Application of Regime Switching and Random Matrix Theory for Portfolio Optimization

By Javed Iqbal

A thesis submitted for the degree of PhD

Centre for Computational Finance and Economic Agents/ Computer Science and Electronics Engineering,

University of Essex

Date of submission (October 2017)

University of Essex

Abstract

Market economies have been characterized by boom and bust cycles. Since the seminal work of Hamilton (1989), these large scale fluctuations have been referred to as regime switches. Ang and Bekaert (2002) were the first to consider the role of regime switches for stock market returns and portfolio optimisation. The key stylized facts regarding regime switching for stock index returns is that boom periods with positive mean stock returns are associated with low volatility, while bear markets with negative mean returns have high volatility. The correlation of asset returns also show asymmetry with greater correlation being found during stock market downturns. In view of the large portfolio losses from correlated negative movements in asset returns during the recent 2007 financial crisis, it has become imperative to incorporate regime sensitivity in portfolio management. This thesis forms an extensive application of regime sensitive statistics for stock returns in the management of equity portfolios for different markets. Starting with the application to a small 3 asset portfolio for UK stocks (in Chapter 4), the methodology is extended to large scale portfolio for the FTSE-100. In chapters 5 and 6, respectively, using stock index data from the subcontinent (India, Pakistan and Bangladesh) and for the Asia Pacific, optimal regime sensitive portfolios have been analysed with the MSCI AC Index (for Emerging and Asia Pacific Markets) being taken as the benchmark index. Portfolio performance has been studied using a dynamic end of month rebalancing of the portfolio on the basis of regime indicators given by market index and relevant regime dependent portfolio statistics. The cumulative end of period returns and risk adjusted Sharpe Ratio from this exercise is compared to the simple Markowitz mean-variance portfolio and market value portfolio. The regime switching optimal portfolio strategy has been found to dominate non-regime

sensitive portfolio strategies in Asia Pacific and 3 asset portfolio for UK stocks cases but not in Subcontinent case (for the first half of out-sample period). In the case of the relationship of the sub-continental indexes vis-à-vis the MSCI benchmark index, the latter has negligible explanatory power for the former especially for the first half of out-sample period. Hence, the regime indicators based on MSCI emerging market index have detrimental effects on portfolio selection based on the sub-continental indexes. As regime sensitive variance–covariance matrices have implications for the selection of optimal portfolio weights, the final Chapter 7 uses the FTSE-100 and its constituent company data to compare and contrast the implications for optimal portfolio management of filtering the covariance matrix using Random Matrix Theory (RMT). While it is found that filtering the variance-covariance matrix using Marchenko-Pasteur bounds of RMT improves optimal portfolio choice in both non-regime and regime dependent cases, remarkably in the latter case for Regime 2 determined variance-covariance matrix, the RMT filter was least needed. This result is given in Chapter 7, Table 7.5-1. This confirms the significance of using Hamilton (1989) regime sensitive statistics for stock returns in identifying the 'true' non-noisy variance-covariance relationships. The RMT methodology is also useful for identifying the centrality, based on eigenvector analysis, of the constituent stocks in their role in driving crisis and non-crisis market conditions. A fully automated suite of programs in MATLAB have been developed for regime switching portfolio optimization with RMT filtering of the variance-covariance matrix.

Key Words: Regime Switching, Asymmetric Correlations, Portfolio Optimization, International Portfolio, Asian Stock Markets, MSCI, Asia/Pacific Stock Markets, Random Matrix Theory, Marchenko-Pasteur Theorem, Correlation Filters,

Acknowledgement

I start in the name of ALLAH, the Most Gracious and the Most Merciful. Alhamdulillah, all praises to Allah for the strengths and His blessing in completing this thesis.

Special appreciation goes to my supervisor, **Professor Sheri Markose** for her supervision and constant support. Her invaluable help of constructive comments and suggestions throughout the experimental and thesis works have contributed to the success of this research. Not forgotten, my appreciation to the Director, **Professor Edward Tsang** for his support and knowledge regarding this topic.

I would like to express my appreciation to all the teaching staff and office staff of CCFEA/CSEE especially **Julie Pearson, Lynda Triolo and Marisa Bostock** for their co-operations.

Sincere thanks to all my friends especially **Dr. Zubair Ahmad, Dr. Zahoor ul Haq, Kashif Rehmat Ullah, Ghazanfar Ali, and Muhammad Furqan Hameedi**, and many others for their kindness and moral support during my study.

Last but not least, my deepest gratitude goes to my beloved parents; **Mr. And Mrs. Abdul Khaliq,** then to my wife **Saeeda Yasmin**, my beautiful children **Sofia Javed, Halima Javed and Muhammad Abdul Hadi** for their endless love, prayers and encouragement.

I would extend my gratitude towards **Bahauddin Zakariya University**, Pakistan, **University of Essex**, UK and **Higher Education Commission of Pakistan** for funding my PhD research during this period. All those who directly or indirectly contributed to this research, your kindness mean a lot to me. **Thank you very much**.

Table of Contents

Table of Figure

Table of Tables

1 Chapter 1: Introduction

1.1 Introduction to Regime Switching

Boom and bust cycles are inherent to economic phenomena associated with business cycles and financial markets. The recent financial and macroeconomic crises respectively called the Great Financial Crisis and the Great Recession, exemplify the sudden economic downturn of enormous proportions after an extensive boom period (Chan, Fry-McKibbin, and Hsiao, 2017). The latter was called the Great Moderation (Bernanke (2009), Bean (2009), Bean et al, (2010), Taylor, (2010), Chan, Fry-McKibbin and Hsiao, (2017)) which between 2003-July 2007 was characterized by low-risk and high asset prices. The popular media characterized the events of Great Moderation as partly 'driven by good luck, including the integration of emerging market countries into the global economy, and partly a dividend from structural economic changes and better policy frameworks. The longer this stability persisted, the more markets became convinced of its permanence, and risk premium became extremely weak. Real short and long-term interest rates were also low due to a combination of loose monetary policy, particularly in the US, and strong savings rates in a few surplus countries' (Taylor, (2010), Chan, Fry-McKibbin and Hsiao, (2017)). This, in turn, lulled both investors and regulators into a sense of complacency. In fact, in some quarters, such as the Nobel Prize-winning macroeconomist Robert Lucas (2003) in his AER Presidential lecture said that modern macroeconomics had solved boom & bust while in fact the West was enduring a large asset price bubble which preceded the largest GDP collapse since the Great Depression.

The Great Financial Crisis began somewhat inconspicuously in late summer 2007 with the failure of two Bear Stearns hedge funds, and then went from bad to worse over the following year despite countless attempts by governments to halt its progress. It is now universally recognized as the worst economic crash since the Great Depression. Indeed, as U.S. economist and New York Times columnist Krugman (2008) indicated, it raises "the prospect of a second Great Depression. Although former Federal Reserve Board chairman Greenspan (2008) has linked it to "a once-in-a-century credit tsunami," and said that Great Financial Crisis is a historical man-made rather than natural phenomenon. It was preceded by a whole series of lesser economic shocks, of growing magnitude, over the last two decades, most notably: the U.S. stock market crash of 1987, the savings and loan crisis of the late 1980s and early '90s, the Japanese financial crisis and Great Stagnation of the 1990s, the Asian financial crisis of 1997-1998, and the New Economy (dot-com) crash of 2000. Yet the Great Financial Crisis has far outreached them all (Foster, 2009).

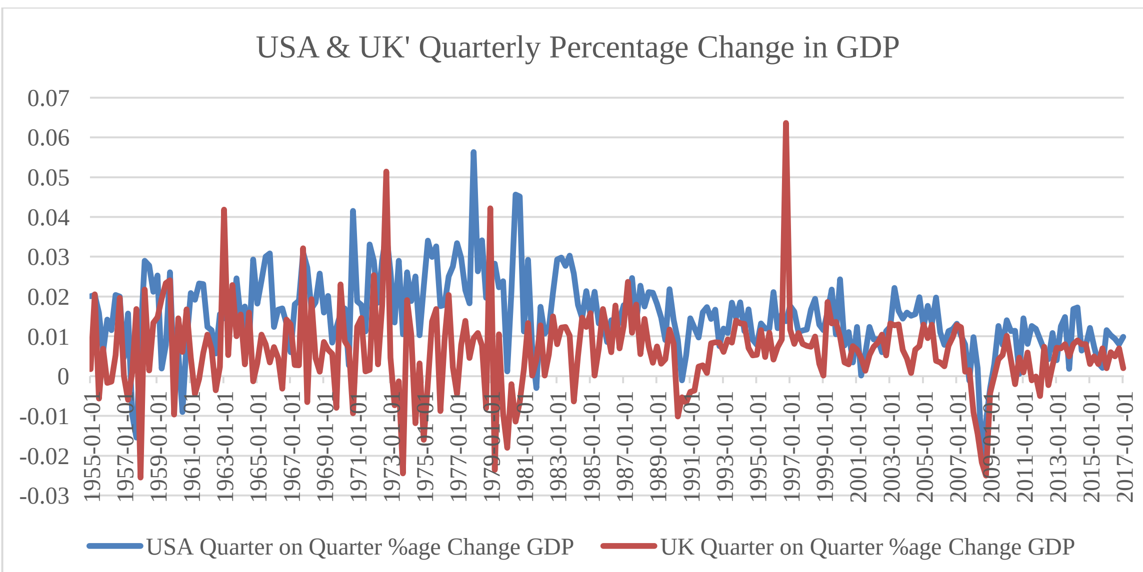


Figure 1.1-1: USA & UK' Quarter on Quarter Percentage Change in GDP from January 1955 to July 2017 (Source: https://fred.stlouisfed.org/series/GDP and https://www.ons.gov.uk/economy/grossdomesticproductgdp/timeseries/ihyq/pn2)

The subsequent economic bust, especially in the US, UK and Eurozone have led to the need to reassess the models used for assessing macroeconomic and financial risk. The need to explicitly incorporate the boom and bust characteristics into economic/financial models has become an active area of research. Figure 1.1-1 is clearly showing the ups and downs in GDP’s of US and UK economies with huge decline in 2007-2009. Since many economic and stock market prices undergo periods of changes or breaks in their behaviour, several substantial economic fluxes can be associated with a range of events. Those driven by institutional changes like deregulation, financial panics from asset price bubbles are endogenous and cause ‘structural breaks.' The alteration may be for a particular period before reverting or switching back to its original or normal behaviour; this is typically termed a regime switch or shift (Brooks, 2007). Previous research shows that the financial and economic data exhibit regimes. This phenomenon is mainly prevalent in financial markets and the macro economy where the data is usually drawn from entirely different distributions while undergoing booms and downturns. There is a compelling evidence of regime shifts in the United States (US) stock returns, emerging market securities and other securities featured in the main indices, (Hamilton and Susmel, 1994 & Assoe, 1998)

Ang & Timmermann (2012) state that financial markets often change their behaviour abruptly. While some changes may be transitory (“jumps”), often the changed behaviour of asset prices persists for many periods. The mean, volatility, and correlation patterns in stock returns changed dramatically at the start of, and persisted through, the global financial crisis of 2008-2009. Similar regime changes, some of which can be recurring (recessions versus expansions) and some of which can be permanent (breaks), are prevalent in fixed income, equities, and foreign exchange markets, and in the behaviour of many macro variables (Ang & Timmermann, 2012). Regime switching models can capture these sudden changes of behaviour, and the phenomenon that the new dynamics of prices and fundamentals persist for several periods after a change.

When applied to financial series, regimes identified by econometric methods often correspond to different periods in regulation, policy, and other secular changes. The interest rate behaviour markedly changed from 1979 through 1982, during which the Federal Reserve changed its operating procedure to targeting monetary aggregates. Other regimes identified in interest rates correspond to the tenure of different Federal Reserve Chairs (Sims and Zha, 2006). In equities, different regimes correspond to periods of high and low volatility, and long bull and bear market periods. Thus, regime switching models can match narratives on changing fundamentals that sometimes can only be interpreted ex-post. The challenge is to identify regime switches that can be used for ex-ante real-time forecasting, optimal portfolio choice, and other economic applications.

There are several reasons why regime switching models have become popular in financial modelling. First, the idea of regime change is natural and intuitive. Indeed, the original application of regime switching in Hamilton's (1989) seminal work was to business cycle recessions, and expansions and the regimes naturally captured cycles of economic activity around a long-term trend. Hamilton's regimes were closely tied to the notion of recession indicators identified as ex-post by some researchers. This thesis identifies closely with the work of Hamilton (1989).

Second, regime switching models parsimoniously capture the stylized behaviour of many financial series including fat tails, persistently occurring periods of turbulence followed by periods of low volatility (ARCH effects), skewness, and time-varying correlations (Ang and Liu, 2007). By appropriately mixing conditional normal (or other types of) distributions, enormous amounts of non-linear effects can be generated (Whitelaw, 2000). Even when the exact model is unknown, regime switching models can provide a good approximation for more complicated processes driving security returns (Ang & Baekert, 2004). Finally, another attractive feature of regime switching models is that they can capture nonlinear stylized dynamics of asset returns in a framework based on linear specifications, or conditionally normal or log-normal distributions, within a regime. This makes asset pricing under regime switching analytically tractable (Ang & Baekert, 2004). In particular, regimes introduced into linear asset pricing models can often be solved in closed form because conditional on the underlying regime, normality (or log-normality) is recovered. It makes incorporating regime dynamics in affine models straightforward. The notion of regimes is closely linked to the familiar concept of good and bad states or states with low versus high risk, but surprising and somewhat counterintuitive results can be obtained from equilibrium asset pricing models with regime changes (Ang & Baekert, 2004). Conventional linear asset pricing models imply a positive and monotonic risk-return relation (e.g., Merton, 1973). In contrast, changes in discrete regimes with different consumption growth rates can lead to increasing, decreasing, flat or non-monotonic risk-return relations as shown by, e.g., Backus and Gregory (1993), Whitelaw (2000), and Ang and Liu (2007). Intuitively, non-monotonic patterns arise because “good” and “bad” regimes, characterized by high and low growth in fundamentals and asset price levels, respectively, may also be associated with greater uncertainty about prospects than more stable, “normal” regimes which are likely to last longer. The possibility of switching across regimes, even if it occurs relatively rarely, induces a significant additional source of uncertainty that investors want to hedge against. Inverse risk-return trade-offs can result in some regimes because the market portfolio hedges against adverse future consumption shock even though the level of uncertainty (return volatility) is high in these regimes. Further non-linearities can be generated because of investors' learning about unobserved regimes.

The financial crises of 2008–2009 was a vivid reminder that financial correlations increase and strengthen during periods of high volatility and assets values fall together. It also highlighted the importance of identifying 'true' correlations in order to quantify the underlying risk of diversified portfolios. Random matrix theory (RMT) has been applied to investigate the statistical properties of the cross-correlations of price changes of global financial indices and stock markets (Nobi et al, 2013). This thesis, utilises the RMT approach to filter variance-co-variance matrices which are subject to regime switches in order to identify the 'non-noisy' correlation matrices that can improve portfolio performance.

Random matrix theory was developed in the context of complex quantum systems in which the precise nature of the interactions between subunits is not known (Mehta (1991), Guhr et al. (1998)). For complex quantum systems, RMT predictions represent an average over all possible interactions (Dyson (1962), Dyson and Mehta (1963a, 1963b)). Deviations from RMT predictions identify non-random properties of the system under consideration, providing clues about the underlying interactions (Mehta (1991), Brody et al (1981)). The eigenvalues of the cross-correlation matrix are compared with the eigenvalues of the random matrix and it has been found that some eigenvalues deviate from the RMT prediction by what is famously called the Marchenko-Pasteur bounds (Plerou et al, (1999), (2002), Laloux et al, (1999), & Daly et al, (2010)). When applying the RMT approach to the stock returns from the constituents of a stock index such as the FTSE-100 index, as undertaken in this thesis, the largest eigenvalue of the variance-covariance matrix of stock returns of the constituents of the FTSE-100. Nobi et al (2013) find that the large eigenvalues occur during crisis periods, which indicates strong interactions among constituents of the stock index during a crisis. The components of the eigenvectors corresponding to the largest eigenvalues in all periods are positive, representing an influence that is common to all stocks in the index (Daly et al, 2010). Larger the value of the eigenvector, the greater than centrality of the stock in the stock market in the determination of the higher eigenvalues, which are known to proxy high crisis conditions. The focus of the thesis is on the RMT filtering of the stock returns variance-covariance matrix and use Plerou et al (1999, 2002) method of filtering explained in Chapter 7.

1.2 Motivation of the research

The supposition that asset returns are normally distributed is a strong assumption, and crucial for the Markowitz' framework to be implemented. It has been accepted by the academia due to its convenience, however, it will rarely be fulfilled in real life. Motivated by the extreme portfolio losses sustained during the Great financial Crisis (see, Khandani and Lo, 2013), there has been a significant increase in the literature stating that asset returns follow a more complicated process than has been assumed previously, and the assumption of normal distribution of returns is often mentioned as one of the largest limitations of the framework. To go beyond the strong assumption of normality, the use of Markov Chains and a Regime Switching Model seem to be the most appropriate solution, as assets returns distribution can be defined with multiple regimes where each regime is associated with a different normal distribution. The joint distribution of returns is then not a normal distribution and is better able to capture the time varying occurrence of fat tails observed in the returns distribution.

Furthermore, the mean-variance framework is mostly based on a buy-and-hold investment strategy and does not allow the investor to alter the portfolio efficiently when new information is at hand. Again, this is seldom the case for investors who want to keep an active eye on the development of financial markets as they might rebalance their portfolios at certain stages due to additional information.

Therefore, by extending the portfolio theory initially introduced by Harry Markowitz (1959) with both the Regime Switching Model and the ideology of rebalancing, one is able to better model the investor's asset allocation problem in financial markets. Hence, this thesis will adopt a generalization of a Capital Asset Pricing Model (CAPM) based portfolio management with an explicit formation of portfolios that are appropriate for the regimes indicated *a priori* by the Hamilton (1989) Regime Switching model regime indicator. Thus, we retain the assumption in CAPM that asset returns are determined by a single factor which is the all share stock price index and hence the switches in regimes are driven by the hidden states creating the boom and busts in the stock market as a whole.

This thesis will analyse the implication of regime switching (RS CAPM) in optimal portfolio allocation in the equity markets. Empirical results show RS CAPM models can explain a larger proportion of the variation in hedge fund and market returns as opposed to orthodox linear models (Ang & Bekaert, 2002 & 2004). This explanatory power is linked to the tendency of markets to synchronize or become more co-dependent in periods of stress. The underlying theme is to continuously rebalance the portfolio from the weights generated by the regime switching (RS CAPM) model and compare them with simple mean-variance portfolio (Non-RS) and market value portfolio (Market value (MV) weighted portfolio).

1.2.1 Contributions of Thesis

This thesis follows the work of authors such as Guidolin and Timmermann (2005(a), 2005(b) 2006, 2007), Guidolin and Ria (2010) and Ang & Baekert (2002, 2004), who have applied similar methods. The RS CAPM model of this thesis is original in a number respects, especially in the construction of the regime dependent variance-covariance matrices.

The regime switching methodology used here is unique in many cases; first, it is based on dynamic CAPM based whereas other studies have used static CAPM. This study uses rolling window based CAPM or dynamic CAPM as it utilises rolling window for computing alphas, betas, mean and sigma and helps in identifying which regime model is in.

Second, it is unique in the sense that it uses RMT methodology to additionally filter the regime based covariance matrices. Third, it uses the Eigenvalue decomposition to have further insight into how stocks behave when regime changes. Due to change in regimes, portfolio selection and weight allocation considerably changes i.e. how they are selected and whether a long or short position is suitable.

Application of Random matrix theory (RMT) to the portfolio optimisation has been done previously but using Regime Switching model alongside RMT filters is a new concept that will further endorse the applicability of the Regime Switching methodology. It is found that the variance-covariance matrices created under Regime 2 crisis state conditions, do not need substantial additional RMT filtering to recover ‘non-noisy’ statistics, indicating the relevance and power of the role of the Hamilton regime switching estimations. This thesis will also look at the largest eigenvalue and its eigenvector from the RMT filters on the variance-covariance matrices for the Regime 1 non-crisis state and the Regime 2 crisis state and highlight the role of the different sectors which contribute more towards the performance of FTSE-100 index during the crisis and non-crisis states.

1.3 Objectives of the thesis

This research focuses on optimal asset allocation in the presence of regime switching in asset returns. By using regime switching model, purpose is to go beyond the unsatisfactory standard mean-variance portfolio theory, which has presented its inadequacy given the normality assumption for asset returns the theory relies on. This thesis will test dataset for a normal distribution and aim to show that asset returns are not normally distributed by using a regime switching model based on Hamilton's (1989, 1994, 2008) approach. Evidence of asymmetric correlation is produced to show that the assets behave differently in different economic conditions. Regimes switching model (RS CAPM) along with simple mean-variance model called Non-Regime Switching (Non-RS) in this thesis and Market Value weighted portfolio (MV) is used to implement the optimal asset allocation for an investor, based on a portfolio of three assets and extending it to 74 Assets. These methods of asset allocation are used to construct an optimal portfolio and compared to show whether RS CAPM strategy outperforms the other two. Do RMT filters also enhance the performance of RS CAPM? This thesis aims to give extensive empirical analyses of the following key questions:

Test for regime existence in returns/excess returns for all the datasets used based on filter/smooth probabilities?

Do assets observe fixed correlations/ covariance's or they show asymmetric correlations?

Do transition probabilities given by Hamilton (1989) Markov-state model give robust a priori regime indicators that correspond with *ex post* characterization of ‘good’ and ‘bad’ regimes?

Do investors allocate their wealth differently when considering regime switching in returns compared to a normal case of a non-regime switching model or Market Value weighted portfolio?

Are the portfolio weight allocations effective with the change in regimes?

Do short selling/no short selling and risk free borrowing and lending compensate the portfolio when regime changes?

Does successful portfolio management depend on appropriate estimation of correlation matrices of asset returns?

How noisy are correlation matrices of stock returns and how does this affect portfolio performance?

Can noisy elements in correlation matrices be filtered?

Does RMT filtering improve the performance of portfolios in terms of end of period cumulated wealth and the Sharpe ratio?

1.4 Structure and Overview of Thesis

In order to pursue these objectives, the rest of the thesis is organized as follows.

Chapter 2 highlights the Literature available on Regime Switching and its applications to different segments of the economy. **Chapter 3** develops the methodology adopted in this thesis and formally discusses all the issues tackled in this thesis. This includes the Hamilton (1989) Markov regime switching model and the RS-CAPM portfolio model with special focus on the construction of the regime dependent variance-covariance matrices and the role of the regime indicator in the rebalancing of the regime sensitive portfolios.

The first three core Chapters 4, 5, & 6 are to be considered pilot study. Chapter 4 forms the basis of extension in this model/work to more markets, initially to Subcontinent (in Chapter-5) which is a developing market and requires us to move to more developed market of Asia-Pacific (Chapter-6) subsequently moving to well established FTSE-100 index (Chapter-7).

Chapter 4 looks at the implications of regime switching (RS CAPM) in optimal asset allocation in the United Kingdom (UK) equity market in the 3-asset case. The data gives significant evidence of regime sensitivity of returns statistics and also of asymmetric correlation. The bull market (Regime 1) had a higher mean, lower volatility, and lower asset correlations compared to the bear market (Regime 2) which vitiates the benefits from diversification when market conditions deteriorated quickly, hence using regime-switching became essential for market timing purposes. Using the maximum likelihood estimation and the Markov switching models, mean, standard deviation and transition probability are estimated for each regime - these coefficients were statistically significant (i.e., at the 0.01% significance level). Both regimes showed evidence of persistence. For the comparison of the portfolio results, the different RS and non-RS portfolio strategies were employed under different scenarios and evaluated in terms of the cumulative portfolio returns and also a risk adjusted Sharpe ratio. The cumulated RS CAPM wealth and its Sharpe ratio were greater than the other two strategies (Non RS and MV approach). The ‘least risk averse’ investor’s end of period cumulated wealth was by far superior to the ‘most risk averse’ investor when compared to all other methods.

Chapter 5 expands the literature on emerging markets by evaluating gains from diversifying into emerging equity markets. The price index data from three stock exchanges, Pakistan Stock Exchange (KSE-100), Bombay Stock Exchange (BSE-100) and Dhaka Stock Exchange (DSEX) is evaluated against MSCI Emerging Market Index. Ang & Bekaert (1999) argued that the standard mean-variance analysis is problematic concerning emerging equity markets hence a need for more robust techniques like Regime Switching (RS CAPM) and MV, etc. The primary focus of this chapter is to ascertain that the RS Strategy is the robust new methodology to achieve portfolio optimisation in emerging markets. The data shows the existence of regimes and hence there is prima facie evidence that RS portfolio methods may be useful. However, the results (as shown in Chapter 5) do not support the assumption completely. It is presented previously in FTSE-100 case that the Regime Switching Strategy (RS CAPM) succeeds, but this chapter does not support the previous argument completely at least for the first half of out-sample period when end of period cumulated wealth is performing poorly and RS CAPM only performs better in later half of out-sample period. The other two strategies performed much better during first half of out-sample period and then fell behind in second half of out-sample period. In conclusion, the methodology works well when applied to more established markets like the UK's FTSE-100. The mediocre performance of the RS CAPM strategy for emerging markets are due to some factors. For example, Pakistan Stock Exchange operates under strict lower/upper cap limits since last seven years. The intervention and influence enjoyed by the governments and its officials (both in Pakistan and Dhaka stock exchanges) are huge and creates an operational barrier. Overall, the RS Strategy offered good regime indicators in the FTSE-100 against the emerging markets of the Subcontinent. This research requires the need for a further query in developing, developed and underdeveloped markets. To address some of the questions raised by Subcontinent study, we have extended the study to more developed/developing Asia Pacific market discussed in next chapter.

Chapter 6: The need for diversification into the Asia-Pacific markets arises from increasing uncertainty around policy issues in the US, the Eurozone and in some emerging markets. These include the US Fiscal Cliff, Eurozone crisis, and potential conflicts in the Middle East. By focusing on economies that are better positioned to withstand the significant drivers of uncertainty, and mitigating exposure to economies that are highly susceptible, investment growth can still be achieved. For example, Asia Pacific, Sub-Saharan Africa, and parts of Latin America are relatively well insulated. Asia Pacific presents an anchor of stability and new pillar of growth for the global economy. While the severity of the fiscal crisis in the Western hemisphere has created headwinds, Asia Pacific continues to be the fastest growing region. This increased vitality and visibility are fuelling a great transition, one that has the potential to create a future defined by the region's consumers, companies, and cultures. This chapter analyses the Asia Pacific and aims to evaluate gains from the Regime Switching (RS CAPM) strategy for portfolio optimisation. The stock Index data from six Asia Pacific stock exchanges is assessed against MSCI Asia Pacific Index. As noted in the previous chapter, the Regime Switching Strategy has been successful in case of FTSE-100 but was not very helpful in the event of a portfolio of Subcontinent indices. The Asia Pacific dataset also support the results found in Chapter 4 and noted when market conditions deteriorated quickly, regime switching became essential for market timing purposes, and it helped improve the performance of the portfolio. On average RS wealth was higher (sometimes even 300%) than the Non RS and MV strategy. All the strategies have seen the decline in cumulated wealth around the time of the credit crisis in 2007 but RS CAPM strategy managed through the crises periods, and it still ended in positive and comparatively higher values. The RS CAPM strategy provided some useful indicators in developed Asia-Pacific countries when compared with developing or emerging markets.

Chapter 7 investigates the statistical properties of the correlation matrices for the constituent stock returns in the FTSE-100 using the random matrix theory (RMT). The implications are analysed for the optimal portfolio management in the standard Markowitz portfolio theory and also in the case of Hamilton two-state regime sensitive portfolio optimization. We use stock returns time series of the firms to compute correlation matrices and their eigenvalue spectra both in a single state (Non-Regime Switching) environment and in a two-state Regime Switching (RS CAPM) Model. In the latter case, there is a correlation matrix associated with the high volatility negative average returns regime and another corresponding to the low volatility regime. Eigenvectors corresponding to eigenvalues deviating from the Marchenko-Pasteur law (Appendix 1) are analysed as they are found to contain market information. Eigenvalues within the Marchenko-Pasteur bounds are taken to be the ‘noisy’ or pure random ones. We ‘deconstruct’ the correlation matrix into non-random and noisy components and filter the noisy ones by using the Plerou et al. (2002) method which replaces the noisy eigenvalues by zeroes. Once the filtered correlation matrix is obtained using RMT, this is used to compute the optimal portfolio weights for the RS

CAPM model and Non-RS model. Remarkably in the RS CAPM model case, under conditions of high volatility (Regime 2), the RMT filter was least needed. The results show improvement in the performance of optimal asset allocation while using RMT filtered correlations when compared with unfiltered correlations in all cases and the filtered RS CAPM strategy has the best performance of all on the basis of end of period cumulated wealth and Sharpe ratio.

The final chapter 8 gives an overall summary of the main findings and conclusions, recommendations, limitations of the investigation and the modelling strategy and, based on that, offers some grounds for further research. **Bibliography** lists the references and **then Appendix** is composed of appendices.

The results are widely encouraging in Chapters 4, 6 & 7 while Chapter 5 results are supportive for a limited time period. the application of RMT widely improves the performance of portfolio as discussed in Chapter 7. the portfolios performance improves a lot due to the Eigenvalue decomposition and applying filtering technique as proposed by Plerou et al (2002).

2 Chapter 2: Literature Review

2.1 Regime Switching Basics

Hamilton (1989) offers an approach on how to model regime shifts in his paper. He studied the behaviour of Gross Domestic Product (GDP) during recessions and booms. He noted that like many other economic variables there was a tendency for the series to behave quite differently during economic downturns. This is an established fact now that there are ups and downs in markets, for example;

- Economic business cycles have boom and bust periods.
- Exchange rates have periods of appreciation and depreciation vis-à-vis other currencies.
- Equity markets are characterized by bull and bear markets.
- Asset prices also have prolonged periods of upward movement followed by downward movements.

A large number of studies find that aggregate stock market returns are predictable. The strength of this predictability, however, has varied considerably over time. The predictable power of many instruments used in the literature to predict excess aggregate equity returns, like dividend yields, term spreads, and default spreads, declined or even disappeared over the 1990s as documented by Welch and Goyal (2008) and Ang and Baekert (2007), among others, and formally tested by Pesaran and Timmermann (2002). One response is that the strength of predictability‒or even the unconditional return distribution (Maheu and McCurdy, 2009) –changes over time and is subject to breaks and parameter instability (see, e.g., Schaller and van Norden, 1997; Paye and Timmermann, 2006; Rapach and Wohar, 2006; Johannes, Korteweg and Polson, 2011). This is the approach of Henkel, Martin and Nardari (2011) who capture the time-varying nature of predictability in a regime switching context. They find that predictability is very weak during business cycle expansions but is very strong during recessions. Another response to the lack of predictability is that predictability was never there, see e.g. Bossaerts and Hillion (1999) and Welch and Goyal (2008).

2.2 Regime Switching-Multiple States

Stylized facts of asset returns show that during bull markets and bear market returns, volatility and correlations behave differently. In the latter, returns are lower, volatility is higher, and asset correlations increases, a phenomenon known as asymmetric correlation. Ang and Bekaert (2002) concluded that there was significant evidence of asymmetric correlation, in their study of several international equities during the period 1975-2000. Correlation between these assets tended to be higher when there where market downturns. On average they were 20% higher than in the normal regime. They statistically rejected at 0.01% significance the equality of volatility across regimes.

The sudden change in the statistics of asset returns led to large losses in portfolios and a widespread failure of asset management during the Great Financial Crisis was signified in Khandani and Lo (2011) query as to “What Happened to Quants?”. The fact that sudden regime shifts will impact financial portfolios was well understood since Ang and Bekaert (2002, 2004).

Ang and Bekaert (2002, 2004) have been the pioneers of asset allocation research in RS framework for portfolio optimization. They classified equity markets into two regimes: a normal/ bull market (Regime 1) and a bear market (Regime 2). In Regime 1, the conditional mean is on average lower, volatility is higher and the assets have higher correlation coefficients compared to the latter. Hess (2006) reported volatility in turbulent times, can be 2.2 times higher than in calmer periods.

Recent studies confirm that the conditional moments of stock returns are business cycle related. This results in the distribution of stock market returns to be time-varying (Hess, 2006). Therefore the optimal portfolio in bear markets is substantially different from the one in bull markets. Ang and Bekaert (2004) noted that the presence of asymmetric correlation has raised doubts about the benefits of diversification especially in market downturns. For example De-Santis and Gerard (1997), stated that severe United States (US) market declines were contagious at the international level. They estimated the gains from international diversification were around 2.11% per year, for the US investor.

Hess (2006) simulated investment decisions based on forecasts of future regimes in a Markov RS model. Modeling regimes this way, was possible due to the persistent characteristic of the states. He concluded RS models were valuable timing signals for portfolio rebalancing. He reported investors should engage in an aggressive portfolio during turbulent periods, this is because in high volatility periods assets tend to co-move closely and therefore investors can suffer startling loses. In calmer times Hess (2006) advocates portfolio managers to hold a broader portfolios (diversify).

Similarly Galagedera and Shami (2004) modelled volatility of the market portfolio return generating process and the slope coefficient of the security return generating process as Markov regime switching processes of order one. A sample of daily returns of thirty securities in the Dow Jones index revealed strong regime-switching behaviour in three securities. In these three securities the low risk state appeared to be more persistent than the high-risk state. A sample of daily returns of the S&P500 index that they used as a proxy for the market portfolio revealed strong volatility switching behaviour with low-volatility regime being more persistent than the high-volatility regime. Modelling switching behaviour in the market volatility and the security beta therefore can provide useful information to the investor. Such information can be used in the construction of portfolios (Galagedera and Shami, 2004).

Campbell et al (2002) considers diversification more important during market downturns. Ang and Chen (2001), however, warned that the diversification may be overestimated in falling markets. Ang and Bekaert (2004) in their study on a sample of international equities, showed that in a tactical asset allocation program with monthly rebalancing of portfolio weights, RS models yielded greater returns on average than holding the market portfolio, as advocated in the Capital Asset Pricing Model (CAPM). Empirical results show RS models can explain a larger proportion of the variation in hedge fund and market returns as opposed to orthodox linear models (Ang and Bekaert, 2004). This explanatory power is linked to the tendency of markets to synchronize or become more dependent in periods of stress.

Often the capital asset pricing model forms the basis of the framework. This includes Harvey and Siddique (2000) where asset pricing models featuring skewness induce co-skewness into expected returns. Guidolin and Timmerman (2008) show that higher order co-moments that are time varying are important for pricing asset returns, while Potì and Wang (2010) show that co-skewness risk is a partial explanation for differences in returns on portfolios. Lambert and Hübner (2013) focus on the US market, and find that differences in co-skewness across regimes can explain the equity home bias, and that co-moment risk is significantly priced by the US market. Such adjustments can occur as risk averse agents alter their skewness and co-skewness preferences, as well as their portfolio allocation depending on the regime. Guidolin and Timmerman (2008) and Fry et al., (2010) show that as risk aversion increases, investors prefer positive skewness and positive co-skewness. This is consistent with changes in the joint distribution of asset returns such as through contagion and structural breaks[1].

RS strategies are superior in modelling time-variations in investment opportunities or the cyclicality displayed in the markets. Static mean variance-covariance analyses (Markowitz framework) to determine optimal portfolio weights, i.e. using the mean and variance of returns, has been severely criticized as it is not responsive or sensitive to sudden movements in the markets (Michaud, 1989). Nor does it take into consideration the time-varying, non-Gaussian and unstable nature of asset distributions' moments (Hamilton and Susmel, 1994). Ang and Bekaert (2004) conclude that RS strategies have the potential to outperform ordinary mean-variance analysis in a practical setting, because they are defensive in bear markets hedging against high correlations and low returns even in the presence of short selling.

Historically, Beta has been considered stable over time for any given asset, but recent evidence states that Beta varies over time see Huang (2000). He considers another stylized fact that Beta is non-stationary by considering two states i.e., a high-risk state and a low-risk state. Huang extends the model of Gibbons (1982) and analyses monthly returns (from April 1986 to December 1993) and concluded that the high-risk state data was inconsistent with

[1] Several papers suggest a role for investor behaviour in crisis periods such as herd behaviour, wake up calls, sudden stops, wealth effects, portfolio rebalancing, credit contractions, self-fulfilling expectations and information asymmetry. See the classic articles by Krugman (1998), Kaminsky and Schmukler (1999), Calvo and Mendoza (2000), Kyle and Xiong (2001), Loisel and Martin (2001) and Yuan (2005). These models are not mutually exclusive to those based on higher order co-moments, and all are consistent with the increasing risk aversion of investors.

CAPM, while the low-risk state was consistent with CAPM. Huang (2003) extended his previous study by considering effects caused by the price limit regulation[2]. He treated the unobservable states and the latent true returns as additional variables by using the simulation-based Bayesian approach, i.e., Gibbs sampler with data augmentation algorithm. He examined the regime-switching model by using daily data on returns of 10 randomly selected stocks and market index from the Taiwan Stock Exchange where price limits were in operation. Despite no formal tests for one-regime against two-regime models, empirical results suggested that the data-generating process can be well characterized by a two-state regime switching model. Furthermore, there was no obvious tendency for the stock to be inconsistent with the CAPM in either high-risk or low-risk regime. His findings also suggest that tendency of one state followed by the same state was very high (regime persistence).

Milidonis and Wang (2007) measured the time-series (stock return) distress costs associated with a selected sample of downgrades from the two types of rating companies. They focused on both the timeliness and accuracy of a selected sample of bond downgrades in order to (a) model changes in daily stock return regimes around the time of downgrades, (b) provide indications of distress costs emerging from regime switches and finally (c) propose a set of risk measures based on the CAPM and the parameters of the regime switching model to quantify these costs. Their results indicated that there was a Moody's downgrading on the day that the market exhibited the highest inclination to switch to the high volatility regime. This represents an early warning to investors who could potentially exploit the expected duration of the high volatility regime following the downgrade. Ang &Timmermann (2012)

[2] Huang (2003) studied RS under price limits which were imposed by Taiwan Stock Exchange in 1997 to allow inter-day price variation on any stock up to 2% on a given day.

analysis of an equilibrium asset pricing model show that regimes in consumption or dividend growth translate into regimes in asset returns.

Seidl's (2012) article discusses an adjusted regime switching model in the context of portfolio optimization and compares the attained portfolio weights and the performance to a classical mean-variance set-up as introduced by Markowitz. The model postulates different asset price dynamics under different regimes, and jumps between regimes are driven by a Markov process. The model is evaluated in an out-of-sample period with a moving window and a forecast of only one period. It is found that with the adjusted regime switching portfolio selection algorithm, the performance of the optimal portfolio is highly improved even where portfolio weights are constrained to realistic values. The model outperforms the classical Markowitz portfolio for both a risky and a risk averse investor. At the classical mean-variance optimization, it can be seen a very smooth run of the different asset weights (Seidl, 2012).

In another paper, Jiang, has shown that in the presence of regime shifts, the optimal dividend policy is given by a threshold strategy set at a level that is a function of the current regime. The policy that maximizes the expectation of the net present value of the paid dividends until the moment of default consists of paying out as dividends, the overflow of the cash reserves above a certain optimal threshold, where this threshold jumps up or down exactly when the regime shifts (Zhengjun Jiang, 2012).

Bae et al (2013) develop stochastic program to optimize portfolios under the regime switching framework and use scenario generation to mathematically formulate the optimization problem and identify regimes and apply this information to a portfolio optimization problem to overcome the limitations of the Markowitz model. Another paper uses a new approach that considers model uncertainty, regime uncertainty, and parameter uncertainty to predict excess stock returns (Xiaoneng Zhu, 2013).

Regime switching model is used by Fu (2014) in optimal asset allocation problem in order to maximize the expected benefits of the portfolio's wealth that contains an option, a stock and a risk-free bond. Using incomplete regime-switching market, he demonstrated both an approximate and an exact solution to the original portfolio optimization problem for power and logarithmic utility functions using functional operator. Fu et al. (2014) concluded that in multiple incomplete regime market, it remains optimal to invest wealth in the same way as in a single-regime market. He conclude that optimal allocation of the wealth in the stock constitute a balance between speculating for profits and hedging the risks involved in the option. As the investor becomes more risk averse, the optimal investment strategy changes gradually from a speculative strategy to a delta hedging strategy (Fu et al. 2014). Another study on regime switching done by Nalewaik (2015) in which he measures the chances or the probability that a random walk in inflation procession reappears. He used Markov switch models that measure the chances of inflation returning back to a high-variance and high-persistence regime, and also uses those models to create prediction intervals. Balcilar et al (2017) paper tested the association between United States crude oil and stock market price by using Markov-Switching vector error-correction model. Foerster (2016) research considered the determinacy and distributional consequences of regime switching in monetary policy. Although switching in the inflation target does not affect determinacy, switches in the inflation response can cause indeterminacy. One study introduced a Monte Carlo type inference in the framework of Markov Switching models to analyse financial time series, namely the Gibbs Sampling (Luca and Frigo 2016). Co-movements and correlations between stocks in terms of crisis periods can be studied to identify global crisis periods or classes of stocks that have the same behaviour (Salhi et al. 2016).

2.3 International Indices and Portfolios Management

International portfolio has been used as a mean of diversifying risk. Many investment companies, hedge funds and even banking sector have become involved with international portfolio mix in order to diversify risk and increase profits from this diversification.

As depicted by De Santis & Gerard (2006), financial systems satisfy financial needs of traders, hedgers, and are used to diversify and pool risk. As a result of financial integration, sharing and diversification of risk and potential for economic growth increases. De Santis and Gerard (2006) evaluated the effect of European Monetary Unit (EMU) on international equity and fixed asset portfolios. They found that financial integration has increased during this period and Home bias over this 4-year period increased for all countries except EMU. This new Borderless global portfolio means that investor has been implementing rational portfolio optimization principle which seems to be his major motive behind internationalization (De Santis & Gerard, 2006).

Grubel and Fadner (1971) noted the same fact that international diversification benefits come from the reduction in the variance of expected returns of these portfolios. This reduction in variance is a factor of exchange rate fluctuations and business cycle affecting investor specific economy (since markets were not integrated during that time period).

A similar study carried out by Lewis (2006) found that despite the fact that international integration has caused correlation to go high but variance of foreign portfolios to decline giving a rationale to invest in foreign markets. Also there was a time when foreign stocks listed in US market were considered as a vehicle for diversification. She found that foreign stocks listed in US have become more correlated over time with US market conditions meaning investors cannot use them as a source of diversification. The key finding from her study is: “diversification of risk is declining due to increased international integration whether the investor holds stocks inside (foreign stocks) or outside the US” (Lewis, 2006).

Similarly Flavin and Panopoulou (2006) stress the importance of co-movement as assets in different countries showing higher co-movement especially in troubled times may erode the benefits of international diversification. Flavin and Panopoulou (2008) used Regime Switching models to exploit the heteroskedasticity inherent in stock returns to identify whether or not increased co-movement occurs between each pair of markets[3] as we move from calm to turbulent periods of market conditions. Moreover, he found that US/Canada market is highly correlated but UK/Italy markets showed low co-movement thus this pair can be used to diversify risk considerably. If the co-movement of markets remain same in turbulent time as in calmer time then investors in such situation gets the maximum benefits (Flavin and Panopoulou, 2006, 2008).

Systematic risk is also known as "un-diversifiable risk" or "market risk." Interest rates, recession and wars all represent sources of systematic risk because they affect the entire market and cannot be avoided through diversification. Systematic risk can only be handled by hedging investment position but even a portfolio of well-diversified assets cannot escape all risk. As shown by Harvey and Bekaert (1995), higher systematic risks are associated

[3] Flavin and Panopoulou (2006) used European countries, USA, Canada etc. to form pairs and then checked their correlation (or as they call it ‘co-movement’)

with lower expected returns in many emerging markets. Ignoring the problems related to the estimation of systematic risks, Assoe (1998) showed that emerging markets go through two regimes whether the market returns are expressed in respective local currencies or in U.S. dollars. Switching between regimes seems to be associated with country-specific events such as monetary shocks and productivity switches that lead to fluctuating confidence in emerging stock markets (Assoe, 1998). In a way, Ang and Bekaert (1999) concluded that "the costs of ignoring regime switching are small for moderate levels of risk aversion;" whereas Das and Uppal (2004) state that "there are substantial differences in the portfolio weights across regimes." Das & Uppal (2004) noted that returns on international equities are characterised by jumps occurring at the same time across countries leading to return distributions that are fat tailed and negatively skewed. Using method of moments and mean variance by ignoring systematic risk and then accounting for systematic risk, they found that the cost of ignoring systematic risk is higher in developed countries than in emerging markets. Thus systematic risk reduces only slightly the gains from international diversification implied by standard mean variance portfolio models (Das & Uppal, 2004).

Fowdar (2008) used MSCI world index and MSCI G-7 index simultaneously vs. African portfolio of country indexes of South Africa, Mauritius, and Botswana etc. and found that as African countries adopt fair accounting practices, a good regulatory framework and sound corporate governance practices, investment in African stock markets will rise due to diversification benefits they offer to the investors.

Majority of research till now was based on the assumptions that all markets are perfectly integrated, individual markets are perfectly segmented or local markets are partially integrated with the degree of integration being constant. Bekaert and Harvey (1995) provide a framework which allows for time-varying conditional market integration. They measured the degree of integration between markets directly from the returns data and their econometric method allows for the degree of integration to change through time. However, they could not find overwhelming evidence pointing to increased integration over time between the sample countries (Bekaert and Harvey, 1995).

Correlation among the international equity returns is not stable in different periods. Longin and Longin and Solnik (1998) suggested that the international correlation of large stock returns, especially negative ones, differs from that of usual returns that is in periods of extreme negative returns that the benefits of international risk diversification are most desired (Longin and Solnik, 1998). Similarly in a study conducted by Ramchand and Susmel (1998) on U.S. and Japanese markets found that variance is time and state varying thus covariance structure between markets is also changing over time. For example, they found that during periods of high U.S. volatility, foreign markets become highly correlated with the U.S. market. This has considerable effect on the formulation of portfolio diversification strategies (Ramchand and Susmel, 1998). Similarly in another study, Longin and Solnik (2001) used extreme value theory to study the inter-dependence of international equity markets. They used 38 years of monthly data for the five largest stock markets to see the evidence of asymmetric correlation. They found that correlation increases in bear markets, but not in bull markets. However, Longin and Solnik (2001) derived a formal statistical method, based on extreme value theory, to test whether the correlation of large returns is higher than expected under the assumption of multivariate normality.

Kallberg, Liu & Pasquariello (2002) researched the relationship between the real estate market and the equity markets in eight developing Asian countries. They employed Granger causality analysis which noted that equity returns impact real estate returns and not vice versa. They also applied the statistical technique of Bai, Lumsdaine and Stock (1998) to analyse the nature of the regime shifts and found that there were structural breaks in both returns and volatility which caused the Regime Shifts in Asian Equity and Real Estate Markets. These regime shifts posed higher risk for real estate securities and increased systemic risk for the stock market (Kallberg, Liu & Pasquariello, 2002).

Now the question arises that regime shifts or preference for higher moments would be able to explain the home bias? "The answer seems to be that both play a role". Guidolin & Timmermann (2008) estimated that in the absence of regimes, a US investor with Mean-Variance Portfolio holds only 30% of the equity portfolio in domestic stocks which rises to 50% in Regime Switching. With the introduction of moment preferences, the allocation to US stocks rises to 70%, describing home biasness of US investor who besides being risk averse also prefers positively skewed (asymmetric) payoffs and dislikes fat tails (kurtosis) (Guidolin & Timmermann, 2008).

Woodward & Marisetty (2005) refined the two-regime dual-beta market model in order to address the transition between the regimes and found that most of the Australian non-linear securities analysed and the US composite airline industry portfolio exhibit smoother rather than abrupt transition between regimes. The amount of time spent in bull and bear markets is important to explain the risk/return trade-off relationship of risky assets (Woodward & Marisetty, 2005). Yin & Zhou (2004) used the Markowitz's Mean-Variance Portfolio Selection with Regime Switching for the discretization of the continuous-time problem, and were able to show that such portfolios are nearly efficient.

Korajczyk & Viallet (1988) compared domestic and international versions of several alternative asset pricing models. Their results indicate that there is some evidence against all of the models, and Multifactor models tend to outperform single-index CAPM-type models in both domestic and international forms. Controlling for regime shifts in the level of capital controls, international versions of the CAPM outperform domestic versions (Korajczyk & Viallet, 1988).

Perez-Quiros and Timmermann (2000), Gu (2005), and Guidolin and Timmermann (2008), among others, fit regime switching models to a small cross section of stock portfolios. On the one hand, these studies show that the magnitude of size and value premiums, among other things, varies across regimes in the same direction. On the other hand, the dynamics of certain stock portfolios react differently across regimes, such as small firms displaying the greatest differences in sensitivities to credit risk across recessions and expansions compared to large firms. Factor loadings of value and growth firms also differ significantly across regimes. Ang and Bekaert (2002) examine portfolio choice for a small number of countries. They exploit the ability of the regime switching model to capture higher correlations during market downturns and examine the question of whether such higher correlations during bear markets negate the benefits of international diversification. They find there are still large benefits of international diversification and the costs of ignoring the regimes is very large when a risk-free asset can be held. Tu (2010) finds that even after taking into account parameter uncertainty, the cost of ignoring the regimes is considerable. This is consistent with the finding in Pettenuzzo and Timmermann (2011) that uncertainty about future regimes can have a large effect on investors' optimal long-run asset allocation decisions which can even change from being upward sloping in the investment horizon in the absence of ‘breaks’ to being downward sloping once uncertainty associated with future regime changes is accounted for.

2.4 Regime Switching in Credit Risk Literature

In the recent banking literature, the relationship between credit risk and the business cycle has been analysed for both (macro) financial stability and (micro) risk management purposes. Indeed, the potential impact of economic developments on banks’ portfolios is relevant for both policy makers, interested in forecasting and preventing banks’ instability due to unfavourable economic conditions, and risk managers, who pay attention to the robustness of their capital allocation plans under different scenarios.

From a macro prudential point of view, many analysts have quantified the effects of macroeconomic conditions on asset quality (see Quagliariello, 2008). As an example, Pesola (2001) shows that shortfalls of GDP growth below forecast contributed to the banking crises in the Nordic countries, while Salas and Saurina (2002) document that macroeconomic shocks are quickly transmitted to Spanish banks’ portfolio riskiness. Similarly, using Italian data, Marcucci and Quagliariello (2008) find that bank borrower’ default rates increase in downturns. Meyer and Yeager (2001) and Gambera (2000) document that a small number of macroeconomic variables are good predictors for the share of non-performing loans in the US. Similarly, Hoggarth et al. (2005) provide evidence of a direct link between the state of the UK business cycle and banks’ write-offs. Analogous evidence is provided in cross-country comparisons by Bikker and Hu (2002), Laeven and Majoni (2003).

However, the vast majority of these studies generally neglect asymmetric effects, i.e., the possibility that the impact of macroeconomic conditions on banks’ portfolio riskiness is dissimilar in different phases of the business cycle. Regime switching models are commonly used for this kind of investigations. The impact of macroeconomic conditions appears therefore to be asymmetric and dependent on the starting creditworthiness of each borrower. In their analysis of the linkage between macroeconomic conditions and migration matrices Bangia et al. (2002) distinguish two states of the economy, expansion and recession, and condition the transition matrix to these states. Their findings suggest that downgrading probabilities, particularly in the extreme classes, increase significantly in recessions. Pederzoli and Torricelli (2005) adopt a similar framework in order to assess the impact of the business cycle on capital requirements under Basel.

Marcucci and Quagliariello (2008) also suggest an innovative four-regime approach with two different threshold variables which allowed them to provide a more comprehensive picture of the behaviour of default rates over changing economic and credit risk conditions. At the aggregate level, they find that banks' portfolio riskiness is mostly affected by the business cycle during downturns and also when portfolio quality is not good. Furthermore, from their results, the impact of the business cycle on credit risk is stronger, the lower the banks' asset quality for models with two or more regimes with one threshold variable (Marcucci and Quagliariello, 2008).

2.5 Regimes in credit spread

Time series of credit spreads undergo successive falling and rising episodes over time. These episodes can be observed in changes in the level and/or the volatility of credit spreads, especially around an economic recession. Across ratings and maturities, the credit spread movements exhibit at least two different regimes in terms of sudden changes in their level and/or volatility. Dionne et al. (2008) use the sequential statistical t-test to test for breakpoints in the level of credit spreads. They detect positive shifts a few months before the beginning of the 2001 recession (March 2001). They also detect other positive shifts after the end of the economic recession (November 2001).

This looks plausible since Dufresne et al. (2001) shows that yields on corporate bonds exhibit persistence and take about a year to adjust to innovations in the bond market. Since low grade bonds are closely related to market factors (Dufresne et al., 2001), they take less time to adjust to new market conditions at the beginning and the end of the cycle. Inspection of the credit spread behaviour at the beginning and the end of the economic cycle reveals that credit spreads have their own cycle. Even though the recession lasts for few months, credit spreads are likely to remain in a period of contraction until the announcement of the recession end (Dufresne et al., 2001).

Davies (2004 and 2008) analyses credit spread determinants using a Markov switching estimation technique assuming two volatility regimes. Alexander and Kaeck (2008) also use two-state Markov chains to analyse credit default swap determinants within distinct volatility regimes. Dionne et al. (2008) use the same period and support the existence of two regimes. Therefore, this thesis also assumes that two state dependent regimes are adequate to capture most of the variation in any series.

Analysis has shown that the optimal number of bond units sold in each regime decreases with the riskiness of the bond perceived by the market, and that the number of bond units sold is smaller for larger investment horizons (Capponi & Jose, 2014).

2.6 Regime Switching and Random Matrix Theory

Financial markets have been known to represent complex adaptive systems, which self-organize into various unexpected dynamical structures according to non-trivial interactions among heterogeneous agents (Lux & Marchesi, 1999, Lux, 1998 & Markose et al. 2011). The study of complex economic systems is not easy because we do not know the control parameters that govern economic systems as these systems typically self-organize. The study of financial markets for their complex dynamics has become prominent with both economists and econo-physicists. Research into financial time series has been given great prominence both for portfolio and risk management. Numerous studies have been devoted to understand the statistical properties of financial time series such as volatility (Engle et al., 1993, 1994), long memory (Engle et al., 1993, Geweke & Porter, 1983.) and asymmetric correlation (Mantegna et al., 1995 & 1996, Plerou et al., 2003, Liu et al., 1999, Cizeau et al., 1997, & Jun et al., 2006).

Empirical correlation matrices are of great importance for risk management and asset allocation. The probability of large losses for a certain portfolio or option book is dominated by correlated moves of its different constituents. The study of correlation (or covariance) matrices has a long history in finance (Gabaix et al., 2003, & Yamasaki et al., 2005) and is one of the cornerstones of Markowitz's theory of optimal portfolios. Given a set of financial assets characterized by their average return and risk, the optimal weight of each asset in the portfolio, such that the overall portfolio provides the best return for a fixed level of risk, or conversely, the smallest risk for a given overall return, is a function of the correlation matrix.

In particular, the analysis of financial data by various methods developed in statistical physics has become a very interesting research area for physicists and economists (Mantegna & Stanley, 1999, Bouchaud & Potters, 2004). There is practical (Elton & Gruber, 1981, Okhrin & Schmidt, 2006, Andersen et al., 2002) as well as scientifically important value in analysing the correlation coefficient between stock return time series because this contains a significant amount of information on the nonlinear interactions in the financial market. The correlation matrix between stock returns, which has unexpected properties due to complex behaviours, such as temporal non-equilibrium, mispricing, bubbles, market crashes and so on, is an important parameter to understand the interactions in the financial market (Noh, 2000).

Markowitz portfolio theory, an intrinsic part of modern financial analysis, relies on the covariance matrix of returns and this can be difficult to estimate. For example, for a time series of length T, a portfolio of N assets requires $(N^2 + N)/2$ covariances to be estimated from NT returns. This results in estimation noise, since the availability of historical information is limited. Moreover, it is commonly accepted that financial covariances are not fixed over time and thus older historical data, even if available, can lead to cumulative noise effects. Thus, it is well understood that in Markowitz portfolio model, realized portfolio returns are far removed from the expected portfolio returns that are maximized given the sample estimates for the variance-covariance matrix. Many methods have been used to improve portfolio performance in terms of realized returns. This chapter is concerned about using Random Matrix Theory (RMT) based filtering of the stock returns correlation matrix to improve the realized returns of the portfolio.

To analyse the correlation matrix, previous studies presented various statistical methods, such as principal component analysis (PCA) (Jackson, 2003), singular value decomposition (SVD) (Gentle, 1998) and factor analysis (FA) (Morrison, 1990). Here, to analyse the actual cross-correlation matrix, random matrix theory (RMT) is employed, which was introduced by Wigner, Dyson and Mehta (Mehta, 1991, Wigner, 1951, Dyson, 1962, Dyson & Mehta, 1963, 1960 & 1971) and Guhr et al. (1998). The RMT can be used for eliminating the deviations from Gaussian noise in the actual correlation matrix (Sengupta & Mitra, 1999, Utsugi et al., 2004, Guhr & Kalberzk, 2003, Ruskin et al., 2004).

RMT, first developed by authors such as Dyson and Mehta to explain the energy levels of complex nuclei has recently been applied by several authors including Plerou et al.(1999) and Laloux et al. (1999) for noise filtering in financial time series, particularly in large dimensional systems such as stock market data. Both groups have analyzed US stock markets and have found that the eigenvalues, of the correlation matrix of returns, were consistent with those calculated using random returns, with the exception of a few large eigenvalues.

Ruskin et al. (2004) studied the dynamics of the correlation matrix of multivariate financial time series by examining the eigenvalue spectrum over sliding time windows. Empirical results for the constituent stock returns of the S&P 500 and the Dow Jones Euro Stoxx 50 indices reveal that the dynamics of the smallest eigenvalues of the correlation matrix, over these time windows, are different from those of the largest eigenvalues. This behavior is shown to be independent of the size of the time window and the number of stocks examined. By partitioning the eigenvalue time series, they then show that negative index returns, (which they call *drawdowns*), are associated with periods where the largest eigenvalue is greatest, while positive index returns, (i.e., *drawups*), are associated with periods where the largest eigenvalue is smallest (Ruskin et al., 2004).

Laloux et al. (1999) and Plerou et al. (1999) analysed the cross-correlation matrix of financial time series using the RMT method. Plerou et al. (1999) found that 94% of the eigenvalues of cross-correlation matrix can be predicted by the RMT, while the other 6% of the eigenvalues deviated from the RMT. In addition, Plerou et al. (2002) applied the RMT method to the S&P 500 stock market and observed that the cross-correlation matrix of stock returns consists of random and non-random parts. They deconstructed the correlation matrix into what is explained by RMT and the residual. This decomposition carries useful information about the financial market. The pattern of eigenvalue deviations from the RMT were in a remarkably constant state over the entire period of 35 years starting from 1962–1996 (Plerou et al., 2002).

In this context one analyses eigenvalue spectra of corresponding covariance matrices. Under the assumption of uncorrelated financial players, it is possible to identify outliers by use of the Marchenko-Pasteur spectrum, a method which has been applied to financial markets in a portfolio optimization framework before (Liu et al., 1999, Cizeau et al., 1997, Yamasaki et al., 2005, Jun et al., 2006, Mantegna & Stanley, 1999, Bouchaud & Potters, 2004, Elton & Gruber, 1981)

Here, we identify outliers of eigenvalues of covariance matrices, obtained from the returns data. The obtained empirical eigenvalue spectrum is compared to the Marchenko-Pasteur spectrum, which allows the identification of clusters of firms which show non-random structure. These clusters can then be examined in more detail and firms which feature irregular behaviour – in comparison to the average behaviour within a cluster – can be identified.

