ESSAYS ON COMOVEMENT

YIXIN LIAO

A thesis submitted for the degree of Doctor of Philosophy in Finance

Essex Business School

University of Essex

July 2018

Abstract

This thesis consists of three essays on comovement between individual stocks and the S&P 500 index, each constituting a separate chapter. These essays provide use with new angles to see the comovement. First, they find that comovement does not always change as people predict. Secondly, they find the motivation to use 3- and 4-factor models to estimate excess comovement. Finally, they use PE ratios to test the reason behind the comovement.

The first essay (Chapter 2) finds that the univariate betas of 36-46% of our sample of 733 added stocks 1976-2015 decrease each year following addition. Moreover, a majority of the 192 deleted stocks increase rather than decrease each year at monthly frequency. This chapter develops a stylised model in which leverage constrained investors like pension funds are index trackers but unconstrained investors like hedge funds employ a betting-against-beta (BAB) strategy to capture this. Decreasing betas can be explained by hedge funds shorting the high beta stocks to be included in the index and this effect more than counters the index tracking effect.

The second essay (chapter 3) finds that returns on the S&P 500 index, small-minus-big, high-minus-low, and momentum factors are cross correlated and hence that 3- and 4-factor models are more appropriate to estimate excess comovement. This chapter finds significant changes in beta even when 3- and 4-factor models are used. It further confirms that momentum plays an essential role in comovement.

The final essay (chapter 4) use a new method to investigate the role sentiment plays in the comovement. The chapter develops an equation to show how the PE ratio pattern should be in an efficient market and find empirical evidence to reject the null that the market is efficient. The chapter confirms that the sentiment plays essential roles in comovement.

Acknowledgements

It is a long journey to finish a thesis for the degree of PhD. I would like to thank people who accompanied me, helped me, and supported me during my studies at the University of Essex.

First, I want to thank my parents. They give their unbounded love to me. They are the power that encourages me to keep moving forward every time I want to give up.

Second, I would like to thank my supervisors, Professor Jerry Coakley, and Professor Neil Kellard. Without them, I could not finish this thesis. Jerry guides and encourages me to solve tricky problems when I was faced with a dead-end. Neil helps me to recognise mistakes of setups in my theoretical model. It is very nice to work with two gentlemen who lead me to success in my PhD.

Besides, I would like to thank my friends, Hao, Jiecheng, Mingchen, Jiazhen, Chenwei, Xuran, Xiaoran, Lili, Hanwen, and Wentong. With them, I am not alone in this country which is 13-hour-flight away from my home but so close to my heart. It is with friendship that I sail across the ocean to achieve my goals.

Contents

List of Figures

List of Tables

Chapter 1. Introduction

Comovement is a broad concept and different kinds of comovement have been studied. This could be the comovement between different national markets. For example, Chelley-Steeley and Steeley (1999) find a rise in comovement between European equity markets after removing exchange control by using impulse and vector autoregressions. Karolyi and Stulz (1996) also find a comovement between Japan and US while Schöllhammer and Sand (1985) find several countries, such as Germany, the UK, the Netherlands and Switzerland, in Europe move together and stocks in all of those countries' market have relationship between US market. Comovement between different securities are also found. For instance, Nieh and Lee (2001) find a short-term significant relationship between stock prices and exchange rates for G-7 countries. Further Natanelov *et al*., (2011) find comovements between agricultural commodities futures prices and crude oil through vector error correction model and threshold cointegration. Researchers also found comovement between individual securities and the index. For example, Vijh (1994) and Barberis, Shleifer, and Wurgler (2005) find comovement among S&P 500 index additions and deletions. Further Boyer (2011) find comovement when S&P 500 value and growth indexes change their memberships. This thesis studies change in comovement between individual stocks with the S&P 500 index. This is a special case of style investing where investors consider investments at the level of different styles to simplify the decision-making process. Barberis and Shleifer (2003) constructed a theoretical model to suggest that individual stocks move closer with the style to which they are added. This means that stocks added to the S&P 500 index should exhibit increasing values of beta after additions. It also implies that comovement decreases between stocks and the style they leave. This is to say that betas decrease after stocks are deleted from the S&P 500 index.

Based on the theoretical model in Barberis and Shleifer (2003), Barberis et al. (2005) report change in betas and sensitivities to the non-S&P 500 index through the univariate and the bivariate regressions. Barberis et al. (2005) find significant increases in betas after stocks are added to the S&P 500 index and significant decreases in betas after deletions. Their findings are consistent with the theoretical model in Barberis and Shleifer (2003) and Vijh (1994) where changes in betas after additions are reported for first time.

Debates focuses on the reason why betas change after changes in the constituents of the S&P 500 index. There are two main theories to explain comovement. The classical theory claims that betas increase after additions simply because fundamentals comove more closely. This explanation dependents on the efficiency of the market. In a completely efficient market, price should always fundamental value, and hence comovement should also reflect the fundamentals. On the other hand, behavioural finance theories try to explain comovement through three different views. First, the category view, suggested by Barberis and Shleifer (2003), suggests that investors put stocks into different categories to simplify decision making. Stocks in the same category should move together because of similar cash flows. After being added to a category, a stock should move more closely with the other members in the category because more common cash flows are invested. The second view is referred to as habitat view by Barberis et al. (2005). In this view, investors choose to trade only a subset of securities. This may due to transaction costs, international barriers, and lack of information. Changes in comovement stem from more common cash flows. The third view is the information diffusion view. This view suggests that stocks in the same group should have similar diffusion rates of information. When stocks are added to the group they should have more similar diffusion rates of information and hence more comovement. Increasing numbers of evidence among other index is also found. Boyer (2011) find excess comovement among changes in S&P 500 value and growth indexes, Greenwood and Sosner (2007) find excess comovement among changes in the Nikkei 225 index, and Claessens and Yafeh (2013) find excess comovement among additions to many national market indexes. These events provide evidence that prices move together for factors that are unrelated to fundamentals.

This thesis posits that the market is not completely efficient. As a result, behavioural views may be better able to explain comovement. Pedersen (2015) gives a simple example to explain why the market is not completely efficient. The market is in between inefficient and efficient so that people pay hedge funds to obtain alphas. Imagine that the market is completely efficient. Then the hedge fund market is inefficient because the hedge fund is not able to earn alpha in an efficient market, so investors should not pay. On the other hand, if the market is completely inefficient, investors should not pay the hedge fund for alpha either. This is because if the market is completely inefficient investors would simply be able to find the alpha by themselves. There is no point in paying for an easy job. At least investors should not pay so much to the hedge fund. Therefore, markets are in the between efficient and inefficient. Besides, the Standard and Pool states specifically that the change in the constituents of the S&P 500 index do not reflect changes in their fundamentals. This indicates that the category view, habitat view, and information diffusion view may be used to explain the comovement. Hitherto, researchers assume that betas always increase after additions and decrease after deletions. This assumption is supported by the extant empirical results except those in Vijh (1994). He reports a negative change in beta after additions during 1975-1979 period. Frazzini & Pedersen (2014) report that unconstrained investors follow a strategy of betting against beta (BAB). This is a potential explanation for negative changes in beta after addition events. Demand for and supply of event stocks from unconstrained investors have impacts on cash flows invested in the event stocks. As a result, the common factor after additions may be lower or higher if some investors follow a different trading strategy which has overlaps with the index tracking strategy.

Building on Chen et al. (2016), this thesis constructs a theoretical model to explain decreasing betas. In this model, the assumption that investors are all simple index trackers is relaxed. Instead, it is assumed that some unconstrained investors trade stocks based on the BAB strategy. Our model implies that defensive stocks should have increasing betas after additions. However, aggressive stocks would have decreasing betas after being added to the index. The empirical analysis divides our sample into two subsamples of increasing and decreasing betas to test hypotheses implied by our theoretical model. The results show that that increasing beta subsamples have significantly lower-than-one pre-addition betas. The average changes are 0.8879, 0.8567, and 0.9540 at the daily, weekly, and monthly frequencies, respectively, during 1976-2015 period. Moreover, dividing our full sample into subsamples with increasing and decreasing betas reveals further interesting patterns in beta changes. Both positive and negative changes in betas are stronger than the overall change in betas which is a result of the offsetting changes in individual beats. More fundamentally, the positive and negative changes are both stronger at lower frequencies which is contrary to the belief that sentiment dissipates and prices revert to fundamentals in the long-run.

Some recent papers argue that the empirical model used by Barberis et al. (2005) to estimate changes in betas is not appropriate. Kasch and Sarkar (2014) argue that 3- and 4-factor models should be used to control size effect, growth effect, and momentum effect. Further, Chen et al. (2016) posit that momentum plays an essential role in comovement. However excess comovement still persists after these factors are taken into account. Moreover, we cannot reject the view that comovement is a result of those 3 behavioural views even if we find insignificant changes in beta through 3- and 4-factor model. This is because we do not reject that returns on portfolios of small-minus-big, high-minus-low, and winners-minus-losers are driven by noise traders. If returns from size, growth, and momentum effects are determined by noise traders, changes in sensitivities to these factors after changes in constituents of the S&P 500 index are also evidence of excess comovement.

The traditional way to test if the reason for comovement is related to fundamental factors is to control fundamental factors that are potentially able to explain comovement. This is time consuming. For example, in this thesis, we support that size, growth and momentum effects can explain parts of comovement but reject the null that these factors can explain comovement completely. To conclude that comovement cannot be explained by other fundamental factors, such as liquidity factor, we need to recognise the factor and undertake the test with controlling the factor. In the 4th chapter, a new methodology is used to test the efficient market hypothesis on whom the null that the reason for comovement is related to fundamental factors is based. Some implications are tested which are derived from the dividend discounted model (DDM). The thesis concludes that the market is not efficient enough to conclude that the comovement is just because fundamentals through rejecting implications of DDM.

1.1. Motivation and contribution

Previous researches make conclusions based on average change in comovement of the overall sample (see Barberis et al., 2005; Boyer, 2012; Green and Hwang, 2009). Changes in comovement of individual stocks are not reported. This motivates us to examine if all stocks added to and deleted from the S&P 500 index have increasing comovement, and decreasing comovement, respectively. Further, Kasch and Sarkar (2014) suggests that size, growth and momentum have influences on comovement. However, they make this conclusion just using daily and weekly data and no formal tests are undertaken to test if these factors have statistically significant influences on comovement. As a result, we examine the influence of size and growth effects, and momentum factor on comovement using daily, weekly, and monthly data separately. Formal tests of power of these factors are also undertaken. Finally, traditional methodology that is used to test the null that comovement is related to fundamentals only is to control potential fundamental factors. This is time consuming because all fundamental factors need to be recognised until excess comovement vanishes. We are motivated to develop a new methodology to test the EMH which is the basis of the null hypothesis. Specific questions this thesis aims to answer are:

1. How does beta change after changes in the constituents of the S&P 500 index?
2. Can the size, growth, and momentum effects explain excess comovement?
3. Is the market efficient enough to support the belief that comovement simply reflects fundamental values moving more closely together?

This thesis makes the following contributions. First, it finds evidence to reject the consensus finding in the literature that beta always increase after additions and decrease after deletions. This finding provides us with a new angle to consider comovement. Second, it constructs a theoretical model to suggest that the overlap between different styles – index tracking and betting-against-beta influences the sign of changes in betas. Third, this thesis examines the correlations between S&P 500 index return and return on small-minus-big factor, between S&P 500 index return and return on high-minus-low factor, and between S&P 500 index return and return on momentum factor. We find low correlations that suggests that estimate of beta from the univariate regression is biased because omission of important variables, and 3- and 4-factor models are more appropriate. Finally, this thesis outlines a simple stylised model for testing the null that comovement is the result of fundamental values moving more closely following additions to and deletions from the index.

1.2. Chapters preview

The thesis is organised as follows. Chapter 2 examines changes in betas using the univariate regression, the bivariate regression, and the univariate regression on the non-S&P 500 index returns from 1976 to 2015. We find significant increases in beta at daily, weekly, and monthly frequencies, respectively. This is consistent with previous research's finding. However, we also find that overall change in beta does not give us the whole picture of the comovement. We find that the univariate betas of 36-46% of our sample of 733 added stocks 1976-2015 decrease each year following addition. Moreover, a majority of the 192 deleted stocks increase rather than decrease each year at monthly frequency. These novel findings are contrary to some of the main stylised factors in the comovement literature. They motivate us to construct a theoretical model to understand them. In the theoretical model, investors are assumed to be either index trackers or unconstrained investors who bet against beta (BAB). Our model suggests that BAB strategy plays an essential role in comovement and is helpful in understanding decreasing betas after additions and increasing betas after deletions.

Chapter 3 re-examine changes in betas by employing the Fama and French 3- and 4-factor models. It confirms that momentum and size factor have some influence on the comovement. It also undertakes an empirical test to examine the effect of momentum. In this chapter, it is argued that nonfundamental factors can still have an impact on comovement even though when size, growth, and momentum factors are considered.

Chapter 4 undertakes a test to examine the null that comovement always reflect the fundamental values using a regression of the change in individual stock PE ratios on that of the S&P 500. We first find that PE ratios are not stationary at least over a one-year window. This is contrary to the belief that price reflects fundamentals. If the market is efficient, prices should move with earnings and hence yield stationary PE ratios. Our regression results provide evidence to reject the view that market is efficient enough to support the belief that comovement stems from fundamental values moving more closely together following index changes.

Chapter 5 concludes with the main findings of the thesis. Ir provides suggestions for future research on the topics studied.

Chapter 2. Comovement is more perplexing than you think!

2.1. Introduction

The comovement literature has established that the stocks added to (or deleted) from the S&P 500 index exhibit subsequent increases (decreases) in their beta coefficients. These synchronised beta increases are referred to as comovement. Classical theories claim that risk factors, such as systematic or market risk, size, book-to-market ratio, and momentum factors, suffice to explain the changes in comovement. On the other hand, behavioural theorists posit that it can be rationalised by non-fundamental factors such category or style changes, preferred habitat, and information diffusion factors. There is a lively debate between the proponents of these opposing views. Barberis, Shleifer and Wurgler (BSW) (2005) pioneered the behavioural approach comovement and sparked a number of related studies supporting their viewpoint. More recently, Chen, Singal and Whitelaw (2016) have challenged the behavioural approach. They stress that momentum has been ignored and that this can explain a large part of the apparent comovement highlighted in the literature.

Interestingly, the underlying assumption of both approaches to comovement is that beta always increases (decreases) after a stock is added to (deleted from) the S&P 500 index. They overlook the potential heterogeneity revealed in the cross section of beta changes of individual stocks following inclusion in the index. This would not matter if the heterogeneity were limited in scope. Moreover, it is not as if the issue has not been raised previously. The first evidence of decreasing betas after addition events is found in the Vijh (1994) pioneering study. He reported a highly significant fall in beta of -0.072 for S&P 500 additions over the 1975-1979 period.[1] Ignoring negative beta changes can have important consequences. First, the average beta changes reported in the literature understate to varying degrees the actual extent of comovement associated with

[1] He also reports a fall in beta of -0.009 for 1985 additions although this result is not statistically significant.

only positive beta changes. Second, there are sound economic and financial reasons why betas might display reversal as well as continuation behaviour in the wake of entry into the S&P 500 for the first time. univariate regression beta changes largely reflect price changes and their associated cycles over one- to three-year periods and it is possible that daily and monthly beta changes may capture these cycles.

The main contribution of this chapter is that it establishes considerable and persistent heterogeneity in the cross section of beta changes of individual stocks following inclusion in and exclusion from the S&P 500 index. This heterogeneity reveals itself along three important dimensions. First, a substantial part of the annual samples of addition events exhibits subsequent decreases in beta. The decreasing beta proportion varies from 26% to 47% depending on the frequency (daily, weekly, or monthly) of the estimation. Similarly, focusing on the average beta change in individual years, our study reveals widespread evidence of. negative beta changes after S&P 500 addition events. It finds it in the daily beta for 10 individual years (over 20% of the total) over the course of our 1976-2015 sample period and in weekly betas for some 13 individual years (32.5%) of the 1976-2015 sample period.[2] The average changes in beta at monthly frequency are negative in 1976, 1979, 1980, 1981, 1984, 1985, 1986, 1991, 1998, 2006, 2008, 2014.

Second, the most compelling element of heterogeneity emerges when the focus is on specific beta changes after dividing total addition events into increasing and decreasing beta subsamples, respectively. For instance, the monthly betas exhibit a highly significant change of 0.1083 over the 1976-2015 sample period. However, the post-addition subsamples are characterised by far more dramatic statistically significant changes. The increasing beta subsample shows a mean beta increase of 0.6166 while the corresponding decreasing beta subsample shows a mean beta drop of 0.4714. These subsample beta changes are more than four and three times the

[2] This sample period is shortened as a 3-year window is employed for estimating monthly betas.

magnitude of the overall beta change of just 0.1083. This pattern is repeated at the weekly and daily frequencies also, albeit the changes are less sharp. The implication is that extant studies that focus on the overall beta changes dramatically understate both the extent and the complex nature of comovement. Finally, our sample of 192 index deletions reveal even stronger evidence of heterogeneity. This is because, while the overall change is statistically significant on 5% level of significance only at daily frequency, this conceals large and significant changes for both increasing and decreasing beta subsamples. For instance, the beta changes for increasing and decreasing beta subsamples are 0.7599 and -0.5939, respectively, at the monthly frequency. Strikingly, a majority of deleted stocks exhibit increasing rather than the expected decreasing beta changes at the monthly frequency.

The chapter's second contribution is that it develops a new stylised model of comovement in which investors can follow two rather distinctive investment styles (categories). Extant studies all focus on S&P 500 index tracking as the basis of comovement. The model also takes account of the impact of the Frazzini and Pedersen (2014) betting against beta (BAB) strategy. In the latter, leverage constrained investors like mutual and pension funds pursue index tracking. They hold portfolios to track the S&P 500 and have to adjust them when the index changes each quarter. However, unconstrained investors like hedge funds follow a BAB strategy. These investors divide all stocks into low and high beta groups based on their historical betas. Since they can use leverage to achieve higher alphas, their portfolios are based on overweighting on low-beta stocks and underweighting (or shorting) on high-beta stocks.

The setup for return on stocks must be adjusted when changes to the index are announced each quarter and there is an overlap in some of the new stocks to be included under each strategy. Ahead of the announced changes, hedge funds implementing their BAB strategy can kill two birds with one stone. They can earn the usual BAB returns by going long on new low beta and shorting new high beta stocks. However, if some of their new low beta stocks are amongst the announced

S&P additions, they can choose to sell them at a profit to S&P 500 index trackers or hold them as part of BAB. This implies that overlapping stock returns are impacted by these two groups of investors. In that case, the excess demand for low beta dual stocks by both index trackers and BAB followers is self-reinforcing. This is consistent with the usual index addition effect. By contrast, for dual stocks with above mean historical betas, the impact of underweighting (shorting) by BAB followers counters the excess demand by index trackers. If this is sufficiently strong, the upshot is the negative beta changes that were originally noted by Vijh (1994).

One may object that the BAB strategy is relatively recent whilst our evidence on both increasing and decreasing beta subsamples goes back to the beginning of our sample. One explanation is the BAB strategy takes advantage of beta as self-correcting-process. It is expected that in the long run betas should converge to one and the value of beta should fluctuate around one. The implication is that, following index inclusion, aggressive stocks may exhibit decreasing betas while defensive stocks may produce increasing betas. Another potential explanation is the defensive equity strategy where investors overweight defensive stocks and underweight aggressive stocks. This strategy is found to produce significant positive risk-adjusted returns.[3] Using this strategy, semi-active investors whose benchmark is the index may overweight defensive stocks and underweight aggressive stocks relative to the index although they also worry about the tracking error. Thus, a larger cashflow is invested in defensive stocks relative to the index while a smaller cashflow is invested in aggressive stocks relative to the index. The larger and smaller cashflow results in higher and lower volatility and hence higher and lower beta.

The BAB strategy explanation also links neatly to the speculative beta theory suggested by Hong and Sraer (2016). They posit that higher-beta stocks have higher disagreement and hence a higher disagreement premium. The latter premium increases the costs of index tracking. To be

[3] See Baker, Bradley, and Wurgler (2011) and Frazzini and Pedersen (2014).

more specific, index tracking investors readily purchase low-beta stocks because of the low or no disagreement premium. On the other hand, the high cost of buying high-beta stocks resulting from the disagreement premium prohibits investors from exactly tracking this part of the index. Hence, the cashflow which is a common factor for excess comovement invested in low-beta and high-beta stocks can be different and could result in increasing and decreasing beta patterns.

The remainder of this chapter is organized as follows. Section two briefly reviews the literature. Section three presents a new stylised theoretical model that has both index tracking and BAB investors. It also develops a list of hypotheses. Section four we describe methodology and data. Results are reported in section five. A conclusion is made in section six.

2.2. The comovement literature

Previous research on additions to the S&P 500 index assumes beta increases for all stock added to the S&P 500 index. The first paper studying the change in beta after additions to the S&P 500 index is Vijh (1994). In this paper, Vijh believes that the beta changes from understated to overstated when the stock is added to the S&P 500 index. Vijh (1994) uses the price pressure hypothesis and the nonsynchronous-trading hypothesis to explain the change in beta. The price pressure hypothesis indicates that betas always increase after stocks are added to the S&P 500 index while the nonsynchronous-trading hypothesis suggests that the sign of change in beta depends on the change in trading frequencies after additions. However, Vijh (1994) believes that the price pressure hypothesis is more important than the nonsynchronous-trading hypothesis and hence betas increase no matter what the change in trading frequencies after additions. He also reports empirical results that betas increase, on average, by 0.08 significantly and 0.037 insignificantly, at the daily and weekly frequencies, during 1975-1989. His results for subsamples at the daily frequency are all significantly positive with the exception that that the average change in betas is -0.072 during 1975-1979 which is not the focus of the paper. The decreasing betas during 1975-1979 indicate that betas do not always increase after additions.

Barberis and Shleifer (2003) construct a theoretical model to study style investing, including index investing. The model implies that a stock should comove more with the index which the stock is added to. Their model assumes that some investors called switchers consider investments at the level of styles. Switchers move funds from a style whose past performance is poor to one which has a stronger performance. It is also assumed that switchers buy an equal number of shares of each asset in the style where the funds are invested. This assumption is not consistent with the S&P 500 index investing because the S&P 500 index is cap-weighted. Index tracking investors would like to construct a portfolio in line with the S&P 500 index to minimise tracking error. Furthermore, stocks added to the index can also be demanded or shorted by investors following other styles. For example, an aggressive stock added to the S&P 500 index can be shorted by investors who are using a defensive equity strategy which results in different demands for each asset in the S&P 500 index.

Barberis et al. (2005) construct an additional theoretical model to explain changes in betas after changes in the constituents of the S&P 500 index. They assume that numbers of shares of each asset in the style demanded are equal, and that returns on individual stocks are determined not only by change in fundamentals but also by change in sentiment about the category where the stock is. This assumption is shared by Chen et al. (2016) who assume that parts of sentiment about the category can influence returns on stocks. These papers also imply that betas increase after stocks are added to the S&P 500 index. Nevertheless, stocks in the same category can be allocated to different categories by different investors following different trading strategies. For instance, stocks added to the S&P 500 index should be put in the S&P 500 index category by index investors. However, other investors following a defensive equity strategy may take the view that some S&P 500 stocks should be put in defensive and aggressive stock categories. The implication is that returns on stocks may be determined by sentiments not only about one category but also about other categories.

Barberis et al. (2005) further introduce a friction-based theory of comovement. It argues that comovement increases after additions because the diffusion rate of information about added stocks are closer to that of information about the S&P 500 index. This theory does not consider different results caused by gradual information diffusion. As Hong & Stein (1999) suggested, gradual information diffusion can result in underreaction in short-term and overreaction in long-term. We believe that event stocks may suffer from different influences of gradual information diffusion although Hong & Stein (1999) support that small, low-analyst-coverage stocks should have more gradual information diffusions.

Empirical tests are undertaken based on the presumption that betas always increase after stocks are added to the S&P 500 index. Barberis et al. (2005) reports significant increases in beta for added stocks from 1976 to 2000 at the daily and weekly frequencies through univariate regressions. Results are stronger when bivariate regressions are used. Kasch & Sarkar (2014) similarly reports significant increases in beta after additions at the daily and weekly frequencies during the 1989-2012 period. Their results are significant at only the daily frequency when the 3- and 4-factor models are used. They show that beta increases are lower when the 3- and the 4-factor models are used. Chen et al. (2016) reports additional increases in beta after additions during the 1976-2012 period. Similar results are also found when other indexes change their constituents (see Boyer, 2011; Claessens and Yafeh, 2012; Greenwood and Sosner, 2007; Greenwood, 2008; Green and Hwang, 2008). These results hold over their full sample periods and also for subsamples based on the increasing beta presumption for stocks added to the index.

Various investment and trading strategies have been shown to impact on betas. Chen et al. (2016) show that momentum influences changes in comovement. It means that profitable trading strategies - including long-small-short-big, long-value-short-growth, and momentum strategies (see Fama & French., 1993 and Jagadeesh & Titman., 1993) – may affect changes in betas. The defensive equity strategy is another profitable strategy (see Black, 1972 and Black, Jensen, and Scholes., 1972) that influence the change in betas. Such a strategy could influence demand for different stocks in the S&P 500 index and hence changes in those stocks' betas. Hong and Sraer (2016) construct a theoretical model to explain the defensive equity strategy. Investors are divided into two groups in their model. These include investors with different opinions on fundamentals and subject to constraints on short sales and investors with the same opinions on fundamentals who are allowed to short sell. Their model implies that aggressive stocks have higher disagreement premiums. This disagreement premium may differentiate the demand for newly added stokes to the S&P 500 index bought by investors engaged in index tracking and hence impact the change in betas. Above papers discussed give us a motivation to construct a theoretical model to understand how investors following different strategies can influence changes in beta.

2.3. Stylised model with BAB and index tracking strategies

This subsection outlines a stylised model of comovement in which the focus is on two distinctive investment styles (categories): S&P 500 tracking and the Frazzini and Pedersen (2014) betting against beta (BAB) strategy. The objective is to develop a model for newly added index stocks that can explain both positive and negative post-addition beta changes. However, it first, outline a conventional model of excess co-movement.

