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Bilateral Trading in Networks

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Bilateral Trading in Networks*

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Abstract

We study an incomplete-information model of sequential bargaining for a single object, with the novel feature that agents are located in a network. In each round of trade, the current owner of the object either consumes it or makes a take-it-or-leave-it offer to some connected trader. Traders may buy in order to consume or to resale to others. We show that the equilibrium price dynamics is non-monotone and that traders that intermediate the object arise endogenously and attain a profit. We also provide insights on how traders' equilibrium payoffs depend on their location in the network.

Keywords: Bargaining, Bilateral Trading, Asymmetric Information, Networks.

JEL Codes: C78, D82, D85

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1 Introduction

We investigate a model of sequential bargaining with incomplete information and the distinctive feature that agents are embedded in a network of trading relationships. There is one indivisible object for sale, initially owned by one of the agents. All traders are risk-neutral and have a high or low monetary valuation for the object, which is private information. In each round of trade the current owner of the object makes a take-it-or-leave-it offer to a connected agent of their choosing. The trader who receives this offer either accepts or rejects it. When a trader becomes a new owner, she can either consume the object and the game ends, or the game enters a new round of trade. The number of rounds is finite, the network is commonly known and all actions are observed.¹

The assumption, that each trader is only able to bargain with a subset of the population, captures the existence of heterogenous match-specific characteristics that may affect the feasibility of exchange. These may include, among others, transaction costs, bonds of trust, reputation, or other legal or geographical constraints. The study of a model with limited bargaining opportunities and resale is of interest for two reasons. First, it captures some realistic aspects of the trade in wide arrays of markets, ranging from over-the-counter markets to markets for artworks and collectibles.² Second, it contributes to our understanding of micro-mechanisms of price formation.

The set of weak-Markov equilibria that we characterize has a simple structure (Proposition 1 and 2). A high-value trader acquires the object to consume it; hence, in equilibrium there are no arbitrage opportunities for high value traders. When a low-value trader acquires the object, she engages in a sequence of offers, until the object is sold. All her offers, but the last, come at prices that only high-value traders are willing to accept. We refer to these offers as *consumption offers* because, once accepted, the object is consumed. Unless there is no time left, consumption offers are always followed by an offer that is accepted by both the high-value and low-value trader. We refer to these offers as *resale offers*, because they come at price equal to the resale value of the low-value trader who receives it. This latter feature implies that low-value traders also have no arbitrage opportunities.

We call *dealers* those traders who get at least one resale offer with positive probability. We call *clients* those traders who obtain only consumption offers. Low-value traders make zero profit. High-

¹We derive the main results under the assumption that agents do not discount the future, but all the results are robust to the introduction of discounting (Section 5.1).

²Financial products such as foreign currencies, swaps, forward rate agreements, and exotic options are almost always traded in over-the-counter markets. These securities are subject to counter-party risk and therefore, bonds of trusts between firms are particularly important and naturally give raise to trading networks. See Allen and Babus (2009) for a survey on networks in financial markets.

value traders make a positive profit if, and only if, they are dealers. Dealers arise endogenously, depending on the network architecture and on the pattern of expected demand. Roughly speaking, traders on the periphery of the network, and traders who have a high expected value, become clients. Traders who provide access to valuable areas of the network become dealers and earn a rent on their location. This result provides a micro-foundation for the prominent sociological theory of structural holes.³

The aforementioned equilibrium properties allow us to prove two main results. Firstly, we show that the equilibrium sequence of demanded prices is non-monotone in time (Proposition 3). While resale offers follow a decreasing trend, consumption offers spike above subsequent and earlier resale offers. Resale offers are decreasing in time because, as time passes, all traders become more pessimistic about the total expected demand in the network. Refused consumption offers represent bad news about the value of the object; as a consequence the value of the object (conditional on still being on the market) must decrease in time. Prices in consumption offers spike, as sellers are attempting to exploit their local market power and to capture the demand of high-value traders to whom they have direct access.

Secondly, we provide insights on how the payoff of different dealers is affected by their location in the network and in the actual flow of trade (Proposition 4). Our main result is that, in general, traders who become dealers earlier obtain a payoff advantage over later dealers. Despite earlier dealers purchasing the object at a higher price than later dealers, they acquire the good with an higher probability. The second effect dominates the first. The decline in price only incorporates the reduction in the expected demand of traders that have rejected consumption offers. The decline in the probability of receiving a resale offer also incorporates the probability that earlier dealers may have high-value and consume the object. A consequence of this is that if a trader is essential in connecting another trader to the initial owner, than the former obtains a higher expected payoff (provided they have the same type).⁴

We finally discuss the efficiency properties of equilibria. When traders' valuations are public information, efficiency is obtained regardless of the trading network (Proposition 5). When the initial seller is connected to all traders, in our simple binary type environment, ex-post efficiency is obtained regardless of the prior information about traders' evaluation (Proposition 6). However, when asymmetric information is combined with limited bargaining possibilities, a seller will sometimes

³See Burt (1992). The theory of structural holes documents the phenomenon that individuals who have bridging position in an economic network tend to obtain a payoff advantage.

⁴A trader i is essential to connect j to the initial owner if trader i lies in every path connecting j to the initial owner.

insist on a high price when she is connected to some high-expected demand trader who, incidentally, provides access to a low-expected demand area of the network. If a high-price is demanded, the low-value trader will be unable to acquire the object and resell it, giving rise to inefficiencies. In spirit of the Coase conjecture, the presence of a deadline provides some commitment power to the seller, and this is key for the source of inefficiency that we have emphasised. Section 5.1 provides an example where the equilibrium outcome becomes ex-post efficient as the number of rounds grows large and the discount rate goes to zero.

Section 2 introduces the model. Section 3 develops a simple example that illustrates the main economic insights of this paper. Section 4 presents our main results. We first present our equilibrium characterization, we then discuss price dynamics (section 4.1), equilibrium payoff ranking (section 4.2), and efficiency (section 4.3). Section 5 checks the robustness of our results to some of the assumptions. Section 5.1 discusses discounting, section 5.2 multiple types, and section 5.3 analyzes a simple network formation game. Section 6 concludes the main text. Appendix A discusses multiplicity of equilibria and Appendix B contains all the proofs. Before proceeding, we review the relevant literature.

1.1 Review of the literature.

Sequential bargaining with asymmetric information between two agents has been extensively studied (see, for example, Fudenberg and Tirole (1983), Sobel and Takahshi (1983), Cramton (1984)). A main message in this literature is that, when the seller makes all the offers, the price at which the object is offered declines over time because, as offers are rejected, the seller becomes more pessimistic about the buyers' evaluation. Our price dynamic is more articulate and it is driven by the combination of asymmetric information, market incompleteness and opportunities for resale. The decline in the price of resale offers, which reflects the tendency of traders to become more pessimistic, is combined with the incentive of sellers to use their trading connections to screen potential buyers, creating spikes in the price demanded for the object. We also show that the combination of asymmetric information, and the incompleteness of trade relationships, may endogenously generate a set traders– essential traders –who block the flow the object. To our knowledge, this form of inefficiency is novel in the bargaining literature.

There a number of papers that study the effect of resale opportunities on market outcomes under incomplete information.⁵ Calzolari and Pavan (2006) study revenue maximizing mechanisms when a monopolist can sell to a single buyer, who in turn can resell to a third party. Zheng (2002)

⁵See Jehiel and Moldovanu (1999) for an analysis of resale markets with complete information and externalities.

investigates the design of a seller-optimal mechanism when winning bidders can attempt to resell the good to the other bidders. Haile (2003) considers a two-stage model in which an auction in the first stage is followed by a resale auction, held by the first-stage winner.⁶ Our model sidesteps many difficulties that are present in these earlier works by considering a simpler informational structure, with two values and common support. On the other hand, we achieve greater generality in modelling the resale market by allowing for an arbitrary trading network. This allows us to investigate novel questions, such as how the location of a trader in the trading network affects her ability to obtain rents.

Finally, our work relates to the emerging literature on markets and networks. A central and open issue in this literature is the understanding of the role of connections in determining the terms of trade and the efficiency of a market. Most of the existing work has focused on the exchange of goods in buyer-seller networks with random matching, in an environment with complete information and absence of resale possibilities (e.g., Calvo-Armengol (2003), Coromina-Bosch (2004), Manea (2011), Polanski (2007)). Kranton and Minehart (2001) consider a buyer-seller network where buyers's valuation is private information, where no resale takes place. Blume et al. (2009) study a complete-information model buyers and sellers connected via intermediaries. Nava (2009) develops a static model of Cournot competition in trading networks under complete information. Gale and Kariv (2007) analyse dynamic exchange in a network under the assumption of complete information.⁷ Finally, Gofman (2011) studies a reduced-form model of bargaining in over-the-counters markets, which are modeled as trading networks. We share with this literature the general approach of using networks to model markets, and similar research questions, such as how the location of a trader in a network affects her payoffs. We contribute to this literature by developing a model that incorporates simultaneously two important features of many markets: asymmetric information and possibility of resales.

2 Model

The economy consists of a set of *traders*, $N \equiv \{1, \dots, n\}$, and two goods, *money* that everyone owns in large quantity, and a single indivisible *object* initially owned by trader 1. Each trader i is risk-neutral and has a binary private monetary evaluation for the good, v_i , normalized to be either zero or one. In our baseline model we assume that traders do not discount the future. Therefore, regardless of the timing, if i consumes the object and pays p_i his utility is $v_i - p_i$. If i does not

⁶See also Garratt and Troger (2006) and Krishna and Hifalir (2008).

⁷See also Gale and Kariv (2009) for a related experiment on trading in networks.

consume the object but pays p_i his utility is $-p_i$. Traders are ex-ante heterogeneous and values are independently distributed. The *common prior* probability that $v_i = 1$ is $\pi_i \in (0, 1)$.

A *trading network* is a graph $G = (N, E)$, where vertices N represent traders, and edges $E \subseteq 2^{N \times N}$ trading relationships. The existence of an edge ij in E indicates that traders i and j can trade. We consider undirected and connected trading networks.⁸

The game consists of a finite number T of rounds of trade. An arbitrary round t develops in three stages and it is illustrated in Figure 1:

- 1st.* The current owner of the object s either (a) makes a *take-it-or-leave-it* offer at price p to one of her neighbors i , or (b) makes no offer and *waits*. If she makes no offer, the game enters the third stage of round t . Otherwise the game proceeds to the second stage.
- 2nd.* Trader i decides whether to *accept* or to *reject* the offer. In case of rejection the game proceeds to the third stage. In case of acceptance trader i becomes the new owner and she transfers an amount p of money to the seller. The game proceeds to the third stage.
- 3rd.* The current owner of the object decides whether to *consume* the object or *not consume*. The game ends if the object is consumed. Otherwise, unless $t = T$, the game proceeds to the first stage of round $t + 1$.

We assume that all actions are observed by all traders, and that everything but the private values is common knowledge. The triple $\Gamma = \langle G, \pi, T \rangle$ represents a *network trading game*, which is a multi-stage extensive form games with observed actions and independent types. In this setting with common prior, independent types and observed action, a system of beliefs specifies, for all non terminal public histories h , a profile of common posterior probabilities $\boldsymbol{\mu}(h) = (\mu_1(h), \dots, \mu_n(h))$, where $\mu_i(h)$ indicates the probability that player i has value one. We will often write $\boldsymbol{\mu}^t = \langle \boldsymbol{\mu}_{-i}^t, \mu_i^t \rangle$ for the profile of beliefs at the beginning of a round t , omitting reference to the particular history.

The adopted solution concept is perfect Bayesian equilibrium (PBE). A perfect Bayesian equilibrium is a strategy profile and a belief system such that the strategies are sequentially rational given the belief system, and the belief system is consistent with Bayesian updating whenever possible (see Fudenberg and Tirole (1991)).

The following analysis is developed under the assumptions that there are only two types, there is no-discounting and the game consists of a finite number of rounds. We emphasize that the

⁸A network is *undirected* if $ij \in E$ if and only if $ji \in E$. A *path* between i to j in G is a non-empty graph where the set of vertices is $\{i, b_1, \dots, b_m, j\} \subseteq N$ and the set of edges is $\{ib_1, b_1b_2, \dots, b_mj\} \subseteq E$. A network is *connected* if there is a path between every pair of traders.

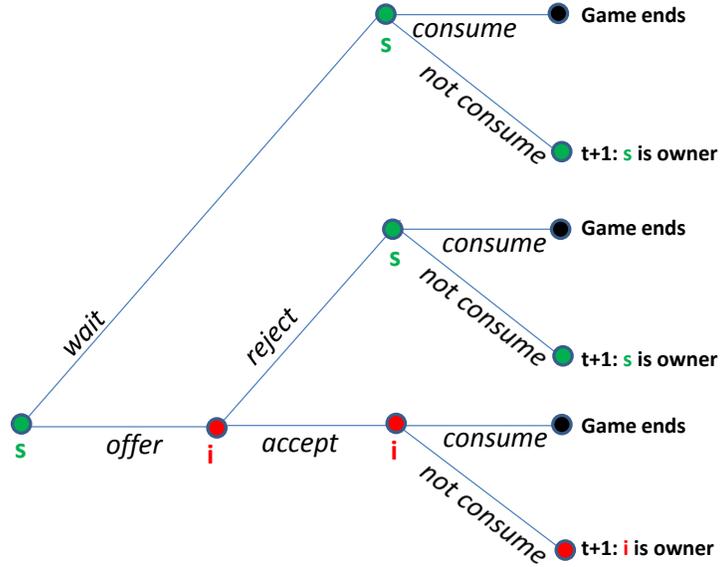


Figure 1: Timing within a period of trade t .

assumption that agents do not discount the future has no main qualitative implications for the equilibrium behavior. In section 5 we discuss formally the implications of relaxing the various restrictions of the model.

3 Illustrative Example

We develop a simple example to illustrate general equilibrium properties of network trading games. The example considers the trading network G depicted in figure 2. There are $n = 5$ traders and $T = 4$ rounds of trade. Trader 1 is the initial owner and the profile of initial beliefs is $\pi = (0, 1/3, 1/2, 1/3, 2/3)$.

3.1 Equilibrium with complete information.

As a benchmark, we briefly discuss the case of complete information. Assume that the profile of values is given and is common knowledge. In this case, all subgame-perfect equilibrium outcomes are Pareto efficient. Furthermore, if there is at least a trader with value one, the object is traded at price 1 and the initial seller extracts all the surplus. For instance, assume that trader 5 has high value and all other traders have low value. In equilibrium the good flows from trader 1 to trader 5, via trader 2 and trader 4, each transaction occurs at price 1, and trader 5 consumes the object.

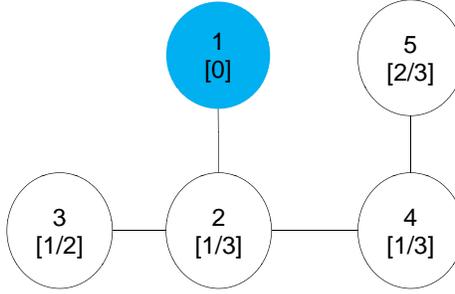


Figure 2: Four rounds, trader 1 is the owner.

