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Time varying cointegration and the UK Great Ratios

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Abstract

We re-examine the great ratios associated with balanced growth models and ask whether they have remained constant over time. We first use a benchmark DSGE model to explore how plausible smooth variations in structural parameters lead to movements in great ratios that are comparable to those seen in the UK data. We then employ a nonparametric methodology that allows for slowly varying coefficients to estimate trends over time. To formally test for stable relationships in the great ratios, we propose a statistical test based on these nonparametric estimators devised to detect time varying cointegrating relationships. Small sample properties of the test are explored in a small Monte Carlo exercise. Generally, we find no evidence for cointegration when parameters are constant, but strong evidence when allowing for time variation. The implications are that in macroeconometric models allowance should be made for shifting long-run relationships, including DSGE models where smooth variation should be allowed in the deep structural relationships.

JEL Codes: C14, C26, C51, O4

Keywords: Time variation, great ratios, cointegration.

*The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England or its committees.

1 Introduction

In his famous 1961 paper,[1] Nicholas Kaldor put forward a set of 'stylised facts' that, he suggested, seemed to describe well long-run growth across all economies. In particular, he noted that: labour productivity and capital per worker had both grown at a roughly constant; the real interest rate, or return on capital, had been stable and that capital and labour had captured stable shares of national income; the ratio of capital to output had been stable; and finally, that among the fast growing countries of the world, there was an appreciable variation in the rate of growth 'of the order of two to five per cent'. He then went on to suggest that any 'sensible' model of economic growth should be one that implies that these stylised facts hold. In the same year Klein and Kosobud (1961) used the phrase 'great ratios' in the title of their paper, to indicate stable and constant relationships between key variables.

This arguably began the practice in economics of identifying stylised facts and great ratios, which is essentially motivated by a belief that the economy is converging towards a balanced growth steady state. To a modern eye it is natural to cast this in terms of stationary combinations of trending series, or in other words cointegrating relationships. However, there is no necessity that the steady state is constant, and indeed evidence that time variation (TV) is endemic in empirical macroeconomic models. In a forecasting context, this was prominently brought to attention by Stock and Watson (1996). Later examples that are more rooted in macroeconomic models include Primiceri (2005), Cogley et al. (2010), Cogley and Sargent (2001) and Cogley and Sargent (2005) using TV VARs on US data, Benati (2008) on UK macroeconomic dynamics, and Sims and Zha (2006) using a regime-switching VAR and Barnett et al. (2014) examining a range of models using UK data. In the DSGE literature, it has been a more neglected issue. Nevertheless, time variation in the preference parameters or volatility of structural shocks of a DSGE model have been modelled by specifying a stochastic process for a small subset of the parameters (Fernandez-Víllaverde and Rubio-Rámirez (2008) and Justiniano and Primiceri (2008)). There are some complex issues to deal with, particularly with regard to expectation formation. Fernandez-Víllaverde and Rubio-Rámirez (2008) assume that agents take into account future parameter variation when forming their expectations. Similar assumptions are made by Schorfheide (2005) Bianchi (2013) and Foerster et al. (2016), but the parameters are modelled as Markov-switching processes. Others (eg Canova (2006), Canova and Sala (2009) and Castelnuovo (2012)) allow for parameter variation by simply estimating models over rolling samples.[2] Galvão et al. (2016) by contrast use methods similar to those employed in this paper. Finally, Kulish and Pagan (2017) consider some different assumptions about belief updating and explore various solution methods.

[1] Kaldor (1961)

[2] Fawcett et al. (2015) allow for variation in trend productivity.

Looking back, it is perhaps remarkable how little the empirical implications of balanced growth have been confronted by the evidence. Klein and Kosobud (1961), writing at the dawn of modern time series econometrics, essentially run log-linear regressions including a deterministic trend. King et al. (1991) cast their analysis in a more modern framework, looking at cointegrating relationships between US data for consumption, investment and income over the period 1949Q1 to 1988Q4 and finding two cointegrating relationships with weak evidence that the balance growth great-ratio restrictions cannot be rejected (although shortly afterwards Serletis (1994) rejected the hypotheses that the log ratios of consumption and investment to output are stationary in Canadian data). Harvey et al. (2003) by contrast found next to no evidence for two cointegrating vectors or the balanced growth restrictions among these variables in the G7 countries (including the USA, and also using the King et al. (1991) definition of output), albeit over a different sample (1972Q1 to 1996Q4). In a mainly forecast related paper, Clements (2016) examines the two main ratios in US data and rejects stationarity for both consumption and investment, although the tests are sensitive to samples. Similar evidence is presented in Franchi and Jusélius (2007) where the Ireland (2004) model, an RBC model with the balanced growth property, is analysed within a VECM framework. Among the conclusions are that a single stochastic trend cannot explain the data and that there is time variation in the time series properties of the data. So the evidence for constant parameter cointegrating vectors was not robust. With structural change in mind, Attfield and Temple (2010) revisited the question for the UK and US for 1955Q1 to 2001Q2 and 2002Q2 respectively. They focussed on nominal shares[3] and allowed for discrete location or mean-shifts (what is often meant by a 'structural break'). With these breaks, the evidence of cointegration is much stronger.[4] But the existence of a limited number of discrete location shifts is not necessarily compelling. In general, structural change may take the form of smooth parameter variation, as discussed below. So in this paper we allow for this possibility in a cointegrating framework. While the notion of a smoothly TV cointegrating relationship is unfamiliar, it is only an extension of existing approaches that allow for discrete shifts in cointegrating relationships, usually in the form of location shifts or breaking trends (see Harris et al. (2016) for a recent example).

To summarise, the UK, as with other economies, has experienced subsstantial structural change over several decades (eg changes in industrial organisation and labour markets, the advent and subsequent decline of North Sea oil and gas and far-reaching changes in taxation and welfare). More recently, the financial crisis and subsequent recession have been associated (for example) with an apparent slow down in TFP growth and a fall in equilibrium unemployment, as well as unprecedentally low interest rates. All this makes constancy of deep structural parameters less plausible. And the casual evidence from the data for

[3] Following Whelan (2003).

[4] Mills (2001) suggests that there is evidence for stationary great ratios in the UK, but that they cannot be found using standard methods.

great ratios is suggestive of changes. Thus in this paper we re-examine the UK evidence, looking for both time variation and equilibration (cointegration). In order to do so, we use nonparametric methods that capture slowly shifting trends and develop a framework that extends the analysis to nonstationarity and cointegration. We find that although when we look for fixed cointegrating vectors the null of cointegration is rejected for all pairs of variables, the contrary holds when we allow for smooth variation. We also find that these relationships are informative for a set of key macroeconomic variables.

In the next section, 2, we briefly examines the data we subsequently model. In Section 3 we set out a benchmark model and experiment with the effects of varying parameters on the data. In Section 4 we discuss approaches to modelling time variation before laying out the econometric methodology, and report some Monte Carlo experiments. Section 5 present results and the final section concludes.

2 Stylised facts and the data

To set the context, Figures 1 to 6 show the growth rate of labour productivity, the growth rate of capital per worker, the ten-year spot real interest rate, the shares of labour and capital in GDP and, finally, the capital to GDP ratio; that is, they show the variables and ratios emphasised by Kaldor (1961).[5] Recession episodes are indicated.[6]

In Figure 1, there is some evidence that the long-run growth rate of labour productivity has fallen over the past 20 years or so. Figure 2 shows a similar picture for the growth rate of capital per worker. Figure 3 shows that real interest rates have fallen since the mid-1980s.

Figures 4 and 5 show the shares in national income of labour and profits ('capital' in the Kaldor sense). The labour share fell from around 1970 to around 1985 but rose again after around 1995 to a level that is somewhere between that of the 1950s and 60s and that of the 1980s. The profit share is not quite the inverse of the labour share, but is close. It appeared to rise between about 1980 and the late 1990s before falling back to a similar level to that seen in the 1950s and 1960s. Finally, Figure 6 reports Kaldor's final stylised fact, that the ratio of capital to output is stable. A casual impression is that this does not hold, instead trending upwards over time.

So a look at Kaldor's stylised facts suggests that these relationships have been shifting over time. Before examining this more rigorously in Section 5, we set out a model in which we can examine how shifts of this kind may be understood.

[5] We examine additional series that a basic model emphasises below.

[6] These are defined as periods of at least two quarters of negative GDP growth.

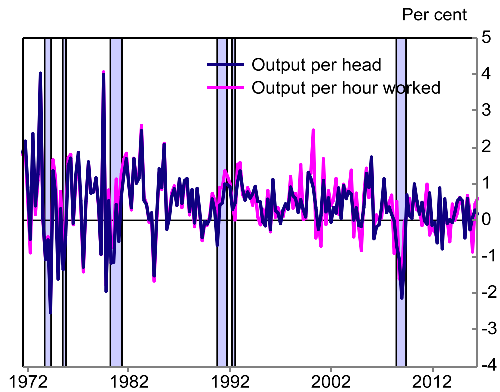

Figure 1: Labour productivity growth

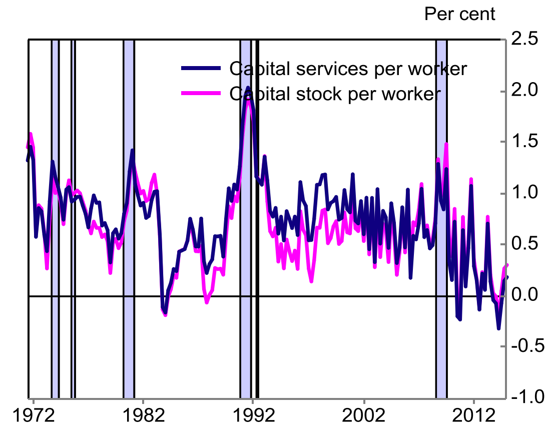

Figure 2: Capital per worker growth

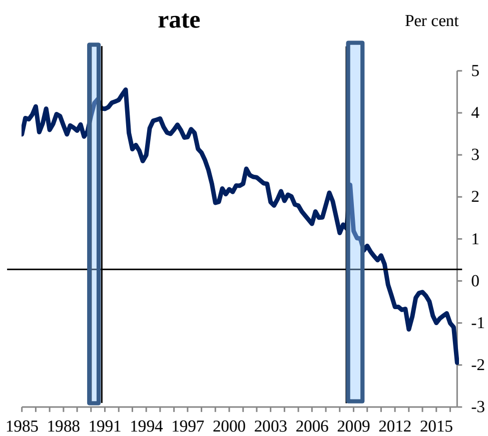

Figure 3: Ten-year real interest rate

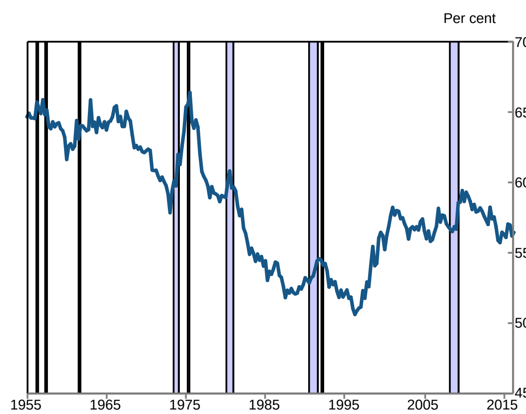

Figure 4: Labour share

3 A benchmark model

In this section we examine a simple benchmark model to explore the consequences for our key ratios of changing structural parameters.