2.3.1. Excess co-movement and index tracking

Following framework analogous to Chen et al. (2016), note that:

$$r_t = b_{rt}f_t + c_{1t}u_{1t} + c_{2t}u_{2t} + e_{rt} \tag{2.1}$$

$$x_{1t} = b_{1t}f_t + u_{1t} + e_{1t} \tag{2.2}$$

$$\mathrm{x_{2t} = b_{2t}f_t + u_{2t} + e_{2t}} \tag{2.3}$$

where r_t represents the return on an individual stock that is changing membership between group 1 (non-index stocks) with group return x_{1t} , and group 2 (index stocks), with group return x_{2t}. In this model, returns on (individual and group) stocks are determined by the fundamental, common return shock, f_t, group-specific non-fundamental return shocks, u_{it}, and an idiosyncratic return shock, e_{it}. Further assume that all return shocks have constant variance, that group-specific non-fundamental return shocks are independent to each other, and that fundamental return shocks, group-specific non-fundamental return shocks and idiosyncratic return shock are independent:

$$var(e_{it}) \equiv \sigma_{eit}^2, var(u_{it}) \equiv \sigma_{eit}^2,\ \ var(f_t) \equiv \sigma_{ft}^2 \tag{2.4}$$

$$cov(u_{1t}, u_{2t}) = 0$$

$$cov(u_{it}, f_t) = cov(e_{it}, f_t) \equiv\ cov\big(u_{it}, e_{jt}\big) \equiv 0\ \ \forall i, j$$

Chen et al. (2016) suggest that the excess comovement hypothesis can be encapsulated by the sensitivities (c_{it}) of individual stocks to the group-specific non-fundamental return shocks, u_{it}. In particular, employing underbars and overbars to designate loadings before and after a stock switches from non-index status to index inclusion, then conventional excess co-movement implies:

$$\underline{c}_{1t} = \underline{c}_1 > 0 \text{ and } \underline{c}_{2t} = \underline{c}_2 = 0; \tag{2.5}$$

and

$\overline{c}_{1t} = \overline{c}_1 = 0$ and $\overline{c}_{2t} = \overline{c}_2 > 0.$ (2.6)

where $\overline{c}_{it}$ and $\underline{c}_{it}$ denotes sensitivity of individual stocks to the group-specific non-fundamental return shocks after and before a stock switches from non-index to index, respectively. Following BSW (2005) and Chen et al. (2016), we assume that sensitivities of individual stocks to the group-specific non-fundamental return shocks do not change during periods before and after a stock switch from non-index to index: $\overline{c}_{it} = \overline{c}_i$ and $\underline{c}_{it} = \underline{c}_i$.

In other words, when a stock is allocated to a particular style, its return is positively correlated with that group's non-fundamental return shock, and uncorrelated with the non-fundamental returns shock of the group it has exited. For example, a stock newly entering the S&P 500 will not only become more prominent, but it will also be purchased by index-trackers, driving an increased correlation between the individual stock and the index.

2.3.2. Allowing for leverage constrained and unconstrained investors

Assume that there are two types of investors in the economy. Firstly, leverage constrained investors like mutual and pension funds pursue index tracking. They hold portfolios to track the index, including the S&P 500, and the non-S&P 500 index, and have to adjust them when the index changes each quarter. However, unconstrained investors like hedge funds follow a BAB strategy. These investors divide all stocks into low and high beta groups based on the mean historical beta. Since they can use leverage to achieve higher alphas, their portfolios are based on overweighting on low-beta stocks and underweighting (or shorting) on high-beta stocks.

The setup for return on stocks must be adjusted when changes to the index are announced and there is an overlap in some of the new stocks to be included under each strategy. Ahead of the announced changes, hedge funds implementing their BAB strategy can kill two birds with one stone. They can earn the usual BAB returns by going long on new low beta and shorting new high beta stocks. However, if some of their new low beta stocks are amongst the announced S&P additions, they can choose to sell them at a profit to S&P 500 index trackers or hold them as part of BAB. This implies that overlapping stock returns are determined by these two groups of investors. In that case, the loadings of individual stocks to the group-specific non-fundamental returns shocks, might usefully be decomposed in the following manner:

$$c_{it} = c_{i,indext} + c_{i,babt} \tag{2.7}$$

where the $c_{i,indext}$ is the sensitivity driven by index trackers, whilst $c_{i,babt}$ reflects the sensitivity due to our unconstrained investors. For example, when a stock is in group 1 (the non-S&P 500 index) its loading to the group-specific non-fundamental shocks is determined by index trackers tracking the non-S&P 500 index and the unconstrained investors. We also assume that sensitivities driven by index trackers and unconstrained investors do not change during periods before and after the stock switches form non-index to index. As a result, in terms of post-addition sensitivities to S&P 500 index tracking behaviour, we would assume that analogously to (2.5):

$$\overline{c}_{2,indext} = \overline{c}_{2,index} > 0. \tag{2.8}$$

In other words, $\overline{c}_{2,indext}$ is always positive due to the pure index inclusion effect. However, given that BAB strategies are conditional on historical betas, the crucial insight is that $\overline{c}_{2,babt}$ can vary in sign depending on the leg (underweighting high beta or overweighting low beta stocks) of the BAB strategy. For dual stocks with low (below mean) historical betas, the implication is:

$$\overline{c}_{2,babt} = \overline{c}_{2,bab} > 0 \Rightarrow \overline{c}_{2,t} = \left(\overline{c}_{2,indext} + \overline{c}_{2,babt}\right) = (\overline{c}_{2,index} + \overline{c}_{2,bab}) > 0. \tag{2.9}$$

This is because the excess demand for low beta dual stocks by both index trackers and BAB followers is self-reinforcing. It explains why the beta increases for subsample with increasing beta are much higher than the mean beta increases. This is consistent with the usual index addition effect. By contrast, for dual stocks with above mean historical betas, underweighting (shorting) by BAB followers implies:

$$\overline{c}_{2,babt} = \overline{c}_{2,bab} < 0. \tag{2.10}$$

In this case, the BAB and index tracking strategies have conflicting impacts. In particular, if the excess demand from index trackers is less than the excess supply from BAB strategists wishing to underweight such stocks then:

$$\overline{c}_{2,t} = \left(\overline{c}_{2,index} + \overline{c}_{2,babt}\right) = \left(\overline{c}_{2,index} + \overline{c}_{2,bab}\right) < 0. \tag{2.11}$$

Thus, our model can explain both beta increases and decreases following index additions.

2.3.3. Hypotheses

Our model allows for two distinct types of investors. How is this likely to affect the empirical evidence for any excess comovement? Typically, such evidence is garnered from a regression such as:

$$r_t = \alpha + \beta x_{2t} + \epsilon_t. \tag{2.12}$$

Note that Chen et al. (2016) show that:

$$\underline{\beta}_{x_2} = \frac{\underline{b}_r \underline{b}_2 \underline{\sigma}_f^2}{\underline{\sigma}_{x2}^2} \quad (2.13)$$

and

$$\overline{\beta}_{x_2} = \frac{\overline{b}_r \overline{b}_2 \overline{\sigma}_f^2 + \overline{c}_2 \overline{\sigma}_{u2}^2}{\overline{\sigma}_{x2}^2} \quad (2.14)$$

Therefore, that the post-addition change in beta is:

$$\overline{\beta}_{x_2} - \underline{\beta}_{x_2} = \frac{\overline{c}_2 \sigma_{u2}^2}{\sigma_{x2}^2} \quad (2.15)$$

From (2.15), it can be seen that the sign of the post-addition change in beta is determined by solely the sign of $\overline{c}_2$. Under (2.6) or (2.9), we encounter the standard result that if a stock is added to the S&P 500 index, the change in its beta is positive. However, if new condition (2.11) holds, then the beta change from (2.15) will be negative.

This leads to the following testable hypotheses that involve refinements and extensions of the comovement hypothesis. The first two hypotheses involve refinements of the basic comovement hypothesis by acknowledging that post-addition beta changes are more heterogenous than hitherto believed. The first hypotheses involve a preliminary test of the impact of the BAB strategy for added stocks.

Hypothesis 1A: Whilst the average post-addition univariate beta changes are positive, the distribution of changes exhibits a sizeable number of negative beta changes;

Hypothesis 1B: The distribution of the bivariate and Chen et al. post-addition beta change changes also exhibit a sizeable number of negative beta changes.

The second hypotheses are a preliminary test of the impact of the BAB strategy for deleted stocks.

Hypothesis 2A: Whilst the average post-deletion univariate beta changes are negative, the distribution of changes exhibit a sizeable number of positive beta changes;

Hypothesis 2B: The distribution of the bivariate and Chen et al. post-deletion beta change changes also exhibit a sizeable number of positive beta changes.

The final hypothesis involves a direct test of the implication of the BAB strategy for pre- and post-addition betas for decreasing and increasing beta subsamples.

Hypothesis 3A: Stocks with high (above 1) historical betas will exhibit negative post-addition beta changes;

Hypothesis 3B: Stocks with low (below 1) historical betas will exhibit positive post-addition beta changes.

The following sections empirically assess these hypotheses.

2.4. Data and empirical results

2.4.1. Data

Data are used to examine effect of S&P 500 index inclusions and deletions over September 1976 to December 2015 and over January 1979 to December 2015, respectively. The list of event firms is from the Compustat North America database. There are 905 inclusion and 878 deletion events in each sample period. Following Barberis et al. (2005), addition events are excluded if the firm is from restructuring or spinning off a firm already in the index or if the firm is involved in a merger or takeover around the event. We also exclude the event firm if the firm's sample in the pre- or post-event window is lower than 30 at the daily and weekly frequencies. For the monthly test, the event is excluded if it happened so close to the end of the sample period that its sample size is too small. This results in 733 addition events at the daily frequency, 726 at the weekly frequency, and 640 at the monthly frequency. The numbers of deletion events are 193, 181, and 143 at the daily, weekly, and monthly frequencies, respectively. [4]

2.4.2. Time series patterns

Figure 1, figure 2, and figure 3 display the patterns of average changes after additions in the univariate regression beta from 1976 to 2015 for the whole sample and the increasing- and decreasing-beta subsamples at the daily, weekly, and monthly frequencies. The figure also reports the percentage of stocks with decreasing beta in each year.

[Insert Figure 2.1 here]

[4] Weekly data are constructed from daily data because we cannot obtain data at weekly frequency.

[Insert Figure 2.2 here]

[Insert Figure 2.3 here]

These figures clearly indicate beta does not always increase after addition events. At the daily frequency, the average changes in beta are negative in 10 individual years. The average weekly changes in beta are negative in 13 years. Finally, average monthly changes in beta are negative in 13 years: 1976, 1979, 1980, 1981, 1984, 1985, 1986, 1991, 1998, 2006, 2008, 2014, and 2015. There is some overlap between negative decreasing betas at both the weekly and monthly frequencies (1979, 1980, 1981, 1985, 1991and 2014) and between the daily and monthly frequencies (1976, 1979, 1980, 2006, and 2014). As the percentage of decreasing beta suggested, the prevalence of decreasing vary as time and data frequencies.

These figures also indicate that increasing and decreasing betas offset each other to a greater or lesser extent each year. The magnitudes of beta changes in both subsamples is much higher than average change in beta for whole sample. This suggests that the relatively small average changes in beta reported in the literature might be due to offsetting large positive and negative beta changes and not because excess comovement is in decline. Moreover, the pattern of average changes in beta does not exhibit a trend. This contraries to the Barberis et al. (2005) view that stronger comovement should be found when more recent data are used because of the increasing importance of index investing.[5]

[5] Their conclusion may have been affected by the fact that their sample ended in 2000 at the height of the dotcom boom.

Figure 2.4, figure 2.5, and figure 2.6 illustrate average change in beta estimated from the univariate regression after deletions from 1979 to 2015 for overall sample, and subsamples with increasing beta and decreasing beta at the daily, weekly, and monthly frequencies. The percentage of sample with increasing beta is also reported.

[Insert Figure 2.4 here]

[Insert Figure 2.5 here]

[Insert Figure 2.6 here]

These figures clearly show that the betas for deleted stocks exhibit far more heterogeneity than hitherto believed. At the daily frequency, there are 9 years (1981, 1982, 1984, 1985, 1987, 1988, 1990, 1991, and 1998) in which none of the deleted stocks has an increasing beta and the overall beta change is positive in 9 years. The overall change in weekly betas is positive in 9 individual years, too. Moreover, all deleted stocks in 1984, 2003, and 2004 have increasing betas. The overall change in monthly betas is positive in no less than 16 individual years: 1983, 1992, 1996, 1997, 1998, 2000, 2001, 2002, 2003, 2004, 2006, 2008, 2011, 2012, 2014, and 2015. Furthermore, all event stocks have increasing beta in 2002, 2003, and 2006. This stronger pattern of counterintuitively increasing comovement for deleted stocks at the monthly frequency contrast with the analogous findings for added stocks

Above figures demonstrate that stock index comovement is a far more complex and nuanced phenomenon than hitherto assumed. Numerous stocks added to the S&P 500 exhibit post-inclusion decreases in comovement with the index. Correspondingly, realtively more deleted stock subsequently increase their comovement with the index. These challenges to the theory of comovement are investigated in the following sections.

2.4.3. Regression results for S&P 500 additions and deletions

We employ three different regression models to obtain estimates of beta before additions and to estimate the average change in beta. These are the univariate and bi-variate models introduced by Barberis et al. (2005). Return on individual stock is regressed on return on the S&P 500 index only and on both return on the S&P 500 index and return on the non-S&P 500 index. Returns on the S&P 500 are from the CRSP Index on the S&P 500 Universe file:

$$r_{j,t} = \alpha_{j,t} + \beta_j r_{SP500,t} + v_{j,t}$$

$$r_{j,t} = \alpha_{j,t} + \beta_{j,SP500} r_{SP500,t} + \beta_{j,nonSP500} r_{nonSP500,t} + v_{j,t}$$

Chen et al. (2016) univariate regression of a stock's return on the non-S&P 500 index return is also used:

$$r_{j,t} = \alpha_{j,t} + \beta_{j,nonSP500} r_{nonSP500,t} + v_{j,t}$$

Returns on non-S&P 500 index are inferred from the identity: $r_{vwcrsp,t} = \left(\frac{CAP_{crsp,t-1} - CAP_{SP500,t-1}}{CAP_{crsp,t-1}}\right) r_{nonSP500,t} + \left(\frac{CAP_{SP500,t-1}}{CAP_{crsp,t-1}}\right) r_{SP500,t}$.

For daily, and weekly data, one-year window before and after event is used to estimate value of beta before and after event, respectively, for each stock. The change in beta is assessed as the difference between beta after the event and that before the event: $\Delta\beta_j = \beta_{j,after\ event} - \beta_{j,before\ event}$. For monthly data pre-event and post-event windows are extended to 3 years. The average changes in beta for each year from 1976 to 2015 are estimated. The t-test is used to examine the significance of the average change in beta. The sample is then divided into increasing and decreasing beta subsample.

We first re-examine the univariate regression and the bi-variate regression reported in Barberis et al. (2005) and then the univariate regression of event stocks' return on the non-S&P 500 index reported in Chen et al. (2016).

[Insert Table 2.1 here]

Table 2.1 reports results of the univariate and the bi-variate regressions (see Barberis et al., 2005). Following Barberis et al. (2005), results for addition events over September 1976-2000 are reported while that for deletions over 1979-2000 are reported. Subsamples over 1976-1987 and over 1988-2000 for additions are then reported, respectively. Our results of additions are consistent with Barberis et al. (2005) and values are very close. The average change in beta after additions is overall 0.16, 0.1178, and 0.0365 at daily, weekly, and monthly, respectively, estimated by the univariate regression. These values are, except the monthly result, highly significant. Results are stronger when the bi-variate regression is used which supports Barberis et al. (2005). For example, beta increases on average by 0.3481, by 0.1947, and by 0.2839 at daily, weekly, and monthly frequency, respectively. Our results also confirm that stock should have less comovement with the group of stocks where it leaves from. The change in sensitivity of stock to the non-S&P 500 index is -0.3351, -0.1222, and -0.2182 on average at daily, weekly, and monthly frequency, respectively.

[Insert Table 2.2 here]

Table 2.2 reports estimators of beta and the sensitivity to the non-S&P 500 index before and after the additions and change in both of them estimated by the univariate regression on the S&P 500 index return, by the univariate regression on the non-S&P 500 index, and by the bivariate regression from 1976 to 2012. The difference between changes in the beta and that in the sensitivity to the non-S&P 500 index. Our results are consistent with Chen et al. (2016). Chen et al. (2016) claims that stocks comove closer not only with the S&P 500 but also with the non-S&P 500 index. This is supported by results of their univariate regression. Our results also support this argument. For instance, the change in the sensitivity to the non-S&P 500 index is 0.0343 in 1976-2012 period and 0.0851 in 2001-2012 period. They also argue that the bivariate regression is not informative about excessive comovement although results from the bivariate regression suggest that stocks comovement less with the non-S&P 500 index.

Nonetheless, above researches are conducted based on the belief that stocks always have increasing beta after addition events while have decreasing beta after deletions. As the time series patterns of change in beta suggested, beta does not always increase after additions. It does not always decrease after deletions either. We extend samples to 2015 and divide our sample into subsamples with increasing and decreasing betas to test hypotheses in section 2.3.3.

2.4.4. Evidence from increasing and decreasing beta subsamples

Table 2.3 reports results of addition events from univariate regression and bivariate model at daily, weekly, and monthly frequencies from 1976 to 2015. It also reports results of increasing- and decreasing-beta subsamples.

[Insert Table 2.3 here]

univariate regression betas increase by 0.1279, 0.0882, and 0.1083 after additions at the daily, weekly, and monthly frequencies, respectively. These increases are all significant at the 1% level. The magnitude of the increases is much higher when the bivariate model is used. The overall change in beta is 0.3545, 0.2230, and 0.2899 at the daily, weekly, and monthly frequencies. At the same time, the sensitivity of added stocks relative to the non-S&P 500 index decreases by 0.3131, 0.1548, and 0.1586 at daily, weekly, and monthly frequencies, respectively. These results are consistent with Barberis et al. (2005) and support the comovement view of beta increases after addition events.

However, these are aggregate results. The results for both individual stocks and for the stock subsamples with of increasing and decreasing betas reveal some really interesting and novel patterns. 269 of 733 stocks have decreasing univariate regression betas after additions at the daily frequency, and 333 of 726, and 299 of 640 at the weekly and monthly frequencies, respectively. For the decreasing-beta subsample, the change of -0.2671, -0.4593, and -0.4714 in beta at the daily, weekly, and monthly frequencies, respectively, are all significant at the 1% level. These changes are much larger in magnitude than the overall changes because they are offset by even larger increasing betas. The increases for the increasing-beta subsample are 0.3569, 0.5521, and 0.6166 at the daily, weekly, and monthly frequencies, respectively, and all are highly significant.

The magnitude of the increasing betas is higher than that of decreasing betas which leads to a positive overall change at all frequencies, in line with most results reported in the literature. However, the overall change is a net change sand hides some very big offsetting changes in betas. Both the increasing- and decreasing-beta subsamples exhibit (absolutely) larger changes as the data frequency decreases from the daily to the monthly levels. For instance, the monthly overall beta increase of 0.1083 stems from the 0.6166 increase for the increasing beta subsample being offset by the -0.4714 decreases for the decreasing beta subsample.

The bivariate model beta increases are approximately triple those of the univariate regression and all highly significant, is consistent with Barberis et al. (2005). They are 0.3545, 0.2230, and 0.2899 at the daily, weekly, and monthly frequencies, respectively. The numbers of stocks with decreasing betas are slightly smaller than those for the univariate regression. The decreasing beta subsample yields falls of 0.3853, 0.9525, and 1.0412 at the daily, weekly, and monthly frequencies, respectively. Much larger changes in both subsamples are found when lower-frequency data are used. The monthly overall beta increase of 0.2901 stems from the 1.1432 increase for the increasing beta subsample being offset by -1.0412 for the decreasing beta subsample. The sensitivity to the non-S&P 500 index decreases absolutely for the increasing-beta sample to while it increases sharply at the monthly level for the decreasing-beta sample. This is evidence against the conclusion of Chen et al. (2016) that stocks comove more with all groups but not just the one they join to. Moreover, significant decreasing beta after inclusion events found in both models support our hypothesis that although the overall change in beta after inclusions is positive there are sizable negative change in beta estimated by both univariate regression and bivariate regression.

Table 2.4 reports the deleted stock results for the univariate regression and bivariate models from 1979 to 2015 at the daily, weekly, and monthly frequencies, respectively.

[Insert Table 2.4 here]

It is notable that only the daily overall change in beta for deletions is significant at the 5% level or better. However, this is far from suggesting an absence of comovement following stock deletions from the S&P 500 as the results from both increasing and decreasing beta subsamples are always statistically significant at the 1% level at all three reported frequencies. The univariate regression results indicate that 89 of 192 stocks have increasing post-deletion betas at the daily frequency. The average increase is 0.3391. Further, 80 of 181 stocks have increasing beta after deletions at weekly frequency which is 0.6351 on average. Moreover, more than half of event stocks have increasing betas at the monthly frequency, with an average change is 0.7599. These changes are offset by those of the decreasing-beta subsamples in all cases.

Table 2.4 indicates that the bivariate results for deletions are different from the univariate regression results in several respects. First, the overall change in beta is significantly negative at the daily frequency but neither at the weekly or at the monthly frequency. Second, relatively fewer deletion event stocks exhibit post-deletion increasing betas from the bivariate regression results at when daily and monthly data are used. Only 52, and 61 stocks have increasing beta at daily, and monthly frequencies, respectively. Third, the bivariate regression beta increases and decreases from the two subsamples are larger in magnitude than those from the corresponding univariate regression. Finally, the non-S&P 500 index sensitivity (beta) changes largely mirror those from the S&P 500 index. As the table 4 suggested, the hypothesis holds that the distribution of post-deletion beta change exhibits a sizable of positive change estimated by the univariate regression and the bivariate regression whilst the average change in post-deletion beta is negative.

Table 2.5 reports changes in the Chen et al. (2016) betas relative to the non-S&P 500 index for S&P 500 additions and deletions from 1976 to 2015.

[Insert Table 2.5 here]

Chen et al. (2016) claim that event stocks comove more with all stocks after additions but not just with the S&P 500 index. The Table 2.5 results appear to support this view for additions at the daily and monthly frequencies. However, we believe that this is resulted by the unconstraint investors and does no reject the excess comovement. Supply of stocks from trackers of non-S&P 500 index will meet the demand from trackers of S&P 500 index trackers. Moreover, our unconstraint investor will demand them if stocks' betas are lower than one. Further, these results hide large offsetting movements from the subsamples with positive and negative change in the sensitivity to the non-S&P 500 index. 341, 351, and 309 event companies have decreasing sensitivity to the non-S&P 500 index at daily, weekly, and monthly frequencies, respectively, which supports the view that stocks comove less with the group they leave. Specifically, sensitivity decreases by 0.3319, 0.4484, and 0.4490 at the daily, weekly, and monthly frequencies, respectively, and these changes are all significant at the 1% level. Decreases are offset by increases in the sensitivities. For instance, 392 stocks' sensitivities increase by 0.3472 on average at daily frequency. Positive changes in sensitivities are 0.4818, and 0.5074 at weekly, and monthly, respectively. These significant increasing sensitivities confirm our hypothesis 3 but do not reject the excessive comovement.

Similar patterns are found in deletion samples. Subsamples with sensitivities' change in different direction offset each other and result in the relative small overall change. Results for subsamples with increasing sensitivities are 0.3979, 1.3773, and 0.7824 at daily, weekly, and monthly, respectively, although overall change in sensitivities are 0.0657, -0.7647, and 0.1869 at corresponding frequencies, respectively. This is because increasing sensitivities are offset by decreasing subsamples whose results are -0.4305, -2.3868, and -0.5482 at daily, weekly, and monthly frequencies, respectively.

Although a sizeable number of added stocks have increasing sensitivities to the non-S&P 500 index, at the same time, some deleted stocks have decreasing sensitivities it cannot reject the excessive comovement. We believe that unconstraint investors play an essential role and have offset influence on the change in sensitivities when beta is lower than one. When excess demand from unconstraint investors is higher than excess supply from non-S&P 500 index trackers, the sensitivities may increase even when stocks are added to the S&P 500 index. In next section, we examine if the betting-against-beta (BAB) strategy can explain our novel findings of comovement.

2.4.5. Betting-against-beta (BAB) strategy

Our main explanation of the negative post-addition betas is the BAB strategy. Here financially unconstrained institutions like hedge funds adopt this strategy whose impact is overlapping with that of index trackers for event stocks.[6] Hedge funds focus in the historical betas of to-be-added stocks. They underweight or short sell high beta stocks and overweight low beta stocks. As our theoretical model suggested, this strategy has influences on loadings on the group-specific non-fundamental shocks which determines the change in beta and sensitivities to the non-S&P 500 index. We assume the mean value of betas of equities is one because the systematic of equity market should be one.

We examine the explanation power of BAB strategy to inclusions and deletions respectively. First, for addition events, stocks have increasing sensitivity to the non-S&P 500 index should have lower-than-one pre-addition betas which is an evidence that these stocks are demanded by investors following BAB strategy. What is more, added stocks that have decreasing beta should have higher-than-one pre-addition betas to prove that they are underweight or shorted by unconstraint investors. Second, deleted stocks with decreasing sensitivities to the non-S&P 500 index should have higher-than-one pre-deletion betas to support the view that they are shorted by BAB strategy players although they are demanded by the non-S&P 500 index trackers. Finally, deleted stocks with increasing beta should have lower-than-one pre-deletion beta. In other words, they are demanded by the BAB strategy players.

---

[6] Other explanations of negative post-addition betas is the long run mean reversion of betas towards one and the defensive beta strategy suggested by Black (1972).

Table 2.6 reports the value of beta before additions for subsamples with increasing sensitivities to the non-S&P 500 index and with decreasing beta after stocks are added to the S&P 500 index from 1976 to 2015 at daily, weekly, and monthly frequencies.

[Insert Table 2.6 here]

As table 2.6 suggested, stocks that have higher-than-one beta have decreasing beta even after they are added to the S&P 500 index. For example, before additions, stocks' betas are 1.2295, 1.3941, and 1.4538 at daily, weekly, and monthly frequencies, respectively. Their corresponding changes in beta are -0.2671, -0.4593, and -0.4714, respectively. These findings support Hypothesis 3A. These highly significant higher-than-one stocks are believed to be shorted by unconstraint investors, and the excess supply results in the decrease in beta. Moreover, stocks that have increasing sensitivities to the non-S&P 500 index have significant lower-than-one betas before additions at daily, and weekly frequencies. Pre-addition betas are 0.9166, and 0.8874 at daily, and weekly frequencies, respectively. This also confirm our explanation to the increasing post-addition sensitivities that unconstraint investors provide excess demands.