3.2 Equilibrium under incomplete information.

Consider the incomplete information case. Recall that $\pi = (0, 1/3, 1/2, 1/3, 2/3)$. Denote p_j^i the price that trader i demands to trader j . The profile of beliefs at t is denoted by μ^t .

Trader 1 has low-value and therefore she sells the object to trader 2. To determine the price that trader 1 asks we need to determine trader 2's strategy when she receives an offer from 1 in the first round of trade.

First, suppose that 2 is a low-value trader. In this case the only reason for purchasing the object is to resale it to other traders. Hence, 2's willingness to pay is the expected payoff that she obtains in the continuation game in which she owns the object, there are three rounds of trade left, and the beliefs are $\mu^2 = (0, 0, 1/2, 1/3, 2/3)$.⁹ We term this payoff trader 2's *resale value*, we denote it by r_2 , and we derive it next.

In the continuation game, illustrated in figure 3(a), there is only one equilibrium. Trader 2 asks price $p_3^2 = 1$ to trader 3, who accepts the offer if she has high-value and rejects it otherwise.¹⁰ In case of rejection, trader 2 makes an offer to 4 at a price which is equal to trader 4's resale value, that is $r_4 = 2/3$. Trader 4 accepts this offer regardless of her value (the reason is explained in the next paragraph). Once trader 4 becomes owner she consumes the object if she has high-value. Otherwise, she asks a price of $p_5^4 = 1$ to trader 5 in the last round of trade. Trader 5 accepts the offer only if she has high-value. We obtain:

$$r_2 = \Pr[3 \text{ accepts } p_3^2] \times p_3^2 + \Pr[3 \text{ rejects } p_3^2] \times \Pr[4 \text{ accepts } r_2] \times r_4 = \frac{1}{2} + \frac{1}{2} \times \frac{2}{3} = \frac{5}{6}$$

Second, suppose that trader 2 has high-value, and take as given the strategy of trader 2 when

⁹Since, in equilibrium, high-value traders always consume the object, once trader 2 makes an offer, it becomes common knowledge that she has a low value, i.e., $\mu_2^2 = 0$

¹⁰If trader 3 has high consumption value she accepts any offer up to one (and then consumes the object) because, by rejecting, she will never receive another offer in the remaining rounds of trade.

she has low-value. By rejecting a price below her resale value, trader 2 signals that she has high-value. In this case her expected payoff is zero because, whenever trader 1 is certain that trader 2 has value one, trader 1's optimal strategy is to wait until the last round and ask a price of one to trader 2. Next, consider an offer which is above trader 2's resale value r_2 . Given trader 2 low-value strategy, if trader 2 rejects an offer above r_2 all traders will believe that the probability that trader 2 has value one lies somewhere in the interval $[0, 1/3]$. Hence, the expected payoff that trader 2 obtains by rejecting that offer is the expected payoff that high-value trader 2 obtains in the continuation game where trader 1 is the owner, there are three rounds of trade left, and beliefs are $\mu^2 = (0, \mu_2^2, 1/2, 1/3, 2/3)$, for some $\mu_2^2 \in [0, 1/3]$. This payoff, which we denote $V_2(\mu_2^2)$, is derived as follows.

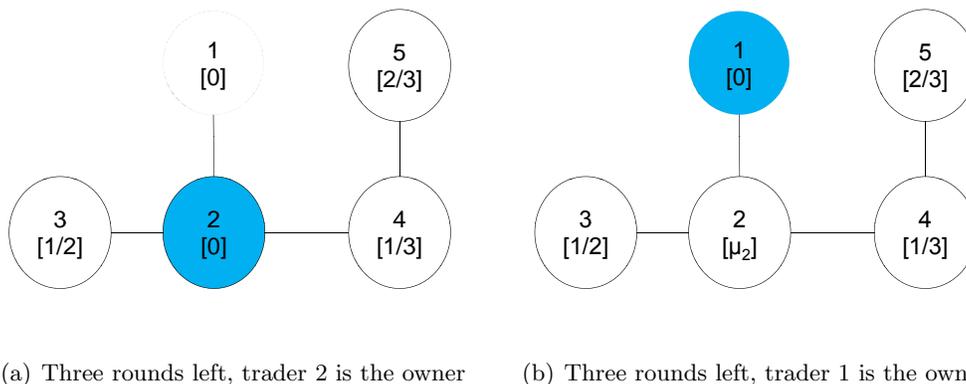


Figure 3: Two different continuation games.

In the continuation game, illustrated in figure 3(b), there is only one equilibrium payoff for trader 2. Trader 1 asks for trader 2's resale value, which is $2/3$. Trader 2 accepts the offer regardless of her type and consumes the object if she has high value. Otherwise she asks a price of 1 to trader 3 and then to trader 4, and each of these offers is accepted only by high-value traders. Concluding, if high-value trader 2 rejects a price above her resale value r_2 , she will buy the object in the subsequent round at a price of $2/3$ and will then consume, obtaining a net payoff of $V_2(\mu_2^2) = 1/3$, for all $\mu_2^2 \in [0, 1/3]$.

It is now possible to verify that a high-value trader 2 strictly prefers to reject every price above $r_2 = 5/6$, because rejection would allow her to buy at a lower price in the subsequent round. Because trader 2 will reject any offer above $5/6$ regardless of her value, and because waiting one round provides player 1 with a payoff of $2/3$, we can conclude that in equilibrium trader 1 will make an offer to trader 2 at price $5/6$.

Summary of the Equilibrium Path. In the first period trader 1 offers the object to 2 at her resale value $5/6$. Trader 2 purchases the object and consumes it if she has high-value. Otherwise, in the second period of trade, trader 2 asks price 1 to trader 3. Trader 3 accepts the offer and consumes the object if and only if she has high-value. Otherwise, trader 2 remains the owner and, in the third period of trade, she offers the object to 4 at trader 4's resale value, which is $2/3$. Trader 4 buys the object regardless of her value. Finally, if trader 4 has a high-value she consumes the object, otherwise, in the last round of trade, she offers the good to trader 5 at a price one. Trader 5 accepts the offer and consumes the good if she has high-value, otherwise the game ends.

There are four important properties that emerge from the description of the equilibrium path, which we now discuss in turn. As we shall show in the next section, these are robust equilibrium properties of network trading games.

Types of offers. There are two types of offers along the equilibrium path: *resale offers* and *consumption offers*. A resale offer is at a price that is accepted both by a high and low-value trader. Trader 2 receives a resale offer in period 1 and trader 4 receives a resale offer in period 3. A consumption offer is at a price that makes a high-value trader indifferent between accepting and rejecting the offer, whereas a low-value trader strictly prefers to reject the offer. Trader 3 receives a consumption offer in period 2 and trader 5 receives a consumption offer in period 4.

Types of traders. There are two types of traders in equilibrium: *clients* and *dealers*. A client receives only consumption offers. Trader 3 and trader 5 are clients. A dealer receives a resale offer with positive probability. Trader 2 and trader 4 are dealers.

Price Dynamics. Consider the sequence of equilibrium offers $(r_2, p_3^2, r_4, p_5^4) = (5/6, 1, 2/3, 1)$. Figure 4 illustrates the pattern of prices over time; the bold points in the figure are the prices of resale offers, the other points are the prices of consumption offers (the point at time zero is trader 1's resale value). The prices associated to resale offers are declining along the sequence of equilibrium offers. This reflects the fact that later dealers are more pessimistic about the profitability of selling the object. However, the price sequence is in general non-monotonic, because each consumption offer is at a price which is higher than every price asked in future resale offers. This reflects the idea that dealers are able to exploit their local market power against some of their directly connected traders.

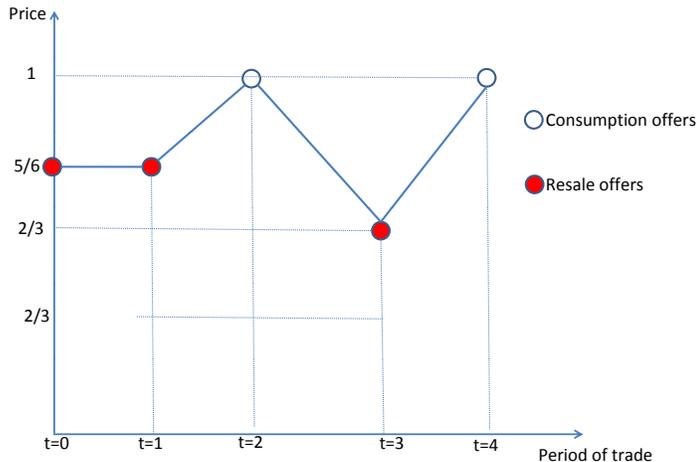


Figure 4: Price Dynamics.

Payoff ranking among traders. Clients and low-value dealers earn zero profits. Dealers with high-value obtain positive expected profits. Furthermore, earlier dealers obtain a higher expected payoff than later dealers. Despite earlier dealers acquire the object at a higher price than later dealers, the former have a higher probability of acquiring the good than the latter. The second effect dominates the former because the decline of the price associated to resale offers incorporates only the decline in the expected demand due to the rejection of consumption offers. Whereas the difference in the probability of acquiring a good also account for the possibility that dealers consume the good themselves. In our example, high-value trader 2 would acquire the object for sure at price $5/6$, obtaining a payoff of $1/6$. High-value trader 4 would acquire the object with probability $1/6$ and, in that event, she would pay a price of $2/3$, obtaining an expected payoff of $1/6(1 - 2/3) = 1/9$.

We conclude with two remarks. First, the equilibrium play with incomplete information generates a number of features that are in stark contrast with the equilibrium properties under complete information. Notably, with asymmetric information the sequence of demanded prices are not constant and dealers obtain a positive profit. Second, in this example the equilibrium outcome is ex-post efficient, i.e. the object is consumed by a high-value trader, if any exists. This does not hold in general and section 4.3 provides general insights on efficiency in these environments.

4 Characterization of Equilibria

We focus on weak-Markov equilibria (see Fudenberg, Levine and Tirole (1983)). That is, we restrict attention to equilibria where two conditions hold. First, the acceptance strategy of a buyer, $a_j^t(p, \boldsymbol{\mu}^t | v_j)$, and the subsequent decision to consume or not, depends only on the price asked, the round of the game, the state of beliefs, and the private information of the buyer. Second,

the offer made by a seller in round t depends only on the round of the game, the state of beliefs, the private information of the seller and, in addition, the offer that she has made in round t .¹¹

Our characterization is recursive. We characterize equilibrium strategies at an arbitrary period t by taking as given continuation payoffs accruing from period $t + 1$ onward. We denote by $V_i^{t+1}[s, (j, p), \boldsymbol{\mu} \mid v_i]$ the continuation payoff that trader i with value v_i obtains in the equilibrium of the continuation game which starts in round $t + 1$ following an offer to j at price p and where trader s is the seller and the profile of posterior beliefs is $\boldsymbol{\mu}$. We write $R_i^{t+1} = V_i^{t+1}[i, (i, p), (\boldsymbol{\mu}_{-i}^t, 0) \mid 0]$ and call it *resale value* of i in round $t + 1$. In words, this is the equilibrium expected payoff to low-value trader i when she is the seller at period $t + 1$, having accepted an offer at price p in t . We also write, for a generic $\mu_i \in [0, 1]$, $V_i^{t+1}(\mu_i) = V_i^{t+1}[s, (i, p), (\boldsymbol{\mu}_{-i}^t, \mu_i) \mid 1]$. In words, $V_i^{t+1}(\mu_i)$ is the equilibrium expected payoff to high value trader i when she refuses an offer at a price of p in round t and this refusal induces public beliefs μ_i . Because the game ends at T we let $V_i^{T+1}[\cdot \mid \cdot] = 0$ for all $i \in N$.

Proposition 1. *Every weak-Markov PBE satisfies the following conditions.*¹²

LOW-VALUE TRADER. *In every round t , whenever applicable, a low-value trader:*

1-L. *Makes an offer (j, p) that maximizes:*

$$E_{v_j}[a_j^t(p, \boldsymbol{\mu}^t \mid v_j)]p + (1 - E_{v_j}[a_j^t(p, \boldsymbol{\mu}^t \mid v_j)])V_i^{t+1}[i, (j, p), \boldsymbol{\mu}^{t+1} \mid 0]. \quad (1)$$

2-L. *Accepts an offer at price p if $p \leq R_i^{t+1}$ and rejects otherwise.*

3-L. *Does not consume the object.*

HIGH-VALUE TRADER. *In every round t , whenever applicable, a high-value trader:*

1-H. *Does not make an offer and waits.*

2-H. *Accepts an offer at price p if $p \leq R_i^{t+1}$, while otherwise plays one of the following:*

- *accepts the offer if $p \leq 1 - V_i^{t+1}(0)$;*

¹¹It is well known that strong Markov equilibria do not always exist in sequential bargaining games with incomplete information. This happens because it may be necessary for the probability of acceptance of some buyer to be constant over some interval. Hence the seller posterior will be the same after an offer in such interval is refused; but for the probability of acceptance to be constant, the seller next offer will have to depend on the current one.

¹²In point 1.L, 3.L, 1H, and 3.H we are selecting some equilibria by breaking indifferences in a special way. This simplifies our narrative, but has no effect on the set of equilibrium payoffs. For example, in 3-H if the resale value of a high value trader is 1, then that trader is indifferent between consuming or selling the object.

- rejects the offer if $p \geq 1 - V_i^{t+1}(\mu_i^t)$;
- accepts with probability λ if there exists a $\lambda \in (0, 1)$ such that

$$p = 1 - V_i^{t+1}(\mu_i^{t+1}), \text{ where } \mu_i^{t+1} = \frac{(1 - \lambda)\mu_i^t}{1 - \lambda\mu_i^t}.$$

3-H. Consumes the object.

The first part of proposition 1 characterizes the equilibrium behavior of low-value traders. A low-value trader only acquires the object in order to resell it. Hence, her willingness to pay equals her resale value (2-L and 3-L). Because the willingness to pay of high-value traders is at least equal to the resale value, along the equilibrium no seller makes an offer at a price below the resale value. Hence, a low-value trader resales the object at a price that maximizes her expected revenues, taking into account that once the offer is accepted her continuation payoff is zero (1-L).

The second part of the proposition characterizes equilibrium strategies of high-value traders. Points 1-H and 3-H state that high-value traders always consume, because they cannot profit from selling the good even though it may be possible for them to reacquire it later at a lower price. In equilibrium there are no arbitrage opportunities. The price differential that a high-value trader may accrue by selling today and buying in the future will not compensate her for the expected loss suffered in case of consumption by some other trader.

2-H describes the equilibrium acceptance strategy of an high-value trader i . Rejecting an offer below the resale value cannot be a best reply for trader i , as that would signal that she has high-value for the object. Consider offers at a price p greater than the resale value of trader i . Recall that if trader i had a low-value, she would reject such offer. In a separating equilibrium, high-value trader i accepts the offer at p . In this case, a rejection would signal that trader i has low-value and trader i would obtain a continuation payoff of $V_i^{t+1}(0)$. Hence, accepting is compatible with equilibrium play only if the payoff from acquiring the object and consuming it, which is $1 - p$, is above $V_i^{t+1}(0)$. In contrast, in a pooling equilibrium, high value trader i would reject the offer. In this case, by rejecting the offer, the beliefs about trader i remain unchanged and his expected payoff is $V_i^{t+1}(\mu_i^t)$. Hence, for pooling to be an equilibrium, it has to be the case that $1 - p$ is below $V_i^{t+1}(\mu_i^t)$.

