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A Theory of Repurchase Agreements, Collateral Re-use, and Repo Intermediation*

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Abstract

We show that repurchase agreements (repos) arise as the instrument of choice to borrow in a competitive model with limited commitment. The repo contract traded in equilibrium provides insurance against fluctuations in the asset price in states where collateral value is high and maximizes borrowing capacity when it is low. Haircuts increase both with counterparty risk and asset risk. In equilibrium, lenders choose to re-use collateral. This increases the circulation of the asset and generates a "collateral multiplier" effect. Finally, we show that intermediation by dealers may endogenously arise in equilibrium, with chains of repos among traders.

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1 Introduction

Gorton and Metrick (2012) argue that the financial panic of 2007-08 started with a run on the market for repurchase agreements (repos). Lenders drastically increased the haircut requested for some types of collateral, or stopped lending altogether. This view was very influential in shaping our understanding of the crisis.[1] Calls for regulation quickly followed.[2] The mere possibility that a run on repos could lead to a financial market meltdown speaks to their importance for money markets. Overall, repo market activity is enormous. Recent surveys estimate outstanding volumes at €5.4 trillion in Europe while calculations vary from \$3.8 trillion to \$5.5 trillion for the U.S.[3] Repos are simple financial instruments used to lend cash against collateral. Repos allow borrowers to carry out leveraged purchases of assets, which are pledged as collateral to obtain cash, or to borrow securities. The main users of repos are large dealer banks and other financial institutions such as money market funds and hedge funds. For these reasons, repo markets have important implications for market liquidity, as Brunnermeier and Pedersen (2009) illustrate. Dealer banks also play a major role as repo intermediaries between cash providers and cash borrowers. Finally, most major central banks implement monetary policy using repos, thus contributing to the size and liquidity of these markets.

Technically, a repo contract is the sale of an asset combined with a forward contract that requires the borrower to repurchase the asset from the lender at a future date for a pre-specified (repurchase) price. The lender requires a haircut defined as the difference between the selling price in a repo and the asset's spot market price. Besides the haircut, a repo differs from a sale of an asset followed by a re-purchase of the same asset in the spot market at a later date because the repo price is pre-specified. While a repo looks very much like a simple collateralized loan, it has two additional and important features. It is a recourse loan and the borrower sells the collateral rather than merely pledging it.[4]

[1] Subsequent studies by Krishnamurty, Nagel, and Orlov (2014) and Copeland, Martin, and Walker (2014) have qualified this finding by showing that the run was specific to the large bilateral segment of the repo market.

[2] See for example Acharya and Öncü (2013) and FRBNY (2010).

[3] The number for Europe is from the International Capital Market Association (ICMA, 2016). The two figures for the US are from Copeland, Davis, LeSueur, and Martin (2014) and Copeland, Davis, LeSueur, and Martin (2012) where the latter adds reverse repo. These numbers are only estimates because many repo contracts are traded over the counter and thus difficult to account for.

[4] The General Master Repurchase Agreement (GMRA, 2011) used mostly by US dealers and the Master Repurchase Agreement (MRA, 1996) used for non-US repos lay out the provisions related to a

The lender thus acquires the legal title to the asset sold and so the possibility to re-use the collateral before the forward contract with the borrower matures.

Repos, as well as the practice to re-use the collateral, known as re-use or re-hypothecation, have attracted a lot of attention from economists and regulators alike.[5] However, a proper understanding of the motivation of traders to enter repos and of the implications of collateral re-use is still needed. We propose a theory of repos that accounts for the basic features of these contracts to answer the following questions. Why are repos used over spot sales of the asset? How does collateral re-use impact leverage in the economy? Finally, why are repos intermediated, i.e. why do borrowers trade through dealer banks rather than directly with lenders and how do dealer banks fund their operations? Our model thus provides a basis to understand repos' potential contribution to systemic risk.

In this paper we analyze a simple competitive economy where some investors have funding needs, but are unable to commit to future payments. To satisfy their needs, they can use the assets they own, by either selling them in the spot market or in repo sales. Repo sales are characterized as loan contracts exhibiting the key features of repos described above. We show that in equilibrium investors prefer to trade repos rather than

default event. In what follows, we briefly outline the recourse features of repo transactions, as summarized by ICMA (2013), the leading trade association for repos. After determining the market value of the collateral, all repo exposures between the two counterparties are netted off. Then, *"whoever owes the residual sum must pay it by the next business day. Either party can be charged interest on late payment."* Hence, the lender has recourse to the balance sheet of the borrower since he can claim any payment due in excess of the market value of the collateral. We embed this recourse loan feature in our model in Section 2. Appendix A provides more details about the default provisions in master agreements for repos and the full documents for these agreements can be found at http://www.sifma.org/services/standard-forms-and-documentation/mra,-gmra,-msla-and-msftas/

[5] Aghion and Bolton (1992) argue that securities are characterized by cash-flow rights but also control rights. Collateralized loans grant neither cash-flow rights nor control rights over the collateral to the lender unless the counterparties sign an agreement for this purpose. As a sale of the asset, a repo automatically gives the lender full control rights over the security as well as over its cash-flows. Re-use rights follow directly from ownership rights. As Comotto (2014) explains, there is a subtle difference between US and EU law however. Under EU law, a repo is a transfer of the security's title to the lender. However, a repo in the US falls under New York law which is the predominant jurisdiction in the US. *"Under the law of New York, the transfer of title to collateral is not legally robust. In the event of a repo seller becoming insolvent, there is a material risk that the rights of the buyer to liquidate collateral could be successfully challenged in court. Consequently, the transfer of collateral in the US takes the form of the seller giving the buyer (1) a pledge, in which the collateral is transferred into the control of the buyer or his investor, and (2) the right to re-use the collateral at any time during the term of the repo, in other words, a right of re-hypothecation. The right of re-use of the pledged collateral (...) gives US repo the same legal effect as a transfer of title of collateral."* To conclude, although there are legal differences between re-use and rehypothecation, they are economically equivalent (see e.g. Singh, 2011) and we treat them as such in our analysis.

spot. Furthermore, in equilibrium investors exercise the option to re-use collateral. This expands the borrowing capacity of investors in the economy through a multiplier effect. Collateral re-use also affects the structure of the repo market: intermediation by safer counterparties, who use repos to fund their purchase of assets, may endogenously arise.

The model features two types of risk averse investors, cash poor investors (natural borrowers) and cash rich investors (natural lenders). Borrowers own an asset, whose future payoff is uncertain. A large variety of possible repo contracts, characterized by different values of the repurchase price, are available for trade. Due to borrowers' inability to commit, they may choose to default on these contracts. The punishment for default is the loss of the asset sold in the repo together with a penalty reflecting the recourse nature of repos and which varies with the borrower's creditworthiness. Hence there is a maximal amount that borrowers can credibly promise to repay, that depends on the future market value of the asset. The recourse nature of repo contracts implies that this maximal amount may exceed the future spot market price of the asset. This amount and the quantity of the asset held by investors determines then their borrowing capacity with a repo sale.

Lenders can re-use the collateral they acquired via repos, e.g. by selling it in the spot market. By returning collateral to the market, re-use allows borrowers to purchase more assets to pledge them again in repo sales to lenders. We find that allowing re-use, through the iteration of these transactions, generates a collateral multiplier effect, thus augmenting the borrowing capacity of borrowers. Hence, the benefits of re-use materialize when the asset is scarce.