There is practical as well as scientifically important value in analysing the correlation coefficient between stock return time series because this contains a significant amount of information on the nonlinear interactions in the financial market and is a parameter in terms of the Markowitz portfolio theory. The cross-correlation matrix between stocks, which has unexpected properties due to complex behaviours, such as temporal non-equilibrium, mispricing, bubbles, market crashes and so on, is an important parameter to understand the interactions in the financial market (Noh, 2000). Here, to analyse the actual cross-correlation matrix, we employ the random matrix theory (RMT), which was introduced by Wigner, Dyson and Mehta (Wigner (1951), Mehta (1991), Dyson (1962), Dyson & Mehta (1963, 1960 & 1971)) and Guhr et al (1998). The RMT method is a useful method for eliminating the randomness in the actual cross-correlation matrix (Sengupta, & Mitra, 1999, Utsugi et al, 2004, and Sharifi et al., 2004).

The RMT method for filtering correlation matrices of asset returns in portfolio management is used by Plerou et al (1999, 2002), Laloux et al. (1999), Sharifi et al. (2004), & Daly et al, (2010) etc. They found that 94% of the eigenvalues of cross-correlation matrix can be predicted by the RMT, while the other 6% of the eigenvalues deviated from the RMT. In addition, Plerou et al. (2002) applied the RMT method to a United States stock market and observed that the cross-correlation matrix of stock markets consists of random and non-random parts, which carry useful information in the financial market. It is possible to identify outliers by use of the Marchenko-Pasteur spectrum, a method which has been applied to financial markets in a portfolio optimization framework (Liu et al., 1999, Cizeau et al., 1997, Yamasaki et al., 2005, Jun et al., 2006, Mantegna, & Stanley, 1999, Bouchaud, & Potters, 2004, Elton, & Gruber, 1981) and gives some solid results. Further details on the methodology of the application of RMT in portfolio management will be given in the Chapter 7.

2.7 Spectral Analysis on Optimisation

The stock market is an institution of considerable interest to the public at large and of real importance to students of a nation's economy. The variables which make up a stock market may not directly affect the mechanism of the economy but they certainly influence the psychological climate within which the economy works. To the extent to which the movements of the economy directly affect the stock market, a feedback situation occurs, although there are reasons to suspect that the strength of the feedback is not strong. The stock market produces large amounts of high quality data derived from well-understood variables. Despite these facts, the stock market has attracted surprisingly little study by professional economists or statisticians. Granger and Morgenstern (2001) also promote the idea that stock market data (and particularly stock exchange "folk-lore") should be investigated by rigorous methods and that the most appropriate statistical techniques to be used in such an investigation are the recently developed spectral methods. These have already been used with considerable success in other fields of research and, although they have required considerable adaptation and improvement before being entirely applicable to economic series, they contend that spectral analysis has now reached a stage of development where it can be used with some confidence on economic series.

Granger and Morgenstern (2001) analyse New York stock price series using a new statistical technique. It was found that short-run movements of the series obey the simple random walk hypothesis proposed by earlier writers, but that the long-run components are of greater importance than suggested by this hypothesis. The seasonal variation and the “business-cycle” components are shown to be of little or no importance and a surprisingly small connection was found between the amount of stocks sold and the stock price series.

Time-series analysis involves the analysis of data so that their characteristics (level of stationarity, length of seasonality, frequency, amplitude, phase,...) can be discovered. The analysis can be done in the time domain through the utilization of the autocorrelation function or in the frequency domain through the use of spectral analysis (Makridakis,1976). Essentially, spectral analysis attempts to decompose a time series into basic components that can be represented as sine and cosine functions. It involves the transformation of a time series into the frequency domain via application of a Fourier transform on the original series. Spectral analysis and its close variants have found little use in social sciences chiefly because they are quite hard to interpret and analyze. In the area of stock market price analysis, there have been few applications since the pioneering work of Granger and Morgenstern (1963), who wanted to promote the idea 'that stock market data should be investigated by rigorous methods and that the most appropriate statistical techniques to be used in such an investigation are the recently developed spectral methods'.

Bertoneche (1979) concludes in his study on Spectral analysis that neither series (used in his study) nor do the estimates suggest deviations from randomness. This means that the various markets are efficient, yielding white-noise at the 95% level of confidence. However, a simple filter rule shows that substantial profits could have been made by a trader in the six

European markets even after accounting for transaction costs, which implies that over the period studied (1969-1976), these markets were quite inefficient.

In an unpublished paper, Stankard (1976) purposely introduced cyclical behaviour into dummy data and attempted to detect it using spectral analysis. Only the very obvious cycles were detected using the technique. Logue and Sweeney (1977) in their study of the foreign exchange market for the French franc and US dollar used spectral analysis and show that this market is quite efficient, yielding white-noise at the 95 % level of confidence. It is only a test of randomness against the alternative hypothesis that non-randomness is of a time dependent source. This conclusion applies not only to spectral analysis but also to its close variants such as serial correlation tests which, aside, from the transformation of data, are analogous to spectral analysis.

Madan, Pistorius & Stadje (2017) discuss financial analysis and decision making relying on quantification and modelling of future risk exposures and consider a new class of such continuous-time dynamic coherent risk measures, called dynamic spectral risk measures (DSRs). Quartile-based coherent risk measures, such as expected shortfall, belong to the most widely used risk measures in risk analysis, and are also known as spectral risk measures. In order to carry out for instance an analysis of portfolios involving dynamic rebalancing, one is led to consider the (strongly) time-consistent extension of such coherent risk measures to given time-grids, which are defined by iterative application of the spectral risk measure along these particular grids. Due to their recursive structure, financial optimisation problems, such as utility optimisation under the entropic risk measure and related robust portfolio optimisation problems, satisfy the dynamic programming principle and admit time-consistent dynamically optimal strategies (see for instance Becherer (2006) and Laeven& Stadje (2014)).

Chaudhuri and Lo (2016) state that economic shocks can have diverse effects on financial market dynamics at different time horizons, yet traditional portfolio management tools do not distinguish between short and long-term components in alpha, beta, and covariance estimators. They apply spectral analysis techniques to quantify stock-return dynamics across multiple time horizons. Using the Fourier transform, they decompose asset-return variances, correlations, alphas, and betas into distinct frequency components. These decompositions allow to identify the relative importance of specific time horizons in determining each of these quantities, as well as to construct mean-variance-frequency optimal portfolios. They contend that their approach can be applied to any portfolio, and is particularly useful for comparing the forecast power of multiple investment strategies.

The frequency domain has long been part of economics (Granger and Hatanaka, 1964; Engle, 1974; Granger and Engle, 1983; Hasbrouck and Sofianos, 1993), and spectral theory has also been used in finance to derive theoretical pricing models for derivative securities (Linetsky, 2002; Linetsky, 2004a; Linetsky, 2004b; Linetsky, 2008). However, econometric and empirical applications of spectral analysis have been less popular in economics and finance, in part because economic time series are rarely considered stationary. However, there has been a recent rebirth of interest in economic applications in response to modern advances in non stationary signal analysis (Baxter and King, 1999; Carr and Madan, 1999; Croux, Forni, and Reichlin, 2001; Ramsey, 2002; Crowley, 2007; Huang, Wu, Qu, Long, Shen, and Zhang, 2003; Breitung and Candelon, 2006; Rua, 2010; Rua, 2012). This rebirth motivates our interest in the spectral properties of financial asset returns.

Financial portfolio optimization is a widely studied problem in mathematics, statistics, financial and computational literature. It adheres to determining an optimal combination of weights that are associated with financial assets held in a portfolio. In practice, portfolio optimization faces challenges by virtue of varying mathematical formulations, parameters, business constraints and complex financial instruments. Empirical nature of data is no longer one-sided; thereby reflecting upside and downside trends with repeated yet unidentifiable cyclic behaviours potentially caused due to high frequency volatile movements in asset trades. Portfolio optimization under such circumstances is theoretically and computationally challenging.

Adam, Houkari and Laurent (2007) deals with risk measurement and portfolio optimization under risk constraints. Rubio, Mestre, and Palomar (2011) study the consistency of sample mean-variance portfolios of arbitrarily high dimension that are based on Bayesian or shrinkage estimation of the input parameters as well as weighted sampling. In an asymptotic setting where the number of assets remains comparable in magnitude to the sample size, they provide a characterization of the estimation risk by providing deterministic equivalents of the portfolio' out-of-sample performance in terms of the underlying investment scenario. The previous estimates represent a means of quantifying the amount of risk underestimation and return overestimation of improved portfolio constructions beyond standard ones. Well-known for the latter, if not corrected, these deviations lead to inaccurate and overly optimistic Sharpe-based investment decisions. Our results are based on recent contributions in the field of random matrix theory. Along with the asymptotic analysis, the analytical framework allows us to find bias corrections improving on the achieved out-of-sample performance of typical portfolio constructions.

The foundations of modern portfolio theory were laid by Markowitz's ground-breaking article **(**Markowitz**,** 1952**)**, where the idea of diversifying a portfolio by spreading bets across a universe of risky financial assets was refined and generalized by the more sophisticated one of combining the assets so as to optimize the risk-return trade off. In practice, Markowitz's mean-variance optimization framework for solving the canonical wealth allocation problem relies on the statistical estimation of the unknown expected values and covariance matrix of the asset returns from sample market observations. In general, the uncertainty inherently associated with imperfect moments estimates represents a major drawback in the application of the classical Markowitz framework. Indeed, the optimal mean-variance solution has been empirically observed to be significantly sensitive to deviations from the true input parameters. In addition, and aside from computational complexity issues, the estimation of the parameters is involved, mainly due to the instability of the parameter estimates through time. Generally, estimates of the covariance matrix are more stable than those of the mean returns, and so many studies disregard the estimation of the latter and concentrate on improving the sample performance of the so-called global minimum variance portfolio (GMVP); (Jagannathan and Ma, 2003).

In the financial literature, the previous source of portfolio performance degradation is referred to as estimation risk. Especially when the number of securities is comparable to the number of observations, estimation errors may in fact prevent the mean-variance optimization framework from being of any practical use. In fact, for severe levels of estimation risk, the naive portfolio allocation rule namely obtained by equally weighting the assets without incorporating any knowledge about their mean and covariance turns out to represent a firm candidate choice (DeMiguel, Garlappi and Uppal, 2009). The consistency and distributional properties of sample optimal mean-variance portfolios and their Sharpe ratio performance has been analyzed and characterized for finite samples and asymptotically (see, (Okhrin, and Schmid, 2006, Kan and Smith, 2008, Schmidt and Schmidt, 2010)).

Plerou et al, (2003) and Laloux et al, (1999) have been reporting on a methodology based on random matrix theory that consists of preserving the stability over time of the covariance matrix estimator by filtering noisy Eigenvalues conveying no valuable information. The cleaning mechanism relies on the empirical fact that relevant information is structurally captured by some few eigenvalues, while the rest can be ascribed to noise and measurement errors and resemble the spectrum of a white covariance matrix (see also Bouchaud and Potters, 2011). By resorting to some recent results from the theory of the spectral analysis of large random matrices, which as in (***Bai, Liu, and Wong, 2009***) and contrary to the random matrix theoretical contributions from statistical physics cited above, are based on Stieltjes transform methods and stochastic convergence theory.

Spectral and co-spectral power, often calculated using either the Fourier or wavelet transform, provide a natural way to study the cyclical components of variance and covariance, two important measures of risk in the financial domain. Specifically, spectral power decomposes the variability of a time series resulting from fluctuations at a specific frequency, while co-spectral power decomposes the covariance between two real-valued time series, and measures the tendency for them to move together over specific time horizons. When the signals are in phase at a given frequency (i.e., their peaks and valleys coincide), the co-spectral power is positive at that frequency, and when they are out of phase, it is negative.

In a recent empirical study, Chaudhuri and Lo (2015) perform a spectral decomposition of the U.S. stock market and individual common stock returns over time. They noticed that measures related to risk and co-movement varied not only across time, but also across frequencies over time. Such changes were especially apparent throughout the 1990s during the advent and proliferation of electronic trading. Studying this connection between technology and market dynamics has become especially important as recent events, including the Flash Crash of 2010, have led many to question the negative impact electronic trading could have on markets. Only by understanding the sources of feedback among these automated trading programs will we be able to construct robust portfolios and implement well-designed policies and algorithms to manage risk. Moreover, identifying asset-return harmonics may have important implications for measuring and managing systematic risk.

In addition to improving passive investment strategies, spectral analysis can also be used to characterize and refine active strategies. The standard tools used for performance attribution originate from the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965). The difference between an investment's expected return and the risk-adjusted value predicted by the CAPM is referred to as "alpha", and Treynor (1965), Sharpe (1966), and Jensen (1968, 1969) applied this measure to quantify the value-added of mutual-fund managers. Since then a number of related measures have been developed including the Sharpe, Treynor, and information ratios. However, none of these measures explicitly depend on the relative timing of portfolio weights and returns in gauging investment skill. In contrast, Lo (2008) proposed a novel measure of active management—the active/passive (AP) decomposition—that quantified the predictive power of an investment process by decomposing the expected portfolio return into the covariance between the underlying security weights and returns (the active component) and the product of the average weights and average returns (the passive component). In this context a successful portfolio manager is one whose decisions induce a positive correlation between portfolio weights and returns. Since portfolio weights are a function of a manager's decision process and proprietary information, positive correlation is a direct indication of forecast power and, consequently, investment skill.

Several authors referred to the scaling as a problem when using SVD, but with few exceptions, most pass over the issue in a few sentences and give no practical solutions of how to handle it (Liu et al., 1999, Cizeau et al., 1997, Yamasaki et al., 2005, Jun et al., 2006, Mantegna, & Stanley, 1999, Bouchaud, & Potters, 2004, Elton, & Gruber, 1981). Some scaled the state and output variables by dividing their values by their associated steady state values. Some made an alternative suggestion that the variables all be scaled so as to exhibit a unity steady state gain. Grosdidier et al. (1985) obtained an upper bound on the minimised condition number by scaling the process transfer function matrix G until the minimised condition number is obtained.

The following Chapter will discuss methodology in detail for the problem at hand keeping in view the research objectives highlighted earlier.

3 Chapter 3: Methodology

3.1 Regime Switching Methodology:

Different econometric methods can be used to estimate regime switching models. Maximum likelihood and Expectation–Maximization (EM) algorithms are outlined by Hamilton (1988, 1989) and Gray (1996). The maximum likelihood algorithm involves a Bayesian updating procedure which infers the probability of being in a regime given all available information up until that time t. An alternative to maximum likelihood estimation is Gibbs sampling, which was developed for regime switching models by Albert and Chib (1993) and Kim and Nelson (1999, Ch. 9).

An important issue in estimating regime switching models is specifying the number of regimes. This is often difficult to determine from data and as far as possible the choice should be based on economic arguments. Such decisions can be difficult since the regimes themselves are often thought of as approximations to underlying states that are unobserved. It is not uncommon to simply fix the number of regimes at some value, typically two, rather than basing the decision on econometric tests. The reason is that tests for the number of regimes are typically difficult to implement because they do not follow standard distributions. To see this, consider the simple two-regime model. Under the null of a single regime, the parameters of the other regime are not identified and so there are unidentified nuisance parameters. An alternative is to use residual tests such as in (Hamilton, 1996). Regime switching models have also been extensively applied to time-varying second moments. In fact, regime switching models themselves generate heteroskedasticity. Under the traditional ARCH and GARCH models of Engle (1982) and Bollerslev (1986), changes in volatility were too gradual and did not capture, despite the additions of asymmetries and other tweaks to the original GARCH formulations, sudden changes in volatilities. Hamilton and Susmel (1994) and Hamilton and Lin (1996) developed regime-switching versions of ARCH dynamics applied to equity returns that allowed volatilities to rapidly change to new regimes. A version of regime switching GARCH was proposed by Gray (1996).

Hamilton (1989) offers an approach on how to model regime changes when the shifts are not directly observable but statistically inferred through observing the behaviour of the series. Parameters of auto regression are viewed as the outcome of a discrete-state Markov process. He uses a first order Markov process, assuming there are two states (denoted by S_t in Equation 1). State 1: high return and low volatility and State 2: low return and high volatility. The conditional transition probabilities are:

Prob $(S_{t=1}|S_{t-1=1}) = P_t$

Prob $(S_{t=2}|S_{t-1=1}) = (1-P_t)$

Prob $(S_{t=2}|S_{t-1=2}) = Q_t$

Prob $(S_{t=1}|S_{t-1=2}) = (1-Q_t)$ (1)

Equation set (1) above, state that if the market is in Regime 1 at time t, then, P_t indicates the probability to remain in Regime 1 in time t+1. $(1-P_t)$ is the complement of this probability when the state changes to Regime 2. Otherwise, if the market is in Regime 2 at time t, then, Q_t indicates the probability it will remain in Regime 2 in time t+1 and $(1-Q_t)$ is to change to Regime 1 as also depicted in Figure 1. State 1 can be defined as the bull market where there are high returns and low volatility and State 2 can be termed bear market that has low returns and high volatility.

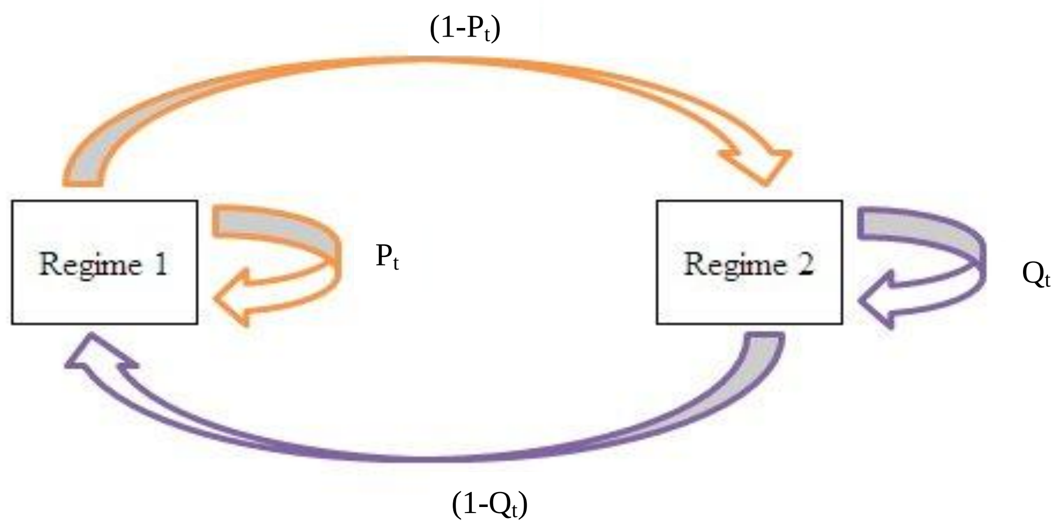


Figure 3.1-1: Transition Probabilities indicating two regimes

Once the regimes have been shown and are related with regime probabilities, the conditional expected returns and variance/covariance need to be calculated.

3.1.1 Maximum Likelihood Estimation:

In the two state RS model, we assume the model is drawn from normal distributions and the mean, variance, correlations are state dependent. The conditional density function for excess returns on market index at time t (r_{mt}) dependent on States {(S_t), S_t =1, 2} and past excess returns (r_{mt-1}) is given in Equation (2). Other variables used in the Equation (2) are; State dependent mean returns (μ_{S_t}) and state dependent variance (σ_{S_t}):

$$f(r_{mt}|s_t, r_{mt-1}) = \frac{1}{\sigma_{S_t}\sqrt{2\pi}} exp\left[-\frac{r_{mt}-\mu_{S_t}}{2\sigma_{S_t}}\right] \qquad (2)$$

The above Equation (2) states that the distributions of the markets' excess returns have variance and mean that are state dependent.

The parameters to be estimated initially for model specification/regime indicator are $\theta = \{\mu_1, \mu_2, \sigma_1, \sigma_2, P_t, Q_t\}$; the transition probabilities P_t and Q_t are estimated using maximum likelihood estimation (MLE). In MLE, θ is considered for all possible values and selected the ones that give the observations the greatest joint probability density (Dougherty, 2002).

The likelihood function is:

$$L\{\mu_{m1}, \mu_{m2}, \sigma_{m1}, \sigma_{m2}, P_t, Q_t | S_t\} = \prod_{t=1}^{T} \sum_{S=1}^{2} \frac{1}{\sqrt{2\pi\sigma_{S_t}^2}} . exp\left[-\frac{1}{2} \frac{(r_{mt} - \mu_{S_t})^2}{\sigma_{S_t}} \right] \quad (3)$$

Where; μ_m is regime-dependent mean of market index, σ_m is regime-dependent volatility of market index as measured by standard deviation, s_t is regime variable of market index and assumes only two values, i.e. $s_t \in [1, 2]$ and t is Time, i.e. $t \in [1, T]$

In this regime-switching model, the portfolio is rebalanced at the end of each month. The switches between regimes depend entirely on the fluctuation of the market index (FTSE-100/MSCI). In addition, the model considers transition probabilities that indicate the probability to change to other regime. The switches between regimes follow the Markov chain[4]. The estimation of a Markov chain process is achieved by considering the joint conditional probability of each of the future states, as a function of the joint conditional probabilities of the current states and the transition probabilities. This is called as filtering by Wang (2003).

As we want to estimate θ, but P_t and Q_t are random variables. To know how the probability of r_{mt} and S_t transit over time, it is modelled through steps used by Wang (2003) and Hamilton (1989):

[4] Given the present state, future states are independent of the past states; therefore the present state fully captures all the information that could influence the future evolution of the process.

The basic filter input is: $P(S_{t-1} = s_{t-1}|r_{mt-1})$ and the output will be: $P(S_t = s_t|r_{mt})$ and $\prod_{t=1}^{T} f(r_{mt}|r_{mt-1})$. The following steps elaborate more on formation of conditional density function;

1. Estimate the conditional probability by

$$P(S_t = s_t|r_{mt-1}) = P(S_t = s_t|S_{t-1} = s_{t-1}) * P(S_{t-1} = s_{t-1}|r_{mt-1}) \quad (4)$$

This is the probability of being in state S_t conditional on the information at time t-1

2. We then calculate the joint conditional density function of r_{mt} and S_t

$$P(S_t = s_t|r_{mt}) = \frac{f(r_{mt}|s_t,\ldots,s_{t-m},R_{mt-1})}{f(r_{mt}|R_{mt-1})} \quad (5)$$

The likelihood function explicitly becomes:

$$L\{\theta\} = \prod_{t=1}^{T} \sum_{S=1}^{2} \frac{\pi_{S_t}^{*}}{\sqrt{2\pi\sigma_{S_t}^{2}}} . exp\left[-\frac{1}{2}\frac{(r_{mt}-\mu_{S_t})^2}{\sigma_{S_t}}\right] \quad (6)$$

The Maximum Likelihood Estimator of $\pi_{S_t}^{*}$ in Equation (6) is the average of the filter probabilities of the state at time t obtained after observing r_{mt}. Equation (6) is also equivalent to:

$$L\{\mu_{m1},\mu_{m2},\sigma_{m1},\sigma_{m2},P_t,Q_t\} = \prod_{t=1}^{T}[\underbrace{\pi_{t1}f(r_{m1})}_{Regime1} + \underbrace{\pi_{t2}f(r_{m2})}_{Regime2}] \quad (7)$$

$$\pi_{t1} = \frac{\pi_1 f\left(r_{mt}|s_t = 1, r_{t-1}\right)}{\pi_1 f\left(r_{mt}|s_t = 1, r_{t-1}\right) + \pi_2 f\left(r_{mt}|s_t = 2, r_{t-1}\right)}$$

and $$\pi_{t2} = \frac{\pi_2 f\left(r_{mt} \middle| s_t = 2, r_{t-1}\right)}{\pi_1 f\left(r_{mt} \middle| s_t = 1, r_{t-1}\right) + \pi_2 f\left(r_{mt} \middle| s_t = 2, r_{t-1}\right)} \quad (8)$$

Equation (8) is the filter probability function. It relates the number of times the series was in regime 1, to the number of times switches occurred. The filter probability for each regime at time t is defined by using the initial values for $(\mu_{m1}, \mu_{m2}, \sigma_{m1}, \sigma_{m2}, P_t, Q_t)$.

Equation (9) below gives the average of the filter probability for all t, the MLE estimator for the transition probabilities for each regime and is defined as:

$$P_t = \pi_1^* = \frac{1}{T}\sum_{t=1}^{T} \pi_{t1} \quad \text{and} \quad Q_t = \pi_2^* = \frac{1}{T}\sum_{t=1}^{T} \pi_{t2} \quad (9)$$

In the same way, the maximum likelihood estimator for the mean and variance in each regime respectively is:

$$\mu_{m1}^* = \frac{1}{T}\sum_{t=1}^{T} \frac{\pi_{t1}}{\pi_1^*} r_{mt} \quad \text{and} \quad \mu_{m2}^* = \frac{1}{T}\sum_{t=1}^{T} \frac{\pi_{t2}}{\pi_2^*} r_{mt} \quad (10)$$

$$\sigma_{m1}^{2*} = \frac{1}{T}\sum_{t=1}^{T} \frac{\pi_{t1}}{\pi_1^*}\left(r_{mt} - \mu_1^*\right)^2 \quad \text{and} \quad \sigma_{m2}^{2*} = \frac{1}{T}\sum_{t=1}^{T} \frac{\pi_{t2}}{\pi_2^*}\left(r_{mt} - \mu_2^*\right)^2 \quad (11)$$

In these formulas, the mean and variance are adjusted by the ratio of the filter probability and the maximum likelihood estimator for each regime.

The process in (8), (9), (10) and (11) is iterated using MATLAB till the values for $\pi_{S_t}^*$, μ_m^* and σ_m^{2*}, for state 1 and 2 stabilize.

3.1.2 CAPM transformed into Regime based CAPM:

In this section, the technical steps needed to transform the classical CAPM model to yield a regime sensitive model for portfolio management is given. Following the CAPM assumption that the stock market index is the only factor that determines the returns on a single asset, the regime switches are modelled to occur only vis-à-vis the stock market index. The development of the corresponding regime sensitive portfolio statistics, especially those for the RS CAPM and variance-covariance matrix are original to this thesis.

The Capital Asset Pricing Model (CAPM) is used to determine a theoretically appropriate required rate of return of an asset, if that asset is to be added to an already well-diversified portfolio, given the asset’s non-diversifiable risk. The model takes into account the asset's sensitivity to non-diversifiable risk (also known as systemic risk), often represented by Beta, as well as the expected return of the market and the expected return of a theoretical risk-free asset (Black, Jensen, and Scholes, 1972).

The CAPM equation is:

$$r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it} \tag{12}$$

Where,

r_{it} = excess return[5] on security i for a given period, at time t

r_{mt} = excess return on market index for a given period, at time t

α_i = intercept term

5 Here ‘excess return’ indicates the return on equity (index) in excess of risk-free rate.

β_i = slope term[6]

ε_i = random error term

Total risk of security i, measured by its variance σ_i^2 equals the following:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \bar{\sigma}_i^2 \quad (13)$$

Where:

σ_m^2 = variance of returns on the market index,

$\bar{\sigma}_i^2$ = specific/ unsystematic risk, an efficient portfolio is one where all the diversifiable risk has been eliminated,

$\beta_i^2 \sigma_m^2$ = systematic / market risk, this is the proportion of total risk that is priced, and it is un-diversifiable.

Therefore, total risk of security *i*, consists of two parts: (1) market risk and (2) specific risk. By specification, the model above shows us that the Beta of asset i, is the sole determinant on the excess return of that asset. It is generally agreed that general market conditions that drive business cycles (GDP growth), also drives regimes in the stock market (all share indices) (Markose and Yang, 2008). The state dependent model for an asset return in Equation (13) is assumed to be driven solely by the Hamilton (1989) two state regimes $\{(S_t), S_t = 1, 2\}$ of the returns on market index:

$$r_{mt} = \mu_{m(S_t)} + \sigma_{m(S_t)} \epsilon_{mt} \quad (14)$$

Where:

States $\{(S_t), S_t = 1, 2\}$ denotes the Hamilton (1989) regimes for the returns on market index.

---

[6] β_i = covariance (security *i* and market index)/ market variance

$\mu_m(s_t)$: denotes the regime-dependent mean of excess return on market index.

$\sigma_m(s_t)$: denotes the regime-dependent conditional volatility, measured by standard deviation.

So the CAPM equation can be transformed into a regime dependent one by introducing state dependence in the CAPM equation for the i asset:

$$r_{it} = \alpha_{i(s_t)} + \beta_{i(s_t)} r_{mt} + \varepsilon_{it} \qquad (15)$$

The CAPM assumptions are assumed to be satisfied. It is assumed that our investor is risk averse and his preferences can be modelled by constant relative risk aversion (CRRA) preferences. Assumptions are to exclude transaction costs, inflation risk etc. Initially portfolio is modelled with no short selling, later extended to incorporate short selling as well.

3.1.3 Risk Aversion:

CAPM assumes investor to be risk averse but one factor to select the mean-variance optimal portfolio for any investor is the degree of risk aversion. This level of aversion to risk can be shown by defining the investor's indifference curve. It shows the required return against the risk taken in a particular investment. In MATLAB and in other readings a typical risk aversion coefficient ranges from 2.0 to 4.0, with the higher number representing more risk averse investor (MATLAB Documentation & Fabozzi, Focardi & Kolm, 2006). The equation used to represent risk aversion in Financial Toolbox™, MATLAB software is;

$$U = E(r) - 0.005*A*\sigma^2$$

Where: U is the utility value, E(r) is the expected return, A is the index of investor's aversion and 'σ' is the standard deviation of returns.

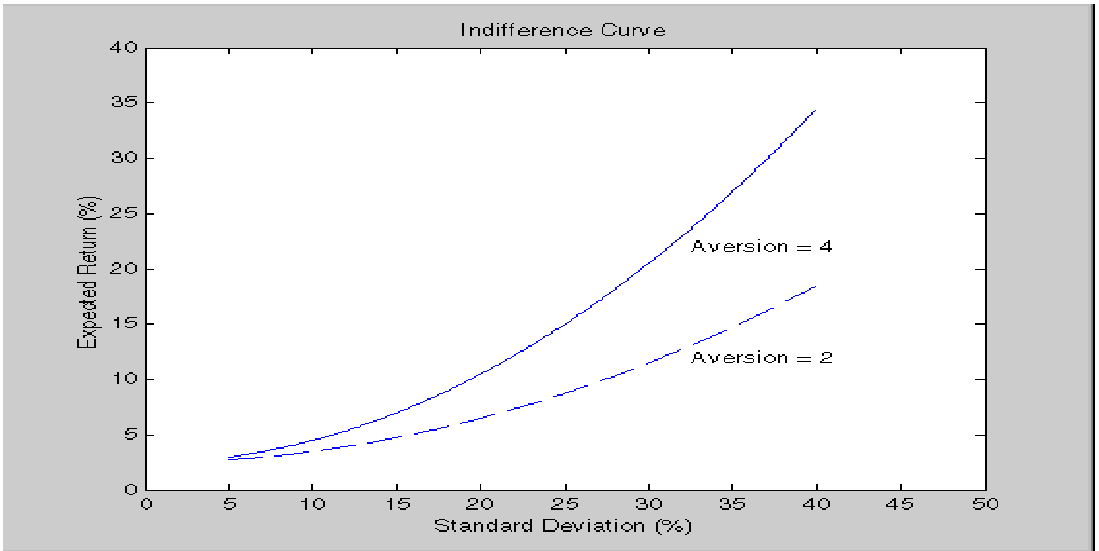


Figure 3.1-2: Indifference curve of expected return to standard deviation of portfolio. Adopted from MATLAB Documentation. http://uk.mathworks.com/help/finance/portfolio-selection-and-risk-aversion.html

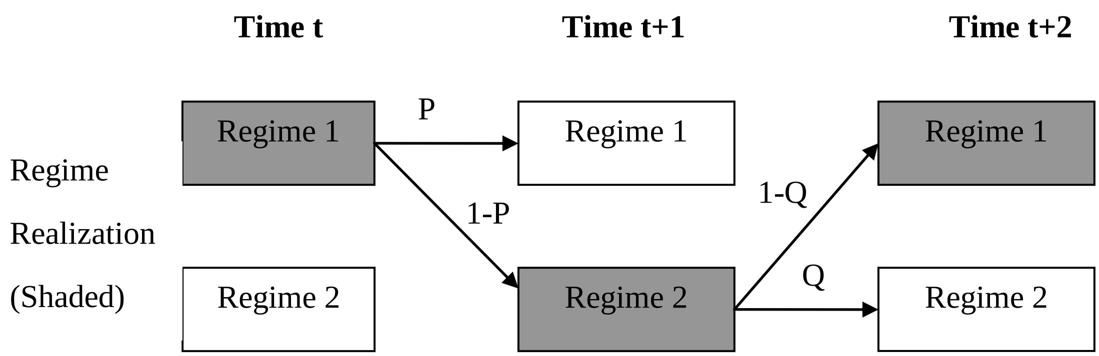
Another variable used in MATLAB for portfolio optimisation is the value of RiskyFraction, which if exceeds 1 (100%), implying that the risk tolerance specified allows borrowing money to invest in the risky portfolio, and that no money is invested in the risk-free asset. This borrowed capital is added to the original capital available for investment (MATLAB Documentation & Fabozzi, Focardi & Kolm, 2006).

3.1.4 Conditional Expected Return and Variance/Covariance for Securities:

Suppose the portfolio manager knows which regime is realized at each point of time, but investor does not know which regime will be realized next time point. If the market is currently in Regime 1, the probability of remaining in Regime 1 at next time point is P, and the probability of transitioning to Regime 2 is (1 - P). Similarly, if the market is currently in Regime 2, the probability of remaining in Regime 2 at next time point is Q, and the probability of transitioning to Regime 1 is (1 - Q). With constant transition probability P and Q, he/she would have same expectations every time when Regime 1 (Regime 2) is realized.

The Markov process and the estimation of conditional probabilities are illustrated in Figure 3.1-3.

Figure 3.1-3: The Transition of Market Regimes: Conditional Mean and Variance



Conditional Expected Return

$$e_{m(1)}: \; P\mu_{m(1)}+(1-P)\mu_{m(2)} \qquad e_{m(2)}: \; (1-Q)\mu_{m(1)}+(Q)\mu_{m(2)} \tag{16}$$

Conditional Expected Variance

$$\Sigma_{m(1)}: \; P[\sigma_{m(1)}]^2+(1-P)[\sigma_{m(2)}]^2+P(1-P)[\mu_{m(1)}-\mu_{m(2)}]^2$$

$$\Sigma_{m(2)}: \; (1-Q)[\sigma_{m(1)}]^2+(Q)[\sigma_{m(2)}]^2+Q(1-Q)[\mu_{m(1)}-\mu_{m(2)}]^2 \tag{17}$$

The shaded box denotes the regime which is realized at the time point. In this example, the market is in Regime 1 at time t, Regime 2 at time t+1 and Regime 1 again at time t+2. Conditional expected return vectors for N securities given that either Regime 1 or Regime 2 is indicated to be the current state are denoted by $e_{m(1)}$ and $e_{m(2)}$ and later denoted by $e(1)$ and $e(2)$ in Equation 19.

Once the Model has been set, it can be seen that we need regime dependent mean and variance along with their transitional probabilities. Classical linear regression models do not take into consideration regime shifts, the auto regression and moving average (ARMA) models do not take into consideration volatility clustering (Brooks, 2007). The generalized autoregressive conditional heteroskedasticity (ARCH and GARCH) models are inappropriate for modelling Asymmetric Correlation; there is greater estimation error in these models compared to models that take into consideration regime switching (Hamilton and Susmel, 1994). That is why this study uses maximum likelihood estimation for the calculation of regime dependent mean, variance and probabilities.

To implement the steps in portfolio optimization problem, the expected return and variance/covariance of securities are required. They can be derived from the market model expressed in Eq. (12) and (13). Since the market is switching between two regimes, the securities are regime-dependent through their relation with market. Let us denote the regime-dependent expected returns of security i as $r_{i(S_t)}$, and $\mu_{m(S_t)}$ is mean market return, then;

$$r_{i(S_t)} = \alpha_i + \beta_i \mu_{m(S_t)} \qquad S_t = 1,2 \qquad (18)$$

Let $\alpha = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{pmatrix}$ and $\beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \end{pmatrix}$, then the regime-dependent expected mean vector for N securities is given by:

$$R_{(S_t)} = \alpha + \beta \mu_{m(S_t)} \qquad S_t = 1,2$$

Then the conditional expected return vectors for *N* securities given that either Regime 1 or Regime 2 is indicated to be the current state:

$$e(1) = PR(1) + (1 - P)R(2)$$

$$e(2) = QR(2) + (1 - Q)R(1) \qquad (19)$$

The variance/covariance matrix has three components. First, the conditional variance of individual assets depends on the asset's exposure to systematic risk through its beta with respect to the market. Therefore, the differences in systematic risk across the different assets and the correlations are completely driven by the variance of the market. However, because the market variance at next time point depends on the realization of the regime, we have two possible variance matrices for the unexpected returns next time period. Second, each asset has an idiosyncratic volatility term V^2 unrelated to its systematic exposure (Note V^2 is the standard error obtained from the OLS regression of Betas). Therefore, the regime-dependent variance for any security will be:

$$\sigma_i^2 = \beta_i^2 \sigma_{m(S_t)}^2 + \overline{\sigma_i^2} \qquad (20)$$

Let $V = \begin{pmatrix} \overline{\sigma}_1 \\ \vdots \\ \overline{\sigma}_N \end{pmatrix}$, then the regime-dependent covariance matrix for *N* securities is given by:

$$\Omega_{S_t} = (\beta_i)[\sigma_{m(S_t)}]^2 + V^2 \qquad S_t = 1,2 \qquad (21)$$

It is straightforward to see in this model that the correlation given by $\Omega(1)$ will be different from the correlation given by $\Omega(2)$ as they come from two different regimes. Finally, the actual covariance matrix takes into account the regime structure, in that it depends on the realization of the current regime and it adds a jump component to the conditional variance matrix, which arises because the conditional means change from one regime to the other. As a consequence, the conditional expected covariance matrix for *N* securities in Regime 1 and Regime 2 can be written as:

$$K(1) = P\Omega(1) + (1-P)\Omega(2) + P(1-P)[\mu(1)-\mu(2)]^2$$
$$K(2) = (1-Q)\Omega(1) + Q\Omega(2) + Q(1-Q)[R(1)-R(2)]^2 \qquad (22)$$

Now, using Eq. (18), (19), (20) (21), and (22), the portfolio optimization steps can proceed.

3.1.5 Market Value Weighted Portfolio:

Apart from index values, stock prices, risk free rate, DataStream also provides the market values (MV) of most of the stocks and stock indices. Once market values are available, weights for the portfolios for each asset can be computed easily by as;

$$w_j = {MV_j}\Big/{(\textstyle\sum_{j=1}^{n} MV_j)} \qquad (23)$$

Where $w_i$is the weight of asset *j* and MV_j is market capitalisation value of asset *i* and $\sum_{j=1}^{n} MV_j$ is the sum of market capitalisation values on that particular time. So for every step of rebalancing the portfolio, the weights for assets are computed and invest according to weights computed in equation (23).

3.1.6 CAPM vs. Multifactor models

Corporate finance theory offers different methods to estimate the cost of equity. Among them, the Capital Asset Pricing Model (CAPM) has become best practice among the financial practitioners (Geginat et al. (2006); Graham and Harvey (2001); Brounen, De Jong and Koedijk (2004)). Fama and French (1993) document that an extension of the CAPM with two others factors related to the firm's size and the firm's book-to-market value better explains variations in average returns across stocks. Likewise, a couple of years later Carhart (1997) expands the Fama/French three factor model by adding a new momentum factor; he documents that its model better explains the returns of mutual fund's portfolios sorted by their previous calendar year's return then the CAPM does. The fact that the CAPM – which is a single factor model – has prevailed in the financial practice over the multifactor models indicates that the quality of models with additional factors does not significantly improve or that a possible improvement of the model's performance – measured by some key statistics such as the coefficient of determination and the model's intercepts – does not justify the extra work needed to determine the additional risk factors. In corporate finance there are lots of models with different complexity that try to estimate expected stock returns. The more factors are used, the more sophisticated the model becomes and consequently the more elaborate and expensive the estimation procedure is. In order to minimize costs, a firm should therefore use the smallest model which suitably describes reality.

3.1.7 Dynamic Updating of Regime Switching Portfolio Weights:

As discussed earlier that this methodology was pioneered by Hamilton but was not meant for portfolio optimisation and was then practically put to use for portfolio optimisation by Ang & Bekaert (2002). They helped identify the potential use of Regime indicator and its implications on Portfolios. To model regime switching, I adopted a model similar to that used by Ang & Bekaert (2004) and Markose & Yang (2008) and extending their work by using rolling window as against static coefficients.

The following steps define in general the process of portfolio formation in all the chapters;

Step 1: Divide Data into in-sample and out-sample (for example, whole sample for Chapter 4 is January1986- June 2017; in-sample is January 1986- December 1995; out-sample is January 1996- June 2017). Monthly data on FTSE-100 and three stocks for the Chapter 4 is used. Monthly data on Subcontinent stock Indices for the Chapter 5 and Asia Pacific stock Indices for Chapter 6 was collected. The MSCI Emerging Markets and MSCI Asia Pacific is used as the market index for Chapter 5 and 6 respectively. For Chapter 7 daily data of FTSE-100 Index and its 74 constituent companies is used (as from the current 100 constituents, 26 companies didn't have data available from start date of dataset). Bank of England's London Interbank Offer Rate (LIBOR) was used as RF rate for Chapters 4 and 7. US Interbank 1 month offer rate from London BBA available on DataStream is taken as the risk free (RF) rate for Chapters 5 and 6 as all price indices used are in common currency (for better comparison purposes) denomination i.e. US Dollars.

Step 2: Load all the in-sample data and do OLS regression of each security with respect to the market excess return and obtain the Alpha and Beta coefficients.

Step 3: Do whole sample and in-sample statistics for Mean, Variance, and Excess kurtosis, Skewness, Correlation and Covariance of the individual Stock Returns and Market Excess Return.

Step 4: Process the in-sample data of the stock returns and excess return of index respectively into regime switching estimator and record the in-sample unconditional parameters$(\mu_{m1}, \mu_{m2}, \sigma_{m1}, \sigma_{m2}, P_t, Q_t)$.

Step 5: Take the filter probabilities (which act as Regime indicator) at the end of the in-sample period and iteratively update at the end of each month (day) to get the inference of which state it is in. If the filter probability is greater than 0.5, it is in Regime 1 otherwise it is in Regime 2.

Step 6: Then calculate the conditional RS mean (μ_{m1}, μ_{m2}), conditional variance $(\sigma_{m1}, \sigma_{m2})$ and conditional covariance (K_1, K_2) of the CAPM Beta based returns of assets using Equations (16), (17), & (22).

Step 7: Use the inputs from Step 5 to the Mean-Variance quadratic programming problem at the end of each month over out-sample period. The aim is to solve the basic quadratic problem of asset allocation, given an initial level of wealth. The portfolio optimisation problem involves:

- Selecting a proportion of the initial investment for each asset i, $\omega_i \in [0,1]$
- The return of each asset r_i is a random variable with $E[r_i] = \mu_i$
- Portfolio return is given by $\sum_{i=1}^{N} \mu_i w_i$
- Portfolio variance is $\sigma^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i \omega_j \sigma_{ij}$ with σ_{ij} : covariance of asset i and j
- The quadratic optimisation problem is:

$$\text{Min} \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i \omega_j \sigma_{ij} \qquad \text{Subject to} \sum_{i=1}^{N} \omega_i r_i = \mu \ \& \ w_i \in [0,1] \ \forall \ i \qquad (24)$$

The above equation (24) is also called Modern Portfolio Theory (MPT) coined by Markowitz (1959) and is used for optimisation of portfolio.

Step 8: Calculate the cumulative return on £1 invested in the RS CAPM, Non-RS and MV (Market value based) portfolios over out-sample period and compare them. This process changes slightly for Chapter 7 as it does not use MV strategy.

Step 9: Repeat step 5, 6, 7 & 8 for different scenarios of RS CAPM vs. Non RS vs. MV portfolios;

1. Short selling allowed with Risk free borrowing & lending
2. Short selling not allowed with Risk free borrowing & lending
3. Short selling allowed without Risk free borrowing & lending
4. Short selling not allowed without Risk free borrowing & lending
5. Based on level of Risk Aversion of Investor as described earlier.

All above scenarios have been tested for data involving Chapter 4, 5 and 6 while Chapter 7 relies only on scenario 1 which is found to be best option and allows short selling and allows risk free borrowing and lending.

As the methodology above indicates, in a dynamic regime sensitive portfolio optimization problem, once the regime indicator signals the state in the next period, the appropriate regime sensitive statistics are fed into the Markowitz portfolio choice model. The steps (5, 6, 7, and 8) are dynamically iterated in each period of the ex-ante (out-sample) portfolio rebalancing period. The following chapters report the results for the specific portfolios under consideration.

4 Chapter 4: Optimal Portfolio Selection with Dynamic Regime Switching Weights using FTSE-100 and its three constituent companies

Abstract

This chapter analyses the implication of regime switching (RS) in optimal asset allocation in the United Kingdom (UK) equity market for the 3 asset case. The purpose of this chapter is to demonstrate the methodology of the CAPM RS model before it is extended to large portfolios for the FTSE-100. In particular, a small portfolio with the risk free asset enables us to make detailed investigations as to how portfolio weights dynamically change to respond to the regime switches in the market. Data showed significant evidence of asymmetric correlation. The bull market (Regime 1) had a higher mean, lower volatility and lower asset correlations compared to the bear market (Regime 2). The latter curtails the benefits from diversification when market conditions deteriorated quickly, hence using regime switching becomes essential for market timing purposes. On average in the out of sample period, the RS (Regime Switching) portfolio showed better performance in terms of the end of period cumulative wealth, which was about 40% higher than the Non-RS (Non Regime Switching or simple Mean-Variance) and MV (Market Value weighted) portfolio strategies. The risk adjustment to return as in the Sharpe ratio is used to further validate the performance of RS and non-RS portfolios. Using Sharpe ratio, the RS CAPM portfolio once again performs better than compared with the other strategies as highlighted by Table 4.6-1 (0.2267 value of Sharpe ratio in RS CAPM strategy, 0.0601 in Non-RS and 0.0601 in MV portfolio strategy). In all the models, it was also easy to see the fall in cumulated wealth round about the time of the commencement of the credit crisis in 2007. But RS Strategy still ended in positive territory and comparatively higher values compared to Non-RS and MV optimization strategy. The reason is RS indicator allows change of portfolio asset weights

beforehand efficiently during market down turns, another reason being, the RS portfolio switches out of the risky assets and allocates more weight into the risk free asset.

Key Words: Regime Switching, Asymmetric Correlations, Portfolio Optimization, FTSE 100

4.1 Introduction

In this chapter the implication of regime switching (RS) in optimal asset allocation in the United Kingdom (UK) equity market is analysed for 3 assets. Initial focus is on data description and then the empirical determination of steps for developing the RS CAPM given in Chapter 3 is followed for the analysis and discussion of results. Total sample data is divided into two periods called in-sample and out-sample and their descriptive statistics are discussed in Section 4.2. There is strong evidence of stylized facts of financial assets in data such as, skewness, kurtosis and asymmetric correlation discussed in section 4.3 and 4.4. The in-sample is used to get initial regime inputs and regime is predicted in out-sample period. Section 4.5 determines the regime probabilities serve as Regime indicator which assumes Regime 1 if probability is more than 0.5 and in Regime 2 if otherwise. The regime Covariance's are calculated and then asset choice and weight allocation is made and lastly portfolio is formed assuming 1GBP investment at the start of January 1996 (start of out-sample period) using the three strategies. End of period cumulated wealth and Sharpe ratio is computed for every portfolio strategy. Section 4.6 derives the portfolio strategies:

(i) Regime Switching CAPM (RS CAPM): Strategy 1 is the regime switching portfolio model. For this the regime sensitive portfolio statistics are used based on the regime indicator at each end of month rebalancing point from the start of the out of sample period in Jan 1996. The portfolio optimization software generates regime switching portfolio weights and are used to construct a monthly portfolio on the basis of these RS weights.

(ii) Traditional Mean Variance portfolio (Non-RS): Strategy 2 is the single-regime model with no regime switching, in which the portfolio is constructed using quadratic programming but the expected returns and covariance are simply the constant statistical moments of the time series data updated on a monthly rolling window basis.

(iii) Market Value based Portfolio (MV): Strategy 3 is to invest into equities according to their market capitalization. The market capitalisation value was imported from DataStream, which is computed from the share price of each asset multiplied by its number of (outstanding/ issued) shares.

The performance among the three strategies is evaluated further on the basis of following scenarios;

A. Cumulated Wealth without Short-Selling Approach with risk free investment
B. Cumulated Wealth without Short-Selling Approach without risk free investment
C. Cumulated Wealth with Short-Selling Approach with risk free investment
D. Cumulated Wealth with Short-Selling Approach without risk free investment

An initial investment of one pound (small investment is considered so that our strategy does not affect the market too much) is made on January 1996 and cumulated wealth in June 2017 is checked. This includes using different combination of scenarios discussed above and an assessment is made of the performance of all strategies and in all scenarios. The cumulated wealth from regime switching (RS CAPM) always significantly outperforming from that of the non-regime switching (Non-RS) and MV strategy.

There is final concluding section that contains a summary of our discussion, as well as recommendations for further analysis.

4.2 Sample Data Description

Data is collected from DataStream for the period January 1986 to June 2017, totally 378 monthly prices, and Market values (MV) for each asset, i.e. British Telecom (BT), British Petroleum (BP) and Barclays (Barc). FTSE-100 index values used here as benchmark index are also obtained from DataStream. The prices were then converted to log returns resulting in the loss of 1 observation giving us 377 log returns for three assets and excess log returns for FTSE 100. Bank of England's monthly Libor is used as risk free asset and the same rate is used as investment rate for short selling purposes, excess returns for FTSE-100 are also calculated using the same risk free rate (Libor). The use of daily data gives ease of handling the data and cross verifying results sometimes manually to check whether research is moving in right direction.

Total Sample is divided into two periods called in-sample and out-sample. The in-sample period is January 1986 to December 1995 (120 observations) and out-sample period starts from January 1996 and ends at June 2017 (258 observations). The in-sample is used to feed initial inputs into the system and afterwards since it's a rolling window, first observation is dropped and a new data point is included. The in-sample is kept proportionately small so as to allow true forecasting over the longer periods of out-sample. One important reason for keeping out-sample long is to allow for the maximum regime switches to check whether it effects rebalancing every time a regime is switched.

4.3 Summary Statistics for 3 assets and the FTSE-100

Table 4.3-1 below shows the whole sample statistics **for British Petroleum (**BP**)**, **British Telecom (**BT**)**, **Barclays (**Barc**)** and FTSE-100 with all of the stocks being positive on average mean return, just near zero and FTSE-100 showing average negative returns also close to zero. As shown by 10.52% standard deviation in Table 4.3-1, Barc has proved to be more risky than the other two stocks and FTSE-100 index. Barc shows more excess kurtosis (10.45) and least negative skewness (-0.3227) than the others which shows that Barc price is fluctuating more than the others and investment in it could be more risky. Analysis of ordinary least square statistics relating to the CAPM for the whole sample show that all have negative small values of intercept with small positive slopes, BP having highest value of 0.4352. R squared values show that BP is more correlated with FTSE-100 that Barc and BT as can also be seen in Correlation values in Table 4.3-1.

Sample Period :	January 1986 to June 2017			
Observations :	Excess Monthly Returns = 378			
Descriptive Statistics				
	BT	BP	Barclays	FTSE
Mean	0.00154725	0.00417866	0.00303546	-0.00020044
Variance	0.00621609	0.00472312	0.01108002	0.00217073
Standard Deviation	0.07884218	0.06872496	0.10526168	0.04659105
Excess Kurtosis	1.95296242	4.14023568	10.45761902	6.33858568
Skewness	-0.68892328	-0.77797314	-0.32275652	-1.23338367
OLS Regression				
Intercept	-0.00073766	-0.00201904	-0.00102926	
Slope	0.34720739	0.43521027	0.27304460	
Standard Error Slope	0.03775101	0.03577043	0.03671852	
R Square	0.34521536	0.41211863	0.38054215	
Covariance Matrix				
BT	0.00619964	0.00174804	0.00323239	0.00215256
BP	0.00174804	0.00471063	0.00221532	0.00205011
Barclays	0.00323239	0.00221532	0.01105071	0.00301734
FTSE	0.00215256	0.00205011	0.00301734	0.00216498
Correlation Matrix				
BT	1.00000000	0.32346551	0.39052201	0.58755031
BP	0.32346551	1.00000000	0.30704540	0.64196466
Barclays	0.39052201	0.30704540	1.00000000	0.61688099
FTSE	0.58755031	0.64196466	0.61688099	1.00000000

Table 4.3-1: Statistical Description of Three Assets and their index (BT, BP, Barclays and FTSE-100)

As can be seen from Table 4.3-1 that BT shows the lowest average return and relatively high volatility. Comparatively BP has the highest average return which may be explained by its high beta value implying that when the average market returns are high, the returns from BP will be on average higher than the market return.

Since benchmark index depicts the overall movement of stock market and that the movement is derived from the collective movement of all its constituents. In Figure 4.3-1 below, the movement of stock prices is in line with the movement in FTSE-100 index. Off course, the degree varies from stock to stock which is shown by the correlations statistics in Table 4.3-1.

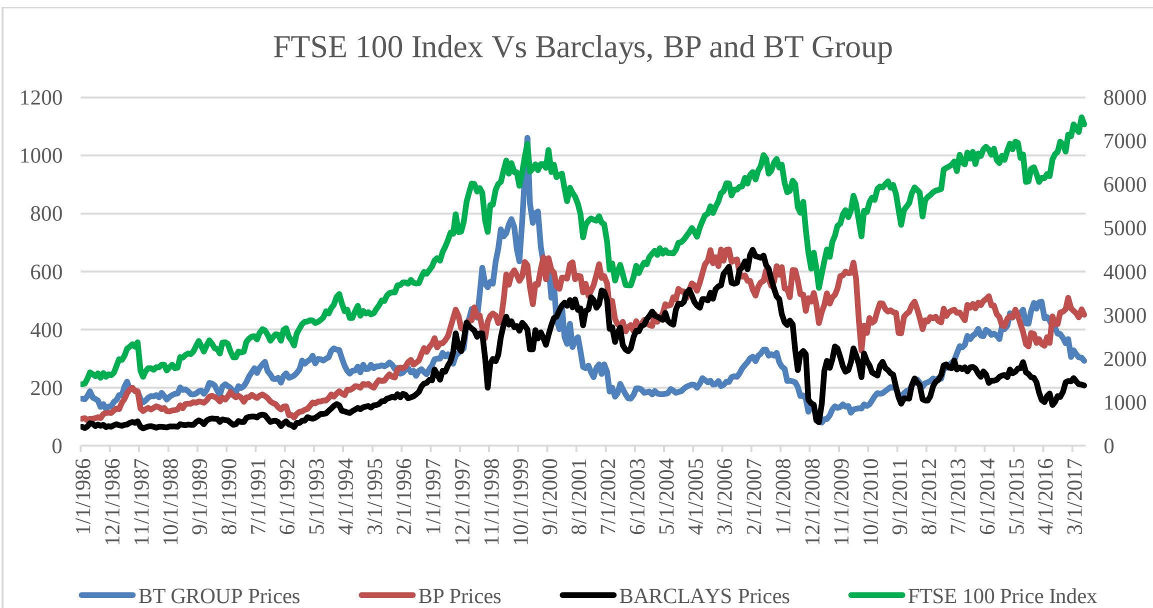


Figure 4.3-1: Prices of Three Stocks (BT, BP, Barclays) versus FTSE 100 Index for whole sample (Note that FTSE 100 (in green) is plotted using Secondary Axis on the right for Comparison purposes).

The in-sample statistics such as mean, standard deviation are used as inputs for the regime predictor and ordinary least square variables (slope, intercept and standard error of slope) are used to compute the initial covariance matrix as highlighted in methodology chapter 3. The statistics show that the mean are very low near zero and Barc being more risky with

7.4% standard deviation. All are negatively skewed in line with FTSE-100 (showing high negative skewness). Barc is highly correlated with FTSE-100 compared with BT and BP (as shown by R squared value of 61.96%).

In-Sample Period: 01/01/1986 to 01/12/1995				
Observations : Excess Monthly Returns = 120				
Descriptive Statistics				
	BT	BP	Barclays	FTSE-100
Mean	0.00352160	0.00886641	0.00773030	-0.00024916
Variance	0.00410152	0.00471438	0.00559522	0.00295495
Standard Deviation	0.06404310	0.06866132	0.07480121	0.05435943
Excess Kurtosis	-0.24262068	9.37975812	0.93692423	10.03476771
Skewness	-0.10047465	-1.88449054	-0.34497360	-1.78678172
OLS Regression				
Intercept	-0.00231013	-0.00512753	-0.00467138	
Slope	0.58523624	0.55020772	0.57206219	
Standard Error Slope	0.03953870	0.03925199	0.03366612	
R Square	0.47539799	0.48297850	0.61966060	

Table 4.3-2: Descriptive Statistics for in-Sample period (January 1986 to December 1995)

The coefficients alpha and beta for each stock was calculated using the OLS regression on the in-sample period as shown in Table 4.3-2. To check for parameter stability and consistency, results of in-sample are compared with out-sample period results, also by varying the period of in-sample and out-sample, our results were consistent.

4.4 Evidence of Asymmetric Correlation

Evidence of asymmetric correlations has been shown by 3 stocks in relation to the FTSE-100 in Table 4.4-1. Here we take approximate sample periods corresponding to the bull and bear market conditions for the FTSE-100. The bear market sample taken after Dotcom bubble busted (January 2001 to December 2003), all stocks have higher correlations with the FTSE-100 compared to the bull market period (sample taken from January 2004 to June 2007). This is consistent with previous studies like Ang and Bekaert (2002, 2004) and concept about Great Moderation and Great Recession as discussed by Chan, Fry-McKibbin and Hsiao (2017). The period July 2007 – Dec 2009 corresponded with the Great Financial Crisis and show higher correlations associated with the bear market as all 3 stocks has clear increase in correlation coefficients. The sample period from January 2009-December 2014 shows further increase in correlations when compared to January 2004-June 2007 sample but is less than the January 2001-December 2003 period except BP whose correlation with FTSE-100 has continuously increased over time.

Correlation Coefficients of 3 Stocks with FTSE-100	Sample: January 2001 – December 2003	Sample: January 2004 – June 2007	Sample: July 2007 – December 2009	Sample: January 2009 – December 2014
	Bear period correlations regime 2	Bull period correlations regime 1	Sample Period Correlations regime 2	Sample Period Correlations regime 2
Barclays	0.793125664	0.566364648	0.600511611	0.663877809
British Petroleum (BP)	0.633042104	0.618245353	0.658768707	0.789605571
British Telecom (BT)	0.727118709	0.486207068	0.529316657	0.543052850

Table 4.4-1: Correlation coefficient value of 3 Stocks with FTSE-100 in different periods (Bull and Bear Periods-assumed based on judgement and previous literature)

Other statistics used for the regime identification are mean and standard deviation shown in Table 4.4-2 below and are also consistent with the above discussion also highlighted by Hamilton (1989) Ang and Bekaert (2004) and Chan, Fry-McKibbin and Hsiao (2017). The bull market period has higher mean returns and low standard deviation (as can be seen in January 2004 to June 2007 period and January 2009-December 2014 sample periods) when compared with regime 2 which has low mean returns and high standard deviations (as can be seen January 2001-December 2003 & July 2007-December 2009 sample periods).

