + \rho_\epsilon^{-1}$.

Moreover, $\hat{\mu}_0(\zeta) + \Delta_\zeta \left(b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} - 2\delta, b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} \right)$ is differentiable in $\hat{\mu}_0(\zeta)$ with

$$\begin{aligned}
& \frac{\partial}{\partial \hat{\mu}_0(\zeta)} \left(\hat{\mu}_0(\zeta) + \Delta_\zeta \left(b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} - 2\delta, b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} \right) \right) \\
= \ & \frac{\left(\int_{b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} - 2\delta}^{b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0}} f(s|\zeta) ds \right) \left(\frac{\partial}{\partial \hat{\mu}_0(\zeta)} \int_{b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} - 2\delta}^{b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0}} s f(s|\zeta) ds \right) - \left(\int_{b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} - 2\delta}^{b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0}} s f(s|\zeta) ds \right) \left(\frac{\partial}{\partial \hat{\mu}_0(\zeta)} \int_{b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} - 2\delta}^{b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0}} f(s|\zeta) ds \right)}{\left(\int_{b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} - 2\delta}^{b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0}} f(s|\zeta) ds \right)^2} \\
= \ & \frac{\mathbb{V}\left[s \middle| b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} - 2\delta, b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0}, \zeta \right]}{\sigma_{s\zeta}^2} \\
> \ & 0 \text{ for } b_\zeta > -\infty
\end{aligned} \tag{85}$$

where the last equality follows by using (84) and combining terms.

Using the implicit function theorem for (83) with (85) and the expression in the last line of (64) in the proof of Lemma 3.6, it follows that

$$\frac{d}{d\hat{\mu}_0(\zeta)} b_\zeta = -\frac{-\frac{\partial}{\partial \hat{\mu}_0} \mathbb{E}[s | s \in [b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} - 2\delta, b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0}], \zeta]}{1 - \frac{\partial}{\partial b} \mathbb{E}[s | s \in [b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0} - 2\delta, b_\zeta + \frac{\gamma}{x_0^U \hat{\rho}_0}], \zeta]} = 1. \tag{86}$$

Since $\frac{d}{d\zeta}\hat{\mu}_0(\zeta) = \frac{\rho_\zeta}{\hat{\rho}_0} > 0$, using the Fundamental Theorem of Calculus, we have

$$b_{\bar{\zeta}} - b_{\mu_0} = \int_{\mu_0}^{\bar{\zeta}} \frac{d}{d\zeta} b_\zeta d\zeta = \frac{\rho_\zeta}{\hat{\rho}_0}(\bar{\zeta} - \mu_0) > 0 \tag{87}$$

as desired.

Using $\hat{\mu}_0(\zeta) = \frac{\rho\mu_0 + \rho_\zeta \zeta}{\hat{\rho}_0}$ and using (83) to express $\psi(s,\zeta)$ (from (26)) in terms of $b_\zeta - \delta + \frac{\gamma}{x_0^U \rho_\epsilon}$ for $\zeta \in \{\bar{\zeta}, \mu_0\}$, we have that for any $s \in [b_{\mu_0}, b_{\bar{\zeta}}]$,

$$p_{PR}(s,\bar{\zeta}) - p_{PR}(s,\mu_0) = \frac{\rho_\zeta}{\hat{\rho}_0 + \rho_\epsilon}(\bar{\zeta} - \mu_0) + \frac{\rho_\epsilon}{\hat{\rho}_0 + \rho_\epsilon}\left(b_\zeta + \delta - \frac{\gamma}{x_0^U \rho_\epsilon} - s\right). \tag{88}$$

and for any $s \in [b_{\mu_0} - 2\delta, b_{\bar{\zeta}} - 2\delta]$,

$$p_{PR}(s,\bar{\zeta}) - p_{PR}(s,\mu_0) = \frac{\rho_\zeta}{\hat{\rho}_0 + \rho_\epsilon}(\bar{\zeta} - \mu_0) + \frac{\rho_\epsilon}{\hat{\rho}_0 + \rho_\epsilon}\left(s - b_{\mu_0} - \delta + \frac{\gamma}{x_0^U \rho_\epsilon}\right). \tag{89}$$

Substituting $s = b_{\mu_0}$ in the RHS of (88), which is decreasing in s, shows that $p_{PR}(s,\bar{\zeta}) - p_{PR}(s,\mu_0) < 0$ and substituting $s = b_\zeta - 2\delta$ in the RHS of (89), which is increasing in s, shows that $p_{PR}(s,\bar{\zeta}) - p_{PR}(s,\mu_0) < 0$ under (29).

