Improving Regulatory Market Risk Management with Heuristic Algorithms

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To my loving wife, Daniela, and our beloved children...

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Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements.

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Publications

Some of the research done in this thesis has been published or are under revision in peer-reviewed journals and conference proceedings:

1. Kleinknecht, M.; Ng, W. L. (2014): "Improving Portfolio Risk Profile with Threshold Accepting", presented at IEEE CIFEr 2014 conference, IEEE CIFEr 2014 conference proceeding, pp. 92-99, presented in Chapter 3

2. Kleinknecht, M.; Ng, W. L. (2015): "Minimising Basel III Capital Requirements with Unconditional Coverage Constraint", Intelligent Systems in Accounting, Finance and Management, 22(2), pp. 1-19, presented in Chapter 3

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Abstract

Recent changes in the regulatory framework for banking supervision increase the regulatory oversight and minimum capital requirements for financial institutions. In this thesis, we research active portfolio optimisation techniques with heuristic algorithms to manage new regulatory challenges faced in risk management.

We first study if heuristic algorithms can support risk management to find global optimal solutions to reduce the regulatory capital requirements. In a benchmark comparison of variance, Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) objective functions combined with different optimisation routines, we show that the Threshold Accepting (TA) heuristic algorithm reduces the capital requirements compared with the Trust-Region (TR) local search algorithm.

Secondly, we introduce a new risk management approach based on the Unconditional Coverage test to optimally manage the regulatory capital requirements, while avoiding to over- or underestimate the portfolio risk. In an empirical analysis with TA and TR optimisation, we show that our new approach successfully optimises the portfolio risk-return profile and reduces the capital requirements.

Next, we analyse the effect of different estimation techniques on the capital requirements. More specifically, empirical and analytical VaR and CVaR estimation is compared with a simulation-based approach using a multivariate GARCH process. The optimisation is performed using the Population-Based Incremental Learning (PBIL) algorithm. We find that the parametric and empirical distribution assumption generate similar results and neither of them clearly outperforms the other. However, portfolios optimised with the simulation approach reduce the capital requirements by about 11%.

Finally, we introduce a global VaR and CVaR hedging approach with multivariate GARCH process and PBIL optimisation. Our hedging framework provides a self-financing hedge that reduces transaction costs by using standardised derivatives. The empirical study shows that the new approach increases the stability of the portfolio while avoiding high transaction costs. The results are compared with benchmark portfolios optimised with a Genetic Algorithm.

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Chapter 1

Introduction

In this chapter, we give a general introduction to the research work in this thesis. We start with an overview of relevant research and market changes that lead to the rationale of our study. Then, the research hypotheses and objectives are presented. The last section of this chapter describes the thesis structure and our contributions to the literature and practice.

1.1 Overview

Financial regulations have been subject to several key changes over the last years. The major focus of these changes is to provide authorities with proper instruments to ensure the stability of financial institutions and prevent them from corporate bankruptcy. As learned from the financial crisis that started in 2007, the bankruptcy of systemically important financial institutions can result in massive disruptions in the entire economy and are thus, too expensive for authorities. The only alternative is a public bailout, which creates moral hazard and does not circumvent banks from taking less risk.

Existing financial regulations did not provide regulators with the appropriate instruments to intervene and to prevent bank failure. To improve the financial regulatory oversight and to abolish several weaknesses in corporate risk management, as revealed in the financial crisis, the Basel Committee on Banking Supervision agreed on the revision of the so called Basel II framework. Thus, the Committee introduced an enhanced regulatory framework, known as Basel III, which is to be fully finalised by the end of 2019.

In general, the Basel III framework consists of three pillars that aim to provide higher transparency, improve the banks' risk management and the ability to absorb financial and economic market shocks. The first pillar regulates the minimum capital a bank is required to provide for its Risk Weighted Assets (RWA). With the latest revision of the Basel III framework, enhanced minimum capital, liquidity and leverage requirements were introduced to pillar one. Pillar two provides key principles of supervisory review and risk management guidance, while the third pillar discusses enhanced disclosures. Significant changes in the regulatory framework can be seen in pillar one and more specifically in the minimum capital requirements for market risk framework and the regulation of "over-the-counter" (OTC) derivatives, which are customised derivative contracts.

The minimum capital requirements for market risk framework regulates the market risk charges a bank has to provide to cover losses arising from market price movements in the trading and banking book. The Committee offers the bank two general methodologies to determine its minimum capital requirements for market risk: the standardised approach and the internal models approach. For the standardised approach there are three main models that can be used: (i) sensitivity bared model, (ii) default risk charge model and (iii) residual risk model.

The internal models approach is provided as an alternative to the standardised approach. It offers banks the opportunity to use their own internal risk models to calculate the minimum capital requirements for market risk and is therefore widely used in practice and literature. For these reasons, Chapters 3 and 4 of this thesis concentrate on the internal models approach. In the internal models approach the bank has to estimate potential losses for each risk factor for the trading book (e.g. equity risk, interest rate risk, credit spread risk, foreign exchange risk and commodities risk) and the banking book (e.g. foreign exchange risk and commodity risk) instruments. The default risk needs separate modelling (Basel Committee on Banking Supervision, 2016).

For the risk estimation process, the bank is required to calculate the Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), often also referred to as Expected Shortfall, of the trading desks. The literature presents several research papers that study VaR and CVaR backtesting models to determine the daily capital charges that need to be reported to the authorities. These papers study the influence of estimation techniques and distribution assumptions on the daily VaR and CVaR level (see Berkowitz and O'Brien, 2002; Kim et al., 2011; Weng and Trück, 2011; Uylangco and Li, 2016; Kellner and Rösch, 2016).

The backtesting of VaR and CVaR models is an important procedure in financial regulations, as the results are reported to the regulatory authorities to determine the minimum capital requirements for market risk. Backtesting is an important feature for post-trade analysis, however, it has no active influence on the pre-trade portfolio composition.

In this thesis, we concentrate on another research path that studies portfolio optimisation techniques to manage the regulatory minimum capital requirements. We classify the existing literature in this research area into active and passive portfolio optimisation techniques to manage the capital charges. Active methods are (i) constraint and (ii) objective function based techniques that directly influence the pre-trade portfolio composition.

The first set of active optimisation methods are constraint based and can easily be added to the portfolio optimisation process. The effects of a regulatory VaR constraint to the mean-variance efficient frontier are presented in the work of Sentana (2003). Cuoco and Liu (2006) optimises the mean-variance utility function subject to a capital requirements constraint that is based on the internal models approach capital charge calculation.

The second subsection of active regulatory based portfolio optimisation methods directly improve the capital charge for market risk via the portfolio objective function. Santos et al. (2012) introduce a minimum regulatory capital objective function that is a direct replication of the internal models approach minimum capital charge for market risk formula. A Multi-Objective Evolutionary Algorithms (MOEA) approach to minimise the regulatory capital requirements and maximise the expected return of a trade portfolio is proposed by Drenovak et al. (2017). However, optimisation methods that directly improve the minimum capital requirements of a portfolio give the risk manager no freedom over the primary objective function.

The dynamic decision rule, as suggested by McAleer et al. (2010), is classified as a passive approach to manage the regulatory capital charges for market risk of a portfolio. The model is intended to assist risk managers to identify if the current portfolio under- or overestimates the risk of the investment. However, it does not directly influence the asset selection during the portfolio optimisation process and assumes that the risk manager is willing to act more conservatively when the number of daily VaR violations is high and to behave more aggressively when the number of violations is small. The passive portfolio optimisation approach gives the risk manager more freedom to decide on how to increase profitability and reduce the risk of a trading portfolio. A drawback of such an approach, however, is that it does not suggest how to change the portfolio composition to adjust it in case of an under- or overestimation of risk. This thesis therefore concentrates on the active regulatory portfolio management under consideration of an optimal risk assessment.

The revision of the Basel III framework introduces significant reforms to the OTC markets to reduce the systemic risk associated with OTC derivatives. The reforms concentrate on five points: (i) trade standardised OTC contracts on exchanges, (ii) clear standardised OTC derivatives through a central counterparty, (iii) report OTC contracts to trade authorities, (iv) increase capital requirements for non-centrally cleared derivatives, and (v) add new margin requirements for non-centrally cleared contracts (Bank for International Settlements, 2015). Points (iv) and (v) significantly increase the costs for non-standardised derivatives (Bank for International Settlements, 2013) and therefore, led to a steep raise in standardised OTC contracts volume (Financial Stability Board, 2016). OTC derivatives are mainly used for the purpose of hedging a firms risk exposure against unfavourable movements in assets prices, commodities prices, foreign exchange rates or interest rates.

There are several hedging techniques proposed in the literature that can be categorised into local and global hedging strategies. Local hedging frameworks aim to reduce the risk of a portfolio for (i) small changes in the underlying asset price (e.g. delta hedging or delta-gamma hedging) or (ii) until the next time step. To determine an optimal hedging strategy, local hedging techniques estimate hedge ratios using ordinary least squares (OLS) regression (Ederington, 1979), conditional heteroskedastic (see Cecchetti et al., 1988; Ortega, 2012; Badescu et al., 2014), error correction (Dark, 2015) or random coefficient (Bera et al., 1997) methods. The OLS estimation technique is criticised as it ignores the time varying structure of conditional distributions. This issue is solved by the other estimation techniques. However, as highlighted by Alexander et al. (2013), these local estimation methods generate high margin and transaction costs and are therefore often too expensive to implement in practice. This is especially true for GARCH estimated local hedging strategies.

An alternative to local hedging techniques and focus of this thesis, are global hedging methods. They optimise the risk associated with the terminal hedging error of the portfolio and the hedging instrument over the entire hedging period.

Several objective functions are discussed in the literature that can be used as risk measure in the hedging process. Quadratic error hedging (Schweizer, 1995) and the extended semi-quadratic error hedging (see Föllmer and Leukert, 2000; François et al., 2014) are risk measures often used in the literature. Quantile hedging (Föllmer and Leukert, 1999) maximises the probability that the terminal value does not exceed a certain threshold. An intuitive risk measure to use in global hedging is VaR, by definition (see Alexander et al., 2004; Cong et al., 2013). However, there are some pitfalls associated with the use of VaR as an objective function e.g. non-coherence and the disregard of losses exceeding the VaR confidence level. The use of CVaR as a risk measure helps to prevent these drawbacks. One of the first who apply a global CVaR hedge with linear optimisation algorithm to a multiple asset optimisation problem are Alexander et al. (2004). A dynamic programming solution to a global CVaR hedging function is proposed in Boda and Filar (2006). Melnikov and Smirnov (2012) provide an one-dimensional optimisation approach to a global CVaR hedge in continuous time by adopting a statistical hypothesis test suggested in Föllmer and Leukert (1999). In a more recent study, Godin (2016) minimises the CVaR of the terminal hedging error in discrete time, using a normal inverse Gaussian distribution to capture fat tails in the return distributions.

Even though, the majority of local hedging strategies with GARCH processes provide evidence that time-varying volatility models improve the efficiency of the hedging strategies (see e.g. Kroner and Sultan, 1993; Chan and Young, 2006), global hedging with conditional heteroskedastic processes is rarely discussed in the literature. To our best knowledge, the first study that applies a GARCH process to a global quadratic hedging problem is provided by Rémillard and Rubenthaler (2013). The results presented in their work show that quadratic error hedging with GARCH model outperforms delta-hedging strategies. The application of GARCH processes in global VaR and CVaR hedging techniques, however, has not been discussed so far and provides opportunities for further research.

1.2 Research Hypotheses and Objectives

The aim of this study is to develop active portfolio optimisation techniques that help the risk management of financial institutions to cope with challenges they encounter with the introduction of new regulatory market and credit risk requirements under the new Basel III regulations. This raises several research opportunities that are of interest both for academia and practice. This thesis aims to answer the following four research questions:

1. Complex VaR and CVaR objective functions can have multiple local extremes over the entire search space. Heuristic optimisation can be used to find an approximate solution to the search problem when other optimisation techniques fail to find an exact solution or are too slow. We are interested if heuristic algorithms can help to improve the trading desk management of a bank with respect to the regulatory minimum capital requirements for market risk. We focus on the question where banks directly optimise their portfolios for VaR and CVaR risk measures as they are part of the calculation of the minimum capital requirements. We identify if heuristic search methods contribute to an optimal portfolio solution with better out-of-sample risk and performance measures and regulatory capital requirements.

2. Banks are required to communicate their daily VaR estimates to the regulatory authorities to determine their regulatory capital charges. If the reported VaR is much higher than the actual losses, the banks' capital requirements and thus, the capital costs are too high and the bank gives away the opportunity for potential profits. Respectively, if the reported VaR is too low and the realised losses exceed the daily VaR level, the bank experiences higher multiplication factors in the minimum capital calculation and potentially a negative reputation in the public. Hence, we explore how a new active

portfolio management approach that we develop, helps to optimally manage the portfolio composition under consideration of regulatory market risk regulations.

3. How do different VaR and CVaR estimation techniques and underlying distribution functions in an active portfolio optimisation approach, influence the number of daily VaR violations and the minimum capital requirements? As highlighted by research papers on VaR and CVaR backtesting models to determine the banks' daily capital level, more advanced downside risk estimations methods have a high number of VaR and CVaR violations while these violations are less extreme when they occur. So far, however, little is known on how the risk measures, distribution functions and estimation techniques in the portfolio optimisation process effect the banks' regulatory requirements.

4. The standardisation of OTC contracts and the increase of transaction costs in derivatives trading caused by new credit risk regulations in the Basel III framework, require risk managers to identify new hedging techniques that can cope with these changes. Existing literature on local hedging shows that the introduction of time-varying volatility processes to local hedging methods outperform other estimation techniques. However, time-varying methods increase the number of necessary transactions and thus, transaction costs. We develop a new hedging approach with conditional heteroskedastic process and investigate its potential to successfully secure an investment with standardised derivatives while avoiding high regulatory capital charges and transaction costs.

Our research work provides new insights into the optimal management of regulatory requirements in the portfolio optimisation process and offers some innovative active optimisation techniques to cope with new challenges caused by the increased regulatory oversight. This research work is primarily interesting to risk, portfolio and trade managers, who are concerned with portfolio optimisation questions under consideration of new regulatory capital requirements for market and credit risk.

1.3 Thesis Structure and Contributions

Following, we present the thesis structure to address the research objectives presented in the Section 1.2 and discuss our contributions to the literature.

In Chapter 2, we begin by outlining the methodology of relevant portfolio risk measures and risk estimation techniques, and provide a hedging literature review.

Chapter 3 first analyses the effect of the Threshold Accepting heuristic algorithm on reducing the minimum capital requirements for market risk for variance, VaR and CVaR portfolio objective functions. The first part of Chapter 3 answers the first research questions in Section 1.2. Related literature (see Sentana, 2003; McAleer et al., 2010; Santos et al., 2012) apply local search algorithms to solve the portfolio objective functions. The main contribution of this analysis is to study the influence of heuristic optimisation on the regulatory capital requirements. In an empirical study, the out-of-sample portfolio statistics of Threshold Accepting optimised portfolios are compared with the Trust-Region local search algorithm. The results highlight the superiority of Threshold Accepting meta-heuristic algorithm to improve the efficiency of trading desk management with respect to regulatory capital charges. Secondly, Chapter 3 introduces a new active portfolio optimisation tool that addresses the second research question in Section 1.2. Our innovative portfolio management tool is based on the Unconditional Coverage test. The Unconditional Coverage constraint avoids over- and underestimation of the portfolio risk and maintains the number of daily VaR violations within optimal boundaries for an efficient capital requirements management. Our contribution extends relevant literature, such as Sentana (2003), McAleer et al. (2010), Santos et al. (2012) and Drenovak et al. (2017), by providing an active portfolio optimisation tool that avoids the drawbacks of a VaR constraint and gives the risk manager control over the primary objective function.

In Chapter 4, we investigate the effect of simulation-based VaR and CVaR estimation with multivariate GARCH process on the daily VaR violations and capital requirements. This chapter looks into the third research question in Section 1.2. In their work, Winker and Maringer (2007) show that objective functions with underlying empirical distribution, on average, exhibit a higher number of daily VaR violations compared with portfolios with normal distribution assumption. Moreover, Winker and Maringer (2007) find that the magnitude of losses exceeding VaR is higher for portfolios with empirical VaR objective function.

Chapter 4 first extends the analysis of Winker and Maringer (2007) to VaR and CVaR objective functions. As a first contribution, we find that empirical CVaR objective functions also have a higher number of daily VaR violations and that these violations have a higher magnitude than losses of CVaR objective functions with normal distribution. As a second contribution, our results in Chapter 4 show that we can reduce the average number and degree of daily VaR violations for empirical VaR and CVaR objective functions, if a Monte Carlo simulation with GARCH-DCC process is used. The portfolio optimisation is performed with the Population-Based Incremental Learning heuristic algorithm, which in case of simulation-based optimisation is more efficient than the Threshold Accepting algorithm used in Chapter 3. The findings in this chapter help to identify the most promising optimisation approach to manage the daily VaR violations and capital requirements.

In Chapter 5, we introduce a global VaR and CVaR hedging approach with multivariate GARCH process to address the forth research question in Section 1.2. The first global hedging strategy with GARCH process is introduced by Rémillard and Rubenthaler (2013) for a global quadratic hedging approach. They demonstrate that global quadratic hedging with GARCH process is superior to a delta hedging technique.

Chapter 5 extends the global VaR and CVaR hedging literature by introducing a multivariate GARCH process to the optimisation approach. Our innovative approach contributes to the hedging literature by securing several instruments with one put option using a time-varying covariance process. Another contribution to the existing global hedging literature, such as Föllmer and Leukert (1999), Alexander et al. (2004), Melnikov and Smirnov (2012), Cong et al. (2013), Rémillard and Rubenthaler (2013), Cong et al. (2014) and Godin (2016), is the practical implementation of standardised derivatives in our hedging approach. Other hedging strategies require options with a specific strike price and maturity to secure the investment. Often, however, there is no standardised options with the exact specification, which has negative impacts on the existing hedging strategies. We show that our hedging approach does not require the option to have specific properties. We find that even larger deviations from the optimal option specifications have no negative impact on the success of our hedging approach. As a third contribution, we provide a detailed comparison between the PBIL and a Genetic Algorithm heuristic algorithm to solve the global VaR and CVaR hedging approach with multivariate GARCH process. Our research extends the existing PBIL literature by applying the algorithm to an optimisation approach with derivatives instruments. In our analysis, we find that the PBIL algorithm is more efficient than a commonly used Genetic Algorithm.

Chapter 5 demonstrates that our global VaR and CVaR hedging approach with multivariate GARCH process opens a promising new research path in global VaR and CVaR hedging. We show that our hedging approach is capable to handle increasing regulatory oversight and capital requirements in OTC markets.

Chapter 6 summarises the research done in this thesis, describes the contributions of the work in more detail and concludes with suggestions for further research.

Chapter 2

Review of Risk Management and Portfolio Optimisation

This chapter outlines the main risk management and portfolio optimisation techniques used in this thesis. Firstly, we review portfolio risk measures that are used throughout our research work. Then, we present methodology described in the literature to evaluate risk. Finally, we present hedging strategies discussed in the literature.

2.1 Risk Measures and Portfolio Optimisation

One of the most common methods used in risk management and portfolio optimisation was introduced by Markowitz (1952). His idea is to measure portfolio risk under consideration of diversification effects between assets. The portfolio risk is determined by minimising the portfolio standard deviation

$$\sigma^{port} = \sqrt{w^{\intercal}\Omega w} \tag{2.1}$$

subject to the constraints

$$\begin{aligned} w_{i_{lb}} \leq w_i \leq w_{i_{ub}}, \forall i \\ \mu^{port} \geq \mu^{target} \\ \kappa_{lb} \leq \kappa \leq \kappa_{ub} \\ w^{\intercal}\mathbf{1} = 1 \end{aligned}$$

where w is an $M \times 1$ vector of weights and M is the number of assets in the investment universe. The asset weight w_i has to maintain a lower $w_{i_{lb}}$ and upper $w_{i_{ub}}$ bound constraint, where $i = 1...M$. Ω is a $M \times M$ covariance matrix of the assets returns and the portfolio return is given by $\mu^{port} = w^{\intercal}r$, where r is an $M \times 1$ vector of expected asset returns. The target portfolio return is defined as μ^{target}. Moreover, κ is the cardinality of the portfolio, that is, the number of instruments in the portfolio. The cardinality is constrained within a lower (κ_{lb}) and upper bound (κ_{ub}). In this study, we consider long portfolios only and thus, the sum of w has to be one. The short selling restriction as this changes the regulatory minimum capital requirements for market risk calculation and makes search space more complex.

Despite variance being probably one of the most common risk measures in risk management, it is not always the most appropriate one to use. Variance only measures the squared deviation of a variate from its mean. However, the risk measure does not consider higher moments like skewness or kurtosis of the underlying return distribution. It therefore can only provide a reliable estimation of the risk if the returns are normally distributed. Several alternative risk measures are proposed in the literature (see e.g. Roy, 1952; Markowitz, 1959; Bawa, 1975) to account for non-normal underlying distribution assumptions and to provide a better measure of risk.

Another risk measure that became very popular in market regulations to risk managers is VaR, commonly understood as the maximum loss in a risk position not exceeded with a certain probability for a holding period. The calculation of VaR relies on the assumption about the distribution of the returns. An intuitive guess is to assume that the future returns are best described by the empirical return distribution. This is often suggested in the quantitative risk management literature (see e.g. Jorion, 2006; Pritsker, 1997; Lucas and Klaasen, 1998). The calculation of VaR with an empirical distribution is easy to implement and has the advantage that dependence across assets are already accounted for. The VaR objective function with underlying empirical distribution H is simply the α-quantile of the empirically distributed returns

$$VaR_{H\alpha} = Q_H(\alpha) \tag{2.2}$$

(Acerbi and Tasche, 2002a). Q denotes the quantile function.

Several alternative tail risk measures have been proposed in the financial literature (see e.g. Roy, 1952; Markowitz, 1959; Bawa, 1975; Acerbi and Tasche, 2002b) with CVaR being the most common expected tail risk measure in quantitative risk management. CVaR is defined as the expected shortfall for all losses exceeding VaR (Acerbi and Tasche, 2002a). Thus, the CVaR objective function with empirical return distribution is

$$CVaR_{H\alpha} = E(\mu^{port} | \mu^{port} \leq VaR_{H\alpha}). \tag{2.3}$$

Future returns are not always best described by their empirical return distribution. Winker and Maringer (2007) show that bond portfolios optimised with VaR, based on an empirical distribution, have serious hidden risk in the out-of-sample period. Portfolios optimised with empirical distribution violate an expected VaR level more often than portfolios optimised with normal distribution. Winker and Maringer (2007) conclude that empirical distributions are good in measuring VaR but are not expedient for the optimisation process. An alternative is to assume normally distributed returns for the VaR optimisation.

Under the assumption that the returns are standard normally distributed, VaR is defined as the inverse of the standard normal distribution function at α

$$VaR_{N\alpha} = -\mu^{port} + \sigma^{port} Q_N(\alpha) \tag{2.4}$$

(Danielsson, 2011).

For a CVaR objective function with underlying standard normal distribution we calculate the density function (φ) for the α-quantile of a standard normal distribution

$$CVaR_{N\alpha} = -\mu^{port} + \sigma^{port} \frac{1}{\alpha} \varphi(Q_N(\alpha)) \tag{2.5}$$

(Rockafellar and Uryasev, 2000).

A drawback of assuming normally distributed returns are specification errors. However, Winker and Maringer (2007) show in their findings that the assumption of normal return distribution in VaR optimisation reduces the hidden risk, which is the risk of exceeding a defined VaR limit in the out-of-sample.

In the following chapters we use the downside risk measures VaR and CVaR to improve the risk management of market portfolios. So far little research has been done on how the underlying return distribution assumption influences the risk management. Thus, we will concentrate on VaR and CVaR risk optimisation with underlying normal and empirical distribution assumption.

2.2 Risk Evaluation Methods

Modelling the volatility of a time-series is an important objective in regulation, valuation, portfolio and risk management. A good volatility forecast is an important tool to improve decision making e.g. in reducing capital requirements, as Chapter 4 demonstrates, or in hedging the expected loss of a portfolio, as Chapter 5 shows.

A basic statistical method for forecasting volatility and correlation of time series is the equally weighted average method. It is one of the first methods used to forecast average volatility over a number of days by simply calculating the equally weighted average of the empirical asset returns. The equally weighted average method can be very inaccurate if the sample size is short.

A better method to forecast volatility and correlation is the equally weighted moving average. The forecast volatility is calculated by taking the equally weighted average of a fixed sample size which is rolled through time. With each new return the oldest data point drops out of the sample. One pitfall of this method, however, is that jumps in the data can lead to an over- or underestimation of the long-term volatility forecast. Moreover, by equally weighting the data and rolling the sample forward through time, jumps in the data can effect the forecast volatility when they enter and exit the sample. To avoid large changes in the forecast volatility and correlation just because data points drop out of the sample by rolling through time, Roberts (1959) introduced the exponentially weighted moving average (EWMA). In his work, he proposes to give more weight on more recent observations and less weight on old return. Thus, the impact of a jump in the data on the forecast average volatility and correlation decreases in time.

The aforementioned moving average models assume the returns to be independent and identically distributed (i.i.d.). Moreover, they assume volatility to be constant over time and only change depending on the estimated sample data. However, as shown by the work of Mandelbrot (1963) or more recently by Cont (2007), large changes in asset prices often cluster together as do small changes. Thus, the volatility of financial asset returns is not i.i.d. but shows a clustering behaviour.

To create dynamic volatility forecasts and to account for volatility clustering behaviour, Engle (1982) introduced the autoregressive conditional heteroscedasticity (ARCH) model. The model assumes that the variance depends on a lagged squared white noise stochastic process which is conditional on the historical data, with mean zero and uncorrelated variances. The ARCH model improves the quality of the volatility forecast. However, it does not consider the empirical conditional volatility and its impact on the forecast conditional volatility.

In his paper, Bollerslev (1986) proposes a generalisation of the ARCH model named generalised autoregressive conditional heteroscedasticity (GARCH), which allows past conditional volatility to influence the forecast conditional volatility. Over the long-run, without jumps in the sample, the conditional volatility converges back to the unconditional volatility of the GARCH model. The unconditional volatility corresponds to the long term average volatility of the conditional volatility.

Several other GARCH variants are proposed in the literature. Glosten et al. (1993) introduces the GJR-GARCH or Threshold GARCH (TGARCH) to forecast volatility. This model accounts for asymmetric effects of price movements on the forecast volatility. To avoid the possibility of negative variance forecasts and to consider asymmetric effects, an alternative is to use the exponential GARCH (EGARCH) model by Nelson (1991). Other non-linear (NGARCH) or non-linear asymmetric (NAGARCH) models are proposed by Higgins and Bera (1992) and Engle and Ng (1993), respectively. To list all GARCH variants is beyond the scope of this thesis. We refer the interested reader to the work of Bollerslev (2009) for a detailed glossary to ARCH and GARCH models.

The aforementioned univariate volatility estimation models are used to capture volatility clustering behaviour. In volatile market situations, however, correlation between assets can also increase as prices tend to move in the same direction. To capture these correlation clustering effects multivariate GARCH models can be used.

In Chapters 4 and 5 we apply multivariate GARCH models to reduce the capital requirements and to hedge the expected absolute loss of a portfolio, respectively. There are several multivariate GARCH models discussed in the literature. A useful classification is provided by Silvennoinen and Teräsvirta (2009) who distinguishes between (i) direct multivariate extensions of univariate GARCH models; (ii) factor and orthogonal models; (iii) conditional correlation models; (iv) semi- or non-parametric models to estimate dependencies. An alternative classification is given by Bauwens et al. (2006). Models in the first category directly model the conditional covariance between assets. These models are multivariate extensions of the univariate GARCH model. The most popular models in this category are the VEC (Bollerslev et al., 1988) and BEKK (Engel and Kroner, 1995) models. However, these models require to estimate a high number of unknown parameters and thus, are rarely used to estimate the dependencies when there are more than three series. An alternative to reduce the number of unknown parameters can be factor and orthogonal models (see e.g. Engel et al., 1990). Factor and orthogonal models are linear combinations of univariate GARCH processes, which, if they are uncorrelated, represent different components that drive the returns. Correlated factors are undesirable as they capture similar characteristics of the series. The third category of multivariate models are conditional correlation models. The idea is to first estimate the conditional variance and correlation before deriving the conditional covariance matrix. The parameters of the model are independent form the number of series to be analysed, which is an computational advantage to other multivariate GARCH models. However, this can also be a weakness if the number of series is large, as the same parameters are used for all series to estimate the conditional correlation matrix. The most common conditional correlation models are the Constant Conditional Correlation (CCC) (Bollerslev, 1990) and Dynamic Conditional Correlation (DCC) (Engle, 2002) model, which is an extension of the CCC model. Semi- and non-parametric correlation models in category four, can be used when there is no information about the structure of the data. Possible misspecification of the data structure can result in inconsistent estimator. A detailed discussion of the recent developments in this category can be found in Linton et al. (2009).

It is difficult or maybe impossible to identify which of the volatility and correlation models have the best out-of-sample forecasting performance. This could be an interesting topic for future research but it is beyond the scope of this thesis.

