Essays on Financial Accelerators and Macroprudential Policy

A thesis submitted for the degree of Doctor of Philosophy

by

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Dedicated to my mother for her selfless support

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Contents

Non-Technical Summary

Abstract

This thesis focuses on the relationship between the real economy and the financial sector which gives rise to various amplification mechanisms known as financial accelerators. Historically, those channels are known to be in the roots of the world's largest crises such as the 2008 Great Recession. In its aftermath, policymakers have undertaken various reforms that introduce macroprudential policy which focuses on the stability of the financial system as a whole. This thesis studies different financial amplification channels and the ability of macroprudential policy to mitigate their impact on the real economy in three chapters.

The first chapter introduces different macroprudential tools into a macroeconomic framework with financial frictions and analyses their ability to mitigate the impact of a crisis originating from the financial sector to the real economy. The main finding of the paper is that sector specific tools can be effective if applied before the occurrence of the crisis, however, broader tools are much more effective once the crisis has spread to the economy.

The second chapter expands the framework of the previous one, in order to provide a realistic representation of the current regulatory setting for capital requirements - the Internal Rating Based approach. The paper then studies the ability of the regulation to lead to procyclical capital requirements and thus amplify the business cycle and reduce social welfare. In order to avoid these consequences, an alternative policy rule is proposed which is able to mitigate the amplification effects.

The third chapter focuses on the founding theory behind the current regulatory framework - the portfolio loss distribution (Vasicek, 2002) and expands it by introducing macroeconomic amplification mechanisms known as financial accelerators. The resulting portfolio distribution shows large losses to be substantially more likely which increases the fragility of the financial system and the amount of capital necessary to maintain its stability.

The Reach of Macroprudential Policy

The Great Recession which started with the burst of the US housing market bubble established financial stability at the forefront of policy discussions. Prior to the crisis, the general view was that responding to fluctuations in asset prices or other financial variables was potentially harmful due to the difficulty in detecting asset price bubbles in real time. However, a growing amount of empirical work has found that large movements in a number of observable variables such as credit and residential investment are reliable advance indicators of house price busts, which in turn are typically associated with substantial falls in output.[1] These observations signal that a countercyclical policy that reacts to such indicators could mitigate the impact of the crisis or even prevent its occurrence. But what is the right policy tool for this purpose?

Macroprudential policy has the objective to limit the accumulation of financial risks, in order to reduce the probability and to mitigate the impact of a potential financial crash and to strengthen the resilience of the financial sector. Thus, purely by its objective, macroprudential policy should be the right approach to lean against the credit cycle. However, because of the recent availability of these tools, the theoretical research which should suggest the appropriate use of them seems not yet fully developed.[2] Moreover, due to the large set of macroprudential tools, the different propagation mechanisms of the different instruments are even less researched.

Following the lessons of the Great Recession, this paper aims to contribute to the debate over the right policy tool to lean against the credit cycle by investigating the role of different types of macroprudential instruments. In particular, whether some macroprudential instruments can be too blunt to address problems in a specific financial sector while others are more appropriate for this purpose. The motivation of this approach is based on the understanding that different policy instruments have different broadness of impact, depending on the level at which they enter the economy. For example, as the capital-asset requirement enters the economy at the bank level, a tighter requirement will most likely tighten

[1]See Kannan et al. (2011), Borio and Lowe (2004) and IMF (2009).
[2]See Angelini et al. (2014).

lending for all types of loans. On the other hand, a loan-to-value requirement restricts the maximum borrowing amount for a given collateral value and thus can be imposed on a specific type of lending such as mortgages. Therefore, in a scenario of a housing market overheating, while capital-asset requirements can lead to higher borrowing costs thus reducing mortgage lending, the instrument would also reduce lending to firms. On the other hand, a tighter LTV limit can be imposed only on mortgages and thus have a more sophisticated direct effect without impeding corporate lending.

For answering the research question a general equilibrium model is employed. The possibility to represent different levels of impact of macroprudential policies is achieved by a detailed banking sector with two types of lending. The banking sector is set-up extending defaulting loans to both households and firms against housing and capital collateral. In addition, while banks are subject to capital requirement at their wholesale level, each sector specific loan type is subject to a LTV requirement. We simulate the crisis as an unexpected increase in the default rate of mortgages which leads to bank capital destruction and transmission of the crisis to the entire banking system and consequently the real economy. The two macroprudential instruments are compared firstly as being permanently tighter prior to the crisis, and secondly as optimized dynamic rules which react after the occurrence of the crisis.

We find that, prior to the crisis, a tighter LTV limit reduces the vulnerability of the banking sector to higher loan defaults but it does so at the cost of lower output. At the same time, a permanently tighter capital-asset requirement seems incapable of mitigating the impact of the financial shock. However, in the crisis aftermath, once we consider optimal dynamic policy setting in terms of social welfare, we find that an optimized capital-asset requirement can be successful in improving welfare and attenuate the transmission of the crisis to the real sector. Apart from being less successful in improving welfare, the dynamic LTV limit is also incapable of reducing the impact of the shock after its occurrence. These results follow from the fact that once the shock has occurred and bank capital is destroyed the crisis is already transmitted to the whole banking system and hence the instrument which operates at this level - the capital asset requirement, is more effective. Alternatively, if the shock could be anticipated, a tightening of the sector specific instrument - the LTV limit to mortgages, could significantly attenuate the impact of the shock.

The conducted research highlights the important feature of macroprudential instruments of having different level of impact ranging from more general to more sector specific. This property allows macroprudential regulators to intervene in a specific type of lending, such as mortgages, without disturbing directly lending to firms. Thus, while an anticipatory use of the more sector specific instrument can be very effective at mitigating the impact of the crisis before its occurrence, the reactive use of the more general instrument can be effective in attenuating the transmission of the crisis after its impact. Hence, a crucial factor in determining the appropriate policy response is the ability of regulators to anticipate and identify the housing market overheating on time and respond to it with tighter LTV limit to mortgages. If such anticipation is not possible a reactive use of the more general - capital asset instrument would mitigate the transmission of financial shocks.

Sectoral Risk-Weights and Macroprudential Policy

This chapter of the doctoral thesis builds on the macroeconomic framework developed in the first chapter and its research findings. The main motivation of the paper is to provide a realistic representation of the existing macroprudential regulation and assess its impact on the financial system in crisis scenarios.

The macroprudential tool under focus is the capital-asset requirements. With the introduction of the Basel II regulatory framework, from 2004 onwards, a major emphasis was put on risk sensitivity - the idea that capital requirements should depend on the type of assets that a bank holds and in particular, that banks with riskier assets should hold larger amount of capital to ensure their solvency. A key aspect of this regulation is the way of measuring the riskiness of banks' assets. While in Basel I, assets' risk was evaluated with the Standardized Approach (SA) - through external fixed ratings, Basel II introduced the Internal Ratings-Based (IRB) approach in which banks can use internal models to estimate their portfolio riskiness which in turn would determine the required regulatory capital to be held. In practise, the risk sensitive requirements are implemented through assigning risk-weights to different assets and then computing a capital over risk-weighted assets ratio[1] that has to comply with the regulatory requirement. While under the SA, the risk-weights are fixed and depending on the asset class, under the IRB approach banks are using their own models to calculate the risk-weights dynamically.

However, in 2008 the Great Recession hit the world's financial system even before the Basel II regulation was fully introduced. As a result, a new regulation was negotiated in the face of Basel III in which the lessons from the crisis were on top of mind and more stringent standards were adopted including higher capital requirements and various capital buffers such as the Countercyclical Capital Buffer (CCB). Nevertheless, regardless of the higher requirements or the time varying buffers, the newly imposed regulation remains highly dependent on the underlying way of measuring risk that is the IRB approach. In empirical studies, the latter has

[1] In bank regulation, the capital over risk-weighted assets ratio is simply referred to as capital-asset ratio, while the ratio of capital over non-weighted assets is referred to as the leverage ratio.

often been criticised for procyclical capital charges that can amplify the financial cycle.[1]

So far, the theoretical general equilibrium literature has analysed macroprudential policy and in particular, capital-asset requirements without introducing the current risk-sensitive approach imposed by regulation. For example, Gerali et al. (2010) introduce capital requirements but regard the assets as equally weighted with a weight of one - thereby corresponding to a leverage ratio. Angelini et al. (2014) study the interaction between capital requirements and monetary policy. However, the latter paper introduces asset risk-weights for the capital requirements according to an ad-hoc rule.

To the best of our knowledge, this paper is the first one to introduce the IRB approach in a general equilibrium framework. It does so by employing the model developed in the previous chapter that features risky and defaulting loans to households and firms. The presence of defaulting loans allows for the calculation of asset risk-weights according to the IRB approach which uses the probability of default (PD) and loss given default (LGD) for a specific type of asset.

It is important to note that, as a component of the regulatory capital-asset ratio, assets risk-weights lead to variability in the capital requirements and hence the tightness of banking regulation and banks' incentive to extend certain types of lending. As a result, failure to represent asset risk-weights realistically, inevitably leads to failure of capturing the relationship between capital requirements and the real economy and hence the impact of macroprudential policy.

After incorporating the current regulatory standards, the paper then proceeds to the policy analysis. The compared policy settings are the following. Risk independent capital requirements that reflect the current approach in the literature[2] in which all risk-weights are constant and equal to one, leading to a leverage ratio requirement. The IRB approach in which the risk-weights for each asset type depend on its PD and LGD, representing current regulation. And finally, an alternative countercyclical macroprudential setting is introduced that sets risk-weights for each type of lending based on sector specific measure of leverage.

The employed macro model also allows for realistic crisis scenarios which orig-

[1] See Markus et al. (2014), Goodhart et al. (2004) and Borio et al. (2001).

[2] See Gerali et al. (2010)

inate from mortgage lending and transmit to the real economy. Being exposed to risky loans, a higher than the expected default proportion of the portfolio with lower than expected collateral value can lead to endogenous bank capital destruction. The lower capital then leads to larger spreads and reduced lending in order to comply with regulatory requirements. The tighter lending and higher spreads in turn further increase default rates and depress collateral prices closing the financial accelerator cycle.

The model parameters are estimated with Euro Area data and a historical variance decomposition identifies the period of the 2008 recession as being subject to shocks from the mortgage lending market leading to larger defaults.

The different policy settings are assessed in terms of their ability to stabilize the economy in two different crisis scenarios originating from the mortgage market. The first scenario represents the bust phase of the crisis in which a higher than expected mortgage defaults destroy bank capital and subsequently tighter lending conditions suppress all types of lending and transmit to the wider economy. The second scenario consists of a simulated boom and bust cycle achieved through unrealized news shocks in the mortgage market. In the latter scenario, a positive shock expected 4 periods in the feature to mortgages risk, causes lenders to expect lower default rates and higher collateral prices thereby relaxing lending conditions and spreads. This in turn leads to increase in leverage and booming collateral prices. However, at period 4 expectations do not materialize and a higher proportion of loans default than expected leading to bank capital destruction and a crisis which is driven entirely by agents' expectations.

Our results show that in both boom and bust phases of the crisis, the IRB approach leads to procyclical capital requirements. In the boom phase, the approach leads to looser capital requirements and thereby to lending conditions that reinforce market exuberance. In the bust phase, higher PD estimates lead to higher risk-weights and tighter capital requirements that depress bank lending and slow down on economic activity. The IRB approach therefore reinforces the financial cycle in the event of a crisis.

By contrast, our macroprudential approach to setting risk-weights leads to countercyclicality in capital requirements in both the boom and bust phases of the crisis – thereby serving to attenuate the financial cycle. As a result, the negative impact of the financial crash to the real economy is smaller and the recovery happens faster.

Finally, the leverage ratio policy setting keeps constant risk-weights equal to one and thus does not vary capital requirements with the business cycle leading to static policy. As a result, the impulse responses in both crisis scenarios lay in between those of the procyclical IRB setting and the countercyclical macroprudential rule.

In order to assess the policies in terms of various shocks and the business cycle, we also compare the second moments of major macroeconomic indicators as well as a measure of social welfare. We find that relatively to the static policy setting of the leverage ratio, the IRB setting of the risk-weights leads to higher variation in the macroeconomic variables and lower social welfare. On the other hand, the macroprudential rule smoothens the business cycle by decreasing the variation in the variables and as a result leads to higher social welfare. Finally, a welfare optimization over the parameters of the macroprudential rule clearly indicates that the countercyclical response to leverage is welfare improving.

The intuition behind these results can be found by reflecting on the purpose of regulatory capital requirements. Bank capital requirements are enforced with a view to ensure that banks hold enough capital to cover the potential Unexpected Losses (UL) associated with their assets. Expected Losses (EL) are to be covered by bank provisioning and credit pricing. While EL are seen as everyday risk costs of lending, UL are rare and large portfolio losses that arise in crisis circumstances. By applying the IRB approach, we estimate the UL of each asset using the same parameters that are used for estimating the EL in bank provisioning - the PD and LGD. The IRB approach therefore leads to a positive relationship between estimates of the EL and UL – thereby leading to procyclical capital requirements. In a situation characterised by optimism in lending markets – the EL will be low, and likewise the IRB approach will tend to estimate lower UL, resulting in lower capital requirements. Conversely, the macroprudential rule relates UL to sectoral measures of leverage – which will tend to make the UL move in the opposite direction of EL and lead to countercyclical regulation.

Analysing further current regulation, Basel III introduces the countercyclical capital buffer (CCB) that allows regulators to require additional amount of capital relative to the risk-weighted assets depending on the phase of the business cycle. However, if risk weights lead to procyclicality in requirements this can negate the effect of the CCB and make it useless.

Another advantage of the suggested approach is related to the research finding of the previous chapter regarding policy broadness. In situations in which a certain type of lending is seen as excessive and risky by policy makers, a broad tool such as the CCB would affect all types of lending and thus harm productive investment. However, the countercyclical risk-weights which respond to sectoral leverage can increase the risk weight for mortgages while reduce the one for firms. As a result, banks will have to hold relatively larger amount of capital for mortgages than for firm loans which will in turn alter their incentives and relative costs of lending.

Our findings tend to support the view that there is room for improvement in the current design of risk-based capital requirements, in particular regarding the IRB approach. As the design of the financial framework evolves, an emphasis on less procyclical mechanisms would be potentially beneficial for the sake of mitigating the banking sector's tendency to exacerbate the real economy impact of financial shocks. The non-risk-based leverage ratio setup could be an improvement in this regard. A macroprudential approach, which encourages banks to continue lending in a recession and discourages banks from lending excessively in the boom phase may be better still.

Underestimating Portfolio Losses

The third chapter further explores financial frictions amplification channels but does so by employing a different approach. The previous two chapters rely on large scale general equilibrium models which can analyse the feedbacks between different markets and financial regulation. However, the size of these models comes at the cost of a linearly approximated solutions around the steady state. While this method can provide reasonable results for simulating small shocks and fluctuations, financial crises are characterized by major deviations in the behaviour of agents and macroeconomic indicators from the one that is observed in normal times, leading to inherently nonlinear economic relationships.[1] Furthermore, the presence of financial accelerators between asset returns, banks' balance sheets, lending contitions, and the real economy further reinforce such effects (Borio, 2012). As a result, macroprudential policy which aims to prevent and mitigate crisis situations cannot be thoroughly analysed in a linearised model.

Reflecting on these conclusions, this paper focuses on a partial equilibrium model that allows for nonlinear solution but at the same time incorporates the crucial for the policy analysis accelerator effects. It does so by starting from the foundation model of the Basel capital regulation and extending it by developing its economic structure at several steps. Namely, the asymptotic single risk factor (ASRF) framework is expanded by incorporating rational behaviour of borrowers, banks, and investors.

The ASRF framework developed by Merton (1974) and Vasicek (2002) presents bank lending as subject to both idiosyncratic and aggregate risk. While the former can be insured, banks remain prone to the latter which introduces endogenously the need of capital buffers to guard against the remaining single risk factor. Furthermore, the simplified structure of the model enables nonlinear solution and derivation of the portfolio loss distribution. In addition, the structure also allows for closed form analytical solution which is particularly appealing to regulators and policymakers which is why the Vasicek model is still employed by the Basel regulation for capital requirements.[2] On the other hand, the structure that allows

[1] See Milne (2009).

[2] The Vasicek formula is the cornerstone of bank capital regulation and is used for the calculation of asset risk-weights in capital requirements. See (BCBS, 2005) and EU Capital

for analytical solution comes at the cost of ignoring important agent behaviour that enables the financial accelerator amplification which is well known in the general equilibrium literature. We argue that the financial accelerator mechanisms are crucial in such environment as they can amplify portfolio losses and their distribution which in turn should be taken into account by bank regulation in order to quantify and guard against bank losses.

This paper bridges the gap between the literature of financial frictions and portfolio value theory. It expands the former with aggregate risk for the expense of the general equilibrium solution and the latter with economic structure and behaviour for the expense of analytical solution. The paper presents a framework that has been specifically designed to analyse the central role of banks and the interaction of their behaviour with the one of borrowers and investors. As a result, the model incorporates important financial accelerator mechanisms between borrowers' net worth, banks' balance sheets and risk premia which are crucial for the analysis of bank regulation. Our representative bank is exposed to a large portfolio of loans with diversifiable idiosyncratic risk due to portfolio size and non-diversifiable aggregate risk.

First, by analysing optimal borrowers' default choice, we derive a positive relationship between the default rate of the portfolio and the loss-given-default of each loan. The relationship arises due to the fact that in adverse aggregate scenarios, the reason for the larger default rate is the cheaper collateral which borrowers prefer to give up rather than repay the loan. As a result, not only that more loans default but banks repossess cheaper collateral which increases their losses in adverse scenarios leading to the first financial accelerator of this paper.

Second, by analysing the process of insurance of idiosyncratic risk by banks, we derive a spread setting behaviour of lenders which creates a negative relationship between borrowing costs and the net worth of borrowers. The relationship arises due to the property of banks to set larger spreads to riskier portfolios with higher loan-to-value. The higher interest rates in turn increase borrowers' owed amount and loan-to-value and thus make them riskier for the bank, leading to the second financial accelerator of this paper.

Requirement Regulation IV - 2013.

Finally, by deriving the loss distribution of the bank's portfolio we analyse the required risk premia by the investors of the bank, which leads to a positive relationship between the costs of funding for the bank and the riskiness of its portfolio. This leads to the third financial accelerator of this paper which also interacts with the other two channels. For example, an increase of the riskiness of the bank's portfolio leads to higher costs of funding to the bank. The latter are passed on to the borrowers as higher interest rates which increases their chance of default. As a result, portfolio riskiness increases as well.

We track how the introduction of each of the three amplification mechanisms affect the portfolio loss distribution - taken independently or all together. Our results illustrate how ignoring of these channels can underestimate portfolio losses and lead to insufficient capital requirements.

In our baseline calibration, the introduction of the accelerator effects leads to an underestimation of the probability of default of a bank of a magnitude between 1.3 and 5.2 times, compared to the bank default probability under the Vasicek loss distribution. This means that the presence of reinforcement effects leads to substantially higher probability of bank default for the same level of capital or conversely if banks want to achieve a certain probability of solvency they would have to hold larger amounts of capital than previously thought.

While the results have important implications for bank capital regulation. They also provide insight into optimal risk management, provisioning and risk pricing by private banks that is consistent with the developing regulatory framework.

Taking into account the currently developing regulation, this paper contributes to the literature by being the first one to summarize bank capital requirements, risk cost provisioning and risk pricing in a single framework consistent with the current regulatory environment.

Chapter I

The Reach of Macroprudential Policy

Abstract

This paper compares different macroprudential tools in their ability to mitigate the impact of a financial crisis originating from the housing market. For this purpose, a financial frictions model is set-up featuring defaulting loans to both households and firms. While banks are subject to capital requirement at their wholesale level, each sector specific loan type is subject to a loan-to-value requirement.

We find that once the crisis has occurred and spread to the whole banking sector, a dynamic capital-asset requirement can attenuate the transmission of the crisis to the real economy. Although a dynamic LTV rule is not successful in the aftermath of the crisis, a lower LTV limit prior to its occurrence can limit the losses of mortgage lending before they spread to the whole banking system and the real economy. In terms of welfare analysis, we find that an optimised capital-asset rule is welfare improving, while the LTV setting is not.

JEL classifications: C68, E44, E58, E61, G21.

Keywords: macroprudential policy, banks, capital requirements, loan-to-value limits.

1 Introduction

The Great Recession which started with the burst of the US housing market bubble established financial stability at the forefront of policy discussions. Over optimism in the housing market lead to persistent increase in house prices which induced relaxed lending standards by banks and substantial increase in subprime mortgages that reinforced housing demand and prices. When the optimistic price expectations failed to materialize and the bubble burst, many households faced a situation in which their mortgages were larger than the value of the houses against which they were underwritten. As a result, the rate of seriously delinquent mortgages[1] increased from 2 percent in the third quarter of 2006 to 10 percent by the first quarter of 2010. As a consequence, banks experienced increasing mortgage default rates that led to higher collateral repossession rates with collateral values much lower than the ones expected when the loans were made. This ultimately lead to severe bank losses in the form of write downs of billions of dollars in bad mortgages. These losses together with the high degree of interconnectedness among financial institutions triggered severe liquidity crisis in the interbank market. Apart from the default of several banks, interbank lending rates increased which ultimately led to reduced access to credit by both households and firms. At this stage, the crisis that started from the mortgage market spread to the real sector where tightened credit conditions and falling house prices forced many borrowers to deleverage and cut consumption and housing purchases.

Although the story of the crisis is complex and involves more than one type of self-reinforcing mechanisms, at the core of the events above was a price bubble and extensive lending in a specific market of the economy. Furthermore, the period prior to the crisis and observed aftermath correspond to a strong empirical evidence of extensive lending and high exposure to risk in the upswing of the business cycle and a downturn characterized by high risk aversion and deleveraging.[2] These movements of leverage often identified as the "leverage cycle"[3] involve a well-known self-reinforcing channel between credit and asset prices. The collateral

[1] According to the National Delinquency Survey of the Mortgage Bankers Association, seriously delinquent mortgages are those more than ninety days past due or in foreclosure.
[2] See Kannan et al. (2011), Borio and Lowe (2004) and IMF (2009).
[3] See Geanakoplos (2010).

channel can be briefly described as an increase of individual's ability to borrow against collateral, following a rise in the collateral value. The increased ability to borrow further increases the asset demand and price, closing the loop of the channel. The risks of the cycle arise because after the credit expansion even a small decrease in the asset price or in the default rate of borrowers can lead to a reversal of the cycle and economy wide distress. "Equity buffers might then prove insufficient to absorb losses and banks may be forced to deleverage. The resulting collective contraction in the supply of credit increases the likelihood of borrower distress, potentially affecting the real economy adversely and amplifying banking sector losses further."(Bank of England, 2011). Moreover, the symptoms above appear to be in the core of many financial crises: "Excessive credit expansion, often in the real estate sector, has characterised the build- up to most financial crises in the past, from the Great Depression, to emerging market crises in Latin America and East Asia, to recent crises in developed countries."(Bank of England, 2011).

Perhaps this is why the crisis restated the debate over the detection of price bubbles, the vulnerability of the financial sector, and the potential policy tools that can lean against the financial cycles. Prior to the 2008 crisis, the general view was that responding to fluctuations in asset prices or other financial variables was potentially harmful due to the difficulty in detecting asset price bubbles in real time (Bernanke and Gertler, 2001). Very often the difficulties with the identification of a price bubble involve the "emergence of seemingly plausible fundamental arguments that seek to justify the dramatic rise in asset prices." (Gelain et al., 2013). Indeed, in a July 1, 2005 media interview, Ben Bernanke, argued that fundamental factors such as strong growth in jobs and incomes, low mortgage rates, demographics, and restricted supply were supporting U.S. house prices. In the same interview, Bernanke stated his view that a substantial nationwide decline in house prices was "a pretty unlikely possibility" (Jurgilas and Lansing, 2013).

However, a growing amount of empirical work has found that large movements in a number of observable variables such as credit, residential investment shares, and current account deficits are reliable indicators of house price busts, which in turn are typically associated with substantial falls in output.[1] These observations signal that perhaps a countercyclical policy that reacts to such indicators could mitigate the impact of the crisis or even prevent its occurrence. But what is the right policy tool for this purpose?

The question of whether or not monetary policy should respond to financial indicators such as asset prices has been numerously investigated and often received opposite answers.[2] There are two main criticisms of this approach. The first one involves the "broadness" of monetary policy, i.e. the concern that MP is too blunt to address imbalances within the financial sector or overheating in a single sector of the economy (housing market).[3] The second caveat is the violation of the Tinbergen principle, stating that: "for each policy objective, at least one policy instrument is needed" (Tinbergen, 1952), i.e. that having a single policy tool to be responsible for more than one objective could lead to policy conflicts. In practise, policymakers have rarely used MP for reacting to asset prices. As an exceptional example, could be seen the case of Sweeden which in 2010 raised its interest rate due to the concern of rapidly raising house prices, at the same time inflation was already low which meant that the Riskbank was not responding to its main objective. The impact of the policy was the triggering of deflation which had bad consequences for the economy and for which the bank was heavily criticised.

Apart from the use of traditional policies, the nature of the Great Recession has led major countries to carry out reforms in the financial regulatory bodies. These reforms gave regulators mandate over financial stability with specific emphasis on systemic risk by using a set of Macroprudential tools. Amongst various definitions, the main objective of macroprudential policy can be summarized as to limit the accumulation of financial risks, in order to reduce the probability and to mitigate the impact of a potential financial crash and to strengthen the resilience of the financial system. Thus, purely by its objective, macroprudential policy should be the right approach to lean against the credit cycles. However, because of their recent availability the theoretical research which should suggest the appropriate use of these new tools seems not yet developed: "Analysis of the

[1] See Kannan et al. (2011), Borio and Lowe (2004) and IMF (2009).

[2] See Bernanke and Gertler (2001), Gilchrist and Leahy (2002), Iacoviello (2005), Kannan et al. (2009).

[3] See Quint and Rabanal (2014), Kohn (2013).

proposals on macroprudential policies has generally lacked the sort of consistent framework that would allow a structured approach. As a result, the process of institutional reform is well ahead of its theoretical and practical underpinning, and faces important challenges." Angelini et al. (2014). Moreover, due to the large set of macroprudential tools, the different propagation mechanisms of the different instruments are even less researched. Thus, most of the existing papers that feature such policy in a general equilibrium framework, focus on a single macroprudential instrument.

Following the lessons of the Great Recession, and the urge for macroprudential policy research by financial regulators, this paper aims to contribute to the debate over the right policy tool to lean against the credit cycle by investigating the role of different types of macroprudential instruments. The motivation of this approach arises from the understanding that different policy tools have different broadness of impact, depending on the level at which they enter the economy. An example of this can be the central bank interest rate as a monetary policy tool which affects the savings and borrowing returns/costs of the entire economy. On the other hand, a macroprudential instrument such as the capital-asset requirement, enters the economy at the level of the banking sector and hence is a more sophisticated tool for responding to financial distress. This understanding is what usually supports the claim that monetary policy is too blunt to address imbalances within the financial sector or overheating in a single sector of the economy.

Taking the same concept further, we investigate whether some macroprudential instruments can be too blunt to address problems in a specific financial sector while others are more appropriate for this purpose. For example, as the capital-asset requirement enters the economy at the bank level, a tighter requirement will most likely tighten lending for all types of loans. On the other hand, a loan-to-value requirement restricts the maximum borrowing amount for a given collateral value and thus can be imposed on a specific type of lending such as mortgages. Therefore, in a scenario of a housing market overheating, similar to the one in Sweden, while monetary policy can decrease house prices it can also deflate the entire economy. Alternatively, capital-asset requirements can lead to higher borrowing costs thus reducing mortgage lending, but at the same time reducing lending to firms. Lastly, a tighter LTV limit can be imposed only on mortgages and thus have even more sophisticated and direct effect.

Investigating the reach of different policies, the main question of this paper is the following: If monetary policy is too blunt to address financial sector overheating then are certain macroprudential instruments too blunt to address a single sector overheating? Or more specifically: If mortgages lending is increasing rapidly should we use capital asset requirement which has the risk of affecting lending to firms, or instead use a LTV requirement to mortgages specifically?

For answering this question a new Keynesian DSGE model is employed. As we will see in the model section, the possibility to represent different levels of impact of macroprudential policies requires a detailed banking sector with two types of lending. The banking sector is set-up extending defaulting loans to both households and firms against housing and capital collateral. In addition, while banks are subject to capital requirement at their wholesale level, each sector specific loan type is subject to a LTV requirement. Apart from the model structure, of equal importance is the origin of the financial distress that the policies will aim to address. It is common in the literature that the crisis is represented as an exogenous destruction of bank capital without modelling defaulting loans and hence the source of destruction of bank capital. To enable a better representation of bank crises and their origins our model employs defaulting loans set-up similar to Quint and Rabanal (2014). We then simulate the main crisis scenario as an exogenous increase in mortgage delinquencies which leads to a larger than the expected default rate of mortgages and bank losses that are absorbed by bank capital.

In order to investigate how the two macroprudential instruments affect the transmission of the crisis, we first analyse the effects of the latter in three cases of static requirements including a benchmark and permanently tighter capital-asset and LTV requirement cases. Then we analyse the ability of dynamic policy rules of the two instruments to improve welfare under various shocks.

Thus, apart from investigating whether an active setting of the policy instruments can mitigate the consequence of the crisis scenario, we can also investigate if a permanently tighter policies can reduce the impact of the crisis before its occurrence.

Our results show that although a tighter LTV limit prior to the crisis reduces the exposure of the banking sector to higher loan defaults, it does so at the cost of lower output. At the same time, a permanently tighter capital-asset requirement seems incapable of mitigating the impact of the financial distress. However, once we consider optimal dynamic policy setting in terms of social welfare, we find that an optimized capital-asset requirement can be successful in improving welfare under various shocks and attenuate the transmission of the crisis to the real sector in its aftermath. Apart from being less successful in improving welfare, the dynamic LTV limit is also incapable of reducing the impact of the shock after its occurrence.

The rest of the paper is organized as follows. Section 2 discusses the related literature and section 3 presents the model and its basic relationships. Section 4 compares the effects of permanently tighter static policies and then conducts welfare analysis of dynamic policy rules. The final section concludes.

2 Related Literature

The importance of the financial sector in economic models and its ability to amplify the business cycle has been well known since the seminal Bernanke et al. (1999) "financial accelerator" - BGG paper, featuring defaulting loans. An alternative approach to modelling financial frictions is the Kiyotaki and Moore (1997) paper which instead of modelling defaulting loans explicitly, introduces a collateral constraint that represents the relationship between collateral value and maximum borrowing amount. The collateral constraint concept is then incorporated by Iacoviello (2005) where the collateral is housing which also enters the utility function of households. The latter is also one of the papers that analyses the question if monetary policy should respond to house prices finding only marginal and somewhat insignificant benefits of such policy setting. A similar result of interest rates responding to asset prices is also obtained by Bernanke and Gertler (2001) and Gilchrist and Leahy (2002).

Kannan et al. (2009), however, find that optimal monetary policy includes reaction to asset prices or credit, this is also one of the first papers to include macroprudential policy represented by a capital-asset requirement to banks, finding that response of the instrument to credit growth can improve welfare.

Another closely related paper which analyses macroprudential policy is Quint and Rabanal (2014) in which the authors use a BGG type of model featuring defaulting loans and also extend their analysis to the open economy. The key departure from Bernanke et al. (1999) is that the lending rate is predetermined which makes lenders exposed to gains/losses from unexpected changes in collateral prices or default rates. This feature is also employed in our model allowing the transmission of unexpected occurrences in a specific type of lending to the whole banking sector. The authors find that macroprudential policy using capital-asset requirement has the potential to stabilize the economy above the reach of monetary policy depending on the type of shock hitting the economy.

A similar paper featuring defaulting loans is the one by Forlati and Lambertini (2011) who analyse how aggregate shocks affect the rate of default on mortgages and how an increase in the rate of default on mortgages transmits to the rest of the economy, emphasizing the role of this relationship in the core of the Great

Recession.

In terms of structure of the banking sector, our model stands close to Gerali et al. (2010) who employ a two-layer banking sector. The wholesale layer maintains the capital position of the banking sector subject to a capital-asset requirement and extends loans to the retail branches. At the retail level, each branch carries out specific type of lending to households and firms against housing and capital collateral.

Angelini et al. (2014) study the interaction between capital requirements and monetary policy using the Gerali et al. (2010) model. The operational objectives of macroprudential policy are introduced through an assumed loss function which the policy rule aims to minimize. The authors find that the capital requirement setting becomes beneficial when the economy is hit by financial rather than supply shocks while the lack of cooperation between the two policies could lead to excessive volatility of the instruments.

Lambertini et al. (2013) is one of the fewer papers that focus on LTV requirement as a macroprudential instrument in a model with expectations-driven business cycles. They find that by itself, monetary policy response to credit aggregates improves welfare of all agents. On the other hand, when implementing LTV policy, the optimal setting for borrowers and savers differ.

In a paper not employing general equilibrium analysis, Geanakoplos (2010) illustrates how the equilibrium supply and demand of credit can determine the collateral level (the reciprocal of the LTV) together with the interest rate. The author argues that while interest rates have been viewed as the main variable affecting borrowing, it is the collateral level required by lenders, which is of equal if not greater importance (especially in times of crisis) than the cost of borrowing. Furthermore, the author claims that if policymakers want to temper the devastating consequences in the downturn of the leverage cycle it is the collateral level that they should be managing and not the interest rate.

From the above papers, ours stands closest to Gerali et al. (2010) regarding the structure of the banking sector. The main and crucial departure, however, is that while in the former, the individual loan spreads arise due to interest rates stickiness in our model they are present due to defaulting loans and the specific expected default rate of each type of loan. Furthermore, we employ predetermined interest rates which make lenders exposed to gains/losses from unexpected changes in collateral prices or default rates, allowing the transmission of unexpected occurrences in a specific type of lending to the whole banking sector.

As we will see, this structure allows for modelling of the crisis as originating from the mortgage market, transmitting to the banking sector and then the rest of the economy. Moreover, it allows for analysis of policy tools that enter the economy at a different level of the transmission path of the crisis and assess their ability to mitigate its impact. This to the best of our knowledge, makes the paper one of the first to compare capital-asset and LTV requirements in model in which bank capital is destroyed endogenously due to loan defaults. The next section explains the model structure in detail.

3 The Model

3.1 Banks

The description starts from the banking sector as it is the one that includes the most important features of the model. That is, a wholesale branch that obtains deposits at the policy rate and then lends funds to retail branches as in Gerali et al. (2010). The spread between the wholesale lending rate and deposit rate arises due to the fact that the wholesale branch also manages the capital position of the banking sector, while facing a quadratic cost for deviating from a capital-asset macroprudential requirement. In addition, bank capital is accumulated out of retained earnings and is used together with deposits to fund loans. At the retail branch level, there are two branches that extend funds to households and firms against housing and capital collateral respectively. In doing so, they use a participation constraint as in Quint and Rabanal (2014) and require that the expected average repossession and repayment amount of a loan equals the cost of funds from the wholesale branch. Due to predetermined lending rates, retail bankers have zero ex-ante profits, but it is possible that ex-post they may realize profits/losses which are added/deducted from the wholesale bank capital. In addition, retail branches face a macroprudential LTV limit which defines the maximum amount they can lend for a given collateral value. The mechanisms which are created by the chosen model structure are analysed in the process.

3.2 The wholesale branch

The wholesale branch collects deposits D at the gross policy rate R which together with the accumulated bank capital K^b is used to fund its loans B, leading to a balance sheet identity:

$$B_t = D_t + K_t^b \tag{3.1}$$

where the two sources of funding are perfect substitutes from the point of view of the balance sheet. Bank capital is accumulated out of retained earnings:

$$K_t^b = (1 - \delta^b)K_{t-1}^b + \Pi_t \tag{3.2}$$

where δ^b represents resources used up in managing bank capital and Π_t is the realized overall profits of all branches, including wholesale and the two retail branches profits:

$$\Pi_t = \Pi_t^{ws} + \Pi_t^h + \Pi_t^f \tag{3.3}$$

The loans B_t are extended to the two retail branches at a gross interest rate of R^b. The wholesale branch maximises profits taking into account a quadratic cost whenever the capital-asset ratio K_t^b/B_t deviates from an exogenous level ν_t which represents regulatory capital requirement.[1] Thus, the wholesale branch maximizes:

$$maxE_0 \sum_{i=0}^{\infty} \Lambda_{0,t} \Big[(R_t^b - 1)B_t - (R_t - 1)D_t - \frac{\kappa_b}{2}\Big(\frac{K_t^b}{B_t} - \nu_b\Big)^2 K_t^b\Big] \tag{3.4}$$

by taking R_t^b and R_t as given and choosing D_t and B_t, subject to the balance sheet identity, leading to the following FOC:

[1] "The optimal leverage ratio in this context can be thought of as capturing the trade-offs that would arise in the decision of how much own resources to hold, or alternatively as a simple shortcut for studying the implications and costs of regulatory capital requirements." (Gerali et al., 2010)

$$R_t^b - R_t = \kappa_b \Big(\nu_b - \frac{K_t^b}{B_t}\Big)\Big(\frac{K_t^b}{B_t}\Big)^2 \tag{3.5}$$

which links the wholesale spread to the degree of leverage B_t/K_t^b, showing the role of bank capital in determining loan supply conditions.

The LHS of the above equation represents the marginal benefit from increasing lending (an increase in profits equal to the spread), while the RHS is the marginal cost from doing so (an increase in the costs for deviating from ν_b). Therefore, the wholesale branch chooses a level of loans which, at the margin, equalizes costs and benefits of reducing the capital-assets ratio. In addition, equation (3.5) creates a positive relationship between the wholesale spread and leverage. If we consider a lower capital-asset ratio than the requirement $\frac{K_t^b}{B_t} < \nu_b$, it would be associated with a higher amount of loans B_t for a given level of bank capital K_t^b and increased costs for deviating from ν_b, hence a positive RHS of the equation above. The branch would be willing to keep that level of B_t only as long as its marginal profit increases by the same amount as the marginal cost, leading to positive LHS and wholesale spread.

Equations (3.1) to (3.5) complete the wholesale branch. It is worth noting at this stage the mechanisms which are incorporated with the equations so far. Since bank capital is accumulated out of retained earnings, the model features a feedback from the real to the financial side of the economy. If macroeconomic conditions deteriorate and bank profits are negatively hit, this would also decrease bank capital and hence, decrease the capital-asset ratio bellow the target, which in turn would lead to higher lending rates and reduced lending that can further worsen economic conditions. "The model can thus potentially account for the type of "credit cycle" typically observed in recent recession episodes, with a weakening real economy, a reduction of bank profits, a weakening of banks' capital position and the ensuing credit restriction." (Gerali et al., 2010).

So far, the banking sector up to the wholesale branch is identical to Gerali et al. (2010), in the next section, however, we will see that incorporating Quint and Rabanal (2014) type of spreads for the two retail branches, introduces the possibility for ex-post realized profits/losses of each branch based on unexpected collateral price movements and leading to a loan default rate different than the one expected by retail branch. This would enable unexpected events in the markets of the goods used as collateral to be passed from retail branch profits to aggregate bank profits and to the capital position of the banking sector, which is very similar to the role of the housing market at the beginning of the Great Recession.

This is a cruicial departure from Gerali et al. (2010) as in the latter the main crisis scenario is represented by an exogenous destruction of bank capital at the wholesale level. Our framework, however, allows for endogenous bank capital destruction which originates from the individual loan markets due to larger than expected loan default rate.

At this point it is also possible to note the different effect of monetary and macroprudential capital-asset requirement. While restrictive monetary policy will simply lead to an increase in the wholesale lending rate R_t^b through equation (3.5), the capital-asset requirement can actually affect the spread between the two rates.

3.3 The retail branches

The retail branches face endogenous loan defaults due to idiosyncratic shock to the collateral value. This feature creates a good motivation for the existence and operation of the retail branches. Since, when issuing loans to households and firms, each branch takes into account the expected return in the events of default and non-default of the loan. We can say that unlike the wholesale branch, each retail branch has the specific expertise for each type of lending in evaluating the relevant collateral and its expected price.

The operation of the two branches is identical with the difference that one of them extends loans to impatient households against housing collateral and the other to firms/entrepreneurs against capital collateral. Hence, if we denote lending to household variables with I superscript and to firms with E, it would be the only difference between the equations describing the lending by each branch, that is we can express them in general form using $j = \{I, E\}$ superscript. The exceptions from this notation is that loans to households are against housing collateral so that: $h_t^I \equiv H_t^I$ at price $q_t^I \equiv q_t^h$, and loans to firms are against capital collateral so that: $h_t^E \equiv K_t^E$ at price $q_t^E \equiv q_t^k$. Ex-ante expected and ex-post realized variables are denoted with a and p superscripts respectively.

The interest rate spread derivation is based on the original BGG financial accelerator and more specifically on the Quint and Rabanal (2014) version of it which has the main differences that:

- there are no agency problems or asymmetric information in the model
- borrowers will only default if they find themselves underwater (if the collateral value is less than the loan repayment amount)
- the one-period lending rate is predetermined and does not depend on the state of the economy

The predetermined interest rate is a realistic assumption due to the fact that many of the loans (mortgages) are with fixed rates.[1] An additional feature that is common in the literature is that loan contracts are set in nominal terms and thus higher inflation would deflate the real amount that borrowers repay.

An important feature of the BGG accelerator is the presence of loan default risk due to the idiosyncratic shock ω_t to the value of the collateral. The shock is log-normally distributed with CDF $F(\omega)$, PDF $f(\omega)$ and mean $E(\omega_t) = 1$,[2] so that there is idiosyncratic but not aggregate risk due to its presence. Because of the specified default condition of borrowers (if underwater), the shock can lead to mortgage defaults with its realization being known at the end of the period. At period t, high enough realizations of ω_{t-1} will induce the borrower to repay his loan in full: $r^j_{t-1}B^j_{t-1}/\pi_t$, where r^j is the gross borrowing rate and B^j the quantity borrowed. Low enough realizations will cause the borrower to default and give up his collateral after the realization of the shock: $\omega^j_{t-1}q^j_t h^j_{t-1}$, where q^j is the collateral price and h^j is its stock. Thus the default condition for borrowers becomes, repay loan if: $r^j_{t-1}B^j_{t-1}/\pi_t \leq \omega^j_{t-1}q^j_t h^j_{t-1}$ and default in the opposite case (being underwater).