	January 2001 - December 2003		January 2004 – June 2007		July 2007 – December 2009		January 2009 – December 2014	
Asset Name	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
BT	-0.0210	0.120538	0.001305	0.049929	-0.0298	0.115231	0.002145	0.083012
BP	-0.0048	0.063361	0.004238	0.053504	-0.0002	0.073805	-0.00427	0.077594
Barclays	-0.0010	0.074782	-0.01008	0.063678	-0.0297	0.23646	-0.01042	0.159011
FTSE-100	-0.0091	0.049869	0.003744	0.029028	-0.0065	0.062694	-0.00004	0.047855

Table 4.4-2: Excess Returns: Mean and Standard Deviations (Values) of 3 Stocks and FTSE-100 in different periods

4.5 Correlation computation and discussion

In this section, statistical properties of the correlation matrices of the 3 monthly stock returns (empirical sample) traded on the FTSE 100 from January 1986 to June 2017 are discussed. Correlations are computed from empirical data returns and discuss whether the properties of the correlation matrices as discussed in RMT hold for the empirical data.

4.5.1 Eigenvalue Analysis

Most of the distribution is consistent with the RMT bounds calculated in different studies (e.g. Plerou et Al., 1999, Daly et al., 2010). This comparison also indicates the presence of several eigenvalues clearly outside the random matrix bound. Particularly interesting is the largest Eigenvalue, suggesting genuine information about the correlations between 3 stocks. Having demonstrated that the bulk of the eigenvalues satisfies RMT predictions, proceed to analyse the eigenvectors of C. First, analyses of the statistics of the eigenvectors are done. In the stock market problem, this eigenvector conveys the fact that the whole market moves together and indicates the presence of correlations that pervade the entire system (Daly et al, 2010).

The Marchenko-Pasteur limits to identify noisy values are given by the Equation (7.3) which is dependent on variable Q= L/N, where L=378 and N = 3. The maximum Eigenvalue limit (λ_{max}) and minimum Eigenvalue limit (λ_{min}) for all the correlation matrices is calculated through Equation (7.3) and their values are;

λ_{min} = 0.829762 and λ_{max} = 1.186111

The limits defined provide us with noisy values. If we carefully observe Table 4.5-1 which is the diagonal matrix of eigenvalues, showing 2.0627 as the largest Eigenvalue and 0.4006 as the smallest Eigenvalue for RS1 CAPM and EmpSample while largest Eigenvalue for RS2 CAPM is 2.4752. EmpSample and RS1 CAPM eigenvalues are same as they generate similar correlation matrices.

EmpSample Eigenvalues			RS1 CAPM Eigenvalues			RS2 CAPM Eigenvalues		
0.4007	0.0000	0.0000	0.4007	0.0000	0.0000	0.4808	0.0000	0.0000
0.0000	0.5366	0.0000	0.0000	0.5366	0.0000	0.0000	0.6439	0.0000
0.0000	0.0000	2.0627	0.0000	0.0000	2.0627	0.0000	0.0000	2.4752

Table 4.5-1: Eigenvalues Diagonal Matrix 'D' for EmpSample, RS1 CAPM & RS2 CAPM

As it can also be observed that no value falls between Marchenko Pasteur limits meaning no noisy value exists in all three scenarios, thus it does not need filtering as shown by Table 4.5-2.

	EmpSample Eigenvalues	RS1 CAPM Eigenvalues	RS2 CAPM Eigenvalues
Number of Noisy Values	0	0	0
Percent of Total (3)	0%	0%	0%

Table 4.5-2: Number of Noisy Eigenvalues shown by Marchenko-Pasteur limits before filtering process

The eigenvector of largest Eigenvalue states that almost all 3 stocks play equally important role in the performance of Index. Largest Eigenvalue suggests that taking long position will be beneficial as shown by Table 4.5-3 below.

0.494302588604273	0.655942139714270	0.570442687959255
0.349085811218492	-0.750762898170348	0.560797795231745
-0.796118111403730	0.0780703533867069	0.600084137948199

Table 4.5-3: Corresponding Eigenvectors 'V' for Eigenvalues for EmpSample & RS1 CAPM

Once the eigenvalues and eigenvectors are computed from the above correlation matrices, the results are examined more closely. The key element was to compute the noisy values given by Marchenko-Pasteur limits. All three correlation matrices show that they did not have any noisy values. Then reconstruct the filtered correlation matrix by using filtered Eigenvalue diagonal matrix (D_{filter}) and corresponding eigenvectors V as follows;

$C_{filter} = V*D_{filter}*V-1$

Once C_{filter} is obtained, we examine it to check whether its diagonal is similar to original matrix with unit values on the diagonal, if not we repeat the process until original diagonal is obtained. The matrix C_{filter}, once obtained is checked for noise again and since the filtration wasn't needed therefore no noisy eigenvalues are detected.

4.6 RS CAPM Empirical Construction

The Capital Asset Pricing Model (CAPM) is used to determine a theoretically appropriate required rate of return of an asset, if that asset is to be added to an already well-diversified portfolio, given the asset’s non-diversifiable risk. The model takes into account the asset's sensitivity to non-diversifiable risk (also known as systemic risk), often represented by Beta, as well as the expected return of the market and the expected return of a theoretical risk-free asset (Black, Jensen, and Scholes, 1972).

As outlined in the Chapter 3, Steps 1 to 3 of the methodology have been discussed earlier in sections 1.2 to 1.4 explaining the in-sample, out-sample and evidence of asymmetric correlation is shown.

Step 4: Obtain the RS Statistics for Data:

This section will look at the RS CAPM and RS covariance computation which involves step 4 to step 6 and involves the usage of conditional RS mean ($\mu1$, $\mu2$), variance ($\sigma1$, $\sigma2$) and covariance (K1, K2) derived from CAPM beta based returns of assets using Equations (16), (17), & (22).

Step 4 uses Hamilton (1989) method of obtaining Maximum likelihood regime sensitive parameters i.e., transition probabilities, means, and standard deviations for each regime as described earlier in Chapter 3 in order to predict the regime. Table 4.5-1 below shows the values of mean and standard deviation used to identify the regimes and their respective transition probabilities.

Regime Indicator	Regime 1		Regime 2		Transition Probability	
	μ_1* %	σ_1*%	μ_2*%	σ_2*%	$P*$	$Q*$[7]
Estimates	.3075	3.970	-.6793	9.903	0.9899	0.5000
Standard Error	0.00011	0.0042	0.0000	0.0002	0.0076	0.1884

Table 4.6-1: Regime Statistics and Transition Probabilities from in-sample Period 01/01/1986-01/12/1995

The values in Table 4.5-1 clearly show evidence of two regimes i.e., $\mu1 > \mu2$, $\sigma1 < \sigma2$. On average Regime 1 expected return and standard deviation are 0.307% and 3.97% while for Regime 2, these values are -0.679% and 9.90%. Regime 1 is much more stable than Regime 2 in a sense that it depicts less volatility as compared to Regime 2. Transition probabilities also show the presence of two different regimes with Regime 1 being more persistent with transition probability P of 0.9899 and Regime 2 is least likely with transition probability of 0.5000.

Step 5: Filter and Smoothed Probabilities:

In addition to the regime parameters reported in Table 4.5-1, complete set of regime probabilities (i.e. filter probabilities and smoothed probabilities) is obtained for out-sample

[7] Statistically significant at 0.01% level of significance.

period as can be seen in Figure 4.5-1. The filter probabilities indicate the process being in some particular regime at time t based on the information available at the time t-1. In contrast the smoothed probabilities indicate the historical regimes the process was in at time t based on whole sample information.

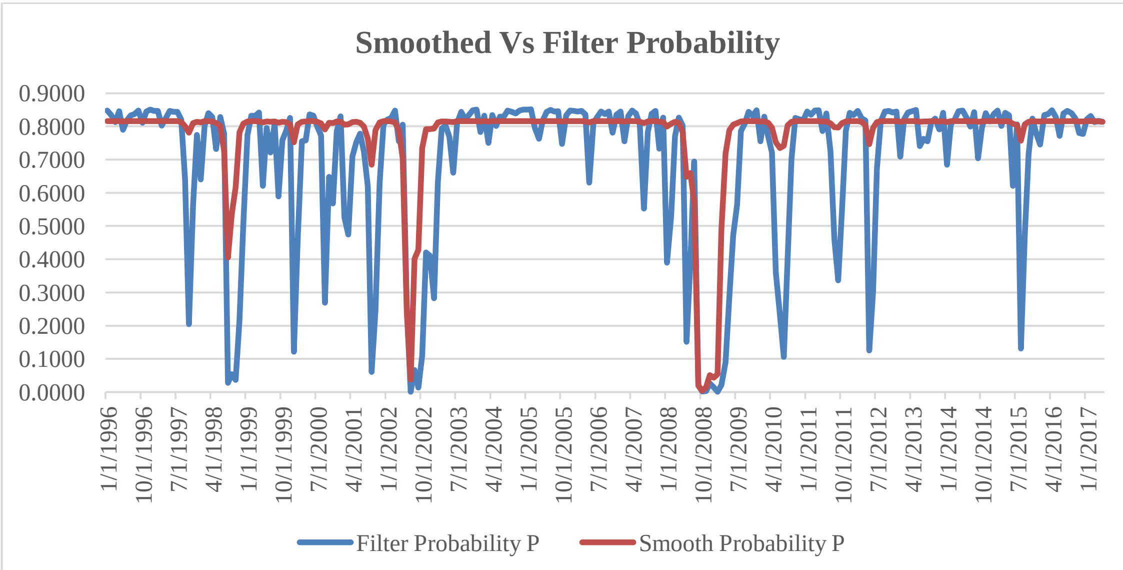


Figure 4.6-1: Smoothed vs. Filtered Probabilities for out-sample period (From January 1996 to June 2017)

Figure 4.5-1 shows the transition probabilities of being in one regime against another regime over out-sample period. The Hamilton RS model used here assumes that if the ex-ante filter probability is greater than 0.50, the market is in regime 1 and that the probability of being in Regime 2 is less than 0.50. We can easily see from figure 4.5-1 that there are many occasions where probability has dipped down from cut-off point of 0.5, hence showing the probability of market being in Regime 2. Filter probabilities (in Figure 4.5-1) show there were 13 instances of less than 0.5 whilst smoothed probabilities, ex post, show only 3 times regime changed from regime 1 to regime 2 that is why filter probabilities are considered more reliable and this thesis uses filter probabilities for the regime prediction.

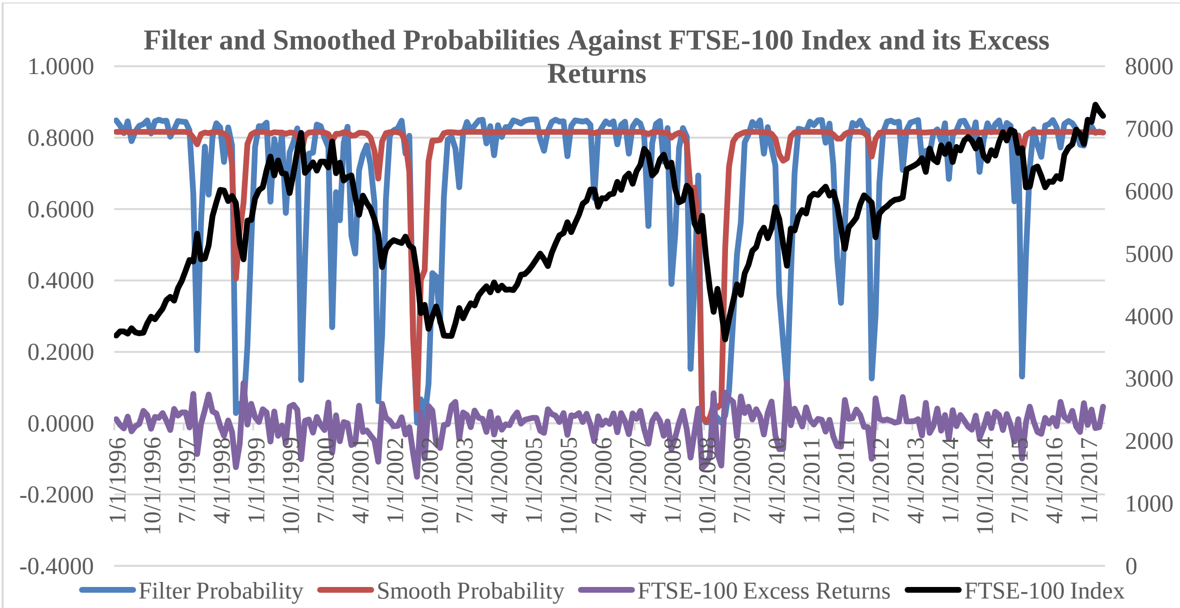


Figure 4.6-2: Filter, Smooth Probabilities, FTSE-100 Index and FTSE-100 Excess Returns (FTSE-100 Index is plotted using secondary axis on the right).

Figure 4.5-2 shows the comparison of filter & smoothed probabilities, FTSE 100 index and the FTSE 100 excess returns and shows that when the market is in Regime 2 (i.e. Filter probability is less than 0.5), FTSE-100 Index bears large negative excess returns. When the market is in Regime 1, the excess returns are generally higher than those in Regime 2.

Step 6: Compute Regime based covariance:

The regime indicators provide guidance as to which regime, market is in and are subsequently used to compute each regime's covariance matrices as outlined in Chapter 3 Equation (22) using the inputs from equations (16) to (21).

Step 7: Equity Portfolio Selection with Regime-switching:

Use the inputs from previous steps to the Mean-Variance quadratic programming problem at the end of each month over out-sample period. The aim is to maximise efficiency of asset allocation provided an initial level of wealth. The portfolio optimisation problem involves decision of allocation of wealth among the portfolio assets and compute the optimal return iteratively by continuously reducing the risk.

Now let us investigate the portfolio selection problem with Regime-switching from a portfolio manager's perspective. According to Markowitz approach of portfolio selection, a portfolio manager should view the return associated with portfolios as random variables, whose probability distribution can be described by their moments, two of which are expected mean and standard deviation. Under the Hamilton Markov Switching, the expected mean and standard deviation vary through time because the random variable could be drawn from two different probability distributions associated with two different regimes. If one were to use the unconditional portfolio efficiency frontiers for the 2 regimes and also of that for the Non-RS, we have following Figure 4.5-3.

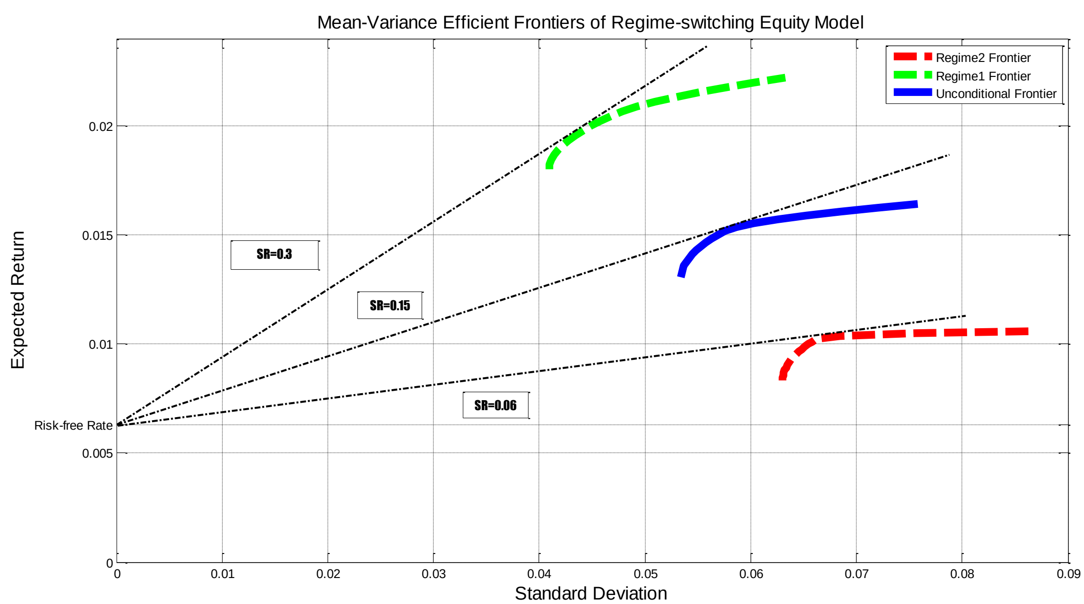


Figure 4.6-3: Mean-variance efficient frontiers RS CAPM model compared with Non-RS efficiency frontier

The dashed curve near the top represents the mean-variance efficient frontier in the Regime 1, while the dashed curve near the bottom represents mean-variance efficient frontier in

Regime 2. The solid curve in the middle is the unconditional frontier implied from a Non regime-switching model based on sample mean and variance for the data period. Of the three frontiers, the one for Regime 1 has the best risk-return trade-off. Intuitively, this is because investor takes into account the likelihood of a bear market regime at next time period is small as regimes are persistent. The Sharpe Ratio[8] along the capital allocation line (the line emanating from the risk-free rate on the vertical axis tangent to the frontier) is 0.3. In the volatile Regime 2, the risk-return trade-off worsens substantially and can only realize a Sharpe Ratio of 0.06. The investor simply has a very different portfolio in the Regime 2, corresponding to the regime-dependent means and covariance. As for the unconditional frontier, the Sharpe Ratio is 0.15 which is between Regime 1 and Regime 2.

Step 8: Calculate the cumulative return:

Calculate the cumulative return on £1 invested in the RS CAPM, Non-RS and MV portfolios over out-sample period and compare them.

Step 9: Repeat step 5, 6, 7 & 8 for different scenarios:

The process is repeated with changes in conditions such as allowing short selling or not and allowing investment in risk free asset or not already discussed in detail earlier. The following sections will discuss in detail every scenario for the data in question.

[8] The Sharpe ratio or Sharpe index or Sharpe measure or reward-to-variability ratio is a measure of the excess return (or risk premium) per unit of deviation in an investment asset or a trading strategy, typically referred to as risk (and is a deviation risk measure), named after Sharpe (1966, 1994).

4.7 Monthly Portfolio Optimization

The results of step 1 to step 8 give us RS CAPM portfolio weights which is used in construction of monthly portfolio. The cumulative wealth for RS CAPM is calculated for out-sample period and are compared with Non-RS and MV strategy.

With 1 GBP investment in out-sample period starting from January 1996. By using the empirical stock return the following month, cumulated wealth is calculated and all the profits are reinvested into the three portfolio strategies.

4.7.1 Cumulated Wealth without Short-Selling Approach

Cumulated wealth is used to measure the performance of all three strategies throughout out-sample period as shown below in Figure 4.6-1 which shows the cumulated wealth of all strategies, without short selling approach.

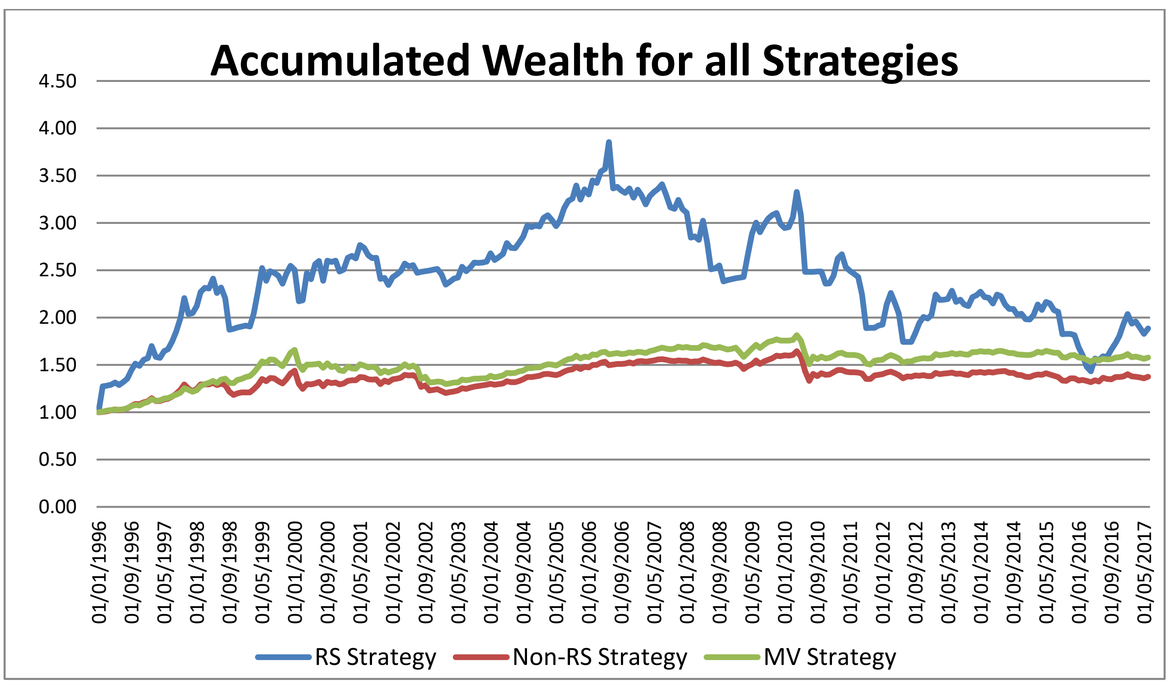


Figure 4.7-1: Cumulated Wealth of Three Portfolio Strategies without Short Selling and No Risk Free Asset

Figure 4.6-1 shows that all the strategies are affected by the shifts in regimes. The cumulated wealth goes up and down along with the shift in regime probabilities. As a whole, all strategies are moving in the same direction (increasing wealth over time) but RS strategy seems to outperform the other strategies with a slight dip in early 2016 but recovering afterwards to end on a relatively higher cumulated wealth against the Non-RS and MV strategies. This performance is largely due to timely change of weights (RS weights) carried out due to change in regime.

By investigating the change in portfolio weights of all three stocks in RS CAPM and Non-RS portfolio strategies, RS weights actively distributes the investment into the stocks on the basis of the inferred regimes and expected returns.

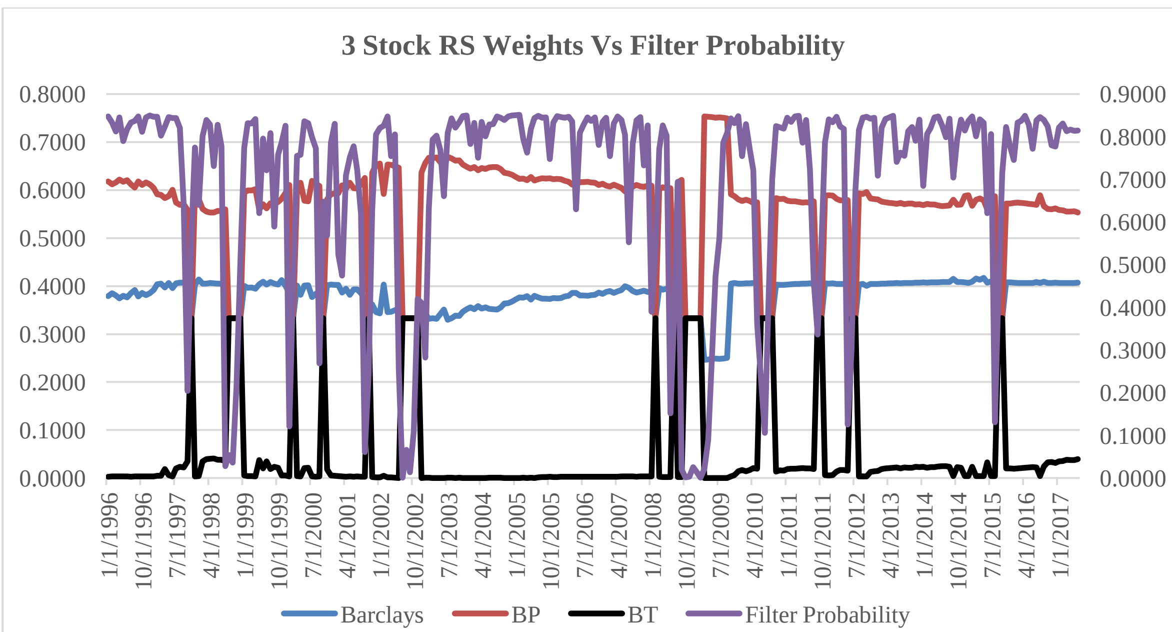


Figure 4.7-2: Filter Probabilities (Plotted on secondary axis on right) versus 3 Stock's RS Weights Scenario 1

The RS CAPM strategy increases the weights of Barc and BP when the market is in regime 1 and reduces their weights when the market is in Regime 2 as can be seen in Figure 4.6-2. RS weight for BT behaves as a protection from bad market conditions. When the market is in Regime 2, it is allocated weight and investment in Barc and BP declines and is allocated nearly zero weight when the market is in good regime (Regime 1) and more RS weight is allocated to Barc and BP.

On the contrary, Non-RS weights do not demonstrate any change to the time dependent investment opportunities captured by probabilities as shown by Figure 4.6-3. Barclays and BP weights do not change significantly when the market was experiencing a major recession. The Non-RS weights show lagging behaviour, i.e., when the market has switched from Regime 1 to Regime 2, Non-RS started reallocation of weights. Interestingly, Barc and BP have been allocated exactly opposite weights with BT mostly allocated zero weight. BT is allocated weight when market changed to Regime 2 and allocated maximum weight only after market has moved back to Regime 1.

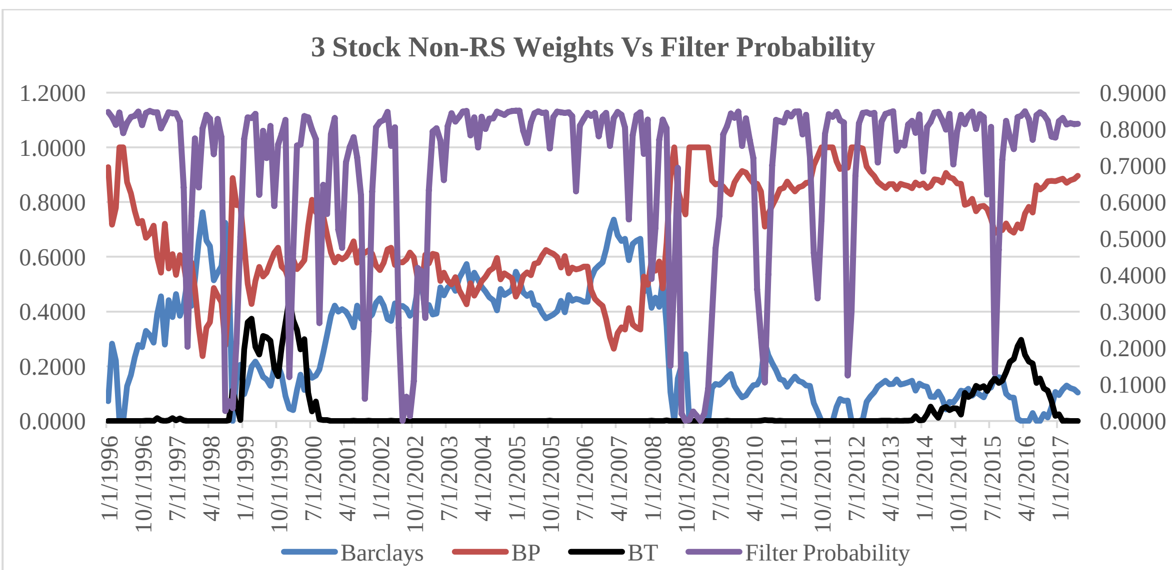


Figure 4.7-3: Filter Probabilities versus 3 Stock's Non-RS Weights

Figure 4.6-4 shows the RS weights versus the stock prices of Barc, BP and BT. RS weights of Barc and BP demonstrate that on several occasions the weights increase/decrease much earlier than the stock prices peaks/collapses.

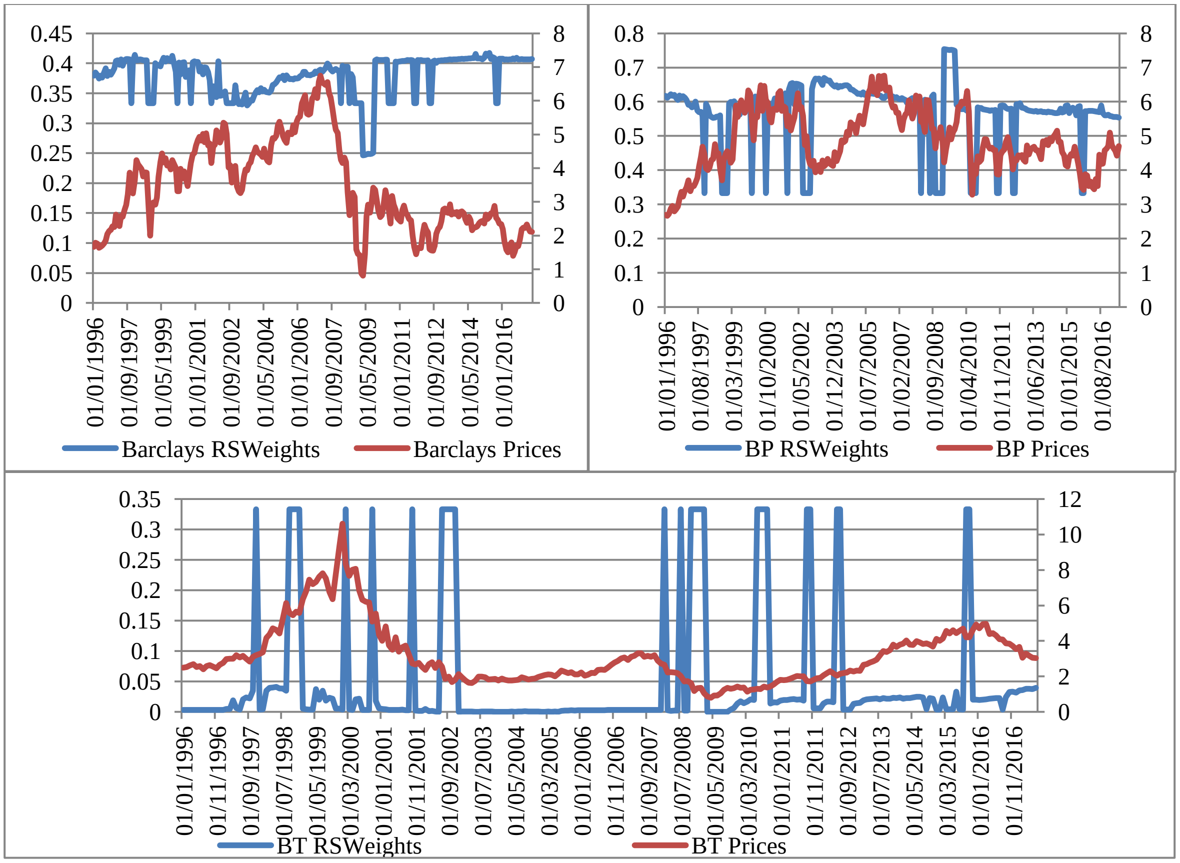


Figure 4.7-4: Stock Prices versus RS Weights

The RS weights change before time due to regime indicator informing of the change to come. The maximum weight allocated to Barc is 0.42 while to BP is 0.75. In contrast to the Barc and BP, the RS weights of BT do not move with its price and mostly has been allocated zero weight which is evident from Eigenvalue analysis discussed in section 4.....

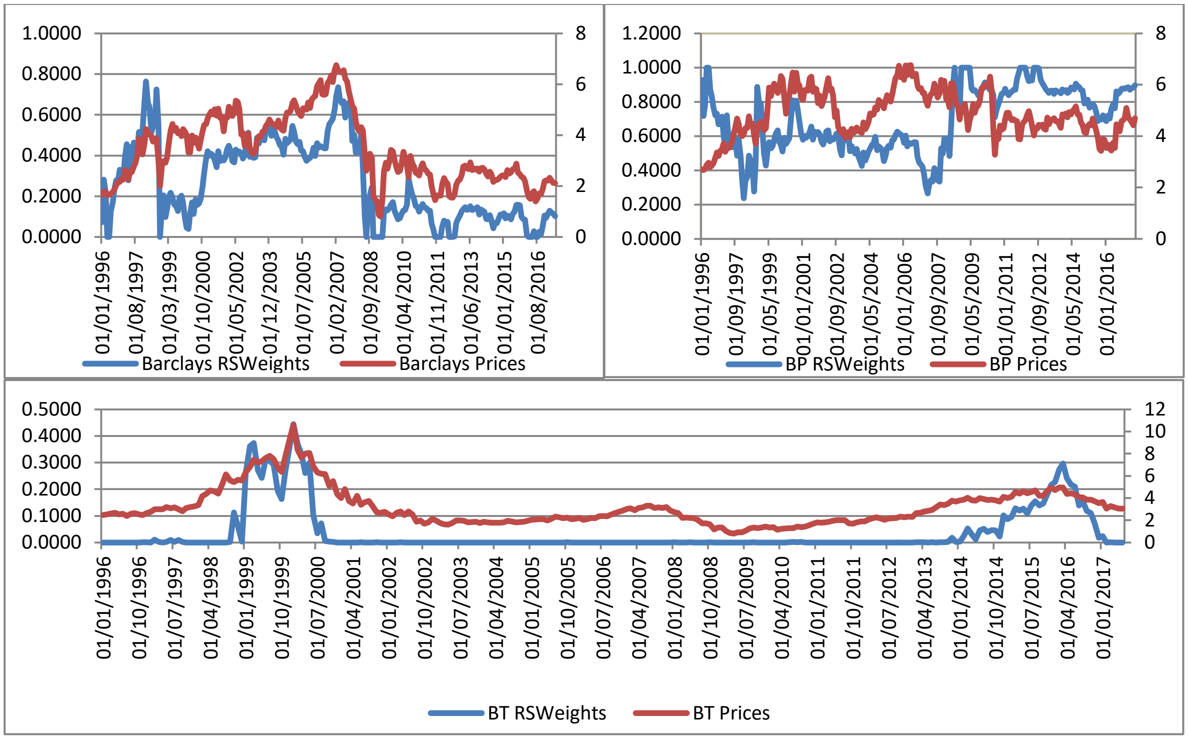


Figure 4.7-5: Stock Prices versus Non RS Weights

Figure 4.6-5 shows the Non-RS weights versus stock prices of the Barc, BP and BT. Non-RS weights, on the other hand, demonstrate the characteristics of a lagging indicator (the Non-RS weights increase/decrease after the prices peaks/collapses), this merely follows the trend of the stock prices one step behind.

The above discussion suggests that regime switching portfolio optimization would benefit from actively rebalancing portfolio weights and would capture effectively the changing trends of equity market. Hence, RS CAPM strategy can be proved as forward looking as compared to other two strategies which are backward looking (relying on past events after they have happened).

4.7.2 Cumulated Wealth with Short-Selling Approach

If short-selling is allowed, the RS CAPM strategy performs even better as compared to the Non-RS and MV strategies as shown by Figure 4.6-6 below. The end of period cumulated wealth is higher than the without short selling approach.

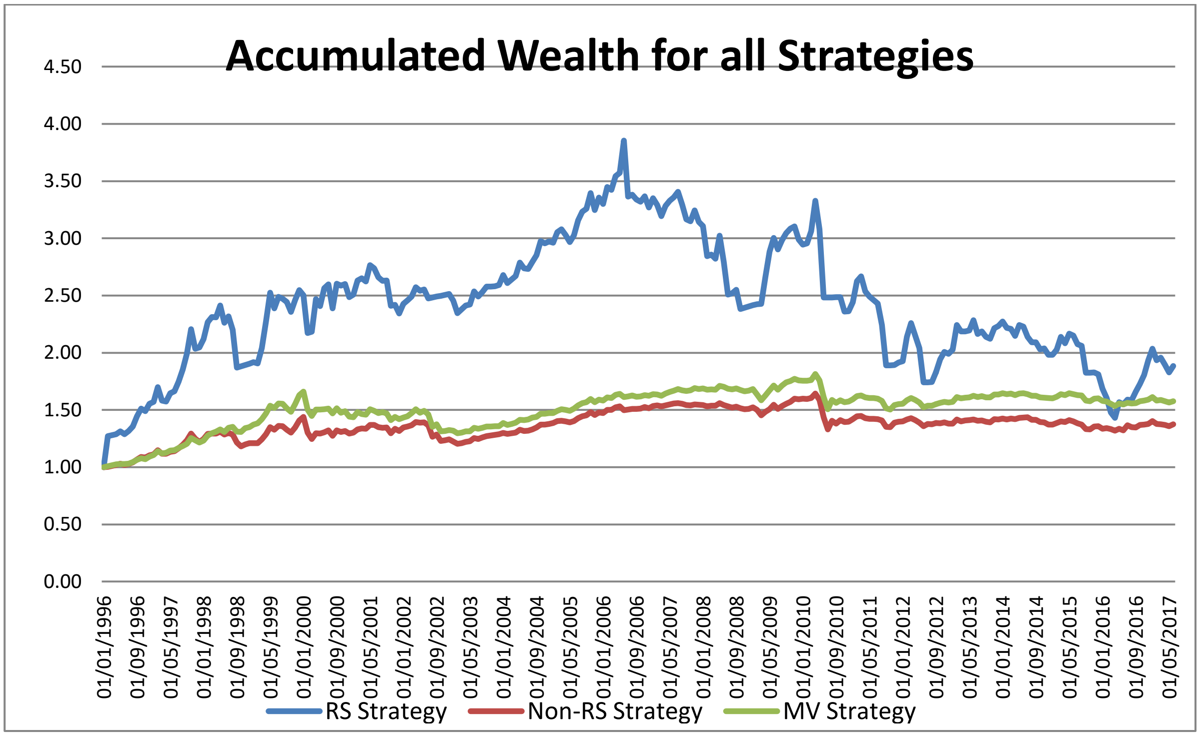


Figure 4.7-6: Cumulated Wealth of Portfolios with Short Selling Approach with Risk Free Asset

Since there is no restriction on short selling, the portfolio weights can go negative or above 1. Short selling is allowed up to 500% (weights range from -5 to 5) short selling. In this case, RS strategy enjoys more flexibility and utilises its capability to infer about regimes and taking the right advantage of its forward looking behaviour by short selling Barc and buying long BP for most of the out-sample period and buying and shorting BT occasionally as can be seen from Figure 4.6-7. The RS weights only took advantage of short selling when the regime switched to bad Regime 2 and during stable periods, RS weights remained more similar to without short selling approach.

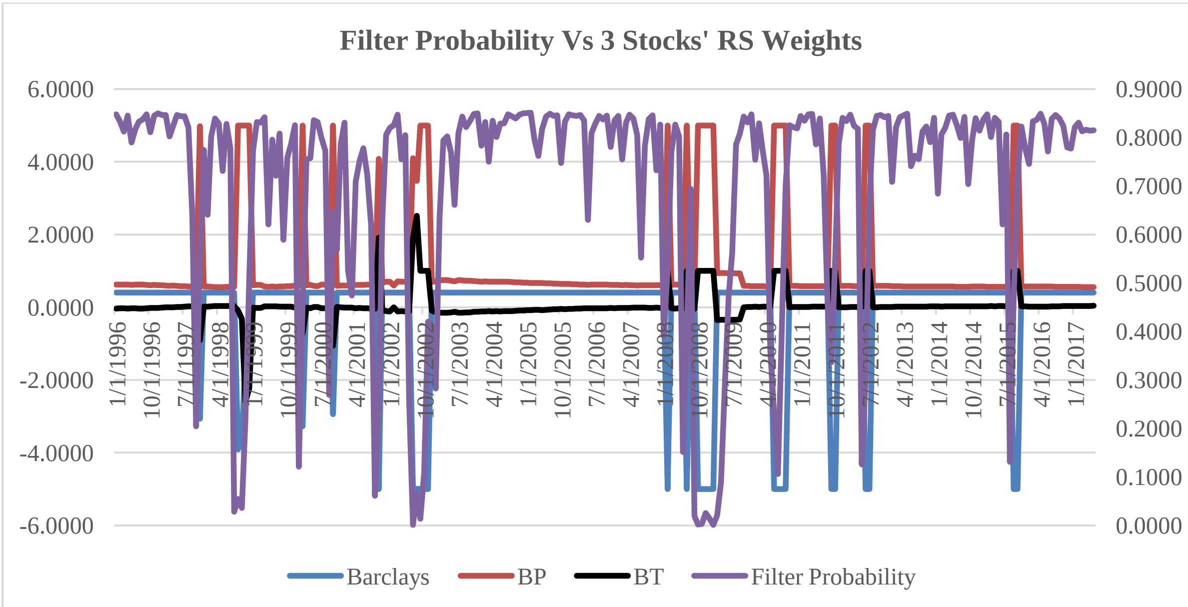


Figure 4.7-7: Filter Probabilities (plotted on secondary axis to the right) versus RS Weights of 3 Stocks

The Non-RS weights instead used BT for short selling regularly and purchased and maintained long position in Barc and BP with untimely changes in weights for all the stocks and ending up with less cumulated wealth than the RS CAPM and MV strategies.

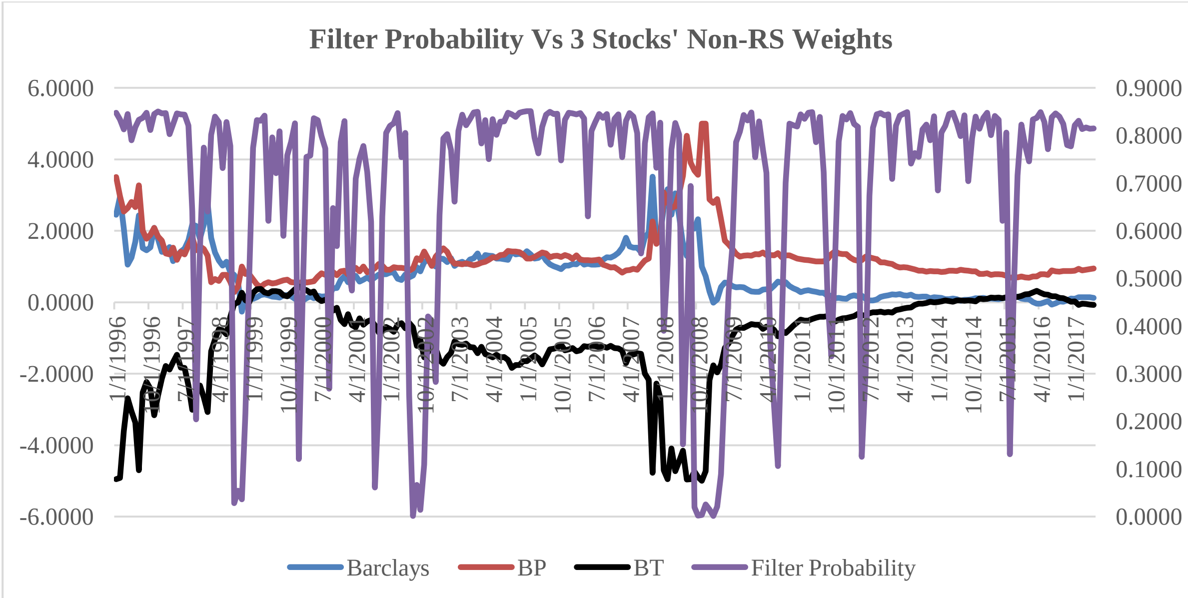


Figure 4.7-8: Filter Probabilities (plotted on secondary axis to the right) versus Non-RS Weights of 3 Stocks

The RS weights of Barc and BP in Figure 4.6-7 show huge decrease/increase when compared with the no short selling approach discussed earlier. But the BT stock on the other hand had been short sold for most of the time in Non-RS weights, shown in figure 4.6-8, and Non-RS do not show any sign of capitalizing on changing investment opportunities with the change in market conditions.

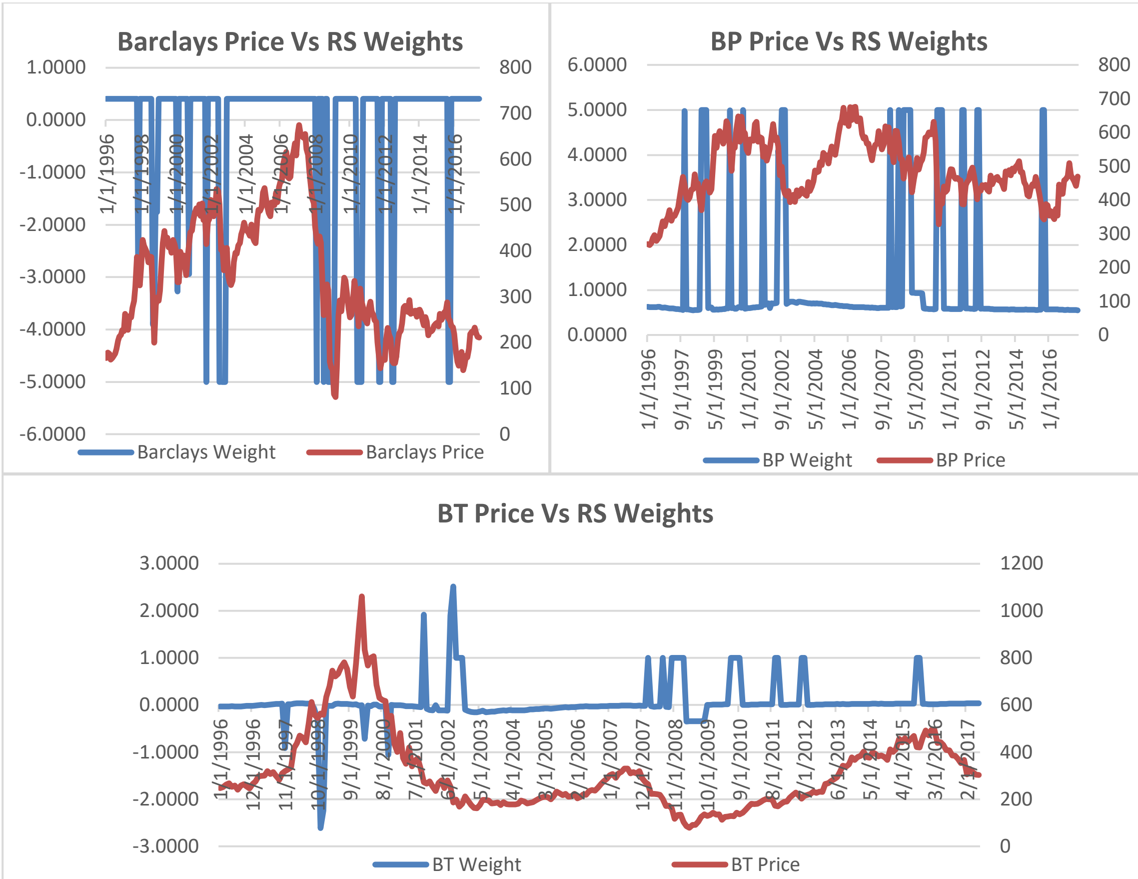


Figure 4.7-9: Stock Prices versus RS Weights

Barc has been short sold when its price was decreasing and purchased when its price increased (Figure 4.6-9, top left). BP has been purchased when its price was increasing and sold when its prices started to fall as can be seen from Figure 4.6-9 (top right). BT (Figure 4.6-9 bottom) was short sold for a very short period of time when its price was decreasing and purchased occasionally when its price was going up, rest of the periods it was nor purchased nor short soled. The short selling opportunity makes the forward looking approach of RS weights of Barc, BP and BT more prominent. When the stock price falls, the RS weights can actively change before the actual decline in stock prices evident from Figure 4.6-9.

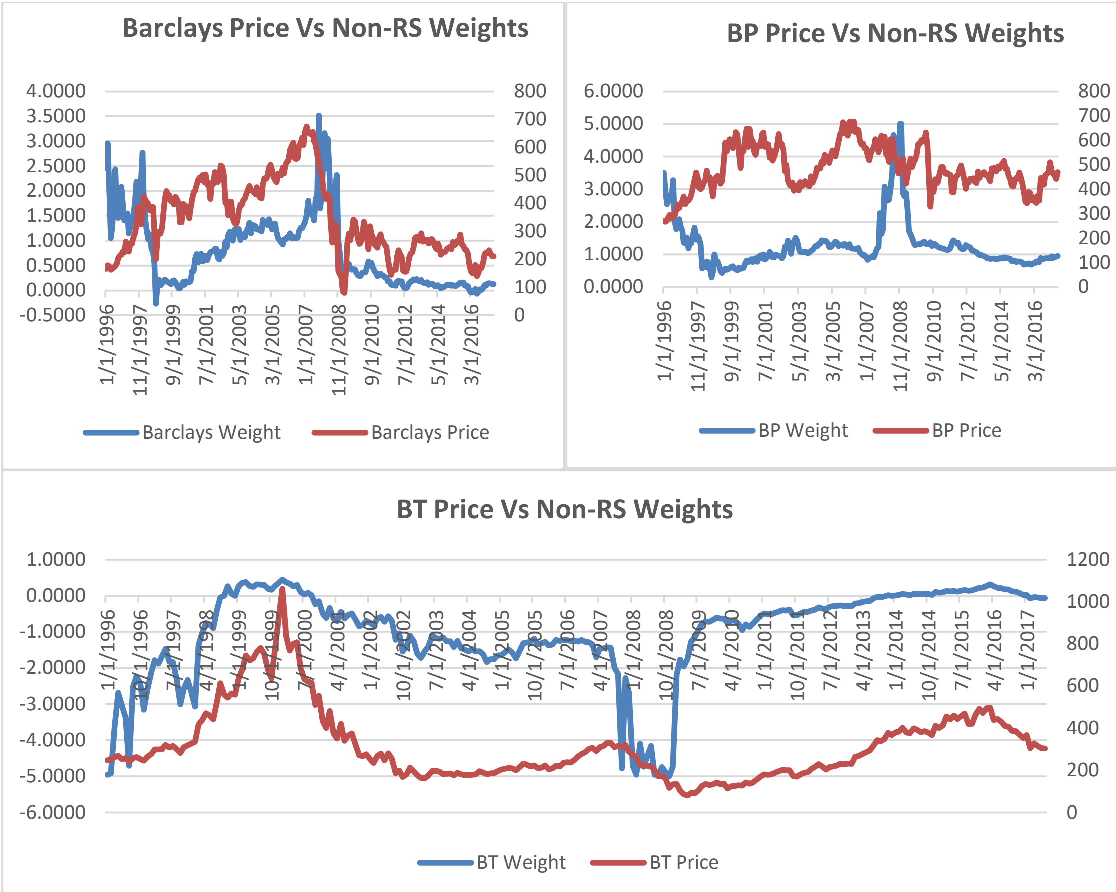


Figure 4.7-10: Stock Prices versus Non-RS Weights

Figures 4.6-10 above shows portfolio weights for Non-RS strategy versus the respective stock' prices during out-sample period. Non-RS weights as shown by figure 14, show clear evidence of being lagging behind as when the actual change in price has occurred only then Non-RS weights change. For example, Barc Non-RS weight in mid-2007 was at maximum (3.52) when actually its price had already started to decline, same is true for BP around mid-2008 when its Non-RS weight is 5 (meaning maximum value used to purchase it) when actually its price has declined significantly. Also according to Non-RS weights for BT, which is being short sold while its price is increasing and vice versa. Such wrong bets deem the Non-RS unsuccessful strategy so that it is performing even lower than the MV strategy.

4.7.3 Market Value Weighted Portfolio Weights

As the MV strategy is not affected by the short selling, so it remains same in both strategies discussed above in sections 4.6.1 and 4.6.2. It can be observed from Figure 4.6-11 that due to high market value associated with BP, it is allocated higher weight throughout the out-sample period. Initially BT enjoyed higher weight due to high market valve when compared to Barc but after 2001 positions switched and BT weight became less than the Barc due to low market values associated with it.

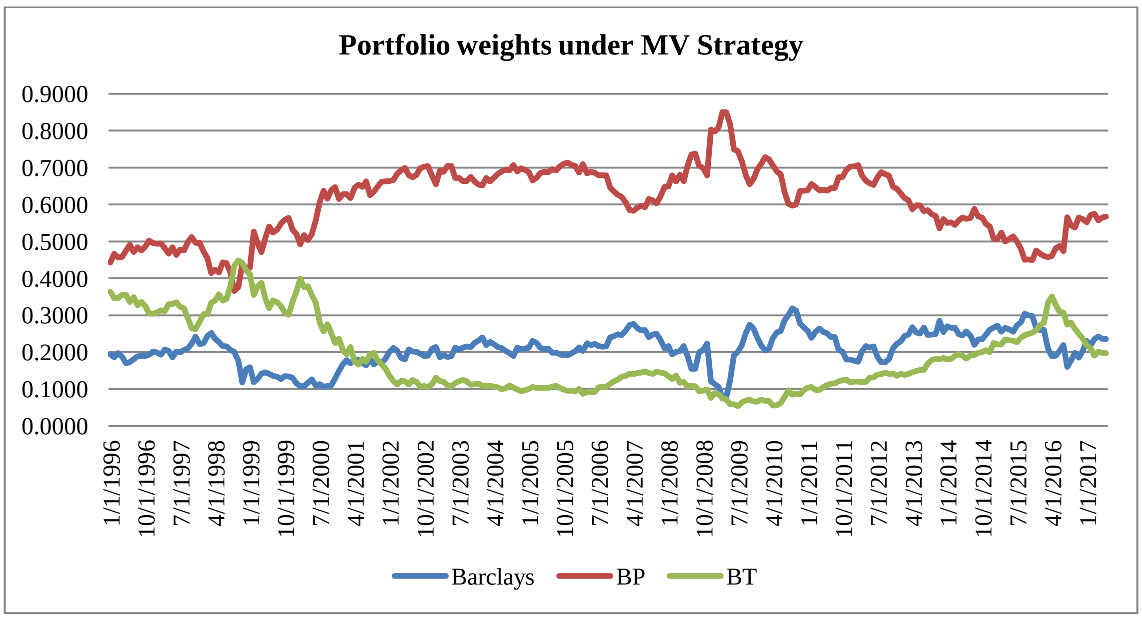


Figure 4.7-11: Market value weights under short selling/without short selling in out-sample period.

4.7.4 Cumulative Wealth with Risk-free Borrowing and Lending

Risk-free asset (Libor) is introduced in the model to provide the investor with an opportunity to freely borrow and lend money at the risk-free rate depending on portfolio strategy and whether short selling is allowed or not. Inclusion of risk free asset also enhances the performance of all the portfolios but once again RS strategy outperforms Non-RS and MV strategies for most of out-sample period. This is evident from Table 6 that in scenario 1 where No short selling is allowed without risk free asset, the cumulated wealth for RS CAPM is 1.174 which is slightly higher than Non-RS cumulated wealth (1.154) but is less than MV strategy (1.388). The RS CAPM cumulated wealth (1.425) is outperforming both Non-RS and MV when risk free asset is introduced in without short selling scenario 2. The point to be noted is that the introduction of risk free asset does not affect much the cumulated of Non-RS strategy which means they fail to capitalise on the risk diversification opportunity arising from the introduction of risk free borrowing and lending. MV strategy has same cumulated wealth as it is not affected by short selling or risk free investment.

Cumulated Wealth and Sharpe ratio for different Strategies	Without Short Selling			With Short Selling		
	RS CAPM Strategy	Non-RS Strategy	MV Strategy	RS CAPM Strategy	Non-RS Strategy	MV Strategy
No Risk Free Asset	**1.174**	**1.154**	**1.388**	**1.915**	**0.975**	**1.440**
Sharpe Ratio	0.2069	0.0599	0.0599	0.2167	0.0601	0.0601
With risk Free Asset	**1.425**	**1.174**	**1.388**	**2.159**	**1.097**	**1.440**
Sharpe Ratio	0.2165	0.0599	0.0599	0.2267	0.0601	0.0601

Table 4.7-1: Cumulated Wealth and Sharpe ratio for different Strategies in different Scenarios on 1st June 2017.

The short selling without risk free asset (scenario 3) generates more cumulated wealth (1.915) than the Non-RS (1.097) and MV (1.440) strategies and performs even better when risk free asset is introduced (scenario 4) with cumulated wealth being 2.159. The key element being usage of risk free asset for borrowing and lending efficiently to maximise the cumulated wealth in out-sample period. This point is strengthened further from the fact that Sharpe ratios calculated for different portfolios show that RS CAPM has highest Sharpe ratios in all scenarios than the Non-RS and MV strategies (as can be seen from Table 4.6-1). The risk free borrowing is used to enhance the performance of RS CAPM by investing more in risky asset as can be seen from Figure 4.6-12. RS CAPM has gone up to 1.3 times investment by borrowing funds at cheap rates to generate more return and minimally to .06 for the investments in risk free asset by lending it and thus generate more returns instantaneously. The sudden switch is in itself explaining that RS CAPM is efficiently and timely switching its borrowing and lending opportunities as compared to Non-RS strategy which is always lagging and in fact has been relying on investment in risk free by lending it and never borrowed to invest more in risky investments to generate more returns.

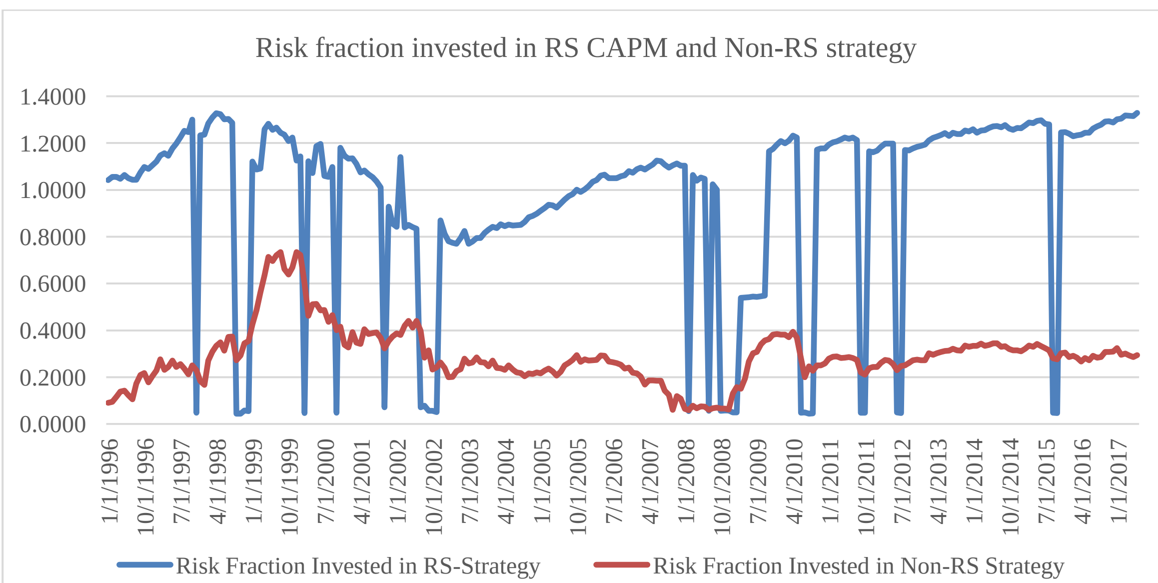


Figure 4.7-12: Portion of investment in risky asset for RS CAPM and Non-RS strategies (With Short-selling)

It can be seen from Figure 4.6-12 that in addition to regime-dependent expectations (i.e. the RS weight change when the regime changes), the RS strategy also exploits sensitive market-timing to risk free borrowing and lending. As explained earlier, the timely change in RS CAPM weights for Barc, BT and BP generated relatively positive returns as compared with Non-RS and MV strategies.

