

Identifying dynamical instabilities in supply networks using generalized modeling

Güven Demirel¹ | Bart L. MacCarthy² | Daniel Ritterskamp³ | Alan R. Champneys³ | Thilo Gross³

¹Management Science & Entrepreneurship Group, Essex Business School, University of Essex, Southend-on-Sea, UK

²Operations Management & Information Systems, Business School, University of Nottingham, Nottingham, UK

³Department of Engineering Mathematics, University of Bristol, Bristol, UK

Correspondence

Bart L. MacCarthy, Operations Management & Information Systems, Business School, University of Nottingham, Nottingham NG7 2RD, UK.
Email: bart.maccarthy@nottingham.ac.uk

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Abstract

Supply networks need to exhibit stability in order to remain functional. Here, we apply a generalized modeling (GM) approach, which has a strong pedigree in the analysis of dynamical systems, to study the stability of real-world supply networks. It goes beyond purely structural network analysis approaches by incorporating material flows, which are defining characteristics of supply networks. The analysis focuses on the network of interactions between material flows, providing new conceptualizations to capture key aspects of production and inventory policies. We provide stability analyses of two contrasting real-world networks—that of an industrial engine manufacturer and an industry-level network in the luxury goods sector. We highlight the criticality of links with suppliers that involve the dispatch, processing, and return of parts or sub-assemblies, cyclic motifs that involve separate paths from a common supplier to a common firm downstream, and competing demands of different end products at specific nodes. Based on a critical discussion of our findings in the context of the supply chain management literature, we generate five propositions to advance knowledge and understanding of supply network stability. We discuss the implications of the propositions for the effective management, control, and development of supply networks. The GM approach enables fast screening to identify hidden vulnerabilities in extensive supply networks.

KEYWORDS

complex networks, nonlinear dynamics, stability, supply chain

1 | INTRODUCTION

Supply networks need to remain functional in the presence of disturbances and disruptions (Tang, 2006). The capability of a network to withstand disturbances and disruptions is related to the concept of stability. This is a term commonly used in a number of disciplines to refer to the capability of a system to remain close or converge back to a steady state following a triggering event (Guckenheimer & Holmes, 1983). In the absence

of stability, even small disturbances may drive a supply network away from a desired or planned state (Venkateswaran & Son, 2007; Wei, Wang, & Qi, 2013). Here, we are interested in the stability of material flows in supply networks, where its loss manifests itself by divergence from an equilibrium state and by oscillations, leading to uncontrolled inventory build-ups and/or stock-outs, production overtime and/or production shutdowns, all of which will typically have very costly consequences (Venkateswaran & Son, 2007). Although firms that constitute a

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supply network would be expected to react to such undesired and costly consequences by revising their policies over time, for instance by changing the replenishment period or shifting orders to other suppliers, costs will still be incurred until these measures show their effect. Worse still, such reactive measures may not lead to the intended outcomes, because individual firms are embedded in complex supply networks, which need to be understood in their entirety (Pathak, Day, Nair, Sawaya, & Kristal, 2007; Surana, Kumar, Greaves, & Raghavan, 2005). It is therefore imperative to understand the stability properties of supply networks.

Supply network stability has been investigated in previous studies, particularly in the context of inventory control policies (Sarimveis, Patrinos, Tarantilis, & Kiranoudis, 2008; Wang & Disney, 2016). Many studies in the literature on supply network dynamics have focused on isolated parts of larger supply networks such as buyer–supplier dyads or retailer–wholesaler–manufacturer triads (Sarimveis et al., 2008; Wang & Disney, 2016). Some recent studies in the context of the bullwhip effect have considered larger networks (Chatfield, 2013; Dominguez, Framinan, & Cannella, 2014), showing that insights from small networks cannot be directly transferred to the larger networks to which they belong. Complex network analysis approaches developed in other disciplines (Newman, 2003) have been applied in supply chain management. However, knowledge and understanding of the stability of large dynamic supply networks is still limited. This may be explained to some extent by the size and nontrivial network structure of large dynamic supply networks, as well as the low visibility that pertains in many such networks (Choi, Dooley, & Rungtusanatham, 2001). An extensive supply network may incorporate hundreds of suppliers, of which only a small fraction are tier-one suppliers directly visible to a focal or prime organization in a network. Even if a focal organization invests resources to analyze the structure of its entire supply network, there are a multitude of operational details that cannot be captured, for example, different suppliers may use different modes of production and different inventory management policies. The problem is further complicated by the intractability of models that seek to capture the dynamics of large networks, especially when nonlinearities are considered.

Supply chain management is not the only field that faces the challenge of modeling and analyzing large dynamic networks. Systems of equal or greater complexity are studied in ecology, which has a long history of mathematical modeling and places substantial emphasis on the study of stability (Grimm & Wissel, 1997; May, 2013). *Generalized modeling* (GM) is an important approach that has been used in ecology and other domains to address the challenge of network modeling when there is uncertainty about the precise mathematical forms of relationships that define a system (Gross & Feudel, 2006; Gross, Rudolf, Levin, & Dieckmann, 2009).

In this article, we apply generalized models to manufacturing supply networks to investigate instabilities emerging from

the pattern of interactions between firms, that is, the network topology. In contrast to the conventional modeling approaches for supply chain dynamics such as control theory, agent-based models, and discrete-event simulation, the GM approach enables the stability of a network to be investigated in the absence of detailed information on operational policies, using information primarily derived from the network structure, that is, who is connected with whom. We take a high-level network view and consider supply as being continuous and instantaneous. This lean modeling approach does not seek to capture instabilities such as the bullwhip effect that may arise at a finer level of granularity, due to the effects of discreteness, delays, and stochasticity.

The article makes four significant contributions to the supply chain management literature. First, we provide stability analyses of two contrasting real-world supply networks—the inbound supply network of an industrial engine manufacturer and an industry-level supply network in the luxury goods sector. Second, we present a set of five propositions on the stability of supply networks. The propositions relate to both the network structure and the material flows on the network and seek to advance the extant knowledge on the stability of supply networks. Cyclic motifs and competition from different product streams in a supply network are identified as having destabilizing effects. Links with suppliers that have bidirectional flows, performing operations such as painting and machining, have a high influence on the rest of the network at the onset of instability but a lower sensitivity to disturbances occurring elsewhere in a network. Limited product availability may have a stabilizing impact in small inbound networks serving a single prime entity but can become destabilizing in industry-level networks formed from the intertwining of separate networks. The more quickly the production rate is adjusted to account for changes in the inventory level, the more likely the supply network is to be stable.

The third contribution of the study is the insights and guidance provided for organizations to manage critical suppliers. Prime entities in supply networks that have the required visibility and power can consider strategic development activities with influential suppliers in their supply networks. Investing in extra buffers is recommended, if organizations are highly sensitive to disturbances elsewhere in the network. Investment in capacity is recommended for suppliers that are at the apex of cycles and/or supply directly or indirectly to multiple prime entities. The fourth contribution is the introduction and development of new conceptualizations for generalized turnover and elasticity parameters, which capture crucial aspects of material flows, inventory management, and production policies in supply networks. These conceptualizations allow the GM approach to be applied in a computationally efficient way that can be automated for fast screening of extensive supply networks. This enables the stability implications of a perceived change in some part of the system to be quickly investigated,

which can facilitate adaptive management decisions to account for stability.

The article is organized as follows. In Section 2, we position the work in the context of the literature. In Section 3, we develop and build a generalized model of supply networks using an illustrative but nontrivial example, a triadic supply network. We use this network to establish a GM-based stability analysis method for supply networks and show how the method can be adapted for real-world supply networks. In Section 4, we present and describe the two real-world supply networks investigated. In Section 5, we present the results of the stability analyses of these networks, discuss the findings in the context of the literature, formulate propositions on supply network stability, and identify opportunities for further research.

2 | LITERATURE REVIEW

This work relates to three strands of literature: the stability of supply networks, the analysis of supply networks using complex networks approaches, and GM of dynamical systems and the stability of ecological networks.

2.1 | Stability of supply networks

Ivanov and Sokolov (2013) define supply chain stability as “the ability to ensure continuity,” which can be captured by different mathematical constructs. In this work, we use the dynamical systems concept of asymptotic stability that corresponds to the ability of a system to return back to the steady state following a disturbance (Guckenheimer & Holmes, 1983). A related concept is that of resilience, described by Christopher and Peck (2004) as “the ability of a system to return to its original state or move to a new, more desirable state after being disturbed.” It is insightful to highlight the differences between stability and resilience. While stability is a basic desirable dynamic property of a supply network without an explicit consideration of performance, resilience explicitly refers to a desired state in the presence of a performance objective that should be achieved within a set time-window (Ivanov & Sokolov, 2013). For instance, if we consider a supply disruption, it is a question of stability whether the inventory levels can return back post-disruption to the pre-established levels over time, rather than for instance oscillating between overstocking and shortages, which are implicitly undesired states, whereas it is a question of resilience whether the desired customer satisfaction level can be achieved within a reasonable time window. This conceptualization is consistent with the equilibrium definitions of stability and resilience in the ecology literature, where stability refers to the capability to return back to a steady state and resilience is concerned with the speed of the return (McCann, 2000).

Stability has been analyzed in the supply chain and operations management literature mostly in the context of inventory dynamics, where dynamic stability is characterized by whether or not a considered inventory level can be sustained over time (Dejonckheere, Disney, Lambrecht, & Towill, 2003; Disney & Towill, 2002; Warburton, 2004; Wei et al., 2013). These studies use variations of the inventory- and order-based production control system (IOBPCS) (Towill, 1982), which are based on linear fractional control rules and constitute generalizations of the standard “inventory order up to level” models (Dejonckheere et al., 2003). Although the models in the literature are predominantly linear, some studies have sought to consider nonlinearities due to the nonnegativity of flows (Laugesen & Mosekilde, 2006; Wang, Disney & Wang, 2012; Warburton, 2004) and the strain on the system due to limited capacity and/or product availability (Spiegler & Naim, 2017; Spiegler, Naim, Towill, & Wikner, 2016; Venkateswaran & Son, 2007). These studies show that nonlinearities greatly influence the stability and the dynamics, leading for instance to chaotic behavior (Laugesen & Mosekilde, 2006).

Thus, conventional modeling approaches mainly use specific and in most cases linear inventory control policies to identify the conditions of instability. Such approaches facilitate analytical tractability in small and serial supply chains but do not scale to allow the analysis of large networks. Therefore, understanding the stability properties of large supply networks with general nonlinear formulations remains an open question, which we address here.

2.2 | Analysis of supply networks using complex network approaches

The structure of supply networks has attracted substantial research interest. Several real supply networks have been documented in the literature. Atalay, Hortacsu, Roberts, and Syverson (2011) present a network consisting of all major supply relations between publicly traded U.S. companies. Saavedra, Reed-Tsochas, and Uzzi (2008, 2009) map the supply network between designers and contractors in the New York garment industry. Supply networks from the automotive and vehicle industries have been documented by several studies (Brintrup, Ledwoch, & Barros, 2016; Choi & Hong, 2002; Lomi & Pattison, 2006; Kito, Brintrup, New, & Reed-Tsochas, 2014). Bode and Wagner (2015) identified two structural dimensions of supply network complexity: horizontal complexity (the number of first-tier suppliers) and vertical complexity (the number of tiers). However, real supply networks deviate from a hierarchical tree topology and involve further aspects such as degree heterogeneity (Atalay et al., 2011; Brintrup et al., 2016; Brintrup, Wang, & Tiwari, 2015), the intertwined nature of supply chain networks (Brintrup et al., 2016; Nair, Narasimhan, & Choi, 2009), and the interrelatedness between

suppliers within the same tier (Brintrup et al., 2015; Choi & Wu, 2009; Lomi & Pattison, 2006).