Then in period t, the cut-off value of $\bar{\omega}^j_{t-1}$, i.e. the ex-post realized threshold value $\bar{\omega}^p_t$ that separates borrowers that default and those that do not can be expressed as: $\bar{\omega}^j_{t-1} \equiv \bar{\omega}^p_t = \frac{r^j_{t-1}B^j_{t-1}}{q^j_t h^j_{t-1}\pi_t}$.

[1]See Quint and Rabanal (2014).

[2]This implies that the log of ω is normally distributed: $log(\omega_t) \sim N(\frac{-\sigma^2_\omega}{2}, \sigma^2_\omega)$.

At period t, the retail branch extends loans at a fixed rate r_t^j without knowing the exact value of the default threshold since it will also depend on period $t+1$ collateral price q_{t+1}^j and next period inflation, hence, it forms its ex-ante expected threshold $\bar{\omega}_t^{j,a}$ value as:

$$\bar{\omega}_t^{j,a} = \frac{r_t^j B_t^j}{E(\pi_{t+1} q_{t+1}^j) h_t^j} \tag{3.6}$$

which is also the expected LTV ratio by the retail branch for loan type j.

Unlike the wholesale branch, when granting credit, retail branches do not maximize profits[1] but simply require that the expected return from a unit of credit equals the cost of funds (the rate at which the funds are obtained from the wholesale branch rate R^b) , leading to the participation constraint:

$$R_t^b = (1-\mu) G(\bar{\omega}_t^{j,a}, \sigma_\omega^j) \frac{E_t(\pi_{t+1} q_{t+1}^j) h_t^j}{B_t^j} + (1 - F(\bar{\omega}_t^{j,a}, \sigma_\omega^j)) r_t^j \tag{3.7}$$

where the two terms on the RHS of the constraint are respectively the expected return in the case of default as the repossessed collateral and the expected return in the case of non-default, as the repayment of the loan. The term $G(\bar{\omega}_t^{j,a}, \sigma_\omega^j) = \int_0^{\bar{\omega}_t^{j,a}} \omega dF(\omega, \sigma_\omega^j)$ is the expected value of the shock, conditional on the shock being less than $\bar{\omega}_t^{j,a}$; and $1 - F(\bar{\omega}_t^{j,a}, \sigma_\omega^j) = \int_{\bar{\omega}_t^{j,a}}^{\infty} f(\omega, \sigma_\omega^j) d\omega$ is the probability that the shock exceeds the ex-ante threshold $\bar{\omega}_t^{j,a}$, i.e. the probability of non-default.[2] Banks can repossess only $1-\mu$ of the collateral as the remainder is paid as a collateral repossession costs which are then transferred to savers who own these repossession agencies.

It is important to note that when granting credit and determining its rate r_t^j, the retail branch is concerned about the future value of the collateral as in the case of default it is the one that will be repossessed. In addition, the formula above leads to the possibility that although the branch has zero expected ex-ante profits, ex-post it may realize profit or loss for a given unexpected collateral price change, as we will see later.

[1] Although the retail branches do not maximize profits, since we consider each bank as composed of one wholesale and two retail branches we can say that each bank operates under monopolistic competition with profit maximization occurring at the wholesale level.

[2] For a shorthand the G and F ex-ante and ex-post functions of loan type j can be denoted as: $G_t^{a,j}$, $G_t^{p,j}$, $F_t^{a,j}$, $F_t^{a,j}$

The next equation is the collateral constraint following Iacoviello (2005) which restricts the maximum amount that the agent can borrow (principal plus interest) for a given expected future collateral value q^j_{t+1}:

$$\frac{r^j_t B^j_t}{\pi_{t+1}} = m^j_t q^j_{t+1} h^j_t \tag{3.8}$$

where, m^j_t is the macroprudential LTV requirement. The intuition behind this constraint however differs from its standard interpretation in the literature.

Firstly, Iacoviello (2005) motivates the constraint as due to the lender's ability to reposes only m^j_t of the collateral value in the event of default, paying the remaining $(1 - m_t)$ of it as transaction costs. Hence, the lender would not lend a higher amount of funds than m^j_t. However, in our model the amount repossessed by banks in the event of default is $1-\mu$ of the collateral and is not necessarily equal to m_t. Instead, when adding the collateral constraint to the BGG interest rate spread, we can think of it as macroprudential policy rule imposed by policymakers and thus m_t is the LTV limit - the maximum amount that the agent can borrow for a given future collateral value.

In other words, in the original motivation of the collateral constraint the LTV limit m^j_t serves as a proxy of default risk when loans defaults are modelled implicitly. Hence, once we model loan defaults explicitly, the original form of the constraint loses its original meaning. While in the Iacoviello scenario, the lender would not lend more than m^j_t due to some default risk and repossession costs, our participation constraint tells us that the lender would be willing to lend more in return for a higher lending rate. Instead what limits borrowing in our case is exogenous macroprudential requirement in the form of a borrowing limit.[1]

It is assumed that due to the high impatience to consume, hence incentive to borrow by borrowers, the collateral constraint will always bind and hold with equality.

[1]The form of the constraint in 3.8 involves the assumption that when restricting borrowing for a future collateral value, regulators are able to form rational expectations of the latter. As alternative, we relaxed this assumption by using a current value constraint. This however lead to instability unless the m^j_t is set as to take into account future price movements, which is essentially equivalent to the future form of the constraint.

3.4 Equations Summary

Before continuing with the rest of the model it would be beneficial to summarize the relationship behind the above lending structure. Firstly, the ex-ante expected default threshold (3.6) and the participation constraint (3.7) create a relationship for the interest rate spread for the j loan type wrt. the wholesale lending rate in the form:

$$\frac{r_t^j}{R_t^b} = \frac{1}{\frac{(1-\mu)G(\bar{\omega}_t^{j,a},\sigma_\omega^j)}{\bar{\omega}_t^{j,a}} + (1 - F(\bar{\omega}_t^{j,a}, \sigma_\omega^j))} \tag{3.9}$$

Then, due to the properties of the log-normal distribution when $E_t(\omega) = 1$, it can be shown that the denominator of the RHS is a decreasing function of the ex-ante threshold $\bar{\omega}_t^{j,a}$ and hence, the spread becomes an increasing function of it.[1]

$$\frac{r_t^j}{R_t^b} = f(\bar{\omega}_t^{j,a}) \tag{3.10}$$

where $f'(\omega_t^a) > 0$. The intuition behind this relationship is that for a larger expected LTV ratio (RHS of equation (3.6)) due to the default condition of borrowers (if underwater), a larger proportion of them is expected to default, and hence the ex-ante threshold increases (LHS of equation (3.6)). Then since the threshold separates the defaulting from non-defaulting loans, the bank would expect a larger default area and smaller non-default area given by $(1-F(\bar{\omega_t^a}, \sigma_\omega))$. In order to compensate for the larger expected defaults, the retail branches increase the loan rate r_t^j, which is their payoff in the non-default case.

In addition, the collateral constraint fixes the expected ex-ante threshold at the level of the LTV limit:

$$\bar{\omega}_t^{j,a} = m_t^j \tag{3.11}$$

hence a higher LTV requirement (increased amount of borrowing for a given expected future collateral value - looser policy) would cause the expected default threshold to increase. The intuition here is that since the collateral constraint is

[1]See appendix 6.1

always binding, the borrowers would always borrow up to the maximum amount allowed by the LTV limit m_t^j. Hence a larger m_t^j would increase the amount of borrowing b_t^j for a given future expected collateral value q_t^j. Everything else equal, the larger borrowing would lead to higher expected default threshold as larger proportion of the possible ω realizations would lead to borrowers being under water and default.

Moreover, from equation (3.10) it follows that this relationship will be passed on to the spread, leading to:

$$\frac{r_t^j}{R_t^b} = f\left(m_t^j\right), \; f'\left(m_t^j\right) > 0 \tag{3.12}$$

In summary, a higher LTV limit (looser macroprudential policy) would cause the expected default threshold to increase, leading to an increased expected default area to which the retail branches will respond by increasing their spread.

This also means that by being able to set m_t^j the macroprudential regulator has perfect control over the loan type j spread from the wholesale lending rate R_t^b. Thus, just like the capital-asset requirement has control over the spread between deposits and wholesale loans $R_t^b - R_t$, the LTV requirement of loan type j has control over the spread between the final loan rate r_t^j and the wholesale one R_t^b.

It is important to emphasize that unlike the capital-asset requirement which affects directly only the wholesale spread, the LTV limit is a borrowing constraint which affects directly borrowers' consumption and housing/capital investment decisions. Interestingly, if we focus solely on the effect of the LTV on retail spreads, the negative relationship implies that if the regulators want to increase the spread they have to allow banks to lend at higher LTV. Our experiments show that the effect of the LTV constraint on households is much stronger than the one on the spread. Hence a tighter LTV would reduce borrowing although it would also lead to lower spread. In that case, we may think of the relationship of equation (3.12) as illustrating that apart from restricting lending to households, a lower LTV would also cause a lower spread.

In terms of broadness of the two macroprudential instruments, we can now see that, a change in the capital-asset requirement, ceteris paribus, would lead to a change of both loan rates, while a change in the LTV requirement of loan type j,

m_t^j can target and affect only its lending rate r_t^j and the borrowing constraint of agent type j.

In period t, as current collateral prices q_t^j become known, the ex-post realized default threshold $\bar{\omega}_t^{j,p}$ becomes known as well:

$$\bar{\omega}_t^{j,p} = \frac{r_{t-1}^j B_{t-1}^j}{\pi_t q_t^j h_{t-1}^j} \tag{3.13}$$

As long as the expectations of the present collateral price from the last period are equal to its present price $E_{t-1}(q_t^j) = q_t^j$, from equations (3.6) and (3.13), it follows that the ex-ante expected threshold from the last period will be equal to the realized ex-post threshold of the current period $\bar{\omega}_{t-1}^{j,a} = \bar{\omega}_t^{j,p}$, leading to equal ex-ante and ex-post G and F functions and zero profits guaranteed by the participation constraint. In other words, as long as the collateral prices are the ones expected, the realized threshold separating the defaulting from non-defaulting loans would be the one expected in the previous period by the bank when setting its lending rate. And in turn, the realized loan default rate and value of repossessed collateral will also be equal to the ones expected in the previous period. For any unexpected collateral price change the equality between all ex-ante variables from last period and the realized ex-post variables from the current period would not hold and the branch j profits will be given by:

$$\Pi_t^j = (1-\mu)G(\bar{\omega}_t^{j,p}, \sigma_{\omega^j}^j) q_t^j h_{t-1}^j \pi_t + (1 - F(\bar{\omega}_t^{j,p}, \sigma_{\omega^j}^j)) r_{t-1}^j B_{t-1}^j - R_{t-1}^b B_{t-1}^j \tag{3.14}$$

calculated as the average repossession value of collateral for the defaulted loans, plus the loan repayment of non-defaulted loans, minus the cost of funds for the bank. In fact, it can be shown that the profits of each branch are a function of the difference between last period's ex-ante expected and current period's ex-post realized thresholds,[1] such that when the two thresholds are equal, profits are zero and when the ex-post is smaller (smaller proportion of loans default than expected) profits are positive:

[1]See appendix 6.2

$$\Pi_t^j = f(\bar{\omega}_{t-1}^{j,a} - \bar{\omega}_t^{j,p}),\ f'() > 0 \tag{3.15}$$

Retail branches' profits/losses are then added/subtracted to the wholesale bank capital which in turn affects the wholesale lending rate. This creates a propagation mechanism from unexpected collateral price movements in a specific loan type to the capital position of the whole banking sector. The latter also allows for spillover effects between the two loan markets. For example, if house prices decline and increase household defaults, the household lending branch would realize losses which would decrease the wholesale bank capital. This in turn would cause the wholesale capital-asset ratio to deviate from the requirement for which the wholesale branch would respond by increasing its lending rate through equation (3.5) that will ultimately increase the borrowing costs for firms.

Moreover, by using equation (3.11) for $\omega_{t-1}^{j,a}$ then expressing similarly $\omega_t^{j,p}$ it can be shown that bank profits depend on the LTV limit:[1]

$$\Pi_t^j = f\left(m_{t-1}^j\left(\frac{q_t^j - E_{t-1}q_t^j}{q_t^j}\right)\right),\ f'() > 0 \tag{3.16}$$

Therefore, for a given lower collateral price than expected $q_t^j < E_{t-1}q_t^j$ and hence larger default threshold, the loss that the retail branch would suffer would be higher for a higher LTV requirement. This could be explained with the fact that while profits can be represented as the difference between the expected default threshold and the realized one, the LTV requirement has the ability to limit both thresholds and hence the difference between expectations and reality in the event of a shock. Moreover, this feature shows that apart from being able to affect loan specific spreads and borrowing constraint, the LTV requirement can also affect the exposure of a retail branch to unexpected events in the collateral market. We will see later in the simulations section that the capital-asset requirement does not have this feature.

[1] In fact, differences between expected and realized inflation also enters (3.16) and can lead to non-zero profits. In addition, changes of the standard deviation of the idiosyncratic shock can also lead to losses due to higher than the expected defaults.

3.5 Savers

Each saver (or patient household) i maximizes expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} (\beta^P)^t \left[(1-\alpha^p)\varepsilon_t^c \, log(C_t^P(i) - \alpha^P C_{t-1}^P) + \varepsilon_t^h \, log(H_t^P(i)) - \frac{(L_t^P(i))^{1+\phi}}{1+\phi} \right] \quad (3.17)$$

which depends on current individual (and lagged aggregate) consumption C_t^P, housing H_t^P and hours worked L_t^P. Pre-multiplying by the habit coefficient α^P offsets the impact of the external habits on the steady-state marginal utility of consumption. The last term is labour disutility where ϕ denotes the inverse elasticity of labour supply. There are two preference shocks ε_t^c affecting the marginal utility of consumption, and ε_t^h the marginal utility of housing.

Budget constraint in real terms:

$$C_t^P(i) + q_t^h \Delta H_t^P(i) + D_t(i) = W_t^P L_t^P(i) + \frac{R_{t-1} D_{t-1}(i)}{\pi_t} + T_t(i) \quad (3.18)$$

which includes current consumption, accumulation of housing (with real house price q_t^h), and real deposits D_t. The income side consist of wage earnings $W_t L_t^P$ (where W_t is the real wage), gross interest income from last period deposits $R_{t-1}D_{t-1}/\pi_t$ (where $\pi_t = P_t/P_{t-1}$ is gross inflation and R_{t-1} is the gross interest rate on deposits), and transfers T_t which include profits from intermediate goods producers, and from debt repossession agencies.

3.6 Borrowers

Borrowers (or impatient households) differ from savers in several aspects. Firstly, their discount factor is smaller than the one of savers ($\beta^I < \beta^P$) which means that they are more impatient to consume. In particular, due to their impatience, in equilibrium, savers are willing to accumulate assets as deposits, and borrowers are willing to offer their housing wealth as collateral to obtain credit. Second, they don't earn profits from goods producers and debt repossession agencies. Lastly, they are subject to a quality ω_j shock to the value of their housing stock which leads to loan default for some of them (explained in detail in the banking sector).

Analogously from savers each borrower i, maximizes expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} (\beta^I)^t \left[(1-\alpha^I)\varepsilon_t^c \, log(C_t^I(i) - \alpha^I C_{t-1}^I) + \varepsilon_t^h \, log(H_t^I(i)) - \frac{(L_t^I(i))^{1+\phi}}{1+\phi} \right] \quad (3.19)$$

where all variables and parameters with the superscript I denote that they are specific to borrowers.

The budget constraint for borrowers differs among those who default and those who repay their loans in full. Hence, aggregating borrowers' budget constraints and dropping the i superscripts, we obtain the following:

$$C_t^I + q_t^h \Delta H_t^I + q_t^h H_{t-1}^I G(\bar{\omega}_t^{p,I}, \sigma_\omega^I) + \frac{(1 - F(\bar{\omega}_t^{p,I}, \sigma_\omega^I)) r_{t-1}^I B_{t-1}^I}{\pi_t} = B_t^I + W_t^I L_t^I \quad (3.20)$$

$$\bar{\omega}_t^{p,I} = \frac{r_{t-1}^I B_{t-1}^I}{q_t^h H_{t-1}^I} \quad (3.21)$$

where the last to terms on the LHS are average repossessed value of collateral of those who default, and repayment of credit of those who don't default.[1] The latter terms are calculated using the ex-post realized threshold separating defaulting from non-defaulting households $\bar{\omega}_t^{p,I}$. When obtaining mortgages, borrowers are subject to a collateral constraint which is imposed by the macroprudential authority and limits the amount of funds they can borrow for a given future expected collateral value according to a LTV requirement:

$$\frac{r_t^I B_t^I}{\pi_{t+1}} \leq m_t^h q_{t+1}^h H_t^I \quad (3.22)$$

The constraint does not have the purpose to ensure that expected value of housing stock can guarantee repayment of debt and interest. In our case the

[1] Since those terms arise from the aggregated budget constraint and not from the individual one, we assume that the individual agent does not take into account the probability of not repaying the loan tomorrow when borrowing today. Similarly we assume that the agent does not consider the probability to default tomorrow when choosing collateral stock today. This is represented by not differentiating those terms wrt. B^I and H^I in the FOC. A similar assumption is made for entrepreneurs. See appendix 6.3

repayment is guaranteed by the retail banks participation constraint. Thus, the purpose of our constraint is solely regulatory limitation on borrowing.

3.7 Enterpreneurs

Or firm (i) is concerned only about deviations of his own consumption $C_t^E(i)$ from aggregate lagged group habits (parameterized by α^E) and maximizes the sum of utility:

$$E_0 \sum_{t=0}^{\infty} (\beta^I)^t \Big[(1-\alpha^E) log(C_t^E(i) - \alpha^E C_{t-1}^E)\Big] \tag{3.23}$$

by choosing consumption C_t^E, physical capital K_t^E, loans from banks B_t^E, degree of capital utilization, and labour inputs from patient and impatient households L_t^P, L_t^I. Just like borrower households, entrepreneurs also have higher impatience to consume which makes them net borrowers willing to pledge the capital used for production as a collateral. Since their budget constraint differs between those that default and those that do not, by aggregating over them and dropping the individual subscript we obtain the following budget constraint:

$$\begin{aligned} &C_t^E + W_t^P L_t^P + W_t^I L_t^I + \frac{(1 - F(\bar{\omega}_t^{E,p}, \sigma_{\omega^E}^E)) r_{t-1}^E B_{t-1}^E}{\pi_t} + \\ &q_t^k [K_t^E - (1-\delta) K_{t-1}^E] + q_t^k K_{t-1}^E G(\bar{\omega}_t^{E,p}, \sigma_{\omega^E}^E) = \frac{Y_t^E}{X_t} + B_t^E \end{aligned} \tag{3.24}$$

where δ is the depreciation rate of capital, q_t^k is the price of capital and $P_t^W / P_t = 1/X_t$ is the relative competitive price of wholesale good Y_t^E produced according to the technology.

$$Y_t^E = A_t^E {K_{t-1}^E}^{\alpha} {L_t^E}^{1-\alpha} \tag{3.25}$$

with A_t^E being stochastic productivity shock. Aggregate labour L_t^E is combined from the labour of both types of households: $L_t^E = (L_t^{E,P})^{\nu} (L_t^{E,I})^{1-\nu}$ where ν measures the labour income share of patient households.[1]

Due to their high impatience, entrepreneurs also become net borrowers, how-

[1] See Iacoviello and Neri (2010).

ever, they use capital as a collateral. Hence the LTV limit imposed on firm loans by the macroprudential authority restricts the borrowed amount for a given expected future capital collateral value:

$$\frac{r_t^E B_t^E}{\pi_{t+1}} \leq m_t^f q_{t+1}^k K_t^E \tag{3.26}$$

where m_t^f is the LTV limit set on loans to firms.

3.8 Capital Producers

Capital producers are simply a modelling device used to derive the price of capital. Being perfectly competitive, they buy last-period undepreciated capital $(1-\delta)K_{t-1}$ at price Q_t^k (nominal price of capital) from entrepreneurs (owners of these firms) and I_t units of final goods from retailers at price P_t. With these inputs the accumulation of capital can be expressed as: $\Delta\bar{x}_t = K_t - (1-\delta)K_{t-1}$. The new stock of effective capital $\bar{x}_t$ is sold back to entrepreneurs at price Q_t^k. In addition, the transformation of the final good into new capital is subject to adjustment costs. Thus capital producers choose $\bar{x}_t$ and I_t to maximize:

$$E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^E (q_t^k \Delta\bar{x}_t - I_t) \tag{3.27}$$

subject to the following constraint:

$$\bar{x}_t = \bar{x}_{t-1} + \left[1 - \frac{\kappa_i}{2}\left(\frac{I_t \varepsilon_t^{qk}}{I_{t-1}} - 1\right)^2\right] I_t \tag{3.28}$$

where κ_i is the adjustment cost of capital transformation, ε_t^{qk} is a shock to the efficiency of investment, and $q_t^k = \frac{Q_t^k}{P_t}$ is the real price of capital.

3.9 Retailers

We follow Bernanke et al. (1999) regarding the structure of the retail goods market which is assumed to be monopolistically competitive. Retail prices are sticky and being indexed to a combination of past and steady-state inflation, with relative weights parameterized by $\imath_p$. If retailers want to change prices beyond this indexation allowance, they face a quadratic adjustment cost parametrised by κ_p.

Thus retailer i chooses $P_t(i)$ to maximize:

$$E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^{P} \left[P_t(i)Y_t(i) - P_t^W Y_t(i) - \frac{\kappa_p}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi_{t-1}^{i_p} \pi^{1-i_p} \right)^2 P_t Y_t \right] \tag{3.29}$$

where, π is steady state inflation, subject to the demand derived from consumers' maximization: $Y_t(i) = (\frac{P_t(i)}{P_t})^{-\varepsilon_t^y} Y_t$, where ε_t^y is a stochastic demand price elasticity.

3.10 Monetary Policy and market clearing

We assume that the central bank sets the deposit interest rate according to the following Taylor rule:

$$R_t = (R)^{(1-\phi_R)} (R_{t-1})^{\phi_R} \left(\frac{\pi_t}{\pi} \right)^{\phi_\pi (1-\phi_R)} \left(\frac{Y_t}{Y_{t-1}} \right)^{\phi_Y (1-\phi_R)} \varepsilon_t^r \tag{3.30}$$

where ϕ_π and ϕ_Y are the weights to inflation and output, R is the steady state policy rate and ε^r is the monetary policy shock.

Finally, employing housing market clearing with fixed supply implies:

$$\bar{H}_t = H_t^P + H_t^I \tag{3.31}$$

3.11 Macroprudential Policy

Following Angelini et al. (2014), we assume that macroprudential regulators set dynamically the capital-asset requirement for the banking sector according to the following rule:

$$\nu_t^b = (\bar{\nu}^b)^{(1-\rho_\nu)} (\nu_{t-1}^b)^{\rho_\nu} \left(\frac{B_t/Y_t}{\bar{B}/\bar{Y}} \right)^{(1-\rho_\nu)\chi_\nu} \tag{3.32}$$

where $\bar{\nu}^b$ is the steady state value of ν^b. We assume that the regulator adjusts the requirements only in response to movements in the loans-to-output ratio, which is in line with the prescriptions of the Basel III regulation. Hence, a positive χ_ν corresponds to a leaning-against-the-wind policy, i.e. when leverage is above its steady-state capital requirements are increased to temper the financial cycle.

LTV limits, on the other hand, are present in much fewer models than the capital-asset requirements. Hence, in defining the measures to which this policy is responding, we focus on market specific indicators which are usually associated with credit fuelled price bubbles. Similarly to the capital-asset requirement and following the LTV setting in Lambertini et al. (2013), we assume that the macroprudential regulator sets the LTV of loan type j according to:

$$m_t^j = (\bar{m}^j)^{(1-\rho_m)}(m_{t-1}^j)^{\rho_m}\left(\frac{x_t^j}{x_{t-1}^j}\right)^{(1-\rho_m)\chi_x} \tag{3.33}$$

where $\bar{m}^j$ is the steady-state LTV and for the variable $x^j = \{q^j, B^j\}$ we investigate two possibilities in which the rule responds to either collateral prices or leverage of the specific loan type j. The motivation behind the response of this requirement to specific loan type j indicators reflects the more sophisticated purpose and reach of this tool. Hence, a LTV requirement for mortgages would respond to changes of house prices or mortgage lending relative to output, where a negative χ_x coefficient represents a countercyclical LTV setting.

3.12 Calibration

Since the goal of our model is mainly qualitative, we calibrate it in line with the existing literature and the models with closest set-up: Gerali et al. (2010) and Quint and Rabanal (2014) which are calibrated and estimated in-line with the Euro area data. Table (3.1) reports the values of the model parameters. The model is calibrated so that a time period represents a quarter. We set the discount factor of patient households to 0.9943 which pins down a quarterly steady-state deposit (policy) interest rate of 0.57 percent (or 2.3 percent annualized). For impatient households, we set the discount factor to 0.975 as in Iacoviello (2005). For the LTV steady-state ratios, we set the one to households m^h to 0.7 and to firms m^f to 0.35 as in Gerali et al. (2010). In the steady-state, the later two values together with the standard deviations of the idiosyncratic shock $\bar{\sigma}_\omega^j$, pin down the default rates of loan type j i.e. loans to households and firms. Hence, similarly to Quint and Rabanal (2014), we set the standard deviations in order to match the average default rates of the two types of loans for the Euro area. Thus a $\bar{\sigma}^h_\omega = 0.17$ gives a default rate of mortgages of 2.5 percent and $\bar{\sigma}^f_\omega = 0.50$ gives a default rate of firm loans of 3.3 percent. Since the LTV's are also the steady state default thresholds, following the participation constraint (3.7), the chosen values so far give us the quarterly borrowing rates of households and firms of $r^I = 1.2$ and $r^I = 1.7$ percent respectively (4.9 and 7 percent annualized). The collateral repossession fees μ are set to 0.2 of the collateral value.

Table 3.1: Callibrated parameters

Parameter	Description	Value
β^P	patient households discount factor	0.9943
β^I, β^E	impatient households and firms discount factor	0.975
ϕ	inverse Frisch elasticity	1
α	capital share in the production function	0.25
δ	capital depreciation rate	0.025
ϵ^Y	$\frac{\epsilon^Y}{\epsilon^Y-1}$ markup in the goods market	6
m^I	households LTV ratio	0.7
m^E	firms LTV ratio	0.35
σ^I_ω	stdev of households' idiosyncratic shock	0.17
σ^E_ω	stdev of households' idiosyncratic shock	0.50
δ^b	bank capital depreciation rate	0.0057
μ	collateral repossession cost	0.2
ν^b	capital-asset requirement	0.09
κ^b	bank capital adjustment cost	10
κ^i	investment adjustment cost	4.1417
κ^p	retail price adjustment cost	33.1332
ϕ^R	TR AR coefficient on R	0.9331
ϕ^π	TR responsiveness to inflation	2
ϕ^Y	TR responsiveness to output	0.24

We set the physical capital share in the production function α to be 0.25 with a depreciation rate δ of 0.025 and the wage share of patient households ν to 0.8 . At the wholesale bank level, we set the bank capital depreciation rate $\delta^b = 0.0057$ such as to ensure a steady-state capital-asset ratio of 9 percent according to the

Basel accords.

Regarding the shock processes in our model we follow Gerali et al. (2010) by assuming AR(1) forms for all of them except for the monetary shock, using the coefficients and standard deviations of the shocks estimated by the authors.

Table 3.2: Shock Processes

Parameter	Description	Value
ρ_{ϵ_y}	retail price mark-up	0.306
σ_{ϵ_y}	retail price mark-up	0.634
ρ_k	capital quality shock	0.543
σ_k	capital quality shock	0.019
ρ_h	housing preferences	0.917
σ_h	housing preferences	0.039
ρ_c	consumption preference	0.396
σ_c	consumption preference	0.027
ρ_A	technology	0.936
σ_A	technology	0.004
σ_r	monetary policy	0.0057
σ_{σ^h}	shock to std of idiosyncratic shock to households	0.11
ρ_{σ^h}	shock to std of idiosyncratic shock to households	0.3
σ_{σ^e}	shock to std of idiosyncratic shock to firms	0.11
ρ_{σ^e}	shock to std of idiosyncratic shock to firms	0.3

4 Simulations

4.1 Banking sector vulnerability

Before considering optimal policy, it is useful to see if the two macroprudential instruments have indeed the ability to influence the economy in the event of a crisis and observe their transmission channels. The main crisis scenario is an increase of the standard deviation of the idiosyncratic shock in the mortgage lending market as in Forlati and Lambertini (2011), which can be thought as representing entering of sub-prime mortgages into the market.

For the same crisis scenario, we will compare a static benchmark with two different static macroprudential policies, that is a tighter LTV requirement for lending to households and a tighter capital-asset requirement for the whole banking sector. Although these policies are static and the amount of tightening of each instrument is arbitrary, the comparison can serve as an example of the different transmission channels and effects of the two. Firstly, as a benchmark we will consider a LTV limit of mortgages of $\bar{m}^h = 0.7$ which is consistent as the EU averages prior to the crisis and a capital-asset requirement of $\bar{\nu}^b = 0.09$ which is consistent with the Basel requirements. Then the two alternative policies would represent a tighter LTV limit $\bar{m}^h = 0.65$ for the same capital requirement and a larger capital requirement $\bar{\nu}^b = 0.11$ for the same LTV limit. In the former case, households would be less leveraged while in the latter the whole banking sector. For all three cases, we will consider an increase from the steady state of the standard deviation of the idiosyncratic shock to mortgages. The size of the shock is chosen as to ensure an increase of the default rate of mortgages from 2.5% to 10.5% which is consistent with the increase of mortgage defaults in the US with the start of the crisis. In all simulations we keep the LTV to firms fixed at $\bar{m}^f = 0.35$.

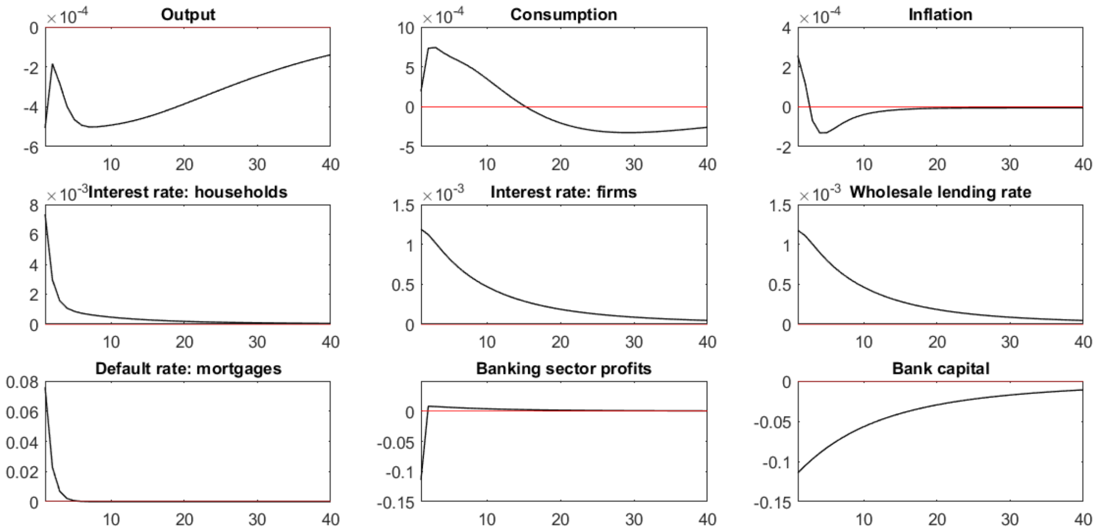
Before comparing the three cases, figure 4.1 presents the responses to the shock in the benchmark setting: $\bar{m}^h = 0.7$, $\bar{\nu}^b = 0.09$. An increase in the standard deviation of the idiosyncratic shock, essentially fattens the tails of the log-normal distribution, thus for the same default threshold ω^p a higher standard deviation implies higher cumulative distribution function and therefore higher default rates on mortgages.

The lower row of the figure presents the starting point of the crisis scenario. On the impact of the shock, the default rate on mortgages increases rapidly to 8% above its steady state. The predetermined lending rates lead to bank losses, since the retail bank which lends to households suffers higher default rates than the ones expected when the interest rate was determined. Even though the shock continues to propagate and the default rate reduces slowly, due to the rational expectations employed, the retail bank now expects this and raises the interest rate to households. As a consequence, the bank does not suffer losses in any other period apart from the surprise moment of the shock. However, the aftermath of the bank losses continues for much longer. As the losses are absorbed by bank capital, the latter falls immediately. The crisis is now transmitted to the wholesale level of the banking sector. There, the endogenous destruction of bank capital reduces the capital-asset ratio above the requirement. As we saw from equation (3.5), to the former event the wholesale branch responds by increasing the wholesale lending rate R^b in attempt to decrease assets and recapitalize to meet the requirement.

As both retail spreads are imposed on top of the wholesale spread, both interest rates to households and firms increase sharply. As a result, aggregate borrowing falls together with investment and consumption by both impatient households and firms leading to a persistent fall in output. Thus, the shock which originated from the mortgage lending propagated though the banking sector tightening borrowing conditions to all types of lending and finally reached the real sector.

What we fail to represent here is the magnitude of the contraction in output and the observed counterfactual initial increase in consumption. Several factors are responsible for this. Firstly, under the assumptions of our model no collateral is lost due to default. In fact, the big winners from the higher defaults are savers as they own the collateral repossession agencies. Due to this the latter experience an income effect with the higher defaults to which they respond by higher saving, consumption and housing demand. The increase in consumption by savers and gradual decrease in consumption by borrowers prevents aggregate consumption from falling initially which also leads to inflation. Due to this, our recession becomes mainly disinvestment driven. A second factor is the assumption that after default the borrowers keep the collateral and simply make a transaction of its amount. This prevents from observing the fire sales effect after higher sales of repossessed collateral which would put downward pressure on its price. Both of these assumptions are inherited from Quint and Rabanal (2014) and we decided to keep them for the sake of simplicity and comparability with the related literature. Nevertheless, the ultimate effect of these features on the model is an underestimation of the cost of defaults for borrowers and relaxing those assumptions would be a good direction of future research.

Figure 4.1: Impulse response to an AR shock to σ_ω^h



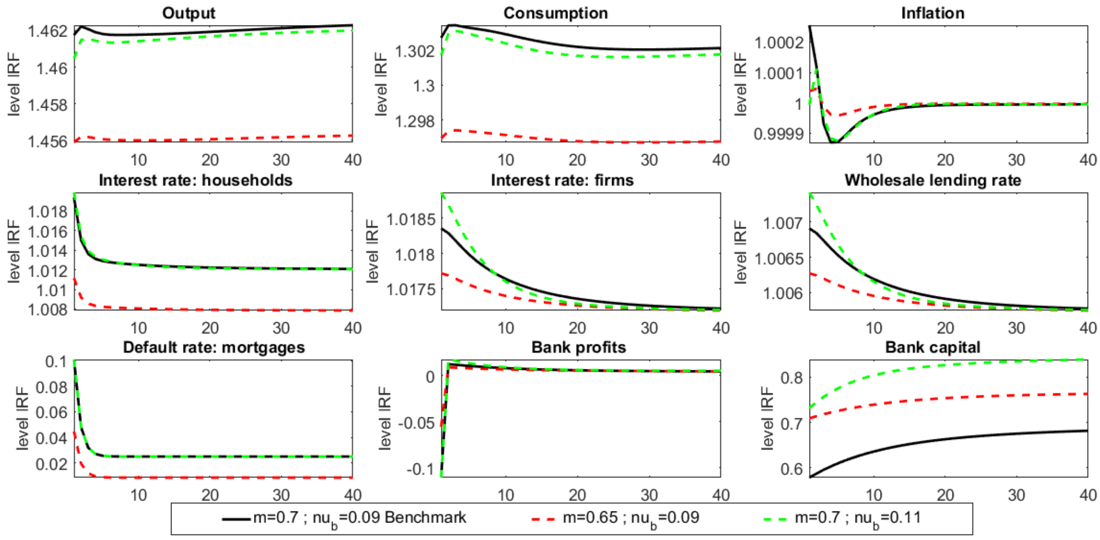
All variables are in percentage deviation from the steady-state except from those that are already in percent. One period corresponds to one quarter. The shock follows an AR(1) process in logs with coefficient of 0.3

We should also note that the magnitude of our results is very sensitive to the parametrisation of the model, nevertheless the direction of the responses and the transmission channels remain robust. Therefore, although the model may not be able to represent quantitatively realistic scenario, it can still fulfil its main goal to qualitatively compare the different transmission mechanisms of the two macroprudential instruments.

Figure 4.2 compares the same scenario of our benchmark static policy setting from above with a tighter LTV limit $\bar{m}^h = 0.65$ for the same capital requirement and a larger capital requirement $\bar{\nu}^b = 0.11$ for the same LTV limit.

Before reporting the results, we should note that the lower LTV limit leads to lower steady-states of the default threshold and hence the default rate of households. The latter also leads to lower steady-states of the interest rate on mortgages and lower output, due to the tighter lending requirement. The higher capital-asset requirement leads to higher steady-state bank capital. These differences in the steady-states force us to report the results in absolute values and levels (not in deviation from steady-state).

Figure 4.2: Impulse response to an AR shock to σ^h_ω



All variables are in absolute values and levels. One period corresponds to one quarter. The shock follows an AR(1) process in logs with coefficient of 0.3

In comparison of the two LTV settings (black and red lines), we notice the following. The bottom row of the figure reveals what became evident in equation (3.16). Apart from the fact that the lower LTV leads to lower steady-state default threshold and hence default rate, it also leads to lower exposure of the banking system to unexpected increases in the default rate. This can be seen in the bottom left corner, as the $m^h = 0.65$ LTV not only leads to 1% steady-state default rate, but the jump in the latter due to the shock is approximately from 1% to 5%. At the same time, regardless of the capital-asset requirement in both $m^h = 0.7$ LTV cases the default rate jumps from 2.5% to 10.5% approximately. In the lower middle graph, we can see that these larger increases in the default rate of the higher LTV cases also lead to higher losses which then lead to larger amount of bank capital destroyed. As the low LTV case gives lowest bank capital destruction, in that setting the increase in the wholesale lending rate and consequently retail rates are the lowest observed. This is turn leads to much lower disturbance of output consumption and investment from their steady-state. However, the ability of the lower LTV to mitigate the impact of the crisis does not come at no cost. Indeed, in that setting both consumption and investment steady-states are lower which in turn leads to lower output. Hence, tighter LTV faces a trade-off between lower output volatility and permanently lower output in the event of a financial shock.

In comparison of the two asset-requirement settings (black and green lines), we notice the following. In the higher capital-asset requirement setting there are no differences in the steady states of the reported variables with the benchmark case apart from the higher bank capital. Consequently, the increase in default rates and the amount of bank losses are identical, hence leading to equal amount of bank capital destroyed. This in turn leads to equal decrease of the capital-asset ratio but bellow different values of the requirement. Hence from equation (3.5), it follows that in case of stricter capital-asset requirement the banking sector will face higher quadratic costs for the same amount of deviation. To this the sector responds by higher wholesale lending rate. The higher wholesale rate increases the borrowing rates for both types of loans and worsen the economic contraction further. Of course, this result is highly sensitive to the assumed quadratic costs for deviating from the requirement. Nevertheless, this does not change the inability of higher capital requirements to mitigate the impact of the crisis.

These results also give us a clue of the potential dynamic policy rules that can attenuate the financial shock. For example, once the crisis has reached the wholesale banking sector a reduction of the capital-asset requirement would lead to lower increase of the wholesale lending rate and contraction of output. But how should these policy rules be chosen? This is what we investigate next.

4.2 Optimal Policy

So far, we saw that different static settings of the macroprudential instruments can have influence on the impact and transmission of a financial shock. Now we consider a dynamic setting of these instruments according to the proposed rules (3.33). Regarding the objective of such rules, the related literature employs two different approaches. The first one assumes ad-hoc loss function including the variances of the instrument and variables such as output and debt. Then the parameters of the policy rules are set such as to minimize the chosen function. The main caveat of this approach is the lack of derivation of the loss function and the weights of the different variances in it. Furthermore, the chosen optimization is conditional on a specific type of shock.

Therefore, we chose the alternative approach of welfare optimization. In doing so, we follow the standard approach in the DSGE literature. The welfare of each agent $j = \{P, I, E\}$ is given by the expected discounted sum of lifetime utility:

$$\Omega_t^j = \max E_t \left[\sum_{i=0}^{\infty} (\beta^j)^i U(C_{t+i}^j, H_{t+i}^j, L_{t+i}^j) \right] \tag{4.1}$$

which at the optimum has the following Bellman form:

$$\Omega_t^j = U(C_t^j, H_t^j, L_t^j) + \beta^j \Omega_{t+1}^j \tag{4.2}$$

Thus, we set the coefficients of the macroprudential policy rules so as to maximize the sum of welfares of the agents in our model. In doing so, we study the ex-ante optimal simple rules based on the second-order approximate solution of the model. We also compute the welfare implied by the different policy rules conditional on the initial state being the deterministic steady-state.

The rules that maximize social welfare are compared both in terms of levels of welfare and consumption-equivalent measure calculated as the percentage increase in steady-state consumption that would make welfare under the benchmark static policy setting equal to welfare under the optimized rule. Table 4.3 reports the results of the optimization.