On the other hand, for any $s \in [b_{\mu_0}, b_{\bar{\zeta}}] \cup [b_{\mu_0} - 2\delta, b_{\bar{\zeta}} - 2\delta]$,

$$p_{FR}(s,\bar{\zeta}) - p_{FR}(s,\mu_0) = \frac{\rho_\zeta}{\rho_\epsilon}(\bar{\zeta} - \mu_0) > 0 \tag{90}$$

□

A.5 Proofs for Section 6.

Proof of Proposition 11. If $\gamma > x_0^U \delta\rho_\epsilon$ the price function

$$p_{FR}(s) = \frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} - \frac{\gamma}{\rho_0 + \rho_\epsilon} - x_0^I \frac{\delta\rho_\epsilon}{\rho_0 + \rho_\epsilon} \tag{91}$$

is a fully-revealing REE price function and if $0 \leq \gamma \leq x_0^U \delta\rho_\epsilon$ then

$$p_{FR}(s) = \frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} - \frac{\gamma}{x_0^U(\rho_0 + \rho_\epsilon)}. \tag{92}$$

is a fully revealing REE price function. □

Proof of Proposition 12. Suppose the I traders' have a symmetric linear strategy $\hat{\theta}^I_{NA}(s,p) = A_I + B_I s - C_I p$. Since I traders do not make any inference from price, the optimal portfolio for each I trader is $\frac{\mathbb{E}[v|s]-p}{\gamma \mathbb{V}[v|s]} = \frac{\rho_0+\rho_\epsilon}{\gamma}\left[\frac{\rho_0\mu_0+\rho_\epsilon s}{\rho_0+\rho_\epsilon} - p\right]$. Comparing this to the linear strategy yields $A_I = \frac{\rho_0\mu_0}{\gamma}, B_I = \frac{\rho_\epsilon}{\gamma}, C_I = \frac{\rho_0+\rho_\epsilon}{\gamma}$.

Given the linear strategy of I traders, the market clearing condition yields $p = \frac{x^I_0 A_I - 1}{x^I_0 C_I} + \frac{B_I}{C_I}s + \frac{x^U_0}{x^I_0 C_I}\hat{\theta}^U(p)$, where $\hat{\theta}^U(p)$ denotes the symmetric strategy of U traders. Given the linear form of p in the market clearing condition, the best reponse of U traders is a linear strategy $\hat{\theta}_U(p) = A_U - C_U p$.

Since U traders make inference from price, the optimal portfolio of the U investor is $\frac{\mathbb{E}[v|p]-p}{\gamma \mathbb{V}[v|p]}$. Using the linear strategies of I and U traders, the market clearing condition yields $s = \frac{x^I_0 C_I + x^U_0 C_U}{x^I_0 B_I}p - \frac{x^I_0 A_I + x^U_0 A_U - 1}{x^I_0 B_I}$, hence conditioning on the market clearing price is equivalent to conditioning on the signal. In particular, $\mathbb{V}[v|s] = (\rho_0+\rho_\epsilon)^{-1}$ and $\mathbb{E}[v|p] = \frac{\rho_0\mu_0}{\rho_0+\rho_\epsilon} + \frac{\rho_\epsilon}{\rho_0+\rho_\epsilon}\left(\frac{x^I_0 C_I + x^U_0 C_U}{x^I_0 B_I}p - \frac{x^I_0 A_I + x^U_0 A_U - 1}{x^I_0 B_I}\right)$

Using the above and the expressions for A_I, B_I, C_I yields $A_U = 1$ and $C_U = 0$ as required. □

Proof of Proposition 13. Suppose the I traders' have the symmetric strategy

$$\hat{\theta}^I(s,p) = \begin{cases} \underline{A}_I + B_I s - C_I p & \text{if } \frac{\underline{A}_I}{C_I} + \frac{B_I}{C_I}s > p \\ 0 & \text{if} \\ \bar{A}_I + B_I s - C_I p & \text{if } \frac{\bar{A}_I}{C_I} + \frac{B_I}{C_I}s < p \end{cases} \tag{93}$$

where $\bar{A}_I > \underline{A}_I$.

