# Discounts, Rationing, and Unemployment

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**Abstract**

How are changes in discount rates transmitted to unemployment, and are they a quantitatively relevant driver of the Great Recession? In this paper I answer these questions in a search and matching model featuring endogenous capital accumulation. I show that endogenous capital amplifies the effects of a rise in discounts on unemployment. When firms make hiring decisions they must also jointly decide how much capital to provide their workers, which determines their labour productivity. Since capital is a long lived asset, a rise in discounts reduces investment, reducing the marginal product of labour and hence incentives to hire. I use the framework of Michaillat (2012) to classify this effect as a rise in *rationing* unemployment. I estimate changes in discount rates during the Great Recession using data on investment, and find a modest but persistent rise in discounts. These discounts cause a 2.3pp rise in unemployment, mainly during the slow recovery, by causing 83% of the observed capital shallowing in the data. However, a decomposition exercise reveals that endogenous capital alone is not enough to allow discounts to explain the whole rise in unemployment during the recession.

*Keywords:* discounts, unemployment, investment, great recession

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# 1 Introduction

What are the sources of the large and persistent disruption in the labour market that the US experienced during the Great Recession? Using the search and matching framework, recent work by Hall (2017) and Kehoe et al. (forthcoming) proposes a rise in "discounts" as a potential explanation for reduced hiring incentives in recessions. In the presence of hiring frictions, firms' hiring decisions are inherently forward looking: firms pay costs today to hire a worker, but receive the benefit of hiring them over the lifetime of the match. If firms discount these future benefits more (if "discounts" rise) this may reduce their incentives to hire, and hence lead to a rise in unemployment.

While there are many reasons that discount rates could rise during a recession, one question the literature on discounts and unemployment has focused on is *how* a given rise in discounts is transmitted into unemployment. A key issue, going back to Mukoyama (2009) is that reasonable fluctuations in discount rates have small effects on unemployment in the standard search and matching model. One reason for this is that empirical job separation rates are very high in US data, meaning that when a firm hires a worker they do not expect them to stay very long. This implies that the effective total discount rate on firms' hiring decisions is actually very high in a calibrated model, and hence changes in the pure discount rate can have limited effects on the incentive to hire.[1]

Both Hall (2017) and Kehoe et al. (forthcoming) propose modifications of the standard model which amplify the effect of changes in discounts on unemployment, and in this paper I propose and test an additional amplification mechanism. The key idea of this paper is that when firms decide to hire an additional worker, they are not just deciding on whether to hire the worker herself, but also whether or not to invest in the capital required to make that worker productive. The basic search model (e.g. Pissarides, 2000) assumes single worker firms and linear production with labour as the only input. However, in reality if a firm wants to expand its workforce whilst maintaining their productivity, it must also invest in the machines and production lines, and in the extreme the entirely new plants, required for them to work with.

Many search models do include capital accumulation, including early contributions by Merz (1995), Andolfatto (1996), and Pissarides (2000). However, this has previously be abstracted away from in search papers investigating the role of discounts. Capital accumulation and discounts interact in a very natural way, which could amplify the effect of discounts on unemployment. Suppose a firm expands its production by simultaneously hiring extra workers and investing in new capital. While these workers may turn over quickly due to high job separation rates, the installed capital which they work with has a much longer productive life. Hence this capital will be used to produce even with the workers who eventually replace those who have left. Put differently, the depreciation rate of capital is much lower than than the separation rate of workers.[2] This lower depreciation rate of capital means that capital is a longer lived investment, and should naturally be much more sensitive to changes in discounts. If a rise in discounts lowers investment, it will then also naturally reduce capital per worker, labour productivity, and the incentive to hire workers. If we interpret the amount of capital as a measure of the number of jobs or positions in a firm, then we can think of this as

[1] Kehoe et al. (forthcoming) show that in a model where wages are chosen via Nash bargaining over the total surplus, both the high job separation rate and the high job finding rate in US data play a role in increasing the effective discount rate of the firm's surplus from hiring. If wages are instead set as exogenous functions of productivity, then only the separation rate determines the effective discount rate, as I show in Section 2.

[2] Standard values of the depreciation rate of capital are typically around 6.5% to 10% per year. Shimer (2005) calculates the worker separation rate to be 3.4% monthly, giving a 34% yearly separation rate. Thus the effective depreciation rate of workers is between 3.4 and 5.2 times higher than that of capital.

a distinction between worker and job flows. While individual worker flows are very high, the total number of jobs within a firm is empirically much more persistent[3] and capital is transferrable across workers even after an individual worker has left.

In this paper I investigate whether, how, and by how much the inclusion of capital changes the propagation of discount rate shocks to unemployment. To this end, I build an otherwise standard search and matching model featuring endogenous capital accumulation, wage rigidity, and a time-varying discount rate. I make three main contributions. I first show using both analytical and numerical comparative statics results that endogenous capital provides a potentially powerful amplification mechanism for the effects of discounts on unemployment. Secondly, I follow Michaillat (2012) and decompose the effect of discounts into "frictional" and "rationing" components. I show that discounts must raise rationing unemployment, while the effect on frictional unemployment is theoretically ambiguous. However, for sensible calibrations of the model frictional unemployment falls when discounts rise. Finally, I perform a quantitative decomposition exercise for the Great Recession. I show that, despite the amplifying role of capital, discounts have only modest effect on unemployment in my model, and mostly contribute to the slow recovery of unemployment rather than the initial peak.

I begin my analysis with analytical results in the steady state of the model. I divide my results into two sections to highlight the additional role of endogenous capital in the model. I first investigate the effects of an increase in discounts on unemployment when capital is held constant. This model corresponds closely to the existing literature on discounts, with the exception that the model features diminishing marginal product of labour. I then move on to the full model, where capital is also allowed to adjust to a change in discounts according to the capital Euler equation.

Using comparative statics results, I first show that a rise in discounts causes unemployment to increase when capital is fixed, since it causes a decrease in the discounted value of posting vacancies, as in the existing literature. I then show that a rise in discounts also increases unemployment when capital is allowed to adjust, and that this increase must be larger than when capital is fixed. Intuitively, unemployment increases more with endogenous capital because an additional effect operates: the rise in discounts lowers the incentive to invest in capital, which reduces the marginal product of labour and hence incentive to hire. Quantitative results from a calibrated version of the model confirm that these differences are large. Following Michaillat (2012) I assume a wage rule linking wages to labour productivity with an empirically reasonable degree of wage rigidity. With this wage rule, doubling the discount rate has a negligible effect on unemployment with fixed capital, while the same exercise leads unemployment to triple when capital is endogenous.

I then investigate the mechanisms through which discounts are transmitted to unemployment, and show that the transmission through capital works fundamentally differently from the effects in the linear search model. I apply Michaillat's (2012) decomposition of unemployment into "rationing" and "frictional" components in my model, thus extending his analysis to a model with capital and variation in discounts. Michaillat defines rationing unemployment as the level of unemployment that would arise in a given model when hiring frictions are removed. Frictional unemployment is then defined as the extra unemployment arising due to the presence of hiring frictions. The combination of wage rigidity and diminishing marginal product of labour mean that in some states of the world rationing unemployment will be positive in my model.

[3]See, for example, Elsby et al. (2017) who show that many establishments exhibit zero net hiring despite nontrivial quit rates.

I first show that the small effect of discounts on unemployment when capital is fixed manifests only as a rise in frictional unemployment. I then show that the effects when capital is endogenous are more interesting. I show that when capital is free to adjust, rationing unemployment must rise when discounts rise. Rising discounts lowers capital intensity and hence the marginal product of labour. When wages are sticky this will cause unemployment to rise due to rationing in the labour market even in the absence of search frictions. However, I also show that the response of frictional unemployment is ambiguous, and it will also rise if wages are sufficiently flexible, while it will fall if wages are sufficiently sticky. Thus, whether the effect of discounts on unemployment is due to search frictions (frictional unemployment) or due to capital shallowing and sticky wages (rationing unemployment) is theoretically ambiguous. I thus move on to a calibrated version of the model, and show that for the baseline calibration, and indeed a wide range of calibrations, frictional unemployment falls when discounts rise. Thus, discounts appear to affect unemployment primarily as a rationing phenomenon.

My final contribution is to perform decomposition exercises for the Great Recession to assess the importance of discounts in generating the observed patterns of unemployment during this period. I use data on investment to estimate the path of discounts. The discount rate estimated in this way rises modestly from an annualised rate of 5.4% pre-crisis to 6.1% by the end of the sample. In contrast to discounts estimated from stock markets, these discounts rise gradually during the sample, but rise permanently. This reflects the fact that the capital-labour ratio in the US has permanently fallen by over 12% post-crisis, and the model requires a modest rise in discounts to explain this.

I first feed this estimated rise in discounts through the model, applying it to the Euler equations for both hiring and investment. I solve the model non-linearly using a perfect foresight approach, and find that the rise in discounts generates a gradual but persistent rise in unemployment by 2.3 percentage points from 5% pre-crisis to 7.3% by the end of the sample. This modest rise in unemployment is driven entirely by a rise in rationing unemployment, and the rise in discounts causes a fall in frictional unemployment by the end of the sample. Intuitively, the estimated rise in discounts alone causes firms to reduce investment, which reduces the capital-labour ratio by 10%, a full 83% of the total decline in the data. This reduces the marginal product of labour, and since wages are sticky this causes a fall in hiring. Since the effects are concentrated towards the end of the recession, this suggests that rising discounts may be more important for explaining the slow recovery than the initial peak rise in unemployment.

While I find that my model with endogenous capital amplifies the response of unemployment to a rise in discounts relative to a baseline model with exogenous capital, the overall effects are still modest. In the data, unemployment rose by 5 percentage points, reaching highs of 10% during the Great Recession. Additionally, if one considers the declines in labour participation over this period, I show that the effective model-consistent unemployment rate rises to over 13%. I thus perform a full decomposition exercise for the Great Recession where I decompose the path for unemployment over this period into components arising from five different shocks: discounts, TFP, job separations, labour force participation, and a residual shock to job creation. These shocks are estimated to exactly replicate the paths for capital, TFP, separations, participation, and employment in the data. I then feed these shocks through the non-linear model individually to assess each shock's contribution to the path of unemployment.

I find that both rising discounts and declining TFP contribute to raising unemployment towards the end of the recession. Declining TFP growth contributes a rise of 2.1pp to unemployment, which is smaller but comparable to the 2.3pp contribution of rising discounts. Combined, the two forces represent a significant drag to unemployment during the recovery, and hence contribute to the slow recovery of unemployment. The effects of rising discounts start slightly sooner, but at unemployment's peak in 2010 discounts are only contributing 0.8pp to the rise. The initial rise in unemployment is instead explained almost entirely by the residual shock to job creation incentives, which is unrelated to discounts in this model.

Finally, I perform several exercises to investigate the role of discounts further, including a comparison of my discounts estimated from capital to discounts estimated from the stock market, as in Hall (2017). I find that discounts estimated from stock markets rise much more at the beginning of the recession, but also recover quickly. This is in contrast to discounts estimated from capital, which rise less but more persistently. Interestingly, the estimated effects on unemployment are of a similar magnitude, but the effects and timings are drastically different: Discounts estimated from stocks increase unemployment at the beginning of the recession and operate via frictional unemployment, while discounts estimated from capital lead unemployment to rise at the end of the recession via rationing unemployment.

**Related Literature.** This paper chiefly contributes to the literature on the role of discounts in driving unemployment over the business cycle. An early contribution by Mukoyama (2009) questioned the ability of discounts to meaningfully drive unemployment. More recently, two papers modify the standard model in order to generate larger movements in unemployment from smaller movements in discounts.

Firstly, Hall (2017) demonstrated that making wages stickier increased the power of variations in discounts. Rather than the Nash Bargaining assumption used in Mukoyama (2009), he uses the alternating offer model of Hall and Milgrom (2008) and shows that it is possible to calibrate the degree of wage stickiness to match the co-movement of discounts and unemployment. Apart from adding endogenous capital, my model differs from Hall's in two ways: 1) I replace the alternating offer model of wages with an exogenous wage rule, with the degree of wage stickiness calibrated to match estimates from Haefke et al. (2013). 2) I measure discounts from the Euler equation for capital investment, while Hall uses the stock market. I show that these two measures of discounts move quite differently during the Great Recession, leading to different implications for unemployment. Given that the stock market measures only a subset of firms while investment data covers the whole economy, these differences raise interesting questions about the heterogeneity of discount rates across firms.

Secondly, Kehoe et al. (forthcoming) show that adding on-the-job human capital accumulation greatly amplifies the effects of a rise in discounts on unemployment. This is because human capital accumulation means that the surplus to a match grows over time as a worker becomes more productive, making the benefits to hiring felt further in the future. This makes the discounted benefits of hiring more sensitive to the discount rate. Making the benefits of a firm-worker match grow over time is similar in spirit to reducing the worker separation rate, which also increases the importance of returns further in the future. My model can be interpreted as giving another reason that firms' hiring decisions should be more forward looking: When firms produce using both capital and labour, hiring decisions are intertwined with investment decisions. Since investment decisions naturally have long horizons due to the relatively low depreciation rates of capital, this spills over to firms' hiring decisions.

In this paper I consider a simple representative-agent matching model where firms produce using both capital and employed workers. However, as discussed above the inclusion of long-lived capital naturally brings to mind the distinction between a firm hiring a worker versus firms opening a new job position. With multi-worker firms this distinction becomes meaningful because some hiring will be "replacement hiring" to replace workers who leave an existing job, while some hiring will be associated with firms expanding their total workforce, and hiring workers to fill new jobs. While individual workers transition very frequently across jobs, firms' total employment levels are much more persistent. Elsby et al. (2017) report that despite substantial turnover of workers, 40% of establishments report no change in *net* employment within one year, and 30% report no net change after three years. Thus, in contrast to the expected tenure of a single worker, net employment at establishments is much more persistent. Given this persistence, it is conceivable that changes in discount rates should have larger effects on firms' net hiring decisions than implied by the high gross worker separation rates. In this paper, the persistence comes from the lower depreciation rate of capital, but it could also come from more detailed specifications of establishment dynamics and adjustment costs.

By focusing on how variations in discounts affect firm investment, I also build on the large literature which investigates how financial frictions affect investment and economic activity. I interpret movements in these frictions through the lens of reduced form discounts affecting the incentive to invest in capital. Early contributions include Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke, Gertler, and Gilchrist (1999).

In this paper I consider the role of discounts in affecting investment in both capital and labour. Existing papers have built frameworks including financial frictions, capital, and labour market matching (for example, Christiano et al., 2011, Mumtaz and Zanetti, 2016) which could be reinterpreted through the lens of discounts. I investigate the importance of capital in amplifying discounts shocks in the search and matching model. In a similar spirit but without variations in discounts, den Haan et al. (2012) argue that capital adjustment increases propagation in a search and matching model with endogenous job separations. Some papers interpret the vacancy posting cost in the search model as the cost of purchasing capital (e.g. Acemoglu and Shimer, 1999). What is crucial for my analysis is the idea that capital is long lived and survives past the separation of any individual worker from the firm, which requires modelling the process for capital explicitly.

Closest to my work are papers which consider discounts and frictions in both investment and hiring. Building on earlier work (Yashiv, 2000, Merz and Yashiv, 2007), Yashiv (2016) estimates a model of joint adjustment costs in capital and labour. He uses a model-free forecasting VAR to relate hiring and investment to their expected future values. He finds that variations in both are driven mostly by their respective expected returns, thus finding an important role for variations in discounts. Interestingly, his estimation also finds that the rates of separation and depreciation do not play a meaningful role in job and investment values. This is supportive of the approach taken here and in Kehoe et al. (forthcoming) to find extensions of the search model to effectively make employment decisions more forward looking.

Hall (2016) extends the linear search and matching model of Hall (2017) to include endogenous capital as well as several shocks. He uses this model to provide a decomposition of the Great Recession, similar in spirit to my exercise, but allowing for separate discounts to capital and labour. The most important difference from my paper is that Hall (2016) does not provide a full model of the labour market, instead using a reduced-form representation of the labour market in which unemployment can only depend on a product market wedge and labour discounts. Thus, the model does not allow the feedback from discount rates to capital to employment via the marginal product of capital, which is the central interest of this paper.

The business cycle accounting approach of Chari et al. (2007) decomposes business cycles into constituent distortions in the equations of the real business cycle model. My time-varying discount rate is similar to the investment wedge in their model, but computed under the assumption of risk-neutrality. They model the distortion in the labour market as a tax on labour income in an otherwise frictionless labour market, whereas I explicitly model search frictions and wage rigidity. Brinca et al. (2016) update the approach to include data for many countries including the Great Recession, and find an important role for the investment wedge during this episode.

By applying Michaillat's (2012) decomposition of unemployment into rationing and frictional components to my model, I also thus extend his work to a model with endogenous capital and discounts as the shock driving unemployment. Michaillat's (2012) model featured diminishing marginal product of labour and a fixed capital stock, making the aggregate production function decreasing returns to scale. I show that his classification continues to be relevant in a model with endogenous capital and constant returns at the aggregate level. Endogenous capital provides a new channel for rationing unemployment to exist, because in my model rationing unemployment can be caused by either low productivity or low capital, since both reduce the marginal product of labour. In Michaillat's (2012) model a rise in discounts can only show up as an increase in frictional unemployment, while in my model it will initially manifest as frictional unemployment, but will manifest as rationing unemployment in the longer term as capital adjusts.

As argued by Michaillat (2012), the distinction between frictional and rationing unemployment is important because these two forms of unemployment have fundamentally different causes and hence different policy implications. Landais, Michaillat, and Saez (2018a, b) use the rationing model to show that when recessions are driven by rationing, optimal unemployment insurance becomes more generous in recessions. This is because if unemployment is driven by rationing, and not by frictional forces, the costs of increasing unemployment insurance, for example that it reduces search effort by job seekers, become less relevant whenever rationing unemployment is high and frictional unemployment is low.

The literature on discounts in search and matching models can also be understood as a reduced-form way to represent the role of financial frictions in making hiring more expensive for firms. There is a large recent literature on this subject, with notable papers including Wasmer and Weil (2004), Petrosky-Nadeau (2014), Petrosky-Nadeau and Wasmer (2015), Quadrini and Sun (2015), Schoefer (2016), and Carrillo-Tudela et al. (2018). Farmer (2012) takes a different approach and removes the assumption of bargained wages, assuming a process for beliefs about the stock market which then drives job creation.

The rest of the paper is structured as follows. In Section 2 I provide two simplified examples to illustrate the main ideas of the paper. In Section 3 I set up the model, and in Section 4 I perform analytical comparative statics. In Section 5 I perform numerical comparative statics, and in Section 6 I analyse the Great Recession using numerical methods. In Section 7 I provide additional results, and in Section 8 I conclude.

## 2 Two Motivating Examples

In this section I present two stylised models to illustrate the key ideas in this paper. I first present a linear search and matching model in the spirit of Hall (2017) and Kehoe et al. (forthcoming), and discuss the features determining the strength of the effect of discounts on unemployment. I then present a model without search frictions but with capital and unemployment driven by sticky wages, and discuss the transmission of discounts in this model.

In both models, I take the extreme case of a fully sticky real wage, which does not respond at all to discounts. This makes the mechanisms transparent and simplifies the exposition. I relax this assumption in the later parts of the paper, where I use an empirically calibrated degree of stickiness.

### 2.1 Basic search model

Consider the steady state of a discrete-time linear search model with single-worker firms. There is a unit mass of workers. Matches produce constant output $z$ and separate with exogenous probability $\rho$. Workers are paid a constant exogenous wage $w$, and firms post vacancies at flow cost $\kappa$. Market tightness, $\theta \equiv v/u$ is the ratio of vacancies, $v$, to unemployment, $u$. Vacancies are filled with probability $q(\theta)$ coming from a constant returns to scale matching technology with $q'(\theta) < 0$. Unemployed workers find jobs at rate $f(\theta) = q(\theta)\theta$ and steady state unemployment is given by $u = \rho/(f(\theta) + \rho)$.

Firms discount the future at rate $r$ per period. In equilibrium, the level of tightness is such that the free entry condition equates the value to the firm of a match to the expected cost of filling a vacancy:

$$\frac{\kappa}{q(\theta)} = \frac{z - w}{r + \rho} \tag{1}$$

This equation gives the intuition for how a rise in discounts increases unemployment. For a fixed wage, the firm discounts the per-period profit $z-w$ at total rate $r+\rho$ capturing both time discounting and the probability that the match will dissolve each period. A rise in the time discount rate ($\uparrow r$) increases the total discount and hence the value of the match to the firm. Thus, market tightness must decrease ($\downarrow \theta$) in order to raise the probability that vacancies are filled ($\uparrow q(\theta)$) to reduce the expected vacancy posting cost and restore the free entry condition.

While the intuition is clear, the quantitative importance of this channel is not yet established. To do so, assume a Cobb Douglas matching function such that the vacancy filling probability is given by $q(\theta) = \psi_0 \theta^{-\psi_1}$, where $\psi_0$ controls match efficiency and $\psi_1$ controls the elasticity of matches with respect to unemployment. Plugging this into (1) and rearranging yields an expression for tightness:

$$\theta = \left( \frac{\psi_0}{\kappa} \frac{z - \bar{w}}{r + \rho} \right)^{\frac{1}{\psi_1}} \tag{2}$$

Consider comparative statics in this equation across values of the discount. While it looks like all the parameters of the model control the response of tightness to discounts, the assumption of a fully fixed wage means that the result is actually much simpler. Suppose the model is calibrated with a steady state discount $r^*$ and values of the other parameters leading to steady state tightness $\theta^*$. To find the level of tightness as we vary $r$ take the ratio of (2) for a generic $r$ versus the calibrated level $r^*$:

$$\frac{\theta}{\theta^*} = \left( \frac{r^* + \rho}{r + \rho} \right)^{\frac{1}{\psi_1}} \tag{3}$$

This shows us that across all calibrations of the model featuring the same steady-state tightness, the responsiveness of tightness to the discount rate is controlled only by the steady state discount, $r^*$, the rate of job separations, $\rho$, and the matching elasticity, $\psi_1$. Thus, features of the calibration such as the fraction of the surplus accruing to the firm $(z - w)$ are actually irrelevant to how powerful discounts are, if these models are calibrated to match the same steady state level of tightness.[4]

With this knowledge in hand, I discuss the power of discounts in a standard calibration of the model. I use a monthly calibration, and I take the monthly job separations rate to be $\rho = 0.0396$, implying a yearly rate of 38%. This is the value used in my full calibration, taken as the total separation rate to both unemployment and non-participation from the 2008M1 CPS data. I choose a steady state discount rate of $r^* = 0.0044$, which corresponds to a yearly rate of 5.37%. This rate is relatively standard, and is calibrated from pre-crisis data in my full model, as detailed in Section 5. I use a standard matching elasticity of $\psi_1 = 0.5$. Following Shimer (2005), with this matching function I can normalise $\theta^* = 1$, and I choose match efficiency to set steady state unemployment to $u^* = 0.05$. As discussed above, the remaining details of the calibration are irrelevant and are omitted.

I present the results of a simple experiment in Table 1. In each column from left to right I give the steady state level of unemployment as the discount rate is increased from its calibrated value up to a maximum value of $r = 2r^*$. The first row gives the results for the baseline calibration, and the remaining rows give the results for various alternative calibrations.

Table 1: Effect of doubling discounts on unemployment in the basic search model

| | $r^*$ | $1.25r^*$ | $1.5r^*$ | $1.75r^*$ | $2r^*$ |
|---|---|---|---|---|---|
| Baseline | 0.050 | 0.051 | 0.052 | 0.054 | 0.055 |
| $\rho \to 0$ | 0.050 | 0.062 | 0.073 | 0.084 | 0.095 |
| $\rho = 0.0198$ | 0.050 | 0.052 | 0.054 | 0.056 | 0.059 |
| $\rho = 0.0792$ | 0.050 | 0.051 | 0.051 | 0.052 | 0.052 |
| $\rho = 1$ | 0.050 | 0.050 | 0.050 | 0.050 | 0.050 |
| $r^* = 0.05$ | 0.050 | 0.057 | 0.063 | 0.069 | 0.076 |

Notes: Columns give steady state unemployment when discount rate, $r$, is raised to given multiples of initial calibrated value, $r^*$. Top row gives results for baseline calibration. Second row gives results for calibration with near-permanent jobs ($\rho = 0.000001$), third row for separation rate half of baseline, fourth for double baseline, fifth row for jobs that last one month only ($\rho = 1$), and final row for calibration with calibrated discount rate of $r^* = 0.05$ per month.

I choose to investigate the effects of doubling discounts because this implies large movements in asset prices. For example, in the absence of other factors, permanently doubling discounts should exactly half the price-dividend ratio in a Gordon Growth model. Hall (2017, Figure 8) reports a similar decline in the S&P500 index during the Great Recession, which I replicate in Figure 7, so this provides a natural benchmark. Additionally, this corresponds roughly to the increase in discounts considered by Hall (2017) during the Great Recession.[5]

[4]This stands in contrast to the comparative static with respect to productivity, $z$. Here the level of the surplus is important because it will determine how large the proportional change in $z - w$ is for a given proportional change in $z$. The proportional change in $z - w$ is larger when the firm's surplus is smaller. This issue is not important for the powerfulness of discounts when the wage is completely fixed.