In the empirical analysis of Chapters 4 and 5, we use the DCC model as the forecast conditional correlations are easy to estimate and have a natural interpretation. The number of series used in the empirical analysis of this thesis does not require to reduce the estimation parameters by using more generalised multivariate GARCH models. In Chapter 4, we demonstrate how multivariate GARCH models can be used to simulate index level movements to improve the portfolio VaR and CVaR forecast to reduce the regulatory capital requirements of financial institutions. Moreover, in Chapter 5 we propose a new hedging approach which applies a multivariate volatility forecasting model and Monte Carlo simulation to reduce the risk of a market portfolio.

2.3 Hedging Techniques

Managing the risk of a portfolio is not just important for banks to reduce their regulatory capital requirements but also to reduce the uncertainty about the expected portfolio value at some future point in time. This is of particular relevance in volatile market situations as experienced during the financial crisis in 2008.

A common approach to secure the value of an equity investment at a certain maturity is to use forwards or futures contracts. The value of the futures contract is equal to the value of the underlying asset at the maturity of the futures contract. Thus, the forward or futures contract perfectly hedges the change in the underlying asset value, at maturity. An investor with a long position in an equity can simply short a futures contract on this equity to secure the future value of his investment. However, there are several drawbacks of using futures to hedge an underlying. This strategy only works if there is a forward or futures market for the asset the investor wants to hedge. The investor has to provide additional capital (e.g. initial and variation margin) for the futures investment, which he might be unable to provide if he faces a budget constraint. Moreover, selling futures as hedging strategy also offsets upside potential of the hedge.

Johnson (1960) assumes that the hedger is not only concerned about securing the investment value but also about its expected returns. He proposes a minimum variance hedging approach using futures to reduce the risk of a portfolio while maintaining upside potential. More sophisticated research on minimum variance hedging has been done by Hill and Schneeweis (1982), Figlewski (1984) and many others. The idea of minimum variance hedging with time-varying covariances was first introduced to by Baillie and Myers (1991). They use the bivariate GARCH model to estimate an optimal futures hedge ratio for some commodities. Several other GARCH processes have been proposed for minimum variance hedging problems (see e.g. Kroner and Sultan, 1993; Ji and Fan, 2011). These papers conclude that time-varying covariance models improve minimum variance hedging compared with static covariance assumptions.

The use of minimum variance hedging as an effective strategy to reduce the risk of a portfolio is only justifiable if the asset returns are normally distributed or the utility function of the investor is quadratic. These assumptions, however, are questioned by several empirical studies (see e.g. Tang and Choi, 1998; Scott and Horvath, 1980). Harris and Shen (2006) shows that minimum variance hedging can even increase negative skewness and kurtosis and thus, lead to portfolios with higher VaR and CVaR measures. Alternative frameworks to overcome the drawbacks of variance are quantile risk measures, which we discuss in more detail in Chapter 5.

More recent literature questions the efficiency of minimum variance hedging and quantile risk hedging using futures and time-varying volatility models (see e.g. Poomimars et al., 2003; Alexander and Barbosa, 2007; Mattos et al., 2008). Their research suggest that the introduction of transaction costs to time-varying volatility hedges clearly reduces the opportunity costs of not hedging (Mattos et al., 2008). The motivation to hedge is reduced even further when considering initial and maintenance margin deposits for selling futures contracts.

Moreover, Alexander and Barbosa (2007) found that minimum variance hedging using futures contracts performs worse than a naive alternative because of the maturity mismatch of the minimum variance hedge. Often, the maturity of the futures does not match the hedging period. The options market, however, offers several instruments with different maturities and thus, reduces the risk of a maturity mismatch.

We address the issue of high transaction costs and low opportunity costs for not hedging in Chapter 5 of this thesis where we propose a self-financing single-option hedging approach to secure the value of an equity portfolio. Our proposed model uses a single long index put option as hedging instrument as there is no margin deposit required for long positions and we reduce the risk of a maturity mismatch.

Chapter 3

Balancing Profitability and Capital Absorption with Heuristic Optimisation and Unconditional Coverage Constraint

In this chapter, we reduce the regulatory capital requirements for the market portfolio of financial institutions using heuristic optimisation methods and a new risk management approach. In particular, we examine how the objective functions described in Section 2.1 can reduce the Basel III market risk capital requirements, using Threshold Accepting (TA) heuristic algorithm. The heuristic optimisation results are compared with the Trust-Region (TR) local search method. Moreover, we propose a new optimisation approach based on the log-likelihood ratio for the Unconditional Coverage (UC) test (LR_{UC}) to manage the regulatory capital requirements. Compared with methods introduced in recent literature, our approach actively manages the portfolio minimum capital requirements while avoiding to over- or underestimate the portfolio risk.

The results of the empirical analysis show that the TA search algorithm applied to a CVaR objective function yields the lowest Basel III market risk capital requirements, in comparison with several different objective functions combined with different optimisation routines. Not only does the TA algorithm outmatch the TR algorithm in all risk and performance measures, but when combined with a 1% CVaR or VaR objective function, it also achieves the best portfolio risk profile. Portfolios optimised with our new capital constraint successfully reduce the Basel III market risk capital requirements. In general, portfolios with VaR and CVaR objective functions and underlying standard normal distribution yield better portfolio risk profiles and have lower capital requirements.

This chapter is organised as follows: Section 3.1 motivates heuristic optimisation algorithms and our new constraint to minimise the regulatory capital requirements. In Section 3.2, we introduce our advocated approach to manage the capital requirements and review the optimisation algorithm and evaluation methods. Section 3.3 demonstrates the superiority of heuristic optimisation to reduce the regulatory capital requirements of a market portfolio. In Section 3.4, we apply the proposed capital requirements approach in an empirical analysis. Finally, Section 3.5 concludes.

3.1 Introduction

Recent financial crises have highlighted several weaknesses in the risk management practices of financial institutions. To prevent future negative impacts on the financial market and the economy, financial regulators have enhanced the regulatory framework with major focus on the capital and liquidity standards.

In 1995, the Basel Committee introduced the market risk rules on minimum capital requirements (Basel Committee on Banking Supervision, 1995). The rules were set to strengthen the stability of financial institutions. Thus, as a result of the financial crisis in 2007, the Committee published a revision of the market risk framework in July 2009 (Basel Committee on Banking Supervision, 2009a), which is now part of the 2010 Basel III framework (Basel Committee on Banking Supervision, 2010).

The market risk framework requires banks to calculate their individual minimum market risk capital requirements to cover potential losses that might arise from their market activity (Basel Committee on Banking Supervision, 1996). The internal model approach to calculate the minimum capital requirements, requires the risk models to meet a series of quantitative and qualitative standards. One essential criteria is to calculate the rolling one-day 1% VaR based on at least 250 days of empirical data (Basel Committee on Banking Supervision, 2009b).

The new Basel III framework increases the minimum capital requirements of financial institutions. Hence, banks are increasingly interested to find ways to decrease their capital requirements. Existing literature suggest to either maximise the return for a given VaR or capital requirement constraint (see e.g. Sentana, 2003; Cuoco and Liu, 2006; Alexander et al., 2007) or to minimise the amount of regulatory capital required to underlie a certain investment (see e.g. McAleer et al., 2010; Santos et al., 2012; Drenovak et al., 2017). In this thesis, we follow the latter approach.

Intuitive objective functions to reduce the capital requirements are downside risk measures, i.e. VaR and CVaR. Often, however, downside risk measures lead to complex optimisation problems with multiple local extremes over the entire multidimensional search space. To find an approximate global solution to the search problem, when other search methods are too slow, heuristic algorithms can be used. Dueck and Winker (1992) proposed heuristic search algorithms in portfolio optimisation and applied the TA algorithm, introduced in Dueck and Scheuer (1990), to a bond portfolio optimisation problem.

In this chapter, we first apply the TA heuristic algorithm to a portfolio allocation problem and compare it against the TR local search method. In contrast to the existing literature, however, we analyse the Basel III market risk capital requirements of the optimised portfolios. As a first contribution of this chapter, we shed more light on the impact of the optimisation approach on the VaR backtesting described by the Basel Committee. In particular, we examine how a combination of TA optimisation algorithm and VaR and CVaR objective functions can reduce the market risk capital requirements.

As a second contribution, we propose a new method based on the UC test to reduce the regulatory capital requirements while optimising the portfolio for some objective function, introduced in Section 2.1. The proposed optimisation process minimises the capital requirements of a portfolio by avoiding to select a portfolio that over- or underestimates the number of daily VaR violations. To determine the optimal number of daily VaR exceedings we impose a LR_{UC} constraint. To solve this non-linear optimisation problem we apply the TA heuristic search method, following the work of Lyra et al. (2015). We use a dynamic rolling window approach for the optimisation of the portfolio weights.

This approach differs from related previous literature in several ways. McAleer et al. (2010) proposes a dynamic decision rule based on the number of daily VaR violations, to consult the risk manager on how conservative or aggressive the current investment is in comparison with the estimated risk. This approach can be classified as a passive risk management approach as the proposed decision rule does not influence the portfolio weight compilation. An active risk management approach to minimise the regulatory capital requirements is introduced by Santos et al. (2012). The proposed objective function minimises the maximum of either the last one day 1% VaR or the 60 days average daily 1% VaR, both for regular and stressed VaR. To determine the optimal portfolio with minimum capital requirements they provide an analytical solution by reformulating the optimisation problem into a convex objective function with a limitation on the maximum number of daily VaR violations. The parameters of the applied GARCH models are calculated for one in-sample period and are adopted throughout the optimisation process. A dynamic extension of this approach is presented by Drenovak et al. (2017), who provide a non-linear multi-objective function to minimise the capital requirements and maximise the expected portfolio return. The optimisation process is performed using a Non-dominated Sorting Genetic Algorithm II (Deb et al., 2002) run in a parallel framework developed by Ivanovic et al. (2015). However, Drenovak et al. (2017) do not include a limitation on the number of daily VaR violations in the optimisation process. This can potentially cause optimal portfolios to have a high number of VaR violations and ultimately can negatively affect the financial stability of the bank.

Our advocated LR_{UC} constraint optimisation approach differs from previous literature as it assists the risk manager to determine the optimal portfolio that avoids over- and underestimation of the portfolio risk and thus, optimises the minimum capital requirements for the portfolio. The new approach incorporates the Basel backtesting rules via the application of the LR_{UC} constraint. It is beneficial for risk managers whose main objective is to optimise the regulatory capital requirements for a given objective function.

In Section 3.3, the Minimum-Variance (MV) and downside risk measures VaR and CVaR with underlying empirical distribution at 1% and 5% significance level are used as objective functions. MV is a standard risk measure used in portfolio optimisation literature. While VaR and CVaR with 1% and 5% significance level have become important risk measures in quantitative risk management literature and regulations (Basel Committee on Banking Supervision, 2009b).

Section 3.4 uses MV and downside risk measures VaR and CVaR with underlying empirical and standard normal distribution at 1% significance level as objective functions. We optimise the portfolios with the TA heuristic algorithm and the common TR local search algorithm to compare our results and to check that the proposed optimisation constraint does not heavily rely on the optimisation procedure used. Other popular heuristic methods proposed in the literature are Particle Swarm Optimisation (Eberhart and Kennedy, 1995), or Ant Colony Optimisation (Dorigo et al., 1999). We refer the interested reader to Maringer (2005) and Gilli et al. (2011) who give a good overview of the most common heuristic search methods.

3.2 Methodology

In the following, we first review the optimisation algorithms used in the optimisation process in Section 3.2.1. Section 3.2.2 describes the evaluation method used in this chapter before we introduce our capital requirements approach in Section 3.2.3.

3.2.1 Search Algorithms

Local search algorithms are standard optimisation processes that are widely used in portfolio optimisation. A common and well known local search method is the TR. The pseudo code for the TR algorithm given in Algorithm 1. To find the local minimum of a constraint minimisation problem $f(\cdot)$ the objective function is calculated for several trial steps s. The trial steps are drawn from a random neighbourhood ($\mathcal{N}^{TA}$) around a current search point w^c (Byrd et al., 1987). If the objective function $f(w^s)$ is smaller than $f(w^c)$, then the current point w^c is updated to w^s. If $f(w^s)$ is not smaller than $f(w^c)$ the current search point is not updated. This process can cause $f(w^c)$ to get stuck at a local minimum when the current solution is at a saddle point (Yuan, 2000).

The other optimisation method that we consider for our portfolio selection problems is the TA algorithm. It was first introduced to portfolio optimisation by Dueck and Winker (1992). The TA is a fast algorithm that even works well for large problem instances. It can easily be implemented and provides robust results for a variety of objective functions and constraints (Gilli and Këllezi, 2002).

Algorithm 1 Trust-Region Algorithm

set n_{rounds}
randomly generate initial current solution w^c
for $r = 1 : n_{rounds}$ **do**
  `generate` $w^n \epsilon \mathcal{N}^{TR}(w^c)$
  `compute` $\triangle = f(w^n) - f(w^c)$
  if $\triangle < 0$ **then**
    $w^c = w^n$
  end if
end for

The TA is a trajectory optimisation method that gradually changes the current solution (Gilli et al., 2011). This is similar to the local search method. However, the TA also accepts solutions that are inferior to the current solution. This is, as long as the difference between the new solution and the current solution is less than a certain threshold τ, where τ is a sequence of thresholds decreasing over time (Gilli et al., 2006).

A neighbourhood function $\mathcal{N}^{TA}$ is used to define new solution in a neighbourhood of the old one. First, a random current solution w^c is set. Then the weight of one asset w_i is slightly reduced by a certain decimal factor. The weight of another asset w_j is selected and increased by the same decimal factor. If the asset weights are within a defined upper and lower bound, the current solution w^c is updated to the new solution w^n; otherwise w^c is not updated.

The pseudo code for the TA algorithm given in Algorithm 2 is based on the model presented by Gilli et al. (2011).

To ensure that the algorithms find an optimal solution, we restart the search process several times. For our analysis, the parameters are set to six restarts, five rounds and 5,000 steps for a total of 25,000 iterations per restart. The asset weights are adjusted by a decimal factor of 0.005. The parameters are chosen after testing several different parameter settings and examining the convergence of the objective function value. In our analysis, the parameters chosen led to fast converging results

Algorithm 2 Threshold Accepting Algorithm

set n_{rounds} and n_{steps}
set threshold sequence τ_r
generate initial current solution w^c
for $r = 1 : n_{rounds}$ **do**
  for $i = 1 : n_{steps}$ **do**
    `generate` $w^n \epsilon \mathcal{N}^{\mathcal{TA}}(w^c)$
    `compute` $\triangle = f(w^n) - f(w^c)$
    if $\triangle < \tau_r$ **then**
      $w^c = w^n$
    end if
  end for
end for

and are the most computationally time efficient calibration. After 50% of the iterations the TA algorithm approaches 86% of the optimal solution.

For the calculation of the threshold sequence we use five percentiles equally distributed from 0.9 to 0. The threshold sequence is calculated as suggested by Gilli et al. (2006). At each round we adjust the threshold to τ_r. As the objective function outcomes are random, we restart the algorithm several times, starting with an equally weighted portfolio, and take the best solution; ideally, the solution should lead to a good out-of-sample result.

To minimise the objective function the TA algorithm can explore the set of possible asset weights that satisfy the constraints. However, a faster approach is to accept all solutions and penalise the ones that violate the constraints. To find the optimal portfolio that satisfies the inequality target return constraint, we extend the objective functions $f(\cdot)$ described in Equations (2.1-2.5) by a penalty function $p(\cdot)$

$$p(\mu^{port}, \mu^{target}) = \begin{cases} c & \text{if } \mu^{target} > \mu^{port} \\ 0 & \text{if } \mu^{target} \leq \mu^{port} \end{cases} \tag{3.1}$$

where $c = \exp(\mu^{target} - \mu^{port}) - 1$ (Bertsekas, 1996). Thus, the optimisation problem $F(\cdot)$ is defined as a combination of one of the risk measures defined in Equations (2.1-2.5) and the penalty function $p(\cdot)$

$$F(w, \mu^{port}, \mu^{target}) = f(w) + p(\mu^{port}, \mu^{target}). \tag{3.2}$$

3.2.2 Evaluation and VaR Backtesting

To determine the market risk capital requirements, the Basel III Committee requires banks to backtest their market risk models. The Committee uses a traffic light scheme to classify the backtested models into three zones depending on the number of portfolio returns exceeding the one-day historical 1% VaR in the sample period. Each zone comes with a different multiplication factor that is used to calculate the market risk capital requirements (Basel Committee on Banking Supervision, 1996). The green zone indicates that the risk model is accurate and, hence, models in this zone have the lowest multiplier. The accuracy of risk models in the yellow zone is questionable. Models in the red zone are determined to be flawed and therefore, have the highest multiplier. The multiplication factor ranges from three in the green zone to four in the red zone. For the yellow zone the multiplier lies between three and four. To determine the boundaries for the three zones, the binomial probabilities are calculated for a given sample size and 99% coverage ratio. The green zone extends up to a cumulated probability of 94.99%, whereas the yellow zone starts at a cumulated probability of 95%. A cumulated probabilty of 99.99% and above indicates the red zone. The regulatory market risk capital requirements (CR) are the maximum of either the one-day 99% VaR at day t before assessment or the last 60 days one-day 99% VaR average ($\overline{VaR}_{60}$) times a multiplier m deduced from the traffic light scheme

$$CR = max\{VaR_t, m \times \overline{VaR}_{60}\} \tag{3.3}$$

(Basel Committee on Banking Supervision, 1996).

In Appendix A, we provide more details about the latest updates in the minimum regulatory capital requirements for market risk calculation and elaborate more on why we apply the Basel II formula (Basel Committee on Banking Supervision, 1996) in our empirical analysis.

In this thesis, the international financial standards proposed by the Basel Committee on Banking Supervision (BCBS) and the Financial Stability Board (FSB) are used in the optimisation and analysis. In essence, this is because these committees provide standardised approaches, methodology and quantification of financial regulation across member countries. The implementation of these standards into national law are carried out by the member countries. The national standards can deviate to some extent from the standards proposed by the BCBS and FSB, i.e., the United States regulations are often more restrictive than the standards proposed by the BCBS and FSB. Fratianni and Pattison (2015) provide a detailed comparison and discussion of Basel III implementation in the European Union and the United States.

Unconditional Coverage Test

A very common test in quantitative risk management is the UC test proposed by Christoffersen (1998). If the number of portfolio returns exceeding the daily VaR estimates is less than a certain significance level, it would indicate that the risk model overestimates risk. Otherwise, if the number of violations are more than the expected number, the risk model is likely to underestimate risk. The UC test is used to assess whether the risk model is acceptable or not.

The parameter η_t is used to determine whether a violation occurred on day t or not. η_t is an i.i.d. Bernoulli sequence and, hence, can take the values 1 or 0. The value 1 indicates a violation on day t and 0 implies no violation. The Bernoulli density function is

$$f^{Bernoulli}(\theta) = \theta^{\eta_t}(1-\theta)^{1-\eta_t} \tag{3.4}$$

where θ is the probability of failure (PF) (Christoffersen, 2003).

The PF can easily be estimated by

$$\hat{\theta} = \frac{\upsilon_1}{W_T} \tag{3.5}$$

where υ_1 is the total number of violations and W_T is the number of observations in the testing period (Danielsson, 2011). The likelihood function for $\hat{\theta}$ is

$$L(\hat{\theta}) = \hat{\theta}^{\upsilon_1}\left(1-\hat{\theta}\right)^{1-\upsilon_1}. \tag{3.6}$$

The log-likelihood ratio for the UC test tests the hypothesis that the expected PF θ equals the observed PF $\hat{\theta}$

$$LR_{UC} = -2\log\left(\frac{\theta^{\upsilon_1}(1-\theta)^{1-\upsilon_1}}{\hat{\theta}^{\upsilon_1}(1-\hat{\theta})^{1-\upsilon_1}}\right) \sim \chi_1^2 \tag{3.7}$$

(Danielsson, 2011). A χ^2-test with one degree of freedom is used to test LR_{UC}.

Independence and Conditional Coverage Test

The independence coverage (IND) test studies if the violations are independently distributed over time. Based on the work of Christoffersen (1998), we calculate the probability for two consecutive violations (θ_{11}) and the probability of a non-violation followed by a violation (θ_{01}) on day t and $t+1$. A non-violation on day t followed by a violation on day $t+1$ is represented by the subscript integers. The first-order Markov chain transition matrix is

$$\Pi_1 = \begin{bmatrix} 1-\theta_{01} & \theta_{01} \\ 1-\theta_{11} & \theta_{11} \end{bmatrix}.$$

For a sample with W_T observations the likelihood function is

$$L\left(\Pi_1\right) = (1-\theta_{01}^{\upsilon_{00}})\theta_{01}^{\upsilon_{01}}(1-\theta_{11}^{\upsilon_{10}})\theta_{11}^{\upsilon_{11}} \tag{3.8}$$

where υ_{ij} is the number of observations where i and j are either 1 or 0 (Danielsson, 2011).

The probabilities of $\hat{\theta}_{01}$ and $\hat{\theta}_{11}$ are estimated with

$$\hat{\theta}_{01} = \frac{\upsilon_{01}}{\upsilon_{00}+\upsilon_{01}}$$

$$\hat{\theta}_{11} = \frac{\upsilon_{11}}{\upsilon_{10}+\upsilon_{11}}$$

(Christoffersen, 2003), which results in the estimated transition matrix

$$\hat{\Pi}_1 = \begin{bmatrix} 1-\hat{\theta}_{01} & \hat{\theta}_{01} \\ 1-\hat{\theta}_{11} & \hat{\theta}_{11} \end{bmatrix}.$$

It is preferable that a violation is not followed by another violation as this indicates that the risk models is unable to adjust for new information. Hence, the probability of a violation tomorrow should not depend on today's observation. Under the assumption of independence the null hypothesis that a violation tomorrow does not depend on today being a violation, can be written as $\theta_{01} = \theta_{11} = \theta$ (Christoffersen, 2003). This gives us the following transition matrix

$$\hat{\Pi}_0 = \begin{bmatrix} 1-\hat{\theta} & \hat{\theta} \\ 1-\hat{\theta} & \hat{\theta} \end{bmatrix}$$

where

$$\hat{\theta} = \frac{\upsilon_{01} + \upsilon_{11}}{\upsilon_{00} + \upsilon_{10} + \upsilon_{01} + \upsilon_{11}}$$

and the likelihood function for the null hypothesis is

$$L\left(\hat{\Pi}_0\right) = (1-\hat{\theta})^{\upsilon_{00}+\upsilon_{10}} \hat{\theta}^{\upsilon_{01}+\upsilon_{11}} \tag{3.9}$$

(Danielsson, 2011). Finally, the likelihood ratio test is given by :

$$LR_{IND} = -2\log\left[\frac{L(\hat{\Pi}_0)}{L(\hat{\Pi}_1)}\right] \sim \chi^2_1 \tag{3.10}$$

(Christoffersen, 2003). The independence coverage test is χ^2 distributed with one degree of freedom.

Furthermore, to test for violation clustering and whether the number of violations significantly deviate from the expected number of violations, we can use the conditional coverage test (CC). The CC is a joint test of LR_{UC} and LR_{IND}. The test statistic can be written as

$$LR_{CC} = LR_{UC} + LR_{IND} \sim \chi^2_2 \tag{3.11}$$

resulting in LR_{CC} being χ^2 distributed with two degrees of freedom (Danielsson, 2011).

3.2.3 Optimal Management of Capital Requirements

To reduce and optimally manage the regulatory capital requirements summarised in Equation (3.3), we introduce a new inequality constraint based on the LR_{UC} ratio in Equation (3.7).

The minimisation problem can be rewritten as

$$\min_{w} F(\cdot) \tag{3.12}$$

subject to the constraints introduced in Section 2.1 and:

$$LR_{UC}^{port} \leq \chi_1^2$$

where LR_{UC}^{port} is the LR_{UC} ratio of portfolio set *port* that needs to be less then or equal to the critical value of a χ^2-test at one degree of freedom. For the LR_{UC}^{port} ratio daily VaR levels, based on the empirical distribution at a 1% significance level, are calculated using a 250 days learning period. The inequality constraint is implemented on the basis of a penalty function similar as the one introduced in Equation (3.1). When the constraint is violated, c is defined as $\exp(LR_{UC}^{port} - \chi_1^2)$.

3.3 Empirical Heuristic Optimisation Results

In this section, we first analyse the efficiency of heuristic optimisation for variance and empirical VaR and CVaR objective functions at different significance levels, to reduce the market risk regulatory capital requirements. Section 3.4 then builds on this work and further investigates the effect of empirical and standard normal based VaR and CVaR optimisation on the regulatory capital requirements.

The empirical analysis in this chapter and the entire thesis focuses on a portfolio of the constituents of the Dow Jones Industrial Average (DJIA) index. The DJIA index is a highly liquid stock market with a long transaction history. Also, the DJIA index is the underlying of several derivatives with different strike prices and maturities, which is relevant for Chapter 5. We consider daily closing prices sampled from 1st January 2003 to 1st January 2013 to cover periods with high and low market volatility. Log-returns are computed for a total of $T = 2610$ days and $M = 30$ equities.

We include data outliers caused by market shocks in our analysis, i.e., the stock market crash in 2008. This follows the approach of related literature, e.g., Santos et al. (2012), Uylangco and Li (2016), Kellner and Rösch (2016), who all include crisis events in their optimisation and analysis. As we use a rolling window optimisation approach the models are trained and tested for periods without (pre-crisis) and with (post-crisis) data outliers. We include crisis events in our optimisation as we are particularly interested in i) the algorithms ability to identify optimal solutions that also hold in the event of a crisis, and ii) the ability of VaR and CVaR objective functions to provide stable results in turbulent market cycles.

A common approach in the financial literature is to distinguish between in-sample and out-of-sample periods (Bailey et al., 2014). The in-sample or learning period, is used in the design of the strategy while in the out-of-sample period the performance of the strategy is tested. In this section, the portfolio optimisation is based on a 1250 days in-sample and tested in a 10 days out-of-sample period. On a rolling window basis the portfolio weights are rebalanced at the end of the 10 days out-of-sample period using a rolled forward in-sample training period. With a total of 2610 days and an in-sample period of 1250 days the entire out-of-sample period consists of 1360 days. Furthermore, daily 2-weeks T-Bill rates from 1st January 2003 to 1st January 2013 are considered as the risk-free interest rate. All financial data are downloaded from DataStream.

For the optimisation we impose the asset weights constraint, target return and short selling constraints proposed in Section 2.1. The out-of-sample risk and performance of the optimal portfolios are analysed using common statistical measures, e.g. mean return, standard deviation, kurtosis, skewness, maximum drawdown. In addition, we calculate the modified Sharpe ratio proposed by Israelsen (2005), which avoids the shortcoming of the traditional Sharpe ratio and provides a consistent way of ranking portfolios. We compare all portfolio optimisation results against the actual performance of the DJIA index and the "naive" equally weighted (EW) portfolio. The latter two are considered as benchmark models and are included to assess potential outperformance of the optimised portfolios.

For the portfolio optimisation process, we use the TA heuristic optimisation and the TR local search algorithm. We consider the latter as the benchmark approach. The algorithms optimise the objective function $F(\cdot)$, which is either the MV objective function (Equation 2.1), the VaR or CVaR function with underlying empirical distribution (Equations 2.2 or 2.3) at a 1% and 5% significance level.

In Section 3.3.1, we first compare the local TR search algorithm with the heuristic TA search algorithm. In Section 3.3.2, we then compare combinations of the optimisation algorithms and objective functions with respect to their impact on the portfolio performance. Finally, in Section 3.3.3, the optimisation models are evaluated based on the Basel III market risk capital requirements.

3.3.1 Comparison of Search Algorithms

Figure 3.1 first compares the dynamic portfolio allocation (weight structure) resulting from the $2^3 = 8$ combinations of optimisation algorithms (TA - upper panels, Trust Region - lower panels), empirical objective functions (CVaR - left panels, VaR - right panels) and confidence level (1% or 5%). A colour map for the portfolio weights is presented Figure 3.2. As discernible, the portfolio composition for CVaR optimised portfolios exhibits far less fluctuation than their VaR analogues, regardless of the confidence level and the optimisation method used.