The impact on firms of the structure of supply networks to which they belong has been discussed with reference to network measures such as clustering coefficient, shortest path length statistics, and centrality (Bellamy & Basole, 2013; Hearnshaw & Wilson, 2013; Kim, Choi, Yan, & Dooley, 2011). In particular, centrality measures have been used to quantify the importance of firms with respect to their positions within supply networks and to assess systemic risk in networks (Brintrup et al., 2015, 2016). At the node level, centrality measures of degree, betweenness, and closeness have been linked with operational load, operational criticality, and informational independence, respectively (Kim et al., 2011). At the system level, Nair and Vidal (2011) show that increased average path length reduces performance robustness when suppliers are subjected to disruptions. The percolation process in physics, where nodes or links are removed (Albert, Jeong & Barabási, 2000), has also been applied to study the topological resilience of supply networks (Kim, Chen, & Linderman, 2015; Thadakamalla, Raghavan, Kumara, & Albert, 2011). Such node and link removals are analogous to catastrophic failures of suppliers, for example, disruptions for Ericsson and Nokia due a fire occurring at a supplier's facility (Chopra & Sodhi, 2004). It is well-known that scale-free networks are very robust against random failures but fragile against targeted attacks (Albert et al., 2000), which has implications for the critical role of hubs in supply networks (Kim et al., 2015; Thadakamalla et al., 2011).

The studies on the impact of supply network structure on performance (e.g., Kim et al., 2015; Nair & Vidal, 2011; Thadakamalla et al., 2011) use synthetic networks, while structures of real-world supply networks tend to be different from those of idealized models (Brintrup et al., 2016; Kito et al., 2014). Therefore, the impact of network structure on performance in real-world supply networks remains an open research question. We address this gap in the context of supply network stability by applying GM, a dynamical systems method from ecology, to two real networks. In doing so, we incorporate a defining characteristic of supply networks, dynamic flow, to the investigation and quantification of node and link importance, differently from the existing studies (e.g., Brintrup et al., 2016).

2.3 | GM and the stability of ecological networks

In socioeconomic systems, there is a great degree of uncertainty about the functional forms that can adequately capture relationships between entities in the system. This problem is amplified in large systems, such as extensive supply networks, where such information and understanding is limited. This raises significant challenges in investigating stability

because a mathematical model with the same structure can behave very differently for different functional forms (Gross, Ebenhöf, & Feudel, 2004). GM was introduced originally in ecology to overcome this challenge (Gross et al., 2009; Gross & Feudel, 2006). Since its introduction, it has become a tool that complements conventional modeling approaches in ecology (Aufderheide, Rudolf, & Gross, 2012; Gross et al., 2009; Gross & Feudel, 2006; Yeakel, Stiefs, Novak, & Gross, 2011) and has been applied in areas such as systems biology (Steuer, Gross, Selbig, & Blasius, 2006; Zumsande, Stiefs, Siegmund, & Gross, 2011), historical analysis (Gross & Feudel, 2006), and social-ecological systems (Lade et al., 2015).

GM uses unspecified or general functions instead of specific functions (Gross & Feudel, 2006). Constructing a generalized model starts with identifying dynamic state variables (stocks), for example, inventory levels, and their inflows and outflows, such as a flow of materials from a supplier to a buyer. The rate of each flow is represented by a mathematical function, which may take a specific form, if known. However, specifying functional forms is generally not possible and almost all flows are expressed as unspecified functions in applications of GM to social systems (e.g., Lade et al., 2015). Once the interactions are defined in this way, the GM approach enables the stability of a network to be characterized in terms of a set of generalized parameters that we explain in Section 3. Once the generalized parameters are established, stability analysis can take two forms. In small systems, the critical conditions that lead to changes in stability, that is, bifurcations, may be solvable analytically (e.g., Gross & Feudel, 2006; Zumsande et al., 2011). In large systems, this is generally not feasible and numerical computation is required (e.g., Gross et al., 2009; Lade et al., 2015). Stability analysis of large systems is undertaken by sampling across a parameter space of generalized parameters, thus enabling the investigation of relationships between generalized parameters and stability (Gross et al., 2009). Although the use of generalized parameters and the stability analysis procedure are standard approaches in GM, their definition, formulation, and interpretation are domain-specific and challenging for specific contexts.

Applications of GM have revealed important insights on the stability of ecological networks. Gross et al. (2009) investigated the stability of food webs with a particular focus on network complexity. Since the seminal work of May (1972), it is known that food webs become unstable with growing size and connectivity, if they are assumed to be randomly connected. However, large complex food webs do exist in reality and there must therefore be factors that contribute to their stability. The ecology literature has proposed various mechanisms that contribute to the stability of large networks, including the abundance of predator-prey relations (Allesina & Pascual, 2008), network modularity and nestedness (Bascompte & Stouffer, 2011; Rohr, Saavedra, &

Bascompte, 2014), and heterogeneity of link strengths (Bascompte, Jordano, & Olesen, 2006). Based on a large ensemble of networks, Gross et al. (2009) use the GM method to show that the variability in the strengths of links between species has a destabilizing role in large webs in contrast to their stabilizing role in small webs. This necessitates that further mechanisms are required for stability, such as having multiple predators for intermediate species and top predators being generalists (Gross et al., 2009).

GM is not the only approach used in ecology to characterize the general stability properties of ecological networks. For instance, Rohr et al. (2014) use structural stability to investigate the stability of all feasible steady states where all species can coexist, instead of a specific steady state. Saavedra, Rohr, Gilarranz, and Bascompte (2014) apply this approach to socio-economic systems where countries are represented as agents that compete for resources. Saavedra et al. (2014) show that stability decreases with increasing global competition for resources and with the heterogeneity in distributions of resources. There are similarities and differences between GM and structural stability analysis. GM does not provide explicit feasibility conditions but instead assumes their existence, while Rohr et al. (2014) and Saavedra et al. (2014) calculate the feasibility as well as stability conditions. GM characterizes the stability for the whole family of nonlinear models in terms of generalized parameters, to which any specific functional form and steady state can be mapped, while Rohr et al. (2014) and Saavedra et al. (2014) consider specific functional forms. Differently from the local stability analysis of GM, structural stability considers global stability, which is a stronger condition that requires the explicit form of the functional relationship, which GM does not consider.

Here, we use for the first time the GM method to study the stability of supply networks. The adoption of a method developed in ecology to supply networks is consonant with the analogies between the two types of systems discussed in Surana et al. (2005) and Saavedra et al. (2008, 2009, 2014); Saavedra, Stouffer, Uzzi, and Bascompte (2011). Here, we define, specify, and interpret concepts of elasticity, sensitivity, and influence, in the context of supply networks. Although the concept of elasticity is widely used in economics, its application to material flows and inventory in supply networks is novel.

2.4 | Theoretical positioning of the study

We seek to fill the theoretical gaps discussed above by identifying through analysis the factors that contribute to the stability of large supply networks in their entirety. On the one hand, inventory dynamics models noted in Section 2.1 capture operational details, such as order batching, discrete delays, and forecast inaccuracy, which do not scale to large networks for the study of their stability properties. Therefore, the impact on stability of

the larger supply networks to which dyads or triads belong is missed by these approaches. On the other hand, complex networks methods used to investigate supply networks (see Section 2.2) either rely on centrality metrics, not capturing the dynamical processes, or they resort to simulation, which is not informative for stability. We move beyond purely structural analyses of supply networks and seek to incorporate dynamic material flows, which are defining characteristics of supply networks. In doing this, we make a compromise between calculating purely structural network metrics and incorporating inventory dynamics at a level of detail, which would result in mathematical intractability for an entire network. The price paid for capturing nonlinearities and scalability to large networks is the exclusion of discreteness, stochasticities, and delays considered in the literature that focuses on much smaller systems at a much finer level of granularity. The principal assumption we make of continuous flow is reasonable and plausible at a high level for supply networks captured in their entirety, which is our focus in this study. This approximation may be poor when local control of flows in a network by a single entity has a strong impact on overall network behavior. Our approach thus provides a high level assessment of the stability properties of supply networks, which is valuable for initial screening of the network to identify suppliers and patterns of supply that are critical for stability. It can be combined with specified inventory models for critical suppliers, if more information is available.

3 | GM FRAMEWORK FOR SUPPLY NETWORK ANALYSIS

Here, we show how a generalized model can be developed to analyze a supply network. We illustrate the approach using the simple but nontrivial triadic supply network illustrated in Figure 1 that captures the interrelatedness between suppliers where a first-tier supplier to one firm supplies at the same time to another first-tier supplier of the same firm. We present a generalized model and its analysis for this triadic network. We then develop the building blocks for a more general GM method to analyze real-world supply networks. We present the main steps here. A fuller mathematical exposition is given in the *Supporting Information Appendix*.

The triadic network comprises three organizations: a prime company and two suppliers. These organizations form the nodes of the network, while product flows constitute the arcs. The supply network produces the final product 6, which is assembled by the prime company (Organization C) from two outsourced parts (Parts 4 and 5). The prime company receives Parts 4 and 5 from suppliers A and B, respectively. However, production of Part 5 by first-tier supplier B depends on the supply of Part 3 by supplier A. The production of Part 5 additionally requires Part 2, while Parts 3 and 4 both require Part 1. Parts 1 and 2 are assumed to be supplied without constraint

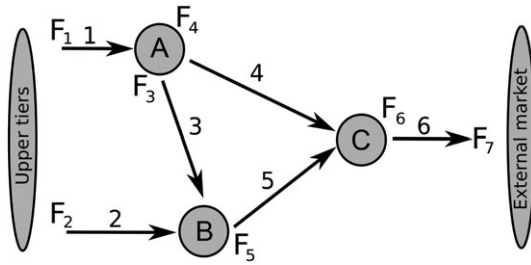


FIGURE 1 Triadic supply network

from the external environment. We assume a constant deterministic demand, which is reasonable for products with stationary demand and low variability.

We use state variables P_1, P_2, \dots, P_6 to denote the total system inventory of products 1 to 6. We do not differentiate between the inventory levels of a given type of part held by different firms. We assume that parts become immediately available at the respective buyer when needed. This abstraction means that we do not seek to capture dynamics that arise from lower level coordination problems such as delay-induced instabilities (e.g., Warburton, 2004; Wei et al., 2013) or periodic reviews and order batching (e.g., Lee, Padmanabhan, & Whang, 1997). The approach enables us to take a lean, “bird’s eye” perspective of flow on the network. We examine higher level network effects critical for real-world networks where detailed information on production, replenishment, and transportation processes is unlikely to be available.

To set up and analyze a generalized model, four steps are necessary. In step one, differential equations with unspecified functions are formulated to describe the time-evolution of the state variables (P_1, P_2, \dots, P_6). In step two, GM uses a simple mathematical transformation, where each variable and function is normalized with respect to their values in the steady state. This provides the first set of generalized parameters: *turnover* parameters that capture the speed of inventory turnover. In step three, the Jacobian matrix (a standard matrix obtainable from a system of differential equations) is calculated for the normalized system. This provides the second set of generalized parameters: *elasticity parameters* that characterize the sensitivity of the unspecified functions to changes in the state variables at the steady state (Gross & Feudel, 2006). Elasticity parameters provide a measure of nonlinearity (see *Supporting Information Appendix*). In step four, the asymptotic stability of the network is determined by calculating the leading eigenvalue of the Jacobian matrix (more precisely, the eigenvalue with the largest real part). The network is unstable, if this eigenvalue has a positive real part and stable otherwise. In large networks, the leading eigenvalue is computed numerically for random samples from the generalized parameter space.