[5]In Table 3 he reports that monthly discounts increase by slightly more than double, 233%, going from the average

Despite the large increase in discounts, the first row of Table 1 shows that this has minimal effects on unemployment in the baseline calibration. Going from the first to last columns shows the effects of doubling discounts from the calibrated value, which only raises unemployment by 0.5 percentage points. This is even under the relatively extreme assumption that wages are fully fixed, and do not fall at all to offset the rise in unemployment.

Studying (3) gives the intuition for this result. The proportional response of tightness to a change in discounts depends not on the change in the discount rate itself, but on the proportional change in the total effective discount rate, including the job separation rate: $r + \rho$. In the baseline calibration the separation rate, which is taken directly from the data, is an order of magnitude larger than the time discount rate: $\rho = 0.0396$ versus $r^* = 0.0042$. Hence increasing the time discount rate will have a small impact on the effective discount rate, since this is dominated by the job separation rate. Since $\rho$ is so large, increasing $r$ from $r^*$ by 100% only increases $r + \rho$ by 9.6%. In other words, with such a high job separation rate, jobs simply don't last very long on average. Hence hiring decisions are not too far from being static, and increasing the discount rate can have only a limited impact on hiring decisions.

To further demonstrate this effect, in rows 2 to 5 of Table 1 I give the results of the same exercise for different values of $\rho$. In row 2 I let $\rho$ approach zero, meaning that jobs become near-permanent. Now the effects of an increase in discounts are much larger: doubling discounts nearly doubles the unemployment rate. In row 5 I increase the separation rate to $\rho = 1$, meaning that all jobs last exactly one month. In this case the effect of raising discounts effectively disappears. However, for reasonable perturbations of the separation rate from its empirical counterpart the effects do not change dramatically from the baseline, as I show in rows 3 and 4 where I half and double the separation rate respectively.

In row 6 I show that the effects of doubling discounts are also larger if I recalibrate the baseline discount rate to be very large. If I set $r^* = 0.05$ (implying a yearly discount rate of nearly 80%) then $r$ is of the same order of magnitude as $\rho$, and so small changes in $r$ can start to have meaningful effects on the overall discount, $r + \rho$. Thus, it is not the high separation rate itself which makes discounts weak at moving unemployment, but rather the fact that it is so much higher than the discount rate.

Having established that the basic linear search model with fully rigid wages is not able to generate meaningful permanent movements in unemployment from large permanent changes in discounts, I now discuss how Hall (2017) and Kehoe et al. (forthcoming) modify the model to amplify the effects. The fact that Hall (2017) finds larger effects is initially surprising, given that the only modification he makes to the standard model is to add wage rigidity. Since I take the limit of fully rigid wages, it might seem that I have made the most extreme possible assumption, and that the small 0.5pp increase in unemployment that I find is the maximum that can be found by increasing wage rigidity. However, Hall's (2017) wage setting protocols actually lead to the real wage increasing when discounts increase. This behaviour can be rationalised under Hall and Milgrom's (2008) alternative offer bargaining. Following a rise in discounts, the real wage also rises, which reduces the share of the surplus accruing to the firm and hence further depresses the incentive to create jobs. Thus, part of Hall's (2017) insight is that it is necessary for the firm's share of the surplus to shrink in order to meaningfully amplify the effect of discounts on unemployment.

state to the state of the world corresponding to the Great Recession in his model. Due to the large effect on stock prices of a permanent doubling of discounts, I will refer to this increase in discounts as "large". When considering dynamics the interpretation of a large change in discounts is modified, as I discuss further at the end of the section.

Kehoe et al. (forthcoming) take a different approach to amplify the effect of discounts. They build a model where workers accumulate human capital on the job. When a worker is hired, their human capital grows over time, allowing them to produce more output over time. This makes matches more "forward looking", since more of the surplus from the match will now be produced further in the future, making the discounted value of the match more sensitive to discounts. In terms of the simple model presented above, this can be understood as reducing the effective job separation rate, which I showed increased the power of discounts.[6]

Overall, this section shows us two things. Firstly, the basic linear search and matching model generates small permanent movements in unemployment from permanent changes in discounts even in the limit of fully rigid wages. Secondly, extensions to the model can amplify the role of discounts either by changing wage-setting protocols to reduce the firm's share of the surplus when discounts rise (Hall, 2017), or by making matches effectively longer lived (Kehoe et al., forthcoming).

One caveat to the above results is that they consider a permanent increase in discounts, while in reality recessions and associated falls in stock prices are temporary. A much larger *temporary* rise in discounts is needed to generate a *temporary* 50% fall in stock prices than the *permanent* rise in discounts required to generate a *permanent* 50% fall. Thus, considering the dynamics of discounts has the potential to amplify their effects on unemployment. In subsection 7.6 I explicitly compute the path of discounts required to match the path of stock prices during the recession, and find that it does have a meaningful but short lived effect on unemployment. However, the required rise in discounts is found to be very large, around a factor of ten, consistent with the results of this section that it takes large movements in discounts to move unemployment in the basic search model.

## 2.2 Pure rationing model

Consider now a model without search frictions but where output is produced using both labour and capital. The production function is $y = zk^{\alpha}l^{1-\alpha}$, where $z$ now denotes total factor productivity (TFP). I abstract from the hours-per-worker margin, and let $l$ denote employment and $u = 1 - l$ unemployment.

I consider a simple model of unemployment where jobs are rationed due to the real wage being stuck above the market-clearing level. For a given wage, firms hire until the marginal product of labour (MPL) equals the real wage. However, I assume the real wage is fixed, and that workers are off their labour supply curves. Without hiring frictions, the firm's optimality condition for labour is the standard static condition equating with wage with the MPL:

$$(1-\alpha)zk^{\alpha}l^{-\alpha} = w \tag{4}$$

For a given wage, TFP, and level of capital, we can solve for equilibrium employment and unemployment as:

$$l = \left(\frac{(1-\alpha)z}{w}\right)^{\frac{1}{\alpha}} k \qquad \Longrightarrow \qquad u = 1 - \left(\frac{(1-\alpha)z}{w}\right)^{\frac{1}{\alpha}} k, \tag{5}$$

where I have assumed for simplicity that parameters are such that unemployment is always non-negative. The expression for $l$ in (5) shows that in this model firms set the level of employment

[6] To see this, suppose that workers initially produce $z$ and are paid $w$ when hired. These both grow at rate $g$ per period as workers accumulate human capital. The discounted value to the firm of a new match is now $J = (z-w)\left(1 + \beta(1-\rho)(1+g) + \beta^2(1-\rho)^2(1+g)^2 + ...\right)$ showing that increasing the growth rate of productivity ($\uparrow g$) is equivalent to reducing the separations rate ($\downarrow \rho$).

optimally proportional to the level of capital. Intuitively, firms hire workers until the marginal product of labour falls to equal the real wage. Reducing the level of capital makes workers less productive and reduces the MPL, and firms respond by reducing hiring by the same proportion to restore the MPL to the real wage.

How does an increase in discounts affect unemployment in this model? If TFP and the real wage are fixed, then it has to be through the level of capital. Just as we expect increased discounts to reduce desired hiring when hiring is frictional, they will also reduce desired investment and capital. An important question is how powerful a channel this is for affecting unemployment. Clearly this will depend on how much capital moves during a recession.

To give a sense of the potential quantitative magnitudes, consider a simple exercise. Suppose that unemployment is initially 5%, so that initial employment is $l = 0.95$. In order to increase the unemployment rate to 10% we need employment to fall to $l = 0.9$, which is a 5.3% reduction from its initial value. Since labour moves proportionally to capital in this simple example, it would only take a 5.3% reduction in capital to push unemployment up to 10%. As I show in the data used for the decomposition exercises of Section 6, in the US detrended capital had fallen by 4.5% from 2008 to 2010, and by more than 15% by 2018. Hence, movements in capital in the Great Recession are large enough to potentially contribute meaningfully to unemployment.

Of course, wages are not fully rigid in reality, which will temper this mechanism, and so incorporating a realistic degree of wage flexibility is important for determining how powerful an effect declines in investment can have on unemployment. Additionally, firms also face matching frictions when hiring, as in the model of the first example. Since these frictions fall in recessions because unemployed workers are abundant, this will also encourage hiring, and limit the ability of declines in capital to increase unemployment. Thus, in order to formally test whether endogenous capital accumulation can meaningfully amplify the effect of discounts on unemployment, in the remainder of the paper I build a full model which blends and extends the mechanisms highlighted in these two simple examples.

# 3 Model

In this section I set up the baseline model to be used throughout the rest of the paper. The model is an extension of the standard search and matching model to include capital, sticky wages, and time-varying discounts. Time is discrete and the horizon is infinite. For simplicity, I restrict myself to deterministic economies. There are two agents in the model: a representative (multi-worker) firm, and a representative household.

The assumption of time-varying discounts is a stand in for any financial or contracting frictions, or preferences, which induce risk-adjusted time preferences to change. Rather than spell these out, I follow Hall (2017) and simply assume that the representative household's discount factor, $\beta_t$, is time varying. $\beta_t$ describes the discount applied to real consumption between $t$ and $t+1$, and it follows a deterministic sequence $\{\beta_t\}_{t=0}^{\infty}$. Since variations in this rate stand in for all risk-adjusted time preferences, I assume that agents are risk neutral, and markets in the model are complete so that I do not have to consider the firm's financial structure.

### 3.1 Firms

The representative firm produces output using a Cobb-Douglas production function $y_t = z_t k_{t-1}^{\alpha} l_{t-1}^{1-\alpha}$, featuring one period time-to-build in both capital and labour. There is no intensive margin for labour, so $l_{t-1}$ refers to employment. $z_t$ is a common productivity shock, which follows a deterministic sequence $\{z_t\}_{t=0}^{\infty}$.

The firm hires labour subject to a standard matching friction. At time $t$ the firm posts $v_t$ vacancies at flow-cost $\kappa$ per vacancy. The firm takes the vacancy-filling probability $q_t$ as given. I assume that the law of large numbers holds, so the firm receives $q_t v_t$ new employees with certainty, who produce from the next period. Workers separate from the firm at the exogenous rate $\rho$, and the firm's labour stock thus evolves as

$$l_t = q_t v_t + (1-\rho) l_{t-1}. \tag{6}$$

The firm additionally accumulates a stock of capital according to

$$k_t = i_t + (1-\delta) k_{t-1}, \tag{7}$$

where $i_t$ is investment and $\delta$ is proportional depreciation. I assume that adjustment costs in capital are external to the firm, so that capital trades at a price $p_t^k$ which the firm takes as given. The price is given by the weakly increasing function $p_t^k = p^k(i_t/k_{t-1})$, which can be derived as the solution to the profit maximisation problem of a competitive capital goods producing sector with convex adjustment costs. Let variables without time subscripts denote steady state values. I normalise the price of capital to one in steady state, by noting that $i/k = \delta$ in steady state, and assuming that $p^k(\delta) = 1$.

Cashflow, $e_t$, is given by output less wages, vacancy posting costs, and investment: $e_t = z_t k_{t-1}^{\alpha} l_{t-1}^{1-\alpha} - w_t l_{t-1} - p_t^k i_t - \kappa v_t$. Plugging in the equations for the evolution of capital and labour, (6) and (7), gives cashflow in terms of the stock variables:

$$e_t = z_t k_{t-1}^{\alpha} l_{t-1}^{1-\alpha} - w_t l_{t-1} - p_t^k (k_t - (1-\delta) k_{t-1}) - \frac{\kappa}{q_t}(l_t - (1-\rho) l_{t-1}). \tag{8}$$

The firm maximises the discounted sum of cashflows, discounted by the households time-varying discount factor, $\beta_t$. Letting $\beta_{-1} = 1$, formally, at time 0 they maximise $\sum_{t=0}^{\infty} \left( \Pi_{j=0}^{t} \beta_{j-1} \right) e_t$ subject to the sequence of constraints (6), (7), and (8). The solution to this problem is characterised by a pair of Euler equations for capital and labour respectively:

$$p_t^k = \beta_t \left( \alpha z_{t+1} k_t^{\alpha-1} l_t^{1-\alpha} + p_{t+1}^k (1-\delta) \right) \tag{9}$$

$$\frac{\kappa}{q_t} = \beta_t \left( (1-\alpha) z_{t+1} k_t^{\alpha} l_t^{-\alpha} - w_{t+1} + \frac{(1-\rho)\kappa}{q_{t+1}} \right) \tag{10}$$

For both assets, the firm trades off the adjustment cost of acquiring one more unit of the asset today with the discounted benefit of the marginal cashflow generated tomorrow, and the adjustment cost saved tomorrow. Search frictions thus naturally act as a labour hiring cost, with it costing the firm $\frac{\kappa}{q_t}$ to acquire one more worker, just as it costs $p_t^k$ to acquire one more unit of capital. Due to the Cobb Douglas assumption, the MPL is simply equal to a constant fraction of measured labour productivity: $(1-\alpha)z_{t+1}k_t^{\alpha}l_t^{-\alpha} = (1-\alpha)y_{t+1}/l_t$.[7] The worker separation rate, $\rho$, acts as a depreciation rate, symmetrically to the capital depreciation rate, $\delta$, in the firm's hiring decision.

### 3.2 Labour market and matching technology

I assume a standard constant returns to scale (CRS) matching function for creating new employment matches. This takes the stock of unemployed workers, defined as $u_t$, and posted vacancies, $v_t$, to create $m_t = m(u_t, v_t)$ new matches. Market tightness is defined as $\theta_t \equiv v_t/u_t$. Given the CRS assumption, this gives the vacancy filling rate as a function $q_t = q(\theta_t) \equiv m(\theta_t^{-1}, 1)$. The job finding rate, $f_t$, is given by $f_t = f(\theta_t) \equiv m(1, \theta_t)$. I maintain as an assumption throughout this paper that the matching function is such that the job finding rate is strictly increasing in tightness.

I initially assume that the labour force is constant at mass one. This gives unemployment as simply $u_t = 1 - l_{t-1}$.[8] In the decomposition exercise for the Great Recession in Section 6 I allow the participation rate to change over time, to capture the large movements in participation and demographics seen over that period.

### 3.3 Wage rule

Following Michaillat (2012), I specify wages in my model in terms of an exogenous wage rule, rather than as the outcome of an endogenous bargaining process. This allows me to directly control the degree of wage rigidity, which is an important object in determining how discount rate shocks are transmitted to the economy.

In particular, consider the market clearing wage in this model, in the absence of matching frictions. Since the labour force is fixed at one, the market clearing wage is simply the marginal product of labour when all workers are employed ($l_{t-1} = 1$) and for a given level of capital and technology: $w_t^{mc} = (1-\alpha)z_t k_{t-1}^{\alpha}$. Recalling that steady-state values of a variable are denoted by an absence of subscripts, the steady state wage is given by $w$. Note that we must have the steady state wage being below the MPL in steady state ($w < w^{mc} = (1-\alpha)zk^{\alpha}$) to compensate the firm for vacancy posting costs. I assume that the wage rule is given by

$$w_t = \omega \left(z_t k_{t-1}^{\alpha}\right)^{\gamma}, \tag{11}$$

with $\omega \equiv w(zk^{\alpha})^{-\gamma}$ controlling the average level of the wage. In response to changes in either $z_t$ or $k_{t-1}$, the wage stickiness parameter $\gamma$ controls how far the wage is able to adjust towards the new market clearing wage. When $\gamma = 0$ the wage is fully rigid and fixed at the steady state wage: $w_t = w$. When $\gamma = 1$ the wage is flexible, and adjusts one-for-one with changes in the market clearing wage: $w_t = w_t^{mc} \times (w/w^{mc})$. Intermediate values of $\gamma$ give partial wage flexibility.

Recent work emphasises that the flexibility of wages in existing matches versus new jobs is different, with wages in new matches responding much more to changes in labour productivity (e.g. Haefke et al., 2013). I do not make the distinction between existing and new wages in my model. Accordingly, when it comes to numerical work I will calibrate wage stickiness to the level of new matches to be conservative.

[7] Iterating forward on (10) yields the standard free entry condition modified to include capital in production and time-varying discounts. At time 0 we have $\frac{\kappa}{q_0} = \sum_{t=0}^{\infty} \left(\Pi_{j=0}^{t}\beta_{j-1}\right)\left((1-\alpha)y_{t+1}/l_t - w_{t+1}\right)$ and similar for $t > 0$.

[8] For simplicity of exposition, I suppose that unemployment is always positive.

### 3.4 Definition of equilibrum

The model is deterministic, so equilibrium is defined as a sequence of prices and allocations which satisfy the restrictions of the model:

**Definition 1.** *Fix initial conditions $(l_{-1}, k_{-1})$ and shock sequences $\{\beta_t, z_t\}_{t=0}^{\infty}$. A sticky-wage equilibrium is defined as a sequence of variables $\{y_t, l_t, k_t, q_t, v_t, u_t, \theta_t, w_t, i_t, p_t^k\}_{t=0}^{\infty}$ satisfying:*

1. *Production function: $y_t = z_t k_{t-1}^{\alpha} l_{t-1}^{1-\alpha}$*
2. *Labour evolution: (6)*
3. *Capital evolution: (7)*
4. *Euler equations: (9) and (10)*
5. *Unemployment definition: $u_t = 1 - l_{t-1}$*
6. *Tightness definition: $\theta_t = v_t/u_t$*
7. *Vacancy filling function: $q_t = q(\theta_t)$*
8. *Wage rule: (11)*
9. *Capital price function: $p_t^k = p^k(i_t/k_{t-1})$*

Since the discount rates are given exogenously, and are not a function of consumption, we can state the equilibrium in the labour and capital markets without reference to market clearing in the goods market. If the capital-production technology behind the function $p^k(i_t/k_{t-1})$ is specified then consumption can additionally be found as the residual from output after vacancy posting and investment costs.

### 3.5 Rationing and frictional unemployment

A key idea in this paper is that discount shocks may be transmitted into unemployment in different ways. To characterise this more concretely, I utilise Michaillat's (2012) distinction between *rationing* and *frictional* unemployment.

Rationing unemployment is defined as the amount of unemployment a model generates even in the absence of matching frictions. Not all models will generate rationing unemployment, since it requires a mechanism to stop the labour market from clearing even when hiring costs vanish. Michaillat (2012) shows that the combination of wage rigidity and diminishing marginal returns to labour delivers rationing unemployment. Intuitively, when hiring frictions vanish firms hire until the wage equals the MPL, and rationing unemployment occurs if the wage is stuck above the market clearing level. Formally:

**Definition 2.** *For a given level of capital, $k_{t-1}$, and productivity, $z_t$, rationing unemployment is the equilibrium level of unemployment in the model when matching frictions are removed ($\kappa = 0$). It is given by*

$$u_t^R = \max\left\{0, 1 - \left(\frac{(1-\alpha) z_t^{1-\gamma} k_{t-1}^{\alpha(1-\gamma)}}{\omega}\right)^{\frac{1}{\alpha}}\right\}. \tag{12}$$

To derive this expression, note that when hiring frictions disappear, the labour optimality condition (10) reduces to the static condition that the wage equals the MPL: $w_t = (1-\alpha)z_t k_{t-1}^{\alpha} l_{t-1}^{-\alpha}$. This is combined with the wage rule, $w_t = \omega\left(z_t k_{t-1}^{\alpha}\right)^{\gamma}$, to find a fully static solution to the level of employment in the absence of rationing frictions, denoted $l_{t-1}^{R}$. The formula for rationing unemployment in (12) is simply the level of unemployment implied by this hiring, or zero if this is greater: $u_t^R = \max\left\{0, 1 - l_{t-1}^{R}\right\}$.

Whether rationing unemployment exists (i.e. is positive) at a given point in time depends on the level of productivity and capital. Inspecting (12), we see that rationing unemployment will be non-zero for low enough productivity or capital. Intuitively, when either is low, the MPL is also low, which reduces the incentives to hire (given that wages are sticky and do not fully adjust) and hence increases rationing unemployment. Similarly, whenever rationing unemployment is non-zero it is decreasing in productivity and capital.

This formula is a generalisation of Michaillat's (2012; equation 15) definition of rationing unemployment to a model with capital. His definition is simply the special case of this formula with capital constant at one ($k_{t-1} = 1$) and noting that his $\alpha$ corresponds to my $1-\alpha$ due to the specification of his production function. An interesting thing to note is that Michaillat's (2012) model had no capital in production, so diminishing MPL translated into decreasing returns to scale in the single production function, labour. My model instead features constant returns in the long run, once capital has been able to adjust. Nevertheless, as I show in the next section, rationing unemployment continues to be well defined in CRS economies.[9]

Finally, frictional unemployment is defined to be the difference between actual unemployment and rationing unemployment. This is the excess amount of unemployment which cannot be explained by rationing alone. Formally:

**Definition 3.** *For a given level of unemployment, $u_t$, and rationing unemployment, $u_t^R$, frictional unemployment is the excess of unemployment over rationing unemployment:*

$$u_t^F = u_t - u_t^R. \tag{13}$$

Unlike rationing unemployment, frictional unemployment need not be positive. Frictional unemployment can, in theory, be negative if rationing unemployment exceeds total unemployment.[10]

[9] The extension to capital reveals a subtlety in how rationing unemployment should be defined, since capital is itself an endogenous object. Should rationing unemployment be simply the level of unemployment in the absence of hiring frictions given the existing capital stock today? Or should we recognise that removing hiring costs would lead to different investment choices and hence a potentially different level of capital? I opt to take the first definition since it is simpler, and allows me to compute rationing unemployment without having to re-solve an entirely model without hiring frictions. However, in practice the dynamics of the results are similar in both definitions. In subsection 7.3 I solve a model of rationing unemployment where capital is also endogenously determined, and show that the results are very similar to the definition of rationing with capital taken from the full model.

[10] Although, in practice this does not happen in any of the calibrated examples in this paper. To understand how rationing unemployment can exceed frictional unemployment, consider a situation where productivity is growing very fast, so that equilibrium unemployment is falling very rapidly. Unemployment may be lower than rationing unemployment because in the full model with hiring costs firms have an incentive to hire now in anticipation of future productivity gains, while in the rationing model the employment decision is static.

# 4 Steady state analysis: analytical results

In this section I provide analytical results to illustrate the main workings of the model. My focus is on how an increase in the discount rate is transmitted into unemployment directly via hiring incentives and indirectly through capital, and to decompose this into frictional and rationing components. I focus on a steady state analysis, and perform comparative statics with respect to the discount rate. Throughout this section, I maintain two basic assumptions on the parameters: Firstly, parameters are such that employment is interior in steady state across any comparative statics considered. That is, $u, l \in (0, 1)$. Secondly, the discount rate is such that the overall discount rate applied to matches is always positive: $r + \rho > 0$.

In order to build intuition, I consider two cases which differentiate between the behaviour of capital. I first consider a version of the model with a fixed capital stock, where capital does not respond to a change in the discount rate. This reduces the model to a search model with labour as the only input and decreasing returns to scale, exactly as in Michaillat (2012). I then consider my full model with endogenous capital, where capital also adjusts to the discount rate through the Euler equation.

In keeping with the idea that the capital stock will evolve gradually in response to shocks, I sometimes casually refer to the model with fixed capital as the "short run" and the model with endogenous capital as the "long run". However, this distinction is arbitrary since there is no concept of dynamics in these exercises. In my numerical exercises I will consider the actual path for capital, which will evolve over time according to agents' optimal decisions.

I perform comparative statics across different values of the discount rate, $r \equiv \beta^{-1} - 1$. Recall that I use variables without a subscript to denote steady state values. Since I will perform comparative statics across different different steady states, I enhance the notation slightly to improve clarity. In this section, I use starred variables to refer to values in the initial steady state: $k^*$, $r^*$ and so on. I use the generic $r$ to denote a particular value of the discount rate. I hold productivity constant across these exercises, treating it as a parameter, and so continue to refer to it simply by $z$.

## 4.1 Fixed capital

In this section I consider the model with a fixed amount of capital, but where the labour market has time to adjust to its new steady state.[11] I thus hold capital at its initial steady state value, $k^*$, and vary the discount rate $r$, studying the effect on the equilibrium. I use a superscript $s$ to denote the new steady state values in the "short run" with fixed capital: $l^s(r), u^s(r), u^{R,s}(r), u^{F,s}(r)$, and so on. I suppress the dependence of these variables on the value of $r$ where it will not cause confusion.

In steady state, we can write the job filling rate as a strictly decreasing function of equilibrium employment: $q = \hat{q}(l)$ with $\hat{q}'(l) < 0$.[12] Intuitively, if steady-state employment is higher then more vacancies must be being posted, reducing the job filling probability.

The steady state level of employment with a fixed capital stock can be found by plugging $\hat{q}(l)$ and the wage rule, (11), into the labour optimality condition, (10), taken in steady state. This gives

[11] Given the high job finding and separation rates in the US, it is often argued that the assumption that the labour market is in steady state provides a reasonable approximation to labour market dynamics (Shimer, 2012).