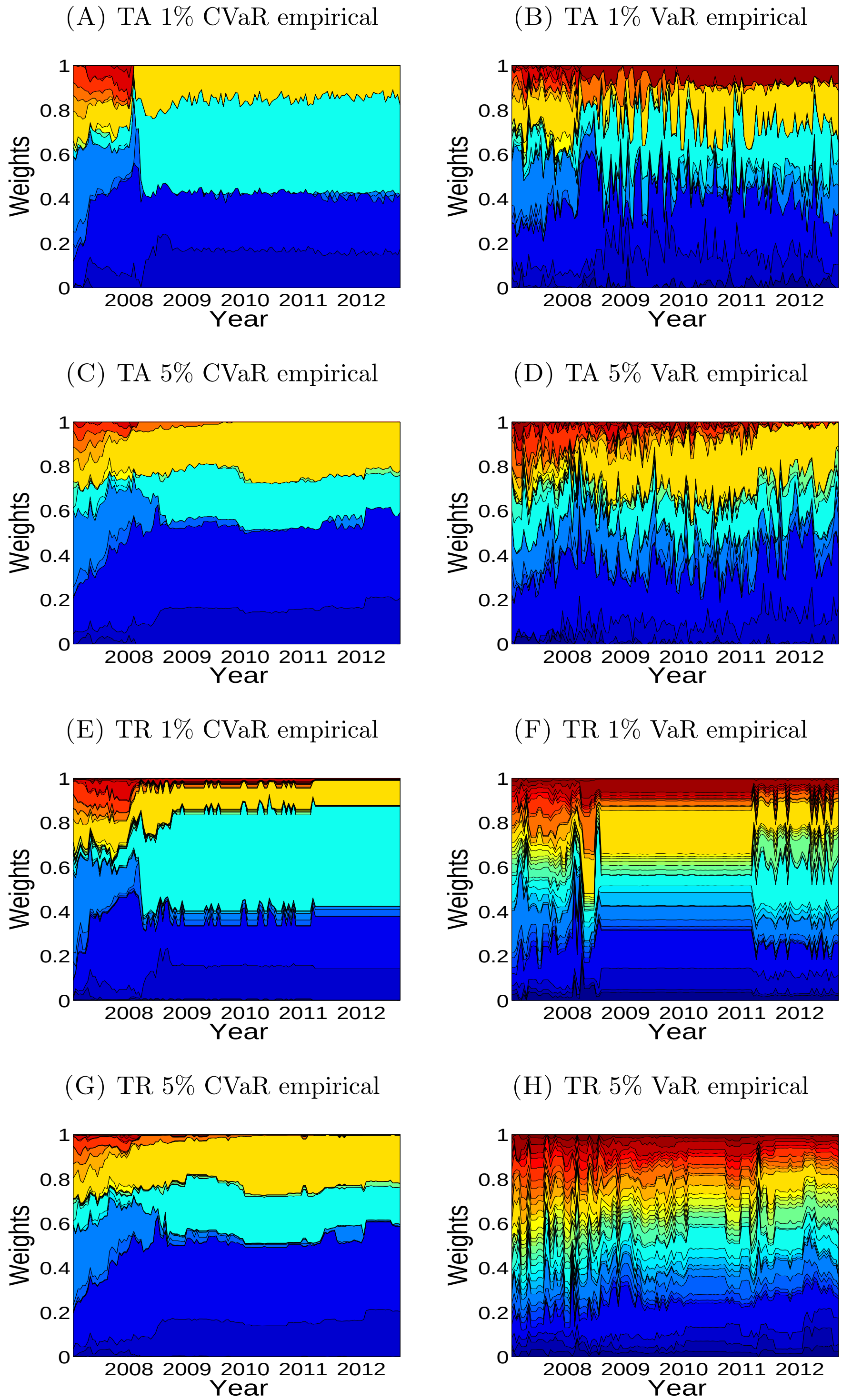

FIGURE 3.1 Comparison of the dynamic portfolio weights for different combinations of VaR and CVaR objective functions with empirical distribtion, TA and TR optimisation algorithm and 1% and 5% significance level. A colour map for the portfolio weights is presented in Figure 3.2.

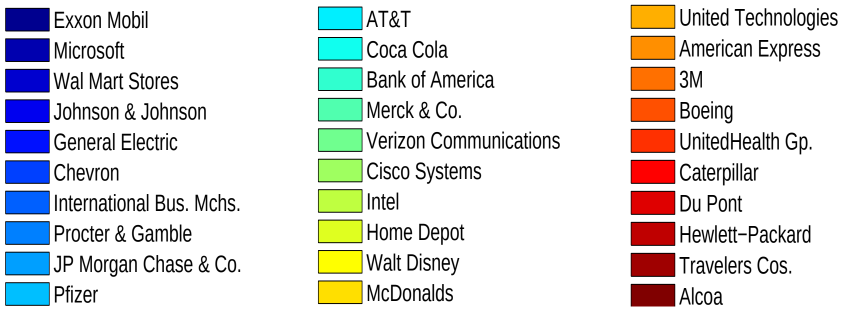

FIGURE 3.2 Colour map for portfolio weights in Chapter 3.

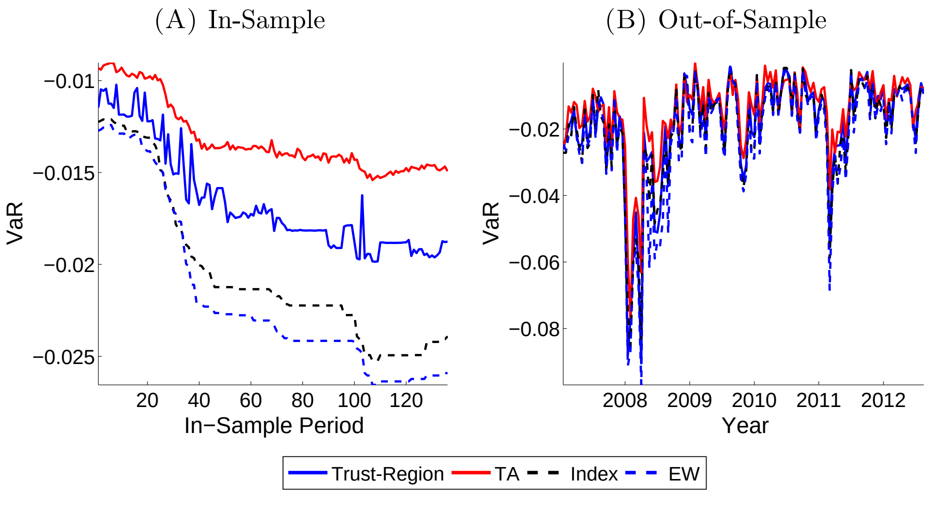

FIGURE 3.3 Empirical VaR at 5% significance level of the $VaR_{H5\%}$ portfolio for TA and local search algorithm in the in-sample (3.3A) and out-of-sample period (3.3B).

(The standard Markowitz optimisation is included in our analysis but not plotted here as it is widely studied in the literature.)

Figure 3.3 compares the TA algorithm with the TR search algorithm for the $VaR_{H5\%}$ portfolio. The figure shows the historical VaR value of the portfolio at a 5% significance level. The optimised portfolios have a lower VaR in the in-sample period than the index and EW portfolio. Moreover, the portfolio optimised with the heuristic algorithm shows to have a lower VaR than the portfolio optimised with the local search algorithm. This finding is observed not only for the in-sample period (Figure 3.3A) but also for the out-of-sample period (Figure 3.3B).

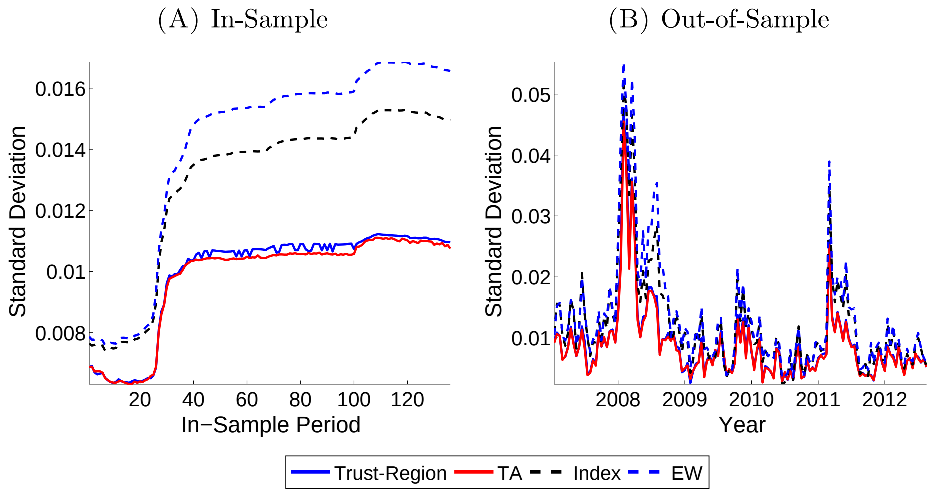

FIGURE 3.4 Standard deviation of the $CVaR_{H1\%}$ portfolio for TA and local search algorithm in the in-sample (3.4A) and out-of-sample period (3.4B).

Figure 3.4 compares the standard deviation of the $CVaR_{H1\%}$ portfolio with TA and TR search method. The optimised portfolios have a lower standard deviation than the index and the EW portfolio in the in-sample and the out-of-sample period. In the in-sample period the TA algorithm produces better statistical results than the TR algorithm (see Figure 3.4A). However, in the out-of-sample period, the difference between both algorithms is marginal (see Figure 3.4B). In general, the TA optimised portfolios show slightly better or at least the same results than the portfolios with local search algorithm. This is observed for CVaR and VaR objective functions with empirical distribution at 1% and 5% significance level. The better results of the TA algorithm compared with the local search method can be explained by the TA's ability to avoid getting stuck at a local optimum in the search space. When solution transitions are successful, the threshold is reduced to explore local optima. This is why TA optimised portfolios show better results, in the in-sample and out-of-sample period.

In Figures 3.3A and 3.4A we see a large change of the VaR and standard deviation measure in the optimised in-sample periods between periods 25 and 35.

The change in the risk measures is explained by the stock market crash between 2008 and 2009, which was due to the great financial crisis that started in 2007. All optimisation periods after the 25 in-sample period include this extreme event in their time series. This is why the VaR and standard deviation values stay on the changed post-crisis level, for all in-sample periods. As both Figures show, the TA algorithm is less affected by the crisis event and provides much better in-sample optimisation results, compared with the $VaR_{H5\%}$ local search, EW and index portfolio. The crisis event can also be seen in Figures 3.3B and 3.4B, for the out-of-sample period. The VaR and standard deviation value quickly change back to pre-crisis levels, in the out-of-sample period. We see that the TA optimised portfolio provides better out-of-sample results before, during and after the crisis, compared with the local search portfolio, EW and index. Thus, we can conclude that the stock market crash in 2008 has no influence on the superiority of the heuristic algorithm over the local search method.

3.3.2 Comparison of Objective Functions

In this section we analyse how the choice of the objective function influences the portfolio statistics and performance measures. The statistics are calculated for portfolios with TA and TR optimisation algorithm for the entire out-of-sample period. The results are displayed in Table 3.1. For completeness, we also provide the results for the index and the EW portfolio.

As expected, the optimised portfolios have a lower standard deviation than the index or the EW portfolio. The lowest standard deviation with 16.45% is reported for the TA algorithm with 5% CVaR objective function. The portfolio mean return for the TA optimised portfolios are all significantly higher compared with their TR optimised equivalents. The TA optimised portfolios also outperform the index and the EW portfolio.

TABLE 3.1 Annualised out-of-sample portfolio results for 1% and 5% VaR and CVaR objective functions with underlying empirical distribution assumption (Equations 2.2 and 2.3). Mean return, standard deviation, maximum drawdown and modified Sharpe ratio are reported in percentage points.

Model		Descriptive Statistics and Performance Measure						
Algorithm	Objective Function	Mean Return	Standard Deviation	Kurtosis	Skewness	Maximum Draw-Down	1% CVaR Historical	Mod. Sharpe Ratio
TA	1% CVaR	0.95	16.69	14.57	0.14	-37.16	-0.6670	2.79
	5% CVaR	0.75	16.45	15.03	0.24	-37.31	-0.6660	1.65
	MV	-0.29	16.69	16.57	0.32	-39.86	-0.6810	-5.15e-4
	1% VaR	1.10	17.88	14.67	0.23	-38.65	-0.7480	3.44
	5% VaR	-0.11	18.35	20.32	0.63	-45.26	-0.7595	-4.37e-4
TR	1% CVaR	-0.15	17.30	15.37	0.25	-40.56	-0.7010	-4.36e-4
	5% CVaR	0.32	16.58	15.10	0.23	-38.58	-0.6760	-1.07e-4
	MV	-0.27	16.70	16.61	0.32	-39.84	-0.6815	-5.03e-4
	1% VaR	-3.71	19.99	13.63	-0.07	-51.97	-0.8710	-3.40e-3
	5% VaR	-3.84	22.42	13.92	-0.04	-54.50	-0.9650	-3.90e-3
	Index	-3.59	23.34	10.25	-0.21	-56.86	-0.9710	-3.80e-3
	EW	-5.23	25.76	10.38	-0.24	-61.34	-1.0905	-5.90e-3

The highest performance can be seen for the 1% VaR objective function followed by the $CVaR_{H1\%}$ and the $CVaR_{H5\%}$ optimised portfolios. The portfolios with TA algorithm and CVaR (1% and 5%) and 1% VaR objective function are the only portfolios with a positive modified Sharpe ratio. All other portfolios and benchmark models have a negative modified Sharpe ratio. The portfolio with the worst modified Sharpe ratio and thus, ranked lowest, is reported for the EW portfolio with the lowest mean return and highest standard deviation. Figure 3.5 shows the entire out-of-sample portfolio development (normalised) for TA optimised portfolios and the two benchmark models.

All TA optimised portfolios have a positive skewness while only three of the portfolios with TR optimisation are positively skewed. A very high skewness with 0.63 can be seen for the $VaR_{H5\%}$ objective function with TA algorithm. The same portfolio is strongly leptokurtical, with a kurtosis of over 20. The TA $CVaR_{H1\%}$ portfolio has with 14.57 the lowest kurtosis of all TA optimised portfolios. Major differences kurtosis and skewness between TA and TR optimised portfolios can be seen for the VaR objective functions. The lowest overall kurtosis can be seen for the index with 10.25.

Figure 3.6A shows the maximum drawdown of the portfolios in the in-sample period. All optimised portfolios have a better maximum drawdown in the in-sample period compared to the index and the EW portfolio which have substantially high draw-downs. For the out-of-sample period, the optimised portfolios outperform the index and the EW portfolio as Table 3.1 shows. The highest maximum drawdown with -37.16% is observed for the TA algorithm with a 1% CVaR objective function, followed again by the 5% CVaR and 1% VaR optimised portfolio with TA algorithm. Again, we see the highest difference between TA and TR optimised portfolios for portfolios with VaR objective functions.

Figure 3.6B displays the 1% empirical CVaR of the TA optimised portfolios. In the in-sample period, the TA $CVaR_{H1\%}$ portfolio clearly outperforms

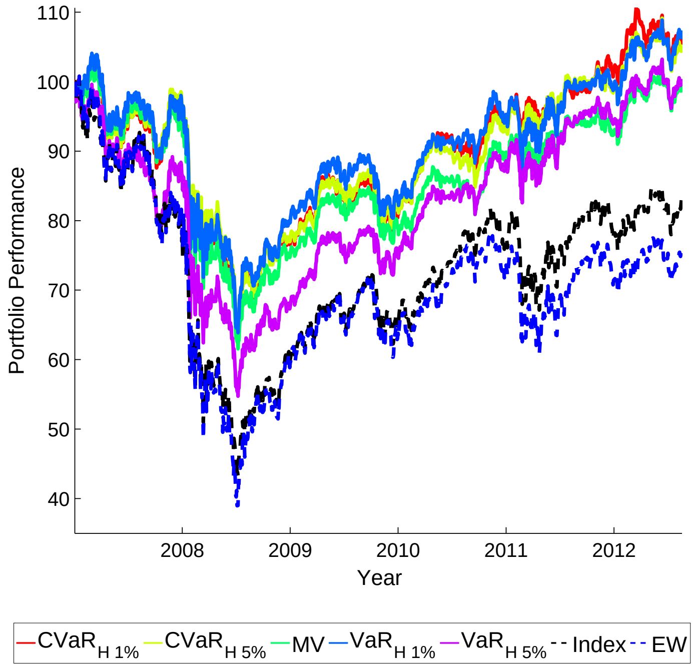

FIGURE 3.5 TA portfolio price development (normalised) in the out-of-sample period.

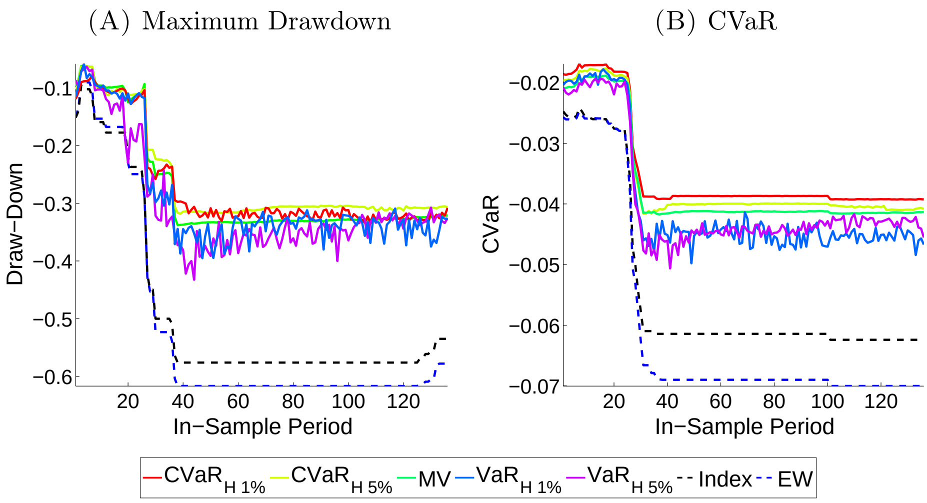

FIGURE 3.6 Maximum draw down (3.6A) and 1% CVaR (3.6B) in the in-sample period.

the other portfolios. As discernible in Table 3.1 in the out-of-sample period the TA $CVaR_{H1\%}$ portfolio has the second lowest annualised empirical CVaR value with -0.6670, following the $CVaR_{H5\%}$ portfolio with -0.6660.

Table 3.1 shows that in general portfolios with the TA optimisation algorithm have better statistics and performance measures compared with the TR search method. The most significant difference for all values can be seen for VaR optimised portfolios. The TA also clearly outperforms the benchmark models.

3.3.3 VaR Backtesting Results

The risk profile of the portfolios is evaluated by the results computed in the VaR backtesting which calculates the daily empirical VaR values for a 1% significance level based on a rolling 250 days interval, as required by the Basel Committee (Basel Committee on Banking Supervision, 2009b). Hence, the first daily VaR value is calculated 250 days after the in-sample period. The VaR values are calculated based on the empirical distribution of each portfolio. The results are evaluated using LR_{UC}, LR_{IND} and LR_{CC} with a χ_1^2 distribution and a 1% significance level. In our study, the Basel III traffic light scheme (Basel Committee on Banking Supervision, 1996), which is used to calculate the Basel III capital requirements for financial institutions, serves as an assessment criterion.

TABLE 3.2 VaR Backtesting Results. The likelihood ratio values for the LR_{UC}, LR_{IND} and LR_{CC} are reported under the corresponding columns. The critical values for a χ^2 test with 1% significance level and one and two degrees of freedom is 6.64 (UC and IND) and 9.21 (CC), respectively. The capital requirements (CR) are reported for portfolios with TA and TR algorithm followed by the relative difference between the objective functions optimised with TA and TR algorithm (relative to the TA optimised portfolio value). The results are calculated for portfolios with 1% and 5% VaR and CVaR objective functions with underlying empirical distribution assumption (Equations 2.2 and 2.3)

Objective	Significance	LR_{UC}		LR_{IND}		LR_{CC}		CR		
Function	Level	TA	TR	TA	TR	TA	TR	TA	TR	+/-
CVaR	1%	7.90	6.38	0.31	0.00	8.21	6.38	0.2230	0.2325	-4.3%
	5%	6.13	7.32	0.40	0.31	6.53	7.63	0.2045	0.2075	-1.5%
VaR	1%	6.87	8.30	0.31	0.26	7.18	8.56	0.2510	0.2550	-1.6%
	5%	4.62	5.96	0.00	0.00	4.62	5.96	0.2220	0.2750	-23.9%
MV		5.10	4.08	0.56	0.70	5.66	4.78	0.2140	0.2145	-0.1%
(Index)		(7.93)		(0.32)		(8.25)		(0.3150)		
(EW)		(5.72)		(0.53)		(6.25)		(0.3440)		

Table 3.2 displays the LR_{UC}, LR_{IND} and LR_{CC} ratio and the capital requirement for a one-day 1% VaR risk model with underlying empirical distribution. The relative difference of the capital requirements for TA and TR optimised portfolios is reported in the last column of the table. The capital requirement values are calculated for a portfolio value of one monetary unit, making the results comparable and interpretable for any portfolio market value. The results differ based on the portfolio optimisation method used, i.e. the combination of objective function, significance level and algorithm. For a better comparison, we also provide the backtesting results for the benchmark models (in parenthesis, as the results are not obtained by optimisation).

The χ^2 distribution with 1% significance level and one degree of freedom is used for the LR_{UC} and LR_{IND} test. For the LR_{CC} test a χ^2 distribution with 1% significance level and two degree of freedom is used. The critical values are 6.64 and 9.21, respectively. The null hypothesis that the fraction of observed violations equals the expected number of violations at the significance level is rejected, if the values in the table exceed the critical value.

As Table 3.2 shows, the backtesting model is below the critical LR_{IND} and LR_{CC} value for all optimised portfolios. Thus, the backtested VaR model is not rejected for any of these portfolios for the investigated sample period. However, the null hypothesis for the LR_{UC} test is rejected for the $CVaR_{H1\%}$ and $VaR_{H1\%}$ portfolio with TA optimisation and for the$CVaR_{H5\%}$ and $VaR_{H1\%}$ portfolio with TR algorithm. The average probability of a VaR violation in these portfolios exceeds the probability of violations we expect for the assumed distribution function and significance level.

As Figure 3.7 shows, the daily negative portfolio returns for the portfolio with 1% VaR objective function are less extreme than for a $VaR_{H5\%}$ objective function. The EW optimised portfolio has the highest capital requirements with 0.3440, which is even worse than the index with 0.3150. The lowest capital requirements

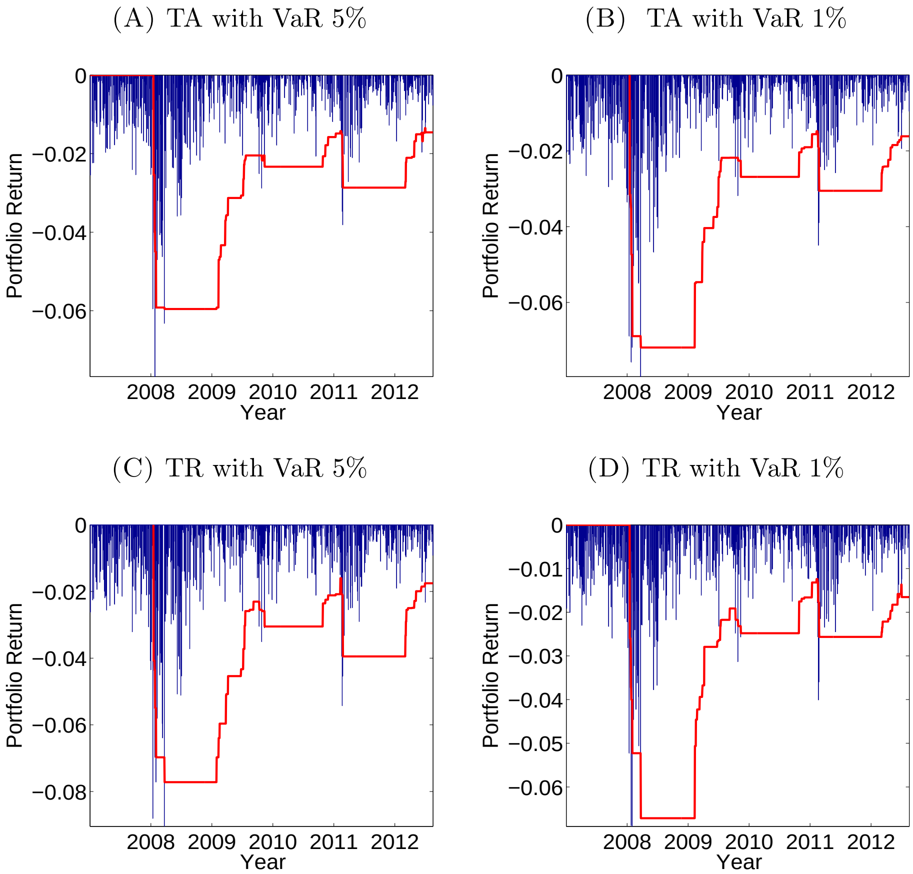

FIGURE 3.7 Daily VaR backtesting in the out-of-sample (blue bars = negative returns, red line = daily VaR level).

with 0.2045 is reported for the $CVaR_{H5\%}$ portfolio with TA search algorithm. The second lowest capital requirements with 0.2075 can be observed for the same objective function but for the local search algorithm. This can be explained by two reasons: i) $CVaR_{H5\%}$ has the second lowest average number of VaR violations (3.80 and 4.05 for TA and TR optimisation, respectively) and ii) the lowest empirical portfolio VaR values. The results show that the 5% significance level captures the tail risk of the portfolio distribution better than the 1% significance level. For portfolios with MV objective function, the capital requirements are slightly lower when the TA algorithm is used (0.2140). The TR algorithm yields higher capital requirements (0.2145). The greatest difference with 23.1%, between TA and TR optimised portfolio, is seen for the empirical 5% VaR objective function. For the $VaR_{H5\%}$ portfolio with local search algorithm the capital requirements are 0.2750, while for the same portfolio with TA algorithm the capital requirements are 0.2220. In general, the capital requirements for market risk are lowest whenever the TA algorithm is used for the portfolio optimisation process. This is true for all objective functions and most significant for the VaR optimised portfolios.

3.4 Portfolio Optimisation with Optimal Capital Requirements Constraint

In the previous section, we demonstrated the superiority of heuristic optimisation compared with the TR local search method. In this section, we apply our proposed indirect capital requirements constraint, described in Section 3.4.1, in an empirical analysis. This is to demonstrate how financial institutions can select an optimal portfolio while minimising the regulatory capital requirements by avoiding to over- or underestimate the risk of their market portfolio.

The analysis is based on the same empirical dataset presented in Section 3.3.

In this section, the portfolios are trained for an in-sample period of 1250 days and tested in a 60 days out-of-sample period. At the end of the out-of-sample period the in-sample period is rolled forward by 60 days. Hence, with an in-sample period of 1250 days and a total of 1361 days our analysis has 22 in-sample and out-of-sample periods.

In the empirical analysis in Section 3.3, transaction costs were not considered in the optimisation of the portfolios. Besides the practical relevance, transaction costs can have a considerable effect on the optimal portfolio weight allocation. Thus, in this section we include proportional transaction costs of one basis point in the optimisation process. Other forms of transaction costs are not included in this analysis; we refer the interested reader to Mansini et al. (2015).

In this analysis, we set the asset weights upper bound to 20% and impose a short selling restriction. Thus, the asset weights of the optimal portfolio have to honour the following boundaries $0 \leq w_i \leq 20\%$ (see e.g. Mostowfi and Stier, 2013; Braun et al., 2015). The asset weights must sum to one.

For the optimisation of the portfolios all objective functions described in Section 2.1 are used. Moreover, we consider daily 3-month T-Bill rates from 30th May 2003 to 31st May 2013 as risk-free interest rates. The risk-free rate is also used as target portfolio return in the optimisation process. We use the DJIA index and an EW portfolio as benchmark models to compare the performance of the optimised portfolios for potential outperformance. For the risk and performance analysis of the portfolios, we use the same measures introduced in Section 3.3.

We first analyse the influence of our new optimal capital constraint on the optimisation process in Section 3.4.1. We then compare portfolios optimised with and without LR_{UC} constraint in Section 3.4.2. Finally, we evaluate the portfolios based on their Basel III market risk capital requirements in Section 3.4.3.

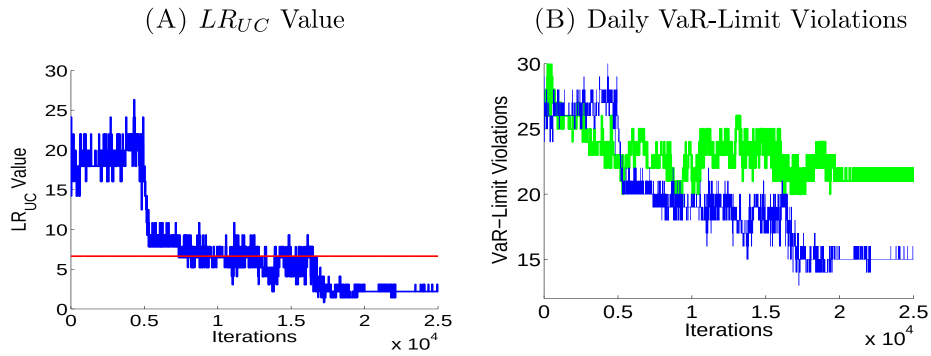

FIGURE 3.8 Optimisation results for the optimal $VaR_{H1\%}$ portfolio in the first in-sample period. The blue and red line in Figure 3.8A shows the development of the LR_{UC} value during the optimisation process and the critical χ^2 value, respectively. Figure 3.8B shows the development of the number of portfolio violations with LR_{UC} constraint (blue line) and without LR_{UC} constraint (green line).