King et al. (1988) consider the restrictions on preferences and technology within macroeconomic models that need to be imposed in order to ensure that the models adhere to Kaldor's (1961) stylised facts and imply balanced growth in the long run. As they observed and remains true today, almost all macroeconomic models today are built on this premise and are compatible with these assumptions. In what follows, we consider a standard model that shows how the great ratios are related to its parameters. We then illustrate how smooth changes in these will imply corresponding changes in the great ratios.

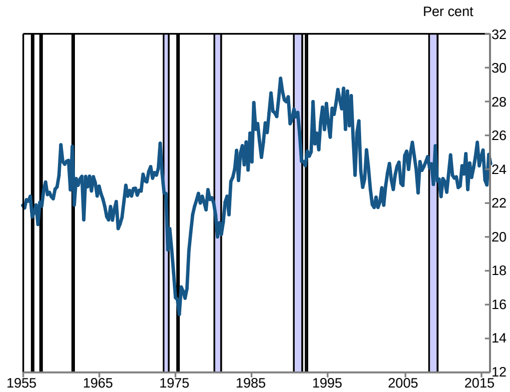

Figure 5: Capital share

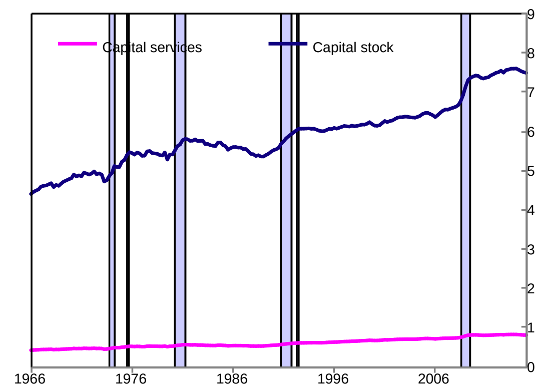

Figure 6: Capital to output ratio

3.1 The model

We start from the first stylised fact in Kaldor (1961) by defining the 'sustained rate of labour productivity growth' as g. That is,

$$\Delta \ln \left(\frac{y}{h}\right) = g \tag{1}$$

where y denotes output (GDP) and h denotes labour input. As shown by Swann (1964) and Phelps (1966), we need to assume that permanent technological change is labour augmenting in order for our model to imply balanced growth (ie, a growing economy in which the great ratios are stationary). Most macroeconomic models do this by using a Cobb-Douglas production function, though any constant returns to scale production function will give this result (King et al. (1988)). Following this literature we assume the following production function for GDP:

$$y = k^{\alpha} \left(Ah\right)^{\gamma} M^{1-\alpha-\gamma} - p_M M \tag{2}$$

where k denotes the capital stock, A denotes labour-augmenting technological change, M denotes imports and p_M denotes the price of imports relative to the GDP deflator. It is easy to show that for a balanced-growth equilibrium in which the rates of growth of output, capital and imports are all equal, we need A to grow at the rate g and that labour input and the relative price of imports do not grow in equilibrium (ie, are stationary). Returning to Kaldor's stylised facts, in this case the growth rate of capital per worker will also equal g, and the ratios of capital and investment to output be stationary.

A problem with this analysis is that it is unclear what is meant by the ratios of the real capital stock or real imports to real output as these three quantities are made up of fundamentally different sets of good and services. And (Attfield and Temple (2010)) the relative prices of these different goods and services are likely to exhibit trends. For example, IT equipment, which has a higher weight in capital goods than in consumption goods, has become substantially cheaper over the years relative to, say, health care, which has a much larger weight in the consumption basket. Similarly, we might expect the relative price of imports to fall as consumers shift from buying relatively expensive domestically-produced goods towards cheaper imported goods.

Attfield and Temple (2010) and Whelan (2003) observe that when relative prices are changing it is not clear how to interpret such a ratio, or even whether it is economically meaningful. We can give an interpretation to real consumption and real output, but it is not at all clear why we should focus on what the share of consumption in output would have been, if relative prices had remained at those of a given base year. Separately, as Whelan (2003) shows, the choice of base year can make large differences to the calculated real shares, suggesting that analysing the ratios of real variables (defined in constant price terms) is problematic. Given these considerations, it makes more sense to consider the ratios of the nominal capital stock and nominal imports to output as, in this case, both the numerator and denominator of the ratio will be measured in the same units and trends in the relative price are likely to be offset by opposing trends in relative demand. So, for the rest of this section we adopt the approach of examining nominal ratios, in line with the stylised facts shown in the previous section and our empirical analysis in later sections.

To examine income shares we need to derive demand curves for capital and labour. Following the standard DSGE literature, we assume that firms are monopolistically competitive and face a demand curve given by

$$y_j = \left(\frac{P_j}{P}\right)^{\frac{\mu}{\mu-1}} y \tag{3}$$

where y_j denotes the output of firm j, P_j denotes firm j's price, P denotes the aggregate price level (GDP deflator) and y denotes aggregate demand. Firm j will then maximise its profits given by $P_j y_j - Wh - r_k P_k k$ subject to its production function and its demand curve. Here W denotes the nominal wage, r_k denotes the real return on capital and P_k denotes the nominal price of capital. Solving this problem and imposing symmetry across firms (ie, all firms produce the same output and sell it at the same price) implies the following for the shares of nominal imports, nominal wages and nominal capital in nominal GDP:

$$\frac{P_M M}{Py} = \frac{1-\alpha-\gamma}{\mu-(1-\alpha-\gamma)}, \tag{4}$$

$$\frac{Wh}{Py} = \frac{\gamma}{\mu-(1-\alpha-\gamma)} \tag{5}$$

and

$$\frac{r_k P_k k}{Py} = \frac{\alpha}{\mu-(1-\alpha-\gamma)}. \tag{6}$$

where P_M denotes the nominal price of imports.

Thus the labour and capital shares will be stationary, as suggested by Kaldor (1961), if the elasticities of output with respect to capital, labour and imports and the mark-up are all constant. Below, we examine the effect on these shares of changing these parameters.

The final two of Kaldor's stylised facts follow from the stationarity of capital's share in output and each other. If we assume that r_k is stationary, then it immediately follows from the stationarity of capital's share that the nominal capital stock to nominal GDP ratio, $\frac{P_k k}{P y}$, will be stationary. King et al. (1988) show that for r_k to be stationary, it must be the case that the intertemporal elasticity of substitution in consumption does not depend on the level of consumption. This implies a utility function of the form

$$U(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} \tag{7}$$

where c denotes consumption. To derive an expression for the real interest rate and the return on capital we suppose that identical, infinitely-lived consumers maximise the present discounted value of their current and (expected) future utility flows discounting utility with a discount factor β subject to a budget constraint. We assume that they hold their wealth as either real bonds (paying the risk-free real interest rate r) or capital (with return r_k and depreciation rate δ). This means we can write the budget constraint for the representative consumer as

$$b_t + p_{k,t}k_t = (1 + r_{t-1})\, b_{t-1} + (1 - \delta + r_{k,t})\, p_{k,t}k_{t-1} + \widetilde{y}_t - c_t \tag{8}$$

where b denotes end-of-period holdings of real bonds, k denotes end-of-period holdings of capital, p_k is the relative (to consumption) price of capital goods and $\widetilde{y}$ denotes other (ie, non-bond and non-capital) income.

Solving the consumer's problem yields the first-order conditions

$$c_t^{-\sigma} = \beta(1 + r_t)E_t c_{t+1}^{-\sigma} \tag{9}$$

and

$$c_t^{-\sigma} = \beta E_t(1 - \delta + r_{k,t})\frac{p_{k,t+1}}{p_{k,t}}c_{t+1}^{-\sigma}. \tag{10}$$

Taking logs and denoting the growth rate of consumption as g_c allows us to rewrite these equations as

$$r = \sigma g_c - \ln\beta \tag{11}$$

and

$$r_k = r + \delta - \pi_k \tag{12}$$

where $\pi_k = \ln\left(\frac{p_{k,t+1}}{p_{k,t}}\right) \forall t$ is the rate of inflation for capital goods relative to consumption goods. Hence

$$\frac{P_k k}{P y} = \frac{\alpha}{\left(\mu - (1 - \alpha - \gamma)\right)\left(\sigma g_c - \ln\beta + \delta - \pi_k\right)}. \tag{13}$$

As well as being one of Kaldor's stylised facts, we can think of the stationarity of the capital to output ratio to be the first of our results on the great ratios. Now, by definition

$$k_t = (1-\delta)\,k_{t-1} + I_t \tag{14}$$

where I denotes investment. Denoting the growth rate of the capital stock by g_k enables us to write the investment to capital ratio as

$$\frac{I}{k} = 1 - \frac{1-\delta}{1+g_k} = \frac{g_k+\delta}{1+g_k}. \tag{15}$$

Clearly, this ratio will be stationary. And the stationarity of both the investment to capital and the nominal capital to nominal GDP ratios implies that the nominal investment to nominal GDP ratio will also be stationary:

$$\frac{P_k I}{Py} = \frac{\alpha\,(g_k+\delta)}{(\mu-(1-\alpha-\gamma))\,(\sigma g_c - \ln(\beta) + \delta - \pi_k)\,(1+g_k)}. \tag{16}$$

We have already shown that given the conditions laid down in King et al. (1988) the ratio of nominal imports to nominal GDP will be stationary. It then follows that the ratio of nominal exports to nominal GDP must be stationary as otherwise the proportional trade balance would not be stationary. Denote this ratio, by x. Finally, as models generally treat government spending as exogenous, we simply assume the ratio $\frac{P_G G}{Py}$ to be stationary.

This leaves us with the ratio of nominal consumption to nominal GDP. By the national accounting identity (ie, expenditure equals output) we have

$$\frac{P_c c}{Py} = 1 + \frac{1-\alpha-\gamma}{\mu-(1-\alpha-\gamma)} - \frac{\alpha\,(g_k+\delta)}{(\mu-(1-\alpha-\gamma))\,(\sigma g_c - \ln(\beta) + \delta - \pi_k)\,(1+g_k)} - \frac{P_G G}{Py} - x \tag{17}$$

That is, given our assumptions on technology and preferences which guarantee the stationarity of the nominal investment to nominal GDP and nominal import to nominal GDP ratios and our assumptions about the stationarity of the trade balance and the nominal government spending to nominal GDP ratio, the ratio of nominal consumption to nominal GDP will be stationary. But we can note that changes in any of our parameters (the elasticities of output with respect to capital, labour and imports, the growth rates of consumption, capital or output, the depreciation rate of capital, the mark-up, the intertemporal elasticity of consumption, the discount factor, the relative inflation rate for capital goods and the shares of government spending and exports in output) will lead to changes in the shares of labour and capital in income and in the great ratios. In our empirical work, we aim to examine whether these shares and ratios evolve smoothly over time in a way that once we allow for this smooth change they remain stationary. In what follows in this section, we examine whether the variation in these shares and ratios that we have observed in the data can be explained by plausible variation in the key parameters described above.