[12] Taking (6) in steady state gives steady state employment as a function of tightness: $l = \frac{f(\theta)}{f(\theta)+\rho}$. Since I assumed that $f(\theta)$ was strictly increasing, $l$ is a strictly increasing function of $\theta$, which can be inverted to give steady-state $\theta$ as a strictly increasing function of steady-state $l$: $\theta = \hat{\theta}(l)$ with $\hat{\theta}'(l) > 0$. Since $q'(\theta) < 0$ we have that $\hat{q}(l) \equiv q(\hat{\theta}(l))$ is decreasing in $l$.

employment as the implicit solution to

$$\frac{\kappa}{\hat{q}(l^s)} = \frac{(1-\alpha)z(k^*/l^s)^\alpha - \omega z^\gamma (k^*)^{\alpha\gamma}}{r+\rho}, \tag{14}$$

which is the standard condition that the expected vacancy posting cost should equal the discounted surplus accruing to the firm. Unemployment is then defined as $u^s = 1 - l^s$. How does unemployment respond to a rise in discounts, i.e. an increase in $r$, when capital is fixed? Discounts enter the equation in only one way: in the denominator discounting the firm's surplus. Implicitly differentiating (14) gives:

$$l^{s\prime}(r) = \frac{-\kappa/\hat{q}(l^s)}{\alpha(1-\alpha)z(k^*)^\alpha l^s(r)^{-\alpha-1} - (r+\rho)\kappa\frac{\hat{q}'(l^s)}{\hat{q}(l^s)^2}} < 0 \tag{15}$$

which is negative because $\hat{q}'(l) < 0$. Since $u^s = 1 - l^s$ we thus have that

$$u^{s\prime}(r) = -l^{s\prime}(r) > 0 \tag{16}$$

Hence, when discounts rise unemployment increases. The intuition is the same as in existing search models. The right hand side of (14) gives the discounted benefit of posting a vacancy in steady state. Raising the discount rate lowers the present value of this stream of future benefits to the firm. To encourage vacancy posting this requires the cost of posting a vacancy to fall on the left hand side, which is achieved by lowering tightness, which increases the job filling rate and also increases unemployment.

Relative to the linear (constant returns to scale) model of Hall (2017), there is also a dampening effect in my model which weakens the immediate effect of discounts on unemployment. This is the effect of diminishing MPL for a fixed level of capital, which is represented by the positive term $\alpha(1-\alpha)z(k^*)^\alpha l^s(r)^{-\alpha-1}$ in the denominator, shrinking the total effect. Intuitively, the initial fall in employment following a rise in discounts raises the MPL and hence also increases hiring incentives. This reduces the total increase in unemployment relative to a constant returns to scale model.

How is this increase in unemployment divided between frictional and rationing unemployment? It turns out that when capital is fixed the increase must be entirely frictional, as rationing unemployment does not change. To see this, recall the definition of rationing unemployment in (12). For a fixed level of capital (and productivity) this is simply a constant,

$$u^{R,s} = \max\left\{0, 1 - \left(\frac{(1-\alpha)z^{1-\gamma}(k^*)^{\alpha(1-\gamma)}}{\omega}\right)^{\frac{1}{\alpha}}\right\}, \tag{17}$$

and hence a rise in discounts can have no effect on rationing unemployment. This is very intuitive: rationing unemployment is simply the level of unemployment which equates the wage to the MPL in the absence of hiring costs. For a given level of capital, the MPL is a fully static object which does not depend on discounts.

Since a rise in discounts causes unemployment to rise without a rise in rationing unemployment, the entire increase must be frictional:

$$u^{F,s}(r) = u^s(r) - u^{R,s}(r) \implies u^{F,s\prime}(r) = u^{s\prime}(r) - u^{R,s\prime}(r) = u^{s\prime}(r) > 0 \tag{18}$$

This is also intuitive. As explained above, the rise in unemployment was due to the discounted benefit of hiring falling relative to vacancy posting costs. This is a purely frictional phenomenon, which is naturally categorised as a rise in frictional unemployment. I summarise the findings from this section in the following proposition.

**Proposition 1.** *When capital is fixed, following a rise in discounts ($\uparrow r \equiv \beta^{-1}-1$): 1) unemployment rises ($u^{s\prime}(r) > 0$), 2) rationing unemployment is constant ($u^{R,s\prime}(r) = 0$), 3) frictional unemployment rises ($u^{F,s\prime}(r) > 0$)*

*Proof.* See text above. □

## 4.2 Flexible capital

In this section I move on to considering my full model, when the level of capital is also free to adjust to a rise in discounts. The addition of endogenous capital is the novelty of this paper, and I show that the transmission of the discount rise to unemployment is very different. I use a superscript $l$ to denote "long-run" steady state values with endogenous capital: $l^l, u^l, u^{R,l}, u^{F,l}$ and so on. I continue to hold productivity constant and consider an increase in $r$, but now allow capital, $k$, to optimally adjust according to the Euler equation, (9).

Recall that with my choice of capital adjustment cost function the price of capital is constant at $p^k = 1$ across all steady states. The capital Euler equation, (9), in steady state gives a solution for the capital-labour ratio in steady state:

$$\frac{k}{l} = \left(\frac{\alpha z}{r+\delta}\right)^{\frac{1}{1-\alpha}} \equiv \mathcal{K}(r). \tag{19}$$

I use $\mathcal{K}(r)$ to summarise the capital-labour ratio as a function of the discount rate (suppressing the dependence on productivity, which I take as constant. (19) shows us that $\mathcal{K}'(r) < 0$, since when agents are more impatient they are less willing to invest in capital, and the capital-labour ratio must fall to raise the MPK. Combining $\mathcal{K}(r)$ with the labour market Euler, (10), and wage rule, (11), gives employment with endogenous capital as the implicit solution to

$$\frac{\kappa}{\hat{q}(l^l)} = \frac{(1-\alpha)z\mathcal{K}(r)^\alpha - \omega z^\gamma (\mathcal{K}(r)l^l)^{\alpha\gamma}}{r+\rho}, \tag{20}$$

where I also used that $k^l = \mathcal{K}(r)l^l$. Just as with fixed capital, this condition states that employment must be such that expected vacancy posting costs equal the discounted sum of surpluses to the firm. However, capital is now endogenous, and is replaced with the optimal value coming from the capital Euler equation. Unemployment with flexible capital is symmetrically defined as $u^l = 1 - l^l$. As before, raising discounts must raise unemployment. This can be seen by differentiating (20), giving

$$l^{l\prime}(r) = \frac{-\frac{\kappa}{\hat{q}(l^l)} + \frac{\alpha\mathcal{K}'(r)}{\mathcal{K}(r)}\left((1-\alpha)z\mathcal{K}(r)^\alpha - \gamma\omega z^\gamma (\mathcal{K}(r)l^l)^{\alpha\gamma}\right)}{\alpha\gamma\omega z^\gamma \mathcal{K}(r)(\mathcal{K}(r)l^l)^{\alpha\gamma-1} - (r+\rho)\kappa\frac{\hat{q}'(l^l)}{\hat{q}(l^l)^2}} < 0. \tag{21}$$

To show that this derivative is negative, note that the denominator is positive because $\hat{q}'(l^l) < 0$. The numerator is negative because 1) $\mathcal{K}'(r) < 0$ and 2) the term in brackets is positive if employment is positive in equilibrium.[13] This shows that employment falls as discounts rise. The definition of

[13] Under the maintained assumption of interior employment, we have that the firm's surplus from posting vacancies

unemployment then trivially shows that unemployment rises as discounts rise:

$$u^{l\prime}(r) = -l^{l\prime}(r) > 0. \tag{22}$$

Intuitively, (20) shows that when capital is endogenous there are now two ways that discounts affect unemployment. Firstly, and commonly with the model with fixed capital, the $r$ in the denominator on the right hand side means that raising discounts directly lowers the firm's discounted surplus from hiring. Secondly, and differently from the model with fixed capital, raising discounts lowers capital intensity ($\mathcal{K}'(r) < 0$) which reduces the MPL through capital shallowing, and hence reduces the firm's surplus when wages are sticky. Notice instead that when capital is fixed, capital intensity actually goes *up* following a rise in discounts, as employment falls while capital is fixed.[14]

How does the increase in unemployment with endogenous capital map into changes in frictional and rationing unemployment? Rationing unemployment with flexible capital is given by the definition, (12), when capital is equal to its long run steady state value:

$$u^{R,l}(r) = \max\left\{0, 1 - \left(\frac{(1-\alpha)z^{1-\gamma}\left(\mathcal{K}(r)l^l(r)\right)^{\alpha(1-\gamma)}}{\omega}\right)^{\frac{1}{\alpha}}\right\}, \tag{23}$$

where I again used that $k^l = \mathcal{K}(r)l^l$ to replace the level of capital. Differently from the model with fixed capital, rationing unemployment now responds to discounts through the response of capital. In particular, if rationing unemployment is non-zero, then raising discounts will lower capital intensity and hence increase rationing unemployment by reducing the MPL and hence the incentive to hire. To see this, differentiate (23) when $u^{R,l}(r) > 0$ to yield

$$u^{R,l\prime}(r) = -(1-\gamma)\left(\frac{(1-\alpha)z^{1-\gamma}}{\omega}\right)^{\frac{1}{\alpha}}\left(\mathcal{K}'(r)l^l(r) + \mathcal{K}(r)l^{l\prime}(r)\right)\left(\mathcal{K}(r)l^l(r)\right)^{-\gamma} > 0. \tag{24}$$

This is positive because $\mathcal{K}'(r) < 0$ and $l'(r) < 0$, which simply means that total capital (not just capital intensity) falls when discounts rise. This is a key novelty of my model relative to Michaillat (2012). In his model there is no capital, and hence no way that discounts can affect rationing unemployment. In my model with capital, discounts can affect rationing unemployment by changing capital intensity.

The degree to which rationing unemployment responds to discounts depends crucially on wage rigidity, as can be seen from the term $(1-\gamma)$ multiplying the derivative in (24). In particular, the less rigid are wages (i.e. the larger is $\gamma$) the less rationing unemployment responds to discounts. In the limit of $\gamma = 1$ rationing unemployment does not respond at all to discounts.[15] On the other hand, when wages are fully rigid (i.e. $\gamma = 0$), rationing unemployment responds strongly to discounts. This is because in this case (23) shows us that rationing unemployment moves linearly with capital, while for $\gamma > 0$ it moves less than linearly.

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is positive: $\frac{\kappa}{\hat{q}(l^l)} = \frac{(1-\alpha)z\mathcal{K}(r)^\alpha - \omega z^\gamma(\mathcal{K}(r)l^l)^{\alpha\gamma}}{r+\rho} > 0$. Since $r+\rho > 0$, this implies that $(1-\alpha)z\mathcal{K}(r)^\alpha - \omega z^\gamma(\mathcal{K}(r)l^l)^{\alpha\gamma} > 0$. Since $\gamma \leq 1$, this also implies that $(1-\alpha)z\mathcal{K}(r)^\alpha - \gamma\omega z^\gamma(\mathcal{K}(r)l^l)^{\alpha\gamma} > 0$, as required.

[14] The difference between the response of the capital-labour ratio in the "short" versus "long run" of the model is consistent with the data during the Great Recession, plotted in Figure 2. In the data, the capital-labour ratio initially rises before eventually permanently falling.

[15] In fact, when $\gamma = 1$, (23) tells us that $u^{R,l} = \max\left\{0, 1 - \left(\frac{(1-\alpha)}{\omega}\right)^{\frac{1}{\alpha}}\right\}$ which is a constant.

Frictional unemployment is defined as before, as the residual unemployment not explained by rationing unemployment:

$$u^{F,l}(r) = u^{l}(r) - u^{R,l}(r) \implies u^{F,l\prime}(r) = u^{l\prime}(r) - u^{R,l\prime}(r). \tag{25}$$

What happens to frictional unemployment following a rise in discounts thus depends on whether total unemployment or frictional unemployment rises more. It turns out that the effect is actually ambiguous, for two reasons.

Firstly, it could be that rationing unemployment is currently zero, and hence unresponsive to the discount rate change. Recall the max operator in the definition of rationing unemployment, (23). If productivity and capital currently are high enough that there would be no unemployment in the absence of matching frictions, then the max "binds" and rationing unemployment is zero, and locally unresponsive to shocks. Since we know that unemployment always increases when discounts increase, in this case frictional unemployment must also increase, since the whole increase in unemployment must be frictional.

Secondly, consider the more interesting case of what happens to frictional unemployment when rationing unemployment is already positive. In this case, frictional unemployment could either rise or fall following a rise in discounts, and this depends crucially on the degree of wage flexibility. To see this, we can compare two limit cases:

As the first limit case, consider the case of fully flexible wages: $\gamma = 1$. Inspecting (23) shows that in this case rationing unemployment is a function only of parameters and is hence unrelated to discounts: $u^{R,l} = \max\left\{0, 1 - \left(\frac{(1-\alpha)}{\omega}\right)^{\frac{1}{\alpha}}\right\}$. Consider a calibration where $\omega > 1 - \alpha$, so that rationing unemployment is then positive.[16] Since unemployment increases when discounts rise even when wages are fully flexible (from (21) we still have $l^{l\prime}(r) < 0$ when $\gamma = 1$) the entire increase in unemployment must be frictional. Intuitively, when wages are very flexible there is limited scope for rationing unemployment, which derives from wage stickiness, and so any increase in unemployment must be frictional.

As the second limit case, consider fully sticky sticky wages: $\gamma = 0$. Continue to consider parameters such that rationing unemployment is positive. In this case, plugging (23) and $l^l \equiv 1 - u^l$ into the definition of frictional unemployment in (25) gives

$$u^{F,l}(r)\Big|_{\gamma=0} = l^{l}(r)\left[\left(\frac{(1-\alpha)z}{\omega}\right)^{\frac{1}{\alpha}} \mathcal{K}(r) - 1\right]. \tag{26}$$

Under the maintained assumption that employment is positive, frictional unemployment must also be positive in this case.[17] This expression reveals that frictional unemployment declines as discounts rise ($u^{F\prime}(r) < 0$) because both $l^{l\prime}(r) < 0$ and $\mathcal{K}'(r) < 0$. Intuitively, when wages are very sticky, rationing unemployment is very powerful. For a fully fixed wage, any fall in capital, and associated fall in MPL, will translate into a large rise in rationing unemployment. Frictional unemployment

[16] Recalling the wage rule (11), a high value of $\omega$ corresponds to a high wage intercept and hence high average wage. For $\omega > 1 - \alpha$ the wage is high enough that unemployment would be positive even in the absence of hiring frictions: i.e. rationing unemployment is positive.

[17] Under the maintained assumption of interior employment, now with $\gamma = 0$, we have that the firm's surplus from posting vacancies is positive: $\frac{\kappa}{\tilde{q}(l^l)} = \frac{(1-\alpha)z\mathcal{K}(r)^{\alpha} - \omega}{r+\rho} > 0$. Since $r + \rho > 0$, this implies that $(1-\alpha)z\mathcal{K}(r)^{\alpha} > \omega$. This implies that the bracket in (26) is positive. Since employment is also positive, $l^l(r) > 0$, (26) show that frictional unemployment must be positive.

falls because, as discussed in Michaillat (2012), search frictions become consequently less important in driving unemployment when unemployment is high. This is because when unemployment is high the vacancy filling rate is high, and it is less costly for firms to fill vacancies, rendering search frictions less important.

Thus, the overall effect of discounts on frictional unemployment is ambiguous when capital is flexible. However, in the numerical sections I will show that for sensible calibrations of the model frictional unemployment will tend to fall following a rise in discounts. I summarise the findings from this section in the following proposition.

**Proposition 2.** *Suppose that parameters are such that rationing unemployment is currently positive ($u^{R,l}(r) > 0$). When capital is flexible, and chosen according to the Euler equation, (9), following a rise in discounts ($\uparrow r \equiv \beta^{-1} - 1$): 1) unemployment rises ($u^{l\prime}(r) > 0$), 2) rationing unemployment rises ($u^{R,l\prime}(r) > 0$), 3) frictional unemployment may either rise or fall ($u^{F,s\prime}(r) \lesseqgtr 0$).*

*Proof.* See text above. □

Finally, given the ambiguous response of frictional unemployment when capital is flexible, one might wonder whether the response of unemployment to a rise in discounts is greater or smaller than when capital is fixed. In fact, as I summarise in the final proposition, it is possible to show that the increase in unemployment is always greater when capital is flexible.

**Proposition 3.** *Following a rise in discounts ($\uparrow r \equiv \beta^{-1} - 1$) from the initial steady state level $r^*$, unemployment increases more when capital is flexible than in the model where capital is fixed. That is, $u^{l\prime}(r^*) > u^{s\prime}(r^*)$.*

*Proof.* Recall that in the initial steady state capital was optimally chosen from the Euler equation, meaning that we have $k^* = \mathcal{K}(r^*)l^*$. Additionally note that when $r = r^*$ we have $l^s(r^*) = l^l(r^*) = l^*$. Plugging these into (15) and (21) gives:

$$l^{s\prime}(r^*) = \frac{-\kappa/\hat{q}(l^*)}{\alpha(1-\alpha)z\mathcal{K}(r^*)^\alpha(l^*)^{-1} - (r^*+\rho)\kappa\frac{\hat{q}'(l^*)}{\hat{q}(l^*)^2}} \tag{27}$$

$$l^{l\prime}(r^*) = \frac{-\frac{\kappa}{\hat{q}(l^*)} + \frac{\alpha\mathcal{K}'(r^*)}{\mathcal{K}(r^*)}\left((1-\alpha)z\mathcal{K}(r^*)^\alpha - \gamma\omega z^\gamma(\mathcal{K}(r^*)l^*)^{\alpha\gamma}\right)}{\alpha\gamma\omega z^\gamma\mathcal{K}(r^*)^{\alpha\gamma}(l^*)^{\alpha\gamma-1} - (r^*+\rho)\kappa\frac{\hat{q}'(l^*)}{\hat{q}(l^*)^2}} \tag{28}$$

Comparing (27) and (28) shows that the fall in employment following an increase in discounts is larger in the long run. This is because 1) in the numerator, (28) contains an extra negative term, and 2) the denominator of (28) is smaller.

Both of these follow from the maintained assumption that unemployment is positive. Under the maintained assumption of interior employment, we have that the firm's surplus from posting vacancies is positive: $\frac{\kappa}{\hat{q}(l^*)} = \frac{(1-\alpha)z\mathcal{K}(r^*)^\alpha - \omega z^\gamma(\mathcal{K}(r^*)l^*)^{\alpha\gamma}}{r^*+\rho} > 0$. Since $r^* + \rho > 0$, this implies that $(1-\alpha)z\mathcal{K}(r^*)^\alpha - \omega z^\gamma(\mathcal{K}(r^*)l^*)^{\alpha\gamma} > 0$. Since $\gamma \leq 1$, this also implies that $(1-\alpha)z\mathcal{K}(r^*)^\alpha - \gamma\omega z^\gamma(\mathcal{K}(r^*)l^*)^{\alpha\gamma} > 0$, as required for point (1). Multiplying both sides by $\alpha(l^*)^{-1}$ gives $\alpha(1-\alpha)z\mathcal{K}(r^*)^\alpha(l^*)^{-1} > \alpha\gamma\omega z^\gamma(\mathcal{K}(r^*))^{\alpha\gamma}(l^*)^{\alpha\gamma-1}$, as required for point (2). □

In summary, the results of this section demonstrated two results. Firstly, adding endogenous capital to the search model amplifies the effect of rising discounts on unemployment. Secondly, endogenous capital also fundamentally changes the transmission mechanism from discounts to unemployment. When capital is fixed, the transmission is through frictional unemployment, while it is through rationing unemployment (with ambiguous response of frictional) when capital is endogenous.

### 4.3 Further discussion

In this section I provide further discussion of two issues. Firstly, what is the role of capital versus the multi-worker firm assumption in driving my results? Pissarides (2000) shows how the basic single-worker-firm search model can be augmented to include capital. Modified to match my notation, we can think of each match producing output $y_t = z_t \tilde{k}_{t-1}^{\alpha}$, where $\tilde{k}_{t-1}$ is capital per match, and total capital is thus $k_t = \tilde{k}_t l_t$. Matches rent capital flexibly each period at rental rate $r_t^k$. Solving the model shows that this model is actually equivalent to my model. Hence all the results of this section go through, and it is not the multi-worker-firm assumption which is important, but simply the addition of capital.[18] We might think of capital sitting in the background of the linear search model as the justification for some exogenous level of labour productivity. However, if discounts are allowed to vary, labour productivity will endogenously adjust, and so models with capital will behave differently from models with exogenous labour productivity.

Secondly, the preceding analysis assumed that there was a single discount rate, $r \equiv \beta^{-1} - 1$, which applied investments in both capital and labour. However, one could also think that firms apply different discount rates to the two investments, and that these discounts vary differently over the cycle. In the appendix I solve such a model and investigate the role of the two discounts separately.

## 5 Steady state analysis: numerical results

In this section I provide numerical results to complement the theoretical results of the last section. I continue to consider comparative statics across steady states, but now calibrate the model to standard values in order to assess magnitudes. In particular, I will be interested in how rationing and frictional unemployment respond to a change in discounts.

### 5.1 Calibration

The model is calibrated at a monthly frequency. Most parameters are calibrated to target moments of the non-stochastic steady state of the model. I choose steady-state productivity, $z$, to normalise output to $y^* = 1$. I normalise the total labour force to $u^* + l^* = 1$. I choose $\alpha$ to match a labour share of income of $w^* l^* / y^* = 2/3$. Since firms extract extra surplus due to the matching frictions, this corresponds to a value of $\alpha = 0.3209$.

On the investment side, I choose values standard to the Real Business Cycle literature, chosen to match postwar US data (see Gomme and Lkhagvasuren, 2013, for a recent discussion). I match an investment to output ratio of $i^*/y^* = 0.18$, which gives $i^* = 0.18 y^* = 0.18$. I take depreciation to be

[18] In the single worker firm setup each worker-firm match chooses the level of capital each period to maximise static output net of capital rental payments: $\pi(z_t, r_t^k) = \max_{\tilde{k}_{t-1}} z_t \tilde{k}_{t-1}^{\alpha} - r_t^k \tilde{k}_{t-1}$. Optimality gives $r_t^k = \alpha z_t \tilde{k}_{t-1}^{\alpha-1}$. Plugging this in yields $\pi(z_t, r_t^k) = (1-\alpha) z_t \tilde{k}_{t-1}^{\alpha} = (1-\alpha) y_t / l_{t-1}$. The optimal choice of capital thus implies that the static output net of capital rental per match is $(1-\alpha) y_t / l_{t-1}$, which is identical to the MPL in the multi worker firm model. The standard free entry condition then gives a condition identical to (10). The household's Euler equation for capital is $p_t^k = \beta_t \left( r_{t+1}^k + (1-\delta) p_{t+1}^k \right)$. Replacing $r_{t+1}^k$ using $r_t^k = \alpha z_t \tilde{k}_{t-1}^{\alpha-1}$ and $k_t = \tilde{k}_t l_t$ yields (9).

6.5% at an annual frequency, giving $\delta = 1-(1-0.065)^{1/12} = 0.0056$ at a monthly frequency. In steady state, investment must equal depreciated capital, giving $k^* = i^*/\delta = 0.18/\delta$. I choose the steady-state discount rate in order to match this steady-state level of capital. Since the price of capital is one in steady state ($p^k(\delta) = 1$), the steady state return on capital is $r^{k,*} = \alpha y^*/k^* + 1 - \delta = 0.0044$, which equates to a 5.37% yearly return on capital. I thus set $\beta = 1/1.0044 = 0.9956$. I choose the commonly used quadratic adjustment cost function for capital, with the adjustment specified over the investment rate, $i_t/k_{t-1}$. Given this assumption, the price of capital is linear in the investment rate:

$$p^k(i_t/k_{t-1}) = 1 + \psi_k\,(i_t/k_{t-1} - \delta) \tag{29}$$

Recall that the investment rate is equal to $\delta$ in steady state. Following Bernanke et al. (1999) and Brinca et al. (2016) I choose the adjustment cost parameter $\psi_k$ to match an elasticity of the price of capital to the investment rate of 0.25 in the steady state. This requires setting $\psi_k = 0.25/\delta = 44.762$.[19] I also perform robustness allowing for no adjustment costs.

Table 2: Calibrated parameters

| Parameter | Interpretation | Value | Source |
|---|---|---|---|
| $z$ | s.s. tfp | 0.3398 | $y^* = 1$ |
| $\alpha$ | capital elasticity | 0.3209 | Labour share 1/3 |
| $\delta$ | depreciation | 0.0056 | Annual dep. 6.5% |
| $\beta$ | s.s. discount | 0.9956 | Annual $r^k = 5.37\%$ |
| $\psi_k$ | capital adj. costs | 44.762 | $\epsilon_{p^k,i/k} = 0.25$ (Brinca et al., 2016) |
| $\rho$ | job separation rate | 0.0396 | BLS, January 2008 value |
| $\psi_0$ | match efficiency | 0.7523 | Normalise $\theta^* = 1$ |
| $\psi_1$ | match elasticity | 0.5 | Petrongolo and Pissarides (2001) |
| $\gamma$ | wage flexibility | 0.7 | Michaillat (2012), Haefke et al. (2013) |
| $\omega$ | wage intercept | 0.6848 | $u^* = 0.05$ |
| $\kappa$ | vacancy posting cost | 0.2246 | $\kappa = 0.32w^*$ (Michaillat, 2012) |

Baseline calibration. Model is calibrated at monthly frequency. See text for detailed discussion of calibration choices.