3.4.1 Capital Constraint Penalty Function Results

The parameters for the TA algorithm are set to six restarts, five rounds and 5,000 steps, yielding a total of 25,000 iterations per restart. This calibration led to fast converging results.

As the number of daily VaR violations is discrete, the LR_{UC} value increases in discrete steps. Hence, in the optimisation process there can be several portfolio sets with the same LR_{UC} penalty value. However, there are no portfolio sets with penalty values in the continuous space between the discrete penalty steps.

Figure 3.8 shows the optimisation results of the $VaR_{H1\%}$ objective function for the first in-sample period, which stands exemplary for all optimised portfolios. The red line in Figure 3.8A represents the χ^2_1 critical value of 6.6349. Portfolio sets with LR_{UC} values above this line are penalised. Figure 3.8B shows the number of daily VaR-limit violations of the LR_{UC} constraint $VaR_{H1\%}$ portfolio for the first in-sample period (blue line). The green line shows the number of daily VaR-limit violations for the $VaR_{H1\%}$ without LR_{UC} constraint.

The LR_{UC} constraint leads to portfolios with optimal number of violations for the in-sample period. Thus, the optimal portfolios cannot maximise the yield for a given portfolio VaR value, as highlighted by Marshall and Prescott (2006) and Winker and Maringer (2007), but is restricted to avoid high number of daily VaR violations. On the other hand, the number of daily VaR violations are not below an optimal level to avoid the portfolio from being to conservative. We extend the analysis of portfolio results with and without the LR_{UC} constraint in the next section.

3.4.2 Comparison of Portfolio Performance

Figures 3.9 and 3.10 show the portfolio weights resulting from the TA optimisation process for the entire out-of-sample period. In each figure, we compare the portfolios with and without LR_{UC} constraint for the MV and VaR and CVaR objective function at 1% significance level with underlying empirical (Figure 3.9) and normal distribution (Figure 3.10).

The portfolio weights for MV, VaR and CVaR standard normally distributed portfolios without LR_{UC} constraint are very similar. This is because the alpha percentile of the standard normal is constant for all the cases analysed as it is the value of the standard normal density function for the alpha percentile. However, the portfolio weights of the same objective functions with LR_{UC} constraint differ to a greater extent. Which can be explained by the capital constraint adding more non-linearity to the optimisation surface.

For portfolios with empirical VaR and CVaR objective function the difference is even larger. The difference between objective functions with underlying standard normal and empirical return distributed can be explained by their density function. Portfolios with standard normal VaR and CVaR, as well as MV objective function assume standard normally distributed returns with expected skewness and kurtosis of zero and three, respectively. However, the skewness and kurtosis

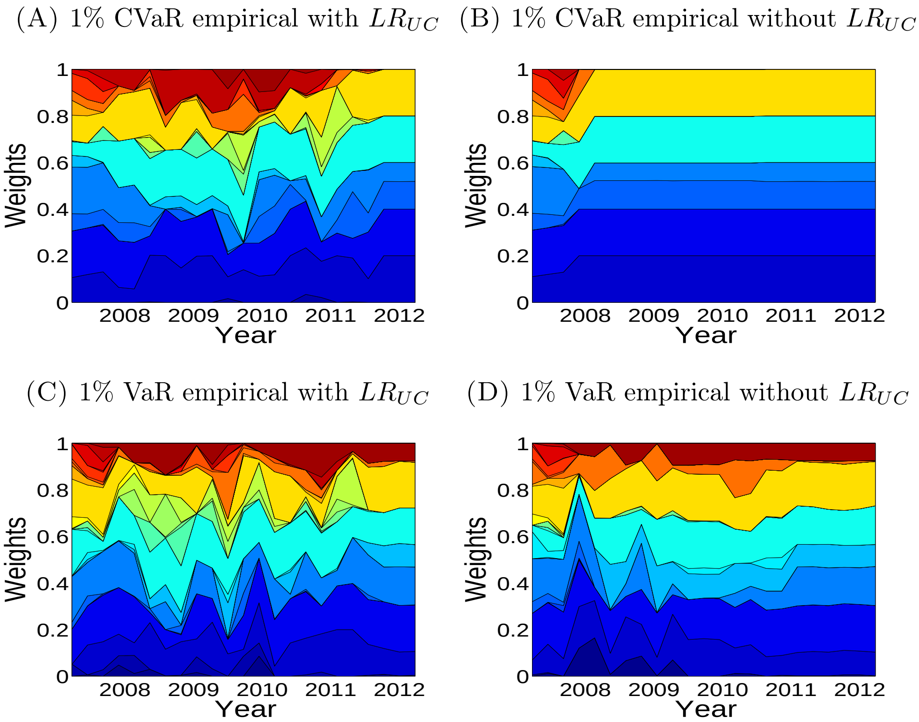

Figure 3.9 Comparison of the dynamic portfolio weights for different combinations of the VaR and CVaR objective function with underlying empirical distribution. A colour map for the portfolio weights is provided in Figure 3.2.

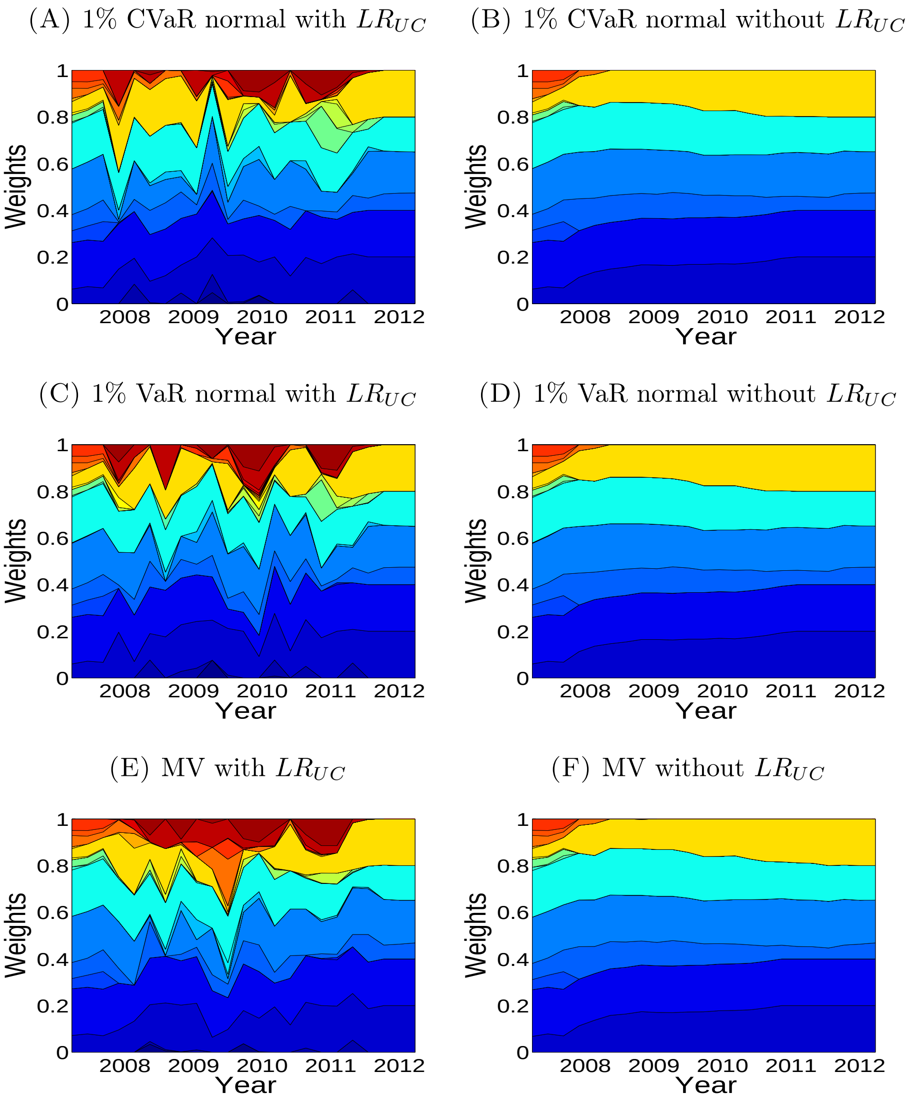

FIGURE 3.10 Comparison of the dynamic portfolio weights for different combinations of the MV, VaR and CVaR objective function with underlying normal distribution. A colour map for the portfolio weights is provided in Figure 3.2.

of the empirical return distribution can deviate from the standard values (Fama, 1965). In situations where the market volatility increases, the empirical return distribution of the assets might change to a large extent. In contrast to the minimum capital requirements approach suggested by Santos et al. (2012), our LR_{UC} constraint adds more volatility to the portfolio weight allocation compared with portfolios without our regulatory constraint. The LR_{UC} constraint increases the risk taking of portfolios that overestimate risk, as can be seen for VaR, CVaR and MV objective functions with normal distribution assumption (see e.g. mean return, standard deviation and modified Sharpe ratio in Table 3.3). If the portfolio underestimates the portfolio risk, the LR_{UC} constraint reduces the portfolio risk, as observable for empirical VaR and CVaR objective functions.

For the optimisation process we use the TR local search and the TA heuristic algorithm. The TA algorithm led to slightly better results. Table 3.3 shows the TA portfolio measures for the entire out-of-sample period. For comparison, we also provide the results for the benchmark models. For the entire out-of-sample period, portfolios optimised with our new constraint have higher mean return values than their equivalents without LR_{UC} constraint. The only exceptions are VaR and CVaR optimised portfolios with underlying empirical distribution. All portfolios have a higher mean return than the index. The only portfolio, however, with higher mean return than the EW portfolio is the $VaR_{H1\%}$ portfolio without capital constraint.

The lowest standard deviation is reported for the $CVaR_{H1\%}$ portfolio without LR_{UC} constraint with 16.95%. Portfolios with MV objective function also report some of the lowest standard deviations with 17.13% and 17.08% with and without LR_{UC} constraint, respectively. The highest standard deviations can be seen for VaR optimised portfolios with underlying empirical distribution. All portfolios have a lower standard deviation value than the benchmark models.

TABLE 3.3 Annualised out-of-sample TA portfolio results based on the daily portfolio returns. The annualised mean return, standard deviation and modified Sharpe ratio are expressed in percentage points.

Model			Descriptive Statistics and Performance Measure								
Algorithm	Objective Function	Return Distribution	Mean Return	Standard Deviation	Mod. Sharpe Ratio	VaR Historical	VaR Normal	CVaR Historical	CVaR Normal	Kurtosis	Skewness
with	CVaR	Historical	5.33	17.27	29.14	-0.5674	-0.4017	-0.7114	-0.4568	18.63	0.67
LR_{UC}		Normal	6.97	17.21	38.78	-0.5374	-0.4003	-0.6952	-0.4542	20.49	0.79
	VaR	Historical	6.37	18.17	33.45	-0.5819	-0.4226	-0.7446	-0.4802	16.22	0.47
		Normal	6.87	17.48	37.61	-0.5373	-0.4068	-0.6956	-0.4617	19.47	0.77
	MV		6.95	17.13	38.86	-0.5358	-0.3985	-0.6892	-0.4521	20.08	0.78
without	CVaR	Historical	6.95	16.95	39.24	-0.5717	-0.3943	-0.7033	-0.4473	17.75	0.55
LR_{UC}		Normal	6.72	17.12	37.51	-0.5377	-0.3982	-0.6932	-0.4519	20.63	0.79
	VaR	Historical	7.89	18.40	41.31	-0.5868	-0.4280	-0.7363	-0.4853	15.55	0.44
		Normal	6.68	17.14	37.22	-0.5381	-0.3988	-0.6951	-0.4526	20.75	0.80
	MV		6.78	17.08	37.95	-0.5366	-0.3973	-0.6897	-0.4509	20.29	0.78
(Benchmark)	Index		3.75	22.99	15.01	-0.7480	-0.5347	-0.9405	-0.6102	11.27	-0.03
	EW		5.83	25.50	21.71	-0.8369	-0.5933	-1.0391	-0.6760	11.28	0.05

The optimised portfolios have a higher modified Sharpe ratio than the benchmark portfolios and are therefore ranked better. This is mainly because of the lower standard deviation of the optimised portfolios compared to the index and the EW portfolio. The highest modified Sharpe ratio with 41.31% can be seen for the portfolio with $VaR_{H1\%}$ objective function and no LR_{UC} constraint. As before, portfolios optimised with capital constraint and VaR and CVaR objective function with underlying normal distribution or MV objective function, have higher modified Sharpe ratios compared to their equivalents without LR_{UC} constraint. The LR_{UC} constraint slightly increases the standard deviation and mean return of the portfolios with normal distribution, as the portfolios without LR_{UC} constraint overestimate the portfolio risk and behave to conservative.

The $VaR_{H1\%}$ portfolio with capital constraint has a higher historical VaR value than the VaR portfolio without our new constraint. The same can not be observed for the other objective functions. The optimised portfolios are better then the benchmark, however, portfolios optimised with the LR_{UC} constraint have the same or slightly worse objective measures than their equivalents without LR_{UC} constraint. The reason is that the portfolio with LR_{UC} constraint is not just optimised for the objective function but also for the number of daily VaR violations in order to reduce the capital requirements. Since the calculation of the capital requirements is mainly based on the daily or 60 days daily average VaR level (Equation 3.3), respectively, portfolios with LR_{UC} constraint are expected to have lower empirical VaR measures than their equivalent counterparts without the new constraint. As Table 3.3 shows, this is true for all objective functions. Portfolios optimised with our new LR_{UC} constraint always have slightly lower empirical 1% VaR values compared to their equivalents without capital constraint.

With respect to the performance of portfolios with LR_{UC} constraint, Table 3.3 shows that the portfolio optimised with $CVaR_{N1\%}$ has a higher mean return value compared with its counterparts with underlying empirical distribution. The

$CaR_{N1\%}$ portfolio with LR_{UC} constraint has the highest mean return closely followed by the MV and $VaR_{N1\%}$ portfolio. The lowest portfolio performance has the $CVaR_{H1\%}$ portfolio. In general, for this analysis portfolios with underlying empirical distribution have lower mean return values than portfolios using the standard normal distribution. The same observation can be made for the modified Sharpe ratio.

The MV portfolio has the lowest standard deviation (17.13%) of all portfolios with LR_{UC} constraint, whereas the $VaR_{H1\%}$ portfolio has the highest values with 18.17%. All LR_{UC} constraint portfolios have better values than the benchmark models. The optimised portfolios are positively skewed and highly leptokurtic. The LR_{UC} constraint portfolio with the highest kurtosis (20.49) and skewness (0.79) is reported for the $CVaR_{N1\%}$ objective function. The lowest kurtosis (16.22) and skewness (0.47) is given by the $VaR_{H1\%}$ portfolio with capital constraint. Both benchmark models have lower kurtosis and skewness values than the optimised portfolios. Objective functions with underlying standard normal distribution have a higher kurtosis and lower skewness than portfolios optimised with empirical distribution.

Compared to the portfolio with $CVaR_{H1\%}$ objective function and LR_{UC} constraint, the $CVaR_{N1\%}$ portfolio with LR_{UC} constraint has a better empirical and normal 1% CVaR value. The same can be observed for the $VaR_{N1\%}$ portfolio. The empirical VaR for the $VaR_{N1\%}$ portfolio (-0.5373) is higher than the $VaR_{H1\%}$ portfolio (-0.5819). For the normal 1% VaR, $VaR_{N1\%}$ is also better than the $VaR_{H1\%}$ portfolio, as Table 3.3 shows.

3.4.3 VaR Backtesting Results

The portfolio risk profiles are evaluated based on the results generated by the backtested VaR model. For all portfolios with and without LR_{UC} constraint, the model calculates the daily VaR values for a 1% significance level based on the empirical portfolio distribution. The VaR values are computed on a 250 out-of-sample days rolling window basis, which is the time period used by the Basel Committee to evaluate the VaR disclosures of the financial institutions. Hence, the first value is calculated for the 251^{st} day of the out-of-sample period.

We use the UC, IND and CC test (see Equations 3.7, 3.10 and 3.11) to identify whether the portfolios under- or overestimate risk (Christoffersen, 2003). Moreover, the Basel III market risk capital requirements (Basel Committee on Banking Supervision, 2009a) serve as a further assessment criterion.

Table 3.4 shows the backtesting results for all portfolios. For comparison purposes, we also provide the backtesting results for the index and EW portfolio. The capital requirements are calculated for one monetary unit to make the results comparable across different portfolio allocations and scalable for any portfolio market value. The critical values for a χ^2 test with 1% significance level and one and two degrees of freedom are 6.64 and 9.21, respectively. The null hypothesis for the UC, IND and CC test is rejected if the corresponding test value in Table 3.4 exceeds the respective threshold.

As Table 3.4 shows, all portfolios have LR_{IND} and LR_{CC} values below the critical values. The LR_{IND} values for the portfolios are close to zero, which means that the violations are independently distributed over the out-of-sample period. The LR_{UC} values indicate that the portfolios have an optimal average number of VaR violations over the testing period. The only exception is the $VaR_{H1\%}$ portfolio without our LR_{UC} constraint. With a LR_{UC} value of 8.23 this portfolio clearly exceeds the critical χ^2 test value of 6.64. We find that this is caused by a relatively high average number of violations with 4.20. By comparison, the equivalent portfolio with our LR_{UC} constraint has an average of 3.63 yearly VaR violations over the entire testing period.

TABLE 3.4 VaR Backtesting Results. The likelihood ratio values for the LR_{UC}, LR_{IND} and LR_{CC} are reported in the corresponding columns. The critical values for a χ^2 test with 1% significance level and one and two degrees of freedom is 6.64 (UC and IND) and 9.21 (CC), respectively. The last columns show the capital requirements (CR) for portfolios with (+) and without (-) LR_{UC} constraint and the relative difference between these portfolios (relative to the LR_{UC} constraint portfolio value).

Objective	Return	LR_{UC}		LR_{IND}		LR_{CC}		CR		
Function	Distribution	+ LR_{UC}	- LR_{UC}	+ LR_{UC}	- LR_{UC}	+ LR_{UC}	- LR_{UC}	+ LR_{UC}	- LR_{UC}	+/-
CVaR	Historical	4.14	4.14	0.00	0.00	4.14	4.14	0.2081	0.2126	-2.2%
	Normal	5.21	5.05	0.00	0.59	5.21	5.64	0.1924	0.2114	-9.9%
VaR	Historical	5.58	8.23	0.52	0.28	6.10	8.50	0.2050	0.2051	-0.0%
	Normal	4.99	5.05	0.59	0.59	5.57	5.64	0.1994	0.2115	-6.0%
MV		3.97	5.05	0.00	0.59	3.97	5.64	0.1921	0.2117	-10.2%
(Index)		(6.62)		(0.41)		(7.03)		(0.2835)		
(EW)		(12.96)		(0.08)		(13.03)		(0.2915)		

Our results are similar to the once presented by Drenovak et al. (2017) who use high and low volatile optimisation periods from 2007 to 2013 for 40 constituents of the S&P 100 index. They report a maximum number of VaR violations for the common mean-VaR portfolio and their mean-regulatory VaR portfolio of 7 and 4 violations in the high volatility sample, respectively. The average number of violations for their proposed VaR optimisation model is 2.93. Even though these results are not directly comparable with our findings, it shows that our model is capable to generate similar results over a longer out-of-sample period and for a lower number of assets considered in the optimisation.

All portfolios have lower capital requirements than the benchmark models. Interestingly, the EW portfolio has the highest capital requirements (0.2915). This is even worse than the index (0.2835) but can be explained by the higher empirical 1% VaR value. Portfolios optimised with LR_{UC} all yield lower capital requirements than the objective functions without our new constraint. Hence, we can conclude that the new constraint successfully reduces the Basel III market risk capital requirements.

The lowest capital requirement is reported for the MV portfolio (0.1921), followed by the $CVaR_{N1\%}$ portfolio (0.1924). Both portfolios use the LR_{UC} constraint in their optimisation. The constraint portfolio with the highest capital requirements (0.2081) uses the $CVaR_{H1\%}$ objective function. However, this is still 2.2% better than the equivalent portfolio without capital constraint (0.2126).

The most significant difference between portfolios with and without LR_{UC} constraint can be observed for the MV objective function. The capital requirements of the portfolio with LR_{UC} are 10.2% lower than the portfolio without constraint. This is followed by the portfolio with the $CVaR_{N1\%}$ objective function, where the LR_{UC} constraint reduces the capital requirements by 9.9%.

Surprisingly, the LR_{UC} portfolios with VaR and CVaR objective function and underlying empirical distribution have higher capital requirements than their counterparts with underlying normal distribution. Thus, the capital requirements for the portfolio with $CVaR_{N1\%}$ objective function is 8.1% lower compared with the $CVaR_{H1\%}$ portfolio. For the VaR objective function the portfolio with normal distribution has 2.8% lower capital requirements.

Our new LR_{UC} capital constraint reduces the Basel III market risk capital requirements by up to 10.2% compared to portfolios that do not use the constraint. The most significant improvement can be seen for the MV and the $CVaR_{N1\%}$ portfolio. However, it should be mentioned that minimising the regulatory capital requirements, as proposed by Santos et al. (2012) and Drenovak et al. (2017), is not the primary objective in our optimisation approach. The suggested LR_{UC} constraint aims to control the number of VaR violations to prevent portfolios from entering the red zone of the Basel III traffic light scheme to avoid potential financial instability and damaging effects on the banks reputation. On the other hand, the LR_{UC} constraint circumvents overestimation of the portfolio risk as this can have negative effects on the banks social and economic factors.

We provide extended test results for the application of our LR_{UC} constraint in Table B.1. For a time series from 30th January 2006 to 29th January 2016, we compare the out-of-sample results of $VaR_{H1\%}$ portfolios with and without the LR_{UC} constraint for a 10-days and 1-day out-of-sample holding period. The results show that the $VaR_{H1\%}$ portfolio with LR_{UC} constraint has on average lower minimum capital requirements, a lower average number of daily VaR violations and a better multiplication factor. The findings support the test results in Section 3.4.3 for an updated time series and different investment horizons. Moreover, the results highlight that our innovative constraint contributes to a better portfolios management under consideration of regulatory requirements.

3.5 Conclusion

In this chapter, we first examined how a combination of TA optimisation algorithm and objective functions described in Section 2.1 can reduce the market risk capital requirements. Then, we introduced a new risk management approach based on the UC test to create portfolios that minimise the regulatory capital requirements while avoiding to over- or underestimate the market portfolio risk.

The first part of our empirical analysis showed that the TA algorithm achieved better risk measures than the TR local search algorithm whenever VaR or CVaR objective functions with underlying empirical distribution is used. This can be observed for 1% and 5% significance levels. The most significant improvement in the risk measure can be seen for VaR based objective functions. Due to the non-linear nature of VaR optimisation problems, the TA algorithm clearly finds better optimal solutions than the TR search method for VaR optimised portfolios.

The portfolio risk profile can be improved when the TA search algorithm is used with an empirical CVaR (1% or 5%) and 1% VaR objective function. For all VaR and CVaR objective functions, the TA optimised portfolios have substantially better risk measures than the TR optimised portfolios. The TA portfolios also clearly outperform the benchmarks in any risk measure. The TA portfolio with 5% CVaR objective function exhibits a lower standard deviation than any other portfolio. It also has a better standard deviation than the MV optimised portfolios. In terms of reducing the probability of significant portfolio losses, the TA optimised portfolios also have the lowest maximum drawdown. In both the in-sample and out-of-sample period the TA portfolios with empirical CVaR (1% or 5%) and 1% VaR objective functions clearly surpass the other portfolios. Furthermore, the best portfolio performance in terms of price development in the out-of-sample period is obtained when the empirical 1% VaR objective function or a CVaR objective function is used with heuristic algorithm.

Our empirical results show that the TA optimisation improves the capital requirements for all portfolio objective functions in our study. We find that the $CVaR_{H5\%}$ portfolio with TA algorithm has the lowest capital charges. The most significant improvement to TR optimised portfolios can be seen for the $VaR_{H5\%}$ portfolio with an improvement of almost 24%.

In the second part of our empirical analysis we studied the contribution of our LR_{UC} constraint to reduce the regulatory capital requirements of financial institutions.

The results of our empirical study show that our new risk management approach reduces the empirical VaR level of a portfolio and optimises the portfolio for the Basel III traffic light scheme. The capital constraint leads to better objective function measures compared with portfolios without LR_{UC} constraint, in the out-of-sample period. For portfolios optimised with empirical VaR or CVaR objective function the results are about the same compared to their equivalents without LR_{UC} constraint. Portfolios with optimal capital constraint have better empirical 1% VaR values than portfolios without LR_{UC} constraint. Except for the $VaR_{H1\%}$ and $CVaR_{H1\%}$ portfolio the results report higher modified Sharpe ratios for portfolios optimised with our optimal capital constraint.

Even though our LR_{UC} constraint does not aim to minimise the capital requirements, our empirical results suggest that for all optimised portfolios we were able to improve the results. Portfolios optimised with LR_{UC} constraint reduce the capital requirements by up to 10.2% compared to the same portfolio without capital constraint, in the out-of-sample period. The most significant difference between portfolios with and without LR_{UC} constraint is seen for portfolios with MV and $CVaR_{N1\%}$ objective function. VaR and CVaR objective functions with underlying standard normal distributions report lower capital requirements than their counterparts with empirical distribution.

Compared with the work of Santos et al. (2012) and Drenovak et al. (2017), portfolios optimised with our proposed constraint achieve similar number of daily VaR violations. However, a benefit of our advocated LR_{UC} constraint is that it manages the regulatory requirements while the risk manager can choose individual objective functions to be optimised.

With the introduction of the new Basel III framework, banks are increasingly interested to find ways to reduce their regulatory capital requirements. Implementing our advocated LR_{UC} based capital requirements approach is a valuable method for financial institutions to optimally manage their market portfolio while indirectly controlling the regulatory capital requirements. Moreover, heuristic optimisation methods provide better optimisation results compared with local search methods and therefore, are useful tools to manage the risk of financial institutions.

In this chapter, we studied the efficiency of standard normal and empirical VaR and CVaR optimisation to reduce the regulatory capital requirements. The use of Monte Carlo simulation and multivariate volatility estimation can moreover improve the risk management and thus the capital requirements of financial institutions. In case of simulation-based optimisation, TA is a computationally expensive optimisation approach as it needs a large number of function evaluations to generate a single solution. Thus, fast converging heuristic optimisation methods are to be preferred in this case.

In the next chapter, we will be examining the influence of simulation-based VaR and CVaR estimation on the number of daily VaR violations and the capital requirements. The results can have implications for the banks internal risk model as it compares different optimisation processes and how they influence the capital requirements.

Chapter 4

The Leverage of Simulation on Regulatory Capital Requirements

In the previous chapter, we introduced a new risk management approach to control the capital requirements for a variety of objective functions, described in Section 2.1. Moreover, we demonstrated how the TA heuristic algorithm can reduce the regulatory capital requirements of financial institutions.

In this chapter, we extend the previous study by examining the influence of simulation-based VaR and CVaR estimation on the number of daily VaR violations and the capital requirements. In their empirical analysis on VaR backtesting models, Uylangco and Li (2016) found that more advanced VaR estimation models experience a higher average number of VaR violations but on average lower capital charges. We study if this conclusion also holds for portfolios with more advanced optimisation models. Moreover, previous literature that focuses on capital requirements focuses on empirical and analytical one-day ahead VaR estimation. In this chapter, we examine the influence of several-days ahead VaR and CVaR estimation methods with different underlying distribution assumptions.

The results of our empirical analysis support the findings of Uylangco and Li (2016), who examined a higher average number of VaR violations for more advanced VaR estimation models with standard normal distribution assumption. In addition to the work of Uylangco and Li (2016), we find that the average number of VaR violations can be reduced for optimisation models with empirical VaR and CVaR objective function when based on the simulated returns. The simulation-based approach gives higher priority to more recent information and thus, provide a better estimation of the future return distribution. In general, the average number of VaR violations is higher for empirical objective functions (Equations 2.2 and 2.3) than for standard normal objective functions (Equations 2.4 and 2.5), which supports the findings of Winker and Maringer (2007). Moreover, our results show that with Monte Carlo simulation the $VaR_{H1\%}$ portfolio has a lower number of average VaR violations than the $VaR_{N1\%}$ portfolio. With regard to the capital requirements, all portfolios, except the $VaR_{N1\%}$, reduce the capital charges when based on the simulated returns distribution. Objective functions with standard normal optimisation have lower capital requirements than their equivalents with empirical distribution. Again, the only exception is the $VaR_{N1\%}$ portfolio. Thus, we can conclude that our forecast based Monte Carlo simulation approach reduces the capital requirements and average number of VaR violations, for most of the portfolios.

This chapter is structured as follows. In Section 4.1, we describe purpose and contributions of this chapter. Section 4.2 introduces the methodology used in the empirical analysis, which follows in Section 4.3. In Section 4.4, we conclude the results of the analysis.

4.1 Introduction

In recent years, VaR has become a popular risk measure in portfolio management and market regulations. As major component of the Basel III mark risk capital requirements formula, VaR directly influence the amount of regulatory capital requirements of a financial institution. It is also an indicator of how efficient the risk model and hence, how stable a bank is. Therefore, it is of interest for financial institutions to reduce the VaR value of their market risk portfolio.