The optimal portfolio for each I trader is

$$\theta_I(s,p) = \begin{cases} \gamma^{-1}(\rho_0+\rho_\epsilon)\left(\rho_0\mu_0 + \rho_\epsilon(s-\delta) - p\right) & \text{if } \rho_0\mu_0 + \rho_\epsilon(s-\delta) > p \\ 0 & \text{if } \rho_0\mu_0 + \rho_\epsilon(s-\delta) \le p \le \rho_0\mu_0 + \rho_\epsilon(s+\delta) \\ \gamma^{-1}(\rho_0+\rho_\epsilon)\left(\rho_0\mu_0 + \rho_\epsilon(s+\delta) - p\right) & \text{if } \rho_0\mu_0 + \rho_\epsilon(s+\delta) < p. \end{cases} \tag{94}$$

. Comparing this to the strategy in (93) yields $\underline{A}_I = \frac{\rho_0\mu_0 - \delta\rho_\epsilon}{\gamma}, \bar{A}_I = \bar{A}_I = \frac{\rho_0\mu_0+\delta\rho_\epsilon}{\gamma}, B_I = \frac{\rho_\epsilon}{\gamma}, C_I = \frac{\rho_0+\rho_\epsilon}{\gamma}$. Note that this demand schedule is sensitive to s since $\rho_0\mu_0 + \rho_\epsilon(s+\delta)$ and $\rho_0\mu_0 + \rho_\epsilon(s-\delta)$ are monotone and linear in s.

Suppose U traders have a symmetric strategy $\hat{\theta}^U(p)$. In this case, the market clearing condition yields

$$p = \begin{cases} \frac{x_0^I A_I - 1}{x_0^I C_I} - \frac{B_I}{C_I} s - x_0^U \hat{\theta}^U(p) & \text{if } \frac{A_I}{C_I} + \frac{B_I}{C_I} s > p \\ \left(\hat{\theta}^U\right)^{-1} \left(\frac{1}{x_0^U}\right) & \text{if } \frac{A_I}{C_I} + \frac{B_I}{C_I} s \le p \le \frac{\bar{A}_I}{C_I} + \frac{B_I}{C_I} s \\ \frac{x_0^I \bar{A}_I - 1}{x_0^I C_I} - \frac{B_I}{C_I} s - x_0^U \hat{\theta}^U(p) & \text{if } \frac{\bar{A}_I}{C_I} + \frac{B_I}{C_I} s < p. \end{cases} \tag{95}$$

If $\frac{A_I}{C_I} + \frac{B_I}{C_I} s > p$ or $\frac{\bar{A}_I}{C_I} + \frac{B_I}{C_I} s < p$, then the linearity of p in s in (95) implies that the optimal portfolio of each the U investor is $\frac{\mathbb{E}[v|p]-p}{\gamma \mathbb{V}[v|p]}$ since U traders make inference from the price. Reasoning similarly to the proof of Proposition 12 shows that p reveals s and market clearing would require $p = \frac{\rho_0 \mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} - \frac{\gamma + x_0^I \delta \rho_\epsilon}{\rho_0 + \rho_\epsilon}$. However, if $\gamma \le x_0^I \delta \rho_\epsilon$, then at this price $\frac{A_I}{C_I} + \frac{B_I}{C_I} s \le p$ or $\frac{\bar{A}_I}{C_I} + \frac{B_I}{C_I} s \ge p$, which rules out an equilibrium with $\hat{\theta}^I(s,p) \ne 0$ for any s.

There are two price functions which lead to market clearing with $\theta^I(s,p) = 0$ and satisfy the requirement that $\frac{\bar{A}_I}{C_I} + \frac{B_I}{C_I} s \le p \le \frac{A_I}{C_I} + \frac{B_I}{C_I} s$. These are the linear fully revealing price function (17) and the non-linear and discontinuous partially revealing price function (14).