4.7.5 Short Selling With Risk Free Borrowing & Lending (Conditional on Risk Aversion Coefficient) the Risk Aversion Factor

A crucial factor in portfolio optimization problems is the risk aversion coefficient (used in MATLAB) of the individual investor based on his level on risk tolerance. All previous analysis is based on default value of risk aversion coefficient (i.e. 3). The higher its value, the more risk averse investor is and vice versa.

Cumulated Wealth for different Strategies	Short Selling With Risk Free Borrowing & Lending (Conditional on Risk Aversion Coefficients)			Without Short Selling With Risk Free Borrowing & Lending (Conditional on Risk Aversion Coefficients)		
	RS CAPM	Non-RS	MV	RS CAPM	Non-RS	MV
Risk Aversion Coefficient= 4	**1.0630**	**0.5953**	**1.1420**	**0.8054**	**0.7966**	**1.0596**
Sharpe Ratio	0.2065	0.0601	0.0601	0.1206	0.0599	0.0599
Risk Aversion Coefficient= 3	**2.159**	**0.975**	**1.440**	**1.4248**	**1.1536**	**1.3885**
Sharpe Ratio	0.2267	0.0601	0.0601	0.2165	0.0599	0.0599
Risk Aversion Coefficient= 2	**2.2988**	**1.2138**	**1.5990**	**1.8252**	**1.3633**	**1.5640**
Sharpe Ratio	0.2975	0.0601	0.0601	0.2205	0.0599	0.0599

Table 4.7-2: Cumulated Wealth and Sharpe ratio for different Strategies conditional on Risk Aversion Coefficients as on 1st June 2017.

When the risk aversion coefficient is changed from 3 to 2 (i.e. the investor is now least risk averse), the investor invest more in risky assets and generates more end of period cumulative wealth (2.298). As shown in the Table 4.6-2, cumulated wealth of RS CAPM strategy is more than the other two strategies. Most risk averse investor (risk aversion coefficient is 4) either using short selling or without short selling generates less returns for the RS CAPM strategy as compared with MV strategy but is higher than Non-RS.

In other scenarios, RS CAPM cumulative wealth and Sharpe ratio is much higher when compared with Non-RS and MV strategies. RS CAPM Strategy' capability to generate right results during wrong moves of market is the key to the success for every future investor and its market timing ability helps it to switch portfolio weights beforehand as compared to other two lagging strategies in turn performing well in Cumulated wealth in out-sample period.

4.8 Conclusion

This chapter discussed the implication of regime switching (RS) in optimal asset allocation in the United Kingdom (UK) equity market in a CAPM framework. We used data on 3 well known UK assets (Barc, BP and BT) that are part of the FTSE-100 index.

Data showed significant evidence of asymmetric correlation. The bull market had a higher sample mean, lower volatility and lower asset correlations compared to the bear market. This minimised the benefits from diversification when market conditions deteriorated quickly, hence using regime switching became essential for market timing purposes. Other stylised facts of asset returns such as volatility clustering, skewness and kurtosis are also observed in data.

Using the maximum likelihood estimation and the Hamilton Markov switching model we estimated the mean, standard deviation and transition probability for each regime, all of these coefficients were statistically significant at the 0.01% significance level except for the Regime 2 transition probabilities (Q). Both regimes showed evidence of persistence especially Regime 1.

To fully appreciate the benefits of RS CAPM vs. Non-RS and MV portfolios asset allocation, cumulated portfolio wealth after investing £1 in out-sample period is computed and compared results for each strategy. Different scenarios are used, including and excluding short selling, allowing the investor to borrow and lend at the risk free rate. In all the scenarios, cumulated RS CAPM Wealth was greater than that of Non-RS and MV end of period cumulated wealth as shown in Table 4.6-1. In all the strategies, it was also easy to see the fall in cumulated wealth round about the time of the commencement of the credit crisis

2007 but RS CAPM recovered quickly due to its timely weight changes which is only possible due to regime indicator which actively guides investor to switch position in portfolio due to change in market conditions.

5 Chapter 5: Regime Switching Portfolio Optimisation for International Indices: Case of Indian Subcontinent

Abstract

This Chapter expands the literature on emerging markets by evaluating gains from diversifying into emerging equity markets. The price index data from three stock exchanges, Pakistan Stock Exchange (KSE-100), Bombay Stock Exchange (BSE-100) and Dhaka Stock Exchange (DSEX) and MSCI AC World EM Asia (MSCI) as benchmark is used. The theme is to continuously rebalance the portfolio from the weights generated by the regime switching (RS CAPM) model and compare them with simple mean-variance portfolio (Non-RS) and market value portfolio (Market value (MV) weighted portfolio). In previous chapter, RS CAPM has been out rightly successful but the dataset used in this chapter support the argument that RS CAPM performs better only at the end after the crises of 2009 as compared with other two strategies. MV strategy is the most successful until 2009 and it is only after that the RS CAPM strategy takes over and outperforms the Non-RS and MV strategies. The justification is lack of correlation of DSEX with MSCI and minimal correlation between each other. Only BSE and KSE are correlated with MSCI with BSE being highly correlated with MSCI benchmark index. Bekaert et al. (1998b) argued that standard mean-variance analysis is somewhat problematic with respect to emerging equity markets that is why we needed some other technique say Regime Switching and MV to compare with Non-RS method and this study proves their conclusion as RS CAPM and Non-RS underperformed compared with MV strategy till 2009. The markets after that have been more active and have been able to attract more international investors due to benefits of diversification. This also confirms that these markets are less efficient and least correlated with that of international markets and have been least affected during the crises periods as compared to other markets.

Key Words: Regime Switching, International Portfolio, Subcontinent Stock markets, MSCI, Optimization etc.

5.1 Introduction to Emerging Indexes

Conventional wisdom gives two rationales for investing in the stock markets of developing countries. The first states that the low correlation of developing-country stock returns with those of developed markets and provides diversification opportunities that enable investors in developed countries to increase the expected return on their portfolio while reducing their risk. The second states that high rates of economic growth in emerging markets provide great absolute investment opportunities. Because the rate of economic growth in most developing countries is expected to exceed the rate of growth in the developed world for many years to come, the typical discussion presumes that long-run stock returns in emerging markets will also exceed those of developed markets (Malkiel and Mei, 1998; Mobius, 1994).

Much of the research in finance focuses on the most efficient markets in the world, in particular, the US and other G-7 markets. Bekaert & Harvey (2002) argue that the conditions of these markets are most likely to be consistent with the assumptions of theoretical models used in this study. Emerging equity markets provide a challenge to existing models and beg the creation of new models. While the data are not nearly as extensive, it is better for the empiricist to use what is available than to use nothing. Such work demands extensive robustness tests given the limited nature of the data. Given the relation between finance and the real economy, the research we do in emerging markets has a chance to make an impact beyond the particular equity markets that we examine. For example, in many of the emerging markets, the impact of a lower cost of capital (and its subsequent impact on economic growth) can be measured not just in dollars—but in the number of people that are elevated from a desperate subsistence level to a more adequate standard of living (Bekaert & Harvey, 2002).

Bekaert et al. (1998) argue that standard mean-variance analysis is somewhat problematic with respect to emerging equity markets. This is because emerging market returns cannot be completely characterized by expected returns, variances, and covariances, as they exhibit significant skewness and kurtosis. Since it is reasonable to assume that investors have preferences pertaining to skewness and kurtosis (see Rubinstein, 1973; Kraus and Lichtenberger, 1976; Scott and Horvath, 1980; Harvey and Siddique, 2000). We should emphasize the return distribution's higher moments.

Because the capital growth rate is affected by the higher moments of the return distribution, optimizing capital growth given a certain risk tolerance implicitly takes all moments of the return distribution into account (Hagelin & Pramborg, 2004).

Emerging equity markets exhibit high degrees of volatility. Absent rebalancing, this volatility can substantially change the portfolio composition over time. To the extent that this change reduces a portfolio's diversification, the portfolio is not only riskier, but it also is likely to earn a lower geometric mean rate of return. Investors in volatile emerging equity markets must be cognizant of the effects that this volatility has on wealth cumulated over time. Cross-sectional diversification has a dramatic effect on the geometric rate of return earned in these markets. This result was first identified by Wilcox (1997) and confirmed by Eaker et al. (2000). Eaker & Grant (2002) show that the degree of diversification also depends on the frequency that portfolio are rebalanced. Their study shows that there are dramatic gains in wealth accumulation as the rebalancing frequency increases from never, to biannually, annually and semi-annually, but that more frequent rebalancing actually decreases the geometric mean return.

Korajczyk (1996) suggest a measure of the deviations from the law of one price (LOP) across potentially segmented capital markets. This measure is applied to stock returns from twenty-four national markets (four developed markets and twenty emerging markets). The measure of market segmentation tends to be much larger for emerging markets than for the developed markets, a result consistent with larger barriers to capital flows into or out of the emerging markets. The measure often tends to decrease through time, a result that is consistent with growing levels of integration. Large values of adjusted mispricing also occur around periods of economic turbulence and periods in which capital controls change significantly. Thus, the adjusted mispricing estimates measure not only the level of deviations from the LOP but also the revaluations inherent in moving from one regime to another.

Demirguc, and Levine (1996) investigate the cross-sectional relation between mispricing and other indicators of capital market development. They find that mispricing (without the bias adjustment) is negatively correlated with the size (market capitalization) and trading volume of the respective markets and is positively related to market volatility and concentration. Levine and Zervos (1993, 1995) find that the mispricing measure proposed is negatively correlated with economic growth and that the levels of adjusted mispricing decline after liberalization of restrictions on capital flows.

In a detailed study, Iqbal (2012) explains the Pakistani stock market on the basis of liquidity, return on investment, volatility and market concentration etc. This thesis will look at the certain aspects of all three Subcontinent indices and try to explain the importance of these indices and to highlight the need for studying with special reference to RS CAPM portfolio. Two of the Subcontinent stock indices (KSE-100 & BSE-100) were characterised in developing markets by MSCI AC World (2017) and DSEX as Frontier Index due to increased market size and performance. The number of companies listed from 2005 to 2011 has increased for India but the Pakistan and Bangladesh stock markets saw decline in listed companies (as can be seen Figure 5.1-1). The reason attributed to this decline in Bangladesh and Pakistan is the imposition of tough regulatory restrictions for the companies to remain listed on stock exchanges.

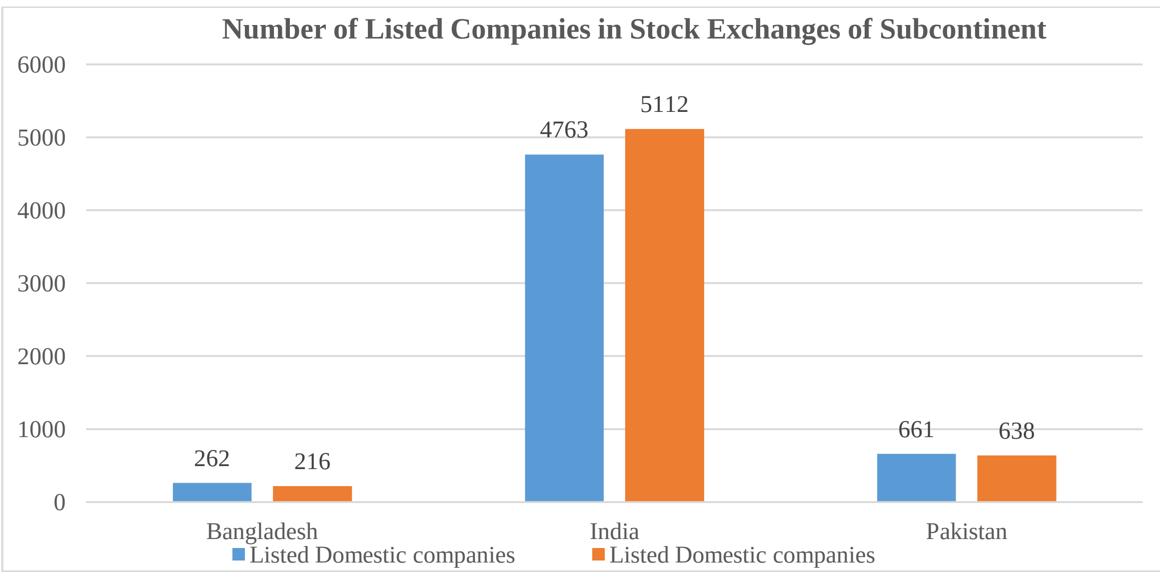


Figure 5.1-1: Number of Listed Companies in Stock Exchanges of Subcontinent (Source: The World Bank, World Development Indicators 2012, Stock Markets pp. 296-300)

Liquidity refers to ease in buying and selling securities. Liquid markets greatly facilitate the role of stock markets as channelling savings to investment for future economic growth (Iqbal, 2012). Two frequently used liquidity measures are ‘Value Traded’ and ‘Turnover Ratio’. The latter being the ratio of dollar value traded to market capitalization.

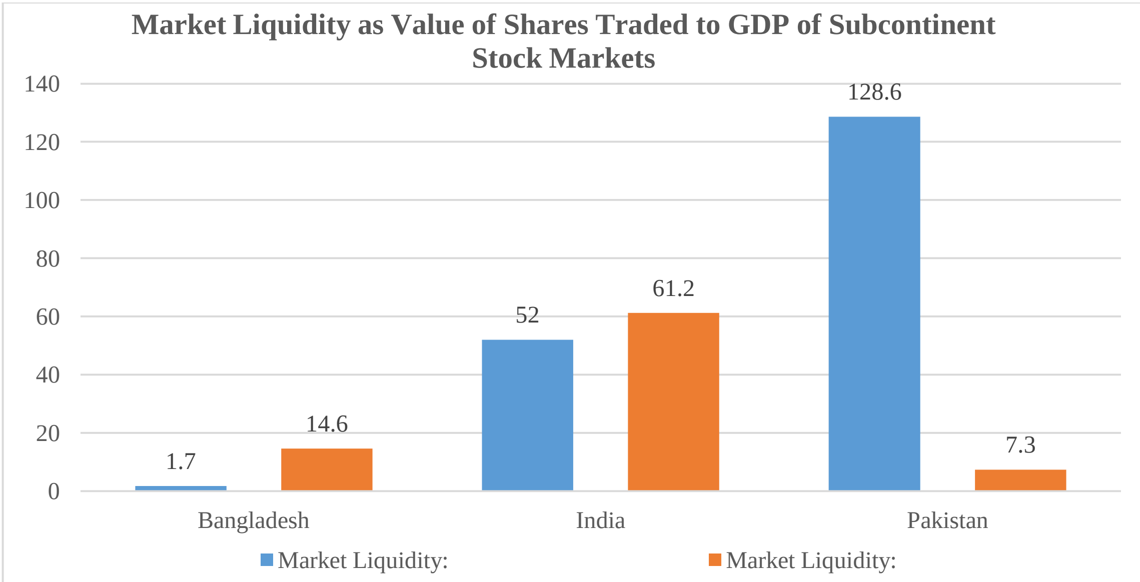


Figure 5.1-2: Market Liquidity as Value of Shares Traded to GDP of Subcontinent Stock Markets (Source: The World Bank, World Development Indicators 2012, Stock Markets pp. 296-300)

Figure 5.1-2 shows the market liquidity measured as value of shares traded to GDP and can be seen that once again Pakistan stock market's liquidity was high as 128.6% in 2005 which declined to 7.3% in 2010, lowest among Subcontinent countries. Comparatively India and Bangladesh stock market became more liquid with Bangladesh stock market showing many times increase from 1.7% to 14.6% and India showing more stable increase in market liquidity (from 2005 to 2010).

Another measure of liquidity is market capitalisation as percentage of GDP and here too Bangladesh stock market is showing almost 3 times increase from 2005 to 2010 with Indian stock market increasing almost 40% but Pakistan stock market once again shows decline in liquidity from 41.9% to 21.6% as shown in Figure 5.1-3.

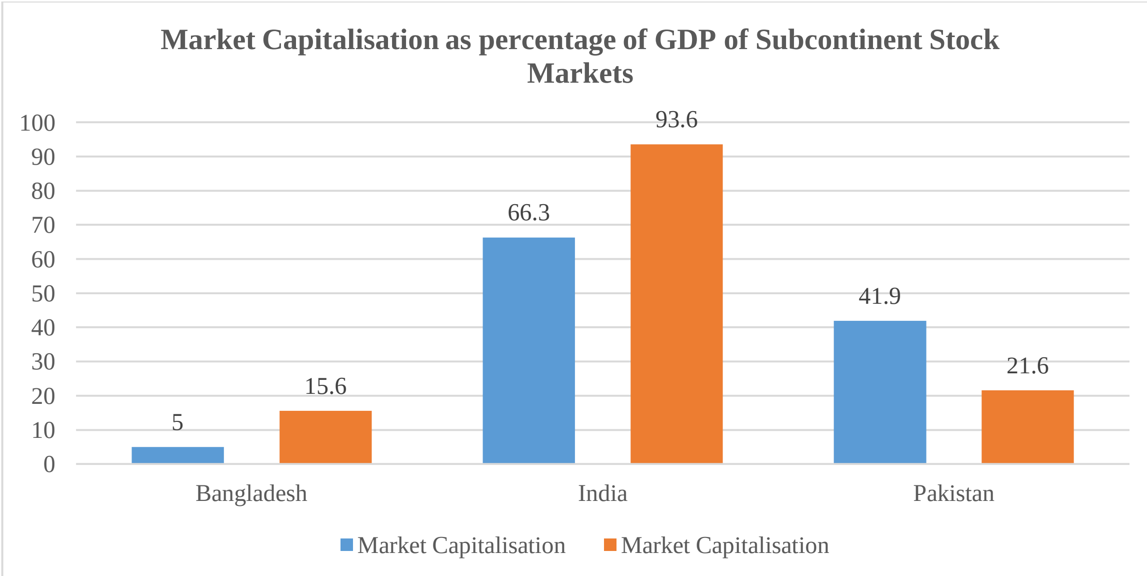


Figure 5.1-3: Market Capitalisation as percentage of GDP of Subcontinent Stock Markets (Source: The World Bank, World Development Indicators 2012, Stock Markets pp. 296-300)

Figure 5.1-4 presents the Turnover Ratio for 3 subcontinent countries. In 2005, the Turnover Ratio of Pakistan's stock market was the highest among the 3 subcontinent countries. Among the main reasons of high trading activity in 2005 are high GDP growth rates, low interest rates, relatively stable political conditions and injection of liquidity in the form of remittances by overseas Pakistanis who are relying more on formal banking channels following a global ban on informal means of money transfer. India has almost 1/4th the Turnover ratio as compared with Pakistan and almost 3 times greater than Bangladesh in 2005 but in 2011, the Bangladesh stock market has grown almost 3 times (on the basis of Turnover ratio) with almost 39% decline in Indian stock market's turnover ratio with Pakistan showing huge decline in Turnover ratio from 376.3% to 28.6%.

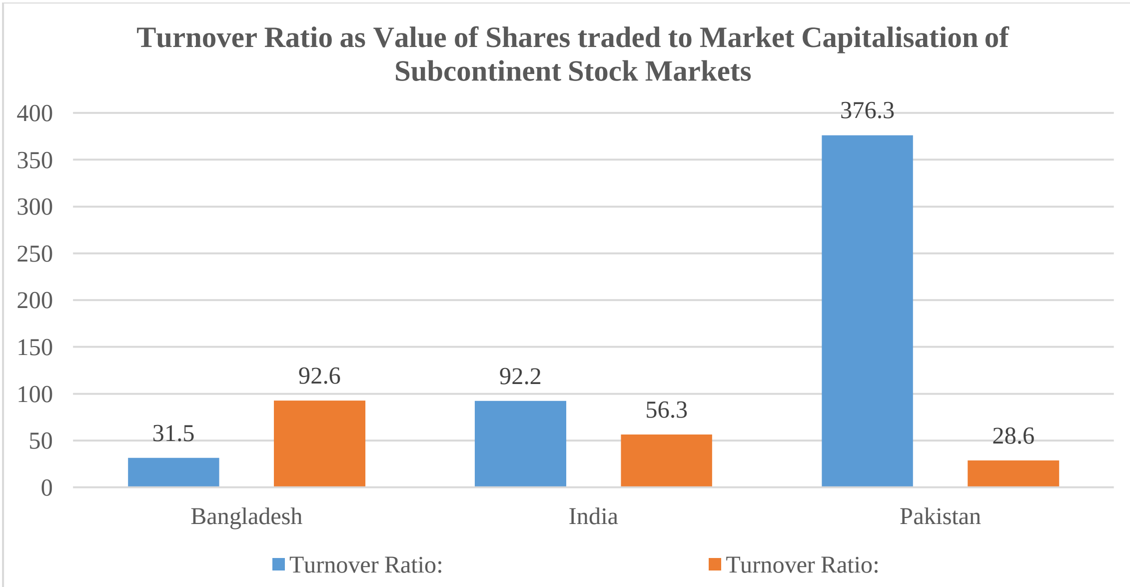


Figure 5.1-4: Comparison of Turnover ratio of Bangladesh, India, & Pakistan's stock market (Source: The World Bank, World Development Indicators 2012, Stock Markets pp. 296-300)

Aggregate return on equity and dividend yield can be used to gauge profitability of investment in emerging markets. Iqbal (2012) concluded on the basis of median return on equity that Pakistan and India are leading the group of developing markets.

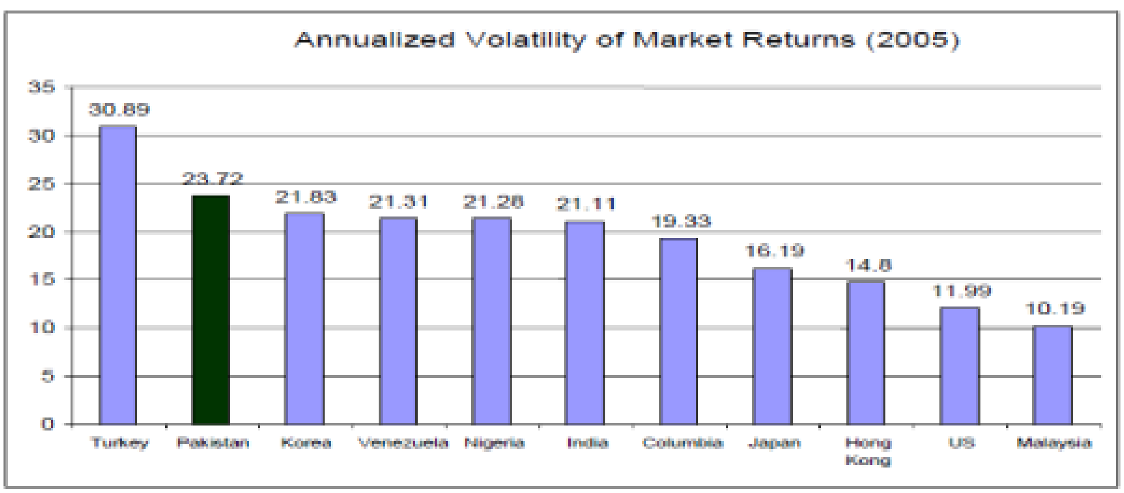


Figure 5.1-5: Comparison of the Volatility of Pakistan & India Stock Market to the selected countries (Source: Financial Indicators, World Bank Group Private Sector Resources Database, 2007)

To know whether the high returns in Pakistan's stock market are associated with high level of risk in 2005, Figure 5.1-5 shows that while volatility in Pakistan's stock market is high (23.72%), the difference from most other emerging markets is not large. India is in the middle and more developed markets are at the other end due to symmetry of information.

Market concentration means the dominance of few stocks in the market. This is an undesirable characteristic of stock markets. Market concentration cause additional risk since the poor performance of a few firms can damage the value of the entire market.

Additionally, in emerging stock markets some sectors may be represented in very different proportions from their share in the economy which may be an indicative of the market not being driven by macroeconomic fundamentals. Market concentration is much higher for Pakistan when compared with India and Bangladesh showing maturity of market (Iqbal, 2012).

5.2 Regime Switching Model using Subcontinent Indices Portfolio:

Regime switching (RS CAPM) is adopted which is similar to that used by Ang & Bekaert (2004) and Markose & Yang (2008) and is discussed in Chapter 3 of methodology. The steps in analysis and discussion will be same as previous Chapter 4 starting with sample data description.

An initial investment of one unit of currency is made on January 1997 and cumulated wealth in July 2017 is checked. This includes using different combination of scenarios and assess the performance of all strategies and in all scenarios, the cumulated wealth from regime switching (RS CAPM) is mostly outperforming from that of the non-regime switching (Non-

RS) and MV strategy. There is final concluding section that contains a summary of our discussion, as well as recommendations for further analysis.

5.3 Sample Data Description:

The data used in this chapter is the monthly price index value of "MSCI AC World EM Asia (MSCI)" used as benchmark index. The sample period is from January 1990 to August 2017. Monthly data on three Subcontinent stock Indices is also drawn from DataStream. The stock indices KSE-100, BSE-100 and DSEX are taken from Pakistan Stock Exchange (PSX), Bombay Stock Exchange (BSE) and Dhaka Stock Exchange (DSE) respectively. The sample period is composed of 332 observations for each stock. US Interbank 1 month offer rate from London BBA available on DataStream is taken as the risk free (RF) rate. The returns are the difference of log prices and excess returns are calculated as the difference between returns and risk free rate leaving 331 returns. All price indices used here are in common currency denomination i.e. US Dollars and that's why US Interbank offer rate is used as risk free asset.

Total Sample is divided into two periods called in-sample and out-sample. The in-sample period is January 1990 to December 1996 (84 observations) and out-sample period starts from January 1997 and end at July 2017 (247 observations).

5.4 Summary Statistics for 3 Subcontinent Indices and the MSCI AC World EM Asia Index

The summary statistics shown by Table 5.4-1 are for whole sample period. The mean is almost zero for all indices as it is less than 1% with standard deviation of all being nearly similar (around 9.25%) with MSCI having least standard deviation of 7.1%. KSE-100 and DSEX show high excess kurtosis and BSE-100 and MSCI show small Excess Kurtosis. The skewness for all is less than 1 and negative except for DSEX having positive skewness of 0.6862.

R squared value for BSE-100 is comparatively strongest among all three indices (0.3057) showing good positive correlation and KSE-100 shows weak positive correlation with DSEX being very weak positively correlated with MSCI. There is weak positive correlation between KSE-100 and DSEX of 0.2029. DSEX and KSE-100 show minimum Covariance in fact almost zero covariance and almost zero correlation. BSE-100 and KSE-100 show highest covariance but is very weak.

Whole Sample : January 1990-August 2017				
Observations: 332				
Descriptive Statistics	**KSE-100**	**BSE-100**	**DSEX**	**MSCI**
Mean	0.0083	0.0073	0.0048	0.0002
Variance	0.0085	0.0086	0.0089	0.0050
Standard Deviation	0.0922	0.0925	0.0945	0.0711
Excess Kurtosis	6.5965	0.9083	7.2268	1.4212
Skewness	-0.9406	-0.0393	0.6862	-0.4878
<u>OLS Regression</u>				
Intercept	-0.0014	-0.0029	0.0001	
Slope	0.1914	0.4247	0.0177	
Standard Error Slope	0.0689	0.0593	0.0711	
R Square	0.0617	0.3057	0.0006	
<u>Covariance Matrix</u>				
KSE-100	0.0085	0.0017	0.0000	0.0016
BSE-100	0.0017	0.0085	0.0001	0.0036
DSEX	0.0000	0.0001	0.0089	0.0002
MSCI	0.0016	0.0036	0.0002	0.0050
<u>Correlation Matrix</u>				
KSE-100	1.0000	0.2029	0.0026	0.2483
BSE-100	0.2029	1.0000	0.0103	0.5529
DSEX	0.0026	0.0103	1.0000	0.0236
MSCI	0.2483	0.5529	0.0236	1.0000

Table 5.4-1: Statistical Description of KSE-100, BSE-100, DSEX and MSCI for Whole Sample Period

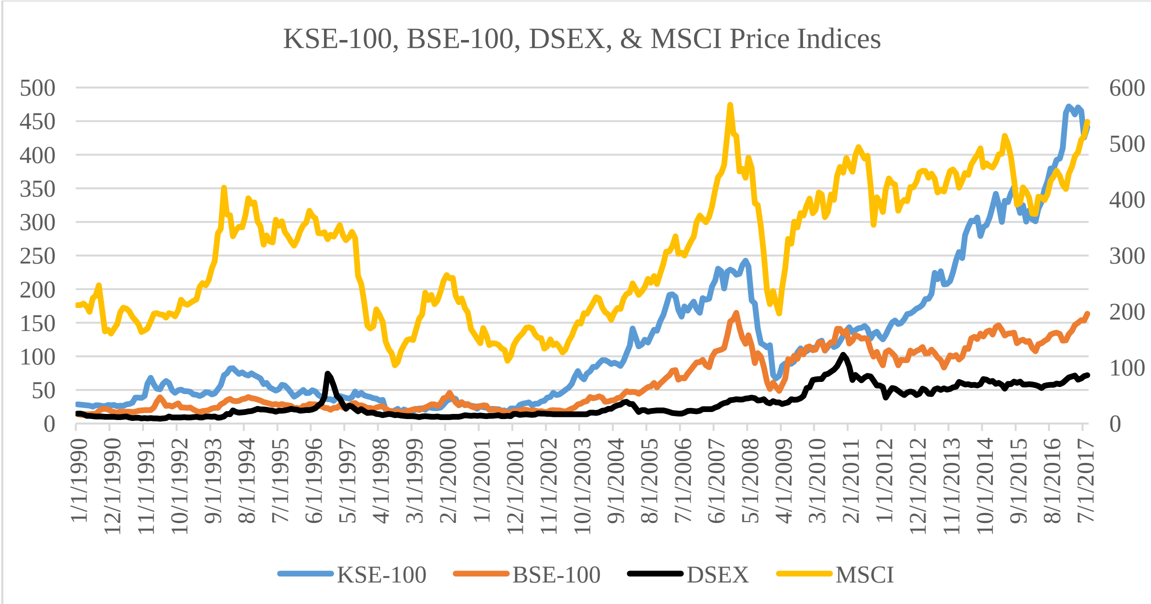


Figure 5.4-1: KSE-100, BSE-100, DSEX, & MSCI Price Indices for whole sample period (MSCI-yellow has been plotted using secondary axis on the right for comparison purposes)

From above Figure 5.4-1, it can be seen that all stock indices are unidirectional except DSEX which behaved oppositely for a good period initially and it is only end where it started behaving more like other indices. KSE-100 has grown in size drastically after 2009 due to stable political regimes. BSE-100 has been shadowing MSCI for whole period and every up and down in MSCI has been well adopted by BSE-100. This co-movement is due to greater integration between BSE-100 and MSCI due to high correlation while KSE-100 and DSEX are helpful in deciding the composition of international portfolios and risk minimization for a rational investor due to their weak correlation with MSCI. Table 5.4-2 below shows highest mean return values for DSEX (1.59%) and standard deviation of 11.54% while lowest mean and standard deviation for MSCI as 0.1% and 6.49% respectively.

In-Sample : January 1990-December 1996				
Observations: 84				
Descriptive Statistics	**KSE-100**	**BSE-100**	**DSEX**	**MSCI**
Mean	0.0020	0.0056	0.0159	0.0010
Variance	0.0079	0.0091	0.0133	0.0042
Standard Deviation	0.0889	0.0952	0.1154	0.0649
Excess Kurtosis	2.9675	1.3201	9.6523	2.6671
Skewness	1.0408	0.4492	2.2305	-0.4108
OLS Regression				
Intercept	0.0007	0.0006	0.0026	
Slope	0.1556	0.0728	-0.1012	
Standard Error Slope	0.0638	0.0650	0.0643	
R Square	0.0454	0.0114	0.0323	
Covariance Matrix				
KSE-100	0.0078	0.0004	0.0004	0.0012
BSE-100	0.0004	0.0089	-0.0010	0.0007
DSEX	0.0004	-0.0010	0.0132	-0.0013
MSCI	0.0012	0.0007	-0.0013	0.0042
Correlation Matrix				
KSE-100	1.0000	0.0503	0.0393	0.2131
BSE-100	0.0503	1.0000	-0.0961	0.1066
DSEX	0.0393	-0.0961	1.0000	-0.1798
MSCI	0.2131	0.1066	-0.1798	1.0000

Table 5.4-2: Descriptive Statistics for the In-Sample period (January 1990 to December 1996)

KSE-100 is weakly correlated to MSCI (0.2131) during in-sample period, with BSE-100 being 0.1066 times correlated and DSEX is weak negatively correlated with MSCI and BSE-100. BSE-100 and DSEX have also negative covariance showing that diversification will definitely help an investor in these markets.

5.5 Correlation computation and discussion

In this section, statistical properties of the correlation matrices of the 3 monthly index returns (empirical sample) from January 1990 to July 2017 are discussed. Correlations are computed from empirical data returns and discuss whether the properties of the correlation matrices as discussed in RMT hold for the empirical data as well as for RS1 CAPM and RS2 CAPM.

5.5.1 Eigenvalue Analysis

Most of the distribution is consistent with the RMT bounds calculated in different studies (e.g. Plerou et Al., 1999, Daly et al., 2010). This comparison also indicates the presence of eigenvalues clearly outside the random matrix bound. Having demonstrated that the bulk of the eigenvalues satisfies RMT predictions.

The Marchenko-Pasteur limits to identify noisy values are given by the Equation (7.3) which is dependent on variable Q= L/N, where L=332 and N = 3. The maximum Eigenvalue limit (λ_{max}) and minimum Eigenvalue limit (λ_{min}) for all the correlation matrices is calculated through Equation (7.3) and their values are;

λ_{min} = 0.8189 and λ_{max} = 1.1992

We use these limits to determine the noisy eigenvalues for the following correlation matrices from empirical sample.

1. Empirical correlation matrix for Non-RS (EmpSample)
2. Regime based correlation matrices further characterised into two sub classes;

a. High volatility based correlation matrix (RS1 CAPM)

b. Low volatility based correlation matrix (RS2 CAPM)

As can be observed in Table 5.5-1 that the largest Eigenvalue for EmpSample and RS1 CAPM is 1.2033 and 1.4440 for RS2 CAPM. As it can also be seen that EmpSample and

RS1 CAPM are same but have 1 Eigenvalue falling within limits and contributing towards 33.33% noisy component as shown by Table 5.5-2.

EmpSample Eigenvalues			RS1 CAPM Eigenvalues			RS2 CAPM Eigenvalues		
0.7970	0.0000	0.0000	0.7970	0.0000	0.0000	0.8182	0.0000	0.0000
0.0000	0.9997	0.0000	0.0000	0.9997	0.0000	0.0000	1.1997	0.0000
0.0000	0.0000	1.2033	0.0000	0.0000	1.2033	0.0000	0.0000	1.4440

Table 5.5-1: Eigenvalues Diagonal Matrix 'D' for EmpSample, RS1 CAPM & RS2 CAPM before filtration

Once the eigenvalues and eigenvectors are computed for the above correlation matrices, the results are examined more closely. The key element was to compute the noisy values given by Marchenko-Pasteur limits.

	EmpSample Eigenvalues	RS1 CAPM Eigenvalues	RS2 CAPM Eigenvalues
Number of Noisy Values	1	1	0
Percent of Total (3)	33.33%	33.33%	0%

Table 5.5-2: Number of Noisy Eigenvalues shown by Marchenko-Pasteur limits before filtering process

RS2 CAPM correlation matrices show that they did not have any noisy values. Then reconstruct the filtered EmpSample and RS1 CAPM correlation matrices by using filtered Eigenvalue diagonal matrix (D_{filter}) and corresponding eigenvectors V as follows;

$C_{filter} = V*D_{filter}*V^{-1}$

Once C_{filter} is obtained, we examine it to check whether its diagonal is similar to original matrix with unit values on the diagonal, if not we repeat the process until original diagonal is obtained. The matrix C_{filter}, once obtained is checked for noise again and is reported in Table 5.5-3 below;

	EmpSample Eigenvalues	RS1 CAPM Eigenvalues	RS2 CAPM Eigenvalues
Number of Noisy Values	0	0	0
Percent of Total	0%	0%	0%

Table 5.5-3: Noisy Eigenvalues shown by Marchenko-Pasteur limits after filtering process

5.5.2 Eigenvector Stock constituents of deviating largest eigenvalues from RMT limits

Literature on Eigenvalue based studies of stock returns suggest that we look into the stock composition of the eigenvector corresponding with the highest Eigenvalue deviation from RMT limits. This helps in identifying sectors and will tell us which companies contributed more to the overall performance of Index (Daly et al, 2007). The following Table 5.5-4 shows the Eigenvalues from all the matrices after filtration process. If the eigenvalues are compared before and after filtering, the maximum Eigenvalue increases in every case after the filtering process indicating an increase in information.

EmpSample Eigenvalues			RS1 CAPM Eigenvalues			RS2 CAPM Eigenvalues		
0.7971	0.0000	0.0000	0.7971	0.0000	0.0000	0.8182	0.0000	0.0000
0.0000	1.1997	0.0000	0.0000	1.1997	0.0000	0.0000	1.1997	0.0000
0.0000	0.0000	1.2733	0.0000	0.0000	1.2733	0.0000	0.0000	1.4440

Table 5.5-4: Eigenvalues for EmpSample, RS1 CAPM & RS2 CAPM after filtration process.

The following Table 5.5-5 shows the eigenvectors for EmpSample and RS1 CAPM showing the weighted contribution of subcontinent indices on MSCI-AP.

EmpSample Eigenvectors			RS1 CAPM Eigenvectors		
0.7064	0.0507	0.7060	0.7064	0.0507	0.706
-0.7073	0.0127	0.7068	-0.7073	0.0127	0.7068

0.0268	-0.9986	0.0448	0.0268	-0.9986	0.0448

Table 5.5-5: Corresponding Eigenvectors 'V' for Eigenvalues for EmpSample and RS1 CAPM

These findings are consistent with Daly et al (2007) who show that largest eigenvalues help describe distinct sectors contributing more towards the performance of constituent index. If the companies highlighted by this eigenvector centrality are used in the formation of portfolios, better performance is bound to follow.

5.6 RS CAPM Empirical Construction

As outlined in the Chapter 3, Steps 1 to 3 of the methodology have been discussed earlier in sections 5.2 to 5.4 explaining the in-sample, out-sample. The step 4 is discussed below;

Step 4: Obtain the RS Statistics for Data:

This section will look at the RS CAPM and RS covariance computation which involves step 4 to step 6 and involves the usage of conditional RS mean (μ1, μ2), variance (σ1, σ2) and covariance (K1, K2) derived from CAPM beta based returns of assets using equations (16), (17), & (22).

Step 4 uses Hamilton (1989) method of obtaining Maximum likelihood regime sensitive parameters i.e., transition probabilities, means, and standard deviations for each regime as described earlier in chapter 3 in order to predict the regime. Table 5.5-1 below shows the values of mean and standard deviation used to identify the regimes and their respective transition probabilities. The results clearly show evidence of regime existence for the data used and can be seen that μ1 > μ2, σ1< σ2, and all the coefficients are statistically significant at 0.01% significance level.

Regime Indicators	Regime 1		Regime 2		Transition Probability	
	μ1	σ1	μ2	σ2	*P*	*Q*
Estimates	0.0003	0.0038	-0.0017	0.01098	0.9900	0.5600
Standard Error	0.0000	0.0000	0.0000	0.0001	0.0064	0.1380

Table 5.6-1: Regime Statistics and Transition Probabilities

Regime 1 is much more stable than Regime 2 in a sense that it depicts less volatility as compared to Regime 2. Transition probabilities show the presence of two different regimes, one being more volatile than the other (regime 1 being more stable than regime 2 as probability of being in regime 1and staying in regime 1 is 0.99).

The simple comparison of regime statistics shows that the volatility in both regimes is much less (0.0038 % & 0.01098%). The volatility for regime indication in Chapter 4 is much larger as compared with this chapter volatilities in both regimes. Another important distinction that can be seen between the Chapter 4 when compared with this Chapter 5 is the value of R squared (Pearson correlation coefficient), i.e., BT, BP and Barc had high correlations with FTSE-100 but here KSE-100, BSE-100 and DSEX had weak correlations in fact DSEX has negative correlation during in-sample period.

Step 5: Filter and Smoothed Probabilities:

In addition to the parameters required for Regime Prediction and their mean-variance, the model can also infer the Regime probabilities i.e. filter probabilities and smoothed probabilities. The filter probabilities indicate the process being in some particular regime at time t based on the information available at the time t-1. In contrast to filtered probabilities, the smoothed probabilities indicate the historical regimes the process was in at time t based on whole sample information and are calculated backwards by using filter and forecasting probabilities.

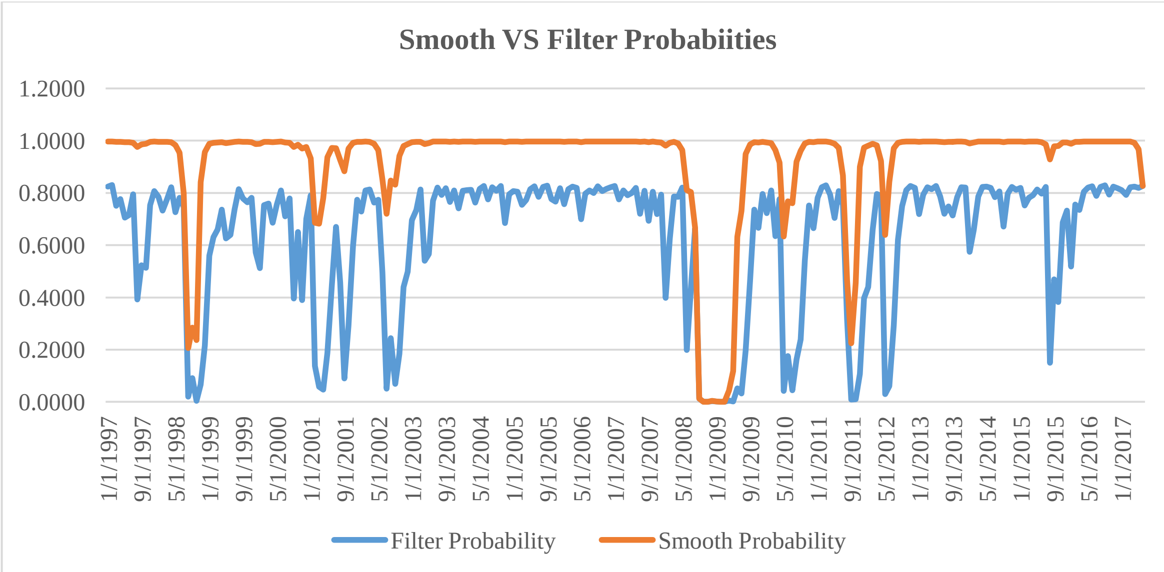


Figure 5.6-1: Smooth & Filter Probability for out-sample period

Figure 5.5-1 shows the transition probabilities of being in one regime against another regime over out-sample period. The RS CAPM model used here assumes that if the probability is greater than 0.50, the market is in regime 1 and that the probability of being in Regime 2 is less than 0.50. Figure 5.5-1 above shows that there are many occasions where probability has come down from cut off, hence showing the probability of being in Regime 2. Looking at the filter probabilities and the cut-off of 0.5, there were 13 regime shifts during out-sample period. Contrary to filter probabilities, smooth probability is suggesting that there had actually been only 3 regime shifts that is why filter probabilities are considered more reliable and this thesis uses filter probabilities for the regime prediction.

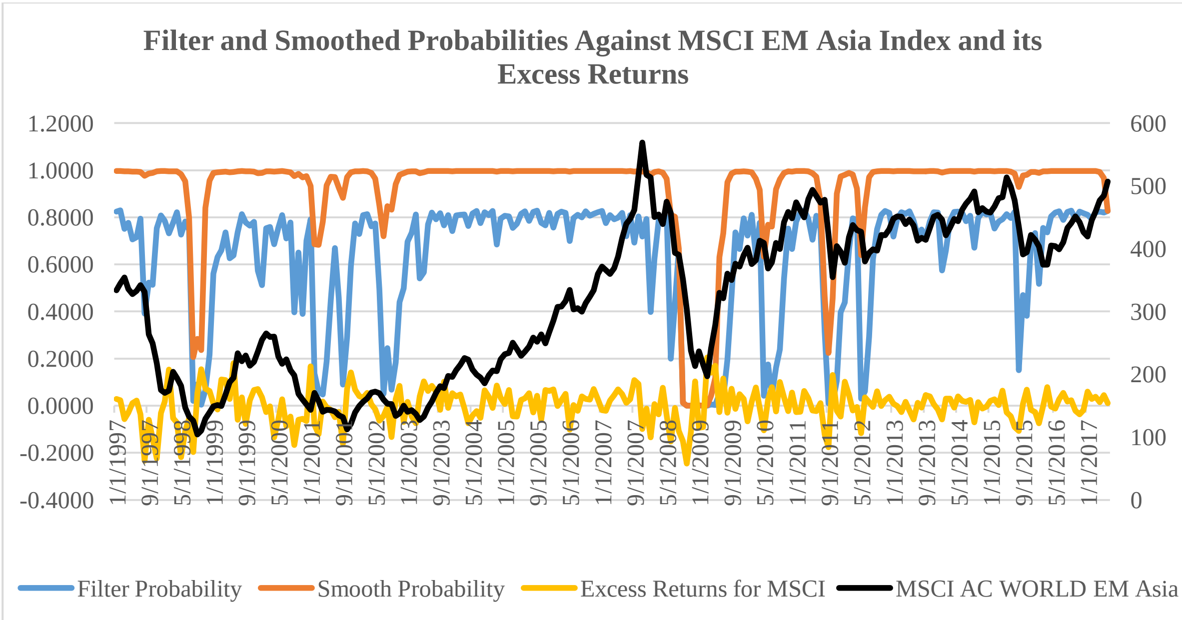


Figure 5.6-2: Filter and Smoothed Probabilities against MSCI EM Asia Index and its Excess Returns (MSCI EM Asia Index is plotted using secondary axis on the right).

Figure 5.5-2 above shows the comparison of filter & smoothed probabilities, MSCI and the MSCI excess returns and shows that when the market is in Regime 2 (i.e. Filter probability is less than 0.5), MSCI Index bears large negative excess returns. When the market is in Regime 1, the excess returns are generally higher than those in Regime 2. It can also be seen that whenever there is decline in MSCI EM Asia, filter probability declines sharply well before time to warn the investor of regime shift. During economic downturns worldwide e.g., Dotcom bubble and post July 2007 financial meltdown, MSCI shows huge decline in returns.

Step 6: Compute Regime based covariance:

The regime indicators provide guidance as to which regime, market is in and are subsequently used to compute each regime's covariance matrices as outlined in Chapter 3 equation 22 using the inputs from equations 16 to 21.

Step 7: Equity Portfolio Selection with Regime-switching:

Use the inputs from previous steps to the Mean-Variance quadratic programming problem at the end of each month over out-sample period. The aim is to maximise efficiency of asset allocation provided an initial level of wealth. The portfolio optimisation problem involves decision of allocation of wealth among the portfolio assets and compute the optimal return iteratively by continuously reducing the risk.

Step 8: Calculate the cumulative return:

Calculate the cumulative return on 1 unit of currency invested in the RS CAPM, Non-RS and MV portfolios over out-sample period and compare them. This process changes slightly for last chapter 7 as it does not use MV strategy.

Step 9: Repeat step 5, 6, 7 & 8 for different scenarios

The process is repeated with changes in conditions such as allowing short selling or not and allowing investment in risk free asset or not already discussed in detail earlier. The following sections will discuss in detail every scenario for the data in question.

5.7 Monthly Portfolio Optimization:

To test the regime switching portfolio optimization and its performance over out-sample period, the following three strategies are employed;

1. Regime switching (RS CAPM)
2. Simple mean-variance optimization (Non-RS)
3. Market Value (MV) weighted portfolio

The in-sample period for our data is January 1990 to December 1996 and out-sample period is from January 1997 to June 2017. Strategy 1 is the RS CAPM proposed earlier. For Strategy 1, presence of regime is judged on the basis of in-sample data and apply the results to get regime switching portfolio weights and construct a monthly portfolio on the basis of RS CAPM weights. Strategy 2 is the single-regime model in which the portfolio is constructed using quadratic programming, but the expected returns and covariance are simply the statistical moments of the time series data. Strategy 3 is to invest into equities according to their market capitalization. The market capitalization of the indices is used to construct the portfolio weights and consequently used to construct MV portfolio.

With 1 unit of currency investment in out-sample data, starting from January 1997, and using the actual stock returns the following month and all the profits were reinvested into the three portfolio strategies. The three different strategies were further divided on the basis of short selling approach and no short selling approach. The No Short selling approach is discussed first and then Short Selling.

5.7.1 Cumulated Wealth without Short-Selling Approach

Cumulated wealth is used to measure the performance of all three strategies throughout out-sample period as shown below in Figure 5.6-1 which shows the cumulated wealth of all strategies, without short selling approach.

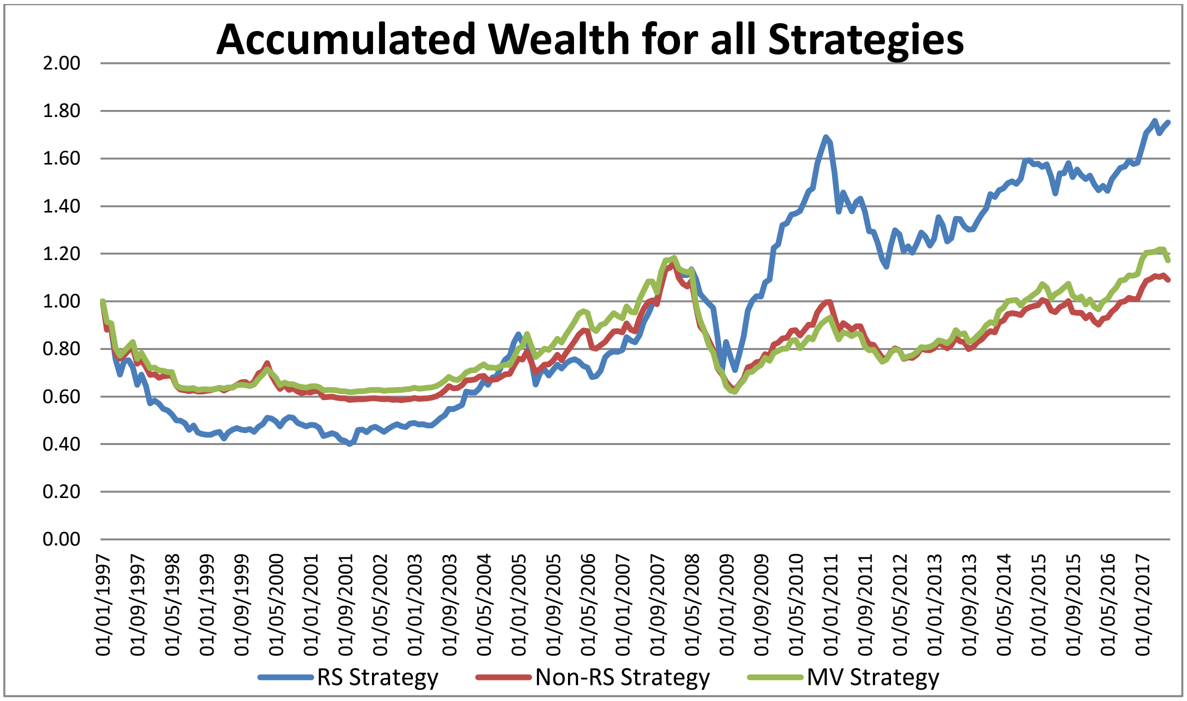


Figure 5.7-1: Cumulated Wealth of Three Portfolio Strategies without Short Selling for out-sample period

Cumulated wealth goes up and down during out-sample period as seen in Figure 5.6-1. As a whole, all strategies are moving in the same direction but RS CAPM strategy seems to underperform the other strategies during the first half of out-sample period except in the second half of out-sample RS CAPM starts picking up and is clearly outperforming the Non-RS and MV strategies towards the end. The reason attributed could be that Subcontinent stock markets started to attract more international investors during this time period and these markets became more integrated with international markets. First half of out-sample period is slow growth period for Subcontinent markets and show less integration with each other in fact negative correlation exist among markets, thus, failed to attract international investors due to the market inefficiencies.

Figure 5.6-2 below shows the portfolio weights of all the Index assets in RS CAPM and it seems that RS CAPM strategy is dynamically allocating resources in the different available options and continuously switching the portfolio weights between KSE-100, BSE-100 and DSEX.

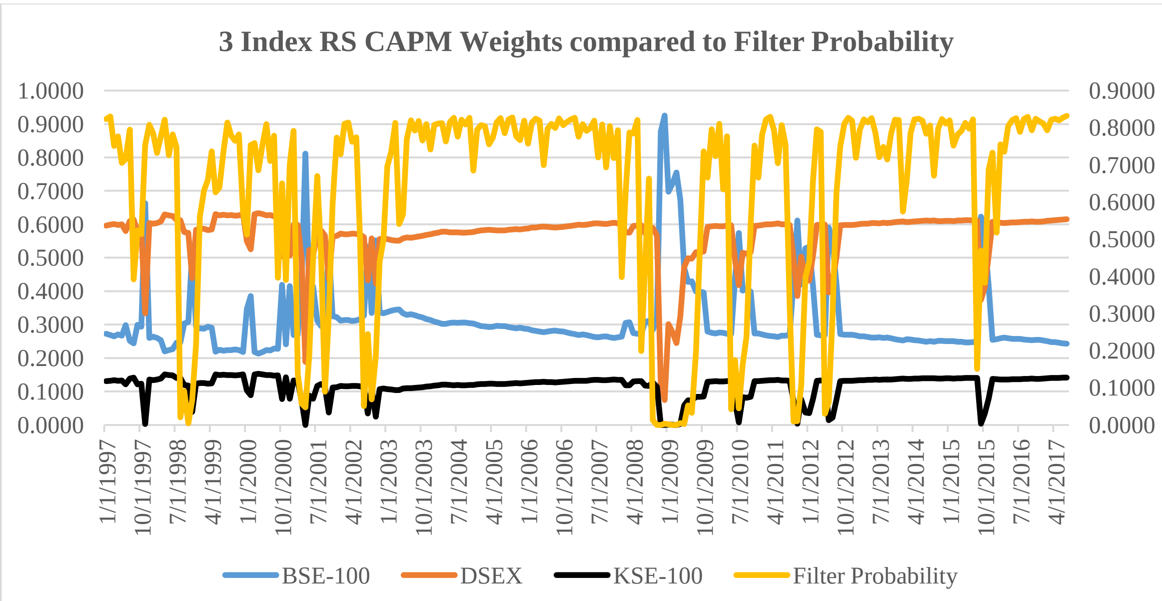


Figure 5.7-2: Filter Probabilities (Plotted on secondary axis on right) versus 3 Index' RS Weights

The RS CAPM strategy invests more in DSEX when the market is in Regime 1 and reduces its weight when the market is in Regime 2 as can be seen in Figure 5.6-2. RS weight for BSE-100 behaves as a protection from bad market conditions i.e., when the market is in Regime 2, it is allocated more weight and investment in KSE-100 becomes zero. The weights switch with every regime change and this timing is key to success in RS CAPM. More weight allocated to DSEX is due to diversification in market risk and as KSE-100 is most unstable of the 3 indices, it is allocated least RS CAPM weight.

The Non-RS weights in Figure 5.6-3, on the other hand, demonstrate the characteristics of a lagging indicator (the non RS weights increase/decrease after the indices peaks/collapses), this merely follows the trend of the indices one step behind. It is obvious that during the Dot Com Bubble, Non-RS strategy allocated all of its resources in BSE-100 and none in KSE-100 and DSEX which negates the portfolio theory's risk diversification principle.

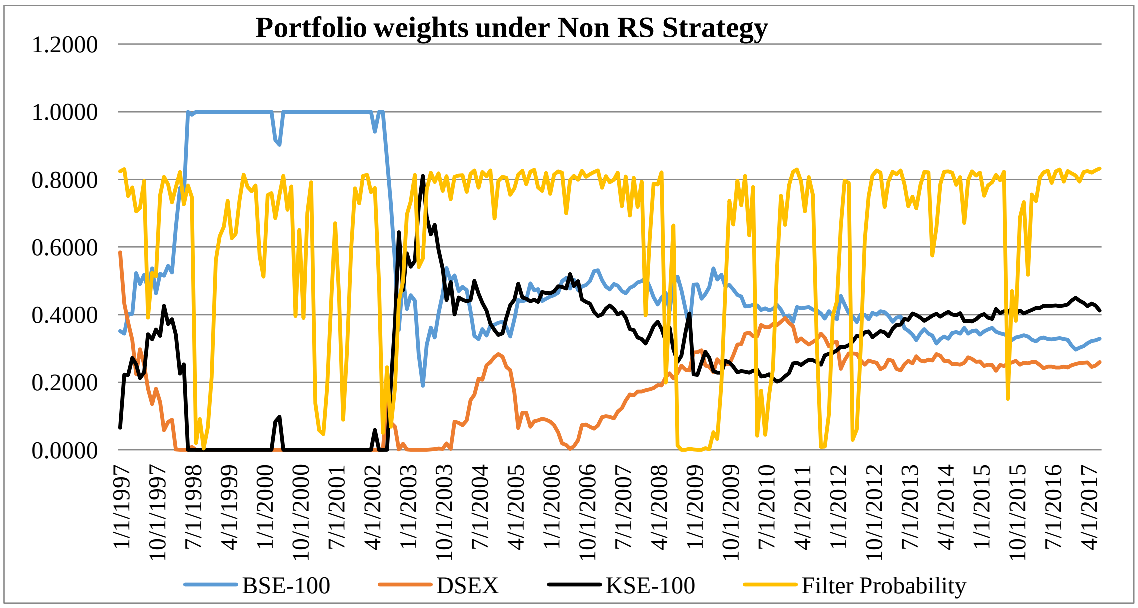


Figure 5.7-3: Portfolio weights under Non RS Strategy for 3 indices and filter probabilities

5.7.2 Cumulated Wealth with Short-Selling Approach

By allowing short-selling, RS CAPM strategy performs much better and cumulated wealth is more when compared with the other two strategies as depicted by Figure 5.6-4 below. The RS strategy picks up more after early 2009 and finishes higher than without short selling's cumulated wealth. Throughout first half of out-sample period, MV and Non-RS strategy performed well by efficiently allocating resources but RS CAPM outperformed after 2009.