We then characterize the repurchase price of the repo contract that investors choose to trade in equilibrium. Risk-averse investors value the ability to borrow but dislike fluctuations in the future asset price. Hence, two motives – a hedging and a borrowing motive – determine the equilibrium repurchase price. In the states where the market value of the asset is low, the ability to borrow is limited. There, the borrowing motive prevails and the repurchase price equals the maximal amount that borrowers can promise to repay. In the other states, where the asset price is high, their borrowing capacity is also high. Hence investors are not constrained and the hedging motive implies that the repurchase price is set at a level that ensures a constant level of consumption. In the absence of re-use, the repurchase price would thus be set at a constant level. When lenders re-use collateral they effectively sell the asset short in the spot market, and are thus exposed to asset price risk: we show that the equilibrium repurchase price also offsets this price exposure of lenders. These hedging and borrowing motives explain why investors prefer repo contracts over spot trades.

We derive comparative statics properties for equilibrium haircuts and liquidity premia. Haircuts increase when collateral is more abundant or when counterparty quality decreases, because riskier borrowers can credibly promise to repay lower amounts. We also show that riskier assets command higher haircuts and lower liquidity premia, since higher risk entails a worse distribution of collateral value across states relative to collateral needs. The effect of collateral re-use on haircuts and liquidity premia is ambiguous. On the one hand, re-use increases the amount of the asset that can be pledged as collateral and hence relaxes the borrowing constraint. This tends to decrease the liquidity premium and to increase the haircut. However, the fact that the asset can be re-used when pledged as collateral makes each unit of the asset more valuable. This tends to increase the liquidity premium. Collateral re-use also modifies the properties of the equilibrium repo contract, increasing the repurchase price, thereby lowering the haircut. These counteracting effects of re-use on the equilibrium spot price and repo price also explain why its overall impact on leverage is ambiguous.

In addition, our paper sheds light on the way in which dealer banks use repos to lever up and fund their activities. Dealers' leverage is closely related to their role in channeling funds between different investors. Dealer banks obtain funds to purchase assets by using these assets as collateral in repos. As a result, they only need to tap into their cash holdings to pay the repo haircut. Since haircuts are usually small, dealer banks can be highly levered. So using repos, dealers can intermediate between cash poor investors, e.g. hedge funds, and cash rich investors, e.g. insurance companies, or money market funds. As a result, dealer banks make for a significant share of the repo market.

To account for these trading patterns, we extend the model by introducing a third type of investors, to whom we refer as dealers. Dealers have limited cash and no asset, but a higher counterparty quality. We show that in this environment, under some conditions we identify, dealers emerge in equilibrium as intermediaries between natural borrowers and natural lenders. Even though they could trade directly, natural borrowers (say, hedge funds) prefer to sell the asset in the spot market to dealers. Dealers in turn pledge it as collateral in a repo with natural lenders (say, insurance companies) to obtain the funds necessary to purchase the asset. The emergence of dealer banks as leveraged intermediaries hinges on their superior counterparty quality.

Finally, we show that with collateral re-use intermediation may also occur via a chain of repo trades. In a repo chain, a natural borrower enters a repo with a dealer bank who in turn enters another repo with the natural lender. Intermediation via a chain of repos can arise when the dealer bank has both a higher counterparty quality than the natural borrower and is better able at re-deploying collateral than the natural lender. Then, through re-use, one unit pledged to the dealer bank can indeed support more borrowing in the chain of transactions. This explains why a natural borrower chooses to trade with dealers even when there are larger gains from trade with natural lenders.

Relation to the literature

Recent theoretical works highlighted some features of repo contracts as sources of funding fragility. As a short-term debt instrument to finance long-term assets, Zhang (2014) and Martin, Skeie, and Thadden (2014) show that repos are subject to roll-over risk. Antinolfi, Carapella, Kahn, Martin, Mills, and Nosal (2015) show that the benefit of an exemption from automatic stay[6] granted to repos may be harmful for social welfare in the presence of fire sales, a point also made by Infante (2013) and Kuong (2016). These papers usually take the trade of repurchase agreements and their specific features as given while we want to understand their emergence as a funding instrument.

One natural question is why borrowers do not simply sell the collateral to lenders? A first strand of papers explains the existence of repos using transaction costs (e.g. Duffie, 1996) or search frictions (e.g. Narajabad and Monnet, 2012, Tomura, 2016, and Parlatore, 2018). Bundling the sale and the repurchase of the asset in one transaction lowers search costs or mitigates bargaining inefficiencies. Bigio (2015) and Madison (2016) emphasize the role of informational asymmetries regarding the quality of the asset to explain repos: their collateralized debt features reduce adverse selection between the informed seller and the uninformed buyer as in DeMarzo and Duffie (1999) or Hendel and Lizzeri (2002). We show that investors choose to trade repos in an environment with symmetric information, where markets are Walrasian, but where collateral has uncertain payoff. One limitation of the works mentioned above is that the borrower chooses to sell repo if he can obtain more cash than in a spot sale of the asset, that is if the haircut is negative. Our analysis rationalizes the use of repos with positive haircuts when investors are risk-averse. In

[6]As shown by Eisfeldt and Rampini (2009) for leases, such benefit is in terms of easier repossession of collateral in a default event.

addition, we account for the possible re-use of collateral in repos by showing its benefits.

To derive the equilibrium repo contract, we follow the competitive approach of Geanakoplos (1996), Araújo, Orrillo, and Páscoa (2000), and Geanakoplos and Zame (2014) where the properties of the collateralized promises traded by investors are selected in equilibrium. Unlike these papers where the only cost of default is the loss of the collateral, our model aims to capture the recourse nature of repo transactions. We thus allow for additional penalties for default, some of them non-pecuniary in the spirit of Dubey, Geanakoplos, and Shubik (2005). While our results on the characterization of repo contracts traded in equilibrium remain valid also in the absence of these additional penalties, the recourse nature of repos is crucial to explain re-use. Indeed, Maurin (2017) showed in a more general environment that the collateral multiplier effect disappears when loans are non-recourse.

Collateral re-use is discussed by Singh and Aitken (2010) and Singh (2011), who claim that it lubricates transactions in the financial system.[7] At the same time, re-use generates the risk that the lender, who receives the collateral, does not or cannot return it when due, as explained by Monnet (2011). Unlike Bottazzi, Luque, and Páscoa (2012) or Andolfatto, Martin, and Zhang (2017), we account for the double commitment problem induced by re-use. The increase in the circulation of collateral obtained with re-use also arises with pyramiding (see Gottardi and Kubler, 2015), where collateralized debt claims are themselves used as collateral. However, the mechanism is different: in pyramiding, no two sided commitment problem arises and the recourse nature of loans also plays no role. We stress the role of collateral re-use in explaining the presence of intermediation in the repo market, as in Infante (2015) and Muley (2016). Unlike in these papers, in our analysis intermediation arises endogenously since direct trade between borrowers and lenders is possible.