When $V_i^{t+1}(\mu_i^t) < V_i^{t+1}(0)$ and trader i receives an offer at a price in $(1 - V_i^{t+1}(0), 1 - V_i^{t+1}(\mu_i^t))$, there is no equilibrium where high value trader i plays a pure strategy. In this case, high value trader i randomizes between acceptance and rejection in such a way that the continuation equilibrium payoff that she obtains in case she rejects, given Bayesian updating, equals the payoff trader i obtains by accepting and consuming the object, which is $1 - p$.

It is worth noting that when $V_i^{t+1}(\mu_i^t) > V_i^{t+1}(0)$, and when high-value i receives an offer above her resale value and in the range $(1 - V_i^{t+1}(\mu_i^t), 1 - V_i^{t+1}(0))$, both acceptance and rejection are compatible with equilibrium play. This gives rise to multiple equilibria since trader i 's behavior is essentially unrestricted. For example, for any two prices in that range, it is a best reply that trader i rejects the lower price and that she accepts the higher price. This possibility, which is discussed extensively in appendix A, motivates the equilibrium selection approach which is proposed next.¹³

To enhance our characterization we impose a continuity restriction on equilibrium play at “near” information sets. More precisely, we require that at every information set where a high-value trader receives an offer, she plays a pure strategy, whenever compatible with equilibrium play, and that she uses the same pure strategy at two information sets that are equal except for the price asked by the seller. This is expressed more formally below.

Definition 1. *A weak-Markov PBE is in pure-strategies whenever possible (PWP) if the equilibrium strategy played by every buyer satisfies two properties:*

- A. *For any price asked in round t to a given trader a best-reply in pure strategies is played whenever compatible with equilibrium play.*
- B. *If accept (reject) is played in equilibrium at price p such that $p > R_i^{t+1}$, then accept (reject) is played also at any price p' such that $p' > p$, whenever compatible with equilibrium play.*

The next proposition supports our equilibrium selection criterium by showing that for every network trading game there exists a weak-Markov PBE in PWP - hereinafter simply a *PWP equilibrium* - and in some network trading game all equilibria are PWP (e.g. the game studied in section 3 has a unique equilibrium).

Proposition 2.

- 1. *A PWP equilibrium exists in every network trading game and in some network trading game all equilibria are in PWP.*
- 2. *In every PWP equilibrium each offer made to trader i in round t is either:*
 - *A resale offer: an offer at a price which equals the resale value of i ;*

¹³Another source of multiplicity is the fact that sellers may have multiple optimal offers. Appendix A also illustrates how the multiplicity of equilibria is linked to possible indifference that potential sellers may have across offers and how such indifference can be broken with the introduction of transaction costs.

- A consumption offer: an offer at a price higher than trader i 's resale value and that makes high value trader i indifferent between accepting the offer and rejecting the offer, i.e., $1 - p = V_i(\mu_i^{t+1})$, where μ_i^{t+1} are the equilibrium updated beliefs in case of refusal.

The next two subsections describes global properties of PWP equilibria: the equilibrium price dynamics, and how the equilibrium payoff of traders depend on their position in the network and in the history of offers.

4.1 Price dynamics

In a PWP equilibrium the sequence of offers and exchanges develops as follows. Whenever a trader acquires the object she consumes it if she has high-value. Otherwise she makes a sequence of offers until the object is sold. In particular, the owner starts by making a sequence of consumption offers to some of her neighbors. If these offers are rejected she makes a resale offer which is accepted for sure. Hence, all possible public histories of the game can be summarized by a list of consumption offers and resale offers, which eventually ends either because the game reaches time T or because a high-value trader acquires the good and consumes it.

We call *trading chain* any public history of offers that may arise in the event in which all traders have low-value and therefore the game reaches time T . Formally, the trading chain is a finite list $(p_1^1, p_2^1 \dots, r_2^1, p_1^2, p_2^2 \dots, r_3^1, \dots)$, where p_i^s indicates the i th consumption offer made by the s th seller, and r_s^{s-1} indicates the resale offer that the $(s-1)$ th seller makes to the s th seller.

If along the equilibrium path each seller has a *unique* optimal offer, the resale offer that the $(s-1)$ th seller proposes to the s th seller is:

$$r_s^{s-1} = \alpha_1 p_1^s + (1 - \alpha_1) \alpha_2 p_2^s + \dots + \prod_{i=1}^k (1 - \alpha_k) r_{s+1}^s, \quad (2)$$

where α_k indicates the probability that the k th consumption offer made by the s th seller is accepted. In words, the resale value of a trader who acquires the object is equal to the expected sale price.

In general, a seller could be indifferent among different optimal offers, in which case equilibria exist where the seller randomizes. Consequently, the trading chain is not unique, and all offers in the chain follow a stochastic process determined by the sellers' equilibrium strategies. In this case, equation 2 must hold for every realization of the random variables. Otherwise, seller s would be making a suboptimal offer at some point.

The following result provides a sharp description of the dynamics of prices in equilibrium.

Proposition 3. *In every PWP equilibrium:*

1. *the price asked in resale offers is decreasing along the trading chain: for all s , $r_{s+1}^s \geq r_{s'+1}^{s'}$ for all $s' \geq s$;*
2. *every consumption offer is at a price greater than the price asked in every subsequent resale offer: for all s , $p_i^s \geq r_{s'+1}^{s'}$ for all $s' \geq s$;*
3. *For every pair of resale offers $(r_{s+1}^s, r_{s'+1}^{s'})$ with $r_{s+1}^s \neq r_{s'+1}^{s'}$ and $s < s'$, there exists some consumption offer demanded by seller $s + 1$, say p_i^{s+1} , which is strictly higher than r_{s+1}^s .*

To understand the first part of proposition 3 consider two consecutive resale offers, the first at period t and the second at period $t + x$. The trader who receives the resale offer at $t + x$ knows that all consumption offers from period $t + 1$ to period $t + x - 1$ have been rejected. Since when a trader rejects a consumption offer all other traders update downward the beliefs that she has a high consumption value, the trader who receives the resale offer at period $t + x$ is more pessimistic about the profitability of reselling the good than the trader who receives the resale offer at period t . Hence, the price asked in resale offers declines overtime.¹⁴

The result that consumption offers are above the subsequent resale offers reflects the ability of sellers to use their local monopoly to demand a high price to some of their neighboring traders, before passing the object to another dealer. While it is clear that consumption offers to clients, which come at price one, are above all subsequent resale offers, it is less obvious that this is the case also for consumption offers made to dealers. To gain an intuition for this phenomenon let's consider a special case in which a seller s makes a consumption offer to i at price p_i and thereafter a resale offer at price r_j to j who, in turn, makes a sequence of consumption offers and then a resale offer to i at price r_i .

We know that the consumption offer to trader i leaves her indifferent between accepting the offer and rejecting it. That is:

$$1 - p_i = \Pr(E)[1 - r_i] \quad \iff \quad p_i = 1 - \Pr(E) + \Pr(E)r_i$$

where $\Pr(E)$ is the probability that trader i receives her resale offer after refusing the consumption offer. This probability is the probability that trader j does not consume the object, times the probability that all those traders who receive a consumption offers from j do not accept such offer.

¹⁴Another force toward the decreasing of resale values is timing. As the deadline approaches, the opportunities of selling the object necessarily decrease.

We can then conclude that the resale value of agent j , r_j , is bounded from above by $1 - \Pr(E) + \Pr(E)r_i$, as every consumption offer is no higher than 1.¹⁵

The last part of the proposition summarizes possible non-monotonicity in the price demanded over time. It follows by combining the observation that equilibrium resale values decline over time (part 1 of proposition 3) and that, in equilibrium, the resale value of an agent equals the expected price at which she will sell the object, expression 2 holds.

4.2 Payoff ranking

We now discuss the equilibrium payoff of different traders. In order to do that we classify all traders (except the initial owner) into three categories.

Definition 2. *In a given equilibrium we say that a trader is A. inactive if she receives no offers with probability one; B. a client if she is active and the probability of getting a resale offer is zero; C. a dealer if there is a positive probability of obtaining at least one resale offer.*

With this taxonomy in mind the following corollary is a direct consequence of proposition 2.

Corollary 1. *In every PWP equilibrium:*

1. *Every trader (except for the initial owner) is either inactive, client or dealer.*
2. *A trader obtains a strictly positive expected payoff if, and only if, she is the initial owner, or she has high consumption value and she is a dealer.*

Low-value traders make zero expected payoff because no offer is ever made at a price below the willingness to pay of a low-value trader. To see that clients obtain zero profit consider the last offer that a client obtains. By proposition 2 this offer must leave the high-value trader indifferent between accepting and rejecting it, and since it is the last offer, it must come at a price of one. This implies that all previous offers must be at price one as well. Finally, high-value dealers obtain a positive expected payoff because, with some probability, they obtain an offer at their resale value. Since every earlier consumption offer must keep them indifferent between accepting and rejecting, such

¹⁵More formally, let $1 - \mu_j$ be the probability that trader j does not consume the object, and let α_j the probability that all those who obtain consumption offers from j do not accept the offer. Hence:

$$p_i = 1 - \Pr(E) + \Pr(E)r_i = 1 - \alpha_j(1 - \mu_j) + \alpha_j(1 - \mu_j)r_i^{s'} \geq 1 - \alpha_j + \alpha_j r_i.$$

Next, note that $1 - \Pr(E) + \Pr(E)r_i^{s'}$ is an upper bound for r_j , because $r_j \leq 1 - \alpha_j + \alpha_j r_i$.

consumption offers must be at a price strictly below one. It follows that a dealer with high-value will, at the beginning of the game, expect to make a positive profit.

Since dealers are the only traders, other than the initial seller, who make a positive profit, we now examine how the position of a dealer in the equilibrium trading chain affects her payoff. For the sake of discussion, consider the case in which the trading chain is deterministic and take a dealer i who immediately precedes dealer j in the trading chain. When comparing their expected equilibrium payoff, there are two countervailing effects at play. The first effect is that, since the price of resale offers decreases along the trading chain, dealer i buys at a higher price than dealer j . The second effect is that the probability that trader i will buy the good is higher than the probability that dealer j will buy the object. As the following proposition illustrates, these two effects can be compared and, generally, earlier traders obtain a payoff advantage.

Proposition 4. *In every PWP equilibrium:*

1. *If all offers to trader j are preceded with certainty by a resale offer to trader i , then the expected utility of a high-value trader i is greater than that of a high-value trader j*
2. *If all resale offers to trader j are preceded with certainty by a resale offer to trader i and $\pi_i \geq \pi_j$, then the expected utility of a high-value trader i is greater than that of trader j*

The main intuition behind the result is that despite later traders pay lower prices than earlier traders, this price differential does not compensate them for the decrease in the probability of obtaining the offer. The decline in price only offsets the expected demand of clients, but does not incorporate the possibility that dealers may consume the object.¹⁶

Proposition 4 and corollary 1 are silent about the relation between the location of traders in the trading network and their payoffs. The following corollary makes some steps in this direction. We say that trader j is *essential* for trader i if j belongs to every path connecting trader i to the initial owner. A trading network G is a *tree* if there is only one path between every pair of traders. A trader is an *end-trader* if she is connected only to another trader and she is not the initial owner.

Corollary 2. *In every PWP equilibrium*

¹⁶In the example of section 3, trader 2 with value one acquires the object at price 5/6 for sure, while trader 4 with value one buys the object at price 2/3 if both 2 and 3 have value zero (which happens with probability 1/2 times 2/3). So, high value trader 2 obtains a higher profit than high value trader 4. Note that the decrease in resale price that occurs among the two dealers equals the probability that the client who gets the offer in between accepts it. So, the decline in the price does not incorporate the probability that the earlier dealer 2 may consume the object. However, this affects negatively the payoff of dealer 4.

1. *Every end-trader obtains zero profit.*
2. *If trader j is essential for trader i , then high-value trader j obtains a higher expected profit than trader i .*
3. *If the trading network is a tree, then in every path starting from the initial owner the expected payoff of high-value traders in the path declines with their distance from the initial owner.*

The corollary points out the importance of the location of a trader in a trading network. In particular, it emphasizes that traders who are essential in connecting other traders to the initial owner, obtain a payoff advantage. This is an economic micro-foundation of the theory of structural holes, which is a prominent theory in the sociological literature of networks (see Burt (1992)).

4.3 Efficiency

If the profile of values is known, a *feasible* outcome of the game is an allocation of the goods (object and money) to the traders that is achievable within T periods of trade. An outcome is *Pareto efficient* if it is feasible and there is no alternative feasible outcome that would make all traders weakly better off and one trader strictly better off. In this setting, an outcome is Pareto efficient if and only if the object is consumed by a trader with value one, whenever at least one exists.

Under incomplete information, a feasible outcome is a mapping from the set of all possible profiles of values into the set of possible feasible outcomes under complete information. Following Holmstrom and Myerson (1983), we say that an outcome is *ex-post Pareto efficient* if it is Pareto efficient in the classic sense for every profile of values.

In what follows we show that inefficient equilibrium outcomes result from the combination of asymmetric information and incompleteness of trading opportunities. Our first result shows that the fact that traders can only bargain with a subset of other traders does not create inefficiency as long as the values of all traders are commonly known.

Proposition 5. *Assume that values are common knowledge. For every network trading game, all subgame perfect equilibrium outcomes are Pareto efficient.*

In every subgame perfect equilibrium the strategy of a seller is to make an offer at the price of one to a trader who has either value one or that has a path to a trader with high-value and that can be reached in the remaining number of rounds. The strategy of a trader receiving an offer is to accept the offer if the price does not exceed one if, and only if, she has either a high-value or

she has a path to a trader with high-value who can be reached in the remaining number of rounds. Hence, subgame perfect equilibrium outcomes are Pareto efficient.

The result that the architecture of the trading network does not affect the efficiency of equilibria breaks down in the presence of asymmetric information. In short, inefficiencies may take place when some trader i , with high *expected-value*, provides monopolistic access to another trader j , with low *expected-value*. In that case, the object may never reach agent j , even when she is the only trader with a high-value for the object. This happens because when trader i has high enough expected value (relative to the expected value of j) the seller prefers to ask her a price of one rather than her resale value. The ability of the seller to convince trader i to accept a price of one rests on the possibility of making that offer close enough to the deadline. In fact, when the deadline approaches, the seller can credibly threat trader i that if she rejects the offer she will not receive any offer at a lower price in the future. The example below illustrates this point.