The labour market is parameterised to stay close to Hall's (2005) strategy. Differently from Hall, and since I will be performing decompositions of the Great Recession in the next section, I calibrate the labour market to pre-crisis values, rather than averages for the whole post war period. In particular, I take the job separation rate to be $\rho = 0.0396$. This is the total rate of separations in January 2008 from employment to both unemployment and non-employment in the BLS' Labor Force Status Flows dataset. This is the appropriate separation rate since it reflects the separation rate experienced by firms when deciding on hiring. In the decomposition exercise in Section 6 I control for changes in participation and discuss my treatment of non-employment further. This value of separations is only slightly above Shimer's (2005) value of 0.034, and equates to a yearly separation probability of 38%. All of the results in this section are robust to using Shimer's value.

I target an unemployment rate of $u^* = 0.05$, equal to the BLS' seasonally adjusted Civilian Unemployment Rate in January 2008. Since I assume a two-state labour market model in this

[19] While this number may seem high, this simply reflects the monthly calibration. Since my periods are only a month long, and investment is measured as a flow while capital is a stock, the investment-capital ratio is lower than a model with a quarterly calibration. If the model was calibrated quarterly the number would instead be $\psi_k = 0.25/\delta_q = 15.0$ where $\delta_q = 1 - (1 - 0.065)^{1/4}$.

section, matching steady state flows implies a monthly job finding rate of $\lambda_w = (1-u^*)\rho/u^* = 0.752$. I assume a Cobb-Douglas matching function, $m_t = \psi_0 u_t^{\psi_1} v_t^{1-\psi_1}$, with standard elasticity $\psi_1 = 0.5$ (Petrongolo and Pissarides, 2001). Following Shimer (2005), this allows me to normalise steady state tightness to $\theta^* = 1$ and I pick the matching efficiency parameter to match the required job finding rate: $\psi_0 = \lambda_w(\theta^*)^{\psi_1 - 1} = 0.752$.

Real wage flexibility is set to $\gamma = 0.7$ following Michaillat (2012). This corresponds to an elasticity of wages to productivity of 0.7, consistent with the empirical evidence from job movers of Haefke et al. (2013) for production and supervisory workers.[20] I set the steady state real wage, $w$, following Michaillat (2012). Based on empirical estimates, he requires that the steady state flow recruiting cost, $\kappa$, is equal to 0.32 of a workers steady state wage: $\kappa = 0.32w^*$. This allows me to solve for $w^*$ and $\kappa$ from the Euler equation for labour, (10), given a value of the steady-state discount rate. This gives values $w^* = 0.7018$ and $\kappa = 0.2246$. In steady state, this means that the wage is 98.2% of the marginal product of labour. I choose the wage intercept, $\omega$, to generate this steady state wage.

In this calibration, all unemployment in steady state is frictional, and rationing unemployment is zero.[21] This outcome is not targeted, and is an endogenous outcome of the model. In this sense, the assumed degree of wage rigidity is low, since it is not high enough to generate any unemployment in the calibrated steady state when search frictions are removed.

### 5.2 Comparative statics across discount rates

The experiment I perform in this section is to hold all other parameters at their calibrated values and vary the discount rate, $r \equiv \beta^{-1} - 1$, computing the new steady-state equilibria of the model. I do so for both the model with capital fixed at its original steady state value (a.k.a. the short run), and the full model with flexible capital (a.k.a. the long run). In both cases, I also decompose the change in unemployment into its frictional and rationing components.

The main results are presented in Figure 1. The four panels plot key variables as I vary the discount rate on the $x$-axis. The discount rate is expressed in annual terms, and is varied between 0% and 10%, where the calibrated steady state value is 5.37%. For each series, steady state values are highlighted by a circle in the plots.

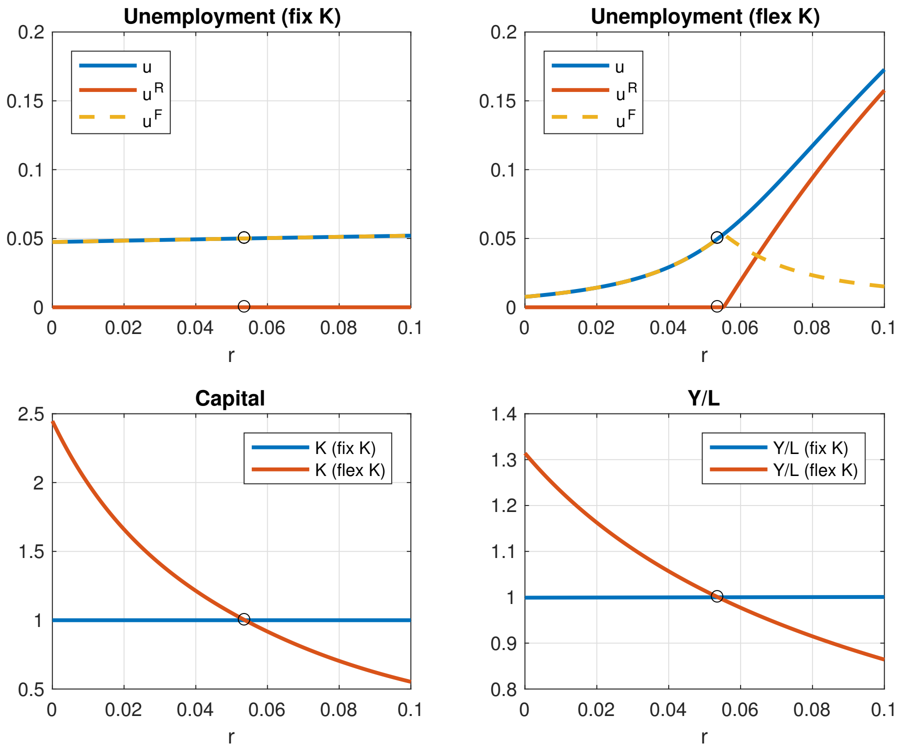
In the top left panel I plot unemployment when capital is fixed at the initial steady state value, as well as its frictional and rationing components. As discussed in the calibration section, in the calibrated steady state unemployment is 5% and is entirely frictional. When capital is fixed, varying the level of discounts has relatively small effects on unemployment: even nearly doubling the annual discount rate from 5.37% to 10% only raises unemployment (the blue solid line) to 5.2%. Additionally, all of this effect is frictional (yellow dashed line), since rationing unemployment (red solid line) is constant when capital is fixed, and hence remains at zero.

In the top right panel we see that the effects when capital is endogenous are very different. Now the effects of discounts on unemployment are much stronger, and raising discounts to 7.4% will raise

[20] This corresponds to the estimates for earnings per person rather than wages, since I hold hours per worker fixed in my model. This is a conservative choice, since they find that wages are less flexible than earnings, with an elasticity of 0.57. This number is also more flexible than estimates of the flexibility of earnings in existing matches. Using the same data, they find an elasticity of 0.39 for all workers. Pissarides (2009) reports numbers in the range of 0.2-0.5 for existing workers in the US.

[21] Inspecting the formula for rationing unemployment in steady state, (17), with the calibrated parameters rationing unemployment would be negative in the absence of the max operator: $u^R = \max\{0, -0.0065\} = 0$. This means that in the absence of hiring frictions firms would try to hire more than the entire workforce, hence leaving zero rationing unemployment.

Figure 1: Comparative statics (Benchmark calibration)




Comparative statics across steady states as adjust discount rate, $r$. Top left panel gives response of unemployment, and rationing / frictional unemployment, in the "short run" when labour markets reach steady state but capital fixed at its initial value. Top right gives the same but when capital has also adjusted to its new steady state value. Bottom left gives steady state capital, and bottom right gives labour productivity.

steady state unemployment to 10%. Furthermore, in contrast to the case of fixed capital now much of the effect is driven by rationing unemployment. In the calibrated steady state all unemployment is frictional, while at 10% unemployment 61% of unemployment is rationing unemployment, and this rises to larger fractions as unemployment rises further. The panel also reveals that, once rationing unemployment becomes positive, frictional unemployment falls as discounts increase. Thus, while the analytical results of Section 4 revealed that the response of frictional unemployment to a rise in discounts was ambiguous when capital was endogenous, for the baseline calibration it falls.

The differences between the fixed and flexible capital models are, by construction, driven by the response of capital to discounts. The bottom two panels illustrate this difference by plotting capital and labour productivity respectively. When capital is allowed to adjust, it responds strongly to discounts, as shown in the bottom left panel. In the bottom right panel we see that this translates into movements in labour productivity. When capital is fixed, labour productivity slightly rises when discounts rise, because employment falls slightly, but this is barely visible in the graph. When capital is endogenous, the rise in discounts required to raise unemployment to 10% does so by causing labour productivity to fall 6.8%.

In summary, these results add quantitative support to the two main results of Section 4. Firstly, in that section I showed that the response of unemployment to discounts is amplified when capital is endogenous. In this section I showed that this effect is quantitatively large across steady states. Secondly, in that section I showed that the transmission mechanism was fundamentally different when capital was endogenous. The only ambiguity from those results concerned the role of frictional unemployment when capital is endogenous, which I showed could either rise or fall when discounts rise. In this section I showed that in the standard calibration frictional unemployment falls when discounts rise, once the increase is large enough for rationing unemployment to become positive. Thus, the model concludes that once endogenous capital is taken into account, discount-led unemployment is a rationing phenomenon.

### 5.3 Further discussion, robustness, and dynamics

In the appendix I provide robustness of these results to two key parameters: the degree of wage rigidity ($\gamma$) and the size of hiring costs ($\kappa$). The results are quite intuitive. When hiring costs are lower, more unemployment is driven by rationing since hiring frictions are less important. Calibrations with lower hiring frictions also entail lower surpluses for firms, making overall unemployment more sensitive to discounts in the long run. Increasing the degree of wage rigidity makes unemployment more sensitive to discounts, and increases the role of rationing. In all cases, the short run responses remain small, and unemployment only responds significantly to discounts in the long run.

In order to demonstrate the analytical result that frictional unemployment could either rise or fall in the long run, I also provide a calibration with very flexible wages and low hiring costs where frictional unemployment rises in the long run. However, this is an extreme calibration, and frictional unemployment robustly falls in the long run across all other calibrations.

Finally, the results of this section ignored dynamics and focused on steady states. In reality, capital and labour will adjust slowly to shocks, and we should account for these dynamics. In the decomposition exercises for the Great Recession in the next section I do precisely this. However, for completeness I also provide transition dynamics for the experiments in this section in the appendix to show that the central messages of this section remain unchanged.

## 6 Discounts in the Great Recession

In this section I estimate discounts during the Great Recession and assess their effects on unemployment. I move on from the comparative statics exercises of the previous section, and now use the full dynamic version of my model. I perform counterfactual exercises by passing estimated shock series through the model, which is solved nonlinearly. After describing the data used for the estimation, I first focus on exercises involving discounts, before moving on to a full decomposition exercise featuring several shocks.

### 6.1 Data

I first describe the data used for the exercises. Since I am focusing on the Great Recession, which falls at the end of the sample for current data, using filtering to extract the business cycle component of the data is problematic. Instead, I use the simpler approach of taking the cyclical component of the data to be the deviation from the pre-crisis trend.

### 6.1.1 National accounts, employment, and productivity

The national accounts data is quarterly, and I thus aggregate all other data to a quarterly frequency before preceding. I take non-seasonally adjusted data, and all data is then seasonally adjusted using a five-quarter moving average filter. Data is converted to per-capita terms using population data from the BLS.

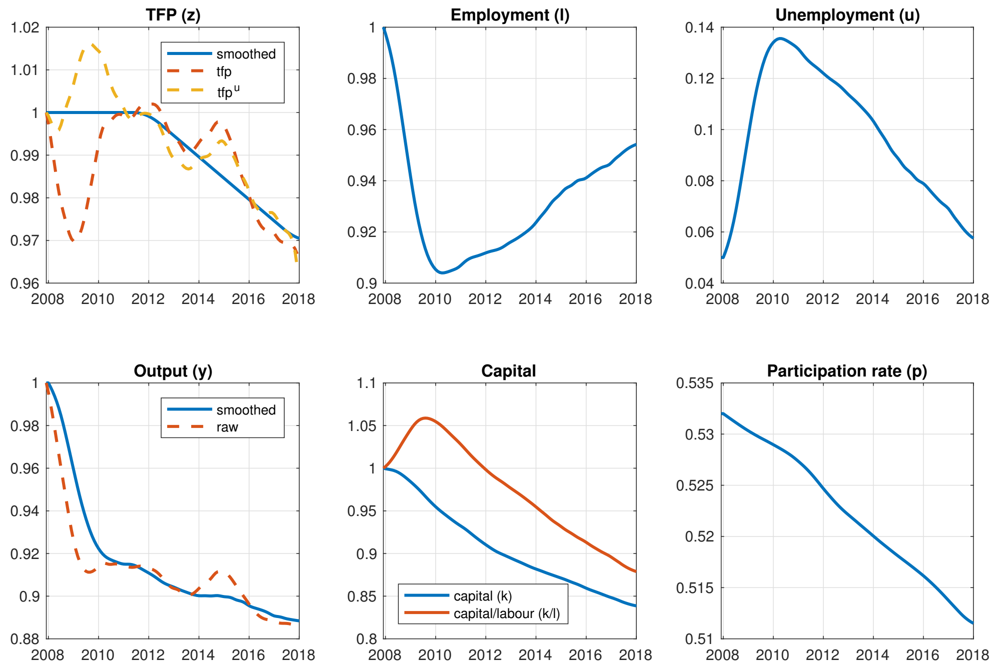
National accounts data are taken from the database maintained by Fernald (2014). I use data from 1950Q1 to 2018Q1. This data has the advantage of having a utilisation adjustment measure, based on the Basu et al. (2006) methodology. The data covers the "Business sector" of the US economy, and accordingly excludes the government and non-profit sectors. I construct indices for output, $y_t$, capital, $k_t$, and factor utilisation, $ut_t$, from this dataset by cumulating forward the quarterly growth rates. Investment is calculated from the capital stock using my assumed depreciation rate.

For employment, $l_t$, in order to match the national accounts data, I must use data on business-sector employment. These are constructed from the "Employment by major industry sector" files from the Bureau of Labor Statistics (BLS) by adding Farm and Non-farm Business sector employment. I next construct the model-relevant notion of TFP, which is defined over capital and employment (not hours, since my model assumes constant hours per worker). To do this, I extract TFP as the residual of a Cobb Douglas production function in capital and employment with the capital share $\alpha$ taken from my calibration. Utilisation adjusted TFP is calculated by dividing TFP by the utilisation index: $z_t^u = z_t/u_t$. All series are per capita, and then detrended using their average growth rates from 1950Q1 to 2008Q1.

The data is plotted in Figure 2, along with data for unemployment and participation (described in the next subsection). Data for these series are deviations from their 2008M1 value. The top centre panel shows a peak decline in employment of nearly 10% by 2010, which never fully recovers even by 2018. In the bottom centre panel we see that capital falls more gradually, but has fallen nearly 5% by 2010, and continues to fall to a deviation of over 15% by the end of the sample. The capital-labour (strictly, capital-employment) ratio initially rises, reflecting that employment falls very quickly, while capital responds more sluggishly. However, by the end of the sample the capital-labour ratio has fallen by 12%.

The top left panel shows adjusted and unadjusted TFP. These two series show a discrepancy at the beginning of the crisis, where raw TFP falls by 3% in 2009 before completely recovering by 2010. Mechanically, this behaviour is because output falls around one year before employment, which eventually catches up. Thus, it appears to be driven by temporary under-utilisation of resources in the first year of the crisis. Accordingly, Fernald's (2014) method to correct for utilisation changes does not find a fall in TFP, but instead finds a temporary rise at the beginning of the crisis, which is also then undone. Given the large discrepancies between the two series, and volatile behaviour of TFP throughout the period, I choose to take a more neutral position when creating data to input into my model. I assume that there is no change in (detrended) TFP in the first three years of the crisis, from 2008M1 to 2012M1. I then allow TFP to linearly fall by 3% up to 2012M1, and then hold it permanently at that value. This delivers TFP which is roughly the average of the two series in the early years of the crisis. I then smooth the series with a 13 month moving average filter. In order for the data inputted into the model to be mutually coherent, I need to adjust one of output, employment, or capital in order for the production function to hold. I choose to adjust output, and recalculate it using actual employment and capital and the smoothed TFP series. Output is

Figure 2: Great recession data




Data for aggregates during Great Recession period. Unless otherwise stated, plots give the data themselves which is also used as the input for model decomposition exercises. Data for TFP, employment, output, and capital are real and detrended with 1950Q1-2008Q1 growth rates, and expressed as deviations from 2008M1 values. Top left panel gives both unadjusted and utilisation adjusted TFP data, and the smoothed series used as input to the model. Bottom right panel gives raw output data, and adjusted series to be consistent with smoothed TFP. Participation rate is the counterfactual participation rate for business employment holding age-specific rates at their 2008 levels, as described in the text, and is detrended using all available data up to 2008 (1981Q1-2008Q1). Unemployment rate is adjusted rate including non-demographic-related decline in participation in unemployment, and is constructed using underlying series which are detrended.

shown in the bottom left panel along with its smoothed version, which falls one year slower than the unadjusted version to correct for the utilisation adjustment. Both measures broadly agree from 2010 onwards, and show a permanent deviation from the pre-crisis trend.

### 6.1.2 Unemployment and participation

During the Great Recession there were large changes in labour market participation, which make analysing the labour market through just the lens of unemployment problematic. To complicate matters further, the change in participation consists of both components related to demographics (population ageing) and also components plausibly endogenous to the business cycle (declines in participation within age groups). In order to construct a model-relevant measure of unemployment, I have to take a stand on these issues. It is potentially important whether this decline in participation is voluntary or involuntary.

The approach I follow in this paper is to construct a counterfactual measure of labour market participation related only to changing demographics. As I describe further in the appendix, I construct this measure using data on participation rates by age from the BLS. I hold participation within each age group at its 2008 level and simulate the change in the demographic makeup of the workforce. This series measures what aggregate participation would be if workers of each age participated as much as they did in 2008, loosely as if the Great Recession "had not happened". Let $p_t^{raw}$ denote the participation rate in the data, and let $p_t^{cf}$ denote this counterfactual rate. The two rates coincide by construction in 2008, when participation is 66%. By 2018 the participation rate had fallen to 63%, but amazingly the counterfactual rate due just to demographics also falls to 63.5%. Hence, by the end of the sample the majority of the aggregate decline in participation is caused simply by demographics.

However, during the middle of the crisis the actual participation rate, $p_t^{raw}$, falls by more than the counterfactual rate, $p_t^{cf}$. I interpret this additional decline as involuntary non-participation by people who want to work but drop out of the labour force during the recession. Since I only have a two-state model of the labour market, I cannot analyse participation decisions directly. Hence, in order to recognise the endogenous nature of participation, I choose to add this extra decline in participation to the definition of unemployment. In terms of the model, this means that some of the workers who are non-participants in the data will be added to the unemployment pool, and will thus be allowed to actively search for jobs.

With measures of employment and participation in hand, I can now compute a measure of unemployment. One final complication is that my data on output and employment is for the Business sector only, while my measure of participation (and indeed standard unemployment data) is for the entire economy. This presents an identification problem, since I do not know if workers are unemployed in the business sector or elsewhere. Indeed, given that employment falls more in the Business sector than in the whole economy (since employment in the Government and Non-profit sectors falls less) one might expect that "Business sector unemployment", to the extent that one can define such a concept, will rise more than the overall Civilian Unemployment Rate. I deal with this by assuming that the evolution of the participation rate is the same in the Business sector as in other sectors. Let $p_t$ denote the participation rate in my model. Unemployment is constructed as participation minus employment, and the unemployment rate in the usual way.

Constructing measures of unemployment for any model requires taking a stand on both the nature of participation, and how unemployment is allocated across the broad sectors of the economy. Making such decisions is unavoidable, and I feel that my approach is a fair way to treat the data given the issues surrounding participation. However, I show in the appendix that all of the results are fully robust to two alternative definitions of unemployment: 1) assuming that unemployment in my model is equal to the Civilian Unemployment Rate in the data, and backing out the required participation rate to make this true, and 2) using the actual, rather than counterfactual, participation rate to construct unemployment.[22]

The top right and bottom right panels of Figure 2 give the constructed unemployment and participation rates respectively. Unemployment rises up to a peak of 13.6% just after 2010, before gradually declining and recovering almost entirely by 2018. The peak unemployment rate is higher than the peak of 10% in the standard definition of unemployment (BLS' Civilian Unemployment Rate, October 2009) since I include some non-participants in the definition of unemployment. Despite this, the dynamics of unemployment are similar, and the large decline in participation attributable to demographics means that the model-consistent unemployment rate almost fully recovers by the end of the sample despite the permanent fall in employment. The participation rate starts at 53.2% in 2008 before steadily declining. Recall that this is participation in the Business sector only, which explains why it is lower than the overall 66% participation in the whole economy, with the remainder explained by participation in the Government and Non-profit sectors.

## 6.2 Exercise 1: IRFs to discount rate shock

The first exercise I perform with the data is to simply measure discounts, and assess their effect on unemployment and other variables in my baseline model. This allows me to transparently demonstrate the role of discounts in the data, and serves as a precursor to the full decomposition exercise. The data requirements of this first exercise are lower, since I only need data on output and capital to measure discounts from the capital Euler equation. Thus, these results will also show that the quantitative magnitudes of the effects of discounts on unemployment are unaffected by, for example, the assumptions on labour participation that I need to make for the full decomposition.[23] Since the model is solved at a monthly frequency, I interpolate the data to monthly frequency using linear splines whenever it needs to be compared directly to model data.

### 6.2.1 Counterfactual simulations and solution method

I bring the model to the data under the assumption of perfect foresight. In particular, in 2007M11 the economy is in the initial steady state from the main calibration. Agents believe that there is zero probability of a change in parameters occurring. However, in 2008Q1 unexpectedly agents learn that the discount rate has changed, and will now follow a deterministic path $(\beta_{2008M1}, \beta_{2008M2}, ...)$. They believe that this is the only shock that will ever occur, and that the economy will eventually settle into a new steady state when $\beta_t$ has reached its new steady state value for long enough.

[22]In particular, this assumption is irrelevant for the impulse response results to discount rates only in subsection 6.2, and only matters for the full decomposition exercise in subsection 6.3. Either alternative assumption on participation means that the rise in unemployment in the data is smaller. However, it turns out that the participation rate has a limited effect on unemployment in the equilibrium of the model, so the roles of the various shocks in explaining this lower level of unemployment are preserved.

[23]In this exercise I do not consider changes in the participation rate. However, for consistency with the full decomposition exercise I recalibrate the model to have a constant participation rate equal to the 2007M11 value from my data. This has no effect on the model, and simply scales the size of the economy. Details are given in subsection 6.3.4.

For an estimated sequence of discounts, I solve the model using the full non-linear system of equations from $t = 2007M11$ up to some sufficiently large $T$. I assume (and verify) that the economy has converged to the new steady state implied by the final value $\beta_T$ in period $T$. The transition is then solved by searching for a sequence of prices and allocations which satisfy all equations from $t = 2007M11$ to $t = T$ including the initial and final steady states. I allow for a 500 period transition, and verify the economy has converged to the final steady state in this time. When bringing the model to the data, my data ends in 2018M1. I assume that data at this date gives the final steady state values of the endogenous variables, and fill in the remaining values up to $T$ accordingly.[24]

### 6.2.2 Estimating discounts from the data

In order to perform this exercise, I first need to estimate discounts from the data. I directly measure discounts using the capital Euler equation. Given data on output and capital, discounts can be directly backed out by solving (9) for $\beta_t$:

$$\beta_t = p_t^k \left( \frac{\alpha y_{t+1}}{k_t} + p_{t+1}^k (1 - \delta) \right)^{-1} \tag{30}$$

where the capital price is inferred from (29) using the investment and capital series. This is used for $\beta_{2008M1}$ onwards, and $\beta_{2007M11} = \beta$ is simply the initial calibrated value. Notice that this procedure relies on the assumption of perfect foresight: in a model with uncertainty one would have to take into account uncertainty when calculating the expectations of next period's output and capital price.

I estimate discounts using the capital Euler equation, and hence using the path for investment. The identifying assumption of the baseline model is that the discount applied to labour and capital is the same, so that this discount is also used in the Euler equation for hiring. Notice that this assumption differs from that of Hall (2017), who measures discounts from the stock market. I discuss this difference further in subsection 7.6, where I show that the time-series properties of the two discounts are in fact quite different.