The structure of the paper is as follows. We present the model and the set of contracts available for trade in Section 2. We characterize the equilibrium and the properties of repo contracts traded in Section 3, where we also derive the properties of haircuts and liquidity premia. Section 4 shows that intermediation arises in equilibrium. Finally, Section 5 establishes the robustness of our findings to alternative specifications of the repurchase price and Section 6 concludes. The proofs are collected in the Appendix.

[7] Fuhrer, Guggenheim, and Schumacher (2016) estimate an average 10% re-use rate in the Swiss repo market over 2006-2013.

2 The Model

In this section we present a simple environment where risk averse investors have funding needs. To accommodate these needs, they can sell an asset in positive net supply and take short positions in a variety of securities in zero net supply. These trades occur in a competitive financial market. Short positions are subject to limited commitment and require collateral. Trade in these securities captures the main ingredients of repo contracts.

2.1 Setting

The economy lasts three periods, $t = 1, 2, 3$. There is a unit mass of investors of each type $i = 1, 2$ and one consumption good each period. For simplification, we will refer to all investors of type i as "investors i." All investors have endowment ω in the first two periods and zero in the last one. Investor 1 is also endowed with a units of an asset while investor 2 has none.[8] Each unit of the asset pays dividend s in period 3. The dividend is distributed according to a cumulative distribution function $G(.)$ with support $\mathcal{S} = [\underline{s}, \bar{s}]$ and mean $\mathbb{E}[s] = 1$. The realization of s becomes known to all investors in period 2, one period before the dividend is paid. As a consequence, price risk arises in period 2.

Let c_t^i denote investor i's consumption in period t. Investors have preferences over consumption profiles $\mathbf{c}^i = (c_1^i, c_2^i, c_3^i)$ described by the following utility functions, respectively for $i = 1, 2$:

$$
\begin{aligned}
U^1(\mathbf{c}^1) &= c_1^1 + v(c_2^1) + c_3^1 \\
U^2(\mathbf{c}^2) &= c_1^2 + u(c_2^2) + \beta c_3^2
\end{aligned}
$$

where $\beta < 1$, $u(.)$ and $v(.)$ are respectively strictly and weakly concave functions. We assume $u'(\omega) > v'(\omega)$ and $u'(2\omega) < v'(0)$.[9] The main role of this specification is to capture the fact that investor 1 wants to borrow short-term in period 1. He wants to borrow because his intertemporal rate of substitution between periods 1 and 2 is lower than that of investor 2. His borrowing should be short-term because investor 2 discounts period 3

[8] This is for simplicity only and we could easily relax this assumption, as none of the results depend on it.

[9] Observe that a special case of the preferences as specified above is $v(.) = \delta u(.)$, where investors only differ with respect to their discount factor.

cash flow more than investor 1 ($\beta < 1$). In addition, the concavity of the investors' utility over date 2 consumption implies that they dislike variability in repayment terms in period 2. We simplify the analysis by assuming that their utility is linear over consumption at the other dates, but linearity plays no essential role in our results.

2.2 Arrow-Debreu equilibrium

To illustrate the basic features of this economy, it is useful to consider its Arrow-Debreu equilibrium allocation $(\mathbf{c}^1_*, \mathbf{c}^2_*)$. Consumption at date 2 is determined by equating the investors' marginal rates of substitution between period 1 and period 2 while investor 2 does not consume in the last period:[10]

$$\begin{cases} u'(c^2_{2,*}) = v'(2\omega - c^2_{2,*}) \\ c^2_{3,*} = 0 \end{cases} \tag{1}$$

where we used the resource constraint in period 2 to substitute for $c^1_{2,*} = 2\omega - c^2_{2,*}$. The prices for period 2 and 3 consumption are respectively $u'(c^2_{2,*})$ and 1, with period 1 consumption as the numeraire. Consumption in period 1 is then obtained from the budget constraints. Thus for investor 2 we have $c^2_{1,*} = \omega - u'(c^2_{2,*})(c^2_{2,*} - \omega)$ and we will assume that

$$\omega \geq u'(c^2_{2,*})(c^2_{2,*} - \omega) \tag{2}$$

in the remainder of the text.

In the Arrow-Debreu equilibrium, investor 1 borrows $u'(c^2_{2,*})(c^2_{2,*} - \omega)$ from investors 2 in period 1 and repays with a net interest rate $r^* = 1/u'(c^2_{2,*}) - 1$ in period 2. In the following we refer for simplicity to this equilibrium allocation as the first best allocation. Observe that consumption in period 2 $(c^1_{2,*}, c^2_{2,*})$ is deterministic even though the asset payoff s is already known. Indeed, risk averse investors prefer a smooth consumption profile.

[10]Intuitively, since $\beta < 1$ investor 2 has a lower marginal utility for period 3 consumption utility than investor 1.

2.3 Financial Markets With Limited Commitment

We assume investors can buy or sell the asset each period in the spot market. They can also take long and short positions in financial securities in the initial period 1, before the uncertainty is realized. Unlike in the Arrow-Debreu framework, agents are unable to fully commit to future promised payments. As we will see, this implies that borrowing positions must be collateralized and the first best allocation cannot always be sustained.

Spot Trades

Let p_1 and $p_2(s)$ denote the period 1 and period 2 spot market price of the asset when the realized payoff is s. We let a_1^i (resp. $a_2^i(s)$) be the asset holdings of investor i after trading in period 1 (resp. period 2 and state s). Note that spot trades could be a way for investor 1 to meet his borrowing needs: he could sell the asset in period 1 to carry only $a_1^1 < a$ into period 2 and then buy it back in period 2 to carry $a_2^1(s) > a_1^1$ into period 3. However, a combination of spot trades alone can never sustain the first best allocation. Indeed, since $p_2(s)$ is a function of the state s, such trades generate undesirable consumption variability in period 2 for both investors.[11]

Repos

In period 1 investors can also trade promises to deliver the consumption good in period 2. We specify these financial securities so as to capture several features of repo contracts, and we will refer to these securities as repos. There are in particular three features of repo contracts we want to match. First, a repo agreement is a sale of an asset combined with a forward repurchase of that asset. The repurchase price is effectively the amount to be repaid by the seller and the asset plays a role as collateral. We thus model securities as collateralized loans. Second, in a repo agreement the lender acquires ownership of the asset and can sell or re-pledge the asset before the repo matures. We then also assume that the collateral backing the security is transferred to the lender who enjoys a right to re-use it. Finally, repos are recourse loans and the non-defaulting counterparty can claim the payment of any remaining shortfall and other expenses. In our environment, default on the securities entails additional costs beyond the loss of the asset pledged as collateral. We now describe in details how we model each of these features.

(i) Collateralized loans - We let $f = \{f(s)\}_{s\in\mathcal{S}}$ denote the payoff schedule for a generic security. An investor selling security f promises to repay $f(s)$ in state s of period

[11] See Online Appendix D.1 for a complete analysis of equilibrium with only spot trades.

2 per unit of security sold. We allow for all possible values of f so that the market for financial securities is complete. As we show below, the investors' inability to commit implies that short positions must be backed by the asset as collateral. Without loss of generality, we set the collateral requirement to one unit of asset per unit of security sold. We refer to the payoff schedule $\{f(s)\}_{s\in\mathcal{S}}$ as the repurchase price, which can be state contingent. Repos usually specify a constant repayment but margin calls or repricing of the terms of trade during the lifetime of a repo are ways in which contingencies can arise.[12] In Section 5 we discuss the implementation of our equilibrium contract using repos with fixed repurchase price.