The inventory levels (P_1, P_2, \dots, P_6) change in time due to production and shipments in response to orders. To

describe this, we use the notation $X + Y \Rightarrow Z$, meaning that product Z is assembled from parts X and Y . Using this notation, the production and ordering processes in the triadic network are described by the following relationships:

$$\begin{array}{ll}
 \text{Input :} & \emptyset \Rightarrow P_1 \quad F_1 \\
 & \emptyset \Rightarrow P_2 \quad F_2 \\
 \text{A :} & P_1 \Rightarrow P_3 \quad F_3 \\
 & P_1 \Rightarrow P_4 \quad F_4 \\
 \text{B :} & P_2 + P_3 \Rightarrow P_5 \quad F_5 \\
 \text{C :} & P_4 + P_5 \Rightarrow P_6 \quad F_6 \\
 \text{Output :} & P_6 \Rightarrow \emptyset \quad F_7,
 \end{array}$$

where \emptyset represents the external environment, that is, sources and sinks in the system. Each of the flows is assigned an auxiliary variable F_i , which denotes the rate at which the respective material flow occurs. The material flows F_1 and F_2 describe the supply of external parts, F_3 - F_6 denote the production of parts and the final product, and F_7 denotes the sale of the final product.

The inventory level of a product increases due to flows where the respective product is manufactured or delivered and decreases due to flows where the respective part is used in assembly processes or sold. The production rate of a product depends on (a) its current inventory level, which is a basic principle of inventory control, (b) the inventory levels of parts required for its production, due to its potential limitation by the unavailability of the requested quantity, and (c) the inventory levels of other parts manufactured by the same firm, due to shared resources. Taking such processes and interdependencies into account, the dynamics are captured by the following system of differential equations.

$$\begin{aligned}
 \frac{d}{dt}P_1 &= -F_3(P_1, P_3, P_4) - F_4(P_1, P_4, P_3) + F_1(P_1) \\
 \frac{d}{dt}P_2 &= -F_5(P_2, P_3, P_5) + F_2(P_2) \\
 \frac{d}{dt}P_3 &= -F_5(P_2, P_3, P_5) + F_3(P_1, P_3, P_4) \\
 \frac{d}{dt}P_4 &= -F_6(P_4, P_5, P_6) + F_4(P_1, P_4, P_3) \\
 \frac{d}{dt}P_5 &= -F_6(P_4, P_5, P_6) + F_5(P_2, P_3, P_5) \\
 \frac{d}{dt}P_6 &= -F_7(P_6) + F_6(P_4, P_5, P_6)
 \end{aligned} \tag{1}$$

In the steady state (denoted by F_i^* and P_j^* , for flow i and inventory level j , respectively), the flows are balanced, that

is, $dP_i/dt = 0$, for $i = 1, 2, \dots, 6$. As materials cannot appear or vanish from the model except in predefined sources and sinks, the material flow must be conserved across the network. Hence, $F_2^* = F_3^* = \dots = F_7^*$, which we denote as F^* , and $F_1^* = 2F^*$. The subsequent steps of the GM analysis use standard methods from dynamical systems theory (see *Supporting Information Appendix*). As an outcome of this procedure, the stability is expressed as a function of the generalized parameters.

3.1 | Generalized parameters

We now introduce the generalized parameters that appear in the triadic network, which are defined analogously for real-world networks (see Section 3.4). We use a statistical sampling approach for the investigation of a network's stability (see Section 3.2) that requires us to identify plausible ranges for generalized parameter values, which is also discussed below.

3.1.1 | Steady-state flow rate

A single free parameter, F^* , characterizes all steady-state flow rates, which must be positive. A dynamical system can be rescaled with respect to time without affecting its dynamical features. Therefore, F^* can assume its value from a large range of positive values. We use the range $(0, 10]$ to study the stability of the triadic network.

3.1.2 | Steady-state inventory level

In contrast to the flow rate, the inventory levels of different products P_1, \dots, P_6 can be different in the steady state. In the following, we use P^* to refer to any one of these inventory levels. We assume quasicontinuous instantaneous flows for the differential equation formulation, which requires the state variables not to be too small. We therefore consider $P^* \in [1, 10]$, but more extensive ranges could also be considered.

3.1.3 | Elasticity to inventory level

The elasticity to inventory level captures the sensitivity of the production rate to the inventory level of the part produced. We collectively denote the elasticity to inventory level parameters as f_i . To explain the concept of elasticity to inventory level, we refer to the IOBPCS model. In a simplified version of this model where work-in-process inventory is not controlled, replenishment is expressed as the sum of the demand forecast and a fraction of the deviation of system inventory from its desired level. As the replenishment amount decreases with increasing inventory level, the elasticity to inventory

level is nonpositive, that is, $f_i \leq 0$ (see *Supporting Information Appendix* for a calculation of the elasticity parameter for a nonlinear generalization of the IOBPCS control rule). Because the elasticity to inventory level describes the impact of the inventory level on its incoming flows, it captures the self-inhibiting role of inventory control. The more negative the elasticity to inventory level is, the more quickly a discrepancy in the inventory level is resolved. The limit value of $f_i = 0$ is observed, when the desired rate of production cannot be achieved because of production capacity or part availability limitations. We consider the range $f_i \in [-2, 0]$.

3.1.4 | Elasticity to supply

The elasticity to supply characterizes the dependence of the production rate of a certain product i on the availability of parts j that are required for its production. We collectively refer to the elasticity to supply parameters as f_S . The elasticity of finished goods sales to their availability is given further consideration in the *Supporting Information Appendix*.

If the desired rate of production set by inventory level and demand can be achieved, changing the level of parts inventory does not impact the production rate, and thus $f_S = 0$. If the inventory level of parts is insufficient to meet a desired production rate, which may be the case when external demand far exceeds the supply for a popular product, the production rate needs to be adjusted according to the inventory level of parts, thus $f_S > 0$. Such strain due to limited material availability is considered by Spiegler et al. (2016). If all parts available in stock are used directly for production, then $f_S = 1$. We note that for deteriorating ingredients in flow manufacturing industries, the production rate is mostly set by the inventory levels of ingredients, that is, f_S is generally positive. Hence the value of elasticity to supply characterizes whether parts availability drives production or not ($f_S > 0$ and $f_S = 0$, respectively). In the following, we focus on the range $f_S \in [0, 2]$.

3.1.5 | Elasticity to coproduction

Many organizations use the same resources to produce different parts. The elasticity to coproduction characterizes the interdependence between production rates of parts manufactured by the same organization using the same resources. In the triadic network, Organization B produces Parts 3 and 4, and we collectively refer to the elasticity of the production rate of Part 3 (or Part 4) to the inventory level of Part 4 (or Part 3) as f_C .

If Parts 3 and 4 use the same limited resources and Organization B runs at full capacity, then an increase in the production rate of Part 3 can only happen at the expense of a decrease in the production rate of Part 4, and vice versa.

Considering that the desired production rate of a part decreases with its inventory level, the elasticity to coproduction is positive. The more capacity allocated to a part, the smaller is its coproduction elasticity to the other part. The more sensitive the desired production rate to the inventory level of a part is, the higher the elasticity to coproduction of the other part (see *Supporting Information Appendix*). This concept is related to the capacitated multiitem lot sizing problem (see Karimi, Fatemi Ghomi, & Wilson, 2003), where the elasticity to coproduction would characterize the dependence of the production rate of an item on the inventory level of the other item at the previous step.

Elasticity to coproduction may also be negative ($f_C \leq 0$). For instance, in a production system with sequence-dependent setup costs, where the setup for Part 4 after Part 3 has a low cost, it might be advantageous to align the production of Part 4 with Part 3, hence making its production dependent on the other product. Similarly, if a product is a by-product of another (for instance in the chemical industry), its production rate is necessarily the same as the production of the primary product.

In summary, a positive value of elasticity to coproduction refers to the sensitivity of the production rate to another part competing for the same resources, while a negative value characterizes the induced production of a part by another. We consider both positive and negative values and use the range $f_C \in [-2, 2]$.

3.2 | Statistical sampling and analysis procedure

We compute the stability of the triadic network numerically across the parameter space of generalized parameters in the plausible ranges identified above, as summarized in Table 1. To investigate the impact of generalized parameters on stability, we use two ensembles (i.e., random samples) of parameter values that each contains 10^6 sets of parameters. In the ensemble of individual parameters, each parameter,

for example, inventory level P_1^* , is drawn independently and uniform randomly from the corresponding range in Table 1, whereas in the ensemble of identical parameters, all parameters of the same type, for example, all inventory levels P_1^*, \dots, P_6^* , are assigned the same value drawn uniform randomly. The ensemble of individual parameters captures the heterogeneity between different firms across the network. However with increasing network size, it becomes difficult to investigate the influence of each parameter. The ensemble of identical parameters is a simplification, because it assumes that the same type of generalized parameter takes the same value for all firms, but it allows the impact of parameters to be investigated in greater depth.

To measure the effect of certain generalized parameter on stability, we first divide the corresponding range in Table 1 into bins of width 0.05. For each bin, we determine the fraction of stable supply networks among all networks where the generalized parameter falls into that bin. This procedure gives us an estimate of the probability of randomly picking a stable network when a generalized parameter falls within a narrow parameter range. This probability has been used in many GM studies (e.g., Gross et al., 2009) and is commonly called the *proportion of stable webs (PSW)*. When uniform distributions are used, PSW provides an estimate for the fraction of the generalized parameter space, where the network is stable. If further information is available, this uncertainty can be reduced by applying nonuniform distributions. In addition, interdependencies between parameters can also be included.

Here, we present the method as a strategic thinking tool, which helps to identify the overall impact of generalized parameters on stability. However, in further studies, model parameters can be calibrated using historical inventory levels and material flow data (see *Supporting Information Appendix*). This could enable a focal organization to gauge the impact of specific policies to improve the likelihood of stability.

3.3 | Stability of the triadic supply network

Figure 2 shows the PSW when each of the generalized parameters is varied.

3.3.1 | Steady-state flow rate

The steady-state flow rate does not have a significant impact on the stability of the network, as shown in Figure 2a for both ensembles. This may seem counterintuitive. It might be thought that a system with large flow must experience high strain and hence be more prone to instabilities. However, this is not true a priori. A supply network where the flow is pushed close to its capacity will generally be less responsive

TABLE 1 Generalized model parameters and their ranges

Parameter	Interpretation	Initial range	Reduced range
Turnover			
F^*	Steady-state flow rates	(0,10]	–
P^*	Steady-state inventory levels	[1,10]	5
Elasticity			
f_S	Elasticity to supply	[0,2]	[0,1]
f_I	Elasticity to inventory level	[-2,0]	[-2,0]
f_C	Elasticity to coproduction	[-2,2]	[-1,1]