### 6.2.3 Measuring discounts: internal rate of return

When discounts are time varying, the month-to-month discount at $t$, $\beta_t$, is not fully informative about, for example, the incentives to invest in capital at time $t$. Since installed capital will persist for many periods, until it is fully depreciated, all of the discount rates over the near future will be important for calculating the total discounted returns from the investment.

In the case of investment incentives, iterating the Euler equation (9) forwards yields the formula

$$p_t^k = \sum_{s=1}^{\infty} \left( \prod_{j=0}^{s-1} \frac{1 - \delta}{1 + r_{t+j}} \right) \alpha z_{t+s} k_{t+s-1}^{\alpha-1} l_{t+s-1}^{1-\alpha}, \tag{31}$$

where I replaced $\beta_t = 1/(1 + r_t)$. Thus, all of $r_t, r_{t+1}, ...$ affect the incentive to invest at $t$ since capital lasts into the infinite future, although it depreciates. The lower is the depreciation rate, $\delta$, the more future discount rates will matter for current investment incentives.

[24] I smooth the transition to the final steady state in the year before and after 2018M1 using a 25 month moving average filter on the data around that date.

Following Hall (2017), I use the internal rate of return to summarise the total discount rate over the life of an investment. This is calculated as the constant discount rate which sets the net present value of the investment to zero:

$$p_t^k \equiv \sum_{s=1}^{\infty} \frac{(1-\delta)^{s-1}}{(1+\hat{r}_{k,t})^s} \alpha z_{t+s} k_{t+s-1}^{\alpha-1} l_{t+s-1}^{1-\alpha} \tag{32}$$

Note that the internal rate of return at $t$, $\hat{r}_{k,t}$, is effectively an average of the rates of return over the life of the investment $(r_t, r_{t+1}, ...)$ weighted by the size of the associated payoffs at each date. Once the model has been solved for paths for the endogenous variables, $\hat{r}_{k,t}$ can be directly calculated period-by-period by solving the (non-linear) equation (32).

In the case of job posting incentives, iterating the FOC (10) forwards and applying the internal rate of return concept yields the formula:

$$\frac{\kappa}{q_t} \equiv \sum_{s=1}^{\infty} \frac{(1-\rho)^{s-1}}{(1+\hat{r}_{l,t})^s} \big((1-\alpha) z_{t+s} k_{t+s-1}^{\alpha} l_{t+s-1}^{-\alpha} - w_{t+s}\big) \tag{33}$$

Note that the internal rate of return for capital, $\hat{r}_{k,t}$, and jobs $\hat{r}_{l,t}$, can differ even though the are derived from the same stream of discounts. This is because the sequence of payoffs for the two assets will differ.

#### 6.2.4 Results

The results for this exercise are given in Figure 3, which plots the paths for key variables. The top left panel gives the path for discounts, expressed as the internal rate of return on capital, in annual terms $((1+\hat{r}_{k,t})^{12} - 1)$. In Figure 13 in the appendix I also plot the monthly discount itself, the internal rate of return on hiring, and the price of capital. The annualised discount in the initial steady state is 5.37%. This is estimated to rise by just under 50% to a peak of 7.5% at the end of 2009, before gradually declining and remaining elevated throughout the whole sample.[25] The final value of the discount, which is achieved just after the end of the sample due to the smoothing of the path for capital, is 6.1%. Hence, the model estimates a permanent change in the US economy, with discounts rising at the beginning of the crisis and never recovering. To understand why, the data in Figure 2 show a permanent decline in the capital-output ratio: output falls by 11% up to 2018, while capital falls by over 16%. This represents an increase in the marginal product of capital, and hence the return on capital, which requires an increase in estimated discounts. In other words, since the financial crisis firms have permanently been investing less.

Returning to the impulse responses in Figure 3, feeding this rise in discounts into the model leads to a gradual but persistent rise in unemployment. Unemployment rises to just under 6% by 2010, and then continues to rise to 7.3% by 2018. Quantitatively, the estimated discounts lead to a meaningful rise in unemployment, but clearly fall short of explaining the entire peak rise in unemployment in 2010. Interestingly, while unemployment in the data rises to a peak in 2010 and then gradually declines (top right panel, Figure 2) the estimated impact of discounts on unemployment is greater towards the end of the sample. Thus, while discounts may provide a modest contribution

[25] The internal rate of return jumps up on impact since it is a weighted average of future discounts, which are significantly above the initial steady state value. In order to generate a gradual increase in the internal rate of return, the monthly discount, $r_t$, which is effectively a measure of the marginal change in the internal rate of return, actually falls on impact.

Figure 3: Response to estimated discount shocks

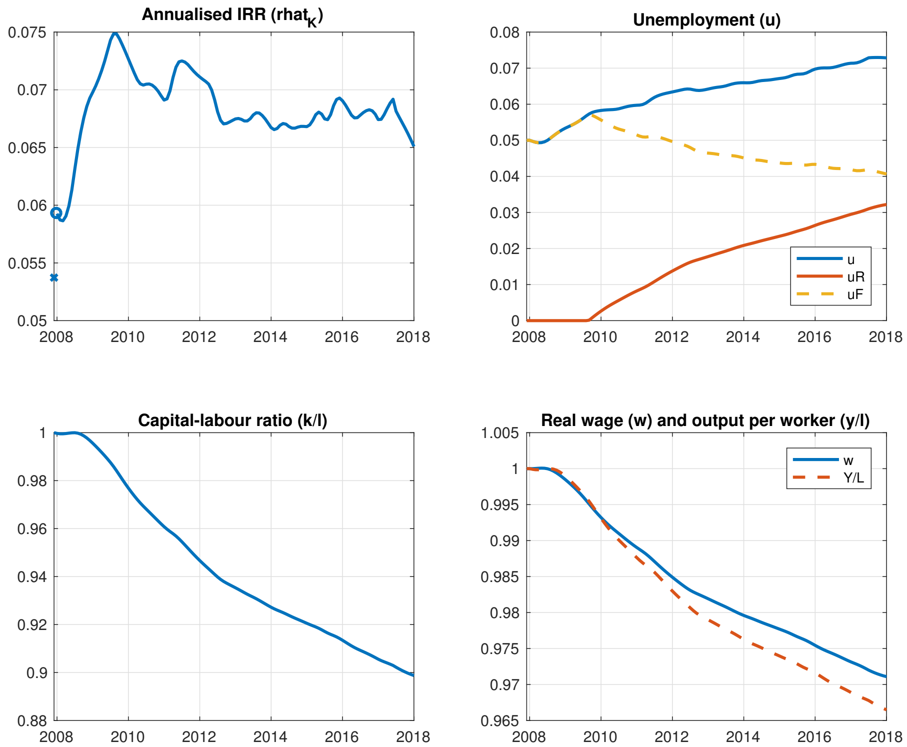



Figure plots response of economy to unexpected change in discounts. In 2008M1 agents learn that discounts have changed and will follow the deterministic path given in the top left panel. Top left panel gives discounts expressed as internal rate of return for capital in annual terms. Top right gives unemployment rate path, and decomposition into rationing and frictional unemployment. Bottom left gives the capital labour ratio and bottom right the real wage and labour productivity, both in deviations from 2007M11 values.

to unemployment in 2010, they appear to contribute more to the slow recovery of unemployment. The effects on unemployment are permanent, reflecting the permanent decrease in investment and capital, and hence the marginal product of labour, following the financial crisis.

How does this decompose into frictional and rationing unemployment? At the beginning of the sample the decrease in capital is not large enough for rationing unemployment to become positive, as per the definition in (12). Hence all unemployment, and the entire rise in unemployment, is initially frictional. However, by 2010 the deterioration is bad enough that rationing unemployment becomes positive, meaning that unemployment would remain positive even in the absence of matching frictions. From then onwards frictional unemployment starts to decline, and by 2018 frictional unemployment has fallen by 0.9 percentage points relative to its initial level and rationing unemployment has risen to over 3.2%. Thus, by the end of the sample, when discounts have their maximum impact, 44% of unemployment is now due to rationing, compared to 0% pre-crisis.

The sources of this rise in rationing unemployment are explored in the bottom two panels. The bottom left panel shows that the capital-labour ratio falls by 10% over the whole sample. This follows immediately from the rise in discounts in the capital Euler equation: a rising required return on capital raises the required MPK, which has to be achieved by lowering the capital-labour ratio if TFP doesn't change. This is transmitted into unemployment via the effects on output per worker, or equivalently the MPL in this Cobb-Douglas framework. The declining capital-labour ratio, also known as *capital shallowing*, lowers output per worker reducing the incentives to hire. Output per worker falls by 3.4% by the end of the sample, while wage rigidity leads the wage to only fall by 2.9%. While this represents a significant fall in wages, it is not enough to prevent unemployment from rising.

Comparing the evolution of the capital-labour ratio to the data in Figure 2 shows that by 2018 the estimated rise in the discount rate alone accounts for fully 83% of the 12% decline in the data. Thus, rises in discounts are crucial for understanding the capital shallowing during the Great Recession, and any implied effects on unemployment. The data also show a rise in the capital-labour ratio during the first two years of the crisis. The impulse response to the estimated discount rate series in Figure 3 instead shows a near monotonic decrease in the capital-labour ratio over the sample period. Thus, other shocks, particularly those which lead employment to fall further in the beginning of the crisis while affecting capital less, must be contributing to the initial discrepancy between the two series. I turn my attention to these other shocks in the full decomposition exercise in the next section.

### 6.3 Exercise 2: Full decomposition

My final core exercise is to investigate the quantitative relevance of discounts relative to other shocks during the Great Recession. I do this using a full decomposition exercise for the Great Recession, where I use the model to extract several shocks from the data. In order to do this, I first need to extend the model to allow it to be brought closer to the data.

#### 6.3.1 Job separation shocks

I allow for a time varying job separation rate, which is modelled as an exogenous process. This is important as a potential source of unemployment at the beginning of the recession, since the separation rate spiked early in the crisis. Instead of assuming a constant separation rate, $\rho$, I allow for a time varying rate, $\rho_t$. This modifies the evolution law for employment to

$$l_t = q_t v_t + (1 - \rho_t) l_{t-1}. \tag{34}$$

Additionally, the first order condition for hiring becomes

$$\frac{\kappa}{q_t} = \beta_t \left( (1 - \alpha) z_{t+1} k_t^{\alpha} l_t^{-\alpha} - w_{t+1} + \frac{(1 - \rho_{t+1})\kappa}{q_{t+1}} \right). \tag{35}$$

I extract the path for the separations rate directly from the data. I take the monthly flow rate from employment to unemployment and non-employment from the BLS' Labor Force Status Flows dataset. I take the raw data for both men and women and seasonally adjust it using a 13 month moving average filter.

### 6.3.2 Adding labour market participation

As I showed in the data section above, the Great Recession saw large changes in labour market participation, which make a two-state model problematic for analysing this period. While a fully-fledged three-state model is beyond the scope of this paper, I extend the model to allow for exogenous changes in participation to improve the accounting of the model.

I continue to normalise the population to one. Recalling that $l_{t-1}$ is employment at $t$ (chosen at $t-1$) and $u_t$ is unemployment at $t$, I now define total non-participation at $t$ as $n_t$. These three states sum up to the whole population: $l_{t-1} + u_t + n_t = 1$. Agents are participants in the labour market if they are employed or unemployed, and I define total participation at $t$ as $p_t \equiv l_{t-1} + u_t$. Accordingly, $p_t + n_t = 1$.

Since the population is normalised to one, $p_t$ is both total participation and the participation rate. I assume that the participation rate evolves exogenously. In terms of flows, I assume that only unemployed workers search for a job, and workers thus cannot transition directly from non-participation to employment.[26] The job separation rate, $\rho_t$, now gives the total probability that a worker separates from a job into either the unemployment or non-participation pools. Since the participation stock is exogenous, for a given $\rho_t$, precisely how I allocate the remaining flows does not affect the equilibrium of the model. Market tightness is still defined as the ratio of vacancies to the unemployment stock: $\theta_t \equiv v_t/u_t$. The unemployment rate is now defined as $\tilde{u}_t \equiv u_t/p_t$. The evolution of employment continues to be given by (6), and together with a path for participation this is sufficient to simulate the labour market.

### 6.3.3 Labour market residual: wage intercept

The final shock I introduce is a reduced-form shock which directly impacts hiring incentives in the labour market. The idea is to treat this shock as a residual, capturing any movements in unemployment which the other shocks are not able to generate.

There are many potential reasons that hiring incentives could change, aside from the shocks already in the model. It could be that discounts for hiring move differently than discounts for capital, or that hiring costs increase in a recession. Alternatively, there could be reasons that the wage does not fall as fast as predicted by my relatively flexible wage rule, perhaps due to downwards nominal wage rigidity and demand-side factors. Taking a stand on which of these forces is the most important is beyond the scope of this paper, and I instead use this single reduced form shock to capture all of these factors.

In order to make this shock easily interpretable, I choose to model the shock as exogenous movements in the intercept in the wage rule. This gives a simple notion of scale to the shock, which can be measured as a percentage change in the real wage. In particular, I replace the wage rule (11) with

$$w_t = \omega_t \left( z_t k_{t-1}^{\alpha} p_t^{-\alpha} \right)^{\gamma} \tag{36}$$

I replace the constant wage intercept, $\omega$, with the time varying version, $\omega_t$. This exogenously raises

[26] This assumption is restrictive, since many workers flow from non-participation to employment in the data. However, as discussed in my data section, I use demographic data to split the reduction in participation during the Great Recession into the reduction coming from the ageing of the population, and a residual. I add the residual reduction in participation into my definition of employment. Thus I allow workers who drop out of the labour force during the Great Recession for non-demographic reasons to search for employment in my model, by adding them to the unemployment pool.

or lowers the wage, and hence changes the incentives to hire by changing the firm's surplus and raising the required marginal product of labour.

Another important modification in the wage rule is the inclusion of the participation rate, $p_t$. Recall that the wage rule allows the wage to move towards the market clearing wage, with $\gamma$ controlling the degree of stickiness. The market clearing wage is defined as the wage required to achieve full employment, which is now when employment is equal to the participation rate, $p_t$. The MPL at full employment is now $z_t k_{t-1}^{\alpha} p_t^{-\alpha}$, which explains the inclusion of $p_t$ in the rule.

Looking at the definitions of rationing unemployment from earlier in the paper, (12), we see that $\omega$ is important for determining the level of rationing unemployment. This is because it controls the level of the real wage, and hence also the level of unemployment that would persist in the absence of matching frictions. I am using $\omega_t$ as a stand in for many shocks, some of which will refer to hiring frictions rather than real wages directly. I thus cannot talk about rationing versus frictional unemployment in this exercise, and will instead focus only on the unemployment rate itself.

#### 6.3.4 Recalibrating the model with lower participation

I recalibrate the model to account for less than 100% participation in the labour market. I take the participation rate at 2007M1 as the initial steady-state participation rate, giving $p = 0.5324$. All other targets in the calibration remain the same. Changing the participation rate in the calibration, while continuing to normalise output at $y = 1$, scales certain parameters (in particular $\omega$, $z$, and $\kappa$) while leaving the targeted moments the same. Since I keep the same calibration targets, the choice of $p$ has no effect on key estimated steady state objects, such as discounts or the split between rationing and frictional unemployment. and simply scales the size of the economy.

#### 6.3.5 Extracting the shocks

In total, there are five shocks in the model: discounts ($\beta_t$), TFP ($z_t$), job separations ($\rho_t$), the wage shock ($\omega_t$), and participation rate ($p_t$). I now describe how I extract these shocks from the data. Several of the shocks can be directly measured from their data counterparts. This is true for TFP, job separations, and the participation rate, and I have described their measurement above.

The paths for discounts and the wage shock are chosen to exactly replicate the paths for employment and capital in the data. The path for discounts is measured from the capital Euler equation, and is identical to the series used in subsection 6.2. Finally, the wage shock is calculated by combining the labour Euler equation, (35), with the wage rule, (36), to eliminate the real wage. The series for $\omega_t$ is chosen to make this equation hold in every period, and can thus be interpreted as being chosen to shift the wage in such a way as to move hiring incentives in the labour Euler to exactly replicate the path for employment.

As a consequence of matching the paths for employment, capital, and TFP, the decomposition also exactly matches the path for output and ratios such as the capital-labour ratio. Note that the decomposition does not use any data on real wages, so will not exactly match wage data by construction.

### 6.4 Results

I decompose the path of any endogenous variable in the model as follows. For each shock, I simulate the non-linear model subject to the path of that shock only, holding all others at their steady state values, to create a counterfactual series. I do this for each of the shocks, and doing so for all shocks at once would trivially exactly replicate the data used to extract the shocks. If the model was linear, adding up the counterfactual (as deviations from steady state) from each separate shock would also recover back the data exactly. Since the solution method is non-linear, this is not exactly true. However, adding up the paths from the shocks very closely recovers the original series, as can be seen from the results below.

I plot the results from the decomposition in Figure 4. The top three panels give the decompositions for unemployment, employment, and output, respectively. For each panel, the solid blue line plots the original data, and the remaining lines give the counterfactual simulations subject to the path of one shock only. The bottom row gives paths for the shocks, apart from TFP and participation, which are already given in Figure 2.

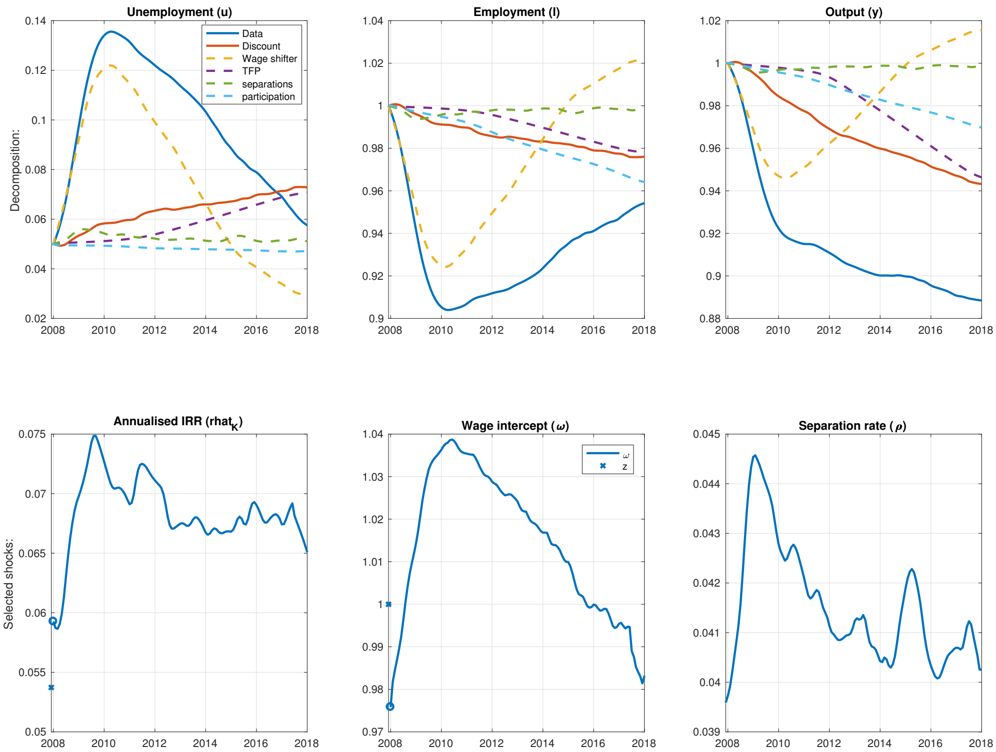
I begin with the decomposition of unemployment, in the top left panel. Recall that, in order to deal with the large decline in participation during the recession, my definition of unemployment contains the declines in participation unrelated to demographics. Unemployment peaks around 2010 at over 13% before gradually declining. However, almost the entirety of the initial rise in unemployment is explained by the residual wage shock, $\omega_t$. By 2010, the rise in discounts contributes just under a percentage point to the rise in unemployment, and the rise in separations accounts (shown in the bottom right panel) for at most 0.6 percentage points, peaking in 2009. The other driving forces (TFP and participation) have negligible effects on unemployment at this time, leaving the wage residual to do the work. The path for the wage shock is plotted in the bottom centre panel. At 2010 this peaks at a rise $\omega_t$ which would, ceteris paribus, raise the real wage by just under 4%.[27] This large increase in the real wage reduces firms' incentives to hire, both by increasing the required equilibrium MPL and by reducing the firms' surplus from posting vacancies, which is why this shock explains so much of the rise in unemployment.

Moving on to the slow recovery of unemployment from 2010 onwards, we see that the importance of the residual wage shock starts to diminish. This mirrors the rising importance of the discount and TFP shocks. The inputted TFP shock series (shown in the top left panel of Figure 2) starts to decline from 2012 onwards, with a final decline of 3%. In anticipation of this, when subject only to the TFP shocks unemployment starts to rise slightly before 2012. At its peak at the end of the sample, the decline in TFP generates just under a 2.1 percentage point rise in unemployment. Additionally, as discounts remain elevated capital continues to decline, leading discounts to generate a peak rise in unemployment of just under 2.3% by the end of the sample. As these shocks gain in importance the estimated contribution of the wage residual falls, and from 2013 onwards they jointly explain more of unemployment than the wage residual. Thus, as emphasised in the results of subsection 6.2, discounts are estimated to be more important for explaining the slow recovery of unemployment than the peak. What we learn from this exercise is that, while the quantitative magnitude of the effect of discounts might be modest, it is of roughly the same importance as the decline in TFP. In fact, discounts are estimated to be slightly more important than TFP, and their effects on unemployment begin earlier in the sample.

For completeness, I also plot the results of the decomposition exercise on employment and output in the top centre and right panels. The relative contributions of the different shocks to these

[27]The initial decline in the estimated wage intercept is explained by the forward looking nature of hiring in the search model. The model requires the wage to rise massively in 2010 to explain the level of unemployment in 2010. However, anticipating this high wage, firms will also want to hire less in 2008 and 2009. Given that unemployment rose only gradually, the initial wage intercept must fall to increase early hiring incentives and smooth the rise in unemployment.

Figure 4: Full decomposition of the Great Recession




Top row plots decomposition of path for key variables attributable to different shocks. Each series is found by simulating the model subject only to that shock series. Model is solved nonlinearly, so adding all responses will not exactly recover the data, but does so approximately. Solid blue line gives the data, and remaining lines the contributions from each shock. Bottom row gives paths for all shocks apart from TFP and participation rate, which are plotted in Figure 2

variables are essentially in line with what is to be expected from the results of the decomposition of unemployment. The only distinction is the contribution of the exogenous decline in participation, $p_t$. This has essentially no effect on unemployment, in the top right panel, while it leads to a nearly 4% decline in employment and 3% decline in output. This is because in general equilibrium the participation rate simply shrinks the size of the economy. Since I allow the real wage to adjust to the change in labour supply in (36) the wage adjusts to correct hiring incentives, leaving the unemployment rate essentially unchanged.[28]

Overall, we learn two things from the full decomposition exercise. Firstly, while discounts are not able to explain the initial increase in unemployment in my model, none of the standard shocks included in my model are either. The model requires a large disruption in hiring incentives to match the path of unemployment, even though the model allows for several other driving forces. Secondly, discounts are more powerful at explaining the slow recovery of unemployment. Quantitatively, discounts are found to be slightly more powerful than the decline in TFP at explaining the slow recovery, with effects that begin earlier in the sample.

# 7 Further exercises

In this section I present additional numerical exercises, designed to give further insight into the main results.

## 7.1 Effect of discounts through each Euler equation

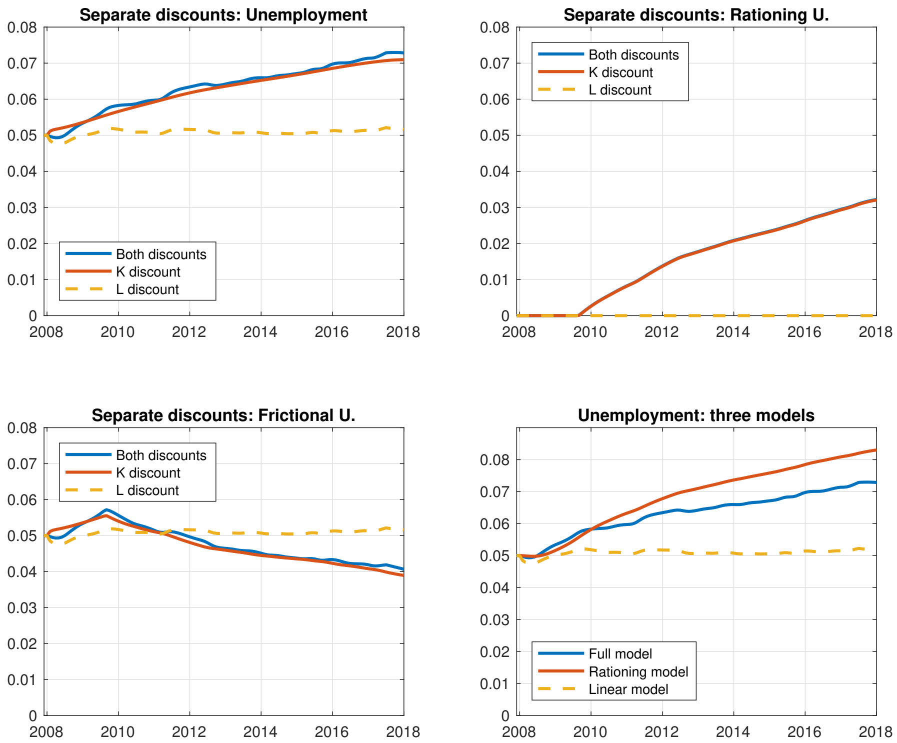
In this section I investigate how the effects of the change in discounts are separately transmitted through the labour and capital Euler equations. As in the theoretical extension in the appendix, I allow discounts to differ in the two Euler equations.[29] I take the same estimated path for discounts from the capital Euler equation as before. In the main exercise I fed this path for discounts through the model, applying the discount to both the capital and labour Euler equation. I now compute the path for unemployment if I only allow capital discounts to change, and hold labour discounts at their steady-state value, and vice versa.