(ii) Ownership transfer - The asset used as collateral is a financial claim. The borrower transfers to the lender both the asset used as collateral and the ownership title to this asset. The lender can then re-use this asset.[13] Specifically, we assume that investor i can re-use a fraction ν_i of the collateral he receives, where $\nu_i \in [0,1]$. We interpret ν_i as a measure of the operational efficiency of a trader in re-deploying collateral for his own trades.[14]

(iii) Recourse loans - In a collateralized loan with re-use there is a double commitment problem. The borrower can fail to pay back the lender, but the lender can also fail to return the collateral. In the following, we describe the punishment faced by investors when they default on their obligation. Besides the loss of the collateral pledged or of the right to receive the repayment due, the defaulting party incurs additional costs since the other party can claim compensation. This captures the recourse nature of repo transactions. We start by specifying the penalty for borrowers and then move on to the case of lenders default.

Borrower Default

When the borrower defaults, the lender can retain or liquidate the collateral. In practice, the lender typically needs to sell the asset at a discount below its fair market

[12] When a trader faces a margin call, he must pledge more collateral to sustain the same level of borrowing. This is equivalent to reducing the amount borrowed per unit of asset pledged.

[13] This distinguishes the situation under consideration from that, for instance, of a mortgage loan where the asset used as collateral is a physical asset and the borrower retains ownership of the collateral

[14] Singh (2011) discusses the role played by collateral desks at large dealer banks in channeling these assets across different business lines. These desks might not be available for less sophisticated repo market participants such as money market mutual funds or pension funds. In Appendix B, we discuss a variant of our model where ν_j is the probability that lender j can access the market to re-use collateral and provide conditions under which our results are robust to this alternative specification.

value, and we model the cost of liquidation as a linear function of the market value of the collateral, that is $\kappa p_2(s)$ per unit of asset, for some $\kappa \geq 0$. Then, the lender can claim the shortfall he faces in a default, equal to the difference (when positive) between the payment due, $f(s)$ and the market value of the collateral, $p_2(s)$, net of the costs associated with the liquidation of the collateral. This is in line with the recourse loan feature of repos and the provisions in the event of default described in standard Repo Master Agreements.[15] We assume that the lender is only able to collect a fraction $\alpha \in [0, 1]$ of this shortfall from the borrower. This partial recovery rate captures various frictions in recouping payments from unsecured claims.[16] We also posit that upon default, an investor i incurs an additional, non-pecuniary cost, equal to a fraction $\pi_i \in [0, 1]$ of the shortfall, measured in consumption units. This non-pecuniary component proxies for legal and reputation costs. It may thus depend on the borrower's type and increases in the size of the default.

To analyze the borrower's incentives to default, consider a trade of one unit[17] of repo contract f between borrower i and lender j. Borrower i prefers to repay rather than default in state s if and only if

$$f(s) \leq p_2(s) + (\alpha + \pi_i) \max\{f(s) - p_2(s)(1 - \kappa), 0\} \tag{3}$$

The borrower will repay whenever the repurchase price $f(s)$ does not exceed the total default cost, given by the expression on the right hand side of (3). The first term in that expression is the market value $p_2(s)$ of the collateral seized by the lender. The second term collects the fraction α of the shortfall recovered by the lender and the non-pecuniary cost $\pi_i \max\{f(s) - p_2(s)(1 - \kappa), 0\}$ for the borrower. We see from equation (3) that a borrower may only choose to default if $f(s) \geq p_2(s)$. Hence (3) can be written as follows:

$$f(s) \leq \frac{1 - (\alpha + \pi_i)(1 - \kappa)}{1 - \alpha - \pi_i} p_2(s) \tag{4}$$

[15] See footnote 4 for a discussion of the recourse features of repo transactions and Appendix A for a detailed summary of the master agreements' provisions in an event of default. According to ICMA (2013) the lender can include "*transactions costs and professional expenses*" when computing the shortfall between the promised payoff and the actual payoff. The losses from the liquidation of the collateral for the lender aim to capture these transactions costs.

[16] For instance, it might take time for the borrower to make these payments. In addition, such claims have a junior status if the borrower files for bankruptcy.

[17] This is without loss of generality since penalties for default are linear in the amount traded, hence incentives to default do not depend on the size of a position.

Lender's Default

We now discuss the punishment faced by a lender (of type j) when he fails to return the collateral. Recall that he can only re-use a fraction ν_j of the asset pledged. We assume that the non re-usable fraction $1-\nu_j$ can be deposited or segregated with a collateral custodian.[18] As a result, the lender may only abscond with the re-usable fraction of the collateral. Absconding with the collateral is a default by the lender.[19] In this event, the borrower gets back the $1-\nu_j$ units of segregated collateral and may also claim any shortfall remaining after the cancellation of his obligation to repay $f(s)$, equal to $\max\{p_2(s)-f(s)-(1-\nu_j)p_2(s),0\}$. Like in a borrower's default, we assume the borrower can only recover a fraction α of the claim and the lender incurs a non-pecuniary cost equal to a fraction $\pi_j \in [0,1]$ of the shortfall.

Hence, the lender prefers to return the non-segregated collateral rather than default if and only if

$$\nu_j p_2(s) \leq f(s) + (\alpha+\pi_j)\max\{\nu_j p_2(s)-f(s),0\} \tag{5}$$

The left hand side of (5) is the benefit of defaulting given by the market value of the re-usable collateral held by the lender.[20] The expression on the right hand side is the cost of defaulting which includes the foregone payment $f(s)$ from the borrower, the fraction α of the shortfall $\max\{\nu_j p_2(s)-f(s),0\}$ recovered by the borrower, and the non-pecuniary cost $\pi_j \max\{\nu_j p_2(s)-f(s),0\}$. Observe that condition (5) can be rewritten as follows:

$$f(s) \geq \nu_j p_2(s) \tag{6}$$

[18] It is easy to understand why the lender segregates the non re-usable collateral. He would not derive ownership benefits from keeping it on his balance sheet and segregation reduces his incentives to default. In the tri-party repo market, BNY Mellon and JP Morgan provide these services. If segregation is not available, incentives for the lender are clearly harder to sustain. This can be seen from equation (5) by taking $\nu_j = 1$. We will also see later that lenders need not segregate more collateral that the fraction $1-\nu_j$. In Appendix B, where ν_j is instead the probability that lender j can access the market, the choice of collateral segregation follows from a trade-off. More segregation strengthens the incentives of the lender but also decreases the re-use rate of collateral.

[19] Standard Repo Master Agreements allow counterparties to distinguish between outright default by the lender and "failure" to deliver the collateral on time. Late delivery of collateral is not necessarily characterized as an event of default because finding the appropriate security to deliver might take time in practice. We focus here on the first one, in which case the constraints imposed by the limited commitment of the lender are more relevant.

[20] A lender might re-use the collateral and not have it on his balance sheet when he must return it to the lender. However, observe that he can always purchase the relevant quantity of the asset in the spot market to satisfy his obligation. When he returns the asset, the lender effectively covers a short position $-\nu_j$.

that is, the lender prefers to return the collateral whenever the repurchase price $f(s)$ exceeds the value of the re-usable collateral he can abscond with.