The assumption about the distribution of the returns is an important decision for the calculation of VaR or other downside risk measures, e.g. CVaR (Acerbi and Tasche, 2002a). In his Modern Portfolio Theory Markowitz (1952) assumes that the returns are normally distributed. However, Fama (1965) doubts this assumption as returns are not well described with mean and variance only. A common approach in the quantitative risk management literature is to use the empirical return distribution (see e.g. Jorion, 2006; Lucas and Klaasen, 1998; Pritsker, 1997). It is a common assumption that the empirical return distribution best describes the future returns. However, an alternative to the parametric and empirical distribution assumption is to generate future returns with Monte Carlo simulation. To improve the quality of the simulated returns, autoregressive conditional heteroscedasticity (ARCH) models such as the Generalised Autoregressive Conditional Heteroscedasticity (GARCH) model (Bollerslev, 1986) and the multivariate Dynamic Conditional Correlation (DCC) model introduced by Engle (2002) can be used.

To improve the VaR estimation and thus, to positively influence the amount of regulatory capital required, several studies in recent literature applied univariate and multivariate GARCH estimation models. McAleer et al. (2010) used several variance estimation methods such as Equally Weighted Moving Average (EWMA), GARCH, Exponential GARCH (EGARCH) and GJR GARCH to estimate VaR for their passive dynamic decision rule to manage regulatory capital charges. Santos et al. (2012) also applies an EWMA approach as well as the multivariate DCC model and a covariance estimation method based on the shrinkage estimator of Ledoit and Wolf (2003) to optimise for their proposed minimum capital requirements objective function. Moreover, Drenovak et al. (2017) examine how the univariate GARCH model reduces their multi-objective optimisation problem. The optimisation model is based on the paper of Ranković et al. (2016) who use an univariate and multivariate GARCH estimation approach.

Another more recent study by Uylangco and Li (2016), analyses different VaR estimation techniques to generate more efficient VaR backtesting models to calculate daily VaR exceeding. They find that Monte Carlo simulations, using the static standard deviation of the empirical returns, and ARMA-GARCH backtesting models show a relatively high percentage of VaR violations but a lower average magnitude of the violations on the capital requirements. They conclude that more sophisticated models improve the VaR estimation compared with empirical and parametric backtesting models. The paper of Uylangco and Li (2016) does not concentrate on portfolio optimisation to reduce the regulatory capital charges but to improve the backtesting model. However, their observation supports the paper of Winker and Maringer (2007), who find that objective functions with underlying empirical distribution function on average have a higher number of VaR violations.

Objective functions with downside risk measures such as VaR or CVaR are non-linear selection problems with multiple local extremes (Alexander et al., 2006). Non-linear optimisation problems can be solved with heuristic methods. Dueck and Winker (1992) were the first to apply the heuristic TA model (Dueck and Scheuer, 1990) to a portfolio optimisation problem. A more advanced heuristic method is the PBIL algorithm (Baluja and Caruana, 1995). It can be classified under the Estimation of Distribution Algorithms and is a hybrid of Genetic Algorithms (GA) (Holland, 1975) and Competitive Learning (see e.g. Zell, 1994). Gosling et al. (2005) provide a comparison between the GA and PBIL algorithm. More heuristic methods are described in the work of Maringer (2005) and Gilli et al. (2011).

In this chapter, we contribute to the existing literature in several ways; First, we study if the observations by Uylangco and Li (2016) also apply to portfolio optimisation problems. This is, we examine if more advanced optimisation problems experience a higher average number of VaR violations but on average lower capital charges. Second, related literature focuses on empirical and analytical one-day ahead VaR estimation. This chapter adds additional knowledge on the ability of CVaR based objective functions to optimally manage the regulatory capital requirements. Moreover, we generate several-days ahead forecasts and simulate multivariate distributed returns to provide a more realistic analysis, as daily trading might not be feasible due to transaction costs.

In an empirical analysis, the correlation between the assets is estimated with the DCC model, as this model proved to successfully capture the dependencies between instruments. We generate several-days ahead forecasts based on the formula of Engle (2002), and compute daily variances and correlations between the assets. Then Monte Carlo simulation and Cholesky decomposition is applied to these forecasts to generate correlated asset returns. Agarwal and Mehra (2014) shows that compared with other decomposition techniques such as QR Decomposition, Singular Value Decomposition (SVD) and Lower-Upper Decomposition (LU), Cholesky decomposition is the most efficient approach in terms of memory storage, computational cost, speed and data reduction. To solve the non-linear portfolio optimisation problems we use the PBIL heuristic algorithm.

The following Section 4.2 presents the methodology used for the empirical study. We first describe the search algorithm used to optimise the objective functions, before we continue with the introduction of the simulation process. Section 4.3 examines the results of the computational study conducted on the empirical sample. In Section 4.4 we conclude with a summary of the results.

4.2 Methodology

In this section, we introduce the methodology used for the empirical analysis in Section 4.3. We first describe the PBIL heuristic algorithm which is used in the optimisation in Section 4.2.1. Then, Section 4.2.2 presents the simulation approach to estimate VaR and CVaR risk measures.

4.2.1 Search Algorithms

For the optimisation process of the objective functions (see Equations 2.2-2.5) we use the PBIL optimisation method introduced by Baluja and Caruana (1995). The PBIL is a hybrid search method that works with a population of candidate solutions. Each candidate solution is a binary vector of length N, where N is the number of assets in the investment universe.

The population evolves over a number of generations using a probability vector. Similar to a competitive learning algorithm, the values in the probability vector are gradually shifted towards representing assets that generate optimal results (Baluja and Caruana, 1995).

Compared to a common GA (Holland, 1992), the PBIL algorithm generates more accurate results while it attains the results faster, both in terms of computational time as well as the number of evaluations. This can be explained by the algorithms ability to focus its search efforts in one region of the search space much faster than the GA (Baluja and Caruana, 1995). Furthermore, the PBIL is a very simple algorithm, which is easy parallelisable and does not need all the subfunctions necessary for GA. This makes the PBIL an excellent search method for our simulation-based portfolio selection problem. Figure 4.1 outlines the PBIL optimisation process while Algorithm 3 provides a more detailed view of the PBIL structure.

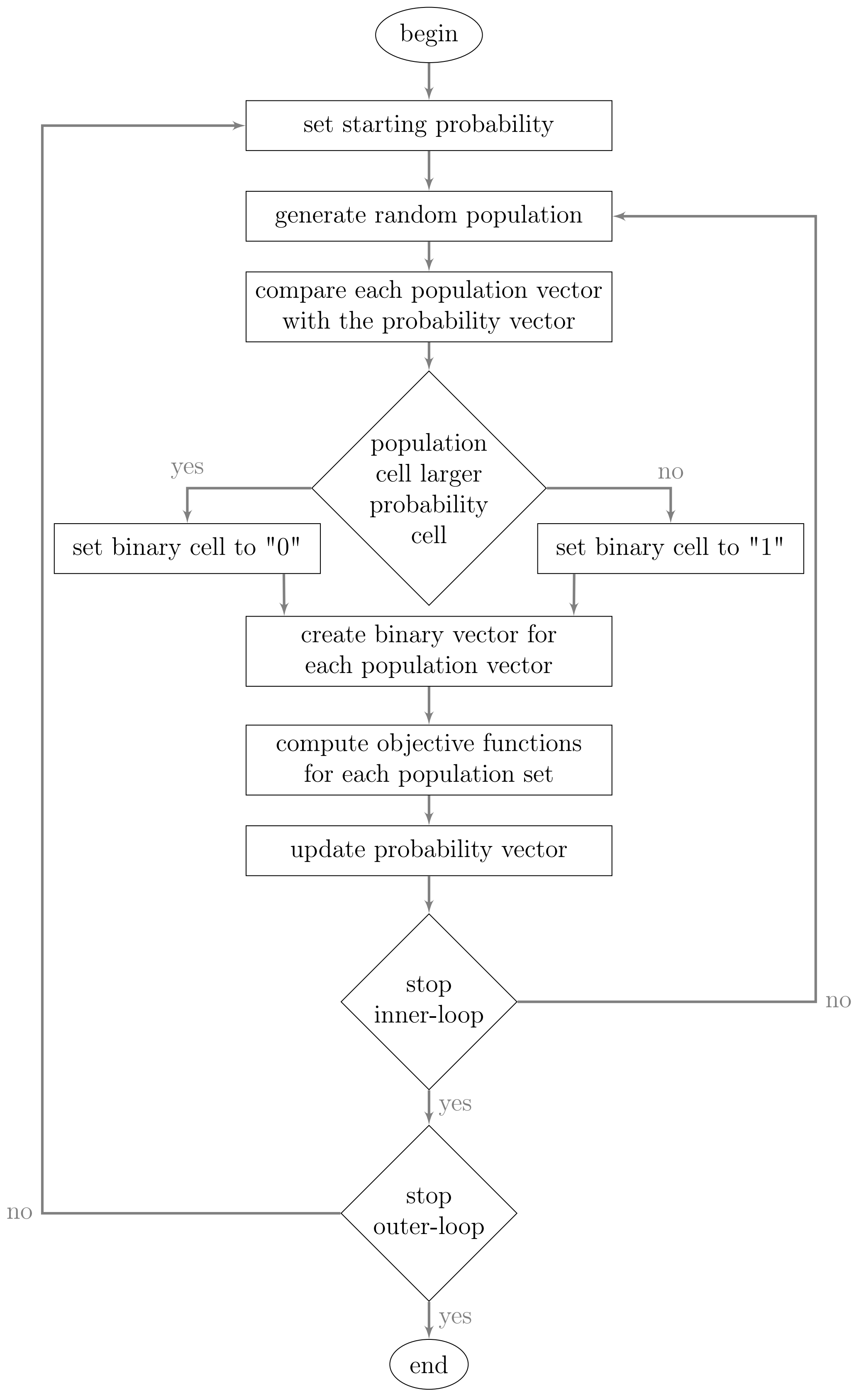

FIGURE 4.1 PBIL flowchart

Algorithm 3 Population-Based Incremental Learning Algorithm

set starting probability vector P
while stopping criteria not met **do**
  generate G random probability vectors
  create binary vector
  for $g = 1$ to G **do**
    compute objective function $F_g = F(w_g, \mu^{port}, \mu^{target})$
  end for
  select survivors $S^{survivor}$
  for $s = 1$ to $S^{survivor}$ **do**
    update probability vector P for $S_s^{survivor}$
  end for
end while

The first step is to define the starting probability vector. Similar to a competitive learning algorithm, the values in the probability vector $P = (P_1, ..., P_M)$ are gradually shifted towards representing instruments that generate optimal results. In the first run of the algorithm the probabilities in vector P are set to 50% as no instrument is preferred over the other (Baluja and Caruana, 1995).

With each generation the algorithm generates a population set of G i.i.d. normal random probability vectors. In the literature, different methods have been proposed to determine the optimal size of G (see e.g. Smith and Smuda, 1995; Mühlenbein, 1989; Goldberg, 1989). Then, the PBIL algorithm generates a population of binary vectors by comparing the random population sets with the probability vector. Each binary vector represents a candidate solution of length M. The candidate solutions are given by setting all instrument positions in the population sets equal to zero if the sample random probability is larger than the probability in P_i, where $i = 1, ..., M$. If the probability of position i in the sample set is smaller or equal to P_i the binary is set to one (Baluja and Caruana, 1995).

The cardinality constraint is easy to implement for the PBIL search method as each candidate solution is represented by a binary vector. Binary vectors with a sum between the cardinality lower and upper bound are considered for further optimisation. To ensure that the algorithm uses the same population size at each generation, we propose to replace population sets that violate the cardinality constraint by another random probability vector.

To implement the portfolio weight constraints we normalise the probabilities of the sample vectors to one, as the sum of the asset weights is one. Asset weights violating the lower bound constraint are set to the value of the lower boundary. Weights exceeding the upper bound constraint are set to the value of the upper boundary. Changes in the asset weight allocation are summarised and equally distributed between the assets that have not violated the constraints.

The objective function values are calculated for each constrained population set w_g, where $g = 1, ..., G$. The results are sorted descending in the result vector F. For a number s_{size} of best binary vectors $S^{survivor}$ the probability vector P is updated, using Equation (4.1) (Baluja and Caruana, 1995).

$$P_i = P_i(1 - LR) + S_s^{survivor} LR \tag{4.1}$$

There are different ways to define the number of s_{size}, e.g. select a random number or the number of best population sets for the α-Quantile of the solution vector F. However, we found that the algorithm generates the best and most stable results when $s_{size} = 2$. Increasing s_{size} makes the algorithm faster but the results are not very stable.

The probability P_i of an instrument represented in a surviving candidate solution is updated by Equation (4.1). The probability of an instrument not represented in the survivor set is decreased by the learning rate LR (Baluja and Caruana, 1995). A high probability P_i increases the likelihood of an instrument to have a binary value of one. Thus, the learning rate influences which part of the function space is explored. If LR is too high the search space is narrowed too fast and the algorithm is unable to exploit the entire function space. If LR is too low the algorithm is unable to focus on the optimal solution space and find the best portfolio allocation (Baluja and Caruana, 1995). Folly and Venayagamoorthy (2009) proposes different techniques to avoid LR from being too high or too low. One way is to linearly increase the LR at every generation by a constant. The algorithm repeats this process until all probabilities in P are below or above certain thresholds (see e.g. Baluja, 1997; Shapiro, 2002).

To find the optimal portfolio that satisfies the inequality target return constraint, we implement the penalty function 3.1 described in Section 3.2.1. Thus, the objective functions can be described using Equation 3.2.

4.2.2 Dynamic Conditional Correlation

To capture the dependencies between the assets to improve the quality of the simulated asset returns, we use the DCC model introduced by Engle (2002). The optimisation of the objective functions described in Section 2.1 is based on empirical and standard normal VaR and CVaR estimation. Chapter 3 analyses the two estimation methods and the influence on the regulatory capital requirements. In this chapter, we extend the previous research by analysing the influence of VaR and CVaR calculation from simulation and how effects the portfolios performance, risk profile and capital requirements. Specifically, we first estimate the volatility (GARCH) and correlation (DCC) model for each asset's daily return and the DJIA daily return, then simulate future returns for the instruments. The VaR and CVaR objective function with simulation is then estimated for the empirical and normal distribution of the simulated returns.

We describe the process to estimate the dependencies between the assets and simulate correlated return series. The model parameters are estimated for the in-sample periods and used to simulate dependent price movements for the out-of-sample periods. The DCC model estimates the conditional correlation between the instruments. The forecast conditional correlations are used to simulate a number of correlated returns.

The conditional covariance matrix H_t at time t is given by Engle (2002)

$$H_t = D_t R_t D_t \tag{4.2}$$

$$R_t = diag(U_t)^{-1/2} U_t diag(U_t)^{-1/2} \tag{4.3}$$

$$U_t = (1 - \hat{a} - \hat{b})\overline{U} + \hat{a}\epsilon_{t-1}\epsilon_{t-1}^{\intercal} + \hat{b}U_{t-1} \tag{4.4}$$

where R_t is the conditional correlation matrix and U_t is an $M \times M$ matrix of covariances. $\overline{U}$ is the unconditional covariance of the standardised correlated residuals $\epsilon_t = D_t^{-1} r_t$. r_t denotes the residuals at time t and D_t is an $M \times M$ diagonal matrix of standard deviations $\sqrt{h_{it}}$ drawn from a univariate GARCH(1,1) model

$$h_{it} = \gamma_i + a_i r_{it-1}^2 + b_i h_{it-1} \tag{4.5}$$

for $i = 1, ..., M$ instruments. As described in Engle (2002) the GARCH variances must be stationary and non-negative. Also, the sum of a_i and b_i need to be less than one and $\gamma_i > 0$.

The parameters $\hat{a}$ and $\hat{b}$ control the influence of ϵ_t on R_t. R_t reverts back to its long term average more slowly if $\hat{b}$ is high. The influence of the latest ϵ_t on the conditional correlation matrix increases with a lower $\hat{b}$ value. To calculate U_t the parameters $\hat{a}$ and $\hat{b}$ need to be estimated. The sum of the DCC parameters $\hat{a}$ and $\hat{b}$ has to be less than one.

Parameter Estimation

Before we can simulate the dependent price movement of the instruments for the PBIL optimisation we need to estimate the model parameters.

The estimation of the DCC parameters is performed in a two step process. First, the GARCH parameters $\phi = (\gamma_1, a_1, b_1, ..., \gamma_M, a_M, b_M)$ are estimated for each asset to calculate the conditional variances. Then, the residuals are standardised by their estimated conditional variances and used to estimate the parameters $\psi = (\hat{a}, \hat{b})$ of the DCC model. The estimation of the GARCH and DCC parameters can be performed by quasi-likelihood $\mathcal{L}$ estimation (Engle, 2002). For the discussion we want to mention an alternative method to estimate the model parameters using heuristics, which is proposed by Winker and Maringer (2009). However, in this research we use the standard quasi-likelihood function for the parameter estimation. The GARCH quasi-likelihood function is

$$\mathcal{L}_1(\phi|r_t) = \sum_{i=1}^{N}\sum_{t=2}^{T}\left(\log(h_{it}) + \frac{r_{it}^2}{h_{it}}\right). \tag{4.6}$$

The final step is to maximise the likelihood function for the DCC process to estimate $\psi = (\hat{a}, \hat{b})$ for the estimated parameters ϕ^* in the first step

$$\mathcal{L}_2(\psi|\phi^*, r_t) = -1/2\sum_{t=1}^{T}\left(M\log(2\pi) + 2\log|D_t| + \log|R_t| + \epsilon_t^\intercal R_t^{-1}\epsilon_t\right). \tag{4.7}$$

The estimated parameters are now used to calculate the conditional covariance matrix (Equation 4.2).

Forecast and Simulation

The estimated parameters are used to forecast and simulate out-of-sample dependent instrument price movements for the heuristic optimisation process.

The GARCH model generates volatility forecast for the next point in time $t+1$, which implies $E_t\left(h_{t+1}|\phi\right) = h_{t+1}$. The same applies to the DCC model. Hence, for $t+1$ the forecast of the conditional correlation is $E_t\left(R_{t+1}|\psi\ \right) = R_{t+1}$.

To generate s-step ahead forecasts of the conditional variance, where $s > 1$, a simple method is given by Engle (2002):

$$h_{t+s} = \sum_{i=0}^{s-2} \gamma(a+b)^i + (a+b)^{s-1} h_{t+1}, \tag{4.8}$$

where a and b are the estimated GARCH parameters in Section 4.2.2.

For the DCC model there is no direct solution to forecast the conditional correlation s-step ahead. This is because the DCC model is a non-linear process. However, R_{t+s} can be forecast if $\overline{U} \approx \overline{R}$ and $E_t\left(U_{t+1}\right) \approx E_t\left(R_{t+1}\right)$ (Engle, 2002). For this approximation, the forecast for $t+s$ is given by

$$E_t\left(R_{t+s}\right) = \sum_{i=0}^{s-2} (1-\hat{a}-\hat{b})\overline{R}(\hat{a}+\hat{b})^i + (\hat{a}+\hat{b})^{s-1} R_{t+1}, \tag{4.9}$$

where $\hat{a}$ and $\hat{b}$ are the parameters estimated for the DCC model in Section 4.2.2.

The forecast of the conditional correlation matrix converges to the unconditional correlation of the residual, in the long run. Also, the influence of R_{t+1} on the forecast conditional correlation decays with ratio $\hat{a}+\hat{b}$ (Engle, 2002).

In a next step, the forecast DCC matrices are used to generate a number (Sim) of correlated random returns. A good method to decompose the forecast correlation matrices is by Cholesky decomposition. Agarwal and Mehra (2014) shows that Cholesky decomposition is superior compared with similar techniques e.g. QR, SVD or LU decomposition. They conclude that Cholesky is the most efficient technique in terms of memory storage, computational cost, speed and data reduction. The decomposed matrices are transposed and multiplied by the i.i.d. normal random variables.

4.3 Out-of-Sample Results

In this section, we analyse the efficiency of the objective functions described in Equations (2.2-2.5) with and without DCC simulation approach. First, Section 4.3.1 compares the performance of VaR and CVaR objective functions with underlying empirical and standard normal distribution for the entire out-of-sample period. Then, Section 4.3.2 compares the backtesting results and capital requirements for portfolios with and without DCC simulated returns. The efficiency of the VaR and CVaR estimation methods is compared using descriptive portfolio statistics. Moreover, we report regulatory evaluation measures to compare the portfolios performance. All results are compared to the "naive" EW portfolio and the DJIA index.

For the computational study we use all constituents of the DJIA index with a minimum of 2524 days of empirical data. This excludes the equity data for the company Visa Inc. with a total of 2185 days of empirical observations, which leaves 29 constituents in our empirical analysis. The period we analyse goes from 30th January 2006 to 29th January 2016. This is to test the portfolios for different market cycles. For the same period, we consider daily 2-weeks T-Bill rates as risk-free interest rates. The risk-free rate also serves as target portfolio return μ^T. We use daily closing prices for a total of $T = 2524$ days of observations. In the empirical analysis we use continuous returns. All empirical data is downloaded from DataStream.

In the analysis, we use the same constraints introduced in Section 3.4. Moreover, we assume the risk manager considers cardinality constraints with a lower bound of five and upper bound of 15 assets. Common proportional transaction costs of one basis point is used in the optimisation. The in-sample and out-of-sample period is selected as in Section 3.3. With a data set of 2524 days, we have

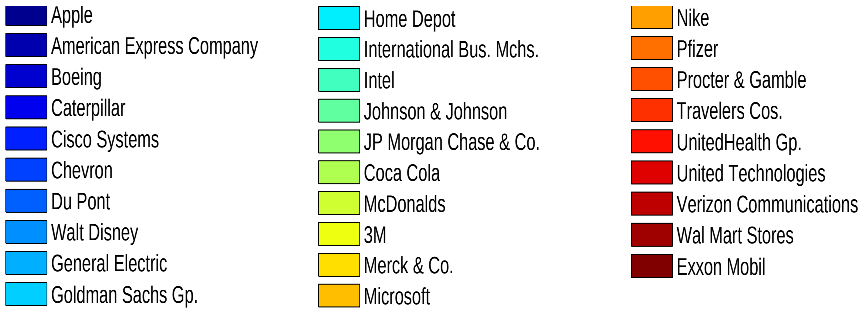

FIGURE 4.2 Colour map for portfolio weights in Chapter 4.

127 in-sample and out-of-sample periods. We use a rolling window analysis to construct out-of-sample ten days ahead forecasts of conditional correlations. For each forecast day we generate 100,000 correlated returns using Cholesky decomposition.

The population size G of the PBIL algorithm is set to 300. The starting learning rate is set to 0.1% and is gradually increased with each repetition by the same rate. The algorithm stops if no probability in P is between 99% and 1%.

4.3.1 Portfolio Performance

For each in-sample period we calculate the optimal weight allocation for the objective functions. Figures 4.3 and 4.4 show the optimal weights for the objective functions optimised with underlying empirical (Figure 4.3) and standard normal (Figure 4.4) distribution function. Each figure compares the portfolio weight allocation for VaR and CVaR estimation with and without simulation approach. Figure 4.2 provides a colour map for the portfolio weights.

The visual analysis clearly shows that portfolios with simulation-based VaR and CVaR estimation have high weight dynamics, regardless if empirically or standard normally optimised. In the simulation approach the variance and covariance of the assets changes more dynamically over time, as more weight is given to recent information. Variance and covariance measures are computed using the GARCH and DCC time-series models. In the forecast period, the simulated variances and

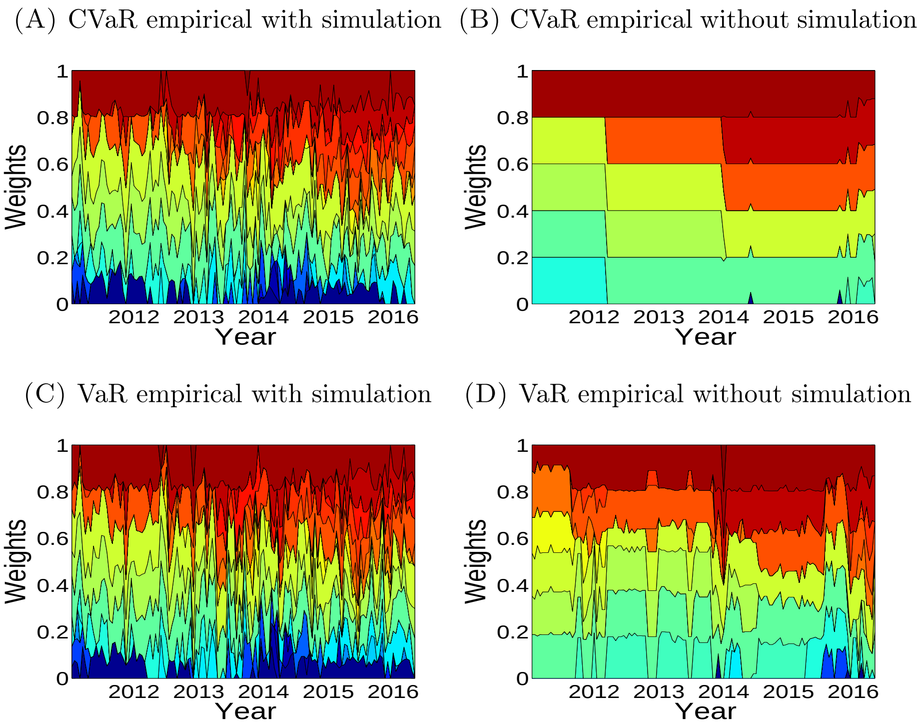

Figure 4.3 Comparison of the dynamic portfolio weights for VaR and CVaR objective functions with underlying empirical distribution at 1% significance level. A colour map for the portfolio weights is presented in Figure 4.2.

correlation gradually move back to their long-term unconditional variance and correlation. In contrast, the VaR and CVaR estimation without simulation only uses the empirical and normal distribution function of the in-sample returns. As the in-sample period rolls forward ten days, the distribution function rarely changes. Therefore the weights of portfolios optimised without simulation change less frequently and to a smaller extent.

The $VaR_{N1\%}$ and $CVaR_{N1\%}$ portfolio without simulation-based estimation show very similar weight distributions. These objective functions have the same standard normal density function value for the alpha percentile. Hence, the optimisation process computes almost identical portfolio weights for the objective functions.

Interestingly, the $CVaR_{H1\%}$ portfolio without simulation-based CVaR estima-

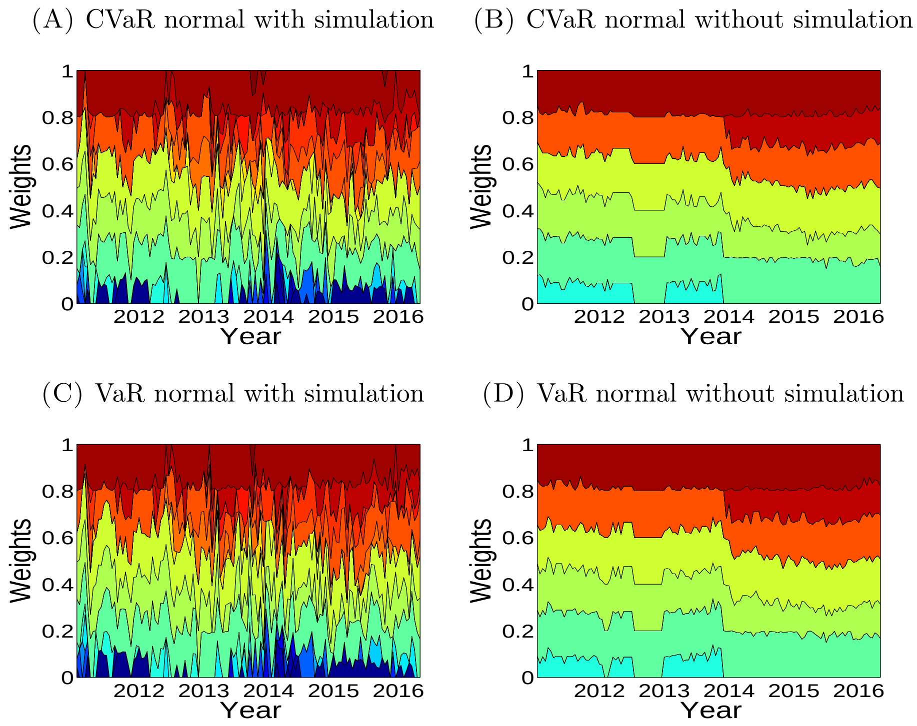

FIGURE 4.4 Comparison of the dynamic portfolio weights for VaR and CVaR objective function with underlying normal distribution at 1% significance level. A colour map for the portfolio weights is presented in Figure 4.2.

tion rarely rebalances its portfolio weights. For most in-sample periods the portfolio allocates a maximum weight of 20% to the optimal assets. This indicates that the algorithm would prefer to give more weight to one or more beneficial asset but is constrained by the upper bound weight constraint of 20%. The combination of only five to six assets seems to be preferred by the portfolio given the empirical sample data. The $VaR_{H1\%}$ portfolio rebalances the portfolio weights more frequently than the $CVaR_{H1\%}$ portfolio. By looking at these two portfolio weight allocations we can see that the changes in the $VaR_{H1\%}$ objective function are more significant than in the $CVaR_{H1\%}$ objective function. Thus, the mean of the expected losses exceeding the empirical VaR estimate seem to remain relatively constant while the VaR estimate changes to a greater extent.