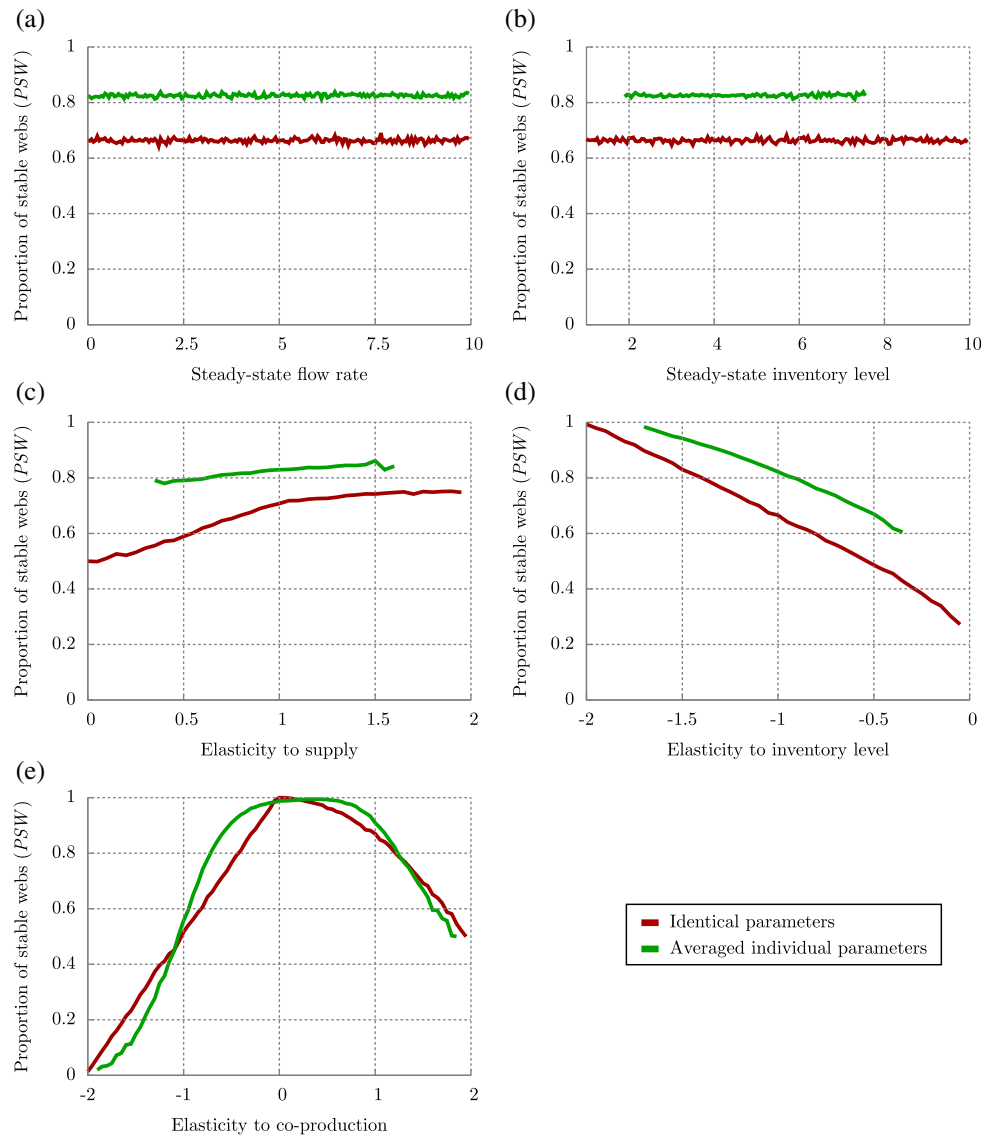


FIGURE 2 PSW as a function of the generalized parameter values in the ensembles of identical (red) and individual (green) parameters [Color figure can be viewed at wileyonlinelibrary.com]

to further attempts to increase the flow rate. In the generalized model, this responsiveness is captured by the elasticity to inventory level. The results below on the elasticity to inventory level confirm that more strained systems are more likely to be unstable. As we draw the elasticity parameters independently of F^* , systems with a high flow rate are not more strained than systems with a low flow rate. Rather, systems with a high flow rate have a higher capacity such that they are on average as strained as systems with a lower flow rate.

3.3.2 | Steady-state inventory level

Similar to the flow, the PSW is not affected by the steady-state inventory levels (Figure 2b). Again this result may seem counter-intuitive as high inventory levels are often used as buffers against disturbances. While it is true that high inventory levels may mitigate the effect of instabilities

by ensuring that demand can be met and thus backlogs can be avoided, the results show that high inventory levels in themselves do not affect the onset of instabilities in this network. We stress that this result does not imply that *inventory management* does not impact stability of the network, but rather that the elasticity to inventory level is a more important factor on stability than steady-state inventory levels.

3.3.3 | Elasticity to supply

The PSW increases with increasing elasticity to supply, but starts to saturate for higher values (Figure 2c). In order to understand this effect, first consider $f_S = 0$, which is observed when supply availability does not limit production. In this case, an increase in the part inventory level does not lead to a higher production rate, resulting in an overflow of inventory, which has a destabilizing effect. The steady state under consideration can however still be stable (PSW ≈ 0.5) as the

excess part inventory leads to less production of the part, because $f_I \leq 0$, which might push the inventory back to its steady-state level. However, for the case $f_S > 0$, where the production is limited by part availability, changes in the part inventory can be compensated by an increased or decreased rate of production, resulting in a higher likelihood of stability.

3.3.4 | Elasticity to inventory level

The PSW is larger for larger negative values of the elasticity to inventory level, and thus f_I has a stabilizing effect (Figure 2d). Consistent with the discussion above, production being more responsive to the current inventory level leads to a higher probability of stability. There is a higher risk of instability for strained systems ($f_I \approx 0$).

3.3.5 | Elasticity to coproduction

The PSW is highest for $f_C \approx 0$, when the production rates of the two products manufactured by the same organization are independent of one another (Figure 2e). The need to shift production from one part to the other due to a capacity constraint, that is, $f_C > 0$, and the dependent production of one part to the other, that is, $f_C < 0$, are both destabilizing.

Generating the above results for the triadic network is straightforward and provides valuable indicators of the likely stability of this small network. The approach can be extended to large networks.

3.4 | Extension of the model and analysis approach to real-world supply networks

To construct generalized models for large real-world supply networks (e.g., consisting of at least 30 firms, which is an order of magnitude bigger than the triad), the procedure developed above can be used but with some adjustments to include specific information available about the networks. Doing so improves the quality and depth of the analysis and its interpretations.

The first adjustment required for a realistic supply network arises from the existence of processing links (Figure 3a), where a product is shipped to a supplier for processing (e.g., painting or machining) and then shipped back to the organization of origin. If an organization (Node j) in the network has regular as well as processing links, we assume that it first assembles an unprocessed product from the parts that arrive via regular links (solid links from top row). Next, the unprocessed product that is sent along the processing link (dotted link) is processed at the relevant processing supplier (Nodes i), without being combined with other parts, and then shipped back as a processed product (solid link heading from left to right). The original organization

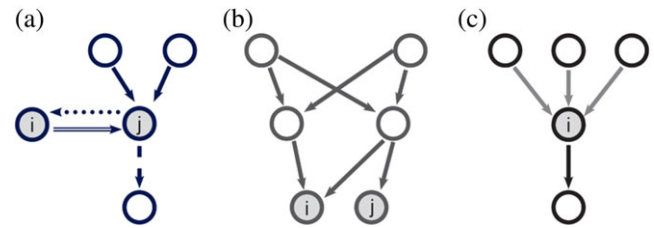


FIGURE 3 Extension to real-world supply networks: (a) processing suppliers, (b) multiple prime companies/retailers, and (c) stockists [Color figure can be viewed at wileyonlinelibrary.com]

then distributes the product using its regular outgoing links (dashed link). This is consistent with the use of processing suppliers in many industrial product supply networks including that of our industrial collaborator that manufactures industrial engines (see Section 4.2.1).

The second adjustment is the existence of multiple prime companies or retailers (Nodes i and j) that sell a product to the external market (Figure 3b), where the material flow can terminate in different consumer products. While the material flow of each individual part is still conserved in the steady state, it is no longer true that all flows are determined by a single flow profile in the network, which may result in non-unique solutions (see *Supporting Information Appendix*). The same problem occurs in other domains and is solved by decomposing the steady-state flow rates into different flux modes (Steuer et al., 2007). Each of these flux modes represents a feasible flow of material through the system. Every feasible flow profile can then be written as a superposition of these flux modes. The flow profile in the steady state is thus formed as a linear combination of flux modes, where the contributions of each flux mode is expressed by a generalized parameter.

The third adjustment is the existence of organizations such as stockists that procure raw materials from different alternative sources and supply them to companies further downstream (Figure 3c). In contrast to a manufacturing firm, for which we assume that each of the incoming parts is complementary for the production of the outgoing product, we assume that all incoming products of a stockist (Node i) are identical. Therefore, an additional flux mode is created for each incoming link.

The statistical analysis of stability can be directly applied to large networks due to the computational efficiency of the approach (see *Supporting Information Appendix*). Further analysis of the defining Jacobian matrix allows the *sensitivity* and the *influence* of nodes and links to be computed (Aufderheide, Rudolf, Gross, & Lafferty, 2013). Influence provides a measure of the propensity of disturbances to a focal part of the network to spread into the wider network. In contrast, sensitivity provides a measure of the propensity of disturbances in the wider network to affect a focal part (see the *Supporting Information Appendix* for formal definitions).

4 | RESEARCH DESIGN: REAL-WORLD SUPPLY NETWORK CASE STUDIES

We apply the GM-based stability analysis to two real networks, which provide a valuable cross-case analysis between networks at different levels.

4.1 | Case selection

We are concerned with multi-tier manufacturing supply networks, where our analysis focuses on material flows rather than supplier–buyer contractual agreements, as for instance in Saavedra et al. (2008, 2009). We use theoretical sampling (McCutcheon & Meredith, 1993) to choose two cases with representative but also contrasting network characteristics that can be anticipated to affect supply network stability.

The first case focuses on a manufacturing supply network that is driven by a focal firm and that has high horizontal and vertical complexity. Products such as machinery, computer and electronics, appliances, and transportation vehicles have complex bill-of-material structures. Due to the high degree of outsourcing, their production can be expected to be organized into multi-tier supply networks with high horizontal and vertical complexity, forming our sampling frame. Although such networks are common in practice, their mapping is generally difficult and time-consuming. One potential source of data is the public reporting of supplier–buyer transactions, which is for instance used in Atalay et al. (2011) for network mapping. However, such information is insufficient to study material flows where all flows must end up in the same final product. Such detailed network mapping (e.g., Choi & Hong, 2002) generally requires close collaboration with powerful prime entities that have the required visibility, information, and resources. An ongoing collaboration between the authors and a prime manufacturer of high-value engineered products facilitated access to such data, which formed the basis of the first case selection. The prime manufacturer is a leading multinational organization that produces industrial engines.

The second case focuses on industry-level networks, which involve all firms in a given industry including prime manufacturers, their suppliers, and upper tier suppliers. Industry-level networks are formed by a combination of individual supply networks considered in case one, but with overlapping and shared supply routes. The intertwining of such individual networks is common (Brintrup et al., 2016; Nair et al., 2009) and can be expected to have implications for stability mainly due to competing demands of different prime manufacturers from shared suppliers. Therefore, the second case provides a contrasting example to the first case, that is, a theoretical replication (Yin, 2009). Industry-level networks have been mapped in the literature, for instance, for the automotive industry by

Kito et al. (2014) and Brintrup et al. (2016). In particular, we are interested in industries where all firms involved are exclusively supplying within that sector where the overall network provides a self-contained unit of analysis. Collaboration with a leading insurance provider for supply networks has provided access to such an industry-level network in the luxury goods sector. Different supply networks within this industry are highly intertwined because each prime manufacturer produces similar products in the same product family using many common sets of parts supplied by a common set of suppliers.

4.2 | Data collection and information on the two real-world supply networks

Generalized models of supply networks essentially require information on who is connected with whom, that is, buyer–supplier relationships. These relationships are summarized in an adjacency matrix that is used to set up the generalized model. Additional information on supplier types and link types (e.g., part manufacturers, stockists, assemblers, processing links) helps in subsequent analysis. For this study, these data and general contextual information were collected for the industrial prime network and provided for the industry-level network. The network information was encoded in R files, which were then used for the stability analysis and visualization. The GM method is used principally to generate insights on network stability with network information of this type. We discuss the potential in further work for specific functional forms to be incorporated in a GM approach when further information is available.

We note that the two supply networks have relatively static structures in the short-to-medium term, which constitutes the timescale we are interested in for the stability analysis we conduct. The suppliers participating in these networks are specialists in the relevant manufacturing and processing technologies and the time to acquire and switch to a new supplier is long. This is in contrast to supply networks where there are many alternative suppliers and the switching cost and time may be low and the network structure is more transient, for example, the garment industry networks analyzed in Saavedra et al. (2008, 2009) and MacCarthy and Jayarathne (2013).