3.2 Some illustrative results

In order to illustrate how changing parameter values alter the great ratios we need a benchmark calibration. We do this by matching the sample means of growth rates and ratios in our model with the data. We first set the rate of labour productivity growth equal to its sample mean of 0.41% per quarter. We also assume balanced growth, setting the growth rates of consumption, the capital stock and GDP each equal to 0.41% per quarter. The real interest rate averaged 2.17%. We set β to 0.9975, which implies a value for σ of 0.72. The rate of inflation of capital goods prices relative to output, π_k, averaged -0.02% per quarter. Using the average investment to capital ratio of 3.71%, together with the average growth rate of 0.41% per quarter, implies a depreciation rate on capital, δ, of 3.32% per quarter.

We then set the mark-up, μ, the elasticity of output with respect to labour, γ, and the elasticity of output with respect to capital, α, in order to match the shares of imports, labour and investment in nominal GDP. The implied mark-up is 1.22, close to the value of 1.2 estimated by Macallan et al. (2008) using UK data for the period 1970 to 2003. We set the implied value of α to 0.19, and of γ to 0.56. Together with our other parameter assumptions, these numbers imply a investment to output ratio of 18.61%, an import share of GDP equal to 26.89% and a labour share of GDP equal to 58.06%, all equal to their mean values. Finally, we likewise set the shares of government spending and exports in GDP to 18.14% and 23.81%, respectively.

We use these calibrated values to illustrate how plausible variation in parameters leads to movements in the great ratios in line with those in the UK data. We start with the real interest rate and look first at varying the growth rate, g, and the discount rate, β. Over our data sample, the standard deviation of the growth rate of productivity was equal to 0.87 percentage points (pp) per quarter. Figure 7 shows how we might expect the steady-state real interest rate to vary as we change the steady-state growth rate between a plausible range of -1% per annum and 4% per annum (ie, well within a $\pm$ one standard deviation range) and we vary the discount rate between 0.9 and 1. As we can see, variations of this magnitude in the steady-state growth rate and the discount rate can explain variation in the steady-state real interest rate between -0.72% to 6.9% per annum. This is enough to explain all the observed variation we have seen in the real interest rate over this period, apart from the extremely negative values it has reached very recently, conditional on our assumed value for the intertemporal elasticity of substitution.

Armed with this knowledge, we now ask whether reasonable variations in our other parameters can explain the variation we have seen in the great ratios over our data sample. In particular, we examine the effects of varying the elasticities of output with respect to capital and labour (α and γ respectively) and the mark-up, μ, on the ratios of nominal consumption, nominal investment, nominal imports and

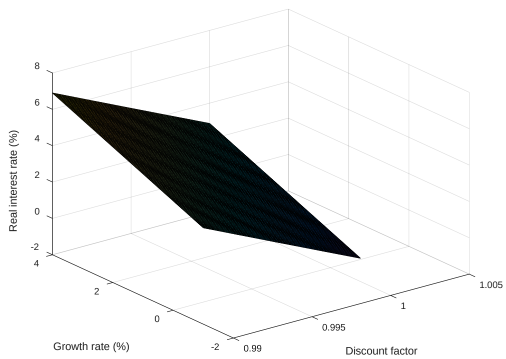

Figure 7: Varying growth and discount rate

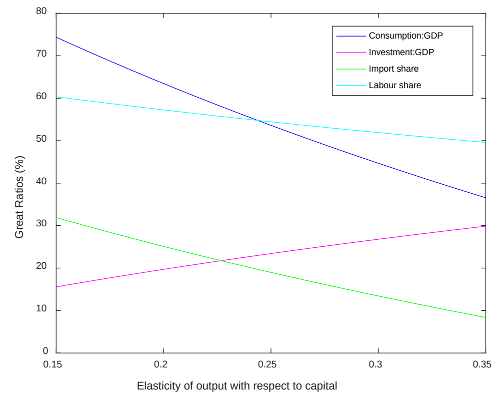

Figure 8: Varying output-capital elasticity

nominal wages to nominal GDP.

We start with the elasticity of output with respect to capital. Varying this elasticity between 0.15 and 0.35 produces the effects on the steady-state great ratios shown in Figure 8. It shows that, as might be expected, increasing the elasticity of output with respect to capital, α, leads to an increase in the investment to output ratio. Increasing this parameter means that firms are using more capital-intensive production techniques; a higher capital to output ratio implies, for a given depreciation rate and growth rate, a higher investment to output ratio, since higher investment is needed to maintain the higher capital stock. The interesting question is whether the rise and subsequent fall in α that we would need to explain the rise and fall in the investment to output ratio in the data is compatible with what we have seen in the share of labour and imports in GDP, as changes in α will lower both these ratios. The share of imports in GDP rose from around 25% in the 1980s to around 35% in the 2010s. The figure shows that a fall in α from around 0.2 to around 0.15 could explain most of this rise. At the same time, such a fall would result in a fall in the investment to output ratio from 19% to around 16%. The labour share fell from around 65% in the 1950s and 60s to around 52% in the 1980s and 90s before rising back to around 55% in the 2000s and 2010s. The initial fall came after the rise in the investment to output ratio suggesting their may have been different factors at work in that case, but the later rise in the labour share corresponded to a fall in the investment to output ratio. The figure suggests that a fall in α from around 0.3 to around 0.23 could explain the rise in the labour share between the 1990s and the 2010s. However, this would only correspond to a fall in the investment to output ratio from 26% to 22% (as opposed to the eight pp fall seen in the UK data over that period). This suggests that something else was moving the labour share.

It also shows that small movements in α can result in large movements in the consumption to output ratio. For example, a fall in α from 0.2 to 0.15 would, other things equal, raise the consumption to output ratio from about 64% to about 74%. As noted above, this change in α would lower the investment to output ratio by around three pp while raising the imports to output ratio by seven pp. Over the period between the 1990s and 2010s, these ratios fell and then rose by three pp and eight pp respectively. At the same time, the ratios of government spending and exports to output rose by about three and six pp respectively, which would result, other things equal, in a nine percentage point fall in the consumption to output ratio. According to the model, the net effect on the consumption to output ratio would be a rise of roughly one pp, which is what we see in the data.

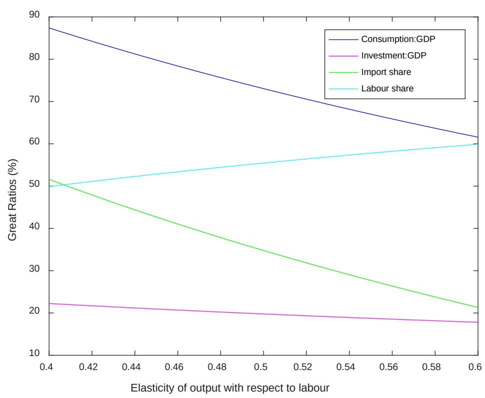

Figure 9: Varying output-labour elasticity

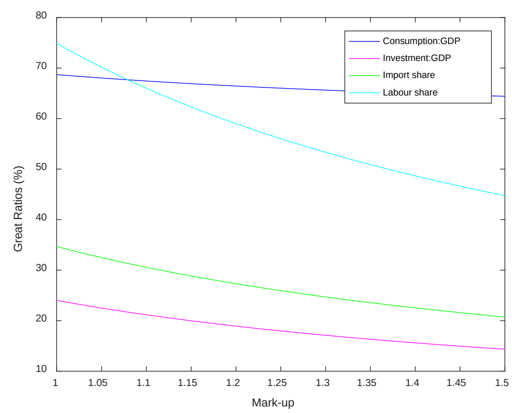

Figure 10: Varying steady-state mark-up

Figure 9 shows the effects of changes in the elasticity of output with respect to labour, γ. It suggests that the rise in the labour share between the 1990s and the 2010s could have resulted from a rise in this elasticity from around 0.5 to around 0.52. Similarly, a fall in γ from around 0.6 to around 0.5 could account for the fall in the labour share between the 1960s and the 1980s.

Finally, Figure 10 shows that as the mark-up increases the investment to output ratio falls and, importantly, plausible movements in the mark-up could explain at least part of the movement that we have seen in the investment to output ratio in the data. It also reveals that plausible variation in the mark-up can explain the variation we have seen in the labour share. But note that a rise in the mark-up will lead to falls in both the labour share and the investment to output ratio; so movements in the mark-up cannot explain opposing movements in these two shares. A rise in the mark-up additionally leads to a fall in the import share of GDP. Importantly, that then implies that a rise in the mark-up will lead to falls in the labour share *and* the import and investment to output ratios; as there was no period in our data sample where these variables were all moving in the same direction, we can assert that movements in the mark-up are not a major driver of movements in these variables.

To sum up the results in this subsection, our calibrated model suggests that movements in the elasticities of output with respect to capital and labour, the mark-up, and the ratios of government spending and exports to GDP could potentially explain movements in the ratios of consumption and investment to output and the shares of labour and imports in output. In the following subsection, we use this intuition to see whether we can match the movements observed in the great ratios in UK data since 1973 Q3 *via* smooth changes in these parameters. If we can, then that would suggest that, once we allow for smoothly time varying parameters, then we would expect the numerator and denominator of each of the great ratios to be cointegrated. This motivates our econometric analysis in Section 5.

3.3 Matching the data

We now use the model developed above to see qualitatively whether plausible movements in our parameters can generate shifts in the great ratios that are consistent with the data. We start by taking the ratios of government spending and exports to output as given, since they are exogenous in the model. We assume balanced growth in steady state, ie, that consumption and the capital stock both grow at the same rate as productivity. Within the model, productivity growth is exogenous. Rather than take the raw series, we removed the volatility by assuming that trend productivity growth was 0.44% between the two business cycle peaks of 1973 Q3 and 2008 Q1 (its average over this time period) and fell to 0.03% from 2002 Q2 to 2016 Q4, the end of our sample period, (its average over this period). This is shown in Figure 11.