TABLE 4.1 Annualised out-of-sample PBIL portfolio results based on the daily portfolio returns. The annualised mean return, standard deviation and modified Sharpe ratio are expressed in percentage points. All values are reported after transaction costs. We calculate the 1% empirical and normal VaR and CVaR objective function for the simulated (with simulation) and historical (without simulation) asset returns.

Model			Descriptive Statistics and Performance Measure								
Algorithm	Objective Function	Return Distribution	Mean Return	Standard Deviation	Mod. Sharpe Ratio	VaR Historical	VaR Normal	CVaR Historical	CVaR Normal	Kurtosis	Skewness
with simulation	CVaR	Historical	8.62	11.79	72.74	-0.3271	-0.2743	-0.4174	-0.3088	5.74	-0.22
		Normal	6.66	11.75	56.29	-0.3162	-0.2733	-0.4235	-0.3088	6.14	-0.22
	VaR	Historical	6.75	11.88	56.43	-0.3354	-0.2763	-0.4131	-0.3122	5.59	-0.17
		Normal	6.35	11.77	53.56	-0.2980	-0.2737	-0.4145	-0.3096	6.04	-0.20
without simulation	CVaR	Historical	5.41	11.85	45.25	-0.3370	-0.2757	-0.4354	-0.3124	5.98	-0.23
		Normal	5.28	11.47	45.63	-0.3110	-0.2669	-0.4122	-0.3024	5.78	-0.21
	VaR	Historical	4.56	12.48	36.19	-0.3386	-0.2904	-0.4484	-0.3298	5.97	-0.21
		Normal	5.07	11.45	43.89	-0.3109	-0.2663	-0.4106	-0.3019	5.81	-0.21
(Benchmark)	Index		5.07	14.56	33.78	-0.3860	-0.3386	-0.5615	-0.3848	6.93	-0.52
	EW		6.11	14.64	41.44	-0.4235	-0.3405	-0.5747	-0.3862	7.42	-0.52

Table 4.1 shows the descriptive statistics and performance measures for the objective functions, with and without simulation-based VaR and CVaR estimation. The values are annualised and calculated for the entire out-of-sample period.

We found that portfolios with simulation-based VaR and CVaR estimation have higher modified Sharpe ratios than their equivalents without simulation-based estimation, as the results in Table 4.1 demonstrate. Our findings show that this difference can be explained by the higher mean return value of portfolios with simulation approach. The simulation-based portfolios outperform portfolios without simulation by up to 4.06%. Our results show that the simulated asset returns provide a more precise distribution function of the future returns than the assumption that the historical asset returns best describe the future returns.

The highest annualised modified Sharpe ratio with 72.74% can be seen for the empirical CVaR objective function, followed by the empirical VaR portfolio with 56.43%. The modified Sharpe ratio of the standard normal CVaR portfolio with simulation-based estimation is a slightly higher than the Sharpe ratio of the $VaR_{N1\%}$ portfolio with simulation approach. This can be explained by the higher mean return after transaction costs of the $CVaR_{N1\%}$ portfolio.

An interesting observation can be made for the portfolio standard deviation, empirical and normal VaR and CVaR value. Portfolios with standard normal distribution assumption report better risk measures when the optimisation is not based on the simulation, compared with the equivalent portfolio with simulation. On the contrary, portfolios with empirical VaR and CVaR objective function perform better with simulation-based estimation. This is because the simulation creates VaR and CVaR estimates that are influenced by more recent information and the empirical objective functions are capable to capture these changes in the return distribution.

In general, all simulation-based VaR and CVaR optimised portfolios have higher mean return and modified Sharpe ratios than their equivalents without simulation-based estimation and the benchmark models. Portfolios with simulation approach and underlying empirical distribution function have better portfolio risk measures than their counterparts without simulation-based VaR and CVaR estimation.

4.3.2 VaR Backtesting Results

In this section, we evaluate the efficiency of the portfolios based on the Basel III capital requirements using Equation (3.3). To calculate the capital requirements the portfolios need to be backtested and classified in one of the three zones of the Basel III traffic light scheme. To backtest the portfolios, we compute daily VaR levels at a 1% significance level based on a 250 days rolling standard normal distribution of the portfolios.

The portfolios are evaluated using the LR_{UC}, LR_{IND} and LR_{CC} test described in Chapter 3.2.2. Moreover, the average and maximum violations and multiplication factors, as well as the average daily capital requirements are used for the analysis. Table 4.2 reports the respective backtesting results for the out-of-sample period.

Table 4.2 reports the backtesting results for the out-of-sample period. For each portfolio the table reports the average LR_{UC}, LR_{IND} and LR_{CC} ratio (see Chapter 3.2.2) and the average and maximum number of VaR violations and multiplication factors. The average daily capital requirements and the relative difference between portfolios with and without simulation-based VaR and CVaR estimation is reported in the last two columns.

On average, all optimised portfolios have LR_{UC}, LR_{IND} and LR_{CC} values below the critical values of 6.64 and 9.21, respectively. The simulation-based $VaR_{H1\%}$ portfolio has the highest test ratios, while the lowest LR_{CC} ratio is reported for the $CVaR_{N1\%}$ portfolio with simulation-based estimation. The EW portfolio fails the LR_{UC} and LR_{CC} test, which can be explained by its high average (4.69) and maximum (15) number of VaR violations. The simulation-based portfolios have an average number of violations between 2.37 ($VaR_{H1\%}$) and 2.84 ($CVaR_{H1\%}$). This is very similar to the portfolios without simulation approach which is between 2.27 ($VaR_{N1\%}$) and 2.86 ($CVaR_{H1\%}$).

The $CVaR_{H1\%}$ without simulation is the only portfolio with a maximum number of eight violations and thus, a maximum multiplier of 4. For all portfolios with simulation approach the average multiplication factor is in the range of 3.06 ($VaR_{H1\%}$) and 3.09 ($CVaR_{N1\%}$). The highest multiplication factor of the simulation-based VaR and CVaR estimated portfolios is reported for the $CVaR_{N1\%}$ portfolio with 3.65. Portfolios with simulation-based VaR and CVaR estimation give higher priority to more recent information and thus, provide a better estimation of the future return distribution. For this reason portfolios with simulation approach have a lower average number of VaR violations and thus, lower multiplication factors.

The results in Table 4.2 show that the average daily capital requirements for simulation-based VaR and CVaR estimated portfolios in general is better than compared to their equivalents without simulation. The highest difference with -4.67% can be seen for the $CVaR_{H1\%}$ portfolios. This can be explained by the high multiplier for the $CVaR_{H1\%}$ portfolio without simulation. For the $VaR_{H1\%}$ and $CVaR_{N1\%}$ the difference is -4.52% and -2.38%, respectively.

Similar to the results in Section 4.3.1, we can see that portfolios with empirical VaR and CVaR optimisation perform better with simulation-based VaR and CVaR estimation.

TABLE 4.2 Out-of-sample portfolio backtesting results and capital requirements (CR). The table reports the average LR_{UC}, LR_{IND} and LR_{CC} ratios of the portfolios for a rolling 250 days period starting at the 251 out-of-sample day. The critical values for a χ^2 test with 1% significance level and one and two degrees of freedom is 6.64 (UC and IND) and 9.21 (CC), respectively. The table also shows the average and maximum VaR violations and multiplication factors, as well as the average daily capital requirements. The relative improvement of simulation-based portfolios over portfolios without simulation approach in terms of lower capital requirements is reported in the last column (e.g. -1% means the portfolio strategy with simulation approach reduces the capital requirements by 1% compared to the equivalent portfolio without simulation).

Model			Backtesting results								
Algorithm	Objective Function	Return Distribution	LR_{UC}	LR_{IND}	LR_{CC}	Mean Violation	Max Violation	Mean Multiplier	Max Multiplier	Mean CR	+/-
with simulation	CVaR	Historical	5.64	2.51	8.15	2.84	6	3.06	3.53	-0.1962	-4.67%
		Normal	5.42	2.17	7.59	2.76	7	3.09	3.65	-0.1900	-2.38%
	VaR	Historical	6.20	2.44	8.64	2.37	6	3.06	3.53	-0.1953	-4.52%
		Normal	5.86	2.15	8.01	2.69	6	3.07	3.53	-0.1988	1.37%
without simulation	CVaR	Historical	5.88	2.82	8.70	2.86	8	3.11	4.00	-0.2054	
		Normal	5.79	2.45	8.24	2.51	6	3.05	3.53	-0.1945	
	VaR	Historical	5.09	2.60	7.68	2.49	7	3.08	3.65	-0.2042	
		Normal	5.79	2.45	8.24	2.27	6	3.05	3.53	-0.1961	
	EW		8.45	2.38	10.85	4.69	15	3.46	4	-0.2457	

4.4 Conclusion

This chapter researches different VaR and CVaR estimation techniques and their influence on the Basel III market risk capital requirements. We extend the existing literature (see e.g. McAleer et al., 2010; Santos et al., 2012; Uylangco and Li, 2016; Drenovak et al., 2017) which focuses on capital requirements by comparing the ability of empirical, standard normal and simulation-based VaR and CVaR portfolio optimisation problems to improve the risk and performance profile of the portfolios. For our empirical analysis we simulate correlated returns for the next ten days using the DCC forecast model and Cholesky decomposition. The portfolio are optimised using the PBIL heuristic algorithm.

The results of the empirical study show that portfolios optimised based on simulated correlated returns yield better portfolio risk profiles and reduce the capital requirements by up to 4.67%.

All simulation-based portfolios have higher modified Sharpe ratios and mean returns than the portfolios without simulation approach. In combination with the simulation of the returns, the empirical VaR and CVaR optimised portfolios perform better than the portfolios using standard normal VaR and CVaR calculation. Without simulation, portfolios with standard normal calculation outperform the empirical distribution based portfolios. Thus, we can support Winker and Maringer (2007) concluding that future returns are not always best described by their empirical return distribution. However, it seems the simulation approach improves the performance of objective functions with empirical distribution.

In their work, Uylangco and Li (2016) find that Monte Carlo simulations and ARMA-GARCH backtesting models show a relatively high percentage of VaR violations but a lower average magnitude of the violations on the capital requirements. We studied if this behaviour can also be observed when Monte Carlo simulation with GARCH-DCC is used in the optimisation rather than for the backtesting model.

Our analysis shows that portfolios with Monte Carlo GARCH-DCC simulation reduce the average capital requirements compared with their equivalents by up to 4.67%. This is true for all objective functions except for the $VaR_{N1\%}$ portfolio. We find that the most significant improvement can be seen for the empirical VaR and CVaR optimised portfolios.

With regard to the average number of violations, our results show that we can support the findings by Uylangco and Li (2016) for optimisation problems with standard normal distribution assumption. We extend the work of Uylangco and Li (2016) to objective functions with empirical distribution and found such objective functions on average have a higher number of VaR violations. This observation is similar to the work of Winker and Maringer (2007). However, we extend the existing literature as the results of our empirical analysis show that these conclusions cannot be applied to portfolios with Monte Carlo GARCH-DCC simulation and empirical optimisation function. In our study, portfolios with simulated return distribution reduce the average number of violations when objective functions with empirical distribution are used. This can be observed both for VaR and CVaR objective functions. Interestingly, portfolios optimised with CVaR objective function yield the highest number of average VaR violations of all objective functions with and without simulation. This is true for portfolios with empirical and standard normal distribution assumption.

The revision of the Basel II Minimum Capital Requirements for Market Risk standards (Basel Committee on Banking Supervision, 2016), increase the regulatory oversight of OTC derivative contracts. In the next chapter, we introduce a new hedging framework based on global VaR and CVaR hedging with GARCH-DCC process, to reduce the transaction costs and regulatory constraints by avoiding non-standardised derivatives.

Chapter 5

Reducing Regulatory Trading Costs with Global VaR and CVaR Hedging

In Chapter 4, we studied the influence of VaR and CVaR estimation methods on the Basel III market risk minimum capital requirements and the number of VaR violations. We found that Monte Carlo GARCH-DCC simulation with heuristic optimisation improve the regulatory capital requirements of the firm. In this chapter, we apply the same VaR and CVaR estimation method to a new hedging framework to secure the investment in several underlying.

The revision of the Basel III framework (Basel Committee on Banking Supervision, 2016) increases the regulatory oversight and transaction costs of OTC contracts. OTC derivatives are often used in risk management for hedging an investment against potential losses. The increasing costs and the standardisation process of OTC contracts caused by the regulators requires firms to seek less perfect hedges. In this chapter, we provide a new self-financing global VaR and CVaR hedging approach with GARCH-DCC process to secure a number of underlying with a long put option. Our approach extends the existing literature of global quantile, VaR and CVaR hedging by introducing a multivariate GARCH process to the hedging framework. Moreover, compared to the work of Alexander et al. (2006), Annaert et al. (2007), Cong et al. (2013), Cong et al. (2014) and Godin (2016), we hedge the underlying without having to define a hedging budget, an optimal strike price or a specific maturity of the option. This study is the first that demonstrates how one long put option can secure the investment in several underlying with global VaR and CVaR hedging by using Monte Carlo GARCH-DCC simulation and heuristic optimisation.

The results of the computational analysis show that our advocated hedging approach is capable of securing the investment while reducing transaction costs. The hedged portfolios yield better performance measures and improve the risk profile of the loss distribution. For an empirical sample with a market maximum drawdown of -52.90%, we test the hedging approach for a number of VaR and CVaR objective functions with multiple constraints. Our results show that our approach is capable to improve not only the maximum drawdown but also the maximum drawdown duration for all hedging strategies. Moreover, the PBIL algorithm finds optimal solutions much quicker than GA optimised hedging strategies.

This chapter is structured as follows. Section 5.1 discusses the contributions and related literature of this chapter. In Section 5.2, we describe the methodology and Section 5.3 shows the results of the computational analysis. Section 5.4 concludes.

5.1 Introduction

Derivative instruments are often used by risk management to protect the firm from potential changes in prices and exchange rates. There are two major categories of derivatives instruments: (i) “exchange-traded” derivatives, which are standardised contracts and (ii) non-standardised OTC derivatives, which are customised contracts to a specific exposure. Before the financial crisis in 2007, OTC derivatives were inexpensive instruments to manage the firms risk exposure.

However, after the crisis regulators identified the OTC market as one of the main drivers that caused the financial crisis. In the following, major changes in the regulation of OTC derivatives increased the costs and reporting requirements for the hedging firm (Financial Stability Board, 2016).

An April 2015 survey of corporate and end-users by the International Swaps Dealers Association (2015) shows that for more than 53% of firms the cost of hedging increased. Moreover, more than 61% of participants determined the increase in cost of hedging as their biggest concern in risk management. The Bank for International Settlements (2013) shows the impact of the regulatory requirements introduced in 2013 by the Basel Committee on the OTC trading costs and capital requirements.

The increase in trading costs for non-standardised OTC derivatives affects the offering of non-standardised contracts by the banks. The increase in volume of standardised OTC derivatives as shown by the Financial Stability Board (2016), also indicates that firms use less perfect hedges to manage the risk of their companies.

In the literature, several local and global hedging strategies are discussed to manage different types of risk. Local hedging techniques aim to secure the portfolio investment for small changes in the asset price or until the next time step. This requires the estimation of a hedging ratio to secure the investment. Local hedging often involves high transaction costs as they only hedge the portfolio risk for small changes of price and time. In contrast, global hedging methods aim to minimise the risk associated with the terminal hedging error for the entire hedging period. Thus, in this chapter, we concentrate on global hedging techniques.

Several global hedging strategies with different objective functions are proposed in the literature. A basic strategy is to minimise the quadratic error of the portfolio loss function (see Bouleau and Lamberton, 1989; Schweizer, 1995). A generalisation of the formula presented by Schweizer (1995) is presented by Rémillard and Rubenthaler (2013), who are the first to apply a GARCH process to the global quadratic hedging problem in discrete time. They show that their proposed quantile hedging framework yield superior results compared with a delta hedging strategy. One disadvantage of quadratic hedging is that it equally penalises both, profit and loss. An alternative are semi-quadratic models (see Föllmer and Leukert, 2000; François et al., 2014) that only penalise losses. The work of Föllmer and Leukert (1999) and Sekine (2000) concentrate on quantile risk hedging, which aims to maximise the probability of successful hedge. Quantile hedging is similar to minimising the VaR of the hedging loss distribution. Alexander et al. (2004) apply a VaR hedge on a derivatives portfolio with a number of options with different maturity and strike price (also see Alexander et al., 2006). Cong et al. (2013) minimise the VaR of a hedge portfolio with one underlying and a bull call spread option strategy. One of the main drawbacks of using VaR in a hedge function is that it disregards extreme losses exceeding the VaR confidence level. The CVaR risk measure captures the magnitude of losses exceeding VaR. In their work, Alexander et al. (2004) also minimise the CVaR of a hedge distribution for a portfolio of derivatives using the simplex linear programming algorithm to solve the optimisation problem. A continuous-time CVaR hedging approach is proposed by Melnikov and Smirnov (2012) who construct an optimal hedging strategy for an insurance contract. Cong et al. (2014) provide an analytical solution to a CVaR hedging problem under some more restrictive assumptions than Melnikov and Smirnov (2012) and find that the CVaR hedging of one underlying is most effective with a bull call spread strategy. Godin (2016) proposes a CVaR based discrete-time hedging method with transaction costs and normal inverse Gaussian return distribution to secure an European call option with an index investment.

As highlighted in the work of Alexander et al. (2004) and Cong et al. (2013), VaR and CVaR optimisation problems can be ill-posed when the decision universe includes derivative contracts. The literature suggests different ways to solve the ill-posed issue of VaR and CVaR optimisation problems. Cong et al. (2013) and Cong et al. (2014) apply a number of constraints on the optimisation function to avoid ill-posed optimisation problems. They assume i) the loss of the hedge function not to exceed the portfolio risk that needs to be hedged, ii) non-negativity of the hedged loss, and iii) the increment of the ceded loss not to exceed the increment of the retained loss. Due to assumption iii) however, the hedger might miss more desirable portfolio compositions, e.g. portfolios which are more robust with respect to model error or portfolios with lower transaction costs. An alternative and more practical approach is proposed by Alexander et al. (2004) and Alexander et al. (2006). As demonstrated in their work, hedging functions with transaction costs solve the problem of ill-posedness and create portfolios that are more robust to model errors. Godin (2016) follows this approach by introducing proportional transaction costs to the optimisation. We extend the cost function of Alexander et al. (2004), Alexander et al. (2006) and Godin (2016) to include additional transaction costs when the maturity of the option does not fit the length of the investment period.

In this chapter, we introduce a multivariate GARCH process to a global VaR and CVaR hedging problem. This contributes to the existing literature in several ways. We are the first who apply a multivariate conditional heteroskedastic method to a global VaR and CVaR hedging problem. Our model extends the work of Rémillard and Rubenthaler (2013), who apply a GARCH process to minimise the quadratic error of the terminal value of an investment. Compared with their model, our VaR and CVaR hedging approach with multivariate GARCH process provides a way to optimise the risk of several underlying with one put option. Moreover, the model we introduce can easily be modified for different distribution assumptions. Our hedging approach adds a new path of research to the existing global VaR and CVaR hedging literature (e.g. Föllmer and Leukert, 2000; Föllmer and Leukert, 1999; Alexander et al., 2004; Godin, 2016) where GARCH processes have not been considered so far.

Another contribution to global hedging techniques proposed so far is that our model neither requires an optimal strike price nor an optimal maturity of the derivative to hedge the underlying. Cong et al. (2013) and Cong et al. (2014) propose a VaR and CVaR hedging strategy that is independent form the market model assumption. However, the model requires that options with specific strike prices exist on the market. Godin (2016) assumes that the strike price of the option matches the underlying level exactly. A drawback of strategies using specific strike prices is that in practice it can be difficult to find an option with the optimised strike price, if it is not determined via the OTC market. Moreover, our advocated hedging framework reduces the difference between forecast and realised loss, as in our self-financing hedging approach the option is not required to end in-the-money. This is different to other hedging strategies, e.g. Föllmer and Leukert (1999) and Melnikov and Smirnov (2012), which require additional constraints on the hedging budget.

We demonstrate the application of our hedging approach for a large computational analysis based on empirical options and stock data. This is different to the vast majority of literature, which uses simulated data to test their hedging strategies. The results of our empirical analysis show that our global hedging approach with multivariate GARCH process improves the stability and profitability of the hedge portfolio after transaction costs.

Our proposed hedging framework is described in Section 5.2. The results of the computational study are analysed and discussed in Section 5.3. Section 5.4 summarises the results of the empirical study and concludes.

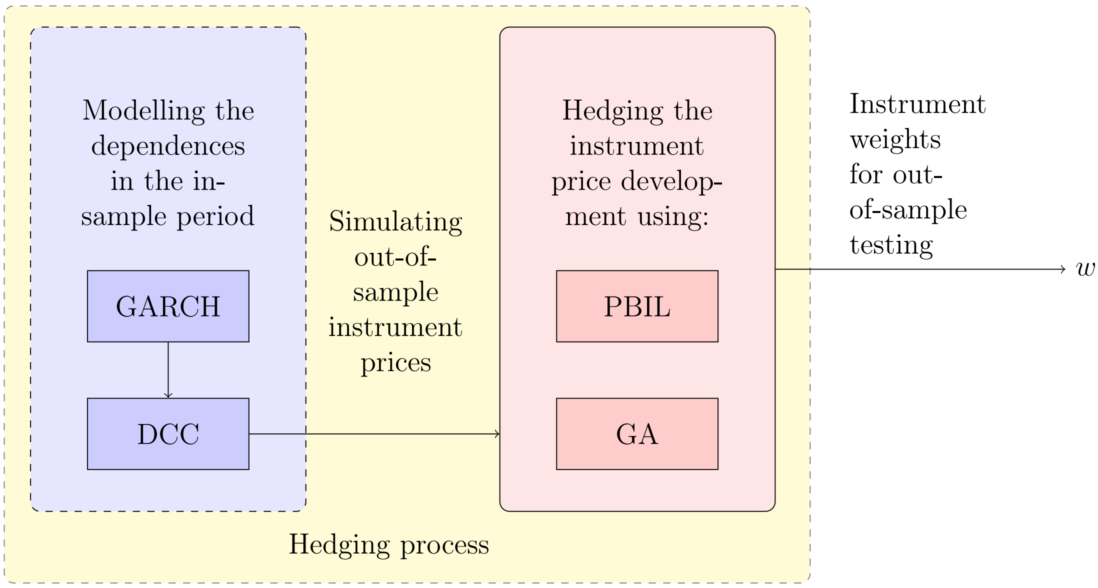

FIGURE 5.1 Hedging framework.

5.2 Methodology

The new Basel III framework increases the cost and regulatory requirements for OTC traded derivative contracts. The augmented use of standardised derivative contracts to hedge the firms risk exposure increases the need of less perfect hedging strategies.

In this section, we describe the methods used for our proposed option hedging framework to secure the invested capital against potential drawbacks in instrument price developments. The hedging approach is visualised in Figure 5.1. The emphasis of the framework is not to outperform alpha portfolio strategies but to improve the stability of the portfolio value. We use one standardised long index put option to hedge a number of equities. This is to reduce the transaction costs and to increase the liquidity of the hedging approach. Similar hedging techniques to reduce transaction costs with index futures are discussed in, i.e., Hull (2015).

The dependencies between the equities and the index put option are estimated for the in-sample periods using GARCH-DCC volatility estimation methods (described in Chapter 4). In Section 5.2.1 we show how we apply this process to simulated option prices. The estimated parameters are then used to simulate out-of-sample price movements of the equities and the index level and to calculate the simulated index option value. In the hedging process the optimal weight allocation between the equities and the index option is determined based on the simulated data. In Section 5.2.2, we describe the hedging function and the risk measures used for the optimisation process. The search methods are discussed in Section 5.2.3. To compare the effectiveness of the different risk measures to secure the portfolio value we introduce some evaluation measures in Section 5.2.4.

5.2.1 Simulation

The estimation of the optimal portfolio weight allocated to the put option is based on simulated out-of-sample price movement of the investment instruments. To simulate the movements we estimate the univariate GARCH volatility and the DCC to capture correlation clustering effects. We described this process in Section 4.2.2.

The simulated returns are used to calculate simulated instrument values. The simulated index put option value V^{Put} is calculated based on the simulated index level Z. In our empirical analysis (see Section 5.3) we use European style index options. The most common method to approximate the price of an European style option is the Black and Scholes (1973) formula

$$V^{Call}(Z,t) = Z\Phi(d_1) - Ke^{r^f(T-t)}\Phi(d_2) \tag{5.1}$$

$$V^{Put}(Z,t) = Ke^{-r^f(T-t)}\Phi(-d_2) - Z\Phi(-d_1) \tag{5.2}$$

$$d_1 = \frac{ln(Z/K) + \left(r^f + 1/2\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}} \tag{5.3}$$

$$d_2 = \frac{ln(Z/K) + \left(r^f - 1/2\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}} \tag{5.4}$$

where Φ is the cumulative normal density function, r^f the risk free interest rate, K the option strike price and T the maturity of the option. In this chapter, we follow the work of Alexander et al. (2004) by using the Black-Scholes formula to estimate the prices of the European style options.

To forecast the price movement of the index option the parameter values can be simulated and used to calculate s-step ahead option values. Parameters K and T need not to be simulated as they remain the same for plain vanilla options, until maturity. The index level Z can be simulated using the DCC model as illustrated in Section 4.2.2. To forecast the volatility term σ a number of methods are discussed in the literature (see e.g. Dumas et al., 1998; Hibbert et al., 2008). For short time horizon τ, we assume r^f to be constant as the expected change in the parameter and thus, the option price is likely to be rather small, compared with other risk factors. The literature suggests several methods to forecast interest rates. Some of the most common are e.g. the Hull and White (1994b) one factor model, Hull and White (1994a) two factor model and the Black and Karasinski (1991) one factor model.

The sensitivity of the option price to a change in a parameter value can be calculated by differentiating Equation (5.2) for each parameter. This analysis can help to identify the parameters with the highest impact on the option price movement.

Existing literature usually assumes the instrument prices to be independent from each other (see Alexander et al., 2004; Alexander et al., 2006). However, forecasting dependences with multivariate GARCH models improves the portfolio risk profile (Switzer and Omelchak, 2009). The model parameters are easy to estimate and the same approach can be used for a large or small number of instruments.

5.2.2 Hedge and Objective Functions

The simulated instrument values in Section 5.2.1 are used to calculate the simulated portfolio values. In this section, we describe the hedging function and the risk measures applied to find the optimal weight allocation to hedge the simulated portfolio values.

For each out-of-sample period τ, the prices of the assets are simulated and hedged with the simulated price movement of one long index put option. The simulated loss of a portfolio in period τ is derived from the combination of the total number of instruments M, which includes the assets and the derivative contract. The simulated change in the instrument values for the out-of-sample period is given by

$$\delta V^{\tau} = V^{\tau} - V^{\tau^0}, \tag{5.5}$$

where $V^{\tau} = [V_1^{\tau}, ..., V_M^{\tau}]$ is a vector of instrument values at the end of each simulated out-of-sample period and V^{τ^0} is a vector of instruments values at the start of the simulated out-of-sample period. The total number of out-of-sample periods is denoted $\tau^{periods}$ and thus, $1 \leq \tau \leq \tau^{periods}$.

The absolute change in the portfolio value Π without transaction costs for the simulated out-of-sample period τ is given by

$$\delta\Pi^{\tau} = w^{\intercal}(\delta V)^{\tau}, \tag{5.6}$$

where $w = [w_1, ..., w_M]$ is a vector of instrument weights. To account for proportional transaction costs, expressed in basis points (bps), we extend Equation (5.6) by the following cost function

$$cost = \begin{cases} bps \times |w_i^{\tau-1} - w_i^{\tau}| \times V_i^{\tau^0} & \text{if } |w_i^{\tau-1} - w_i^{\tau}| > 0 \\ bps \left(w_i^{\tau-1} \times V_i^{\tau-1} + w_i^{\tau} \times V_i^{\tau^0} \right) & \text{if } w_i^{\tau} \text{ is option} \\ 0 & \text{else} \end{cases} . \tag{5.7}$$

If the magnitude of the difference between the optimal weight in instrument i for the previous period $\tau - 1$ and the estimated weight in instrument i for the current period τ is greater than zero, then the absolute difference is multiplied by the proportional transaction costs in bps and the instrument value $V_i^{\tau^0}$ at the beginning of the current period. To hedge the instrument values we select an at-the-money put option with maturity closest to the rebalancing period of the portfolio. Often however, there is no option with an exact maturity match. Therefore, the derivative is sold at the end of the previous period instrument price and a new option is bought at the start of the current period.