4.2.1 | Inbound supply network of a prime manufacturer for an engineered product

The data for the inbound supply network did not exist in the form needed and was collected by the research team in close collaboration with the prime manufacturer. The principal contact person, the Supplier Development and System Purchasing Manager, facilitated data collection as well as access to functions and individuals within the business and their supporting information systems. The primary data collection was carried out mainly

during a one-week site visit by three members of the research team to the prime's main manufacturing assembly facility in 2014. A list of questions was sent to the contact person the week before the visit. The site visit started with a presentation of the research project. The data collection was organized around semi-structured interviews with 13 managers and employees from different business functions including the purchasing and supply, production planning, and finance functions. At the end of each day, a review meeting was held with the contact person to go through the list of questions and schedule extra meetings, where needed. Following the visit, requests for clarification and further data were made by emails and phone calls. Two months after the visit, a site visit report was submitted to the organization for validation. The results of the GM analysis were presented about 12 months later and discussed with the firm.

The data collection covered various, mostly quantitative, aspects of the supply network and operations management practices of the prime manufacturer. Questions asked within interviews covered product information, inventory management policies, questions regarding the supply network including basic information about suppliers and subsuppliers (company name, location, part/process supplied, lead time), performance data, and history of relationships with suppliers. Validity and reliability were ensured by having multiple investigators and multiple sources of information. Access was provided to company documents such as brochures, product data sheets, ERP systems data, performance data, and risk registers with explanations where needed. Meeting notes were held by each member of the research team and a single short report was produced at the end of each day. The mapping of the supply network was achieved with close assistance of managers of the machining and fabrication sub-assembly (see below). Although the key information on first- and upper tier suppliers (company name and location, part supplied or activity done, and connection with other suppliers) was accessible, it was not kept centrally. With the help of the managers, we sought information from employees with responsibility for specific parts, and in a number of cases contacting suppliers for upstream information. Collected information was encoded into fields for company name, location, part supplied or activity done, and connection with other suppliers. Theoretical saturation was reached when all material flow paths for important parts up to the raw material level were covered.

The manufacturer specializes in offering customized engines tailored to specific customer needs. A build-to-order inventory positioning strategy is used because the volume of demand for final products is low and the variety and customization of these products are high. The product build is organized into four different subassemblies: casting, machining and fabrication, engine drive, and systems. The machining and fabrication subassembly supply network was selected for

the analysis. This choice was because the prime manufacturer is the design owner and this supply network requires a high degree of active management to guarantee coordinated supply. The network involves suppliers with different roles (raw material suppliers, part manufacturers, processing suppliers, assemblers, and stockists), which is organized into a multi-tier network. Despite the level of control that the prime can exercise over its suppliers, supply network visibility is still limited because of its extent and depth, its geographical spread, and the limited resources available to monitor beyond the first tier. The information on the performance and the operational policies of upper tiers were not registered or documented and the burden of managing upper tiers was mainly passed onto the first-tier suppliers.

The inbound supply network map (Figure 5) is organized around the prime manufacturer (Node 27). It consists of 55 organizations, six of which are stockists. The organizations are linked by 74 product flows including six processing links. While the prime manufacturer has 17 incoming links, the suppliers at the highest tier have none. In addition to the high horizontal and vertical complexity and degree heterogeneity, the network involves a further aspect: interrelatedness between suppliers, that is, some of the suppliers appear at multiple tiers. For example, the aggregator (Node 1) represents both a tier-two and tier-three supplier, and triadic motifs are present in the network (e.g., formed by Nodes 10, 22, and 27) as well as a pentadic motif (e.g., formed by Nodes 1, 27, 45, 49, and 54). The mean degree (average number of links per node) is 1.3455, the average path length of the undirected network is 3.5434, and the (undirected) clustering coefficient is 0.0268.

4.2.2 | Industry supply network in the luxury goods sector

The data on this case was provided by one of our industrial partners that is an external stakeholder, an insurance provider, making risk assessments of supply networks. The insurance company has policies that provide coverage for supply networks in their entirety. Our contact in the insurance company was the Global Supply Chain Product Leader. There were strong issues of commercial confidentiality related to this case. The communication with the company was organized around two meetings and several follow-up email and phone conversations for clarification that spanned over a time period of more than one year. The first meeting involved an explanation of the research project by two members of the research team, who attended the meeting and discussion of the supply network provided. The second meeting focused on the presentation of the results of the analysis and the discussion of the policy implications of the approach. Data triangulation was provided by supporting

documents, that is, an industry report prepared by a third party and brochures on the supply network insurance products and supply network risk assessment tools. We note that we have relied on the supply network information provided and cross-checking was not possible. Given the use by the organization of the information for risk assessment, it is likely that the information is reliable.

The second case is an industry-level supply network from the luxury goods sector. The network comprises a set of companies that are concentrated in the same geographical region. The network as a whole has a global monopoly for this luxury product, which has a considerably high and growing global demand. The network consists of 36 prime companies, several first-tier and many secondary and tertiary part suppliers, with some being processing suppliers. There is a common trend of vertical integration, where prime companies extend their domain of activities to include more production, by acquisition of suppliers or bringing the production of parts in-house. This is documented by the public records of company acquisitions. There is still a high degree of variation in prime company profiles in terms of their involvement in production and their degree of vertical integration. The extended supply networks of different prime companies significantly overlap, forming a highly intertwined network. Consequently, stock-outs due to unavailability of parts and capacity problems are common and have been identified as a major threat for the sector. Another consequence is the challenge of supply allocation for suppliers with limited capacity that have to balance the supply between multiple competing buyers. This has led to several disputes between companies regarding capacity allocation to customers.

The supply network is shown in Figure 7. It consists of 107 organizations. There are 32 suppliers that either manufacture parts from raw materials or receive basic parts from the external environment. In total, these parts are exchanged along 185 links including 5 processing links. The mean degree is 1.7290, the average path length of the undirected network is 3.4914, and the (undirected) clustering coefficient is 0.0386.

5 | RESULTS AND DISCUSSION

In the following, we first investigate the stability of the two networks introduced in Section 4, compute the influence and sensitivity in both networks, and compare the influence and sensitivity of nodes with measures of network centrality. We discuss the results of the GM analysis of these networks in the context of the literature and formulate five propositions on the dynamic behavior of supply networks. We finish with a discussion and elaboration of the general insights and the theoretical, managerial, and policy implications of the study. We conclude by noting future research directions.

5.1 | Inbound supply network of an engineered product

To compute PSW, we used the reduced parameter range (Table 1), set the flux mode strengths range to $(0, 5]$, and computed PSW for 10^6 parameter instances randomly generated from the ensemble of identical parameters.

The results (Figure 4) show some similarities to the triadic network (Figure 2). However, the overall PSW in the inbound network of the engineered product is lower than in the triadic network. As in the triadic network, PSW increases as the magnitude of the elasticity to inventory level increases and neither the steady-state flow rates nor the inventory levels are major factors influencing stability. Plotted over the elasticity to coproduction range, PSW still peaks at zero elasticity but its decline for positive values is sharper. Thus, stability suffers if interdependencies exist between the production of different parts by a single supplier. PSW remains largely constant across different values of the elasticity to supply. Therefore, adapting the production rate to the inventory levels of parts in this network does not have the stabilizing impact observed for the triadic network.

Going beyond the analysis applied to the triadic network, we also analyze which manufacturers and parts have the strongest influence on network dynamics leading to instability and which are most sensitive to disturbances in the network. To compute these measures, we initialize the system by drawing the generalized parameters from a subset of the reduced range ($f_S \in [0, 0.5]$, $f_I \in [-2, 0]$, $f_C \in [-0.1, 0.1]$, $P^* = 5$), with flux mode strengths in the range $(0, 5]$, in order to increase the number of stable systems so that we have a sufficiently high number of cases where the network is at the onset of instability.

The analysis reveals that the most influential and sensitive organization is the prime manufacturer (Node 27), closely followed by a stockist (Node 2) that is of comparable importance for the network (see Figure 5). These organizations are also the nodes with highest degrees (Figure A2 in Appendix A). In addition, the influence of upper tier organizations tends to be less than their sensitivity to disturbances. In contrast, the influence of lower tier organizations tends to be greater than their sensitivity. We observe that the outgoing links from stockists (e.g., Node 2) tend to be highly influential. The incoming parts and the suppliers of these parts for stockists are of lower influence but of higher sensitivity compared to other parts flowing from suppliers to manufacturers at the same tier. Furthermore, we observe that raw material suppliers who supply to more than one buyer in the system (e.g., Nodes 8 and 33) have high sensitivity. Conversely, the processing nodes (Nodes 13, 29, and 38) that can be identified from bidirectional links have larger influence but smaller sensitivity than others with similar degree. We denote these as “processing nodes” because of the nature of the flows they receive.

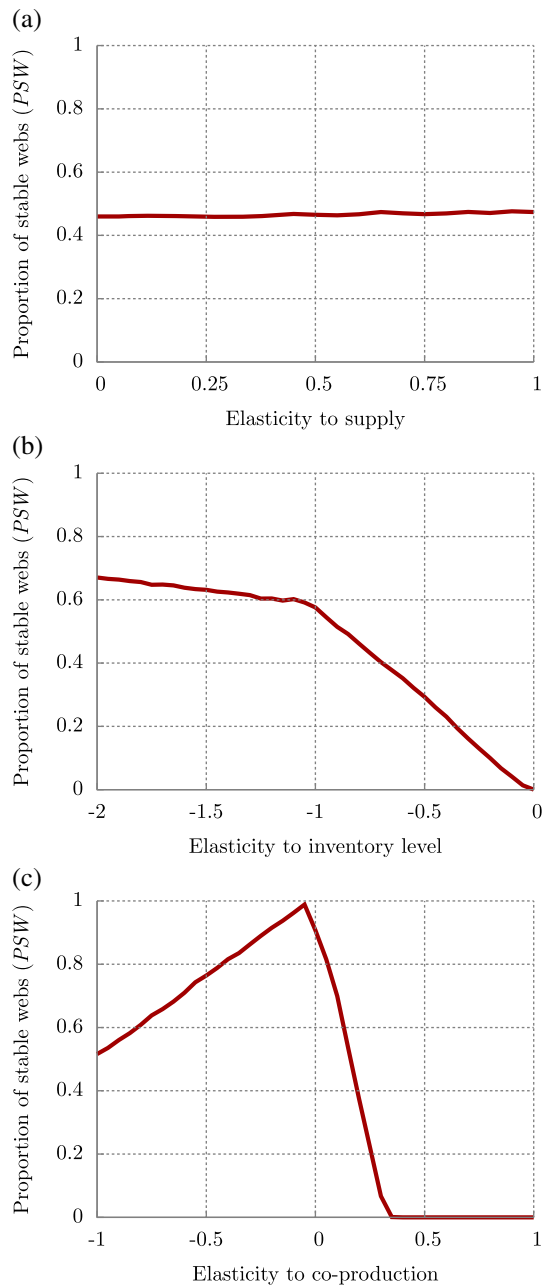


FIGURE 4 PSW for the inbound supply network of an engineered product [Color figure can be viewed at wileyonlinelibrary.com]

5.2 | Industry supply network in the luxury goods sector

We use the same ensemble with the same parameter ranges used in Section 5.1. The results for PSW (Figure 6) show similarities to the inbound supply network for the engineered product. However, PSW for the industry network declines with increasing elasticity to supply, albeit this is a relatively weak effect. Hence, the impact of elasticity to supply changes from being stabilizing in the triadic network to being destabilizing in the industry-level network. The extended inbound

network of a single industrial manufacturer is an intermediate case in which elasticity to supply does not have a significant effect on stability.