Example 1. *There are three traders, trader 1 is the initial owner, who has a link with trader 2, who has a link with trader 3 (i.e. $N = \{1, 2, 3\}$ and $E = \{12, 23\}$). The initial profile of prior beliefs is $\pi = (0, \pi, 1/2)$. Suppose for simplicity that $T = 2$ (nothing would change for $T > 2$). We consider two cases.*

Case 1. $\pi < 1/2$. *When $\pi < 1/2$, there is only one equilibrium in which trader 1 asks the resale value to trader 2, which is equal to $1/2$; trader 2 consumes if she has high value, otherwise she asks a price of 1 to trader 3, who, in case she has high consumption value, accepts the offer and consumes the object. So, even if the initial trader is not connected to all traders, for all profile of initial beliefs $\pi = (0, \pi, 1/2)$ where $\pi < 1/2$, every equilibrium outcome is ex-post efficient.*

Case 2. $\pi > 1/2$ *When $\pi > 1/2$, every equilibrium is payoff equivalent and has the following structure: with probability λ trader 1 waits in the first round and in the second round asks a price of one to trader 2, and with the remaining probability trader 1 asks a price of one to trader 2 in the first period, and if the offer is rejected, trader 1 consumes the object. In both cases, trader 2 accepts the offer of 1 if and only if she has high value. Hence, the equilibrium outcome is ex-post inefficient: trader 3 does not consume the object in the event in which she is the only trader with high-value.*

The inefficiency emphasized by example 1 is typical in network trading games. The next proposition shows that in every situation in which the initial owner cannot bargain directly with all the other traders, inefficiency arises for set of priors which has positive measure.

Proposition 6. *There exists a T^* such that for every $T > T^*$ and for every $\pi \in (0, 1)^{n \times n}$ there is*

at least one ex-post efficient equilibrium outcome of network trading game $\langle G, \pi, T \rangle$ if, and only if, the initial seller is connected to all other traders.

If the initial owner is linked to all other agents, then, as long as the time is large enough, she asks a price of one to each trader, sequentially, and, if they all reject the respective offer, the initial owner consumes the object. The equilibrium outcome is ex-post efficient. However, whenever there is a trader who is essential to connect the initial owner to another set of traders, one can find profiles of initial beliefs which are sufficiently optimistic about the value of the essential trader so that the flow of trade will never reach the traders in latter set, leading to inefficient outcomes.

This may suggest that increasing the connectivity of a trading network would help the emergence of efficient equilibrium outcomes, as connectivity makes each trader "less" essential. The following example shows that this is not always the case.¹⁷

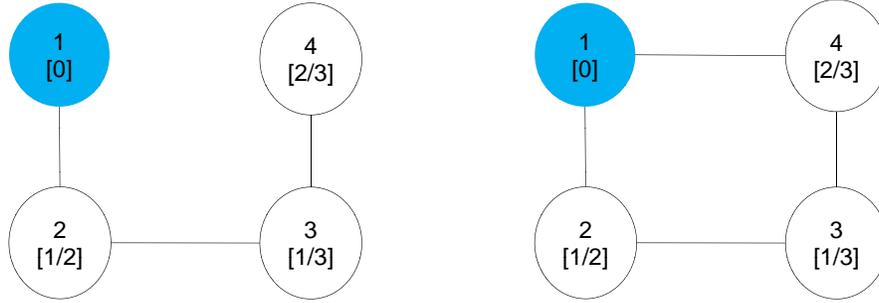
Example 2. *There are $n = 4$ traders and the initial profile of beliefs is $\pi = (0, 1/2, 1/3, 2/3)$. Suppose that $T = 3$, but the conclusion will not change if $T > 3$. First consider the network trading game depicted in figure 5(a). Here, there is a unique equilibrium and the equilibrium outcome is efficient: trader 1 sells to trader 2 at a price of $2/3$, trader 2 consumes if she has high value, otherwise she resells the good for the same price to trader 3, who consumes if she has high value, otherwise she offers the good at a price of one to trader 4. Trader 4 acquires the good and consumes it if she has high-value.*

Consider now the new network obtained by adding a link from trader 1 to trader 4. The new network trading game is depicted in figure 5(b). In this case all equilibrium outcomes are payoff equivalent and in every equilibrium trader 1 will ask a price of 1 to each of his neighbors, and in case they both reject she consumes the object. So, whenever trader 4 is the only trader who has high-value, the equilibrium outcome is inefficient.

5 Extensions

We consider a number of extensions to our model. Section 5.1 shows that our results are robust to the introduction of discounting, and shows that the outcome of the game becomes approximately ex-post efficient when the time-horizon is large and discounting is small. Section 5.2 studies a simple three player and finite-horizon game where there are three types of traders. We highlight

¹⁷This phenomenon is often referred to as *Braess paradox*, following the terminology of the literature on transportation networks.



(a) Three rounds of trade, $T = 3$, and trader 1 is the owner (b) Three rounds of trade, $T = 3$, and trader 1 is the owner.

Figure 5: Braess Paradox in Network Trading Games.

the complications that arise when valuations may take more than two values and we hint that the main qualitative properties of the equilibrium characterization with two types should be robust to the introduction of multiple types. Finally, in section 5.3 we present a simple three player network formation game.

5.1 Discounting

We modify the model by assuming that if trader i consumes the object and pays p within period t his utility is $\delta^{t-1}(v_i - p)$, with $\delta \in (0, 1)$. Similarly, a trader who receives a payment p in round t obtains utility $\delta^{t-1}p$.

The equilibrium characterization of Proposition 1 and Proposition 2 are easily extended to a network trading game with discounting. In fact, adopting the same exact notation, both propositions continue to hold if we discount by δ all continuation payoffs that appear in the statements.¹⁸ With this modification, corollary 1, corollary 2, proposition 3 and proposition 4 are still valid in the case of discounting. Intuitively, discounting will only reinforce the result that resale prices decrease because the expected demand in the network declines, not only because of learning, but also because the object depreciates over time. A similar intuition applies to the result that expected utilities of high-value dealers decline along the trading chain.

The introduction of discounting, however, may decrease the revenue that the initial seller obtains

¹⁸In 1-L $V_i^{t+1}[i, (j, p), \boldsymbol{\mu}^{t+1} | 0]$ is replaced with $\delta V_i^{t+1}[i, (j, p), \boldsymbol{\mu}^{t+1} | 0]$. In 2-L R_i^{t+1} is replaced with δR_i^{t+1} . In 3-H $V_i^{t+1}(0)$ is replaced with $\delta V_i^{t+1}(0)$. Finally in 3-H and in Proposition 2 $V_i^{t+1}(\mu_i^t)$ is replaced with $\delta V_i^{t+1}(\mu_i^t)$.

and it may modify the efficiency properties of equilibrium outcomes. These two effects, the loss of bargaining power for sellers of a durable good, and the tendency toward efficiency of bargaining with large time horizon and small discounting, have been known, for some time, as the Coase conjecture, and have been formally obtained in several models of bargaining (see, for example, Bulow (1982) and Fudenberg, Levine and Tirole (1985)). We now illustrate that these effects are present also in network trading games with discounting.

We develop a three-player example where the initial seller is connected to a middleman, who in turn is connected to a third trader. We know that when the time horizon is finite and traders do not discount the future, if the expected value of the middleman is higher than that of the third trader, then, in every equilibrium, the initial seller waits until the deadline and asks to the middleman a high-price (see example 1, case 2). The presence of a deadline, together with no discounting, allow the initial seller to credibly commit not to sell the good in case of a refusal of her offer. Hence, the initial seller can extract the surplus of the high-value middleman. We have also seen that this is a source of inefficiency, since the third trader may be the only trader with high value. The following proposition shows that when traders discount the future, the initial seller gradually loses her bargaining power against the middleman as the time-horizon gets larger. In particular, when time is large enough, the object is immediately sold at the resale price. This implies that, as discounting goes to one, the equilibrium outcome of the game is approximately ex-post efficient.

Proposition 7. *Consider a network trading game with discounting. Let $N = \{1, 2, 3\}$, $E = \{12, 23\}$ and $\pi_1 = 0$. For any $1 < \delta < 1$ and $\pi_2 > \delta\pi_3$ there exists T^* such that for any $T > T^*$ the trading game $\langle G, T, \pi, \delta \rangle$ has an equilibrium where (a) in round one, trader 1 asks to trader 2 a price $1 - \delta(1 - \delta\pi_3)$ and trader 2 accepts and consumes if, and only if, she has value 1; (b) if the offer is refused, in round two trader 1 asks to trader 2 her resale value, which is equal to $\delta\pi_3$, and trader 2 accepts the offer; (c) in round three, trader 2 asks price one to trader 3, who accepts if, and only if, she has value one; (d) if the offer is rejected there are no other offers until the end of the game.¹⁹*

We conclude this section remarking that, under discounting, the network trading game becomes continuous at infinity, because per-period payoffs are uniformly bounded. Following Fudenberg and Levine (1986), we can then find an equilibrium in the infinite-horizon game by considering the limit of a model with the deadline and discounting. With this remark in mind, we conjecture that proposition 7 could be generalized to arbitrary network trading game: when the time horizon is infinite and discounting vanishes, there is an equilibrium outcome of the trading game that converges to an ex-post efficient outcome.

¹⁹When $\pi_2 < \delta\pi_3$ the object is immediately sold at the resale value.

5.2 Multiple types

We consider an example where there are three traders located in a line, i.e., $N = \{1, 2, 3\}$ and $E = \{12, 23\}$. We let $T = 3$ and assume that there are three possible types, $(1, v, 0)$, with corresponding priors $(\pi_i^h, \pi_i^m, \pi_i^l)$. For simplicity we further assume that the initial owner has value zero (i.e., $\pi_1^l = 1$) and that $\pi_i^h = \pi_i^m = \pi_i^l = 1/3$ for $i = 2, 3$. Finally we assume that $v = 1/2 + \epsilon$ for some small but positive ϵ . We now describe the equilibrium path using five observations.

First observation. Suppose trader 2 becomes the owner at the end of the first round (or the second). If she has high-value, trader 2 consumes and obtains a payoff of 1. If she is medium-value, then she waits one round, asks a price of one to trader 3 in the last round, and she consumes if the offer is rejected. Because in the last round a trader accepts every price up to her valuation, the continuation payoff of the medium-value trader 2 is $1/3 + 2v/3$. If trader 2 has low-value, she makes immediately an offer at price v to trader 3, and trader 3 accepts if she has high or medium-value (note that $2v/3 > 1/3$ because $v > 1/2$). Hence the continuation payoff of low-value trader 2 is $2v/3$.

Next, observe that in the construction above both low-value 2 and medium-value 2 try to resell the good to trader 3 at price 1. However, in equilibrium, low-type 2 makes an immediate offer at price v , whereas medium value trader 2 waits that the deadline approaches before making an offer to trader 3 at price 1. In fact, there is no equilibrium in which the medium-value 2 asks a price of one immediately and such an offer is accepted by the high-value 3. If that were the case then the low-value 2 would mimic the high-value and ask a price of one as well; subsequently lowering the price to v in case of rejection. The fact that signalling occurs also in the selling stage represents an additional complexity which is absent in the two-type model.

Second observation. The strategy of trader 2 when she receives an offer from trader 1 in the second round is summarized in Figure 6. This strategy represents a best reply for trader 2 for the following reasons. First, since $2v/3$ is the resale value of 2's low-type, we can specify that everyone accepts prices below $2v/3$ by assuming that an out-of-equilibrium rejection signals high-value. Consider now an offer at a price $p > 1/3 + 2v/3$. If trader 2 follows the strategy and rejects such offer, then beliefs do not change and trader 1 (who is for sure low-value) will ask in the last round a price $v < p$. Hence, by refusing, all types of trader 2 are better off. Lastly, for prices in the range $(2v/3, 1/3 + 2v/3)$ we know that the refusal would signal that 2 has low-value. Hence, following a rejection from 2, trader 1 will not make any other offer to 2. Therefore accepting is a

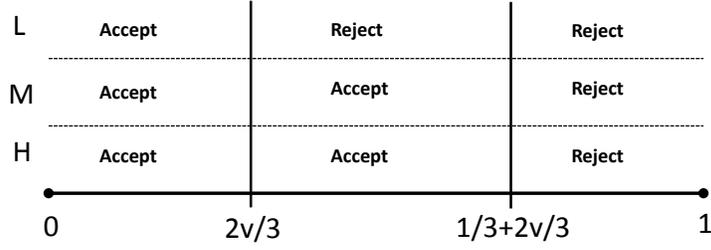


Figure 6: Strategy of 2 with offer at $t=2$.

best-reply for 2.

Given the strategy outlined above, and assuming that the posterior beliefs that 2 is high-value and medium-value are respectively μ^h and μ^m , at the beginning of the second round trader 1 will behave as follows. She will ask either price $2v/3$ (i.e., the resale value of the low-value), or $1/3 + 2v/3$ (i.e. the resale value of the medium-value), or will wait one round and then ask a price of 1 (in the final round player 2 will accept all offers up to 1 if she has value one). Which offer is optimal for trader 1 depends on the beliefs μ^h and μ^m . Observe that waiting is optimal for trader 1 whenever $\mu^h \geq 2/3$; when $\mu^h + \mu^m = \frac{2v}{2v+1}$ trader 1 is indifferent between the resale value of the medium-value trader and the resale value of the low value-trader. If posteriors were equal to prior beliefs, the initial owner would find it optimal to ask price $1/3 + 2v/3$.

Third observation. Let's now consider the strategy of player 2 when she receives an offer from trader 1 in the first round. This is summarized in Figure 7. First observe that, for the same reasons outlined in our second observation, accepting every offer below $2v/3$ and rejecting every offer above $1/3 + 2v/3$, regardless of her type, is a best reply. However, there is no pure strategy which is compatible with equilibrium play, when the offer is at a price in the range $(2v/3, 1/3 + 2v/3)$.²⁰

We now construct a mixed strategy equilibrium where: (a) the low-value trader always rejects the offer with probability one, (b) the medium-value trader and the high-value trader accept the

²⁰It cannot be the case that the low type refuses the offer and the high and medium type accepts the offer; for otherwise, both the medium value and high value trader strictly prefers to refuse the offer, signal low valuation and get an offer at a price equal to the resale of the low-type in the next round. The other cases can be ruled out with similar arguments.

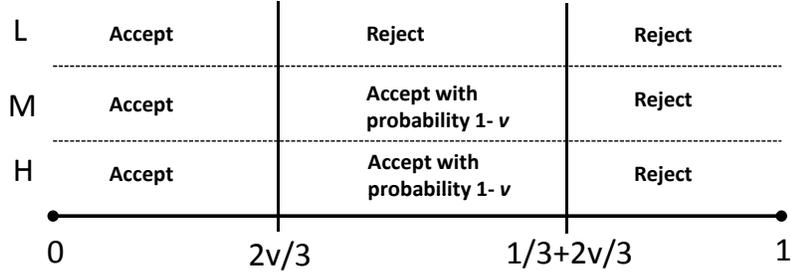


Figure 7: Strategy of 2 with offer at $t=1$.

offer with the same probability λ , and, (c) upon rejection, trader 1 is indifferent between asking the resale value of trader 2's low-value, $2v/3$, and the resale value of trader 2's medium-value, $1/3 + 2v/3$. This construction mimics the construction of a mixed strategy in the model with two-types.

Following our second observation above, for trader 1 to be indifferent between asking a price of $2v/3$ and asking a price of $1/3 + 2v/3$ upon rejection of his first offer, we must have that (i) $\mu^h \leq 2/3$ and (ii) $\mu^h + \mu^m = \frac{2v}{2v+1}$. Because the medium-value and the high-value accept with the same probability λ , Bayes rule imply that

$$\mu^h = \mu^m = \frac{1 - \lambda}{2(1 - \lambda) + 1}.$$

Therefore $\mu^h + \mu^m = \frac{2(1-\lambda)}{2(1-\lambda)+1}$, and by setting $1 - \lambda = v$ we guarantee that both conditions (i) and (ii) above are satisfied.