The resulting absolute periodic change in the simulated portfolio value with transaction costs is thus

$$\delta\Pi^{\tau}_{cost} = w^{\intercal}(\delta V)^{\tau} - cost. \tag{5.8}$$

We assume the optimal portfolio is subject to an inequality absolute target constraint. This is, the absolute expected gain $\delta\Pi^{\tau}_{cost}$ has to be equal or greater than an absolute target profit $\delta\Pi^{\tau}_{target}$

$$\delta\Pi^{\tau}_{cost} \geq \delta\Pi^{\tau}_{target}, \tag{5.9}$$

where $\delta\Pi^{\tau}_{target} = \Pi^{\tau^0} r^{\tau}_{f}$. r^{τ}_{f} represents a risk free rate of returns for period τ and Π^{τ^0} is the portfolio value at the beginning of period τ.

For the portfolio loss distribution in Equation (5.8), we calculate the normal and empirical VaR and CVaR objective functions, subject to the portfolio constraints described in Section 2.1.

5.2.3 Search Algorithms

For some hedging problems it is possible to provide an analytical solution to the optimisation problem (see e.g. Ahn et al., 1999; Li and Xu, 2008; Cong et al., 2013; Cong et al., 2014). A linear programming technique is used by Alexander et al. (2004) and Alexander et al. (2006), who apply a simplex linear programming algorithm to solve VaR and CVaR objective functions of derivative portfolios with linear constraints. Rémillard and Rubenthaler (2013) and Godin (2016) use dynamic programming (DP) to solve their hedging approach.

DP is easy to implement and can be a good algorithm to find an exact global optimal solution to the hedging problem. However, the optimisation process uses a lot of memory and computation time. In our advocated global VaR and CVaR hedging approach with Monte Carlo simulated GARCH-DCC returns, we apply a PBIL heuristic algorithm. The PBIL algorithm is a reasonable fast and memory efficient heuristic optimisation approach that is able to handle the constraints described in Section 5.2.2 in the hedging approach. To our best knowledge, this study is the first that applies PBIL heuristic algorithm to a hedging approach with VaR and CVaR objective functions. In the empirical analysis, we compare the optimisation efficiency of PBIL with a standard GA optimisation. The PBIL

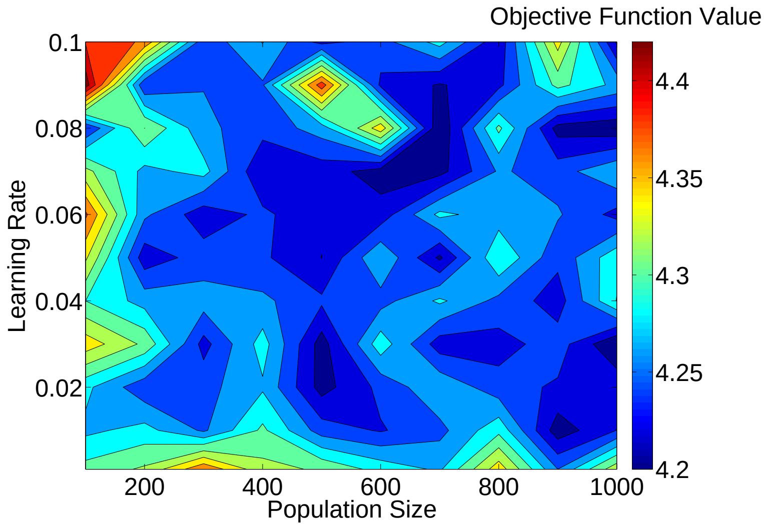

FIGURE 5.2 Sensitivity of the absolute $CVaR_H$ objective function value to changes in the learning rate and the population size.

algorithm is described in Section 4.2.1. In this section, the PBIL input parameters are chosen using sensitivity analysis.

To find reasonable model calibration regions for the model configuration, we conduct a sensitivity analysis from the learning rate and the population size. Figure 5.2 illustrates the $CVaR_H$ objective function value, expressed as expected absolute loss, to different LR values and population sizes. In this example, we aim to reduce the objective function value. Low objective function values are coloured dark blue while high values have a red colour.

The results of the sensitivity analysis show that population sizes between 100 and 300 have relatively high objective function values for all LR values. Population sizes from about 700 and higher generate better results for the observed hedging strategy. In general, higher population sizes generate better results as it increases the probability to find optimal candidate solutions. However, if LR is too low or too high the algorithm does not find optimal solutions regardless of the population size. The objective function value is relatively low if LR is between 2% and 8%.

To find an optimal LR value, Folly and Venayagamoorthy (2009) suggest to linearly increase LR by a constant value at every generation. This process is repeated until the probabilities in P exceed a specific threshold (see e.g. Baluja, 1997; Shapiro, 2002). We set the starting learning rate LR to 0.1% and gradually increase it by the same rate until no probability in P is between 1% and 99%.

In our analysis, the population size is set to 700 as the results of the sensitivity analysis suggest this to be a good population size for all LR values.

To evaluate the performance of the PBIL algorithm we use GA optimisation as a benchmark search method. The number of generations and the population size of the GA is set to 700. The number of children generated is 500 and the mating pool size is set to 450. The probability to undergo mutation is 20% and the algorithm restarts six times. In our optimisation, this setting led to fast converging results.

5.2.4 Out-of-Sample Evaluation

To evaluate the effectiveness of the PBIL and GA optimisation method for the hedging problems, we use standard portfolio measures like the modified Sharpe ratio and standard deviation. In addition, we calculate the maximum drawdown and the maximum drawdown duration of the portfolios to assess the algorithm's capability to secure the instrument values against potential losses.

To evaluate the trading activity required to hedge the portfolios we compute the turnover of each portfolio

$$Turnover = 1/\tau^{periods} \sum_{\tau=1}^{\tau^{periods}} \sum_{i=1}^{M} \left(|w_i^{\tau} - w_i^{\tau^0}| \right), \tag{5.10}$$

where w_i^{τ} is the portfolio weight in instrument i for period τ and $w_i^{\tau^0}$ is the portfolio weight before rebalancing. The defined turnover is the average fraction of wealth traded in each period (DeMiguel et al., 2009).

5.3 Empirical Results

In this section, we present a detailed empirical examination of the proposed hedging approach. The purpose of this analysis is to study if the hedging framework discussed in Section 5.2 is able to reduce hedging costs and provide stability to the portfolio investment. Moreover, we provide a comparative analysis for PBIL and GA heuristic algorithm in option based portfolio optimisation problems. To evaluate the effectiveness of the proposed hedging approach with the heuristic algorithm we compare the results with unhedged portfolios with PBIL optimisation.

The empirical study uses daily closing prices provided by DataStream for the DJIA index and ten randomly selected DJIA constituents (Verizon Communications, General Electric, Boeing, Microsoft, Travelers Cos., Walt Disney, 3M, Hewlett-Packard, Home Depot, Bank of America). We reduce the investment universe to ten assets due to computational time restrictions. Moreover, the empirical DJIA index put option data is downloaded from OptionMetrics' Ivy DB US database. We consider the same data and computational approach as described in Chapter 3 and Chapter 4.

The empirical data is divided in an in-sample and out-of-sample period. The in-sample period consists of 1250 observations and is used to train the algorithm. The estimated parameters of the in-sample period are tested in an ten days out-of-sample period (Bailey et al., 2014). At the end of one test period the in-sample period is rolled forward by ten days. For a total of 1750 observations the analysis consists of 50 in-sample and out-of-sample periods.

5.3.1 Descriptive Statistics and Hedging Results

Figure 5.3 shows the evolution of the $CVaR_H$ hedging function value for the simulated data based on the first in-sample period using PBIL and GA optimisation algorithms. The figure shows the best optimisation of the PBIL and GA algorithm. The objective function with PBIL search method evolves much quicker to its optimum compared with the GA optimisation. After 30 generations the PBIL algorithm already finds 81% of the optimal objective function value while the GA optimisation approach only finds 72%.

The PBIL optimisation process quickly concentrates on the most promising weight allocations, as illustrated in Figure 5.4A. High asset weights are coloured red while low asset weights are dark blue. The highest fluctuation of the asset weights can be seen for the first generations in the PBIL optimisation. With continuous optimisation the weights of the optimal portfolio stay almost constant. The weight allocation for the GA optimisation process looks different. For entire generations the weights of the assets constantly change, as shown in Figure 5.4B.

Table 5.1 shows the mean return, standard deviation, modified Sharpe ratio, skewness and kurtosis of the hedged and unhedged portfolios, for the entire out-of-sample period. Objective functions referenced with an asterisk (*) indicate unhedged PBIL optimised portfolios.

In general, we see that all hedged portfolios have better performance measures than their unhedged equivalents. The unhedged portfolios have negative mean return values between -3.29% and -14.79%, with the highest mean return for the normal VaR and the lowest for the empirical VaR objective function. The hedged portfolios have higher mean returns than the unhedged portfolios. On average, the hedged portfolios with PBIL optimisation have an annualised mean return of -1.00% and with GA algorithm -2.04%. The highest mean return is reported for the normal VaR with PBIL algorithm with 3.23%. The same hedging strategy with GA search method has a mean return of -3.07%. The $CVaR_{N1\%}$ and $VaR_{H1\%}$ portfolio with GA optimisation have positive mean returns with 0.62% and 0.18%, respectively. All other portfolios have negative mean returns. Of all hedged portfolios the empirical CVaR with GA algorithm has the lowest mean

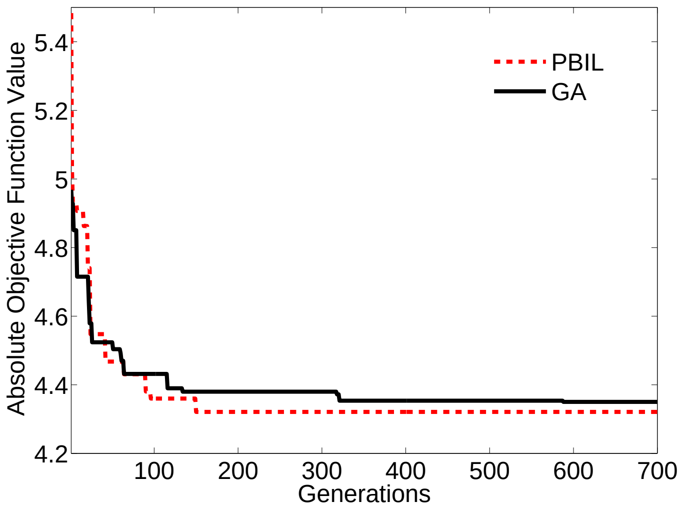

FIGURE 5.3 Evolution of the absolute objective function value for the $CVaR_H$ hedging strategy with GA and PBIL search method.

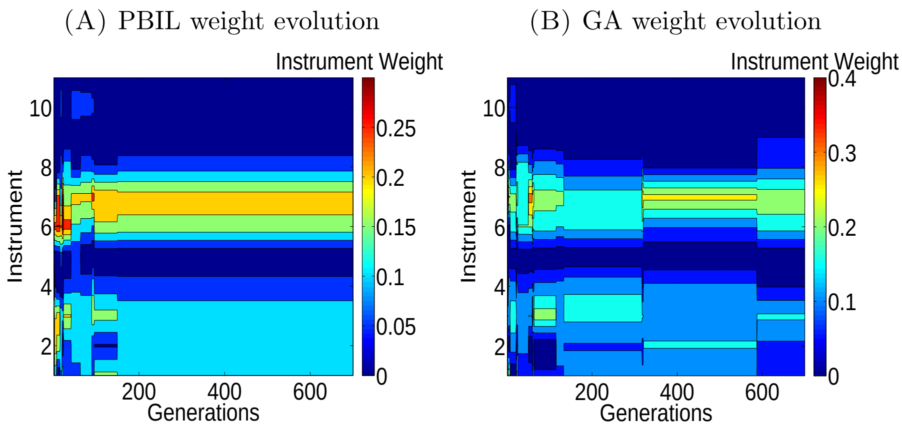

FIGURE 5.4 Evolution of the weights for the $CVaR_H$ hedging strategy with GA and PBIL search method.

TABLE 5.1 The table shows the descriptive statistics of the hedged and unhedged (*) portfolios, after transaction costs. The mean return, standard deviation, modified Sharpe ratio are annualised results and shown as percentage value.

Algorithm	Objective Function	Mean Return	Standard Deviation	Sharpe Ratio	Skewness	Kurtosis
GA	$CVaR_{H1\%}$	-5.90	19.12	-1.17	-1.66	11.62
	$CVaR_{N1\%}$	0.62	18.33	2.23	-1.93	12.65
	$VaR_{H1\%}$	0.18	18.46	-0.01	-1.10	9.78
	$VaR_{N1\%}$	-3.07	19.48	-0.64	-1.81	11.85
PBIL	$CVaR_{H1\%}$	-2.39	18.87	-0.49	-0.83	11.09
	$CVaR_{N1\%}$	-3.84	18.78	-0.76	-1.43	10.00
	$VaR_{H1\%}$	-0.82	19.02	-0.20	-1.01	9.70
	$VaR_{N1\%}$	3.23	18.24	16.55	-1.50	11.43
	$CVaR^{*}_{H1\%}$	-6.99	26.78	-1.93	-1.83	8.90
	$CVaR^{*}_{N1\%}$	-6.43	26.46	-1.76	-1.85	9.43
	$VaR^{*}_{H1\%}$	-14.79	26.74	-4.01	-1.82	8.76
	$VaR^{*}_{N1\%}$	-3.29	26.99	-0.94	-1.93	9.45

return with -5.90%.

Our proposed option based hedging approach reduces the standard deviation for all hedge strategies compared with the unhedged portfolios. This is because the option reduces the price fluctuation of the portfolios. The average standard deviation of the hedged portfolios is 18.73% and 18.85% with PBIL and GA search method, respectively. For the unhedged portfolios the average standard deviation is 26.74%. Again, best standard deviation of the hedged portfolios is given by the PBIL optimised normal VaR hedging strategy with 18.24%. The same hedging strategy with GA algorithm has the highest standard deviation of all hedged portfolios with 19.48%. The normal CVaR portfolio with GA optimisation has the second lowest standard deviation with 18.33%.

Due to the negative mean returns, only two optimised portfolios have positive modified Sharpe ratios. The highest modified Sharpe ratio is given by the PBIL optimised $VaR_{N1\%}$ portfolio with 16.55%. The only other portfolio with positive modified Sharpe ratio is the normal CVaR portfolio with GA algorithm with 2.23%. The modified Sharpe ratio of all other portfolios is negative. The portfolio with the lowest rank and thus, the lowest modified Sharpe ratio is given by the unhedged $VaR_{H1\%}$ with -4.01%.

All portfolios are left-skewed with skewness between -1.93 for the unhedged $VaR^*_{N1\%}$ and -0.83 for the empirical CVaR hedged portfolio with PBIL algorithm. The highest difference between a hedged and an unhedged portfolio can also be seen for the empirical CVaR objective function. The skewness of the hedged $CVaR_{H1\%}$ portfolio with PBIL algorithm is 54.6% higher than for the unhedged $CVaR^*_{H1\%}$ portfolio, with -0.83 and -1.83 for the hedged and unhedged portfolio, respectively. All hedged portfolios have a higher skewness than their unhedged equivalents. The only exception is the GA optimised normal CVaR portfolio with -1.93. On average, the skewness of PBIL optimised hedged portfolios is 36.0% and for portfolios with GA algorithm 12.7% higher than the skewness of the unhedged portfolios.

The hedged portfolios have a higher kurtosis than the unhedged portfolios. The kurtosis of the unhedged portfolios is in the range of 8.76 and 9.45 for the $VaR^*_{H1\%}$ and $VaR^*_{N1\%}$, respectively. The average kurtosis of the hedged portfolios with PBIL algorithm is 15.6% higher than the kurtosis of the unhedged portfolios. For GA optimised portfolios on average the kurtosis is 25.4% higher than for the unhedged portfolios. The highest kurtosis of the hedged portfolios is reported for the GA optimised normal CVaR hedging strategy with 12.65. The lowest kurtosis is given by the PBIL $VaR_{H1\%}$ portfolio with 9.70. The kurtosis of the GA optimised portfolios is higher compared with the kurtosis of portfolios with PBIL search method.

The annualised empirical and normal VaR and CVaR portfolio values at 1% significance level for the hedging strategies and unhedged portfolios are shown in Table 5.2. The maximum drawdown and the maximum drawdown duration is shown in the last two columns of the table. The maximum drawdown is expressed

Table 5.2 Annualised empirical and normal CVaR and VaR values for the heded and unhedged (*) portfolios, after transaction costs. The maximum drawdown (DD) is the maximum percentage drop from a peak and the maximum drawdown duration is the length of this drop period, expressed in days.

Algorithm	Objective Function	CVaR Historical	CVaR Normal	VaR Historical	VaR Normal	Max DD	Max DD Duration
GA	$CVaR_{H1\%}$	-0.89	-0.41	-0.66	-0.33	-22.66	138
	$CVaR_{N1\%}$	-0.88	-0.38	-0.59	-0.30	-20.19	138
	$VaR_{H1\%}$	-0.80	-0.38	-0.56	-0.30	-17.43	138
	$VaR_{N1\%}$	-0.91	-0.41	-0.66	-0.33	-22.59	138
PBIL	$CVaR_{H1\%}$	-0.82	-0.39	-0.57	-0.32	-17.33	138
	$CVaR_{N1\%}$	-0.84	-0.40	-0.58	-0.32	-22.44	138
	$VaR_{H1\%}$	-0.81	-0.39	-0.56	-0.31	-16.21	15
	$VaR_{N1\%}$	-0.83	-0.37	-0.58	-0.29	-17.06	89
	$CVaR^*_{H1\%}$	-1.16	-0.57	-0.93	-0.46	-64.87	210
	$CVaR^*_{N1\%}$	-1.18	-0.56	-0.89	-0.45	-59.23	210
	$VaR^*_{H1\%}$	-1.17	-0.58	-0.95	-0.47	-65.68	210
	$VaR^*_{N1\%}$	-1.19	-0.56	-0.93	-0.45	-63.40	210

as cumulated return value and the maximum drawdown duration as number of days. The statistics are calculated for the entire 500 days out-of-sample period.

As Table 5.2 shows, the proposed hedging approach significantly reduces the objective function values of the hedged portfolios compared with the unhedged portfolios. Furthermore, portfolios with PBIL algorithm have slightly better objective function values than their equivalents with GA search method. Only in a few cases portfolios with GA optimisation perform better than portfolios with PBIL algorithm. On average, the PBIL algorithm improves the objective function values relative to the unhedged portfolios by 32.9%. The GA search method performs slightly worse with an average improvement of the objective function values by 30.2%. The most significant improvement in the objective function values can be seen for the empirical VaR values with on average 38.1% and 33.2% for PBIL and GA optimised portfolios, respectively.

A good indicator for the efficiency of our proposed single-option hedge is the maximum drawdown and the maximum drawdown duration of the portfolios. The primary interest is to secure the investment from drawdowns in prices. Secondly, we want to reduce the drawdown duration as long drawdown periods show that the model is unable to adjust to new market situations. In the entire out-of-sample period, the maximum drawdown of the index level is -52.90% with a maximum drawdown duration of 221 days. All unhedged portfolios have about the same maximum drawdown. The $VaR^*_{H1\%}$ has the lowest maximum drawdown of all portfolios, with -65.68%. This is even lower than for the index level. The maximum drawdown duration of the unhedged portfolios is 210 days long. Thus, even though the unhedged portfolios are optimised for the objective functions given in Equations (2.2-2.5) they cannot significantly reduce the maximum drawdown. This is because of the transaction costs which reduce the performance of the optimised portfolios. Moreover, in this optimisation we considered ten equity instruments while the index consists of 30 constituents and thus, achieves a better diversification effect.

All hedged portfolios are able to improve the maximum drawdown and the drawdown duration. The most significant improvement in drawdown duration can be seen for the PBIL optimised $VaR_{H1\%}$ hedging strategy with a duration of 15 days. The second shortest maximum drawdown duration is reported for the $VaR_{N1\%}$ portfolio with PBIL algorithm with 89 days. All other hedged portfolios have a maximum drawdown duration of 138 days.

The best maximum drawdown of the hedged portfolios can be seen for the PBIL optimised empirical VaR hedging strategy with -16.21%. This is followed by the normal VaR and the empirical CVaR hedge with PBIL optimisation with -17.06% and -17.33%, respectively. The lowest maximum drawdown of the hedged portfolios can be seen for the $CVaR_{H1\%}$ and $VaR_{N1\%}$ hedging strategy with GA optimisation with -22.66% and -22.59%, respectively. On average, hedging strategies with PBIL search method improve the maximum drawdown by 71.0%. Portfolios with GA optimisation on average reduce the maximum drawdown values by 67.2%.

Our empirical analysis provides a comparison between coherent and non-coherent risk measures with combinations of empirical and normal distribution assumptions. The results of our study add knowledge to the findings of Godin (2016) using a CVaR objective function with normal inverse Gaussian return distribution. We demonstrate that the VaR objective function with normal distribution assumption and PBIL optimisation has the best portfolio risk-return profile, followed by the empirical VaR hedging strategy. Both hedging strategies also have the highest maximum drawdown and drawdown duration.

The PBIL and GA heuristic optimisation algorithms are able to provide a global solution for convex and non-convex risk measures. Our optimisation approach even allows for non-convex penalty functions and thus, provides an improvement to the hedging approach introduced by Alexander et al. (2003). Our hedging framework is easily applicable to other hedging problems with different objective functions and hedging instruments. This is a clear advantage over the analytical VaR and CVaR hedging solution provided by Cong et al. (2013) and Cong et al. (2014), respectively.

5.3.2 Weight Allocation of the Hedged Portfolios

In this section, we analyse the weight allocation, turnover and transaction costs of the hedged portfolios with GA and PBIL optimisation and compare them with the unhedged PBIL optimised portfolios.

Figures 5.6 and 5.7 show the portfolio weight allocation for the entire out-of-sample period with PBIL search method. The y-axes show the fractional weights of the portfolio for a one unit investment. The colour map is given in Figure 5.5. The x-axes show the out-of-sample years. Figure 5.6 shows the optimal weight

Table 5.3 The table reports the portfolio turnover (in %) and transaction costs (in bps) for GA and PBIL hedged and unhedged portfolios. The relative improvement for portfolios with GA and PBIL algorithm over the unhedged portfolios is displayed below the turnover rate and transaction costs, respectively.

		Objective Function			
		$CVaR_{H1\%}$	$CVaR_{N1\%}$	$VaR_{H1\%}$	$VaR_{N1\%}$
Turnover (in %)	GA hedged	55.41	64.67	47.98	57.68
	PBIL hedged	58.80	56.81	52.76	55.14
	unhedged	52.13	50.34	51.81	50.86
Relative Difference (in %)	GA to unhedged	6.29	28.47	-7.39	13.41
	PBIL to unhedged	12.79	12.85	1.83	8.42
Transaction Cost (in bps)	GA hedged	27	28	28	30
	PBIL hedged	30	29	28	28
	unhedged	26	25	26	25
Relative Difference (in %)	GA to unhedged	3.84	12.00	7.69	20.00
	PBIL to unhedged	15.38	16.00	7.69	12.00

allocation of portfolios with underlying empirical distribution assumption. The weight allocation of portfolios optimised for Equations (2.2-2.5) are displayed in Figure 5.7. Panels 5.6A, 5.6B, 5.7A and 5.7B show the portfolio weights for CVaR hedged and unhedged portfolios with PBIL search method, respectively. Similarly, Panels 5.6C, 5.6D, 5.7C and 5.7D illustrate the weights for VaR hedged and unhedged portfolios with PBIL search method.

The turnover rates and transaction costs, expressed in percentage and bps, respectively, are calculated for the optimised portfolios and reported in Table 5.3. The second column of Table 5.3 shows the optimisation method and the first two rows show the objective function as given by Equations (2.2-2.5). Rows three to five report the turnover rates and rows six to eight the transaction costs of the strategies, for the entire out-of-sample period.

The highest turnover of the unhedged portfolios is given by the empirical CVaR and VaR portfolio with 52.13% and 51.81%, respectively. They also have the

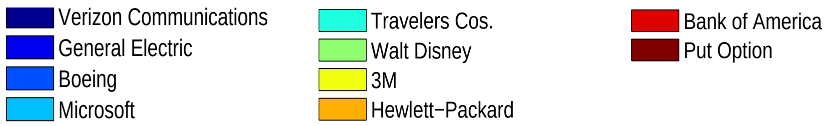

FIGURE 5.5 Colour map for portfolio weights in Chapter 5.

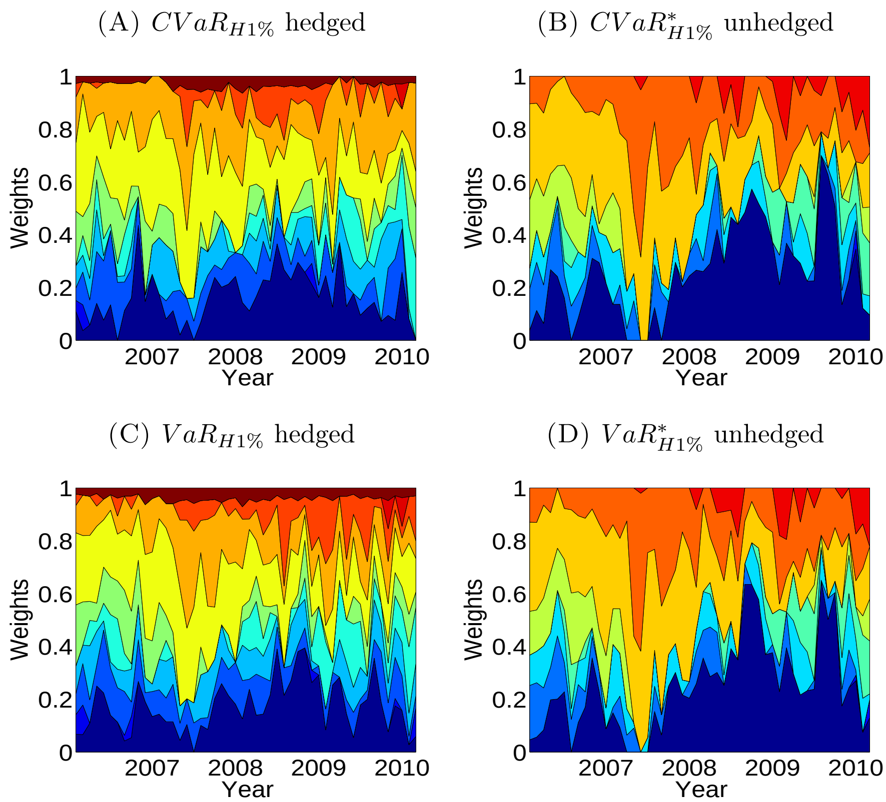

FIGURE 5.6 Comparison of the dynamic weights for PBIL hedged and unhedged (*) portfolios with VaR and CVaR objective functions and underlying empirical distribution. A colour map for the portfolio weights is provided in Figure 5.5.

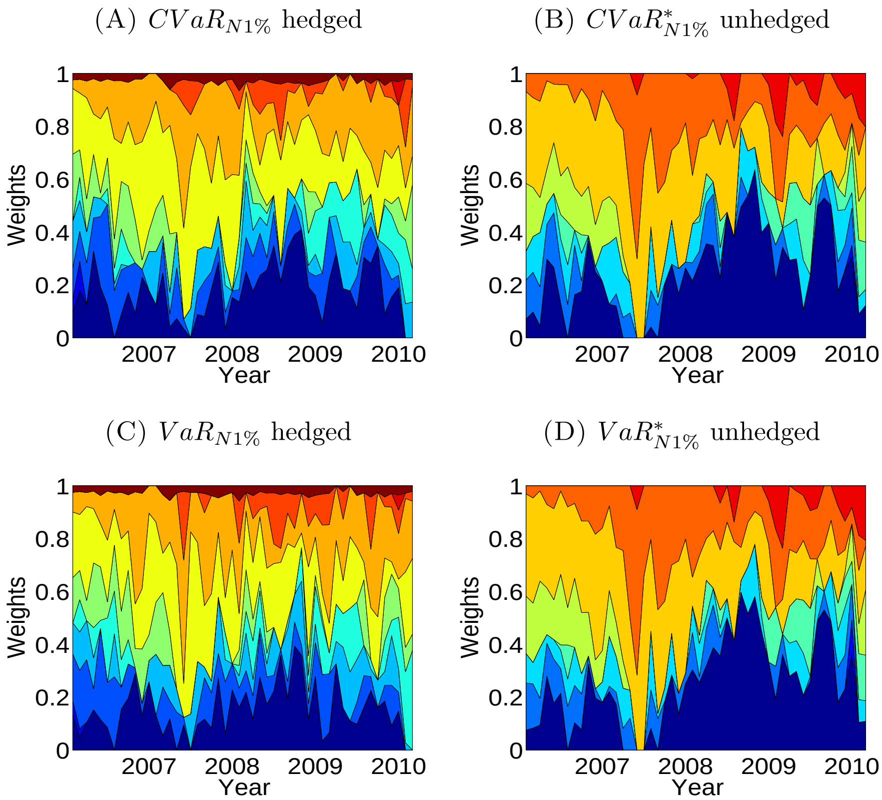

FIGURE 5.7 Comparison of the dynamic weights for PBIL hedged and unhedged (*) portfolios with VaR and CVaR objective functions and underlying normal distribution. A colour map for the portfolio weights is provided in Figure 5.5.

highest transaction costs of the unhedged portfolios in the out-of-sample period with 26 bps.