Analogous patterns are also found for deletions. Table 2.7 reports estimators of beta before deletions for increasing post-deletion beta and decreasing sensitivities, respectively, at daily, weekly, and monthly frequencies from 1979 to 2015. Deleted stocks with increasing sensitivities to the non-S&P 500 index are aggressive at daily, weekly, and monthly frequencies. Pre-deletion betas are 1.2805, 1.2783, and 1.3594 at daily, weekly, and monthly frequencies, respectively. Although deleted stocks that have increasing beta are insignificantly different from neutral stocks we still find evidences to support the role of BAB strategy in the change in comovement from the sensitivities to the non-S&P 500 index. Further, added stocks with increasing beta are defensive on average. And pre-addition betas are 0.8879, 0.8567, and 0.9540 at daily, weekly, and monthly frequencies, respectively. They are highly significant different from one except that monthly result is significant only at 10% level. This finding cannot reject the hypothesis 3B: stocks with lower-than-mean betas should have increasing beta.

[Insert Table 2.7 here]

To sum up, the results have more complicated patterns when sample is divided into subsamples with increasing and decreasing betas. The results from the univariate regression indicate that changes in comovement conform with an important implication of the BAB strategy.

2.5. Conclusions

Dividing whole sample into increasing- and decreasing-beta subsamples, we find change in beta of each subsample is stronger than overall change. These changes are stronger as lower frequency data are used. Some stocks have increasing sensitivity to the non-S&P 500 index after they are added to the S&P 500 index, however, this may because the more efficient information diffusion after additions.

It is more appropriate to examine the comovement in the S&P 500 index into increasing- and decreasing-beta subsamples because both of subsamples offset each other and make overall average change in beta low. It means that results of the full sample, to some extent, are misleading.

Changes in beta and in sensitivity to the non-S&P 500 index can be self-adjustment to themselves. When S&P 500 index changes its constituent, aggressive and defensive stocks adjust their beta to neutral, but this process overcorrect the value of beta and hence that aggressive and defensive stocks turn to each other. This is also the case for sensitivity to the non-S&P 500 index.

Our theoretical model can explain the decreasing beta after additions, increasing beta after deletions, and increasing sensitivity to the non-S&P 500 index. However, we believe some other factors also have impacts on comovement because our theoretical model fails to explain that some stocks have increasing slopes of S&P 500 index return and non-S&P 500 index return.. Some may argue that more risk factors should be considered. For example, the estimator of change in beta could be examined using the 4-factor model. However, it is still a matter of debate whether factors in this kind of model reflect risk or frictions in markets. As a result, we cannot conclude that there is not excess comovement even when we find lower comovement from 3- and 4-factor regressions. In next chapter, we will test changes in slopes through 3- and 4-factor regressions to examine influences of small minus big capitalisations of stocks, high minus low price-to-earnings ratios of stocks, and momentum factors on changes in comovement.

Figure 2.1 Changes in slopes and percentage of decreasing slopes after additions (daily)

Univariate regression on returns of S&P 500 index: $r_{j,t} = \alpha_j + \beta_j r_{SP500,t} + v_{j,t}$ is examined, for each stock j added to the S&P 500 index, for pre- and post-addition. Daily data are used. Changes in slope are then calculated and average changes in slope for each year during 1976-2015 are finally reported. Samples are divided into two subsamples with positive changes in slopes and with negative changes in slopes. Percentage of decreasing beta is calculated through dividing number of observations in subsample with decreasing slopes by total number observations in the overall sample.

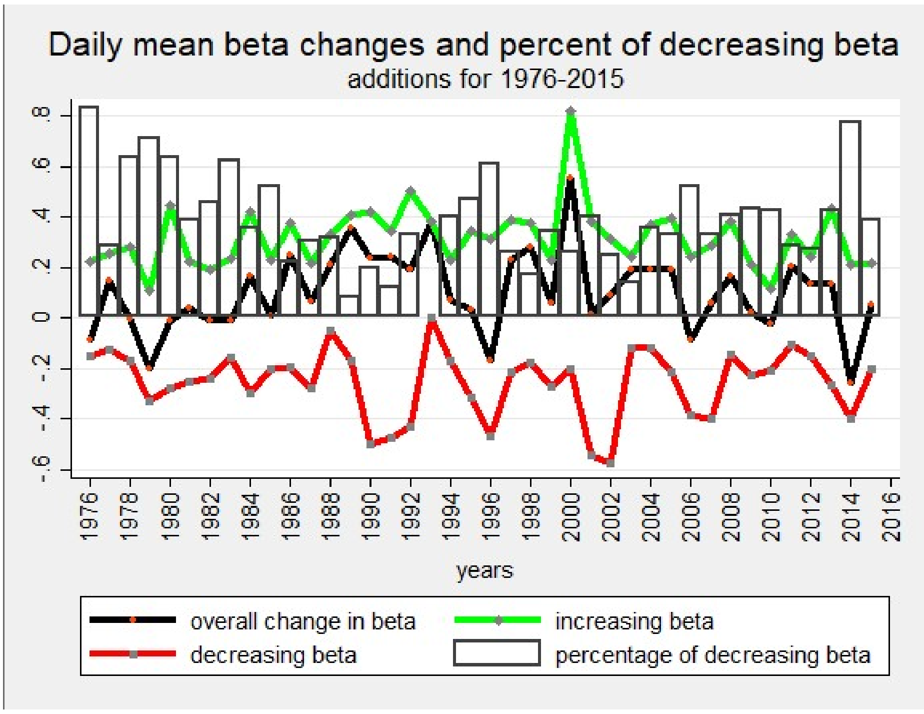
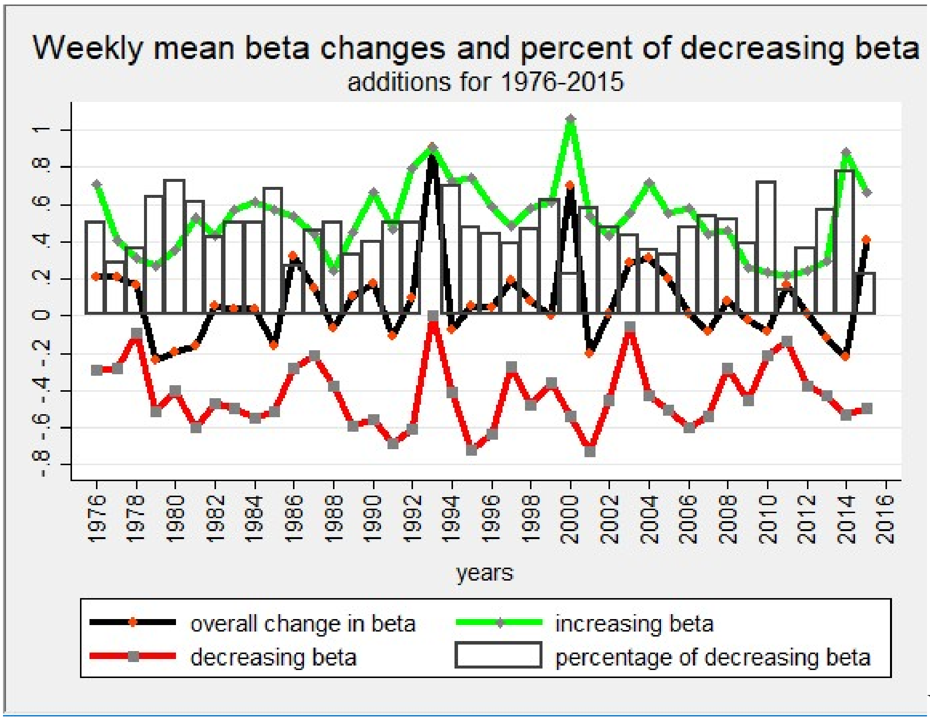


Figure 2.2 Changes in slopes and percentage of decreasing slopes after additions (weekly)

Univariate regression on returns of S&P 500 index: $r_{j,t} = \alpha_j + \beta_j r_{SP500,t} + v_{j,t}$ is examined, for each stock j added to the S&P 500 index, for pre- and post-addition. Weekly data are used. Changes in slope are then calculated and average changes in slope for each year during 1976-2015 are finally reported. Samples are divided into two subsamples with positive changes in slopes and with negative changes in slopes. Percentage of decreasing beta is calculated through dividing number of observations in subsample with decreasing slopes by total number observations in the overall sample.



W

Figure 2.3 Changes in slopes and percentage of decreasing slopes after additions (monthly)

Univariate regression on returns of S&P 500 index: $r_{j,t} = \alpha_j + \beta_j r_{SP500,t} + v_{j,t}$ is examined, for each stock j added to the S&P 500 index, for pre- and post-addition. Monthly data are used. Changes in slope are then calculated and average changes in slope for each year during 1976-2015 are finally reported. Samples are divided into two subsamples with positive changes in slopes and with negative changes in slopes. Percentage of decreasing beta is calculated through dividing number of observations in subsample with decreasing slopes by total number observations in the overall sample.

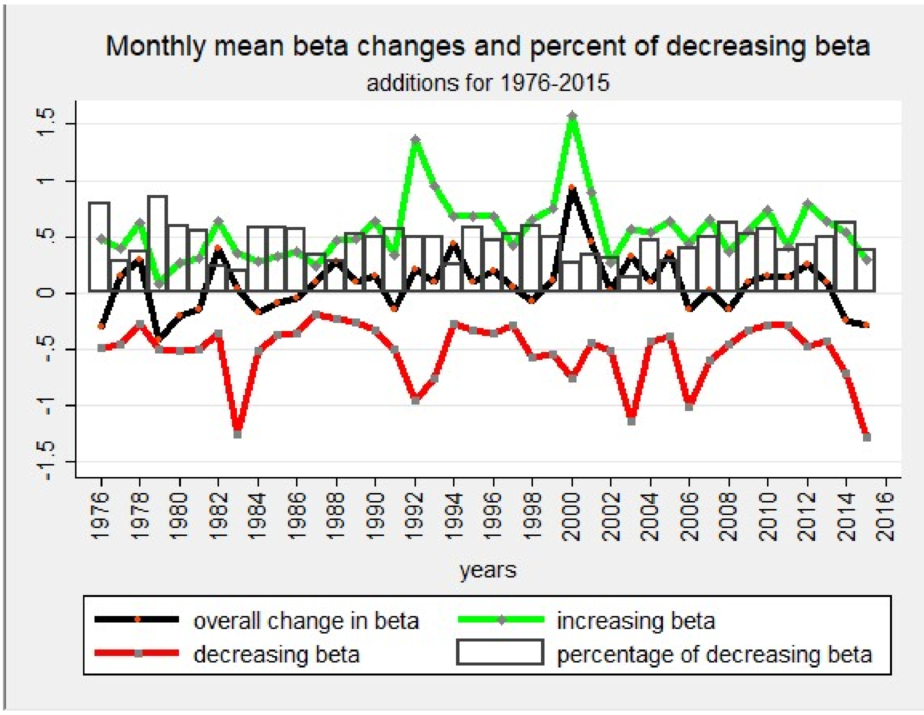


Figure 2.4 Changes in slopes and percentage of increasing slopes after deletions (daily)

Univariate regression on returns of S&P 500 index: $r_{j,t} = \alpha_j + \beta_j r_{SP500,t} + v_{j,t}$ is examined, for each stock j added to the S&P 500 index, for pre- and post-addition. Daily data are used. Changes in slope are then calculated and average changes in slope for each year during 1976-2015 are finally reported. Samples are divided into two subsamples with positive changes in slopes and with negative changes in slopes. Percentage of increasing beta is calculated through dividing number of observations in subsample with increasing slopes by total number observations in the overall sample.

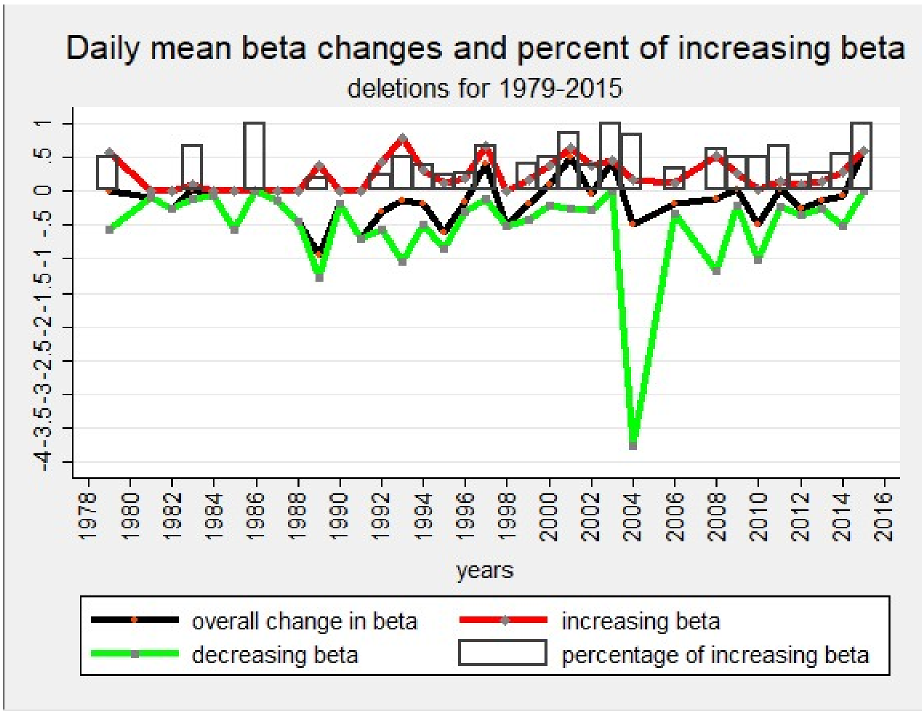


Figure 2.5 Changes in slopes and percentage of increasing slopes after deletions (weekly)

Univariate regression on returns of S&P 500 index: $r_{j,t} = \alpha_j + \beta_j r_{SP500,t} + v_{j,t}$ is examined, for each stock j added to the S&P 500 index, for pre- and post-addition. Weekly data are used. Changes in slope are then calculated and average changes in slope for each year during 1976-2015 are finally reported. Samples are divided into two subsamples with positive changes in slopes and with negative changes in slopes. Percentage of increasing beta is calculated through dividing number of observations in subsample with increasing slopes by total number observations in the overall sample.

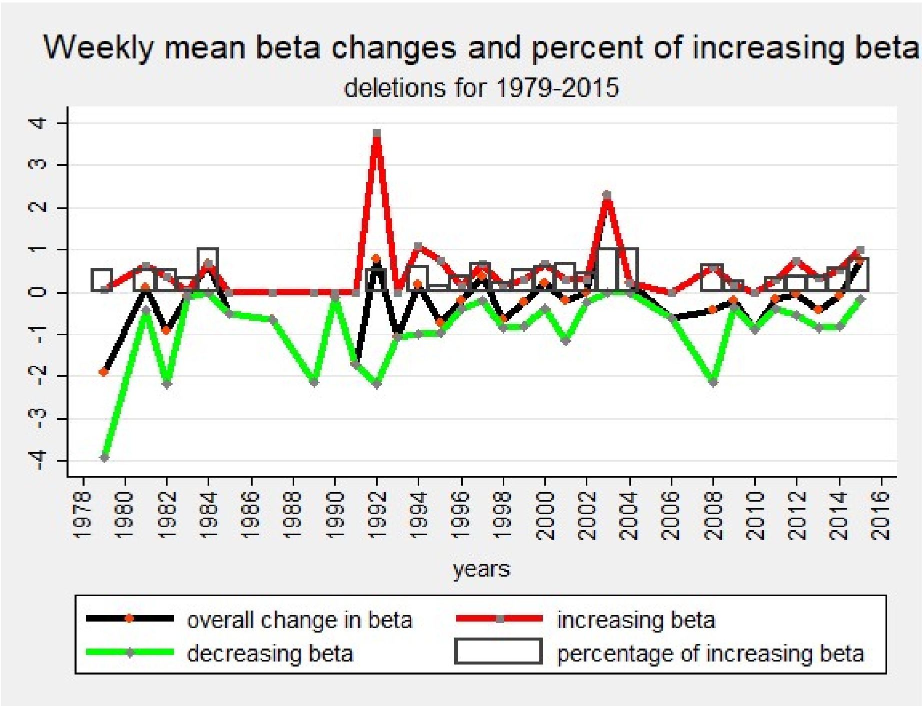


Figure 2.6 Changes in slopes and percentage of increasing slopes after deletions (monthly)

Univariate regression on returns of S&P 500 index: $r_{j,t} = \alpha_j + \beta_j r_{SP500,t} + v_{j,t}$ is examined, for each stock j added to the S&P 500 index, for pre- and post-addition. Monthly data are used. Changes in slope are then calculated and average changes in slope for each year during 1976-2015 are finally reported. Samples are divided into two subsamples with positive changes in slopes and with negative changes in slopes. Percentage of increasing beta is calculated through dividing number of observations in subsample with increasing slopes by total number observations in the overall sample.

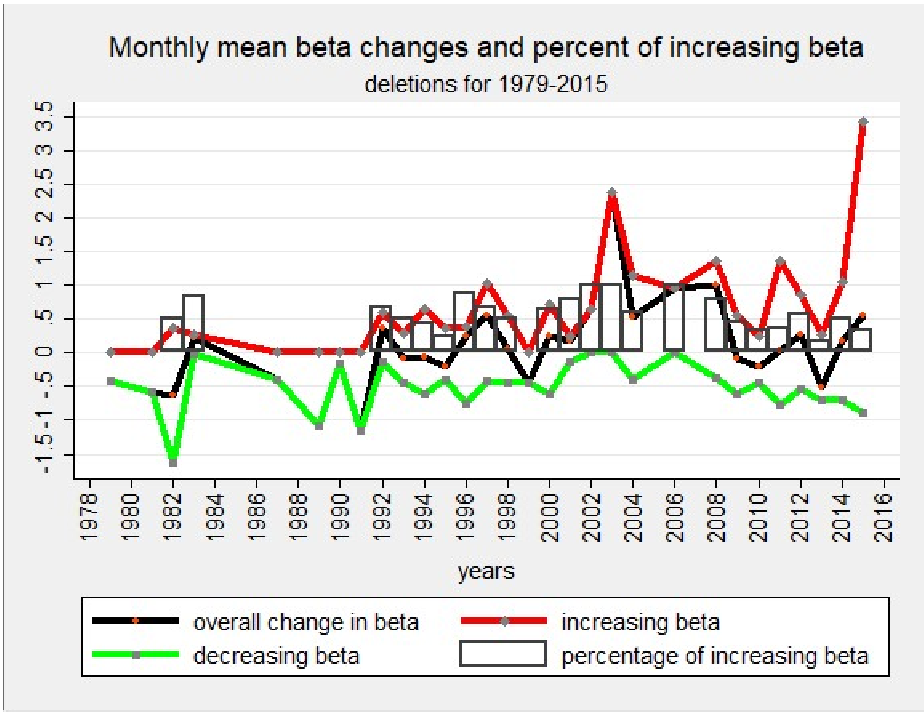


Table 2.1

Re-examination of changes in comovement of stocks added to and deleted from the S&P 500

We re-examine the average change in slopes and fit of regressions of returns on stocks added to and deleted from the S&P 500 index on returns on the S&P 500 index and on the non-S&P 500 index. Event stocks added to and deleted from the S&P 500 index during 1976-2000 that are not involved in mergers, takeovers, acquisitions, and bankruptcies are included by the sample. The univariate regression:

$$r_{j,t} = \alpha_j + \beta_j r_{sp500,t} + v_{j,t}$$

and the bi-variate regression:

$$r_{j,t} = \alpha_j + \beta_{j,SP500} r_{SP500,t} + \beta_{j,nonSP500} r_{nonSP500,t} + v_{j,t}$$

are used separately to examine values of slopes and fitness for the pre- and post-event period, respectively for each event stock j. Returns on the S&P 500 index ($r_{sp500,t}$) are from the CRSP Index on the S&P 500 Universe file while returns on the non-S&P 500 index are inferred from the identity: $r_{vwcrsp,t} = \left(\frac{CAP_{crsp,t-1} - CAP_{SP500,t-1}}{CAP_{crsp,t-1}}\right) r_{nonSP500,t} + \left(\frac{CAP_{SP500,t-1}}{CAP_{crsp,t-1}}\right) r_{SP500,t}$

Returns on the value-weighted CRSP NYSE, AMEX, and Nasdaq index (r_{vwcrsp}) and total capitalisation (CAP_{crsp}) are from the CRSP stock Index file. Change in slope and fitness is calculated as difference of slopes and fitness between post- and pre-event periods, respectively. Pre- and post-event windows are 12 months before and after the announcement month, respectively, for daily and weekly tests. For monthly tests, 36 months are used for pre- and post-event windows. For the univariate regression, the average change in slope ($\Delta\overline{\beta}$) and fit ($\Delta\overline{R^2}$) are reported. For the bivariate regression, the average change in slopes ($\Delta\overline{\beta}_{SP500}$ and $\Delta\overline{\beta}_{nonSP500}$). Panel A, B, and C reports daily, weekly, and monthly results, respectively. Standard errors are also reported in brackets.

Sample		N	Univariate		Bivariate	
			$\Delta\overline{\beta}$(s.e.)	$\Delta\overline{R^2}$(s.e.)	$\Delta\overline{\beta}_{SP500}$(s.e.)	$\Delta\overline{\beta}_{nonSP500}$(s.e.)
Panel A: daily returns						
Additions	1976-2000	464	0.16*** (0.0203)	0.0499*** (0.0059)	0.3481*** (0.0246)	-0.3351*** (0.0298)
	1976-1987	204	0.0545*** (0.0226)	0.0477*** (0.0091)	0.2986*** (0.0347)	-0.335*** (0.041)
	1988-2000	260	0.2427*** (0.0306)	0.0516*** (0.0077)	0.3869*** (0.0343)	-0.3352*** (0.0425)
Deletions	1979-2000	92	-0.1886*** (0.0578)	-0.0136** (0.0072)	-0.6778*** (0.1167)	0.7102*** (0.1286)
Panel B: weekly returns						
Additions	1976-2000	460	0.1178*** (0.0327)	0.0345*** (0.0088)	0.1947*** (0.0612)	-0.1222** (0.061)
	1976-1987	202	0.0307 (0.0409)	0.0338** (0.0145)	0.139* (0.1014)	-0.1326* (0.1005)
	1988-2000	258	0.186*** (0.0484)	0.035*** (0.0107)	0.2384*** (0.0749)	-0.1141* (0.0754)
Deletions	1979-2000	85	-0.1862* (0.1355)	-0.0231** (0.0106)	0.2382 (0.2897)	-1.563* (0.9403)
Panel C: monthly returns						

Additions	1976-1998	340	0.0365 (0.032)	0.0084 (0.0113)	0.2839*** (0.0624)	-0.2182*** (0.0582)
	1976-1987	187	-0.0209 (0.0403)	0.0451*** (0.0155)	0.1921** (0.0852)	-0.134** (0.0789)
	1988-1998	153	0.1066** (0.051)	-0.0366** (0.0158)	0.3962*** (0.0911)	-0.3212*** (0.0857)
Deletions	1979-1998	48	-0.0165 (0.0929)	0.0035 (0.0223)	0.2622 (0.2538)	-0.2232 (0.2504)

*****, **, and * represent significance at 1%, 5%, and 10% level of significance.**

Table 2.2

Differences between changes in slope of S&P 500 return and changes in slope of non-S&P 500 return

For each event stock j, univariate regression on returns of non-S&P 500 index:

$$r_{j,t} = \alpha_j + \beta_{j,nonSP500} r_{nonSP500,t} + v_{j,t}$$

and univariate regression on returns of S&P 500 index:

$$r_{j,t} = \alpha_j + \beta_{j,SP500} r_{SP500,t} + v_{j,t}$$

are examined separately for pre- and post-event periods, respectively to obtain slops of returns on non-S&P 500 index before and after events ($\overline{\beta}_{nonSP500,pre}$ and $\overline{\beta}_{nonSP500,post}$), and slopes of returns on S&P 500 index before and after events ($\overline{\beta}_{SP500,pre}$ and $\overline{\beta}_{SP500,post}$). Changes in slopes of returns on non-S&P 500 index and in slopes of returns on S&P 500 index ($\Delta\overline{\beta}_{nonSP500}$ and $\Delta\overline{\beta}_{SP500}$) are then calculated. The difference of changes (Diff. of diff) in slopes are finally calculated: $\Delta\overline{\beta}_{SP500} - \Delta\overline{\beta}_{nonSP500}$. These results are reported in Panel A.

Bivariate regression:

$$r_{j,t} = \alpha_j + \beta_{j,SP500} r_{SP500,t} + \beta_{j,nonSP500} r_{nonSP500,t} + v_{j,t}$$

is also examined for pre- and post-event periods, respectively. Slopes of returns on non-S&P 500 index before and after events and slopes of returns on S&P 500 index before and after events are reported in Panel B. Changes in slopes and difference of changes in slopes are also reported.

Sample								
Panel A: Univariate regressions								
	N		Non-S&P 500			S&P 500		Diff. of diff.
		$\overline{\beta}_{nonSP500,pre}$(s.e.)	$\overline{\beta}_{nonSP500,post}$(s.e.)	$\Delta\overline{\beta}_{nonSP500}$(s.e.)	$\overline{\beta}_{SP500,pre}$(s.e.)	$\overline{\beta}_{SP500,post}$(s.e.)	$\Delta\overline{\beta}_{SP500}$(s.e.)	$\Delta\overline{\beta}_{SP500} - \Delta\overline{\beta}_{SP500}$(s.e.)
1976-1987	204	1.2548*** (0.0354)	1.2581*** (0.0404)	0.0033 (0.0286)	0.9581*** (0.0318)	1.0127*** (0.0339)	0.0545*** (0.0226)	0.0512*** (0.0167)
1988-2000	260	1.2522*** (0.0430)	1.2663*** (0.0460)	0.1411 (0.0326)	0.9799*** (0.0367)	1.2226*** (0.0449)	0.2427*** (0.0306)	0.2286*** (0.0259)
2001-2012	228	1.0339*** (0.0356)	1.1190*** (0.0330)	0.0851*** (0.0256)	1.0736*** (0.0369)	1.1568*** (0.0302)	0.0832*** (0.0257)	-0.0018 (0.0178)

1976-2012	692	1.1811*** (0.0229)	1.2153*** (0.0237)	0.0343** (0.0171)	1.0043*** (0.0207)	1.139*** (0.0222)	0.1347*** (0.0161)	0.1004*** (0.0129)
Panel B: Bivariate regressions								
1976-1987	204	0.8906*** (0.0407)	0.5557*** (0.0400)	-0.3350*** (0.0410)	0.3494*** (0.0415)	0.6480*** (0.0446)	0.2986*** (0.0347)	0.6336*** (0.0709)
1988-2000	260	1.0040*** (0.0457)	0.6689*** (0.0554)	-0.3352*** (0.0425)	0.2757*** (0.0259)	0.6626*** (0.0308)	0.3869*** (0.0343)	0.7220*** (0.0711)
2001-2012	228	0.9401*** (0.0641)	0.6514*** (0.0592)	-0.2887*** (0.0478)	0.1223*** (0.0485)	0.5129*** (0.0446)	0.3906*** (0.0446)	0.6793*** (0.0896)
1976-2012	692	0.9495*** (0.0298)	0.6297*** (0.0309)	-0.3198*** (0.0255)	0.2469*** (0.0226)	0.6090*** (0.0230)	0.3621*** (0.0221)	0.6819*** (0.0450)

Note: *, **, and * represent significance at 1%, 5%, and 10% level of significance.**

Table 2.3 Increases and decreases in comovement of stocks added to the S&P 500 index during 1976-2015

Univariate regression on returns of S&P 500 index: $r_{j,t} = \alpha_j + \beta_j r_{SP500,t} + v_{j,t}$ and bivariate regression: $r_{j,t} = \alpha_j + \beta_{j,SP500} r_{SP500,t} + \beta_{jnon,SP500} r_{nonSP500,t} + v_{j,t}$ are examined separately for each stock j added to the S&P 500 index. Changes in slopes are then calculated. Samples are divided into subsamples with increasing beta and subsamples with decreasing beta based on sign of change in slope of S&P 500 return. For daily and weekly tests, pre- and post-event windows are 12 months before and after announcement month, while for monthly test, pre- and post-event windows are 36 months before and after announcement month. Panel A, B, and C report daily, weekly, and monthly results, respectively.