Finally, in equilibrium, the seller must randomize in the subsequent round between asking price $2v/3$ and price $1/3 + 2v/3$, in such a way that both the high-value trader 2 and the medium-value trader 2 are indifferent between accepting and rejecting the offer in the first period. Let $\gamma(p)$ be the probability with which trader 1 plays price $2v/3$ given the rejection of a price $p \in (2v/3, 1/3+2v/3)$. To make the medium-value of trader 2 indifferent we must have

$$\frac{1}{3} + \frac{2}{3}v - p = \left(\frac{1}{3} + \frac{2}{3}v - \frac{2}{3}v\right)\gamma(p),$$

or equivalently, $\gamma(p) = 1 + 2v - 3p$. And it is easy to check that $\gamma(p)$ also makes the high-value of trader 2 indifferent between accepting and rejecting.

Fourth observation. Given the strategy of trader 2, the initial owner in the first round will either ask the resale value of the low-value trader 2, $2v/3$, or the resale value of the medium-value trader 2, $1/3 + 2/3v$, or she will wait one round. Given the specified priors, the optimal strategy for trader 1 is to demand a price $1/3 + 2/3v$ in the first round.

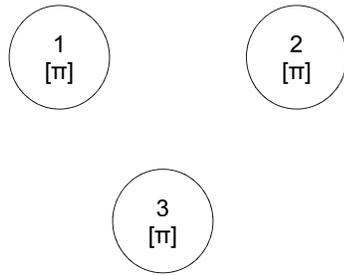
Summarizing. In the first round trader 1 asks the resale value of the medium-value of trader 2. Trader 2 rejects if she has low-value, while she accepts with probability $\lambda = 1 - v$ if she has either a high or medium-value. In case of rejection, in the second round trader 1 asks again $1/3 + 2/3v$ with probability one. Trader 2 rejects if she has low-value, but she accepts if she has either high or medium-value. In the former case, in the last round trader 1 consumes the object. In the latter case there are two possibilities. If trader 2 has high-value, she consumes the object. If trader 2 has medium-value, then she demands a price of 1 to trader 3, who accepts the offer if she has high-value. Otherwise, trader 2 consumes.

5.3 Trading Network Formation

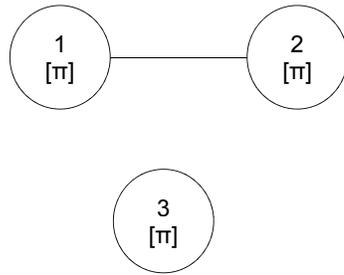
The presence of a trading relationship indicates the feasibility of a direct exchange. A connection may be generated by some costly investment incurred by the two parties, such infrastructure building or repeated meetings to built up trust. In this section, by focusing on the three player case, we considers a simple network-formation game that takes place ex-ante.

Assume that at $t = -1$ the trading network G is formed. Each link has a cost c , that is equally shared among the pair of traders. At $t = 0$, with equal probability the object is allocated to one of the trader, say trader i , and the identity of the initial owner is revealed and becomes common knowledge. Furthermore, types are realized, but they remain private information. Afterwards, the trading network game $(G, T, \boldsymbol{\pi}, i)$, where i indicates the identity of the initial owner, is played. Define $U_j(G)$ the expected utility of j at time $t = -1$. This utility depends on the equilibrium selected in each of the games $(G, T, \boldsymbol{\pi}, i)$, $i \in N$.

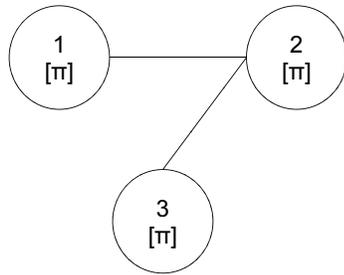
With three traders, there are four distinct network architectures: the empty network, the partial network, the star network and the complete network. These are depicted in figure 8. We assume throughout that $T \geq 2$ and that $\pi_i = \pi$ for $i = 1, 2, 3$ and some $\pi \in (0, 1)$. In each of these networks, regardless of the initial owner, there is always an equilibrium which is ex-post efficient. In what follows, in case of multiplicity we always select the efficient equilibrium in order to compute the expected utility of each trader. We now describe the four cases in more details:



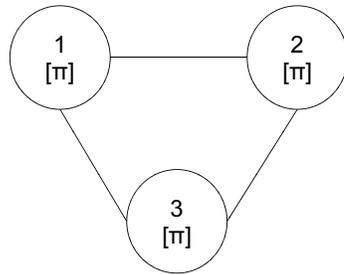
(a) The empty network.



(b) The partial network.



(c) The star network.



(d) The complete network.

Figure 8: Networks with three traders.

- In the empty network (figure 8(a)) the expected payoff to each trader is the probability of becoming the owner times the probability of having a high-value, i.e., $\pi/3$. Total welfare is π .
- In the partial network (figure 8(b)) the expected payoff of the isolated trader (trader 3 in the figure) is $\pi/3$. The payoff of each of the other two traders is $[1 - (1 - \pi)^2]/3 - c/2$. For example, if trader 2 is the owner, she consumes if she has high-value, and otherwise she asks a price of 1 to trader 1, who accepts if she has high-value. Total welfare is $\pi/3 + 2[1 - (1 - \pi)^2]/3 - c$
- Consider the star network (figure 8(c)). The expected payoff to the central trader (trader 2 in the figure) is:

$$\frac{1}{3}[1 - (1 - \pi)^3] + \frac{2}{3}(1 - \pi)^2\pi - c.$$

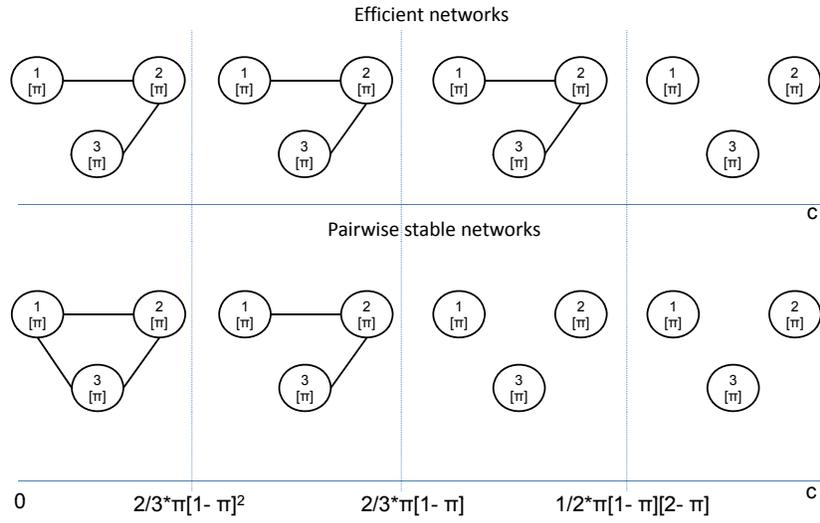
The first part is the payoff of the central trader in the event she is the initial owner: she consumes if she has high-value, and otherwise she asks a price of 1 in turn to each of the other two traders, who accept when they have high-value. The second part of the expression is the central trader's payoff when the owner is one of the other traders: whenever the owner is a low-value, trader 2 buys at a price of π and when she has high-value she consumes the object. Hence, the expected payoff of each of the peripheral traders is $[\pi + (1 - \pi)\pi]/3 - c/2$. Total welfare is $1 - (1 - \pi)^3 - 2c$

- In the complete trading network (figure 8(d)) each trader obtains an expected payoff of $[1 - (1 - \pi)^3]/3 - c$ and total welfare is $1 - (1 - \pi)^3 - 3c$

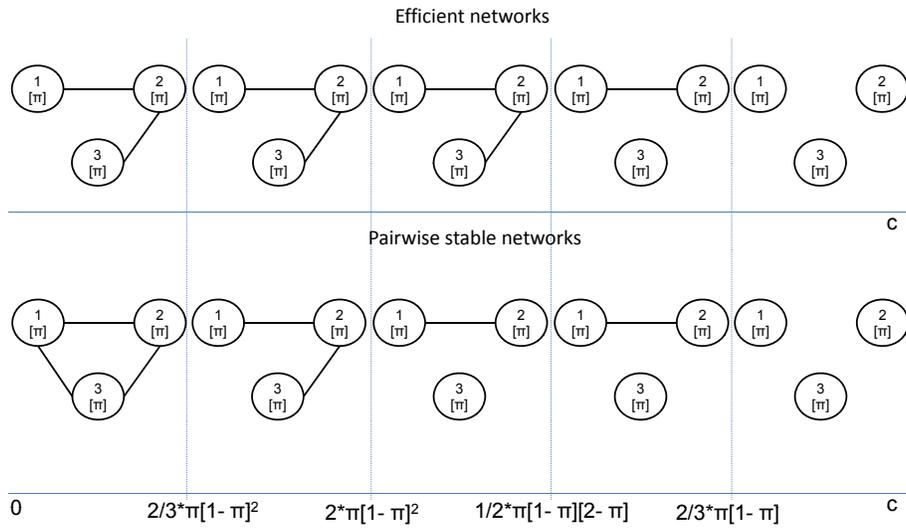
We can now characterize efficient networks, that is networks that maximize total welfare. We also characterize *pairwise stable* networks: a network G so that no individual trader wants to delete any of the link she has, and no pair of disconnected traders want to form an additional link (given the expected payoffs described above). Figure 5.3 summarizes the characterization of efficient and pairwise stable networks. The following proposition, which follows from simple algebra and hence we state without proof, summarizes the main economic insights of this exercise.

Proposition 8. *When the cost of forming a trading link is sufficiently low, the pairwise-stable trading network is over-connected as compared to the socially-efficient trading network. When the cost of forming a trading link is moderate, the pairwise stable trading network is under connected as compared to the socially efficient trading network.*

When the cost of forming a link is low, the socially efficient network has to be connected to allow for the good to flow from the initial owner to every other traders. Because in the star network



(a) Three traders, $T \geq 2$ and $\pi < 2/3$.



(b) Three traders, $T \geq 2$, $\pi > 2/3$.

Figure 9: Efficient networks *vs.* Pairwise Stable Networks.

there is always an ex-post efficient equilibrium outcome, this is the efficient network architecture. However, the star is not pairwise stable for low costs. In the star each of the peripheral traders leaves some rent to the central player whenever they are allocated the object and they have a low value. By forming a link between them they can extract such rent. Therefore, as the cost of forming a link is sufficiently low, they strictly prefers to get directly connected. This over-connectivity therefore reflects the tendency to form direct connections to extract rents that otherwise would be left to dealers.

When the cost of forming a link is moderate, there is a different source of inefficiency. Suppose for example that $\pi < 2/3$ and that $c \in (2/3\pi(1 - \pi), 1/2\pi(1 - \pi)(2 - \pi))$. In this case, the efficient network is the star network. However, the pairwise stable network is the empty network. The star is not pairwise stable because the cost of the link with the central trader is too high from the view point of the peripheral traders, given that the central trader extracts rents from the peripheral agents. That is, peripheral traders do not internalize the value that trading links create to other traders, thereby inducing under-connectivity as compared to the efficient trading network.

6 Conclusions

In this paper we study a sequential trading model with incomplete information, where traders are located in a network and there is one object for sale.

We characterize a class of weak-Markov equilibria and provide two main results. First, we show that the price dynamics is non-monotone, and we describe the pattern. Second, we show that dealers, who intermediate the object, arise endogenously and earn a profit. The rents that dealers earn are determined by their position in the trading chain, which in turns depend on their position in the network. Furthermore, we provide insights on how the combination of asymmetric information and the incompleteness of trading relationships determine inefficient equilibrium outcomes. We check the robustness of our results by considering the implications of introducing discounting, infinite horizon, and multiple types.

In our model, sellers can only bargain with a single trader at a time. Further research may focus on different and possibly more general trading mechanisms. Furthermore, in our model there is only one object for sale. The presence of multiple objects would introduce competition among owners and also represents an interesting avenue for further research.

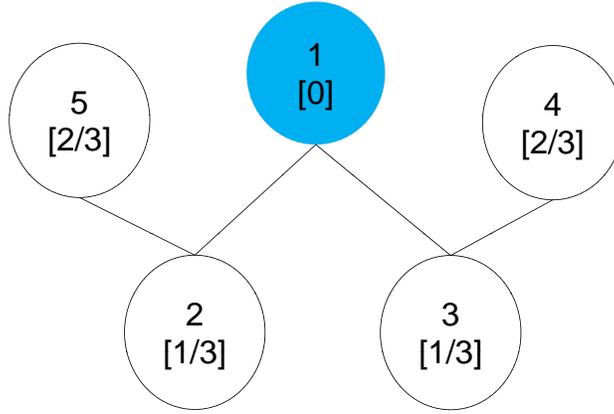


Figure 10: Number of rounds $T = 6$, trader 1 is the initial owner.

Appendix A

In this section we illustrate the consequences of having multiple equilibria using an example. The example considers the trading network G depicted in figure 10. There are $n = 5$ traders, $T = 6$ rounds of trade and the initial profile of beliefs is $\pi = (0, 1/3, 1/3, 2/3, 2/3)$.

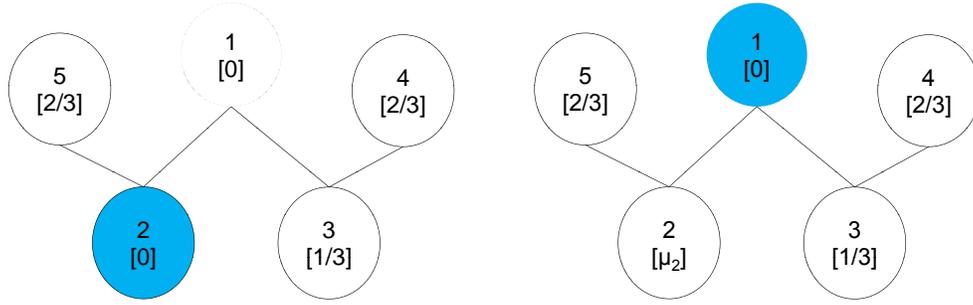
Trader 1 has a low-value and therefore she will make an offer to one of her neighbor. The optimal offer will depend on the acceptance strategy of 2 and 3. We restrict attention, without loss of generality, to the case in which in the first period trader 1 makes an offer to trader 2.

Following proposition 1, to determine the equilibrium strategy of low value trader 2 we need to derive the resale value of trader 2. In this case, the resale value of trader 2 is unique and equal to $R_2^2 = 8/9$. In fact, in the continuation game in figure 11(a) trader 2 will make a consumption offer at a price of 1 to trader 5, and in case of rejection will sell the object to trader 1 at trader 1's resale value, which, at that point, is equal to $2/3$. Hence, the strategy of low-value 2 when she obtains an offer in the first period from trader 1 is to accept every offer at a price below R_2^2 and to reject offers at higher prices.