No-Default Repos

The environment described extends the framework considered in standard models of collateralized lending, as for instance Geanakoplos (1996), to allow for recourse loans. When a borrower defaults, in addition to the loss of the collateral, he incurs pecuniary and non-pecuniary costs. We assume these costs are sufficiently low and the non-pecuniary cost is not too low compared to the recovery rate. Specifically, for every i:

$$\pi_i + \alpha < 1 \tag{7}$$

$$\alpha(u'(\omega) - v'(\omega)) \leq \pi_i v'(\omega) \tag{8}$$

Assumption (7) implies that a borrower always defaults if the loan is not collateralized. In our environment unsecured borrowing is equivalent to a repo collateralized by an asset with zero value. Under (7) the no-default condition for the borrower, (3), never holds when $p_2(s) = 0$. The second property, condition (8), then ensures that in equilibrium investors prefer to trade default-free repo contracts. When the recovery rate α is positive, the recourse feature of loans in our environment implies that borrowers could make higher payments to lenders with contracts inducing default.[21] However, by doing so, borrowers incur a non pecuniary penalty which is a deadweight loss. As we will show in the proof of Proposition 1, under (8) such deadweight loss always outweighs the benefits of increasing the income pledged through default. To summarize, investors will use the asset as collateral to sustain borrowers' incentives, and choose to trade securities that do not induce default.

We can now define the set of repo contracts $\mathcal{F}_{ij}$ that can be sold by investor i to investor j so that no default occurs. This set depends on the period 2 spot market price $\mathbf{p}_2 = \{p_2(s)\}_{s\in\mathcal{S}}$. To simplify notation, we let $\theta_i := (\alpha + \pi_i)\kappa/(1 - (\alpha + \pi_i)(1 - \kappa))$. From conditions (4) and (6) we obtain:

$$\mathcal{F}_{ij}(\mathbf{p}_2) = \left\{ f \mid \forall\, s \in [\underline{s}, \bar{s}]\,,\ \ \nu_j p_2(s) \leq f(s) \leq \frac{p_2(s)}{1 - \theta_i} \right\} \tag{9}$$

[21] It is easy to verify that, for f large enough, the actual payment to the lender after a borrower defaults, given by $(1 - \alpha(1 - \kappa))p_2(s) + \alpha f(s)$, exceeds the maximum amount a borrower can repay without defaulting, given by the right hand side of (4).

The upper bound of this set, $\frac{p_2(s)}{1-\theta_i}$, is the maximal amount that investor i can promise to repay by selling one unit of a repo contract. It is increasing in θ_i, which we can interpret as a measure of the creditworthiness or counterparty quality of investor i. Observe that the set $\mathcal{F}_{ij}(\mathbf{p}_2)$ is convex and that all contracts have the same collateral requirement given our normalization. Hence, for any combination of multiple contracts sold by i, there exists an equivalent trade of a single repo contract. We can thus focus our attention on equilibria where at most one contract is sold by each agent and we use $f_{ij} \in \mathcal{F}_{ij}(\mathbf{p}_2)$ to denote the (unique) contract sold by investor i to investor j.

Investors' optimization problem.

We can now write the optimization problem of an investor i. Let $q_{ij}(f_{ij})$ be the unit price of contract f_{ij}.[22] The collection of these repo prices is $\mathbf{q}_{ij} = \{q_{ij}(f_{ij}) \mid f_{ij} \in \mathcal{F}_{ij}(\mathbf{p}_2)\}$. Given the spot prices and the prices of the repo contracts, investor i chooses which contract to sell in $\mathcal{F}_{ij}(\mathbf{p}_2)$, which contract to buy in $\mathcal{F}_{ji}(\mathbf{p}_2)$, the volume of trade for the two contracts as well as the trades of the asset in the spot market. Let b^{ij} (resp. l^{ij}) denote the amount sold (resp. bought) by investor i to investor j using the chosen contract f_{ij} (resp. f_{ji}), that is the amount borrowed and lent. These contracts must be such that investor i does not strictly benefit from trading any other existing contract at the prices he faces. The quantities traded of the two contracts as well as the spot trades must be a solution of the following problem:

$$\max_{a_1^i, a_2^i(s), b^{ij}, l^{ij}} E\left[U^i(c_1^i, c_2^i(s), c_3^i(s))\right] \tag{10}$$

$$\begin{aligned}
\text{subject to} \qquad c_1^i &= \omega + p_1(a_0^i - a_1^i) + q_{ij}(f_{ij})b^{ij} - q_{ji}(f_{ji})l^{ij} && (11)\\
c_2^i(s) &= \omega + p_2(s)(a_1^i - a_2^i(s)) - f_{ij}(s)b^{ij} + f_{ji}(s)l^{ij} && (12)\\
c_3^i(s) &= a_2^i(s)s && (13)\\
a_1^i + \nu_i l^{ij} &\geq b^{ij} && (14)\\
b^{ij} &\geq 0 && (15)\\
l^{ij} &\geq 0 && (16)\\
a_2^i(s) &\geq 0 && (17)
\end{aligned}$$

[22] Even in the absence of default the price may depend on the identities of the agents trading the contract, to the extent that investors may have different re-use abilities.

Equation (11) is the budget constraint in period 1 for investor i, where the resources available are $\omega + p_1 a_0^i$. Equation (12) is the budget constraint in period 2 for every realization of s, with the resources available given by the endowment ω, the value of the investor's asset holdings $p_2(s)a_1^i$ and the net value of the repo positions $f_{ji}(s)\ell^{ij} - f_{ij}(s)b^{ij}$. Equation (13) is the budget constraint in period 3. The collateral constraint of investor i is specified in (14). When investor i sells contract f_{ij} (that is $b^{ij} > 0$), he must pledge as collateral one unit of the asset per unit of repo contract sold. He can satisfy this requirement either by acquiring the asset in the spot market (that is $a_1^i > 0$), or in the repo market (that is $l^{ij} > 0$). In the latter case however, only a fraction ν_i of the asset purchased can be re-used.

It is important to observe that, when investor i buys but does not sell a repo contract (that is $l^{ij} > 0$ and $b^{ij} = 0$), the collateral constraint may be satisfied with $a_1^i < 0$ if $\nu_i > 0$. Indeed, with re-use investor i can sell in the spot market an asset that he acquired by purchasing a repo contract. When the repo matures, the investor would then buy the asset to satisfy his obligation to return it to the repo seller, thus covering his short position. Hence, equation (14) shows that a lender can use repo trades to take a short position in the asset. However, investors cannot engage in naked short sales of the asset.

We can now define a competitive equilibrium (in short a repo equilibrium) in the environment described:

Definition.