Samples	univariate regression		bivariate		
Panel A: daily returns	N	$\Delta\overline{\beta}$(s.e.)	N	$\Delta\overline{\beta}_{SP500}$(s.e.)	$\Delta\overline{\beta}_{nonSP500}$(s.e.)
overall change	733	0.1279*** (0.0155)	733	0.3545*** (0.0219)	-0.3131*** (0.0248)
Increasing beta	464	0.3569*** (0.0145)	546	0.6079*** (0.0179)	0.5455*** (0.0227)
Decreasing beta	269	-0.2671*** 0.0157	187	-0.3853*** (0.0267)	0.3656*** (0.0422)
Panel B: weekly returns					
overall change	726	0.0882*** (0.0257)	726	0.2230*** (0.0535)	-0.1548*** (0.0495)
Increasing beta	393	0.5521*** (0.0262)	407	1.14447*** (0.0528)	-0.9251*** (0.0489)
Decreasing beta	333	-0.4593*** (0.0230)	319	-0.9529*** (0.0504)	0.8281*** (0.0582)
Panel C: monthly returns					
overall change	640	0.1083*** (0.0296)	640	0.2899*** (0.0584)	-0.1586*** (0.0484)
Increasing beta	341	0.6166*** (0.0320)	390	1.1432*** (0.0535)	-0.8031*** (0.0446)
Decreasing beta	299	-0.4714*** (0.0241)	250	-1.0412*** (0.0615)	0.8469*** (0.0623)
*****, **, and * represent significance at 1%, 5%, and 10% level of significance.**					

Table 2.4

Decreases and increases in comovement of stocks deleted from the S&P 500 index during 1979-2015

Univariate regression on returns of S&P 500 index: $r_{j,t} = \alpha_j + \beta_j r_{SP500,t} + v_{j,t}$ and bivariate regression: $r_{j,t} = \alpha_j + \beta_{j,SP500} r_{SP500,t} + \beta_{jnon,SP500} r_{nonSP500,t} + v_{j,t}$ are examined separately for each stock j deleted from the S&P 500 index. Changes in slopes are then calculated. Samples are divided into subsamples with increasing beta and subsamples with decreasing beta based on sign of change in slope of S&P 500 return. For daily and weekly tests, pre- and post-event windows are 12 months before and after announcement month, while for monthly test, pre- and post-event windows are 36 months before and after announcement month. Panel A, B, and C report daily, weekly, and monthly results, respectively.

Samples	Univariate regression		bivariate		
Panel A: daily returns	N	$\Delta\overline{\beta}$(s.e.)	N	$\Delta\overline{\beta}_{SP500}$(s.e.)	$\Delta\overline{\beta}_{nonSP500}$(s.e.)
overall change	192	-0.0919** (0.0451)	192	-0.5947** (0.0963)	0.6077*** (0.0995)
Increasing beta	89	0.3391*** (0.0389)	52	0.6504*** (0.1540)	-0.5597*** (0.1275)
Decreasing beta	103	-0.4644*** (0.0552)	140	-1.0572*** (0.0926)	1.0412*** (0.1069)
Panel B: weekly returns					
overall change	181	-0.1332 (0.0853)	181	0.0996 (0.1462)	-0.9238** (0.4629)
Increasing beta	80	0.6351*** (0.1023)	93	0.9820*** (0.2359)	-2.0384** (0.8233)
Decreasing beta	101	-0.7418 (0.0925)	88	-0.8330*** (0.0968)	0.2540 (0.3512)
Panel C: monthly returns					
overall change	143	0.1351* (0.0782)	143	-0.2067 (0.1757)	0.3402** (0.1610)
Increasing beta	77	0.7599*** (0.0892)	61	1.5748*** (0.1850)	-1.1772*** (0.1538)
Decreasing beta	66	-0.5939*** (0.0536)	82	-1.5319*** (0.1572)	1.4689*** (0.1712)
***, **, and * represent significance at 1%, 5%, and 10% level of significance.					

Table 2.5

Changes in slopes of non-S&P 500 returns for stocks added to and deleted from the S&P 500 index Univariate regression on returns of non-S&P 500 index: $r_{j,t} = \alpha_j + \beta_j r_{nonSP500,t} + v_{j,t}$ is examined for each stock j deleted from the S&P 500 index. Changes in slopes are then calculated. Samples are divided into subsamples with positive changes in slopes and subsamples with negative changes in slopes based on sign of change in slope of non-S&P 500 return. For daily and weekly tests, pre- and post-event windows are 12 months before and after announcement month, while for monthly test, pre- and post-event windows are 36 months before and after announcement month. Panel A, B, and C report daily, weekly, and monthly results, respectively.

Samples	Additions		Deletions	
Panel A: daily returns	N	$\Delta\overline{\beta}_{nonSP500}$(s.e.)	N	$\Delta\overline{\beta}_{nonSP500}$(s.e.)
overall change	733	0.0312* (0.0164)	192	0.0657* (0.0442)
positive change	392	0.3472*** (0.0142)	115	0.3979*** (0.0389)
negative change	341	-0.3319*** (0.0158)	77	-0.4305*** (0.0587)
Panel B: weekly returns				
overall change	726	0.0321* (0.0230)	181	-0.7647*** (0.2953)
positive change	375	0.4818*** (0.0221)	78	1.3773*** (0.1397)
negative change	351	-0.4484*** (0.0208)	103	-2.3868*** (0.4465)
Panel C: monthly returns				
overall change	640	0.0456** (0.0247)	143	0.1869*** (0.1869)
positive change	331	0.5074*** (0.0240)	79	0.7824*** (0.0861)
negative change	309	-0.4490*** (0.0204)	64	-0.5482*** (0.0491)

*****, **, and * represent significance at 1%, 5%, and 10% level of significance.**

Table 2.6 Changes in slopes of univariate regressions, and pre-addition slopes

Univariate regressions of returns of each stock j added to the S&P 500 index on returns of non-S&P 500 index, $r_{j,t} = \alpha_j + \beta_{j,nonSP500} r_{nonSP500,t} + v_{j,t}$, and on returns of S&P 500 index, $r_{j,t} = \alpha_j + \beta_{j,SP500} r_{SP500,t} + v_{j,t}$, are examined separately for pre- and post-event periods, respectively. Changes in slopes of returns on non-S&P 500 index and returns on S&P 500 index are then calculated. Samples with positive changes in slope of non-S&P 500 return and negative changes in slope of S&P 500 return are reported in the table. Average value of slope of S&P 500 return before additions, and average changes in slope of non-S&P 500 return and in slope of S&P 500 return are reported.

Univariate regressions						
	N	Non-S&P500		N	S&P 500	
Sample		$\beta_{SP500,pre}$(s.e.)	$\Delta\overline{\beta}_{nonSP500}$(s.e.)		$\beta_{SP500,pre}$(s.e.)	$\Delta\overline{\beta}_{SP500}$(s.e.)
daily	392	0.9166*** (0.0274)	0.3472*** (0.0142)	269	1.2295*** (0.0339)	-0.2671*** (0.0157)
weekly	375	0.8874*** (0.0309)	0.4818*** (0.0221)	333	1.3941*** (0.0353)	-0.4593*** (0.0230)
monthly	331	1.0096 (0.0314)	0.5074*** (0.0240)	299	1.4538*** (0.0327)	-0.4714*** (0.0241)

*****, **, and * represent significance at 1%, 5%, and 10% level of significance.**

Table 2.7 Changes in slopes of univariate regressions, and pre-deletion slopes

Univariate regressions of returns of each stock j deleted from the S&P 500 index on returns of non-S&P 500 index, $r_{j,t} = \alpha_j + \beta_{j,nonSP500} r_{nonSP500,t} + v_{j,t}$, and on returns of S&P 500 index, $r_{j,t} = \alpha_j + \beta_{j,SP500} r_{SP500,t} + v_{j,t}$, are examined separately for pre- and post-event periods, respectively. Changes in slopes of returns on non-S&P 500 index and returns on S&P 500 index are then calculated. Samples with negative changes in slope of non-S&P 500 return and positive changes in slope of S&P 500 return are reported in the table. Average value of slope of S&P 500 return before additions, and average changes in slope of non-S&P 500 return and in slope of S&P 500 return are reported.

Univariate regressions						
	N	Non-S&P500		N	S&P 500	
Sample		$\beta_{SP500,pre}$(s.e.)	$\Delta\overline{\beta}_{nonSP500}$(s.e.)		$\beta_{SP500,pre}$(s.e.)	$\Delta\overline{\beta}_{SP500}$(s.e.)
daily	77	1.2805*** (0.0735)	-0.4305*** (0.0587)	89	1.0102 (0.0627)	0.3391*** (0.0386)
weekly	103	1.2783*** (0.0734)	-2.3868*** (0.4465)	80	0.9547 (0.0915)	0.6351*** (0.1023)
monthly	64	1.3594*** (0.0859)	-0.5482*** (0.0491)	77	0.9938 (0.0847)	0.7599*** (0.0892)

*****, **, and * represent significance at 1%, 5%, and 10% level of significance.**

Chapter 3. Comovement using 3- and 4-factor models

3.1. Introduction

Chapter 2 examines the exact patterns of changes in comovement and concludes that there is a sizable number of decreases in comovement for stocks added to the S&P 500 index and a sizable number of increases in comovement for stocks deleted from the S&P 500 index. In this chapter, the attention moves onto how methodologies of regressions influence estimators of comovement. This chapter focuses on how three- and four-factor regressions influence measures of comovement and tries to answer if fundamental factors, such as small minus big capitalisations of stocks, high minus low price-to-earnings ratios of stocks, and momentum can explain excess comovement completely. This chapter does not base research on evidence found on the previous one, so that it can focus on effects of regression methodologies and fundamental factors purely.

In a path-breaking study, Vijh (1994) found that the increase in daily and weekly betas for S&P 500 additions averaged 0.21 and 0.13, respectively, using the CRSP value-weighted returns over the 1985-1989 period.[7] He attributed most of these increases to the price pressure or excess volatility caused by index trading strategies.[8] The Barberis, Shleifer and Wurgler [hereafter BSW] (2005) seminal study shifted the focus of the debate to changes in return comovement of added and deleted stocks whereas the prior debate had centred mainly on price changes.[9] They find evidence of excess comovement or beta increases after index addition and decreasing comovement following deletions. Since S&P additions apparently convey no new information about fundamentals, these comovement findings are a puzzle. Virtually all of the extant comovement studies either rely on the BSW (2005) stylised model or no model at all as in the case of three-factor tests of comovement. A notable exception is the Chen, Singal and Whitelaw (2016) study that developed a new theoretical model that underlines the role of univariate regressions.

---

[7] Vijh (1994) described stock betas as being overstated and did not use the comovement concept.
[8] See Wurgler (2011) for an interesting discussion of increasing index linked investment.
[9] See for example Harris and Gurel (1986), Shleifer (1986), and Wurgler and Zhuravskaya (2002).

The excess comovement debate mainly focuses on the reasons for the beta increases of new stocks added to the index. Classical theory asserts that changes in fundamentals are the cause of the increases while behavioral theory emphasises the role of noise traders and sentiment on the presumption that changes in the constituents of a stock index are not informative about their fundamentals. BSW (2005) were the first to propose a non-fundamental explanation for excess comovement instead of one based on added stock fundamentals. Building on the Barberis and Shleifer (2003) concept of style investment, they propose the category rationale for excess comovement. The underlying idea is that some investors treating the S&P 500 as a category are noise traders with correlated sentiment. Index changes then induce correlated demand shocks for added stocks that impact on their prices and returns. This category or index common factor in returns is unrelated to a stock's cash flows or discount rates and is what explains excess comovement for S&P inclusions and deletions.[10] BSW (2005) extended the methodology for examining comovement from univariate regressions to bivariate regressions with both the S&P 500 and non-S&P 500 (rest of the market) indexes. Several studies of other country indexes have supported the BSW findings. Greenwood and Sosner (2007) found similar findings for the Nikkei 225 index.[11] Claessens and Yafeh (2011) in a comprehensive study of 40 stock markets find support for the BSW univariate findings in the vast majority of their developed and emerging market sample indexes.

Recent studies have proposed an interesting new explanation for excess comovement. They argue that that new entries to the S&P 500 index are momemtum stocks and that this explains their post-entry beta increases. In this vein, Chen et al. (2016) develop a new univariate model that departs from the BSW model which shows that the bivariate regression provides little

[10] BSW actually propose three sentiment- or friction-based views of comovement where the category view is the one that contrasts most sharply with the fundamental view. Note that Greenwood (2008) and Claessens and Yafeh (2011) combine the category and habitat views into a demand-driven view of comovement.

[11] See also Greenwood (2008) who studied changes in the Nikkei 225 index weights.

information about comovement. Their model shows that univariate regressions of the added stock on the non-S&P and S&P indexes provide the relevant information on comovement. They compare the beta change of event-stocks with that of momentum-matched-stocks and do not find significant differences when using the Dimson adjustment for nonsynchronous trading. They interpret this as evidence of momentum effects being able to explain much of the changes in comovement. Similarly, Kasch and Sarkar (2014) criticise the asset pricing basis of the BSW methodology and instead argue for employing asset pricing regressions that condition on other common factors. They claim that their factor regression results show that momentum can resolve the apparent comovement puzzle since the betas are insignificant in their three factor model regressions. The problem with both of these studies is that, notwithstanding the plausibility of their empirical results, momentum is not embedded in a theoretical model of comovement and, as such, their results are not straightforward to interpret.

The first contribution of this chapter is that it provides a motivation for using the 3- and 4-factor model to examine excess comovement. Specifically, this chapter outlines how omitted variable bias can influence the estimator of comovement. This chapter reports cross correlations for returns on the S&P 500 index, the small-minus-big factor, the high-minus-low factor, and the momentum factor. Some of these correlations suggest that the estimator of beta from the univariate regression may be subject to omitted variable bias. However, the low cross correlations between the univariate regression betas and the other factor loadings suggest that multicollinearity is not an issue for the 3- and 4-factor model estimators.

The second contribution is that the chapter provides novel evidence on excess comovement when it re-examines beta changes after S&P500 addition and deletion events using data from 1988 up to 2014. The univariate and bivariate comovement results are consistent with those in BSW (2005) for the earlier part of our sample period up to 1990 but the updated monthly results from the four-factor model indicate evidence of comovement with a strongly significant univariate beta change of 0.26 for the 1988-2000 subsample period. The beta changes at the daily (significant) and weekly (insignificant) frequencies employing the three-factor regression are similar to those found by Kasch and Sarkar (2014). However, the monthly results from these models are novel and revealing. They are significant for both the full sample period and the two sub-periods, 1988-2000 and 2001-2014, respectively. The magnitude of the full-sample monthly comovement increases for added stocks from the Fama and French 3-factor and Carhart 4-factor models (0.23 and 0.20, respectively) exceeds the significant increase of 0.18 from the univariate regression.[12] Note that these increases are conditioned on the effects of small-minus-big (*SML*), high-minus-low (*HML*) and additionally on momentum (*MOM*) for the four-factor model. Excess comovement refuses to go away even when conditioning on momentum in a four-factor asset pricing framework.

The final contribution is that the role of momentum is more nuanced in our findings than it is in those of Chen et al. (2016) and Kasch and Sarkar (2014). The daily four-factor model loading on momentum is significantly negative at -0.24 for the full sample and the two sub-periods but the *SMB* daily loadings are always significantly also. Daily and monthly excess comovement are also significant for the full sample and for all the sub-periods. By contrast the monthly momentum loadings are of smaller magnitude (compared to those the daily frequency) and less significant (at the 5% compared to the 1% critical value) for both the full sample and 1988-2000 and only marginally significant for the 2001-2012 period. Thus while momentum matters for the comovement puzzle at the daily frequency, it cannot explain the puzzle at the monthly level where the beta changes are everywhere larger and more significant. These findings are contrary at variance with both the Chen et al. (2016) and Kasch and Sarkar (2014) conclusions.

The remainder of the chapter is organized as follows. Section two reviews the existing literature. In section three, we discuss why 3- and 4-factor model regressions are more appropriate

[12] Kasch and Sarkar (2014) do not report monthly beta changes.

than the model typically used to assess comovement. Section four describes our data, methodology and analyses the empirical results. A final section concludes.

3.2. Fundamentals versus other factors

Following the BSW (2005) study, there appeared to be a consensus in the literature on comovement. Behavioural finance proponents suggested that irrational investors, frictions, style investing and investor habitats are able temporarily to delink prices from fundamentals and that these can explain comovement following index additions and deletions. However, debate around the reasons for beta increases for S&P additions has recently resurfaced. Proponents of classical theory claim that betas increase because of changes to actual or expected fundamentals prior to index inclusions.

3.2.1. Competing models

Excess comovement studies employ the univariate regression regression framework in testing for beta changes in stocks added to the market index. The basic idea is that investors focus on style, category, or index to simplify investment decisions. Often these concepts are not formally modelled but Barberis and Shleifer (2003) develop a style investment model. In this model, noise traders can influence security prices due to limits to arbitrage. While Vijh (1994) assumes the effect of the price pressure is on individual stocks, Barberis and Shleifer (2003) consider the question based on style investing. The implication is that assets in the same style comove due to the same factors, including price pressure where relevant.

Barberis and Shleifer (2003) assume that one group of investors, switchers, makes decisions based on a momentum strategy as suggested by Jegadeesh and Titman (2001) and the other group makes decisions based on fundamentals. The switchers are assumed to compare performances of different investment styles and switch funds from poorly performing styles to styles enjoying good performance. In other words, investment styles are influenced by noise traders and not just by fundamentals. This is one of the critical assumptions in the new models developed in this paper. The Barberis and Shleifer (2003) model has several interesting implications. First, the covariance between a stock and a style increases after it is added to that style. Thus a stock added to the S&P 500 should move more closely this index which is similar to the prediction of the Vijh (1994) classical model. Second, stocks that are in the same style move more closely than their fundamentals do. Third, stocks that are in different styles move less closely than their fundamentals do. The latter two predictions imply that changes in comovement are not caused by fundamentals alone.

BSW (2005) build on Barberis and Shleifer (2003) by constructing a model of excess comovement based on notions of category and habitat and employ univariate and bivariate regressions. Both are based on the univariate regression as the bivariate regression simply employs a broader (than just the S&P 500) definition of the market index. They also recognise that market frictions such as slow information diffusion may play a role. The BSW model assumes that risky assets are divided into different categories and noise traders invest funds in or withdraw funds from these categories depending on their sentiment. It also assumes that returns on assets depend on market-wide, group-specific and idiosyncratic fundamental shocks and on noise trader sentiment. This model has two implications. First, in their univariate model, the beta coefficient of the index to which a stock is added increases after the stock is reclassified. Second, in their bivariate model, the beta coefficient of the index to which a stock is added increases while the beta of the index the stock exits decreases after the asset is reclassified. The absolute magnitudes of beta increases and decreases are predicted to be equal. These two implications are similar to what

Wurgler (2011) later refers to as the index inclusion and detachment effects of index-linked investment.

Chen et al. (2016) build a model that shares some of the BSW (2005) assumptions but also introduces new assumptions. They seek to show that the BSW bivariate model does not contain any information about excess comovement and that its regression coefficients are very sensitive to time variation in other characteristics of the return process. They employ different assumptions and specifications. They do not specify what is included in common fundamental shocks. Further, they assume that non-fundamental shocks are group-specific and correlations of non-fundamental shock across groups are zero. By contrast, BSW (2005) assume that non-fundamental shocks such as those to sentiment are correlated across groups. The Chen et al. (2016) assumption is not consistent with the view that noise traders move funds from one group to the other one (see Barberis and Shleifer, 2003). The implication is that the demand of one group increases when that of the other group decreases. Chen et al. (2016) also use other stylised assumptions such as loadings of unity on the non-fundamental group shock and fundamental shocks. The implication of their model is that univariate regressions of stock returns on S&P and non-S&P returns are more informative than the BSW bivariate regressions about comovement. This chapter does not address the theoretical role of bivariate models but instead focuses on multi-factor models of comovement and, in particular, the four-factor model that includes momentum.

3.2.2. Contrasting results

Vijh (1994) regresses stock returns on the CRSP value-weighted index return to estimate the beta for pre- and post-event windows and then averages the difference between pre and post-event betas calculated using data from 1975 to 1989. Daily and weekly data are used for the test and a significant increase in beta is found at both frequencies for the whole sample. This finding is consistent with the prediction of Vijh's (1994) model. However, the beta changes vary over the sample and the magnitude of change in beta and its significance for daily data is higher than for weekly data. Vijh (1994) interprets this as the effect of the price pressure disappearing in the long term.

BSW (2005) estimate changes in the S&P beta using univariate regressions and changes in the S&P and non-S&P beta employing bivariate regressions. Their empirical tests use data from 1976 to 2000 at the daily, weekly (455 additions in both cases) and monthly (324 additions) frequencies.[13] Their univariate regression results are similar to those of Vijh (1994) in showing small to moderate beta increases for added stocks. However, they also produced three novel results. First, their bivariate results offered the strongest support for excess comovement. For instance, over their full 1976-2000 sample period they found that the mean daily S&P betas of added stocks increased by 0.326 while the corresponding non-S&P betas fell by 0.319. Second, they establish that their monthly 1988-1998 subsample produces stronger results with S&P betas and non-S&P beta increases of 0.375 and -0.348, respectively. This they attribute to the S&P 500 index become more popular within the investor community in the later years of their sample. Finally, using Dimson forward and lagged betas, they establish that slow information diffusion accounts for around one third of their univariate beta increases and up to two thirds of the larger bivariate increases.

The BSW (2005) results have been replicated in many studies and for a range of stock indices across the world by Claessens and Yafeh (2012). They employ data on forty developed and emerging markets over a 10-year sample span and find beta increases for stocks added to a major index in most markets. Their test results are very similar to BSW. They support the BSW category/habitat views and they also find that information-related factors play a role. Some studies

[13] The addition numbers are smaller at the monthly frequency as this entails a three-year post-implementation window from 1976 -1998.

have focused on other major stock market indices. For instance, Greenwood and Sosner (2007) find support for changes in the Nikkei 225 index. Greenwood (2008) finds a strong positive relation between overweighting and the comovement of a stock with other stocks in the Nikkei 225 index.[14] Changes in beta coefficients have been found not only in relation to indexing style. For example, Green and Hwang (2008) find that stocks have higher beta coefficients with low-priced stocks and lower coefficients with high-priced stocks after splits. Boyer (2011) finds that value index coefficients increase while growth index coefficients decrease after stocks are reclassified from a growth to a value index in the USA.

The empirical results discussed above provide evidence that changes in the coefficients cannot be explained fully by fundamentals. As changes in constituents of the S&P 500 do not represent changes in fundamentals, BSW (2005) claim that the changes in beta after changes in the S&P 500 constituent stocks is a reflection of the fact that sentiment-based, category or habitat theories may explain the changes in comovement. More recent supportive evidence is provided by Claessens and Yafeh (2012) and Kumar et al. (2013). Recent research provides evidence that support classical theories. For example, Kasch and Sarkar[15] (2014) claim empirical results discussed above are explained by changes in loadings on common factors in returns, including *SMB*, *HML*, and *MOM*. The univariate regression, Fama and French (1993) three-factor model and Carhart (1994) four-factor model are used to investigate the changes in comovement. Kasch and Sarkar (2014) find the average change in the beta is insignificant for S&P 500 addition events through the Fama-French three-factor- and Carhart four-factor-models with the CRSP value-weighted index[16] return as a proxy of market return. However, the daily average change in the market beta is significantly positive in the 3-factor and 4-factor models with the S&P 500 index

[14] Note that is a cross-sectional study and thus a cleaner test of comovement than the time series studies.
[15] They drop the first two months after the month of inclusion announcement while post-inclusion time interval in Barberis et al. (2005) is from the first month after the month of inclusion announcement.
[16] Note that the CRSP value-weighted index is not a natural category for investors and, moreover, has no entries or exits as it includes all quoted stock in the USA.

return which is consistent with Kasch and Sarkar (2014). We interpret these findings as consistent with the view that changes in comovement are explained not by fundamentals alone.

3.3. Three- and four-factor models

Most previous research uses the univariate regression to estimate beta and its post-addition change to examine comovement. This may suffer from omitted variable bias that the estimator of beta would be biased if a relevant variable is omitted. Kasch & Sarkar (2014) argue that 3- and 4-factor models should be used to take other fundamental factors into account. However, they do not specify how omitted variable bias can affect estimates of comovement. Instead, they establish how the estimation of comovement changes when different regressions are used. This section gives a theoretical example to show how omitted variable bias might affect our estimates of comovement.