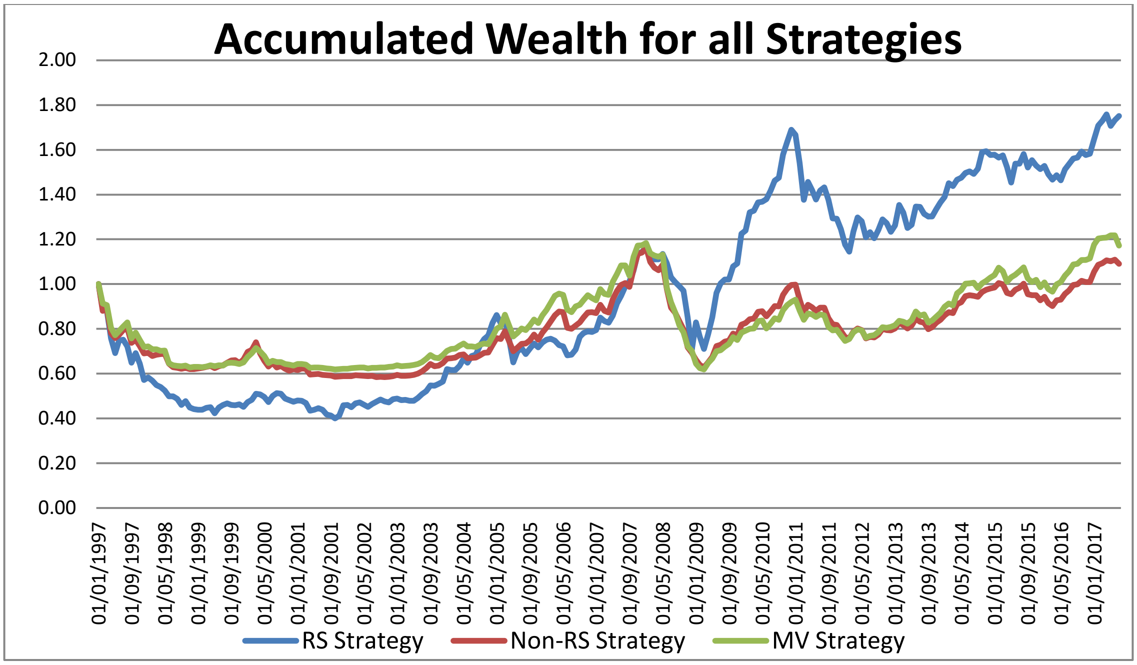


Figure 5.7-4: Cumulated Wealth of Portfolios with Short Selling Approach for out-sample period

Since there is no restriction on short selling, the portfolio weights can go negative or above 1, in fact we allowed it to go from -5 to 5. In this case, RS strategy enjoys more flexibility and utilises its capability to infer about regimes and taking the right advantage of its forward looking behaviour by short selling and buying long as can be seen from the figure 5.6-5 below.

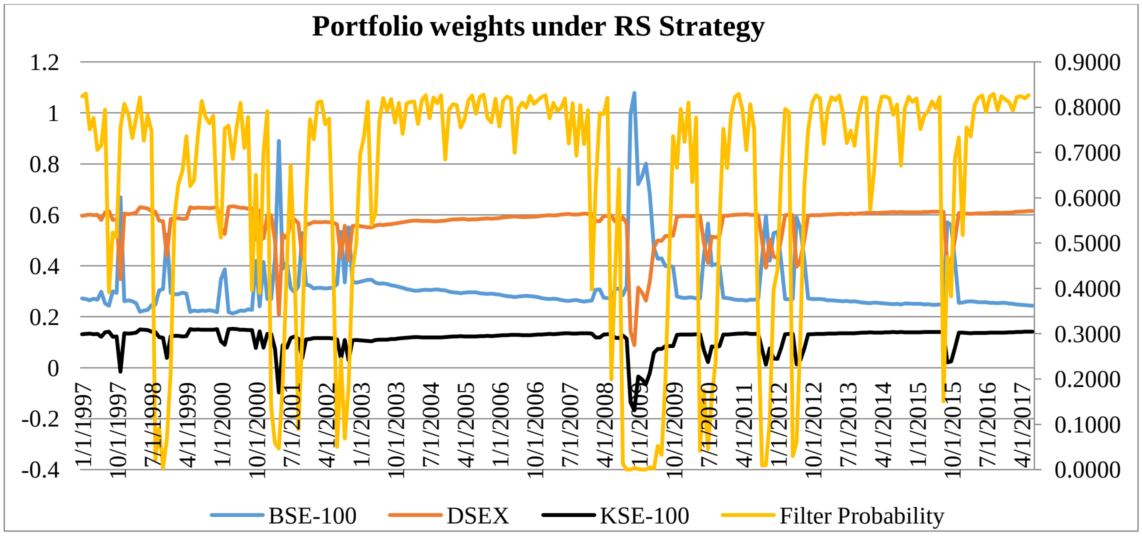


Figure 5.7-5: Portfolio weights of Indices under RS Strategy and filter probabilities (plotted at secondary axis on the right)

While RS strategy is seen many a times buying long BSE-100 and DSEX and Short selling KSE-100 occasionally when the market is in Regime 2. RS CAPM does not take full advantage of allowed short selling range because increasing the RS CAPM weight to that extent would have made the portfolio too risky to generate optimal cumulated wealth. RS CAPM is predicting it properly but somehow failed to allocate right wieghts into the right assets before 2009 which has been observed only in this chapter, as the previous and next chapters show that RS CAPM outrightly performs better than Non-RS and MV strategies.

Figure 5.6-6 below show weights for Non-RS strategy which capitalises fully on short selling in the beginning and allocates maximum weight to BSE-100 and minimal possible weight to KSE-100 by short selling when the market was in Regime 2. DSEX is also short sold for a breif period of Dotcom bubble but Non-RS strategy confines itself to simple weights as if short selling is not allowed after early 2003 and then slightly changed weights.

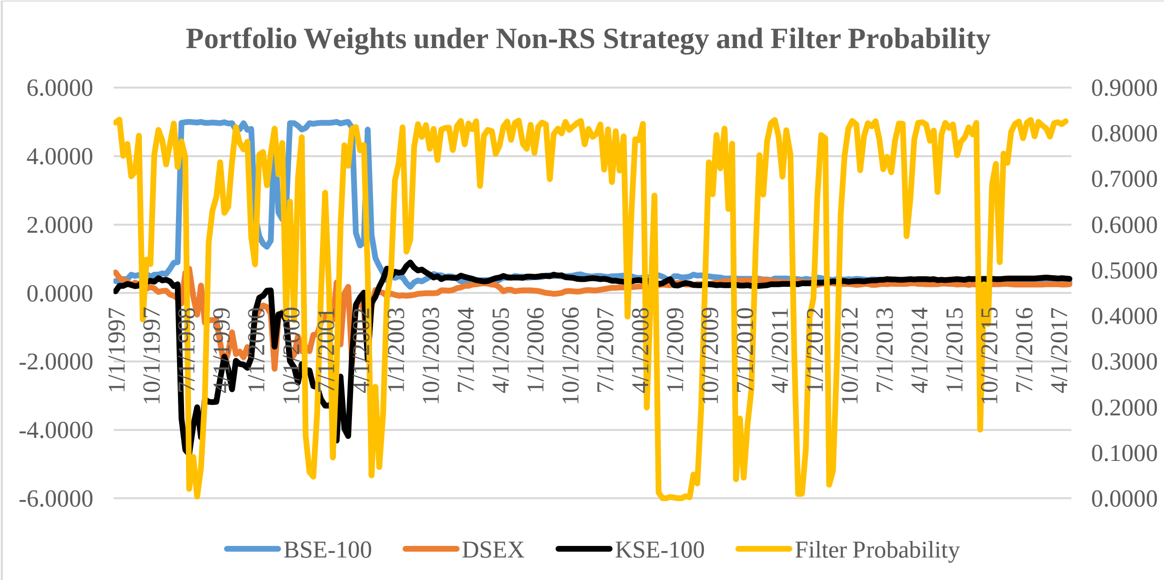


Figure 5.7-6: Portfolio weights of Indices under Non-RS Strategy and Filter Probability (Plotted on Secondary Axis on the right)

5.7.3 Market Value Weighted Portfolio Weights

MV Strategy is using same weights either without short selling or short selling and is not able to utilise the potential offered by the market to borrow and use funds productively by shortselling or investing more as shown by Figure 5.6-7. It is obvious that MV increases its weight in KSE-100 when market is in stable Regime 1 and decreased in KSE-100 when market is in Regime 2. When Regime 1 is stable, DSEX and BSE-100 are allocated more weights and vice versa which is also risky in a sense that it wouldn't be able to predict sudden change which is done before time by RS CAPM and outperformed the Non-RS and MV strategies after 2009.

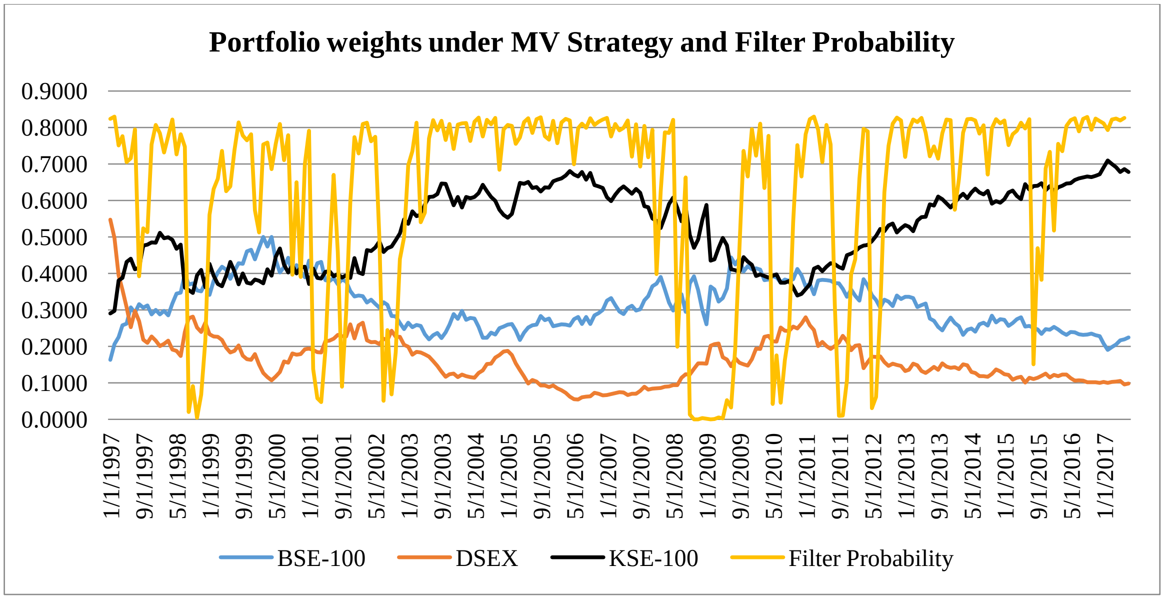


Figure 5.7-7: Portfolio weights of Indices under MV Strategy and Filter Probability

The short-selling opportunity makes the forward looking approach of RS CAPM Strategy weights more prominent. That is, when the stock index falls, the RS CAPM weights can actively change before the actual decline in indices. For the Non-RS weights show clear evidence of being a lagging indicator.

5.7.4 Cumulative Wealth with Risk-free Borrowing and Lending

Risk-free asset (US Interbank 1 month offer rate) is introduced in the model to provide the investor with an opportunity to freely borrow and lend money at the risk-free rate depending on portfolio strategy and whether short selling is allowed or not. Inclusion of risk free asset also enhances the performance of all the portfolios but once again RS strategy outperforms Non-RS and MV strategies for most of out-sample period. This is evident from Table 5.6-1 that in scenario 1 where No short selling is allowed without risk free asset, the cumulated wealth for RS CAPM is 1.801 which is higher than Non-RS cumulated wealth (1.170) but is less than MV strategy (1.272). The RS CAPM cumulated wealth (1.815) is outperforming both Non-RS and MV when risk free asset is introduced in without short selling scenario 2.

The point to be noted is that the introduction of risk free asset does not affect the cumulated wealth of Non-RS and MV strategy which means they fail to capitalise on the risk diversification opportunity arising from the introduction of risk free borrowing and lending.

Cumulated Wealth and Sharpe ratio for different Strategies	**Without Short Selling**			**With Short Selling**		
	RS CAPM Strategy	Non-RS Strategy	MV Strategy	RS CAPM Strategy	Non-RS Strategy	MV Strategy
No Risk Free Asset	**1.8018**	**1.1706**	**1.2729**	**1.8942**	**1.0737**	**1.2844**
Sharpe Ratio	0.1526	0.1163	0.1163	0.1557	0.1159	0.1165
With risk Free Asset	**1.8954**	**1.1804**	**1.2729**	**1.9742**	**1.0937**	**1.2844**
Sharpe Ratio	0.1536	0.1163	0.1163	0.1563	0.1159	0.1165

Table 5.7-1: End of Period Cumulated Wealth and Sharpe ratio for different Strategies in different Scenarios.

The short selling without risk free asset (scenario 3) generates more cumulated wealth (1.894) than the Non-RS (1.073) and MV (1.284) strategies and performs even better when risk free asset is introduced (scenario 4) with cumulated wealth being 1.974. The key element being usage of risk free asset for borrowing and lending efficiently to maximise the cumulated wealth in out-sample period. This point is strengthened further from the fact that Sharpe ratios calculated for different portfolios show that RS CAPM has high Sharpe ratios in all scenarios than the Non-RS and MV strategies (as can be seen from Table 5.6-1).

Another factor that enhances the capability and efficiency of RS CAPM strategy is when compared on grounds of Risk Allocation capability against Non-RS Strategy. Since it judges the Regime changes, it can allocate more in the Return generating assets as shown by the Figures 5.6-8 & 5.6-9. Non-RS Strategy relied more on Risk free assets to generate nominal returns but did not allocate to Indices and when it did went up to invest more, that was recession period so decision going totally wrong and then had to resort to its original conservative policy.

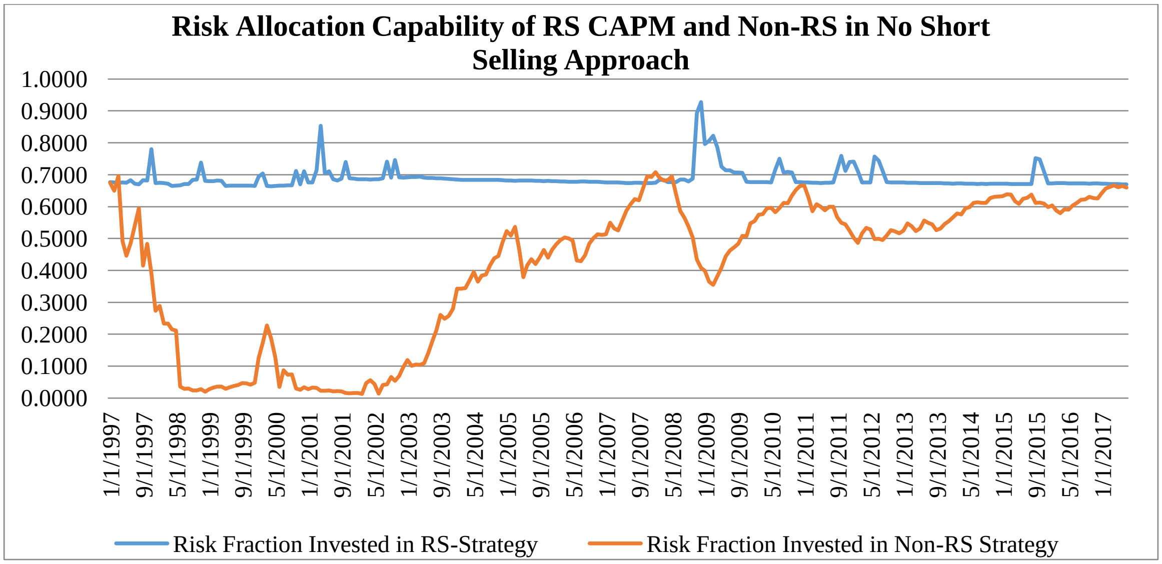


Figure 5.7-8: Risk Allocation Capability of RS CAPM and Non-RS in No Short Selling Approach

RS CAPM with short selling approach has gone up to 1.1 times investment by borrowing funds at cheap rates to generate more return and minimally to .06 for the investments in risk free asset by lending it and thus generate more returns instantaneously. The sudden switch is in itself explaining that RS CAPM is efficiently and timely switching its borrowing and lending opportunities as compared to Non-RS strategy which is always lagging and in fact has been relying on investment in risk free by lending it and never borrowed to invest more in risky investments to generate more returns.

Once again, RS is using majority of its funds to invest in return oriented indices but Non-RS strategy is conservative in approach by investing more of its funds in risk free asset.

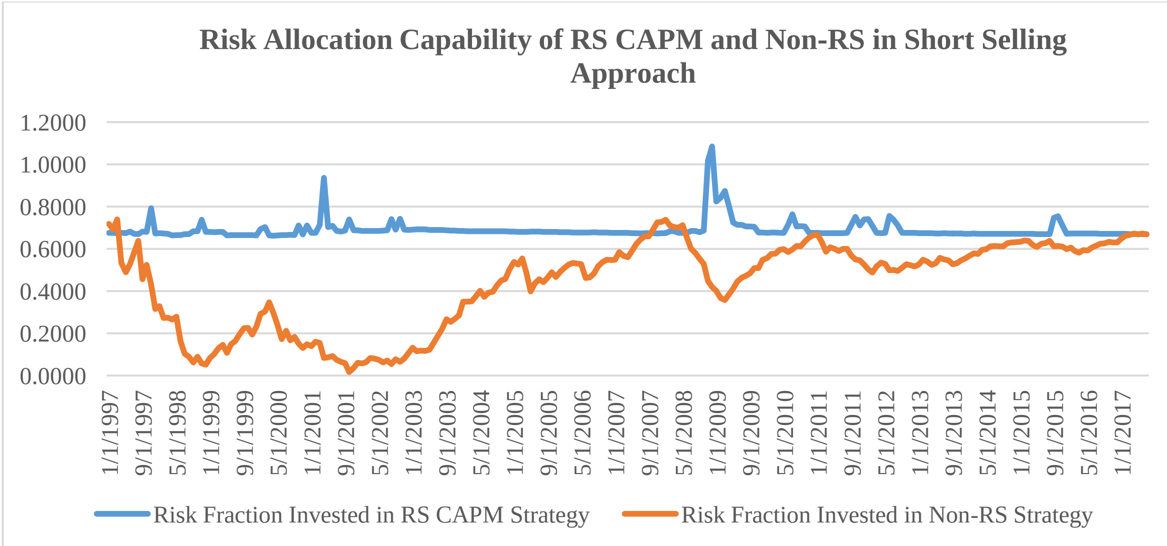


Figure 5.7-9: Risk Allocation Capability of RS CAPM and Non-RS in Short Selling Approach

5.7.5 Short Selling With Risk Free Borrowing & Lending (Conditional on Risk Aversion Coefficient) the Risk Aversion Factor

The previous section 5.6.4 is based on medium level of risk aversion having coefficient at 3. When the risk aversion coefficient is changed from 3 to 2 (i.e. the investor is now least risk averse), RS CAPM generates highest end of period cumulative wealth (2.068) due to its freedom to invest more in risky assets (due to regime realisations) and generate more returns. As shown in the Table 5.6-2, cumulated wealth of RS CAPM strategy is much more than the other two strategies in both case when short selling is allowed or not (both are computed when risk free borrowing and lending is allowed). Overall the RS CAPM end of period cumulated wealth performs superbly compared to the other scenarios.

Cumulated Wealth for different Strategies	Short Selling With Risk Free Borrowing & Lending (Conditional on Risk Aversion Coefficients)			Without Short Selling With Risk Free Borrowing & Lending (Conditional on Risk Aversion Coefficients)		
	RS CAPM	Non-RS	MV	RS CAPM	Non-RS	MV
Risk Aversion Coefficient= 2	**2.0680**	**0.9932**	**1.3122**	**1.9942**	**1.1325**	**1.2910**
Sharpe Ratio	0.1563	0.1159	0.1159	0.1563	0.1159	0.1159
Risk Aversion Coefficient= 3	**1.8954**	**1.1804**	**1.2729**	**1.9742**	**1.0937**	**1.2844**
Sharpe Ratio	0.1536	0.1163	0.1163	0.1563	0.1163	0.1163
Risk Aversion Coefficient= 4	**1.7208**	**1.0916**	**1.2456**	**1.6635**	**1.1642**	**1.2381**
Sharpe Ratio	0.1526	0.1161	0.1161	0.1525	0.1161	0.1161

Table 5.7-2: End of Period Cumulated Wealth and Sharpe ratio for different Strategies conditional on Risk Aversion Coefficients.

Most risk averse investor (risk aversion coefficient is 4) still generates higher returns using the RS CAPM Strategy as compared with the other two strategies. All strategies show decline in end of period cumulative return as most risk averse investor relies more on investing in risk free asset, RS CAPM performed once again better than other strategies. RS CAPM Strategy’ capability to generate right results during wrong moves of market is the key to the success for every future investor and its market timing ability helps it to switch portfolio weights beforehand as compared to other two lagging strategies in turn performing well in Cumulated wealth in out-sample period.

5.8 Conclusion

In this paper, the target was to come up with the proposition that RS CAPM Strategy is the robust new methodology to achieve portfolio optimisation which we failed to achieve as it failed to perform better throughout (as was the case in Chapter 4). It is widely observed that the system was acknowledging the existence of Regimes and was trying to deal accordingly as we have seen from the portfolio weights calculations done through RS CAPM strategy and its steadiness made it profitable at the latter stages (though it was under performing in start). The weights calculated through MV & Non-RS strategies remained almost same by allowing short selling or without it due to which they suffered huge losses at end. The methodology is found to be robust when applied to more established UK market such as FTSE-100 and its listed stocks as was the case in Chapter 4. There are number of other factors which caused the RS CAPM Strategy to underperform, the key example could be of KSE-100 and DSEX which are operating under strict lower cap limits from last eight years and how much influence government and its officials has on that specific market creating an opportunity for manipulation under supervision.

As discussed Section 5.1 stock indices in emerging countries like Pakistan, Bangladesh and India are dominated by trade in top 10 companies, they show higher contribution in market index due to their huge trading volume and number of trades. Also volatility is found to be higher compared with the developed markets. We think the factors discussed in section 5.1 are responsible for RS CAPM strategy being underperforming.

As a whole RS CAPM Strategy provided some good regime indicators in FTSE-100 when compared with developing/emerging markets of the Subcontinent. This research implicates the need for further query in developed, developing and under developed markets. To address some of the questions highlighted above, we have extended the study to more developed Asia Pacific market discussed in next chapter.

6 Chapter 6: Regime Switching Portfolio Optimisation for International Indices: Case of Asia Pacific

Abstract:

This Chapter expands the literature on other developing and developed markets of Asia Pacific, evaluating gains from Regime Switching (RS CAPM) strategy for portfolio optimisation. The stock Index data from six Asia Pacific stock exchanges, Koreas SE Comp, Nikkei 225 Stock Average, FTSE Bursa Malaysia (KLCI), Jakarta SE Composite, Shanghai SE All Share Index and S&P ASX 200 Australia is used against MSCI AC World Asia Pacific index. Continuously rebalancing the portfolio from the weights generated by the Regime Switching (RS CAPM) model and compare them with simple mean-variance portfolio (Non-RS) and market value portfolio (Market value (MV) weighted portfolio). Previously, Regime Switching Strategy has been successful in case of FTSE-100 (Chapter 4) and has not been that robust in the case of portfolio of subcontinent indices (Chapter 5). This data supports the results found in chapter 4 and noted when market conditions deteriorated quickly, regime switching (RS CAPM) became essential for market timing purposes and it helped improve the performance of portfolio. On average, RS CAPM wealth was higher (almost 1.5 times or more) than the Non-RS and MV strategy. All the strategies have seen decline in cumulated wealth around the time of the credit crisis in 2007 but RS CAPM strategy managed to survive the crises periods and it still ended in positive and comparatively higher values.

Key Words: Regime Switching, International Portfolio, Asian Stock markets, MSCI, Asia Pacific, and Optimization etc.

6.1 Introduction and Literature Review:

Global uncertainty is increasing. The back-and-forth negotiations surrounding policy decisions on the US Fiscal Cliff, Euro zone crisis, and potential conflicts in the Middle East may be intriguing for political scientists, but for global business executives, they are cause of major concern. Any escalation of one of these major drivers of global risk could seriously thwart hard-earned recovery, and plunge the globe into another recession. With that said, emerging markets continue to show promise for 2013, even in the face of increased global risk. By focusing on economies that are better positioned to withstand the major influencers of uncertainty, and mitigating exposure to economies that are highly susceptible, growth can still be achieved. For example, economies in: Asia Pacific, Sub-Saharan Africa, and parts of Latin America are relatively well insulated.

Asia Pacific presents an anchor of stability and new pillar of growth for the global economy. While the severity of the financial crisis in the Western hemisphere has created headwinds, Asia Pacific still continues to be the fastest growing region. This increased vitality and visibility is fuelling a powerful transition, one that has the potential to create a future defined by the region's consumers, companies and cultures. The ability of market research to offer high value solutions that can sustain, stimulate and rebalance growth across the developed, developing and emerging markets in the region will be key in future.

The Asia Pacific region is a geographical appellation that many still feel with justification will be the dynamic economic arena for this century. Accepting this premise and acknowledging the importance of the role of finance in that development brings with it the imperative to gain a greater understanding of the unique financial characteristics of the region. This chapter has two major pursuits. The first goal is to provide some basics on the various sub markets of the region. The vital roles played by stock markets of pricing capital, issuing new shares, providing a liquidity-creating secondary feature, serving as a vehicle for asset transfer and providing a linkage to international capital markets and are as important to emerging markets as to developed countries. However, fixed income and capital markets are still not as well developed in emerging markets and therefore an even heavier capital sourcing burden is placed on emerging stock markets. The Asia Pacific region derivatives markets (futures and options) play their risk-transfer role in equity and fixed income areas and are integral to the scene which is almost non-existent in the Subcontinent market[9].

Weber (2007) aims at identifying the impacts between key financial markets in the Asian Pacific region. "More specifically, the focus is on determining causal inter linkages between daily data of the exchange rate, the money market rate and the stock index in the post-crisis period 1999-2006. The markets concerned are characterised by the absence of serious barriers and frictions, so that reactions to economic news and mutual influences are taking place even within the same day. This short time window brings the need of a thorough understanding of the structural interdependence to the fore". For the same reasons, empirical approach (employed here) also takes volatility effects into account, which play an important role for the functioning of financial systems and the realisation of regimes (Regime 1 is low volatility regime compared with high volatility Regime 2).

Most prominently, the Asian financial crisis in 1997/98 has brought topics such as contagion and volatility transmission on the agenda (Weber, 2007). "The years since then have witnessed a fast economic recovery in some countries as well as the establishment of policy concepts directed at fostering financial stability. The task of constructing a sound system of

[9] The derivative market in Pakistan and India is relatively new and Bangladesh is just starting to develop derivative instruments. (source: respective country's securities and exchange commissions)

financial markets has reached high priority in international politics. Therefore, it is as well the more stable periods, which call for a better understanding of the short-run interactions between different financial assets. For example, identifying the relevant effects is crucial for conducting monetary policy in a solid and foresighted fashion. By the same token, organising the currency management especially in South-East Asia, a frequently discussed question, deserves detailed information on the mechanisms of shock propagation. Another important task, building regional capital markets for efficient factor allocation and stable development, depends on the role of stock exchanges in receiving and generating economic signals" (Weber, 2007).

All markets naturally follow the rules of supply and demand, so that every theoretical foundation should come across along these lines to distinguish different markets: According to Stiglitz (1999), while talking about the equity market influences, one should consider the role of stocks as growth indicators: Reflecting expectations about the value of future cash-flows, they could work as signals for the performance of the economy, thus the inflows of Asia Pacific market are comparatively very high compared to Subcontinent market. Also market size has increased drastically over last two decades for Asia Pacific market when compared with other world markets.

The effects from the stock market can be structured in a similar way: Although normally denied, equity developments have a signalling function for the monetary policy. Another mechanism probably works through the tendency of investors to switch to relatively safe bonds or money market assets in times of economic difficulties. These properties obviously distinguish different markets. The stock markets of Asia Pacific are much more developed and information symmetry is prevalent in this market.

"Impacts on the equity index naturally are propagated through the formation of expectations, but remain theoretically indefinite in their overall direction: Taking the example of currency depreciation, the fear of capital outflows and monetary tightening would have a negative influence on the equity performance. Hopes of strengthening exports or rising retail prices would produce the contrary result (see Cappiello and De Santis 2005). Obviously, not all mentioned effects can be of the same importance in every country model. In the particular context of post-crisis Asian-Pacific financial markets the focus should be on the stock indices to understand the differences between these markets and others". The chapter formally starts with the sample data used in this Asia Pacific chapter.

6.2 ASEAN Market Consolidation

The Association of Southeast Asian Nations (ASEAN) encompasses 600 million people across 10 countries, with a combined GDP of $2.3 trillion. In 2015, the ASEAN Economic Community (AEC) has come into effect to form a single market and production base with a free flow of goods, services, investment, and skilled labour. The AEC will be the fourth most populous bloc in the world behind China, India, and the European Union. The AEC is and will be a game changer. The region's companies will face unprecedented access to markets—and unprecedented competition. Organizations that do not have a regional game plan, or understand how to build brands, will fall behind.

Everyone wants a piece of ASEAN. With limited growth opportunities elsewhere, Southeast Asia remains one of the world's few unsullied growth stories. ASEAN's population is projected to reach more than 650 million people by 2020, with half under the age of 30. By

2030, 51 percent of the population (not including Myanmar, Laos, and Brunei) will be in the middle class, according to the Brookings Institute. This young population is educated and technology savvy. And as its members move into the middle class, they will continue to want more products and services and will demand more from the brands they buy. They are also among the world's most optimistic consumers.

The big players are already scaling up their presence to capture new opportunities, as indicated by U.S. multinationals' plans for the region. In a recent American Chamber of Commerce survey, 90 percent of respondents expect their trade and investment to rise in ASEAN by 2015, and 73 percent say ASEAN's contribution to global profits will rise over the same period. One big reason: Growth is slow back home, and ASEAN remains one of the brightest sparks. Southeast Asian companies that plan ahead can emerge as regional champions. Years of growth have left many of the region's companies cash rich compared to their Northeast Asian or Western peers. Forward-thinking Southeast Asian CEOs are putting that cash to good use, snatching up competitors at home and across the region. The first half of 2013 saw 183 merger and acquisition (M&A) deals worth $27.1 billion, up 10 percent by volume and 6 percent by value over the same period in 2012, according to Mergermarket. Most of these were in-country acquisitions, and the bulk of the cross-border activity consisted of outbound deals initiated by Southeast Asian companies expanding outside their home market.

Companies across Southeast Asia are going to have to work harder to defend their home turf against a growing number of global and regional competitors. Many domestic players in this region have historically focused on their home markets, where they often enjoyed minimal competition.

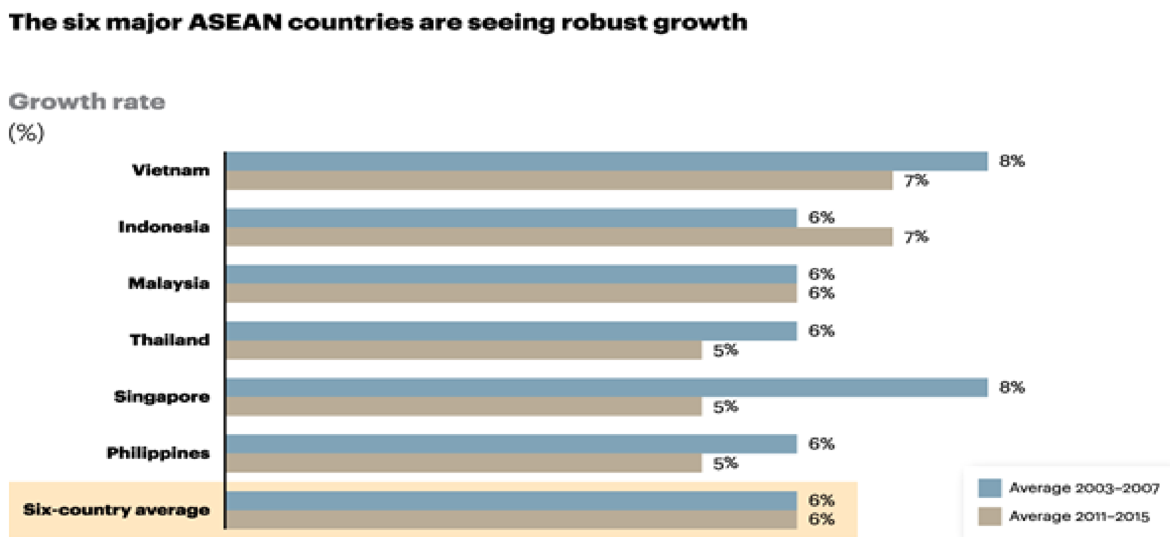


Figure 6.2-1: ASEAN Growth Rate; Source: OECD Development centre's medium term projection framework in South East Asian Economic outlook

But then after there are situations where lack of communication and rules have left asking questions e.g. Singapore’s financial regulator and central bank has said that Stock markets in Singapore, Malaysia and Thailand should develop linked up post-trade systems to speed integration of financial markets across the Association of Southeast Asian Nations. Countries in the 10-member bloc, of which Indonesia and Thailand are the largest economies, have been working towards a more integrated capital market in the region, which is home to about 620 million people. Exchanges in Singapore, Malaysia and Thailand in 2012 created a so-called ASEAN Trading Link. This has involved the creation of an electronic “order routing” system, allowing brokers in Malaysia and Singapore to connect their clients more easily to trading on each other’s exchanges. Before the system went live, an investor in Malaysia wanting to trade shares in Singapore would typically have to telephone a local broker, who would contact a broker in Singapore — each time incurring fees. The system was supposed to bypass that, thus lowering barriers to entry for investors. However, the system has not attracted much interest and bankers, notably Piyush Gupta, chief executive of DBS, Singapore's biggest bank by assets, have suggested that post-trade linkages need to be built before market participants will engage in cross-border trading on any scale. Ravi Menon, managing director of the Monetary Authority of Singapore, said progress on ASEAN financial integration had been "disappointing" and was lagging behind efforts to integrate trade within the region. He suggested that member countries should build on the ASEAN Trading Link "by further broadening and deepening post-trade linkages". That should involve establishing clearing, settlement and custody links which would make the ASEAN Trading Link a "full-fledged end-to-end platform across the three ASEAN markets", Mr Menon told a meeting of the ASEAN Banking Council in Singapore. Each exchange in the region has its own clearing and settlement systems but there are no links between them. Hard economics hit regional consumer dream Multinationals drawn by hopes of a long consumption boom have been jolted as household debt, sluggish wage rises and political uncertainties drag on spending growth in Thailand, Indonesia and Malaysia.

Many studies have focused on the integration or segmentation of financial or stock markets during pre and post first liberalization and financial crisis, mainly for developing or emerging countries. The stock markets have also undergone the 1997-1998 Asian financial crisis and world recession in the early and end of 2000s. The findings of previous literature on the impact of stock market liberalization on stock market integration of emerging countries, reveal that there is little to no evidence of market segmentation, but an increasing level of market integration after the first stock market liberalization (Tai, 2007, Baharumshah et al, 2003 Lin, 2005, Guo, 2005, Gerard et al, 2003). The countries were found segmented before liberalization (*Auzairy et al., 2012)*.

6.3 Regime Switching Model using Asia Pacific Indices Portfolio

Regime Switching model (RS CAPM) is adopted similar to that used by Ang & Bekaert (2004), and Markose & Yang (2008) and explained in detail in Chapter 3 of methodology. This Chapter will follow the same steps devised in Chapter 3.

Monthly data on Asia Pacific stock Indices from June 1992 to August 2017 is downloaded from DataStream and the sample data is composed of following indices;

1. Korea Stock Exchange Composite Index (KSECI)
2. Japan Nikkei 225 Stock Average (Nikkei)
3. FTSE Bursa Malaysia KLCI (KLCI)
4. Jakarta Stock Exchange Composite Index (JSECI)
5. Shanghai Stock Exchange All Share Index & (SSE-A)
6. S & P ASX 200 Australia (ASX)

6.4 Sample Data Description:

Monthly index values of MSCI AC World Asia Pacific (MSCI-AP) and monthly index data on six Asia Pacific stock Indices from June 1992 to August 2017 are downloaded from DataStream. The names of stock indices are listed above. The sample period is composed of 303 observations for each stock. US Interbank 1 month offer rate from London BBA available on DataStream is taken as the risk free (RF) rate. Returns are the difference of log prices and excess returns are calculated as the difference between returns and risk free rate.

All the indices are used in common currency denomination i.e. US Dollars and that's why US Interbank offer rate is used as risk free asset.

Total Sample is divided into two periods called in-sample and out-sample. The in-sample period is June 1992 to May 1999 (84 return observations) and out-sample period starts from June 1999 and end at July 2017 (218 return observations).

6.5 Summary Statistics for 6 Asia Pacific Indices and the MSCI AC World Asia Pacific Index

Table 6.4-1 shows the relationship between all six stock indices and MSCI-AP. Highest mean of returns is of ASX with Nikkei showing minimum mean return and MSCI-AP having negative but near zero return. Highest standard deviation is shown by SSE-A, also has the 2nd highest Excess Kurtosis and a positive Skewness. The lowest standard deviation is of Nikkei and ASX with Nikkei showing the least values in third and fourth moments among all indices. Nikkei also has positive small Skew.

All indices have small negative values of intercept with MSCI-AP and have positive slopes with Nikkei being highest (0.7894). Nikkei is highly correlated with MSCI-AP having R Squared value of 0.79 and SSE-A has minimum correlation based on R Squared value of 0.0143 with MSCI-AP in fact SSE-A has least correlation with all other indices (with 0.1696 being highest with KLCI). Nikkei and SSE-A have least covariance and correlation among them during whole sample period (0.0355). All other indices have positive correlation (ranging from weak to strong) with each other. ASX has the 2nd highest correlation of 0.7574 with MSCI-AP. The MSCI-AP correlation with individual indices would probably be affecting the portfolio allocation decision.

Whole Sample : June 1992-July 2017							
Total Log Return Observations: 302							
Descriptive Statist	**KSECI**	**NIKKEI**	**KLCI**	**JSECI**	**SSE-A**	**ASX**	**MSCI-**
Mean	0.0036	0.0008	0.0019	0.0036	0.0027	0.0042	-0.0001
Variance	0.0100	0.0037	0.0066	0.0121	0.0133	0.0039	0.0029
S Deviation	0.0998	0.0606	0.0812	0.1099	0.1154	0.0622	0.0536
Excess Kurtosis	4.3279	0.9531	6.3508	5.3367	5.6505	1.9044	1.2532
Skewness	-0.2454	0.0330	-0.2886	-0.9631	0.0645	-0.6515	-0.1628
OLS Regression							
Intercept	-0.0015	-0.0008	-0.0006	-0.0009	-0.0003	-0.0029	
Slope	0.3709	0.7894	0.2394	0.2338	0.0555	0.6528	
Standard Error	0.0388	0.0241	0.0500	0.0471	0.0533	0.0350	
R Square	0.4775	0.7978	0.1316	0.2298	0.0143	0.5737	
Covariance Matrix							
KSECI	0.0099	0.0032	0.0030	0.0053	0.0012	0.0036	0.0037
NIKKEI	0.0032	0.0037	0.0008	0.0020	0.0002	0.0020	0.0029
KLCI	0.0030	0.0008	0.0066	0.0043	0.0016	0.0019	0.0016
JSECI	0.0053	0.0020	0.0043	0.0120	0.0010	0.0031	0.0028
SSE-A	0.0012	0.0002	0.0016	0.0010	0.0133	0.0012	0.0007
ASX	0.0036	0.0020	0.0019	0.0031	0.0012	0.0039	0.0025
MSCI-AP	0.0037	0.0029	0.0016	0.0028	0.0007	0.0025	0.0029
Correlation Matrix							
KSECI	1.0000	0.5365	0.3761	0.4880	0.1022	0.5854	0.6910
NIKKEI	0.5365	1.0000	0.1535	0.3088	0.0355	0.5271	0.8932
KLCI	0.3761	0.1535	1.0000	0.4828	0.1696	0.3789	0.3628
JSECI	0.4880	0.3088	0.4828	1.0000	0.0812	0.4614	0.4794
SSE-A	0.1022	0.0355	0.1696	0.0812	1.0000	0.1719	0.1195
ASX	0.5854	0.5271	0.3789	0.4614	0.1719	1.0000	0.7574
MSCI-AP	0.6910	0.8932	0.3628	0.4794	0.1195	0.7574	1.0000

Table 6.5-1: Statistical Description of Six Assets along with MSCI-AP for Whole Sample Period.

Major swings in the ASX, KSECI, SSE-A and KLCI indices during 1993 to 1997 and 2005 to 2011 is observed in Figure 6.4-1, JSECI and Nikkei seem to show no huge change in value during these periods and afterwards. SSE-A seems to be fluctuating hugely even in other periods e.g., between 2014 & 2015. This high correlation, as noted in international portfolio literature, seems to reduce the benefits of diversification (Ang & Bekaert, 2004). This co-movement is due to greater integration between them and MSCI-AP, as noted in Table 6.4-1. It is also helpful in deciding the composition of international portfolios and risk minimization of a rational investor.

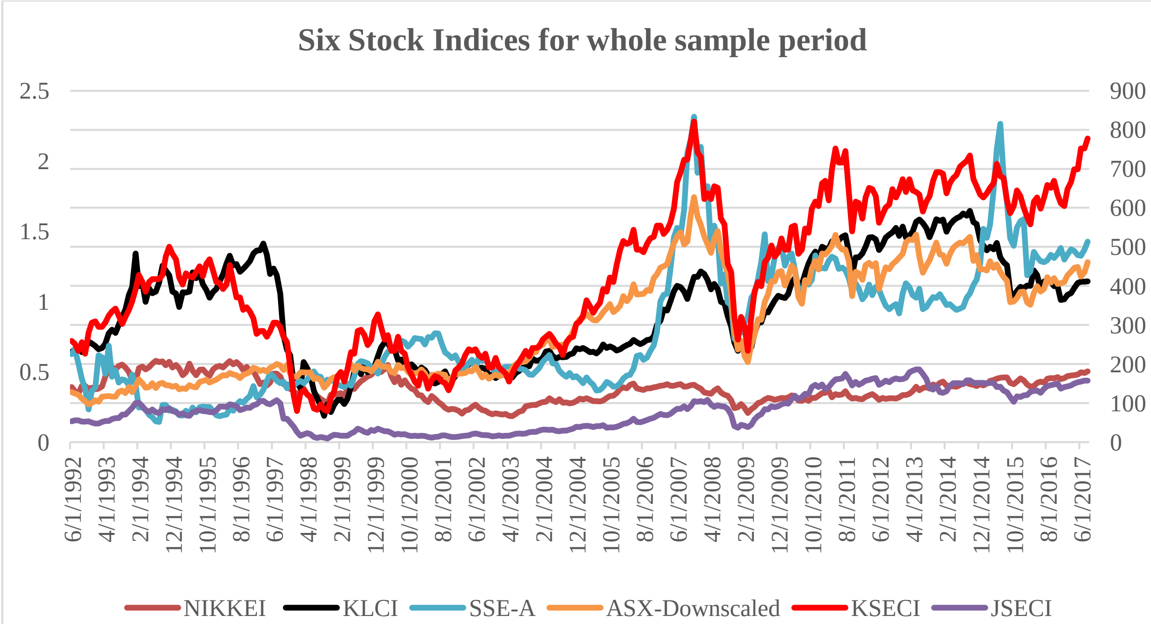


Figure 6.5-1: Six Stock Indices for whole sample period (KSECI and JSECI are plotted on Secondary axis on the left and ASX is divided by 10 to get better visual plotted on Primary axis on the right)

The in-sample statistics are shown in Table 6.4-2. Mean for all the indices including MSCI-AP except ASX are small negative values nearly zero with highest standard deviation in SSE-A and lowest in ASX. All show positive excess kurtosis except ASX which has small negative excess value. ASX, Nikkei, and MSCI-AP have positive skew while remaining have negative skew. All indices have negative intercept of small values with small positive slope except SSE-A which has small negative slope. Nikkei is still highly correlated with MSCI-AP with SSE-A being negatively correlated with MSCI-AP. SSE-A, in fact, has small negative correlation with all indices except KLCI with which it has small positive correlation. All other assets have weak to moderate positive correlations with each other.

In-Sample : June 1992-May 1999							
Observations: 84							
Descriptive Statistics	KSECI	NIKKEI	KLCI	JSECI	SSE-A	ASX	MSCI-AP
Mean	-0.0016	-0.0005	-0.0030	-0.0087	-0.0035	0.0042	-0.0026
Variance	0.0173	0.0054	0.0166	0.0212	0.0297	0.0032	0.0037
Standard Deviation	0.1316	0.0736	0.1287	0.1456	0.1723	0.0564	0.0607
Excess Kurtosis	4.0714	0.5369	2.2601	3.4234	2.8626	-0.3641	1.1917
Skewness	-0.1858	0.7323	-0.1087	-0.8473	0.3430	-0.0691	0.6128
OLS Regression							
Intercept	-0.0021	-0.0022	-0.0022	-0.0012	-0.0027	-0.0052	
Slope	0.2552	0.7639	0.1188	0.1547	-0.0459	0.6187	
Standard Error Slope	0.0509	0.0231	0.0591	0.0568	0.0606	0.0500	
R Square	0.3054	0.8573	0.0633	0.1376	0.0169	0.3299	
Covariance Matrix							
KSECI	0.0171	0.0043	0.0051	0.0077	-0.0011	0.0026	0.0044
NIKKEI	0.0043	0.0054	0.0004	0.0021	-0.0018	0.0017	0.0041
KLCI	0.0051	0.0004	0.0164	0.0083	0.0023	0.0020	0.0019
JSECI	0.0077	0.0021	0.0083	0.0210	-0.0003	0.0026	0.0032
SSE-A	-0.0011	-0.0018	0.0023	-0.0003	0.0293	-0.0002	-0.0013
ASX	0.0026	0.0017	0.0020	0.0026	-0.0002	0.0031	0.0019
MSCI-AP	0.0044	0.0041	0.0019	0.0032	-0.0013	0.0019	0.0036
Correlation Matrix							
KSECI	1.0000	0.4543	0.3024	0.4058	-0.0472	0.3547	0.5527
NIKKEI	0.4543	1.0000	0.0476	0.1956	-0.1437	0.4235	0.9259
KLCI	0.3024	0.0476	1.0000	0.4459	0.1048	0.2844	0.2516
JSECI	0.4058	0.1956	0.4459	1.0000	-0.0108	0.3165	0.3709
SSE-A	-0.0472	-0.1437	0.1048	-0.0108	1.0000	-0.0178	-0.1302
ASX	0.3547	0.4235	0.2844	0.3165	-0.0178	1.0000	0.5744
MSCI-AP	0.5527	0.9259	0.2516	0.3709	-0.1302	0.5744	1.0000

Table 6.5-2: In-Sample Descriptive Statistics for 6 Indices along with MSCI-AP for in-sample period.

6.6 Correlation computation and discussion

In this section, statistical properties of the correlation matrices of the 6 monthly index returns (empirical sample) from June 1992 to August 2017 are discussed. Correlations are computed from empirical data returns and discuss whether the properties of the correlation matrices as discussed in RMT hold for the empirical data.

6.6.1 Eigenvalue Analysis

Most of the distribution is consistent with the RMT bounds calculated in different studies (e.g. Plerou et Al., 1999, Daly et al., 2010). This comparison also indicates the presence of several eigenvalues clearly outside the random matrix bound. Having demonstrated that the bulk of the eigenvalues satisfies RMT predictions, analyses of the statistics of the eigenvectors are done. In the stock market problem, this eigenvector conveys the fact that the whole market moves together and indicates the presence of correlations that pervade the entire system (Daly et al, 2010).

The Marchenko-Pasteur limits to identify noisy values are given by the Equation (7.3) which is dependent on variable Q= L/N, where L=302 and N = 6. The maximum Eigenvalue limit (λ_{max}) and minimum Eigenvalue limit (λ_{min}) for all the correlation matrices is calculated through Equation (7.3) and their values are;

λ_{min} = 0.7380 and λ_{max} = 1.3018

We use these limits to determine the noisy eigenvalues for the following correlation matrices from empirical sample.

1. Empirical correlation matrix for Non-RS (EmpSample)
2. Regime based correlation matrices further characterised into two sub classes;

a. High volatility based correlation matrix (RS1 CAPM)

b. Low volatility based correlation matrix (RS2 CAPM)

The Table 6.6-2 is showing eigenvalues for all three matrix types and is showing that many follow RMT and some fall in Marchenko-Pasteur bounds showing they are noisy and need filtering.

EmpSample Eigenvalues	0.4046	0.4117	0.4847	0.8833	1.0367	2.7790
RS1 CAPM Eigenvalues	0.4855	0.4940	0.5816	1.0600	1.2440	3.3348
RS2 CAPM Eigenvalues	0.5259	0.5352	0.6301	1.1483	1.3477	3.6127

Table 6.6-1: Eigenvalues for EmpSample, RS1 CAPM & RS2 CAPM before filtration

The eigenvectors for EmpSample are shown below in Table 6.6-2.

0.8172	0.2346	-0.0054	0.0822	0.1696	0.4915
-0.5058	0.4498	-0.1148	0.4191	0.4348	0.4049
-0.1540	0.2058	-0.6017	-0.5210	-0.3989	0.3759
-0.2099	0.0352	0.7606	-0.4179	-0.0873	0.4405
-0.0111	0.1101	0.1413	0.5821	-0.7807	0.1396
-0.0918	-0.8288	-0.1622	0.1809	0.0775	0.4895

Table 6.6-2: EmpSample Eigenvectors 'V'

Noisy Eigen Values shown by Marchenko-Pasteur limits before filtering as shown in Table 6.6-3 depicts that EmpSample and RS1 CAPM show 33.33% noise and RS2 CAPM shows 16.66% noise which is removed when filtered using Plerou et al (2002) method.

	EmpSample Eigenvalues	RS1 CAPM Eigenvalues	RS2 CAPM Eigenvalues
Number of Noisy Values	2	2	1
Percent of Total (6)	33.33%	33.33%	16.66%

Table 6.6-3: Number of Noisy Eigenvalues shown by Marchenko-Pasteur limits before filtering process

Once the eigenvalues and eigenvectors are computed for the above correlation matrices, the results are examined more closely. The key element was to compute the noisy values given by Marchenko-Pasteur limits. Then reconstruct the filtered correlation matrix by using filtered Eigenvalue diagonal matrix (D_{filter}) and corresponding eigenvectors V as follows;

$C_{filter} = V*D_{filter}*V-1$

Once C_{filter} is obtained, we examine it to check whether its diagonal is similar to original matrix with unit values on the diagonal, if not we repeat the process until original diagonal is obtained. The matrix C_{filter}, once obtained is checked for noise again and is reported in Table 6.6-4 below;

	EmpSample Eigenvalues	RS1 CAPM Eigenvalues	RS2 CAPM Eigenvalues
Number of Noisy Values	0	0	0
Percent of Total	0%	0%	0%

Table 6.6-4: Noisy Eigenvalues shown by Marchenko-Pasteur limits after filtering process

6.6.2 Eigenvector Stock constituents of deviating largest eigenvalues from RMT limits

Literature on Eigenvalue based studies of stock returns suggest that we look into the stock composition of the eigenvector corresponding with the highest Eigenvalue deviation from RMT limits. Table 6.6-1 shows the λ_6 (Maximum Eigenvalue) for each type of correlation matrix. If the eigenvalues are compared before and after filtering, the maximum Eigenvalue increases in every case after the filtering process indicating an increase in information.

These findings are consistent with Daly et al (2007) who show that largest eigenvalues help describe distinct sectors contributing more towards the performance of constituent index. If the companies highlighted by this eigenvector centrality are used in the formation of portfolios, better performance is bound to follow.

6.7 RS CAPM Empirical Construction

As outlined in the Chapter 3, Steps 1 to 3 of the methodology have been discussed earlier in sections 1.2 to 1.4 explaining the in-sample, out-sample.

Step 4: Obtain the RS Statistics for Data:

This section will look at the RS CAPM and RS covariance computation which involves step 4 to step 6 and involves the usage of conditional RS mean (μ1, μ2), variance (σ1, σ2) and covariance (K1, K2) derived from CAPM beta based returns of assets using Equations (16), (17), & (22). Step 4 uses Hamilton (1989) method of obtaining Maximum likelihood regime sensitive parameters i.e., transition probabilities, means, and standard deviations for each regime as described earlier in Chapter 3 in order to predict the regime. Table 6.5-1 below shows the values of mean and standard deviation used to identify the regimes and their respective transition probabilities.

Regime Indicators	**Regime 1**		**Regime 2**		**Transition Probability**	
	μ1	**σ1**	**μ2**	**σ2**	***P***	***Q***
Estimates	0.00104	0.00318	-0.00143	0.00678	0.8000	0.7000
Standard Error	0.00002	0.00049	0.00004	0.00030	0.00704	0.28147

Table 6.7-1: Regime Statistics as Regime Indicators for MSCI-AP for Asia Pacific Market

The values in Table 6.5-1 clearly show evidence of two regimes i.e., μ1> μ2, σ1< σ2, and. On average Regime 1 expected return and standard deviation are 0.104% and 0.318% while for Regime 2, these values are -0.143% and 0.678%. Regime 1 is much more stable than Regime 2 in a sense that it depicts less volatility as compared to Regime 2. Transition probabilities also show the presence of two different regimes with Regime 1 being more persistent with transition probability of .8000 and Regime 2 is most likely with transition probability of 0.7000. Please note that the coefficients in Table 6.5-1 are statistically significant at 0.01% significance level.

Step 5: Filter and Smoothed Probabilities:

In addition to the parameters required for Regime Prediction and their mean-variance, the model can also infer the Regime probabilities i.e. filter probabilities and smoothed probabilities. The filter probabilities indicate the process being in some particular regime at time t based on the information available at the time t-1. In contrast to filtered probabilities, the smoothed probabilities indicate the historical regimes the process was in at time t based on whole sample information. Figure 6.5-1 shows the smooth and filter probabilities for out-sample period.

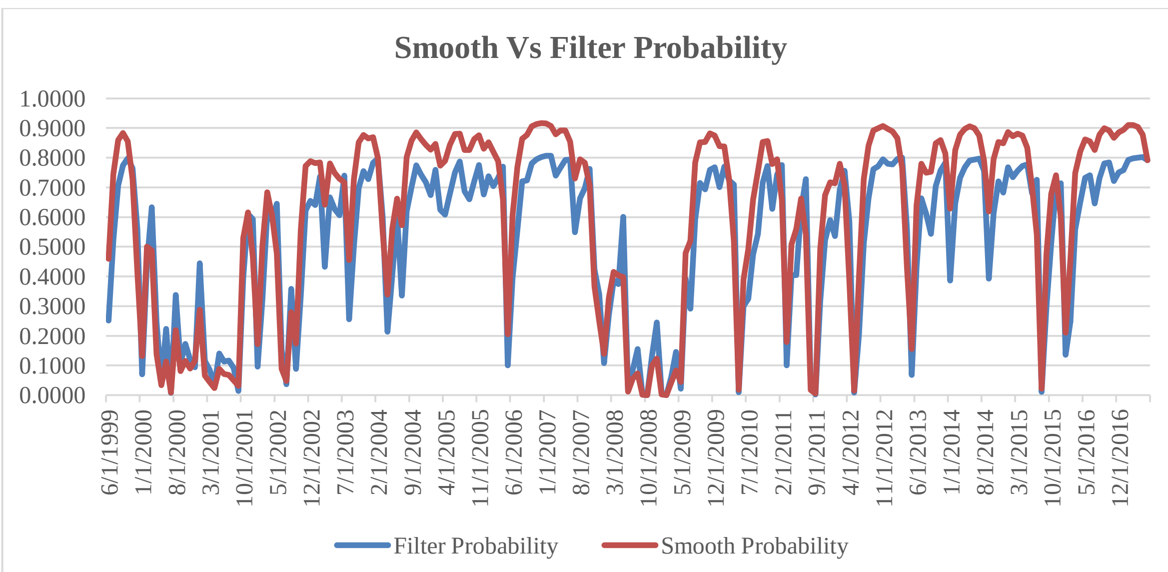


Figure 6.7-1: Smooth & Filter Probability for out-sample period

By using a cut off of 0.5 probability, we assume that the probability of being in Regime 1 is greater than 0.50 and the probability of being in Regime 2 is less than 0.50. We can easily see from Figure 6.5-1 above that there are many occasions where probability has come down from cut-off point, hence showing the probability of being in Regime 2. Looking at the filter probabilities and the cut-off of 0.5, there were almost 20 regime shifts during the entire period of almost eighteen years and Regime 2 persisted for some time as highlighted by

0.7000 probability. Contrary to filter probabilities, Smooth probability is suggesting that there had been 18 regime shifts.

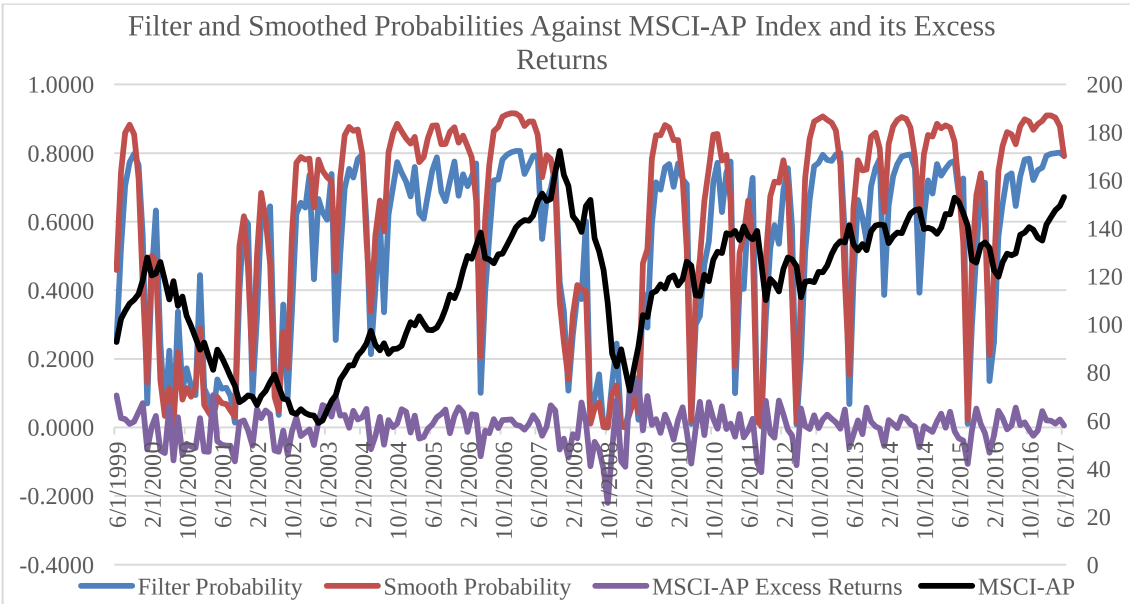


Figure 6.7-2: Filter Probability, Smooth Probability, MSCI-AP Index and its Excess Returns in out-sample period

Figure 6.5-2 above shows the filter, smooth probability against MSCI-AP and its excess returns and it can be seen that the MSCI-AP decline faced huge negative returns in periods of Dotcom bubble and 2007 credit crises, interesting to note is transition probabilities show transition to Regime 2 from Regime 1 well before time and stayed there until the recovery started in MSCI-AP. RS CAPM is therefore a good predictor of regimes and is helpful for investors in deciding the portfolio options.

Step 6: Compute Regime based covariance:

The regime indicators provide guidance as to which regime, market is in and are subsequently used to compute each regime's covariance matrices as outlined in Chapter 3 Equation (22) using the inputs from Equations (16) to (21).

Step 7: Equity Portfolio Selection with Regime-switching:

Use the inputs from previous steps to the Mean-Variance quadratic programming problem at the end of each month over out-sample period. The aim is to maximise efficiency of asset allocation provided an initial level of wealth. The portfolio optimisation problem involves decision of allocation of wealth among the portfolio assets and compute the optimal return iteratively by continuously reducing the risk.

Step 8: Calculate the cumulative return:

Calculate the cumulative return on 1 unit invested in the RS CAPM, Non-RS and MV portfolios over out-sample period and compare them.

Step 9: Repeat step 5, 6, 7 & 8 for different scenarios

The process is repeated with changes in conditions such as allowing short selling or not and allowing investment in risk free asset or not already discussed in detail. The following sections will discuss in detail every scenario for the data in question.