A repo equilibrium is a system of spot prices p_1, $\mathbf{p_2} = \{p_2(s)\}_{s\in\mathcal{S}}$, repo prices $\mathbf{q}_{12}$, $\mathbf{q}_{21}$, a pair of repo contracts $(f_{12}, f_{21}) \in \mathcal{F}_{12}(\mathbf{p_2})\times\mathcal{F}_{21}(\mathbf{p_2})$ and an allocation $\{c_t^i(s), a_1^i, a_2^i(s), \ell^{ij}, b^{ij}\}$ for $i = 1, 2$, $j \neq i$, $t = 1, 2, 3$ and $s \in \mathcal{S}$ such that

1. *$\{c_t^i(s), a_1^i, a_2^i(s), \ell^{ij}, b^{ij}\}_{t=1..3,s\in\mathcal{S}}^{j\neq i}$ solves problem (10) with contracts (f_{ij}, f_{ji}), $j \neq i$, for investor $i = 1, 2$.*

2. *Spot markets clear: $a_1^1 + a_1^2 = a$ and $a_2^1(s) + a_2^2(s) = a$ for any s. Repo markets clear: $b^{ij} = l^{ji}$ for $i = 1, 2$ and $j \neq i$.*

3. *For every other contract $\tilde{f}_{ij} \in \mathcal{F}_{ij}(\mathbf{p}_2)\setminus\{f_{ij}\}$ the price $q_{ij}(\tilde{f}_{ij})$ is such that investors i and j do not wish to trade this contract, for $i = 1, 2$ and $j \neq i$.*

The equilibrium selects the repo contracts that agents trade. Condition 3. ensures that the market for other repo contracts clear with a zero level of trade.

3 Repo markets with re-use and the collateral multiplier

In this section, we characterize the equilibrium repo contracts and the resulting equilibrium allocation. We will show that investors 1 satisfy all their funding needs by selling a repo contract and that lenders always want to re-use collateral. Various authors (see for instance Singh and Aitken, 2010) have stressed the importance of collateral re-use for repo transactions where the collateral is sold to the lender. Our model helps to precisely characterize the benefits of re-use and its effects on repo contracts, in the presence of limited commitment.

3.1 The collateral multiplier

We begin the analysis by showing that re-use gives rise to a collateral multiplier, which we denote by M. By re-using collateral, investors 2 put back some assets in circulation, that investors 1 can buy and use to back further loans. The process can then be iterated over several rounds of trade. The existence of the collateral multiplier is similar to the money multiplier in models with fractional reserve banking.

To determine the value of the multiplier we need to consider the sequence of trades that may occur in period 1 with re-use. Let $f(s)$ denote the payoff of the repo contract that is traded in equilibrium. In the first round, investor 1 sells the asset in a repo to some investor 2, against the promise to pay $f(s)$ in state s in period 2. At the end of the first round of trade, an investor 2 who purchased a unit of asset in a repo re-uses a fraction ν_2 by selling it spot. This amount is purchased by an investor 1 who then resells it in a second repo. Observe that with this second repo, the additional payoff that an investor 2 obtains in period 2 is equal to $\nu_2\,(f(s) - p_2(s))$: while investors 1 pledge an additional $\nu_2 f(s)$ in each state s, investors 2 have to purchase ν_2 units of the asset at price $p_2(s)$ in order to make good on their promises to return the collateral.

Re-using ν_2 units of the asset thus allows an investor 1 to pledge $\nu_2\,(f(s) - p_2(s))$ additional units in each state s of period 2. Whenever this amount is positive, re-using the collateral expands the ability to borrow of investors 1. Then, at the end of this second round, an investor 2 has $(\nu_2)^2$ units of the asset which can again be re-used as above. Iterating this process over infinitely many rounds, we obtain the amount that one unit of asset allows investors 1, using repos with payoff $f(s)$, to pledge in state s at date 2, net of the price paid to repurchase the collateral in the spot market:

$$f(s) + \sum_{r=1}^{\infty} (\nu_2)^r \left(f(s) - p_2(s)\right) = \left(\frac{1}{1-\nu_2}\right) f(s) - \frac{\nu_2}{1-\nu_2} p_2(s) \tag{18}$$

The term $1/(1-\nu_2)$ is the "physical" multiplier that describes the increase in the amount of collateral in circulation generated by the above sequence of trades. The second term is the cost that must be paid to purchase the $\nu_2/(1-\nu_2)$ units of the asset that are re-used.

Recall that $\frac{p_2(s)}{1-\theta_1}$ is the maximal amount that investors 1 can promise to repay when they sell one unit of an asset in a repo. Setting $f(s)$ equal to this value in (18), we obtain the *borrowing capacity* of investors 1 in state s, per unit of asset held, when lenders always re-use the collateral received :

$$\left(\frac{1}{1-\nu_2}\right) \frac{p_2(s)}{1-\theta_1} - \frac{\nu_2}{1-\nu_2} p_2(s) = \frac{1-\theta_1}{1-\nu_2} \left[\frac{1}{1-\theta_1} - \nu_2\right] \frac{p_2(s)}{1-\theta_1} \tag{19}$$

The collateral multiplier is the factor M that increases the maximum amount that can be pledged with one unit of the asset, when there is re-use, that is

$$M \equiv \frac{1-\theta_1}{1-\nu_2} \left[\frac{1}{1-\theta_1} - \nu_2\right] \tag{20}$$

We see that the multiplier is greater than 1 and strictly increasing in ν_2 as long as $\theta_1 > 0$. The fact that repos are recourse loans thus plays a crucial role to ensure that re-use increases the borrowing capacity. If the only punishment for default were the loss of collateral, re-use would have no effect.[23]

[23] In line with our result, Maurin (2017) proved in a more general setting that when loans are non recourse, re-use is redundant unless the market for financial securities is incomplete.

3.2 The equilibrium repo contract

We now derive the repo contracts traded in equilibrium. To do so, it is useful to begin by determining the conditions under which the equilibrium allocation is the same as in the Arrow-Debreu equilibrium, that is, limited commitment constraints do not bind and first best is attained. Intuitively, this can happen when the asset payoff is sufficiently high that investors 1 can pledge enough income in period 2.

Recall that, at the first best allocation, investor 1 borrows in period 1 by promising to repay $\omega - c^1_{2,*}$ in period 2. Since each investor 1 has a units of collateral, in a repo equilibrium with re-use the maximum income in state s at date 2 that can be pledged by this investor is $aMp_2(s)/(1-\theta_1)$. When in every state s this amount exceeds $\omega - c^1_{2,*}$, the first best is attained.

This is no longer possible when in some states the value of the collateral is so low that this payment falls short of $\omega - c^1_{2,*}$. In this case, in equilibrium investors 1 sell all the asset they have in a repo, only one contract is traded in equilibrium and lenders always re-use the collateral they receive. In the remainder of this section we simply refer to this contract as f and to its price as $q := q_{12}(f)$. The sequence of trades described earlier pins down investors 1's holdings in period 1. Their spot position, $a^1_1 = a/(1-\nu_2)$ is the sum of their initial holdings $a^1_0 = a$ and of the amount of collateral $\nu_2 a/(1-\nu_2)$ re-used by investors 2, that they buy in the spot market. Since investors 1 pledge all their asset in repos, we have $b^{12} = a/(1-\nu_2)$. At the end of period 2, investors 1 end up holding all the asset in equilibrium, that is $a^2_1 = a$. Substituting these values into the expression of the budget constraint (12), we obtain the value of their consumption in period 2:

$$c^1_2(s) = \omega + p_2(s)(\frac{a}{1-\upsilon_2} - a) - f(s)\frac{a}{1-\upsilon_2} \tag{21}$$

Note that in period 2, although they end up holding all the asset, investors 1 are net sellers in the spot market. This is because investors 2 need to buy $\nu_2 a/(1-\nu_2)$ units to cover the short positions they entered when re-using collateral.