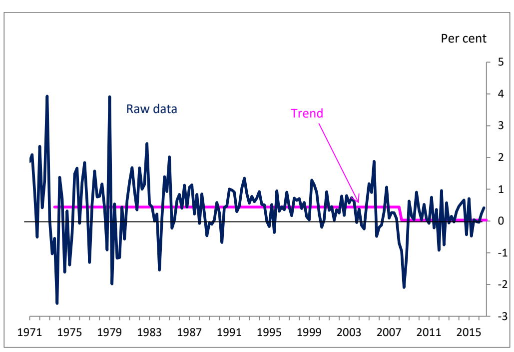

Figure 11: Productivity growth

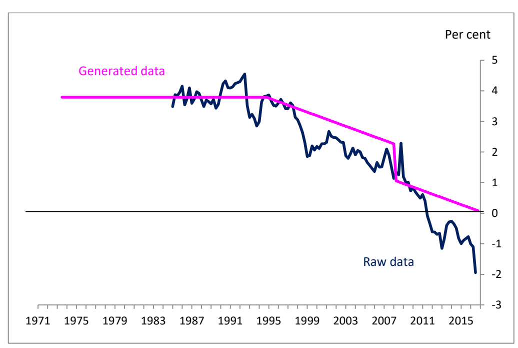

Figure 12: Real interest rate

Our results above suggest that, in order to match the fall in the real interest rate since around 1995, we need to smoothly raise the discount factor, while holding the coefficient of relative risk aversion, σ, constant (which we do at the value of 0.72). We set the discount factor at 0.9937 up to 1994 Q4. Given our assumed trend productivity growth rate, this value implies a steady-state real interest rate equal to

3.79%, the average ten-year spot real interest rate derived from Index-linked gilts between 1985 Q1 (the earliest point at which this data is available) and 1994 Q4. We then increase the discount rate linearly to a value of 1 in 2016 Q4. As shown in Figure 12, this leads to a smooth fall in the model-implied steady-state real interest rate, with a jump at the point where productivity growth fell. However, for 'reasonable' values of the discount rate (ie, $\beta \leq 1$) the model is not capable of generating a negative steady-state real interest rate (as the data currently suggest might be the case).

We again assume that the depreciation rate on capital, δ, is fixed at 3.32% per quarter and that the rate of inflation of capital goods prices relative to output, π_k, is zero. We also fix the elasticity of output with respect to labour, γ, at 0.53. Our results of the previous section suggest that, to match the fall in the investment to output ratio since the early 1990s, we need a gradual decline in the elasticity of output with respect to capital. Between 1973 Q3 and 1990 Q2 (business cycle peaks), we set α equal to 0.22, which implies an investment to output ratio of 0.203 (close to its average value over this period). From 1990 Q3 to 2016 Q4 we reduce α linearly down to a value of 0.15. At the same time, we increase the steady-state mark-up, μ, from 1.2, its assumed value prior to 1990 Q2, to 1.26. The result is that we are roughly able to match the fall in the investment to output ratio and the concomitant rise in the import to output ratio, as shown in Figures 13 and 14 below. We are also able to match the average labour share of 0.56 over our sample period as shown in Figure 15.

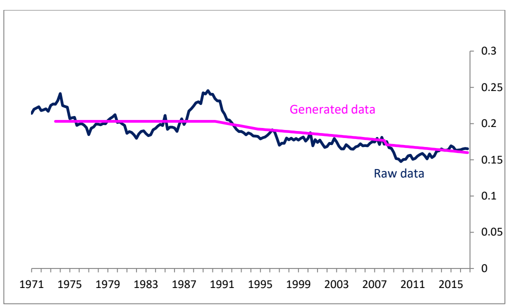

Figure 13: Investment to output ratio

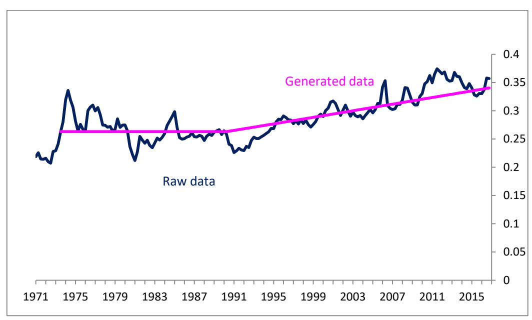

Figure 14: Import share

So our model suggests that plausible movements in the elasticity of output with respect to capital and the steady-state mark-up, together with the movements we saw in the ratios of government spending and exports to output, can explain the broad evolution of the ratios of consumption and investment to output and the shares of labour and imports in output. In other words, smooth movements in deep parameters would likely result in smooth changes to these shares and ratios. In the empirical part of the paper, we examine whether these shares and ratios have indeed moved such that there exist cointegrating relationships between the main macroeconomic variables that define these shares and ratios.

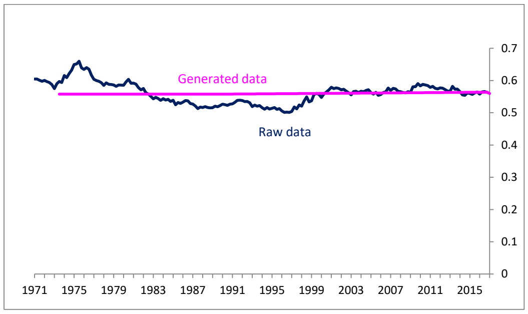

Figure 15: Labour share

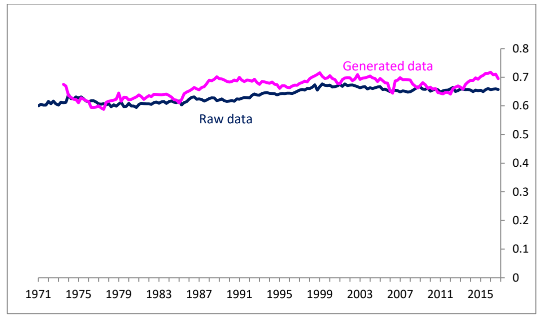

Figure 16: Consumption to output ratio

4 Econometric Methodology

As discussed above, there is widespread evidence for parameter time variation or structural change, which collectively may be described as instabilities in macroeconomic models. The proposed ways of dealing with this constitute a vast literature with a diverse range of approaches. Much of the focus has been on forecasting (comprehensively surveyed in Rossi (2013)) but inference in the presence of parameter instability as well as extensions to structural models have also received some attention. There is little consensus on the appropriate methods for parameter instabilities. Unforecastable permanent exogenous parameter shifts such as abrupt location shifts are one type of 'structural change'; another is regime shifts triggered by endogenous processes as in smooth transition models or probabilistic shifts between discrete regimes as in Markov switching models; finally, there are models which consider smooth deterministic or stochastic time series processes for the parameters. Random walks and long memory processes belong to the last category, which sees stochastic trends as a succession of structural breaks. If there is a clear idea of what the driving parameter process is, then the appropriate method is easily applied. However, it is often the case that we do not know which particular parameter model should be chosen, and the problem is more serious than choosing a model for an observed time series, since parameters are by definition latent.

In practice, state space models and linear filters, such as the Kalman filter, are often used to filter parameter variation as random, known, and persistent process (often a random walk). An alternative approach has been proposed by Giraitis et al. (2014), who make use of nonparametric kernel methods to model parameter drifts, allowing a certain level of agnosticism about the driving parameter process, and delivering consistent and asymptotically normal time varying estimators in a wide class of deterministic and stochastic processes. The advantage of the method is that it is robust to misspecification in the parameter process, and hence, it is relevant in a range of macroeconomic and financial contexts. Because the theory developed in Giraitis et al. (2014) is limited to stationary time series, we adopt their approach but we also provide some extensions below to deal with parameter time variation in nonstationary series. Our motivation is that the focus of this paper is on the great ratios which can be seen as long-run macroeconomic relations. Such long-run relationships have been often dealt with in the literature through standard fixed-parameter tests for cointegrating relationships. In the next section, we propose a time varying parameter extension to cointegration testing that incorporates the residuals from the kernel-type regressions proposed by Giraitis et al. (2014).

4.1 Cointegration with TVPs

We next establish the econometric methodology for inference in a simple cointegrating regression model in the presence of time varying parameters. The analysis is conducted by, first, extending the kernel estimators of Giraitis et al. (2018) to a cointegrating regression setup and proving consistency; and, second, proposing a cointegration test which can detect cointregation when the parameters are not constant.

The model we consider is a linear regression of the form

$$y_t = x_t'\beta_t + u_t \tag{18}$$

where x_t' is a unit root process, u_t is a homoskedastic martingale difference and β_t is a $k \times 1$ vector of time varying parameters. We assume that β_t satisfies

$$\sup_{|s|<s_0} \|\beta_t - \beta_{t-s}\| = O_p\left(\left(\frac{s_0}{t}\right)^{\gamma}\right) \text{ for some } 0 < \gamma \leq 1. \tag{19}$$

Condition (19) implies that the sequence of parameters drifts slowly with time, a property that is sufficient for consistent estimation of β_t. This covers deterministic piecewise differentiable processes assumed in the work of Dahlhaus on locally stationary processes (e.g. Dahlhaus (2000) or Dahlhaus and Polonik (2006)). Condition (19) also includes stochastic parameter processes exhibiting a degree of persistence necessary for consistent estimation of stochastically driven time variation. These include bounded random walk processes, as well as some fractionally integrated processes. In addition, parameters satisfying (19) can feature a combination of deterministic trends and breaks.

Under the parameter time variation framework of (19), an extremum estimator for β_t is derived by minimising an objective function $\widehat{\beta}_t = \arg\min_\beta \sum_{j=1}^{T} k_{tj} u_j^2$:

$$\widehat{\beta}_t = \left(\sum_{j=1}^{T} k_{tj} x_j x_j'\right)^{-1} \left(\sum_{j=1}^{T} k_{tj} x_j y_j\right)$$

where the weights k_{tj} are generated by a kernel, $k_{tj} := K\big((t-j)/H\big)$, where $K(x) \geq 0$, $x \in \mathbb{R}$ is a bounded function and H is a bandwidth parameter such that $H \to \infty$, $H = o(T/\log T)$. The kernel estimator $\widehat{\beta}_t$ is a simple generalisation of a rolling window estimator of the form

$$\widehat{\beta}_t = \left(\sum_{j=t-H}^{t+H} x_j x_j'\right)^{-1} \left(\sum_{j=t-H}^{t+H} x_j y_j\right).$$

We assume that K is a non-negative bounded function with a piecewise bounded derivative $\dot{K}(x)$ such that $\int K(x)dx = 1$. For example,

$$\begin{aligned}
K(x) &= (1/2)I(|x| \leq 1), \quad \text{flat kernel,} \\
K(x) &= (3/4)(1-x^2)I(|x| \leq 1), \quad \text{Epanechnikov kernel,} \\
K(x) &= (1/\sqrt{2\pi})e^{-x^2/2}, \quad \text{Gaussian kernel.}
\end{aligned}$$

If K has unbounded support, we assume in addition that

$$K(x) \leq C\exp(-cx^2), \quad |\dot{K}(x)| \leq C(1+x^2)^{-1}, \quad x \geq 0, \qquad \text{for some } C > 0,\ c > 0. \tag{20}$$

When x_t is stationary, β_t is bounded away from zero, and for simplicity if $\gamma = 1/2$, Giraitis et al. (2018) show that $\hat{\beta}_t - \beta_t = O_p\left(\left(\frac{1}{H}\right)^{1/2}\right) + O_p\left(\left(\frac{H}{T}\right)^{1/2}\right)$. Further, if x_t is a unit root process and β_t is deterministic, then Phillips et al. (2017) have shown consistency and derived rates for $\widehat{\beta}_t$.

We wish to test the hypothesis that u_t is an $I(0)$ process. To do so we extend the cointegrating KPSS test with a statistic based on the kernel estimate $\widehat{\beta}_t$.[7] We define the model's residuals by

$$\hat{u}_t = y_t - x_t'\widehat{\beta}_t$$

and the KPSS test statistic by

$$CI = \frac{T^{-2}h\sum_{j=1}^{T} S_j^2}{\hat{s}^2}$$

where $h = H/T$, $\hat{s}^2$ is an estimate of the long run variance of $\hat{u}_t$ and $S_{[Tr]} = \sum_{j=1}^{[Tr]} \hat{u}_j$.