Figure 7 shows that the most highly connected organization (Node 41) in the industry-level supply network is both the most sensitive and the most influential. Note that in the network visualization in Figure 7, organizations are aligned according to their degree, that is, organizations with higher degree are closer to the center. These central (high degree) organizations tend to be both influential and sensitive. Beyond the effects of node degree, we observe network structural effects that also existed for the inbound network of the prime manufacturer. For instance, organizations with processing links (e.g., Nodes 22 and 79) tend to be more influential and less sensitive than organizations with a similar degree (e.g., Nodes 57 and 74), which is discussed further in regard to a generic proposition presented below (Proposition 1). A general observation for both networks is that upper tier organizations tend to be more sensitive than influential, although, both their sensitivity and influence are relatively low. Lower tier organizations are in general more influential than sensitive, with both influence and sensitivity values being relatively large.

5.3 | Comparison of influence and sensitivity with measures of network centrality

While network centrality measures quantify importance purely from a structural point of view, the influence and sensitivity measures quantify the importance of nodes under dynamic conditions and account for both network structure and flows on the network. We compare the two classes of measures in order to gain insights on the interaction between the network structure and network dynamics. We consider degree, closeness, and betweenness centrality measures, which are also used in the supply chain management literature in the context of risk assessment (see Section 2.2).

The results summarized in Table 2 and plotted in Figure A1 in Appendix A show that the correlations of all measures are reasonably high overall. Both influence and sensitivity are very highly correlated with degree and betweenness centrality measures for the inbound supply network, indicating that high-degree nodes and nodes located on shortest paths (the stockist and the prime company) are highly influential and sensitive, while nodes that are not central with respect to these aspects do not induce or suffer from instability to the same extent in this network.

The correlation is lower for the industry-level network, which comprises a collection of individual supply networks. While the correlation with degree and betweenness centrality decreases in comparison to their values for the inbound network, the closeness centrality correlation is similar for this

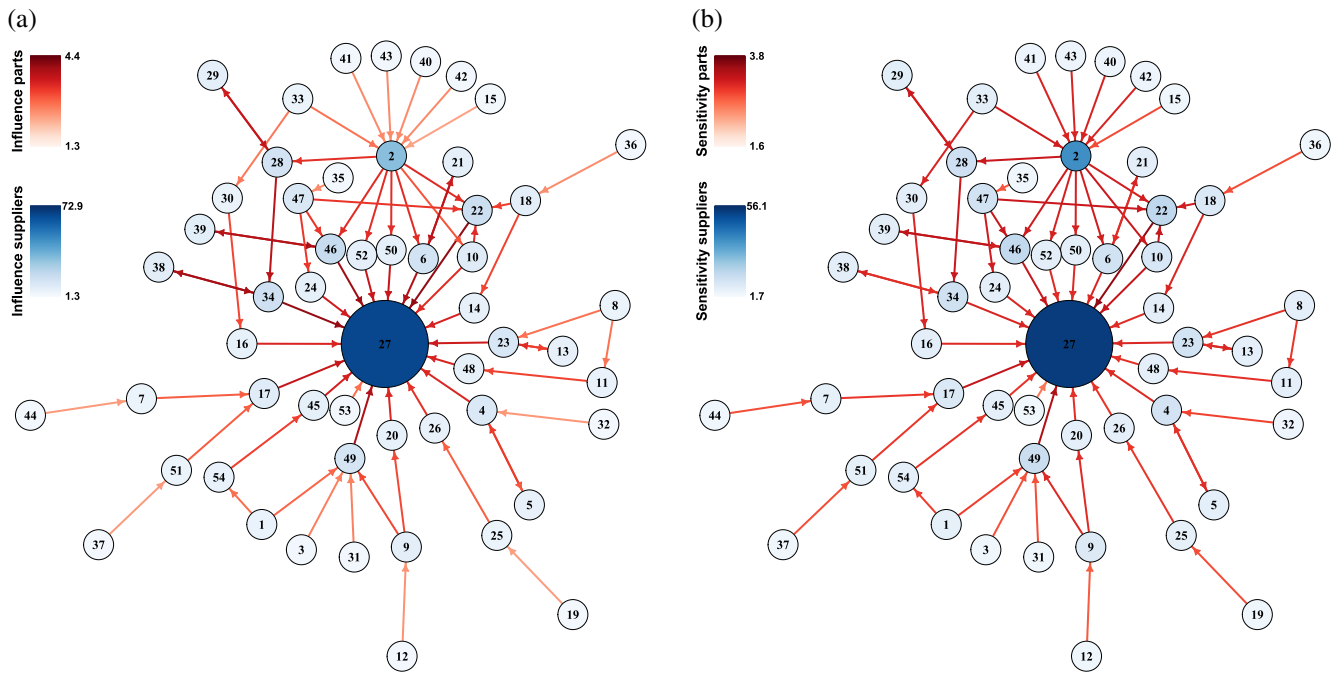


FIGURE 5 Influence and sensitivity of parts and suppliers in the inbound supply network of an engineered product. (a) Influence of supplier (blue) and parts (red) on instability. (b) Sensitivity of suppliers and parts toward perturbations. Node 55 (external market) omitted from figures for clarity [Color figure can be viewed at wileyonlinelibrary.com]

network. Investigating these patterns in more detail, more insights are evident. For the industry-level network, degree centrality identifies Node 41 as an outlier. However, GM identifies Nodes 22, 76, and 79 also as being highly influential (Figure A3 in Appendix A). From our analysis and discussion in Section 5.2, it is evident that the sensitivity and influence of Nodes 22 and 79 are increased by the processing link between them. Thus links, especially processing links, to a node with high influence and sensitivity, may increase the sensitivity and influence of the node itself. This also explains the high influence and sensitivity of Node 76.

The analysis confirms that node-based centrality metrics may explain the spread of disturbances in supply networks to some extent. In particular, high-degree centrality corresponds to a firm receiving many types of parts, processing/assembling products, and/or delivering the output products to many customers. Considering that managing each of these connections implies a burden for a focal organization, degree centrality may be interpreted as a measure of operational load in supply networks (Kim et al., 2011). Our results show that the node degree is correlated with the sensitivity to disturbances, which is consistent with the interpretation of degree as capturing operational load in supply networks. Problems arising in high-degree nodes can be expected to spread to the rest of the network. This is consonant with the impact of high-degree nodes, that is, hubs, on epidemic spreading in complex networks (Pastor-Satorras, Castellano, Van Mieghem, & Vespignani, 2015) and network resilience

(Albert et al., 2000; Kim et al., 2015). In both networks studied, high-degree centrality indicates high influence and sensitivity. However, the dynamic measures of influence and sensitivity that capture flows on a network provide more subtle and important insights for the onset and impact of instability in supply networks that we discuss below.

5.4 | Identifying organizations, network motifs, and operations policies critical for network stability

Analyses of the inbound supply network of the industrial engine manufacturer and the industry-level supply network in the luxury goods sector highlight specific features of organizations and specific network motifs, that is, specific subgraphs that define patterns of interactions between nodes, that are important for stability and that are influential for, and/or sensitive to, the spreading of disturbances when instabilities emerge. Sensitive organizations and sensitive links between organizations are highly likely to be affected by disturbances in the network. Influential organizations and influential links between organizations are highly likely to trigger problems elsewhere in the network.

We observe in both networks that links with suppliers we denote as “processing suppliers,” which receive, process, and return subassemblies (e.g., suppliers that perform operations such as painting and machining), are characterized by high influence but low sensitivity. Disturbances emanating from

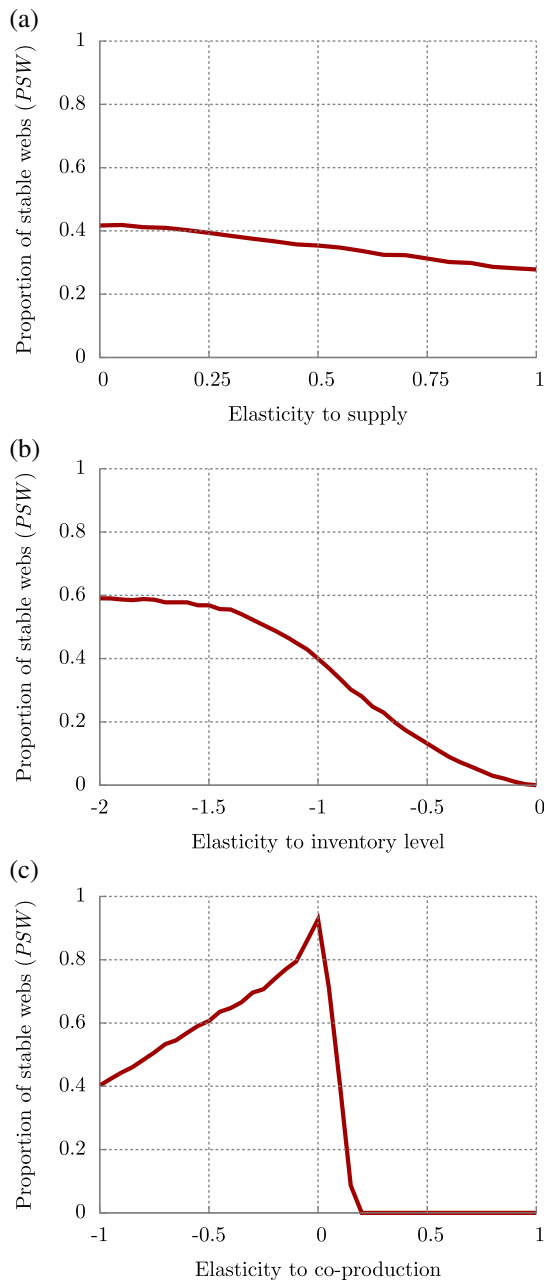


FIGURE 6 PSW for the industry supply network in the luxury goods sector [Color figure can be viewed at wileyonlinelibrary.com]

these links will quickly affect the material flow even if such processing suppliers themselves might have low degree. Such nodes prove to be disruptive, if instability originates within that processing link. Due to their low sensitivity, processing suppliers are more likely to withstand instabilities originating elsewhere. Although the disturbances elsewhere will eventually affect them, they are not particularly sensitive. This is supported by the experience of our industrial collaborator with the disruptions caused by suppliers that perform operations such as machining or painting at different tiers of the supply network being common. The above discussion leads us to formulate Proposition 1.

Proposition 1 *Disturbances originating in links with processing suppliers that receive, process, and return subassemblies tend to influence the flows in the rest of the network strongly but such suppliers are likely to withstand disturbances originating elsewhere in the network.*

The GM analysis provides a valuable initial screening method to identify a set of suppliers for scrutiny and potential supplier development and/or the need for network redesign to achieve stronger vertical integration. Highly sensitive organizations that are strongly affected, resulting in a surplus or lack of products, can be advised to invest in flexibility, extra buffers, and resource redundancy (e.g., Kamalahmadi & Parast, 2016 for related approaches). Influential organizations may need strong, active monitoring and management by prime entities because the problems they face may trigger risks throughout the network. Processing suppliers and organizations that are highly connected, for example, stockists and assemblers, constitute such examples. Considering that influential suppliers may be positioned deep in the network and may lack the necessary resources to cope with disturbances, the prime network entities need to ensure such nodes are visible, apply a strategic supply network development approach and, where appropriate, invest in developing their capabilities before problems arise (Krause, Handfield, & Scannell, 1998). Vertical integration of such suppliers may also be considered, which is a trend noted in the context of the luxury goods sector analyzed.