For low realizations of $p_2(s)$, the borrowing constraint binds and the repurchase price of the chosen contract equals the maximal amount that can be pledged by selling one unit of the asset in a repo, $p_2(s)/(1-\theta_1)$. Substituting this value for $f(s)$ into (21) yields the equilibrium value of consumption in those states:

$$c_2^1(s) = \omega - aMp_2(s)/(1-\theta_1) \tag{22}$$

In the other states, where the borrowing constraint does not bind, $f(s)$ is set at the value that ensures that the total payment made to investors 2, net of the cost of repurchasing the collateral, equals $\omega - c_{2,*}^1$, so that $c_2^1(s) = c_{2,*}^1$.

The equilibrium spot price is then determined by investor 1's first order condition:

$$p_2(s) = s/v'(c_2^1(s)) \tag{23}$$

This, together with (22), implies that $p_2(s)$ is strictly increasing in s. For any given value of ν_2, we denote then by $s^*(\nu_2)$ the lowest state for which the borrowing constraint does not bind, obtained as a solution of the following equation:

$$c_{2,*}^1 = \omega - aM\frac{p_2(s^*(\nu_2))}{1-\theta_1} = \omega - aM\frac{s^*(\nu_2)}{(1-\theta_1)v'(c_{2,*}^1)} \tag{24}$$

For all $s \geq s^*(\nu_2)$, equilibrium consumption equals the first best level $(c_{2,*}^1, c_{2,*}^2)$, while for $s \leq s^*(\nu_2)$, $c_2^1(s)$ is given by (22) and $c_2^2(s) = 2\omega - c_2^1(s)$. Observe that this threshold $s^*(\nu_2)$ is decreasing in a, θ_1, and ν_2. Hence, when the amount of asset, the creditworthiness of investor 1 or the re-use capacity increases, the first best consumption level is attained in more states.

Substituting the values obtained for $c_2^1(s)$ in equation (23) yields the following expression for the equilibrium spot price in period 2

$$\begin{cases} p_2(s)v'\left(\omega - aM\dfrac{p_2(s)}{1-\theta_1}\right) = s & \text{if} \quad s < s^*(\nu_2) \\ p_2(s)v'(c_{2,*}^1) = s & \text{if} \quad s \geq s^*(\nu_2) \end{cases} \tag{25}$$

We can now state our main result:

Proposition 1. ***Equilibrium***. *Let $\nu_1, \nu_2 \in [0, 1)$, $\theta_1 > 0$. If $s^*(v_2) > \underline{s}$, there is a unique equilibrium allocation and a unique repo contract sold by investor* 1 *with payoff:*

$$f(s) = \begin{cases} \dfrac{p_2(s)}{1-\theta_1} & \text{if } s < s^*(\nu_2) \\ \dfrac{p_2(s^*(\nu_2))}{(1-\theta_1)} + \nu_2(p_2(s) - p_2(s^*(\nu_2))) & \text{if } s \geq s^*(\nu_2) \end{cases} \tag{26}$$

where $p_2(s)$ is defined in (25) and $s^*(\nu_2)$ in (24). Investors 2 re-use all the collateral received by selling it spot. If $s^*(\nu_2) \leq \underline{s}$, or equivalently if $\nu_2 \geq \nu^*$ for some $\nu^* < 1$, the first-best allocation is always attained in equilibrium.

Two main forces shape the characteristics of the equilibrium repo contract: investor 1's desire to borrow in period 1 (the *borrowing motive*) and the aversion of both investors to risk in their portfolio return in period 2 (the *hedging motive*). When the value of the asset is low, for $s \leq s^*(v_2)$, the repurchase price is equal to the maximal amount investors 1 can promise to repay, thus exhausting their borrowing capacity. Hence, $f(s)$ is increasing in s and is only determined by investor 1's *borrowing motive* because investors are borrowing constrained.

On the other hand, when the collateral value is high, for $s > s^*(v_2)$, the borrowing capacity of investor 1 exceeds his borrowing needs. Then, the repurchase price is set at a level that allows investors to perfectly hedge the price risk affecting the value of their portfolio in those states. In high states, the *hedging motive* thus pins down the repurchase price. Note that the repo contract provides hedging against the asset price in two ways. First, while the maximum income pledgeable in a repo sale varies with $p_2(s)$, the first term in the expression of $f(s)$ in the last line of (26) is constant and equal to the pledgeable income in state $s^*(v_2)$. Second, the repurchase price offsets the price exposure of investor 2 who must cover the short position she entered when re-using collateral. To unwind her position, she must buy the asset back, which exposes her to price risk. Thanks to the term $\nu_2 [p_2(s) - p_2(s^*(\nu_2))]$ in (26), the repurchase price offsets the cost of unwinding the short positions when $s > s^*(\nu_2)$.

Altogether, both the *borrowing motive* and the *hedging motive* generate variability of the repurchase price. The blue curve in Figure 1 plots the repurchase price of the equilibrium repo contract when $v(x) = \delta x$, for $\delta \in (0, 1)$.

By construction, the repo contract specified in (26) is such that borrowers never

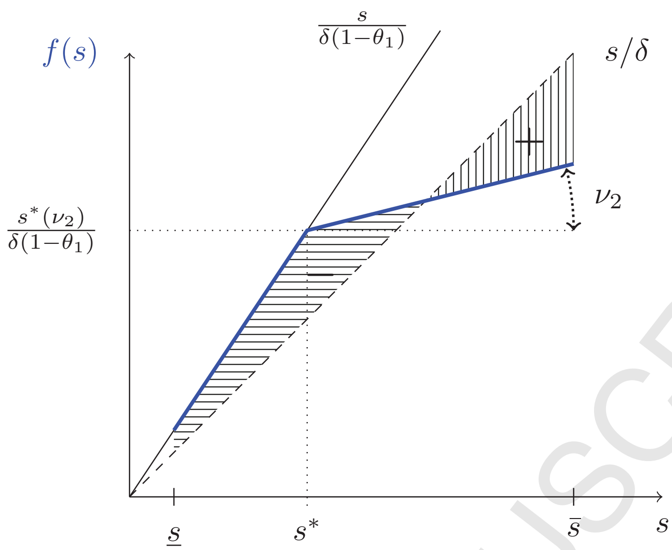

Figure 1: Repo contract ($v(x) = \delta x$).

want to default. With re-use however, we must also verify that the lenders' incentives are satisfied and that they are willing to comply with their promise to return the asset pledged as collateral. It is immediate to see that the payoff of the repo contract $f(s)$ is always higher than the value of the re-usable fraction of the collateral $\nu_2 p_2(s)$ investor 2 can abscond with. Hence lenders never want to default with this contract since their no-default constraint (6) is satisfied for any value of ν_2.

As we already noticed, the higher is ν_2, the higher the collateral multiplier M and hence the borrowing capacity of investors 1. The final claim in the proposition states that, when the re-usable fraction of the collateral is sufficiently high ($\nu_2 \geq \nu^*$), the first-best level of consumption can be attained even in the lowest state $\underline{s}$. One can obtain the expression for ν^* simply by setting $s^*(\nu_2) = \underline{s}$ in equation (24):

$$\nu^* = \frac{s^*(0) - \underline{s}}{s^*(0) - (1 - \theta_1)\underline{s}}.$$

In this case, several repo contracts or a combination of repo and spot trades can support the equilibrium allocation. In contrast, when collateral is scarce or ν_2 sufficiently low that $s^*(\nu_2) \geq \underline{s}$, there is a unique equilibrium repo contract and investor 1 sells all his asset using this repo.