The results are given in the first three panels of Figure 5. The top left panel shows the path for the unemployment rate in three experiments: The blue line repeats the exercise of the last section where both discounts are allowed to vary. The red line shows unemployment if only the discount in the capital Euler is allowed to vary and the discount in the labour Euler is left at the steady state value. Finally, the yellow line shows the inverse experiment where the labour discount is varied according to the estimated discount and the capital discount is left at steady state. The plot shows that the effect of the full model is driven almost entirely by the capital discount. Allowing just the capital discount to vary generates a path for unemployment very close to the path when both discounts vary. Conversely, allowing just the labour discount to vary generates very little movement in unemployment.

[28] Due to capital adjustment costs capital adjusts slowly to the fall in participation. This leads the MPL to be slightly elevated during the transition, which increases hiring incentives and causes unemployment to temporarily fall slightly. In steady state the unemployment rate is invariant to the exogenous level of participation in my model.

[29] Specifically, I allow for separate discount factors $\beta_{K,t}$ and $\beta_{L,t}$ and associated discount rates $r_{K,t} \equiv \beta_{K,t}^{-1} - 1$ and $r_{L,t} \equiv \beta_{L,t}^{-1} - 1$. These lead to the following modified versions of the Euler equations (9) and (10): $p_t^k = \beta_{K,t}\left(\alpha z_{t+1} k_t^{\alpha-1} l_t^{1-\alpha} + p_{t+1}^k(1-\delta)\right)$, and $\frac{\kappa}{q_t} = \beta_{L,t}\left((1-\alpha) z_{t+1} k_t^{\alpha} l_t^{-\alpha} - w_{t+1} + \frac{(1-\rho)\kappa}{q_{t+1}}\right)$.

Figure 5: IRFs to separate discounts, and alternative models




Figures plots response of economy to unexpected change in discounts. First three panels show responses of baseline model to discounts (blue line) and change in capital (red line) and labour (dashed yellow) discounts only. Top left panel gives unemployment rate, and top right and bottom left give rationing and frictional unemployment respectively. Bottom right panel compares the response of unemployment across three models: baseline (blue), rationing only (red), and linear search (dashed yellow).

Thus, while the last section showed that even up to the end of the crisis, both rationing and frictional unemployment play a role in generating unemployment, this exercise makes clear that this is not to be confused with thinking that both capital and labour discounts are driving unemployment. Rather, capital discounts are effectively the whole driving force for changes in unemployment. In fact, capital discounts drive essentially all of the movements in both rationing and frictional unemployment, as shown in the top right and bottom left panels of Figure 5 respectively. As argued in the simple model of Section 2, the separation rate is simply too high for hiring discounts to matter in this model.

### 7.2 Linear search model

In order to check that the offsetting effect of decreasing MPL isn't dampening ability of discounts in the hiring Euler equation to generate unemployment in the last section, in this section I solve the simple linear search and matching model without capital, and with output formed as a linear function of labour: $y_t = z_t l_t$. I calibrate this model to match the same steady state as the full model, keeping the same form of wage rigidity. I feed the path for discounts estimated from the full model through the labour Euler equation of this model.

The results are plotted in the bottom right panel of Figure 5, and show that the linear search and matching model also generates very little response of unemployment to the estimated discounts. In fact, comparing the path to the path for unemployment in the full model with capital but where only labour discounts are varied in the top left panel reveals them to be visually indistinguishable.

### 7.3 Comparison to a pure rationing model

What is the role of labour market frictions in transmitting increases in discounts to unemployment in this model? Given that rationing unemployment drives the increase in unemployment in the previous exercises, do we need labour market frictions to model unemployment during this episode? My next exercise shows that this is the case, and that both matching and capital play meaningful roles in driving unemployment. I construct a pure rationing model, without any search frictions. I recalibrate the intercept in the wage rule, $\omega$, so that the model generates 5% unemployment in the initial steady state.

This model thus generates all unemployment as rationing unemployment, coming from wage stickiness alone. The model is identical to the full model, with the only difference being that hiring costs are assumed to be zero ($\kappa = 0$) and labour optimality thus reduces to the standard static first order condition, $w_t = (1-\alpha) z_t k_{t-1}^{\alpha} l_{t-1}^{-\alpha}$. Unemployment in this model corresponds closely to Michaillat's (2012) concept of rationing unemployment in my full model, with two differences. Firstly, the calibration of the model is different. The full model had no rationing unemployment in steady state, while in this model all unemployment is rationing, and it is calibrated to match the same steady state level of unemployment. Secondly, recall that rationing unemployment was defined in (12) as the level of unemployment when matching frictions were removed, but holding the level of capital at the equilibrium level from the model with matching frictions. In this exercise, capital is determined optimally inside the rationing model itself.

Since the capital Euler equation in this model is identical to the full model, the estimated paths for discounts are the same. I therefore feed the previously estimated discount rate series through the capital Euler equation of this model, and calculate the implied path of unemployment. This is plotted in the bottom right panel of Figure 5, along with the path for unemployment from the full model. We learn two things from this exercise. Firstly, taking a model without search frictions overstates the role of discounts in increasing unemployment relative to the full model. The pure rationing model generates an increase in unemployment of 3.3pp by the end of the sample, while the increase in the full model is only 2.3%. Hence, search frictions actually play an important *dampening* role on unemployment during the recession. During the recession the abundance of unemployed workers increases the vacancy filling probability, reducing hiring costs for firms. Secondly, notice that the increase in unemployment in the pure rationing model (3.3pp) is very close to the increase in rationing unemployment from the main model (3.2pp), demonstrating that Michaillat's (2012) rationing unemployment construct remains useful even in a model with an endogenous state variable such as capital.

### 7.4 Role of capital adjustment costs

In this section I investigate the role of capital adjustment costs in driving the quantitative magnitude of my results. To do this, I repeat the exercise of subsection Section 6.2, but where I remove capital adjustment costs by setting $\psi = 0$. I first remeasure the path for discounts from the capital Euler equation, as in (30), but without adjustment costs. I then input this path for discounts into the model, and plot paths for key variables in Figure 6. As before, the alternative discount measures are given in Figure 14 in the appendix.

Starting with unemployment, in the top right panel of Figure 6, we see that removing adjustment costs does change the estimated effect of discounts on unemployment. Unemployment is still estimated to increase more towards the end of the sample, but now rises slightly less, with a peak increase of 1.7pp rather than the 2.3pp of the baseline model with adjustment costs. Additionally, unemployment initially falls during 2008, before starting its rise.

To understand why this is, we must consider how removing adjustment costs changes the nature of estimated discounts. The internal rate of return on capital, plotted in the top left panel of Figure 6 behaves differently from the model with adjustment costs, only increasing gradually towards its final steady state value. With adjustment costs (top left panel of Figure 3) the IRR overshoots its final value at the beginning of the crisis. In fact, since the IRR essentially averages discount rates in the near future, it hides important dynamics at the beginning of the crisis when adjustment costs are switched off. In the bottom left panel of Figure 6 we see that the capital-labour ratio initially rises before starting to fall from mid-2009 onwards. This is caused by an initial *fall* in estimated monthly discounts during that period, which is plotted in Figure 14 in the appendix. This fall is masked in the IRR on capital, which averages it out with later rises.

The initial rise in the capital-labour ratio increases the marginal product of labour, which encourages hiring and explains the initial fall in unemployment. Why does the model estimate this behaviour when adjustment costs are ignored, and not when they are included? To understand this, we must return to the equation from which discounts are estimated, (30). This shows that two forces drive estimated discounts: the output-capital ratio ($y_{t+1}/k_t$, which drives the MPK) and the evolution of the price of capital, $p_t^k$. When adjustment cost are switched off, the price of capital is fixed at one in all periods: $p_t^k = 1$. This means that the estimated discount is driven only by the output-capital ratio. As can be seen from the data in Figure 2, this initially falls in the crisis, because output falls faster than capital, driven by the fact that labour fell quickly. This initial fall in the output-capital ratio lowers the estimated MPK, requiring a fall in estimated discounts in (30).

This is also true in the model with adjustment costs, but there the behaviour of the price of capital dominates, adding upwards pressure to estimated discounts at the beginning of the sample. Recall that adjustment costs mean that the price of capital falls whenever investment falls, as per (29). Even though capital itself falls gradually during the crisis, investment falls much more quickly. Thus, the estimated price of capital falls instantly, and only starts to recover in mid-2009, as shown in the bottom right panel of Figure 13.

Thus, capital adjustment costs play an important role in moving forward the estimated date at which discounts start to rise: roughly, with capital adjustment costs there is pressure for discounts to rise whenever investment falls, while without adjustment costs they only rise when capital falls. Finally, capital adjustment costs then also play a role in how estimated discounts affect capital in the simulation of the model. Without capital adjustment costs, capital adjusts each period to the level dictated by the current discount. However, with adjustment costs capital adjusts more slowly.

Figure 6: Results with no capital adjustment costs ($\psi_k = 0$)

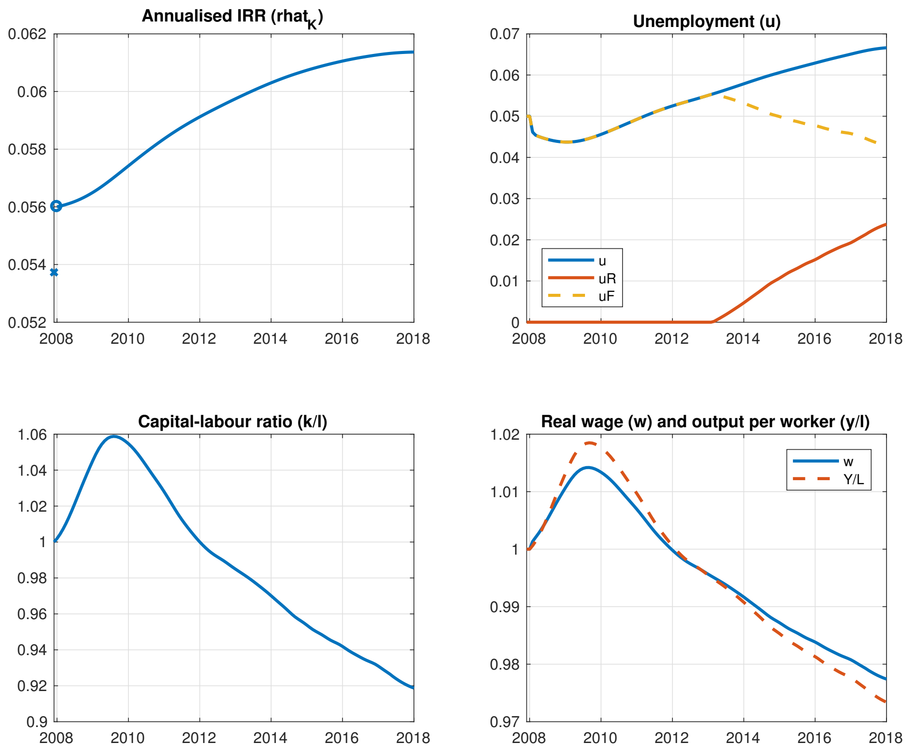



Figure plots response of economy to unexpected change in discounts, when capital adjustment costs are set to zero ($\psi_k = 0$). In 2008M1 agents learn that discounts have changed and will follow the deterministic path given in the top left panel. Top left panel gives discounts expressed as internal rate of return for capital in annual terms. Top right gives unemployment rate path, and decomposition into rationing and frictional unemployment. Bottom left gives the capital labour ratio and bottom right the real wage and labour productivity, both in deviations from 2007M11 values.

### 7.5 Comparison of model to stock market data

In this paper I took the approach of measuring discounts from investment data (i.e. the capital Euler equation) rather than from stock prices, as in Hall (2017). A valid question is to what extent these two measures of discounts co-move. For example, do investment and stock markets capture similar patterns of discounts during the Great Recession, or are they measuring different forces? Alternatively, one could simply ask if my measure of discounts implies sensible movements in stock prices in the model.

To answer these questions, I compute stock prices in my model and compare them to the data. In the absence of a theory of firms' capital structures I simply equate cashflow, $e_t$ from (8), with dividends. I then compute stock prices using the standard asset pricing formula

$$p_t = \beta_t \left(e_{t+1} + p_{t+1}\right) \tag{37}$$

which is simply the risk neutral pricing equation for equity modified to allow for a time-varying discount factor. Dividing by $e_t$ gives a formula for the price-dividend ratio in the model:

$$\frac{p_t}{e_t} = \beta_t \frac{e_{t+1}}{e_t} \left(\frac{p_{t+1}}{e_{t+1}} + 1\right) \tag{38}$$

This shows that two forces move the price-dividend model: the discount factor, $\beta_t$, and the expected rate of dividend growth, $e_{t+1}/e_t$. In terms of the full model used in the decomposition, this means that the estimated path for the discount factor will have a direct effect on the price-dividend ratio, while all other shocks will only have an indirect effect by moving the expected dividend growth rate. Using this formula I compute the price-dividend ratio in the model in response to the various shocks.

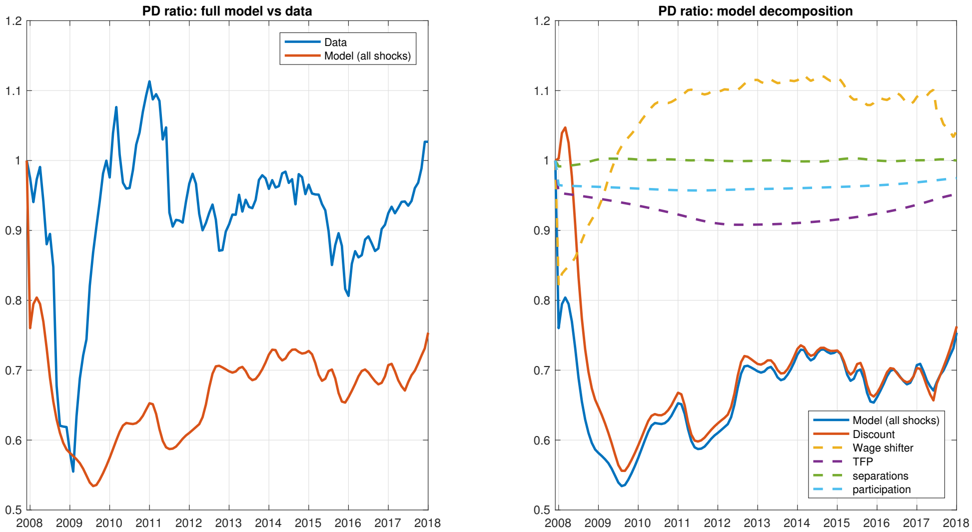
In the left panel of Figure 7 I compare the model-implied price-dividend ratio, subject to all the estimated shocks, to the data. For consistency with earlier work, I construct the data for the price-dividend ratio exactly as in Hall (2017). I take monthly data for the S&P500 portfolio from Robert Shiller's website (`http://www.econ.yale.edu/~shiller/`) and remove a linear time trend. Recall that no data on stock prices was used in the estimation, so any fit of the data is not by construction.

The plot reveals both interesting similarities and differences between the two series. In the data the price-dividend ratio falls by nearly 45% by 2009, before quickly recovering all off these losses by 2010. In the model, surprisingly the initial fall in the price-dividend ratio is very similar to that in the data. While it starts slightly sooner, the ratio also falls by around 45% by mid 2009. This serves as a validation that the estimated discount rates are in some sense reasonable, since they imply a stock market bust in line with the data.

However, it is in the behaviour later in the sample that the two series greatly diverge. Following a speedy recovery in 2010, the price-dividend ratio in the data does fluctuate, but has broadly recovered to its pre-crisis value. In the model, the ratio remains severely depressed throughout the whole sample, and is still 25% below its pre-crisis value in 2018. Thus, the model estimation is capturing forces which are clearly unrelated to the stock market in the latter part of the recession.

To understand what these forces are, in the right panel of Figure 7 I plot the implied price-dividend ratio in the model when it is subject only to one shock at a time. While other shocks do move the ratio, by moving the dividend growth rate, the figure shows that it is the estimated path for discounts which drives the ratio from mid-2009 onwards. Thus, the model predicts the price-

Figure 7: Price-Dividend ratio in the model and data




Left panel plots the price-dividend ratio from the data versus the model. Data are constructed as in Hall (2017), see text for details. Model gives the ratio when all shocks are inputted into the model, so that it exactly replicates the data on output, employment, capital, and productivity. Right panel decomposes the ratio from the model into the components attributable to each shock.

dividend ratio to remain low throughout the sample because it estimates discounts which remain continuously elevated (see the bottom left panel of Figure 4). Recalling that these discounts are estimated from the capital Euler equation, the reason for the divergence of the price-dividend ratio in the model and data becomes clear. In the data, the stock market had recovered by 2010, while the level of investment has never recovered following the recession. While TFP has fallen, this alone is not enough to explain the fall in investment, so the model estimates that discount rates must have persistently risen to explain the lower level of investment. In the model, this means that stock markets remain depressed to match the lower investment rates.

Thus, the data on investment and stock prices seem to contain different information when it comes to the estimation of discounts. The brief but severe stock market crash in the data suggests a large but short lived rise in discounts. On the other hand, the prolonged and gradual decline in investment rates suggests a persistent rise in discounts. In the next section I investigate their differing implications for unemployment.

### 7.6 Estimating discounts from the stock market

In this section I estimate the path for discounts required to exactly replicate the price-dividend ratio in the data, given in the left panel of Figure 7. I do this through the lens of my model. In particular, I only allow for shocks to the discount rate, and choose the sequence $\{\beta_t\}_{t=0}^{\infty}$ so that the model's implied path for $\{p_t/e_t\}_{t=0}^{\infty}$ exactly matches the data.

The results are plotted in Figure 8. The right panel shows the path for discount rates required

Figure 8: Unemployment when discounts estimated from the stock market


Model behaviour when path for discount rates is estimated to match the path of the price-dividend ratio in the data (and no other shocks assumed). Left panel plots the unemployment rate, the middle plots the capital-labour ratio, and the right gives the required discount path, presented as the annualised internal rate of return on labour.

to match the stock price data. I plot the annualised discount rate measured as the internal rate on return on labour. I plot the raw discount series, and internal rate of return on capital, in the appendix. Interestingly, since the estimated increase in discounts is so short lived, it does not register in the internal rate of return on capital, which is why I plot the internal rate of return on labour instead. Unsurprisingly the model requires a short but very sharp rise in discounts to match the brief but severe crash in stock prices. The discount rate peaks in early 2009 at a value over eight times its initial steady state level. By the beginning of 2010 it has fully recovered back to its initial level, around which it then fluctuates.

The left panel plots the response of the unemployment rate to this sequence of shocks, which mirrors closely the path of discounts. The very large increase in discounts leads unemployment to rise by nearly two percentage points, peaking at 6.9% in early 2009, before recovering by early 2010. The large increase in unemployment might seem to stand in contrast to my previous results, which showed that without movements in capital, changes in discounts did not have large effects on unemployment. This is still the case, the difference here being that the estimated temporary rise in discounts required to cause the temporary stock market crash is very large.

If we compare the path of unemployment here to that implied by discounts estimated from capital, given in Figure 3, the dynamics are drastically different. Both lead to rises in unemployment of around two percentage points, but at different points at the recession. Matching the early crash and recovery of the stock market, discounts estimated from stock prices generate a short burst of unemployment at the beginning of the recession. Matching the gradual and sustained decreases in investment, discounts estimated from capital lead to a gradual rise in unemployment which is greatest at the end of the sample.

In fact, not only are the timings of the rise in unemployment implied by each discount series different, but so are the transmission mechanisms. The left panel of Figure 8 also plots the decomposition of unemployment into its frictional and rationing components. This shows that the rise in unemployment driven by stock market discounts is entirely frictional. This is in contrast to the rise driven by capital discounts, shown in the top right panel of Figure 3, which was entirely accounted for by rationing unemployment.

The difference is not due to differences in the models – indeed the two experiments are carried out in the same model. Instead, it is due to the different timing structures of the shocks. The increase in discounts measured from stock markets is very short lived, and is completely reversed within a year. Given the high adjustment costs in capital, firms do not find it optimal to adjust their capital stocks in response to such a short lived shock. In fact, the capital stock (not shown) barely moves in response to the shock. Consequently, since employment falls, the capital labour ratio actually increases in response to the rise in discounts, as shown in the centre panel of Figure 8. Without a fall in capital, rationing unemployment does not rise, and consequently the rise in unemployment must be frictional.

This is in contrast to the response of the economy to the gradual increase in discounts estimated from capital. Since this increase in discounts is persistent, firms do find it optimal to decrease investment in capital in response to this shock, and consequently the capital-labour ratio falls, as I showed in the bottom left panel of Figure 3. This fall in capital means that rationing unemployment increases.

Overall, this exercise reveals interesting differences between the information content of discounts extracted from stock markets versus capital data. Both suggest that discounts increased, but at different times and with different degrees of persistence. Consequently, they explain different parts of the unemployment dynamics during the Great Recession, and through different mechanisms.

# 8 Conclusion

In this paper I have investigated the role of discounts in driving unemployment, with a particular focus on the mechanisms through which the two are linked. Firms' hiring decisions are inherently forward looking, with hiring costs paid today, and the benefits of an extra employee spread over the future. If, for whatever reason, firms start discounting these future benefits more heavily, this naturally reduces incentives for firms to hire, and thus time-varying discount rates could be a potential driver of unemployment over the business cycle.

While Mukoyama (2009) initially argued that the standard search model cannot generate quantitatively meaningful effects of discounts on unemployment, Hall (2017) and Kehoe et al. (forthcoming) provide extensions of the standard model which amplify the response of unemployment to changes in discounts. In this paper I investigate a third potential mechanism, paying particular attention to capital formation.

The standard search model features single worker firms with linear production functions. Thus, workers are not related in any way, nor is their capability to produce tied to any other decisions the firm makes. The central idea of this paper is that hiring decisions are not made in isolation, because workers only produce output in tandem with other factors of production. Thus, when a firm makes the decision to hire, they do not simply have to decide whether hire another worker. They must also decide if they are willing to commit to the extra infrastructure required to make that worker productive, be that a new factory, plant, or production line. While individual workers might transition quickly out of jobs, these investment decisions are much more longer lived, and thus potentially more sensitive to changes in discounts.

To investigate this idea, I extend the basic linear search model to incorporate endogenous capital accumulation, so that output is produced from a Cobb Douglas aggregate of capital and labour. I incorporate an empirically plausible level of wage rigidity, following Hall's (2017) insight that wages need to be sufficiently sticky for discounts to be powerful. I begin with comparative statics results, both analytical and quantitative. I show that endogenous capital greatly amplifies the effect of discounts on unemployment. When discounts rise firms invest in less capital, which reduces the marginal product of labour and hence the incentive to hire.

Thus, the model captures in a simple way the main idea that discounts affect investment in the capital required to make workers productive. Conventional discount rates imply that capital is a much longer lived investment than an individual worker, and investment thus responds more strongly to discounts than hiring, all else equal. The changes in investment then feed into the incentives to hire, and thus the paper also connects with Kehoe et al.'s (forthcoming) insight that making hiring decisions more forward looking is important for amplifying the effect of discounts on unemployment.

To show that this additional effect on unemployment is fundamentally different from the mechanism in the standard search model, I extend Michaillat (2012) and decompose unemployment in my model into both frictional and rationing components. I show that the short run effects of discounts on unemployment, defined as the effects before capital adjusts, operate as frictional unemployment and are weak. On the other hand, the long term effects once capital adjusts are much stronger, and are driven by rationing unemployment.

Having investigated the mechanisms, I then assess the quantitative importance of my new channel. I do so for the Great Recession, through a variety of numerical exercises. Ultimately, I find that while endogenous capital does amplify the effect of discounts on unemployment, the quantitative magnitude is not large enough to explain the bulk of unemployment in the Great Recession. However, elevated discounts are found to play a modest role in generating the slow recovery of unemployment, of a quantitatively similar magnitude to the slowdown in TFP growth. Finally, I show that stock prices and investment contain different information about discounts, that affect unemployment at different times and through different channels.