To determine the equilibrium behavior of trader 2 when she has high-value we need to determine $V_2^2(0)$ and $V_2^2(1/3)$. That is, we need to characterize equilibria in the continuation game in figure 11(b). Note that, regardless of the belief about 2, i.e. $\mu_2 \in [0, 1/3]$, trader 1 is indifferent between the following two equilibrium paths:

1. Trader 1 makes a resale offer to trader 2 at $8/9$, who accepts the offer. In this case high value trader 2 gets a payoff of $1/9$.²¹

²¹If trader 2 has a low value she will ask a price of 1 to trader 5 and, upon rejection, will sell the object to trader 1 at her resale value which is, at that point, $2/3$. So, trader 2's expected profit (or trader 2's resale value) is



(a) Five rounds of trade left and the owner is trader 2. (b) Five rounds of trade left and the owner is trader 1.

Figure 11: Two different continuation games.

2. Trader 1 makes a resale offer to trader 3, which is also equal to $8/9$. In the ensuing continuation equilibrium, trader 2 acquires the good when both trader 3 and trader 5 have low value, which happens with probability $2/9$ and in that case she pays a price of $2/3$; her expected profit is therefore $2/27$.²²

Since trader 1 is indifferent between the two paths, the following strategy is optimal: if $\mu_2 = 0$ in the continuation game in figure 11(b) then trader 1 "plays" path 1 with probability $\sigma(0)$, and path 2 with the remaining probability, whereas if $\mu_2 = 1/3$ then trader 1 "plays" path 1 with probability $\sigma(1/3)$, and path 2 with the remaining probability. Hence, for $\mu_2 \in \{0, 1/3\}$, we have that:

$$V_2^2(\mu_2) = \frac{1}{9}\sigma(\mu_2) + \frac{2}{27}[1 - \sigma(\mu_2)] \in \left[\frac{1}{9}, \frac{2}{27}\right].$$

Note that $R_2^2 \leq 1 - V_2^2(\mu_2)$. We can distinguish two cases.

Case 1. Equilibria where $\sigma(0) > \sigma(1/3)$. In this case $V_2^2(0) > V_2^2(1/3)$. Proposition 1 and the definition of PWP equilibria imply that in every PWP equilibrium we have that: high value trader 2 accepts every offer at a price $p \in [8/9, 1 - V_2^2(0)]$, mixes for offers at a price $p \in (1 - V_2^2(0), 1 - V_2^2(1/3)]$, and rejects all offers at an higher price.

There are however equilibria which are not PWP. Consider for example an offer in the first period at a price $p \in [8/9, 1 - V_2(0)]$ and suppose that trader 2 accepts that offer with probability

$2/3 + 1/3 \times 2/3 = 8/9$.

²²Indeed, once accepted the offer from trader 1, trader 3 consumes if she has high value, otherwise she asks a price of 1 to trader 4, who accepts the offer if she has high value. Upon rejection, trader 3 resales the object to trader 1 who then asks a price of $2/3$ to trader 2.

1/2. Then, in case of rejection, at the beginning of the second period all traders believe that trader 2 has high value with probability $\mu_2 = 1/5$. For trader 2 to be indifferent between accepting and rejecting the offer at price p in the first period it has to be the case that $1 - p = V_2^2(1/5)$. This is possible under the conjecture that, in case of rejection of offer p , in the second round of trade trader 1 plays path 1 with probability $\sigma(1/5)$, where $\sigma(1/5)$ solves:

$$V_2^2(1/5) = \sigma(1/5)\frac{1}{9} + [1 - \sigma(1/5)]\frac{2}{27} = 1 - p,$$

and it is easy to verify that such value of $\sigma(1/5)$ exists. Restriction A in the definition of PWP equilibria rules out these equilibrium play by postulating that whenever there is a best reply which is in pure strategy traders will coordinate on that.

Case 2. Equilibria where $\sigma(0) < \sigma(1/3)$. In this case $V_2^2(0) < V_2^2(1/3)$. Proposition 1 and the definition of PWP equilibria implies that there are two types of PWP equilibria: in one equilibrium high value trader 2 accepts every offer at a price $p \in [8/9, 1 - V_2^2(1/3)]$ and rejects offers at a higher price; in the other equilibrium high value trader 2 accepts every offer at a price $p \in [8/9, 1 - V_2^2(0)]$ and rejects offers at a higher price.

There are however equilibria which are not PWP. In these equilibria some offers at a price within the range $[1 - V_2^2(1/3), 1 - V_2^2(0)]$ are accepted, and other offers in the same range are rejected. Restriction B in the definition of PWP rules out these equilibrium plays by imposing consistent behaviour between information sets that differs only for the price asked by the seller.

We conclude the discussion with two observations. First, despite the restrictions that the definition of PWP imposes on the equilibrium play, in this network trading game there are multiple PWP equilibria. Second, in this example multiple equilibria are induced by the indifference that trader 1 has between offers. A small introduction of trader specific transaction costs will break this indifference leading to a unique equilibrium outcome. Suppose that if trader 1 sells to trader 2 then trader 1 has to pay a transaction cost of τ_{12} , while if trader 1 sells to trader 3, then she pays a transaction cost of τ_{13} , where $\tau_{12} > \tau_{13}$ and they are both very small. There are no other transaction costs in the economy. Note that now, in the continuation game depicted in figure 11(b), trader 1 strictly prefers path 2 to path 1. Hence, if in the first period trader 1 makes an offer to trader 2 and the offer is rejected, in the second period trader 1 will make a resale offer to trader 3, i.e., $\sigma(\mu_2) = 0$ for all $\mu_2 \in [0, 1/3]$. So, in this context, the introduction of small transaction costs leads to a unique equilibrium, which is a PWP equilibrium. The equilibrium path is the following: trader 1 makes a consumption offer to trader 2 at $p_2^1 = 1 - V_2^2(0) = 25/27$, and, upon rejection, trader 1 asks the resale value (which is approximately) $r_3^1 = R_3^3 = 8/9$ to trader 3, who accepts the offer. If trader 3 has low value then she makes a consumption offer to trader 4 at a price of

$p_4^3 = 1 - V_4^4 = 1$, and, if rejected, she resells back the good to trader 1 at $r_1^3 = R_1^5 = 2/3$, who, in turn, sells it to trader 2 at $r_2^1 = R_2^6 = 2/3$. If trader 2 has a high value she consumes; otherwise she asks a price of $p_5^2 = 1$ to trader 5.

Appendix B

Proof of Proposition 1. We prove the proposition by induction. We show that all equilibria in round T have the desired properties; we then show that these properties hold in every weak-Markov PBE of every game starting in round t , given that these properties are satisfied in every weak-Markov PBE of every game starting from $t + 1$ to T .

First consider round T . Recall that $V_i^{T+1}[\cdot | \cdot] = 0$. It is always optimal for a low-value trader to make some offer at price one. Not making any offer is also a best reply only if $\mu_j = 0$ for all j who are i 's neighbors. Payoffs are clearly unaffected by this possibility. Accepting an offer at price above $R_i^{T+1} = 0$ provides negative payoff, hence refusing is the only action compatible with equilibrium play. In round T the low-value trader is indifferent between consuming and not consuming. It is always optimal for a high-value trader to wait and consume since $\mu_j^T \leq 1$ for every j . Again, making an offer at price 1 is also a best reply when $\mu_j^T = 1$ for some j neighbor of i , but has no effect on equilibrium payoffs. For a high value trader it is optimal to accept every price below or equal to one. Note that there is no equilibrium where price 1 is refused by the high-value, because the seller would have an incentive to slightly undercut his price.

Next, assume that properties 1-L, 2-L, 3-L and 1-H, 2-H, 3-H hold for every weak-Markov PBE of every game starting in round $t + 1$. We now prove that these properties hold for every game starting at t .

Consider 3-H. Consuming at the end of round t is always optimal for a high-value trader; otherwise according to induction hypothesis she will wait and consume in $t + 1$. Consider 3-L. Consuming gives 0 payoff to a low-value trader, while $R_i^{t+1} \geq 0$ is the continuation value from not consuming. Hence, unless $R_i^{t+1} = 0$, a low-value trader strictly prefers not to consume the object.

Next, consider 2-L. Suppose trader i accepts an offer in round t , she does not consume in the last stage of round t , and she makes an offer at the beginning of round $t + 1$, then, Bayesian updating together with our induction hypothesis, imply that $\mu_i^{t+1} = 0$. Hence, accepting an offer at time t and not consuming, gives a payoff to i of:

$$V_i^{t+1}[i, (i, p), (0, \mu_{-i}^t) | 0] - p = R_i^{t+1} - p$$

On the other hand, rejecting the offer of a seller s provides $V_i^{t+1}[s, (i, p), (\mu_i^{t+1}, \mu_{-i}^t)|0] \geq 0$. Hence, it is always optimal for trader i to refuse every price strictly greater than R_i^{t+1} . We now show that a low value trader i accepts offers at prices $p < R_i^{t+1}$. For a contradiction, suppose that trader i refuses with positive probability some offer at price p where $p < R_i^{t+1}$. Since $p < R_i^{t+1}$ accepting gives a strictly positive payoff. Therefore rejecting can be a best reply only if $V_i^{t+1}[s, (i, p), (\mu_i^{t+1}, \mu_{-i}^t)|0] > 0$. We now show that this is impossible.

To see this, we write the continuation payoff of trader i as follows:

$$V_i^{t+1}[s, (i, p), (\mu_i^{t+1}, \mu_{-i}^t)|0] = \sum_{e \in E} \Pr[e] (R_i(e) - p(e))$$

where E is the set of events under which i will receive an offer at price $p(e)$ that she will accept. Since $V_i^{t+1}[s, (i, p), (\mu_i^{t+1}, \mu_{-i}^t)|0] > 0$ there must exist some $e \in E$ such that

$$R_i(e) - p(e) > 0,$$

where $R_i(e)$ is the resale value of trader i when she becomes owner after accepting offer $p(e)$. In other words, for $V_i^{t+1}[s, (i, p), (\mu_i^{t+1}, \mu_{-i}^t)|0] > 0$, trader i must receive, with some probability, an offer at a price below his resale value. However, note that when e is realized, by our induction hypothesis, trader i accepts all prices lower or equal to $R_i(e)$, regardless of her value. Hence, the seller offering $p(e)$ could strictly improve her payoff by slightly increasing the price, a contradiction to sequential rationality.

Consider now 2-H. Note first that since the low type always accepts an offer below or equal to the resale value, there is no equilibrium where the high type refuses such an offer with positive probability. Otherwise she would signal that she has high value and get zero payoff. Next, suppose that i receives an offer at a price p such that $p > R_i^{t+1}$. Recall that a low value trader always rejects such an offer. Suppose that $1 - p \geq V_i^{t+1}(0)$, i.e. the continuation value of i given that everyone believes she is a low type. Then accepting is a best reply because by rejecting she induces $\mu_i^{t+1} = 0$ and therefore she obtains $V_i^{t+1}(0)$, while if she accepts and consumes she obtains $1 - p$. Suppose that $1 - p \leq 1 - V_i^{t+1}(\mu_i^t)$. In this case rejecting is a best reply because acceptance provides payoff $1 - p$, which is below the continuation value from rejection given belief updating. Finally accepting with probability λ is a best reply only if the trader is indifferent between acceptance and rejection, given that upon rejection beliefs are updated according to Bayes rule.

Next consider 1-L. To see that in equilibrium every low-value seller must maximize the objective in expression 1, it is sufficient to recall that, when proving 2-L we have shown that the continuation payoff of every low-value trader who has sold an object is always zero.

We finally consider 1-H. To prove that a high-value trader prefers to wait in round t and subsequently consume, we need to show that her continuation payoff for the game just starting after any of her offers in t cannot be above one, i.e., for every offer (j, p) that i can make in t we have that $V_i^t[i, \boldsymbol{\mu}^t|1] \leq 1$, where $V_i^t[i, \boldsymbol{\mu}^t|1]$ is given by expression 1. First, note that if the offer (j, p) is refused with probability 1, then the induction hypothesis implies that trader i consumes in the subsequent round and therefore $V_i^t[i, \boldsymbol{\mu}^t|1] = 1$. Hence, suppose that the offer (j, p) is accepted with some positive probability. We distinguish two cases.

First, assume that $p > R_j^{t+1}$. If the offer is rejected the seller waits and consumes in round $t + 1$. If the offer is accepted, then the price must be less than one, since we are assuming by induction that no trader obtains a continuation greater than one if she has value zero, and we are assuming that high-value traders consume immediately when they are owner in every period $t' > t$. Moreover, since we have proved above that a low-value trader rejects every offer above her resale value, if j accepts the offer, she must be an high type and therefore she consumes in period $t + 1$. Overall, this implies that $V_i^t[i, \boldsymbol{\mu}^t|1] \leq 1$.

Second, suppose the offer is at a price $p = R_j^{t+1}$. Note that asking $p < R_j^{t+1}$ is suboptimal for the seller given that the offer is accepted for sure and that in a weak-Markov equilibrium the continuation for the seller is the same regardless of the price asked, when the price below or equal to R_j^{t+1} . In this case the offer is accepted and therefore the payoff of the high-value seller i by not consuming and imitating a zero type seller is:

$$V_i^t[i, \boldsymbol{\mu}^t|1] = R_j^{t+1} + V_i^{t+1}[j, (j, p), (\mu_{-i, -j}^t, \mu_i^{t+1}, \mu_j^{t+1})|1].$$

If i never gets another offer along the equilibrium path, then her payoff is equal to R_j^{t+1} , which by the induction hypothesis is not greater than one. So, suppose that there exists a positive probability that i gets another offer along the equilibrium path.

Since the induction hypothesis implies that in every equilibrium starting from $t + 1$ onward each seller with value 1 consumes the good whenever she acquires it, then from $t + 1$ onward someone who buys the good and does not consume is believed to be a low-value trader. Because low-value traders reject every offer above their continuation value, if i sells the good and she obtains another offer at $t' > t$ the price asked to her will be higher or equal to the resale value at t' (we have seen that no seller asks a price below the resale value). Therefore i will accept that offer and consume afterwards.

Taking this into consideration, assume that, starting from the acceptance of j in round t , there are k subsequent histories of play that lead to another offer being made to i . Let's denote by

$R_i(1), \dots, R_i(k)$ the continuation values of the low type i after these histories. Along each history a number of offers is made to other players. For each of the continuation history define o^1, \dots, o^k the set of players that receive an offer. Let $r(o^x)$ indicate the probability that all traders in o^x have value zero. Let $\Pr(o^x)$ indicate the probability that the history x ensues. Given the above, and given that the probability that all offers are refused in o^x is less or equal than $r(o^x)$ we know that: $V_i^{t+1}[j, (i, p), \mu^{t+1}, | 1] \leq \sum_j \Pr(o^x)r(o^x)[1 - R_i(x)]$ (note that we have removed the probability that j consumes and this can only strengthen the inequality).