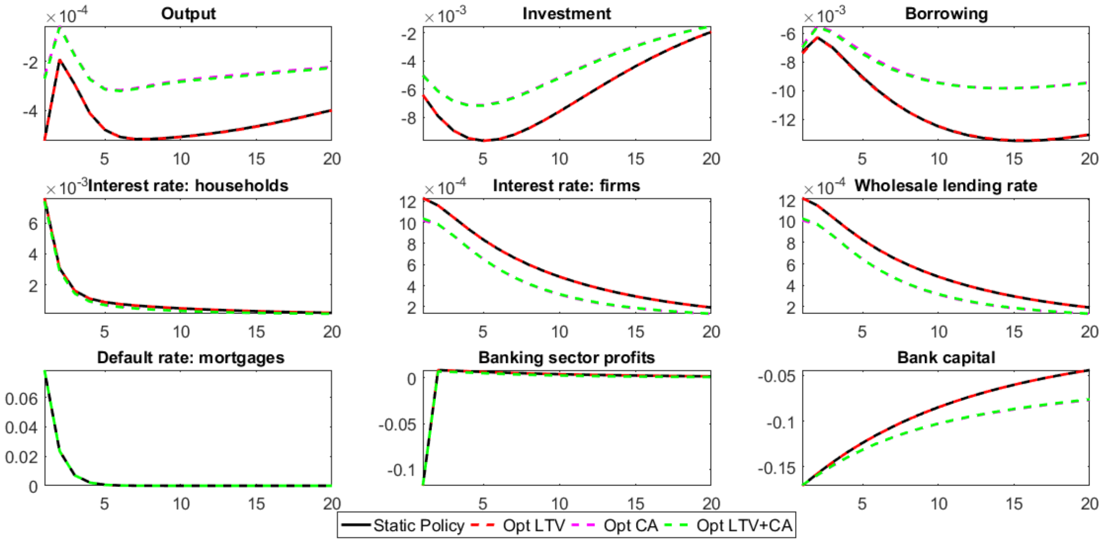
Table 4.3: Optimal Policy

Optimal Policy	Welfare	CE
Optimal LTV: $\rho_m = 0.326$ $\chi_q = -0.252$	-435.0019	0.00031
Optimal CA: $\rho_\nu = 0.0942$ $\chi_\nu = 4.998$	-432.7923	0.82422
LTV+CA: $\rho_m = 0.647$ $\chi_q = -0.609$; $\rho_\nu = 0.171$ $\chi_\nu = 4.902$	-432.7884	0.82565

The results suggest that by itself a dynamic LTV setting responding to house prices has little impact on welfare. On the contrary, dynamic capital-asset requirement setting improves welfare significantly and finally, optimizing using both active policies leads to a marginal improvement relatively to the capital-asset requirement alone.

The figure below presents impulse responses to a shock to the standard deviation of the idiosyncratic shock and compares the different policy settings.

Figure 4.3: Impulse response to an AR shock to σ_ω^h



All variables are in percentage deviation from the steady-state except from those that are already in percent. One period corresponds to one quarter. The shock follows an AR(1) process in logs with coefficient of 0.3

The four cases overlap in two pairs since, as we saw from table 4.3, the optimal LTV setting improves the benchmark case with static policy insignificantly and the optimal capital-asset requirement by very little. Thus, the only major improvement arises from optimizing over the capital-asset requirement rule.

The success of the countercyclical requirement arises from relaxation of the requirement in the periods after the shock. This leads to a lower increase in the wholesale lending rate and hence retail rates, since banks are not forced to return to the requirement as fast as before. The lower retail rates are less noticeable for households, as in their case the initial increase is due not only to higher wholesale rate but due to the higher defaults to which the mortgage lending branch responds by increasing its interest rate. Nevertheless, the attenuation of interest rates leads to lower decline in borrowing, investment and output.

In the first two graphs, at the bottom row, we see that none of the dynamic policies can affect the impact of the crisis and the amount of bank capital destroyed. This is simply because in our model banks losses and gains can occur only due to expectations mismatch that arise from shocks. In that setting, we can only analyse the ability of the instruments to mitigate the impact once the crisis has happened. This is why the LTV setting has so little impact. Once the shock has happened and bank-capital is destroyed, the crisis has already reached the wholesale level of the banking sector at which stage a sector specific requirement such as the LTV is insufficient to lean against the crisis.

Nevertheless, we saw from figure 4.2 that a lower LTV prior to the shock can in fact attenuate the destruction of bank capital. Unfortunately, we cannot capture this quality of LTV requirements in our optimal policy analysis in the absence of news shocks or a mechanism for gradually build-up of the crisis to which the LTV can respond.

In a final note regarding the robustness of the results above, we should say that our optimal welfare analysis using second order approximation is sensitive to the parameters of the exogenous shock processes which we did not estimate. Interestingly, in a preliminary version of this model we experimented with alternative approach in which the policy rules aim to minimize a loss function as in Angelini et al. (2014). Those results also suggested that the capital-asset requirement is much more effective in dealing with the crisis once it has occurred.

5 Conclusion

This paper presented a financial frictions model with a detailed banking sector featuring collaterized defaulting loans. The employed framework allowed us to represent different levels of reach of two macroprudential instruments: capital-asset and LTV requirements, and analyse their potential to mitigate the impact of a crisis originating from the housing market. In particular, we found that while the capital-asset requirement affects the whole banking sector and hence has a broader impact on the economy, the LTV limit can be imposed on a specific type of lending and thus be more precise in its effect. Moreover, we found that the LTV limit has the ability to affect the exposure to losses of a particular type of lending to unexpected increase in loan defaults.

The crisis scenario is an increase of the default rate of mortgages, leading to banks' losses which are transmitted from the particular type of lending to the whole banking sector and finally to the real side of the economy. Our static policy analysis of the two instruments showed that a permanently tighter LTV limit, prior to the crisis, can significantly reduce the realized bank losses and hence the following output contraction, this however came at the cost of permanently lower output due to the tighter borrowing limit. A permanently tighter capital-asset requirement, on the other hand, proved to be incapable of mitigating the impact of the shock while in some cases it could even worsen the crisis by forcing banks to recapitalize and reduce assets faster.

Our analysis continued with dynamic policy rules for the two macroprudential instruments. The coefficients of the rules were chosen such as to maximize the social welfare of the agents in the economy. Although the structure of our model did not allow for endogenous build up of the crisis and anticipatory response of the instruments, we were able to analyse the ability of the optimized rules to mitigate the effect of the crisis after its occurrence. In this scenario, we found that the optimized countercyclical capital-asset requirement rule can significantly improve welfare under various shocks and also attenuate the tightening of credit conditions by banks after suffering losses. On the other hand, the optimized LTV limit was incapable of improving welfare and mitigate the aftermath of the crisis once it has hit the economy and banks had suffered losses.

6 Appendix

6.1 Spread expression

Given the spread equation (3.9) we have that the denominator is the following function of the ex-ante threshold $\bar{\omega}^{j,a}$:

$$X(\bar{\omega}^{j,a}) = \frac{(1-\mu)G(\bar{\omega}_t^{j,a}, \sigma_\omega^j)}{\bar{\omega}_t^{j,a}} + (1 - F(\bar{\omega}_t^{j,a}, \sigma_\omega^j)) \tag{6.1}$$

or expressed with integrals:

$$X(\bar{\omega}^{j,a}) = \frac{(1-\mu)\int_0^{\bar{\omega}_t^{j,a}} \omega f(\omega) d\omega}{\bar{\omega}_t^{j,a}} + 1 - F(\bar{\omega}^{j,a}) \tag{6.2}$$

where $f(\omega)$ is the PDF and $F(\omega)$ is the CDF of the log-normal distribution. In fact, the second therm in the RHS which is the probability of non-default, expressed as 1 - the probability of default, where the latter is just the CDF evaluated at $\bar{\omega}^{j,a}$. Then it is straightforward to see that as the CDF is increasing function in $\bar{\omega}^{j,a}$ then:

$$\frac{d(1-F(\bar{\omega}^{j,a}))}{d\bar{\omega}^{j,a}} < 0 \tag{6.3}$$

is a decreasing function in $\bar{\omega}^{j,a}$. Then calculating the derivative of the of $X(\bar{\omega}^{j,a})$ wrt $\bar{\omega}^{j,a}$ we obtain:

$$\begin{aligned} \frac{dX(\bar{\omega}^{j,a})}{d\bar{\omega}^{j,a}} &= \frac{(1-\mu)}{\bar{\omega}^{j,a}} \bar{\omega}^{j,a} f(\bar{\omega}^{j,a}) - f(\bar{\omega}^{j,a}) - \\ &\frac{(1-\mu)\int_0^{\bar{\omega}^{j,a}} \omega f(\omega) d\omega}{(\bar{\omega}^{j,a})^2} + \frac{d(1-F(\bar{\omega}^{j,a}))}{d\bar{\omega}^{j,a}} \end{aligned} \tag{6.4}$$

which simplifies to:

$$\frac{dX(\bar{\omega}^{j,a})}{d\bar{\omega}^{j,a}} = -\mu f(\bar{\omega}^{j,a}) - \frac{(1-\mu)\int_0^{\bar{\omega}^{j,a}} \omega f(\omega) d\omega}{(\bar{\omega}^{j,a})^2} + \frac{d(1-F(\bar{\omega}^{j,a}))}{d\bar{\omega}^{j,a}} \tag{6.5}$$

which is negative, meaning that $X(\bar{\omega}^{j,a})$ is decreasing fuction of the ex-ante threshold $\bar{\omega}^{j,a}$. Then as we have from equation (3.9) the spread is:

$$\frac{r_t^j}{R_t^b} = \frac{1}{\frac{(1-\mu)G(\bar{\omega}_t^{j,a},\sigma_\omega^j)}{\bar{\omega}_t^{j,a}} + (1 - F(\bar{\omega}_t^{j,a}, \sigma_\omega^j))} = \frac{1}{X(\bar{\omega}^{j,a})} \tag{6.6}$$

meaning that the spread is an increasing function of the ex-ante threshold such that:

$$\frac{r_t^j}{R_t^b} = f(\bar{\omega}^{j,a}), f'() > 0 \tag{6.7}$$

which is equation (3.10).

6.2 Profits expression

Starting from the equation (3.13) of profits, dividing by the borrowing quantity b_{t-1}^j and substituting the ex-post threshold (3.14), we obtain profits per unit of loans as:

$$\frac{\Pi_t^j}{b_{t-1}^j} = (1-\mu)G(\bar{\omega}_t^{j,p}, \sigma_{\omega^j}^j)\frac{r_{t-1}^j}{\bar{\omega}_t^{j,p}} + (1 - F(\bar{\omega}_t^{j,p}, \sigma_{\omega^j}^j))r_{t-1}^j - R_{t-1}^b \tag{6.8}$$

then from evaluating the participation constraint (3.7) in period $t-1$ and substituting the ex-ante threshold (3.6) in period $t-1$, $\bar{\omega}_{t-1}^{j,a}$ in it we have that:

$$R_{t-1}^b = (1-\mu)G(\bar{\omega}_{t-1}^{j,a}, \sigma_\omega^j)\frac{r_{t-1}^j}{\bar{\omega}_{t-1}^{j,a}} + (1 - F(\bar{\omega}_{t-1}^{j,a}, \sigma_\omega^j))r_{t-1}^j \tag{6.9}$$

which can be substituted in (6.8) leading to:

$$\begin{aligned} \frac{\Pi_t^j}{b_{t-1}^j} = r_{t-1}^j \Bigg[& \frac{(1-\mu)G(\bar{\omega}_t^{j,p}, \sigma_{\omega^j}^j)}{\bar{\omega}_t^{j,p}} + (1 - F(\bar{\omega}_t^{j,p}, \sigma_{\omega^j}^j)) - \\ & \left(\frac{(1-\mu)G(\bar{\omega}_{t-1}^{j,a}, \sigma_\omega^j)}{\bar{\omega}_{t-1}^{j,a}} + (1 - F(\bar{\omega}_{t-1}^{j,a}, \sigma_\omega^j)) \right) \Bigg] \end{aligned} \tag{6.10}$$

Then using the formulation of $X(\bar{\omega}^{j,a})$ in (6.1), the last equation becomes:

$$\Pi_t^j = b_{t-1}^j r_{t-1}^j \left[-\left(X(\bar{\omega}_t^{j,a}) - X(\bar{\omega}_{t-1}^{j,p}) \right) \right] \tag{6.11}$$

And since we have showed in 6.1 that $X(\bar{\omega}^{j,a})$ is a decreasing function in

$\bar{\omega}^{j,a}$, then for any $\bar{\omega}^{j,a}_{t-1} = \bar{\omega}^{j,p}_{t}$ the above expression would be zero, and for any $\bar{\omega}^{j,a}_{t-1} > \bar{\omega}^{j,p}_{t}$ we would have that $X(\bar{\omega}^{j,p}_{t}) > X(\bar{\omega}^{j,a}_{t-1})$ and that $\Pi^{j}_{t} > 0$ leading to:

$$\Pi^{j}_{t} = f(\bar{\omega}^{j,a}_{t-1} - \bar{\omega}^{j,p}_{t}),\ f'() > 0 \tag{6.12}$$

6.3 Model first order conditions

Patient households (Savers)

PHHs choose: C^{P}_{t}, H^{P}_{t}, and L^{P}_{t} to maximize:

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} (\beta^P)^t U_t = E_0 \sum_{t=0}^{\infty} (\beta^P)^t \Bigg[& (1-\alpha^p)\varepsilon^c_t \ log(C^P_t(j) - \alpha^P C^P_{t-1}) + \\ & \varepsilon^h_t \ log(H^P_t(j)) - \frac{(L^P_t(j))^{1+\phi}}{1+\phi} \Bigg] \end{aligned} \tag{6.13}$$

subject to:

$$C^P_t(j) + q^h_t \Delta H^P_t(j) + D_t(j) = W_t L^P_t(j) + \frac{R_{t-1} D_{t-1}(j)}{\pi_t} + T_t(j) \tag{6.14}$$

If we denote marginal utility of consumption with:

$$U_{C^P_t} = \Lambda^P_t = \frac{(1-\alpha^p)\varepsilon^c_t}{C^P_t - \alpha^p C^P_{t-1}} \tag{6.15}$$

then substituting eq(31) for C_t and C_{t+1} into eq(30) and differentiating wrt. D_t we obtain the following Euler equation:

$$\Lambda^P_t = \beta^P \Lambda^P_{t+1} \frac{R_t}{\pi_{t+1}} \tag{6.16}$$

Then differentiating the infinite sum of discounted utility wrt. H^P_t gives the demand for housing:

$$\Lambda^P_t q^h_t = \frac{\varepsilon^h_t}{H^P_t} + \beta^P \Lambda^P_{t+1} q^h_{t+1} \tag{6.17}$$

Finally differentiating wrt. leisure L^P_t, we obtain the labour supply:

$$\Lambda_t^P = \frac{(L_t^P)^\phi}{W_t^P} \tag{6.18}$$

Impatient households (Borrowers)

IHHs choose: C_t^I, H_t^I, and L_t^I to maximize:

$$E_0 \sum_{t=0}^{\infty} (\beta^I)^t U_t = E_0 \sum_{t=0}^{\infty} (\beta^I)^t \Big[(1-\alpha^I)\varepsilon_t^c \, log(C_t^I(i) - \alpha^I C_{t-1}^I) + \\ \varepsilon_t^h \, log(H_t^I(i)) - \frac{(L_t^I(i))^{1+\phi}}{1+\phi} \Big] \tag{6.19}$$

subject to the budget constraint:

$$C_t^I + q_t^h \Delta H_t^I + \frac{(1-F_t^p) r_{t-1}^I B_{t-1}^I}{\pi_t} + q_t^h H_{t-1}^I G_t^p = B_t^I + W_t L_t^I \tag{6.20}$$

and collateral constraint:

$$r_t^I B_t^I / \pi_{t+1} = m_t^h q_{t+1}^h H_t^I \tag{6.21}$$

If we denote marginal utility of consumption with:

$$U_{C_t^I} = \Lambda_t^I = \frac{(1-\alpha^I)\varepsilon_t^c}{C_t^I - \alpha^I C_{t-1}^I} \tag{6.22}$$

By constructing a Lagrangian with the collateral constraint and S_t^I being its shadow value we obtain:

$$\mathcal{L}^I = E_0 \sum_{t=0}^{\infty} (\beta^I)^t [U_t + S_t^I (m_t^h q_{t+1}^h H_t^I - \frac{r_t^I B_t^I}{\pi_{t+1}})] \tag{6.23}$$

substituting the budget constraint for C_t^I and C_{t+1}^I and differentiating wrt. B_t^I we obtain the following Euler equation:

$$\Lambda_t^I = \frac{\beta^I \Lambda_{t+1}^I r_t^I}{\pi_{t+1}} + \frac{S_t^I r_t^I}{\pi_{t+1}} \tag{6.24}$$

Differetiating wrt H_t gives the following housing demand:

$$\Lambda_t^I q_t^h = \beta^I \Lambda_{t+1}^I q_{t+1}^h + \frac{\varepsilon^h}{H_t^I} + S^I m_t^h q_t^h \tag{6.25}$$

Lastly, labour supply:

$$\Lambda_t^I = \frac{(L_t^I)^\phi}{W_t^I} \tag{6.26}$$

Entrepreneurs

Choose consumption C_t^E, physical capital K_t^E, loans from banks B_t^E, degree of capital utilization, and labour inputs from patient and impatient households L_t^P, L_t^I to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta_I^t \left[(1-\alpha^E) log(C_t^E(i) - \alpha^E C_{t-1}^E) \right] \tag{6.27}$$

subject to:

$$\begin{aligned} &C_t^E + W_t^P L_t^P + W_t^I L_t^I + \frac{(1-F_t^{p,E}) r_{t-1}^E B_{t-1}^E(i)}{\pi_t} + q_t^k [K_t^E - (1-\delta) K_{t-1}^E] + \\ &q_t^k K_{t-1}^E G_t^{p,E} = \frac{Y_t^E}{X_t} + B_t^E \end{aligned} \tag{6.28}$$

with production function:

$$Y_t^E(i) = A_t^E K_{t-1}^E(i)^\alpha L_t^E(i)^{1-\alpha} \tag{6.29}$$

where: $L_t^E = (L_t^P)^\nu (L_t^I)^{1-\nu}$, subject to a budget constraint:

$$r_t^E B_t^E / \pi_{t+1} \leq m_t^f q_{t+1}^k K_t^E \tag{6.30}$$

Denoting marginal utility of consumption as:

$$\Lambda_t^E = \frac{(1-\alpha^e)}{C_t^E - \alpha^e C_{t-1}^E} \tag{6.31}$$

Constructing Lagrangian with S^E being the shadow value of the collateral constraint, then differentiating wrt. K_t^E leads to:

$$\Lambda_t^E q_t^k = \Lambda_{t+1}^E \beta^E \left(q_{t+1}^k (1-\delta) + r_{t+1}^k \right) + S^E m_t^f q_{t+1}^k \tag{6.32}$$

where r_t^k is the rental rate of capital: $r_t^k = \frac{\alpha Y_t^E}{K_{t-1}^E} \frac{1}{X_t}$

For labour demand we have MP of each labour type equal to its MC:

$$W_t^P = \frac{\nu(1-\alpha)Y_t^E}{L_t^P X_t} \qquad W_t^I = \frac{(1-\nu)(1-\alpha)Y_t^E}{L_t^I X_t} \tag{6.33}$$

Finally the Euler equation is:

$$\Lambda_t^E = \frac{\Lambda_{t+1}^E \beta^E r_t^E}{\pi_{t+1}} + \frac{S^E r_t^E}{\pi_{t+1}} \tag{6.34}$$

Capital Producers

Using the discount factor of entrepreneurs (as being owned by them), capital producers maximize:

$$E_0 \sum_{t=0}^{\infty} \Lambda_t^E (\beta^E)^t \left[q_t^k \Delta x_t - I_t \right] \tag{6.35}$$

by choosing Δx_t and I_t subject to the following constraint:

$$\Delta x_t = \left[1 - \frac{\kappa_i}{2} \left(\frac{I_t \varepsilon_t^k}{I_{t-1}} - 1 \right)^2 \right] I_t \tag{6.36}$$

Where, $\Delta x_t = K_t - (1-\delta)K_{t-1}$. Differentiating wrt. I_t we obtain:

$$\Lambda_t^E \left[q_t^k \frac{\partial \Delta x_t}{\partial I_t} - 1 \right] + \Lambda_{t+1}^E \beta^E \left[q_{t+1}^k \frac{\partial \Delta x_{t+1}}{\partial I_t} \right] = 0 \tag{6.37}$$

for the partial derivatives we obtain:

$$\frac{\partial \Delta x_t}{\partial I_t} = 1 - \frac{\kappa_i}{2} \left(\frac{I_t \varepsilon_t^k}{I_{t-1}} - 1 \right)^2 - \kappa_i \left(\frac{I_t \varepsilon_t^k}{I_{t-1}} - 1 \right) \frac{I_t \varepsilon_t^k}{I_{t-1}} \tag{6.38}$$

$$\frac{\partial \Delta x_{t+1}}{\partial I_t} = \kappa_i \left(\frac{I_{t+1} \varepsilon_{t+1}^k}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \varepsilon_{t+1}^k \tag{6.39}$$

substituting the last two into 52 we obtain the optimality condition:

$$1 = q_t^k\left[1 - \frac{\kappa_i}{2}\left(\frac{I_t\varepsilon_t^k}{I_{t-1}} - 1\right)^2 - \kappa_i\left(\frac{I_t\varepsilon_t^k}{I_{t-1}} - 1\right)\frac{I_t\varepsilon_t^k}{I_{t-1}}\right] + \beta^E E_t\left[\frac{\Lambda_{t+1}^E q_{t+1}^k \varepsilon_{t+1}^k}{\Lambda_t^E}\kappa_i\left(\frac{I_{t+1}\varepsilon_{t+1}^k}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \tag{6.40}$$

Retailers

Thus retailers choose $P_t(j)$ to maximize:

$$E_0\sum_{t=0}^{\infty}\Lambda_{0,t}^P\left[P_t(j)Y_t(j) - P_t^W Y_t(j) - \frac{\kappa_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}^{i_p}\pi^{1-i_p}\right)^2 P_t Y_t\right] \tag{6.41}$$

subject to: $Y_t(j) = (\frac{P_t(j)}{P_t})^{-\epsilon_t^y} Y_t$.

Thus the part of the infinite sum that includes $P_t(j)$ is:

$$\sum^R = \Lambda_t^P\left[Y_t(j)(P_t(j) - P_t^W) - \frac{\kappa_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}^{i_p}\pi^{1-i_p}\right)^2 P_t Y_t\right] + \Lambda_{t+1}^P\beta^P\left[Y_{t+1}(j)(P_{t+1}(j) - P_{t+1}^W) - \frac{\kappa_p}{2}\left(\frac{P_{t+1}(j)}{P_t(j)} - \pi_t^{i_p}\pi^{1-i_p}\right)^2 P_{t+1}Y_{t+1}\right] \tag{6.42}$$

Differentiating wrt. $P_t(j)$ and imposing $P_t(j) = P_t$ leads to:

$$\Lambda_t^P\left[-\epsilon_t^y Y_t + \frac{\epsilon_t^y Y_t}{X_t} + Y_t - \kappa_p(\pi_t - \pi_{t-1}^{i_p}\pi^{1-i_p})P_t Y_t\frac{1}{P_{t-1}(j)}\right] + \Lambda_{t+1}^P\beta^P\left[\kappa_p(\pi_{t+1} - \pi_t^{i_p}\pi^{1-i_p})P_{t+1}Y_{t+1}\frac{P_{t+1}(j)}{P_t^2(j)}\right] = 0 \tag{6.43}$$

which after dividing by Y_t and Λ_t^P simplifies to:

$$1 - \epsilon_t^y + \frac{\epsilon_t^y}{X_t} - \kappa_p(\pi_t - \pi_{t-1}^{i_p}\pi^{1-i_p})\pi_t + \frac{\Lambda_{t+1}^P\beta^P}{\Lambda_t^P}\kappa_p(\pi_{t+1} - \pi_t^{i_p}\pi^{1-i_p})\frac{Y_{t+1}}{Y_t}\pi_{t+1}^2 = 0 \tag{6.44}$$

where we use that $1/X = P_t^W/P_t$ and $\pi_t = P_t/P_{t-1}$

The profits of retailers that are transferred back to savers are:

$$J_t^R = Y_t(1 - \frac{1}{X_t}) - \kappa_p(\pi_t - \pi_{t-1}^{i_p}\pi^{1-i_p})^2 \tag{6.45}$$

6.4 Steady Sate

Patient Households

Steady state inflation is set to zero. The discount factor β^P is exogenously set and then the deposit rate is set to $R = 1/\beta^P$

Budget constraint:

$$C^P + D = W^P L^p + RD + J \tag{6.46}$$

Housing demand:

$$\frac{q^h}{C^p} = \frac{1}{H^P} + \frac{\beta^P q^h}{C^P} \tag{6.47}$$

Labour supply:

$$\frac{1}{C^P} = \frac{(L^P)^\phi}{W^P} \tag{6.48}$$

Impatient Households

The LTV limit m^h pins down the steady state default threshold $\bar{\omega}^I$ which in turn pins down the steady state default rate and repossession value: F^I and G^I functions. In addition, at the steady state all ex-ante and ex-post variables are equal. As the F, G functions and threshold are known this also pins down the borrowing rate r^I through the participation constraint.

Euler equation:

$$\frac{1}{C^I} = \frac{\beta^I r^I}{C^I} + S^I r^I \tag{6.49}$$

QR use the borrowing rate r^I to endogenously determine the discount factor β^I through the Euler equation. However, this is not directly possible in our case due to the collateral constraint which shadow value appears in the Euler equation.

Housing demand:

$$\frac{q^h}{C^I} = \frac{\beta^I q^h}{C^I} + \frac{1}{H^I} + S^I m^h q^h \tag{6.50}$$

Budget constraint:

$$C^I + (1 - F^I) r^I B^I + q^h H^I G^I = B^I + W^I L^I \tag{6.51}$$

Labour supply:

$$1/C^I = (L^I)^\phi / W^I \tag{6.52}$$

Collateral constraint:

$$r^I B^I = q^h H^I m^h \tag{6.53}$$

Entrepreneurs

As with impatient households, the ss borrowing rate by entrepreneurs is pinned down by the LTV limit m^f. However, again due to the collateral constraint term in the Euler equation, determining β^E as in QR is not straightforward.

Euler equation:

$$\frac{1}{C^E} = \frac{\beta^E r^E}{C^E} + S^E r^E \tag{6.54}$$

Budget constraint:

$$C^E + W^P L^P + W^I L^I + (1 - F^E) r^E B^E + q^k K^E (\delta + G^E) = Y^E / X + B^E \tag{6.55}$$

Production:

$$Y^E = (K^E)^\alpha ((L^P)^\nu (L^I)^{1-\nu})^{1-\alpha} \tag{6.56}$$

Collateral constraint:

$$r^E B^E = q^k K^E m^f \tag{6.57}$$

Optimal choice of capital:

$$\frac{q^k}{C^E} = \frac{\beta^E}{C^E}(q^k(1 - \delta) + r^k) + S^E m^f q^k \tag{6.58}$$

$$r^k = \frac{\alpha Y^E}{K^E X} \tag{6.59}$$

Labour demand:

$$W^P = \frac{\nu(1-\alpha)Y^E}{L^P X} \;\; ; \;\; W^I = \frac{(1-\nu)(1-\alpha)Y^E}{L^I X} \tag{6.60}$$

Capital producers

In ss it follows that: $K^E\delta = I$ and $q^k = 1$

Retailers

$X = \varepsilon^y/(\varepsilon^y - 1)$, where ε^y is stochastic demand price elasticity.

$J^R = Y^E(1 - 1/X)$

Wholesale bank

Steady state capital asset ratio equal to requirement: $K^b/B = \nu^b$.

Wholesale rate equal to policy rate: $R^b = R$

Balance sheet: $B = B^I + B^E = D + K^b$.

Profits: $K^b\delta^b = \Pi^{ws} = (R-1)(B-D) = (R-1)K^b$ so we set $\delta^b = R^b - 1$

Market clearing

Housing market: $1 = H^I + H^P$.

Aggregate output: $Y^E = C^P + C^I + C^E + q^k\delta K^E + \delta^b K^b$.

Chapter II

Sectoral Risk-Weights and Macroprudential Policy

Co-authored with A. Hodbod (ECB) and S. Huber (University of Amsterdam)

Abstract

This paper introduces risk-weighted capital-asset requirements in a general equilibrium model and analyses the implications of different risk-weighting methods for financial distress in the event of a crisis. In particular, we compare a static risk-weight setting with the use of Internal Ratings-Based approach (IRB), and a macroprudential setting of the risk-weights that responds to sectoral measures of leverage. The different methods are compared in a crisis scenario originating from the housing market and mortgages that affects the banking sector and transmits to the wider economy. We aim to represent both boom and bust phases of the crisis by simulating an unrealized news shock that leads to gradual build up and rapid crash in the economy. Our results suggest that relatively to the static risk-weights, the IRB model induces procyclicality of regulatory capital requirements and thus amplifies both boom and bust phases of the financial cycle. On the other hand, a macroprudential control over the risk-weights that responds to sector specific leverage, leads to countercyclicality of regulatory capital requirements and thus attenuates the financial cycle and improves welfare.

JEL Classifications: C68, E44, E58, E61.

Keywords: macroprudential policy, risk-weights, countercyclical capital requirements, IRB approach, DSGE.

1 Introduction

Since the introduction of the Basel II bank regulatory framework from 2004, a major emphasis was put on risk-sensitivity - the idea that capital requirements should depend on the type of assets that a bank holds and in particular that banks with riskier assets should hold larger amount of capital to ensure their solvency. A key aspect of this regulation is the way of measuring the riskiness of banks' assets. While in Basel I, assets' risk was evaluated with the Standardized Approach (SA) - through external fixed ratings, Basel II introduced the Internal Ratings-Based (IRB) approach in which banks can use internal models to estimate their portfolio riskiness which in turn would determine the required regulatory capital to be held. In practice, the risk-sensitive requirements are implemented through assigning risk-weights to different assets and then computing a capital over risk-weighted assets ratio[1] that has to comply with the regulatory requirement. While under the SA, the risk-weights are fixed and depending on the asset class, under the IRB approach, banks are using their own models to calculate the risk-weights dynamically.

However, in 2008 the Great Recession hit the world's financial system even before the Basel II regulation was fully introduced. As a result, a new regulation was negotiated in the face of Basel III in which the lessons from the crisis were on top of mind and more stringent standards were adopted including higher capital requirements and various capital buffers such as the Countercyclical Capital Buffer (CCB). Nevertheless, regardless of the higher requirements or the time varying buffers, the newly imposed regulation remains highly dependent on the underlying way of measuring risk that is the IRB approach. It is important to note that, as a component of the regulatory capital-asset ratio, assets risk-weights lead to variability in the capital requirements and hence the tightness of banking regulation and banks' incentive to extend certain types of lending. As a result, failure to represent asset risk-weights realistically, inevitably leads to failure of capturing the relationship between capital requirements and the real economy and hence the impact of macroprudential policy.

[1] In bank regulation, the capital over risk-weighted assets ratio is simply referred to as capital-asset ratio, while the ratio of capital over non-weighted assets is referred to as the leverage ratio.

This paper analyses the implications of different methods of risk-weighting for capital-asset requirements in terms of their ability to attenuate the business cycle. Due to its wide application, the IRB approach has been extensively investigated by empirical studies which often find it as the cause of procyclical capital charges that can amplify the financial cycle.[1] On the other hand, the theoretical general equilibrium literature has analysed macroprudential policy and in particular capital-asset requirements without introducing the current risk-sensitive approach imposed by regulation. For example, Gerali et al. (2010) introduce capital requirements but regard the assets as equally weighted with a weight of one - thereby corresponding to a leverage ratio. Angelini et al. (2014) study the interaction between capital requirements and monetary policy. However, the latter paper introduces asset risk-weights for the capital requirements according to an ad-hoc rule.

To the best of our knowledge, this paper is the first one to introduce the IRB approach in a general equilibrium framework. It does so by employing a New-Keynesian general equilibrium model with financial frictions of the type of Bernanke et al. (1999), hereafter referred to as BGG. The main departure from the BGG set-up is that interest rates are predetermined as in Quint and Rabanal (2014). However, unlike the latter paper, our model includes a banking sector and bank capital requirements. Concerning the basic structure of the banking sector, our paper is closely related to Gerali et al. (2010) and Angelini et al. (2014). However, we extend the banking sector by adding defaulting loans and asset risk-weights in the bank capital requirements. The presence of defaulting loans allows for the calculation of asset risk-weights according to the IRB approach which uses the probability of default (PD) and loss given default (LGD) for a specific type of asset. The employed macro model also allows for realistic crisis scenarios which originate from mortgage lending and transmit to the real economy. Being exposed to risky loans, a higher than the expected default proportion of the portfolio with lower than expected collateral value can lead to endogenous bank capital destruction. The lower capital then leads to larger spreads and reduced lending in order to comply with regulatory requirements. The tighter lending and higher

[1] See Markus et al. (2014), Goodhart et al. (2004) and Borio et al. (2001).

spreads in turn further increase default rates and depress collateral prices closing the financial accelerator cycle.

After incorporating the current regulatory standards, the paper then proceeds to the policy analysis. The compared policy settings are the following. Risk independent capital requirements that reflect the current approach in the literature[1] in which all risk-weights are constant and equal to one, leading to a leverage ratio requirement. The IRB approach in which the risk-weights for each asset type depend on its PD and LGD, representing current regulation. And finally, an alternative countercyclical macroprudential setting is introduced that sets risk-weights for each type of lending based on sector specific measure of leverage.

The model parameters are estimated with Euro Area data and a historical variance decomposition identifies the period of the 2008 recession as being subject to shocks from the mortgage lending market leading to larger defaults.

The different policy settings are assessed in terms of their ability to stabilize the economy in two different crisis scenarios originating from the mortgage market as well as in terms of social welfare. The first scenario represents the bust phase of the crisis in which a higher than expected mortgage defaults destroy bank capital and subsequently tighter lending conditions suppress all types of lending and transmit to the wider economy. The second scenario consists of a simulated boom and bust cycle achieved through unrealized news shocks in the mortgage market. In the latter scenario, a positive shock expected 4 periods in the feature to mortgages risk, causes lenders to expect lower default rates and higher collateral prices, thereby relaxing lending conditions and spreads. This in turn leads to increase in leverage and booming collateral prices. However, at period 4 expectations do not materialize and a higher proportion of loans default than expected leading to bank capital destruction and a crisis which is driven entirely by agents' expectations.

Our results show that in both boom and bust phases of the crisis the IRB approach leads to procyclical capital requirements. In the boom phase, the IRB approach leads to looser capital requirements and thereby to lending conditions that reinforce market exuberance. In the bust phase, higher PD estimates lead to higher risk-weights and tighter capital requirements that depress bank lending and

[1]See Gerali et al. (2010).

slow down economic activity. The IRB approach therefore reinforces the financial cycle in the event of a crisis.

By contrast, our macroprudential approach to setting risk-weights leads to countercyclicality in capital requirements in both the boom and bust phases of the crisis – thereby serving to attenuate the financial cycle. As a result, the negative impact of the financial crash to the real economy is smaller and the recovery happens faster.

Finally, the leverage ratio policy setting keeps constant risk-weights equal to one and thus does not vary capital requirements with the business cycle leading to static policy relative to the other two settings. As a result, the impulse responses in both crisis scenarios lay in between those of the procyclical IRB setting and the countercyclical macroprudential rule. Our results are evident both in a financial crisis scenario as well as measured by social welfare outlining the risks of procyclical regulation.

This paper is structured as follows. Section 2 outlines the model design, section 3 explains our approach to calibrating or estimating the model parameters, section 4 sets out our results and provides accompanying policy analysis, and the final section concludes.

2 Model

This paper employs a dynamic stochastic general equilibrium model with financial frictions and a banking sector. The model is used as a laboratory for the comparison of the IRB approach versus a macropudential asset risk-weight setting rule in terms of their effect on the economy during recessions and general business cycle dynamics. The model is populated by entrepreneurs, patient and impatient households, and monopolistically competitive banks and firms. This section describes the agents in the model as well as the direct impact of the different policy settings. Due to their central role in the paper, we start with describing the banking sector.

2.1 Banks

The banking sector consists of a wholesale branch and two retail branches. The wholesale branch manages the capital-asset position of the bank as it accumulates bank capital out of retained earnings and pays a quadratic cost whenever it deviates from a risk-weighted capital-asset requirement. As bank capital can only be accumulated through retained earnings, the supply of credit is constrained as imposed by the capital regulation. The two retail branches obtain funds from the wholesale branch and lend them to households and firms respectively. The two types of loans are non-recourse with pre-determined interest rates - this allows for unexpected changes in the collateral prices to be transmitted to the loan default rates. These unexpected changes then lead to profits/losses that affect the capital-asset position of the banking sector.

Wholesale branch

The wholesale branch collects deposits D at the gross policy rate R which together with the accumulated bank capital K^b is used to fund its loans B, leading to a balance sheet identity

$$B_t = D_t + K_t^b \tag{2.1}$$

where the two sources of funding, K^b and B, are perfect substitutes. Bank capital is accumulated through retained earnings with law of motion given by:

$$K_t^b = (1-\delta^b)K_{t-1}^b + \Pi_t$$

where δ^b is the depreciation rate of bank capital, and should be interpreted as the costs of managing bank capital. Π_t denotes the realized overall profits of all bank branches, including the profits of the wholesale Π_t^{ws} and the two retail branches profits Π_t^h and Π_t^f

$$\Pi_t = \Pi_t^{ws} + \Pi_t^h + \Pi_t^f$$

The overall loans B_t in the economy consist of the loans B_t^I and B_t^E that the two retail branches lend to households and firms, respectively. The retail branches obtain the funds to lend from the wholesale branch at the gross interest rates $R^{b,I}$ and $R^{b,E}$ respectively.

The wholesale branch maximizes profits taking into account a quadratic cost QC_t whenever the risk-weighted capital-asset ratio K_t^b/RWA_t deviates from an exogenous level ν^b which represents the regulatory capital requirement.[1]

$$QC_t = \frac{\kappa_b}{2}\left(\frac{K_t^b}{RWA_t} - \nu^b\right)^2 K_t^b$$

where RWA_t denotes the risk-weighted assets and is given by the weighted sum of each asset type. The asset specific weights w_t^I and w_t^E represent a regulatory instrument that allows for adjusting the risk-weight of a specific asset class.

$$RWA_t = \mathrm{w}_t^I B_t^I + \mathrm{w}_t^E B_t^E \tag{2.2}$$

Thus the wholesale branch maximization problem is given by:

$$\max_{\{D_t, B_t^I, B_t^E\}} E_0 \sum_{i=0}^{\infty} \Lambda_{0,t}\Big[(R_t^{b,I}-1)B_t^I + (R_t^{b,E}-1)B_t^E - (R_t-1)D_t - QC_t\Big]$$
$$\text{s.t.} \quad B_t = D_t + K_t^b$$

[1] The quadratic cost for deviating from the regulatory requirement can be thought of as a simple shortcut for studying the implications and costs of regulatory capital requirements as in Gerali et al. (2010). In reality, similar trade-offs would arise from banks' decision of how much own resources to hold.

As a result, the wholesale branch maximizes its profits subject to the balance sheet identity (2.1) and the quadratic cost for deviation from the regulatory requirements by taking $R_t^{b,I}$, $R_t^{b,E}$ and R_t as given. Using the FOCs, we can write

$$R_t^{b,j} - R_t = \kappa_b\Big(\nu^b - \frac{K_t^b}{RWA_t}\Big)\Big(\frac{K_t^b}{RWA_t}\Big)^2 \mathrm{w}_t^j \qquad for \quad j \in \{I, E\} \tag{2.3}$$

Equation (2.3) links the interest rate spread $R_t^{b,j} - R_t$ for each loan type $j \in \{I, E\}$ to the degree of deviation of the capital-asset ratio from its requirement ν^b, as well as to the loan specific risk-weight w_t^j. The LHS of equation (2.3) represents the marginal benefit from increasing lending of type j (an increase in profits equal to the interest rate spread), while the RHS represents the marginal cost from doing so (an increase in the costs for deviating from ν^b). Therefore, the wholesale branch chooses a level of each type of lending j which, at the margin, equalizes costs and benefits of changing the capital risk-weighted asset ratio.

The retail branches

The retail branches face endogenous loan defaults due to an idiosyncratic shock to the collateral value of borrowers and the non-recourse contract with predetermined interest rates. Unlike the wholesale branch, each retail branch has the necessary and specialized expertise for it's type of lending - that is to evaluate expected collateral prices and default rates.

The operation of the two branches is identical with the difference that one of them extends loans to impatient households against housing collateral and the other to firms (entrepreneurs) against capital collateral. Hence, if we denote lending to household variables with I superscript and to firms with E, it would be the only difference between the equations describing the lending by each branch, hence we can express them in general form using $j = \{I, E\}$ superscript. The exceptions from this notation is that loans to households are against housing collateral so that: $h_t^I \equiv H_t^I$ at price $q_t^I \equiv q_t^h$, and loans to firms are against capital collateral so that: $h_t^E \equiv K_t^E$ at price $q_t^E \equiv q_t^k$. Ex-ante expected and ex-post realized variables are denoted with a and p superscripts respectively.

The retail interest rate derivation is based on the original BGG financial accelerator and more specifically on the Quint and Rabanal (2014) version of it which has the main differences that interest rates are predetermined and there are no agency problems or asymmetric information in the model. With these assumptions the problem that the retail branch faces is that knowing the rate at which obtains funds from the wholesale branch $R^{b,j}$, it has to set interest rate on loans r^j such as to insure that it does not suffer losses, taking into account the expected proportion of loans that will default.