In this paper I investigated the idea that hiring might fall when discounts rise because hiring requires the creation of costly, long-lived, capital for workers to produce with. In practice, much job creation happens at young firms (Haltiwanger et al., 2013), suggesting an important role for the creation of entirely new firms in driving unemployment. Future work could investigate this idea further, for example by considering the interplay between firm entry, unemployment, and discounts in a full model of firm dynamics.

# References


[1] Acemoglu, Daron, and Robert Shimer, 1999. *Efficient Unemployment Insurance.* Journal of Political Economy, 1999, vol. 107, issue 5, 893-928.

[2] Andolfatto, David, 1996. *Business Cycles and Labor Market Search.* American Economic Review 86(1): 112-32

[3] Basu, Susanto, John G. Fernald and Miles S. Kimball, 2006. *Are Technology Improvements Contractionary?* American Economic Review, Vol. 96, NO. 5, December 2006 (pp. 1418-1448)

[4] Bernanke, Ben, and Mark Gertler, 1989. *Agency Costs, Net Worth, and Business Fluctuations.* The American Economic Review, Vol. 79, No. 1 (Mar., 1989), pp. 14-31

[5] Bernanke, Ben S, Gertler, Mark & Gilchrist, Simon, 1999. *The financial accelerator in a quantitative business cycle framework.* Handbook of Macroeconomics, in: J. B. Taylor & M. Woodford (ed.), Handbook of Macroeconomics, edition 1, volume 1, chapter 21, pages 1341-1393 Elsevier.

[6] Brinca, P., V. V. Chari, Patrick J. Kehoe, Ellen R. McGrattan, 2016. *Accounting for Business Cycles.* Handbook of Macroeconomics, Volume 2, 2016, Pages 1013-1063

[7] Carrillo-Tudela, Carlos, Michael Graberb, and Klaus Waelde, 2018. *Unemployment and vacancy dynamics with imperfect financial markets.* Labour Economics, Volume 50, March 2018, Pages 128-143.

[8] Chari, V. V., Patrick J. Kehoe, Ellen R. McGrattan, 2007. *Business Cycle Accounting.* Volume 75, Issue 3, May 2007, Pages 781-836.

[9] Christiano, Lawrence J, Trabandt, Mathias & Walentin, Karl, 2011. *Introducing financial frictions and unemployment into a small open economy model.* Journal of Economic Dynamics and Control, Elsevier, vol. 35(12), pages 1999-2041.

[10] den Haan, Wouter, J., Garey Ramey, and Joel Watson. 2000 *Job Destruction and Propagation of Shocks.* American Economic Review, 90 (3): 482-498.

[11] Elsby, Michael, Ryan Michaels and David Ratner, 2017. *Vacancy Chains.* 2017 Meeting Papers 888, Society for Economic Dynamics.

[12] Farmer, Roger, 2012. *Confidence, Crashes and Animal Spirits.* Economic Journal, Royal Economic Society, vol. 122(559), pages 155-172, 03.

[13] Fernald, John, 2014. *A Quarterly, Utilization-Adjusted Series on Total Factor Productivity.* FRBSF Working Paper 2012-19.

[14] Gomme, Paul and Damba Lkhagvasuren, 2013. *Calibration and Simulation of DSGE Models.* Handbook of Research Methods and Applications in Empirical Methods in Macroeconomics, Nigar Nasimzade and Michael Thornton, eds. (Edward Elgar), pp. 575-592.

[15] Haefke, Christian, Marcus Sonntag, and Thijs van Rens, 2013. *Wage Rigidity and Job Creation.* Journal of Monetary Economics, 60(8).

[16] Hall, Robert E, 2016. *Macroeconomics of Persistent Slumps.* National Bureau of Economic Research Working Paper 22230.

[17] Hall, Robert, 2017. *High Discounts & High Unemployment.* American Economic Review 2017, 107(2): 305–330.

[18] Hall, Robert E., and Paul R. Milgrom, 2008. *The Limited Influence of Unemployment on the Wage Bargain.* American Economic Review 98 (4): 1653-74.

[19] Haltiwanger, John, Ron Jarmin, and Javier Miranda, 2013. *Who Creates Jobs? Small versus Large versus Young.* The Review of Economics and Statistics, Vol. XCV(2), May 2013.

[20] Kehoe, Patrick, Viriliu Midrigan, and Elena Pastorino, forthcoming. *Debt Constraints and Employment.* Journal of Political Economy, forthcoming.

[21] Kiyotaki, Nobuhiro & Moore, John, 1997. *Credit Cycles.* Journal of Political Economy, University of Chicago Press, vol. 105(2), pages 211-48, April.

[22] Landais, Camille, Pascal Michaillat, Emmanuel Saez, 2018a. *A Macroeconomic Approach to Optimal Unemployment Insurance: Theory.* American Economic Journal: Economic Policy 10(2), May 2018.

[23] Landais, Camille, Pascal Michaillat, Emmanuel Saez, 2018b. *A Macroeconomic Approach to Optimal Unemployment Insurance: Applications.* American Economic Journal: Economic Policy 10(2), May 2018.

[24] Merz, Monika, 1995. *Search in the Labor Market and the Real Business Cycle.* Journal of Monetary Economics 36(2): 269-300

[25] Merz, Monika, and Eran Yashiv, 2007. *Labor and the Market Value of the Firm.* American Economic Review 97 (4): 1419-31.

[26] Michaillat, Pascal, 2012. *Do Matching Frictions Explain Unemployment? Not in Bad Times.* American Economic Review, American Economic Association, vol. 102(4), pages 1721-50, June.

[27] Mukoyama, Toshihiko, 2009. *A Note on Cyclical Discount Factors and Labor Market Volatility.* `https://sites.google.com/site/toshimukoyama/` (accessed December 21, 2018)

[28] Mumtaz, Haroon & Zanetti, Francesco , 2013. *The Effect of Labor and Financial Frictions on Aggregate Fluctuations.* Economics Series Working Papers 690, University of Oxford, Department of Economics.

[29] Petrongolo, Barbara & Christopher A. Pissarides, 2001. *Looking into the Black Box: A Survey of the Matching Function.* Journal of Economic Literature, American Economic Association, vol. 39(2), pages 390-431, June.

[30] Petrosky-Nadeau, Nicolas, 2014. *Credit, Vacancies and Unemployment Fluctuations.* Review of Economic Dynamics, Elsevier for the Society for Economic Dynamics, vol. 17(2), pages 191-205, April.

[31] Petrosky-Nadeau, Nicolas and Wasmer, Etienne, 2015. *Macroeconomic dynamics in a model of goods, labor, and credit market frictions.* Journal of Monetary Economics, Elsevier, vol. 72(C), pages 97-113.

[32] Pissarides, Christopher A., 2000. *Equilibrium Unemployment Theory, 2nd Edition.* MIT Press Books, The MIT Press, edition 1, volume 1, number 0262161877, June.

[33] Pissarides, Christopher A. 2009. *The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?* Econometrica 77 (5): 1339-69.

[34] Quadrini, Vincenso, and Sun, Qi, 2015. *Credit and Hiring.* Mimeo, available at `http://www-bcf.usc.edu/~quadrini/papers/UPpap.pdf`. Last accessed 30/03/2015.

[35] Rogerson, Richard, and Robert Shimer. 2010. *Search in Macroeconomic Models of the Labor Market.* Handbook of Labor Economics, Elsevier.

[36] Schoefer, Benjamin, 2015. *The Financial Channel of Wage Rigidity.* Mimeo, available at http://scholar.harvard.edu/files/schoefer/files/jmp_bschoefer.pdf. Last accessed 30/03/2015.

[37] Shimer, Robert, 2005. *The Cyclical Behavior of Equilibrium Unemployment and Vacancies.* American Economic Review, American Economic Association, vol. 95(1), pages 25-49, March.

[38] Shimer, Robert, 2012. *Reassessing the ins and outs of unemployment.* Review of Economic Dynamics. Volume 15, Issue 2, April 2012, Pages 127-148

[39] Wasmer, Etienne and Philippe Weil, 2004. *The Macroeconomics of Labor and Credit Market Imperfections.* American Economic Review, Vol. 94, No. 4, September 2004 (pp. 944-963)

[40] Winkler, Fabian, 2015. *The Role of Learning for Asset Prices, Business Cycles and Monetary Policy.* Mimeo, available at http://personal.lse.ac.uk/winklerh/jmp.pdf. Last accessed 27/03/2015.

[41] Yashiv, Eran, 2000. *Hiring as Investment Behavior.* Review of Economic Dynamics 3 (3): 486-522.

[42] Yashiv, Eran, 2016. *Capital values and job values.* Review of Economic Dynamics 19 (2016) 190–209.

# Appendices

## A Appendix to Section 4: Separate discounts

In this section I analyse the case of separate discounts. I focus on the full model with endogenous capital, where the distinction is meaningful, and allow the discount rates on capital, $r_K \equiv \beta_K^{-1} - 1$, and labour, $r_L \equiv \beta_L^{-1} - 1$, to differ.

In this case, the equilibrium is described by the following modified versions of (19) and (20):

$$\frac{k}{l} = \left(\frac{\alpha z}{r_K + \delta}\right)^{\frac{1}{1-\alpha}} \equiv \mathcal{K}(r_K) \tag{39}$$

$$\frac{\kappa}{q(l^l)} = \frac{(1-\alpha) z \mathcal{K}(r_K)^\alpha - \omega z^\gamma (\mathcal{K}(r_K) l^l)^{\alpha\gamma}}{r_L + \rho}. \tag{40}$$

Now, the discount on labour only appears in the denominator of the job value in (40). The discount on capital only directly affects the capital-labour ratio via (39), and then indirectly affects the job value. Following the arguments of Section 4, an increase in either discount rate will reduce employment and hence increase unemployment. But how does the transmission vary for the two discounts? Differentiating (40) with respect to each discount shows how employment responds to each:

$$l^l_{r_L} = \frac{-\frac{\kappa}{q(l^l)}}{\alpha\gamma\omega z^\gamma \mathcal{K}(r_K)(\mathcal{K}(r_K) l^l)^{\alpha\gamma - 1} - \frac{q'(l^l)\kappa(r_L+\rho)}{q(l^l)^2}} \tag{41}$$

$$l^l_{r_K} = \frac{\frac{\alpha \mathcal{K}'(r_K)}{\mathcal{K}(r_K)}\left((1-\alpha) z \mathcal{K}(r_K)^\alpha - \gamma\omega z^\gamma (\mathcal{K}(r_K) l^l)^{\alpha\gamma}\right)}{\alpha\gamma\omega z^\gamma \mathcal{K}(r_K)(\mathcal{K}(r_K) l^l)^{\alpha\gamma - 1} - \frac{q'(l^l)\kappa(r_L+\rho)}{q(l^l)^2}}. \tag{42}$$

As should be expected, these two responses sum up to the total response of employment in the model with a common discount given in (21). This reveals how each discount operates on total employment and hence on unemployment. The denominators are common, and each derivative differs only in the numerator. The magnitude of the effect of the labour discount is controlled by search frictions, via the term $-\frac{\kappa}{q(l^l)}$. The magnitude of the effect of the capital discount is controlled by the responsiveness of the capital-labour ratio to the capital discount, $\mathcal{K}'(r_K)$, moderated by the degree of wage rigidity, $\gamma$.

How do the two frictions affect rationing and frictional unemployment? It turns out that rationing unemployment depends on both the capital and labour discounts:

$$u^{R,l}(r_K, r_L) = \max\left\{0, 1 - \left(\frac{(1-\alpha) z^{1-\gamma}\left(\mathcal{K}(r_K) l^l(r_K, r_L)\right)^{\alpha(1-\gamma)}}{\omega}\right)^{\frac{1}{\alpha}}\right\}. \tag{43}$$

Thus, there is not an exact separation between the capital and labour discounts in affecting frictional and rationing unemployment, as one might expect. Intuitively, this is because rationing unemployment is defined as the level of unemployment which would prevail in the absence of search frictions, but for the current level of capital. In the long run, the level of capital is an endogenous object which depends on both discounts: $k(r_K, r_L) = \mathcal{K}(r_K) l^l(r_K, r_L)$. The capital discount affects total capital by changing the capital-labour ratio, while the labour discount affects the level of capital by changing the level of labour, and hence indirectly the level of capital for a given capital-labour ratio. Additionally, the capital discount will affect frictional unemployment since this is defined as the residual level of unemployment net of rationing unemployment.

# B Appendix to Section 5

## B.1 Robustness and role of key parameters

In this section I discuss the role of two parameters: the level of hiring costs, $\kappa$, and the degree of wage rigidity, $\gamma$. This serves to both illuminate the mechanisms of the model, and to provide robustness to these key parameters.

In Figure 9 I plot the short and long run responses of unemployment to discounts across three values of the hiring cost. In panel (a) I plot the responses with hiring costs equal to 10% of the steady-state wage, in panel (b) with the baseline hiring costs of 32% of the wage, and in panel (c) with high hiring costs of 50% of the wage. These numbers span the range of estimates discussed in Michaillat (2012). In all three cases, wage rigidity is held at its baseline value and all other parameters are recalibrated to match the same steady state moments.

Starting in the top left panel, we see that lower hiring costs have little to no effect on the short run response of total unemployment to discounts relative to the baseline calibration. However, the split between rationing and frictional unemployment is changed. With lower hiring costs half of the unemployment in steady state is rationing unemployment. Intuitively, with lower hiring costs search frictions become less important. Since the model is recalibrated to maintain the same steady state unemployment of 5%, this leads to more of the total being attributed to rationing, i.e. wage rigidities. The increase in unemployment in the short run remains small and driven by frictional unemployment.

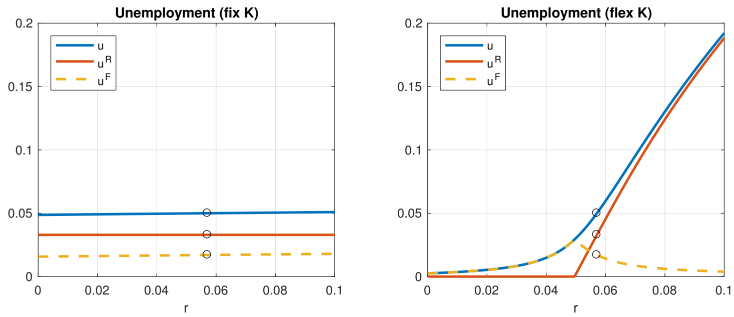
The top right panel shows the long run effect with lower hiring costs. Again rationing unemployment becomes more important with lower hiring costs, and now drives a much larger fraction of the increase in unemployment in the long run. Additionally, the total increase in unemployment for a given increase in discounts is larger. Recall that unemployment responds more in the long run due to capital shallowing, as the reduction in investment lowers the capital-labour ratio and hence the MPL. The calibration with lower hiring costs also is calibrated to have higher average wages (higher $\omega$) in order to encourage firms to still post only enough vacancies to yield 5% unemployment. This economy thus has a lower share of the labour surplus going to firms, and hence unemployment responds more to a given change in the MPL since this represents a larger percentage change in their surplus.

The results are reversed in panel (c) in the model with higher hiring costs. Again the short run responses are essentially unchanged, but now frictional unemployment remains a larger driver of long run unemployment for even higher levels of discounts than in the baseline. Finally, the overall response of long run unemployment is now smaller reflecting the higher firm surplus in this economy.

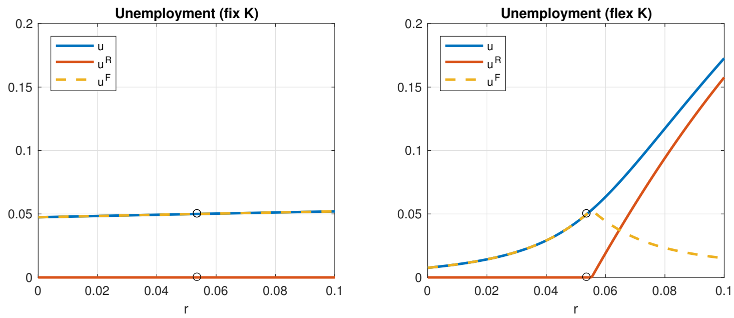
In the first three rows of Figure 10 I give the results of changing the level of wage rigidity. I continue to pick the other parameters to match the targets in the baseline calibration. Row (a) displays the results for more flexible wages ($\gamma = 0.9$) where the wage can adjust 90% of the way to its market clearing value. Row (b) gives the results for the baseline level of wage rigidity ($\gamma = 0.7$), and row (c) for high wage rigidity ($\gamma = 0.5$) where wages can only adjust 50% of the wage to their

Figure 9: Role of hiring costs

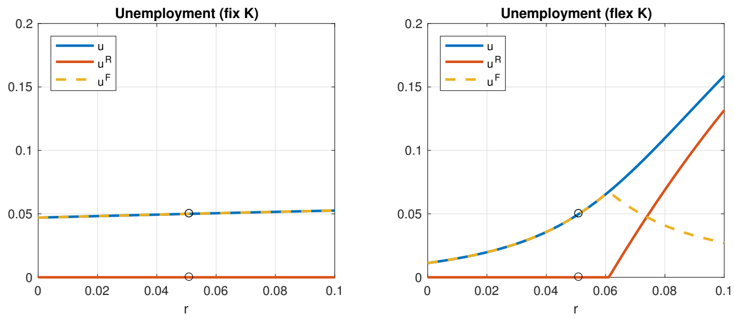
(a) Low hiring costs ($\kappa/w_{ss} = 0.1$)




(b) Baseline hiring costs ($\kappa/w_{ss} = 0.32$)




(c) High hiring costs ($\kappa/w_{ss} = 0.5$)




Comparative statics across steady states as adjust discount rate, $r$, in different calibrations. In all panels, left plot is for fixed initial capital, and right is when capital has adjusted to new steady state value. Hold wage flexibility at baseline and change hiring costs, with all other parameters adjusted to achieve the same calibration targets.

market clearing level.

As with the results from varying hiring costs, varying the degree of wage rigidity has little to no effect on the short run response of unemployment to discounts. This can be seen by looking the left panels of Figure 10, which show no significant differences across the three levels of wage rigidity.

On the other hand, the long run responses show much more dramatic differences. Comparing the right panels of rows (b) and (c) shows that increasing wage rigidity 1) increases the total response of unemployment to a rise in discounts, and 2) increases the role of rationing unemployment while decreasing the role of frictional unemployment. The increased total response of unemployment is intuitive, since more wage rigidity stops wages falling to encourage hiring and offset the negative effects of the rise in discounts. The increased role of rationing unemployment reflects the differing source of unemployment when wages become more rigid. Rationing unemployment becomes more sensitive to discounts the more rigid wages are, since larger movements in labour are required to equalise the MPL with a wage which moves less. Symmetrically, row (a) shows the reduced total effect and reduced role of rationing unemployment when wages are less rigid.

Row (d) presents the result of a related exercise. I recalibrate the model to feature very flexible wages ($\gamma = 0.99$) and low hiring costs ($\kappa/w_{ss} = 0.1$). The objective is to present an example where frictional unemployment increases in the long run in response to a rise in discounts, even when rationing unemployment is positive. Note that in all the previous examples frictional unemployment falls in this case, but that the effect is theoretically ambiguous as per Proposition 2. As shown in the bottom right panel, for this combination of parameters frictional unemployment rises in the long run. However, this is only true for extreme parameter values, and so one contribution of the numerical results here is to show that for reasonable parameter values we should generally expect frictional unemployment to fall in the long run in response to a rise in discounts.

## B.2 Dynamics

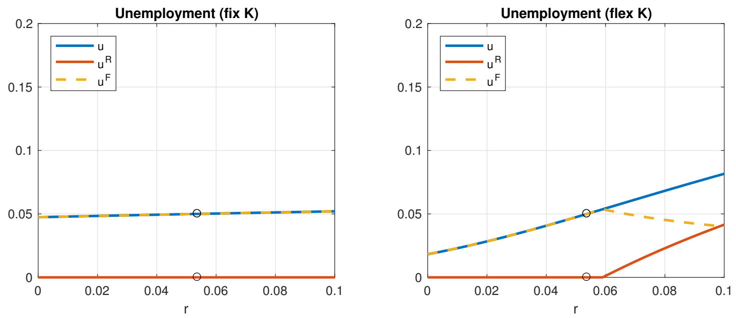
This section performed comparative statics across steady states, but revealed an interesting time dimension to the results by comparing what I call the short and long run steady states. However, one cannot take the time dimension of these comparative statics results exactly literally, since the short run steady state of this analysis does not take into account anticipation effects. In it, it is assumed that agents believe that capital will remain fixed indefinitely, while we know that capital will in fact fall later, once it is allowed to adjust. Since hiring is a forward looking investment, allowing for these anticipation effects could lead unemployment to increase more in the short run, as firms anticipate a lower MPL in the future.

This issue is irrelevant for the decomposition exercises of Section 6, which do take into account all model dynamics. However, it is also interesting to study the transition path to the long run steady state in the earlier comparative statics exercises. Importantly, this does not change at all the main messages of this section, which are that 1) the response of unemployment to discounts is amplified by reductions in capital and 2) following a rise in discounts the rise in unemployment before capital adjusts is driven by frictional unemployment, while after capital adjusts the rise in unemployment is driven more by rationing. Both of these remain true even if unemployment is allowed to adjust through anticipation effects. The only difference is that unemployment might respond more in the short run, but still by less than in the long run. This larger increase will all be frictional, since the definition of rationing unemployment remains static in any case.

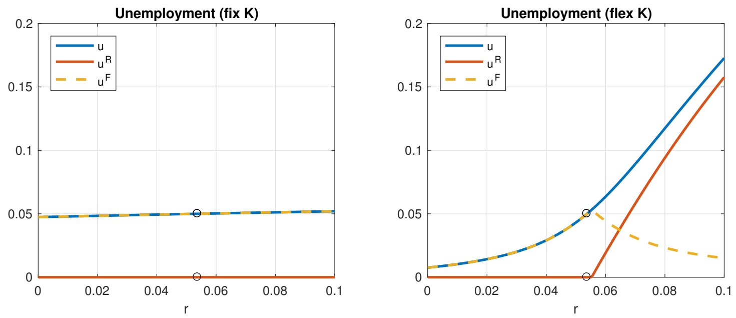
To illustrate these effects, I explicitly compute the equilibrium transition path from the calibrated

Figure 10: Role of wage rigidity

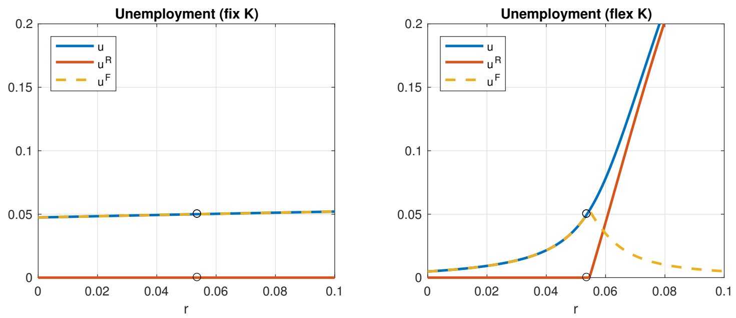
(a) Low wage rigidity ($\gamma = 0.9$)



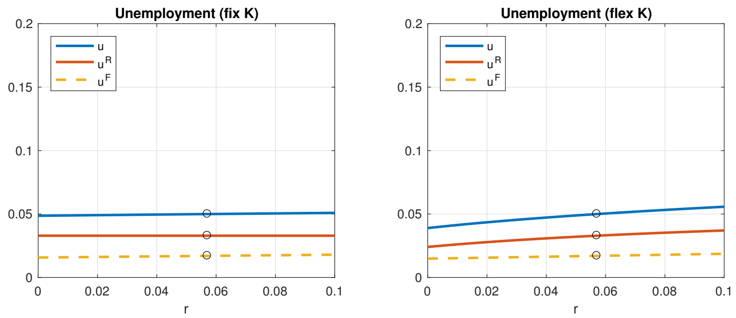

(b) Baseline wage rigidity ($\gamma = 0.7$)




(c) High wage rigidity ($\gamma = 0.5$)




(d) Flexible wages ($\gamma = 0.99$) and low hiring costs ($\kappa/w_{ss} = 0.1$)




Comparative statics across steady states as adjust discount rate, $r$, in different calibrations. In all panels, left plot is for fixed initial capital, and right is when capital has adjusted to new steady state value. Top three hold hiring costs relative to wage at baseline and change wage rigidity, with all other parameters adjusted to achieve the same calibration targets.

steady state to a steady state with higher discounts. I choose an increase in discounts which leads to an increase in unemployment to 10% in the final steady state. I start in the initial calibrated steady state, and then unexpectedly and permanently raise the discount rate to $r = 0.00596$. I then solve for the path of all endogenous variables, following the solution procedure of Section 6.