Network properties such as the level of nestedness and compartmentalization have been shown to impact stability in ecological networks (Bascompte & Stouffer, 2011; Rohr et al., 2014; Saavedra et al., 2011). Specific types of network motifs may strongly influence stability of the larger networks to which they belong, irrespective of the dynamical properties of the rest of the system (Aufderheide et al., 2012; Zhigulin, 2004). In this study, we observe in particular the destabilizing role of cycles in supply networks. Elasticity to coproduction captures the interdependence between the rates of production of parts manufactured by a single organization. As the absolute value of the elasticity to coproduction increases, the likelihood of stability decreases in all the three networks studied. If the same organization produces multiple parts that end up in the same final product through assembly processes further downstream in a supply network, this leads to a cyclic motif. Hence, the elasticity to coproduction is closely linked to cycles. The real-world supply networks analyzed here contain an abundance of cycles, a feature which has also been noted for some other supply networks reported in the literature (Brintrup et al., 2016; Lomi & Pattison, 2006). In particular, there is an increased awareness of triadic relationships in supply networks (Choi & Wu, 2009). In our GM analysis, we observe the potentially detrimental effects such relationships have on network stability, which

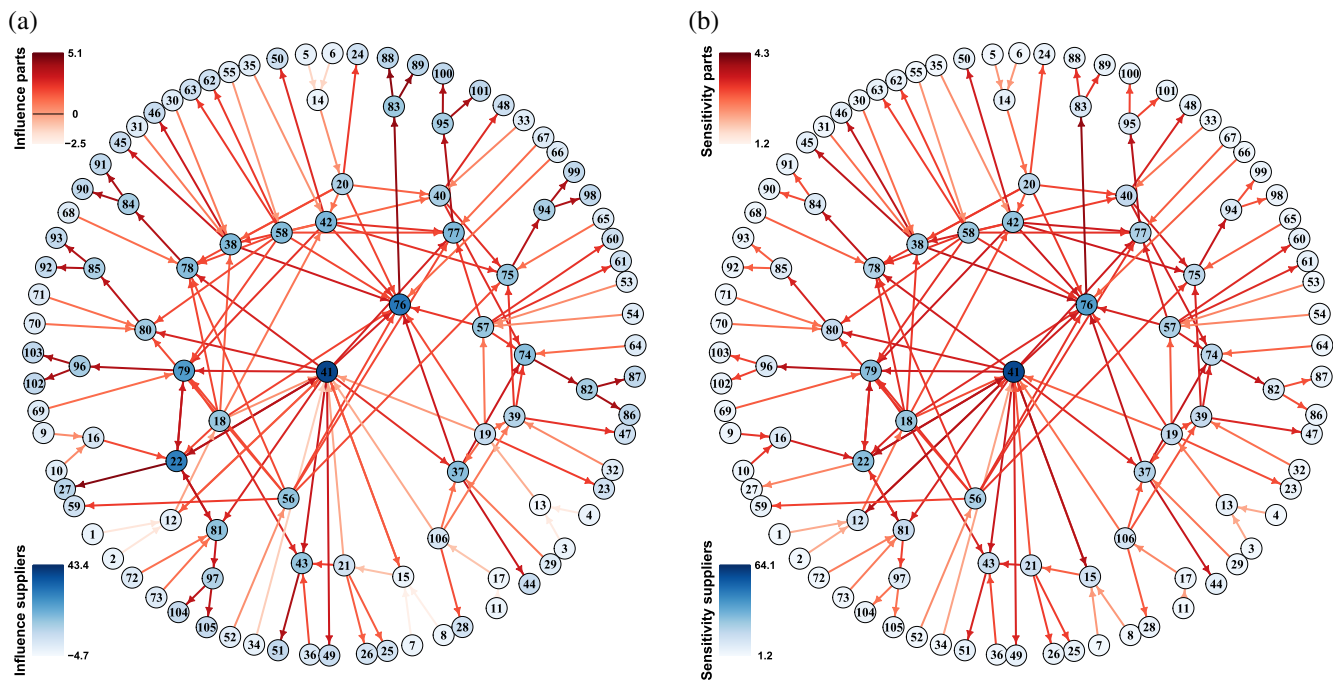


FIGURE 7 Influence and sensitivity of parts and suppliers in the industry supply network in the luxury goods sector. (a) Influence of supplier (blue) and parts (red) on instability. (b) Sensitivity of suppliers and parts toward perturbations. Node 107 (external market) omitted from figures for clarity [Color figure can be viewed at wileyonlinelibrary.com]

agrees with the anecdotal experience of our industrial partners (Section 4.2.1) that triadic supply relationships create coordination problems. This occurs because the common supplier at the apex of a cyclic motif needs to balance the capacity demands from multiple customers, and thus the coproduction elasticity affects stability even if they are for different end products, as in the industry-level network studied. This can be more generally interpreted as competition for limited resources. Investigating socioeconomic systems in a context where individual countries compete for limited financial resources, Saavedra et al. (2014) show that network stability decreases with the level of competition. Our results show similar insights about the impact of competition for limited resources in a manufacturing supply network context. This can in particular explain the source of the decreased stability for the industry-level network in comparison with the inbound supply network of an engineered product for a single manufacturer. These observations lead us to formulate Propositions 2 and 3.

Proposition 2 *Cyclic motifs, which involve two separate paths of material flows from a common supplier at the apex of the cycle to a common firm further downstream, destabilize material flows.*

Proposition 3 *When different end-product streams share a common supply network, competition between them has a destabilizing effect on material flows in the network.*

These propositions highlight the importance of effective capacity planning and management in supply networks, which is particularly challenging for a disparate industry-level network in comparison to a coordinated prime-driven network of a large industrial player. Furthermore, they point supply chain managers to ways of identifying suppliers critical for stability due to capacity constraints. Suppliers that are located at the apex of cyclic network motifs and those suppliers providing parts for multiple prime manufacturers or

TABLE 2 Correlation between classical network measures and sensitivity and influence for the inbound supply network (SN1) and industry supply network (SN2)

Correlation	Sensitivity SN1	Influence SN1	Sensitivity SN2	Influence SN2
Betweenness centrality	0.930	0.953	0.572	0.363
Closeness centrality	0.652	0.689	0.643	0.693
Degree centrality	0.998	0.983	0.630	0.461

multiple retailers are critical for stability. Sufficient capacity needs to be in place for these suppliers to minimize the interdependence between production rates of different products, thereby removing the need for compromise between different prime entities. Considering that many real supply networks are highly intertwined (Atalay et al., 2011; Brintrup et al., 2016), this constitutes a major challenge for stability. For instance, the disruption in the supply of a specific paint pigment from a common supplier deep in the network led to an industry-wide cascade in the automotive industry (Park, Hong, & Roh, 2013). Strategies such as strong aggregate capacity planning processes, multisourcing, and investment in capacity “cushions,” may be advocated to address or mitigate the effects of such instabilities.

Elasticity to supply captures the effect of the inventory level of parts on the production rate. An elasticity value of zero corresponds to the case where desired production orders can be satisfied with parts available in stock, while a positive value corresponds to limited part availability that constrains production. We find that limited part availability has a positive effect on the likelihood of stability for the triadic network. However, the results for the industry-level network indicate a negative impact. The contrast in these effects supports the nontrivial impact of the product limitation on stability conditions reported in Venkateswaran and Son (2007), although we note a fundamental difference in the contexts. We consider a supply network with product assembly and instantaneous flows while Venkateswaran and Son (2007) consider a single manufacturer with multiple production stages and material delays. Venkateswaran and Son (2007) compare the stability conditions under limited versus unlimited product availability, showing that an otherwise stable state can be destabilized by the strain due to product availability constraints, while the opposite is also possible. Hence, the stability implications of product availability constraints are nontrivial. This also resonates with a finding from ecology: a factor that is stabilizing for small food webs can become destabilizing for larger food webs (Gross et al., 2009). This leads us to formulate Proposition 4.

Proposition 4 *Constraints on part availability may be stabilizing for small isolated networks for a single prime entity but may destabilize industry-level supply networks that are comprised of multiple intertwined product flows.*

If systemic material shortages exist at an industry level, supply networks may be prone to instability and potentially to collapse. Such industry level instabilities may ultimately result in network disintegration and/or degeneration to smaller and less intertwined networks. The industry-level network investigated here was found overall to be less stable than the inbound network, which in turn is less stable than

the triadic network. This echoes the findings in ecology that larger food webs are more prone to instability and require special structural and functional properties to ensure stability (May, 1972; May, 2013; McCann, 2000). Clearly, we need to be cautious in inferring the topological causes or predictors of instability in supply networks. However as a testable proposition, it signals a direction for further empirical studies of real networks (of which there are still only a limited number reported in the literature).

In the analyses, we find that the steady-state flow rates and inventory levels do not have a strong direct effect on network stability. This is consistent with findings in ecology (Gross et al., 2009) that turnover parameters do not have a significant effect on stability. In inventory control models, the conditions that lead to the loss of stability have been shown to be independent of the steady-state flow and inventory levels (e.g., Warburton, 2004). Thus, we argue that the buffering role of inventories does not strongly impact the onset of instabilities. However, buffer inventories may clearly minimize the impact of disturbances and disruptions once they occur. This is in agreement with the lean perspective of inventories, that is, the existence of inventory may mask fundamental operational problems. However, we do observe that stability is influenced by the dependence of the material flow on the inventory level, that is, elasticity to inventory level, which captures how quickly the production is adjusted to cover for deviation from desired inventory levels. This stabilizing effect of the elasticity to inventory level results from the self-limitation of production, that is, the more that is produced the less the need to produce more. This is in agreement with the known effect that negative feedback loops are stabilizing in dynamical systems (Sterman, 2000). However, this observation should be interpreted with caution given the nongranular level at which we study network behavior. Here, we do not capture delay induced instabilities, which could counteract the stabilizing role of elasticity to inventory level. Based on these observations, we formulate Proposition 5.

Proposition 5 *The steady-state level of inventory and material flows do not strongly impact the onset of stability. The more quickly the production rate is adjusted to account for changes in the inventory level, the more likely the supply network is to be stable.*

Propositions 4 and 5 explain the impact of part availability and inventory abundance on supply network stability. Supply networks need to be monitored for deviations from their designed or intended states (Ivanov & Sokolov, 2013; Ivanov, Sokolov, & Kaeschel, 2010). If such deviations occur, for instance due to changes in demand or due to disruptions, the control policies need to be adapted to ensure

stability, for example by more promptly adjusting production rates or by elevating the significance of part limitation constraints by working closely with existing suppliers or using backup suppliers. However, in supply networks, such a response may be difficult to automate as may be possible in some engineered systems and requires active managerial involvement (Ivanov & Sokolov, 2013). The GM approach enables the stability implications of a perceived change in some part of the system to be quickly investigated, which can facilitate adaptive management decisions to account for stability. In particular, within a software implementation of the methodology, it is straightforward to rerun an analysis on a reconfigured network to assess the benefits or disbenefits of adaptive strategies. For example, the effect on network vulnerability of adding dual sourcing to a critical supply line, or cutting out an existing aggregator, can be assessed.

5.5 | Contributions to theory, managerial and policy implications, and further research directions

We have applied a methodology emanating originally from ecology, GM, to study supply network stability. We have shown how GM concepts and constructs can be defined, applied, and interpreted in a supply network context. We introduce material flow related elasticity concepts to the supply chain management domain, which is consonant with their widespread use in economic theory (Nievergelt, 1983). The approach goes beyond purely structural network analysis approaches (e.g., Bellamy & Basole, 2013; Kim et al., 2011) by incorporating not just network structure but also the material flows on supply networks. This is significant because material flow is a defining characteristic of such systems.

Through the application of contemporary dynamical systems theory and network science concepts and tools, we generate quantitative information and indicators that can help direct improvements in a supply network. The approach is relatively parsimonious in the required input information and can therefore be applied in many practical contexts, where only limited resources can be invested in supply network monitoring. The method can be used as a triage approach to identify hidden potential vulnerabilities in a network about which a focal organization might not otherwise be aware.