The default decision is taken by the borrower after the realization of the idiosyncratic shock ω_t to the value of his collateral. The latter is log-normally distributed with CDF $F(\omega)$, PDF $f(\omega)$ and mean $E(\omega_t) = 1$,[1] so that there is idiosyncratic but not aggregate risk due to its presence. Thus, after the realization of the shock, the borrower will decide to default and give up the collateral or repay the loan, depending on which of the two amounts is smaller. At period t, high enough realizations of ω_{t-1} will induce the borrower to repay his loan in full: $r^j_{t-1}B^j_{t-1}/\pi_t$, where r^j is the gross borrowing rate and B^j the quantity borrowed. Low enough realizations will cause the borrower to default and give up his collateral after the realization of the shock: $\omega^j_{t-1}q^j_t h^j_{t-1}$, where q^j is the collateral price and h^j is its stock. Thus, the default condition for borrowers becomes, repay loan if:

$$\frac{r^j_{t-1}B^j_{t-1}}{\pi_t} \leq \omega^j_{t-1}q^j_t h^j_{t-1} \tag{2.4}$$

and default in the opposite case (being underwater). Therefore, in period t, the cut-off value of $\bar{\omega}^j_{t-1}$, i.e. the ex-post realized threshold value $\bar{\omega}^{j,p}_t$ that separates borrowers that default and those that do not can be expressed as:

$$\bar{\omega}^j_{t-1} \equiv \bar{\omega}^{j,p}_t = \frac{r^j_{t-1}B^j_{t-1}}{q^j_t h^j_{t-1}\pi_t}$$

At period t, the retail branch extends loans at a rate r^j_t without knowing the exact value of the default threshold, since it will also depend on the period $t+1$ collateral price q^j_{t+1} and next period inflation. Hence, the retail branch forms ex-ante expections of $\bar{\omega}^{j,a}_t$

[1]This implies that the log of ω is normally distributed: $log(\omega_t) \sim N(\frac{-\sigma^2_\omega}{2}, \sigma^2_\omega)$.

$$\bar{\omega}_t^{j,a} = \frac{r_t^j B_t^j}{E(\pi_{t+1} q_{t+1}^j) h_t^j} \tag{2.5}$$

Note that $\bar{\omega}_t^{j,a}$ is also the retail branch's expected LTV ratio of loan type j. Unlike the wholesale branch, retail branches do not maximize profits[1] but simply require that the expected return from a unit of credit equals the cost of funds (the rate at which the funds are obtained from the wholesale branch rate $R^{b,j}$). This leads to the following participation constraint:

$$R_t^{b,j} = (1-\mu) G_t^{j,a} \frac{E_t(\pi_{t+1} q_{t+1}^j) h_t^j}{B_t^j} + (1 - F_t^{j,a}) r_t^j \tag{2.6}$$

where the RHS of (2.6) consists of the expected return in the case of default (i.e. the repossessed collateral) and the expected return in the case of non-default (i.e. the repayment of the loan). $G_t^{j,a} \equiv G(\bar{\omega}_t^{j,a}, \sigma_{\omega,t}^j) = \int_0^{\bar{\omega}_t^{j,a}} \omega dF(\omega, \sigma_\omega^j)$ denotes the expected value of the idioscratic shock, conditional on the shock being less than $\bar{\omega}_t^{j,a}$; and $1 - F_t^{j,a} \equiv 1 - F(\bar{\omega}_t^{j,a}, \sigma_{\omega,t}^j) = \int_{\bar{\omega}_t^{j,a}}^{\infty} f(\omega, \sigma_\omega^j) d\omega$ being the probability that the shock exceeds the ex-ante threshold $\bar{\omega}_t^{j,a}$, i.e. the probability of non-default. Banks can repossess only a fraction $1-\mu$ of the collateral as the remainder is assumed to be lost as a cost of default.

Rearranging the participation constraint (2.6) yields:

$$\frac{r_t^j}{R_t^{b,j}} = \frac{1}{\frac{(1-\mu) G_t^{j,a}}{\bar{\omega}_t^{j,a}} + (1 - F_t^{j,a})} \tag{2.7}$$

where the retail spread of each type of loan $j \in \{I, E\}$ is expressed as a function of the expected default threshold $\bar{\omega}_t^{j,a}$. Due to the properties of the log-normal distribution with $E_t(\omega) = 1$, it can be shown that the denominator of the RHS of (2.7) is a decreasing function in the ex-ante threshold $\bar{\omega}_t^{j,a}$, and hence, the interest rate spread becomes an increasing function of $\bar{\omega}_t^{j,a}$.[2]

$$\frac{r_t^j}{R_t^b} = f(\bar{\omega}_t^{j,a}); \quad f'(\bar{\omega}_t^{j,a}) > 0$$

[1]Although the retail branches do not maximize profits, since we consider each bank as composed of one wholesale and two retail branches we can say that each bank operates under monopolistic competition with profit maximization occurring at the wholesale level.

[2]See appendix 6.2

The intuition behind this relationship is the following: For a larger expected LTV ratio (RHS of equation (2.5)), a larger proportion of loans is expected to default, and hence the ex-ante threshold $\bar{\omega}_t^{j,a}$ increases. Since the threshold separates the defaulting from non-defaulting loans, the bank would expect a larger default area and a smaller non-default area given by $(1 - F_t^{j,a})$. In order to compensate for the larger expected defaults, the retail branches increase the loan rate r_t^j.

Bank profits

The participation constraint (2.6) ensures that retail branches make zero profits in expectation terms. However, due to the predetermined interest rate and as a consequence of shocks, the participation constraint does not always hold ex-post. This can occur due to the aggregate risk that cannot be insured by the retail branches. For example, an unexpected increase of the collateral price would lead to lower ex-post threshold than the one expected last period when the loan was issued: $\bar{\omega}_t^{j,p} < \bar{\omega}_{t-1}^{j,a}$. Hence, a smaller fraction of borrowers would be below the threshold and default. The decrease in the default rate and the price increase of the repossessed collateral would lead to positive profits for the respective retail branch and these profits would be accumulated as bank capital.

Thus ex-post profits of loan type j are given by:

$$\Pi_t^j = (1 - \mu) G_t^{j,p} q_t^j h_{t-1}^j \pi_t + (1 - F_t^{j,p}) r_{t-1}^j B_{t-1}^j - R_{t-1}^{j,b} B_{t-1}^j$$

that is, the sum of the average repossession value of collateral for the defaulted loans and the loan repayment of non-defaulted loans, minus the cost of funds for the bank.

It can be shown that the profit of each branch is a function of the difference between last period's ex-ante expected and current period's ex-post realized thresholds.[1] Whenever the two thresholds are equal, profits are zero. When the ex-post threshold is smaller than expected (i.e. a smaller proportion of loans default than expected) profits are positive.

$$\Pi_t^j = f(\bar{\omega}_{t-1}^{j,a} - \bar{\omega}_t^{j,p}), \; f'() > 0$$

[1] See appendix 6.3

At this stage our banking sector is completed and we can see the structure of the spreads that occur and the factors that affect them. While the wholesale spreads: $R^b - R$ and $R^{b,I} - R^b$, $R^{b,E} - R^b$ are affected by the capital-asset position of the banking sector relative to the regulatory requirements and composition of the loan portfolio. The two retail spreads: $r^I - R^{b,I}$ and $r^E - R^{b,E}$ are mainly affected by the expected collateral values of each type of loan. The figure below illustrates the spreads structure.

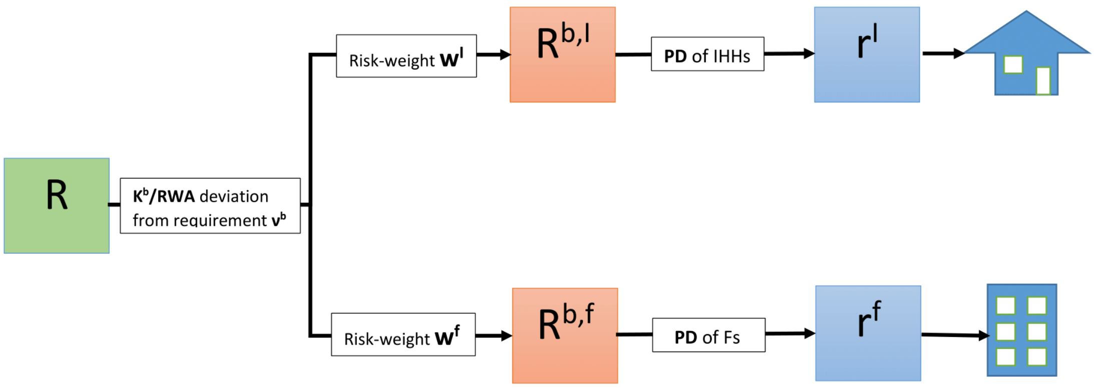


Figure 2.1: Interest rate spreads structure

In terms of policy we can distinguish the following direct effects:

- **Monetary Policy:** Changes policy rate R which affects all interest rates (even those to deposits) without affecting any of the spreads.
- **Macroprudential Capital-Asset requirement:** Changes the wholesale spread $R^b - R$ which affects all lending rates.
- **Macroprudential Risk-Weight setting:** Changes the individual spread of a specific type of loan, e.g. $R^{b,I} - R^b$ while having a smaller effect on the other spread $R^{b,E} - R^b$, thus changing the relative costs of borrowing of the two loan types.

The above two-level representation of spreads can also be interpreted from the perspective of the Basel capital regulation. While the retail level spread arises due to provisioning of expected losses by retail branches, the wholesale level spread arises due to capital regulation which aims to address the possibility of unexpected losses which are covered by bank capital.

2.2 Households

Savers

Each saver (or patient household) i maximizes expected lifetime utility subject to the budget constraint

$$\max E_0 \sum_{t=0}^{\infty} (\beta^P)^t \left[(1-\alpha^p)\varepsilon_t^c \, log(C_t^P(i) - \alpha^P C_{t-1}^P) + \varepsilon_t^h \, log(H_t^P(i)) - \frac{(L_t^P(i))^{1+\phi}}{1+\phi} \right]$$
$$\text{s.t.} \quad C_t^P + q_t^h \Delta H_t^P + D_t = W_t^P L_t^P + \frac{R_{t-1} D_{t-1}}{\pi_t} + T_t$$

Expected lifetime utility depends on current individual (and lagged aggregate) consumption C_t^P, housing H_t^P and hours worked L_t^P.[1] The last term is labour disutility where ϕ denotes the inverse elasticity of labour supply. There are two preference shocks present, ε_t^c affects the marginal utility of consumption, and ε_t^h the marginal utility of housing.

The patient household spends his income on current consumption, accumulation of housing (with q_t^h denoting real house prices), and on saving via real deposits D_t. The income side consists of wage earnings $W_t L_t^P$ (where W_t is the real wage), and gross interest income from last period deposits $R_{t-1} D_{t-1}/\pi_t$, where $\pi_t = P_t/P_{t-1}$ is gross inflation and R_{t-1} denotes the gross interest rate on deposits. The term T_t represents transactions including profits from intermediate goods producers and from debt repossession agencies.

Borrowers

Borrowers (or impatient households) differ from savers in several aspects. First, their discount factor is smaller than the one of savers ($\beta^I < \beta^P$) which means that they are more impatient to consume. Due to their impatience, in equilibrium, savers are willing to accumulate assets as deposits, and borrowers are willing to offer their housing wealth as collateral to obtain loans. Second, the borrowers don't earn profits from goods producers. And third, borrowers are subject to an

[1] Pre-multiplying by the habit coefficient α^P offsets the impact of external habits on the steady-state marginal utility of consumption.

idiosyncratic quality shock ω^j to the value of their housing stock which leads to loan default for some of them.
Analogously to savers, each borrower i, maximizes expected lifetime utility subject to the budget and collateral constraints[1]

$$\max E_0 \sum_{t=0}^{\infty} (\beta^I)^t \left[(1-\alpha^I)\varepsilon_t^c \, log(C_t^I(i) - \alpha^I C_{t-1}^I) + \varepsilon_t^h \, log(H_t^I(i)) - \frac{(L_t^I(i))^{1+\phi}}{1+\phi} \right]$$

$$\text{s.t.} \quad C_t^I + q_t^h \Delta H_t^I + q_t^h H_{t-1}^I G_t^{I,p} + \frac{(1-F_t^{I,p}) r_{t-1}^I B_{t-1}^I}{\pi_t} = B_t^I + W_t^I L_t^I \tag{2.8}$$

$$\bar{\omega}_t^{p,I} = \frac{r_{t-1}^I B_{t-1}^I}{q_t^h H_{t-1}^I}$$

The budget constraint for borrowers differs among those who default and those who repay their loans. Aggregating borrowers' budget constraints[2] and dropping the i superscripts, yields (2.8).

2.3 Firm sector

Entrepreneurs

Entrepreneurs maximize the sum of expected lifetime utility subject the budget constraint, production function and the collateral constraint

$$\max_{\{C_t^E, K_t^E, B_t^E, L_t^P, L_t^I\}} E_0 \sum_{t=0}^{\infty} (\beta^I)^t \left[(1-\alpha^E) log(C_t^E(i) - \alpha^E C_{t-1}^E) \right]$$

$$\text{s.t.} \quad C_t^E + W_t^P L_t^P + W_t^I L_t^I + \frac{(1-F_t^{E,p}) r_{t-1}^E B_{t-1}^E}{\pi_t} + q_t^k [K_t^E - (1-\delta) K_{t-1}^E]$$

$$+ q_t^k K_{t-1}^E G_t^{E,p} = \frac{Y_t^E}{X_t} + B_t^E \tag{2.9}$$

$$Y_t^E = A_t^E {K_{t-1}^E}^{\alpha} {L_t^E}^{1-\alpha} \tag{2.10}$$

The entrepreneur i's utility depends on the deviations of his consumption $C_t^E(i)$ from the aggregated lagged level.[3] The entrepreneur chooses consumption C_t^E,

[1] All variables and parameters with the superscript I indicate that they are specific to borrowers.

[2] We make the assumption that the households are members of a dynasty and insure themselves after the realization of the shock, thus becoming ex-post identical ensuring representative agent solution.

[3] Group habits are parameterized by α^E.

physical capital K_t^E, loans from banks B_t^E, and labour (L_t^P, L_t^I). Entrepreneurs have the same discount factor as borrower households, such that entrepreneurs become net borrowers in equilibrium, willing to pledge capital used for production as a collateral.

The depreciation rate of capital is denoted by δ, q_t^k denotes the price of capital and $P_t^W/P_t = 1/X_t$ is the relative competitive price of the wholesale good Y_t^E that is produced according to the Cobb-Douglas production technology (2.10), where A_t^E denotes a stochastic productivity shock. Aggregate labour, denoted by L_t^E, is given by $L_t^E = (L_t^P)^\nu (L_t^I)^{1-\nu}$, where ν measures the labour income share of patient households.

Capital Producers

Capital producers are a modeling device to derive the price of capital. Capital producers are perfectly competitive. To produce capital, capital producers buy two inputs. First, last-period undepreciated capital $(1-\delta)K_{t-1}$ at price Q_t^k (the nominal price of capital) from entrepreneurs. Second, I_t units of the final consumption good from retailers at price P_t. The accumulation of capital is given by $\Delta\bar{x}_t = K_t - (1-\delta)K_{t-1}$. The new stock of effective capital $\bar{x}_t$ is sold back to entrepreneurs at price Q_t^k. In addition, the transformation of the final good into new capital is subject to adjustment costs κ_i. Capital producers maximization problem is given by

$$\begin{aligned} \max_{\{\bar{x}_t, I_t\}} \quad & E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^E (q_t^k \Delta\bar{x}_t - I_t) \\ \text{s.t.} \quad & \bar{x}_t = \bar{x}_{t-1} + \left[1 - \frac{\kappa_i}{2}\left(\frac{I_t \varepsilon_t^{qk}}{I_{t-1}} - 1\right)^2\right] I_t \end{aligned} \tag{2.11}$$

where ε_t^{qk} denotes a shock to investment efficiency, and $q_t^k \equiv \frac{Q_t^k}{P_t}$ the real price of capital.

Retailers

We follow Bernanke et al. (1999) regarding the structure of the retail good market. We assume monopolistic competition. Retail prices are sticky and are indexed to a combination of past and steady-state inflation, with relative weights parameterized by $\imath_p$. Whenever retailers want to change prices beyond this indexation allowance, they face a quadratic adjustment cost parameterized by κ_p. Retailer i chooses $P_t(i)$ to maximize subject to the consumers demand function (2.12)

$$\max_{P_t(i)} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^{P} \left[P_t(i)Y_t(i) - P_t^W Y_t(i) - \frac{\kappa_p}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - \pi_{t-1}^{i_p}\pi^{1-i_p}\right)^2 P_t Y_t \right]$$

$$\text{s.t.} \quad Y_t(i) = (\frac{P_t(i)}{P_t})^{-\varepsilon_t^y} Y_t \tag{2.12}$$

where π denotes steady state inflation, and ε_t^y the stochastic demand price elasticity.

2.4 Policy

Monetary Policy

The central bank sets the deposit interest rate according to the following Taylor rule:

$$R_t = (R)^{(1-\phi_R)}(R_{t-1})^{\phi_R}\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi(1-\phi_R)}\left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_Y(1-\phi_R)}\varepsilon_t^r \tag{2.13}$$

where ϕ_π and ϕ_Y denote the weights of inflation and output, R the steady state policy rate and ε^r the monetary policy shock. Changes in policy rate R_t will affect all interest rates equally, without affecting any of the interest rate spreads as shown in Figure (2.1).

Macroprudential policy

Equation (2.3) allows the analysis of how different macroprudential instruments impact the asset specific interest rate spreads. In turn, the asset specific interest rate spreads determine the borrowing costs of households and firms and hence the volumes of loans to different sectors of the economy. For convenience, equation (2.3) is repeated here

$$R_t^{b,j} - R_t = \kappa_b\Big(\nu^b - \frac{K_t^b}{RWA_t}\Big)\Big(\frac{K_t^b}{RWA_t}\Big)^2 \mathrm{w}_t^j \qquad for \quad j \in \{I, E\}$$

Keeping everything else constant, an increase of the capital-asset requirement ν^b increases the interest rate spread $R^{b,j} - R$ for all loan types j. The impact of this instrument is not asset type specific, it affects the spread of both loan types alike. This is because, ν^b changes the requirement for the capital-asset ratio without changing the risk-weighting of the different types of lending that compose the RWA denominator.

In contrast, an increase of the risk-weight w_t^j of a specific loan type $j \in \{I, E\}$ will have a stronger impact on interest rate spread $(R^{b,j} - R^b)$ of the loan type j relative to $j' \neq j$. However, the interest rate spread of loan type j' will also be affected through an increase in the risk-weighted assets (RWA) defined by (2.2). This creates the possibility for macroprudential policy to conduct tailored interventions in order to influence bank lending behaviour. For example, by increasing the risk-weight on mortgages and maintaining or decreasing the risk-weight for corporate loans, the macroprudential regulator can alter the relative cost of the two types of lending.

Under our suggested macroprudential approach, the policy maker sets risk-weight w_t^j for asset j according to a Taylor-type rule that responds to credit-to-GDP measures. According to (ESRB, 2014), the credit-to-GDP ratio is an empirically sound basis for designing macroprudential interventions. Fluctuations in this ratio are historically associated with episodes of financial instability whereby the banking sector can destabilise the real economy. In our setup, the macroprudential Taylor-type rule takes the form

$$\mathrm{w}_t^j = (\bar{\mathrm{w}}^j)^{(1-\rho_{\mathrm{w}})}(\mathrm{w}_{t-1}^j)^{\rho_{\mathrm{w}}}\left(\frac{B_t^j/Y_t}{\bar{B}^j/\bar{Y}}\right)^{\chi_{\mathrm{w}}(1-\rho_{\mathrm{w}})}$$

The risk-weight of loans to households and firms are set according to the deviation of the loan-specific measure of leverage (B_t^j/Y_t) from its steady state, where the parameters χ_{w} and ρ_{w} represent the responsiveness of the instrument to the sectoral leverage measure and its autoregressive properties. In Section 4, we discuss our macroprudential setting of risk-weights in further detail and compare its results to the leverage ratio capital requirements and the current regulatory setting known as the Internal ratings-based approach - IRB.

2.5 Market clearing and shock processes

The equilibrium in the goods market can be expressed by the resource constraint, i.e. the aggregated budget constraint of the entrepreneurs, equation (2.9), where C_t denotes aggregate consumption and is given by $C_t = C_t^E + C_t^I + C_t^P$, while output is given by the production function $Y^E = A_t^E K_{t-1}^E L_t^E$. The assumption that the housing stock exists in fixed supply, $\bar{H} = 1$, leads to the house market clearing condition:

$$\bar{H} = H_t^P + H_t^I \tag{2.14}$$

Shock processes

The shock processes we employ, are specified in Table 3.3, and have an AR(1) form. The scenario with news shocks is simulated by a negative shock to the expected exogenous term four periods in the future. Then at period 4 a positive shock is simulated and the two impulse responses are added.[1] This cancels the shock itself and the resulting responses of the variables are entirely due to changes in expectations. In particular, the shock to idiosyncratic risk of mortgages takes the form $\sigma_t^i = \bar{\sigma^i} + \rho^{\sigma^i}(\sigma_{t-1}^i - \bar{\sigma^i}) - \epsilon_{t-4}^{\sigma^i}$ and $\sigma_{t+4}^i = \bar{\sigma^i} + \rho^{\sigma^i}(\sigma_{t+3}^i - \bar{\sigma^i}) + \epsilon_{t+4}^{\sigma^i}$.

[1] Adding up the impulse responses is possible due to the linear solution of the model.

3 Calibration and Estimation

3.1 Calibration

Table (3.1) summarizes the calibration of the model parameters. Some model parameters are calibrated to match data or have been taken directly from the literature. The model is calibrated so that each period represents a quarter.

The discount factor of patient households is set to 0.9939 which pins down a quarterly steady state policy (deposit) interest rate of 0.60 percent (2.5 percent annualized), which is consistent with the policy rate average of our data sample. Discount factors for impatient households and entrepreneurs are calibrated such that we match steady state quarterly borrowing rates of 0.98 and 1.1 percent (4 and 4.5 percent annualized), respectively. These borrowing rates are consistent with the average borrowing rates for mortgages and corporate loans in our data sample.

For the calibration of the LTV steady-state ratios, we follow Gerali et al. (2010). We set the LTV of households loans (i.e. mortgages) $\bar{\omega}^I$ to 0.7 and for entrepreneurs $\bar{\omega}^E$ to 0.35. In the steady-state, the two LTVs together with the standard deviations of the idiosyncratic shock $\bar{\sigma}_\omega^j$ pin down the default rates of loan type j. Hence, similarly to Quint and Rabanal (2014), we set the standard deviation of households' idiosyncratic shock $\bar{\sigma}_\omega^I$ such that we match the average default rate of mortgages for the Euro area of 2.5 percent. For firms we calibrate the standard deviation of entrepreneurs' idiosyncratic shock $\bar{\sigma}_\omega^E$ to 0.47 to match a default rate of 2.5 percent.[1]

The collateral repossession cost parameters of households and firms of (μ^I, μ^E) are implied by the interest rates, LTV ratios and standard deviations of idiosyncratic shocks. The calibration values for the capital share, frisch elasticity, depreciation rates, and mark-ups are taken from the literature. We follow Gerali et al. (2010) and set the capital share to 0.25 and the depreciation rate to 0.025. As common in the literature, we assume a mark-up of 20% in the good market, and hence set ϵ^Y to 6. For the calibration of the markup in the labor market, we

[1]Due to data availability, we cannot differentiate between default rates of mortgages and corporate loans in the data. The average default rate of all types of loans is 2.5 percent for the Euro area.

follow Gerali et al. (2010) and set ϵ^Y to 5, implying a mark-up of 15%.

The capital-asset requirement ν^b is set to 0.08, consistent with the the Basel II regulation. The parameter δ^b, the bank capital depreciation rate, is set to 0.0061.[1]

Parameter	***Description***	***Value***
β^P	Patient households discount factor	0.9939
β^I	Impatient households discount factor	0.9902
β^E	Entrepreneurs discount factor	0.9890
ϕ	Inverse Frisch elasticity	1
α	Capital share in the production function	0.25
δ	Capital depreciation rate	0.025
ϵ^Y	$\frac{\epsilon^Y}{\epsilon^Y-1}$ markup in the goods market	6
ϵ^l	$\frac{\epsilon^l}{\epsilon^l-1}$ markup in the labour market	5
$\bar{\omega}^I$	Households LTV ratio	0.7
$\bar{\omega}^E$	Firms LTV ratio	0.35
$\bar{\sigma}^I_\omega$	Stdev of households' idiosyncratic shock	0.17
$\bar{\sigma}^E_\omega$	Stdev of households' idiosyncratic shock	0.47
δ^b	Bank capital depreciation rate	0.0061
μ^I	Collateral repossession cost, households	0.093
μ^E	Collateral repossession cost, firms	0.049
ν^b	capital-asset requirement	0.08

Table 3.1: Calibration of model parameters

[1] In our model, banks make profits in the steady state and the depreciation rate δ^b is set such that it consumes the steady state profits so that bank capital stays constant at the steady state.

3.2 Data

The dataset includes 12 variables for the Euro Area with quarterly frequency covering the time period 2000:1 to 2014:4. Data is collected on real consumption, real investment, real house prices, real loans to households and firms, real deposits, real wages, inflation, interest rates to households and firms and the policy (deposit) rate. Variables involving a trend component (i.e. consumption, investment, house prices, wages, borrowing of households and firms, and deposits) are made stationary using the HP filter (smoothing parameter set to 1600) and are transformed to log deviations from their HP-filtered trend. Interest rates and the inflation rate are de-meaned.[1] The time-series of the variables are shown in Figure (3.2).

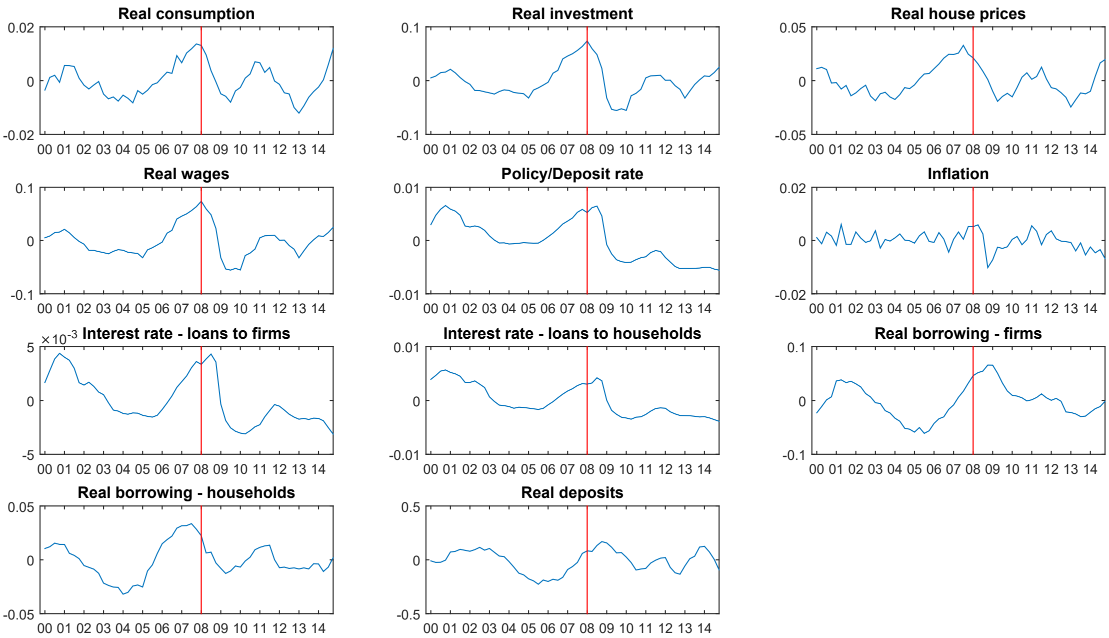


Figure 3.2: Transformed data series

[1] A full description of the data is provided in Appendix 6.1

3.3 Estimation (Metropolis-Hastings algorithm)

Model parameters that cannot be calibrated are estimated using Bayesian methods. The tables below report the estimation results, followed by the estimation method.

	Parameter	Prior Distribution			Posterior Dist.	
		Distribution	Mean	Stdev.	Mean	Mode
κ_b	Bank capital adj. cost	Gamma	10	5	0.79	0.78
κ_i	Capital adj. cost	Gamma	2.5	1	3.74	2.79
κ_p	Retailers' price adj. cost	Gamma	50	20	38.51	39.20
ι_p	Retailers' price index	Beta	0.5	0.15	0.18	0.16
α_h	Habit coefficient	Beta	0.5	0.1	0.49	0.49
ϕ^R	TR AR coeff.	Beta	0.75	0.1	0.70	0.71
ϕ^π	TR inflation coeff.	Gamma	2	0.5	1.48	1.38
ϕ^Y	TR output coeff.	Normal	0.1	0.15	0.08	0.14

Table 3.2: Estimated structural parameters

	Parameter	Prior Distribution			Posterior Dist.	
		Distribution	Mean	Stdev.	Mean	Mode
ρ_c	Cons. pref.	Beta	0.8	0.1	0.97	0.98
ρ_h	Housing. pref.	Beta	0.8	0.1	0.73	0.75
ρ_k	Capital adj. cost	Beta	0.8	0.1	0.65	0.64
ρ_A	Technology	Beta	0.8	0.1	0.97	0.98
$\rho_{\sigma i}$	HHs idiosync.	Beta	0.8	0.1	0.10	0.10
$\rho_{\sigma e}$	Es idiosync.	Beta	0.8	0.1	0.93	0.93
σ_c	Cons. pref.	Inv. gamma	0.01	0.05	0.007	0.007
σ_h	Housing. pref.	Inv. gamma	0.01	0.05	0.03	0.02
σ_k	Capital adj. cost	Inv. gamma	0.01	0.05	0.01	0.01
σ_A	Technology	Inv. gamma	0.01	0.05	0.01	0.01
σ_r	Monetary Policy	Inv. gamma	0.01	0.05	0.001	0.0017
$\sigma_{\sigma i}$	HHs idiosync.	Inv. gamma	0.01	0.05	0.55	0.55
$\sigma_{\sigma e}$	Es idiosync.	Inv. gamma	0.01	0.05	0.007	0.0067

Table 3.3: Estimated exogenous processes' parameters

We are using a Monte-Carlo based optimization technique for computing the mode with 10 parallel chains for the Metropolis-Hastings algorithm with 20000 replications each. The scale parameter of the jumping distribution's covariance matrix is set to 0.4 which leads to an average acceptance ratio of 33%.

3.4 Historical Variance Decomposition

After completing the model parametrisation we analyse its ability to represent and identify the fluctuations in the historical data.[1] Estimating the model with real data allows the conduct of a historical variance decomposition that assesses the importance of different shocks by determining the relative share of variance that each structural shock contributes to the total variance of each variable. Figures (3.3)-(3.7) visualize the variance decomposition for the following variables: real consumption, interest rates charged on mortgages, real house prices, real investment, and interest rates charged for corporate loans.

In Figure (3.3), the variance decomposition of consumption shows that the model identifies the productivity shock and the shock to idiosyncratic risk in mortgage lending to be the main drivers of the build-up and fall in real consumption during the Great Recession. The main channel through which the shock to idiosyncratic risk of mortgages can have a procyclical effect on consumption is through lending and house prices. Figure (3.4) demonstrates that this idiosyncratic risk shock contributed negatively to mortgages interest rates in the build up phase, and positively in the crash period, while figure (3.5) shows that the same shock contributed positively to house prices in the build up phase, and negatively in the crash period. The variance decomposition of investment indicates that the dynamics of real investment can be well explained by shocks to idiosyncratic firm default risk as seen in Figure (3.6). The channel works as follows: A lower firm idiosyncratic default risk shock leads to lower expected default rates of firm loans, and hence lower interest rates as shown in Figure (3.7). The Lower interest rates in turn lead to higher investment.

In summary, the model is able to identify both the build up-phase and the crash of the recent crisis as originating from mortgage and firm lending. In the build up phase, lower mortgage risk leads to lower mortgage interest rates, higher house prices and higher consumption. At the same time, a lower firm lending risk leads to lower firm interest rates and higher investment. The 2008 crash is explained as a rapid increase in the risk of both types of lending (mortgages and

[1] See Appendix 6.5 for comparison of the responses of the parametrized model to the related models in the literature.

firm loans) leading to higher interest rate spreads, and a decline in both types of borrowing and house prices. As a result consumption, investment and output all decline. The results of the variance decomposition motivate the comparison of different policies in a crisis scenario. In particular we simulate the crisis as originating from a shock to the idiosyncratic risk in the mortgage market.

Figure 3.3: Variance decomposition - Real consumption

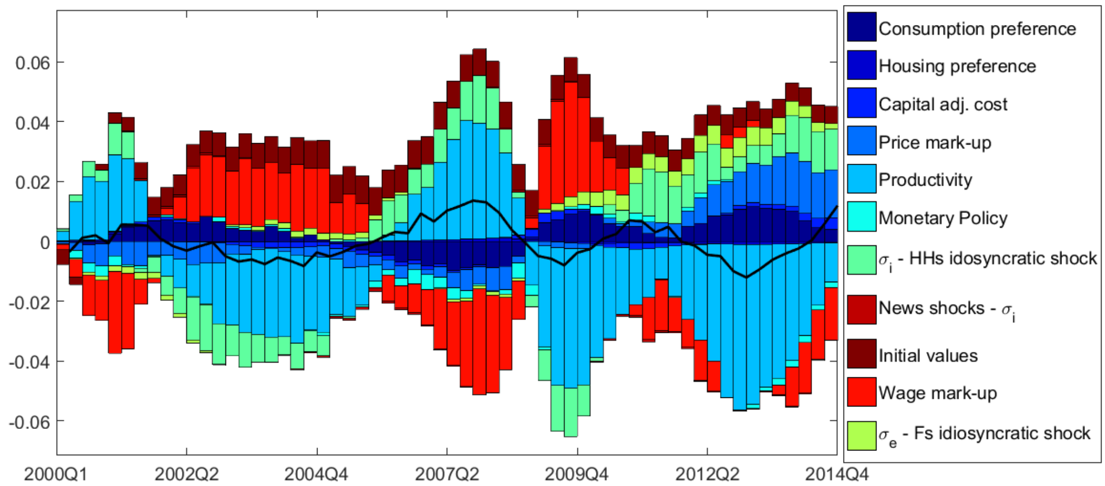


Figure 3.4: Variance decomposition - Interest rate - Households

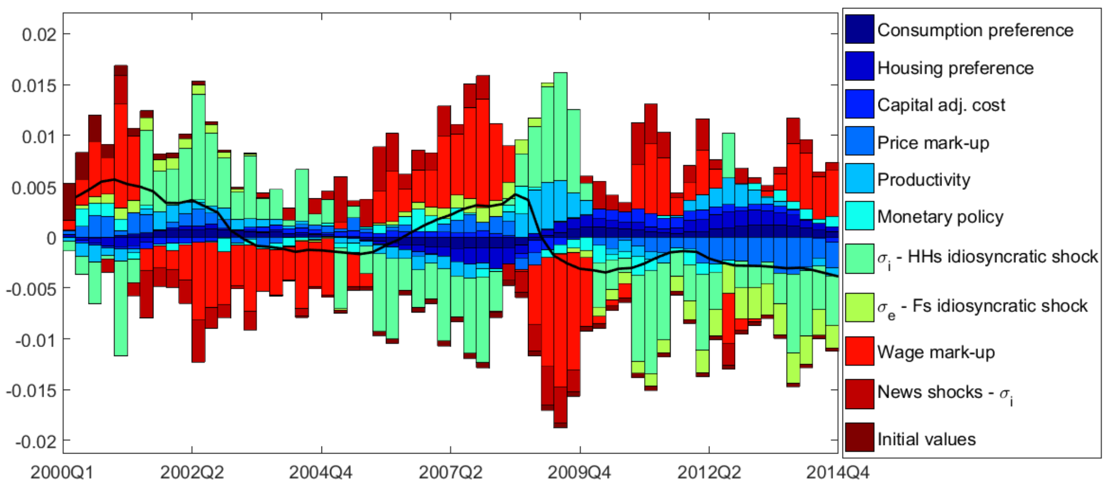


Figure 3.5: Variance decomposition - Real house prices

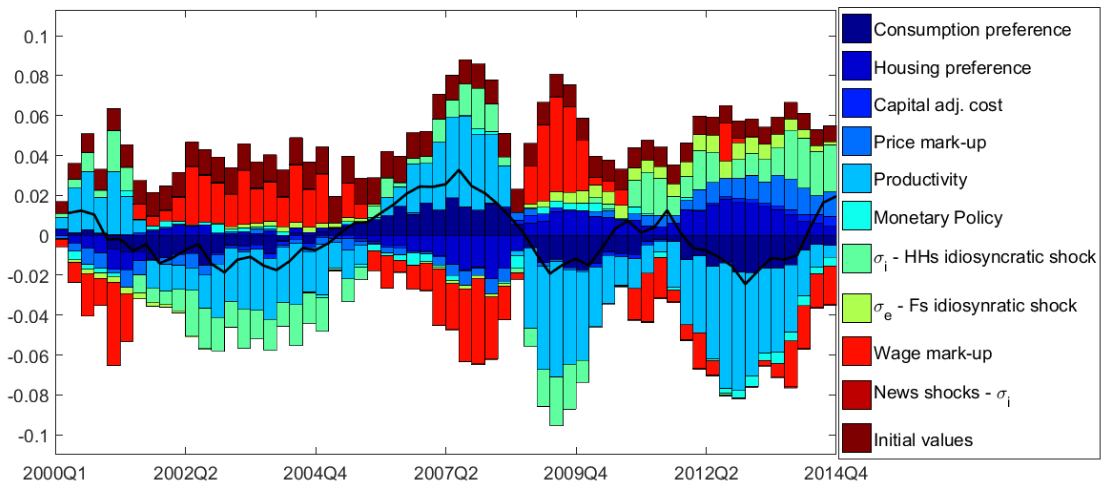


Figure 3.6: Variance decomposition - Real investment

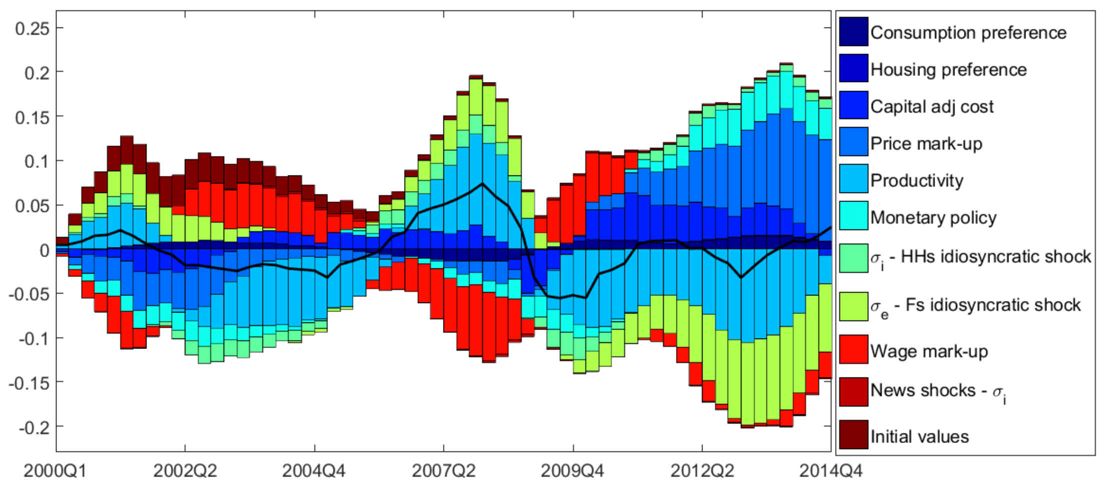
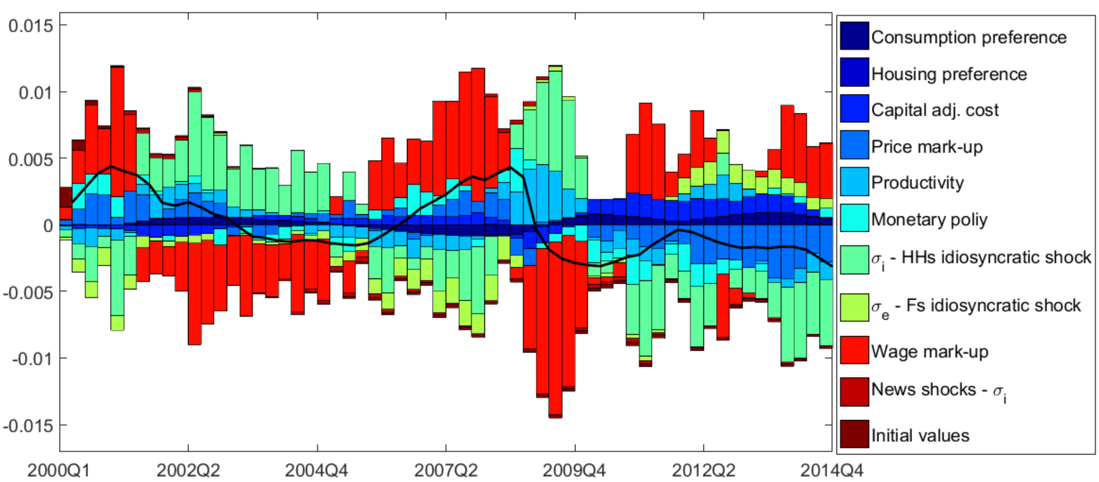


Figure 3.7: Variance decomposition - Interest rate - Firms



4 Policy analysis

In this section, we analyse and compare three alternative risk-weight setting policies. First, the leverage ratio in subsection 4.1. Second the internal ratings-based (IRB) approach which was introduced by Basel II in subsection 4.2. And finally, a novel sectoral macroprudential risk-weight setting rule in subsection 4.3. The impacts of the three policy instruments and their effectiveness in stabilizing the economy are compared under two different scenarios, in subsections 4.4 and 4.5 respectively. The first scenario represents the crash phase of the crisis, and the second consists of a simulated boom and bust cycle. Finally, we compare the welfare and business cycle implications of each policy setting in section 4.6.