With the linear price function (17), U traders' optimal portfolio is $\frac{\mathbb{E}[v|p]-p}{\gamma \mathbb{V}[v|p]}$ since U traders make inference from the price. Given this price function and the optimal portfolio of U traders, the strategy $\hat{\theta}^U(p) = \frac{1}{x_0^U}$ is U traders' best response as required.

With the non-linear price function (14), U traders' optimal portfolio is

$$\begin{cases} \frac{\mathbb{E}[v|p]-p}{\gamma \mathbb{V}[v|p]} & \text{if } p \ne \bar{p} \\ \gamma^{-1} \rho_0 \left(\mu_0 + \frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon} \Delta \left(b - 2\delta + \frac{\gamma \theta^U}{\rho_0}, b + \frac{\gamma \theta^U}{\rho_0} \right) - p \right) & \text{if } p = \bar{p} \end{cases} \tag{96}$$

where $\bar{p} = \mu_0 + \frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon} \left(\Delta \left(b + \frac{\gamma}{x_0^U \rho_0} - 2\delta, b + \frac{\gamma}{x_0^U \rho_0} \right) - \frac{\gamma}{x_0^U \rho_0} \right) - \frac{\gamma}{x_0^U} \frac{1}{\rho_0 + \rho_\epsilon}$.

Given the price function and the optimal porfolio of U traders, the strategy $\hat{\theta}(p) = \frac{1}{x_0^U}$ is U traders' best response. □

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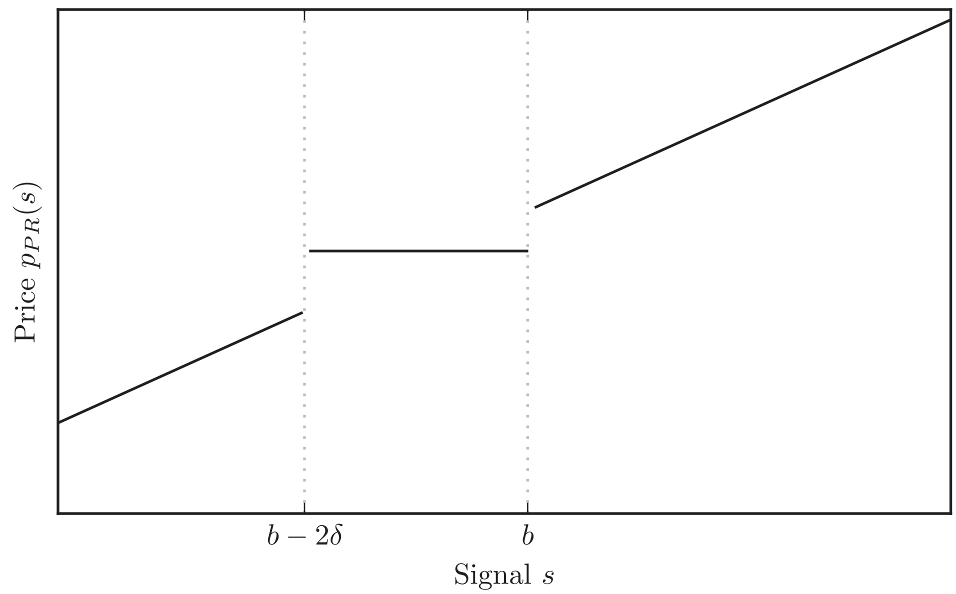
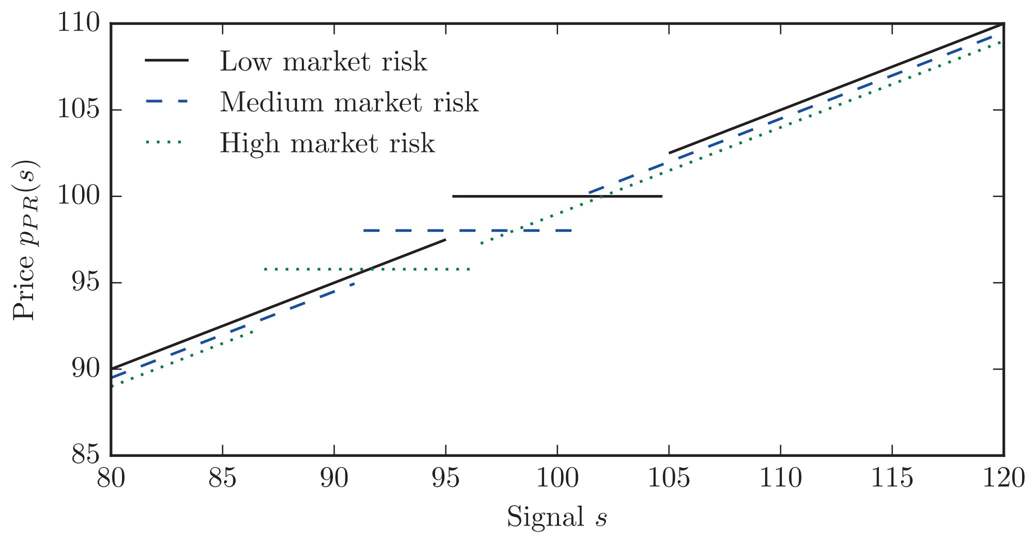