The study provides new insights contributing to theoretical understanding of supply networks. Propositions 1-5 provide testable relationships for future investigation to underpin and further develop the understanding of supply network stability. Propositions 1-3 provide indicators to identify hidden vulnerabilities to instability in supply networks, highlighting the criticality of links with suppliers that involve the dispatch, processing and return of parts or sub-

assemblies, cyclic motifs that involve separate paths from a common supplier to a common firm downstream, and competing demands of different end products at specific nodes. Such “rules of thumb” are particularly valuable in focusing the attention of managerial decision makers on supply network stability, given the widespread use of such heuristics in practice (Katsikopoulos, 2011). The approach supports strategic supply network development activities and capacity planning at the supply network level. Propositions 4 and 5 provide guidelines on the impact of part availability limitations and inventory control policies on supply network stability, which can be used within an adaptive management system. Proposition 4 highlights that lessons learned from small networks may not be transferable to extensive supply networks. This is a significant observation given the trends of elongation and increasing complexity of supply networks (Maruchek, Greis, Mena, & Cai, 2011).

The GM methodology provides a platform for systemic risk management of supply networks. There are clear benefits of such approaches for prime focal companies, such as large industrial firms that seek to design, manage, and control their supply networks, and also large retailers and brand owners that are reliant on globally dispersed supply networks (MacCarthy & Jayarathne, 2013). Furthermore, there is a growing supply chain insurance market, where insurers seek to understand and quantify their risk exposure. Policy makers in government and industry bodies are also interested in understanding the vulnerabilities of critical supply networks that contribute to employment and economic growth. The network approach developed here is consistent with the risk management agenda proposed for financial networks (Haldane & May, 2011). In this context, our approach provides an effective and efficient method that offers practical help to organizations seeking to design, manage, or risk-assure their supply networks.

The approach has significant potential for further study of adaptation in supply networks, particularly with respect to planning, control, and governance mechanisms (Choi et al., 2001; Ivanov et al., 2010; Pathak et al., 2007). It enables the impact on stability of a wide spectrum of policies adopted by supply network members that are typically beyond the reach or influence of a focal organization to be investigated. Experimentation with generalized elasticity parameters can provide insights on network stability characteristics under different policies, indicating whether and where the adaptation of plans and possibly restructuring of a network is required.

The method can be further developed to enable decision support and may be augmented with tools from network visualization, data and statistical science, and decision science. There are opportunities for further refinement of the GM approach, particularly in tailoring and calibrating the parameters for specific contexts. Developing models that

incorporate work-in-process and transport inventories to capture the impact of material delays is a natural step forward but requires hurdles of model size and model tractability to be overcome. Generalized models can also be combined with more granular and specific modeling approaches, such as control theory, agent-based modeling, and discrete-event simulation, enabling more detailed information for critical suppliers to be incorporated, if available.

The computational efficiency of GM makes it possible to process large libraries of different network topologies to identify robust general network design rules. We draw an analogy with ecology. Evolution tends to produce ecosystems that are stable against perturbations and are thus robust and resilient. The analogy with the desire of industry to design and operate robust and resilient supply networks is too obvious to ignore. Constructing metrics, including vulnerability indices, to quantify the stability of a supply network to different potential risks is a potential area for development. Such metrics can be aided by the GM framework presented. A relevant area for further research is the study of supply network resilience to catastrophic failures (e.g., Chopra & Sodhi, 2004), which could be studied by analyzing the leading eigenvalue of the generalized Jacobian matrix that characterizes the recovery time.

Finally, the ecology literature presents interesting findings that may have further implications for supply network stability. For instance, the variability in link strengths has been shown to have a strong impact on the stability of ecological networks (Bascompte et al., 2006; Gross et al., 2009; McCann, 2000). The investigation of whether such phenomena hold in supply networks is a promising area for further work.

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SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of this article.

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APPENDIX A. | FIGURES FOR THE COMPARISON OF SENSITIVITY/INFLUENCE WITH CLASSICAL NETWORK MEASURES

Figures A1, A2, and A3 below are noted in Sections 5.1 and 5.3.

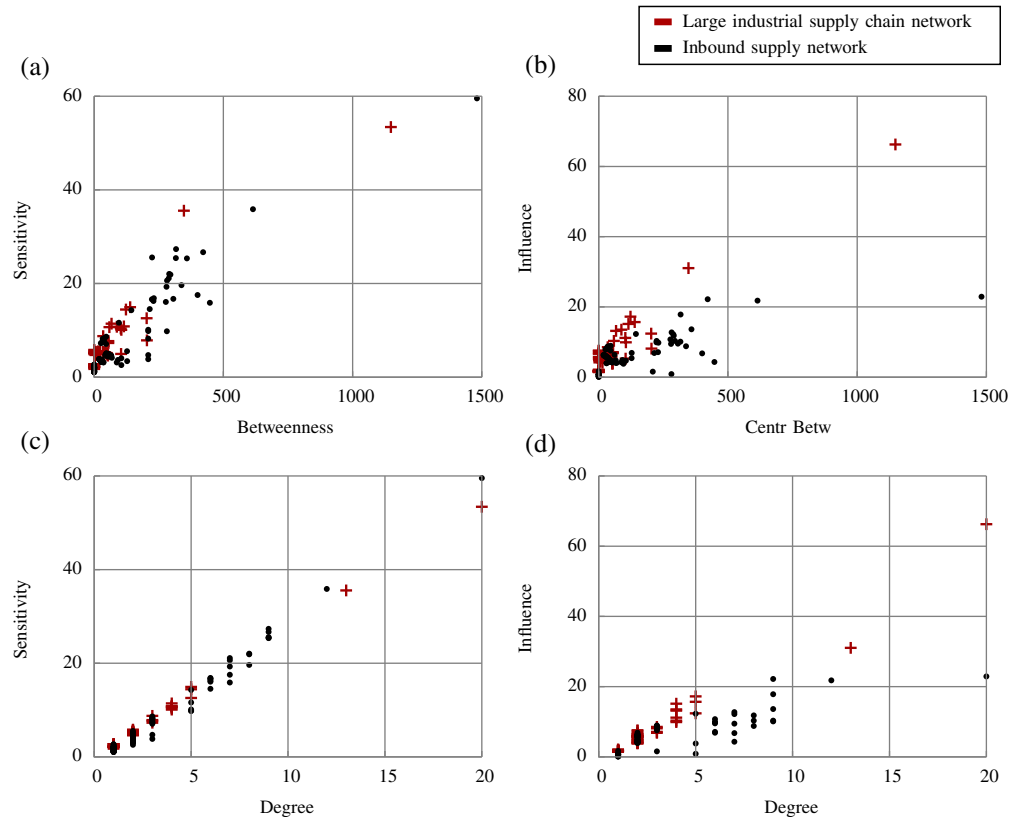


FIGURE A1 Relation between sensitivity/influence of nodes and classical network measures for the large industrial supply chain network and the inbound supply network [Color figure can be viewed at wileyonlinelibrary.com]

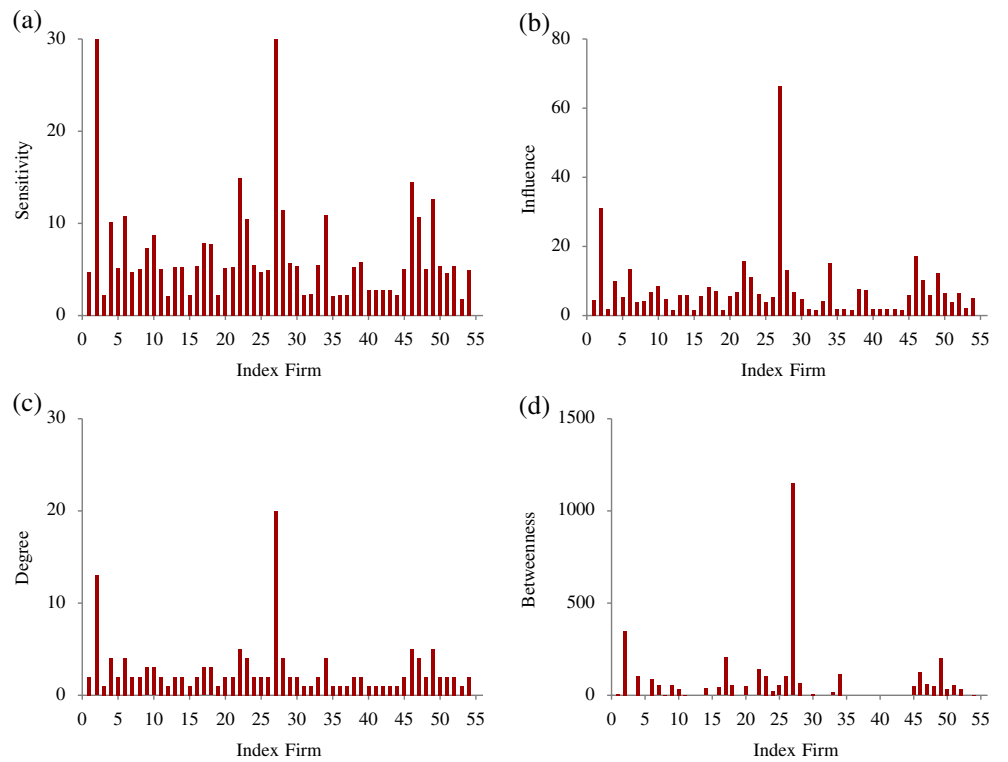


FIGURE A2 Comparison between sensitivity/influence and most significant network measures (degree and betweenness centrality) for each node in the inbound supply network [Color figure can be viewed at wileyonlinelibrary.com]

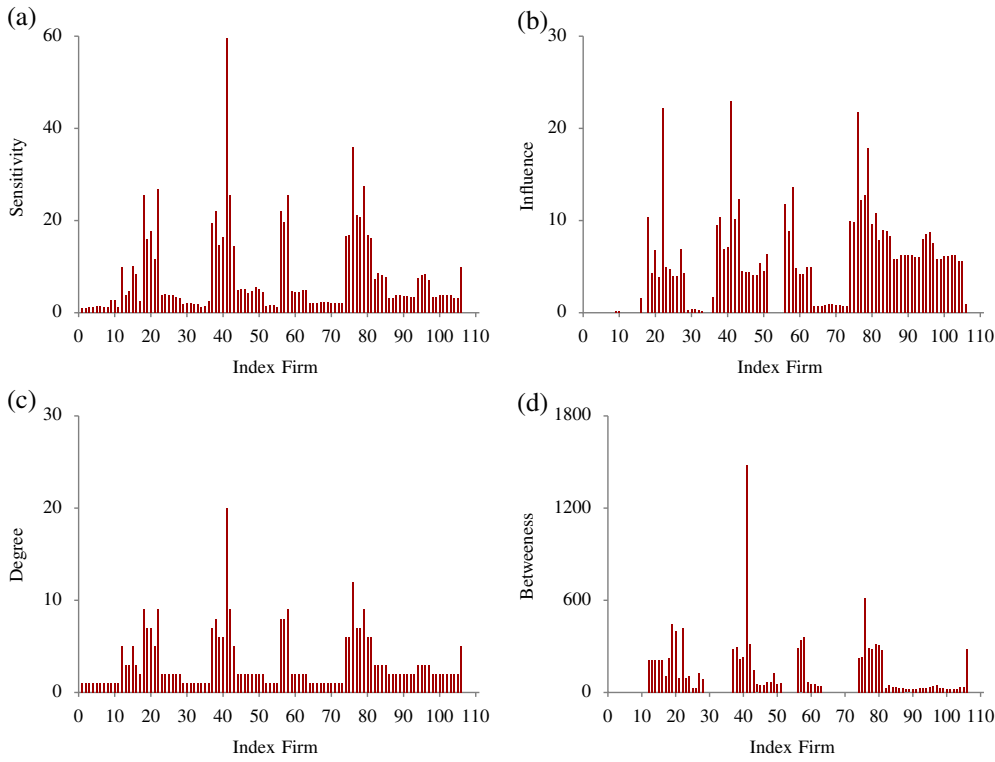


FIGURE A3 Comparison between sensitivity/influence and most significant network measures (degree and betweenness centrality) for each node in the large industrial supply chain network [Color figure can be viewed at wileyonlinelibrary.com]