4.1 Leverage Ratio

The related literature that seeks to analyse the effects of macroprudential capital requirements in general equilibrium models, often regards the regulatory capital requirement as a leverage ratio, thereby abstracting from any risk-weighting of assets.[1] Such a setup is equivalent to a setting in which the risk-weights are constant and equal to one, i.e. $w_t^I = w_t^E = 1$.

Hence in the case of the leverage ratio, the risk-weighted assets in equation (2.2) equal the total assets of the bank, $RWA_t^{LR} = B_t^I + B_t^E$. As a result, the wholesale interest rate spreads are determined by the deviation of the capital-asset ratio $\frac{K_t^b}{B_t^I + B_t^E}$ from the requirement ν^b in equation (2.3).

4.2 Internal ratings-based (IRB) risk-weighting

While risk-weights are often not taken into account in the related literature, in practice large banks ($> EUR\ 100bn$ in assets) in general, calculate their risk-weighted assets following the Internal ratings-based (IRB) approach.[2] In contrast, our model allows for risky defaulting loans and hence can be used as a means to study the impact of the IRB approach on financial stability and the real economy.

[1] For example, see Angelini et al. (2014) and Gerali et al. (2010).

[2] The current Basel III reform introduces both, a leverage ratio and asset risk-weight based constraints on bank capital.

The purpose of the IRB framework is to guarantee financial stability by imposing a bank capital requirement that is sufficient to absorb any unexpected losses arising from the assets of a bank. The capital charge that the bank has to hold for each loan type is proportional to the loan's probability of incurring unexpected losses. Below, we discuss the IRB approach in detail and then introduce it in our theoretical model presented in section 3.

According to the IRB approach, expected losses (EL) should be covered by bank provisions – and are entered on the bank's balance sheet directly as a cost associated with its lending. In our model, bank provisioning is represented by the retail level of the banking system. As a result. retail bankers set the interest rate spread by taking into account the probability of default. Unexpected losses (UL) arise in exceptional circumstances, and hence are not taken into account by bank provisioning at the retail branch level. In the model, the unexpected losses are taken into account on the wholesale level of the banking system. The wholesale branch makes sure that the capital-asset requirement is met, i.e. assets that are riskier / more prone to generating unexpected losses require the bank to hold more capital to absorb those unexpected losses through the setting of risk-weights.

The IRB framework allows banks to calculate the risk-weight of a specific loan type in order to ensure it has enough capital to cover the unexpected loss region shown in Figure (4.8). The expected loss (EL) per unit of a loan is defined as the expected annual probability of default (PD) times the loss-given-default (LGD), $EL = PD \cdot LGD$. The expected total losses ($TL = EL + UL$) are rather higher than the pure EL, as some unexpected losses (UL) are also likely to occur in some scenarios – where systematic factors (e.g. large economy-wide recessions) make the realised annual default rate higher than the expected PD. To model the UL, and thereby derive capital requirements, one must therefore condition the PD and LGD – to increase them beyond their simple historical average levels. In the IRB approach, the conditioning of the PD is designed to increase the unconditioned PD to the point where the bank is able to absorb the unexpected losses on its assets in all but the absolute most severe (top 0.1%) negative scenarios that may occur in the following year.

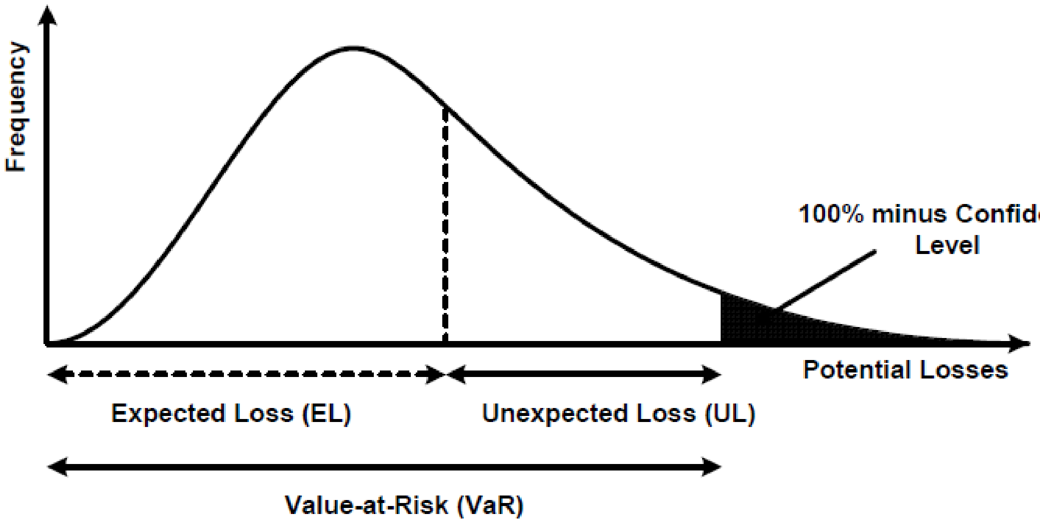


Figure 4.8: Loss distribution of a loan portfolio

Hence, Unexpected losses can be expressed as:

$$UL = TL - EL = LGD^c \cdot PD^c - PD \cdot LGD \tag{4.1}$$

where PD^c denotes the conditional probability of default and LGD^c the conditional loss-given-default. Hence, the risk-weight that would ensure enough capital to cover the unexpected losses of loan type j can be calculated as

$$\mathrm{w}_t^j = \frac{1}{\nu^b} UL_t^j \tag{4.2}$$

where ν^b is the regulatory risk-weighted capital-asset ratio requirement. As a result, the risk-weight of a particular loan type becomes a function of the respective default probability PD and loss-given-default LGD.

In our theoretical model, we are able to use the true model values for the PD and LGD, thus eliminating any estimation errors. In terms of our notation, the PD is simply $F_t^{j,a}$ and the expected loss in the event of default of loan type j is given by:

$$EL_t^j = \frac{r_t^j F(\bar{\omega}_t^{j,a}, \sigma_\omega^j)}{\pi_{t+1}} - \frac{(1-\mu^j) G(\bar{\omega}_t^{j,a}, \sigma_\omega^j) q_{t+1}^j H_t^j}{B_t^j} \tag{4.3}$$

The expected losses in (4.3) are expressed as the value of foregone interest minus the value of repossessed collateral. We calculate the loss-given-default as $LGD = \frac{EL}{PD}$, and the conditional PD^c, and LGD^c values according to the Basel methodology.[1] Using the latter, we then calculate the total losses as $TL = LGD^c \cdot PD^c$. Finally, using equation (4.1), we compute the loan specific, time varying risk-weight according to equation (4.2).

4.3 Macroprudential risk-weighting

In practice, the IRB risk-weight setting approach presented in the previous section, could create a positive relationship between the risk-weight of a particular type of loan and its probability of default which can make capital requirements procyclical. For example, in the boom phase of the economy, asset prices are high and lending conditions are lax, hence the default probability of loans decreases, leading to lower risk-weights. Similarly, in the downturn, asset prices are low and lending conditions tighten, the default rate of loans increases leading to higher risk-weights. In both phases of the credit cycle the IRB approach may lead to risk-weights that reinforce economic fluctuations thereby increasing financial fragility. This procyclicality of capital requirements is consistent with the empirical evidence found by Markus et al. (2014) and Goodhart et al. (2004).

As an alternative policy setting we introduce macroprudential interventions that aim to attenuate the business cycle and minimize its vulnerability to financial distress.[2] For this purpose we employ a Taylor-type rule that sets the risk-weight of a loan type responding to an indicator. We have chosen the indicators following the regulatory guidelines and set our instruments to respond to credit-to-GDP measures (ESRB, 2014). Therefore, in our macroprudential setting we substitute the risk-weights of equation (4.2) with the following Taylor-type rule:

$$\mathrm{w}_t^j = (\bar{\mathrm{w}}^j)^{(1-\rho_{\mathrm{w}})}(\mathrm{w}_{t-1}^j)^{\rho_{\mathrm{w}}}\left(\frac{B_t^j/Y_t}{\bar{B}^j/\bar{Y}}\right)^{\chi_{\mathrm{w}}(1-\rho_{\mathrm{w}})} \tag{4.4}$$

The risk-weight of loans to households and firms are set according to the deviation of the loan-specific measure of leverage (B_t^j/Y_t) from its steady state, where the parameters χ_{w} and ρ_{w} represent the responsiveness of the instrument to the leverage measure and its autoregressive properties. In the following sections of

[1] See BCBS (2005), for the LDG^c we use the unconditional LDG increased by 10% as a downturn estimate.

[2] The Capital requirements regulation - CRR IV allows for regulatory setting of higher risk-weights due to "financial stability considerations", see Article 124(4)(b).

the crisis simulations, we use countercyclical[1] values for the parameters governing the responsitiveness of the macroprudential rule to leverage that lead to a realistic response of policymakers to credit indicators.[2] In the appendix (6.6), we present the same responses with an optimal rule in which the parameters are set so as to maximize social welfare.[3] We refrain from using optimal MaP policy rule in our main simulations, firstly because it leads to unrealistically volatile policy instruments and second, since it has clear welfare advantage over the other settings that represent static policy and policy pinned down by the IRB regulation. Moreover, optimal policy in terms of social welfare is not the main goal of this paper which focuses on comparing the current approach in the literature with the imposed regulation and a proposed alternative setting of risk-weights.

With the end of this section we complete the description of the risk-weight setting under the different policy regimes. In summary, under the leverage ratio, there is no risk-weighting and as a results both risk-weights are constant and equal to one ($w_t^I = w_t^E = 1$) hence the risk-weighted assets equal the total assets of the bank and the lending spreads are determined by the deviation of the capital-asset ratio from the requirement ν^b (equation 2.3). Alternatively, under the IRB approach risk-weights are set according to equation 4.2. As a result, each asset enters the capital requirement multiplied by its corresponding weight which ultimately leads to asset specific spread (equation 2.3). Finally, under the macroprudential setting that we propose, risk-weights for each asset type are set according to equation 4.4 and respond to sectoral measure of leverage. In the next section, we compare the different policy settings under a bust phase and then the boom and bust cycle of a crisis scenario.

4.4 Analysis in the crisis scenario - *Bust Phase*

This scenario allows us to represent the crash phase of the crisis, and is therefore suitable to assess different policies in terms of their effect in the aftermath of a crisis. We study the impulse responses to an unexpected increase in the standard

[1] We refer to countercyclical setting in the sense of countercyclical capital requirements which are achieved through procyclical risk-weights and a positive value for the parameter χ_{w}

[2] $\rho_{\mathrm{w}} = 0.1103$ and $\chi_{\mathrm{w}} = 1.9483$

[3] Leading to values of: $\rho_{\mathrm{w}} = -0.2023$ and $\chi_{\mathrm{w}} = 4.9587$.

deviation of the idiosyncratic shock to mortgages. In Section 3.4., the variance decomposition identified this shock to be a driving factor at the peak of the crisis. The direct impact of this shock consists of increasing the proportion of loans below the ex-ante default threshold. This leads to a larger default rate for mortgages than was expected by the banks when the loans were issued. This in turn leads to losses to banks and the destruction of bank capital – resulting in the capital-asset ratio falling below the regulatory requirement.

Figures (4.9) and (4.10) highlight the differences in the impulse responses to the shock due to the different policy settings of capital requirements.[1] In the leverage ratio (LR) case (static and equal risk-weights), the destruction of bank capital reduces the capital-asset ratio and the capital to risk-weighted asset ratio below the regulatory requirements. In order to adjust their balance sheet to ensure the regulatory requirement is met, banks increase the wholesale interest rate spread, thereby leading to higher interest rates on loans. The higher interest rates depress economic activity and lead to a long recession.

In contrast to the leverage ratio case, the IRB approach increases the risk-weights of mortgages as the estimate of default probability increases. The risk-weights decrease following the process of household deleveraging (which results in the default probability falling). The higher risk-weight on mortgages leads to a higher value for the risk-weighted assets (RWAs). This in turn leads to a larger decline in the Capital/RWA measure and hence to a higher increase in spreads and interest rates. Ultimately, this results in a larger decline in investment and output following the shock, and a slower recovery.

Finally, the macroprudential approach to setting risk-weights (MaP) has a countercyclical effect – as it decreases the risk-weights on both types of lending as a result of the de-risking effect of the lower sectoral leverage levels in the bust phase of the crisis. This leads to lower risk-weighted assets (RWA) and a higher Capital/RWA ratio, and thereby to a relatively lower increase in spreads and interest rates on bank lending. Ultimately, this results in the stimulation of investment and thereby to a relatively fast recovery.

[1] Figures (4.9) and (4.10) shows the responses of the variables in percentage deviation from steady-state values except for the responses of variables denoted with *. These variables are plotted as absolute responses due to different steady states or variables already being in percentage form.

Figure 4.9: IRF - unexpected shock to σ^i

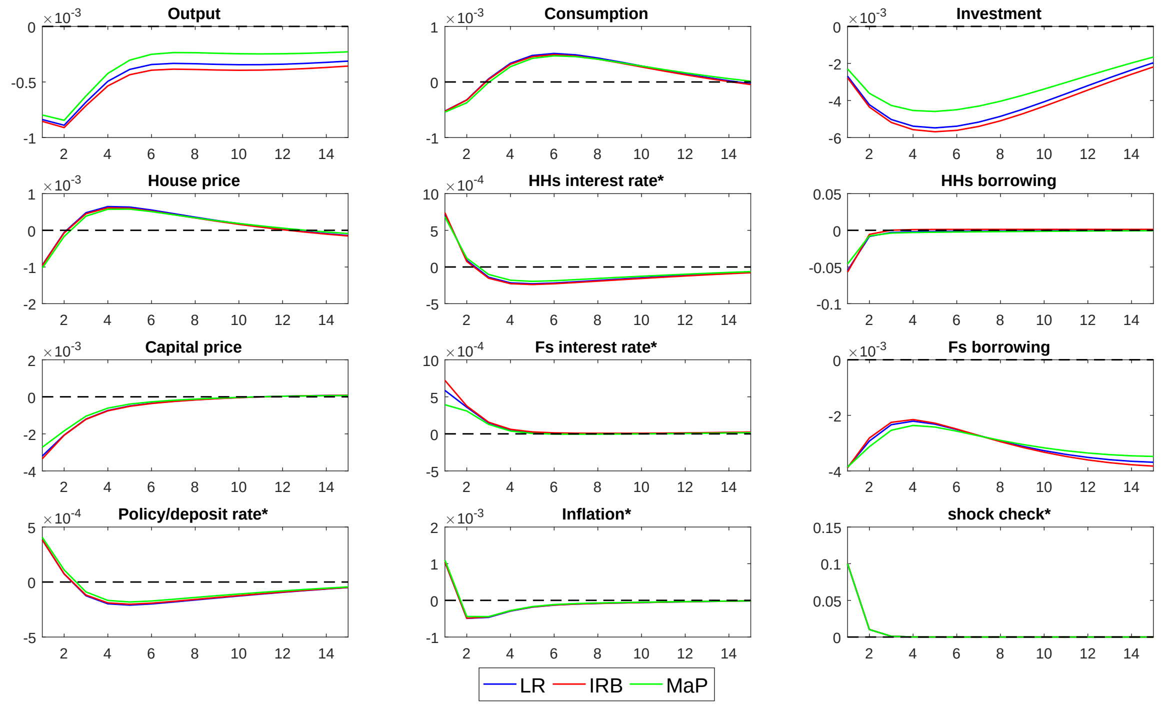
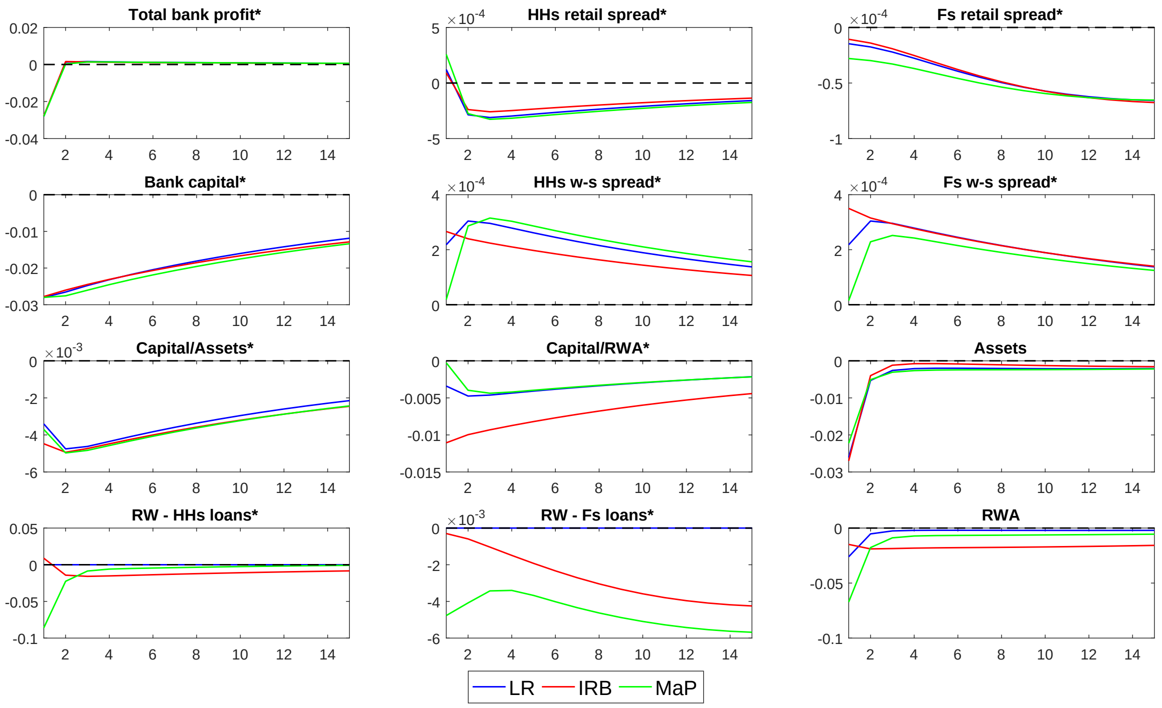


Figure 4.10: IRF - unexpected shock to σ^i



4.5 Analysis in the crisis scenario - *Boom and Bust*

In this crisis scenario we aim to represent both the build-up and crash phases of the crisis and thus to examine how the different policy approaches perform in terms of their amplifying or attenuating effects on the full cycle. The scenario is simulated as a positive news shock in the initial period whereby the agents in the economy expect the default rate of mortgages 4 periods in the future to decrease. This thereby leads to optimism and buoyancy in both lending and asset markets. However, when period 4 arrives, the shock does not occur and agents' expectations of lower default rates do not materialize. As a result, the default rate of mortgages is higher than expected and banks realize losses – thereby leading to a destruction of bank capital.

Figures (4.11) and (4.12) show the various impacts on agents' behaviour associated with the positive news shock. In the leverage ratio case (static and equal risk-weights), optimism leads to higher borrowing and decreases in the capital-assets and capital-risk-weighted-assets ratios. Banks respond to these decreased regulatory capital ratios by increasing the wholesale spread in order to stay in line with the regulatory requirement. However, the higher wholesale spread to mortgages is not enough to offset the lower retail spread which is driven by the lower default probability in the boom phase. As a result, mortgages face lower interest rates and sectoral leverage is increased further.

Unlike the results of the leverage ratio approach, the IRB approach results in decreases to the risk-weights on loans due to lower PD estimates in the optimistic phase. As a result, risk-weighted assets (RWA) decline and the Capital/RWA measure increases – leaving the impression that banks are better capitalised when, in reality, the pure Capital/Asset measure has decreased. During this phase, IRB banks decrease their wholesale spreads and further reinforce lower interest rates and higher sectoral leverage.

As in the previous scenario, the macroprudential approach to setting risk-weights has a countercyclical effect during the boom phase of the crisis – as it increases risk-weights on both types of lending in response to the increases in leverage in both sectors. This leads to higher risk-weighted assets (RWA) and a lower Capital/RWA ratio - and hence to an increase in wholesale spreads, leading to higher interest rates and lower borrowing than is observed under the other capital measurement approaches.

At period 4 the positive shock does not materialize, and the economy faces less favourable financial conditions than expected. From that point forward, the crisis proceeds in a similar way to the bust phase in section 4.3. The difference between the scenarios is that the negative shock here is driven by unmaterialized expectations rather than actual changes in financial outcomes.[1]

In the leverage ratio setting, the destruction of bank capital reduces the Capital/Assets and Capital/RWA ratios below the regulatory requirement. In order to meet their regulatory requirement, banks increase wholesale spreads – resulting in higher interest rates to loans. The higher rates depress economic activity and lead to a relatively long recession.

Unlike the leverage ratio case, the IRB approach increases the risk-weight on mortgages at the point where the negative shock arises – due to higher resulting estimates of PDs. Subsequently, risk-weights then fall as households deleverage, and PDs decline. The higher the risk-weight to mortgages leads to a larger measure of risk-weighted assets which in turn leads to larger decline of the Capital/RWA ratio and hence to a greater increase in spreads and interest rates on lending.

In the case of our macroprudential approach to setting risk-weights, lending conditions are tight before the shock – due to the stricter capital requirements that result from high risk-weights during the phase where sectoral leverage is increasing. The destruction in bank capital is therefore lower when the shock hits, and therefore the negative impacts of the shock are also lower. After the shock, the economy faces relatively favourable credit conditions in comparison to the IRB and leverage ratio regulatory cases - and the economic recovery is therefore faster, as investment can be sustained through the cycle.

[1] Note that in the unrealized news shock (boom and bust) scenario the dynamics are entirely driven by expectations while the impulse response of the shock remains flat.

Figure 4.11: IRF - unrealized news shock to σ^i at period 4

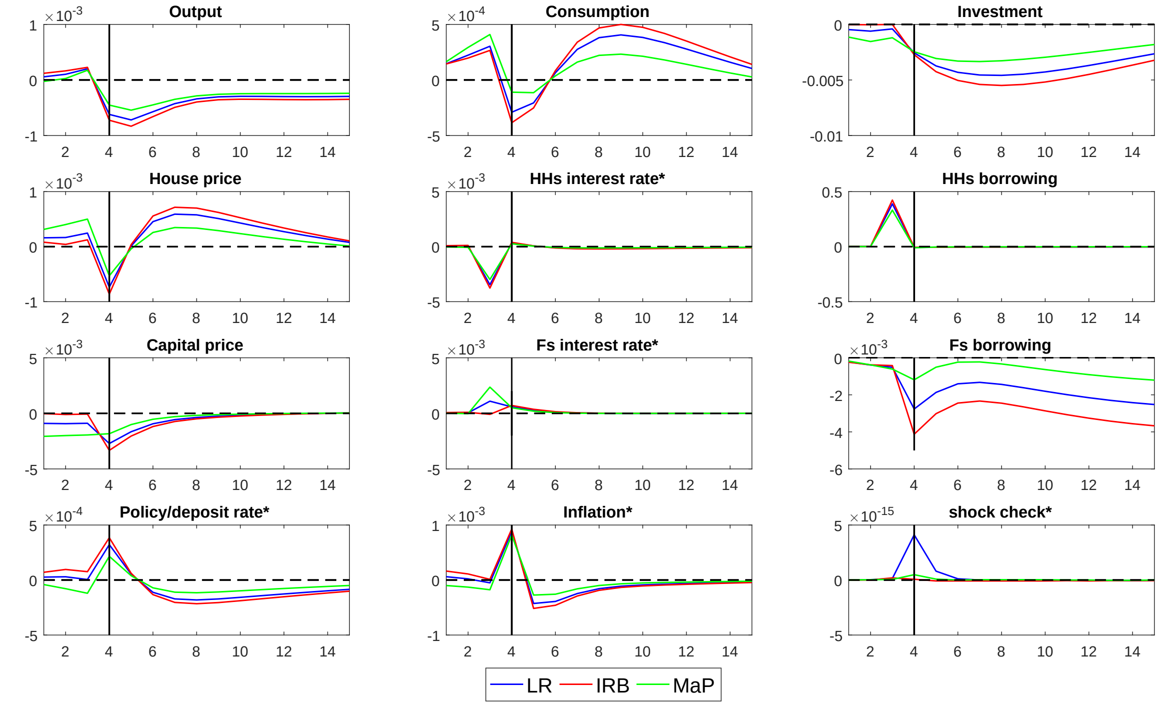
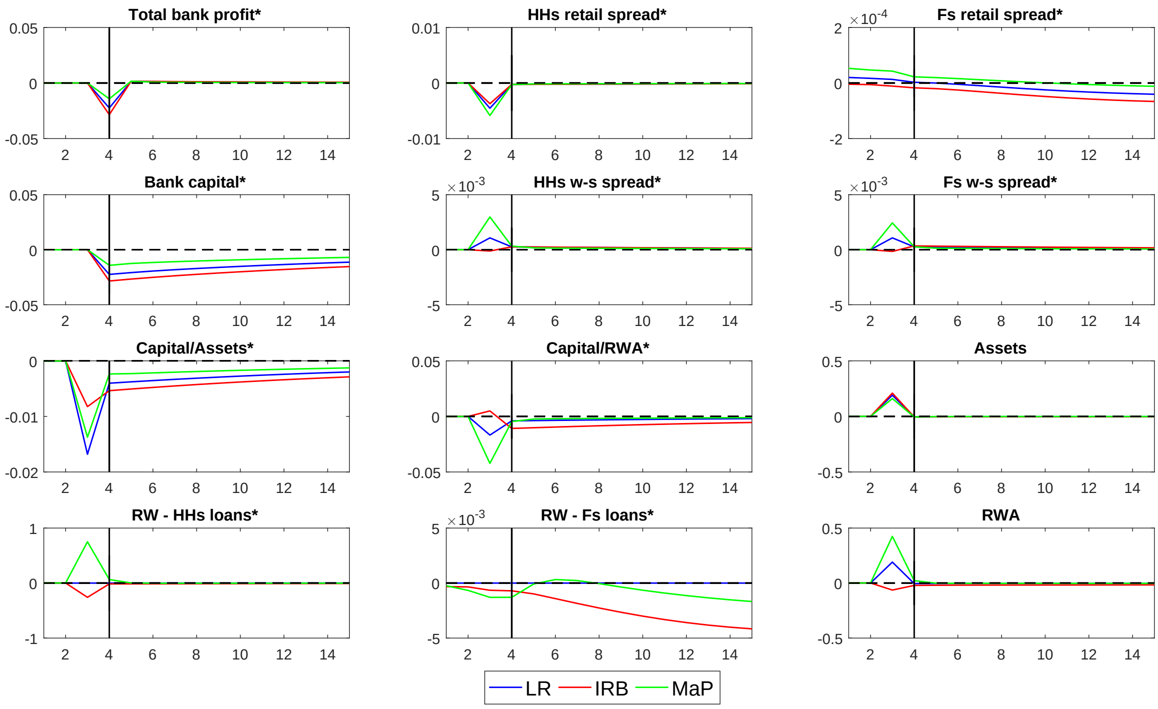


Figure 4.12: IRF - unrealized news shock to σ^i at period 4



4.6 Procyclicality and Welfare

In the previous sections, we compared the responses of the three different policy settings in two different crisis scenarios. Although that the scenarios represent closely the 2008 financial crisis, they cannot be used as a proof of procyclicality of the existing regulation in terms of all possible shocks that can occur in a longer simulation of the model. In order to represent the ability of the different settings to smoothen or amplify the business cycle, we report the variation of the main macroeconomic variables. In addition, we report the social welfare in terms of lifetime utility and consumption equivalence in each of the settings.

The welfare of each agent $j = \{P, I, E\}$ is given by the expected discounted sum of lifetime utility:

$$\Omega_t^j = \max E_t \left[\sum_{i=0}^{\infty} (\beta^j)^i U(C_{t+i}^j, H_{t+i}^j, L_{t+i}^j) \right] \tag{4.5}$$

which at the optimum has the following Bellman form:

$$\Omega_t^j = U(C_t^j, H_t^j, L_t^j) + \beta^j \Omega_{t+1}^j \tag{4.6}$$

Finally, we also compute an optimal macroprudential rule by setting the coefficients of the macroprudential policy rules so as to maximize the sum of welfares of the agents in our model. In doing so we study the ex-ante optimal simple rules based on the second-order approximate solution of the model. We also compute the welfare implied by the different policy rules conditional on the initial state being the deterministic steady-state. The policies are compared both in terms of levels of welfare and consumption-equivalent (CE) measure calculated as the percentage increase in steady-state consumption that would make welfare under the leverage ratio (static policy setting) equal to welfare under each of the policy settings. The table below reports the results.

Table 4.4: Variation and Welfare

Risk-weight setting	Standard Deveation				Welfare	Con. Eq.
	Income	**Cons.**	**Invest.**	**Borr.**	**Wel**	**CE**
Leverage Ratio	0.1101	0.0901	0.0298	0.9615	-347.07	0%
IRB	0.1105	0.0903	0.0306	0.9602	-351.94	-1.98%
Macroprudential $\rho_{\mathrm{w}} = 0.1103$ $\chi_{\mathrm{w}} = 1.9483$	0.1096	0.0899	0.0276	0.8844	-345.62	0.59%
Optimal Macroprudential $\rho_{\mathrm{w}} = -0.2023$ $\chi_{\mathrm{w}} = 4.9587$	0.1094	0.0897	0.0270	0.7662	-344.04	1.23%

As we can see, relatively to the static policy setting of the leverage ratio, the IRB setting of the risk-weights leads to higher variation in the macroeconomic variables and lower social welfare. On the other hand, the macroprudential rule smoothens the business cycle by decreasing the variation in the variables and as a result leads to higher social welfare. Finally, the optimal macroprudential rule leads to higher responsiveness of the risk-weights to leverage: $\chi_{\mathrm{w}} = 4.9587$ than the one in the non-optimal rule $\chi_{\mathrm{w}} = 1.9483$.[1]

[1] The higher responsiveness leads to better smoothing and higher welfare, however, an extremely responsive setting of the risk-weights is unrealistic from the point of view of a policy that is actually implementable. For this reason we use the macroprudential rule with the lower responsiveness for our main simulations and report all the crisis scenarios repeated with the optimal rule in appendix (6.6).

5 Conclusion

Bank capital regulation has evolved through time to incorporate risk-sensitivity, i.e. the idea that different asset classes contribute to credit risk to a different degree and hence capital charges should be proportionate to the riskiness of each asset class. This framework is incorporated by the introduction of risk-weights in capital-asset requirements. However, there has been an ongoing debate on the correct approach to measure the riskiness of assets that would allow for bank regulation that attenuates the financial cycle.

While most studies on this topic rely on empirical evidence and econometric models, we incorporate different methods to asset risk-weighting in a general equilibrium macro model. This approach allows us to investigate the effect of the different methods for setting risk-weights on the financial cycle, the macroeconomy, and on the resilience of the financial system in a crisis. We compare two active policy approaches. The Internal Ratings Based approach (IRB) sets risk-weights for each asset on the basis of estimates for PDs and LGDs. Alternatively, our macroprudential approach sets risk-weights for each asset type in a countercyclical manner – linking risk-weights to measures of leverage in each sector.

Our results show that in both boom and bust phases of the crisis the IRB approach leads to procyclical capital requirements. In the boom phase, the IRB approach leads to looser capital requirements and thereby to lending conditions that reinforce market exuberance. In the bust phase, higher risk estimates lead to higher risk-weights and tighter capital requirements that depress bank lending and push down on economic activity. The IRB approach therefore reinforces the financial cycle in the event of a crisis.

By contrast, our macroprudential approach to setting risk-weights leads to countercyclicality in capital requirements in both the boom and bust phases of the crisis – thereby serving to attenuate the financial cycle. The negative impact of the financial crash to the real economy is smaller and the recovery happens faster. These results are evident both in the specific crisis simulations as well as social welfare analysis. In the latter, the IRB approach leads to decrease of welfare due to amplifying the business cycle while our suggested setting leads to welfare improvement through attenuating economic fluctuations.

The intuition behind these results can be found by reflecting on the purpose of regulatory capital requirements. Bank capital requirements are enforced with a view to ensuring that banks hold enough capital to cover the potential Unexpected Losses (UL) associated with their assets. Expected Losses (EL) are to be covered by bank provisioning and credit pricing. In applying of the IRB approach we estimate the UL of each asset using the same parameters that are used for estimating the EL in bank provisioning - the PD and LGD. The IRB approach therefore leads to a positive relationship between estimates of the EL and UL – thereby leading to procyclical capital requirements. In a situation characterised by optimism in lending markets – the EL will be low, and likewise the IRB approach will tend to estimate lower UL, resulting in lower capital requirements.

Conversely, the macroprudential rule relates UL to sectoral measures of leverage – which will tend to make the UL move in the opposite direction of EL. Hence in an optimistic scenario where lending conditions are loose, EL is low and leverage is growing - the macroprudential capital rule sets tighter capital requirements to mitigate the potential boom situation.

Finally, our macroprudential approach to setting risk-weights is an alternative means of adjusting capital requirements for the state of the financial cycle in comparison to the Countercyclical Capital Buffer approach that is set out within Basel III. The macroprudential risk-weights we apply adjust to the sectoral financial cycle – thereby providing a more precise tool for reacting to sector-specific bubbles and credit booms. The sectoral risk-weights that we put forward in this analysis could focus macroprudential controls over capital requirements on those sectors that are experiencing booms, whilst avoiding unwarranted impacts on other non-bubble sectors. This would potentially concentrate the impact of the tool where it needs to be to stabilise capital flows, and reduce unintended spillover costs to other sectors.

6 Appendix

6.1 Data description

- **Consumption:** Household and NPISH final consumption expenditure, chain linked volumes (2010), seasonally adjusted and adjusted data by working days. Transformation: log deviation from HP-filtered mean. Source: Eurostat.

- **Investment:** Gross fixed capital formation, chain linked volumes (2010), seasonally adjusted and adjusted data by working days. Transformation: log deviation from HP-filtered mean. Source: Eurostat.

- **House prices:** Residential Property Valuation, new and existing dwellings, neither seasonally nor working day adjusted. Transformation: deflated by HICP inflation, log deviation from HP-filtered mean. Source: ECB.

- **Wages**: Labour cost index, whole economy excluding agriculture, fishing and government sectors, working day and seasonally adjusted. Transformation: deflated by HICP inflation, log deviation from HP-filtered mean. Source: Eurostat.

- **Inflation:** Harmonised Index of Consumer Prices (HICP), seasonally adjusted, not working day adjusted. Transformation: deviation from mean. Source: ECB.

- **Policy Rate:** Euribor 3-month - historical close, average of observations through period. Transformation: in gross quarterly form, deviation from mean. Source: ECB.

- **Borrowing rate - households:** Annualised agreed rate (AAR) / Narrowly defined effective rate (NDER), Credit and other institutions (MFI except MMFs and central banks), Lending for house purchase excluding revolving loans and overdrafts, convenience and extended credit card debt, Up to 1 year initial rate fixation, New business coverage, Households and NPISH. Transformation: in gross quarterly form, deviation from mean. Source: ECB.

- **Borrowing rate - firms:** Annualised agreed rate (AAR) / Narrowly defined effective rate (NDER), Credit and other institutions (MFI except MMFs and central banks) reporting sector - Loans other than revolving loans and overdrafts, convenience and extended credit card debt, Up to 1 year initial rate fixation, Up to and including EUR 1 million amount, New business coverage, Non-Financial corporations. Transformation: in gross quarterly form, deviation from mean. Source: ECB.

- **Borrowing volume - households:** Lending for house purchase, households and NPISH, outstanding amounts at the end of the period (stocks), neither seasonally nor working day adjusted .Transformation: deflated by HICP inflation, log deviation from HP-filtered mean. Source: ECB.

- **Borrowing volume - firms:** Loans to non-financial corporations, outstanding amounts at the end of the period (stocks), neither seasonally nor working day adjusted. Transformation: deflated by HICP inflation, log deviation from HP-filtered mean. Source: ECB.

- **Deposits:** Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector - Deposits with agreed maturity, Over 1 and up to 2 years maturity, All currencies combined - Euro area (changing composition) counterpart, Households and NPISH, denominated in Euro, neither seasonally nor working day adjusted. Transformation: deflated by HICP inflation, log deviation from HP-filtered mean. Source: ECB.