The turnover rate and transaction costs for hedging strategies with underlying normal distribution are lower for the PBIL algorithm. Compared with the turnover of the GA optimised portfolio the $CVaR_{N1\%}$ and $VaR_{N1\%}$ hedge with PBIL search method reduces the turnover by 12.15% and 4.40%, respectively. The transaction costs for the $VaR_{N1\%}$ PBIL portfolio is reduced by 6.67% from 30 bps to 28 bps, compared with its equivalent with GA optimisation. Only the transaction costs for the $CVaR_{N1\%}$ strategy are lower with GA search method. For the hedged portfolios, the empirical CVaR and VaR portfolios with GA optimisation have the lowest transaction costs and turnover rates.

The most significant increase in turnover rate and transaction costs compared with the unhedged portfolios can be seen for the GA optimised $CVaR_{N1\%}$ and $VaR_{N1\%}$ hedging strategy. The turnover rate for the $CVaR_{N1\%}$ hedge increases by 28.47% and the transaction costs for the $VaR_{N1\%}$ hedge by 20.00%. Compared with the unhedged portfolios the turnover rate and transaction costs for hedging strategies with PBIL algorithm increase not as much as for the GA optimisation method. The maximum increase in turnover rate and transaction costs is reported for the $CVaR_{N1\%}$ hedging strategy with 12.85% and 16.00% when the PBIL algorithm is used. On average, the GA optimised hedging strategies increase the turnover rate and transaction costs compared with the unhedged portfolios by 10.19% and 10.88%, respectively. With PBIL algorithm, the turnover rate increases by 8.97% and the transaction costs by 12.77%.

Compared with the GA optimised hedges the transaction costs on average increase by 2.85% when the PBIL algorithm is used. However, the average turnover rate for hedging strategies with PBIL algorithm is reduced by 0.12% compared with the turnover rate of GA optimised portfolios. As the results in Table 5.1 show, the average mean return of portfolios with PBIL optimisation after transaction

TABLE 5.4 The table shows the average number of instruments and the average weight allocated to the option in percent for each hedging strategy and algorithm, for the entire out-of-sample period.

		Objective Function			
		$CVaR_{H1\%}$	$CVaR_{N1\%}$	$VaR_{H1\%}$	$VaR_{N1\%}$
Average Number of Assets	GA	7.34	6.92	8.14	7.02
	PBIL	7.42	7.02	8.06	8.08
	unhedged	5.24	5.24	5.30	5.24
Average Option Weight (in %)	GA	3.33	2.94	4.02	2.95
	PBIL	3.29	2.89	4	3.25

costs is higher than the average mean of all hedging strategies for GA optimised portfolios.

The average number of instruments represented in each portfolio over the entire out-of-sample period is reported in Table 5.4. Moreover, the table shows the average weight allocated to the option for GA and PBIL hedged portfolios. The optimal weight allocation of the hedged and unhedged portfolios with PBIL optimisation is shown in Figures 5.6 and 5.7.

The lowest average number of instruments represented in one of the hedging strategies is reported for the GA optimised $CVaR_{N1\%}$ portfolio with 6.92 instruments on average. With PBIL algorithm the average number of instruments is 7.02 for the same hedging strategy. The empirical VaR hedging strategy has the highest average number of instruments with 8.14 for GA and 8.06 for PBIL. The average number of instruments in the unhedged portfolios is 5.30 for the empirical VaR and 5.24 for the three remaining objective functions. The higher average number of instruments in the hedged PBIL and GA portfolios can be explained by the negative correlation between the equities and the index put option. The underlying instrument of the option is the DJIA index. The index and the put option have a high negative correlation. In our approach, we hedge a number of

DIJA constituents with the index put option. Thus, an optimal negative correlation can be generated by replicating the index correlation with the equities in the portfolio.

The average weight allocated to the put option differs by the hedging strategy and algorithm. In the entire out-of-sample period, the empirical VaR hedging strategy with GA algorithm has the highest average weight allocated to the option with 4.02%. The equivalent hedging strategy with PBIL algorithm on average allocates 4.00% to the option. The empirical CVaR hedging strategy requires the second highest investment in the option both with GA and PBIL search method with 3.33% and 3.29%, respectively. Hedging strategies with underlying normal distribution have a lower share invested in the option. The normal VaR and CVaR hedge on average allocate 2.95% and 2.94% for GA optimisation and 3.25% and 2.89% for the PBIL search method. In general, the hedging strategies with GA algorithm have a higher share invested into the option than PBIL optimised hedges. The only exception is the $VaR_{N1\%}$ strategy.

The higher average option weight for the empirical VaR and CVaR hedging strategy is caused by heavier tails assumed using the empirical distribution assumption. The expected downside risk measures for the empirical distribution are lower than for the normal distribution, in the in-sample periods. Thus, the optimisation algorithm increases the average weight of the option for the empirical VaR and CVaR hedging strategy to improve the downside risk of the portfolio. VaR and CVaR hedged portfolios with underlying normal distribution expect lower downside risk and thus, allocate less weight to the option.

In contrast to the work of Alexander et al. (2004) and Godin (2016), we propose a transaction cost function that is more practically relevant if the maturity of derivative hedging instrument does not match the length of the portfolio rebalancing period. Our empirical analysis shows, that the proposed hedging approach slightly increases the transaction costs compared with their unhedged equivalents.

However, the results show that the hedge significantly improves the portfolio results after transaction costs. Compared with the papers presented by Ahn et al. (1999) and Annaert et al. (2007), it is not necessary to determine the optimal strike price of the option to find the optimal hedging ratio for the portfolio. The optimal weight allocated to the put option is determined in the VaR and CVaR optimisation process and the option is not required to end in-the-money.

The empirical results show that our proposed hedge is able to efficiently secure a number of equities with a single long index put option. Instead of having to create an OTC option with increasing costs, lower liquidity or trade volume, we reduce the transaction costs by hedging all assets with a single long index put option. The proposed hedging framework reduces the option pricing risk while it increases the trade volume and market liquidity compared to an OTC option hedge.

5.4 Conclusion

Recent surveys by the ISDA (International Swaps Dealers Association, 2015) show that one of the major concerns in risk management is the increase in cost of hedging, caused by changes in the regulatory oversight of OTC derivatives contracts. Moreover, the new Basel III framework for non-standardised OTC contracts requires firms to find strategies to hedge their risk using standardised derivatives.

In this chapter, we contribute to the existing literature by introducing a self-financing global VaR and CVaR hedging approach with multivariate GARCH process. We extend the work of Rémillard and Rubenthaler (2013) and provide a hedging approach that is able to secure a number of underlying with one put option. Our hedging model minimises the VaR and CVaR of the terminal value of the portfolio by using a GARCH-DCC process to simulate the future hedging loss distribution. This extends the existing global hedging literature like Föllmer and

Leukert (2000), Föllmer and Leukert (1999), Alexander et al. (2004) and Godin (2016), and opens a new research path for global hedging with GARCH processes.

As another contribution to the existing literature (Föllmer and Leukert, 1999; Melnikov and Smirnov, 2012; Cong et al., 2013; Cong et al., 2014; Godin, 2016), our global VaR and CVaR hedging model provides a more practical approach to minimise the terminal value of the investments, as it requires no optimal strike price and maturity of the option, which often is hard to find for standardised derivatives. We provide a cost function that recognises such option selection issues. The optimisation of our global hedging approach is performed using the more computationally efficient PBIL algorithm compared with the dynamic programming algorithm used by Rémillard and Rubenthaler (2013) and Godin (2016).

The results of our empirical analysis show that the self-financing hedge improves the descriptive statistics of the portfolios after transaction costs. The hedged portfolios have a higher mean return and lower standard deviation compared with their unhedged equivalents. The best mean return and standard deviation is reported for the normal VaR hedging strategy with PBIL optimisation algorithm.

Unhedged portfolios are much more left-skewed than their hedged equivalents. The highest skewness is reported for the empirical CVaR hedging strategy and PBIL optimisation. Compared with the hedged portfolios with GA search method, all hedging strategies with PBIL algorithm have higher skewness values. In contrast, PBIL optimised hedging strategies have lower kurtosis values than portfolios with GA algorithm. However, all hedging strategies increase the kurtosis compared with the unhedged portfolios.

Our analysis shows that the proposed hedging approach successfully secures the investment in the underlying. Compared with the unhedged portfolios the hedging strategies significantly improve the normal and empirical VaR and CVaR values. Moreover, the maximum drawdown and drawdown duration of the hedged portfolios ameliorate substantially. In general, the hedging strategies with PBIL optimisation improve the maximum drawdown and drawdown duration slightly more than GA optimised hedge functions.

The turnover rates of the PBIL optimised hedging strategies increases on average by 8.97% compared with the unhedged portfolios. However, compared with GA optimised hedging strategies the turnover rate of portfolios with PBIL algorithm decrease by 0.12%. VaR and CVaR hedging strategies with underlying normal distribution and PBIL optimisation have up to 12.15% lower turnover rate than their equivalents with GA search method.

The transaction costs for hedging strategies with PBIL optimisation are between 7.69 % and 16.00% higher compared with their equivalent unhedged objective functions. For hedging strategies with GA optimisation the transaction costs range from 3.84% to 20.00%.

Our advocated global hedging approach secures a number of underlying with a standardised index long put option. We show that with increasing regulatory requirements and transaction costs for non-standardised OTC derivatives, our global VaR and CVaR hedging framework is a cost efficient technique that improves stability and profitability of the investments, after transaction costs. The framework is easily adoptable to other distribution and objective functions, and can be applied to hedge a variable number of instruments. Our self-financing global hedging approach requires no additional budget constraints. A natural extension to the proposed hedging approach is to use asymmetric conditional variance and correlation models. More research has to be conducted on how different types of options and option strategies can contribute to the hedging approach. Compared with the GA search method the PBIL algorithm requires only a few number of model parameters for the optimisation process. Future studies could research mechanisms to automatically set the parameters, e.g. based on the evolution of the underlying objective function.

Chapter 6

Conclusive Remarks

The research in this thesis develops new portfolio optimisation tools in combination with heuristic optimisation methods to manage the increasing market risk regulatory requirements of financial institutions under the Basel III regulations. This chapter provides a summary of this study.

This final chapter is structured as follows. The presented work is recapitulated in Section 6.1. We then list the contributions of this thesis in Section 6.2 before looking at further research in Section 6.3.

6.1 Summary

The ongoing revision process of the Basel III framework and the thereby caused changes to the regulatory requirements for financial firms, lead to increasing costs and capital requirements for their risk management processes. In Chapter 1, we discuss the changes in the regulatory environment and the need of financial institutions to actively manage their market risk exposure. This discussion leads to the research objectives of this thesis. We outline relevant risk management literature in Chapter 2 before proceeding with the research studies we have conducted to contribute in this field of study.

The research we undertake in this thesis provides new regulatory risk management tools and aims to understand the influence of portfolio optimisation decisions on the regulatory requirements. In all studies, multi-constraint portfolio optimisation problems with heuristic optimisation are carried out for an empirical sample and conclusions about the influence on the financial regulatory requirements of the company are made.

In the first research study, presented in Chapter 3, the effect of heuristic optimisation on the regulatory market risk capital requirements is examined in an empirical analysis. We extend this optimisation problem to manage the regulatory risk of the portfolio by proposing a new regulatory risk constraint to manage the number of daily VaR violations of a portfolio.

The first part of research Chapter 3 analyses the Threshold Accepting (TA) heuristic optimisation algorithm and its effect on reducing the market risk capital requirements by finding better portfolio compositions for ill-posed VaR and CVaR optimisation problems. The results are compared with the Trust-Region (TR) local search method in an empirical analysis for the 30 constituents of the Dow Jones Industrial Average (DJIA) index. Our computational experiment demonstrates the superiority of the heuristic optimisation over the local search method for ill-posed optimisation problems. Portfolios optimised with the TA algorithm outmatch portfolios with TR search method in all risk and performance measures. We identified the most promising objective function to be a CVaR risk measure with underlying empirical distribution.

Based on the findings in the first part of Chapter 3, we propose a new regulatory risk constraint that is based on the Unconditional Coverage (UC) log-likelihood ratio. This methodology is discussed in the second part of Chapter 3. The purpose of this new risk constraint is to avoid under- and overestimation of the regulatory portfolio risk while optimising the portfolio for some objective function. We identify the regulatory portfolio risk as the number of daily VaR violations, as this has a direct effect on the portfolios market risk capital requirements. Our new risk constraint is a unique approach that is able to manage the portfolio regulatory requirements in the ongoing revision process of the Basel III framework. In an empirical analysis of 30 DJIA constituents, we find that our new approach leads to better objective function measures for all portfolios. The findings suggest that portfolios with our constraint approach perform best with standard normal VaR and CVaR risk measure. Interestingly, the empirical results suggest that for all optimised portfolios we were able to improve the regulatory market risk capital requirements.

The second research study is presented in Chapter 4. In this research work, we examine the question if more advanced VaR and CVaR estimation methods have a positive impact on the daily VaR violations and the capital requirements. To answer this question, a Monte Carlo simulation approach is proposed that uses GARCH and DCC modelling to create several-days ahead VaR and CVaR estimates for different underlying distribution assumptions. The Monte Carlo approach uses Cholesky decomposition to generate correlated random variables. For the optimisation we apply the PBIL heuristic algorithm. The results of our empirical analysis on the DJIA constituents show that more advanced estimation models are able to reduce the regulatory capital requirements for VaR and CVaR objective functions. We find that the average number of VaR violations can be reduced for optimisation models with empirical VaR and CVaR objective function when the more advanced approach is used. Objective functions with standard normal distribution assumption, however, have a higher average number of daily VaR violations with the Monte Carlo GARCH-DCC simulation approach.

Chapter 5 presents the third research work of our thesis. In risk management, firm often use OTC derivatives to hedge their exposure against potential losses. With the Basel III framework, however, regulatory oversight and transaction costs significantly increase for OTC contracts. Thus, firms seek new hedging strategies to reduce the regulatory and transaction costs. In this chapter, we provide a new global hedging approach with multivariate GARCH process. We introduce a GARCH-DCC process to the global VaR and CVaR hedging approach to model future returns and find an optimal solution to the VaR and CVaR minimisation problem. For the optimisation process, we use a PBIL heuristic algorithm. The optimisation results of the PBIL algorithm are compared with a GA optimisation algorithm, which we use as a benchmark for the PBIL search method. We apply our self-financing global hedging framework to several underlying and hedged them with an index put option. In an empirical study of DJIA constituents and DJIA index options, we test the models ability to secure the investment in several underlying while reducing the transaction costs. The empirical examination shows that portfolios hedged with our approach yield better performance measures and improve the stability of the hedged portfolios. Moreover, our global hedging model improves the maximum drawdown and maximum drawdown duration of the secured portfolio. The results show that PBIL algorithm is capable to find good solutions for option based downside risk hedging problems. In our setting, the algorithm outperforms the GA search method in efficiency and quality of the solution.

6.2 Contributions

This thesis contributes to the fields of heuristic optimisation, regulatory finance, portfolio optimisation and hedging. We provide new active risk management tools that use heuristic optimisation techniques to optimally manage new challenges in risk and portfolio management that are caused by changes in the regulatory market and credit risk requirements. The major contributions of this thesis are:

1. In Chapter 3, we provide the first empirical evidence for the significance of applying heuristic optimisation algorithms to financial regulatory portfo-

lio optimisation problems of trading desks. Our results show that heuristic optimisation algorithms reduce the regulatory capital requirements for market risk of ill-posed VaR and CVaR portfolio optimisation problems. The findings highlight the importance to apply heuristic algorithms in regulatory risk management. A Multi-Objective Evolutionary Algorithms (MOEA) approach to find a Pareto-optimal solution set that optimises for the regulatory capital requirements and the expected portfolio return is applied by Drenovak et al. (2017). The multi-objective portfolio optimisation problem is solved by using a Non-dominated Sorting Genetic Algorithm II (Deb et al., 2002) that is run under a parallel framework developed by Ivanovic et al. (2015). We demonstrate that a meta-heuristic algorithm can efficiently solve financial regulatory optimisation problems.

2. We provide a regulatory risk constraint that is based on the UC log-likelihood ratio. This new constraint leads to an optimal number of daily VaR violations to avoid under- and overestimation of the regulatory portfolio risk. By doing this, it incorporates the Basel III backtesting rules while the risk manger is able to optimise for some preferred objective function. Compared with the dynamic decision rule proposed by McAleer et al. (2010), our model is an active risk management approach. The trader simply optimises for the desired objective function while the constraint sets boundaries to keep the model Basel III conform. This is different to the model by McAleer et al. (2010) where the trader has to decide on a number of initial model parameters, which are difficult to estimate. Other than our UC constraint model and the model presented by McAleer et al. (2010), Santos et al. (2012) directly minimise the capital requirements of the portfolio. They analytically solve the minimum capital requirements objective function and provide a convex solution with a daily VaR violations constraint. The UC constraint

provides a more flexible optimisation approach and allows for different objective functions. The MOEA approach to minimise the capital requirements and maximise the expected portfolio return proposed by Drenovak et al. (2017) is an extension to the model presented by Santos et al. (2012). However, the missing constraint on the number of daily VaR violations can lead to optimal portfolios with a high number of VaR violations. Each violation of the daily VaR limit is a risk to the stability of the portfolio and the bank. Our proposed UC constraint optimally manages the daily VaR violations of the portfolio. It can even be applied to a MOEA portfolio optimisation problem with several contradicting objective functions. With the most recent changes in the regulatory framework the objective function provided by Santos et al. (2012) and Drenovak et al. (2017) are outdated. Our UC constraint approach is applicable to all revised versions of the Basel III minimum capital requirements for market risk estimations.

3. We demonstrate that more advanced VaR and CVaR estimation models reduce the regulatory portfolio capital requirements even for several-days ahead forecasts. We show that the average number of daily VaR violations can be reduced for empirical VaR and CVaR objective functions when a Monte Carlo simulation approach is used. These findings can have implications on the internal risk management approach of financial institutions. Our results extend the findings of Sentana (2003), Cuoco and Liu (2006), Alexander et al. (2007), McAleer et al. (2010), Santos et al. (2012) and Drenovak et al. (2017) by examining the influence of more advanced VaR and CVaR estimation models on the number of daily VaR violations and market risk capital requirements.

4. We introduce a self-financing global VaR and CVaR hedging approach with multivariate GARCH process. The introduction of GARCH processes in

global VaR and CVaR hedging is a new research path that extends the existing global hedging literature (see Föllmer and Leukert, 2000; Föllmer and Leukert, 1999; Alexander et al., 2004; Godin, 2016). Our model extends the global hedging approach of Rémillard and Rubenthaler (2013), who apply a GARCH process to a global quadratic hedging approach. Compared with their framework, our model aims to VaR and CVaR hedge several underlying with one put option. We use a multivariate GARCH process to model the future returns and find an optimal solution of the global hedging problem.

Our global hedging framework neither requires an optimal strike price nor an optimal maturity of the derivative to hedge the underlying. This is different to Cong et al. (2013), Cong et al. (2014) and Godin (2016), who assume an option strike price that matches the price of the underlying. In practice however, it can be very difficult to find options with the exact strike price. Our propose global VaR and CVaR hedging approach finds the optimal hedging solution even when the maturity or strike price of the option is not optimal. We can easily extend our model for different distribution and objective functions. Moreover, we provide a more practical transaction costs function for situations where the maturity of the option does not fit the length of the investment period.

6.3 Future Research

The ongoing revision process of the Basel III framework creates several new research needs to assist the decision process of firms risk management and financial regulators. This thesis provides new methods and insights to regulatory portfolio optimisation problems with heuristic algorithms. There is a wide range of opportunities to carry out more research on these models and provide further contributions to the literature.

Given our research in portfolio optimisation with minimum capital requirements objective functions, we find that such models are difficult to apply in a simulation-based optimisation approach. A natural extension to our proposed optimal regulatory risk constraint and similar minimum capital requirements objective functions provided in the literature, is a simulation-based regulatory risk constraint for portfolio optimisation.

Some recently proposed changes in the calculation of the minimum capital requirements for market risk need further examination. A study introduced by Kellner and Rösch (2016) shows that model risk increases with these changes. In our future research we will introduce a Multi-Objective Evolutionary Algorithm that reduces the model risk, based on findings by Skolpadungket et al. (2016) to handle model risk in portfolio selection using Multi-Objective Genetic Algorithm with Sharpe ratio errors.

Ranković et al. (2016) propose a different mean-VaR optimisation approach that is based on the actual number of shares of an asset not on the portfolio weight allocated to this asset. They argue their approach is more relevant to asset managers facing regulatory VaR constraints. Drenovak et al. (2017) also apply this approach in their minimum capital requirements framework with Multi-Objective Evolutionary Algorithms portfolio optimisation. An extension to our study could be to apply the active portfolio framework, proposed by Ranković et al. (2016), in our optimisation process.

Ranković et al. (2016) and Drenovak et al. (2017) use the parallel NSGA-II algorithm developed by Deb et al. (2002) and extended by Ivanovic et al. (2015). As our research results in Chapters 4 and 5 demonstrate, PBIL is a valid alternative to Genetic Algorithm (GA) and can easily run in a parallel framework. A parallel Multi-Objective PBIL (MO-PBIL) is introduced by Brown et al. (2014) and extended by Carmona Cortes and Rau-Chaplin (2016). A more detailed analysis between the NSGA-II and MO-PBIL could provide new information on how efficient the algorithms are to solve optimisation problems with focus on regulatory requirement.

Aside from studying the efficiency of NSGA-II and MO-PBIL, another research path could be to improve these models even further. Both algorithms use k-means clustering to determine Pareto-optimal sets to start the next generation. The number of clusters is a constant that needs to be determined at the beginning of the optimisation process. An extension could be to dynamically set the number of clusters for the optimisation process. A dynamic k-means algorithm is proposed by Tao et al. (2016) and could be used for such purpose.

An extension to our VaR and CVaR hedging approach with GARCH-DCC estimation can be to model the error terms of the GARCH processes with non-Gaussian distribution functions, as suggested in Godin (2016). Such distributions might provide a better estimate of the returns distribution. Another option can be Copula-GARCH models that show good performance for hedging equities in local hedging frameworks (see Hsu et al., 2008; Lee, 2009).

Other research opportunities are related to the integration of regulatory instruments and their effects on the economy. The majority of the literature focuses on the costs and benefits of capital requirements. Papers such as Miles et al. (2013) estimate the optimal capital requirements taking into account the costs and benefits for both the institution and the economy. Also, there is some literature that examines the impact of total loss-absorbing capital (TLAC) instruments, see e.g. Prescott (2012) and Nguyen (2013). The pricing of TLAC instruments seems to open up another interesting research path, see e.g. Berg and Kaserer (2015).

The introduction of liquidity capital requirements (LCR) to the Basel III framework also gives rise to a new research path that studies the costs and benefits of this new instrument to the stability of institutions and potential interactions with the capital requirements. Covas and Driscoll (2014) and Cornett et al. (2011) study the effects on the bank itself, while Perottia and Suarez (2011) examines the impact on the economy. Krause and Giansante (2012) address the issue of how the minimum capital and liquidity requirements affect the systemic risk and the likelihood of bank failures. They are the first who consider the network structure of interbank loans as well as the balance sheet structure of individual banks.

Another research direction can be other supervisory tools (e.g. buffers and macroprudential policies) that are introduced in the new Basel III framework and which are to support the more complex capital and liquidity regulatory requirements. Van Den End and Kruidhof (2013) and Aiyar et al. (2016) are just two exemplary papers that study the effectiveness of macroprudential policies. These simpler rules are more pro-cyclical and can help to discourage arbitrage behaviour by the bank, e.g. regulatory and tax arbitrage. The discussion on regulatory arbitrage is also connected to research work on the regulation of the shadow banking system, see e.g. Lengwiler and Maringer (2015).

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Appendix A

Minimum Capital Requirements for Market Risk

As specified in detail in Section 3.2.2, the Basel II formula for calculating the minimum capital charge for market risk is given by

$$CR = max\{VaR_t, m \times \overline{VaR}_{60}\}. \tag{A.1}$$

With the introduction of the Basel III accord (Basel Committee on Banking Supervision, 2009b) the regulatory capital requirements for market risk formula was extended by a stressed VaR ($sVaR$) calculation. The regulatory authorities avoid to determine specific stress scenarios to calculate the $sVaR$ term. With the introduction of the new stressed VaR term, the firm's minimum capital requirements for market risk is reformulated as

$$CR = max\{VaR_t, m \times \overline{VaR}_{60}\} + max\{sVaR_t, m \times \overline{sVaR}_{60}\}. \tag{A.2}$$

The latest update of the Basel III accord (Basel Committee on Banking Supervision, 2016) led to significant changes in the calculation of the minimum regulatory capital requirements for market risk and is now mainly based on the portfolio's Expected Shortfall (ES), also referred to as Conditional Value-at-Risk (CVaR). In this thesis, we focus on the modellable risk factors and thus, the regulatory minimum capital charge for market risk is given by

$$CR = max\{IMCC_t, m \times \overline{IMCC}_{60}\}, \tag{A.3}$$

where

$$IMCC = p(IMCC(C)) + (1-p)\left(\sum_{i=1}^{R} IMCC(C_i)\right), \tag{A.4}$$

$$IMCC(C) = ES_{R,S} \times \frac{ES_{F,C}}{ES_{R,C}}, \tag{A.5}$$

and

$$IMCC(C_i) = ES_{R,S,i} \times \frac{ES_{F,C,i}}{ES_{R,C,i}}. \tag{A.6}$$

The value of p is 0.5 and is the relative weight assigned to the bank's internal risk model. The multiplication variable m can take values between 1.5 and 2.0. As for the previous models, the multiplication factor is based on the outcome of the backtesting of the bank's daily 99% VaR based on the full set of risk factors (F) in the current period (C). Variable R is the reduced set of risk factors. The ES for the reduced set of risk factors ($ES_{R,C}$) needs to explain a minimum of 75% of the ES value with full set of risk factors ($ES_{F,C}$). $ES_{R,S}$ is the portfolio-wide stressed ES value for a reduced set of risk factors. $ES_{R,C,i}$, $ES_{F,C,i}$ and $ES_{R,S,i}$ are the respective ES values for each of the risk classes.

For the minimum capital charge calculation the regulatory liquidity-adjusted ES with a 97.5th percentile is computed as follows (Basel Committee on Banking Supervision, 2016):

$$ES = \sqrt{(ES_T(P))^2 + \sum_{j\geq 2}\left(ES_T(P,j)\sqrt{\frac{(LH_j - LH_{j-1})}{T}}\right)^2}, \tag{A.7}$$

where LH_j is the liquidity horizon $j = 1, 2, ..., 5$ that varies by the type of portfolio position P and can take the values $LH_j = [10, 20, 40, 60, 120]$. Variable T is the length of the base liquidity horizon. $ES_T(P)$ is the ES for portfolio positions with shocks to all risk factors. The ES for portfolio positions with shocks for each instrument is given by $ES_T(P, j)$.

To our best knowledge, the latest Basel III update on the minimum capital requirements for market risk formula has not been applied in relevant literature, so far. Related research, conducted by Santos et al. (2012) and Drenovak et al. (2017), apply Formula A.1 and A.2 in their work.

Drenovak et al. (2017) show that for their Multi-Objective Evolutionary Algorithm (MOEA) optimisation problem the introduction of the stressed VaR term has no effect on the model evaluation compared with models that use the Basel II formula in Equation A.1. They find that stressed return series and volatilities have no effect on the models risk and performance ranking. The only exception are stress scenarios with significant shift in the correlation matrix. Drenovak et al. (2017) highlight that the implementation of stressed VaR term into the optimisation model can be ignored, when stress tests without significant correlation changes are included in the optimisation process. This reduces model complexity, computational time and improves the explanatory strength of the optimisation model. The results of the analysis in Drenovak et al. (2017) support the findings in Santos et al. (2012). Santos et al. (2012) conclude that the implementation of the stressed VaR in the capital requirements calculation has no effect on the general model evaluation, when compared with models that apply Equation A.1. Their conclusion even holds for large changes in the stressed correlation matrix.

In this thesis, we therefore implement the Basel II minimum capital requirement calculation for market risk formula, given in Equation A.1, and exclude the stressed VaR term in the optimisation process. In our future research, we will investigate our advocated LR_{UC} constraint for the new regulatory minimum capital requirement formula (Basel Committee on Banking Supervision, 2016), presented in Equation A.3.

Appendix B

Extended Test Results to Chapter 3

TABLE B.1 Extended test results to Chapter 3 for $VaR_{H1\%}$ portfolio with (+) and without (-) LR_{UC} constraint. The table shows the optimisation results for portfolios with 10-days and 1-day investment horizon for a test period from 30th January 2006 to 29th January 2016.

	10-days		1-day	
	+ LR_{UC}	- LR_{UC}	+ LR_{UC}	- LR_{UC}
Mean CR	0.1903	0.2025	0.2797	0.2907
Mean Multiplier	3.42	3.55	3.33	3.42
Max Multiplier	4	4	4	4
Mean Violation	4.12	4.41	4.17	4.28
Max Violation	10	10	12	11