Proposition 1 The asymptotic distribution of the test statistic CI defined above is given by the following expression:

$$T^{-2}h\sum_{j=1}^{T} S_j^2 = T^{-1}\sum_{j=1}^{T}\left(T^{-1/2}h^{1/2}S_j\right)^2 \Longrightarrow Q^2$$

where $Q = \sqrt{2}\int_{-1}^{1} K(s)dB^*_{y,(s+1)/2}$.

The proof of this proposition is given in Appendix A.

[7] We use KPSS rather than one of the many alternatives as if the null is non-stationarity the test misbehaves under the null. In that case the residual does not reflect the unit root error. This occurs because the kernel causes the residual to be more stationary than the error. With the null of stationarity, the KPSS test does not suffer from this problem.

4.2 Monte Carlo Exercise

We next provide a small Monte Carlo exercise to study the finite sample properties of our time varying extension to the KPSS test. We simulate data using as the data generating process the model in (18) where β_t is a bounded random walk

$$\beta_t = \sum_{i=1}^{t} v_t/\sqrt{t},\ v_t \sim \mathcal{N}(0,1).$$

Based on these simulated samples, Table 1 compares the rejection probabilities of our KPSS test at 95% under the null of cointegration ($u_t \sim I(0)$) and alternative of no cointegration ($u_t \sim I(1)$) under a range of sample sizes and proportional bandwidths. The table shows that the test is more or less correctly sized for all the sample sizes and bandwidths considered. The power is increasing in the sample size and relatively tight bandwidths are preferred.

Finally, we note that our method may seem to bear a resemblance to bandpass filters and specifically the Hodrick Prescott (HP) filter, which is often said to remove stochastic trends from time series. But Phillips and Jin (2015) have demonstrated that while the HP filter can only remove the stochastic trend if the smoothing parameter λ is optimally selected, this will not usually correspond to the conventionally selected values (it is not the frequency of the data that matters but the value in relation to the sample size). Moreover, our method estimates the TV cointegrating parameters which the HP filter can not, and we have established a methodology for testing for cointegration.

5 Results

5.1 Great ratios

Next we examine whether the great ratios may be considered to constitute cointegrating relationships when they are allowed to evolve. In each case, we consider models of the form

$$y_t = \alpha_t + x_t + \varepsilon_t \tag{21}$$

where y_t and x_t indicate the log of the variables Y_t and X_t respectively. We use a bandwidth equal to $T^0.5$. The cointegrating vector associated with the set $\{y_t, x_t\}$ is restricted to be $\{1, -1\}$ so the cointegrating relationship defines the log great ratio $y_t - x_t - \alpha_t$. In the fixed coefficient case $\alpha_t = \alpha\ \forall\ t$. Figures

Band-width	Rejection probabilities under the null				Rejection probabilities under the alternative			
	T=200	T=400	T=600	T=800	T=200	T=400	T=600	T=800
0.20	0.047	0.049	0.064	0.048	0.605	0.740	0.828	0.891
0.25	0.054	0.050	0.045	0.037	0.663	0.732	0.830	0.890
0.30	0.065	0.058	0.061	0.057	0.631	0.770	0.804	0.874
0.35	0.042	0.052	0.060	0.053	0.597	0.762	0.819	0.861
0.40	0.054	0.060	0.048	0.043	0.608	0.711	0.752	0.824
0.45	0.055	0.047	0.053	0.055	0.577	0.672	0.733	0.768
0.50	0.058	0.057	0.045	0.062	0.548	0.618	0.667	0.756
0.55	0.061	0.052	0.055	0.061	0.506	0.587	0.645	0.713
0.60	0.056	0.047	0.043	0.059	0.495	0.549	0.593	0.653
0.65	0.066	0.039	0.038	0.041	0.466	0.498	0.545	0.565
0.70	0.042	0.037	0.042	0.049	0.392	0.391	0.491	0.509
0.75	0.061	0.058	0.042	0.052	0.352	0.393	0.395	0.442
0.80	0.040	0.046	0.054	0.052	0.303	0.342	0.364	0.365
0.85	0.053	0.061	0.048	0.041	0.272	0.276	0.350	0.331
0.90	0.043	0.044	0.039	0.040	0.273	0.289	0.292	0.313

Table 1: Power and size of the TV extension to the KPSS test

17 to 25 show the unconditional mean (the *fixed cointegrating coefficient* , fixed α), the actual data for the great ratios or shares (*ratio*) and the time varying mean (the α_t, *TV cointegrating coefficient*) in the left hand panels. In the right hand panels the potential cointegrating residuals are illustrated. In the theoretical section we restrict attention to the nominal case, but we also tested real ratios (and labour to nominal output ratios are not meaningful). For reasons of space we do not report the corresponding figures, but do present formal tests.

Speaking somewhat loosely, it is clear from the charts that in most cases the great ratios do not look stationary, with the possible exception of the profit share. This consequently applies also to the fixed-coefficient residuals. This is particularly marked for the capital stock and servives By contrast, the TV coefficient residuals are more plausible candidates for stationarity.

The formal evidence is reported in Tables 2 and 3 for variables defined in real and nominal terms respectively. They report KPSS statistics for cointegration, maintaining the null of cointegration. Rejections of the null at the 5% level are indicated in the tables. For the fixed parameters, cointegration in the nominal ratios and shares is rejected in six out of nine cases and in six out of seven real cases. In the nominal case, stationarity (cointegration) of the fixed trade ratios cannot be rejected at 5%, and the same holds for the capital services ratio, which is somewhat surprisingly from the informal evidence Figure 25: it is rejected in the real case. By contrast, where time variation is allowed in no case is the null rejected for either the real or nominal cases.

Ratio	TV KPSSS	Fixed KPSS
C/Y	0.14	0.41**
I/Y	0.10	0.37**
G/Y	0.06	0.22**
X/Y	0.06	0.08
M/Y	0.06	0.36**
K/Y	0.08	0.17**
KS/Y	0.07	0.19**

*** indicates rejection of the null of cointegration at 5%.*

C = consumption, Y = output, I = investment, G = government consumption, X = exports, M = imports, K = capital stock, KS = capital services

Table 2: KPSS test statistics from TV and fixed parameter cointegration: real variables

Ratio	TV KPSSS stat	Fixed KPSS
Labour share	0.06	0.42**
Profit share	0.04	0.15**
C/Y	0.13	0.44**
I/Y	0.07	0.39**
G/Y	0.05	0.19**
X/Y	0.05	0.11
M/Y	0.04	0.15
K/Y	0.09	0.17**
KS/Y	0.07	0.13

*** indicates rejection of the null of cointegration at 5%.*

Table 3: KPSS test statistics from TV and fixed parameter cointegration: nominal variables