(a) Partial revelation price function p_{PR}



(b) For differing levels of risk aversion $0 \leq \gamma \leq x_0^U \delta \rho_\epsilon$

Figure 1: Equilibrium price function

The parameter values for all plots in the paper are $\rho_0 = 0.1$, $\rho_\epsilon = 0.1$, $\mu_0 = 100$ and $\delta = 5$. For each market risk scenario, $x_0^U = 0.99$. Low, medium, and high market risk correspond to $\gamma = 0$ (risk-neutral), $\gamma = 0.1$ and $\gamma = 0.2$ respectively.

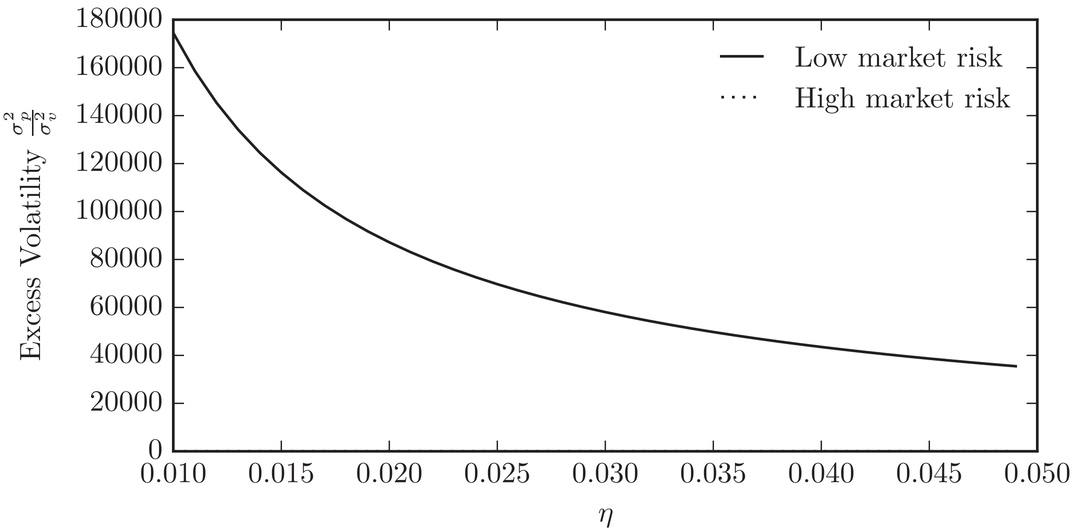


Figure 2: Relative volatility of prices with $\sigma^2_{p_{PR},\eta} = \text{Var}(p_{PR}|b - \eta < s < b + \eta)$, $\sigma^2_{p_{FR},\eta} = \text{Var}(p_{FR}|b - \eta < s < b + \eta)$ for small $\eta > 0$. Parameter values are as in Figure 1(b).