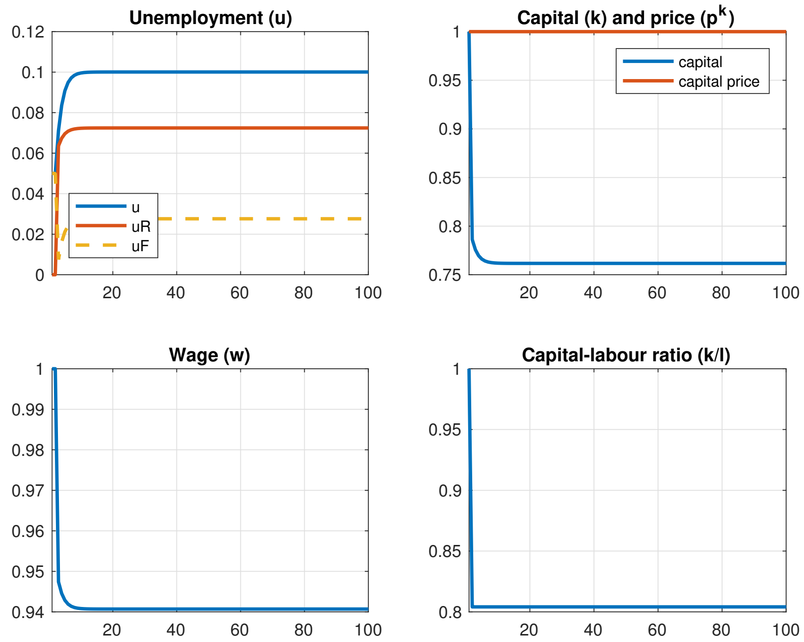
I plot the results in Figure 11. Panel (a) gives the results with no capital adjustment costs ($\psi = 0$) and Panel (b) gives the results with the calibrated level of capital adjustment costs. By construction, both converge to the same final steady state. However, their paths are quite different due to the different speeds at which capital adjusts to the shock.

Without capital adjustment costs, the capital-labour ratio adjusts instantly to the new steady state level implied by the higher discount rate. This can be seen in the bottom right plot of Panel (a). This follows immediately from the capital Euler equation (9) when the capital price is fixed at one. Consequently, the economy converges very fast to the long run equilibrium with flexible capital. All dynamics are due only to the slow response of labour due to matching frictions, and all variables including unemployment have converged to the long run steady state within a year.

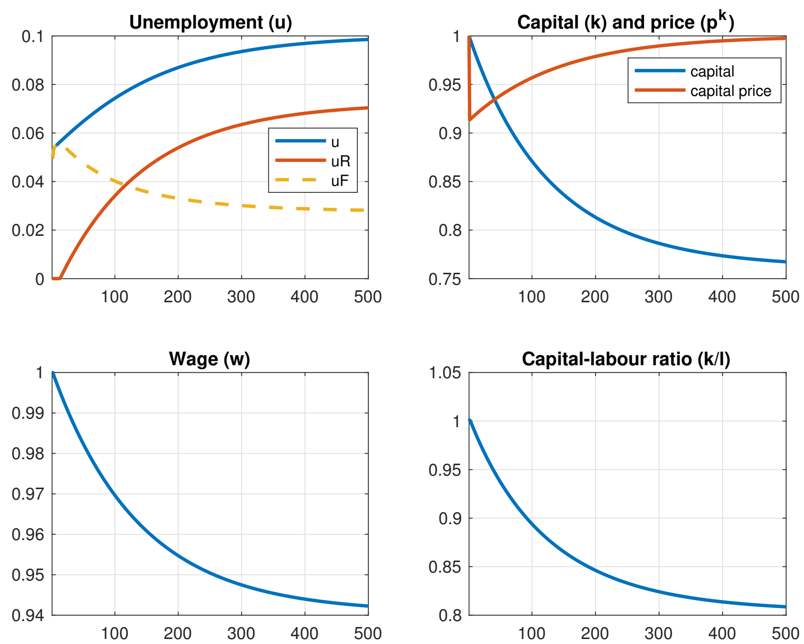
Conversely, with the calibrated level of adjustment costs the transition to the long run steady state takes much longer. Since capital is slow to adjust the price of capital adjusts rather than the capital-labour ratio to satisfy the capital Euler equation. Panel (b) shows that the capital-labour ratio now gradually adjusts to the higher discount rate, and consequently so does the whole economy. Unemployment now reacts more slowly, and it takes around 10 years to converge halfway to the new steady state.

Figure 11: Transition to permanently higher discounts

(a) No capital adjustment costs ($\psi = 0$)




(b) With capital adjustment costs ($\psi = \delta/0.25$)




Transition to steady state with higher discounts leading to $u_T = 10\%$. Top panel is for model with no capital adjustment costs, and bottom for the calibrated level of adjustment costs.

# C Data appendix

In this section I give the details of how the counterfactual participation rate and unemployment are constructed from the data.

## C.1 Participation and demographic adjustment

The BLS provides data on labour force participation both for the whole population, and for different age groups. Denote the whole economy participation rate by $p_t^{raw}$. Let $lf_t$ denote the total labour force, $pop_t$ the population, and index age groups by $i$. Then by construction we can decompose total participation to

$$p_t^{raw} = \frac{lf_t}{pop_t} = \frac{\sum_i lf_{i,t}}{pop_t} = \sum_i p_{i,t} s_{i,t} \tag{44}$$

with $s_{i,t} = pop_{i,t}/pop_t$ being the population shares of each age group, and $p_{i,t} = \frac{lf_{i,t}}{pop_{i,t}}$ the participation rates of each group. I construct a counterfactual overall participation rate, which gives the level of participation predicted by demographic change only, assuming that the participation rate within every age group is permanently fixed at its 2008 level, $p_{i,2008}$. Denote this counterfactual series by $p_t^{cf}$. It is given by

$$p_t^{cf} = \sum_i p_{i,2008} s_{i,t}. \tag{45}$$

Note that $p_{2008}^{cf} = p_{2008}^{raw}$ by construction. For this exercise I use the most detailed population breakdown available in the data. I take the whole population to be all those 16 years and older. The population groups are: 16 to 19, 20 to 24, 25 to 29, 30 to 34, 35 to 39, 40 to 44, 45 to 49, 50 to 54, 55 to 59, 60 to 64, 65 to 69, 70 to 74, and 75 plus. The raw and counterfactual participation rates are plotted for the whole sample in Figure 12.

## C.2 Model-consistent unemployment

Rather than take unemployment directly from the data, I construct unemployment as the level consistent with my data and assumptions on employment and participation. For a participation series $p_t$, we have $u_t + l_{t-1} = p_t \Rightarrow u_t = p_t - l_{t-1}$. Here, consistent with the model, I have normalised the total population to $pop_t = 1$ and hence $u_t$ and $l_{t-1}$ should be understood to be per-capita unemployment and employment.

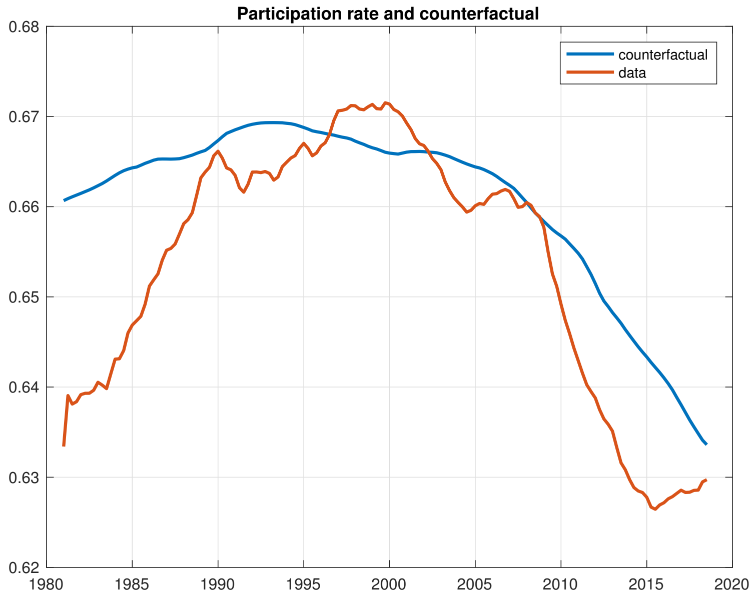
In the previous section I constructed a measure of counterfactual participation for the whole economy from the BLS data. Since the rest of my data is for the Business Sector only, I finally need to construct a measure of participation for just the Business Sector. Unfortunately, the data is only given for the whole economy. In the absence of direct data on this, I back out this series under two assumptions. Firstly, I assume that unemployment in the Business Sector was at the economy-wide level, 5%, just before the crisis in 2007M11. Secondly, I assume that from then onwards participation in the Business Sector evolves proportionately to participation in the whole economy.

Let $p_t$ denote my participation series. The first assumption allows me to back out participation in 2007M11 as

$$p_{2007M11} = u_{2007M11} + l_{2007M10}, \tag{46}$$

where $u_{2007M11} = 0.05 l_{2007M10}/(1 - 0.05)$. The second assumption then allows me to construct participation for the rest of the sample as $p_t = p_{2007M11} \times (p_t^{cf}/p_{2007M11}^{cf})$. Finally, unemployment

Figure 12: Counterfactual participation rate




Compares the actual participation rate in the data with the counterfactual rate coming just from demographic changes.

for the rest of the sample is given by $u_t = p_t - l_{t-1}$.

# D Different discount measures and the capital price

Plots in this appendix give the alternative discount measures referenced in the text for various decomposition exercises. Additionally, the price of capital in each case is given.

Figure 13: Alternative discount measures

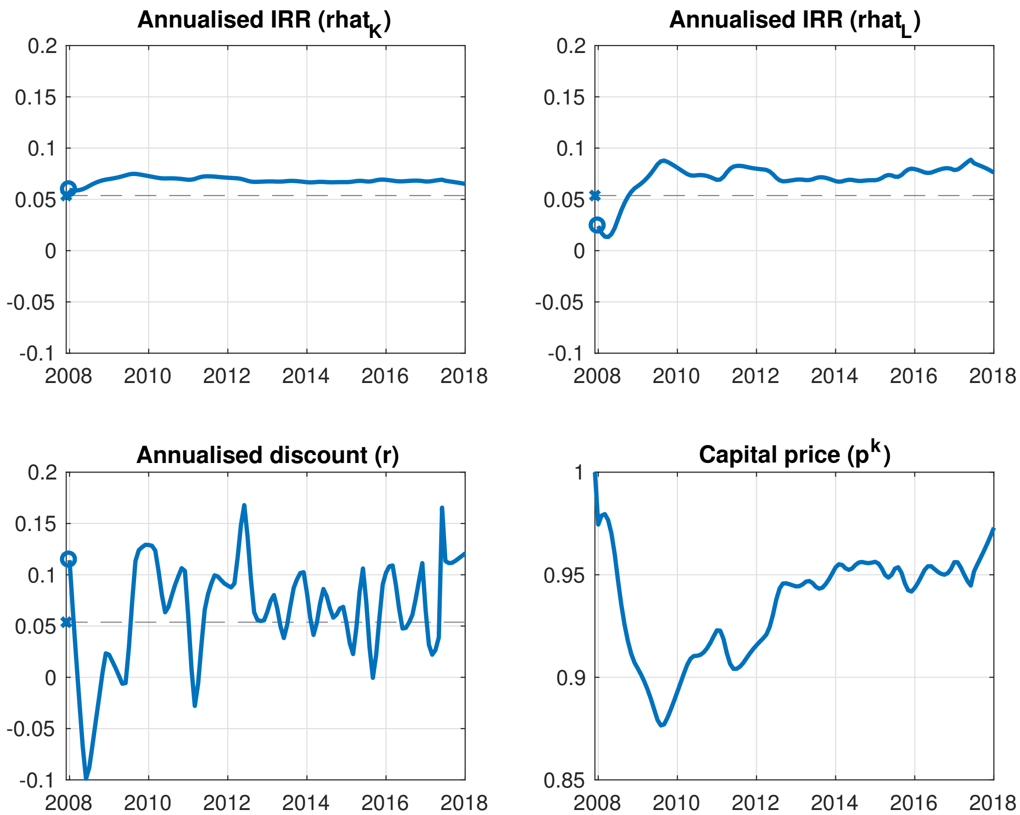



Figure plots estimated discounts from the full decomposition measured three ways. Bottom left panel plots the monthly discount, $r_t \equiv \beta_t^{-1} - 1$. The plot is annualised by actually plotting $(r_t - 1)^{12} - 1$. Top left panel plots the internal rate of return on capital, again annualised, and the top right panel plots the internal rate of return on labour. Bottom right panel plots the price of capital.

Figure 14: Alternative discount measures: model without adjustment costs ($\psi = 0$)

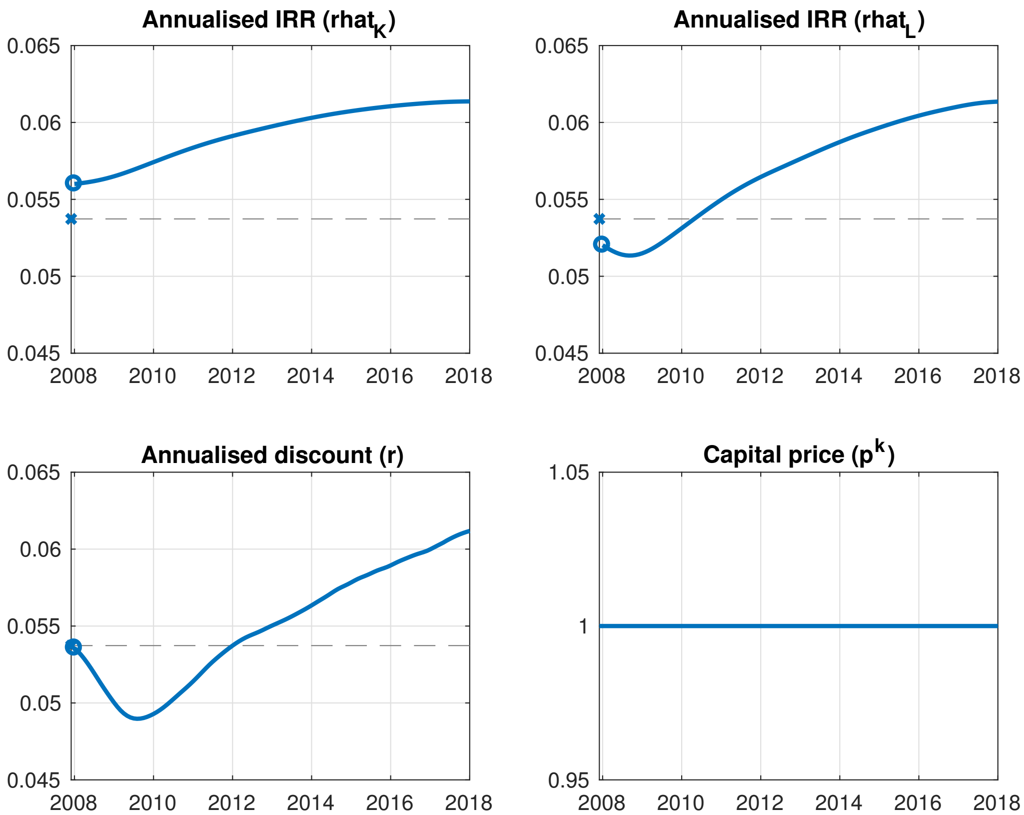



Figure plots estimated discounts from the model without adjustment costs ($\psi = 0$) measured three ways. Bottom left panel plots the monthly discount, $r_t \equiv \beta_t^{-1} - 1$. The plot is annualised by actually plotting $(r_t - 1)^{12} - 1$. Top left panel plots the internal rate of return on capital, again annualised, and the top right panel plots the internal rate of return on labour. Bottom right panel plots the price of capital.

Figure 15: Alternative discount measures: discounts from stock market

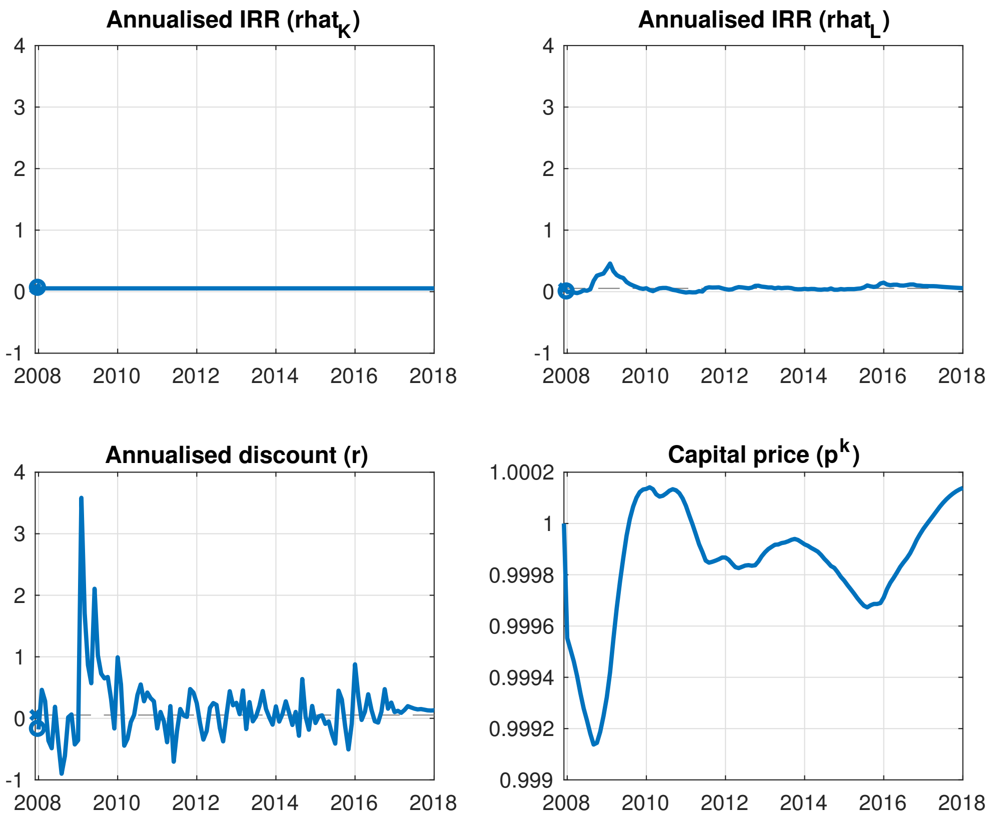
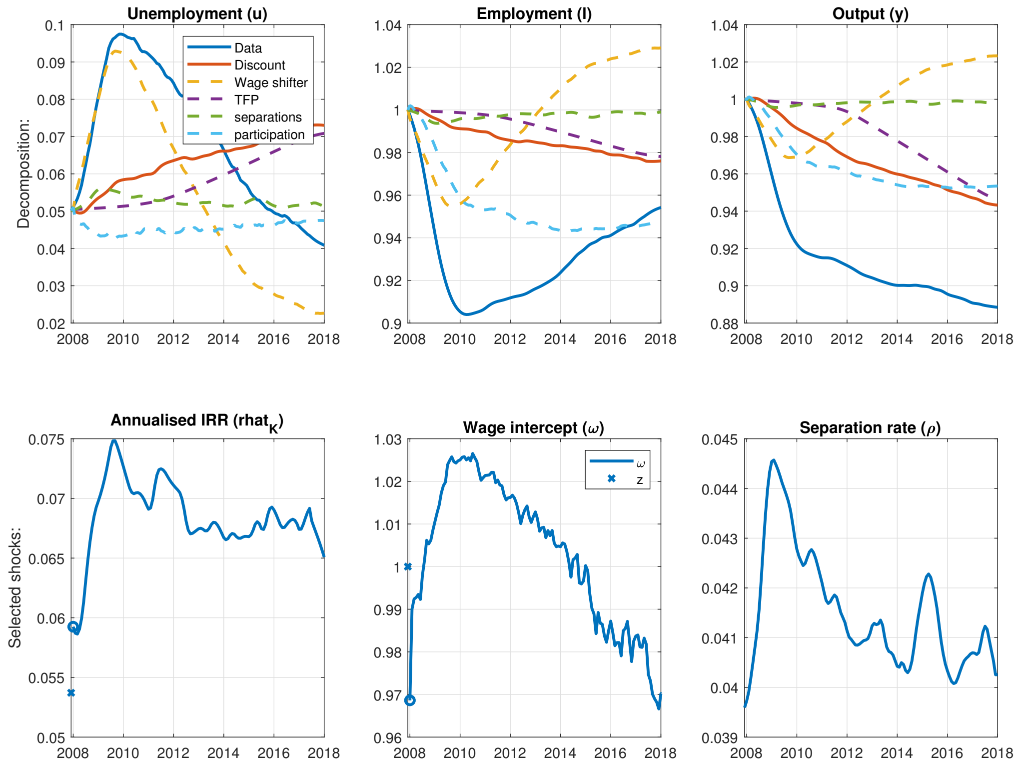



Figure plots estimated discounts from the exercise where discounts are chosen to match stock prices, measured three ways. Bottom left panel plots the monthly discount, $r_t \equiv \beta_t^{-1} - 1$. The plot is annualised by actually plotting $(r_t - 1)^{12} - 1$. Top left panel plots the internal rate of return on capital, again annualised, and the top right panel plots the internal rate of return on labour. Bottom right panel plots the price of capital.

## D.1 Decomposition under different participation assumptions

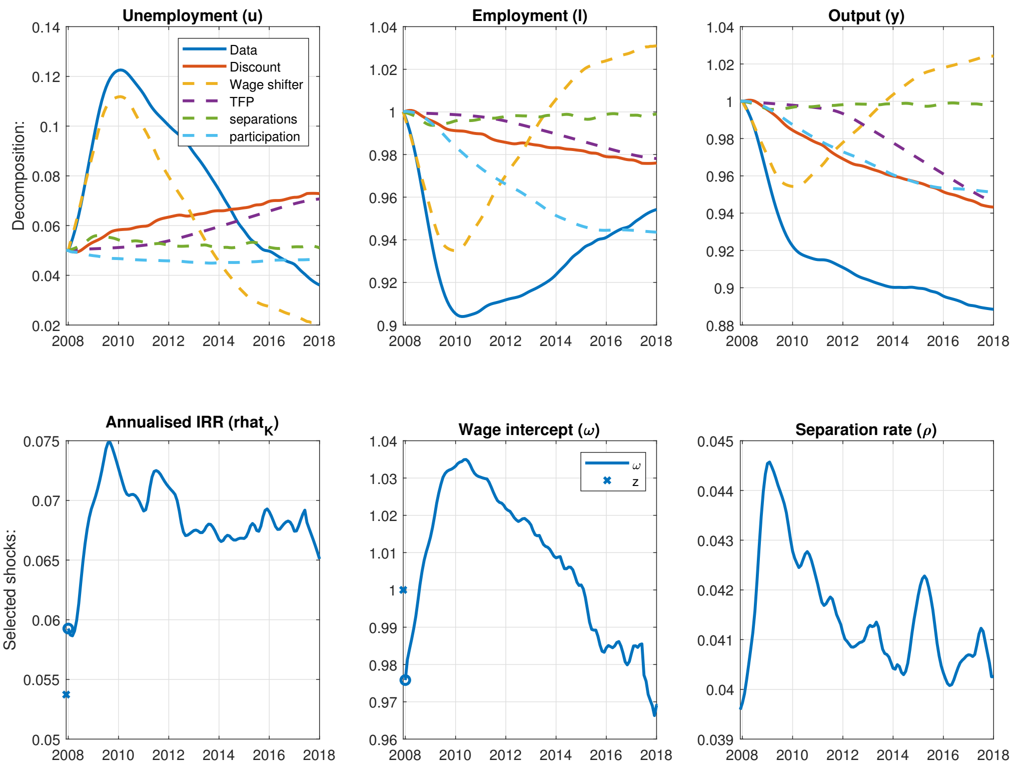
In this section I redo the full decomposition exercise of subsection 6.3 under two alternative assumptions on participation. In the first exercise I choose participation to match the standard whole-economy civilian unemployment rate data, which peaks at the lower 10% rather than my adjusted series which also includes some non-participants. Secondly, I take the raw participation series, $p_t^{raw}$ as my basis for computing unemployment rather than the counterfactual series. The results are given in Figures 16 and 17. All of the qualitative properties of the main results are preserved: discounts contribute the same amount to the unemployment rate in all cases by construction. The relative importance of discounts versus the other shocks is also preserved, with discounts contributing late in the crisis and with a similar quantitative magnitude to TFP. The initial run up n unemployment is smaller when matching the civilian unemployment rate data, but it is still almost entirely explained by the residual shock to hiring incentives.

Figure 16: Full decomposition: Participation to match civilian unemployment rate data




Top row plots decomposition of path for key variables attributable to different shocks. Each series is found by simulating the model subject only to that shock series. Model is solved nonlinearly, so adding all responses will not exactly recover the data, but does so approximately. Solid blue line gives the data, and remaining lines the contributions from each shock. Bottom row gives paths for all shocks apart from TFP and participation rate, which are plotted in Figure 2

Figure 17: Full decomposition: Participation from “raw” participation data




Top row plots decomposition of path for key variables attributable to different shocks. Each series is found by simulating the model subject only to that shock series. Model is solved nonlinearly, so adding all responses will not exactly recover the data, but does so approximately. Solid blue line gives the data, and remaining lines the contributions from each shock. Bottom row gives paths for all shocks apart from TFP and participation rate, which are plotted in Figure 2