When the univariate regression is used to examine the beta, the regression is given by:

$$r_t = \alpha + \beta_1 r_{sp,t} + \varepsilon_t \tag{3.1}$$

where r_t denotes return on an individual stock, $r_{sp,t}$ represents the return on the S&P 500 index, and β_1 examines the comovement. The OLS estimator of β_1 from equation 3.1 is simply the ratio of the covariance between the individual stock return and the S&P 500 index return to the variance of S&P 500 index return, $\frac{\sigma_{r_t,r_{sp,t}}}{\sigma^2_{r_{sp,t}}}$. However, this is not the case when a relevant variable is omitted. For example, suppose that the 'true' regression model should be:

$$r_t = \alpha + \beta_1 r_{sp,t} + \beta_2 r_{o,t} + \varepsilon_t \tag{3.2}$$

where $r_{o,t}$ denotes a potential important variable in the regression. If 3.1, rather than 3.2, is estimated then the estimator of β_1 is:

$$\beta_1 = \frac{1}{1-\rho_{r_{sp}r_o}}\left(\frac{\sigma_{r_i,r_{sp}}}{\sigma^2_{r_{sp}}} - \rho_{r_{sp}r_o}\frac{\sigma_{r_o}}{\sigma_{r_{sp}}} \times \frac{\sigma_{r_i r_o}}{\sigma^2_{r_o}}\right)^{17} \tag{3.3}$$

where $\rho_{r_{sp}r_o}$ represents the correlation between $r_{sp,t}$ and $r_{o,t}$.

Equation (3.3) shows how the estimator of β_1 is influenced by the correlation between the explanatory variables and by the ratios of volatilities. However, the correlation coefficient between independent variables plays a key role. If the variables are uncorrelated or the correlation coefficient is zero, the coefficient estimate of β_1 will be the same in (3.1) and (3.2). Moreover, multicollinearity will be an issue if the correlation between the variables is too high.

The example suggests that estimator of comovement is biased only when explanatory variables exhibit non-zer cross correlation. In other words, we can still use univariate regression to estimate betas even when the univariate regression suffers from the omitted variable bias if S&P 500 index return is not correlated with other potentially important variables, and if other variables are not cross correlated. This is because estimator of beta from the univariate regression is still unbiased when potential explanatory variables are not correlated. Kasch & Sarkar (2014) claim that 3- and 4-factor models are more appropriate for estimating betas because they take SMB, HML, and MOM into account. However, based on a matching exercise, BSW (2005) suggests that size has little influences on change in comovement. As a result, there is no point in using 3- and 4-factor models if we cannot find evidence of that estimator of comovement from the univariate regression is biased.

Cross correlations provide us with a useful tool to examine whether 3- and 4-factor models are more appropriate. Correlations between returns on the S&P 500 index, on SMB, on HML, and on MOM should be non-zero to justify the use of 3- and 4-factor models. This is because estimator of beta from the univariate regression is no different from that from the 3- and 4-factor models if

[17] See appendix.

these factors are not correlated. At the same time, correlations should not be too high to avoid multicollinearity issues.

[Insert Table 3.1 here]

Table 3.1 reports cross correlations between monthly returns on S&P 500 index and other factors from 1988 to 2014. It shows that correlations are low but non-zero. This indicates that the estimator of comovement from univariate regression may possibly be biased and that 3- and 4-factor models may be more appropriate for estimating change in comovement. However, while two of the correlation coefficients between the S&P 500 return and the factor loadings are negative (HML and MOM) and negative, the corresponding coefficient for SMB is positive. Finally, the relatively low correlation values imply that 3- and 4-factor model do not suffer from multicollinearity.

However, this does not mean that excess comovement is zero but instead suggests that 3- and 4-factor models may provide more accurate estimators of change in comovement. We cannot simply conclude that comovement is a result of fundamental values even if we find that change in comovement is insignificant after 3- and 4-factor models are used. This is because there is no theory to state whether SMB, HML, and MOM are fundamental factors or are influenced by sentiment or both. Moreover, changes in the loadings on SMB, HML, and MOM after changes in membership of the S&P 500 index may provide extra evidence of excess comovement. For example, the size of a company should not change just because its stock is added to the S&P 500 index. As a result, a significant change in loading on SMB may also be an evidence of excess comovement. A theoretical model may be required to help us to understand results of 3- and 4-factor models.[18]

Finally, nonzero correlations support the assumption in chapter 2 that overlaps between styles happen. These overlaps affect demand to and supply of event stocks and hence cashflow to these stocks. The MOM has the highest absolute correlation with the S&P 500 index return. This suggests that MOM may plays a more important role relative to SMB and HML. This is consistent with Chen et al. (2016).

3.4. Data, methodology and empirical results

3.4.1. Data

Data over the January 1988-June 2014 period are used for examining the beta changes in stocks that were added to or deleted from the S&P 500 index. The list of event stocks is from the Compustat North America database. There are 572 inclusion and 144 deletion events over the period. Following BSW (2005), addition events are excluded if the firm results from restructuring or spinning off a firm already in the index or if the firm is involved in a merger or takeover around the event. We also exclude the event firm if the firm's sample in the pre- or post-event window is less than 30 at the daily and weekly frequencies. For the monthly test, the event is excluded if we do not have data for the full 36-month post-event window. These criteria yield a daily sample of 515 events, a weekly sample of 509 and a monthly sample of 397 events. Deletion events are excluded if bankruptcy, merger and takeover of the firm happen around the event or the data are not available. The final deletion sample includes 144 events with daily, 140 with weekly, and 90 with monthly data.

[18] A potential theoretical model is given in the Appendix.

The prices of the event stocks, the level of the S&P index and the level of the value-weighted CRSP NYSE, AMEX and Nasdaq indexes are from the CRSP database. Log returns on the stock, the S&P 500, and the value-weighted CRSP market index are calculated. The non-S&P 500 return index uses the formula, $R_{nonSP500,t} = \left(R_{vwcrsp,t} - \frac{CAP_{SP500,t-1}}{CAP_{vwcrsp,t-1}} R_{SP500,t}\right) \times \frac{CAP_{vwcrsp,t-1}}{CAP_{vwcrsp,t-1} - CAP_{SP500,t-1}}$.[19] Total capitalisation of the S&P 500 index, $CAP_{SP500,t-1}$, and of the value-weighted CRSP market index, $CAP_{vwcrsp,t-1}$, are from the CRSP database. The Fama-French 3 factors and momentum factor are taken from the Fama-French Data Library.

3.4.2. Methodology

Following BSW (2005), univariate and bivariate model regressions are used for tests of beta coefficient changes relative to the S&P 500 index and the S&P and non-S&P 500 indexes, respectively. The following equations give the univariate and bivariate regressions, respectively, where $R_{j,t}$ denotes the added (deleted) stock return, and $R_{s\&p\ 500,t}$ and $R_{nonSP,t}$ are S&P-500 and non-S&P-500 returns, respectively.

$$R_{j,t} = \alpha_j + \beta_{j,sp500,1} R_{s\&p\ 500,t} + \varepsilon_{j,t}$$

$$R_{j,t} = \alpha_j + \beta_{j,SP500,2} R_{SP500,t} + \beta_{j,nonSP} R_{nonSP,t} + e_{j,t} \tag{3.4}$$

The beta coefficients for both models are estimated using one-year pre- and post-addition windows for daily and weekly data. The average change in all added (deleted) firm coefficients is estimated

[19] The formula is inferred from the identity, $R_{vwcrsp,t} = \frac{CAP_{SP500,t-1}}{CAP_{vwcrsp,t-1}} R_{SP500,t} + \frac{CAP_{vwcrsp,t-1} - CAP_{SP500,t-1}}{CAP_{vwcrsp,t-1}} R_{nonSP500,t}$.

and *t*-statistics are calculated for a significance test. As BSW (2005) suggest, the magnitude and significance of the average coefficient change should be smaller at lower frequencies (daily and weekly) because noise trader sentiment should disappear in the long-term when monthly data are employed with a three-year window.

As Kasch and Sarkar (2014) suggested, SMB, HML and MOM can have influences on change in comovement, we undertake empirical tests for added stocks using the three- and four-factor regressions as follows.

$$r_{j,t} = \alpha_j + \beta_{j,sp500,3} r_{sp500,t} + \beta_{j,smb} r_{smb,t} + \beta_{j,hml} r_{hml,t} + \varepsilon_{j,t}$$

$$r_{j,t} = \alpha_j + \beta_{j,sp500,4} r_{sp500,t} + \beta_{j,smb} r_{smb,t} + \beta_{j,hml} r_{hml,t} + \beta_{j,mom} r_{mom,t} + \varepsilon_{j,t} \qquad (3.5)$$

Like most recent studies, we follow BSW in defining window length and in excluding the month in which the event is announced and implemented due to noise. This is different from what Kasch and Sarkar (2014) do. Kasch and Sarkar (2014) drop the first two months after the month of inclusion announcement which may cause that the effect of the event fade.

To estimate effects of the SMB, HML and MOM factors statistically we use a t-test to examine the following hypotheses:

Hypothesis 1: The changes in sensitivity to the size and value factors affects the change in comovement.

$$H_0: \beta_{j,sp500,3} - \beta_{j,sp500,1} = 0$$

$$H_A: \beta_{j,sp500,3} - \beta_{j,sp500,1} \neq 0$$

Hypothesis 2: The change in loading on the momentum factor affects the change in comovement

$$H_0: \beta_{j,sp500,4} - \beta_{j,sp500,3} = 0$$

$$H_A: \beta_{j,sp500,4} - \beta_{j,sp500,3} \neq 0$$

3.4.3. Empirical results

We first analyse the traditional univariate regression and bivariate regression results to check the impact of including recent data on S&P 500 additions and deletions. We then discuss the results reported from implementing regression tests implied by our theoretical multi-factor models

Univariate regression and bivariate regression results

Table 3.2 reports the results from estimating the univariate regressions and bivariate regressions given by equation (3.4). It gives the 1988-2014 average change in the slope coefficient across all events in the sample and the average change in the R^2.

[Insert Table 3.2 here]

The univariate results support the BSW (2005) prediction that the beta coefficients at all frequencies increase after stocks are added to the S&P 500 index and that they decrease after stocks are deleted except at the weekly frequency. The average increase in the daily and monthly betas over the full sample period is 0.17 in both cases while that in the weekly betas is 0.10. These increases in both the betas and the R^2 are all highly significant. Interestingly, there is no evidence of a weaker effect at the monthly frequency.

The findings for the 1988-2000 subsample show average increases in the daily, weekly and monthly beta coefficients and R^2.[20] Both the average beta and R^2 changes are significant at the monthly frequency when more recent data are available. However, the significant monthly results are caused by the both economically and statistically significant results for the 65 stocks added in 1999 and 2000. This suggests that the BSW view needs to be revised in the light of the additional data up to 2000.

The results for the 2001-2014 subsample also confirm that comovement increases after stocks are added to the S&P 500 index but, surprisingly, the results are now strongest at the monthly frequency with a significant average increase of 0.13, followed by those at the daily frequency while those at the weekly frequency are insignificant. The average change in the daily slope coefficient is 0.079 which is just one third of that for the 1988-2000 subsample. While Chen et al. (2016) find that the average change in the daily slope coefficient over the 2001-2012 period is 0.071, their findings cannot be interpreted as evidence of the decreasing importance of index investing. Chang, Hong and Liskovich (2015) find that shorting of index members has increased over the years and that mutual funds with large stocks in their portfolios supply liquidity for index trackers which should imply smaller beta changes.[21] Moreover, the average monthly change in the R^2 is a significant (at the 1% level) 0.102 for the 2001-2012 period which exceeds that for the 1988-2000 period by a factor of 3. The daily change in the R^2 is also stronger in the more recent relative to the 1988-2000 period.

[20] Note that the monthly beta change is not significant if the 1999-2000 observations are not included as in BSW (2005).

[21] Note that this study refers to the Russell 1000 and 2000 indices. It is plausible to assume similar behaviour for S&P 500 index changes.

The bivariate regression results show that average changes in the S&P beta and the non-S&P beta confirm the BSW (2005) prediction that the S&P betas increase while the non-S&P 500 betas decrease after additions to the S&P 500. Table 3.2 shows the average change in the daily S&P 500 and non-S&P 500 coefficients across all addition and deletion events for the whole sample is 0.407 and -0.329, respectively, 0.235 and -0.142, respectively, at the weekly frequency and 0.3009 and -0.1262, respectively, the monthly frequency. The results from the bivariate model are statistically stronger than that from the univariate model in line with the BSW (2005) results. Finally, our bivariate results – like theirs – show only weak evidence of sentiment reversion at lower frequencies.

New three- and four-factor loadings

Table 3.3 and 3.4 report changes estimated through the 3- and 4-factor models in beta and in the SMB, and HML and MOM loadings.

[Insert Table 3.3 here]

[Insert Table 3.4 here]

The changes in beta from the 3- and 4-factor models are smaller in magnitude and not always as significant as in the univariate results with the very important exception of results at the monthly frequency. The smaller changes in comovement suggest that SMB, HML, and MOM factors are able to explain comovement, but they cannot explain the comovement completely. This is because daily and monthly overall change in comovement is still significantly positive from 3- and 4-factor regression. For example, daily change in comovement is 0.08 and 0.1033 from 3- and 4-factor regression, respectively. Further, monthly change in comovement is 0.2347 and 0.2011 from 3- and 4-factor regression, respectively. These figures are all significant at 1% level of significance. Changes in comovement estimated at the weekly frequency are insignificant as in Kasch and Sarwar (2014). The novel finding here is that monthly beta changes are larger in magnitude than those in the univariate regressions despite being conditioned on the SMB, HML and MOM portfolios. The results for the two sub-periods are also statistically significant at the 1% critical value in both models.

The decrease in the loading on SMB is highly significant at the daily frequency for the 3- and 4-factor models consistent with the Kasch and Sarkar (2014) empirical findings. This may because the constituents of the S&P 500 index overlap with those of the SMB portfolio and thus cash flows are invested to both at the same time and not because fundamental factors are more important. The HML results are only significant in the 3- the 4-factor models for the full sample and the 1988-2000 sub-period at the monthly frequency.[22] The full sample loading on MOM in the 4-factor model decreases significantly by 0.240, 0.122, and 0.14 at the daily, weekly and monthly frequencies, respectively. Both the daily and monthly MOM loadings are significant for both sub-periods.

To sum up, the most interesting results are those at the daily and monthly frequencies in both the 3- and 4-factor models. At these frequencies, excess comovement measured by beta increases is everywhere significant and the results are notably economically stronger (at least double) at the monthly compared to the daily frequency. This may reflect that both the SMB and MOM loadings are stronger at the daily than the monthly frequency and these have offsetting unconditional correlation coefficients. These results suggest that excess comovement cannot be

[22] Note that we are ignoring the weekly results as they are highly inconsistent and mostly insignificant.

driven by transient investor sentiment given the strength of the monthly changes. Note also that some of the changes in the MOM may be due to gradual information diffusion. In this respect, our results run contrary to the Kasch and Sarwar (2014) and Chen et al. (2016) premise that excess comovement can be explained by traditional risk loadings.

Differences in changes in beta

Table 3.5 reports the average difference between the change in beta estimated by univariate regression and that estimated by the 3-factor model and the average difference between the change in beta estimated by 3-factor model and that estimated by the 4-factor model.

[Insert Table 3.5 here]

The table suggests SMB, HML, and MOM factors have some influences on comovement. First, the difference of change in comovement between 3-factor regression and univariate regression is significant at daily, weekly, and monthly frequency, respectively. When daily and weekly data are used, the difference is significantly negative (-0.090, -0.058) implying that change in comovement decreases when SMB and HML are considered. This means that comovement can be explained partly by SMB and HML factors together. However, this is not the case when monthly data are used. The difference is significantly positive, 0.058, suggesting that comovement is higher even when SMB and HML factors are considered. This is interesting because if SMB and HML factors can explain comovement the difference should be negative. As the difference between 3-factor regression and univariate regression suggested, SMB and HML factors may explain parts of comovement at daily and weekly frequencies. This confirms that sentiment views should also have some influences on comovement.

Second, the difference of change in comovement between 4-factor and 3-factor regressions is significant at daily and monthly frequencies. However the difference is positive, 0.023, at daily frequency. This suggests that MOM factor is not helpful for explaining comovement at daily frequency. On the other hand, the monthly difference is -0.034, implying that MOM factor is able to explain parts of comovement. These findings suggest that MOM factor cannot explain comovement completely and other factors, such as nonfundamental factors should be taken into account.

To sum up, the loadings of SMB, HML, and MOM clearly do have an impact on comovement. However, what is fascinating is that the loadings from both the 3- and 4-factor models serve to strengthen the evidence of excess comovement at the monthly frequency relative to the findings from the univariate regression. It is implied that SMB, HML, and MOM can just explain comovement partly. This is because changes in comovement do not become to zero when 3- and 4-factor regressions are used. Further, the difference of change in comovement between 3-factor regression and univariate regression is not always negative.

3.5. Conclusions

This chapter first uses a theoretical example to show how omitted variable bias can influence change in comoovement. It also uses pair correlations between S&P 500 index returns, SMB,HML, and MOM to support that 3- and 4-factor regression should be used to examine slope of S&P 500 index return more accurately. The results of three- and four-factor models for the sample of stocks added to the S&P 500 over the 1988-2014 period are reported. Regressions using the three- and four-factor regressions produce novel results. At the daily and monthly frequencies, excess comovement - as measured by beta increases - is everywhere significant and the loadings on MOM are also mostly significant at these frequencies also. However, the monthly excess comovement results in the 4-factor model are double the magnitude of those at the daily frequency. They suggest that excess comovement is not driven by transient investor sentiment given the strength of the monthly changes. Clearly the factor loadings do have an impact on comovement and momentum plays a strong role in our results. However, the later at variance with those of Chen et al. (2016) and Kasch and Sarwar (2014) who suggest that momemntum can explain the puzzle. Our conclusion is that excess comovement refuses to go away and remains a puzzle!

Some may argue that other fundamental factors could also influence slopes of S&P 500 index return, and 5-factor regression could be used to examine the slope more accurately. However, in this chapter, we just want to test if SMB, HML and MOM are able to explain excess comovement. Further, it is time consuming to find all fundamental factors that can explain excess comovement together completely. As a result, next chapter want to use a new methodology to test the basis of the null that changes in comovement can be explained by fundamentals only.

Table 3.1 cross correlations

This table reports cross correlations between monthly return on the S&P 500 index, on small minus big portfolio (SMB), on high minus low portfolio (HML), and on momentum portfolio (MOM) from 1988 to 2014 Returns on SMB, HML and MOM are from Kenneth R. French Data Library.

	$\boldsymbol{r_{sp}}$	SMB	HML	MOM
$\boldsymbol{r_{sp}}$	**1**			
SMB	**0.0590**	**1**		
HML	**-0.1240**	**-0.3044**	**1**	
MOM	**-0.2687**	**0.0656**	**-0.1747**	**1**

Table 3.2

Changes in comovement of stocks added to and deleted from the S&P 500 index

The sample includes stocks added to and deleted from the S&P 500 index from 1988 to 2014. Stocks that are involved in mergers, acquisitions, takeovers, and bankruptcies are excluded. For each event stock j, univariate regression of returns on event stock on returns on the S&P 500 index: $r_{j,t} = \alpha_j + \beta_j r_{SP500,t} + v_{j,t}$

and bivariate regression of returns on event stock on returns on the S&P 500 index and returns on the non-S&P 500 index:

$r_{j,t} = \alpha_j + \beta_{SP500,j} r_{SP500,t} + \beta_{nonSP500,j} r_{nonSP500,t} + v_{j,t}$

are examined separately for pre- and post-event windows, respectively. Changes in parameters are calculated as differences of corresponding values of parameters between periods before and after events. For univariate regression, average change in slope ($\Delta\overline{\beta}$) and fit of the regression ($\overline{\Delta R^2}$) are reported. For bivariate regression, average change in slopes ($\Delta\overline{\beta}_{SP500}$ and $\Delta\overline{\beta}_{nonSP500}$) are reported. For daily and weekly tests, 12 months before and after announcement month are pre- and post-windows, however, for monthly test, 36 months are used. Panel A, B, and C report daily, weekly, and monthly results, respectively.

Panel A: Daily						
Sample		**N**	**Univariate**		**Bivariate**	
			$\Delta\overline{\beta}$**(se)**	$\overline{\Delta R^2}$ **(se)**	$\Delta\overline{\beta}_{SP500}$ **(se)**	$\Delta\overline{\beta}_{nonSP500}$ **(se)**
Additions	1988-2014	515	0.1706*** (0.0220)	0.0726*** (0.0072)	0.4069*** (0.0304)	-0.3287*** (0.0339)
	1988-2000	265	0.2574*** (0.0345)	0.0517*** (0.0084)	0.4114*** (0.0396)	-0.3543*** (0.0476)
	2001-2014	250	0.0786*** (0.0255)	0.0948*** (0.0119)	0.4020*** (0.0466)	-0.3017*** (0.0484)
Deletions	1988-2014	144	-0.149*** (0.0571)	-0.0351*** (0.0102)	-0.5842*** (0.1162)	0.5880*** (0.1162)
Panel B: Weekly						
Additions	1988-2014	509	0.1009*** (0.0349)	0.0436*** (0.0090)	0.2352*** (0.0661)	-0.1420** (0.0606)
	1988-2000	263	0.1872*** (0.0546)	0.0361*** (0.0115)	0.2404*** (0.0886)	-0.1022 (0.0856)
	2001-2014	246	0.0086 (0.0417)	0.0517*** (0.0140)	0.2298** (0.0987)	-0.1845** (0.0859)
Deletions	1988-2014	140	-0.0831 (0.0765)	-0.0369*** (0.0141)	-0.5356*** (0.1580)	0.4538*** (0.1521)
Panel C: Monthly						
Additions	1988-2012	397	0.1764*** (0.0482)	0.0674*** (0.0117)	0.3009*** (0.1033)	-0.1262* (0.0846)
	1988-2000	207	0.2191*** (0.0756)	0.0352** (0.0164)	0.3926*** (0.1457)	-0.1998** (0.1178)

	2001-2012	190	0.1299** (0.0580)	0.1024*** (0.0162)	0.2010* (0.1464)	-0.0461 (0.1217)
Deletions	1988-2011	90	0.2284** (0.1069)	-0.028* (0.0214)	-0.0316 (0.2356)	0.2269 (0.2073)

***, ** and * denote significant differences from at the 1%, 5% and 10% levels in one-sided test.
se: standard error

Table 3.3

Changes in comovement by using the three-factor regression

Three-factor regression on returns of S&P 500 index, small minus big market capitalisation stocks, and high minus low book-to-market ratio stocks: $r_{j,t} = \alpha_j + \beta_{SP500,j,3} r_{SP500,t} + \beta_{smb,j} r_{smb,t} + \beta_{hml,j} r_{hml,t} + v_{j,t}$ is examined for each event stock j. Regression is examined for pre- and post-evetn window, respectively, and changes in slopes ($\Delta\overline{\beta}_{SP500,j,3}$, $\Delta\overline{\beta}_{smb}$, and $\Delta\overline{\beta}_{hml}$) are calculated. For daily and weekly tests, pre- and post-event windows are 12 months before and after announcement month, respectively, while for monthly tests, 36 months are used. Panel A, B, and C reports daily, weekly, and monthly results, respectively.

Sample		N	Three-factor model		
Panel A: daily returns			$\Delta\overline{\beta}_{SP500,3}$(s.e.)	$\Delta\overline{\beta}_{smb}$(s.e.)	$\Delta\overline{\beta}_{hml}$(s.e.)
Additions	1988-2014	515	0.0808*** (0.0275)	-0.2261*** (0.0363)	-0.0686* (0.0488)
	1988-2000	265	0.0791** (0.0461)	-0.3181*** (0.0586)	-0.021 (0.0703)
	2001-2014	250	0.0826*** (0.0287)	-0.1286*** (0.0407)	-0.119** (0.0674)
Panel B: weekly returns					
Additions	1988-2014	509	0.044 (0.0414)	-0.1431** (0.0615)	-0.1941*** (0.0773)
	1988-2000	263	0.0749 (0.0664)	-0.2458*** (0.0955)	-0.2003* (0.1208)
	2001-2014	246	0.0109 (0.0479)	-0.0333 (0.0755)	-0.1874** (0.0946)
Panel C: monthly returns					
Additions	1988-2012	397	0.2347*** (0.0559)	-0.0990* (0.0666)	0.1817** (0.0850)
	1988-2000	207	0.3133*** (0.0860)	-0.0586 (0.0990)	0.3296*** (0.1161)
	2001-2012	190	0.1489** (0.0694)	-0.1430* (0.0880)	0.0206 (0.1240)
***, ** and * denote significant differences at the 1%, 5% and 10% levels in one-sided test.					

Table 3.4

Changes in comovement by using 4-factor regression

Four-factor regression on returns of S&P 500 index, small minus big market capitalisation stocks, high minus low book-to-market ratio stocks, and momentum portfolio: $r_{j,t} = \alpha_j + \beta_{SP500,j,3} r_{SP500,t} + \beta_{smb,j} r_{smb,t} + \beta_{hml,j} r_{hml,t} + \beta_{mom,j} r_{mom,t} + v_{j,t}$ is examined for each event stock j. Regression is examined for pre- and post-evetn window, respectively, and changes in slopes ($\Delta\overline{\beta}_{SP500,j,3}$, $\Delta\overline{\beta}_{smb}$, $\Delta\overline{\beta}_{\text{hml}}$, and $\Delta\overline{\beta}_{mom}$) are calculated. For daily and weekly tests, pre- and post-event windows are 12 months before and after announcement month, respectively, while for monthly tests, 36 months are used. Panel A, B, and C reports daily, weekly, and monthly results, respectively.

Sample		N	Four-factor model			
Panel A: daily returns			$\Delta(\overline{\boldsymbol{\beta}}_{SP500,4}$(s.e.)	$\Delta\overline{\boldsymbol{\beta}}_{smb}$(s.e.)	$\Delta\overline{\boldsymbol{\beta}}_{hml}$(s.e.)	$\Delta\overline{\boldsymbol{\beta}}_{mom}$(s.e.)
Additions	1988-2014	515	0.1033*** (0.0263)	-0.1842*** (0.0349)	-0.0264 (0.0495)	-0.2399*** (0.0431)
	1988-2000	265	0.1307*** (0.0442)	-0.2375*** (0.0585)	0.0079 (0.0759)	-0.2575*** (0.0686)
	2001-2014	250	0.0744*** (0.0272)	-0.1278*** (0.0361)	-0.0627 (0.0626)	-0.2212*** (0.0512)
Panel B: weekly returns						
Additions	1988-2014	509	0.0556 (0.0428)	-0.0986 (0.0642)	-0.1224 (0.0847)	-0.1216** (0.0631)
	1988-2000	263	0.1025 (0.0677)	-0.1815* (0.1004)	-0.1621 (0.1304)	-0.0425 (0.1005)
	2001-2014	246	0.0055 (0.0509)	-0.01 (0.0781)	-0.08 (0.1063)	-0.2062*** (0.074)
Panel C: monthly returns						
Additions	1988-2012	397	0.2011*** (0.0556)	-0.0930* (0.0674)	0.1950** (0.0873)	-0.1397** (0.0585)
	1988-2000	207	0.2664*** (0.0842)	-0.0532 (0.1002)	0.3599*** (0.1222)	-0.1817** (0.0958)
	2001-2012	190	0.1299** (0.0711)	-0.1362* (0.0891)	0.0152 (0.1237)	-0.0941* (0.0634)
***, ** and * denote significant differences at the 1%, 5% and 10% levels in one-sided test.						