6.8 Monthly Portfolio Optimization

The results of step 1 to step 8 give us RS CAPM portfolio weights which is used in construction of monthly portfolio. The cumulative wealth for RS CAPM is calculated for out-sample period and are compared with Non-RS and MV strategy.

With 1 unit investment in out-sample period starting from June 1999. By using the empirical stock return the following month, cumulated wealth is calculated and all the profits are reinvested into the three portfolio strategies.

The three different strategies were further divided on the basis of short selling approach and no short selling approach. The No Short selling approach is discussed first and then Short Selling.

6.8.1 Cumulated Wealth without Short-Selling Approach

Cumulated wealth is used to measure the performance of all three strategies throughout out-sample period as shown below in Figure 6.6-1 which shows the cumulated wealth of all strategies, without short selling approach. With 1 unit investment in out-sample data, starting from June 1999, and using the actual prices on the following month, we calculated the stock returns and all the profits were reinvested into the three portfolio strategies.

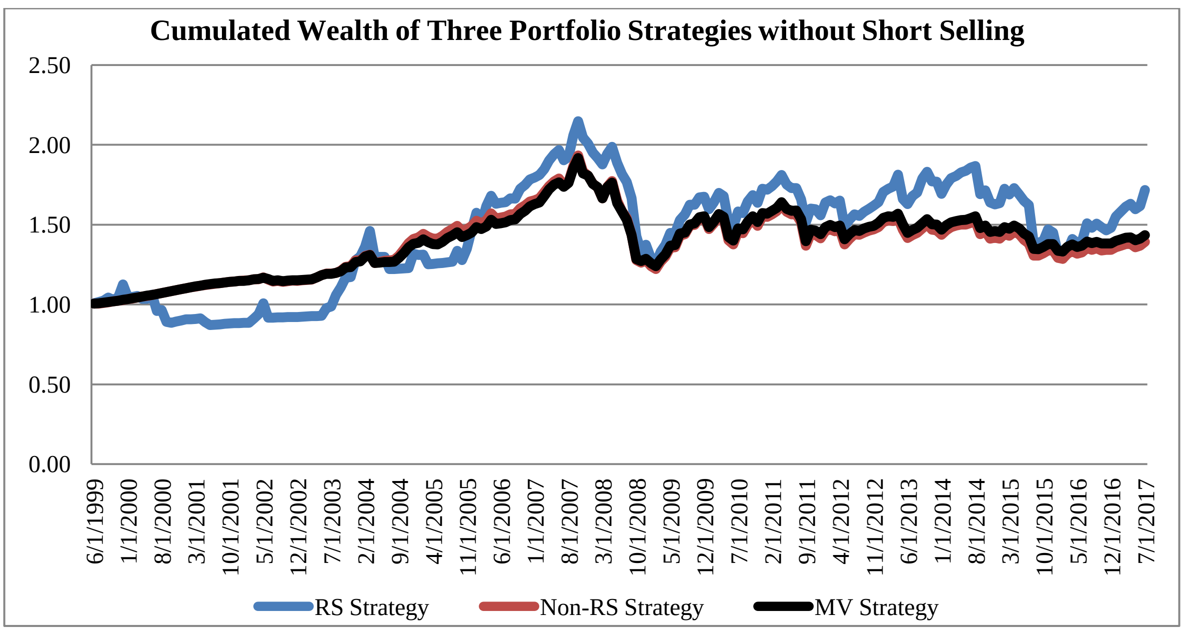


Figure 6.8-1: Cumulated Wealth of Three Portfolio Strategies without Short Selling Approach

Figure 6.6-1 above shows the cumulated wealth of all strategies using without short selling approach. It can easily be seen that all the strategies are affected by the shifts in regimes. The cumulated wealth goes up and down along with the shift in Regime probabilities. As a whole, all strategies are moving in the same direction but RS CAPM strategy seems to outperform the other strategies after 2005 when all other strategies are seemingly underperforming. As it can be seen in the Figure 6.6-1, before 2005, RS CAPM strategy does not perform well as compared to others when the regime changed during and after Dotcom bubble but picks up afterwards and RS CAPM strategy outperformed the other two strategies and closed above the Non-RS & MV strategies' cumulated wealth.

RS CAPM strategy has efficiently allocated resources in the right options at the right time, as can be seen in Figure 6.6-2. It is very obvious that most of the portfolio allocation has been in the ASX though some weight has been allocated to Nikkei and SSE-A during the turbulent times until 2005, after that all the investment has been in ASX and after 2013 ASX investment is reduced and Nikkei has been introduced into the portfolio. Thus this heavy reliance on two major economies in formation of portfolio in out-sample period would have definitely affected the performance of RS CAPM strategy. These economies felt most of the crises starting from 2007 onwards that started from US economy and spread like a contagion to whole world with some economies affected more while some were affected less. The other supportive argument behind this phenomenon could be that since short selling is not allowed, the other six indices could not be longed due to market conditions and the correlation between ASX and SSE-A was low making it a viable diversification option at the start and then RS CAPM reliance on ASX and Nikkei afterwards needs detailed investigation to understand the underlying factors better.

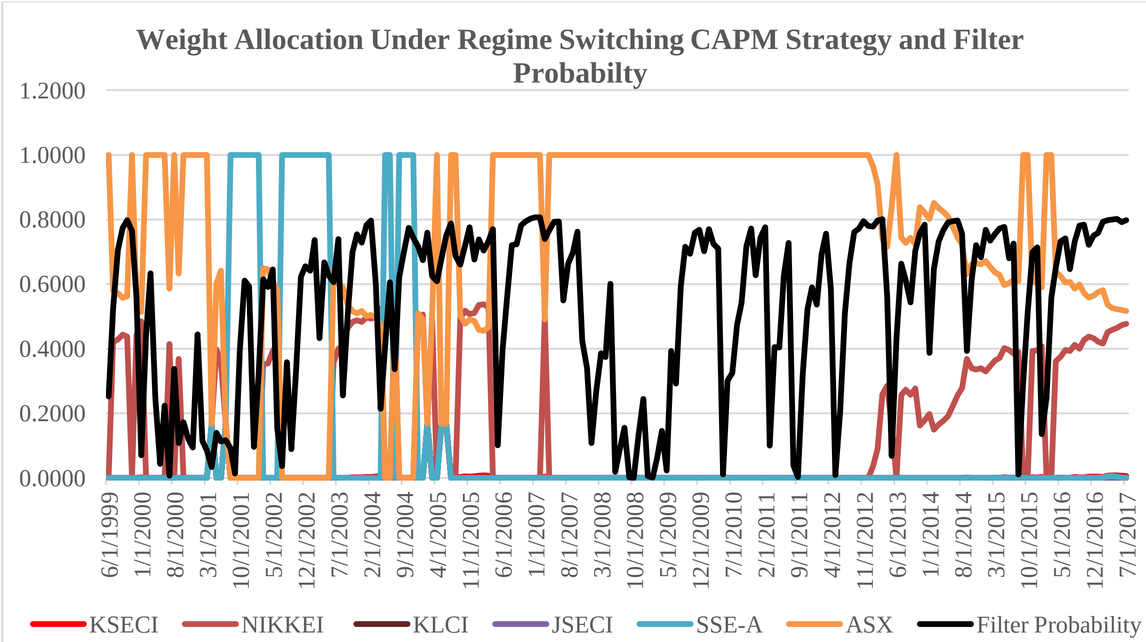


Figure 6.8-2: Portfolio weights of Indices under RS CAPM Strategy and Filter Probability when short selling is not allowed

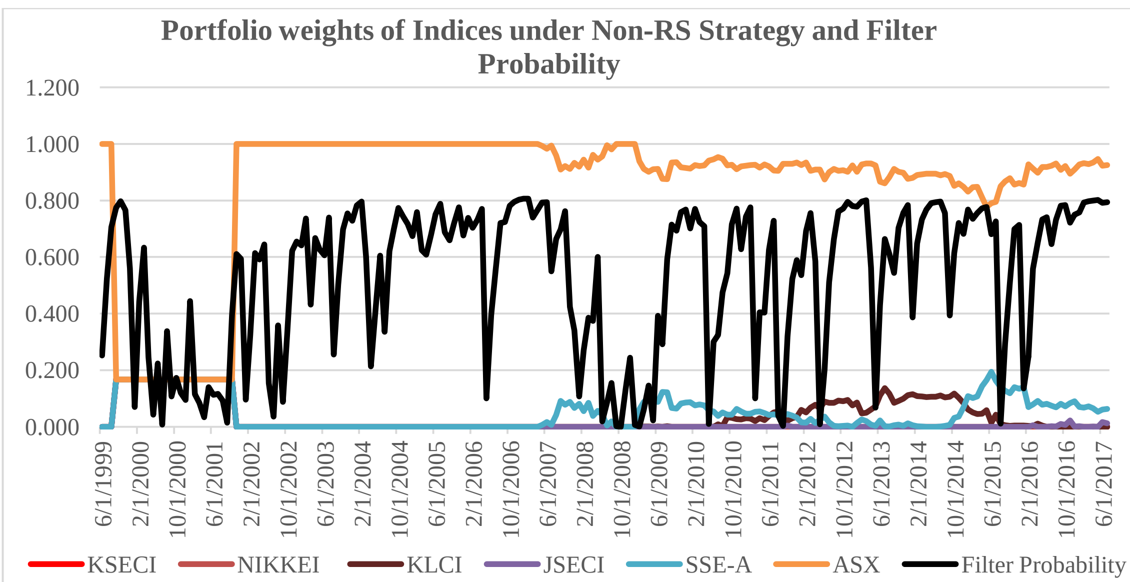


Figure 6.8-3: Portfolio weights of Indices under Non-RS Strategy and Filter Probability when short selling is not allowed

The Non-RS weights shown in Figure 6.6-3, on the other hand, demonstrate the characteristics of a lagging indicator and merely follow the trend of indices one step behind. This strategy also relied heavily on ASX and SSE-A from start till end of 2001 and towards the end after mid-2007 and all the weight is allocated to ASX. KLCI has been allocated some weight between 2010 and 2015. Non-RS has not been able to change weights timely.

6.8.2 Cumulated Wealth with Short-Selling Approach:

Having allowed short selling, RS CAPM strategy performs even better as compared to the other two strategies as shown in Figure 6.6-4 below and ending cumulated wealth has increased as compared to Non-RS and MV strategies. Initial decline in cumulated wealth noticed in without short selling approach has been overturned when short selling is allowed and despite of ups and downs in portfolio wealth, RS CAPM has been well ahead in terms of cumulated portfolio wealth.

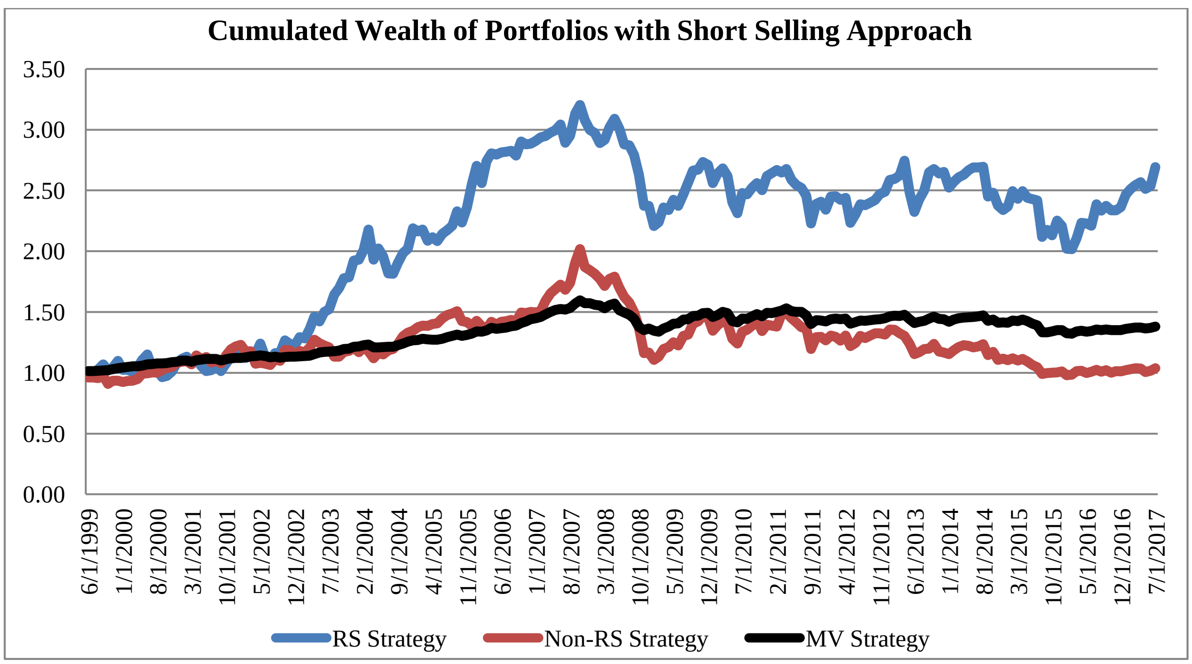


Figure 6.8-4: Cumulated Wealth of all Portfolio Strategies with Short Selling Approach during out-sample Period.

Portfolio weights can go negative or above 1 as there is no restriction on short selling (this thesis allowed weights ranging from -5 to 5). In this case, RS CAPM strategy enjoys more flexibility and utilizes its capability to infer about regimes and taking the right advantage of its forward looking behaviour by short selling and buying long as can be seen from the Figure 6.6-5 below. RS CAPM has exploited well on short selling as it has used ASX for long position and Nikkei and JSECI as short position. SSE-A, KSECI and KLCI have also been used only for long position though for a brief period but have been included in portfolio.

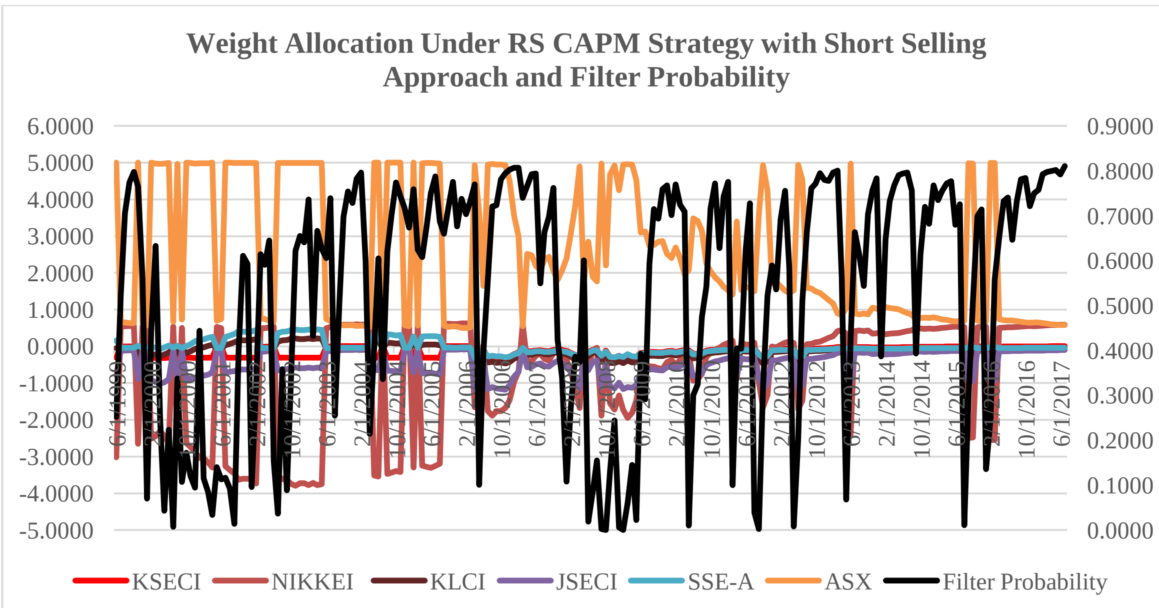


Figure 6.8-5: Weight Allocation of Indices under RS CAPM Strategy with Short Selling Approach and Filter Probability (Filter probability is plotted on secondary axis on right)

While RS CAPM strategy is seen many a times buying long and Short selling different indices but Non-RS strategy have not been able to predict the expected changes in market and has not used full potentioal of allowing short selling especially towards end as cen be seen in Figure 6.6-6. Non-RS has throughout relied heavily on ASX for long position taking and for breif periods relied on KSECI and KLCI. SSE-A, JSECI and Nikkei are short sold to generate profits but Non-RS has not succeded at all.

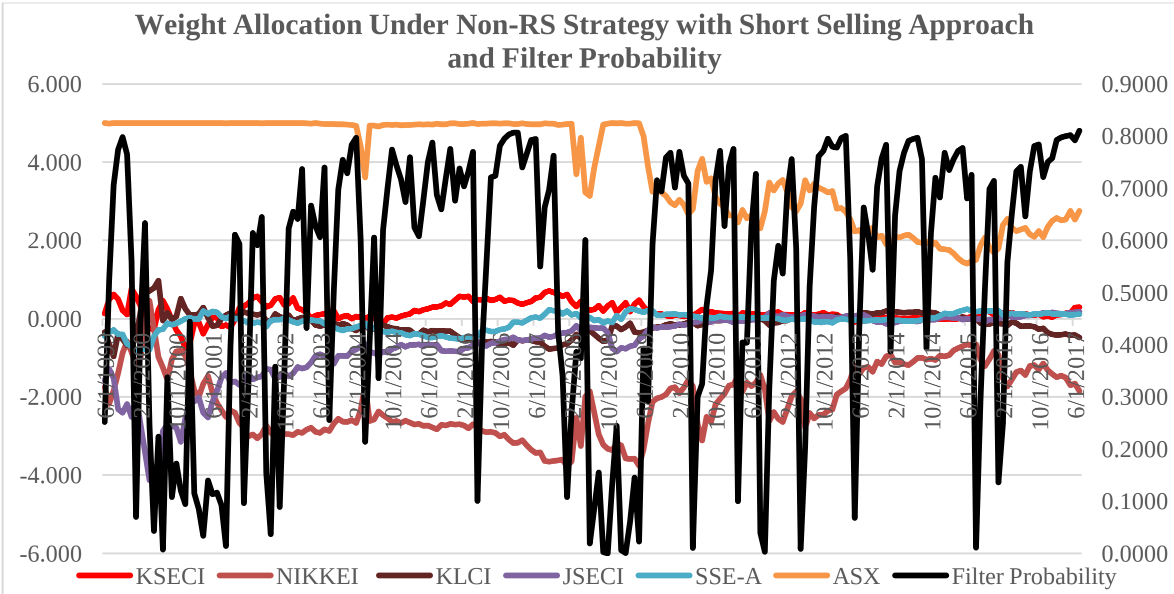


Figure 6.8-6: Weight Allocation of Indices under Non-RS Strategy with Short Selling Approach and Filter Probability (Filter probability is plotted on secondary axis on right)

6.8.3 Market Value Weighted Portfolio Weights

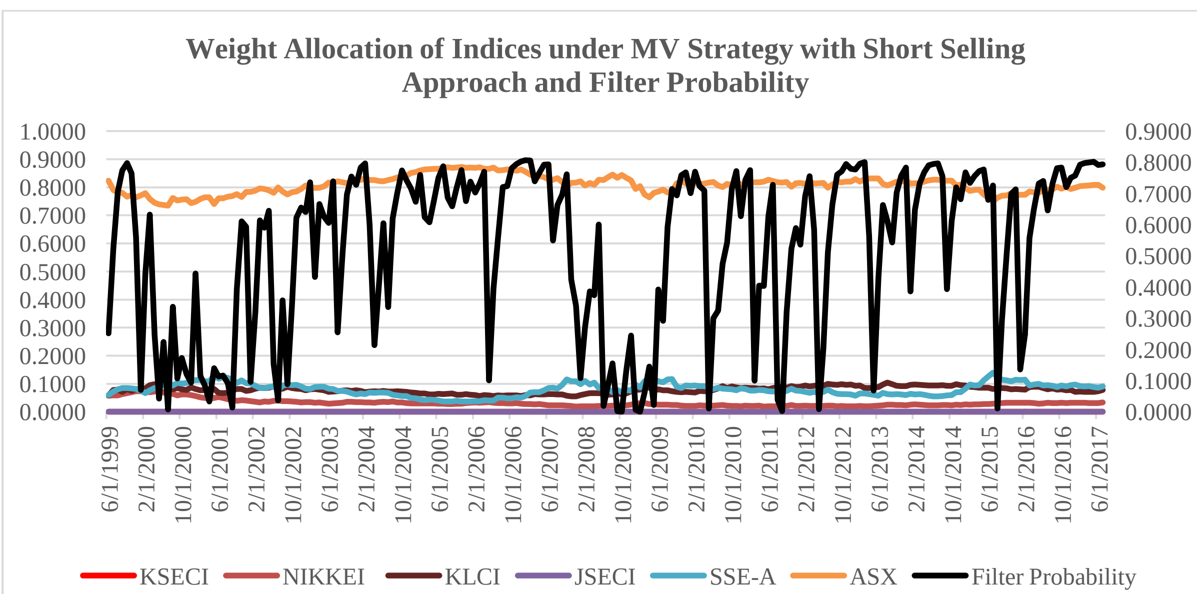


Figure 6.8-7: Weight Allocation of Indices under MV Strategy with Short Selling Approach and Filter Probability (Filter probability is plotted on secondary axis on right)

Figure 6.6-7 shows the MV strategy weights which are similar in without short selling or with short selling approaches and is not able to utilise the potential offered by the market to borrow and use funds productively. MV weights favouring ASX as most weight is allocated to it, obviously due to its large Market values in Dollar terms compared to other indices. Some weight is allocated to indices based on their market values which is counterproductive to this strategy.

6.8.4 Cumulative Wealth with Risk-free Borrowing and Lending

Risk free asset is introduced in the model to provide the investor with an opportunity to freely borrow and lend money at the risk free rate depending on portfolio strategy and whether short selling is allowed or not. Inclusion of risk free asset also enhances the performance of all the portfolios but once again RS CAPM strategy outperforms Non-RS and MV strategies for most of out-sample period. This is evident from Table 6.6-1 that in scenario 1 where No short selling is allowed without risk free asset, the cumulated wealth for RS CAPM is 1.6258 which is higher than Non-RS cumulated wealth (1.392) but is slightly higher than MV strategy (1.490). The RS CAPM cumulated wealth (1.716) is outperforming both Non-RS and MV when risk free asset is introduced in without short selling scenario 2.

Cumulated Wealth and Sharpe ratio for different Strategies	Without Short Selling			With Short Selling		
	RS CAPM Strategy	Non-RS Strategy	MV Strategy	RS CAPM Strategy	Non-RS Strategy	MV Strategy
No Risk Free Asset	**1.6258**	**1.3923**	**1.4901**	**2.5102**	**1.0363**	**1.3806**
Sharpe Ratio	0.1898	0.0512	0.0512	0.2007	0.0634	0.0634
With risk Free Asset	**1.7163**	**1.4795**	**1.5341**	**2.6906**	**1.0563**	**1.4306**
Sharpe Ratio	0.1998	0.0522	0.0522	0.2199	0.0634	0.0634

Table 6.8-1: End of Period Cumulated Wealth and Sharpe ratio for different Strategies

The short selling without risk free asset (scenario 3) generates more cumulated wealth (2.510) than the Non-RS (1.036) and MV (1.380) strategies and performs even better when risk free asset is introduced (scenario 4) with cumulated wealth being 2.690. The key element being usage of risk free asset for borrowing and lending efficiently to maximise the cumulated wealth in out-sample period. This point is strengthened further from the fact that Sharpe ratios calculated for different portfolios show that RS CAPM has high Sharpe ratios in all scenarios than the Non-RS and MV strategies (as can be seen from Table 6.6-1).

Introduction of risk free asset does not have more effect on the cumulated wealth of Non-RS and MV strategy (in both with short selling or without short selling scenarios) which means they fail to capitalise on the risk diversification opportunity arising from the introduction of risk free borrowing and lending. Another point to note here is that the introduction of short selling has negative effect on Non-RS and MV strategies due to wrong choice of assets and allocation of wrong weights to those assets.

Another Factor that enhances the capability and efficiency of RS CAPM strategy is when compared on grounds of Risk Allocation capability against Non RS Strategy. The borrowing at risk free rate is used to enhance the performance of RS CAPM by investing more in risky asset as can be seen from Figure 6.6-8 & Figure 6.6-9. Since RS CAPM judges the Regime changes, it can allocate more in the return generating (risky) assets sometimes by assuming more risk. Non-RS Strategy relied more on Risk free assets to generate nominal returns but did not allocate to Indices and when it did went up to invest more, that was recession period so decision going totally wrong and then had to resort to its original conservative policy.

RS CAPM has gone up to almost 1.67 times investment by borrowing funds at cheap rates to generate more return and invested more in risky assets and minimally to 0.00 times in without short selling approach by lending it at risk free rate and thus generate more returns instantaneously.

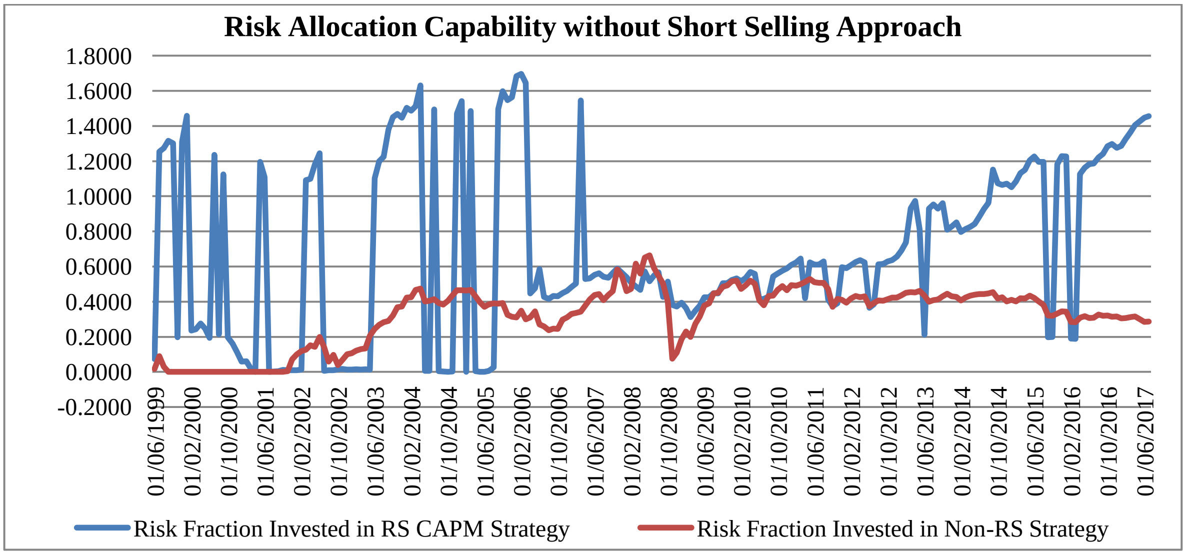


Figure 6.8-8: Risk Allocation Capability of RS CAPM and Non-RS strategies without Short Selling Approach

With short selling allowed RS CAPM uses maximally 1.55 times borrowing ability by investing more in risky assets and lowest borrowing used is 0.165 times and rest is used for the investments in risk free asset by lending it and thus generate more returns instantaneously. Non-RS on the contrary, is relying heavily on investment in risk free asset in both scenarios of short selling or without short selling. Non-RS is more conservative in short selling approach by using maximum risky investment of 0.32 as compared to without selling approach where it is maximally investing in risky assets of up to 0.63 times and rest in risk free assets.

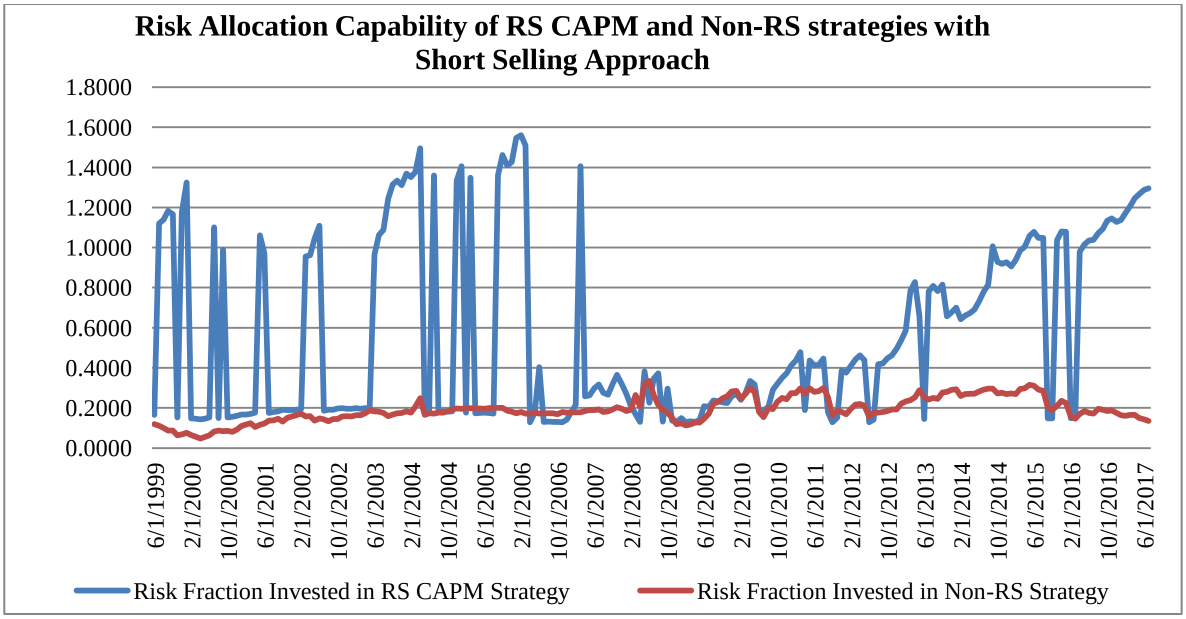


Figure 6.8-9: Risk Allocation Capability of RS CAPM and Non-RS strategies with Short Selling Approach with Risk Free Asset

The sudden switch is in itself explaining that RS CAPM is efficiently and timely switching its borrowing and lending opportunities as compared to Non-RS strategy which is always lagging and in fact has been relying on investment in risk free asset by lending it and never borrowed to invest more in risky investments to generate more returns.

It is concluded that RS CAPM is using majority of its funds to invest in return oriented indices but Non-RS strategy is conservative in approach by investing more of its funds in Risk free asset.

6.8.5 Short Selling With Risk Free Borrowing & Lending (Conditional on Risk Aversion Coefficient) the Risk Aversion Factor

As discussed earlier, default value of risk aversion coefficient is 3 and higher the value, the more risk averse investor is and vice versa. Previous sections 6.6.1-6.6.4 used default value of risk aversion coefficient. Risk aversion coefficient tests are run in two scenarios with and without short selling allowing risk free borrowing and lending.

Cumulated Wealth for different Strategies	Short Selling With Risk Free Borrowing & Lending (Conditional on Risk Aversion Coefficients)			Without Short Selling With Risk Free Borrowing & Lending (Conditional on Risk Aversion Coefficients)		
	RS CAPM	Non-RS	MV	RS CAPM	Non-RS	MV
Risk Aversion Coefficient= 2	**3.1585**	**0.7839**	**1.3292**	**1.9265**	**1.2981**	**1.3640**
Sharpe Ratio	0.2453	0.0634	0.0634	0.2078	0.0522	0.0522
Risk Aversion Coefficient= 3	**2.6906**	**1.0563**	**1.4306**	**1.7163**	**1.4795**	**1.5341**
Sharpe Ratio	0.2199	0.0634	0.0634	0.1998	0.0522	0.0522
Risk Aversion Coefficient= 4	**2.3926**	**1.1587**	**1.4029**	**1.7027**	**1.4244**	**1.4546**
Sharpe Ratio	0.2116	0.0634	0.0634	0.1996	0.0522	0.0522

Table 6.8-2: End of Period Cumulated Wealth and Sharpe Ratio for Different Strategies Conditional on Risk Aversion Coefficients.

When the risk aversion coefficient is changed from 3 to 2 (i.e. the investor is now least risk averse), in both scenarios, RS CAPM generates maximum return (3.158) and highest Sharpe ratio (0.245). As shown in the Table 6.6-2, cumulated wealth of RS CAPM strategy is much more than the other two strategies in every scenario. Most risk averse investor (risk aversion coefficient is 4) generates still higher returns (on the basis of cumulated wealth and Sharpe ratio in Table 6.6-2) for the RS CAPM Strategy as compared with the other two strategies. All strategies show decline in end of period cumulative return as most risk averse investor relies more on investing in risk free asset, RS CAPM performed once again better than other strategies. RS CAPM Strategy' capability to generate right results during wrong moves of market is the key to the success for every future investor and its market timing ability helps it to switch portfolio weights beforehand as compared to other two lagging strategies in turn performing well based on cumulated wealth and Sharpe ratio.

6.9 Conclusion

The stock Index data from six Asia Pacific stock exchanges, KSECI, Nikkei, KLCI, JSECI, SSE-A and ASX is used for portfolio optimisation by using MSCI-AP as benchmark index. In this chapter, the objective was to reinforce the proposition that RS CAPM Strategy is the robust new methodology to achieve portfolio optimisation which has been achieved mostly has been seen from the portfolio weight calculations done through RS CAPM strategy and cumulated wealth achieved through portfolio optimisation. The weights calculated through MV & Non-RS strategies remained stagnant due to which they suffered against RS CAPM strategy. The methodology is found to be robust when applied to established Asia Pacific and UK (FTSE-100 and its listed stocks) markets.

This data set also supports the results found in Chapter 4 and noted when market conditions deteriorated quickly, Regime Switching CAPM became essential for market timing purposes and it helped improve the performance of portfolio. All the strategies have seen decline in cumulated wealth around the time of the credit crisis in 2007 but RS CAPM strategy managed to survive the crises periods and ended in positive and comparatively higher values.

As a whole RS CAPM Strategy provided some good indicators in Developed countries when compared with emerging markets such as Subcontinent markets discussed in Chapter 5. Further detailed studies need to be carried out to understand the reasons of RS CAPM strategy failing during the first half in Subcontinent.

7 Chapter 7: Portfolio Optimization Using Random Matrix Theory on Regime Sensitive Correlation Matrices

Abstract

This Chapter investigates the statistical properties of the correlation matrices for individual stock returns in the FTSE-100 using the random matrix theory (RMT). RMT applied to correlation matrices of stock returns is a filtering method to distinguish between noise and information in these matrices in order to improve portfolio selection (Plerou et al, 2002, Laloux et al, 1999, Daly et al, 2007). The filtering method in RMT relies on the Marchenko-Pasteur bounds for the eigenvalues of the correlation matrices which determines whether the correlation matrix contains noisy elements or not. The implications of this are analysed for the optimal portfolio management in the standard Markowitz portfolio theory (Non-RS) and also in the case of Hamilton two state regime sensitive portfolio optimization (RS CAPM). Note, the CAPM based Hamilton Regime Switching portfolio matrices characterize a form of filtering based on high and low volatility regimes. This Chapter uses stock returns time series of the FTSE-100 firms to compute correlation matrices and their eigenvalue spectra both in a single state (Non Regime Switching) environment and in a two state Regime Switching Model. In the latter case, there is a correlation matrix associated with the high volatility negative average returns regime and another corresponding to the low volatility regime. Eigenvectors corresponding to eigenvalues deviating from the Marchenko-Pasteur bounds are analysed as they are found to contain market information on the centrality of the FTSE-100 firms in the determination of portfolio returns. Eigenvalues within the Marchenko-Pasteur bounds are taken to be the ‘noisy’. We differentiate the correlation matrix into non-noisy and noisy components and filter the noisy ones by using the Plerou et al. (2002) method which replaces the noisy eigenvalues by zeroes. The RMT filtering is done on the sample correlation matrix for the Non-RS case and on the respective

Regime 1 and Regime 2 correlation matrices obtained from the RS-CAPM model. Remarkably in the RS CAPM model case, especially under conditions of high volatility (Regime 2), the RMT filter was least needed as Hamilton RS model works as a filter in itself. Once the filtered correlation matrices are obtained using RMT, this is used to compute the optimal portfolio weights for the RS CAPM model and Non-RS model. The results show improvement in the performance of optimal asset allocation while using RMT filtered correlations when compared with unfiltered correlations in all cases and the filtered RS CAPM case has the best performance of all based on end of period cumulative wealth and Sharpe ratio.

Keywords: Random Matrix Theory, Marchenko-Pasteur Theorem, Correlation Filters, Regime Switching, Portfolio Optimization.

7.1 Introduction

Financial markets have been known to represent complex adaptive systems, which self-organize into various unexpected dynamical structures according to non-trivial interactions among heterogeneous agents (Lux & Marchesi, 1999, Lux, 1998 & Markose et al. 2011). The study of complex economic systems is not easy because we do not know the control parameters that govern economic systems as these systems typically self-organize. The study of financial markets for their complex dynamics has become prominent with both economists and econo-physicists. Research into financial time series has been given great prominence both for portfolio and risk management. Numerous studies have been devoted to understand the statistical properties of financial time series such as volatility (Engle et al., 1993, 1994), long memory (Engle et al., 1993, Geweke & Porter, 1983.) and asymmetric correlation (Mantegna et al., 1995 & 1996, Plerou et al., 2003, Liu et al., 1999, Cizeau et al., 1997, & Jun et al., 2006).

Empirical correlation matrices are of great importance for risk management and asset allocation. The probability of large losses for a certain portfolio or option book is dominated by correlated moves of its different constituents. The study of correlation (or covariance) matrices has a long history in finance (Gabaix et al., 2003, & Yamasaki et al., 2005) and is one of the cornerstones of Markowitz's theory of optimal portfolios. Given a set of financial assets characterized by their average return and risk, the optimal weight of each asset in the portfolio, such that the overall portfolio provides the best return for a fixed level of risk, or conversely, the smallest risk for a given overall return, is a function of the correlation matrix.

In particular, the analysis of financial data by various methods developed in statistical physics has become a very interesting research area for physicists and economists (Mantegna & Stanley, 1999, Bouchaud & Potters, 2004). There is practical (Elton & Gruber, 1981, Okhrin & Schmidt, 2006, Andersen et al., 2002) as well as scientifically important value in analysing the correlation coefficient between stock return time series because this contains a significant amount of information on the nonlinear interactions in the financial market. The correlation matrix between stock returns, which has unexpected properties due to complex behaviours, such as temporal non-equilibrium, mispricing, bubbles, market crashes and so on, is an important parameter to understand the interactions in the financial market (Noh, 2000).

Markowitz portfolio theory, an intrinsic part of modern financial analysis, relies on the covariance matrix of returns and this can be difficult to estimate. For example, for a time series of length T, a portfolio of N assets requires $(N^2 +N)/2$ covariances to be estimated from NT returns. This results in estimation noise, since the availability of historical information is limited. Moreover, it is commonly accepted that financial covariances are not fixed over time and thus older historical data, even if available, can lead to cumulative noise effects. Thus, it is well understood that in Markowitz portfolio model, realized portfolio returns are far removed from the expected portfolio returns that are maximized given the sample estimates for the variance-covariance matrix. Many methods have been used to improve portfolio performance in terms of realized returns. This chapter is concerned about using Random Matrix Theory (RMT) based filtering of the stock returns correlation matrix to improve the realized returns of the portfolio.

To analyse the correlation matrix, previous studies presented various statistical methods, such as principal component analysis (PCA) (Jackson, 2003), singular value decomposition (SVD) (Gentle, 1998) and factor analysis (FA) (Morrison, 1990). Here, to analyse the actual cross-correlation matrix, random matrix theory (RMT) is employed, which was introduced by Wigner, Dyson and Mehta (Mehta, 1991, Wigner, 1951, Dyson, 1962, Dyson & Mehta, 1963, 1960 & 1971) and Guhr et al. (1998). The RMT can be used for eliminating the deviations from Gaussian noise in the actual correlation matrix (Sengupta & Mitra, 1999, Utsugi et al., 2004, Guhr & Kalberzk, 2003, Ruskin et al., 2004).

RMT, first developed by authors such as Dyson and Mehta to explain the energy levels of complex nuclei has recently been applied by several authors including Plerou et al. (1999) and Laloux et al. (1999) for noise filtering in financial time series, particularly in large dimensional systems such as stock market data. Both groups have analyzed US stock markets and have found that the eigenvalues, of the correlation matrix of returns, were consistent with those calculated using random returns, with the exception of a few large eigenvalues.

Ruskin et al. (2004) studied the dynamics of the correlation matrix of multivariate financial time series by examining the eigenvalue spectrum over sliding time windows. Empirical results for the constituent stock returns of the S&P 500 and the Dow Jones Euro Stoxx 50 indices reveal that the dynamics of the smallest eigenvalues of the correlation matrix, over these time windows, are different from those of the largest eigenvalues. This behavior is shown to be independent of the size of the time window and the number of stocks examined. By partitioning the eigenvalue time series, they then show that negative index returns, (which they call drawdowns), are associated with periods where the largest eigenvalue is greatest, while positive index returns, (i.e., drawups), are associated with periods where the largest eigenvalue is smallest (Ruskin et al., 2004).

Laloux et al. (1999) and Plerou et al. (1999) analysed the cross-correlation matrix of financial time series using the RMT method. Plerou et al. (1999) found that 94% of the eigenvalues of cross-correlation matrix can be predicted by the RMT, while the other 6% of the eigenvalues deviated from the RMT. In addition, Plerou et al. (2002) applied the RMT method to the S&P 500 stock market and observed that the cross-correlation matrix of stock returns consists of random and non-random parts. They deconstructed the correlation matrix into what is explained by RMT and the residual. This decomposition carries useful information about the financial market. The pattern of eigenvalue deviations from the RMT were in a remarkably constant state over the entire period of 35 years starting from 1962–1996 (Plerou et al., 2002).

In this context one analyses eigenvalue spectra of corresponding covariance matrices. Under the assumption of uncorrelated financial players, it is possible to identify outliers by use of the Marchenko-Pasteur spectrum, a method which has been applied to financial markets in a portfolio optimization framework before (Liu et al., 1999, Cizeau et al., 1997, Yamasaki et al., 2005, Jun et al., 2006, Mantegna & Stanley, 1999, Bouchaud & Potters, 2004, Elton & Gruber, 1981)

Here, we identify outliers of eigenvalues of covariance matrices, obtained from the returns data. The obtained empirical eigenvalue spectrum is compared to the Marchenko-Pasteur spectrum, which allows the identification of clusters of firms which show non-random structure. These clusters can then be examined in more detail and firms which feature irregular behaviour – in comparison to the average behaviour within a cluster – can be identified.

This chapter will investigate the statistical properties of the correlation matrices for individual stock returns in the FTSE-100 using the random matrix theory (RMT) and analyse the implications of this for the optimal portfolio weights in standard Markowitz portfolio theory and also for Hamilton two state regime sensitive portfolio optimization. In Section 7.2, the methodology of RMT is presented relating to the Marchenko-Pasteur law for correlation matrices. Section 7.2 also describes the RMT based filtering methods used for the correlation matrix of stock returns. We use stock returns time series of the firms to compute correlation matrices and their eigenvalue spectra both in a single state (Non Regime Switching) environment and in a two state Regime Switching CAPM Model. In the latter case, there is a correlation matrix associated with each high volatility negative returns regime and low volatility regime. We deconstruct the correlation matrix into non-random and noisy components and filter the noisy one by using the Plerou et al. (2002) method. Remarkably in the RS model case, under conditions of high volatility, the RMT filter was least needed. Once the filtered correlation matrix is obtained using RMT, this is used to compute the optimal portfolio weights using RS CAPM model and Non-RS model. The results show improvement in the performance of optimal asset allocation while using RMT filtered correlations when compared with the respective cases optimal portfolios with unfiltered correlation matrices.

7.2 Methodology:

We start with the introduction of RMT which was proposed by Wigner, Dyson, and Mehta (Wigner (1951), Mehta (1991), Dyson (1962), Dyson & Mehta (1963, 1960 & 1971)).

7.2.1 Random Matrix Theory

Random Matrix Theory (RMT) is an active research area of modern Mathematics with input from Theoretical Physics, Mathematical Analysis and Probability, and with numerous applications, most importantly in Theoretical Physics, Number Theory, and Combinatorics, and further in Statistics, Financial Mathematics, Biology and Engineering & Telecommunications. Although origins of RMT could be traced back to works by Wishart (1928) and James (1954-1964) in the field of Statistics, the real start of the field is usually attributed to highly influential papers by Eugene Wigner in 1950's motivated by applications in Nuclear Physics.

7.2.2 Why random matrix theory for stock return correlation matrix?

How can we identify the correlated cluster/group of stocks when there is randomness in the measured correlations (*C*), either in the form of correlations that change in time, or by the finite length used to compute the correlation matrix elements? The problem of understanding the properties of matrices with random entries is one which has a rich history originating from 1950 nuclear physics from the work of Wigner (1950), and later on by Dyson and Mehta (1963). In the case of nuclear physics, the problem was to understand the energy levels of complex nuclei, when model calculations failed to explain experimental data. The problem was tackled by Wigner (1950), who made the bold assumption that the interactions between the constituents comprising the nucleus are so complex that they can be modelled as random. Wigner assumed that the Hermitian (*H*) describing a heavy nucleus has, in the matrix representation, elements H_{ij} which can be assumed as mutually independent random numbers. Based on this assumption alone, Wigner derived properties for the statistics of eigenvalues of H, which were in remarkable agreement with experimental data.

RMT predictions represent an average over all possible interactions. Deviations from the universal predictions of RMT identify system-specific, non-random properties of the system under consideration, providing clues about the underlying interactions. The class of matrices Wigner considered are real symmetric matrices, whose elements are distributed according to a Gaussian probability distribution.

Here, this chapter reviews how this framework can be used to quantify and understand the correlations between different stocks. Denoting the correlation matrix by *C*, it is noted that *C* is indeed consistent with a real-symmetric random matrix. From the scientific side, agreement of the eigenvalue statistics of *C* with RMT results imply that *C* has entries that contain a considerable degree of randomness.

7.2.3 Deviations from RMT predictions:

Deviations from RMT indicate properties that are specific to the system and arise from the presence of collective modes. One approach is to study the eigenvalue distribution of *C* computed from time series. We can therefore compare the empirical distribution $P(\lambda)$ with the prediction for uncorrelated time series.

The main goal of RMT is to provide understanding of the diverse properties (most notably, statistics of matrix eigenvalues) of matrices with entries drawn randomly from various probability distributions traditionally referred to as the random matrix ensembles. Three classical random matrix ensembles are the Gaussian Orthogonal Ensemble (GOE), the Gaussian Unitary Ensemble (GUE) and the Gaussian Symplectic Ensemble (GSE) (Mehta et al. 1971). They are composed respectively of real symmetric, complex Hermitian and complex self-adjoint quaternion matrices with independent, normally distributed mean-zero entries whose variances are adjusted to ensure the invariance of their joint probability density with respect to Orthogonal (respectively, Unitary or Symplectic) similarity transformations.

7.2.4 RMT for Correlation Matrix

The process of generating a random matrix involves the following steps: we use *N* (number of companies) data sets having *L* (different for every Chapter) data points following *iid* (0, 1) process. Let the matrix be represented by the symbol *G*. Here, the *G* is a matrix ($N \times L$) with the random elements and the correlation matrix *C* is defined by;

$$C_{random} = \frac{1}{L} G G^T \qquad (7.1)$$

Where G^T is the transpose of *G*, and the correlation is C_{random}.

If N $\longrightarrow \infty$ and L $\longrightarrow \infty$, the eigenvalues spectrum of random matrix is calculated by using;

$$P_{random(\lambda)} = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_{+} - \lambda)(\lambda_{+} - \lambda)}}{\lambda} \qquad (7.2)$$

Where the eigenvalues λ lie within $\lambda_{-} <= \lambda <= \lambda_{+}$, and $Q = L/N$, and the maximum and minimum eigenvalues of the random matrix C_{random}, are given by Marchenko-Pasteur limits[10];

$$\lambda_{\pm} = 1 + \frac{1}{Q} \pm 2\sqrt{\frac{1}{Q}} \qquad (7.3)$$

[10] Marchenko-Pasteur theorem is shown in Appendix 1.

In random matrix theory, the Marchenko–Pasteur distribution, or Marchenko-Pasteur law, describes the asymptotic behaviour of singular values of large rectangular random matrices. The theorem is named after Ukrainian mathematicians Vladimir Marchenko and Leonid Pasteur who proved this result in 1967. The Marchenko–Pasteur law also becomes the free Poisson law in free probability theory, having rate λ and jump size α.

By comparing the Eigenvalue spectrum of the correlation matrix with the analytical results, obtained for random matrix ensembles, significant deviations from RMT Eigenvalue predictions provide genuine information about the correlation structure of the system. This information has been used to reduce the difference between predicted and realized risk of different portfolios in this thesis as used by Daly et al (2010).

Similarly, eigenvectors are a special set of vectors associated with a linear system of equations (i.e., a matrix equation) that are sometimes also known as characteristic vectors, proper vectors, or latent vectors (Marcus and Minc, 1988). The determination of the eigenvectors and eigenvalues of a system is extremely important in physics and engineering, where it is equivalent to matrix diagonalization and arises in such common applications as stability analysis, the physics of rotating bodies, and small oscillations of vibrating systems, to name only a few. Each eigenvector is paired with a corresponding Eigenvalue. Mathematically, two different kinds of eigenvectors need to be distinguished: left eigenvectors and right eigenvectors. However, for many problems in physics and engineering, it is sufficient to consider only right eigenvectors. The term "eigenvector" used without qualification in such applications can therefore be understood to refer to a right eigenvector (Marcus and Minc, 1988).

Some basics about eigenvalues and eigenvectors suggest eigenvalues are invariant with respect to similarity transformations meaning if two matrices have different sets of eigenvalues, they must not be similar to each other. A large part of basic linear algebra deals with the question of distinguishing matrices and linear operators up to similarity. If *L* (number of observations-number of rows in this chapter) and *N* (number of companies-number of columns in this chapter) have a finite length, then the Eigenvalue spectrum shows gradual decrease from the theoretical values of the largest Eigenvalue predicted by the RMT. Similarly, eigenvectors are used to find bases with respect to which the matrix of an operator is "nice", for example in diagonalizing matrices or putting them in canonical forms. Through basis of eigenvectors, we get enough data to reconstruct the operator in a very simple way.

For example, in quantum mechanics, a measurement (like if you want to measure the energy or angular momentum of a system) corresponds to a linear operator, and the result of a measurement is ‘always’ an Eigenvalue. The eigenvectors correspond to the "pure" states, the ones where the outcome of the measurement can be known with certainty (whereas in general, the outcome can only be predicted probabilistically). So there is a sense in which eigenvalues and eigenvectors are fundamentally important.

Research has suggested recently that some real correlation information may be hidden in the RMT defined random part of the Eigenvalue spectrum. The correlation structure of multivariate financial time series was studied by investigation of the Eigenvalue spectrum of the equal-time cross-correlation matrix (Daly et al., 2007, 2010). By filtering the correlation matrix through the use of a sliding window, behaviour of the largest Eigenvalue can be examined over time. The largest Eigenvalue moves counter to that of a band of small eigenvalues, due to Eigenvalue repulsion (Daly et al., 2007, 2010, Plerou et al, 2002). A decrease in the largest Eigenvalue, with a corresponding increase in the small eigenvalues, relates to a redistribution of the correlation structure across more dimensions of the vector space spanned by the correlation matrix. Hence, additional eigenvalues are needed to explain the correlation structure in the data. Conversely, when the correlation structure is dominated by a smaller number of factors (e.g. the "single-factor model" of equity returns), the number of eigenvalues needed to describe the correlation structure in the data is reduced. This means that fewer eigenvalues are needed to describe the correlation structure of 'draw-downs' than that of 'draw-ups' (Daly et al., 2007, 2010, Plerou et al, 2002).

7.2.5 RMT Filtering of Returns Correlation Matrix

We used the RMT filtering method used by Plerou et al. (1999, 2002), keeping in mind available methods for filter purposes introduced in RMT. All three filtering methods discussed here are based on a common procedure of replacing the "noisy" eigenvalues of the correlation matrix, while maintaining its own trace. The noisy eigenvalues are taken to be those that lie within the maximum and minimum limits of the Marchenko-Pasteur bounds (M-P) result given in equation (7.3). The correlation matrix being filtered is first deconstructed via the Eigen decomposition theorem. Once you have correlation matrix, one can decompose it as;

$$C = V D V^{-1} \qquad (7.4)$$

Where D is matrix of diagonal eigenvalues and V is the matrix of their corresponding eigenvectors.

The noisy values in D are identified through Marchenko-Pasteur limits and D is then composed of;

$$D = D_{noisy} + D_{nonnoisy} \quad (7.5)$$

Since D_{noisy} needs to be filtered through Plerou et al. (2002) method which defines λ being noisy values and will be filtered like;

$$D_{noisy} \text{ has } \lambda \text{ replaced by } 0 \; if \; (\lambda_{-} < \lambda < \lambda_{+}) \quad (7.6)$$

$$\text{Therefore,} \quad D_{filter} = D_{filtered} + D_{nonnoisy} \quad (7.7)$$

Use the Eigen decomposition theorem to re-compute the filtered correlation matrix.

$$C_{filter} = V\, D_{filter}\, V^{-1} \quad (7.8)$$

The noisy eigenvalues identified in Equation (7.6) are subsequently replaced using one of the three methods outlined here, and the matrix is rebuilt using the Eigen decomposition theorem (Equation (7.8)), resulting in the new filtered correlation matrix, C_{filter}.

The first filtering method examined is that of Laloux et al. (1999), which replace the noisy eigenvalues with their mean, thus maintaining the trace of C (trace is maintained when the sum of eigenvalues of C and C_{filter} is same).

The second filtering method is that implemented by Plerou et al. (2002). This method replaces the noisy eigenvalues by zeroes and, after C_{filter} is built, replaces its main diagonal with that of the original matrix C, again preserving the trace.

The third filtering method is used by Daly et al. (2010) and is adapted from that of Sharifi et al. (2004). To maximise the Krzanowski stability of the filtered matrix, while also maintaining its trace, the method of Sharifi et al. (2004) replaces the noisy eigenvalues with positive values that are equally and maximally spaced, and have sum equal to the sum of those replaced. To achieve maximal spacing, it is assumed that the smallest replacement eigenvalue should be very close to zero. It was found, in Daly et al. (2010), that the optimal parameter value for reducing realised portfolio risk involved some reduction in stability[11]. This chapter uses the method explained in Plerou et al. (2002) as this method was found to provide stability as well as provided ease of implementation when compared with method three. Method 1 could not be used as the average of noisy eigenvalues for some of the correlation matrices still fell in Marchenko-Pasteur limits.

7.2.6 Application of RMT Filtered Correlation Matrix in Markowitz Portfolio Theory

Here we use the RMT filtered correlation matrices to select the optimal portfolio of all stocks. The purpose of MPT (Markowitz Portfolio Theory) is to minimize the portfolio risk in a given portfolio return, which can be quantified by the variance defined as follows.

$$\Omega = \sum_{i=1}^{N} \omega_i \sum_{j=1}^{N} \omega_j C_{ij}\, \sigma_i\, \sigma_j \qquad (7.9)$$

Where ω_i is the portfolio weight of stock *i*, which can be calculated using two Lagrange multipliers, σ_i and σ_j is the standard deviation of stock *i* and *j*, and C_{ij} is the correlation coefficient between stock *i* and stock *j*. In this Chapter, we use the no short-selling constraint for portfolio weights i.e. we assume that all the weights are non-negative numbers. We also normalize portfolio weights in such a way that $\sum_{i=1}^{N} \omega_i = 1$ .The portfolio return, μ, is calculated by;

$$\mu = \sum_{i=1}^{N} \omega_i\, \mu_i \qquad (7.10)$$

Where μ_i is the mean value of stock i.

[11] Stability is discussed keeping in view the correlation matrix before and after filter by Daly et al (2010).

7.2.7 How to compute CAPM Correlation Matrix:

Since we consider regime switching portfolio optimization in which the regime dependency is determined from the stock index returns, the CAPM relationship is used to obtain the returns for each stock in the portfolio. Hence the CAPM beta and CAPM correlation matrix have to be obtained in the standard settings. First determine the CAPM beta for stock. The formula is the cost of equity equals the risk-free rate of return plus the beta multiplied by the risk premium.

$$E_c = R_f + \beta(R_p) \tag{7.11}$$

1. Solve for the Beta based on the inputs or assumptions, As we already have empirical data, Beta is computed for all the stocks using one-factor CAPM model, widely acknowledged in the financial literature as a pricing model, which influences all stocks in the market and is defined by;

$$r_i(\mathrm{t}) = \alpha_\mathrm{i} + \beta_\mathrm{i} R_{Market}(\mathrm{t}) + \epsilon_\mathrm{i}(\mathrm{t}) \tag{7.12}$$

2. Where R_{Market} is the FTSE-100 market index return, α_i and β_i are the regression coefficients of stock *i*. β coefficient is used as a measurement to quantify the relationship between returns of stock *i* and market index returns.
3. For each stock return *i*, its Beta is equal to the correlation coefficient times the standard deviation of the stock divided by the standard deviation of the index.

$$\beta = \rho({}^{\sigma_i}/_{\sigma_{index}}) \tag{7.13}$$

Where ρ is the Correlation coefficient for all the stocks (used in sample) and FTSE-100 index and CAPM based Correlation matrix is computed.

7.2.8 The RMT Portfolio Mechanism:

The following steps outline the whole process in a more simplistic way and Chapter will proceed on these steps.

Step 1: Compute the returns for all the stocks and excess returns for the market index.

Step 2: The whole sample data is divided into in-sample and out-sample period. The correlation matrix is computed for in-sample to proceed for simple MPT or Non-RS portfolio. The correlation matrix is;

(i) Empirical correlation matrix for Non-RS (EmpSample)

Correlation matrices based on regimes are computed which are used for RS CAPM based portfolio as proposed in Chapter 3 and are;

(ii) Empirical Regime 1 CAPM Correlation matrix (RS1 CAPM)

(iii) Empirical Regime 2 CAPM Correlation matrix (RS2 CAPM)

Note that the above matrices are computed from empirical data.

Step 3: Since, the correlation are composed of following components as suggested by RMT;

$$C_{original} = C_{non\text{-}noisy} + C_{noisy}$$

Eigenvalues λ_i and their respective eigenvectors V_i are calculated from the above three correlation matrices.

Step 4: Compute the Marchenko-Pasteur limits using Equation (7.3) since we know data matrix size ($N \times L$).

Step 5: Identify C_{noisy} from eigenvalues λ_i by using Marchenko-Pasteur limits.

Step 6: The next step is filtering noisy values using Plerou et al method so that you have $C_{original}$ transformed into C_{filter} and obtain the following three matrices.

(i) Filtered Correlation matrix for Non-RS (EmpSample)

(ii) Filtered Regime 1 CAPM Correlation matrix (RS1 CAPM)

(iii) Filtered Regime 2 CAPM Correlation matrix (RS2 CAPM)

Step 7: Once the filtered correlation matrices are obtained, once again their eigenvalues and eigenvectors are calculated and checked whether they still have 'noisy' elements or not?

Step 8: The eigenvector for the highest Eigenvalue is discussed for its implications for portfolio formation have been widely accepted.

Step 9: The portfolio is formed based on unfiltered and filtered correlation matrices (after changing correlation matrices to covariance matrices) for comparison purposes. Optimised portfolios after daily rebalancing in out-sample period for two scenarios of filtered Non-RS, and RS CAPM vs. two scenarios of unfiltered Non-RS and RS CAPM are compared to measure the impact of RMT correlation filters on portfolio performance.