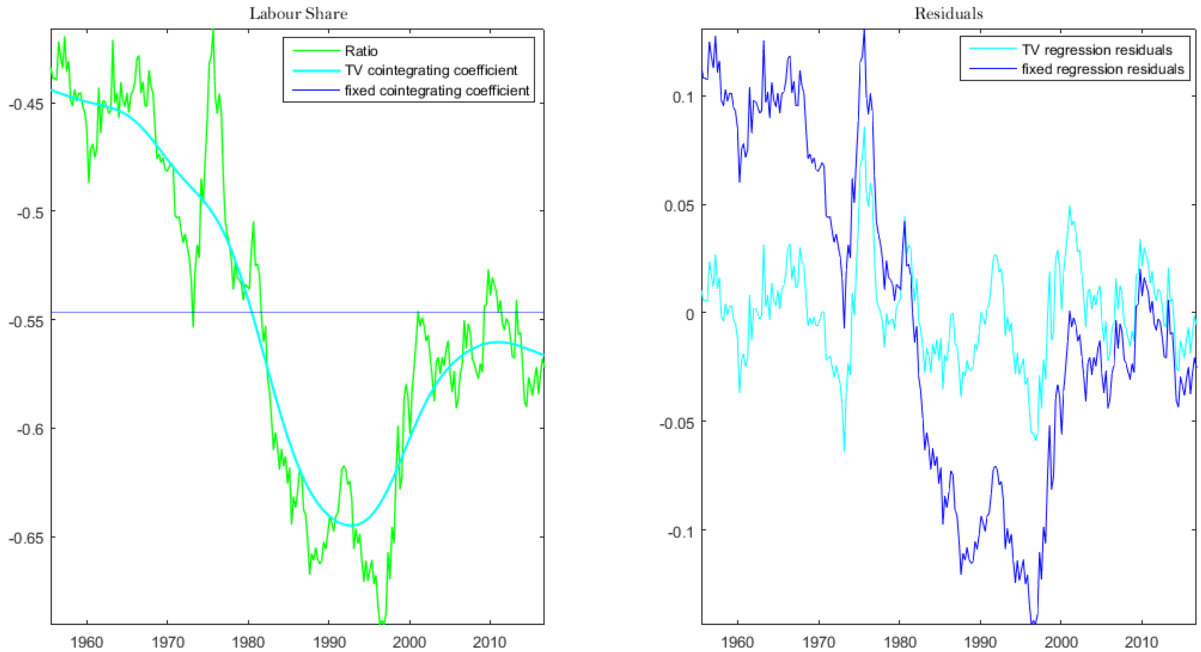

Figure 17: Labour share

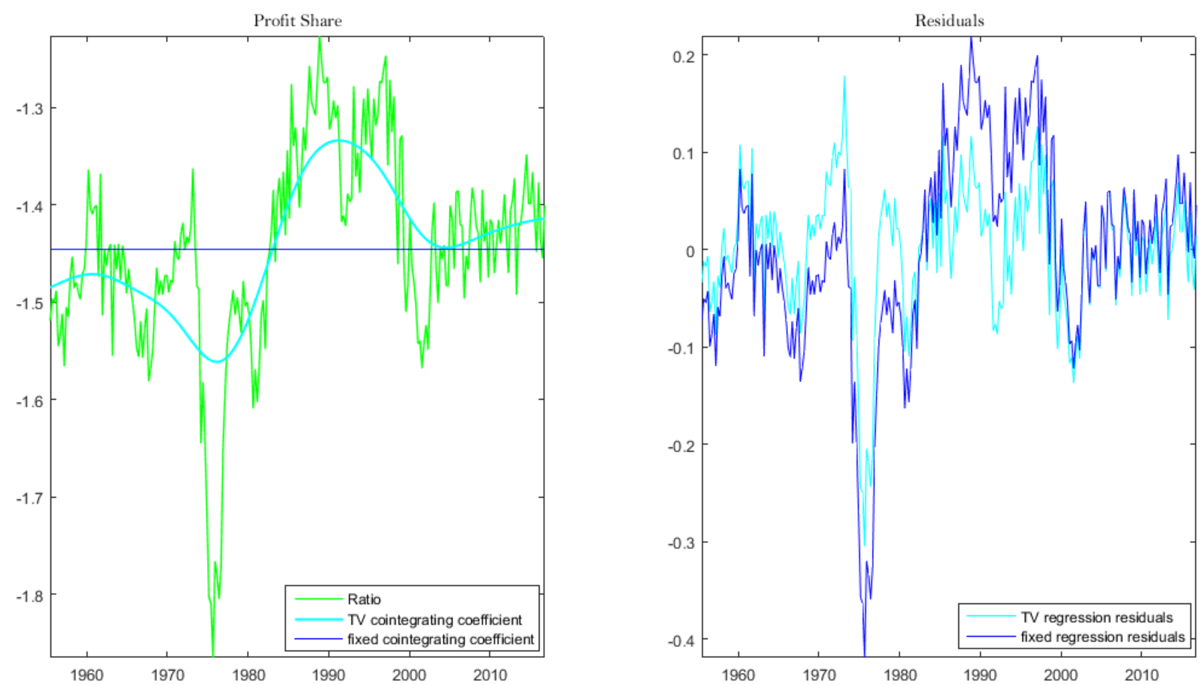

Figure 18: Profit share

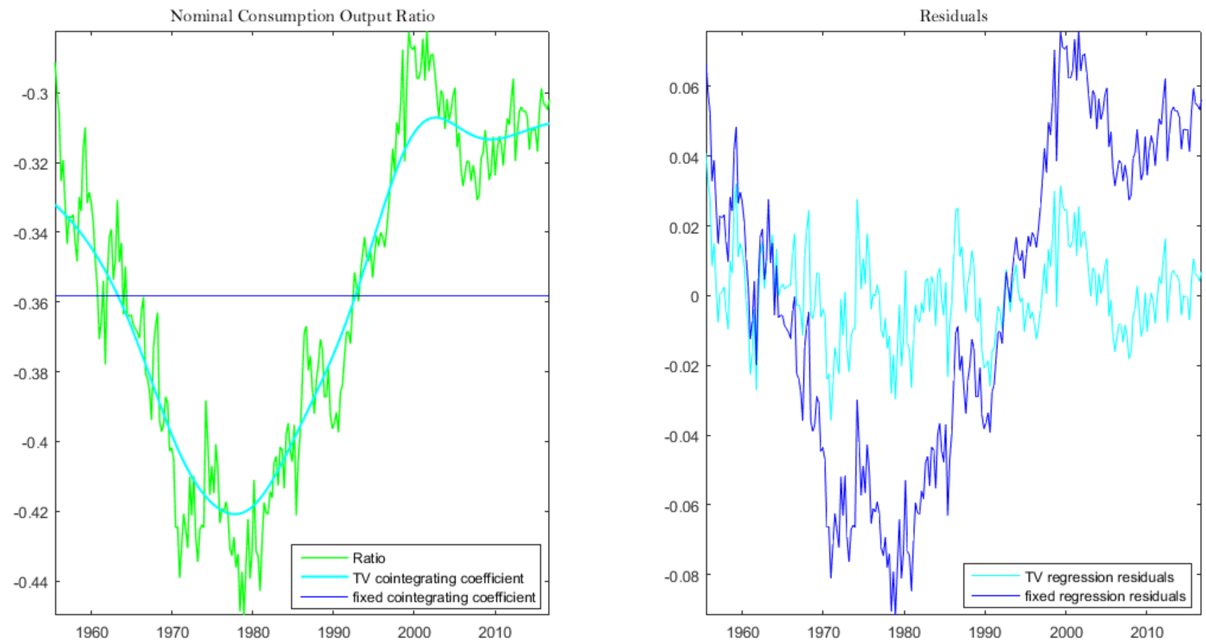

Figure 19: Consumption-output ratio (nominal)

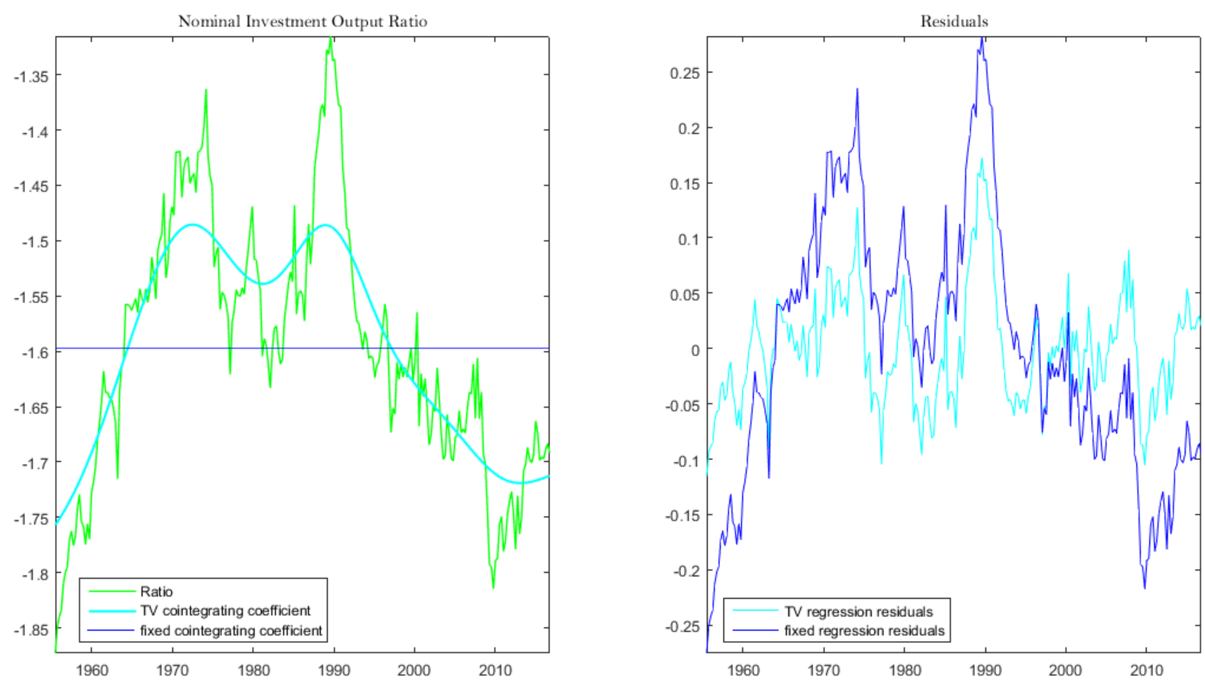

Figure 20: Investment-output ratio (nominal)

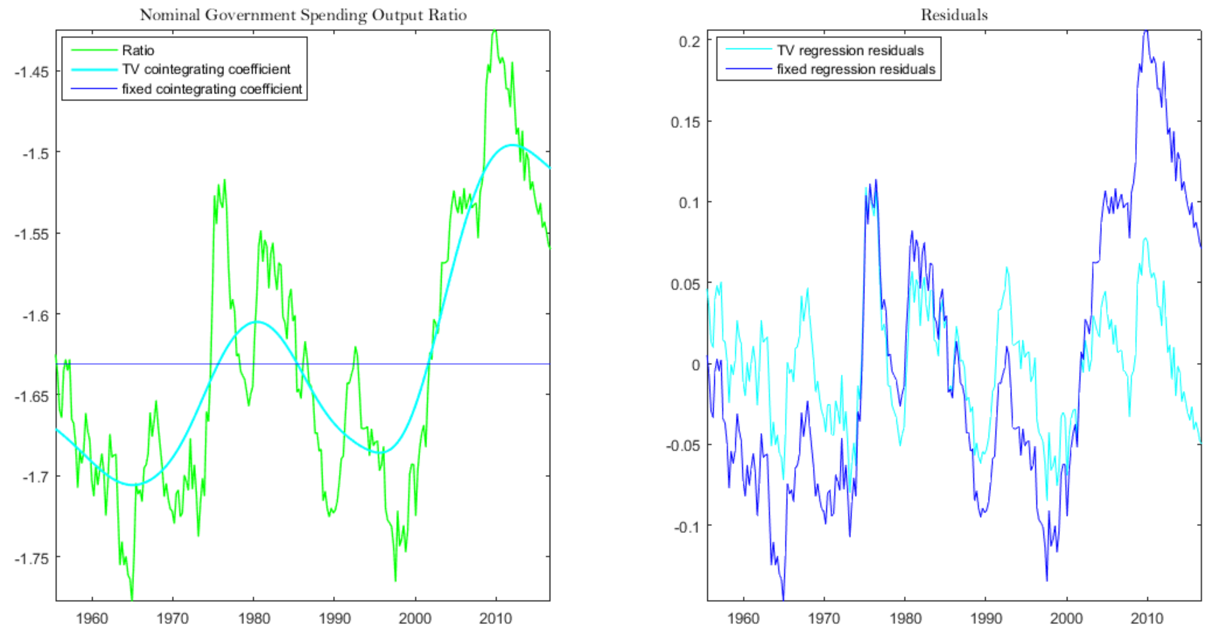

Figure 21: Government consumption-output ratio (nominal)

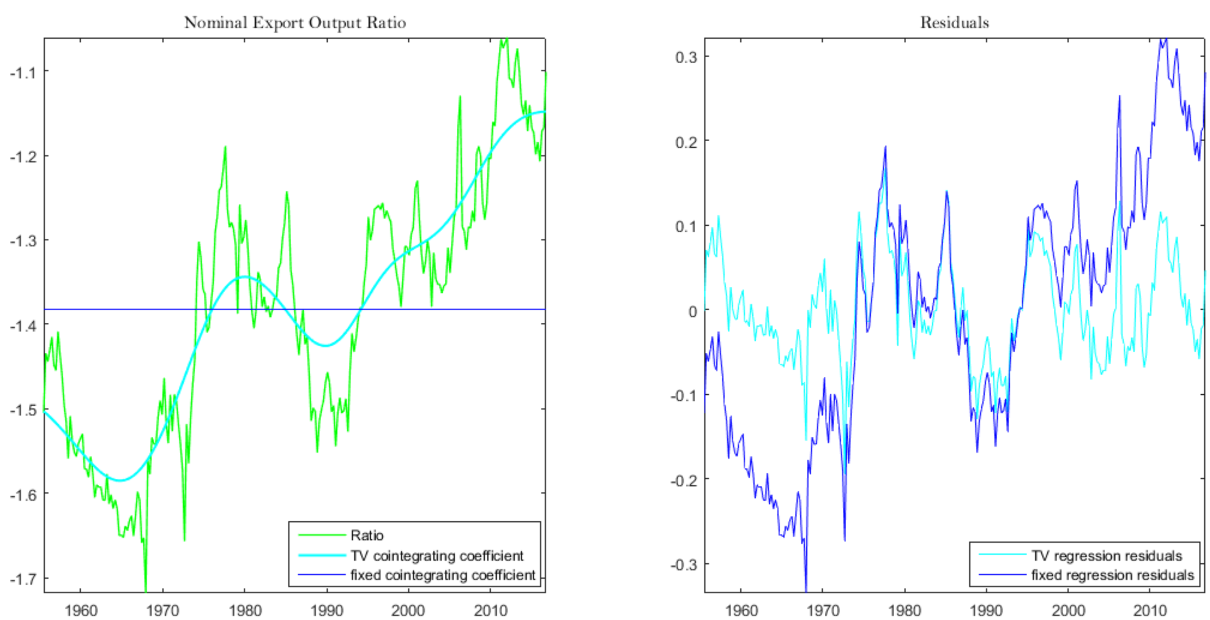

Figure 22: Export-output ratio (nominal)