Then, we observe that $R_j^{t+1} \leq \sum_x \Pr(o^x)r(o^x)R_i(x) + [1 - \sum_x \Pr(o^x)r(o^x)]$ since the price at which i sells the good in t to j , i.e. R_j^{t+1} , cannot exceed an upper bound given by the case in which, with probability $\Pr(o^x)r(o^x)$ the good is sold at price $R_i(x)$ for all x , while with probability $1 - \sum_j \Pr(o^x)r(o^x)$ it is sold at price 1 (which, by our induction hypothesis, is the maximum price at which the good could be ever sold). These considerations imply that $R_j^{t+1} + V_i^{t+1}(j, (j, p), \mu^{t+1} | 1) \leq 1$. ■

Proof of Proposition 2. We first show that an equilibrium in PWP always exist. We do this by induction. We show that a PWP equilibrium exists in the last round and then we show that we can construct a PWP equilibrium for a game starting in round t taking as given the existence of a PWP equilibrium in round $t + 1$. In doing so we use the one-shot deviation principle for dynamic incomplete information games (see Hendon et al. (1996)).

The equilibria we construct is as follows. The strategy profile satisfies the properties defined in Proposition 1 at every information set. Furthermore, for the case of the high value trader who receives an offer in round t we define her strategy (i.e. acceptance probability) as follows:

$$\begin{cases} 1 & \text{if } p \leq L = \max\{R_i^{t+1}, 1 - V_i^{t+1}(0)\} \\ \lambda_i(p) & \text{if } L < p \leq U = \max\{L, 1 - V_i^{t+1}(\mu_i^t)\} \\ 0 & \text{if } p > U \end{cases} \quad (3)$$

The defined strategy profile satisfied proposition 1 and in addition satisfies restriction A and restriction B in the definition of PWP equilibria.

To see that the prescribed strategy profile constitute an equilibrium when a the game starts in round T is straightforward. For a game starting in round T and for any potential seller, we have $V_i^{T+1}(\mu_i) = R_i^{T+1} = 0$ for all μ_i and for all i . Hence, it is a best reply for a high-value trader is to accept every offer up to a price of one.

Therefore it remains to show that the specified strategy is a best reply in round t , taking as given the continuation equilibria in round $t + 1$. In fact, since the strategy profile satisfies the properties

in proposition 1, we only need to show that that it is always well defined. The only concern in this case is the existence, for every price in $(1 - V_i^{t+1}(0), 1 - V_i^{t+1}(\mu_i^t))$ (whenever this interval is not empty), of a $\mu_i^* < \mu_i^t$ such that $p = 1 - V_i^{t+1}(\mu_i^*)$. In fact, when i receives an offer in the range $(1 - V_i^{t+1}(0), 1 - V_i^{t+1}(\mu_i^t))$ the only best reply is to randomize and for this to be possible there must exist a $\mu_i^* < \mu_i^t$ such that $p = 1 - V_i^{t+1}(\mu_i^*)$. Existence is guaranteed because the equilibrium payoff correspondence, as a function of the prior beliefs, has a closed graph (see Fudenberg and Tirole (1991)).

We now prove the second part of the statement. Consider two cases: (A) $V_i^{t+1}(0) \geq V_i^{t+1}(\mu^t)$ and (B) $V_i^{t+1}(0) < V_i^{t+1}(\mu^t)$.

Case (A). Three subcases are possible: (i) $R_i^{t+1} \geq 1 - V_i^{t+1}(\mu_i^t)$, (ii) $1 - V_i^{t+1}(\mu_i^t) \geq R_i^{t+1} \geq 1 - V_i^{t+1}(0)$, and (iii) $R_i^{t+1} < 1 - V_i^{t+1}(0)$. In case (i) the only *PWP* best reply requires the high-value buyer to reject an offer for a price greater than R_i^{t+1} . In case (ii) the only *PWP* best reply requires the high-value to reject any offer strictly above $1 - V_i^{t+1}(\mu_i^t)$ and mix between acceptance and rejection in $(R_i^{t+1}, 1 - V_i^{t+1}(\mu_i^t)]$. Note that in the latter instance $p = 1 - V_i^{t+1}(\mu_i^*)$ for some $\mu_i^* \leq \mu_i^{t+1}$. In case (iii), the only *PWP* best reply specifies acceptance in $(R_i^{t+1}, 1 - V_i^{t+1}(0)]$, mixing in $(1 - V_i^{t+1}(0), 1 - V_i^{t+1}(\mu_i^t)]$, and rejection in $(1 - V_i^{t+1}(\mu_i^t), 1]$. These observations together with the strategy of the low-value trader and the Markov property of equilibria imply that a seller can never find it optimal to sell the good at a price which is different from R_i^{t+1} or $1 - V_i^{t+1}(\mu_i^{t+1})$, where μ_i^{t+1} is computed using Bayes rule.

Case (B). Three subcases are also possible: (i) $R_i^{t+1} \geq 1 - V_i^{t+1}(0)$, (ii) $1 - V_i^{t+1}(0) \geq R_i^{t+1} \geq 1 - V_i^{t+1}(\mu_i^t)$, and (iii) $R_i^{t+1} < 1 - V_i^{t+1}(\mu_i^t)$. Case (i) is the same as above. In case (ii) there are two possible *PWP* best replies. First, to reject every offer above resale up to price 1. Second, to accept every offer up to $1 - V_i^{t+1}(0)$ and to reject every offer above. In case (iii) the only *PWP* best reply is to accept every price up to $1 - V_i^{t+1}(0)$ and to reject every price above. These observations together with the strategy of the low-value buyer and the Markov property imply that a seller can never find it optimal to sell the good at a price which is different from R_i^{t+1} or $1 - V_i^{t+1}(\mu_i^{t+1})$, where μ_i^{t+1} is computed using Bayes rule. ■

Proof of Proposition 3. We prove part 2 of the proposition only. Part 1 follows immediately from 2. The proof is by induction. The Proposition holds at $T - 1$, because every resale offer at T will be at zero. Suppose that the conjecture holds at every $t' > t$ and consider a consumption offer p_i^d at time t .

First we introduce some notation. Following the consumption offer (i, p) from seller s in round t many continuation trading chains can ensue in equilibrium. Among all those trading chains, we consider the following set of trading chains: each of the trading chain in the set differs from the other in at least one offer, and each of the trading chain leads to a seller different from s becoming the next owner. Say that there are k of these trading chains. For trading chain $x \in \{1, \dots, k\}$, name $o^x = \{i_1^x, \dots, i_m^x\}$ the set of traders receiving a consumption offer in the trading chain before the first resale offer and name d^x the first trader in the chain that receives a resale offer (the first future seller after s). For each $y = 1, \dots, m$, let $r(i_y^x)$ indicate the equilibrium probability that i_y^x refuses the consumption offer. Let $r(o^x)$ indicate the equilibrium probability that all the consumption offers to $\{i_1^x, \dots, i_m^x\}$ are rejected. Finally, let $p(i_y^x)$ denote the price of the consumption offer to i_y^x , $R(d^x)$ the resale value of d_x when she receives the resale offer, and $\Pr(o^x)$ indicate the probability that trading chain x is played in equilibrium.

The first observation is that s , starting from $t + 1$, expects to obtain the same continuation payoff along each of the trading chain $x = 1 \dots k$. That is, for each $x, y \in \{1, \dots, k\}$:

$$[1 - r(i_1^x)]p(i_1^x) + \dots + r(o^x)R(d^x) = [1 - r(i_1^y)]p(i_1^y) + \dots + r(o^y)R(d^y). \quad (4)$$

Condition 4 is simply a standard indifference condition for mixed strategies: if that condition did not hold, it would not be possible for both trading chains to occur with strictly positive probability in equilibrium, because seller s would find profitable to deviate from the prescribed equilibrium strategy at some point where the two trading chains differ.

The second observation is that for every $x = 1 \dots k$ the following holds:

$$R(d^x) \leq [1 - r(i_1^x)]p(i_1^x) + \dots + r(o^x)R(d^x) \leq (1 - r(o^x)) + r(o^x)R(d^x), \quad (5)$$

where the first inequality follows because, by our induction hypothesis $R(d^x) \leq p(i_y^x)$ for all $y = 1 \dots m$, and the second inequality follows because $p(i_y^x) \leq 1$ for all $y = 1 \dots m$. Now, combining the inequalities in 5 with the indifference condition 4 we obtain that for every $x, y \in \{1, \dots, k\}$ the following holds

$$R(d^x) \leq (1 - r(o^y)) + r(o^y)R(d^y) \quad (6)$$

The third observation is the following. A trading chain $x \in \{1, \dots, k\}$ is such that either: (i) after that d^x as received a resale offer there is a positive probability that in the future trader i receives a resale offer, or (ii) this probability is zero. Name the trading chains which satisfy (i) trading chains $\{1, \dots, z\}$, while the one that satisfy (ii) are $\{z + 1, \dots, k\}$. Next, for each $x = 1 \dots z$, let o_1^x, \dots, o_h^x the set of paths starting from d^x and leading to a resale offer to i . As before, $r(o_j^x)$

represents the probability that all elements of the path o_j^x , excluding future sellers (i.e. dealers) but possibly including i , refuse their offers. Then, denote the resale price to i following path o_j^x by $R(i_j^x)$ and the probability that the path o_j^x ensues as $\Pr(o_j^x)$. We know that for every $x = 1, \dots, z$ and $j = 1, \dots, m$, the following holds:

$$R(d^x) \leq 1 - r(o_j^x) + r(o_j^x)R(i_j^x)$$

Therefore we conclude, using the previous observations 1 and 2, that for every $x = 1, \dots, k$ and every $y = 1, \dots, z$, and every $j = 1, \dots, m$ we have:

$$R(d^x) \leq (1 - r(o_j^y)r(o_j^y)) + r(o_j^y)r(o_j^y)R(i_j^y). \quad (7)$$

The following fourth observation concludes the proof. We show that the consumption offer that i obtains at t is $p_i \geq (1 - r(o_j^y)r(o_j^y)) + r(o_j^y)r(o_j^y)R(i_j^y)$ for at least one path $y = 1 \dots z$ (who recall passes via d^y and then reaches i , where y may be i).

As a first step, observe that if $z = 0$, then that means, by construction, that i only receives consumption offers i from in future periods, and so i is a client, and each consumption offer is equal to 1, including the one in period t , i.e., $p_i = 1$. In this case the claim follows. So, suppose $z > 0$, i.e., with some probability i receives a resale offer. In this case, we have that $1 - p_i$ must equal the continuation payoff of i given that he rejects the offer, which is the probability that he obtains a resale offer in the future times the net profit in that case which is 1 minus the price she pays (her resale value).²³ Formally:

$$\begin{aligned} 1 - p_i &= \sum_{j=1}^z \Pr(o^j)r(o^j) \Pr(v_{dy} = 0) \left[\sum_{w=1}^{h(j)} \Pr(o_w^j)r(o_w^j)(1 - R(i_w^j)) \right] \\ &\leq \sum_{j=1}^z \Pr(o^j)r(o^j) \left[\sum_{w=1}^{h(j)} \Pr(o_w^j)r(o_w^j)(1 - R(i_w^j)) \right] \\ &\leq \max_{j=1, \dots, z} r(o^j) \left[\max_{w=1 \dots h} r(o_w^j)(1 - R(i_w^j)) \right] \end{aligned}$$

where the first inequality follows because we have removed the probability that dealers consume which is below one, and the second inequality follows because we have selected the maximum value among all trading considered trading chains.

²³ Note that if trader i receives another consumption offer before receiving the resale, then at that consumption offer she must be indifferent between accepting and rejecting, where rejecting equals the probability that she receives an offer at the resale value in the future. In the next formula, for sake of exposition, we consider the case where if i rejects the consumption offer, in every path in which he receives a resale offer, she does not receive other consumption offers. The other case can be proved in the same way by taking into account the aforementioned argument, and so details are not provided.

Rewriting the above equation we then obtain that there exists a $y = 1 \dots k$ and $j = 1 \dots m$ so that $p_i \geq (1 - r(o^y)r(o_j^y)) + r(o^y)r(o_j^y)R(i_j^y)$. This, together with inequality 7 completes the proof of the proposition. \blacksquare

Proof of Proposition 4. Let's consider all the different histories of offers which start from the initial seller and in which trader i gets a resale offer for the first time and name these different resales $R_i(1), \dots, R_i(k)$. If this set is empty, than by assumption both i and j makes a zero payoff and the result holds. So suppose it is not empty. Let the path x be the path that leads to resale offer $R_i(x)$, where $x = 1 \dots k$. In the path x , we denote by o^x the traders in that path who receives an offer, excluding both trader i and trader j . $r(o^x)$ indicates the probability that all traders in o^x refuse the respective consumption offer or, they do not consume if they have received a resale offer; $r(i^x)$ and $r(j^x)$ denote the probability that i and j refuse an offer along the path x , whenever they are included in the path (otherwise set these numbers equal to one). Finally, $\Pr(o^x)$ is the probability that the path occurs in equilibrium.

We can write the expected utility of trader 1 at the beginning of the game, condition on having value 1, as follows:

$$U_i(1) = \sum_x \Pr(o^x)r(o^x)r(j^x) (1 - R_i(x)).$$

Next, for each path x , let's consider the different paths that start when i receives the resale offer at the price $R_i(x)$ and that lead to a resale offer to j . If this set is empty then the result follows. Suppose it is not empty. Call the price of these resale offers to j as $R_j^x(1), \dots, R_j^x(l)$. For each $R_j^x(y)$ let o_y^x indicate that set of traders in the path that leads to the resale offer to j , excluding j himself. Let $\Pr(o_y^x)$ the probability that the path occurs in equilibrium, conditional on path x occurring. Let $r(o_y^x)$ be the probability that all element in the path refuse the offer (or in case of sellers that they have value zero conditional on the path occurring). Let $U_j(1)$ indicate the interim utility of a trader j with value 1 at the start of the game.

We know that

$$U_j(1) = \sum_{x=1}^k \Pr(o^x)r(o^x)(1 - \pi_i) \left[\sum_{y=1}^l \Pr(o_y^x)r(o_y^x)(1 - R_j^x(y)) \right].$$

We now note that, from the proof of proposition 3 (observation 2, inequality 6), for every $x = 1, \dots, k$, with associated $y = 1, \dots, l$, $k, l > 0$, we have that

$$R_i(x) \leq (1 - r(o_y^x)) + r(o_y^x)R_j^x(y),$$

and therefore

$$1 - R_i(x) \geq r(o_y^x)(1 - R_j^x(y)).$$

Observe that the fact that j might receive a consumption offer before his resale along the path o_y^x may only relax the bound. So,

$$U_i(1) \geq \sum_x \Pr(o^x)r(o^x)r(j^x) \left[\sum_y \Pr(o_y^x)r(o_y^x)(1 - R_j^x(y)) \right].$$

Now note that in the first part of the statement of the proposition we have that $r(j^x) = 1$, and therefore we conclude that

$$U_i(1) \geq \sum_x \Pr(o^x)r(o^x) \left[\sum_y \Pr(o_y^x)r(o_y^x)(1 - R_j^x(y)) \right] > U_j(1).$$

To prove the second part of the statement, consider that $r(j^x) \geq (1 - \pi_j)$. Then, the assumption that $\pi_i \geq \pi_j$ is sufficient to show that $U_i(1) > U_j(1)$. This concludes the proof of the proposition. ■

Proof of Proposition 5. Let $d(i, j|G)$ be the geodesic distance between i and j . Consider the following strategy profile. In period $t = 1, \dots, T$, if trader i receives an offer (i, p) she accepts if and only if $p \leq 1$ and there exists a $j \in N$ such that $v_j = 1$ and $d(i, j|G) \leq T - t - 1$; otherwise she rejects the offer. If trader i is the owner in period t then she consumes whenever $v_i = 1$ or $v_j = 0$ for all j such that $d(i, j|G) \leq T - t$. Otherwise, trader i asks a price of 1 to a trader l such that $d(l, j|G) \leq T - t + 1$, where $v_j = 1$.