Table 3.5

Difference of change in comovement between univariate regression and 3-factor regression, and between 3- and 4-factor regressions

Changes in comovement from univariate regression, 3-factor regression, and 4-factor regression are reported by Tables 3.2-3.3. The difference between change in slope of S&P 500 index from univariate regression and change in slope of S&P 500 index from 3-factor regression ($\Delta\overline{\beta}_{j,SP500,3} - \Delta\overline{\beta}_{j,SP500}$) is calculated. The difference between change in slope of S&P 500 index return from 3-factor regression and change in slope of S&P 500 index return from 4-factor regression ($\Delta\overline{\beta}_{j,SP500,4} - \Delta\overline{\beta}_{j,SP500,3}$) is also calculated.

Sample		N		
Panel A: daily returns			$\Delta\overline{\boldsymbol{\beta}}_{j,sp500,3} - \Delta\overline{\boldsymbol{\beta}}_{j,sp500}$ **(s.e)**	$\Delta\overline{\boldsymbol{\beta}}_{j,sp500,4} - \Delta\overline{\boldsymbol{\beta}}_{j,sp500,3}$ **(s.e.)**
Additions	1988-2014	515	-0.0898*** (0.0242)	0.0225** (0.0121)
	1988-2000	265	-0.1782*** (0.0409)	0.0515*** (0.0162)
	2001-2014	250	0.0040 (0.0232)	-0.0082 (0.0180)
Panel B: weekly returns				
Additions	1988-2014	509	-0.0569** (0.0309)	0.0116 (0.0166)
	1988-2000	263	-0.1122** (0.0527)	0.0276 (0.0250)
	2001-2014	246	0.0023 (0.0297)	-0.0054 (0.0217)
Panel C: monthly returns				
Additions	1988-2012	397	0.0583** (0.0318)	-0.0335** (.0190)
	1988-2000	207	0.0943** (0.0433)	-.0469** (0.0281)
	2001-2012	190	0.0190 (0.0468)	-0.0190 (0.0252)
***, ** and * denote significant differences at the 1%, 5% and 10% levels in one-sided test.				

Chapter 4. A PE ratio approach for comovement

4.1. Introduction

In Chapter 3, it is concluded that SMB, HML, and MOM cannot explain excess comovement completely. More fundamental factors could be added to the multifactor regressions to try to explain excess comovement until comovement can be explained completely. However, this is time consuming, and hence that a more convenient way is needed to test the null that fundamentals can explain excess comovement completely.

The reasons for comovement changes are widely discussed. Classical theories posit that comovement changes because fundamentals change while behavioural theories use sentiment- or friction-based views to explain this phenomenon. The usual empirical way to test the reason for comovement is to control for changes in fundamentals. The traditional way is to examine effects of events which do not have any influence on fundamental values. For example, changes in the constituents of the S&P 500 index are designed to improve representativeness (see Barberis, Shleifer, and Wurgler, 2005). Changes in members of the S&P growth & value indexes are because of changes in companies' value ratio (see Boyer, 2011). Splits of shares induce large changes in prices without fundamental changes (see Green and Hwang, 2009). These events cannot be explained by fundamental factors alone. Researchers also control characteristics which are potential factors for explaining these effects. For instance, the size of companies is usually controlled for in these event studies. To control for size, the difference in difference method is used. Companies which have identical sizes but are not involved in the event are used for comparison with event companies. If changes in comovement of matched companies are different from that of event companies, we can reject the effect of the size factor. The matching exercise is a useful way of excluding potential specific explanations of comovement. However, it cannot reject the effect of fundamentals on comovement. This is because it cannot conclude that all fundamental factors do not change when an event happens.

Many researchers exclude the influence of size, industry, thin trading, or liquidity on comovement. Classical critics find that momentum plays a key role. Kasch and Sarkar (2014)find changes in comovement become insignificant when 3- and 4-factor models and the CRSP index are used. Fama & French (2015) argue that the 5-factor model which captures the size, value, profitability, and investment patterns outperforms the 3-factor model in Fama & French (1993). Profitability and investment patterns might be other potential explanations of comovement although they have not been tested so far.

Another way of challenging the roles sentiment or friction play in comovement is to find counter examples to the implications of a theoretical model with nonfundamental shocks. For example, Chen et al. (2016) construct a model where group-specific nonfundamental shocks are assumed to determine returns on individual stocks and specific groups. This model challenges the Barberis et al. (2005) approach and implies that that their bivariate regression is not informative about excess comovement. Further, they posit that regressions of individual stocks' return on the non-S&P 500 index return are informative about excess comovement. This model suggests that slope coefficients from the regressions on the non-S&P 500 index return should decrease after stocks are added to the S&P 500 index. However, Chen et al. (2016) report increases in the slope coefficients. This finding challenges the influence of sentiment on comovement. They argue that stocks have stronger comovement with all securities and not just the group they join.

This motivates the derivations of some implications from an efficient market model such as the dividend discounted model (DDM). The DDM assumes that rational investors should pay for what the security produces. Specifically, the price of a security should be the sum of the present value of all its future dividends. If anyone is willing to pay more than the price, rational investors are willing to short sell the security to make a profit. This arbitrage trade is also a key to making price equal to the equilibrium value.

This chapter develops a simple stylised model from the DDM which has implications about PE ratio patterns. The model indicates that PE ratios should always be stationary. We find that this is not the case. When the random walk without drift model is used, 95.7% to 93.3% individual stocks' PE ratios are found to be nonstationary. Some 97.9% to 98.8% of the corresponding S&P 500 index PE ratios are also nonstationary. These findings n can reject the null that sufficient numbers of rational investors participate in the market to make it efficient. In other words, we confirm the influences of sentiment and friction through rejecting the impact of rational investors.

Our results are consistent with other literatures. Shiller (1981) derives the volatility of prices from a simple efficient market model and suggests that actual prices is more volatile than their fundamentals. Coakley and Fuertes (2006) report that PE ratios drift away from fundamentals in the short-run but mean revert in the long run. Our results are consistent with these findings. We also find that individual PE ratios are not cointegrated with the S&P 500 PE ratios. This provides indirect supports the influence of sentiment and friction on comovement. Our proposed new test of comovement provide additional evidence of excess comovement.

In section 2, we review the literature. We then discuss our methodology in section 3. Data and results are reported in section 4. A final section concludes.

4.2. Literature review

This chapter discusses reasons why beta change after the constituents of the S&P 500 index change, and it likes to review literature of the efficient market hypothesis (EMH), behavioural theories, and PE ratios. This is because this literature can help readers to understand our methodology to examine the null that comovement reflects fundamentals only.

The null is the EMH that prices at any time reflect all available information fully. It implies a frictionless market where all available information is costlessly available to every traders, there are no transaction costs, and all participants have the same belief about the implications of current information for the current price and distributions of future prices of each security (see Fama., 1970). Fama (1970) further argues that these implications are not necessary but sufficient conditions for market efficiency. He argues that the market will still be efficient if investors take transaction costs and costs of information into account when pricing a security, and if sufficient numbers of investors have ready access to available information. The argument in Fama (1970) has the implication that rational investors play an essential role in efficient markets. These rational investors have to be smart enough to update their beliefs once new information is available. Moreover, the number of rational investors must be sufficient so that influences of irrational investors are offset in prices. As a result, sufficient numbers of rational investors are the necessary condition for efficient markets.

Other literature, based on sentiment and frictions, argues that the market is not efficient completely and noise traders play roles. Barberis et al. (2005) introduces three friction- or sentiment-based views. The first is referred by Barberis et al. (2005) as category view. In this view, investors divide assets into different categories and consider investments at category level to simplify decision making. It is also studied by Barberis & Shleifer (2003) who refers it as style investing. It argues that if style investors are noise traders with correlated sentiment, and if their trading affects prices, the common demands for stocks added to the S&P 500 index results in event stocks moving closer with their counterparty in the same categories even though their fundamentals are not correlated. The second view is habitat view where investors hold a preferred subset of all available securities. And the common demands for these specific preferred securities would induce a higher correlation between them. The last view is the information diffusion view.

This suggests that some stocks may have a faster rate of information diffusion while some others have slower rates. This may because of transaction costs or costs of information.

The three sentiment- or friction-based views provide some potential explanation of comovement from the view point of behavioural theories. However, they do not reject the EMH. In the other words, comovement can still be caused by fundamentals. As Fama (1970) suggested, if sufficient numbers of rational investors participate in the market, rational investors can take account of all available information, including effects of noise traders, transaction costs, and costs of information. It implies that rational investors with correlated beliefs will have offset effect to any influences on price by other irrational investors. As a result, when noise traders demand added stocks at the same time prices of event stocks will still be the equilibrium prices. This is because when the bid price from noise traders is higher than rational investors' expected value of the stock rational investors will sell it and demand is fulfilled once this happens. Hence, it is important to test if investors are rational.

Previous research emphasises that change in the constituents of the index does not reflect any fundamentals. Barberis et al. (2005) find increasing betas after stocks are added to the index. Green and Hwang (2009) find that stocks have increasing correlations with low priced stocks and decreasing correlations with high priced stocks after stock splits. Boyer (2011) finds that stocks move closer with the value index after they added from the growth index to the value index. These events truly do not reflect any change in fundamentals. However, this does not mean that rational investors' expectations about event stocks are still the same. Rational investors' expectation may change because other factors happen at the same time. Researchers try to control for the effects of other factors. For example, Barberis et al. (2005) compare change in betas between event companies and those with identical characteristics but not added to the index to support the view that effects of companies' characteristics are limited on comovement. However, supporters of classical theories argue that momentum is another essential factor which is not controlled in

Barberis et al. (2005). When known fundamental factors are controlled, other researchers may find more fundamental factors. This is just like that it changes from the univariate regression to the 3- and 4-factor model and now the 5-factor model to estimate the expected return on an asset.

A more efficient way of rejecting the null that comovement reflects the fundamentals is to reject the EMH. And a more efficient way to reject the EMH is to reject the necessary condition that sufficient numbers of rational investors participate in the market. Classical theories define rational investors through illustrating how investors should make decisions. However behavioural theories challenge them through showing how investors really make decisions. Classical theories claim that rational investors should have a concave utility function and worry about total wealth, while prospect theory asserts that investors pay more attention to gains and losses and investors worry more about losses than gains. Classical theories claim that rational investors should construct an overall portfolio to gain the advantages of diversification, while behavioural theories argue that investors suffer from mental account biases and like to allocate assets into different liars with different objectives of returns and risks. Classical theories claim that rational investors should always update their expectation once new information is available, while behavioural theories suggest that investors suffer from status quo bias that investors are unwilling to make changes. (See Kahneman & Tversky., 1979, Shefrin & Thaler., 1988, and Kahneman et al., 1991).

Some may argue that these findings provide evidence of investors having biases on average but ignore the question of whether sufficient rational investors exist. Hence models that track how rational investors price securities are needed. These models, such as the dividend discounted model, focus on prices but not returns which are determined by univariate regression, 3-, and 4-factor models. The implications from these models are helpful to understand how rational investors influence the market. If these influences do not exist, we can reject the necessary condition of the EMH.

Shiller (1981) derives several implications about volatility of equity prices from the dividend discounted model (DDM). After obtaining expected volatility of prices from the DDM, he compares it with the data and finds that volatility of stock prices is too high relative to fundamentals. It motivates us to derive some implications about EMH from the DDM so that we can test the EMH which is the basis of the null that comovement can be explained by fundamentals completely.

The PE ratio is interesting because it links prices to fundamentals. From the DDM, we derive an equation of expected value of PE ratios. This equation suggests that the PE ratio is an indicator of the growth of the company. When a company is in a specific period of the life cycle, it should have a stable growth. In the other word, PE ratios must be stationary until the company move to the next period of the life cycle. Coakley & Fuertes (2005) documents asymmetries in the time evolution of value ratios through a non-linear model. They find that PE ratios exhibit short-term continuation. They confirm that price can drift away from fundamentals due to role of sentiment.

4.3. Methodology

This section adopts a new approach to examine the reasons of comovement. We start with the dividend discount model (DDM) to show what happens if the efficient market hypothesis (EMH) holds. We then test if the implication of the EMH is the case to examine the validity of the EMH.

A standard method of determining the price of a share of a stock at the beginning of time period t is to sum present values of all future cash flows generated by the stock:

$$p_t = \sum_{i=0}^{\infty} \gamma^{k+1} E_t d_{t+k} \qquad 0 < \gamma < 1 \qquad (4.1)$$

where $E_t D_{t+k}$is the expectation of dividend at time t+k based on the information available at time t, and γ is the constant discount factor. This model implies that investors are willing to pay what they can obtain through holding the stock. However, investors are not able to predict all future dividends precisely based on current available information. As a result, some variations are needed:

$$p_t = \sum_{k=0}^{\infty} \gamma^{k+1} d_t (1 + E_t g)^{k+1} \qquad 0 < \gamma < 1 \tag{4.2}$$

where $E_t g$ is the expectation of growth of dividend based on the information available at time t. As equation (4.2) suggests, investors use expected growth of dividends to price a stock as they cannot predict all future cash flows. Equation (4.2) can then be restated as:

$$p_t = \frac{(1+E_t g) d_t}{E_t (r-g)} \qquad r > g \tag{4.3}$$

where r is the required rate of return on capital. If we substitute the product of earnings and the pay-out rate for the dividend, we have:

$$p_t = \frac{(1+E_t g) b e_t}{E_t (r-g)} \qquad r > g \tag{4.4}$$

where b represents the pay-out rate determined by the company, and e_t is the earnings at time t. As DeAngelo (2006) suggests, there is a trade-off between retention and distribution based on the life cycle theory. Young companies with high costs of raising external capital, and attractive investment opportunities would like to have high retentions so that the company can grow fast. On the other hand, mature companies with relatively low costs of raising capital but shrinking investment opportunities are more willing to distribute earnings. Companies determine their retention rate based on their life cycle to maximize the worth they can create. Based on equation (4.2), investors should make expectations of growth and the pay-out rate according to where the company is located in the life cycle in order to price that company's stock. Equation (4.2) further has an implication for the PE ratio:

$$p/e_t = \frac{(1+E_t g)b}{E_t(r-g)} \qquad r > g \tag{4.5}$$

As equation (4.5) suggested, the best expectation of the PE ratio should be $\frac{(1+E_t g)b}{E_t(r-g)}$. This indicates that if the company stays in a specific stage of its life cycle and its required rate of return on capital never change, it has a constant growth and pay-out rate, and hence a constant PE ratio. We further assume the required rate of return on capital does not change in the short term (one year). This is because rational investors in efficient markets construct portfolios based on long-run returns and risks. Even for those investors who have tactical asset allocation that focus on short-term returns, the portfolio should not drift away from the strategic asset allocations in the long-run. As a result, PE ratios should be a stationary variable that fluctuates around its expected value:

$$p/e_t = \frac{(1+E_t g)b}{E_t(r-g)} + \varepsilon_t \qquad r > g \tag{4.6}$$

$$E(\varepsilon_t) = 0,\ Var(\varepsilon_t) = \sigma_\varepsilon$$

where ε_t represents a random factor resulted from investors' tactical asset allocations or unexpected financial events. This is true especially for event stocks that are added to the S&P 500 index. After added to the S&P 500 index, event stocks have more attentions and faster diffusion rate of information. These factors make investors have more similar view about expected required return on capital and growth of the company, and hence a similar expected price. As a result, these companies should have stationary PE ratios in an efficient market. We use the augmented Dickey-Fuller test to test the random walk model, and the random walk with drift model. Lags are selected by means of information criteria. Equation (4.6) provides us with a way of testing the efficient market hypothesis which is the basis of null that comovement reflects the fact that fundamental values move more closely. If the efficient market hypothesis is rejected, the null that comovement reflects fundamentals is rejected.

A more direct methodology for testing the null that excess comovement does not exist is to run the regression:

$$p/e_{i,t} = \alpha_i + \beta_i \, p/e_{sp,t} + \epsilon_{i,t} \qquad (4.7)$$

We regress individual stocks' PE ratio on the S&P 500 index PE ratio for pre- and post-addition windows, respectively. We then estimate the change in the slope coefficient from equation (4.7). If the efficient market hypothesis holds, there is not any change in the slope coefficient. This is because price always reflect fundamentals under EMH. As equation (4.5) suggests, PE ratios have constant expected values. This is the case even when fundamental values of individual stocks and the S&P 500 index have stronger relationships. As a result, we reject the null that comovement is a reflection of the fundamentals if we find any change in the slope coefficient in equation (4.7).

We can examine equation (4.7) only when PE ratios are stationary. If we find PE ratios are not stationary we will test the first difference of the PE ratios and see if there is cointegration between individual stocks' PE ratios and S&P 500 index PE ratios. If they are cointegrated, a error correction model (ECM) will be used to investigate the short-run relationship but equation (4.7) will be still used to model the long-run relationship. If there is no cointegration, but the first differences of PE ratios are stationary, we will regress the change rate of individual PE ratios on the S&P 500 PE ratios. For the same reason, slope coefficients estimated from these regressions should not change either. If we find a significant change in the slope coefficient, we reject the null.

4.4. Data and results

4.4.1. Data

We construct daily PE ratios by dividing daily prices by quarterly earnings.[23] Most of the movement in PE ratios stems from price changes since many large companies tend to smooth quarterly earnings. Prices of individual stocks and the S&P 500 index are from the CRSP data base. Earnings of individual companies are from the Compustat data base while earnings of the S&P 500 index are from Shiller's webpage. We exclude stocks whose observations are less than 30 for each pre- and post-addition window. We have 649 added stocks covering the 1976-2015 period.

4.4.2. Results

Unit root tests

Table 4.1 reports percentages of stationary individual stocks' PE ratios and corresponding S&P 500's PE ratios for pre- and post-addition windows. Results are reported for the levels and the first differences of PE ratios.

[Insert Table 4.1 here]

[23] Earnings are only reported quarterly.

As Table 4.1 suggests, majorities of stocks' PE ratios are nonstationary during both 1-year pre- and post-addition periods. The S&P 500 PE ratio in most of corresponding periods is not stationary either. When the random walk without drift model is used, only 4.3% of the individual stocks' PE ratios are stationary before stocks are added to the index. After stocks are added to the index, this increases to 6.78% which is still low enough to reject the null that individual stocks' PE ratios are stationary. For the S&P 500 PE ratios in the corresponding periods, we can reject the null at the 5% level of significance during pre- and post-addition periods. Only 1.23%, and 2.11%, respectively before and after stock additions are stationary of the S&P 500 PE ratios. This finding contradicts the predicted patterns of PE ratios be in an efficient market where sufficient numbers of rational investors play their role and offset influences on securities' prices of irrational investors. Our results support Coakley & Fuertes (2005) who report the PE ratio of the S&P Composite Stock index exhibits continuation in the short-run. This may because the numbers of rational investors are not high enough to offset influences of noise traders. As Coakley & Fuertes (2005) suggested, the market need a long time period for prices revert to the fundamentals. Our results indicate that one year is not enough for rational investors to correct the price. Combining our results and results in Coakley & Fuertes (2005), we believe that the market is overall efficient in the long run (several decades) which consists in a huge number of short, inefficient periods. In the other word, the market is not efficient all the time. When the market is not efficient, there is no basis to conclude that comovement can be explained by fundamentals only. It means that nonfundamental factors, besides fundamental factors, also have influence on comovement. As a result, we believe that sentiment- or friction-based views in Barberis et al. (2005) are also helpful for explaining comovement.

Johansen methodology

We then employ the Johansen methodology to examine if individual stocks' PE ratios are cointegrated with the S&P 500 PE ratio. As the univariate regression suggests, only systematic or market risk has influences on returns on individual stocks. Its influence should be reflected in prices. Therefore, their PE ratios should be cointegrated. We find that 9.7% and 13.73% of companys' PE ratios are cointegrated with the S&P 500 PE ratio during pre- and post-addition periods, respectively. Based on equation 4.6, PE ratios can indicate how good investors believe the prospects of economy or the company could be. Values of PE ratios depend on values of required returns and expected values of growth rate. The higher PE ratio suggests the higher expected growth rate or lower required rate of return that implies lower risk profile. The higher expected growth and lower risk profile both implies better prospects. When investors have positive expectations about the overall economy, they also have relatively good expectations about individual companies. In the other word, companies cannot grow at an explosive rate during a recession. As a result, in an efficient market, company's PE ratios should be cointegrated with market's PE ratios. The low percentages of cointegration implies that market is not efficient enough to conclude that comovement can be explained by fundamentals fully. Instead, category views and habitat views should also have powers to explain the comovement. . As a result, the low percentages of cointegration support the sentiment- or friction-based view from a different angle.

Regressions in first differences

Table 4.1 indicates that the first differences of PE ratios are all stationary. Hence a weak form of equation (4.7) can be undertaken where first differences are substitutes for the levels. It means that investors update their belief about individual companies when they update that about the overall economy. And this relationship should not change when the S&P 500 index changes its constituents. Table 4.2 reports results of the regressions of first difference of individual added stocks' PE ratios on first difference of corresponding period S&P 500 PE ratios.

[Insert Table 4.2 here]

The changes in the slope coefficients are insignificantly different from zero on average. The overall changes in the slope coefficients are significant at the 5% level of significance only during the 1988-2000 period. However, this does not mean changes in slope coefficients are insignificant. After dividing samples into subsamples with positive and negative changes in slope coefficients, we find significant changes. From 1976 to 2015, 335 stocks have 25.1957 of increasing slope coefficients while 314 stocks have -37.7666 of decreasing slope coefficients. During 1976-1987, 1988-2000, and 2001-2015, significantly positive and negative changes coexist and offset each other, which leads to insignificant overall changes.

This finding indicates that prices exhibit different patterns from those implied by efficient market model implies. The slope coefficients can be used as indications that how investors expect these companies to perform in the future relative to the overall market. The structure of the industry where these stocks are, and structures of these companies should be stable. In the other word, the slope coefficient should not change when the stock is added to or deleted from the S&P 500 index because investors will not change their expectations of the stock for additions or deletions. Increases and decreases in the slope coefficient imply that investors change expectations of the stock relative to the overall market. It is abnormal that rational investors have huge different view about these companies after they added to the S&P 500 index. This finding also supports that noise traders play essential roles in the comovement because rational investors are not strong enough to offset them.

To sum up, nonstationary PE ratios in short term suggest that rational investors are not be able to induce prices to revert to fundamentals in the short-run when securities are misvalued.

Further, a majority of added stocks' PE ratios are not cointegrated with the corresponding period S&P 500 PE ratio. This suggests the inefficient market in the short-run form another angle. Regressions of first differences of PE ratios also reject the null that market is efficient when samples are divided into subsamples with positive and negative changes in the slope coefficients. Overall, we believe that sentiment- or friction-based views are appropriate for comovement.

4.5. Conclusions

From the DDM we derive an equation which implies patterns of PE ratios in an efficient market setting where Fama (1970) assumes that sufficient numbers of rational investors exist. Our stylised model suggests that PE ratios should be stationary if the EMH hypothesis holds. However, we find that we can reject this hypothesis at the usual significance levels.This is is consistent with Coakley & Fuertes (2005) who find continuation or trend following behaviour of PE ratios in the short term.

Our results also reject cointegration between individual stock PE ratios and the corresponding S&P 500 PE ratio. This not only means that we need an alternative regression to equation (4.7) but also means that investors, at least sometimes, do not form expectations about individual stocks based on the overall market factor. Rather, they may focus on subsets of stocks or a specific style. Regressions of first differences of stock on S&P PE ratios exhibit significant changes in slope coefficients. These are difficult to explain by fundamentals alone. In this chapter er, we reject the null that comovement reflects fundamental values and instead support the view that sentiment- and friction-based views are more appropriate for explaining comovement.

Table 4.1 ADF tests for level and 1st difference of PE ratios

For each event stock, stationarity of PE ratios for pre- and post-event period are examined, respectively. Level of PE ratio and the first difference of PE ratio are tested through models of random walk without drift and random walk with drift. The level and the first difference of the PE ratios of the S&P 500 in corresponding periods are also examined. The percentages of stationary level of PE ratio and the first difference of PE ratio are reported.

		individual stocks		**S&P 500 in corresponding periods**	
level					
		Pre	post	pre	post
	N				
random walk without drift	649	4.3077	6.7873	1.2308	2.1116
random walk with drift	649	7.3846	11.3122	10.4615	11.3122
1st difference					
random walk without drift	649	100	100	100	100
random walk with drift	649	100	100	100	100

Table 4.2 regressions of first differences of PE ratios

For each stock j, the first differences of stock's PE ratios are regressed on the first differences of the S&P 500 index PE ratios. The regression is examined for pre- and post-event periods and change in slope is calculated. Overall average change in slope is reported. Samples are divided into subsamples with positive and negative change in slope and corresponding results are also reported. Results of subperiods, 1976-1987, 1988-2000, and 2001-2015, are also reported.

		1976-2015	**1976-1987**	**1988-2000**	**2001-2015**
overall					
	N	649	165	228	256
	$\Delta\overline{\beta_i}$	-5.2668	8.3098	-9.5828**	-10.1734
	(s.e.)	(4.5293)	(9.7356)	(0.0218)	(8.6356)
increasing					
	N	335	84	124	127
	$\Delta\overline{\beta_i}$	25.1957***	40.3263***	16.8680***	23.3190***
	(s.e.)	(4.4667)	(15.0701)	(2.6258)	(5.6757)
decreasing					
	N	314	81	104	129
	$\Delta\overline{\beta_i}$	-37.7666***	-24.8925**	-41.1203***	-43.1466***
	(s.e.)	(7.6507)	(11.1526)	(8.9512)	(15.7002)

*****, **, and * represent significance at 1%, 5%, and 10% level of significance.**

Chapter 5. Conclusion

This thesis consists of three essays on changes in comovement after the S&P 500 index changes some of its members at its. The aim of the thesis is to answer two questions. First, does comovement with the S&P 500 index always increase and decrease after a stock is added to and deleted from the S&P 500 index, respectively? Second, can fundamental factors explain the change in comovement completely? Chapter 2 tries to find how comovement really change when a stock is added to and deleted from the S&P 500 index. Chapter 3 tries to answer if SMB, HML, and MOM are able to explain change in comovement? Chapter 4 tries to test the null that fundamental factors are able to explain comovement fully through examining implications of EMH which is the basis of the null. This chapter summarizes the main findings in these essays and gives suggestions for future research.