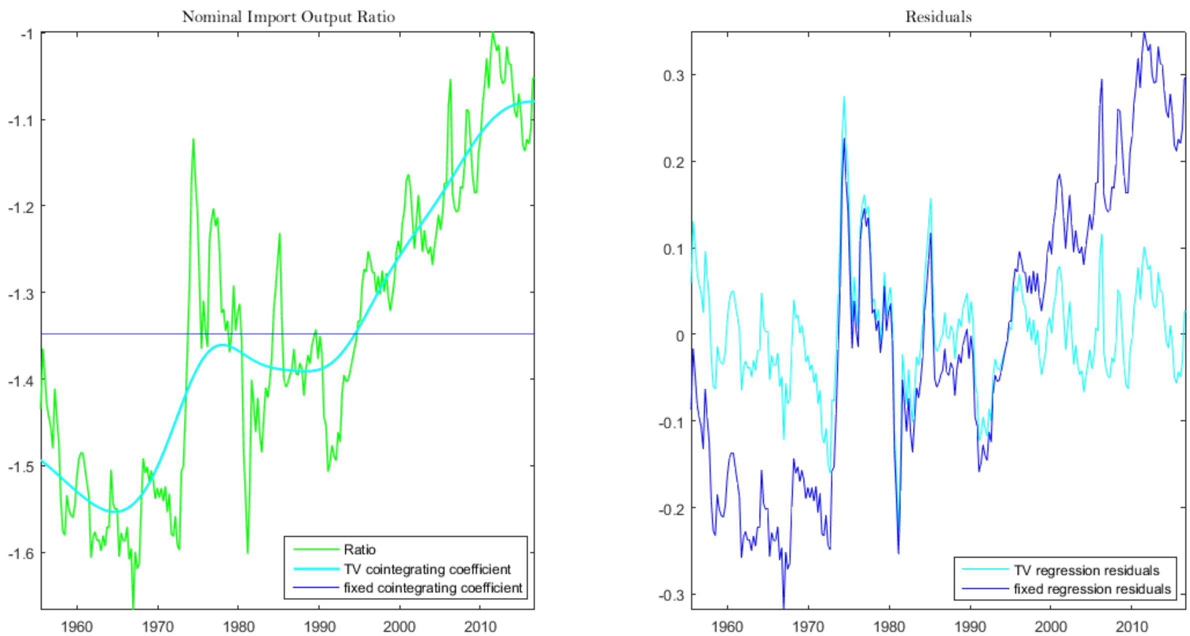

Figure 23: Import-output ratio (nominal)

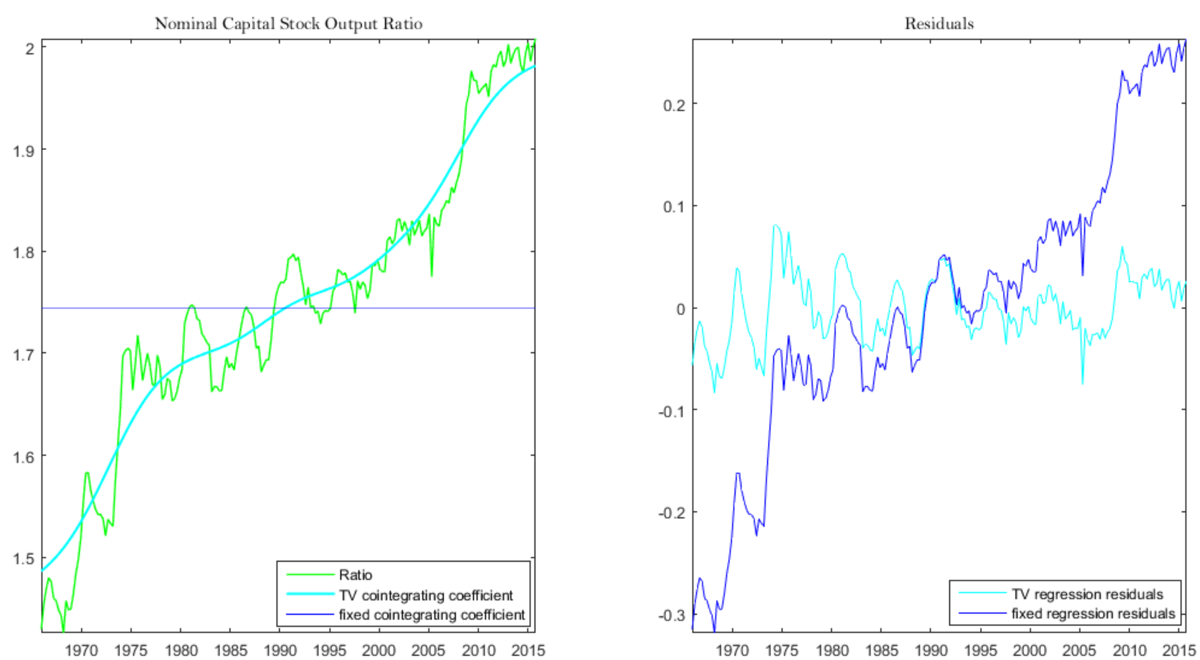

Figure 24: Capital-output ratio (nominal: stock)

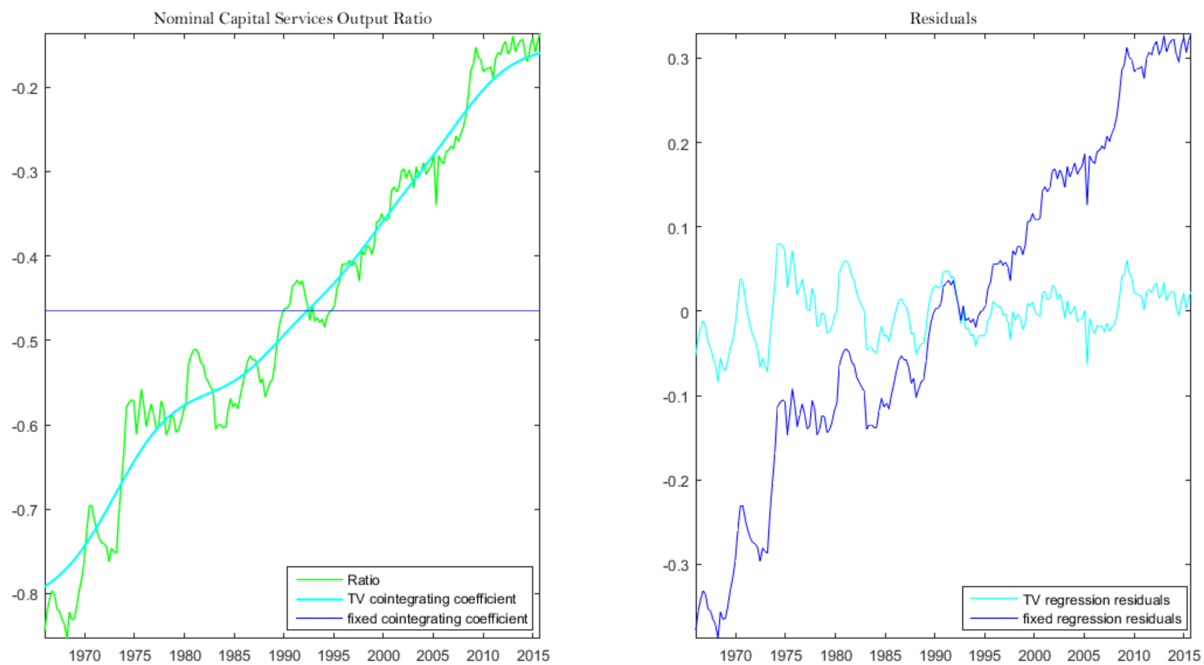

Figure 25: Capital-output ratio (nominal: services)

5.2 A VECM

We have established that time varying relationships between variables suggested by macroeconomic considerations exist where we cannot reject the null of stationarity, implying that they are cointegrating relationships. If that were the case, some of these relationships should act as attractors in a VECM representation of subsets of the data, which constitutes an informal test of our approach. As an example, we estimated a system explaining growth in output (Δy_t), consumption (Δc_t), the capital stock (Δk_t) and employment (Δe_t)[8] by lags of the growth terms and the estimated lagged cointegrating residuals from the time varying ratios for consumption and output ($c_t - y_t$), capital and output ($k_t - y_t$) and the labour share ($w_t + e_t - y_t$ where w_t is the real wage).[9]

All the cointegrating relationships are of the form $x_t^* - z_t^* - a_t^*$ where $*$ indicates the long-run value and a_t^* is the time varying mean of the ratio. The equilibrating error term is $\epsilon_{xz,t} \equiv x_t - z_t - a_t^*$. Thus the errors are defined as $\epsilon_{1,t-1} \equiv c_t - y_t - a_{1,t}^*$, $\epsilon_{2,t-1} \equiv k_t - y_t - a_{2,t}^*$ and $\epsilon_{3,t-1} \equiv w_t + e_t - y_t - a_{3,t}^*$.

$$\begin{aligned}
\Delta y_t &= \beta_{0,1} + \sum_{i=1}^{p} \beta_{1,1i}\Delta y_{t-i} + \sum_{i=1}^{p} \beta_{1,2i}\Delta c_{t-i} + \sum_{i=1}^{p} \beta_{1,3i}\Delta k_{t-i} + \sum_{i=1}^{p} \beta_{1,4i}\Delta e_{t-i} \\
&\quad - \lambda_{1,1}\epsilon_{1,t-1} - \alpha_{1,2}\epsilon_{2,t-1} - \alpha_{1,3}\epsilon_{3,t-1} + \varepsilon_{1,t} \\
\Delta c_t &= \beta_{0,2} + \sum_{i=1}^{p} \beta_{2,1i}\Delta y_{t-i} + \sum_{i=1}^{p} \beta_{2,2i}\Delta c_{t-i} + \sum_{i=1}^{p} \beta_{2,3i}\Delta k_{t-i} + \sum_{i=1}^{p} \beta_{2,4i}\Delta e_{t-i} \\
&\quad - \alpha_{2,1}\epsilon_{1,t-1} - \alpha_{2,2}\epsilon_{2,t-1} - \alpha_{2,3}\epsilon_{3,t-1} + \varepsilon_{2,t} \\
\Delta k_t &= \beta_{0,3} + \sum_{i=1}^{p} \beta_{3,1i}\Delta y_{t-i} + \sum_{i=1}^{p} \beta_{3,2i}\Delta c_{t-i} + \sum_{i=1}^{p} \beta_{3,3i}\Delta k_{t-i} + \sum_{i=1}^{p} \beta_{3,4i}\Delta e_{t-i} \\
&\quad - \alpha_{3,1}\epsilon_{1,t-1} - \alpha_{3,2}\epsilon_{2,t-1} - \alpha_{3,3}\epsilon_{3,t-1} + \varepsilon_{3,t} \\
\Delta e_t &= \beta_{0,4} + \sum_{i=1}^{p} \beta_{4,1i}\Delta y_{t-i} + \sum_{i=1}^{p} \beta_{4,2i}\Delta c_{t-i} + \sum_{i=1}^{p} \beta_{4,3i}\Delta k_{t-i} + \sum_{i=1}^{p} \beta_{4,4i}\Delta e_{t-i} \\
&\quad - \alpha_{4,1}\epsilon_{1,t-1} - \alpha_{4,2}\epsilon_{2,t-1} - \alpha_{4,3}\epsilon_{3,t-1} + \varepsilon_{4,t}
\end{aligned} \tag{22}$$

Interpreted as a VECM, there are three cointegrating relationships between the four variables $\{y_t, c_t, k_t, e_t\}$. w_t is treated as an exogenous variable in the relevant sense (w_t is not equilibrated in the system). In the standard VECM decomposition $\Pi = \alpha\beta'$ where the cointegrating set is $\{y_t, c_t, k_t, e_t\}$ the restrictions on β' are written

$$\begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \\ \beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \tag{23}$$

[8] All these series are I(1).