7.3 Empirical Analysis and Results:

The data is obtained from DataStream for FTSE-100 index and its current constituent 100 companies as on 30th June 2017. The constituent' data availability from start date of 03 January 2000 is checked, which leaves us with 74 companies that had a long record of trading history in London Stock Exchange. The data start date is 03 January 2000 and end date is 30 June 2017 which covers 4565 trading days. The in-sample period is taken from Jan 3, 2000 – Dec 31, 2005 and out-sample is from Jan 1, 2006- Jun 30, 2017. The complete list of constituent companies used in empirical sample and companies that are dropped is given at the end in Appendix-2.

The major steps are in following order; first, data descriptive statistics for empirical sample returns are discussed for whole sample. Second, the different correlation matrices are computed as discussed in section 7.2.8 using the Non-RS, and RS CAPM. Third, Eigenvalue and eigenvector analysis of correlation matrices (in section 7.5.2) is discussed.

Fourth, the performance of the portfolios without filtering is discussed using the Non-RS and RS CAPM strategies and compared with the performance of portfolios when RMT filtering is applied using Marchenko-Pasteur and Plerou et al method. Conclusion is drawn at the end.

7.4 Statistical Data Description

We first analyse the sample data statistics. The Appendix 3 gives the whole sample statistics for the mean variance skewness and kurtosis for each of the 74 stock returns and for the FTSE-100 returns. Highest mean return is associated with Randgold Resources (0.0009) and lowest mean return is for Royal Bank of Scotland Group (-0.0005). The highest standard deviation is of Ashtead Group (4.26%) and lowest standard deviation is associated with National Grid (1.36%). Majority show negative skew and high values of excess kurtosis, Ashtead Group having very high excess kurtosis and highest negative skew of -9.152. All are positively correlated with FTSE-100 with Scottish Mortgage being highly correlated (on the basis of R Squared value of 0.5665) and Ashtead having lowest correlation of 0.0715 with FTSE-100.

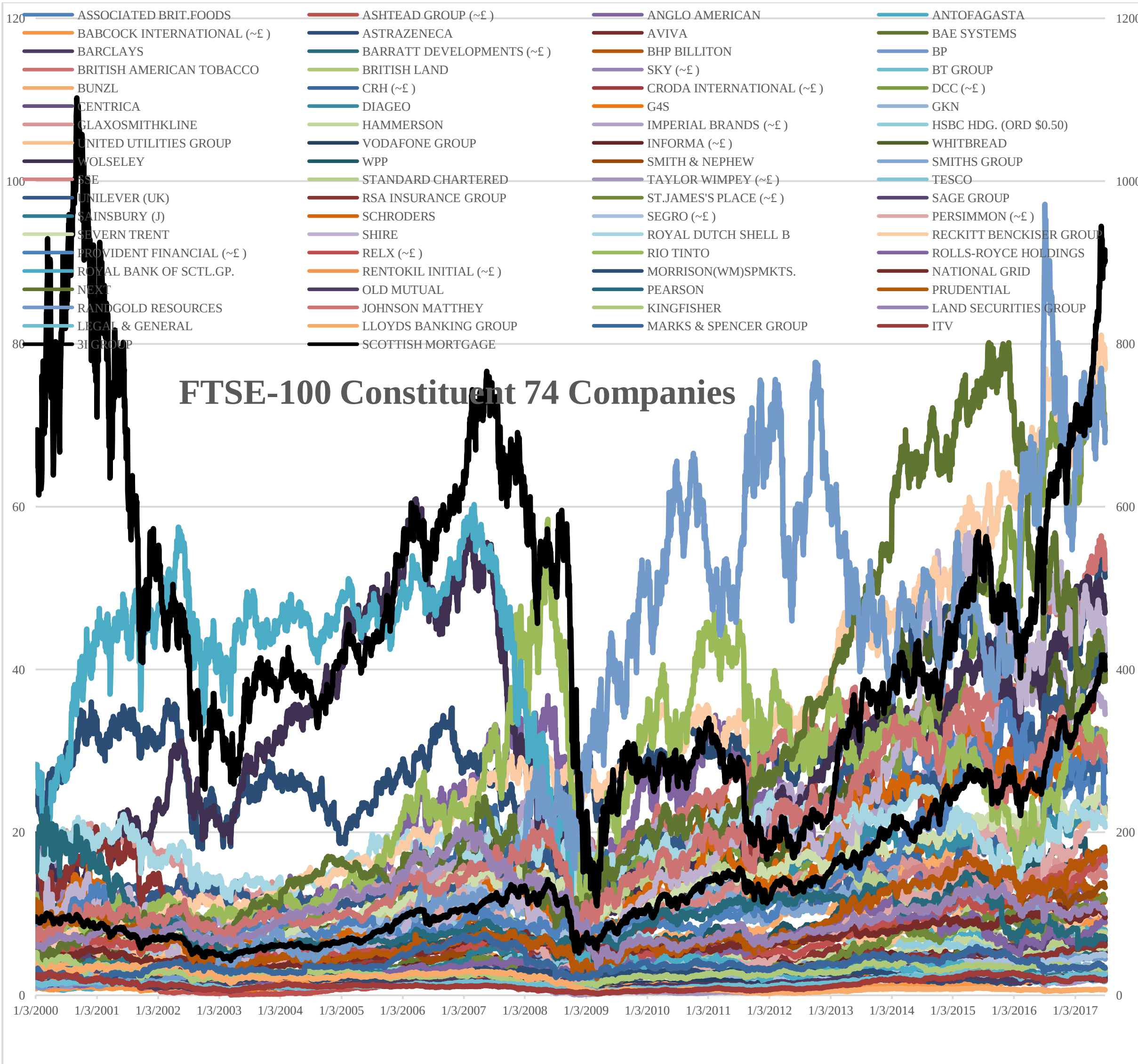


Figure 7.4-1: FTSE-100' 74 Constituent companies (3 I Group and Scottish Mortgage (in Black colour) are on secondary Axis on the Right)

Figure 7.4-1 shows that all the assets are moving in the same direction due to correlations (weak or strong) between all of them. All stocks have prices going up and down showing existence of regimes. Since Figure 7.4-1 is not clear, please refer to Appendix 10.7 for detailed graphic representation of FTSE 100 constituents. FTSE-100 price index and its excess returns show the similar ups and downs confirming the existence of regimes (Figure 7.4-2). Figure 7.4-2 also shows that between 2001 and 2003 sample period and between

2008 and 2009, it faced huge decline in excess returns due to international factors of Dotcom and Great Financial crises during the same period.

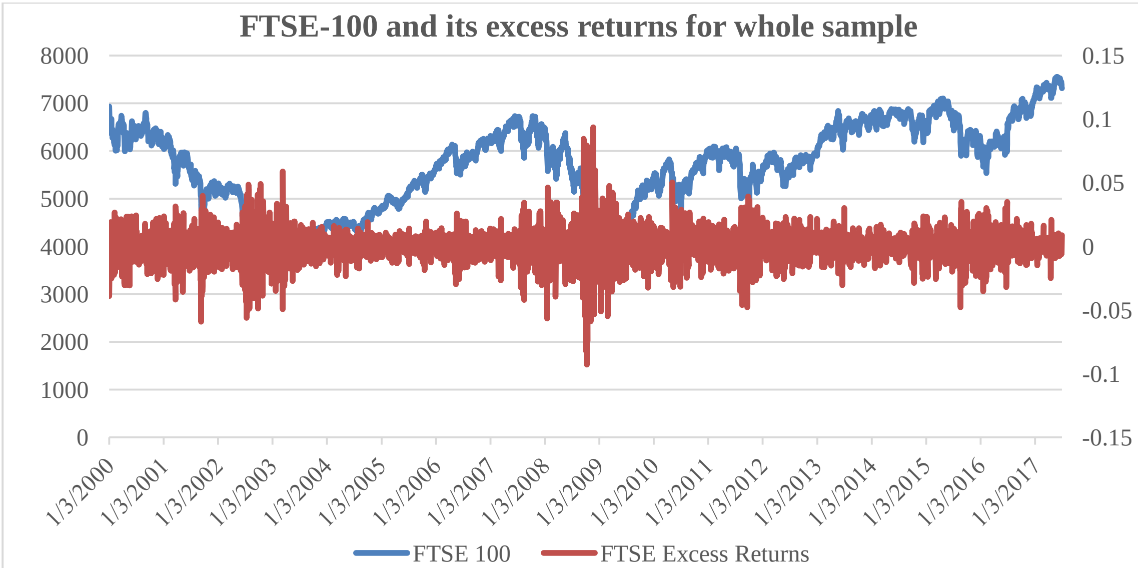


Figure 7.4-2: FTSE-100 and its excess returns for whole sample period

7.5 Correlation computation and discussion

In this section, statistical properties of the correlation matrices of the 74 daily stock returns (empirical sample) traded on the FTSE 100 from 3 January 2000 to 30 June 2017 are discussed. Correlations are computed from empirical data returns and discuss whether the properties of the correlation matrices as discussed in RMT hold for the empirical data.

7.5.1 Eigenvalue Analysis

Most of the distribution is consistent with the RMT bounds calculated in different studies (e.g. Plerou et Al., 1999, Daly et al., 2010). This comparison also indicates the presence of several eigenvalues clearly outside the random matrix bound. Particularly interesting is the largest Eigenvalue, which Plerou et Al. (1999) found was approximately 25 times larger than the value predicted for a random correlation matrix suggesting genuine information about the correlations between different stocks. Having demonstrated that the bulk of the eigenvalues satisfies RMT predictions, proceed to analyse the eigenvectors of *C*. First, analyses of the statistics of the eigenvectors are done. An examination of the eigenvectors corresponding to the eigenvalues which deviate from the random-matrix bound shows systematic deviations from the Gaussian prediction. In particular, the largest Eigenvalue is strongly non-Gaussian, tending to be uniform, suggesting that all companies participate towards the index performance. This notion can be accurately quantified by the concept of inverse participation ratios, borrowed from the localization theory, where research finds that all components participate approximately equally to the largest eigenvector. This implies that every company is connected with every other company. In the stock market problem, this eigenvector conveys the fact that the whole market moves together and indicates the presence of correlations that pervade the entire system (Daly et al, 2010).

The Marchenko-Pasteur limits to identify noisy values are given by the Equation (7.3) which is dependent on variable $Q = L/N$, where L=4565 and N = 74. The maximum Eigenvalue limit (λ_{max}) and minimum Eigenvalue limit (λ_{min}) for all the correlation matrices is calculated through Equation (7.3) and their values are;

$\lambda_{min} = 0.7615$ & $\lambda_{max} = 1.2709$

We use these limits to determine the noisy eigenvalues for the following correlation matrices from empirical sample.

1. Empirical correlation matrix for Non-RS (EmpSample)
2. Regime based correlation matrices further characterised into two sub classes;
    a. High volatility based correlation matrix (RS1 CAPM)

b. Low volatility based correlation matrix (RS2 CAPM)

Noisy Eigen Values shown by Marchenko-Pasteur limits before filtering			
	EmpSample Eigenvalues	RS1 CAPM Eigenvalues	RS2 CAPM Eigenvalues
Number of Noisy Values	18	13	2
Percent of Total (74)	24.32%	17.57%	2.7%

Table 7.5-1: Number of Noisy Eigenvalues shown by Marchenko-Pasteur limits before filtering process

Once the eigenvalues and eigenvectors are computed for the above correlation matrices, the results are examined more closely. The key element was to compute the noisy values given by Marchenko-Pasteur limits. Two correlation matrices show that they had many noisy values except RS2 CAPM which had only 2 noisy values. Remarkably in the RS CAPM model case, especially under conditions of high volatility (Regime 2), the RMT filter was least needed as Hamilton RS model works as a filter in itself and demonstrates that regime dependent correlation matrix provides a 'natural' filter for the noisy values for the bad state. The noisy values in EmpSample and RS1 CAPM are given respectively by 24.32% and 17.57% as shown in Table 7.5-1. If these noisy values are not treated within these matrices, they will impact negatively on the portfolio formation by giving wrong information on the correlation of assets and hence the performance of the portfolio will be severely affected.

To treat this noise we use the filtering method of Plerou et al. (2002) by filling in zeros in the place of noisy values.[12] Then the filtered correlation matrices are constructed by using Eigenvalue decomposition technique and making sure that the diagonal of filtered matrix is

[12] This method was chosen because the insertion of the averaged noisy eigenvalues used by Laloux et al (1999) still produced noise (as the average was 1.032 in case of CAPM) and values fall within the Marchenko-Pasteur limits.

same as of original matrix, in order to maintain its trace. The following steps show the process of filtering and reassembling of C_{filter}.

1. From $C_{original}$ we get Eigenvalue diagonal matrix D and corresponding eigenvectors matrix V.
2. The noisy eigenvalues are identified as within Marchenko Pasteur limits, i.e. greater than 0.7615 and less than 1.2709 and are replaced with "0".
3. Then reconstruct the filtered correlation matrix by using filtered Eigenvalue diagonal matrix (D_{filter}) and corresponding eigenvectors V as follows;

$$C_{filter} = V*D_{filter}*V^{-1}$$

4. Once C_{filter} is obtained, we examine it to check whether its diagonal is similar to original matrix with unit values on the diagonal, if not we repeat the process until original diagonal is obtained.

The matrix C_{filter}, once obtained is checked for noise again and is reported in Table 7.5-2 below;

Noisy Eigenvalues shown by Marchenko-Pasteur limits after Filtering process			
	EmpSample Eigenvalues	RS1 CAPM Eigenvalues	RS2 CAPM Eigenvalues
Number of Noisy Values	13	0	0
Percent of Total	17.57%	0.00%	0.00%

Table 7.5-2: Noisy Eigenvalues shown by Marchenko-Pasteur limits after filtering process

The new filtered correlation matrices obtained are analysed to check the impact of the filtering process, as it can be seen from Table 7.5-2 that RS1 CAPM and RS2 CAPM have been completely filtered and show no noisy values anymore. The point of concern is EmpSample which shows decline in noisy values of only 6.75% and still has too much noise in it which will definitely affect its portfolio performance (performance as to portfolio formation is discussed later in next section).

7.5.2 Eigenvector Stock constituents of deviating largest eigenvalues from RMT limits

Literature on Eigenvalue based studies of stock returns suggest that we look into the stock composition of the eigenvector corresponding with the highest Eigenvalue deviation from RMT limits. This helps in identifying sectors and will tell us which companies contributed more to the overall performance of Index (Daly et al, 2007). The following Table 7.5-3 shows the highest Eigenvalue from all the matrices before and after filtration process.

Table 7.5-3 show the λ_{74} (Maximum Eigenvalue) for each type of correlation matrix. If the eigenvalues are compared before and after filtering, the maximum Eigenvalue increases in every case after the filtering process indicating an increase in information.

Unfiltered		Filtered		Unfiltered		Filtered		Unfiltered		Filtered	
EmpSample		**EmpSample**		**RS1 CAPM**		**RS1 CAPM**		**RS2 CAPM**		**RS2 CAPM**	
$\lambda_{74sample}$	**22.398**	$\lambda_{74sample}$	**29.071**	λ_{74RS1}	***46.150***	λ_{74RS1}	***61.005***	λ_{74RS2}	***76.931***	λ_{74RS2}	***77.914***

Table 7.5-3: Largest eigenvalues before and after filtering process for all correlations

Another important factor that needs to be highlighted here is the composition of top ten companies and is vital to portfolio formulation by rebalancing and optimising with the help of Regime Switching model and Non Regime Switching model. If the RMT filters improve the informative part of data, it will help form better portfolios.

As it can be seen from table 7.5-4 below; if we are to select these top 10 companies from the largest eigenvalues in all the scenarios and form portfolios on the basis of largest Eigenvalue' constituent eigenvector assuming them to be weights, we would have been wrong in picking these organisations. The filters have changed the ranking and identities of the top ten organisations.

In the case of EmpSample correlation matrix, both filtered and unfiltered cases show a dominance of financial companies and only one company is replaced by the filtering process (i.e. LEGAL & GENERAL is replaced with ROYAL DUTCH SHELL B). But in case of the RS1 CAPM, and RS2 CAPM correlation matrices, the composition of top ten contributors remains same before filtering. The top 10 contributors to largest Eigenvalue changes completely for the RS1 CAPM and 9 out of 10 contributors change for RS2 CAPM after filtering, a scenario, which is expected to improve the portfolio performance. Before filtering, RS1 CAPM and RS2 CAPM were relying on similar set of organisations with slight change in weights and rank order, whereas after filtering which there is a big change in the in the constituent companies for Regime 1 and Regime 2 cases. Two companies, Barclays and British American Tobacco are same in RS1 CAPM and RS2 CAPM after filtering which were not in top 10 contributors before filtering.

In general, the ten largest contributor names and their corresponding industry/sectors come from three major sectors;

1. Banking and Financial Services (Prudential, Aviva, Standard Chartered, Legal & General Etc.)
2. Metals & Mining (Rio Tinto, BHP Billiton, Anglo American etc.)
3. Investment Management (Old Mutual, Schroder's etc.)

The few exceptions are from: fast moving consumer goods (food, beverage, retails etc.): Tesco, NEXT, Kingfisher, Whitbread, Marks and Spencer's. Some companies are from Pharmaceuticals and Oil and Gas sectors.

TOP 10 CONTRIBUTORS IN EIGENVECTOR OF HIGHEST/MAXIMUM EIGENVALUE FOR ALL MATRIX TYPES (UNFILTERED VS. FILTERED)										
UNFILTERED EMPSAMPLE	PRUDENTIAL	ANGLO-AMERICAN	AVIVA	OLD MUTUAL	STANDARD CHARTERED	BHP BILLITON	NEXT	SCHRODERS	LEGAL & GENERAL	RIO TINTO
	0.1579	**0.1567**	**0.1498**	**0.149**	**0.1472**	**0.1468**	**0.1449**	**0.1443**	**0.1429**	**0.1423**
FILTERED EMPSAMPLE	SCHRODERS	NEXT	PRUDENTIAL	ANGLO AMERICAN	AVIVA	OLD MUTUAL	ROYAL DUTCH SHELL B	BHP BILLITON	STANDARD CHARTERED	RIO TINTO
	0.1585	**0.1584**	**0.1441**	**0.1416**	**0.137**	**0.1343**	**0.1338**	**0.1332**	**0.1323**	**0.1323**
UNFILTERED RS1 CAPM	GKN	BT GROUP	WHITBREAD	ROLLS-ROYCE HOLDINGS	RIO TINTO	KINGFISHER	MARKS & SPENCER GROUP	WOLSELEY	SMITHS GROUP	PROVIDENT FINANCIAL
	0.1409	**0.1406**	**0.1394**	**0.1351**	**0.1349**	**0.1348**	**0.1323**	**0.1322**	**0.1316**	**0.1308**
FILTERED RS1 CAPM	SKY	PEARSON	BARCLAYS	AVIVA	ASSOCIATED BRIT.FOODS	UNILEVER (UK)	SHIRE	HAMMERSON	BRITISH AMERICAN TOBACCO	BP
	0.1269	**0.1268**	**0.1266**	**0.1262**	**0.126**	**0.1259**	**0.1259**	**0.1258**	**0.1254**	**0.1242**
UNFILTERED RS2 CAPM	GKN	BT GROUP	WHITBREAD	RIO TINTO	KINGFISHER	ROLLS-ROYCE HOLDINGS	MARKS & SPENCER GROUP	WOLSELEY	SMITHS GROUP	PROVIDENT FINANCIAL
	0.114	**0.114**	**0.114**	**0.114**	**0.114**	**0.114**	**0.114**	**0.114**	**0.114**	**0.114**
FILTERED RS2 CAPM	BARCLAYS	BRITISH AMERICAN TOBACCO	ASTRAZENECA	3I GROUP	NATIONAL GRID	TESCO	G4S	WHITBREAD	BUNZL	LAND SECURITIES GROUP
	0.1133	**0.1133**	**0.1133**	**0.1133**	**0.1133**	**0.1133**	**0.1133**	**0.1133**	**0.1133**	**0.1133**

Table 7.5-4: Top 10 contributors in eigenvector of highest/maximum Eigenvalue for all matrix types (Unfiltered vs. Filtered)

These findings are consistent with Daly et al (2007) who show that largest eigenvalues help describe distinct sectors contributing more towards the performance of constituent index. If the companies highlighted by this eigenvector centrality are used in the formation of portfolios, better performance is bound to follow.

The largest eigenvalue in case of unfiltered RS1 CAPM λ_{74} is 46.15 and unfiltered RS2 CAPM λ_{74} is 76.93 and look at the corresponding eigenvector which shows the components that contribute to the performance of overall market. In this case ten largest contributor names and their corresponding industry/sectors when analysed for both the regimes are same as the companies in both cases are same and come from the following sectors;

1. Metals & Mining
2. Consumer Goods
3. Investment Management
4. Telecom sector
5. Oil and Gas

If we look at the filtered RS1 CAPM λ_{74} (61.005) and RS2 CAPM λ_{74} (77.914) and their constituent companies, both are using different sectors i.e. RS1 CAPM is relying mostly on Media, Insurance, Banking, consumer goods and oil & mining sector based companies while RS2 CAPM is relying more traditional organisations from Investment, Consumer Goods, Banking, Pharmaceutical, Distribution and Outsourcing, Construction, Power and Mining sector for the information base of the market and seems to be much more diversified than RS1 CAPM.

7.6 Portfolio Optimisation:

Once the data is gathered, sorted and filtered, the ultimate task is to form optimal portfolios. To form portfolio, we use three different categories of RMT filtered correlation matrices analysed in previous section. Optimised portfolios after daily rebalancing in out-sample period for two scenarios of filtered Non-RS, and RS CAPM vs two scenarios of unfiltered

Non-RS and RS CAPM are compared to measure the impact of RMT correlation filters on portfolio performance. The MPT for Non-RS is explained in Chapter 2 and chapter 6 and Regime Switching Method/Theory (RS CAPM) for Optimisation is elaborated in Chapter 2.

We started with an initial investment of 1 GBP to see how well different methods respond. It can be seen from the Figure 7.6-1 that RS CAPM performs well and shows the most cumulated wealth. As it can also be observed that the RMT filter has improved the performance in both cases when compared with the simple unfiltered portfolios.

The cumulative end of period return for the EmpSample before filtering is 4.21 and after filtering is 5.52 showing improvement in performance after the application of RMT filters. The accumulated wealth for the RS CAPM before filtering is 5.91 and after filtering is 7.37 which shows increase in performance after the application of RMT filters. Note that the performance increased in all cases but RS CAPM shows more percentage increase than the other two strategies.

Our argument here is that RMT performs better as it uses Marchenko-Pasteur limits to filter out the noise. Since Non-RS fails to address the existence of two regimes in market, therefore, underperforming when compared with RS CAPM. RS CAPM is the best technique to capture ups and downs in the market and perform according to the situation thus helping to reshape the portfolios well before time to prevent shock to a given portfolio.

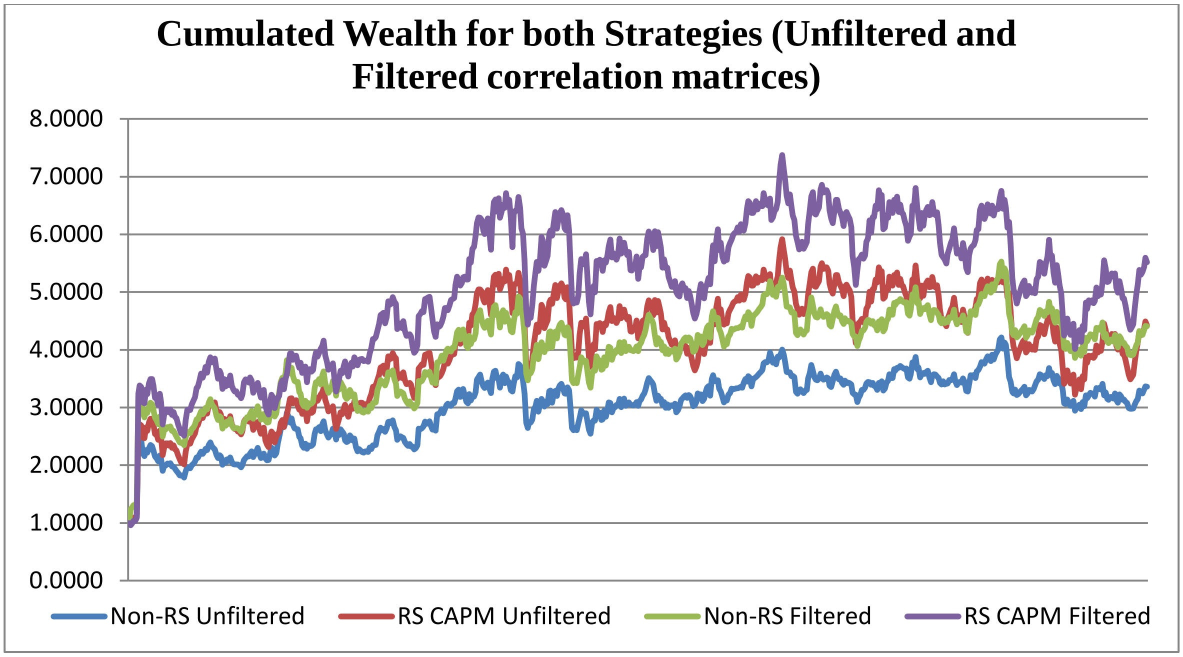


Figure 7.6-1: Cumulated wealth using Random Matrix Theory on Non-RS and RS CAPM models

The Sharpe ratio for RS CAPM, when computed throughout out-sample period, is consistent around 0.2 and is falling for Non-RS strategy as can be seen from Figure 7.6-2.

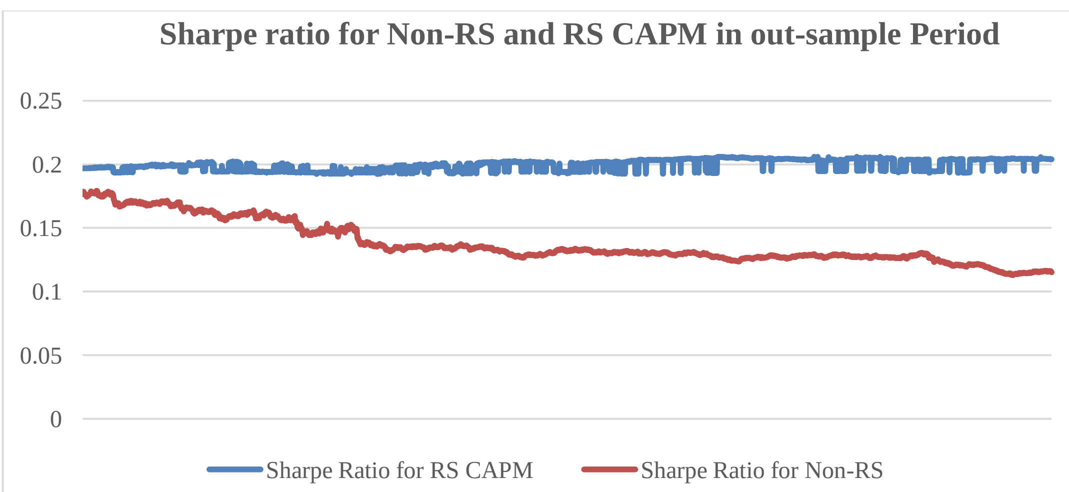


Figure 7.6-2: Sharpe ratio for Non-RS and RS CAPM in out-sample Period

7.7 Conclusion:

In this chapter we investigate the statistical properties of the correlation matrices for individual stock returns in the 74 constituent companies of FTSE 100 using the random matrix theory (RMT). We use stock returns time series of the firms to compute correlation matrices and their Eigenvalue spectra both in a single state (Non Regime Switching) environment and in a two state Regime Switching Model (RS CAPM). In the latter case, there is a correlation matrix associated with the high volatility negative returns regime and another corresponding to the low volatility regime. The eigenvalues in both regimes are significantly different from each other. Indeed, what is now becoming a stylized fact is that the Eigenvalue of Regime 1 with low volatility and higher average stock returns is less than Regime 2. In Table 7.5-3, the maximum eigenvalues were around 76.931 for Regime 2 compared to 46.15 for Regime 1 in the unfiltered case. Filtering increases these restive eigenvalues to 77.91 and 61.005 respectively. In the unfiltered case the top ten eigenvector stock constituents for the both regimes were similar, while they grew divergent after filtering.

Eigenvectors corresponding to eigenvalues deviating from the Marchenko-Pasteur law are analysed as they are found to contain market information. Largest eigenvalues have been discussed in detail (Daly et al, 2007) which shows that the largest Eigenvalue and their corresponding eigenvectors tend to highlight the sectoral groupings. Many authors (Plerou et al, 2002, Laloux et al, 1999, Daly et al, 2007) have discussed the largest Eigenvalue and have shown the largest Eigenvalue is at least 25 times greater than the average of remaining Eigenvalue which is also observed in this study. Eigenvalues within the Marchenko-Pasteur bounds are taken to be the ‘noisy’ or pure random ones. We ‘deconstruct’ the correlation matrix into non-random and noisy components and filter the noisy ones by using the Plerou et al. (2002) method. Once the filtered correlation matrix is obtained using RMT, this is used to compute the optimal portfolio weights for the RS model and Non-RS model. The results show improvement in the performance of optimal asset allocation while using RMT filtered correlations when compared with unfiltered correlations.

The filtering process used on Marchenko-Pasteur limits improves the performance of all the correlation matrices but improves more in regime based cases. Since this noise is almost eliminated, it is bound to impact the performance of correlation matrices when used in optimisation process.

Before filtering, RS1 and RS2 were relying on similar set of firms with slight change in weights and rank order but after the application of RMT filter, the organisation set totally changes for both RS1 & RS2. The sectorial grouping is different not only from unfiltered companies but they are different among themselves except for two companies after the filtration process. The EmpSample for Non-RS seems to be the worst performer but still RMT filter improves its performance as well. Key results can be that filtering changes the informative part of data and better informed portfolio are formed after the application of RMT filter.

Further, we analyse the deviations from RMT, and find that (i) the largest eigenvalue and its corresponding eigenvector represent the influence of the entire market on all stocks, and (ii) using the rest of the deviating eigenvectors, we can partition the set of all stocks studied into distinct subsets whose identity corresponds to conventionally-identified business sectors.

The previous chapters highlight the importance of using regime switching models for portfolio rebalancing and optimisation and successfully show that the portfolios formed using this approach performed better. After using RMT filter the performance of RS CAPM is still better thus it can be said the potential for improving the portfolio performance increases when both RMT and RS CAPM are used collectively.

8 Chapter 8: Conclusion & Suggestions

8.1 Conclusion

Market economies have been characterized by boom and bust cycles. Since the seminal work of Hamilton (1989), these large scale fluctuations have been referred to as regime switches. Ang and Bekaert (2002) were the first to consider the role of regime switches for stock market returns and portfolio optimisation. The key stylized facts regarding regime switching for stock index returns is that boom periods with positive mean stock returns are associated with low volatility, while bear markets with negative mean returns have high volatility. The correlation of asset returns also show asymmetry with greater correlation being found during stock market downturns. In view of the large portfolio losses from correlated negative movements in asset returns during the recent 2007 financial crisis, it has become imperative to incorporate regime sensitivity in portfolio management.

This thesis forms an extensive use of regime sensitive statistics for stock returns in the management of equity portfolios for different markets. Starting with the application to a small 3 asset portfolio for UK stocks (in Chapter 4), the methodology is extended to large scale portfolio for the FTSE-100. In chapters 5 and 6, respectively, using stock index data from the subcontinent (India, Pakistan and Bangladesh) and for the Asia Pacific, optimal regime sensitive portfolios have been analysed with the MSCI AC Index (for Emerging and Asia Pacific Markets) being taken as the benchmark index. Portfolio performance has been studied using a dynamic end of month rebalancing of the portfolio (for Chapters 4, 5 & 6) on the basis of regime indicators given by market index and relevant regime dependent portfolio statistics. The cumulative end of period returns and risk adjusted Sharpe Ratio from this exercise is compared to the simple Markowitz mean-variance portfolio and market value portfolio. The regime switching optimal portfolio strategy has been found to dominate non-regime sensitive portfolio strategies in Asia Pacific (Chapter 6) and 3 asset portfolio for UK stocks (Chapter 4) cases but not in Subcontinent case (at least for the first half of out-sample period-Chapter 5). In the case of the relationship of the Sub-continental indexes vis-à-vis the MSCI benchmark index, the latter has negligible explanatory power for the former especially for the first half of out-sample period. Hence, the regime indicators based on MSCI emerging market index have detrimental effects on portfolio selection based on the Sub-continental indexes. As regime sensitive variance–covariance matrices have implications for the selection of optimal portfolio weights, the final Chapter uses the FTSE-100 and its constituent company data to compare and contrast the implications for optimal portfolio management of filtering the covariance matrix using Random Matrix Theory (RMT). While it is found that filtering the variance-covariance matrix using Marchenko-Pasteur bounds of RMT improves optimal portfolio choice in both non-regime and regime dependent cases, remarkably in the latter case for Regime 2 determined variance-covariance matrix, the RMT filter was least needed. This result is given in Chapter 7, Table 7.5-1. This confirms the significance of using Hamilton (1989) regime sensitive statistics for stock returns in identifying the ‘true’ non-noisy variance-covariance relationships. The RMT methodology is also useful for identifying the centrality, based on eigenvector analysis, of the constituent stocks in their role in driving crisis and non-crisis market conditions.

8.2 Suggestions

The following is suggested for future researchers;

- Developing and Emerging markets need further empirical studies to validate stylised facts already established in developed markets. Similarly, regime switching models need to be checked for these markets.
- RMT filters explained in Chapter 7 could also be used on these markets for further empirical validation.
- Other methods employed in Regime Switching Models can be compared with the Model presented in thesis for empirical verification and suggest which method is suitable for different markets.

9 Chapter 9: Bibliography/References

"Greenspan Testimony on Sources of Financial Crisis," Wall Street Journal, October 23, 2008.

Adam, Alexandre, Houkari, Mohamed and Laurent, Jean-Paul (2007), Spectral risk measures and portfolio selection, unpublished work, this version: July, 19th 2007

Albert, James, and Siddhartha Chib (1993), "Bayes Inference via Gibbs Sampling of Autoregressive Time Series Subject to Markov Mean and Variance Shifts," Journal of Business and Economic Statistics, 11, 1-15.

Alexander, Carol, Andreas Kaeck, 2008, Regime dependent determinants of credit default swap spreads, In Journal of Banking & Finance, Volume 32, Issue 6, 2008, Pages 1008-1021, ISSN 0378-4266, https://doi.org/10.1016/j.jbankfin.2007.08.002.

Andersen, J.V., D. Sornette, P. Simonetti, Phys. Rep. 335, 19 (2002)

Ang, A. and Bekaert, G. (2002), "International Asset Allocation with Regime Shifts." Rev. Finance. Stud. 15(4): 1137-1187.

Ang, A. and Chen, J. (2001) "Asymmetric Correlations of Equity Portfolios ", Journal of Financial Economics, 63 (3). 637-657.

Ang, A., G. Bekaert, (2004), "How do regimes affect asset allocation", Financial Analysts Journal, 60, pp. 86-99

Ang, Andrew & Bekaert, G., 1999, "International Asset Allocation with Time-Varying Correlations," NBER Working Papers 7056, National Bureau of Economic Research, Inc.

Ang, Andrew & Bekaert, Geert, 2007, Stock Return Predictability: Is it there? Published by Oxford University Press on behalf of The Society for Financial Studies.

Ang, Andrew & Liu, Jun, 2007. "Risk, return, and dividends," Journal of Financial Economics, Elsevier, vol. 85(1), pages 1-38

Ang, Andrew & Timmermann, Allan, (2012), "Regime Changes and Financial Markets," Annual Review of Financial Economics, Annual Reviews, vol. 4(1), pages 313-337

Assoe, K.G. (1998), Regime-switching in emerging stock market returns, Multinational Finance Journal 2(2): 101-32.

Backus, David K., Gregory, Allan W. (1993), Theoretical Relations between Risk Premiums and Conditional Variances, Journal of Business & Economic Statistics, Vol. 11, No. 2. (Apr., 1993), pp. 177-185.

Bae, Geum II, Woo Chang Kim, John M. Mulvey, 2013, Dynamic asset allocation for varied financial markets under regime switching framework, European Journal of Operational Research,

Baharumshah, A. Z., T. Sarmidi and Hui T. B., 2003, "Dynamic linkages of Asian stock markets: An analysis of pre-liberalization and post liberalization eras," Journal of the Asia Pacific Economy, vol. 8, pp. 180–209, 2003.

Bai, Jushan, Lumsdaine, Robin L. Stock, James H., 1998, Testing For and Dating Common Breaks in Multivariate Time Series, Review of Economic Studies (1998) 65, 395-432

Bai, Z., Liu, H. and Wong, W.-K, 2009 “Enhancement of the applicability of Markowitz’s portfolio optimization by utilizing random matrix theory,” Mathematical Finance, vol. 19, pp. 639–667, October 2009.

Balcilar, Mehmet, Rangan Gupta and Clement Kyei, 2017, Predicting Stock Returns and Volatility with Investor Sentiment Indices: A Reconsideration Using a Nonparametric Causality-In-Quantiles Test, Bulletin of Economic Research, pp. 0307-3378 DOI: 10.1111/boer.12119

Bangia A., Diebold F. X., Kronimus A., Schagen C. and Schuermann T. (2002), “Ratings migration and the business cycle, with application to credit risk portfolio stress testing”, Journal of Banking and Finance, 26, 445-474.

Baxter, M., and R. King, 1999, “Measuring business cycles: Approximate band-pass filters for economic time series,” Review of Economics and Statistics, 81(4), 575–593.

Bean, Charles, 2009, The Great Moderation, the Great Panic and the Great Contraction, Text of the Schumpeter Lecture by Mr Charles Bean, Deputy Governor for Monetary Policy and Member of the Monetary Policy Committee, Bank of England, at the Annual Congress of the European Economic Association, Barcelona, 25 August 2009. Source: BIS Review 101/2009

Bean, Charles, Matthias Paustian, Adrian Penalver and Tim Taylor, 2010, “Monetary Policy after the fall”, Federal Reserve Bank of Kansas City, Jackson Hole Symposium.

Becherer, D.: Bounded solutions to backward SDEs with jumps for utility optimization and indifference hedging. Ann. Appl. Probab. 16, 2027–2054 (2006)

Bekaert, G. C.R. Harvey, (2002), Research in emerging markets finance: looking to the future, Emerging Markets Review 3, 429–448

Bekaert, G., Harvey, C., 1995. Time–varying world market integration. Journal of Finance 50, 403-444.

Bernanke, Ben S., 2009, “Asia and the Global Financial Crisis”, at the Federal Reserve Bank of San Francisco’s Conference on Asia and the Global Financial Crisis, Santa Barbara, California.

Bernanke, Ben S., 2011, “Global imbalances-Links to Economic and Financial Stability”, at the Banque de France Financial Stability Review Launch Event, Paris, France.

Bertoneche, Marc L., 1979, Spectral Analysis Of Stock Market Prices, Journal of Banking and Finance, 3 (1979) pp. 201-208.

Bikker J. A. and Hu H. (2002), “Cyclical Patterns in Profits, Provisioning and Lending of Banks”, DNB Staff Reports, 86.

Black, Fischer, Jensen, Michael C. and Myron S. Scholes, 1972, The Capital Asset Pricing Model: Some Empirical Tests, Studies In The Theory of Capital Markets, Praeger Publishers Inc.,

Bollerslev, T., 1986, Generalized autoregressive conditional heteroskedasticity, Journal of Econometrics 31, 307-327.

Bossaerts, Peter & Hillion, Pierre, 1999, Implementing Statistical Criteria to Select Return Forecasting Models: What Do We Learn?, The Review of Financial Studies, Volume 12, Issue 2, 1 April 1999, Pages 405–428, https://doi.org/10.1093/rfs/12.2.405

Bouchaud, J.-P. and M. Potters, Theory of Financial Risk and Derivative Pricing: From Statistical Physics to Risk Management (Cambridge University Press, Cambridge, USA, 2004)

Bouchaud, J.-P. and Potters, M., 2011, "Financial applications of random matrix theory: a short review," in Chapter 40 of Handbook of Random Matrix Theory, G. Akemann, J. Baik, and P. D. Francesco, Eds. Oxford, UK: Oxford Univesity Press, 2011.

Breitung, J., and B. Candelon, 2006, "Testing for short- and long-run causality: A frequencydomain approach," Journal of Econometrics, 132(2), 363–378.

Brody, T. A., J. Flores, J. B. French, P. A. Mello, A. Pandey and S. S. M. Wong, Rev. Mod. Phys. 53, 385 (1981).

Brooks, C. (2007), "Introductory Econometrics for Finance", Springer.

Campbell, R., K. Koedijk, and P. Kofman, (2002), "Increased Correlation in Bear Market", Financial Analysts Journal, Jan-Feb: 87-94

Cappiello, Lorenzo, De Santis, R. A. 2007, The Uncovered Return Parity Condition, European Monetary Union." Working Paper (812).

Capponi, Agostino; Figueroa-Lopez, Jose E., 2014, Dynamic Portfolio Optimization with a Defaultable Security and Regime Switching, ARXIV, http://adsabs.harvard.edu/abs/2011arXiv1105.0042C

Carr, P., and D. Madan, 1999, "Option valuation using the fast Fourier transform," Journal of Computational Finance, 2(4), 61–73.

Chan, Joshua C.C., Hsiao, Cody Yu-Ling, Fry-McKibbin, Renée A. 2017. "A Regime Switching Skew-normal Model for Measuring Financial Crisis and Contagion," CAMA Working Papers 2013-15, Centre for Applied Macroeconomic Analysis, Crawford School of Public Policy, The Australian National University. http://joshuachan.org/papers/rssn.pdf

Chaudhuri, S., and A. Lo, 2015, "Spectral analysis of stock-return volatility, correlation, and beta.," IEEE Signal Processing and Signal Processing Education Workshop (SP/SPE), pp. 232–236.

Chaudhuri, Shomesh E. and Lo, Andrew W. (2016), Spectral Portfolio Theory, unpublished version, this Draft: June 2, 2016.

Chen, S. W. & Huang, N. C. (2007) Estimates of the ICAPM with regime-switching betas: evidence from four Pacific Rim economies. Applied Financial Economics, 17, 313-327.

Chen, S. W. (2008) Identifying US turning points revisited: the panel model with the regime switching approach. Applied Economics Letters, 15, 893-897.

Cizeau, P., Liu, Y., M. Meyer, C.-K. Peng, and H. E. Stanley, (1997), "Correlations in Economic Time Series," Physica A 245, 437-440

Collin-Dufresne, Pierre, Goldstein, Robert S., Martin, J. Spencer, 2001, The Determinants of Credit Spread Changes, The Journal of Finance, 56-6, Blackwell Publishers, Inc.

Croux, C., M. Forni, and L. Reichlin, 2001, "A measure of comovement for economic variables: Theory and empirics," Review of Economics and Statistics, 83(2), 232–241.

Crowley, P., 2007, “A guide to wavelets for economists,” Journal of Economic Surveys, 21(2), 207–267.

Daly, J., Crane, M. and Ruskin, H. J., 2010, Random matrix theory filters and currency portfolio optimisation, 7th International Conference on Applications of Physics in Financial Analysis IOP Publishing, Journal of Physics: Conference Series 221 (2010) 012003 doi:10.1088/1742-6596/221/1/012003

Das, Sanjiv, Ranjan, Uppal. R. (2004). "Systemic Risk and International Portfolio Choice." The Journal of Finance 59(6): 2809-2834.

Davies, A., 2004. Credit spread modelling with regime switching techniques. Journal of Fixed Income. (http://www.sciencedirect.com/science/article/pii/S1386418107000353)

Davies, Andrew, 2008, Credit spread determinants: An 85 year perspective, In Journal of Financial Markets, Volume 11, Issue 2, 2008, Pages 180-197, ISSN 1386-4181, https://doi.org/10.1016/j.finmar.2007.10.002.

De Santis, Roberto A. and Bruno Gérard, 2006, ECB Working Paper Series, NO 626 / MAY 2006 http://www.ecb.int or from the Social Science Research Network electronic library at http://ssrn.com/abstract_id=900398

DeMiguel, V., Garlappi, L. and Uppal, R., 2009, “Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy?” The Review of Financial Studies, vol. 22, pp. 1915–1953, December 2009.

Demirguc-Kunt and Levine, R., 1996, Stock Markets, Corporate Finance, and Economic Growth World Bank Economic Review, 10-2, 223-239

De-Santis, G. and Gerard, B. (1997), “International Asset Pricing and Portfolio diversification with Time-Varying Risk”. Journal of Finance, Vol. 12, Issue 3, April pp189-204.

Dionne, Georges, Pascal François, and Olfa Maalaoui Chun, 2008, Detecting Regime Shifts in Corporate Credit Spreads, Cahier de recherché / Working Paper 09-2 http://www.cirpee.org/fileadmin/documents/Cahiers_2009/CIRPEE09-29.pdf

Dougherty, C. (2007) “Introduction to econometrics”. Oxford University Press, Oxford.

Dyson F. J., J. Math. Phys. 3, 140 (1962).

Dyson, F. J. and Mehta, M. L. J., J. Math. Phys. 4, 713 (1963b).

Dyson, F. J. and Mehta, M. L. J., Math. Phys. 4, 701 (1963a).

Eaker, M., Grant, D.M., Woodard, N., 2000. Realized rates of returns in emerging markets, Journal of Portfolio Management 26(3), 41-49.

Elton, E. and M. Gruber, (1981), Modern Portfolio Theory and Investment Analysis (Wiley, New York,)

Engle, R., 1974, “Band Spectrum Regression,” International Economic Review, 15(1), 1–11.

Engle, R.F., 1982, Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation, Econometrica 50, 987-1008.

Fabozzi, Frank J., Sergio M. Focardi, Petter N. Kolm, 2006, Financial Modelling of the Equity Market: From CAPM to Cointegration, Wiley Publishers, pp. 33, 94, and 301.

Flavin, Thomas and Panopoulou, Ekaterini, 2006, International Portfolio Diversification and Market Linkages in the Presence of Regime-Switching Volatility (June 2006). IIIS Working Paper No. 167. Available at SSRN: https://ssrn.com/abstract=923553 or http://dx.doi.org/10.2139/ssrn.923553

Flavin, Thomas J., Ekaterini Panopoulou, 2008, "Detecting shift and pure contagion in East Asian equity markets: A Unified Approach, IIIS Discussion Paper, No.236/ and Flavin, Thomas J., Ekaterini Panopoulou, 2010, "Detecting shift and pure contagion in East Asian equity markets: A Unified Approach, Pacific Economic Review, Volume 15, Issue 3,Pages 401–421

Foerster, Andrew T. 2016, Monetary Policy Regime Switches and Macroeconomic Dynamics, International Economic Review, 57-1, 1468-2354

Foster, John Bellamy, 2009, A Failed System: The World Crisis of Capitalist Globalization and its Impact on China, Monthly Review-An Independent Socialist Magazine, Volume 60, and Issue 10 (March 2009)

Fowdar, S. (2008). "International Portfolio Diversification: Assessing The Benefits Of Investing In Africa" EABR & TLC Conferences Proceedings downloaded from http://www.cluteinstitute-onlinejournals.com/Programs /Rothenburg_2008/Article%20254.pdf

Fry, RenÃ©e, Martin, Vance and Tang, Chrismin, (2010), A New Class of Tests of Contagion With Applications, Journal of Business & Economic Statistics, 28, issue 3, p. 423-437, https://EconPapers.repec.org/RePEc:bes:jnlbes:v:28:i:3:y:2010:p:423-437.

Fu, J; Wei, J; Yang, H, 2014, Portfolio optimization in a regime-switching market with derivatives, European Journal of Operational Research, 2014, v. 233 n. 1, p.184-192

Gabaix, X., P. Gopikrishnan, V. Plerou, and H. E. Stanley, "A Theory of Power-Law Distributions in Financial Market Fluctuations," Nature 423, 267-270 (2003).

Galagedera, Don U.A. and Roland Shami (March, 2004), "Association between Markov regime-switching market volatility and beta risk: Evidence from Dow Jones industrial securities", Working Paper, Department of Econometrics and Business Statistics, Monash University.

Gambera M. (2000), "Simple Forecasts of Bank Loan Quality in the Business Cycle", Federal Reserve Bank of Chicago, Supervision and Regulation Department, Emerging Issues Series, S&R-2000-3.

Gerard, B., K. Thanyalakpark and J. Batten, 2003, "Are the East Asian markets integrated? Evidence from the ICAPM," Journal of Economics and Business, vol. 55, pp. 585-607, 2003.

Geweke, J. & Porter-Hudak, S. 1983. The estimation and application of long memory time series models. Journal of Time Series Analysis, 4, 221-238.

Gibbons, M. R., (1982), "Multivariate tests of financial models", Journal of Financial Economics, Vol.10, pp.3-27

Granger, C., & Morgenstern, O. (2001). Spectral Analysis of New York Stock Market Prices, Essays in Econometrics pp. 85-105, Cambridge University Press. available at https://www.cambridge.org/core.

Granger, C., and M. Hatanaka, 1964, Spectral Analysis of Economic Time Series. Princeton University Press, Princeton, NJ.

Granger, C., and R. Engle, 1983, "Applications of Spectral Analysis in Econometrics," Handbook of Statistics, 3, 93–109.

Granger, C.W.J. and M. Hatanaka, 1964, Spectral analysis of economic time-series (Princeton University Press, Princeton, N J).

Granger, C.W.J. and O. Morgenstern, 1963, Spectral analysis of New York stock market prices, Kyklos, 1-27.

Granger, C.W.J. and O. Morgenstern, 1970, Predictability of stock market prices (Heath- Lexington Books, Lexington, MA).

Grant, Jeremy, June 12, 2015, Singapore urges closer Asean markets integration, https://www.ft.com/content/50d42aa6-10d1-11e5-9bf8-00144feabdc0
Gray, S. F., 1996, "Modelling the Conditional Distribution of Interest Rates as a Regime-Switching Process," Journal of Financial Economics, 42, 27–62.

Grosdidier, P., Morari, M., Holt, B.R., (1985), Closed-loop properties from steady-state gain information, Ind. Engng Chem. Fundam., 24 (1985), pp. 221-235

Grubel, H. G. and K. Fadner (1971). "The Interdependence of International Equity Markets." The Journal of Finance 26(1): 89-94.

Gu, L. 2005. Asymmetric risk loadings in the cross section of stock returns, University of Columbia, Working Paper.

Guhr, T., A. Müller-Groeling, and H. A. Weidenmüller, Phys. Rep. 299, 190 (1998).

Guhr, T., A. Muller-Groeling, H.A. Weiden muller, Phys. Rep. 299, 190 (1998)

Guhr, T., B. Kalberzk, J. Phys. A: Math. Gen. 36, 3009 (2003)

Guidolin, M. and A. G. Timmermann (2008). "International Asset Allocation under Regime Switching, Skew and Kurtosis Preferences", Review of Financial Studies, Oxford University Press for Society for Financial Studies, vol. 21(2), pages 889-935, April.

Guidolin, M. and A., Timmermann, 2005a, "Strategic Asset Allocation and Consumption Decisions under Multivariate Regime Switching", Federal Reserve Bank of St. Louis working paper No. 2005-002A.

Guidolin, M. and A., Timmermann, 2005b, Size and Value Anomalies under Regime Switching. Federal Reserve Bank of St. Louis working paper No. 2005-007A.

Guidolin, M. and A., Timmermann, 2006a, an Econometric Model of Nonlinear Dynamics in the Joint Distribution of Stock and Bond Returns. Journal of Applied Econometrics, 21, 1-22.

Guidolin, M. and A., Timmermann, 2006b, Term Structure of Risk under Alternative Econometric Specifications. Journal of Econometrics, 131, 285-308.

Guidolin, M. and A., Timmermann, 2006c, International Asset Allocation under Regime Switching, Skew and Kurtosis Preferences. Federal Reserve Bank of St. Louis working paper No. 2005-018A

Guidolin, Massimo & Allan Timmermann, 2008. "Size and Value Anomalies under Regime Shifts," Journal of Financial Econometrics, Society for Financial Econometrics, vol. 6(1), pages 1-48,

Guidolin, Massimo and Ria, Federica, Regime Shifts in Mean-Variance Efficient Frontiers: Some International Evidence (December 1, 2010). Manchester Business School Research Paper No. 609. Available at SSRN: https://ssrn.com/abstract=1723146 or http://dx.doi.org/10.2139/ssrn.1723146

Guo, F., 2005, Essays on East Asian capital markets: Integration and implications for economic activity. Unpublished PhD, City University of New York, New York, U.S.A., 2005.
Hagelin, N. and Pramborg, B. (2004), Hedging Foreign Exchange Exposure: Risk Reduction from Transaction and Translation Hedging. Journal of International Financial Management & Accounting, 15: 1–20. doi:10.1111/j.1467-646X.2004.00099.x

Hamilton J. D. (1994), Time Series Analysis, Princeton: Princeton University Press.

Hamilton, J. D. (1988), A Neoclassical Model of Unemployment and the Business Cycle, Journal of Political Economy, Vol. 96, No. 3.

Hamilton, J. D. (1989), A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle, Econometrica, 57, 357-384.

Hamilton, J. D. and Susmel (1994), "Autoregressive Conditional Heteroskedasticity and Changes in Regime," Journal of Econometrics, vol. 64, pp 307-333.

Hamilton, J.D., 2008. "Oil and the macroeconomy." Durlauf, S., Blume, L. (Eds.), The New Palgrave Dictionary of Economics, 2nd ed., Palgrave MacMillan Ltd

Hamilton, James D. (1996), "Specification Testing in Markov-Switching Time-Series Models," Journal of Econometrics, 70, 127-157.

Hamilton, James D. and Gang Lin, 1996, Stock Market Volatility and the Business Cycle, Journal of Applied Econometrics, Vol. 11, No. 5, Special Issue: Econometric Forecasting (Sep. - Oct., 1996), pp. 573-593

Harvey, Campbell R. and Siddique, Akhtar, 2000, Conditional Skewness in Asset Pricing Tests, The Journal of Finance, Vol. LV, No. 3.3

Hasbrouck, J., and G. Sofianos, 1993, "The trades of market makers: An empirical analysis of NYSE specialists," The Journal of Finance, 48(5), 1565–1593.

Henkel, Sam James, J. Spencer Martin, Federico Nardari, 2011, Time-varying short-horizon predictability, In Journal of Financial Economics, Volume 99, Issue 3, 2011, Pages 560-580, ISSN 0304-405X, https://doi.org/10.1016/j.jfineco.2010.09.008.

Hess, Martin K., 2006, Timing and diversification: A state-dependent asset allocation approach. The European Journal of Finance, Vol. 12, Issue. 3

Hoggarth, Glenn, Steffen Sorensen, and Lea Zicchino, 2005, Stress tests of UK banks using a VAR approach, The Bank of England's working paper Series, no. 282

http://content.time.com/time/magazine/article/0,9171,1865965,00.html

https://www.atkearney.com/consumer-goods/article?/a/countdown-to-2015-creating-asean-champion

Huang, H. C., (2000), "Test of regimes-switching CAPM", Applied Financial Economics, Vol.10, pp. 573-578

Huang, H. C., (2003), "Test of regimes-switching CAPM under price limits", International Review of Economics and Finance, 12, 305-326

Huang, N., M. Wu, W. Qu, S. Long, S. Shen, and J. Zhang, 2003, "Applications of Hilbert Huang transform to non-stationary financial time series analysis," Applied Stochastic Models in Business and Industry, 19(3), 245–268.

Huang, Y. L. & Huang, C. H. (2007) The persistence of Taiwan's output fluctuations: an empirical study using innovation regime-switching model. Applied Economics, 39, 2673-2679.

Iqbal, Javed, 2012, Stock Market in Pakistan: An Overview, Journal of Emerging Market Finance, vol. 11, 1: pp. 61-91. , First Published March 13, 2012

Jagannathan, R. and Ma, T., 2003 "Risk reduction in large portfolios: Why imposing the wrong constraints helps," Journal of Finance, vol. 58, no. 5, pp. 1651–1684, 2003.

Jensen, M., 1968, "The performance of mutual funds in the period 1945–1964," The Journal of Finance, 23(2), 389–416.

Jensen, M., 1969, "Risk, the pricing of capital assets, and the evaluation of investment portfolios," The Journal of Business, 42(2), 167–247.

Johannes, Michael, Korteweg, Arthur and Polson, Nicholas, 2011, "Sequential Learning, Predictability, and Optimal Portfolio Returns," Journal of Finance.

Jun, W.C. et al., Phys. Rev. E 73, 066128 (2006)

Kallberg, Jarl G. Liu, C. H., Pasquariello, Paolo, (2002). "Regime Shifts in Asian Equity and Real Estate Markets." Real Estate Economics 30(2): 263-291.

Kan, R. and Smith, D., 2008, "The distribution of the sample minimum-variance frontier," Management Science, vol. 54, no. 7, pp. 1364–1380, July 2008.

Khandani, Amir E. & Lo, Andrew W., 2011. "What happened to the quants in August 2007? Evidence from factors and transactions data," Journal of Financial Markets, Elsevier, vol. 14(1), pages 1-46

Kim, Chang Jin, and Charles R. Nelson (1999), State-Space Models with Regime Switching, Cambridge, Massachusetts: MIT Press

Korajczyk, R. and C. Viallet (1988). "An empirical investigation of international asset pricing", INSEAD.

Korajczyk, Robert A., 1996, a measure of stock market integration for Developed and emerging markets, the World Bank Economic review, vol. 10, no. 2. 267-289

Krugman, Paul 2008, "Desperately Seeking Seriousness," New York Times, October 26, 2008.

Krugman, Paul, 2008, The Return of Depression Economics and the Crisis of 2008, Publishers, W. W. Norton & Company

Laeven L. and Majoni G. (2003), "Loan Loss Provisioning and Economic Slowdowns: Too Much, Too Late?" Journal of Financial Intermediation, 12, 2, 178-197.

Laeven, R.J.A., Stadje, M.: Robust portfolio choice and indifference valuation. Math. Oper. Res. 39, 1109–1141 (2014)

Laloux, L., Cizeau, P. Bouchaud, J., and Potters, M., 1999, "Noise dressing of financial correlation matrices," Physycal Review Letters, vol. 83, no. 7, pp. 1467–1470, August 1999.

Laloux, L., Cizeau, P., Bouchaud, J.-P., Potters, M., Phys. Rev. Lett. 83, 1467 (1999)

Lambert, Marie, and Georges Hübner, 2013, Comoment risk and stock returns, Journal of Empirical Finance 23, 191–205.

Levine, R., and S.J. Zervos (1993), "What we have learned about Policy and Growth from Cross-country Regressions?" American Economic Review, Vol. 83, 426-430.

Levine, R., and S.J. Zervos (1995), "Stock Market Development and Long Run Growth," World Bank Economic Review, Vol. 82, No. 4, 942-963.

Lewis, K. K. (2006). "Is the International Diversification Potential Diminishing? Foreign Equity Inside and Outside the US". http://knowledge.wharton.upenn.edu/wp-content/uploads/2013/09/1349.pdf

Lin, M.-C., 2005, "Capital market integration and market liberalization in Asian emerging markets," International Journal of Management, vol. 22, no. 2, pp. 249-265, 2005.
Linetsky, V., 2002, "Exotic Spectra," RISK, pp. 85–89.

Linetsky, V., 2004a, "The Spectral Decomposition of the Option Value," International Journal of Theoretical and Applied Finance, 7, 337–384.