6.2 Spread expression

Given the spread equation (2.7) we have that the denominator is the following function of the ex-ante threshold $\bar{\omega}^{j,a}$:

$$X(\bar{\omega}^{j,a}) = \frac{(1-\mu)G(\bar{\omega}_t^{j,a}, \sigma_\omega^j)}{\bar{\omega}_t^{j,a}} + (1 - F(\bar{\omega}_t^{j,a}, \sigma_\omega^j)) \tag{6.1}$$

or expressed with integrals:

$$X(\bar{\omega}^{j,a}) = \frac{(1-\mu)\int_0^{\bar{\omega}_t^{j,a}} \omega f(\omega) d\omega}{\bar{\omega}_t^{j,a}} + 1 - F(\bar{\omega}^{j,a}) \tag{6.2}$$

where $f(\omega)$ is the PDF and $F(\omega)$ is the CDF of the log-normal distribution. In fact, the second therm in the RHS which is the probability of non-default, expressed as 1 - the probability of default, where the latter is just the CDF evaluated at $\bar{\omega}^{j,a}$. Then it is straightforward to see that as the CDF is increasing function in $\bar{\omega}^{j,a}$ then:

$$\frac{d(1 - F(\bar{\omega}^{j,a}))}{d\bar{\omega}^{j,a}} < 0 \tag{6.3}$$

is a decreasing function in $\bar{\omega}^{j,a}$. Then calculating the derivative of the of $X(\bar{\omega}^{j,a})$ wrt $\bar{\omega}^{j,a}$ we obtain:

$$\begin{aligned} \frac{dX(\bar{\omega}^{j,a})}{d\bar{\omega}^{j,a}} &= \frac{(1-\mu)}{\bar{\omega}^{j,a}} \bar{\omega}^{j,a} f(\bar{\omega}^{j,a}) - f(\bar{\omega}^{j,a}) - \\ \frac{(1-\mu)\int_0^{\bar{\omega}^{j,a}} \omega f(\omega) d\omega}{(\bar{\omega}^{j,a})^2} &+ \frac{d(1 - F(\bar{\omega}^{j,a}))}{d\bar{\omega}^{j,a}} \end{aligned} \tag{6.4}$$

which simplifies to:

$$\frac{dX(\bar{\omega}^{j,a})}{d\bar{\omega}^{j,a}} = -\mu f(\bar{\omega}^{j,a}) - \frac{(1-\mu)\int_0^{\bar{\omega}^{j,a}} \omega f(\omega) d\omega}{(\bar{\omega}^{j,a})^2} + \frac{d(1 - F(\bar{\omega}^{j,a}))}{d\bar{\omega}^{j,a}} \tag{6.5}$$

which is negative, meaning that $X(\bar{\omega}^{j,a})$ is decreasing fuction of the ex-ante threshold $\bar{\omega}^{j,a}$. Then as we have from equation (2.7) the spread is:

$$\frac{r_t^j}{R_t^b} = \frac{1}{\frac{(1-\mu)G(\bar{\omega}_t^{j,a},\sigma_\omega^j)}{\bar{\omega}_t^{j,a}} + (1 - F(\bar{\omega}_t^{j,a}, \sigma_\omega^j))} = \frac{1}{X(\bar{\omega}^{j,a})} \tag{6.6}$$

hence, the spread is an increasing function of the ex-ante threshold such that:

$$\frac{r_t^j}{R_t^b} = f(\bar{\omega}^{j,a}), f'() > 0 \tag{6.7}$$

6.3 Profits expression

Starting from the equation of profits then dividing by the borrowing quantity b_{t-1}^j and substituting the ex-post threshold, we obtain profits per unit of loans as:

$$\frac{\Pi_t^j}{b_{t-1}^j} = (1-\mu)G(\bar{\omega}_t^{j,p}, \sigma_{\omega^j}^j)\frac{r_{t-1}^j}{\bar{\omega}_t^{j,p}} + (1 - F(\bar{\omega}_t^{j,p}, \sigma_{\omega^j}^j))r_{t-1}^j - R_{t-1}^b \tag{6.8}$$

then from evaluating the participation constraint (2.6) in period $t-1$ and substituting the ex-ante threshold (2.5) in period $t-1$, $\bar{\omega}_{t-1}^{j,a}$ in it we have that:

$$R_{t-1}^b = (1-\mu)G(\bar{\omega}_{t-1}^{j,a}, \sigma_\omega^j)\frac{r_{t-1}^j}{\bar{\omega}_{t-1}^{j,a}} + (1 - F(\bar{\omega}_{t-1}^{j,a}, \sigma_\omega^j))r_{t-1}^j \tag{6.9}$$

which can be substituted in (6.8) leading to:

$$\begin{aligned} \frac{\Pi_t^j}{b_{t-1}^j} = r_{t-1}^j \Bigg[& \frac{(1-\mu)G(\bar{\omega}_t^{j,p}, \sigma_{\omega^j}^j)}{\bar{\omega}_t^{j,p}} + (1 - F(\bar{\omega}_t^{j,p}, \sigma_{\omega^j}^j)) - \\ & \left(\frac{(1-\mu)G(\bar{\omega}_{t-1}^{j,a}, \sigma_\omega^j)}{\bar{\omega}_{t-1}^{j,a}} + (1 - F(\bar{\omega}_{t-1}^{j,a}, \sigma_\omega^j)) \right) \Bigg] \end{aligned} \tag{6.10}$$

Then using the formulation of $X(\bar{\omega}^{j,a})$ in (17), the last equation becomes:

$$\Pi_t^j = b_{t-1}^j r_{t-1}^j \left[- \left(X(\bar{\omega}_t^{j,a}) - X(\bar{\omega}_{t-1}^{j,p}) \right) \right] \tag{6.11}$$

And since we have showed in 6.2 that $X(\bar{\omega}^{j,a})$ is a decreasing function in $\bar{\omega}^{j,a}$, then for any $\bar{\omega}_{t-1}^{j,a} = \bar{\omega}_t^{j,p}$ the above expression would be zero, and for any $\bar{\omega}_{t-1}^{j,a} > \bar{\omega}_t^{j,p}$ we would have that $X(\bar{\omega}_t^{j,p}) > X(\bar{\omega}_{t-1}^{j,a})$ and that $\Pi_t^j > 0$ leading to:

$$\Pi_t^j = f(\bar{\omega}_{t-1}^{j,a} - \bar{\omega}_t^{j,p}), \ f'() > 0 \tag{6.12}$$

6.4 Model first order conditions

Patient households (Savers)

PHHs choose: C_t^P, H_t^P, and L_t^P to maximize:

$$E_0 \sum_{t=0}^{\infty} (\beta^P)^t U_t = E_0 \sum_{t=0}^{\infty} (\beta^P)^t \Bigg[(1-\alpha^p)\varepsilon_t^c \, log(C_t^P(j) - \alpha^P C_{t-1}^P) + \\ \varepsilon_t^h \, log(H_t^P(j)) - \frac{(L_t^P(j))^{1+\phi}}{1+\phi} \Bigg] \tag{6.13}$$

subject to:

$$C_t^P(j) + q_t^h \Delta H_t^P(j) + D_t(j) = W_t L_t^P(j) + \frac{R_{t-1} D_{t-1}(j)}{\pi_t} + T_t(j) \tag{6.14}$$

If we denote marginal utility of consumption with:

$$U_{C_t^P} = \Lambda_t^P = \frac{(1-\alpha^p)\varepsilon_t^c}{C_t^P - \alpha^p C_{t-1}^P} \tag{6.15}$$

then substituting eq(31) for C_t and C_{t+1} into lifetime utility and differentiating wrt. D_t we obtain the following Euler equation:

$$\Lambda_t^P = \beta^P \Lambda_{t+1}^P \frac{R_t}{\pi_{t+1}} \tag{6.16}$$

Then differentiating the infinite sum of discounted utility wrt. H_t^P gives the demand for housing:

$$\Lambda_t^P q_t^h = \frac{\varepsilon_t^h}{H_t^P} + \beta^P \Lambda_{t+1}^P q_{t+1}^h \tag{6.17}$$

Finally differentiating wrt. leisure L_t^P, we obtain the labour supply:

$$\Lambda_t^P = \frac{(L_t^P)^\phi}{W_t^P} \tag{6.18}$$

Impatient households (Borrowers)

IHHs choose: C_t^I, H_t^I, and L_t^I to maximize:

$$E_0 \sum_{t=0}^{\infty} (\beta^I)^t U_t = E_0 \sum_{t=0}^{\infty} (\beta^I)^t \Big[(1-\alpha^I)\varepsilon_t^c \, log(C_t^I(i) - \alpha^I C_{t-1}^I) + \varepsilon_t^h \, log(H_t^I(i)) - \frac{(L_t^I(i))^{1+\phi}}{1+\phi} \Big] \tag{6.19}$$

subject to the budget constraint:

$$C_t^I + q_t^h \Delta H_t^I + \frac{(1-F_t^p) r_{t-1}^I B_{t-1}^I}{\pi_t} + q_t^h H_{t-1}^I G_t^p = B_t^I + W_t L_t^I \tag{6.20}$$

and collateral constraint:

$$r_t^I B_t^I / \pi_{t+1} = m_t^h q_{t+1}^h H_t^I \tag{6.21}$$

If we denote marginal utility of consumption with:

$$U_{C_t^I} = \Lambda_t^I = \frac{(1-\alpha^I)\varepsilon_t^c}{C_t^I - \alpha^I C_{t-1}^I} \tag{6.22}$$

By constructing a Lagrangian with the collateral constraint and S_t^I being its shadow value we obtain:

$$\mathcal{L}^I = E_0 \sum_{t=0}^{\infty} (\beta^I)^t [U_t + S_t^I (m_t^h q_{t+1}^h H_t^I - \frac{r_t^I B_t^I}{\pi_{t+1}})] \tag{6.23}$$

substituting the budget constraint for C_t^I and C_{t+1}^I and differentiating wrt. B_t^I we obtain the following Euler equation:

$$\Lambda_t^I = \frac{\beta^I \Lambda_{t+1}^I r_t^I}{\pi_{t+1}} + \frac{S_t^I r_t^I}{\pi_{t+1}} \tag{6.24}$$

Differetiating wrt H_t gives the following housing demand:

$$\Lambda_t^I q_t^h = \beta^I \Lambda_{t+1}^I q_{t+1}^h + \frac{\varepsilon^h}{H_t^I} + S^I m_t^h q_t^h \tag{6.25}$$

Lastly, labour supply:

$$\Lambda_t^I = \frac{(L_t^I)^\phi}{W_t^I} \tag{6.26}$$

Entrepreneurs

Choose consumption C_t^E, physical capital K_t^E, loans from banks B_t^E, degree of capital utilization, and labour inputs from patient and impatient households L_t^P, L_t^I to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta_I^t \Big[(1-\alpha^E) log(C_t^E(i) - \alpha^E C_{t-1}^E) \Big] \tag{6.27}$$

subject to:

$$\begin{aligned} & C_t^E + W_t^P L_t^P + W_t^I L_t^I + \frac{(1-F_t^{p,E}) r_{t-1}^E B_{t-1}^E(i)}{\pi_t} + q_t^k [K_t^E - (1-\delta) K_{t-1}^E] + \\ & q_t^k K_{t-1}^E G_t^{p,E} = \frac{Y_t^E}{X_t} + B_t^E \end{aligned} \tag{6.28}$$

with production function:

$$Y_t^E(i) = A_t^E K_{t-1}^E(i)^\alpha L_t^E(i)^{1-\alpha} \tag{6.29}$$

where: $L_t^E = (L_t^P)^\nu (L_t^I)^{1-\nu}$

subject to a budget constraint:

$$r_t^E B_t^E / \pi_{t+1} \leq m_t^f q_{t+1}^k K_t^E \tag{6.30}$$

Denoting marginal utility of consumption as:

$$\Lambda_t^E = \frac{(1-\alpha^e)}{C_t^E - \alpha^e C_{t-1}^E} \tag{6.31}$$

Constructing Lagrangian with S^E being the shadow value of the collateral constraint, then differentiating wrt. K_t^E leads to:

$$\Lambda_t^E q_t^k = \Lambda_{t+1}^E \beta^E \Big(q_{t+1}^k (1-\delta) + r_{t+1}^k \Big) + S^E m_t^f q_{t+1}^k \tag{6.32}$$

where r_t^k is the rental rate of capital: $r_t^k = \frac{\alpha Y_t^E}{K_{t-1}^E} \frac{1}{X_t}$
For labour demand we have MP of each labour type equal to its MC:

$$W_t^P = \frac{\nu(1-\alpha)Y_t^E}{L_t^P X_t} \qquad W_t^I = \frac{(1-\nu)(1-\alpha)Y_t^E}{L_t^I X_t} \tag{6.33}$$

Finally the Euler equation is:

$$\Lambda_t^E = \frac{\Lambda_{t+1}^E \beta^E r_t^E}{\pi_{t+1}} + \frac{S^E r_t^E}{\pi_{t+1}} \tag{6.34}$$

Capital Producers

Using the discount factor of entrepreneurs (as being owned by them), capital producers maximize:

$$E_0 \sum_{t=0}^{\infty} \Lambda_t^E (\beta^E)^t \Big[q_t^k \Delta x_t - I_t \Big] \tag{6.35}$$

by choosing Δx_t and I_t subject to the following constraint:

$$\Delta x_t = \left[1 - \frac{\kappa_i}{2} \left(\frac{I_t \varepsilon_t^k}{I_{t-1}} - 1 \right)^2 \right] I_t \tag{6.36}$$

Where, $\Delta x_t = K_t - (1-\delta)K_{t-1}$. Differentiating wrt. I_t we obtain:

$$\Lambda_t^E \left[q_t^k \frac{\partial \Delta x_t}{\partial I_t} - 1 \right] + \Lambda_{t+1}^E \beta^E \left[q_{t+1}^k \frac{\partial \Delta x_{t+1}}{\partial I_t} \right] = 0 \tag{6.37}$$

for the partial derivatives we obtain:

$$\frac{\partial \Delta x_t}{\partial I_t} = 1 - \frac{\kappa_i}{2} \left(\frac{I_t \varepsilon_t^k}{I_{t-1}} - 1 \right)^2 - \kappa_i \left(\frac{I_t \varepsilon_t^k}{I_{t-1}} - 1 \right) \frac{I_t \varepsilon_t^k}{I_{t-1}} \tag{6.38}$$

$$\frac{\partial \Delta x_{t+1}}{\partial I_t} = \kappa_i \left(\frac{I_{t+1} \varepsilon_{t+1}^k}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \varepsilon_{t+1}^k \tag{6.39}$$

substituting the last two into 52 we obtain the optimality condition:

$$1 = q_t^k \left[1 - \frac{\kappa_i}{2} \left(\frac{I_t \varepsilon_t^k}{I_{t-1}} - 1 \right)^2 - \kappa_i \left(\frac{I_t \varepsilon_t^k}{I_{t-1}} - 1 \right) \frac{I_t \varepsilon_t^k}{I_{t-1}} \right] +$$
$$\beta^E E_t \left[\frac{\Lambda_{t+1}^E q_{t+1}^k \varepsilon_{t+1}^k}{\Lambda_t^E} \kappa_i \left(\frac{I_{t+1} \varepsilon_{t+1}^k}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right]$$

Retailers

Thus retailers choose $P_t(j)$ to maximize:

$$E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^P \left[P_t(j) Y_t(j) - P_t^W Y_t(j) - \frac{\kappa_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}^{i_p} \pi^{1-i_p} \right)^2 P_t Y_t \right] \tag{6.40}$$

subject to: $Y_t(j) = (\frac{P_t(j)}{P_t})^{-\epsilon_t^y} Y_t$.

Thus the part of the infinite sum that includes $P_t(j)$ is:

$$\begin{aligned} \sum^R = \Lambda_t^P \left[Y_t(j)(P_t(j) - P_t^W) - \frac{\kappa_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}^{i_p} \pi^{1-i_p} \right)^2 P_t Y_t \right] + \\ \Lambda_{t+1}^P \beta^P \left[Y_{t+1}(j)(P_{t+1}(j) - P_{t+1}^W) - \frac{\kappa_p}{2} \left(\frac{P_{t+1}(j)}{P_t(j)} - \pi_t^{i_p} \pi^{1-i_p} \right)^2 P_{t+1} Y_{t+1} \right] \end{aligned} \tag{6.41}$$

Differentiating wrt. $P_t(j)$ and imposing $P_t(j) = P_t$ leads to:

$$\begin{aligned} \Lambda_t^P \left[-\epsilon_t^y Y_t + \frac{\epsilon_t^y Y_t}{X_t} + Y_t - \kappa_p (\pi_t - \pi_{t-1}^{i_p} \pi^{1-i_p}) P_t Y_t \frac{1}{P_{t-1}(j)} \right] + \\ \Lambda_{t+1}^P \beta^P \left[\kappa_p (\pi_{t+1} - \pi_t^{i_p} \pi^{1-i_p}) P_{t+1} Y_{t+1} \frac{P_{t+1}(j)}{P_t^2(j)} \right] = 0 \end{aligned} \tag{6.42}$$

which after dividing by Y_t and Λ_t^P simplifies to:

$$1 - \epsilon_t^y + \frac{\epsilon_t^y}{X_t} - \kappa_p (\pi_t - \pi_{t-1}^{i_p} \pi^{1-i_p}) \pi_t + \frac{\Lambda_{t+1}^P \beta^P}{\Lambda_t^P} \kappa_p (\pi_{t+1} - \pi_t^{i_p} \pi^{1-i_p}) \frac{Y_{t+1}}{Y_t} \pi_{t+1}^2 = 0 \tag{6.43}$$

where we use that $1/X = P_t^W / P_t$ and $\pi_t = P_t / P_{t-1}$

The profits of retailers that are transferred back to savers are:

$$J_t^R = Y_t(1 - \frac{1}{X_t}) - \kappa_p(\pi_t - \pi_{t-1}^{i_p}\pi^{1-i_p})^2 \tag{6.44}$$

6.5 Model responses to standard shocks

In this section, we report the behaviour of the estimated model in terms of responses to technology and monetary shocks. In doing so we compare our results with Gerali et al. (2010) as a closely related model. The responses of our model are closely related to the benchmark simulations - BK of Gerali et al. (2010). However, a major difference arises due to the different structure of the models in terms of bank capital. In Gerali et al. (2010), the interest rate setting at the retail level is driven entirely by sticky prices, due to the absence of defaulting loans in the model. On the other hand, our model features defaulting loans and hence banks are facing not only a proportion of non-repaid loans but are exposed to the value of the repossessed collateral. As a result, in our setting bank profits and bank capital are driven not only by the interest rate margin but the proportion of defaulted loans and the value of the repossessed collateral. Therefore, a lower than expected default rate with higher than expected value of reposessed collateral leads to positive profits by banks and higher capital[1] - which can be seen in the technology shock. On the contrary, for the same shock Gerali et al. (2010) reports countercyclical bank capital which is counterfactual.

[1]See section 2.1 for bank profits equations.

Figure 6.13: IRF - Technology shock

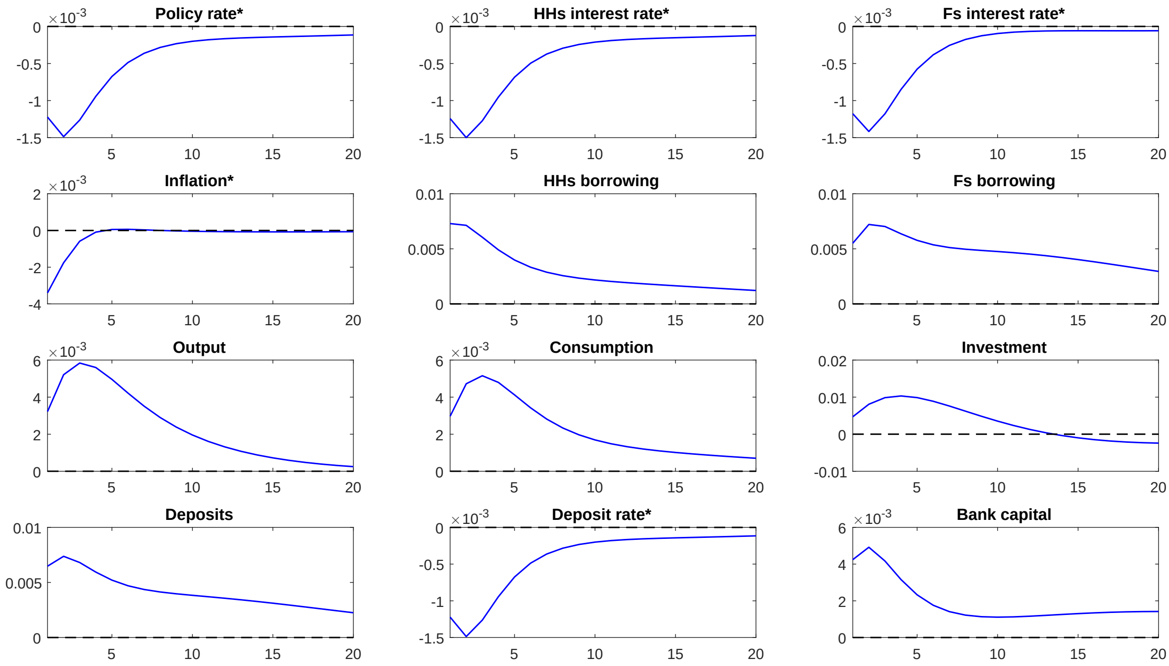


Figure 6.14: IRF - Technology shock - (Gerali et al., 2010)

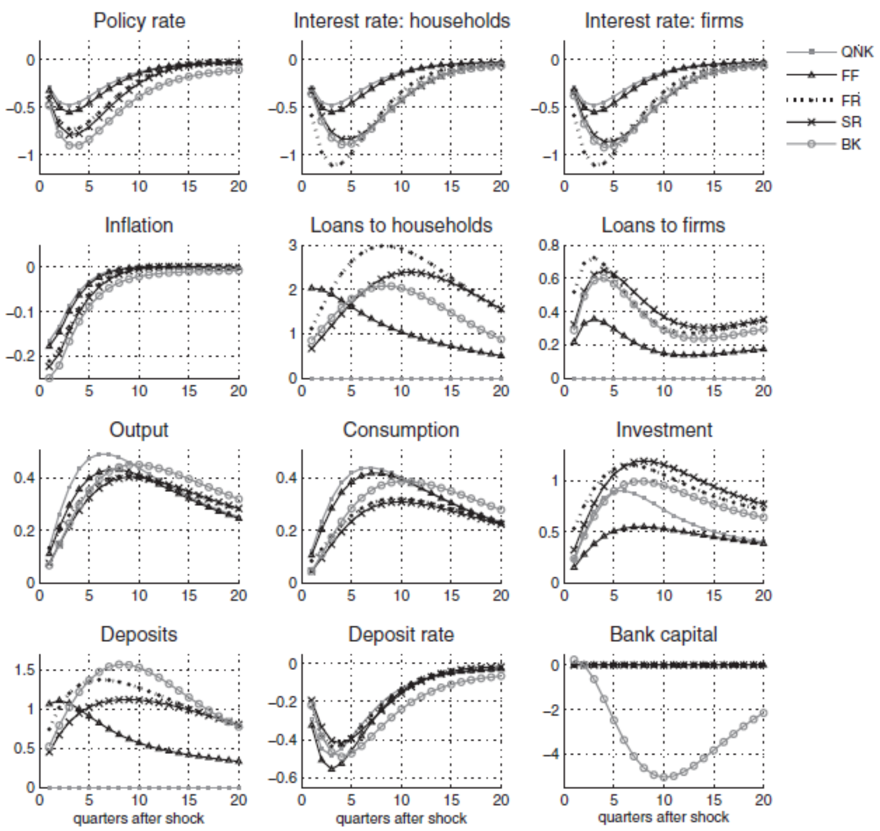


Figure 6.15: IRF - Monetary Policy shock

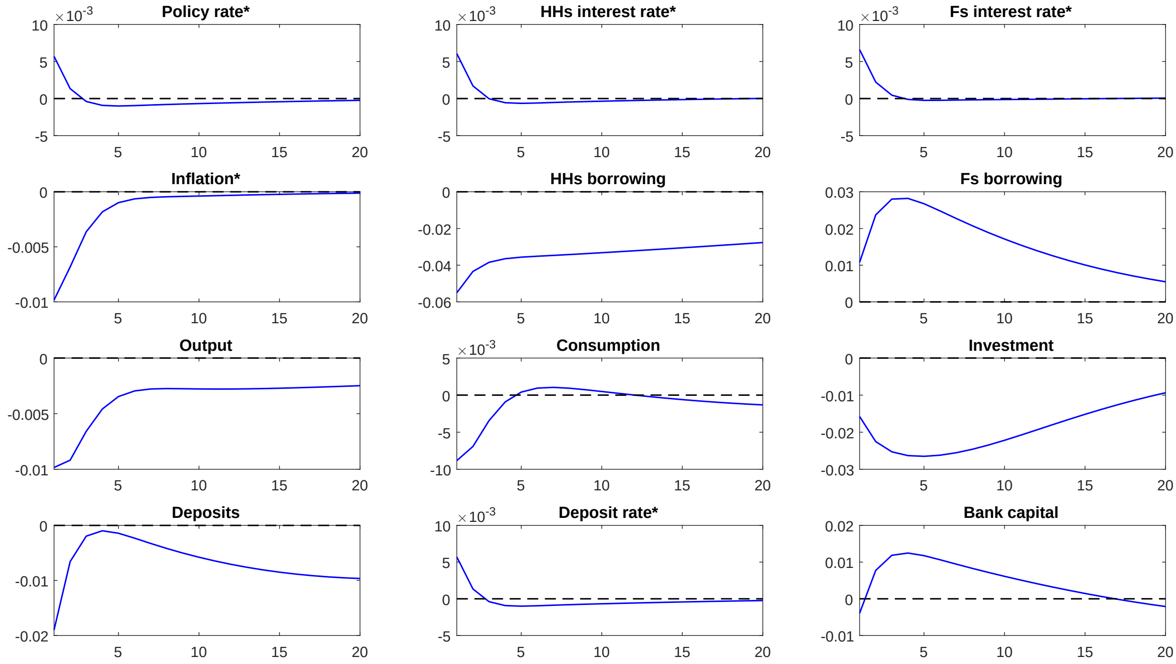
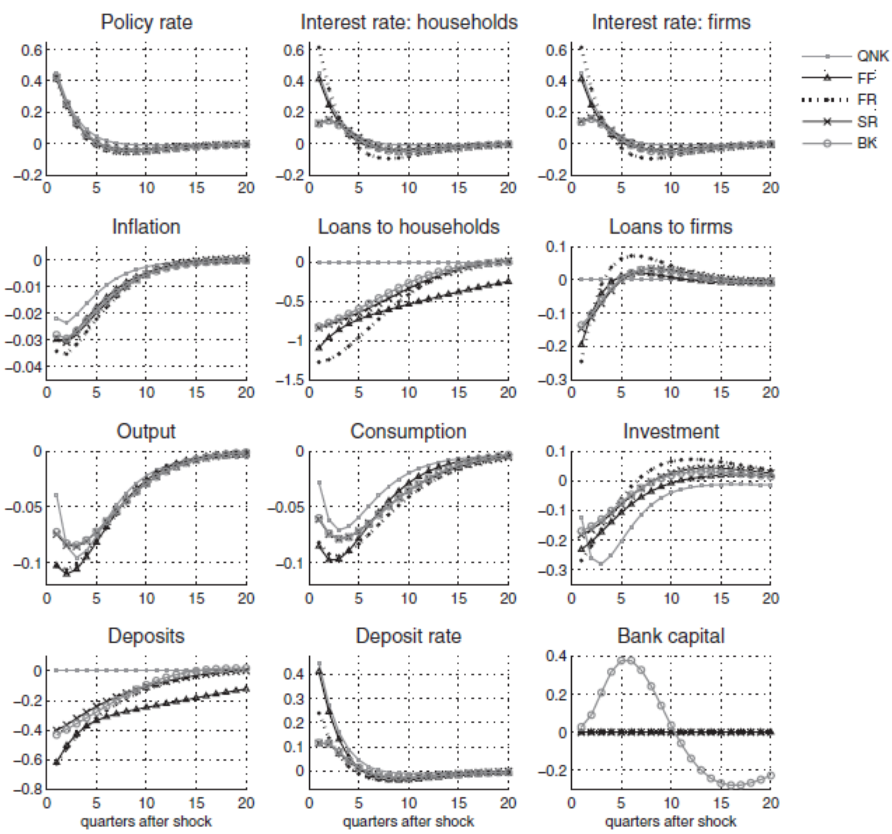


Figure 6.16: IRF - Monetary Policy shock - (Gerali et al., 2010)



6.6 Optimal Macroprudential Rule

Figure 6.17: IRF - unexpected shock to σ^i

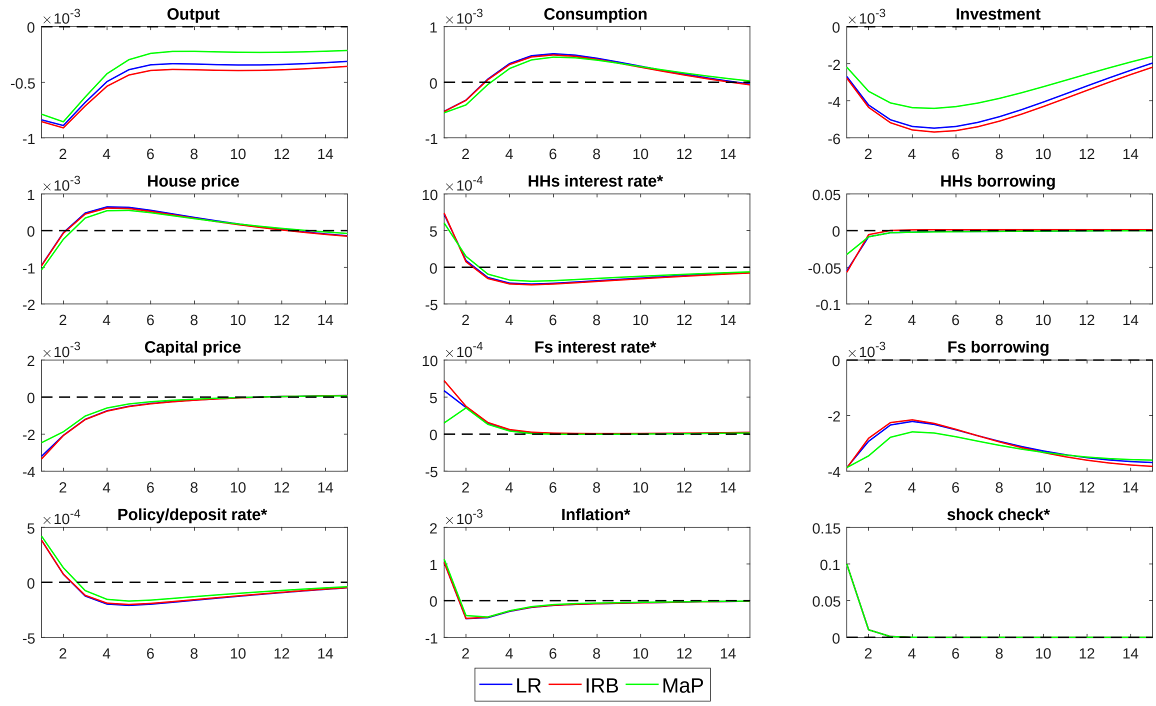


Figure 6.18: IRF - unexpected shock to σ^i

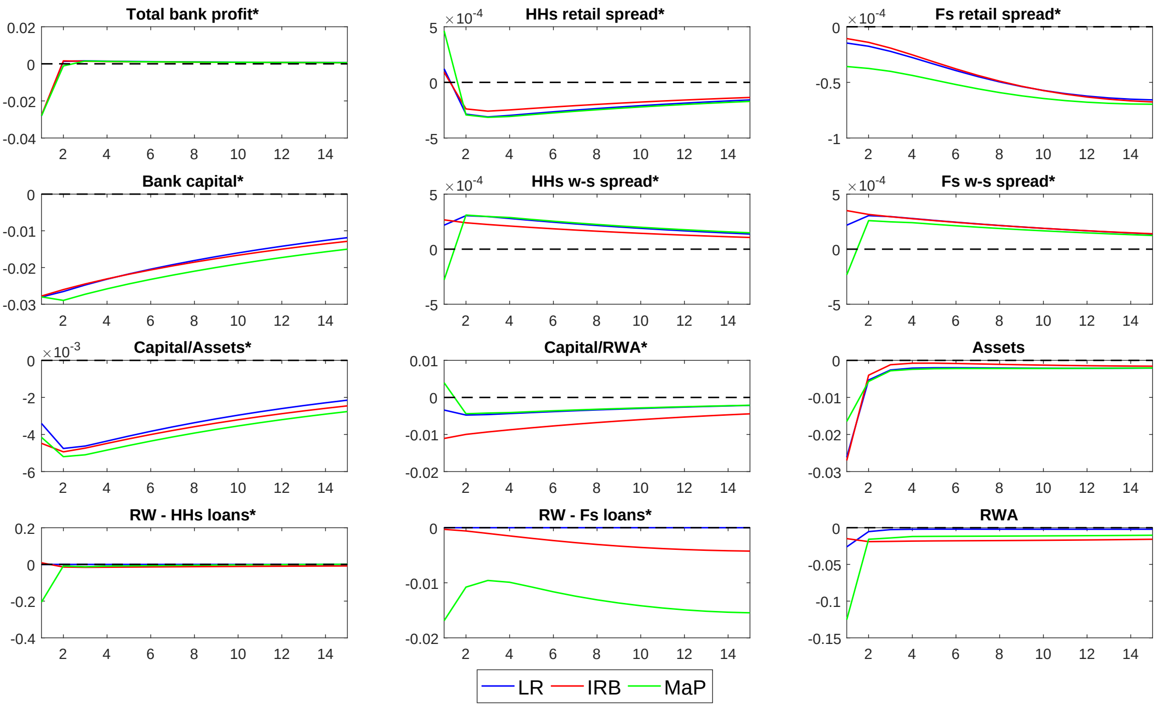


Figure 6.19: IRF - unrealized news shock to σ^i at period 4

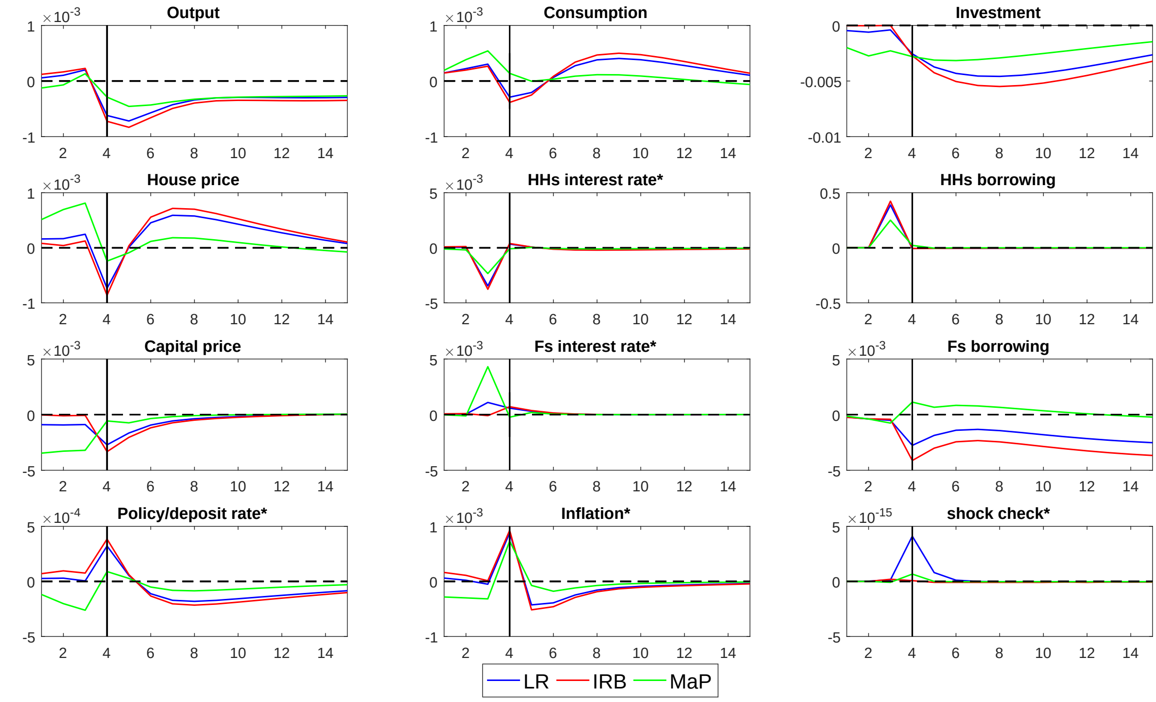
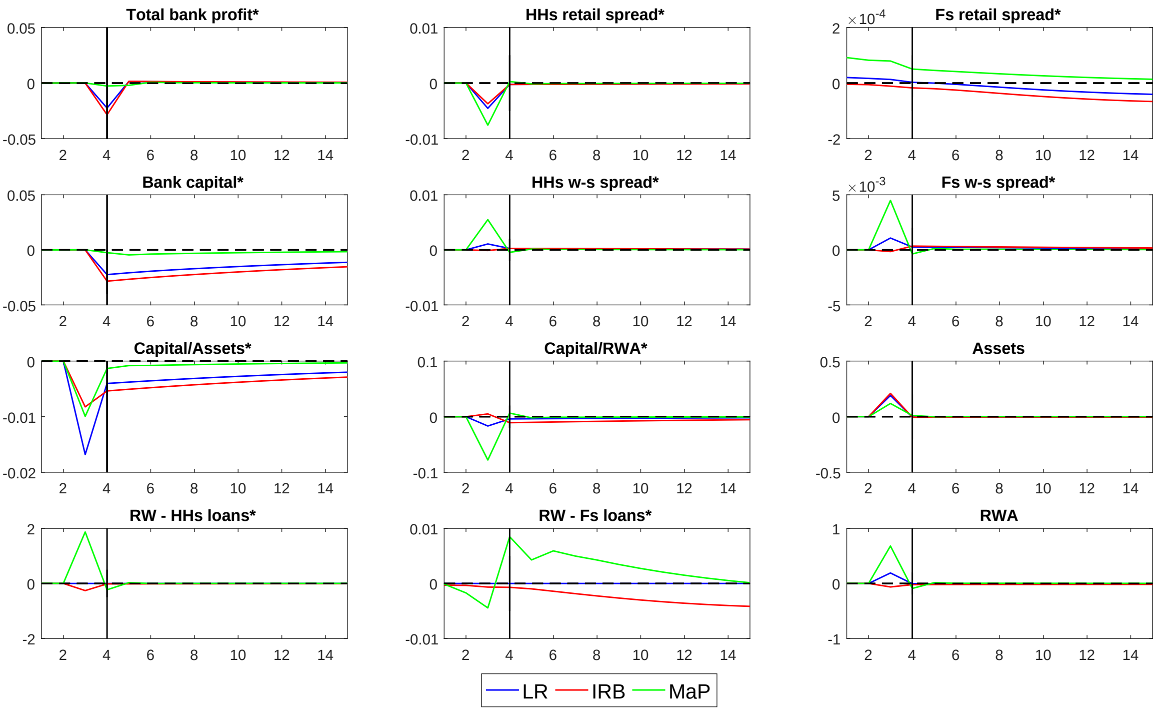


Figure 6.20: IRF - unrealized news shock to σ^i at period 4



Chapter III

Underestimating Portfolio Losses

Abstract

This paper starts from the foundation model of the Basel capital regulation and extends it by developing its economic structure at several steps. Namely, the asymptotic single risk factor (ASRF) framework is expanded by incorporating rational behaviour of borrowers, banks, and investors. Each stage introduces a specific feedback between the portfolio loss distribution and the behaviour of the agents. First, the optimal default choice of borrowers, creates a positive relationship between the probability of default (PD) and loss given default (LGD), which are both related negatively to borrowers' net worth. Second, the insurance of risk by banks, creates a negative relationship between spreads and borrowers' net worth. Third, the required risk premia by investors, creates a positive relationship between the riskiness of the credit portfolio of the bank and its funding costs. Taken independently or together, each of these effects leads to a financial accelerator that reinforces adverse shocks and their impact on the financial system. Finally, the resulting framework is able to give insight into the relationship between bank capital and cost of funds. In summary, our results are strongly of favour of higher capital requirements. First, the illustrated financial accelerators lead to much riskier financial system than previously thought and second insufficient capital levels can incentivise bankers to engage in riskier lending. Our results have implications for risk management practices of private banks as well as for capital regulation by central banks

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JEL classifications: C61, E44, E58, E61, G21, G28.

Keywords: ASRF, financial accelerators, macroprudential policy, capital requirements, credit risk, portfolio loss.

1 Introduction

Financial crises are characterized by major deviations in the behaviour of agents and macroeconomic indicators from the one that is observed in normal times. As a result, economic relationships become inherently nonlinear when subject to large adverse shocks as emphasized by authors such as Milne (2009). Furthermore, there can be powerful interactions between asset returns, banks' balance sheets, lending conditions, and the real economy that reinforce such effects (Borio, 2012).

Conventional linearized general-equilibrium models are not very suitable for studying such unusual conditions and macroprudential policies aimed to address them. First, because by being linearized they do not capture the effects of the nonlinear relationships and second, because they underestimate the role of banks and their behaviour in contributing to these nonlinearities. On the other hand, the models that focus solely on banks and the value of their portfolio, abstract from important economic interactions which play a key role in crisis scenarios.

In the class of general equilibrium models, the literature in financial frictions is the one that focuses on banks and their interaction with the real economy. The cornerstone of this literature is the so called "financial accelerator" first illustrated by Bernanke et al. (1999) - BGG and Kiyotaki and Moore (1997) and later developed by Gertler and Kiyotaki (2010). In simple words, the accelerator is a reinforcement channel between the net worth of an agent and lending conditions which can amplify the business cycle in both good and bad times. The literature usually focuses on macroprudential policy that aims to mitigate the impact of this amplification mechanism. However, due to their size as a general equilibrium frameworks, these models are usually victims of various simplifications that enable their solution under rational expectations. For example, the absence of risk in lending activity is present in Kiyotaki and Moore (1997) and Iacoviello (2005) due to the presence of binding constraints that exclude the possibility of defaulting loans. In Bernanke et al. (1999), lenders are not exposed to credit risk as they can vary interest rates due to state-contingent contracts and thus guard themselves from unexpected losses. On the other hand, the Gertler and Kiyotaki (2010) class of models introduce financial crises as exogenous destruction of bank capital which does not arise as a consequence of risky bank lending. A newer generation of DSGE models by Quint and Rabanal (2014) and Forlati and Lambertini (2011) amend the BGG framework with fixed interest contracts. In such environments, banks can diversify idiosyncratic risk through spreads, but become prone to aggregate shocks due to the non-contingent contracts, and as a result banks' profits/losses become endogenously determined by borrowers' net worth and aggregate risk. However, even in the latter models, the analysis of large aggregate shocks is not possible due to the linearized solution methods. This prevents the derivation of bank loss distribution which is crucial for analysing policies that aim to cover bank losses in rare extreme scenarios, such as capital requirements.

Another strand of literature focuses solely on banks and portfolio value theory. The framework developed by Merton (1974) and Vasicek (2002) presents bank lending as subject to both idiosyncratic and aggregate risk. While the former can be insured, banks remain prone to the latter which introduces endogenously the need of capital buffers to guard against the remaining single risk factor. Furthermore, the simplified structure of the model enables nonlinear solution and derivation of the portfolio loss distribution. In addition, the structure also allows for closed form analytical solution which is particularly appealing to regulators and policymakers which is why the Vasicek model is still employed by the Basel regulation for capital requirements.[1] On the other hand, the structure that allows for analytical solution comes at the cost of ignoring important agent behaviour that enables the financial accelerator amplification which is well known in the general equilibrium literature. We argue that the financial accelerator mechanisms are crucial for the design of macroprudential policy, as they can amplify portfolio losses and affect their distribution which in turn should be taken into account by bank regulation, in order to quantify and guard against bank losses.

This paper bridges the gap between the literature of financial frictions and portfolio value theory. It expands the former with aggregate risk for the expense of the general equilibrium solution and the latter with economic structure and behaviour for the expense of analytical solution. The paper presents a framework that has been specifically designed to analyse the central role of banks and the

[1] The Vasicek formula is the cornerstone of bank capital regulation and is used for the calculation of asset risk-weights in capital requirements. See (BCBS, 2005) and EU Capital Requirement Regulation IV - 2013.

interaction of their behaviour with the one of borrowers and investors. As a result, the model incorporates important financial accelerator mechanisms between borrowers' net worth, banks' balance sheets and risk premia which are crucial for the analysis of bank regulation. Our representative bank is exposed to a large portfolio of loans with diversifiable idiosyncratic risk due to portfolio size and non-diversifiable aggregate risk.

First, by analysing optimal borrowers' default choice, we derive a positive relationship between the default rate of the portfolio and the loss-given-default of each loan. The relationship arises due to the fact that in adverse aggregate scenarios, the reason for the larger default rate is the cheaper collateral which borrowers prefer to give up rather than to repay the loan. As a result, not only that more loans default but banks repossess cheaper collateral which increases their losses in adverse scenarios, leading to the first financial accelerator of this paper.

Second, by analysing the process of insurance of idiosyncratic risk by banks, we derive a spread setting behaviour of lenders which creates a negative relationship between borrowing costs and the net worth of borrowers. The relationship arises due to the risk-pricing behaviour of banks to set larger spreads to riskier portfolios with higher loan-to-value. The higher interest rates in turn increase borrowers' owed amount and loan-to-value and thus make them riskier for the bank, leading to the second financial accelerator of this paper.

Finally, by deriving the loss distribution of the bank's portfolio we analyse the required risk premia by the investors of the bank, which leads to a positive relationship between the costs of funding for the bank and the riskiness of its portfolio. This leads to the third financial accelerator of this paper which also interacts with the other two channels. For example, an increase of the riskiness of the bank's portfolio leads to higher costs of funding for the bank. The latter costs are passed on to the borrowers as higher interest rates which increases their chance of default. As a result, portfolio riskiness increases as well, thus closing the accelerator cycle.

We track how the introduction of each of the three amplification mechanisms affects the portfolio loss distribution - taken independently or all together. Our results illustrate how ignoring of these channels can underestimate portfolio losses and lead to insufficient capital regulation.

After calibration of the model to match key US banking sector statistics, we find that non-recognizing the illustrated financial accelerators can lead to an underestimation of the probability of default of a bank of a magnitude between 1.3 and 5.2 times for a given level of capital. And vice versa, if banks want to achieve a certain probability of default corresponding to their target rating, they would need higher amounts of capital.

Finally, we employ the developed framework for obtaining valuable insight regarding the relationship of bank capital level and the risk-taking incentives for the bank. Our observation is that, per se, the level of bank capital doesn't affect the underlying portfolio riskiness and hence, banks' cost of funding. However, bank capital affects the shares in which the underlying risk is borne between debt holders and equity holders. As a result, under deposit insurance, banks have clear incentives to engage in riskier lending when the level of capital is low and larger proportion of the risk is taken by depositors while being priced as risk-free.