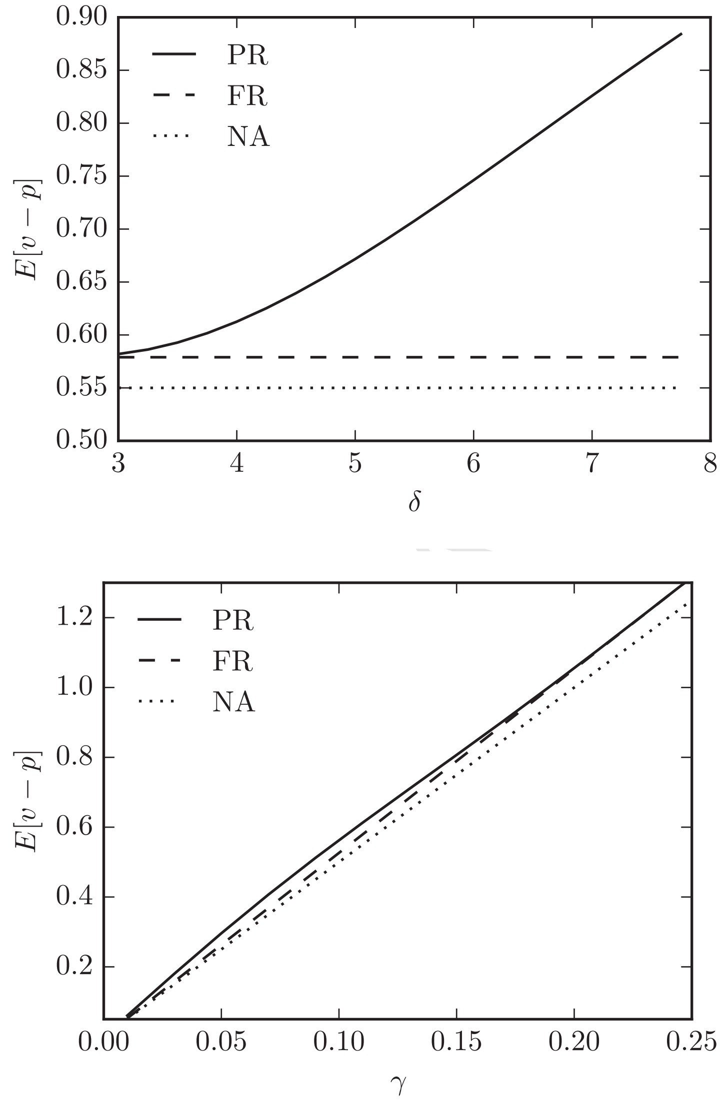


Figure 3: The equity premium as a function of δ and γ.

All figures in this section have the following parameter values: $x_0^U = 0.95$, $\mu_0 = 100$, $\rho_0 = \rho_\epsilon = 1/10$. Plots with varying γ have $\delta = 4$ and plots with varying δ have $\gamma = 0.11$.