Linetsky, V., 2004b, "Spectral Expansions for Asian (Average Price) Options," Operations Research, 52, 856–867.

Linetsky, V., 2008, "Spectral Methods in Derivatives Pricing," in Handbooks in Operations Research and Management Science, ed. by J. R. Birge, and V. Linetsky. Elsevier, Amsterdam, vol. 15, pp. 223–299.

Lintner, J., 1965, "The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets," The Review of Economics and Statistics, 47(1), 13–37.

Liu, Y., P. Gopikrishnan, P. Cizeau, M. Meyer, C.-K. Peng, and H. E. Stanley, (1999), "The Statistical Properties of the Volatility of Price Fluctuations," Phys. Rev. E 60, 1390-1400

Lo, A., 2008, "Where do alphas come from?: A new measure of the value of active investment management," Journal of Investment Management, 6(2), 1–29.

Logue, D.E. and R.J. Sweeney, 1977, 'White-Noise' in imperfect markets: The case of the franc/dollar exchange rate, The Journal of Finance XXXII, no. 3, 761-768.

Longin, F. and B. Solnik, 1998, 'Correlation Structure of International Equity Markets during Extremely Volatile Periods', Les Cahiers de Recherche 646.

Longin, F., and B. Solnik, (2001), “Correlation Structure of International Equity Market during Extremely Volatile Periods”, Journal of Finance, vol. 56, no. 2 (April): 649-676

Lucas, R.E.J. (2009). ‘Trade and the diffusion of the industrial revolution’, American Economic Journal: Macroeconomics, vol. 1(1), pp. 1–25.

Lucas, Robert E., Jr. 2003. “Macroeconomic Priorities.”A.E.R.93 (March 2003): 1–14.

Lux, T. (1998), the Socio-Economic dynamics of speculative markets: interacting agents, chaos, and fat tails of return distribution. J. Econ. Behav. Organizat. 33, 143165

Lux, T., M. Marchesi, (1999), Scaling and criticality in a stochastic multi-agent model of a financial market, Nature 397, 498

Madan, D., Pistorius, M. & Stadje, M., (2017), On dynamic spectral risk measures, a limit theorem and optimal portfolio allocation, Finance Stoch (2017) 21:1073–1102 DOI 10.1007/s00780-017-0339-1. www.Springerlink.com

Maheu, John M. & McCurdy, Thomas H., 2009. "Do High-Frequency Measures of Volatility Improve Forecasts of Return Distributions?" Working Paper Series 19_09, The Rimini Centre for Economic Analysis.

Makridakis, S., 1976, A survey of time-series, International Statistical Review 44, no. 1.29-70.

Malkiel, B., and J. P. Mei. 1998. Global Bargain Hunting: The Investor’s Guide to Profits in Emerging Markets. New York: Simon Schuster.

Mantegna, R. N. and H. E. Stanley (1996), Turbulence and Financial Markets, Nature 383, 587

Mantegna, R. N. and H. E. Stanley, (1995), "Scaling Behaviour in the Dynamics of an Economic Index," Nature 376, 46-49

Mantegna, R.N., H.E. Stanley, (1999), an Introduction to Econo-physics: Correlation and Complexity in Finance (Cambridge University Press, Cambridge, UK,)

Marcucci, Juri and Quagliariello, Mario, (2008), Is bank portfolio riskiness procyclical: Evidence from Italy using a vector autoregression, Journal of International Financial Markets, Institutions and Money, 18, issue 1, p. 46-63, https://EconPapers.repec.org/RePEc:eee:intfin:v:18:y:2008:i:1:p:46-63.

Marcus, Marvin & Minc, Henryk, 1988, Introduction to linear algebra, Dover Publications (New York) ISBN 0486656950

Mark R. Eaker & Dwight Grant, 2002, the wealth effects of portfolio rebalancing in emerging equity markets, Journal of Multinational Financial Management 12 (2002) 79–88

Markose, S.M and J. Yang, (2008), “Optimal Portfolio Selection with Dynamic Regime Switching Weights”, Mimeo Economics Department and Centre For Computational Finance and Economic Agents.

Markose, Sheri, Simone Giansante, Mateusz Gatkowski, Ali Rais Shaghaghi (2011), “Too Interconnected to Fail: Financial Contagion and ystemic Risk in Network Model of CDS and Other Credit Enhancement Obligations of US Banks”, Discussion Paper Series, Department of Economics, University of Essex, Markowitz H., “Portfolio selection,” Journal of Finance, vol. 1, no. 7, 1952.

Markowitz, H.M. (1959). Portfolio Selection: Efficient Diversification of Investments. New York: John Wiley & Sons. (Reprinted by Yale University Press, 1970)

Markowitz, H.M., 1952, Journal of Finance 7, 77

Mehta, M. L., Random Matrices (Academic Press, Boston, 1991).

Mehta, M.L. (1991), Random Matrices (Academic Press, Boston)

Mehta, M.L. and F.J. Dyson, (1963), J. Math. Phys. 4, 713

Mehta, M.L., 1971, Commun. Math. Phys. 20, 245

Mehta, M.L., Nuclear Phys. 18, 395 (1960)

Merton, Robert C. (1973), An Intertemporal Capital Asset Pricing Model, Econometrica, Vol. 41, No. 5 (Sep., 1973), pp. 867-887

Meyer A. P. and Yeager T. J. (2001), “Are small Rural Banks Vulnerable to Local Economic Downturns?”, Federal Reserve Bank of St. Louis Review, 25-38.

Michaud, Richard O., The Markowitz Optimization Enigma: is 'Optimized' Optimal?. Financial Analysts Journal, 1989. Available at SSRN: https://ssrn.com/abstract=2387669 or http://dx.doi.org/10.2139/ssrn.2387669

Milidonis, Andreas and Shaun Wang (September 28, 2007), “Estimation of Distress Costs Associated with Downgrades Using Regime Switching Models”; Working Paper, Manchester Business School (MBS), The University of Manchester.

Mobius, M. 1994. The Investor’s Guide to Emerging Markets. Columbus, OH: Mc-Graw-Hill.

models, Journal of Time Series Analysis 4, 221-238.

Nalewaik, Jeremy J. (2015). “Regime-Switching Models for Estimating Inflation Uncertainty,” Finance and Economics Discussion Series 2015-093. Washington: Board of Governors of the Federal Reserve System, http://dx.doi.org/10.17016/FEDS.2015.093

Nobi, A. and Maeng, S.E. and Ha, G. G. and Lee, J.W., 2013, Random matrix theory and cross-correlations in global financial indices and local stock market indices, Journal of Korean Physical Society, volume, 62, pages, 569-574

Noh, J.D., Phys. Rev. E 61, 5981 (2000)

Noor A. Auzairy, Rubi Ahmad, Catherine S.F. Ho, and Ros Z. Z. Sapian, 2012, Integration of Asian Stock Markets, World Academy of Science, Engineering and Technology International Journal of Economics and Management Engineering Vol:6, No:12.
Oh, G., C. Eom, F. Wang, W.-S. Jung, H.E. Stanley, and S. Kim, (2011), Statistical properties of cross-correlation in the Korean stock market, The European Physical Journal B 79, 55–60

Oh, G., et al., J. Korean, Phys. Soc. 48, 197 (2006)

Okhrin, Y. and Schmid, W., 2006, “Distributional properties of portfolio weights,” Journal of Econometrics, vol. 134, pp. 235–256, August 2006.

Okhrin, Y. and W. Schmidt, J. Economy. 134, 235 (2006)

Paye, Bradley S. & Timmermann, Allan, 2006. "Instability of return prediction models," Journal of Empirical Finance, Elsevier, vol. 13(3), pages 274-315, June.

Pederzoli C. and Torricelli C. (2005), “Capital requirements and business cycle regimes: Forward-looking modelling of default probabilities”, Journal of Banking and Finance, 29, 3121-3140.

Perez-Quiros, Gabriel, and A. G. Timmermann, 2000, Firm Size and Cyclical Variations in Stock Returns, Journal of Finance, Volume 55, Issue 3, Pages 1229–1262

Pesaran, M. Hashem & Timmermann, Allan, 2002. "Market timing and return prediction under model instability," Journal of Empirical Finance, Elsevier, vol. 9(5), pages 495-510, December.

Pesola, J. (2001), “The role of macroeconomic shocks in banking crises”, Bank of Finland, discussion paper no. 6.

Pettenuzzo, Davide & Timmermann, Allan, 2011. "Predictability of stock returns and asset allocation under structural breaks," Journal of Econometrics, Elsevier, vol. 164(1), pages 60-78, September.

Plerou, V., Gopikrishnan, P., Rosenow, B., Amaral, L. and Stanley, H., 2003, “Universal and nonuniversal properties of cross correlations in financial time series,” Physycal Review Letters, vol. 83, no. 7, pp. 1471–1474, August 2003.

Plerou, V., B. Rosenow, P. Gopikrishnan, L.A.N. Amaral, H.E. Stanley, Int. J. Theoret. Appl. Finance 3, 399 (2000)

Plerou, V., Gopikrishnan, P., Rosenow B., Amaral, L.A.N., Stanley, H.E. Phys. Rev. Lett. 83, 1471 (1999)

Plerou, V., Gopikrishnan, P., Rosenow, B. Amaral, L.A.N., Stanley, H.E. Guhr, T., Phys. Rev. E 64, 066126 (2002)

Potì, Valerio and Wang, Dengli,2009, The Coskewness Puzzle (December 1, 2009). Journal of Banking and Finance, Forthcoming. Available at SSRN: https://ssrn.com/abstract=821315 or http://dx.doi.org/10.2139/ssrn.821315

Quagliariello M. (2008), “Does Macroeconomy Affect Bank Stability? A Review of the Empirical Evidence”, forthcoming in the Journal of Banking Regulation.

Ramchand, L. and R. Susmel (1998). "Volatility and cross correlation across major stock markets." Journal of Empirical Finance 5(4): 397-416.

Ramsey, J., 2002, “Wavelets in economics and finance: Past and future,” Studies in Nonlinear Dynamics & Econometrics, 6(3), 1–27.

Rapach, David E. & Wohar, Mark E. 2006. "Structural Breaks and Predictive Regression Models of Aggregate U.S. Stock Returns," Journal of Financial Econometrics, Society for Financial Econometrics, vol. 4(2), pages 238-274.

Rua, A., 2010, “Measuring comovement in the time-frequency space,” Journal of Macroeconomics, 32(2), 685–691.

Rua, A., 2012, “Wavelets in economics,” Economic Bulletin and Financial Stability Report Articles, pp. 71–79.

Rubinstein, Mark E., A Mean-Variance Synthesis of Corporate Financial Theory, The Journal of Finance, 28-1, 1540-6261

Rubio, Francisco, Mestre, Xavier and Palomar¸ Daniel P., 2011, Performance analysis and optimal selection of large mean-variance portfolios under estimation risk, October 18, 2011 unpublished draft.

Salas, V., Saurina, J. (2002), “Credit risk in two institutional settings: Spanish commercial and saving banks”, Journal of Financial Services Research, 22(3), December, 203-224.

Salhi, Khaled, Deaconu, Madalina. Lejay, Antoine, Champagnat, Nicolas, Navet, Nicolas, (2016), Regime switching model for financial data: Empirical risk analysis, Physica A: Statistical Mechanics and its Applications Physica A, 461, 148–157

Schaller, Huntley and van Norden, Simon, Fads or Bubbles? (January 1997). Working Paper 97-2. Available at SSRN: https://ssrn.com/abstract=57941 or http://dx.doi.org/10.2139/ssrn.57941

Schmidt, F. and Schmidt, R., 2010, “Statistical inference for Sharpe's ratio,” in Interest Rate Models, Asset Allocation and Quantitative Techniques for Central Banks and Sovereign Wealth Funds, A. Berkelaar, J. Coche, and K. Nyholm, Eds. Palgrave Macmillan, 2010.

Scott, Robert C. and Horvath, Philip A., 1980, On the Direction of Preference for Moments of Higher Order Than The Variance, The Journal of Finance, 35-4

Seba, P. et al., Phys. Rev. Lett. 91, 19 (2003)

Seidl, Immanuel, 2012, Markowitz versus Regime Switching: an Empirical Approach, The Review of Finance and Banking, Volume 04, Issue 1, Year 2012, Pages 033—043

Sengupta, A.M. and P.P. Mitra, Phys. Rev. E 60, 3 (1999)

Sharifi, S., M. Crane, A. Shamaie, H. Ruskin, Physica A 335, 629 (2004)

Sharifi, S., M. Crane, A. Shamaie, H. Ruskin, Random matrix theory for portfolio optimization: a stability approach, In Physica A: Statistical Mechanics and its Applications, Volume 335, Issues 3–4, 2004, Pages 629-643, ISSN 0378-4371, https://doi.org/10.1016/j.physa.2003.12.016.

Sharpe, W. F. (1966). "Mutual Fund Performance". Journal of Business. 39 (S1): 119–138.

Sharpe, W., 1964, “Capital asset prices: A theory of market equilibrium under conditions of risk,” The Journal of Finance, 19(3), 425–442.

Sharpe, W., 1966, “Mutual fund performance,” The Journal of Business, 39(1), 119–138. 41

Sharpe, William F. (1994). "The Sharpe Ratio". The Journal of Portfolio Management. 21 (1): 49–58.

Sheri Markose et al (2011), ICT Integrated Solutions for Systemic Risk Modelling and Governance of Complex Monetary and Financial Networks (ISR-FinGov), Small or medium-scale focused research project (STREP)

Sims, Christopher, A., and Zha, Tao 2006. "Were There Regime Switches in U.S. Monetary Policy?" American Economic Review, 96(1): 54-81.

Stiglitz, Joseph, 1999. "The Financial System, Business Cycles and Growth," UADE Textos de Discusión 2_1999, Instituto de Economía, Universidad Argentina de la Empresa.

Tai, C.-S., 2007 "Market integration and contagion: Evidence from Asian emerging stock and foreign exchange markets," Emerging Markets Review, vol. 8, pp. 264–283, 2007.

Taylor, John B., 2010, "Macroeconomic lessons from the Great Deviation", in D Acemoğlu and M Woodford (eds.), NBER Macroeconomics Annual, vol. 25, 387–95.

Treynor, J., 1965, "How to rate management of investment funds," Harvard Business Review, 43(1), 63–75.

Tu, Jun (2010), Is Regime Switching in Stock Returns Important in Portfolio Decisions? Management Science 2010, 56:7, 1198-1215

Utsugi, K. Ino, M. Oshikawa, Phys. Rev. E 70, 026110 (2004)

Weber, Enzo, 2007, Volatility and Causality in Asia Pacific Financial Markets, SFB 649 Discussion Paper 2007-004, Freie Universität Berlin, Germany, http://sfb649.wiwi.hu-berlin.de ISSN 1860-5664

Welch, Ivo & Goyal, Amit, 2008, "A Comprehensive Look at The Empirical Performance of Equity Premium Prediction," Review of Financial Studies, Oxford University Press for Society for Financial Studies, vol. 21(4), pages 1455-1508, July.

Whitelaw, Robert F., 2000, Stock Market Risk and Return: An Equilibrium Approach, The Review of Financial Studies, Volume 13, Issue 3, 1 July 2000, Pages 521–547, https://doi.org/10.1093/rfs/13.3.521

Wigner, E.P., 1951, Ann. Math. 53, 36 (1951)

Wigner, E.P., 1951, Proc. Cambridge Philos. Soc. 47, 790 (1951)

Wilcox, S. (1997), AGE AND GENDER IN RELATION TO BODY ATTITUDES. Psychology of Women Quarterly, 21: 549–565. doi:10.1111/j.1471-6402.1997.tb00130.x

Wishart, J., Biometrika 20, 32 (1928)

Woodward, G. and V. Marisetty (2005). "Introducing non-linear dynamics to the two-regime market model: Evidence." Quarterly Review of Economics and Finance 45(4-5): 559-581.

Xiaoneng Zhu, J. Z. (2013). Predicting stock returns: A regime-switching combination approach and economic links. Journal of Banking & Finance, 37, 4120–4133.

Yamasaki, K. et al., Proc. Natl. Acad. Sci. 102, 9424 (2005)

Yin, G. and X. Zhou (2004). "Markowitz's mean-variance portfolio selection with regime switching: from discrete-time models to their continuous-time limits." Automatic Control, IEEE Transactions on 49(3): 349-360.

Zhengjun Jiang, M. P. (2012). Optimal dividend distribution under Markov regime switching. Finance Stoch, 16, 449–476.

10 Chapter 10: Appendices

10.1 Appendix 1: Marchenko-Pasteur Theorem

In random matrix theory, the Marchenko–Pasteur distribution, or Marchenko-Pasteur law, describes the asymptotic behaviour of singular values of large rectangular random matrices. The theorem is named after Ukrainian mathematicians Vladimir Marchenko and Leonid Pasteur who proved this result in 1967.

If X denotes a $M \times N$ random matrix whose entries are independent identically distributed random variables with mean 0 and variance $\sigma^2 < \infty$, let

$$Y_N = N^{-1} X X^T \qquad (1)$$

And let $\lambda_1, \lambda_2, \dots, \lambda_M$ be the eigenvalues of Y_N (viewed as random variables). Finally, consider the random measure

$$\mu_M(A) = \frac{1}{M} \# \{\lambda_j \in A\}, \quad A \subset \mathbb{R}. \qquad (2)$$

Theorem: Assume that $M, N \to \infty$ so that the ratio $M/N \to \lambda \in (0, +\infty)$. Then $\mu_M \to \mu$ (in weak* topology in distribution), where

$$\mu(A) = \begin{cases} (1 - \frac{1}{\lambda}) \mathbf{1}_{0 \in A} + \nu(A), & \text{if } \lambda > 1 \\ \nu(A), & \text{if } 0 \le \lambda \le 1, \end{cases} \qquad (3)$$

And $$d\nu(x) = \frac{1}{2\pi\sigma^2} \frac{\sqrt{(\lambda_+ - x)(x - \lambda_-)}}{\lambda x} \mathbf{1}_{[\lambda_-, \lambda_+]} \, dx \qquad (4)$$

With $$\lambda_\pm = \sigma^2 (1 \pm \sqrt{\lambda})^2. \qquad (5)$$

The Marchenko–Pasteur law also arises as the free Poisson law in free probability theory, having rate λ and jump size α.

10.2 Appendix 2: List of FTSE-100 74 selected Constituent Companies used in Empirical Sample

List of FTSE-100 Selected Constituent companies selected in the Sample for Chapter 7							
ASSOCIATED BRIT.FOODS	BARRATT DEVELOPMENTS	CRODA INTERNATIONAL	HSBC HDG. (ORD $0.50)	SSE	SAINSBURY (J)	RELX	PEARSON
ASHTEAD GROUP	BHP BILLITON	DCC	UNITED UTILITIES GROUP	STANDARD CHARTERED	SCHRODERS	RIO TINTO	PRUDENTIAL
3I GROUP	BP	CENTRICA	VODAFONE GROUP	TAYLOR WIMPEY	SEGRO	ROLLS-ROYCE HOLDINGS	RANDGOLD RESOURCES
ANGLO AMERICAN	BRITISH AMERICAN TOBACCO	DIAGEO	INFORMA	TESCO	PERSIMMON	ROYAL BANK OF SCTL.GP.	JOHNSON MATTHEY
ANTOFAGASTA	BRITISH LAND	G4S	WHITBREAD	SCOTTISH MORTGAGE	SEVERN TRENT	RENTOKIL INITIAL	KINGFISHER
BABCOCK INTERNATIONAL	SKY	GKN	WOLSELEY	UNILEVER (UK)	SHIRE	MORRISON (WM) SPMKTS.	LAND SECURITIES GROUP
ASTRAZENECA	BT GROUP	GLAXOSMITHKLINE	WPP	RSA INSURANCE GROUP	ROYAL DUTCH SHELL B	NATIONAL GRID	LEGAL & GENERAL
AVIVA	BUNZL	HAMMERSON	SMITH & NEPHEW	ST.JAMES'S PLACE	RECKITT BENCKISER GROUP	NEXT	LLOYDS BANKING GROUP
BAE SYSTEMS	CRH	IMPERIAL BRANDS	SMITHS GROUP	SAGE GROUP	PROVIDENT FINANCIAL	OLD MUTUAL	MARKS & SPENCER GROUP
BARCLAYS							ITV

10.3 Appendix 3 Sample Statistics for Chapter 7.

Descriptive Statistics	Mean	Variance	Standard Deviation	Excess Kurtosis	Skewness	Intercept	Slope	Standard Error	R Squared
ASSOCIATED BRIT.FOODS	-0.0035	0.0005	0.0225	1.1209	0.8215	-0.0047	-0.0166	0.0200	0.0004
ASHTEAD GROUP	-0.0240	0.0005	0.0231	0.7340	0.4262	0.0044	0.3762	0.0177	0.2173
3I GROUP	-0.0245	0.0008	0.0282	-1.8857	0.3758	-0.0041	0.0191	0.0200	0.0008
ANGLO AMERICAN	0.0038	0.0004	0.0208	2.6146	1.5834	-0.0031	-0.3893	0.0180	0.1881
ANTOFAGASTA	-0.0003	0.0024	0.0489	3.6121	-1.3768	-0.0045	0.2334	0.0158	0.3728
BABCOCK INTERNATIONAL	0.0007	0.0011	0.0336	3.1236	1.4336	-0.0046	-0.0331	0.0199	0.0035
ASTRAZENECA	-0.0083	0.0007	0.0270	-0.5188	0.3338	-0.0059	-0.1513	0.0195	0.0478
AVIVA	0.0084	0.0011	0.0335	5.3464	-2.1013	-0.0044	-0.0267	0.0199	0.0023
BAE SYSTEMS	0.0023	0.0007	0.0274	0.3793	-0.2532	-0.0047	0.0629	0.0199	0.0085
BARCLAYS	-0.0152	0.0005	0.0232	-0.7064	-0.3800	-0.0075	-0.1890	0.0194	0.0552
BARRATT DEVELOPMENTS	-0.0156	0.0006	0.0247	0.1745	0.3874	-0.0015	0.2000	0.0193	0.0699
BHP BILLITON	0.0103	0.0005	0.0222	-1.0065	-0.1113	-0.0056	0.1013	0.0198	0.0145
BP	-0.0153	0.0004	0.0190	-0.8677	0.2246	-0.0021	0.1605	0.0197	0.0266
BRITISH AMERICAN TOBACCO	-0.0010	0.0002	0.0137	1.2023	-1.2572	-0.0049	-0.3153	0.0194	0.0536
BRITISH LAND	-0.0272	0.0007	0.0273	-0.5026	-0.0831	-0.0060	-0.0521	0.0199	0.0058
SKY	-0.0170	0.0008	0.0283	0.5184	-1.2584	-0.0055	-0.0513	0.0199	0.0060
BT GROUP	0.0025	0.0015	0.0389	2.0275	-1.2679	-0.0050	0.1496	0.0190	0.0969
BUNZL	0.0114	0.0012	0.0348	-0.8903	0.2703	-0.0050	0.0309	0.0199	0.0033
CRH	0.0003	0.0022	0.0469	0.0761	-0.0469	-0.0046	-0.0886	0.0195	0.0496
CRODA INTERNATIONAL	0.0024	0.0043	0.0657	-0.3864	0.1228	-0.0047	0.0319	0.0198	0.0126
DCC	-0.0119	0.0005	0.0232	-0.3682	0.3713	-0.0071	-0.2109	0.0193	0.0684
CENTRICA	-0.0277	0.0009	0.0305	-1.2224	0.1533	-0.0125	-0.2858	0.0177	0.2182
DIAGEO	0.0072	0.0013	0.0362	2.3919	-1.2057	-0.0053	0.1040	0.0196	0.0407
G4S	0.0172	0.0012	0.0350	1.9545	1.7177	-0.0082	0.2068	0.0184	0.1502
GKN	0.0002	0.0006	0.0247	2.7804	-1.3356	-0.0046	0.0561	0.0199	0.0055
GLAXOSMITHKLINE	-0.0021	0.0004	0.0205	-0.6220	0.4759	-0.0043	0.1408	0.0197	0.0238
HAMMERSON	-0.0223	0.0012	0.0344	-1.6933	0.2168	-0.0051	-0.0229	0.0199	0.0018
IMPERIAL BRANDS	0.0271	0.0017	0.0416	3.8398	1.7124	-0.0051	0.0171	0.0199	0.0014
HSBC HDG. (ORD $0.50)	0.0142	0.0031	0.0556	7.7670	2.7190	-0.0012	-0.2375	0.0141	0.4998
UNITED UTILITIES GROUP	0.0022	0.0004	0.0204	0.5282	-0.9835	-0.0050	0.1846	0.0196	0.0405
VODAFONE GROUP	-0.0007	0.0005	0.0215	0.4826	0.8668	-0.0046	-0.0146	0.0200	0.0003
INFORMA	0.0193	0.0013	0.0356	-0.1154	0.0949	-0.0065	0.0982	0.0196	0.0350
WHITBREAD	0.0356	0.0021	0.0461	2.0499	1.2370	-0.0037	-0.0267	0.0199	0.0044
WOLSELEY	0.0193	0.0029	0.0538	2.0351	1.2085	0.0007	-0.2728	0.0123	0.6180
WPP	0.0033	0.0003	0.0164	-0.4417	0.6913	-0.0041	-0.1446	0.0198	0.0160

SMITH & NEPHEW	-0.0114	0.0005	0.0233	-1.4822	-0.4806	-0.0024	0.1982	0.0193	0.0611
SMITHS GROUP	-0.0113	0.0010	0.0324	-0.5760	-0.6159	-0.0052	-0.0490	0.0199	0.0072
SSE	-0.0125	0.0001	0.0122	1.2728	-1.1478	-0.0145	-0.7908	0.0171	0.2673
STANDARD CHARTERED	0.0123	0.0006	0.0244	0.4849	-0.0102	-0.0087	0.3315	0.0180	0.1881
TAYLOR WIMPEY	0.0149	0.0005	0.0229	3.8020	1.6465	-0.0005	-0.2742	0.0188	0.1130
TESCO	-0.0142	0.0004	0.0201	1.5518	-1.5106	-0.0070	-0.1659	0.0196	0.0319
SCOTTISH MORTGAGE	0.0128	0.0002	0.0134	-2.2565	0.2150	-0.0062	0.1217	0.0199	0.0076
UNILEVER (UK)	-0.0028	0.0006	0.0250	1.7493	1.2399	-0.0041	0.1762	0.0194	0.0555
RSA INSURANCE GROUP	0.0225	0.0020	0.0442	1.8778	1.2142	-0.0027	-0.0851	0.0196	0.0406
ST.JAMES'S PLACE	-0.0030	0.0007	0.0262	-1.1579	0.0370	-0.0049	-0.1134	0.0197	0.0253
SAGE GROUP	0.0040	0.0002	0.0143	4.4767	-1.9319	-0.0055	0.2181	0.0197	0.0279
SAINSBURY (J)	0.0123	0.0006	0.0248	0.4586	1.4263	-0.0065	0.1562	0.0195	0.0432
SCHRODERS	0.0174	0.0014	0.0372	1.5805	1.6380	-0.0038	-0.0457	0.0199	0.0083
SEGRO	0.0000	0.0013	0.0362	-0.9014	-0.2288	-0.0046	0.2557	0.0173	0.2455
PERSIMMON	0.0147	0.0011	0.0336	-0.2842	-0.9780	-0.0057	0.0748	0.0198	0.0181
SEVERN TRENT	-0.0085	0.0001	0.0110	-0.6422	-0.3555	0.0043	1.0452	0.0157	0.3809
SHIRE	-0.0010	0.0002	0.0124	3.1343	1.2404	-0.0053	-0.6951	0.0177	0.2114
ROYAL DUTCH SHELL B	-0.0126	0.0005	0.0223	1.3335	-0.4747	-0.0059	-0.1027	0.0198	0.0150
RECKITT BENCKISER GROUP	-0.0117	0.0006	0.0243	0.6534	0.9720	-0.0101	-0.4707	0.0158	0.3756
PROVIDENT FINANCIAL	-0.0003	0.0009	0.0303	-1.0975	-0.0772	-0.0045	0.3166	0.0171	0.2631
RELX	0.0098	0.0013	0.0362	0.1703	0.5885	-0.0048	0.0175	0.0200	0.0012
RIO TINTO	0.0280	0.0016	0.0405	0.2513	-0.5369	-0.0016	-0.1086	0.0194	0.0556
ROLLS-ROYCE HOLDINGS	0.0020	0.0020	0.0442	4.4629	2.0244	-0.0040	-0.2916	0.0144	0.4763
ROYAL BANK OF SCTL.GP.	-0.0102	0.0014	0.0368	-1.3844	-0.1744	-0.0031	0.1488	0.0191	0.0861
RENTOKIL INITIAL	0.0078	0.0009	0.0297	1.5281	0.9494	-0.0050	0.0444	0.0199	0.0050
MORRISON (WM) SPMKTS.	0.0046	0.0007	0.0256	2.2306	-0.8231	-0.0051	0.1141	0.0197	0.0245
NATIONAL GRID	-0.0178	0.0004	0.0208	-0.3042	-0.9020	0.0022	0.3818	0.0181	0.1811
NEXT	-0.0024	0.0007	0.0257	0.2601	0.0315	-0.0056	-0.4123	0.0164	0.3222
OLD MUTUAL	-0.0017	0.0001	0.0097	2.8607	1.4853	-0.0040	0.3710	0.0196	0.0373
PEARSON	-0.0138	0.0005	0.0233	-0.0658	-0.6322	-0.0098	-0.3801	0.0176	0.2256
PRUDENTIAL	-0.0256	0.0006	0.0238	0.3956	-0.9713	-0.0069	-0.0903	0.0198	0.0132
RANDGOLD RESOURCES	0.0097	0.0002	0.0154	-0.4431	0.4334	-0.0053	0.0754	0.0199	0.0039
JOHNSON MATTHEY	0.0074	0.0002	0.0148	-1.7737	0.2512	-0.0069	0.3088	0.0194	0.0595
KINGFISHER	0.0086	0.0004	0.0196	2.4411	-1.0775	-0.0033	-0.1531	0.0197	0.0259
LAND SECURITIES GROUP	0.0000	0.0000	0.0001	9.0000	-3.0000	-0.0076	-162.7837	0.0176	0.2258
LEGAL & GENERAL	0.0000	0.0000	0.0001	9.0000	-3.0000	-0.0076	-162.7837	0.0176	0.2258
LLOYDS	0.0048	0.0009	0.0299	-0.6644	0.4056	-0.0061	0.3069	0.0174	0.2411

BANKING GROUP									
MARKS & SPENCER GROUP	-0.0040	0.0005	0.0233	-1.1155	-0.3022	-0.0039	0.1843	0.0194	0.0529
ITV	-0.0209	0.0005	0.0226	-0.2183	0.4987	-0.0008	0.1811	0.0195	0.0479
FTSE 100 Excess Returns	0.0146	0.0006	0.0249	2.1003	1.0549				

10.4 : Appendix 4 Code Used for Experiments

% **

% Codes for the Thesis "**Application of Regime Switching and Random Matrix Theory % for Portfolio Optimization**", by Sheri Markose and Javed Iqbal.

%**

% This is the parent program that calls all the other codes.

% The workspace is saved in the current path with the name 'Results'

function www = three_regime_fit % comment out this line to see all the outputs in the workspace

clear;

pdirectory = pwd; % Obtain the current directory (same as the m files)

cd DAT % Change to the directory where the data is

% There should be 3 files for each stock:

% prices, returns and mv. Minus the libor and FTSE files

filespi = dir([pwd '*pi8605.dat']); % Files for the prices

filesmv = dir([pwd '*mv8605.dat']); % Files for the mv

filesret = dir([pwd '*returns8605.dat']); % Files for the returns

filesret = filesret(3:end); % Remove the filenames of the FTSE and Libor

%OBS: The filenames for the Libor and FTSE should be capitalised so that

%they are the first and second files.

% Number of stocks to be analysed

numstocks = length(filespi);

% Check that the number of files for the three series are the same

if numstocks ~= length(filesret) && numstocks ~= length (filesmv)

error('You do not have all the necessary files in the DAT folder')

end

%% load Price Index for three stocks

ftse = load('FTSE100MonthlyExcessReturns8605.dat');

N = size(ftse,1);

```
lib = load('LiborMonthlyReturns8605.dat');
% Allocate memory for pi, sr and mv
pi = zeros(N,numstocks); sr = pi; mv = pi;
for k = 1:numstocks
% The price series for each stock becomes a column in p
pi(:,k) = load(filespi(k).name);
sr(:,k) = load(filesret(k).name);
mv(:,k) = load(filesmv(k).name);
end
cd(pdirectory) % Change to original directory
%% Calculate In-sample alphas and betas
s_coeff = zeros(numstocks,3); % Allocate memory for coefficients
for k = 1:numstocks
s_coeff(k,:) = ols(sr(1:120,k)-lib(1:120),[ones(120,1) ftse(1:120)]);
end
% Prepare the OLS coefficients with matrix
A = s_coeff(:,1);
B = s_coeff(:,2);
V = diag(s_coeff(:,3));
% Initiation some variables for storage
optrs = cell(2);
optnon = cell(2);
optmv = cell(1);
W_rs=1;
W_non=1;
W_mv=1;
% Out-of-sample performance test
```

```
for i = 120:N-1

data = ftse(1:i)';

[Regime_Out] = regime_fit(data); % call the regime-switching regression function regime_fit

% estimates for regime-switching regression

u1 = Regime_Out.par(1);

u2 = Regime_Out.par(2);

c1 = Regime_Out.par(3);

c2 = Regime_Out.par(4);

P = 1-Regime_Out.par(5);

Q = 1-Regime_Out.par(6);

filtone(i-119) = Regime_Out.filt(i,1); % filter prob of regime1

% regime-dependent expected returns and covariance matrix

R1 = A + B*u1;

R2 = A + B*u2;

E1 = P*R1 + (1-P)*R2;

E2 = (1-Q)*R1 + Q*R2;

T1 = B*B'*c1^2 + V;

T2 = B*B'*c2^2 + V;

K1 = P*T1 + (1-P)*T2 + P*(1-P)*(R1-R2)*(R1-R2)';

K2 = (1-Q)*T1 + Q*T2 + Q*(1-Q)*(R1-R2)*(R1-R2)';

bound = repmat([-2;2],1,3); % no short selling constraint. change to [] for resuming short selling

r = lib(i);

num_port = 50;

% regime realization

if filtone(i-119)>0.5

    returns_rs = E1 + r;
```

```
    covariance_rs = K1;
else
    returns_rs = E2 + r;
    covariance_rs = K2;
end
% construct portfolio with quadratic programming
[PortRisk_rs, PortReturn_rs, PortWts_rs] = frontcon (returns_rs, covariance_rs, num_port, [], bound);
 plot(PortRisk_rs,PortReturn_rs);
[RiskyRisk_rs, RiskyReturn_rs, RiskyWts_rs, RiskyFraction_rs, OverallRisk_rs, OverallReturn_rs] = portalloc (PortRisk_rs, PortReturn_rs, PortWts_rs,r,r,3);
L = isnan(RiskyWts_rs);
if sum(L)>1
    RiskyWts_rs = [1/3,1/3,1/3];
end
optrs(i-119,1)={RiskyWts_rs};
optrs(i-119,2)={RiskyFraction_rs};
% calculate and store the accumulated wealth for RS strategy
PR_rs = RiskyFraction_rs*(RiskyWts_rs*sr(i+1,:)') + (1-RiskyFraction_rs)*r;
W_rs = W_rs*(1+PR_rs);
WT_rs(i-119)=W_rs;
[returns_non, covariance_non,NumEffObs] = ewstats(sr(1:i,:));
[PortRisk_non, PortReturn_non, PortWts_non] = frontcon (returns_non, covariance_non, num_port, [],bound);
plot(PortRisk_non,PortReturn_non)
[RiskyRisk_non, RiskyReturn_non, RiskyWts_non, RiskyFraction_non, OverallRisk_non, OverallReturn_non] = portalloc(PortRisk_non, PortReturn_non, PortWts_non,r,r,3);
optnon(i-119,1)={RiskyWts_non};
optnon(i-119,2)={RiskyFraction_non};
```

```
% calculate and store the accumulated wealth for non-RS strategy
PR_non = RiskyFraction_non*(RiskyWts_non*sr(i+1,:)') + (1-RiskyFraction_non)*r;
W_non = W_non*(1+PR_non);
WT_non(i-119)=W_non;
RiskyWts_mv = (mv(i,:))/sum(mv(i,:));
optmv(i-119,1)={RiskyWts_mv};
% calculate and store the accumulated wealth for Market-cap strategy
PR_mv = RiskyFraction_non*(RiskyWts_mv*sr(i+1,:)') + (1-RiskyFraction_non)*r;
W_mv = W_mv*(1+PR_mv);
WT_mv(i-119)=W_mv;
ER_mv(i-119)=PR_mv - r; % what for u need excess returns
end
% accumulate wealth
www = [WT_rs',WT_non',WT_mv'];
% the estimates which are updated at each iteration
AVR_rs=mean(cell2mat(optrs(:,1)));
AVF_rs=mean(cell2mat(optrs(:,2)));
ALLR_rs=cell2mat(optrs(:,1));
ALLF_rs=cell2mat(optrs(:,2));
AVR_non=mean(cell2mat(optnon(:,1)));
AVF_non=mean(cell2mat(optnon(:,2)));
ALLR_non=cell2mat(optnon(:,1));
ALLF_non=cell2mat(optnon(:,2));
AVR_mv =mean(cell2mat(optmv(:,1)));
optmv(:,1);
%% Dates for the plots
% Needs Financial Toolbox
```

```
% x_data = datemnth('1 Jan 1986', 1:120, 0, 0, 0);

% %% plot various results

% figure(1);

plot(1:i-119,WT_rs,'r',1:i-119,WT_non,'g',1:i-119,WT_mv,'b',1:i-
119,Regime_Out.smooth(120:i,1),'k');

plot(x_data,WT_rs,'r',x_data,WT_non,'g',x_data,WT_mv,'b',x_data,Regime_Out.smooth(12
0:i,1),'k');

datetick('x','mmmyy')

figure(2);

 plot(x_data,filtone','r',x_data,Regime_Out.smooth(120:i,1),'k');

 datetick('x','mmmyy')

 figure(3);

subplot(2,2,1);

 [AX,H1,H2] = plotyy(x_data,pi(120:i,1),x_data,ALLR_rs(:,1));

 datetick(AX(1),'x','mmmyy'),  set(AX(2),'XTick',[])

 subplot(2,2,2);

 [AX,H1,H2] = plotyy(x_data,pi(120:i,2),x_data,ALLR_rs(:,2));

 datetick(AX(1),'x','mmmyy'), set(AX(2),'XTick',[])

 subplot(2,2,3)

 [AX,H1,H2] = plotyy(x_data,pi(120:i,3),x_data,ALLR_rs(:,3));

 datetick(AX(1),'x','mmmyy'),  set(AX(2),'XTick',[])

save Results
```

10.5 : Appendix 5 Transaction Costs and Implications on Results

The CAPM assumption that there are no transaction costs helps grow portfolio cumulated wealth throughout the thesis. If we include transaction cost for each buy and sell for every stock, this would decline the cumulated wealth significantly as portfolios are rebalanced on daily basis thus incurring cost every time portfolio is rebalanced. Following Table shows the impact of 1% transaction cost for out-sample period based on daily rebalancing and if we restrict portfolio positions to be rebalanced only at regime switches.

Chapter Number	**Number of times Transaction Cost will be incurred if rebalancing occurs daily**	**Impact on end of period Cumulated Wealth**	**Number of times Transaction Cost will be incurred if rebalancing occurs fewer times keeping in view Regime Switches**	**Impact on end of period Cumulated Wealth**
Chapter 4	258 days*3 Stocks	1% Transaction Cost reduces Cumulated Wealth from 2.159 to 1.523	24 Regime Switches *3 Stocks	1% Transaction Cost reduces Cumulated Wealth from 2.159 to 1.9325
Chapter 5	247 days*3 Stocks	1% Transaction Cost reduces Cumulated Wealth from 1.9742 to 1.3572	22 Regime Switches*3 Stocks	1% Transaction Cost reduces Cumulated Wealth from 1.9742 to 1.6753
Chapter 6	218 days*6 Stocks	1% Transaction Cost reduces Cumulated Wealth from 1.7163 to 1.1254	30 Regime Switches*6 Stocks	1% Transaction Cost reduces Cumulated Wealth from 1.7163 to 1.5232
Chapter 7	4565 days*74 Stocks	1% Transaction Cost reduces ending Cumulated Wealth from 9.43 to 5.325	24 Regime Switches *74 Stocks	1% Transaction Cost reduces ending Cumulated Wealth from 9.43 to 7.65

10.6 : Appendix 6 Results of Filtration Methods on Sample Data only

Correlation Matrix	ASSOCIATED BRIT.FOODS	AGGREKO	AMEC	ANGLO AMERICAN	ANTOFAGASTA	ARM HOLDINGS	ASTRAZENECA	AVIVA	BAE SYSTEMS	BARCLAYS	BG GROUP	BHP BILLITON	BP	BRITISH AMERICAN TOBACCO	BRITISH LAND
ASSOCIATED BRIT. FOODS	22.398314	0	0	0	0	0	0	0	0	0	0	0	0	0	0
AGGREKO	0	3.3728695	0	0	0	0	0	0	0	0	0	0	0	0	0
AMEC	0	0	2.8015774	0	0	0	0	0	0	0	0	0	0	0	0
ANGLO AMERICAN	0	0	0	2.6025404	0	0	0	0	0	0	0	0	0	0	0
ANTOFAGASTA	0	0	0	0	2.2792554	0	0	0	0	0	0	0	0	0	0
ARM HOLDINGS	0	0	0	0	0	1.6667509	0	0	0	0	0	0	0	0	0
ASTRAZENECA	0	0	0	0	0	0	1.4255339	0	0	0	0	0	0	0	0
AVIVA	0	0	0	0	0	0	0	1.2953707	0	0	0	0	0	0	0
BAE SYSTEMS	0	0	0	0	0	0	0	0	1.1106834	0	0	0	0	0	0
BARCLAYS	0	0	0	0	0	0	0	0	0	1.0837824	0	0	0	0	0
BG GROUP	0	0	0	0	0	0	0	0	0	0	1.0467467	0	0	0	0
BHP BILLITON	0	0	0	0	0	0	0	0	0	0	0	1.0113982	0	0	0
BP	0	0	0	0	0	0	0	0	0	0	0	0	0.9667941	0	0
BRITISH AMERICAN TOBACCO	0	0	0	0	0	0	0	0	0	0	0	0	0	0.9590627	0
BRITISH LAND	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1670516

λ_{max} 1.27085

λ_{min} 0.761571

Noisy Eigenvalues which Fall between λ_{max} and λ_{min} are highlighted in Red

If we take only diagonal matrix of Eigenvalues which shows that out of 15 sample points, 06 are noisy values. If we apply the filtration methods one by one and see if it completely filters the diagonal matrix or not?

1. Plerou et al.: This method states that replace noisy Eigenvalues by '0' and it can be seen that '0' is non noisy in this case as it becomes less than λ_{min}

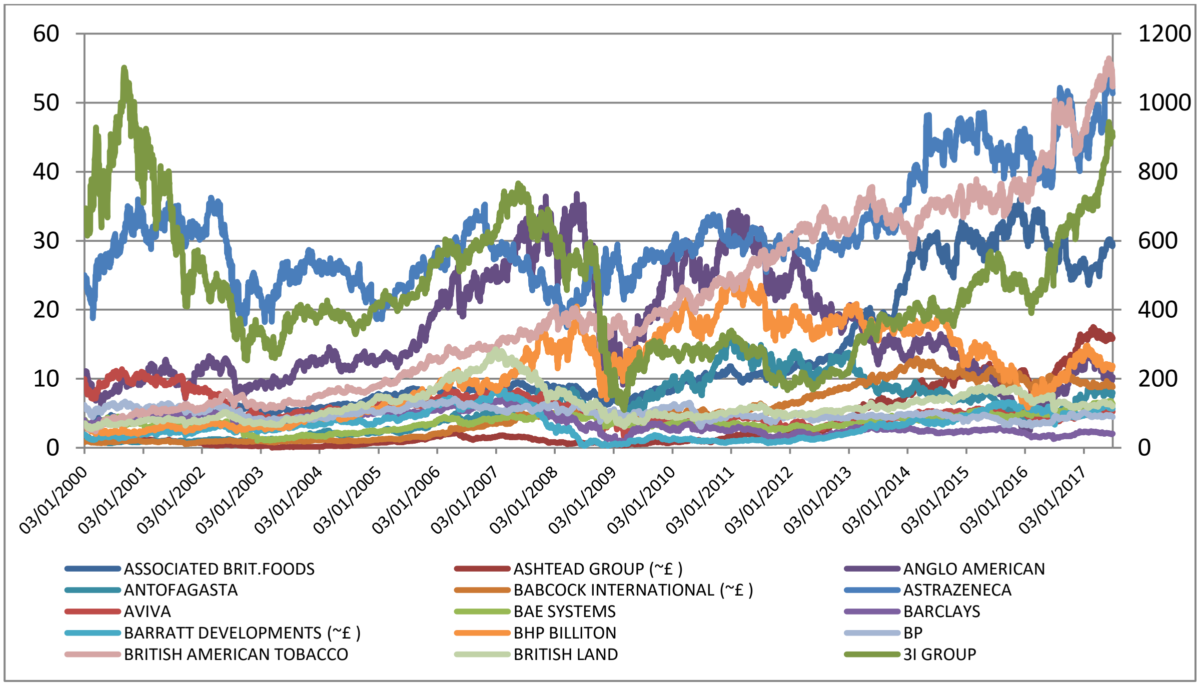
2. Laloux et al.: This method states that replace all noisy values by mean of Eigenvalues. In the above sample data mean becomes λ_{mean} = 2.94585 which when put in noisy Eigenvalues gives non noisy values as it is more than $\lambda_{max.}$ when it is put in total sample of 74 companies λ_{mean} becomes 1.2695 which is still noisy so this study does not use this filtration method.

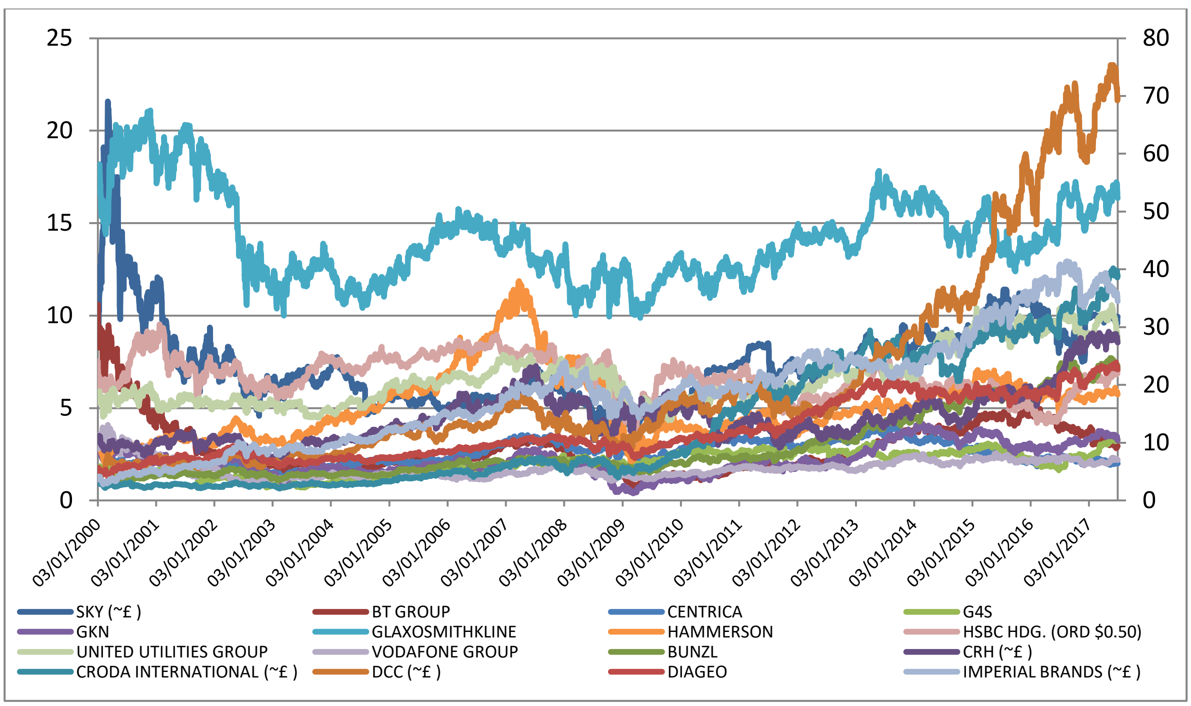
3. Daly et al./ Sharifi et al.: This method replaces the noisy eigenvalues with positive values that are equally and maximally spaced, and have sum equal to the sum of those replaced. To achieve maximal spacing, it is assumed that the smallest replacement eigenvalue should be very close to zero. For above sample points equal spacing of 0.3969 was used keeping in view sum should remain same but it can be seen in row 3 of following table that their is one noisy Eigenvalue in filtered diagonal matrix.

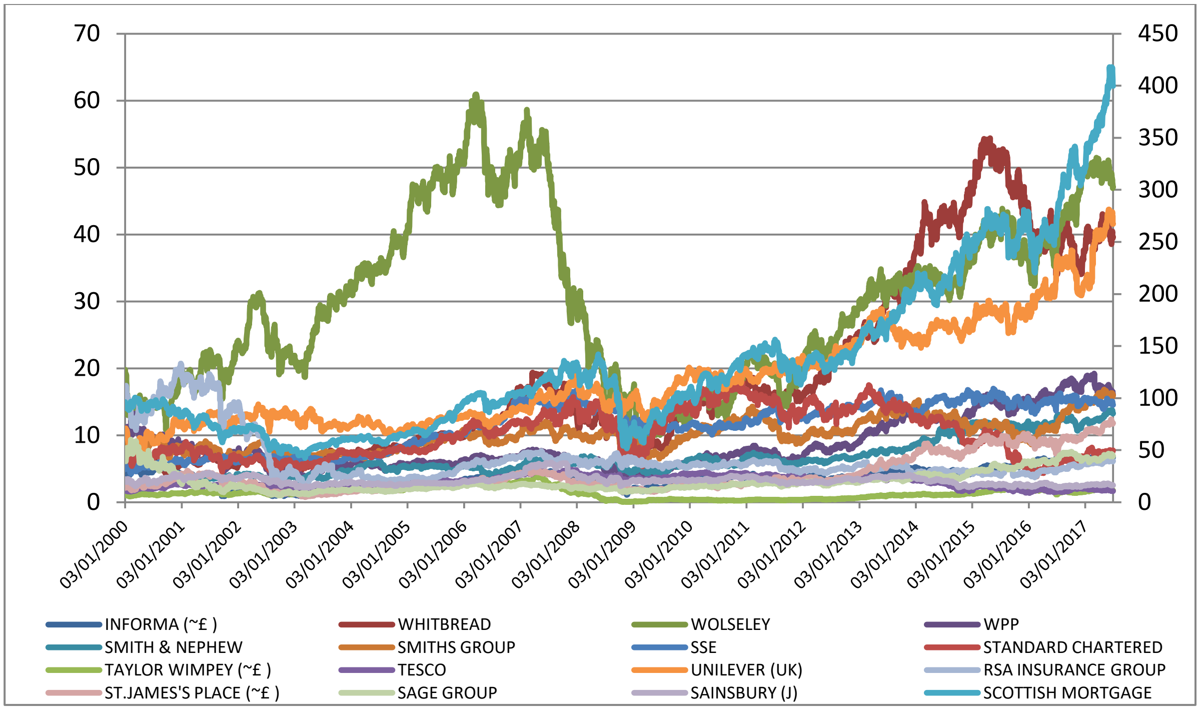
EigenValues before and after filtration method of Sharifi et Al.															Sum
22.3983	3.37287	2.80158	2.60254	2.27926	1.66675	1.42553	1.29537	1.11068	1.08378	1.04675	1.0114	0.96679	0.95906	0.16705	44.18773
0.16705	0.56395	0.96085	1.35775	1.75465	2.15155	2.54845	2.94535	3.34225	3.73915	4.13605	4.53295	4.92985	5.32675	5.72365	44.18027

The above discussion gives a clear understanding of why Plerou et al (2002) method of filtration was adopted in this thesis.

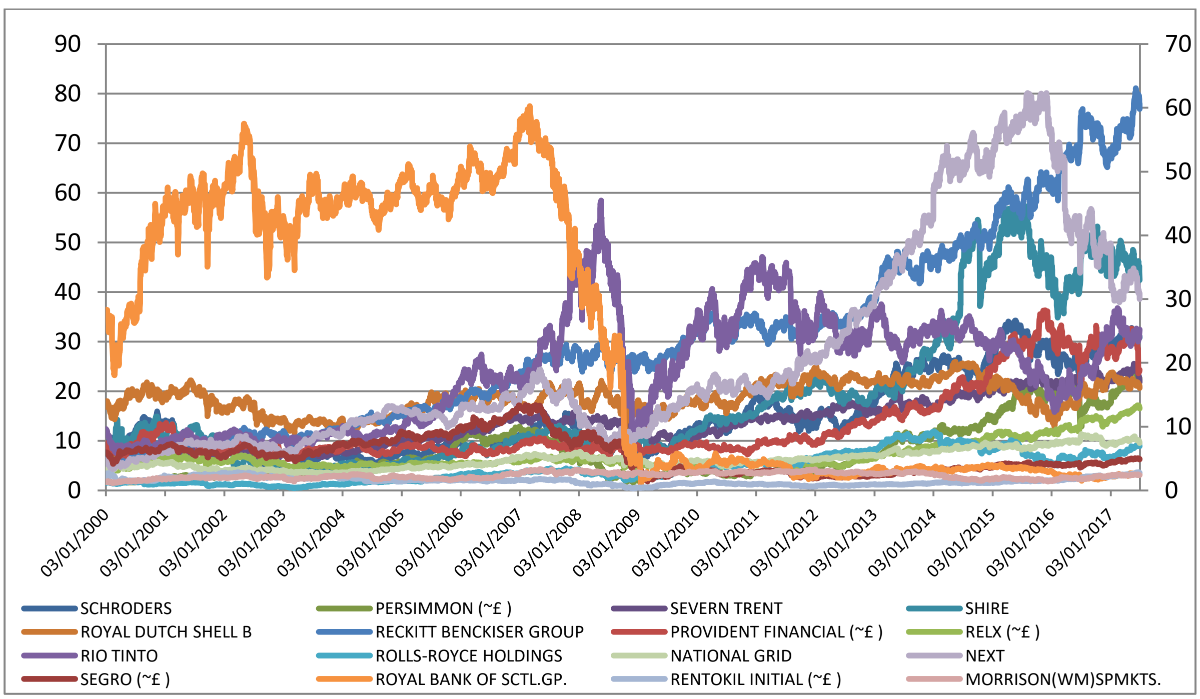
10.7 : Appendix 7 Graphs of FTSE 100 Constituents (Fixing Figure 7.4-1)



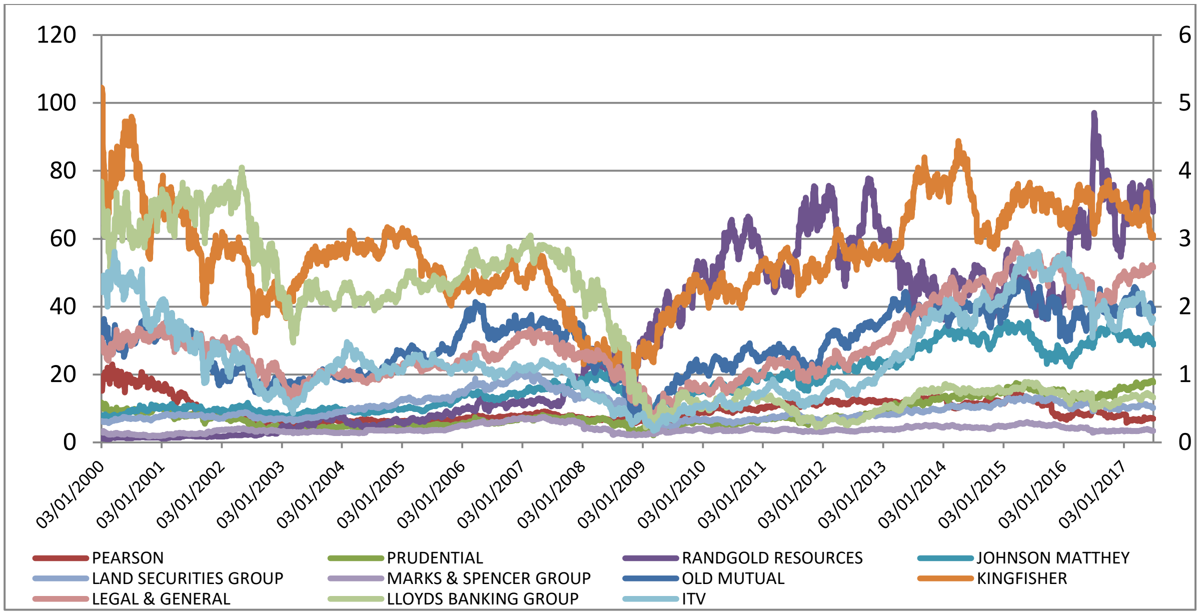




70
60
50
40
30
20
10
0
450
400
350
300
250
200
150
100
50
0
03/01/2000
03/01/2001
03/01/2002
03/01/2003
03/01/2004
03/01/2005
03/01/2006
03/01/2007
03/01/2008
03/01/2009
03/01/2010
03/01/2011
03/01/2012
03/01/2013
03/01/2014
03/01/2015
03/01/2016
03/01/2017
INFORMA (~£)
WHITBREAD
WOLSELEY
WPP
SMITH & NEPHEW
SMITHS GROUP
SSE
STANDARD CHARTERED
TAYLOR WIMPEY (~£)
TESCO
UNILEVER (UK)
RSA INSURANCE GROUP
ST.JAMES'S PLACE (~£)
SAGE GROUP
SAINSBURY (J)
SCOTTISH MORTGAGE



90
80
70
60
50
40
30
20
10
0
70
60
50
40
30
20
10
0
03/01/2000
03/01/2001
03/01/2002
03/01/2003
03/01/2004
03/01/2005
03/01/2006
03/01/2007
03/01/2008
03/01/2009
03/01/2010
03/01/2011
03/01/2012
03/01/2013
03/01/2014
03/01/2015
03/01/2016
03/01/2017
SCHRODERS
PERSIMMON (~£)
SEVERN TRENT
SHIRE
ROYAL DUTCH SHELL B
RECKITT BENCKISER GROUP
PROVIDENT FINANCIAL (~£)
RELX (~£)
RIO TINTO
ROLLS-ROYCE HOLDINGS
NATIONAL GRID
NEXT
SEGRO (~£)
ROYAL BANK OF SCTL.GP.
RENTOKIL INITIAL (~£)
MORRISON(WM)SPMKTS.


120
100
80
60
40
20
0
6
5
4
3
2
1
0
03/01/2000
03/01/2001
03/01/2002
03/01/2003
03/01/2004
03/01/2005
03/01/2006
03/01/2007
03/01/2008
03/01/2009
03/01/2010
03/01/2011
03/01/2012
03/01/2013
03/01/2014
03/01/2015
03/01/2016
03/01/2017
PEARSON
PRUDENTIAL
RANDGOLD RESOURCES
JOHNSON MATTHEY
LAND SECURITIES GROUP
MARKS & SPENCER GROUP
OLD MUTUAL
KINGFISHER
LEGAL & GENERAL
LLOYDS BANKING GROUP
ITV