In summary, our results are strongly of favour of higher capital requirements. First, the illustrated financial accelerators lead to much riskier financial system than previously thought and second insufficient capital levels can incentivise bankers to engage in riskier lending. While the results have important implications for bank regulation. They also provide insight into optimal risk management, provisioning and risk pricing by private banks that is consistent with the existing regulatory framework.

The paper proceeds as follows. Section 2 describes the starting point of the model, namely the ASFR. The following sections 3,4 and 5 expand the model by respectively introducing rational behaviour of borrowers, banks, and bank investors. Finally, section 6 uses the developed framework to quantify the resulting loss underestimation and analyse different approaches to bank provisioning and the relationship between bank cost of funding and the level of capital.

2 The ASRF and the financial accelerator

The ASRF - asymptotic single risk factor framework developed by Vasicek (2002) is the conceptual foundation of the internal rating-based approach defined in Basel II. The main implication of the model is that the default rate in a large homogeneous portfolio of non-contingent loans can be expressed as a result of a single aggregate risk factor. While each individual default can be driven by a combination of idiosyncratic and aggregate shocks, on a portfolio level the idiosyncratic risk can be diversified and thus the portfolio loss is determined solely by the realization of the aggregate factor that cannot be diversified.

A representative bank holds a portfolio of one period, non-state contingent loans extended to a large number of borrowers indexed by $i = 1, ...n$. The total size of the portfolio B_t consists of the individual exposures b_t^i, where $B_t = \sum b_t^i$. At period t, the bank and borrower i sign a contract and agree on the loan amount b_t^i, a non-contingent gross interest rate r_t^i and collateral $r_{t+1}^{i,k} k_t^i$ - consisting of return $r_{t+1}^{i,k}$ and capital amount k_t^i. At the time of repayment $t+1$, the return on collateral is observed, after realization of aggregate z_{t+1} and idiosyncratic shocks ω_{t+1}^i thus becoming $r_{t+1}^{i,k} = z_{t+1}\omega_{t+1}^i r_t^k$.[1] Borrower i makes the rational choice of defaulting and giving up the collateral after realization of its return $z_{t+1}\omega_{t+1}^i r_t^k k_t^i$, or repaying the loan $r_t^i b_t^i$, by choosing to give up the lesser amount. Hence, the borrower would default if the realized return falls below a predetermined default threshold:

$$z_{t+1}\omega_{t+1}^i < \frac{r_t^i b_t^i}{r_t^k k_t^i} = ltv_t^i \tag{2.1}$$

we can denote the RHS of the equation above as the loan-to-value of borrower i, ltv_t^i which is known at the time of signing the contract. After taking a logarithm of both sides of equation (2.1), the default condition takes the discrete-time version of Merton (1974):

$$\ln z_{t+1} + \ln \omega_{t+1}^i < \ln ltv_t^i \tag{2.2}$$

[1] We assume that the return on capital after the realization of uncertainty r_t^k is known, identical for all agents and exogenously determined by macroeconomic conditions.

The distribution of the shocks z_{t+1} and ω^i_{t+1} is assumed to be lognormal, hence the logarithm of the shocks are normally distributed as following:

$$\ln z_{t+1} = N(0, (\varsigma\sqrt{\rho})^2) \;\; ; \;\; \ln \omega^i_{t+1} = N(0, (\varsigma\sqrt{1-\rho})^2) \tag{2.3}$$

Thus, the overall individual risk factor Ω^i_{t+1} is distributed as:

$$\Omega^i_{t+1} = lnz_{t+1} + ln\omega^i_{t+1} = N(0, (\varsigma)^2) \tag{2.4}$$

where both $\varsigma > 0$ and $\rho > 0$. The main implication of this risk structure is that each pair of overal individual risk factors Ω^i_{t+1}, Ω^j_{t+1} (for all $i \neq j$) is cross-correlated due to the pressence of the aggregate factor z_{t+1} and the coefficient of correlation is ρ. The parameter ς determines the standard deviation of the aggregate shock Ω^i_{t+1} or the overall riskiness of the environment. As we will see, the non-zero cross-correlation of individual risk factors results in the inability of banks to fully diversify risk.

The unconditional probability of individual default is the probability of the overal risk factor Ω^i_{t+1} falling below the $\ln ltv^i_t$ threshold.

$$PD^i_t = P(\Omega^i_{t+1} < \ln ltv^i_t) = \Phi\left(\frac{\ln ltv^i_t}{\varsigma}\right) \tag{2.5}$$

where Φ denotes the standard normal cdf.

The conditional probability of default is defined for a given realization of the aggregate risk factor z_{t+1} and is calculated as the probability that the idiosyncratic shock ω^i_t falls below the LTV threshold for the given realization of the aggregate factor z_{t+1}.

$$PD^i_t(z_{t+1}) = P\Big(\ln \omega^i_t < \ln\Big(\frac{ltv^i_t}{z_{t+1}}\Big)\Big) = \Phi\left(\frac{\Phi^{-1}(PD^i_t)\varsigma - \ln z_{t+1}}{\varsigma\sqrt{1-\rho}}\right) \tag{2.6}$$

The conditional and unconditional probabilities of individual default are the key elements for deriving the portfolio loss distribution, incorporated in Basel. Under the assumptions of perfect granularity and equal probability of default (equal LTV for all loans) Vasicek shows that:

1) The unconditional expectation of the portfolio default rate equals the unconditional probability of individual default:

$$E_t[DR_{t+1}] = PD_t^i \tag{2.7}$$

2) The realized ex-post default rate DR_{t+1} for a given state of the aggregate risk factor z_{t+1} equals the conditional probability of individual default:

$$DR_{t+1}|z_{t+1} = PD_t^i(z_{t+1}) \tag{2.8}$$

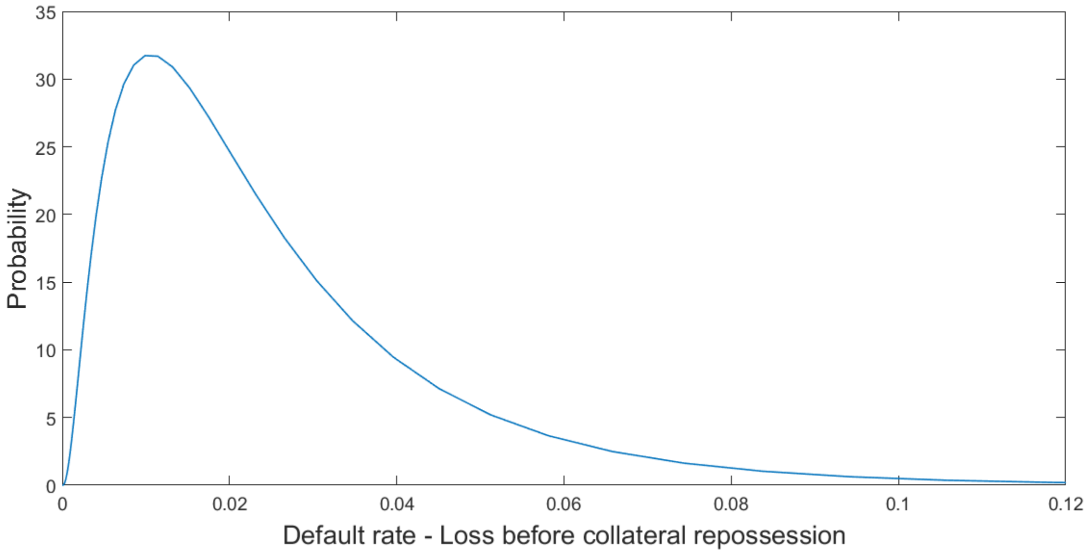
Combining equation (2.8) and the distribution of the aggregate risk factor z_{t+1}, we can evaluate the probability of each realization of the aggregate shock and its corresponding portfolio default rate.[1] Thus, the CDF of the portfolio default rate Π_t, can be expressed as a function of the unconditional individual probability of default PD_t^i and the cross-correlation risk factor ρ.

$$\Pi_t(x) = P(DR_{t+1} < x) = \Phi\left(\frac{\Phi^{-1}(x)\sqrt{1-\rho} - \Phi^{-1}(PD_t)}{\sqrt{\rho}}\right) \tag{2.9}$$

Equation (2.9) is the one which is used for calculating capital requirements in Basel II. It provides the distribution of portfolio losses, before collection of collateral, for a large portfolio consisting of loans with similar PD. The distribution is then used to calculate the necessary regulatory amount of capital k^* in order for the probability that the portfolio losses exceed k^* to be less than 0.1%. The figure below plots the corresponding probability density function.

[1] This is possible since the portfolio default rate is a function of the random variable z_{t+1} which distribution is known, as a result the default rate can also be expressed as a random variable which distribution can be derived, see Vasicek (2002).

Figure 2.1: Distribution of the portfolio default rate (LTV $= 0.7$, $\rho = 0.1$, $\varsigma = 0.18$)



While the equation above provides the distribution of the default rate DR_{t+1}, i.e. loss rate before collection of collateral, it does not provide the distribution of the portfolio losses L_{t+1} that take into account collateral repossession and the realized loss-given-default LGD for each of the defaulted loans such that: $L_{t+1} = DR_{t+1}LGD_{t+1}$.[1]

The Basel documentation does not provide any theoretical framework for the calculation of the LGD which is needed for measuring losses after collateral repossession L_{t+1} and the necessary amount of capital to cover them. Instead, the documentation specifies that banks can either use their own methodology for estimating the LGD of a portfolio (upon supervisory approval) or use a static LGD value, e.g. 40%.

In the next section, we show that the assumptions made so far are sufficient in order to derive the value of the repossessed collateral and hence the loss rate L_{t+1} of the portfolio and its distribution.

[1]For each defaulted loan the bank does not suffer a loss-given-default of the full amount of the exposure as it repossesses the collateral after realization of its return and thus recovers part of the loss.

3 Borrowers - optimal default choice

It is important to note that the necessary component to derive the individual PD, the portfolio DR, and its distribution is the loan-to-value ratio of the loans in the portfolio. We can interpret it as a distance-to-default measure in the sense that the lower value of the collateral backing an exposure (higher LTV) the wider range of shocks combinations that could bring the borrower to default. This is also true for every given aggregate state z_{t+1} as a wider range of idiosyncratic realizations could bring the borrower to default.

The model so far considers the borrowers' default choice only for the purposes of calculating the proportion of borrowers that default. We continue forward by evaluating the repossessed collateral that the bank recovers from each defaulted loan which is necessary for deriving the portfolio loss rate.

Considering the expected return from lending to borrower i, conditional on the aggregate state z_{t+1}. We know that with probability $1 - PD_t^i(z_{t+1})$ the borrower repays and the bank receives the contract interest rate r_t^i, while with probability $PD_t^i(z_{t+1})$ the borrower defaults and the bank receives the collateral after the realization of the shocks. Hence the expected return becomes:

$$Return_t^i(z_{t+1}) = \Big(1 - PD_t^i(z_{t+1})\Big) r_t^i + PD_t^i(z_{t+1}) \frac{\mu z_{t+1} E_t\Big(\omega_{t+1}^i | \omega_{t+1}^i < \frac{ltv_t^i}{z_{t+1}}\Big) r_t^k k_t^i}{b_t^i} \tag{3.1}$$

where the parameter $\mu < 1$, represents costs that the bank pays associated with the collateral repossession and sale. After substituting equation (2.1) for the LTV ratio and rearranging, we can express expected return from lending as:

$$Return_t^i(z_{t+1}) = r_t^i \left[1 - PD_t^i(z_{t+1}) \left(1 - \frac{\mu z_{t+1} E_t\Big(\omega_{t+1}^i | \omega_{t+1}^i < \frac{ltv_t^i}{z_{t+1}}\Big)}{ltv_t^i} \right) \right] \tag{3.2}$$

The term $\frac{\mu z_{t+1} E_t\left(\omega_{t+1}^i | \omega_{t+1}^i < \frac{ltv_t^i}{z_{t+1}}\right)}{ltv_t^i}$ is the expected value of the repossessed collateral or simply the expected recovery rate. Hence, one minus the recovery rate is the amount that the bank loses - the loss-given-default (LGD)[1]:

[1]We define both PD, LGD and all loss rates normalized by the assets size leading to per

$$LGD_t^i(z_{t+1}) = 1 - \frac{\mu z_{t+1} E_t\left(\omega_{t+1}^i | \omega_{t+1}^i < \frac{ltv_t^i}{z_{t+1}}\right)}{ltv_t^i} \tag{3.3}$$

As a result, the conditional loss-given-default can also be expressed as a function of the LTV threshold. This implies that the PD and LGD are related through the default threshold. We already know that the PD is increasing in the LTV but how is the loss-given-default related to the default threshold?

As the LTV increases the bank starts to repossess more collateral but this collateral is of lesser value relatively to the amount lent, leading to increase in both the numerator and denominator of the recovery rate. It can be shown that the deterioration of collateral value dominates and the recovery rate decreases for higher LTVs.[1] As a result, for a given realization of the aggregate shock, the conditional LGD is an increasing function of the LTV. The conditional statement regarding the shock $\omega_{t+1}^i < \frac{ltv_t^i}{z_{t+1}}$ is crucial for this result as for a given aggregate state, the bank repossess collateral only from individual borrowers with low enough realizations of the idiosyncratic shock that has made their collateral of lesser value than the amount of the loan and induced them to default. This is possible due to the fact that the liability of the borrower is limited only to the value of the collateral. As a result, in adverse realizations of the risk factors, the borrower passes on its loss on the investment project to the bank by giving up the cheap collateral.[2]

Finally, the loss rate is simply the probability of default times the loss given default:

$$L_t^i(z_{t+1}) = LGD_t^i(z_{t+1}) PD_t^i(z_{t+1}) \tag{3.4}$$

In the previous section, we used the assumptions of large and homogeneous portfolio to apply the law of large numbers and conclude that the conditional probability of default of loan i for the aggregate state z_{t+1}, $PD_t^i(z_{t+1})$ is also the realized aggregate portfolio default rate for the same aggregate state. Analogously,

unit of loan terms.

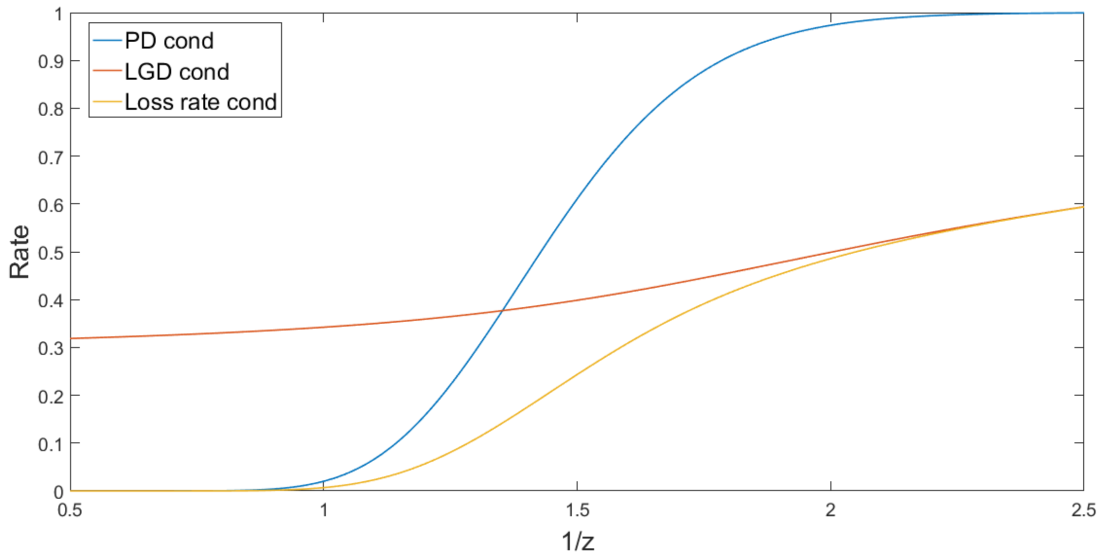
[1]See Appendix 8.1.

[2]In fact, the expression for the value of repossessed collateral is well known in the financial accelerator literature. BGG uses it in order to calculate the expected return from lending and derive a participation constraint for banks that set interest rates in a manner that ensures a certain level of expected return.

we conclude that the expected individual loss-given-default, $LGD_t^i(z_{t+1})$ is also the realized aggregate portfolio LGD. Finally, we can say that the expected individual loss rate, $L_t^i(z_{t+1})$ is also the aggregate portfolio realized loss rate. Therefore, we can drop the i superscripts and consider each of these individual characteristics expressed in expected terms as aggregate portfolio characteristics in realized terms. The characteristics of each individual loan can also be considered aggregate due to the homogeneity of the portfolio ($r_t^i = r_t; ltv_t^i = ltv_t$).

Due to the derived relationships, if we fix the portfolio and its LTV and consider different realizations of the aggregate shock. As the scenario becomes adverse, not only that larger proportion of the loans default, but the LGD would also increase as banks repossess cheaper collateral. Figure 3.2 plots the realized default rate $PD_t(z_{t+1})$, loss given default $LGD_t(z_{t+1})$ and loss rate $L_t(z_{t+1})$ for different realizations of the aggregate risk factor z_{t+1}.

Figure 3.2: Conditional PD, LGD and Loss rate (LTV = 0.7, $\rho = 0.1$, $\varsigma = 0.18$, $\mu = 0.7$)



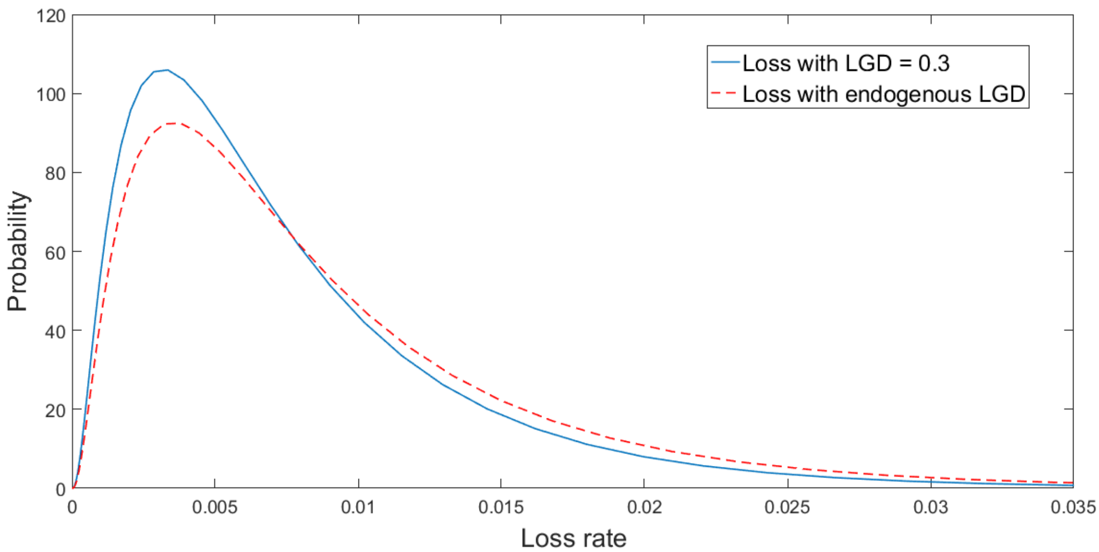
The explanation of this result lies in the contract between the bank and the borrower. In adverse scenarios, more and more borrowers default, however, the reason for their default is the losses that have been realized on their investment projects which also increase. Due to the limited liability contract, the borrowers pass those losses to the bank by giving up the deteriorated collateral.

As a result, both the default rate and loss given default increase, reinforcing each other, and leading to the first financial accelerator of this paper. This accelerator is a positive relationship between the default rate of a portfolio and the loss-given-default of each defaulted loan and is a result of the contract type and the rational default choice of the borrower. Not taking this effect into account and treating LGD as fixed (as it is allowed by the Basel standards) would lead to underestimation of the portfolio losses especially for adverse scenarios.

Using the distribution of the aggregate shock z_{t+1}, we can use the fact that the realized portfolio loss rate is a function of it and compute the distribution of the loss rate.

Figure 3.3 shows the distributions of portfolio losses after repossession of collateral for the endogenous LGD derived above and a static LGD[1].

Figure 3.3: Loss rate distribution (LTV $= 0.7$, $\rho = 0.1$, $\varsigma = 0.18$, $\mu = 0.7$)



As we can see, using a static LGD can overestimate the likelihood of relatively low losses and underestimate the likelihood of particularly high losses that occur in the very bad scenarios. Such result would have significant implications for capital requirements, since the required capital is calculated as the necessary amount k^* in order for the probability that the portfolio losses exceed k^* to be less than 0.1%, i.e. P(bank default $<$ 0.1%). That is the level that cuts the 0.1 percentile of the

[1] The chosen static LGD value of 30% is purely illustrative at this stage of the paper and does not affect any of the main results. In the following sections the model is calibrated with US banking data.

right tail of the loss distribution - black area in the figure below.

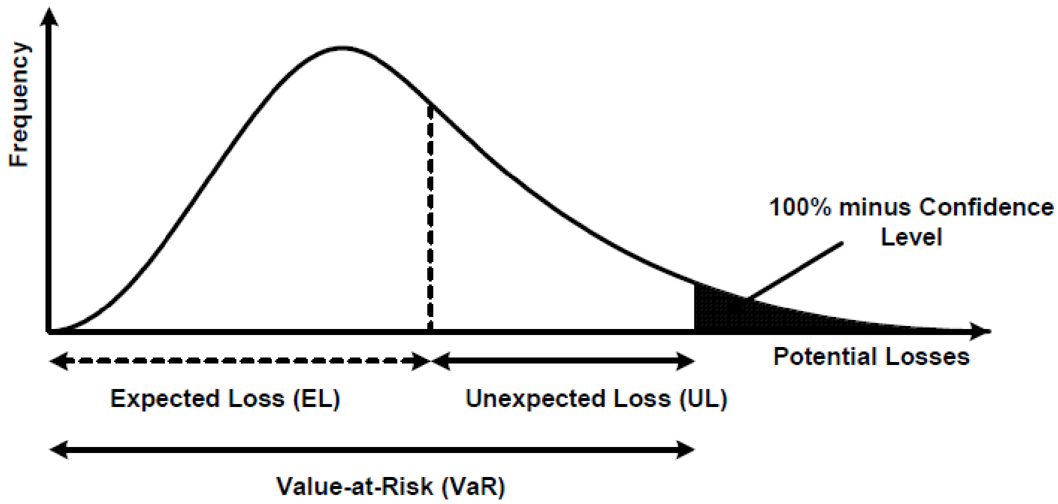


Figure 3.4: Loss distribution of a loan portfolio, source: (BCBS, 2005)

To summarize, we saw that due to limited liability borrowers can transfer risk from their investment projects to the bank. However, we have not analysed the optimal behaviour of the bank in such circumstances. As we will see in the next section, the borrowers would be charged for their riskiness by the bank and moreover, this will further change their default incentives and the portfolio loss distribution.

4 Banks - risk insurance

So far, we showed that under a fixed interest rate contract with limited liability, we can derive the probability distribution of the portfolio loss that consists of default rate and loss-given-default as a function of the portfolio LTV. In that process, we have completely ignored the behaviour of the bank. If idiosyncratic risk is diversifiable how can the bank insure against it? Does bank behaviour for risk insurance affect borrowers' default incentives and the loss distribution? We focus on these questions in this section.

Considering idiosyncratic risk, we know that a proportion of the borrowers would default due to low realized return on their collateral. The borrowers receiving a higher return would simply repay the loan and thus the maximum return that the bank can make from a loan is limited to the size of the interest rate r_t^i. Under these circumstances, the only way that the bank can compensate the expenses from defaulting loans is by charging a spread over its cost of funds. The insurance against idiosyncratic risk is well known in the financial frictions literature and we incorporate those mechanisms into the portfolio value theory.[1] In the previous section, from equations (3.2 - 3.4), we showed that the realized return from lending conditional on the aggregate state z_{t+1} can be expressed as the contract rate r times one minus the loss rate $L_t(z_{t+1})$.

$$Return_t(z_{t+1}) = r_t\left[1 - PD_t(z_{t+1})LGD_t(z_{t+1})\right] = r_t\left[1 - L_t(z_{t+1})\right] \tag{4.1}$$

We denote the cost of funds for the bank is r^b and we regard it as fixed for now. Then for each aggregate state z_{t+1}, the net loss for the bank would be the excess of r^b over the return from lending:

$$Loss_t(z_{t+1}) = r^b - r_t\left[1 - L_t(z_{t+1})\right] \tag{4.2}$$

[1] Under a variable interest rate contract, BGG introduces a participation constraint in which banks charge a spread that would give them expected return from lending equal to their cost of funds. The BGG constraint holds in both expectation and realized terms as the bank can vary the interest rate after the signing of the contract. Quint and Rabanal (2014) and Forlati and Lambertini (2011) introduce a participation constraint for fixed interest contract which holds only in expectation terms. The latter approach is closest to the one analysed here.

We further assume perfect competition in the banking sector, which implies that the bank operates under zero profits and that any interest rate above the one sufficient to insure against risk would exclude the bank from the market. This translates into interest rate setting of the lending rate r_t such that the return from lending covers the cost of funds $r^b = r_t(1 - L_t^A)$ for a provisioned loss rate from defaults L_t^A:

$$r_t = \frac{r^b}{1 - L_t^A} \tag{4.3}$$

Substituting the lending rate setting into the net loss we obtain:

$$Loss_t(z_{t+1}) = r^b - \frac{r^b\Big[1 - L_t(z_{t+1})\Big]}{1 - L_t^A} = r^b\Big[1 - \frac{1 - L_t(z_{t+1})}{1 - L_t^A}\Big] \tag{4.4}$$

From equation (4.4), we can see that for any realized loss rate higher than the one provisioned $L_t(z_{t+1}) > L_t^A$, the bank would realize net losses that are absorbed by capital. On the other hand, a lower than provisioned loss rate would lead to profits to equity holders. But what loss rate should the banks provision?

While capital requirements are very heavily regulated and supervised, the provisioning practices of banks are regarded as accounting standards and are not defined in the Basel framework.[1] According to Basel, provisioning should cover losses in "normal" times.

For now, let's assume that the bank provisions loss rate of amount $L_t^A = L_t(1)$ for the most likely state of the world $E[z_{t+1}] = 1$. This is equivalent to considering and insuring against solely idiosyncratic risk and completely ignoring aggregate risk at the stage of provisioning. In that setting, the responsibilities of provisioning and bank capital are completely separated with each taking care of idiosyncratic and aggregate risk respectively. The above provisioning practice implies the following interest rate setting:

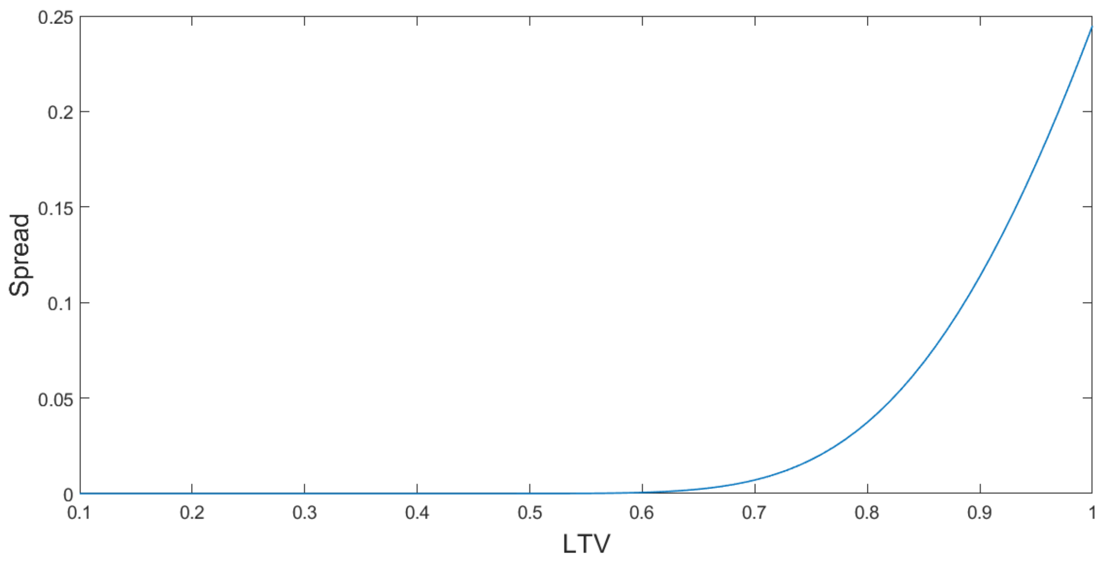
$$r_t = \frac{r^b}{1 - L_t(1)} \tag{4.5}$$

As a result, the bank would charge a spread over its cost of funds r^b that

[1] This is about to change with the introduction of the International Financial Reporting Standard (IFRS) 9 from 2018, in which loss provisioning should be forward looking and consider possible macroeconomic developments.

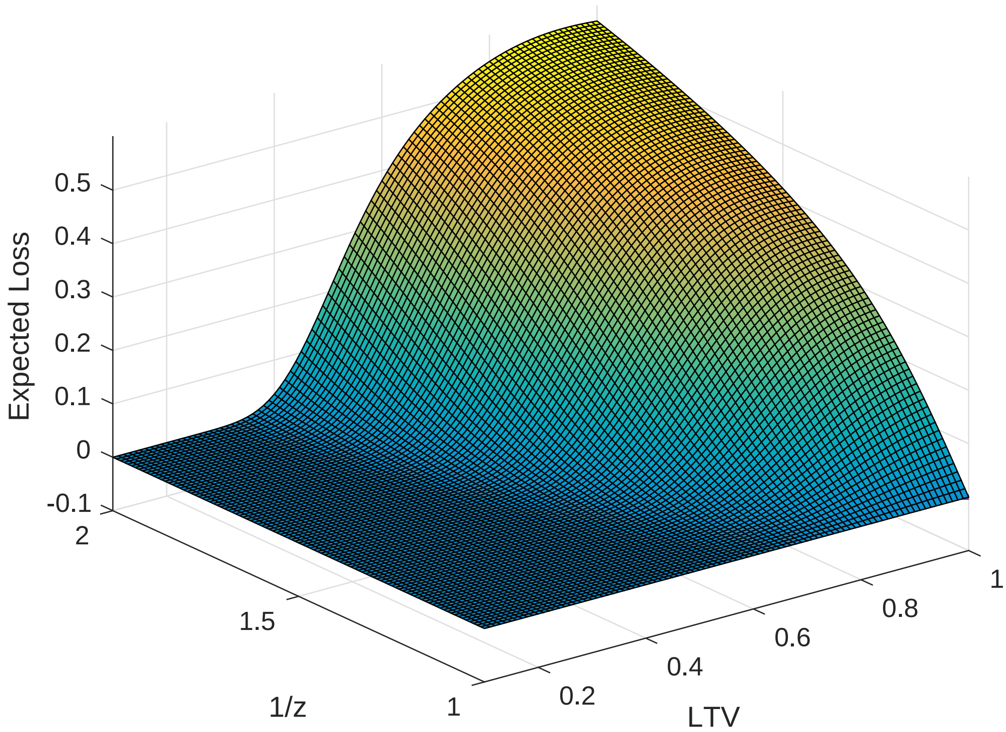
depends on the provisioned loss rate. As we know from the previous sections, the loss rate $L_t(1)$ can be expressed as PD times LGD which in turn depend on the LTV ratio. Hence, the spread over the cost of funds r^b is also determined by the LTV of the portfolio and is an increasing function of it. The intuition is that a higher LTV increases the proportion of defaulting loans and collateral repossession. To compensate for this and ensure covering of its funding costs, the bank has to charge more the fewer loans that are being repaid. Figure 4.5 plots the spread $r^i - r^b$, LTV curve.

Figure 4.5: Spread-LTV curve ($\rho = 0.1$, $\varsigma = 0.18$, $\mu = 0.7$)



For no aggregate risk: $z_{t+1} = 1$, due to the full insurance against idiosyncratic risk during the provisioning and setting of r_t stage, the expected net loss becomes zero. However, for any aggregate shock, the bank would experience profits or losses. Figure 4.6 shows the expected loss of a loan for a given range of LTV's and adverse realizations of the aggregate shock z, before realization of the idiosyncratic shocks.

Figure 4.6: Expected Loss ($\rho = 0.1$, $\varsigma = 0.18$, $\mu = 0.7$)



Here we observe that for low LTV loans, even very adverse realizations of the aggregate shock do not lead to expected losses. On the other hand, for high LTV loans, even slightly adverse realizations lead to large expected losses. Although that are insured by higher spread, the bank would also need to hold higher amount of capital for high LTV loans in order to cover potential losses. This is in line with the risk-sensitive framework introduced in the Basel II capital regulation which sets higher risk-weights to riskier loans and hence lead to higher capital charge for the bank.

Now we have considered bank behaviour in interest rate setting, but not considered how this behaviour interacts with the borrowers' default choice. While the LTV has been regarded as constant for the interest rate setting in equation (4.3), in fact it depends on the interest rate itself as $ltv_t = \frac{r_t b_t}{r_t^k k_t}$. This is because the LTV that would affect the default decision of the borrower is the one including loan amount plus interest. This means that as the bank charges higher LTV with a higher interest rate, the LTV itself increases. The intuition is that the insurance of idiosyncratic risk leads to an additional cost for borrowers which in turn makes them riskier for the bank.

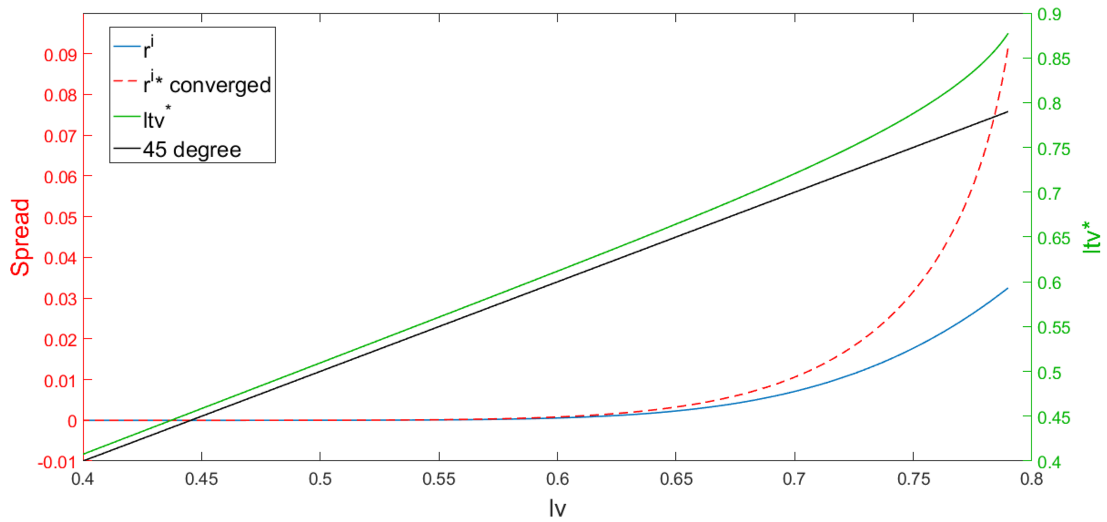
This reinforcing cycle is the second financial accelerator of this paper. It arises due to the effect of interest rate setting by banks on borrowers' default behaviour. As we will see, the presence of this accelerator has also an effect on the portfolio loss distribution.

If we define $lv_t = \frac{b_t}{r_t^k k_t}$ as the loan-to-value before interest, then $ltv_t^i = r_t^i lv_t^i$ and the interest rate setting becomes:

$$r_t^* = \frac{r^b}{1 - \int_0^{lv_t r_t^*} f(\omega)d\omega + \frac{\mu \int_0^{lv_t r_t^*} \omega f(\omega)d\omega}{lv_t r_t^*}} \tag{4.6}$$

The above expression is an implicit equation for the interest rate as a function of the LTV before interest lv_t^i, that takes into account the effect of interest rate setting on borrowers and their default incentives. For every LTV before interest - lv_t^i there exists an interest rate r^{i*} that satisfies the above equation and leads to a borrower LTV: $ltv^{i*} = lv_t^i r^{i*}$. Figure 4.7 shows the spread with and without the financial accelerator,r^{i*} and r^i for different values of lv and the resulting ltv^* that is consistent with the interest rate setting behaviour of the bank.

Figure 4.7: Risk pricing accelerator ($\rho = 0.1$, $\varsigma = 0.18$, $\mu = 0.7$)

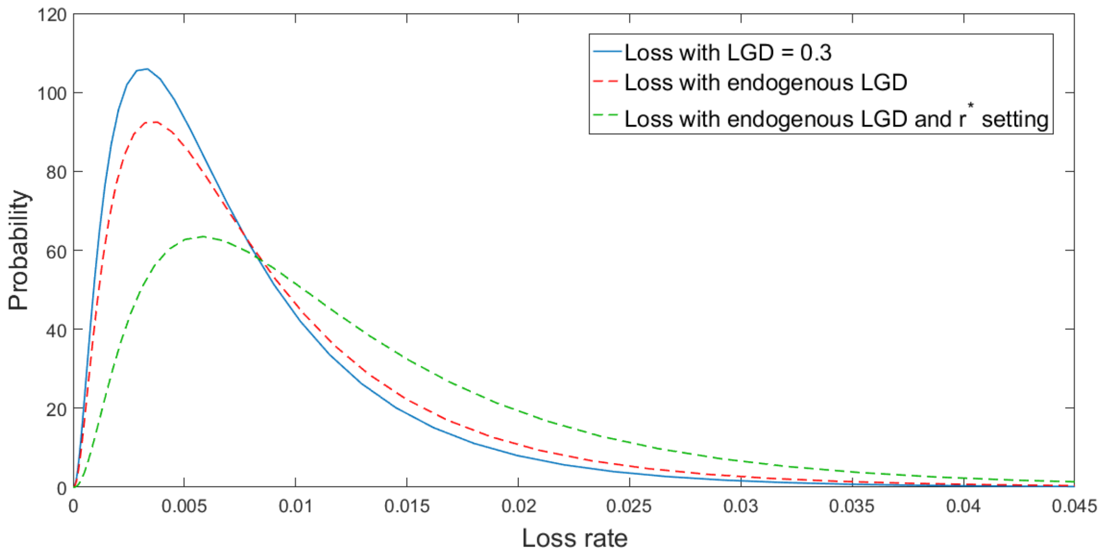


As we can see for every lv, the rate that takes into account its own effect on the borrower's default incentive r^*(red line) is higher than the one that does not

r^i (blue line). As a result the ltv^* is also higher than the lv. In summary, once the bank prices adequately borrower's risk, both the risk itself and its price end up higher than the initial level.

Moreover, this reinforcement channel has impact on the loss distribution that works through leverage. As banks set higher rates to high LTV loans, this increases borrowers' leverage and their incentives to default, thus increasing portfolio losses for every aggregate state. Figure 4.8 adds the loss rate distribution resulting from the interest rate setting behaviour of banks to the ones with static LGD and endogenous LGD.

Figure 4.8: Loss rate distribution($lv = 0.7, \rho = 0.1$, $\varsigma = 0.18$, $\mu = 0.7$)



As we can see, once we add the financial accelerator resulting from bank behaviour, to the one due to borrowers' default choice, the resulting distribution shows that larger losses are more likely and smaller losses less likely compared to the distributions that do not include those reinforcement channels. As we will see later, this underestimation of large losses can lead to insufficient capital requirements to the banking sector.

It is worth noting that the endogenously obtained accelerator is similar to the ones in the financial frictions literature e.g. Kiyotaki and Moore (1997) and Iacoviello (2005). However, while these papers impose an ad-hoc constraint limiting the LTV at which banks are willing to lend, in our case banks are willing to lend to riskier loans but for a higher price. Furthermore, this risk pricing by banks results in additional amplification which is not present in the papers above.

5 Investors - risk premia

So far, we have assumed that the cost of funds for the bank r^b is fixed. In practice, this cost would equal the weighted sum of the cost of equity r^e and cost of debt r^d for the bank. Assuming that the bank funds its loans B_t with debt D_t and equity (bank capital) K_t^b, we obtain a balance sheet identity.

$$B_t = K_t^b + D_t \tag{5.1}$$

Thus, the cost of funds for the bank r_t^b can be expressed as:

$$r_t^b = kr_t^e + (1-k)r_t^d \tag{5.2}$$

where k denotes the capital per unit of assets or the capital-asset ratio $\frac{K_t^b}{B_t}$.

In addition, we make the further assumption that in the state in which the realized loss rate equals the provisioned one $L(z_{t+1}) = L_t(1)$, the obtained return from lending is fully used to pay the required returns to equity and debt holders, while the level of capital remains constant. However, away from this state the riskiness of the portfolio is passed on to the providers of funds to the bank. The equity holders will bear the Profits/Losses of the portfolio of loans, while debt holders would receive decreased returns, in case that the amount of realized net loss is greater than the amount of capital k and the bank defaults. As we showed in the previous section, the net loss $Loss_{t+1}(z_{t+1})$ can be expressed as funding cost minus return from lending, where the return is the loan rate r_t times one minus the interest forgone due to the realized loss rate L_t.

$$Loss_{t+1}(z_{t+1}) = r_t^b - r_t\left[1 - L_t(z_{t+1})\right] \tag{5.3}$$

where the contract lending rate r_t is set as to ensure zero loss in the deterministic steady state $z_{t+1} = 1$ according to the provisioned loss rate $L_t(1)$:

$$r_t = \frac{r^b}{1 - L_t(1)} \tag{5.4}$$

Having the conditional net loss for any realization of the aggregate shock, we can derive the net loss PDF - $f(Loss)$. Then using the net loss distribution, we can evaluate the required risk premia by equity holders and debt holders, taking into account the respective risk that they take by investing into the bank. For that purpose, we assume risk neutral investors and as such they perceive as risk only the expected return from investing and not its higher moments.