[9] Lower case indicating logs.

Estimating this system with $p = 1$ results in significant loadings (the $\alpha_{i,j}$) in all equations (Table 4).[10] Restricting the two loadings with p-values in excess of 10% leaves the results essentially unchanged.

	Loadings					
	y			c		
	$\alpha_{1,1}$	$\alpha_{1,2}$	$\alpha_{1,3}$	$\alpha_{2,1}$	$\alpha_{2,2}$	$\alpha_{2,3}$
estimate	−0.130	−0.056	0.049	−0.128	−0.056	0.126
p-value	0.04	0.03	0.14	0.06	0.05	0.00
	k			e		
	$\alpha_{3,1}$	$\alpha_{3,2}$	$\alpha_{3,3}$	$\alpha_{4,1}$	$\alpha_{4,2}$	$\alpha_{4,3}$
estimate	−0.006	−0.000	0.003	−0.077	−0.004	0.033
p-value	0.08	0.87	0.06	0.00	0.62	0.00
Restricted estimates: $\chi^2_3 = 2.43$						
	y			c		
	$\alpha_{1,1}$	$\alpha_{1,2}$	$\alpha_{1,3}$	$\alpha_{2,1}$	$\alpha_{2,2}$	$\alpha_{2,3}$
estimate	−0.091	−0.042	–	−0.128	−0.056	0.127
p-value	0.11	0.08	–	0.07	0.06	0.00
	k			e		
	$\alpha_{3,1}$	$\alpha_{3,2}$	$\alpha_{3,3}$	$\alpha_{4,1}$	$\alpha_{4,2}$	$\alpha_{4,3}$
estimate	−0.006	–	0.003	−0.076	–	0.031
p-value	0.08	–	0.05	0.00	–	0.00

Sample 1971Q2 to 2015Q4.

In $\alpha_{i,j}$ for i = 1, 2, 3 and 4 refer respectively to the ECMs for output, consumption, capital and employment; and j = 1, 2 and 3 the residuals from the consumption to output ratio, capital to output ratio and the labour share.

Table 4: Estimated loadings in a VECM

If the great ratios were time-invariant, then a VECM constituting these series and additionally real wages w_t should reveal the same three cointegrating vectors as in equation 5.2. An unrestricted VAR selected a VECM lag of $p = 1$ (following AIC and SIC) and the Johansen trace and eigenvalue tests both suggest one cointegrating vector exists at the 5% level. The vector has no obvious interpretation and the hypothesis of zero coefficients on w and e cannot be rejected. This rejects the constant great ratio hypothesis.

[10] There is no evidence of autocorrelation although there is non-normality, driven by some large outliers.

6 Conclusions

There is a long-standing belief that the economy is characterised by stable great ratios which are consistent with balanced growth models including the steady-states of DSGE models. Yet there is widespread evidence in macroeconometric and forecasting models for parameter variation and structural change. We use a benchmark DSGE model to explore how plausible variation in structural parameters leads to variation in the great ratios similar to the one found in UK data. This motivates the use of a frequentist nonparametric methodology for allowing time variation using persistent but bounded random coefficients to identify trends and estimate cointegrating relationships in these series. Generally, we find no evidence for cointegration where parameters are constant, but strong evidence when we explicitly allow for time variation. Moreover, the estimated relationships are informative with respect to a set of key macroeconomic variables. This implies that practical macroeconometric models could be built allowing for this variation, including forecasting applications.[11]. Moreover, the clear implication is that DSGE models assuming constant deep parameters are unable to correctly represent the data, a conclusion also drawn by Franchi and Jusélius (2007). In this paper, we have not attempted to estimate the time varying DSGE parameters; rather, the purpose of our paper was to demonstrate the need to relax the restrictive assumption of parameter constancy and explicitly allow for time variation in order to improve the data fit of structural models.

[11]See Clements (2016) for an example where great ratios are used to improve long-run forecasts *via* exponential tilting.

A Appendix: proofs

Proof of Proposition 1 We sketch a derivation of the asymptotic distribution of CI. We consider the proof of Theorem 1 of Shin (1994). Note that

$$T^{-1/2}S_{[Tr]} = T^{-1/2}\sum_{j=1}^{[Tr]}\hat{u}_j = T^{-1/2}\sum_{j=1}^{[Tr]}u_j - T^{-3/2}\sum_{j=1}^{[Tr]}x_j T\left(\hat{\beta}-\beta\right)$$

The time varying version of that is

$$T^{-1/2}h^{1/2}S_{[Tr]} = T^{-1/2}h^{1/2}\sum_{j=1}^{[Tr]}\hat{u}_j = T^{-1/2}h^{1/2}\sum_{j=1}^{[Tr]}u_j - T^{-3/2}\sum_{j=1}^{[Tr]}x_j Th^{1/2}\left(\hat{\beta}_j-\beta_j\right).$$

But $T^{-1/2}h^{1/2}\sum_{j=1}^{[Tr]}v_{1j} = O_p\left(h^{1/2}\right) = o_p\left(1\right)$. So we focus on $T^{-3/2}\sum_{j=1}^{[Tr]}x_j Th^{1/2}\left(\hat{\beta}_j-\beta_j\right)$. We have by Phillips et al. (2017) that for $j=[Tr]$,

$$Th^{1/2}\left(\hat{\beta}_j-\beta_j\right) = Th^{1/2}\left(\hat{\beta}_{[Tr]}-\beta_{[Tr]}\right) \Longrightarrow \Delta_r^{-1}\Gamma_r$$

where

$$\Delta_r = B_{x,r}^2$$
$$\Gamma_r = \sqrt{2}B_{x,r}\int_{-1}^{1}K(s)dB_{y,(s+1)/2}^{*}$$

So

$$Th^{1/2}\left(\hat{\beta}_j-\beta_j\right) = Th^{1/2}\left(\hat{\beta}_{[Tr]}-\beta_{[Tr]}\right) \Longrightarrow \sqrt{2}B_{x,r}^{-1}\int_{-1}^{1}K(s)dB_{y,(s+1)/2}^{*}$$

So

$$T^{-3/2}\sum_{j=1}^{[Tr]}Z_j Th^{1/2}\left(\hat{\beta}_j-\beta_j\right) = T^{-1}\sum_{j=1}^{[Tr]}\frac{Z_j}{T^{1/2}}Th^{1/2}\left(\hat{\beta}_j-\beta_j\right) \Longrightarrow \sqrt{2}\int_0^1 B_{x,r}\left(B_{x,r}^{-1}\right)\left[\int_{-1}^{1}K(s)dB_{y,(s+1)/2}^{*}\right]dr = \sqrt{2}\int_{-1}^{1}K(s)dB_{y,(s+1)/2}^{*} \equiv Q$$

Then,

$$T^{-2}h\sum_{j=1}^{T}S_j^2 = T^{-1}\sum_{j=1}^{T}\left(T^{-1/2}h^{1/2}S_j\right)^2 \Longrightarrow Q^2$$

proving the result.

B Appendix: data definitions

The bulk of our data was sourced from the ONS. Below, we provide a list of these series together with their ONS codes.

Variable name	ONS Code
Real GDP	ABMM
Real consumption	HFC1
Real investment	NPQT
Real government spending	NMRY
Real exports	IKBK
Real imports	IKBL
Nominal GDP by expenditure	YBHA
Nominal GDP by income	CGCB
Nominal GDP by output	ABML
Nominal consumption	ABJQ+HAYE
Nominal investment	NPQS
Nominal government spending	NMRP
Nominal exports	IKBH
Nominal imports	IKBI
Employment	MGRZ
Total hours worked	YBUS
Compensation of workers	DTWM
Gross operating surplus of UK firms	CGBZ+DMUQ
GDP deflator	ABML/ABMM

Table 5: Variables and ONS codes

We constructed two measures of labour productivity: GDP per head (ABMM/MGRZ) and GDP per hour worked (ABMM/YBUS). The quarterly growth rates were calculated as the change in the log of each series. For the capital stock, KBUSNH, and capital services, VBUSNH, we used two measures constructed within the Bank of England.[12] Capital stock and capital services per worker were then calculated as KBUSNH/MGRZ and VBUSNH/MGRZ, respectively. Again, the quarterly growth rates were calculated as the change in the log of each series. For the capital to output ratios, we needed nominal series for capital stock and capital services. To obtain a price index for capital, we used the implicit deflator for investment spending, ie, NPQS/NPQT. Hence, the nominal capital stock and nominal capital services were calculated as KBUSNH*NPQS/NPQT and VBUSNH*NPQS/NPQT, respectively.

[12] See Oulton and Srinivasan (2003).

The ratios of nominal capital stock to nominal GDP and nominal capital services to nominal GDP were then given by KBUSNH*NPQS/(NPQT*ABML) and KBUSNH*NPQS/(NPQT*ABML), respectively.

The short-run real interest rate was defined as the Official Bank Rate less the annualised quarterly change in the GDP deflator (AMBML/ABMM). A time series for the Official Bank Rate can be found on the Bank of England's website at `https://www.bankofengland.co.uk/boeapps/database/`.

The long-run real interest rates was defined as the ten-year spot real interest rate series derived from Index-linked gilt yields. This data is available on the Bank of England's website at `https://www.bankofengland.co.uk/statistics/yield-curves`.

The labour share is given by compensation of workers divided by nominal GDP by income, ie, DTWM/CGCB. Similarly, capital's share is given by the gross operating surplus of UK firms divided by nominal GDP by income, ie, (CGBZ+DMUQ)/CGCB.

For our great ratios, we divided each of nominal consumption, investment, government spending, exports and imports by nominal GDP by expenditure. That is, we defined the consumption to output ratio by (ABJQ+HAYE)/YBHA, the investment to output ratio by NPQS/YBHA, the government spending to output ratio by NMRP/YBHA, the export to output ratio by IKBH/YBHA and the import to output ratio by IKBI/YBHA.

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