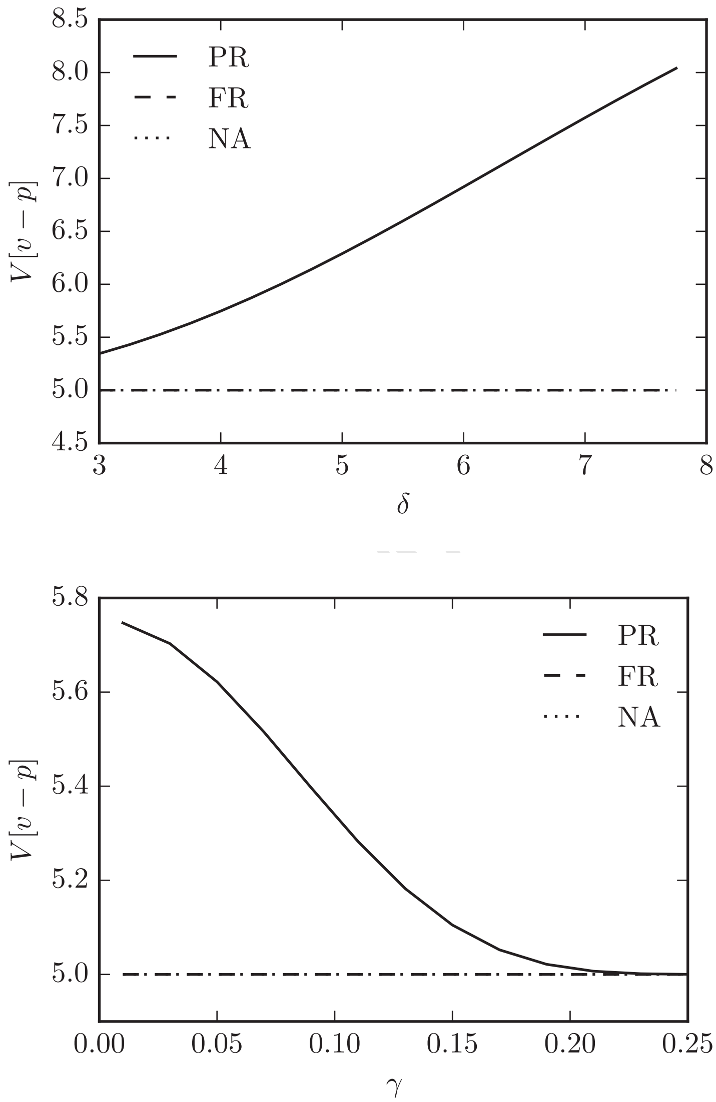


Figure 4: Return variance as a function of δ and γ.