5.1. Summary of main findings

Chapter 2 re-examines changes in beta after additions to and exclusions from the S&P 500 index. Our results are consistent with theory of Barberis et al. (2005). In this essay, we find interesting patterns of change in beta when the full sample is divided into two subsamples based on their changes in beta. We find that added stocks do not always have increasing betas while deleted stocks do not always have decreasing betas. These opposing changes offset each other, resulting in overall changes in betas that are much weaker. Some of overall changes are even insignificant. We believe that patterns from overall sample are misleading because they camouflage noise trader influences on comovement. Our findings of positive and negative changes in betas suggest that sentiment and friction do affect stocks at the style levels. The theoretical model in Chapter 2 relaxes the assumption that investors just follow S&P 500 and non-S&P 500 styles. Instead, it assumes that some investors may bet against beta where stocks with higher-than-mean betas are underweighted or shorted while stocks with lower-than-mean betas are over-weighted. This model explains why some added stocks can have decreasing betas while some deleted stocks can have increasing betas. The model also emphasises the role of overlaps between the different styles. Another contribution of this chapter is that, prior to the S&P 500 index changes its members, it establishes that stocks with increasing betas are defensive while those with decreasing betas are aggressive.

Chapter 3 constructs a theoretical model to help us to understand changes in betas estimated from the 3- and 4-factor model. This model suggest that sentiment and friction still play essential roles in comovement when size, value, and momentum factors are considered. Our model suggests that estimators of betas from 3- or 4-factor models are linear combinations of parameter from univariate regressions. This indicates that noise traders and rational investors coexist and both influence comovement. The extent of excess comovement depends on who has stronger power to affect prices. However excess comovement persists. More importantly, changes in loadings on SMB factors are also influenced by sentiment and friction. Chapter 3 also examines the effect of size, value, and momentum factors on comovement. We find these factors have some impact during the 1988-2000 period. However, they do not always impact on comovement.

Chapter 4 provides a new framework for considering reasons why comovement changes as S&P 500 index changes its members. From the DDM, an equation is derived to imply patterns of PE ratios in an efficient market where sufficient rational investors operate. The DDM implies that PE ratios should be stationary in an efficient market. This implies that prices will be pulled back towards their fundamentals by rational investors once they drift away. This implication is contrary to what we find in chapter 4. We find that a majority of stocks' PE ratios are nonstationary. This is consistent with Coakley & Fuertes (2005) and suggests that rational investors need a long term to correct prices. In other words, noise traders survive in the short-run and play essential roles in comovement. Our proposed new regressions show that slope coefficients from first difference regressions ofstock PE on S&P PE ratios have similar patterns to the corresponding beta change.

Positive and negative changes in slope coefficients are significant. However, they offset each other so that the overall changes are insignificant.

5.2. Suggestions for future research

When studying comovement, it is wise to be aware that comovement can increase or decrease following both addition and exclusion events. Some may argue that this is evidence that noise traders having different views offset each other, and hence do not have any influence. This is not the case. We still find significant changes in beta when the full samples are studied. Dividing samples into increasing- and decreasing-beta subsamples is helpful to understand how betas really move. It enables us to get some insights into what characteristics induce betas to increase. Future research could use matching samples exercises to find specific potential explanations for comovement. Profitability and investment patterns could be potential factors that we should control for in future research. Some research suggests that beta itself is not constant over time. This provides new angle to consider the problem. Kalman filters, and GARCH models could be used to investigate how beta changes before stocks switch groups.

References

Baffes, J., 1994. Does comovement among exchange rates imply market inefficieny?. *Economics Letters,* Volume 44, pp. 273-280.

Barberis, N. & Shleifer, A., 2003. Style investing. *Journal of Financial Economics,* Volume 68, pp. 161-199.

Barberis, N., Shleifer, A. & Wurgler, J., 2005. Comovement. *Journal of Financial Economics,* Volume 75, pp. 283-317.

Basak, S. & Pavlova, A., 2013. Asset prices and institutional investors. *American Economic Review,* Volume 103, pp. 1728-58.

Black, F., 1972. Capital market equilibrium with restricted borrowing. *The Journal of Business,* Volume 45, pp. 444-455.

Bodurtha, J. N., Kim, D.-S. & Lee, C. M., 2015. Closed-end country funds and U.S. market sentiment. *The Review of Financial Studies,* Volume 8, pp. 879-918.

Boudoukh, J., Richardson, M. & Whitelaw, R., 1994. A tale of three schools: insights on autocorrelations of short-horizon stock returns. *The Review of Financial Studies,* Volume 7, pp. 539-573.

Boyer, B. H., 2011. Style-related comovement: fundamentals or labels?. *The Journal of Finance,* Volume 66, pp. 307-332.

Chelley-Steeley, P. L. & Steeley, J. M., 1999. Changes in the comovement of European equity markets. *Economic Inquiry,* 37(3), pp. 387-573.

Chen, H.-L. & Bondt, W. D., 2004. Style momentum within the S&P-500 index. *Journal of Empirical Finance,* Volume 11, pp. 483-507.

Chen, H., Noronha, G. & Singal , V., 2005. The price response to S&P 500 index additions and deletions: evidence of asymmetry and a new explanation. *The Journal of Finance,* Volume 59, pp. 1901-1930.

Chen, H., Singal, V. & Whitelaw, R. F., 2016. Comovement revisited. *Journal of Financial Econometrics,* Volume 121, pp. 624-644.

Cheung, Y.-w. & Lai, K. S., 1999. Macroeconomic determinants of long-term stock market comovements among major EMS countries. *Applied Financial Economics,* Volume 9, pp. 73-85.

Claessens, S. & Yafeh, Y., 2012. Comovement of newly added stocks with national market indices: evidence from around the world. *Review of Finance,* Volume 17, pp. 203-227.

Coakley, J. & Fuertes, A.-M., 2006. Valuation ratios and price deviations from fundamentals. *Journal of Banking & Finance,* 30(8), pp. 2325-2346.

De Long, J. B., Shleifer, A., Summers, L. H. & Waldmann, R. J., 1990. Noise trader risk in financial markets. *Journal of Political Economy,* Volume 98, pp. 703-738.

Demarzo, P. M., Kaniel, R. & Kremer, I., 2004. Diversification as public good: community effects in portfolio choice. *The Journal of Finance,* Volume 59, pp. 1677-1716.

Denis, D. J. & Kadlec, G. B., 1994. Corporate events, trading activity, and the estimation of systematic risk: evidence from equity offerings and share repurchases. *The Journal of Finance,* Volume 49, pp. 1787-1811.

Dimson, E., 1979. Risk measurement when shares are subject to infrequent trading. *Journal of Financial Economics,* Volume 7, pp. 197-226.

Elliott, W. B., Van Ness, B. F., Walker, M. D. & Warr, R. S., 2008. What drives the s&p 500 inclusion effect? an analytical survey. *Financial Management,* Volume 35, pp. 31-48.

Fama, E. F., 1970. Efficient capital markets: a review of theory and empirical work. *The Journal of Finance,* Volume 25, pp. 383-417.

Fama, E. F. & French, K. R., 1993. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics,* Volume 33, pp. 3-56.

Fama, E. F. & French, K. R., 1996. Mutilfactor explanations of asset pricing anomalies. *The Journal of Finance,* Volume 51, pp. 55-84.

Frazzini, A. & Pedersen, L. H., 2014. Betting against beta. *Journal of Financial Economics,* 111(1), pp. 1-25.

Green, T. C. & Hwang, B.-H., 2009. Price-based return comovement. *Journal of Financial Economics,* Volume 93, pp. 37-50.

Greenwood, R., 2008. Excess comovement of stock returns:evidence from cross-sectional variation in Nikkei 225 weights. *The Review of Financial Studies,* Volume 21, pp. 1153-1186.

Greenwood, R. M. & Sosner, N., 2007. Trading patterns and excess comovement of stock returns. *Financial Analysts Journal,* Volume 63, pp. 69-81.

Hameed, A., Morck, R., Shen, J. & Yeung, B., 2015. Information, analysts, and stock return comovement. *The Review of Financial Studies,* Volume 28, pp. 3153-3187.

Hong, H. & Sraer, D. A., 2016. Speculative betas. *The Journal of Finance,* 71(5), pp. 2095-2144.

Hong, H. & Stein, J. C., 1999. A unified theory of underreaction, momentum trading, and overreaction in asset markets. *The Journal of Finance,* 54(6), pp. 2143-2184.

Hou, K. & Moskowitz, T. J., 2005. Market frictions, price delay, and the cross-section of expected returns. *The Review of Financial Studies,* Volume 18, pp. 981-1020.

Jegadeesh, N. & Titman, S., 1993. Returns to buying winners and selling losers: implications for stock market efficiency. *The Journal of Finance,* 48(1), pp. 65-91.

Jegadeesh, N. & Titman, S., 2001. Profitability of momentum strategies: an evaluation of alternative explanations. *The Journal of Finance,* Volume 56, pp. 699-720.

Johnson, T. c., 2002. Rational momentum effects. *The Journal of Finance,* Volume 57, pp. 585-608.

Kadlec, G. B. & Patterson, D. M., 1999. A transactions data analysis of nonsynchronous trading. *The Review of Financial Studies,* Volume 12, pp. 609-630.

Karolyi, G. A. & Stulz, R. M., 1996. Why do markets move together? An investigation of U.S.-Japan stock return comovements. *The Journal of Finance,* 51(3), pp. 951-986.

Kasch, M. & Sarkar, A., 2014. *Is there an s&p 500 index effect?.* [Online] Available at: https://ssrn.com/abstract=2171235

Kumar, A. & Lee, C. M., 2006. Retail investor sentiment and return comovements. *The Journal of Finance,* Volume 61, pp. 2451-2486.

Lasse, P. H., 2015. *Efficiently inefficient: how smart money invests and market prices are determined.* 1st ed. Woodstock: Princeton University Press.

Lee, C. M., Shleifer, A. & Thaler, R. H., 1991. Investor sentiment and the closed-end fund puzzle. *The Journal of Finance,* Volume 46, pp. 75-109.

Lo, A. W. & MacKinlay, C. A., 1990. An econometric analysis of nonsynchronous trading. *Journal of Econometrics,* Volume 45, pp. 181-211.

Mase, B., 2008. Comovement in the FTSE 100 index. *Applied Financial Economics Letters,* Volume 4, pp. 9-12.

Natanelov, V., Alam, M. J., Mckenzie, A. M. & Huylenbroeck, G. V., 2011. Is there co-movement of agricultural commodities futures prices and crude oil?. *Energy Policy,* 39(9), pp. 4971-4984.

Nieh, C.-C. & Lee, C.-F., 2001. Dynamic relationship between stock prices and exchange rates for G-7 countries. *The Quarterly Review of Economics and Finance,* 41(4), pp. 477-490.

Pedersen, L. H., 2015. *Efficiently inefficient: how smart money invests and market prices are determined.* 1st ed. Woodstock: Princeton University Press.

Pindyck, R. S. & Rotemberg, J. J., 1993. The comovement of stock prices. *The Quarterly Journal of Economics,* Volume 108, pp. 1073-1104.

Pirinsky, C. & Wang, Q., 2006. Does corporate headquarters location matter for stock returns?. *The Journal of Finance,* Volume 61, pp. 1991-2015.

Rigobon, R., 2002. The curse of non-investment grade countries. *Journal of Development Economics,* Volume 69, pp. 423-449.

Sagi, J. S. & Seasholes, M. S., 2007. Firm-specific attributes and the cross-section of momentum. *Journal of Financial Economics,* Volume 84, pp. 389-434.

Schöllhammer, H. & Sand, O., 1985. The independence among the stock markets of major European countries and the United States: an empirical investigation of interrelationships among national stock price movements. *Management International Review,* Volume 25, pp. 17-26.

Spanos, A. & McGuirk, A., 2002. The problem of near-multicollinearity revisited: erratic vs systematic volatility. *Journal of Econometrics,* Volume 108, pp. 365-393.

Tang, K. & Xiong, W., 2012. Index investment and the financialization of commodities. *Financial Analyst Journal,* Volume 68, pp. 54-74.

Vijh, A. M., 1994. S&P 500 trading strategies and stock betas. *The Review of Financial Studies,* Volume 7, pp. 215-251.

Wahal, S. & Yavuz, M. D., 2013. Style investing,comovement and return predictability. *Journal of Financial Economics,* Volume 107, pp. 136-154.

Wurgler, J., 2010. *On the ecnomic consequences of index-linked investing.* [Online] Available at: https://ssrn.com/abstract=1667188

Appendix

Proof for equation 3.3

The probability limits of the coefficients are

$$\beta = (X^T X)^{-1}(X^T Y) \Rightarrow \beta_1 = \frac{\sigma_{r_i, r_{sp}} \sigma_{r_o}^2 - \sigma_{r_i r_o} \sigma_{r_{sp} r_o}}{\sigma_{r_{sp}}^2 \sigma_{r_o}^2 - \sigma_{r_{sp} r_o} \sigma_{r_{sp} r_o}} = \frac{\sigma_{r_i, r_{sp}} \sigma_{r_o}^2 - \rho_{r_{sp} r_o} \sigma_{r_o} \sigma_{r_{sp}} \sigma_{r_i r_o}}{\sigma_{r_{sp}}^2 \sigma_{r_o}^2 (1 - \rho_{r_{sp} r_o})}$$

$$\beta_1 = \frac{1}{1 - \rho_{r_{sp} r_o}} \left(\frac{\sigma_{r_i, r_{sp}}}{\sigma_{r_{sp}}^2} - \rho_{r_{sp} r_o} \frac{\sigma_{r_o}}{\sigma_{r_{sp}}} \times \frac{\sigma_{r_i r_o}}{\sigma_{r_o}^2} \right)$$

Stylised three- and four-factor models

A stylised model of comovement is required to interpret and understand the implications of the empirical results reported in the extant literature and, in particular, to help interpret the key role of the *SMB*, *HML*, and *MOM* factors highlighted in recent studies (see Kasch & Sarkar, 2014; Chen et al., 2016). This requires three- and four-factor model settings in contrast to the extant models used to investigate the features of the univariate and bivariate regressions in BSW (2005). Our model yields several novel testable predictions about comovement.

Model assumptions

Chen et al. (2016) construct a simple model to explain excess comovement. Their theoretical model has implications about univariate and bivariate regressions. However, three- and four-factor models are required to understand roles of characteristics such as size (*SMB*), value (*HML*) and momentum in excess comovement. Our model shares assumptions with Chen et al. (2016) except that we extend the fundamental factor to a multifactor context which includes returns on market risk, *SMB*, *HML*, and *MOM* portfolios and other fundamental factors

Let r_t be the overall return on an stock that switches from one group to another group, and let $r_{sp,t}$, $r_{smb,t}$, $r_{hml,t}$, and $r_{mom,t}$ denote the part of returns on the S&P 500 index, on the *SMB*, on the *HML*, and on the *MOM* portfolios, respectively.

$$r_t = b_{sp,t}f_{sp,t} + b_{smb,t}f_{smb,t} + b_{hm\ ,t}f_{hml,t} + b_{mom,t}f_{mom,t} + c_t u_{sp,t} + e_{r,t}$$

$$r_{sp,t} = f_{sp,t} + u_{sp,t} + e_{sp,t}$$

$$r_{smb,t} = f_{smb,t} + e_{smb,t}$$

$$r_{hm\ ,t} = f_{hml,t} + e_{hml,t}$$

$$r_{mom,t} = f_{mom,t} + e_{mom,t} \quad (0.1)$$

where f denotes the fundamental, common factors, u denotes non-fundamental group-specific factors, and e denotes idiosyncratic fundamental factors. It is assumed that the returns on added stocks are affected by the fundamental common factors including returns on market risk, the *SMB*, *HML*, and *MOM* portfolios and other fundamental factors. These factors are extended from the fundamental shock in Chen et al. (2016). The S&P 500 index is taken as a proxy for market risk since it needs to be a readily tradable asset category to test the excess comovement hypothesis in the spirit of BSW (2005). The model shares the Chen et al. (2016) assumption that added stock returns are also affected by an index (group-specific) nonfundamental factor and idiosyncratic risk. The return on the S&P 500 index is determined by the return on systematic risk, index nonfundamental risk and idiosyncratic risk. The return on other portfolios is determined by returns on the specific factor risk and idiosyncratic risk.

It is further assumed that fundamental factors are not correlated with non-fundamental factors, and that idiosyncratic fundamental factors are not correlated with common fundamental factors:

$$cov(u_t, f_{i,t}) = cov(u_t, e_{j,t}) = 0 \ \forall i, j$$

$$cov(e_{j,t}, f_{i,t}) = 0 \ \forall \ i, j \qquad (\ 0.2)$$

These assumptions are shared with Chen et al. (2016). We add an additional assumption that fundamental factors derived from the overall fundamental factor in (2016) are not correlated to each other:

$$cov(f_{j,t}, f_{i,t}) = 0 \ \forall \ i, j \qquad (\ 0.3)$$

Behavioural finance theories imply that excess comovement is driven by changes to group-specific (category) non-fundamental shocks. The sensitivity to non-index, group-specific non-fundamental shocks goes to zero while that to index, group-specific non-fundamental factors increases from zero after a stock is added to the S&P 500 index. Underbars and overbars are employed to denote stock values before and after index addition. We further assume sensitivities to fundamentals do not change when the stock is added to the index but may change during sub-periods in line with Chen et al. (2016). This implies the following restrictions on the coefficients of the main equation in (3.1):

$$\underline{c}_t = 0 \quad \overline{c}_t > 0 \qquad (\ 0.4)$$

Measures of excess comovement

Excess comovement is defined as the fraction of the group return variance that can be explained by the non-fundamental shocks and can be expressed as

$$\frac{\sigma^2_{u,sp}}{\sigma^2_{f,m}} \qquad (0.5)$$

calculated for the windows prior to and following index additions. The focus is on the univariate regression and, for reasons of algebraic tractability, on the three- and four-factor models. Consider the following three return regressions:

$$r_t = \alpha_1 + \beta_1 r_{sp,t} + \varepsilon_t$$

$$r_{j,t} = \alpha_j + \beta_{j,sp500,3} r_{sp500,t} + \beta_{j,smb} r_{smb,t} + \beta_{j,hm}\ r_{hml,t} + \varepsilon_{j,t}$$

$$r_{j,t} = \alpha_j + \beta_{j,sp500,4} r_{sp500,t} + \beta_{j,smb} r_{smb,t} + \beta_{j,hml} r_{hml,t} + \beta_{j,mom} r_{mom,t} + \varepsilon_{j,t} \qquad (0.6)$$

where the final two regressions are the three- and four-factor models, respectively.

The probability limits of these regression coefficients to returns on the S&P 500 index are:

$$\underline{\beta}_1 = \frac{\underline{b}_{m,t}\underline{\sigma}^2_{f,m}}{\underline{\sigma}^2_{f,m}} \quad \overline{\beta}_1 = \frac{\overline{b}_{m,t}\overline{\sigma}^2_{f,m} + \overline{c}_t\overline{\sigma}^2_{u,sp}}{\overline{\sigma}^2_{f,m}}$$

$$\underline{\beta}_{j,sp500,3} = \alpha \times \left[\underline{\delta}_m\underline{\beta}_1 - \underline{\theta}_m\underline{\beta}_3 - \underline{\varphi}_m\underline{\beta}_2\right]$$

$$\overline{\beta}_{j,sp500,3} = \alpha \times \left[\overline{\delta}_m\overline{\beta}_1 - \overline{\theta}_m\overline{\beta}_3 - \overline{\varphi}_m\overline{\beta}_2\right]$$

$$\underline{\beta}_{j,sp500,4} = \gamma \times \left[\underline{\vartheta}_m\underline{\beta}_1 - \underline{\pi}_m\underline{\beta}_2 - \underline{\tau}_m\underline{\beta}_3 - \underline{\omega}_m\underline{\beta}_4\right]$$

$$\overline{\beta}_{j,sp500,4} = \gamma \times \left[\overline{\vartheta}_m\overline{\beta}_1 - \overline{\pi}_m\overline{\beta}_2 - \overline{\tau}_m\overline{\beta}_3 - \overline{\omega}_m\overline{\beta}_4\right] \qquad (0.7)$$

where β_2, β_3 and β_4 are slope coefficients from univariate regressions where the stock return is regressed on *SMB*, *HML* and *MOM*, respectively, and under- and over-bars denote pre- and post-addition periods, respectively.

Following Chen et al (2015), we assume also that sensitivities to the S&P 500 index common factor, the variances of the non-fundamental factors, variances of the fundamental factors, and correlations between returns on groups are constant over time. However, sensitivities to the other common factors, such as *SMB*, *HML* and *MOM*, are assumed to change following additions.

$$\underline{b}_{sp,t} = \overline{b}_{sp,t} \equiv b_{i,t} \quad \underline{\rho}_{i,t} = \overline{\rho}_{i,t} \equiv \rho_{i,t}$$

$$\underline{\sigma}^2_{u,sp} = \overline{\sigma}^2_{u,sp} \equiv \sigma^2_{u,sp} > 0$$

$$\underline{\sigma}^2_{e,i} = \overline{\sigma}^2_{e,i} \equiv \sigma^2_{e,i} \quad \underline{\sigma}^2_{f,i} = \overline{\sigma}^2_{f,i} \equiv \sigma^2_{f,i} \quad i = sp, smb, hml, mom \qquad (0.8)$$

The assumptions yield these univariate estimators of changes in comovement:

$$\overline{\beta}_1 - \underline{\beta}_1 = \frac{c_t \sigma^2_{u,sp}}{\sigma^2_{f,m}} > 0 \qquad (0.9)$$

This estimator demonstrates that univariate regressions are still informative about the excess comovement after the fundamental factor is extended to a multifactor context. It indicates that the change in beta is positive after the stock is added to the index and is determined by the sensitivity to the group or style nonfundamental factor which is consistent with Chen et al. (2016).

The estimator of a beta change in comovement for the three-factor model is:

$$\overline{\beta}_{j,sp500,3} - \underline{\beta}_{j,sp500,3} = \alpha \times \left[\delta_{sp}\left(\overline{\beta}_1 - \underline{\beta}_1\right) - \theta_{sp}\left(\overline{\beta}_3 - \underline{\beta}_3\right) - \varphi_{sp}\left(\overline{\beta}_2 - \underline{\beta}_2\right)\right] \qquad (0.10)$$

where: $\alpha = \frac{1}{1+2\rho_{sp,smb}\rho_{sp,hml}\rho_{smb,hml}-\rho^2_{sp,smb}-\rho^2_{sp,hml}-\rho^2_{smb,hml}}$

$$\delta_{sp} = 1 - \rho^2_{smb,hml} > 0$$

$$\theta_{sp} = \frac{\rho_{sp,hm}\,\sigma_{hml} - \rho_{sp,smb}\rho_{smb,hml}\sigma_{hml}}{\sigma_{sp}} = \left(\rho_{sp,hml} - \rho_{sp,smb}\rho_{smb,hml}\right)\frac{\sigma_{hml}}{\sigma_{sp}}$$

$$\varphi_{sp} = \frac{\rho_{sp,smb}\sigma_{smb} - \rho_{sp,hml}\rho_{smb,hml}\sigma_{smb}}{\sigma_{sp}} = \left(\rho_{sp,smb} - \rho_{sp,hml}\rho_{smb,hml}\right)\frac{\sigma_{smb}}{\sigma_{sp}}$$

The above estimator indicates that the change in comovement in the three-factor model is a linear combination of changes in index comovement and the loadings on size and value factors. In other words, the changes in size and value loadings impact on the change in beta. The impact depends not only on the changes in sensitivities of a stock return to the *SMB* and *HML* factors but also on the importance of these changes. Specifically, if the *HML* factor has a higher sensitivity to return on the S&P 500 index and a lower correlation with the *SMB* factor, the change in sensitivity of the stock to the *HML* factor will have a larger weight. On the other hand, if the *SMB* factor has a higher sensitivity to the S&P 500 return and lower correlation with the *HML* factor, the change in sensitivity to the *SMB* factor will be more important.

The effect of the nonfundamental factor is still relevant even though changes in the stock's characteristics are considered. The first term in equation (0.10) indicates the effect of change in nonfundamental factor which always has a non-zero weight. The implication is that the nonfundamental factor effects on the change in beta may be offset the changes in fundamentals. This leads to the possibility that the change in beta estimated by the three-factor model goes to zero. Contrarywise, if the index return, size factor, and value factor are all uncorrelated, the estimator of change in comovement from the three-factor model will be the same as that from the univariate regression.

The estimator of change in comovement for the four-factor model is:

$$\overline{\beta}_{j,sp500,3} - \underline{\beta}_{j,sp500,3} = \gamma\left[\vartheta_m\left(\overline{\beta}_1 - \underline{\beta}_1\right) - \pi_m\left(\overline{\beta}_2 - \underline{\beta}_2\right) - \tau_m\left(\overline{\beta}_3 - \underline{\beta}_3\right) - \omega_m\left(\overline{\beta}_4 - \underline{\beta}_4\right)\right]$$

where $\gamma =$

$$1 \Bigg/ \left(\begin{array}{c} 1 + 2\rho_{smb,mom}\rho_{smb,hml}\rho_{hml,mom} + 2\rho_{sp,mom}\rho_{hm\ ,mom}\rho_{sp,hml} + 2\rho_{sp,smb}\rho_{sp,mom}\rho_{smb,mom} \\ +2\rho_{sp,smb}\rho_{sp,hml}\rho_{smb,hml} + \rho^2_{sp,smb}\rho^2_{hml,mom} + \rho^2_{sp,hml}\rho^2_{smb,mom} + \rho^2_{sp,mom}\rho^2_{smb,hml} \\ -\rho^2_{sp,smb} - \rho^2_{sp,hm} - \rho^2_{sp,mom} - \rho^2_{smb,hml} - \rho^2_{smb,mom} - \rho^2_{hm\ ,mom} \\ -2\rho_{sp,smb}\rho_{smb,hm}\ \rho_{hml,mom}\rho_{sp,mom} - 2\rho_{sp,smb}\rho_{smb,mom}\rho_{sp,hm}\ \rho_{smb,mom} \\ -2\rho_{sp,hml}\rho_{sp,mom}\rho_{smb,hml}\rho_{smb,mom} \end{array}\right)$$

$$\vartheta_m = 1 + 2\rho_{smb,hml}\rho_{hm\ ,mom}\rho_{smb,mom} - \rho^2_{smb,mom} - \rho^2_{hml,mom} - \rho^2_{smb,hml}$$

$$\pi_m = \left(\rho_{sp,sm} + \rho_{sp,hml}\rho_{hm\ ,mom}\rho_{smb,mom} + \rho_{sp,mom}\rho_{smb,hml}\rho_{hml,mom} - \rho_{sp,mom}\rho_{smb,mom} - \rho^2_{hml,mom}\rho_{sp,smb} - \rho_{sp,hm}\ \rho_{sm\ ,hml}\right)\frac{\sigma_{smb}}{\sigma_{sp}}$$

$$\tau_m = \left(\rho_{sp,hm} + \rho_{sp,mom}\rho_{smb,mom}\rho_{smb,hml} + \rho_{sp,smb}\rho_{smb,mom}\rho_{hm\ ,mom} - \rho_{sp,smb}\rho_{smb,hml} - \rho^2_{smb,mom}\rho_{sp,hml} - \rho_{sp,mom}\rho_{hml,mom}\right)\frac{\sigma_{hml}}{\sigma_{sp}}$$

$$\omega_m = \left(\rho_{sp,mom} + \rho_{sp,smb}\rho_{smb,hml}\rho_{hml,mom} + \rho_{sp,hml}\rho_{smb,mom}\rho_{smb,hml} - \rho_{sp,smb}\rho_{smb,mom} - \rho^2_{smb,hml}\rho_{sp,mom} - \rho_{hml,mom}\rho_{sp,hml}\right)\frac{\sigma_{mom}}{\sigma_{sp}} \quad (0.11)$$

The solution of the four-factor model is also a linear combination of estimators from univariate regressions. The above equations indicate that changes in factor loadings influence the change in comovement. Their relative impact is determined by correlations between the factors and ratios of volatility of each factor to that of the index return. The estimator also suggests that changes in comovement are still affected by the nonfundamental factors. Thus the change in comovement may be zero just because the effect of nonfundamental factors offsets the effect of fundamentals. However, the estimator of comovement from the four-factor model will be the same as that from the univariate regression only if all the factors are uncorrelated.