Starting from the equity holders, we assume that they require a rate of return r_t^e which net of the expected loss in the events of: profit, loss and default for the bank, ensures them the risk-free rate r^f:

$$r_t^e - \frac{\int_{r^b - r}^{kr^e} Loss_{t+1} f(Loss) dz + kr^e \int_{kr^e}^{r^b} f(Loss) dz}{k} = r^f \tag{5.5}$$

Thus, for a fixed level of capital, for any increase in the riskiness of the credit portfolio, the equity holders would require a higher risk premium. Note that the loss that equity holders can bear is limited to the amount of capital they have invested.

Debt holders, on the other hand, bear much lesser risk from lending to the bank. Their return would only deviate from the required rate r^d, if the losses are so large that the amount of capital is not enough to absorb them and the bank defaults. Hence, they would require a rate of return which net of the expected loss in the event of bank default ensures them the risk-free rate:

$$r_t^d - \frac{\int_{kr^e}^{r^b} (Loss_{t+1} - kr^e) f(Loss) dz}{1 - k} = r^f \tag{5.6}$$

The loss that debt holders can bear is limited to the amount of debt plus interest. In addition, for a fixed level of capital a riskier credit portfolio would lead to a higher required risk premium for debt.

In summary, equations (5.5) and (5.6) show that under the assumption of risk insurance by investors, both sources of funding become more costly as the credit portfolio becomes riskier. As a result, the cost of funding for the bank r_t^b

is increasing in the riskiness of the portfolio.

The required returns of debt and capital also give us valuable intuition for the relevance of the capital structure of the bank to its cost of funding. In theory, we can use the above equations to track how the cost of debt and equity change with the amount of capital k. However, given the set of assumptions made so far,[1] the Modiliagni-Miler theorem holds and as a result, the weighted sum of the costs of equity and debt gives us a cost of funds for the bank r^b which is independent of the level of capital:

$$r_t^b = kr_t^e + (1-k)r_t^d = r^f + \int_{r^b - r}^{r^b} Loss_{t+1} f(Loss) dz \tag{5.7}$$

Combining the interest setting rule $r_t = \frac{r_t^b}{1-L_t(1)}$ that provisions loss rate of $L_t(1)$ we obtain a net loss function for every aggregate state:

$$Loss_t(z_{t+1}) = r^b - \frac{r^b\Big[1 - L_t(z_{t+1})\Big]}{1 - L_t(1)} = r^b\Big[1 - \frac{1 - L_t(z_{t+1})}{1 - L_t(1)}\Big] \tag{5.8}$$

As we can see from equations (5.7) and (5.8), the cost of funds of the bank is increasing in the riskiness of the portfolio and in turn, the portfolio loss increases in the cost of funds for every aggregate state. The resulting interaction is the third financial accelerator of this paper and works in combination with the previously described amplification mechanisms. An increase of portfolio riskiness leads to higher costs of funding for the bank driven by investors' behaviour. In addition, bank behaviour of interest rate setting passes the higher costs of funding to borrowers which in turn become more prone to defaults and increase the riskiness of the portfolio, thus closing the acceleration cycle.

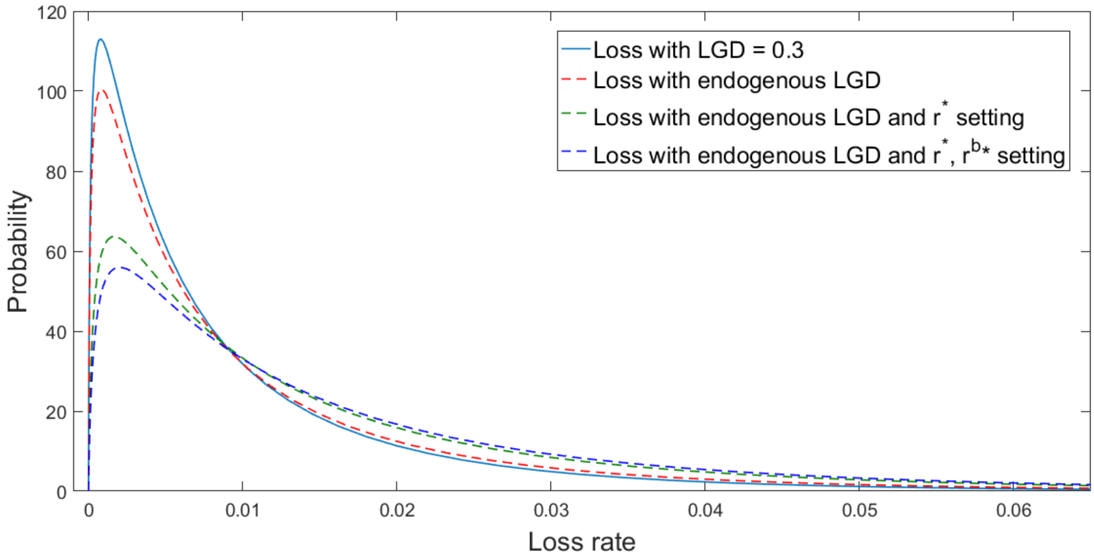
The tightening of lending conditions to banks by investors is similar to the Gertler and Kiyotaki (2010) financial accelerator. However, instead of imposing a quantity constraint on the amount of funds that investors are willing to provide, we derive an endogenous constraint in which investors are still willing to lend to riskier banks but at a higher cost.

Just like the previous amplification channels, the one due to investors' be-

[1]Namely, we haven't assumed any preferential tax treatment or state insurance for debt/deposits.

haviour would also amplify the vulnerability of portfolio losses to aggregate shocks. As a result, the absence of the channel would underestimate the portfolio loss distribution and hence undermine the calculation of sufficient capital requirements and prudent bank regulation. Figure 5.9 plots the resulting loss rate distribution $f(L)$, once the risk premia required by investors is taken into account.

Figure 5.9: Loss rate distribution($lv = 0.7, \rho = 0.1$, $\varsigma = 0.18$, $\mu = 0.7$)



From the figure above, we can track how the distribution of the loss rate $f(L)$ (in terms of forgone interest) is evolving, as we have expanded the economic structure of the model. Starting from the baseline model, by introducing borrowers' default behaviour, banks' interest rate setting and investors' risk premia, we have incorporated three different financial accelerators that taken independently or all together increase the riskiness of the portfolio, leading to a distribution with less likely small losses and more likely large losses.

Equilibrium

Having derived the endogenous cost of funding for banks r_t^b, our model is completed and we can define the equilibrium conditions. For a given portfolio ltv before interest lv and capital of the bank k, our equilibrium is characterized by the set of variables $\{r^*, ltv^*, Loss^*, f(Loss^*), r^{e*}, r^{d*}, r^{b*}\}$ that satisfy the following conditions:

- loan interest rate r^* set according to cost of funding r^{b*} and the ltv before interest lv - equation (4.6).
- conditional net loss $Loss^*_{t+1}(z_{t+1})$ with distribution $f(Loss^*_{t+1})$ consistent with the equilibrium cost of funding r^{b*} and loan rate r^* - equation (5.8).
- bank cost of funding r^{b*} as a weighted sum of the cost of debt and equity r^{d*}, r^{e*} which are consistent with the capital ratio of the bank k and the net loss distribution $f(Loss^*_{t+1})$ - equations (5.5) and (5.6).

Appendix 8.2 describes the algorithm used for computing the equilibrium.

So far, we have considered the provisioning and level of capital of the bank as fixed. Furthermore, the simple provisioning rule employed so far completely ignores aggregate risk and passes it on to borrowers. On the other hand, the completed model allows to study how the interest rates, borrowers' and bank's probability of default vary as we change the levels of provisioning and capital. Is there an optimal level of provisioning and capital in terms of risk and lending costs? We analyse the optimal properties of the model in the next section.

6 Model Analysis

6.1 Provisioning

So far, under the employed provisioning and interest rate setting rule, we assumed that the bank sets interest rates so that the revenues from lending $r_t(1-L_t^A)$ under provisioned loss rate L_t^A are enough to cover the costs of funds for the bank r_t^b. For simplicity, we assumed, that during the provisioning stage, the bank completely ignores aggregate risk and and provisions a loss rate $L_t^A = L_t(E[z_{t+1}]) = L_t(1)$, for the most likely state of the world $E[z_{t+1}] = 1$. Leading to interest rate rule:

$$r_t = \frac{r^b}{1 - L_t(1)} \tag{6.1}$$

In addition, from the previous section, we saw that by ignoring aggregate risk at the stage of provisioning, the bank passes this risk to its investors which in turn require higher return for their investment. As a result, the higher cost of funds is passed on to the lending rate and the final borrowers, thus leading to higher leverage and risk.

In this section, we investigate what would happen if the bank considers both idiosyncratic and aggregate risk at the stage of provisioning, implying a provisioned loss rate of $L_t^A = E[L_t(z_{t+1})] = \int L_t(z_{t+1})f(L)dz$ and corresponding loan rate setting:

$$r_t = \frac{r^b}{1 - \int L_t(z_{t+1})f(L)dz} \tag{6.2}$$

As we will see later, the expected loss rate $E[L_t(z_{t+1})]$ is higher than the loss rate in the expected state $L_t(E[z_{t+1}])$ due to the properties of the loss distribution. This leads to higher provisioned loss rate by the bank and hence higher lending rate to borrowers. Substituting the rate setting into the conditional net loss leads to:

$$Loss_t(z_{t+1}) = r_t^b - r_t(1 - L_t(z_{t+1})) = r_t^b - \frac{r_t^b(1 - L_t(z_{t+1}))}{1 - \int L_t(z_{t+1})f(L)dz} \tag{6.3}$$

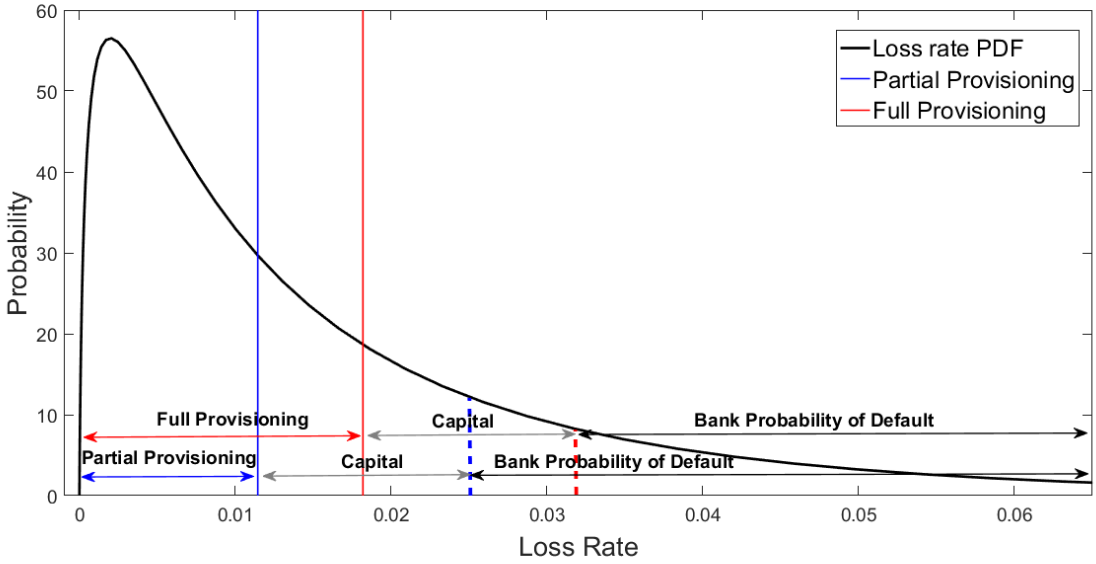
As a result, the expected net loss is zero $E[Loss_t(z_{t+1})] = 0$. Hence, substituting it into the cost of funds equation (5.7) implies that the investors would require a risk-free return for lending to the bank $r_t^b = r^f$.

In summary, if the bank disregards aggregate risk at the stage of provisioning (Partial Provisioning) it would set lower spread from its cost of funds, however, the aggregate risk would be passed on to investors which would set higher cost of funds for the bank. On the other hand, if the bank considers all risks at the stage of provisioning (Full Provisioning), it would set higher spread from its cost of funds, however, its investors would receive a risk-free portfolio for which they would require a risk-free return. In both types of provisioning, the final lending rate to the borrower is the same and hence, borrower's leverage and loss rate distribution is unchanged. What changes is the level of provisioning and the cost of funds for the bank.

What about bank probability of default? Under partial and full provisioning the loss rate distribution is the same. However, under full provisioning the provisioned threshold above which losses are absorbed by capital is higher. Hence, for the same level of capital, the threshold above which losses are large enough to lead to bank default is also higher. This leads to a lower probability of default of the bank - PD^b for the same level of capital - Figure 6.10.

Note that the partial provisioning case is not an unrealistic outcome. The default threshold of the bank also separates how the exposure to the loss distribution is shared between shareholders and debtholders. Since shareholders' liability is limited to the level of capital and hence the default threshold, then a lower threshold and higher default probability transfers risk from shareholders to debtholders. Considering that the provisioning strategy depends on bank management which is under direct control of shareholders and the possibility of unadequate pricing of default risk by debtholders due to deposit insurance, banks may be incentivised to provision lower level of losses.

Figure 6.10: Bank default threshold under different provisioning



6.2 Quantifying Loss Underestimation

The model equilibrium from section 5 allows us to calculate some model statistics after a baseline calibration. Moreover, we can compare the equilibrium characteristics of the model with all three financial accelerators, under partial and full provisioning, with the equilibrium implied by the Basel capital regulation and the Vasicek (2002) loss distribution that does not consider accelerator effects.

Calibration

The model is calibrated to the US, given the fact that the limited liability contract is more common in the US rather than the EU. We start the calibration by setting the loan-to-value ratio: $LTV = 0.7$ to a value that is standard in the literature. It is the main parameter that sets portfolio riskiness as it changes individual PD and LGD and also the probability of default of the bank and hence, we conduct robustness checks of the results for different values of the LTV.[1] The bank capital-asset ratio is set to the weighted average leverage ratio of the top US banks $k = 5.75\%$. The risk-free rate is set to the current US 12 month Treasury bond yield $r^f = 1.15\%$.

[1] See Appendix 8.3.

We then proceed by setting the standard deviation parameters determining the distribution of aggregate and idiosyncratic risk: $\rho = 15\%$ and $\varsigma = 19\%$. The former parameter governs the correlation between the collateral values and is set to match the value set by the Basel regulation for mortgages. The latter parameter governs the aggregate level of risk in the economy and is set, so as to lead to an unconditional individual probability of default $PD = 4.19\%$. Finally, the parameter for the costs of collateral repossession $\mu = 64\%$ is set, so as to lead to unconditional loss-given-default $LGD = 40\%$. The calibrated value of the PD is obtained as the long run average delinquency rate of US mortgages while the value for the LGD is chosen as the value set under the Basel FIRB approach.[1]

Baseline Results

Under both partial and full provisioning, the final lending rate to the borrowers is the same $r_t = 2.9\%$. However, under partial provisioning banks set lower spread from their cost of funds $r_t - r_r^b = 2.9 - 1.65 = 1.25\%$, but by setting lower provisions, banks transfer higher risks to their investors which in turn charge them a higher spread from the risk-free rate $r_t^b - r^f = 1.65 - 1.15 = 0.5\%$. Under full provisioning, banks provision larger losses and charge a higher spread from their cost of funds $r_t - r_r^b = 2.9 - 1.15 = 1.75\%$ and as a result provide a risk-free portfolio to their investors which require a risk-free return $r_t^b = r^f = 1.15\%$. Since the final lending rate is the same under both types of provisioning, then for a given LTV, borrowers' leverage and riskiness is the same leading to borrowers' probability of default $PD = 4.19\%$ and loss-given-default $LGD = 40\%$ under both partial and full provisioning. However, due to the higher default threshold of the bank under full provisioning, bank probability of default is lower under full provisioning: 1.23% than under partial provisioning 1.65%.

In addition, we can compare the equilibrium characteristics of the loss distribution with the financial accelerators with the Basel (Vasicek, 2002) distribution that does not consider such effects (Figure 5.9). This is can be done in two ways. First, we can directly use the PD and LGD resulting from the starting LTV and

[1]For a detailed summary of the calibrated parameters see Appendix 8.4.

the financial accelerators, as inputs into the Vasicek distribution employed by the Basel capital regulation. This results in a bank probability of default of 0.98%. Second, we can use the starting LTV without the application of the accelerators to obtain simplified PD,LGD and corresponding Basel bank probability of default estimate of 0.32%. The table below sumarizes the obtained results.

Table 6.1: Model results under baseline calibration

Variable	Acceler., PP	Acceler., FP	Basel	Basel/simple
lending rate r_t	2.9%	2.9%		
cost of funds r_t^b	1.65%	$1.15\% = r^f$		
borrower PD	4.19%	4.19% →	4.19%	3.0%
borrower LGD	40%	40% →	40%	39%
Bank PD	**1.65%**	**1.23%**	**0.98%**	**0.32%**

Reflecting on the results, it is important to note that using the simplified lower PD and LGD values without the financial accelerators as inputs of the Basel formula is not an unrealistic quantification. While our baseline unconditional PD and LGD have been calibrated to match a 30 year average, in practice banks are allowed to use as little as 5 yeas of default history for obtaining PD and LGD estimates. In addition, the final lending rate to borrowers is not surprisingly low, given the historically low risk-free rate and the strong assumptions of perfect competition in the banking sector and risk neutral investors. Finally, while we conduct robustness checks for different portfolio riskiness set by the LTV, in practice banks' loan portfolios are divided into pools with similar characteristics and hence, our LTV represents a crude average of the overall bank portfolio riskiness.

Taking this into account, we can conclude that non-recognizing the illustrated financial accelerators can lead to an underestimation of the probability of default of a bank of a magnitude between 1.3 and 5.2 times for a given level of capital. And vice versa, if banks want to achieve a certain probability of default corresponding to their target rating they would need higher amounts of costly capital. Translated into bank ratings, our most conservative underestimation corresponds to a downgrade with 1 notch from BB+ to BB and the most severe to a downgrade with 4 notches from BBB to BB.[1] Furthermore, our results show robustness in a wide range of portfolio riskiness with possible increase of the underestimation effect for riskier portfolios.[2]

6.3 Cost of bank funds and risk-taking under Deposit Insurance

In the previous section, we derived the cost of equity r_t^e and debt r_t^b for the bank by taking into account the risk to which equity and debt holders are exposed to, by providing funds to the bank which has a portfolio with a given level of riskiness - equations (6.4) and (6.5).

$$r_t^e - \frac{\displaystyle\int_{r^b-r}^{kr^e} Loss_{t+1} f(Loss)dz + kr^e \int_{kr^e}^{r^b} f(Loss)dz}{k} = r^f \tag{6.4}$$

$$r_t^d - \frac{\displaystyle\int_{kr^e}^{r^b} (Loss_{t+1} - kr^e) f(Loss)dz}{1-k} = r^f \tag{6.5}$$

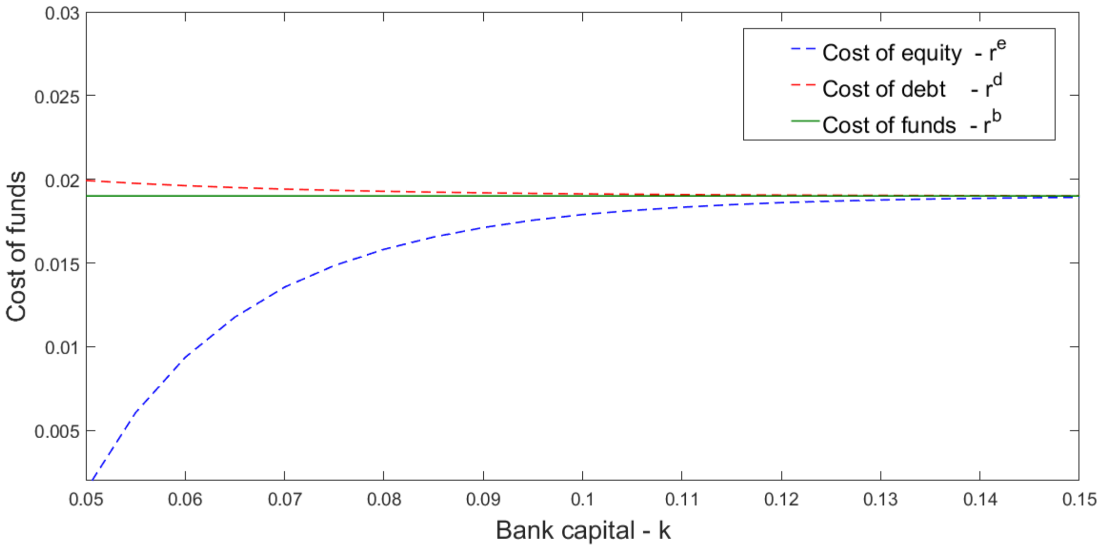
Furthermore, we showed that without any preferential treatment of debt or equity the weighted sum of debt and equity gives an aggregate cost of funds for the bank r_t^b that does not depend on the level of capital k leading to irrelevance of the capital structure of the bank - equation (6.6).

$$r_t^b = kr_t^e + (1-k)r_t^d = r^f + \int_{r^b-r}^{r^b} Loss_{t+1} f(Loss)dz \tag{6.6}$$

This means that as bank capital is increasing the costs of equity and debt are changing and offsetting the change of the funding composition. Figure 6.11 plots the two costs and their weighted sum for different levels of capital.

[1] According to the Fitch ratings system for banks.
[2] See appendix 8.3.

Figure 6.11: Cost of debt and equity for different levels of capital



From the figure above, we can see that the cost of debt r^d declines with the level of capital. This occurs since, as bank capital increases, the default threshold of the bank increases and there are fewer states of the loss distribution for which the bank defaults and debt holders suffer losses. Or in simpler words, debt holders are exposed to smaller share of the loss distribution and as a result require lower risk premium.

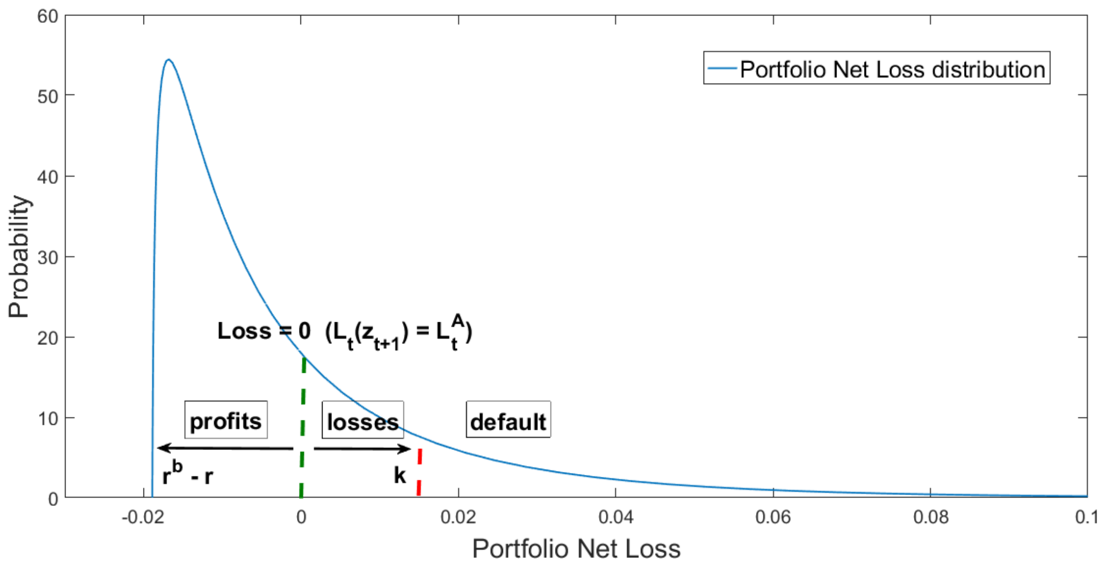
However, the result for the cost of equity is more controversial. While the Modigliani-Miller theorem holds and the overall cost of funds r^b does not depend on capital, the cost of equity is lower than the cost of debt and is increasing with the level of capital. This contradicts economic reality since capital is more costly than debt and contradicts the literature that explains the observed data showing that bank equity cost decreases with the level of capital.

To analyse those differences, we consider how bank performance affects the return of investors. Unlike the loss rate $L_t(z_{t+1})$, the net loss $Loss_{t+1}$ can take negative values, i.e. the bank can make profits. The highest amount of profits the bank can make is limited by the spread between the lending rate to borrowers and borrowing rate from investors $r_t - r_t^b$ which occurs for a loss rate of zero (no loans default). This is true since the net loss is:

$$Loss_t(z_{t+1}) = r_t^b - r_t(1 - L_t(z_{t+1})) \tag{6.7}$$

Due to the behaviour of setting the lending rate r_t, the bank would experience zero net profits/losses at loss rate equal to the one provisioned $L_t(z_{t+1}) = L_t^A$. For loss rate higher than the provisioned the bank would experience net losses that would be absorbed by equity holders' capital. If capital is not enough to absorb the losses then the bank would default and debt holders would also suffer.

Figure 6.12: Net Loss distribution (indicative level of capital - k)



As we showed in section (6.1), under full provisioning, the bank insures against both types of risk and as a result the expected net loss is zero $E[Loss_t(z_{t+1})] = 0$ and the aggregate lender to the bank requires a risk-free return $r_t^b = r^f$. The net loss can take both positive and negative values. However, equity holders are exposed entirely to the negative realizations of the loss (profits) and partially to the positive realizations (losses) - only up to the amount of capital. As a result, the default threshold of the bank (red line in figure 6.12) is at the level of losses equal to the level of capital. On the other hand, debt holders are exposed only to the positive realizations of the loss above the default threshold - in the scenarios in which the bank defaults and they lose their deposits.

As a result, equity holders would require lower than the risk-free return and as capital increases they become exposed to a larger share of the positive realization of the net loss and require higher return that converges to the risk-free rate for very high levels of capital. On the other hand, debt holders are exposed to only positive realizations of the net loss and require higher than the risk-free return. As capital increases, their exposure to risk decreases and their required return also converges to the risk-free rate.

The reason for the counterfactually lower cost of equity than cost of debt lies in the very few assumptions made so far. In particular, given our assumption of risk neutral investors that do not have mean-variance preferences. Equity holders which have higher mean and variance of their return than debt holders consider only the first moment and as a result require lower premium. Note, however, that this can be easily changed to match the empirical facts by introducing risk-averse investors. Furthermore, none of our qualitative results of financial accelerator mechanisms rely on the assumption of risk-neutrality. On the contrary, risk-aversion would only make the aggregate investor requiring higher premium which would reinforce the quantitative effect of the accelerators.

This however, would not change the puzzling result of equity costs that are increasing with the level of capital. The explanation of this result of our model is very simple. For a given portfolio riskiness defined by the LTV, the net loss distribution is fixed and the level of capital determines the distribution of the loss between debt holders and equity holders. Then with or without risk-averse investors, a higher level of capital would expose equity holders to larger share of the loss distribution and debt holders to a smaller share, which implies increasing cost of equity with the level of capital.

Deposit Insurance

A main result of the empirical papers explaining the relationship between the cost of equity and capital level relies on the assumption of risk-free debt due to deposit insurance. If the underlying riskiness of bank assets β^A, is be decomposed into risk of equity β^E and debt β^D we obtain:

$$\beta^A = k\beta^E + (1-k)\beta^D \tag{6.8}$$

Then a riskless debt would imply $\beta^D = 0$ and that for a given level of asset riskiness, the equity riskiness is negatively related to the level of capital as for a higher levels of capital the same assets risk is distributed over a larger equity cushion.

$$\beta^E = \frac{1}{k}\beta^A \tag{6.9}$$

However, if deposits are guaranteed by the state and as a result perceived by debt holders as risk-free, this does not mean that the deposits are in fact riskless $\beta^D \neq 0$. Because as a consequence of insuring deposits, the state bears the risk to which these deposits are entitled which is then implicitly transferred to taxpayers. Or in other words, transferring a larger share of the loss distribution to debt holders does not make the underlying risk of the portfolio disappear - leading to incorrect specification of equation (6.9).

Several papers[1] estimate bank equity beta's and then find a negative relationship between bank capital and equity beta and hence negative relationship between bank capital and equity risk premium. However, none of those papers accounts for the banks' portfolio riskiness and as a result it is unclear whether higher capital levels makes equity less risky or that higher capital induces banks to engage in less risky lending.

At this stage, our paper can provide novel insight into the observed empirical results. Considering our conclusion that bank capital level, per se, does not change portfolio riskiness but only the distribution of risk between equity and debt holders. Then our model can potentially explain the observed empirical results in the following way. As capital levels increase and equity holders are exposed to a greater share of the portfolio risk, they are incentivised to reduce bank portfolio riskiness. This incentive would be even stronger under deposit insurance, simply because the lower is the level of capital, the larger share of the portfolio risk is taken by the state while being priced at a constant risk-free rate. This would make equity holders prone to allow risky lending, as the cost of funding of the bank would be less responsive to it while at the same time receive higher margins in the good times while being insured in the bad times - Figure 6.12.

While a thorough optimal capital analysis is out of the scope of this paper, nevertheless its comprehensive structure can still provide valuable insight on the

[1]See Kashyap et al. (2010) and Miles et al. (2011).

topic. The main conclusion is strongly in favour of higher capital requirements especially under deposit insurance. In such circumstances, lower capital levels may induce risky lending by banks and inconsistent perception of risk by society which is sooner or later borne by taxpayers in the events of large crises. Moreover, the benefits of extensively complicated risk-sensitive capital requirements under the Basel regulations are in question, due to the natural incentives of equity holders to enforce safer lending when having more "skin in the game".

7 Conclusion

This paper started from the already established framework for bank capital regulation and expanded it by introducing optimal agent behaviour leading to feedback channels known as financial accelerators. The ASRF model employed by the Basel capital regulation was extended by bridging the gap between the finance-risk literature and the economics of financial frictions.

First, the optimal default choice of borrowers was employed for deriving the cost of repossessed collateral leading to a portfolio loss distribution that takes into loss-given-default. The former showed that in bad states, not only that a larger share of borrowers default but the bank repossesses cheaper collateral reinforcing the losses.

Second, risk insurance by banks is introduced, leading to an interest rate setting behaviour that requires higher rates from riskier borrowers with lower net worth. As a result, the higher interest further reduces borrower net worth and ability to repay closing the amplification cycle. In addition, the spread setting by banks involves the process of provisioning which is an important building block of bank behaviour.

Third, introducing required risk premia by investors allowed us to endogenise the cost of funds for the bank and derive the net loss distribution. An amplification channel resulted, as investors would require higher cost of funding from banks with riskier portfolios further worsening the riskiness of the bank.

The resulting novel framework encompassed bank portfolio losses, interest rate setting, provisioning and funding costs into a single regulatory consistent model. Quantifying the effect of the introduced financial accelerators showed that non-recognizing the illustrated financial accelerators can lead to an underestimation of the probability of default of a bank of a magnitude between 1.3 and 5.2 times for a given level of capital. And vice versa, if banks want to achieve a certain probability of default corresponding to their target rating they would need higher amounts of capital.

Finally, the extensive framework is used for gaining novel insight into the question of bank capital level and cost of funding. A common understanding in the literature is that under deposit insurance higher capital levels reduce equity riskiness. However, our model is able to illustrate that capital level, per se, does not change portfolio riskiness but only how the underlying risk of the bank is distributed between equity holders and debt holders. As a result, increasing capital levels leads to larger exposure to risk and higher required premium by equity holders but only as long as we consider portfolio riskiness for fixed. In fact, under deposit insurance, the lower is the level of capital the larger share of the portfolio risk is taken by the state while still being priced at the risk-free rate by depositors. As a result deposit insurance induces equity holders to invest as little "skin in the game" as possible exploiting the cheap insurance. Furthermore, higher capital requirements and exposure to the portfolio risk by equity holders induces them to require safer lending by the bank.

In summary, our results are strongly of favour of higher capital requirements. First, the illustrated financial accelerators lead to much riskier financial system than previously thought and second insufficient capital levels, under deposit insurance, can incentivise bankers to engage in riskier lending.

8 Appendix

8.1 LGD - LTV relationship

The recovery rate in the event of default is:

$$Recovery(z_{t+1}) = \frac{\mu z_{t+1} E_t\Big(\omega^i_{t+1}|\omega^i_{t+1} < \frac{ltv^i_t}{z_{t+1}}\Big)}{ltv_t} \tag{8.1}$$

Using the relationship between conditional and partial expectation:

$$E_t\Big(\omega^i_{t+1}|\omega^i_{t+1} < \frac{ltv^i_t}{z_{t+1}}\Big) = \frac{\int_0^{\frac{ltv_t}{z_{t+1}}} \omega f(\omega) d\omega}{\int_0^{\frac{ltv_t}{z_{t+1}}} f(\omega) d\omega} \tag{8.2}$$

We can express the recovery rate as:

$$Recovery(z_{t+1}) = \frac{\mu z_{t+1} \int_0^{\frac{ltv_t}{z_{t+1}}} \omega f(\omega) d\omega}{ltv_t \int_0^{\frac{ltv_t}{z_{t+1}}} f(\omega) d\omega} = \frac{\mu z_{t+1} \int_0^{\frac{ltv_t}{z_{t+1}}} \omega f(\omega) d\omega}{ltv_t PD(z_{t+1})} \tag{8.3}$$

where $f(\omega)$ is the PDF of the log-normal distribution. The probability of default $PD(z_{t+1})$ is simply the CDF of the distribution evaluated at the ltv/z_{t+1} threshold, which is increasing in the ltv. If we denote the remaining part of the recovery rate with

$$X(ltv) = \frac{\mu z_{t+1} \int_0^{\frac{ltv_t}{z_{t+1}}} \omega f(\omega) d\omega}{ltv_t} \tag{8.4}$$

then its derivative is:

$$\frac{dX(ltv)}{dltv} = \frac{\mu}{ltv} ltv f(ltv) - f(ltv) - \frac{\mu \int_0^{ltv} \omega f(\omega) d\omega}{(ltv)^2} \tag{8.5}$$

which simplifies to:

$$\frac{dX(ltv)}{dltv} = (\mu - 1) f(ltv) - \frac{(1-\mu) \int_0^{\bar{\omega}^{j,a}} \omega f(\omega) d\omega}{(\bar{\omega}^{j,a})^2} \tag{8.6}$$

which is negative since $\mu < 1$, meaning that $X(ltv)$ is decreasing function of the LTV. As a result the recovery rate is a decreasing function of the LTV.

8.2 Equilibrium

For solving the equilibrium we use the following algorithm.

1. Set starting cost of funding equal to the risk-free rate: $r^b = r^f$

2. Find r satisfying:

$$r_t = \frac{r^b}{1 - \int_0^{lv_t r_t} f(\omega)d\omega + \frac{\mu \int_0^{lv_t r_t} \omega f(\omega)d\omega}{lv_t r_t}} \tag{8.7}$$

3. Compute the portfolio loss rate for the domain of realizations of the aggregate shock z_{t+1}

$$Loss_{t+1}(z_{t+1}) = r^b - r_t \left[1 - L_t(z_{t+1})\right] \tag{8.8}$$

4. Find the PDF of the portfolio Loss - $f(Loss_{t+1})$

5. Compute the costs of debt and equity:

$$r_t^e = r^f + \frac{\displaystyle\int_{-\infty}^{k} Loss_{t+1} f(Loss)dLoss}{k} \tag{8.9}$$

$$r_t^d = r^f + \int_k^1 \frac{Loss_{t+1}}{(1-k)} f(Loss)dLoss \tag{8.10}$$

6. Compute the costs of funds for the bank: $r^b_{new} = kr^e + (1-k)r^d$

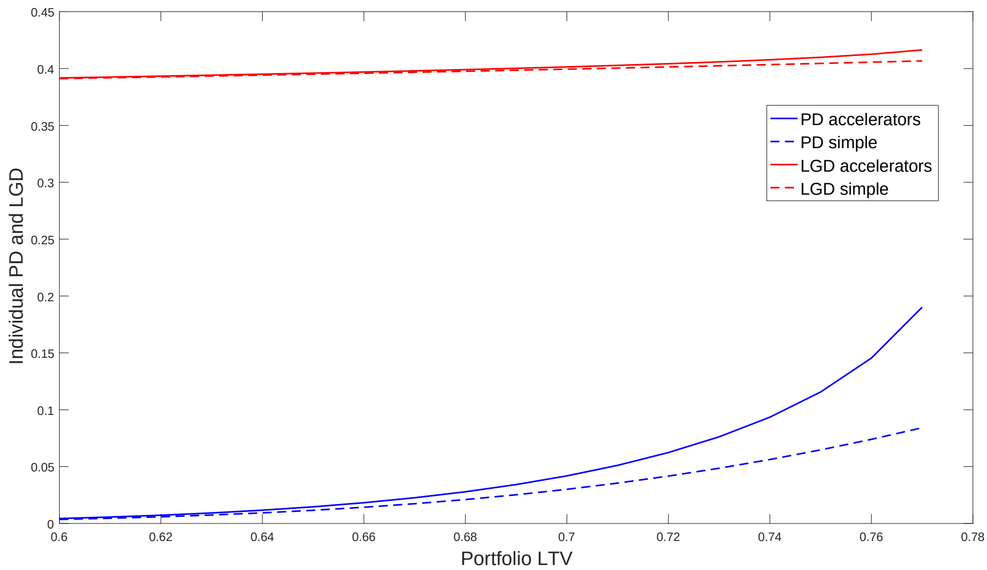
7. If $\mid r^b_{new} - r^b \mid < 1x10^{-8}$ then stop;

   otherwise update $r^b = r^b_{new}$ and continue from point 2.

8.3 Robustness checks

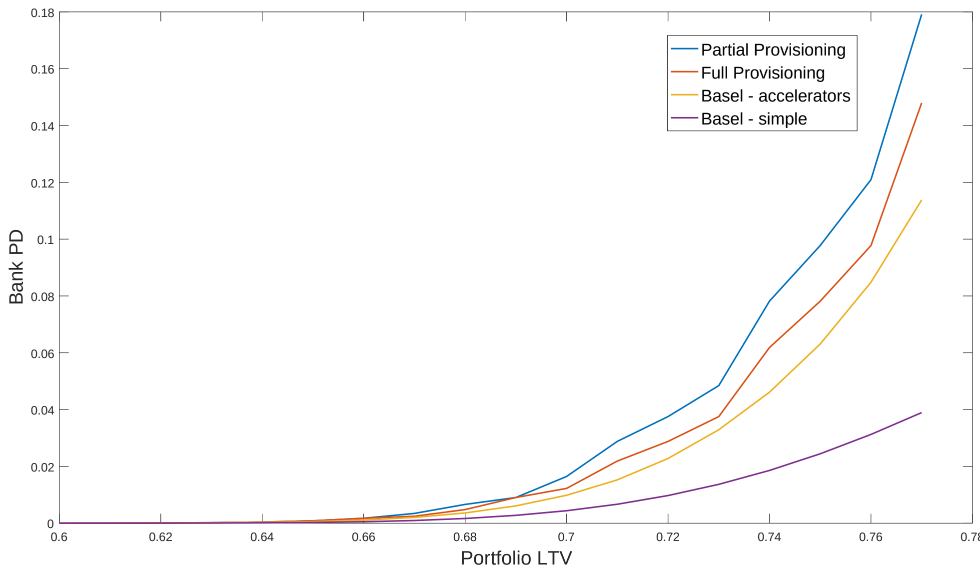
Although that the model produces reasonable results for the baseline calibration, this may not be true under different values for the parameter governing the portfolio riskiness - the LTV. In order to verify that, we conduct robustness checks by evaluating the model equilibrium for different values of the LTV. We evaluate the individual unconditional PD and LGD both in the case of taking and not taking into account the financial accelerators. The figure below plots the results.

Figure 8.13: Individual PD and LGD under different LTV values



In addition, we compute the bank probability of default under: partial and full provisioning and under the Basel formula with our without financial accelerators.

Figure 8.14: Bank probability of default under different LTV values



We are able to evaluate the equilibrium in the range of $0.5 < LTV < 0.77$. For lower LTV values the individual and bank probabilities of default become zero and for higher values, our solution algorithm has hard time converging. Nevertheless, in practice bank portfolios would be divided into pools of different riskiness and as a result an aggregate LTV that is too high or too low is highly unlikely. Moreover, banks' credit policy or macroprudential regulation often has upper cap on the LTV limit restricting extensively risky lending.

Taking this into account, we can see that for higher LTV values, both individual PD and LGD increase and this is more pronounced once the accelerator effects are taken into account. An interesting observation is the low responsiveness of the LGD to risk which seems counterfactual. A possible cause is the static cost of collateral repossession that we have picked to match the data in the baseline calibration. In practice, these costs would also vary with the business cycle and riskiness and endogenising these costs is a possible direction of future research.

Bank probability of default also increases rapidly with the risk of the portfolio. This is more pronounced for the settings taking into account the financial accelerator and less for the Basel case without accelerators. This leads to a possible increase of the reported magnitude of bank probability of default underestimation for riskier portfolios.

8.4 Calibration

Table 8.2: Baseline parameters calibration

Param	Value	Description	Source
LTV	70%	loan-to-value ratio	Standard value in the literature (Gerali et al., 2010), subject to robustness checks.
k	5.75%	bank capital-asset ratio	Federal Deposit Insurance Corporation. (2016) Weighted average leverage ratio of top 8 US banks.
r^f	1.15%	risk-free rate	Bloomberg (2017) US 12 month Treasury bond yield.
ρ	15%	Correlation between collateral values.	Basel asset correlation for mortgages.
ς	19%	Aggregate level of risk in the economy.	Set as to give the targeted PD and LGD values.
PD	4.19%	Unconditional individual probability of default.	Long run average delinquency rate of US mortgages. Federal Reserve Board, long run average 1987Q1 to 2016Q4.
LGD	40%	Unconditional individual loss-given-default.	Fixed LGD value under the Basel Foundation-IRB approach.

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