To verify that this strategy profile constitutes a subgame perfect equilibrium, one can proceed by induction with respect to t , starting from $t = T$. It is also immediate to verify that the equilibrium outcome is Pareto Efficient. Furthermore, every other subgame perfect equilibrium has a strategy profile that is different from the one above only when there are indifferences. For example, when trader i is the owner at t , she has $v_i = 0$ and she cannot reach any other trader with value 1 in the remaining period, we have prescribed above that trader i consumes. Of course, prescribing to sell and a price of 0, or to randomize between the two actions, will not change the equilibrium outcome. In fact, a tedious argument shows that every subgame perfect equilibrium outcome is equivalent to a subgame perfect equilibrium outcome derived with the above strategy. This completes the proof of the proposition. ■

Proof of Proposition 6. Suppose G is such that the initial owner is linked to all other traders, and let $T^* = n - 1$. Then the following is an ex-post efficient equilibrium: trader 1 asks a price of 1 to

each of his neighbors sequentially, each of the trader accepts the offer if and only if they have high value, and, if they all reject the offer, trader 1 consumes the good.

Next, let $i = 1$ be the initial seller and let $N_1(G)$ indicate the set of traders in G connected to 1. Suppose that G is such that $N_1(G) \neq N \setminus \{1\}$. We show that for every T , there exists some profile of π such that under T and π every equilibrium is ex-post inefficient. If $T < |N_1(G)|$ (where $|\cdot|$ indicates the cardinality of the set), then every equilibrium is ex-post inefficient because some traders cannot be reached. So, let $T \geq |N_1(G)|$, and assume that it is sufficiently large. Let $\pi_j = \pi_H$ for all $j \in N_1(G)$ and let $\pi_j = \pi_L$ for all $j \in G/N_1(G)$. The initial seller can always play the following strategy: wait till $|N_1(G)|$ periods are left and then she asks a price of 1 to each of her neighbors in sequence. This strategy will provide an ex-ante payoff of $1 - (1 - \pi_H)^{|N_1(G)|}$.

Suppose now that there exists an equilibrium outcome that is Pareto efficient. Then, there must exist at least one trader j linked to the initial seller who receives an offer at her resale value with probability one. Since j is getting a resale offer with probability one, the highest consumption offer that she can accept before receiving the resale offer is at a price:

$$p^* = 1 - (1 - \pi_L)^{n-|N_1(G)|-2}(1 - \pi_H)^{|N_1(G)|-1}(1 - \pi_L) < 1$$

Hence, an upper bound to the expected payoff that the initial seller can make in every equilibrium in which at least one of the traders linked to her receives a resale offer is:

$$1 - (1 - \pi_H)^{|N_1(G)|-1} + (1 - \pi_H)^{|N_1(G)|-1}\pi_H p^* + (1 - \pi_H)^{|N_1(G)|}[1 - (1 - \pi_L)^{n-|N_1(G)|-1}].$$

Substituting the upper bound for p^* in the expression above, after some elaboration we get that the upper bound in revenue from the strategy above is lower than $1 - (1 - \pi_H)^{|N_1(G)|}$ whenever

$$1 - (1 - \pi_L)^{n-|N_1(G)|-1} \left[1 + \pi_H(1 - \pi_H)^{|N_1(G)|-2} \right] < 0,$$

which is always satisfied for sufficiently small π_L . When the condition above holds the initial seller is better off by asking a price of one to all her neighbors and an inefficiency arises. ■

Proof of Proposition 7. Consider first any arbitrary subgame starting in round $T - 1$ with trader 1 as owner. For any belief profile μ_2^{T-1} such that $\mu_2^{T-1} \geq \delta\pi_3$ trader 1 asks price one in round $T - 1$ and makes no subsequent offer in case the offer is refused. Player 2 accepts only if she has value one. Note that if 2 is the owner at the beginning of $T - 1$ and she has value zero she makes an offer at price one to 3. Trader 3 accepts if she has value one and rejects otherwise. If 3 rejects then 2 makes no other offer. Observe that the same happens in any round t when 2 has low-value and she is the owner (recall that it is known that 1 has value zero).

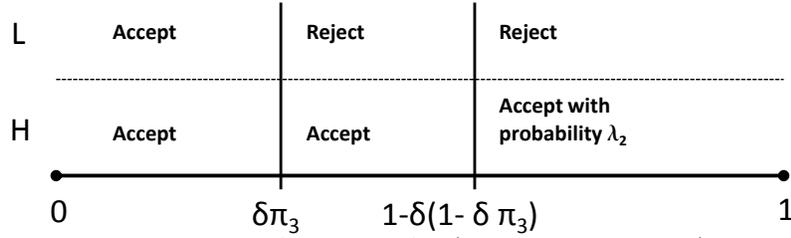


Figure 12: Strategy of 2 (low and high-value).

Consider now a subgame where 1 is the owner and there are three periods left. Focus on the case where $\mu_2^{T-2} > \delta\pi_3$. In fact, when $\mu_2^{T-2} \leq \delta\pi_3$, then it is optimal for 1 to sell at the resale value to 2 and the proposition holds. Moreover, even in a subgame starting in any previous round with some $\pi_2 \leq \delta\pi_3$ the proposition trivially holds.

First, we obtain the equilibrium acceptance strategy of player 2 using our Proposition 1 (modified taking into account the remarks of subsection 5.1) and the continuation payoffs $V_2^{T-1}(0)$, $V_2^{T-1}(\mu_2^{T-2})$ and R_2^{T-1} . We know that $V_2^{T-1}(0) = 1 - \delta\pi_3$ because in this case 1 prefers to ask to 2 the resale value. We know that $R_2^{T-1} = \delta\pi_3$. Finally we know that $V_2^{T-1}(\mu_2^{T-2}) = 0$, because $\mu_2^{T-2} > \delta\pi_3$. Hence, when there are two rounds left player 1 will ask price one to 2. Given the considerations above, we know that the equilibrium strategy for 2 is the one depicted in figure 12 where (omitting player 2 subscripts from now on) $\lambda^{T-2} = \frac{\mu^{T-2} - \delta\pi_3}{\mu^{T-2}(1 - \delta\pi_3)}$, because in round $T - 1$ player 1 must be indifferent between asking price one and asking the resale value of 2 (equal to $\delta\pi_3$), and therefore we must have $\mu^{T-1} = \delta\pi_3$.

Once the acceptance strategy of 2 has been specified, we can now compute the optimal strategy for player 1. Given the acceptance strategy of 2, trader 1 can focus on three offers: (A) asking the resale value, in which case the offer is accepted for sure; (B) asking $1 - \delta V^{T-1}(0) = 1 - \delta(1 - \delta\pi_3)$, which is accepted with probability 1 only by the high-value; (C) asking price one, which is accepted by the high-value with probability λ^{T-2} . Note that when (B) is refused 1 will subsequently ask the resale value of 2, that is $\delta\pi_3$ while when (C) is refused he will ask again price one (he is indifferent at that point between asking one and the resale). We conclude that player 1 obtains the following payoffs in the three offers that she considers:

- A. $\delta\pi_3$
- B. $\mu^{T-2}(1 - \delta) + \delta^2\pi_3$
- C. $\frac{1}{1 - \delta\pi_3}[\mu^{T-2} - \delta\pi_3 + \delta^2\pi_3 - \delta^2\pi_3\mu^{T-2}]$

First, note that B is always preferable to A when $\mu^{T-2} > \delta\pi_3$. Second, observe that there exists $\bar{\mu}^{T-2} = \frac{\pi_3(1-\delta^2\pi_3)}{1+\pi_3-2\delta\pi_3}$ such that when $\mu^{T-2} = \bar{\mu}^{T-2}$ then 1 is indifferent between B and C, when $\mu^{T-2} > \bar{\mu}^{T-2}$ she prefers C and when $\mu^{T-2} < \bar{\mu}^{T-2}$ she prefers B.

Now consider any subgame starting at $T-3$. If $\mu^{T-3} \leq \bar{\mu}^{T-2}$ then, there is an equilibrium where in round $T-2$ the seller follows strategy B; it can be shown that this is optimal also at $T-3$. This is true because in that case we have $V^{T-2}(0) = V^{T-2}(\mu^{T-3})$. Therefore, if $\mu^{T-3} \leq \bar{\mu}^{T-2}$, the proposition holds. This allows also to conclude that if $\pi_2 \leq \bar{\mu}^{T-2}$, then in any game starting earlier than $T-3$ the proposition holds.

We then focus on situations where $\mu^{T-3} > \bar{\mu}^{T-2}$. We first construct the strategy of player 2. We now note that $V_2^{T-3}(0) = V_2^{T-2}(0)$ and $R_2^{T-3} = R_2^{T-2}$ and observe that for any earlier round these continuation payoffs continue to keep the same value. Moreover, we know that since C is optimal at $T-2$ with $\mu^{T-2} = \mu^{T-3}$, we will have $V^{T-2}(\mu^{T-3}) = 0$. Therefore, the strategy of 2 is exactly the same as the one depicted in figure 12, with the exception that the probability with which 2 mixes, now λ^{T-3} , must be such that $\mu^{T-2} = \bar{\mu}^{T-2}$. In fact, the equilibrium is constructed with 2 mixing in such a way that in $T-2$, trader 1 will be indifferent between B and C. Note that, for this to happen, by Bayes rule we must have $\bar{\mu}^{T-2} = \frac{(1-\lambda^{T-3})\mu^{T-3}}{1-\lambda^{T-3}\mu^{T-3}}$.

Given that trader 2 randomizes with equal probability in the interval of prices $(1-\delta(1-\delta\pi_3), 1]$, the optimal strategy of trader 1 is, again, either A, B or C. It is evident that the payoff of 1 for taking actions A and B are the same as they are in $T-2$. Therefore, consider the payoff of 1 by making offer C. We have:

$$\begin{aligned} & \lambda^{T-3}\mu^{T-3} + (1-\lambda^{T-3}\mu^{T-3})\delta(\bar{\mu}^{T-2}(1-\delta) + \delta^2\pi_3) = \\ & = \frac{1}{1-\bar{\mu}^{T-2}}[\mu^{T-3} - \bar{\mu}^{T-2} + \delta(1-\delta)\bar{\mu}^{T-2}(1-\mu^{T-3}) + \delta^3\pi_3(1-\mu^{T-3})] \end{aligned}$$

It is true that B is always preferred to A. Moreover there exists an $\bar{\mu}^{T-3}$ such that when $\mu^{T-3} = \bar{\mu}^{T-3}$ then seller 1 is indifferent among B and C, when $\mu^{T-3} > \bar{\mu}^{T-3}$ she prefers C and when $\mu^{T-3} < \bar{\mu}^{T-3}$ she prefers B.

Plain algebra shows that for any $\delta \in (0, 1)$ and any fixed π_3 we always have $\bar{\mu}^{T-3} > \bar{\mu}^{T-2}$. Moreover, for some δ , there exists now an interval of π_2 with $\pi_2 > \delta\pi_3$, such that B would be preferred to C. In other words, the set of π_2 for which, for any given δ , 1 does not play B (and therefore the theorem not hold) is shrinking as the number of rounds has increased.

To complete the proof, we show that, for any fixed $\delta \in (0, 1)$ and $\pi_3 \in (0, 1)$, as time gets large, the values that $\pi_2 > \delta\pi_3$ can take such that trader 1 prefers to follow strategy C, shrinks to the empty set. Using the equilibrium construction developed so far, we must prove that, in any general

round t , if the start with $\mu^{T-t} > \bar{\mu}^{T-t+1}$ then we get that the threshold value $\bar{\mu}^{T-t}$ grows over time and it becomes higher than 1 when t is sufficiently large (i.e. the deadline is sufficiently far away).

Note that R^{T-t+1} and $V^{T-t+1}(0)$ are stationary, and, when $\mu^{T-t} > \bar{\mu}^{T-t+1}$, then $V^{T-t+1}(\mu^{T-t}) = 0$ (as in $T-t+1$, trader 1 follows strategy C under that belief profile).²⁴ Therefore the strategy of buyer 2 can be defined as in figure 12 in any round, using $\bar{\mu}^{T-t+1}$, δ , t , and π_3 only. This implies that (i) the seller contemplates only strategies A, B and C above, (ii) and the value of B is fixed and greater than A and (iii) the value of C is determined using μ^{T-t} , $\bar{\mu}^{T-t+1}$, δ , t , and π_3 only. We can then equate B to C and solve for μ^{T-t} in order to obtain $\bar{\mu}^{T-t}$.

To do this, recall that trader 2, when she is offered high prices, must mix as to induce belief $\bar{\mu}^{T-t+1}$ in round $T-t+1$. Therefore, Bayes rule implies that λ^{T-t} is such that

$$\bar{\mu}^{T-t+1} = \frac{(1 - \lambda^{T-t})\mu^{T-t}}{1 - \lambda^{T-t}\mu^{T-t}}.$$

We can now compute the value of C in round $T-t$ and by equating C with B we get the threshold $\bar{\mu}^{T-t}$ in round $T-t$. In particular:

$$\bar{\mu}^{T-t} = \bar{\mu}^{T-t+1} \frac{1 - \delta(1 - \delta) - \delta^2\pi_3}{\delta + (1 - \delta)^2\bar{\mu}^{T-t+1} - \delta^3\pi_3} + \frac{\delta^2(1 - \delta)\pi_3}{\delta + (1 - \delta)^2\bar{\mu}^{T-t+1} - \delta^3\pi_3}.$$

Plain algebra shows that $\bar{\mu}^{T-t} > \bar{\mu}^{T-t+1}$ for all $\delta \in (0, 1)$. To see that after some finite number of rounds it must become optimal for the seller to follow the line of action B (and therefore the theorem holds) consider that the worst case scenario is one where $\pi_3 = 0$ (i.e., there is no value from reselling). In this case the threshold simplifies to:

$$\bar{\mu}^{T-t} = \bar{\mu}^{T-t+1} \frac{1 - \delta(1 - \delta)}{\delta + (1 - \delta)^2\bar{\mu}^{T-t+1}}.$$

By solving this difference equation one gets, for some arbitrary constant C :

$$\bar{\mu}^{T-t} = \frac{\delta}{\delta - \delta \left(\frac{\delta}{1 - \delta + \delta^2} \right)^t + \left(\frac{\delta}{1 - \delta + \delta^2} \right)^t - \delta \left(\frac{\delta}{1 - \delta + \delta^2} \right)^t C + \delta^2 \left(\frac{\delta}{1 - \delta + \delta^2} \right)^t C},$$

which, because $(1 - \delta + \delta^2) > \delta$, converges to 1. This implies that when $\pi_3 > 0$ the set of π_2 , for which the best reply of the initial seller is to follow strategy C becomes null as the number of rounds gets large. ■

²⁴Note that if $\mu^{T-t} \leq \bar{\mu}^{T-t+1}$ then we have an equilibrium where the good flows immediately, and this is the case also if the number of round increases.

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