The impact of correlation between factors

The solutions of the three- and four-factor models imply that the relative influence of the *SMB*, *HML* and *MOM* factors on comovement depends on their correlation with the S&P500 index. If they move in the same direction as the S&P500 index, they have a negative effect on changes in comovement. However, they may lead to an increase in excess comovement if they are inversely correlated with the S&P 500 index. Consider that investors adopt the simple 1/*N* (where *N* is the number of distinct styles) heuristic in their approach to asset allocation. Then funds are equally invested in different investment styles, such as passive index investment, value, growth and momentum styles. Under this scenario, different styles would comove with the S&P 500 index which is a popular benchmark for passive investment. However, given the growing popularity of index tracking, more funds are invested in S&P 500 index products and less in other investment style funds. This lowers the correlation between the S&P 500 index and other styles and the ratio of the style volatility to that of the index will thus be less than one. Therefore a low correlation implies relatively more funds being invested in the S&P 500 index and thus higher excess comovement. On the other hand, if the volatility ratio exceeds one, a lower correlation indicates less investment in the S&P 500 index and hence lower excess comovement. This is because more funds are being invested in other investment styles.

To sum up, our theoretical model suggests that the change in fundamentals may have influences on the excess comovement, however these influences depend on the preferences of investors. If the passive investment is more favoured, fundamentals correspondingly have less impact on excess comovement.

Recent research underlines the role that momentum plays in changes in comovement (see Chen et al., 2016 and Kasch & Sarkar., 2014). This suggests that the importance of the change in the momentum loading is partially determined by the magnitude of the correlation between the momentum and *SMB* factors. Hong and Stein (1999) suggest that momentum is driven by the gradual diffusion of information and that information about smaller firmsdiffuses more gradually. This would lead to a positive correlation between the *SMB* and momentum factors. Moreover, when a stock is added to the index, it is plausible to assume the information about this stock diffuses faster because the size of the company increases and/or analysts pay more attention to the stock. Hence, the sensitivity of the stock to the momentum factor decreases after the stock is added to the S&P 500 index. Thus one cannot simply conclude that comovement changes are driven by fundamentals should one find that the change in momentum loading influences comovement changes. This is because the change in loading on momentum is also driven by nonfundamentals.

This section provides proofs of the theoretical model.

Assume the driving processes for returns before the stock is added to the S&P 500 index are:

$$r_t = \underline{b}_{sp,t} f_{sp,t} + \underline{b}_{smb,t} f_{smb,t} + \underline{b}_{hml,t} f_{hml,t} + \underline{b}_{mom,t} f_{mom,t} + \underline{c}_t u_{sp,t} + e_{r,t}$$

$$r_{sp,t} = f_{sp,t} + u_{sp,t} + e_{sp,t}$$

$$r_{smb,t} = f_{smb,t} + e_{smb,t}$$

$$r_{hml,t} = f_{hml,t} + e_{hml,t}$$

$$r_{mom,t} = f_{mom,t} + e_{mom,t}$$

$$var(e_{i,t}) \equiv \underline{\sigma}_{e,i}^2 \quad var(u_{sp,t}) \equiv \underline{\sigma}_{u,sp}^2 \quad var(f) \equiv \underline{\sigma}_f^2 \quad \underline{c}_t = 0$$

and after the stock is added to the S&P 500 index:

$$r_t = \overline{b}_{sp,t} f_{sp,t} + \overline{b}_{smb,t} f_{smb,t} + \overline{b}_{hml,t} f_{hml,t} + \overline{b}_{mom,t} f_{mom,t} + \overline{c}_t u_{sp,t} + e_{r,t}$$

$$r_{sp,t} = f_{sp,t} + u_{sp,t} + e_{sp,t}$$

$$r_{smb,t} = f_{smb,t} + e_{smb,t}$$

$$r_{hml,t} = f_{hml,t} + e_{hm\ ,t}$$

$$r_{mom,t} = f_{mom,t} + e_{mom,t}$$

$$var(e_{i,t}) \equiv \overline{\sigma}_{e,i}^2 \quad var(u_{sp,t}) \equiv \overline{\sigma}_{u,sp}^2 \quad var(f) \equiv \overline{\sigma}_f^2$$

$$\overline{c}_{m,t} > 0 \quad \overline{b}_{hm\ ,t} f_t < \underline{b}_{hm\ ,t} f_t \neq 0 \quad \overline{b}_{mom,t} f_t < \underline{b}_{mom,t} f_t \neq 0$$

Univariate regressions

We run four univariate regressions

$$r_t = \alpha_1 + \beta_1 r_{sp,t} + \varepsilon_t$$

$$r_t = \alpha_2 + \beta_2 r_{smb,t} + \varepsilon_t$$

$$r_t = \alpha_3 + \beta_3 r_{hml,t} + \varepsilon_t$$

$$r_t = \alpha_4 + \beta_4 r_{mom,t} + \varepsilon_t$$

the probability limits of the slope coefficients are

$$\beta_1 = \frac{cov(r_t, r_{sp,t})}{var(r_{sp,t})} \quad \beta_2 = \frac{cov(r_t, r_{smb,t})}{var(r_{smb,t})}$$

$$\beta_3 = \frac{cov(r_t, r_{hml,t})}{var(r_{hml,t})} \quad \beta_4 = \frac{cov(r_t, r_{mom,t})}{var(r_{mom,t})}$$

estimators before and after the stock is added to the S&P 500 index

$$\underline{\beta}_1 = \frac{cov\begin{pmatrix}\underline{b}_{sp,t}f_{sp,t} + \underline{b}_{smb,t}f_{smb,t} + \underline{b}_{hm\ ,t}f_{hm\ ,t} + \underline{b}_{mom,t}f_{mom,t} + \underline{c}_t u_{sp,t} + e_{r,t},\\ f_{sp,t} + u_{sp,t} + e_{sp,t}\end{pmatrix}}{var(f_{sp,t} + u_{sp,t} + e_{sp,t})} = \frac{\underline{b}_{sp,t}\underline{\sigma}^2_{f,sp}}{\underline{\sigma}^2_m}$$

$$\underline{\beta}_2 = \frac{cov\begin{pmatrix}\underline{b}_{sp,t}f_{sp,t} + \underline{b}_{smb,t}f_{smb,t} + \underline{b}_{hml,t}f_{hm\ ,t} + \underline{b}_{mom,t}f_{mom,t} + \underline{c}_t u_{sp,t} + e_{r,t},\\ f_{smb,t} + e_{smb,t}\end{pmatrix}}{var(f_{smb,t} + e_{smb,t})} = \frac{\underline{b}_{smb,t}\underline{\sigma}^2_{f,\ smb}}{\underline{\sigma}^2_{smb}}$$

$$\underline{\beta}_3 = \frac{cov\begin{pmatrix}\underline{b}_{sp,t}f_{sp,t} + \underline{b}_{smb,t}f_{smb,t} + \underline{b}_{hml,t}f_{hml,t} + \underline{b}_{mom,t}f_{mom,t} + \underline{c}_t u_{sp,t} + e_{r,t},\\ f_{hml,t} + e_{hml,t}\end{pmatrix}}{var(f_{hml,t} + e_{hml,t})} = \frac{\underline{b}_{hml,t}\underline{\sigma}^2_{f,\ hml}}{\underline{\sigma}^2_{hml}}$$

$$\underline{\beta}_4 = \frac{cov\begin{pmatrix}\underline{b}_{sp,t}f_{sp,t} + \underline{b}_{smb,t}f_{smb,t} + \underline{b}_{hml,t}f_{hml,t} + \underline{b}_{mom,t}f_{mom,t} + \underline{c}_t u_{sp,t} + e_{r,t},\\ f_{mom,t} + e_{mom,t}\end{pmatrix}}{var(f_{mom,t} + e_{mom,t})} = \frac{\underline{b}_{mom,t}\underline{\sigma}^2_{f,\ mom}}{\underline{\sigma}^2_{mom}}$$

similarly

$$\overline{\beta}_1 = \frac{cov\begin{pmatrix}\overline{b}_{sp,t}f_{sp,t} + \overline{b}_{smb,t}f_{smb,t} + \overline{b}_{hml,t}f_{hml,t} + \overline{b}_{mom,t}f_{mom,t} + \overline{c}_t u_{sp,t} + e_{r,t},\\ f_{sp,t} + u_{sp,t} + e_{sp,t}\end{pmatrix}}{var(f_{sp,t} + u_{sp,t} + e_{sp,t})}$$

$$= \frac{\overline{b}_{sp,t}\overline{\sigma}^2_{f,sp} + \overline{c}_t\overline{\sigma}^2_{u,sp}}{\overline{\sigma}^2_{sp}}$$

$$\overline{\beta}_2 = \frac{cov\begin{pmatrix}\overline{b}_{sp,t}f_{sp,t} + \overline{b}_{smb,t}f_{smb,t} + \overline{b}_{hml,t}f_{hml,t} + \overline{b}_{mom,t}f_{mom,t} + \overline{c}_t u_{sp,t} + e_{r,t},\\ f_{smb,t} + e_{smb,t}\end{pmatrix}}{var(f_{smb,t} + e_{smb,t})} = \frac{\overline{b}_{smb,t}\overline{\sigma}^2_{f,smb}}{\overline{\sigma}^2_{smb}}$$

$$\overline{\beta}_3 = \frac{cov\begin{pmatrix}\overline{b}_{sp,t}f_{sp,t} + \overline{b}_{smb,t}f_{smb,t} + \overline{b}_{hml,t}f_{hml,t} + \overline{b}_{mom,t}f_{mom,t} + \overline{c}_t u_{sp,t} + e_{r,t},\\ f_{hml,t} + e_{hml,t}\end{pmatrix}}{var(f_{hml,t} + e_{hml,t})} = \frac{\overline{b}_{hml,t}\overline{\sigma}^2_{f,hml}}{\overline{\sigma}^2_{hml}}$$

$$\overline{\beta}_3 = \frac{cov\begin{pmatrix}\overline{b}_{sp,t}f_{sp,t} + \overline{b}_{smb,t}f_{smb,t} + \overline{b}_{hml,t}f_{hml,t} + \overline{b}_{mom,t}f_{mom,t} + \overline{c}_t u_{sp,t} + e_{r,t}, \\ f_{mom,t} + e_{mom,t}\end{pmatrix}}{var(f_{mom,t} + e_{mom,t})} = \frac{\overline{b}_{mom,t}\overline{\sigma}^2_{f,mom}}{\overline{\sigma}^2_{mom}}$$

Following Chen et al (2015), we assume also that sensitivities to the S&P 500 index common factor, the variances of the non-fundamental factors, variances of the fundamental factors, and correlations between returns on groups are constant over time. However, we assume sensitivities to the other common factors, such as SMB, HML and MOM, change during additions.

$$\underline{b}_{sp,t} = \overline{b}_{sp,t} \equiv b_{i,t} \quad \underline{\rho}_{i,t} = \overline{\rho}_{i,t} \equiv \rho_{i,t}$$

$$\underline{\sigma}^2_{u,sp} = \overline{\sigma}^2_{u,sp} \equiv \sigma^2_{u,sp} > 0$$

$$\underline{\sigma}^2_{e,i} = \overline{\sigma}^2_{e,i} \equiv \sigma^2_{e,i} \quad \underline{\sigma}^2_{f,i} = \overline{\sigma}^2_{f,i} \equiv \sigma^2_{f,i} \quad i = sp, smb, hml, mom$$

then

$$\overline{\beta}_1 - \underline{\beta}_1 = \frac{\overline{c}_t\overline{\sigma}^2_{u,sp}}{\sigma^2_{sp}} > 0$$

$$\overline{\beta}_2 - \underline{\beta}_2 = \frac{(\overline{b}_{smb,t} - \underline{b}_{smb,t})\sigma^2_{f,smb}}{\sigma^2_{smb}}$$

$$\overline{\beta}_3 - \underline{\beta}_3 = \frac{(\overline{b}_{hml,t} - \underline{b}_{hml,t})\sigma^2_{f,hml}}{\sigma^2_{hml}}$$

$$\overline{\beta}_4 - \underline{\beta}_4 = \frac{(\overline{b}_{mom,t} - \underline{b}_{mom,t})\sigma^2_{f,mom}}{\sigma^2_{mom}}$$

Three-factor regression

The three-factor regression is

$$r_{j,t} = \alpha_j + \beta_{j,sp500,3}r_{sp500,t} + \beta_{j,smb}r_{smb,t} + \beta_{j,hml}r_{hml,t} + \varepsilon_{j,t}$$

The probability limits of slope coefficients are

$$\beta = (X^T X)^{-1}(X^T Y)$$

$$\underline{\beta}_{j,sp500,3} = \underline{\alpha} \times \left[\underline{\delta}_{sp}\underline{\beta}_1 - \underline{\theta}_{sp}\underline{\beta}_3 - \underline{\varphi}_{sp}\underline{\beta}_2\right]$$

$$\underline{\alpha} = \frac{1}{1 + 2\underline{\rho}_{sp,smb}\underline{\rho}_{sp,hml}\underline{\rho}_{smb,hml} - \underline{\rho}^2_{sp,smb} - \underline{\rho}^2_{sp,hml} - \underline{\rho}^2_{smb,hml}}$$

$$\underline{\delta}_{sp} = 1 - \underline{\rho}^2_{smb,hml}$$

$$\underline{\theta}_{sp} = \frac{\underline{\rho}_{sp,hml}\underline{\sigma}_{hm} - \underline{\rho}_{sp,smb}\underline{\rho}_{smb,hml}\underline{\sigma}_{hml}}{\underline{\sigma}_{sp}}$$

$$\underline{\varphi}_{sp} = \frac{\underline{\rho}_{sp,smb}\underline{\sigma}_{smb} - \underline{\rho}_{sp,hml}\underline{\rho}_{smb,hml}\underline{\sigma}_{smb}}{\underline{\sigma}_{sp}}$$

$$\overline{\beta}_{j,sp500,3} = \overline{\alpha} \times \left[\overline{\delta}_m\overline{\beta}_1 - \overline{\theta}_m\overline{\beta}_3 - \overline{\varphi}_m\overline{\beta}_2\right]$$

$$\overline{\alpha} = \frac{1}{1 + 2\overline{\rho}_{sp,smb}\overline{\rho}_{sp,hml}\overline{\rho}_{smb,hml} - \overline{\rho}^2_{sp,smb} - \overline{\rho}^2_{sp,hml} - \overline{\rho}^2_{smb,hml}}$$

$$\overline{\delta}_m = 1 - \overline{\rho}^2_{smb,hml}$$

$$\overline{\theta}_m = \frac{\overline{\rho}_{sp,hml}\overline{\sigma}_{hml} - \overline{\rho}_{sp,smb}\overline{\rho}_{smb,hml}\overline{\sigma}_{hml}}{\overline{\sigma}_{sp}}$$

$$\overline{\varphi}_{sp} = \frac{\overline{\rho}_{sp,smb}\overline{\sigma}_{smb} - \overline{\rho}_{sp,hml}\overline{\rho}_{smb,hml}\overline{\sigma}_{smb}}{\overline{\sigma}_{sp}}$$

Using the same assumption in univariate regressions, we can get estimators of excess comovement.

$$\overline{\beta}_{j,sp500,3} - \underline{\beta}_{j,sp500,3} = \alpha \times \left[\delta_{sp}(\overline{\beta}_1 - \underline{\beta}_1) - \theta_{sp}(\overline{\beta}_3 - \underline{\beta}_3) - \varphi_{sp}(\overline{\beta}_2 - \underline{\beta}_2)\right]$$

Four-factor regression

The four-factor regression is

$r_{j,t} = \alpha_j + \beta_{j,sp500,4} r_{sp500,t} + \beta_{j,smb} r_{smb,t} + \beta_{j,hml} r_{hml,t} + \beta_{j,mom} r_{mom,t} + \varepsilon_{j,t}$ The probability limits of slope coefficients are

$$\beta = (X^T X)^{-1}(X^T Y)$$

$$\underline{\beta}_{j,sp500,4} = \underline{\gamma} \times \left[\underline{\vartheta}_{sp}\beta_1 - \underline{\pi}_{sp}\beta_2 - \underline{\tau}_{sp}\beta_3 - \underline{\omega}_{sp}\beta_4\right]$$

$\underline{\gamma} =$

$$1 \Bigg/ \begin{pmatrix} 1 + 2\underline{\rho}_{smb,mom}\underline{\rho}_{smb,hml}\underline{\rho}_{hml,mom} + 2\underline{\rho}_{sp,mom}\underline{\rho}_{hm\ ,mom}\underline{\rho}_{sp,hml} + 2\underline{\rho}_{sp,smb}\underline{\rho}_{sp,mom}\underline{\rho}_{smb,mom} \\ +2\underline{\rho}_{sp,smb}\underline{\rho}_{sp,hml}\underline{\rho}_{smb,hm} + \underline{\rho}^2_{sp,smb}\underline{\rho}^2_{hm\ ,mom} + \underline{\rho}^2_{sp,hm}\underline{\rho}^2_{smb,mom} + \underline{\rho}^2_{sp,mom}\underline{\rho}^2_{smb,hml} \\ -\underline{\rho}^2_{sp,smb} - \underline{\rho}^2_{sp,hm} - \underline{\rho}^2_{sp,mom} - \underline{\rho}^2_{smb,hml} - \underline{\rho}^2_{smb,mom} - \underline{\rho}^2_{hml,mom} \\ -2\underline{\rho}_{sp,smb}\underline{\rho}_{smb,hml}\underline{\rho}_{hm\ ,mom}\underline{\rho}_{sp,mom} - 2\underline{\rho}_{sp,smb}\underline{\rho}_{smb,mom}\underline{\rho}_{sp,hml}\underline{\rho}_{smb,mom} \\ -2\underline{\rho}_{sp,hml}\underline{\rho}_{sp,mom}\underline{\rho}_{smb,hm}\ \underline{\rho}_{smb,mom} \end{pmatrix}$$

$$\underline{\vartheta}_{sp} = 1 + 2\underline{\rho}_{smb,hml}\underline{\rho}_{hml,mom}\underline{\rho}_{smb,mom} - \underline{\rho}^2_{smb,mom} - \underline{\rho}^2_{hm\ ,mo} - \underline{\rho}^2_{smb,hm}$$

$$\underline{\pi}_{sp} = \Big(\underline{\rho}_{sp,sm} + \underline{\rho}_{sp,hml}\underline{\rho}_{hm\ ,mom}\underline{\rho}_{smb,mom} + \underline{\rho}_{sp,mom}\underline{\rho}_{smb,hml}\underline{\rho}_{hml,mom} - \underline{\rho}_{sp,mom}\underline{\rho}_{smb,mom} - \underline{\rho}^2_{hml,mom}\underline{\rho}_{sp,smb} - \underline{\rho}_{sp,hml}\underline{\rho}_{smb,hml}\Big)\frac{\underline{\sigma}_{smb}}{\underline{\sigma}_{sp}}$$

$$\tau_{sp} = \Big(\underline{\rho}_{sp,hml} + \underline{\rho}_{sp,mom}\underline{\rho}_{smb,mom}\underline{\rho}_{smb,hml} + \underline{\rho}_{sp,smb}\underline{\rho}_{smb,mom}\underline{\rho}_{hml,mom} - \underline{\rho}_{sp,smb}\underline{\rho}_{smb,hml} - \underline{\rho}^2_{smb,mom}\underline{\rho}_{sp,hm} - \underline{\rho}_{sp,mom}\underline{\rho}_{hm\ ,mom}\Big)\frac{\underline{\sigma}_{hml}}{\underline{\sigma}_{sp}}$$

$$\underline{\omega}_{sp} = \Big(\underline{\rho}_{sp,mom} + \underline{\rho}_{sp,smb}\underline{\rho}_{smb,hml}\underline{\rho}_{hml,mom} + \underline{\rho}_{sp,hml}\underline{\rho}_{smb,mom}\underline{\rho}_{smb,hml} - \underline{\rho}_{sp,smb}\underline{\rho}_{smb,mom} - \underline{\rho}^2_{smb,hml}\underline{\rho}_{sp,mom} - \underline{\rho}_{hml,mom}\underline{\rho}_{sp,hml}\Big)\frac{\underline{\sigma}_{mom}}{\underline{\sigma}_{sp}}$$

$$\overline{\beta}_{j,sp500,4} = \overline{\gamma} \times \left[\overline{\vartheta}_{sp}\beta_1 - \overline{\pi}_{sp}\beta_2 - \overline{\tau}_{sp}\beta_3 - \overline{\omega}_{sp}\beta_4\right]$$

$$\overline{\gamma} =$$

$$1 \Bigg/ \left(\begin{array}{c} 1 + 2\overline{\rho}_{smb,mom}\overline{\rho}_{smb,hml}\overline{\rho}_{hm\ ,mom} + 2\overline{\rho}_{sp,mom}\overline{\rho}_{hml,mom}\overline{\rho}_{sp,hm} + 2\overline{\rho}_{sp,smb}\overline{\rho}_{sp,mom}\overline{\rho}_{smb,mom} \\ +2\overline{\rho}_{sp,smb}\overline{\rho}_{sp,hml}\overline{\rho}_{smb,hml} + \overline{\rho}^2_{sp,smb}\overline{\rho}^2_{hm\ ,mom} + \overline{\rho}^2_{sp,hml}\overline{\rho}^2_{smb,mom} + \overline{\rho}^2_{sp,mom}\overline{\rho}^2_{smb,hm} \\ -\overline{\rho}^2_{sp,smb} - \overline{\rho}^2_{sp,hm} - \overline{\rho}^2_{sp,mom} - \overline{\rho}^2_{smb,hml} - \overline{\rho}^2_{smb,mom} - \overline{\rho}^2_{hml,mom} \\ -2\overline{\rho}_{sp,smb}\overline{\rho}_{smb,hm}\ \overline{\rho}_{hml,mom}\overline{\rho}_{sp,mom} - 2\overline{\rho}_{sp,smb}\overline{\rho}_{smb,mom}\overline{\rho}_{sp,hml}\overline{\rho}_{smb,mom} \\ -2\overline{\rho}_{sp,hml}\overline{\rho}_{sp,mom}\overline{\rho}_{smb,hm}\ \overline{\rho}_{smb,mom} \end{array} \right)$$

$$\overline{\vartheta}_{sp} = 1 + 2\overline{\rho}_{smb,hml}\overline{\rho}_{hm\ ,mom}\overline{\rho}_{smb,mom} - \overline{\rho}^2_{smb,mom} - \overline{\rho}^2_{hml,mom} - \overline{\rho}^2_{smb,hml}$$

$$\overline{\pi}_{sp} = \Big(\overline{\rho}_{sp,sm} + \overline{\rho}_{sp,hm}\ \overline{\rho}_{hml,mom}\overline{\rho}_{smb,mom} + \overline{\rho}_{sp,mom}\overline{\rho}_{smb,hm}\ \overline{\rho}_{hml,mom} - \overline{\rho}_{sp,mom}\overline{\rho}_{smb,mom} - \overline{\rho}^2_{hml,mom}\overline{\rho}_{sp,smb} - \overline{\rho}_{sp,hm}\ \overline{\rho}_{smb,hml}\Big)\frac{\overline{\sigma}_{smb}}{\overline{\sigma}_{sp}}$$

$$\overline{\tau}_{sp} = \Big(\overline{\rho}_{sp,hml} + \overline{\rho}_{sp,mom}\overline{\rho}_{smb,mom}\overline{\rho}_{smb,hml} + \overline{\rho}_{sp,smb}\overline{\rho}_{smb,mom}\overline{\rho}_{hml,mom} - \overline{\rho}_{sp,smb}\overline{\rho}_{smb,hml} - \overline{\rho}^2_{smb,mom}\overline{\rho}_{sp,hm} - \overline{\rho}_{sp,mom}\overline{\rho}_{hml,mom}\Big)\frac{\overline{\sigma}_{hml}}{\overline{\sigma}_{sp}}$$

$$\overline{\omega}_{sp} = \Big(\overline{\rho}_{sp,mom} + \overline{\rho}_{sp,smb}\overline{\rho}_{smb,hml}\overline{\rho}_{hml,mom} + \overline{\rho}_{sp,hm}\ \overline{\rho}_{smb,mom}\overline{\rho}_{smb,hml} - \overline{\rho}_{sp,smb}\overline{\rho}_{smb,mom} - \overline{\rho}^2_{smb,hml}\overline{\rho}_{sp,mom} - \overline{\rho}_{hml,mom}\overline{\rho}_{sp,hml}\Big)\frac{\overline{\sigma}_{mom}}{\overline{\sigma}_{sp}}$$

Using the same assumption in univariate regressions, we can get estimators of excess comovement.

$$\overline{\beta}_{j,sp500,4} - \underline{\beta}_{j,sp500,4} = \gamma\left[\vartheta_m\left(\overline{\beta}_1 - \underline{\beta}_1\right) - \pi_m\left(\overline{\beta}_2 - \underline{\beta}_2\right) - \tau_m\left(\overline{\beta}_3 - \underline{\beta}_3\right) - \omega_m\left(\overline{\beta}_4 - \underline{\beta}_4\right)\right]$$