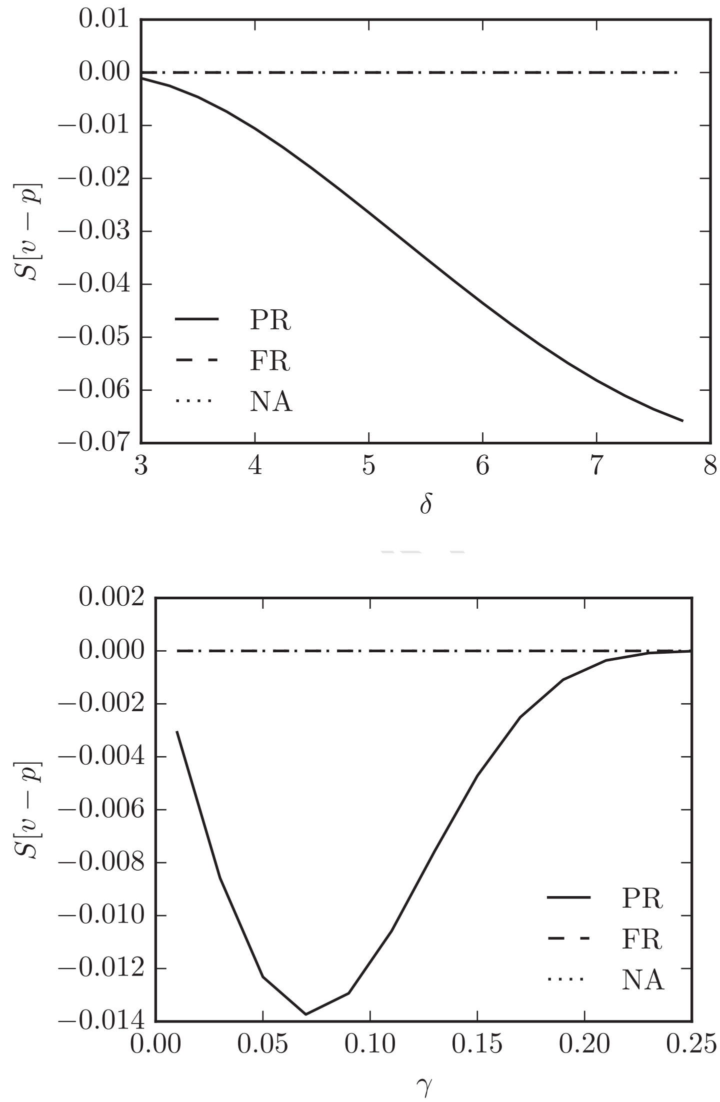


Figure 5: Return skewness as a function of δ and γ

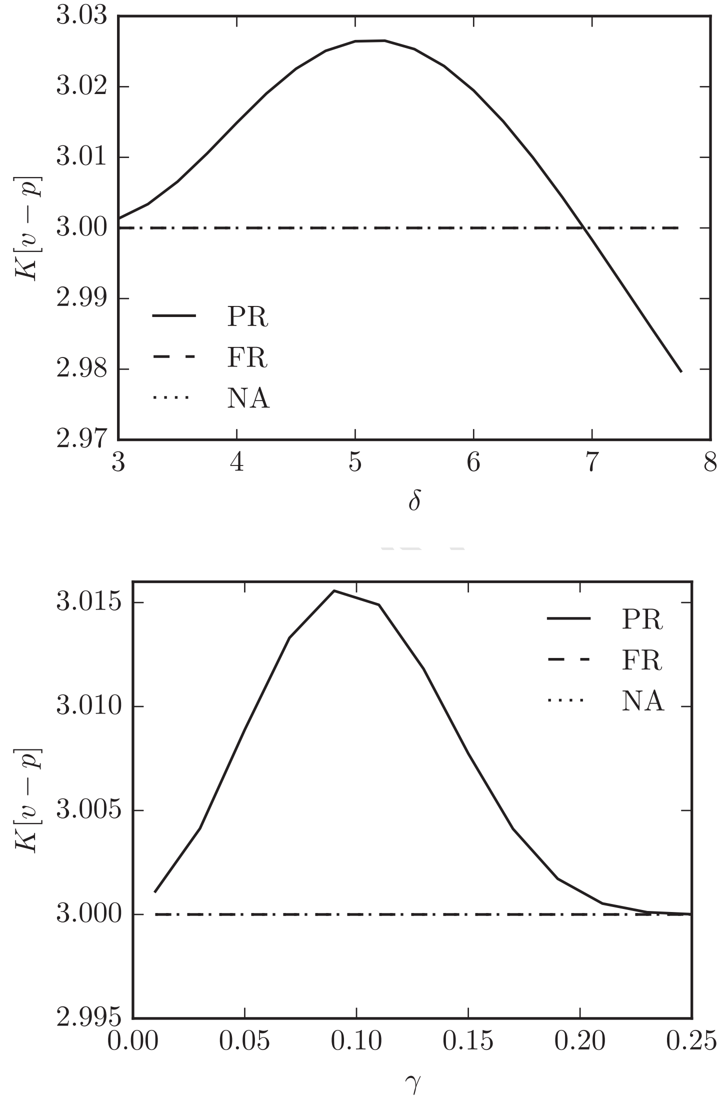


Figure 6: Return kurtosis as a function of δ and γ.

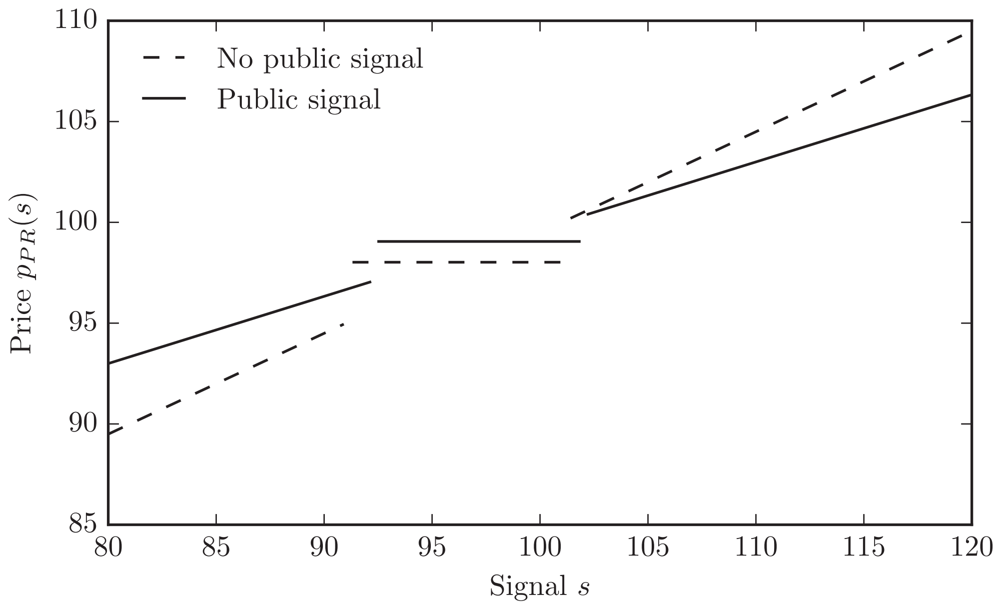


Figure 7: Effect of a public signal ζ that confirms the mean ($\zeta = \mu_0$). Parameters are the same as those of Figure 1(b) with $\gamma = 0.1$. The public signal has precision $\rho_\zeta = 0.1$.