As we also show in the proof of Proposition 1, when investors trade the repo contract f they do not want to trade other repo contracts and there is no other equilibrium where a different contract is traded.

Finally, and for completeness notice that the repo rate r is determined as follows:

$$1 + r = \frac{\mathbb{E}[f(s)]}{q}. \tag{27}$$

Since borrowers are constrained and $u'(c_2^2(s)) > u'(c_{2,*}^2)$ this rate is lower than in the first best allocation: $1 + r < 1 + r^*$.

3.3 Haircuts and liquidity premium

In this section, we derive the properties of the liquidity premium and the haircut in the repo equilibrium. We define the liquidity premium $\mathcal{L}$ as the difference between the spot price p_1 of the asset in period 1 and the spot price $\hat{p}_1$ of another (virtual) asset that cannot be used as collateral but has the same payoff. The liquidity premium thus captures the value of the asset over and above its holding value when this asset facilitates borrowing. It is equal to the shadow price of the collateral constraint and we can also refer to it as the collateral premium. Using the equilibrium characterization and the investors' first order conditions, we can show[24] that the liquidity premium is given by:

$$\mathcal{L} = \frac{1}{1-\nu_2}\mathbb{E}[(f(s) - p_2(s))(u'(c_2^2(s)) - v'(c_2^1(s))] \tag{28}$$

The expression for the liquidity premium has an intuitive interpretation. Observe first that it is positive only if investors are constrained, that is when $u'(c_2^2(s)) > v'(c_2^1(s))$ for some states. This arises when $\underline{s} < s^*(v_2)$. The term $f(s) - p_2(s)$ is the additional amount investor 1 can pledge when using the asset as collateral rather than in a spot trade in the states where investors are constrained. Finally, the liquidity premium is proportional to the physical multiplier $1/(1 - \nu_2)$ since a unit of the asset pledged as collateral can be re-used in further trades.

[24] In the repo equilibrium with re-use, the collateral constraint binds both for lenders and borrowers. Still, since $u'(c_2^2(s)) \geq v'(c_2^1(s))$ for all s, the value of $\hat{p}_1$ is always determined by investor 2's marginal utility. Hence the collateral constraint reflects the shadow price of the constraint for this investor.

The repo haircut is the difference between the spot price and the repo price of the asset in period 1. One unit of the asset can be bought in the spot market at price p_1 and sold at the equilibrium repo price q. So to purchase 1 unit of the asset, an investor needs $p_1 - q$, which is the down payment or haircut:[25]

$$\mathcal{H} \equiv p_1 - q = \mathbb{E}[(p_2(s) - f(s))v'(c_2^1(s))] \tag{29}$$

The second equality in (29) follows from the first order condition of investor 1 with respect to spot and repo trades. As Figure 1 shows, the borrowing and hedging motives have opposite effects on the size of the haircut. In the region $s < s^*(v_2)$, where investor 1 is constrained, the repurchase price is equal to $p_2(s)/(1-\theta_1)$ while the asset trades at price $p_2(s)$. From expression (29) we see that these low states thus contribute negatively to the haircut. On the other hand, when $s \geq s^*(v_2)$ the difference $p_2(s) - f(s)$ is first negative and then positive for s sufficiently large. When this happens, the difference contributes positively to the haircut. These two cases correspond respectively to the regions with horizontal and vertical lines in Figure 1. The overall sign of the haircut depends on the probability mass attributed to the two regions by the distribution of s. Finally, observe that the haircut is not uniquely pinned down when investors are not borrowing constrained ($s^*(\nu_2) \leq \underline{s}$) since as we already noticed, several repo contracts support the first best equilibrium allocation.

With re-use, investor 1 finds it profitable to buy the asset spot in order to re-use it in a repo. These trades increase investor 1's income in period 1 by $-p_1+q = -\mathcal{H}$, that is minus the haircut. When $\mathcal{H} < 0$, these transactions relax investor 1's borrowing constraint to capture some of the unexploited gains from trade. It may thus seem that buying spot to sell in a repo is not desirable when $\mathcal{H} > 0$ since in that case the period 1 income of investor 1 decreases. But this line of argument ignores the gains from transferring income across states in period 2. In state s in period 2, the corresponding income generated from these trades for investor 1 is $p_2(s) - f(s)$ which is the value of the asset minus the repurchase price. This gain is negative for low values of s but from expression (29) we see that, when $\mathcal{H} > 0$, it must be positive for s above some threshold. In words, these trades allow investor 1 to reduce his income in the low states where his marginal utility for consumption is low (and the one of investor 2 is high) while increasing his income

[25] An alternative but analogous definition states the haircut in percentage terms: $(p_1 - q)/q$.

in the high states. Therefore, re-use with $\mathcal{H} > 0$ allows investor 1 to smooth (albeit imperfectly) his consumption across states in period 2. The benefit of this additional smoothing effect in period 2 compensates the cost of the reduction in investor 1's income in period 1. Proposition 1 indeed shows that when $s^*(\nu_2) > \underline{s}$ lenders always want to re-use the collateral they are pledged, and this is true independently of the sign of the haircut. Finally, our model predicts that the benefits of re-use are larger when collateral is most scarce and there is evidence that this is indeed the case (see Fuhrer, Guggenheim, and Schumacher, 2016).

3.3.1 Collateral scarcity and counterparty quality

In this section we study the impact of collateral scarcity, counterparty quality, and re-use on the level of the liquidity premium and the haircut.

Proposition 2. *$\mathcal{L}$ is decreasing and $\mathcal{H}$ is increasing in the amount of collateral a. $\mathcal{H}$ decreases in counterparty quality θ_1 while the effect of θ_1 on $\mathcal{L}$ is ambiguous. The effect of ν_2 on $\mathcal{H}$ and $\mathcal{L}$ is ambiguous.*

When a increases, more asset can be sold in a repo. Note that $s^*(v_2)$ declines when a increases. Hence there are fewer states where $s < s^*(v_2)$, and in those states the wedge in marginal utilities $u'(c_2^2(s)) - v'(c_2^1(s))$ is reduced. As more gains from trade are realized, the shadow price of the collateral constraint, that is $\mathcal{L}$, goes down. This result is standard in models where investors are borrowing constrained and use an asset as collateral or as a mean of payment.[26] The decline in $s^*(v_2)$ when a increases also implies that the haircut decreases since there are less states where the repurchase price is equal to the maximal income that can be pledged in a repo sale, which contributes negatively to the haircut as we saw in Figure 1.

We now discuss the comparative statics effects related to the more novel features of our model, counterparty quality and re-use. As argued above, increasing counterparty quality and re-use (through an increase in ν_2) expands the borrowing capacity, like an increase in a. However, Proposition 2 shows that the overall effect of increasing either θ_1 or ν_2 on the haircut and liquidity premium are different from that of an increase in a.

[26]See Geanakoplos and Zame (2014) for the canonical model of endogenously incomplete markets with collateral and Lagos (2010) for a monetary environment where the asset facilitates decentralized trades.

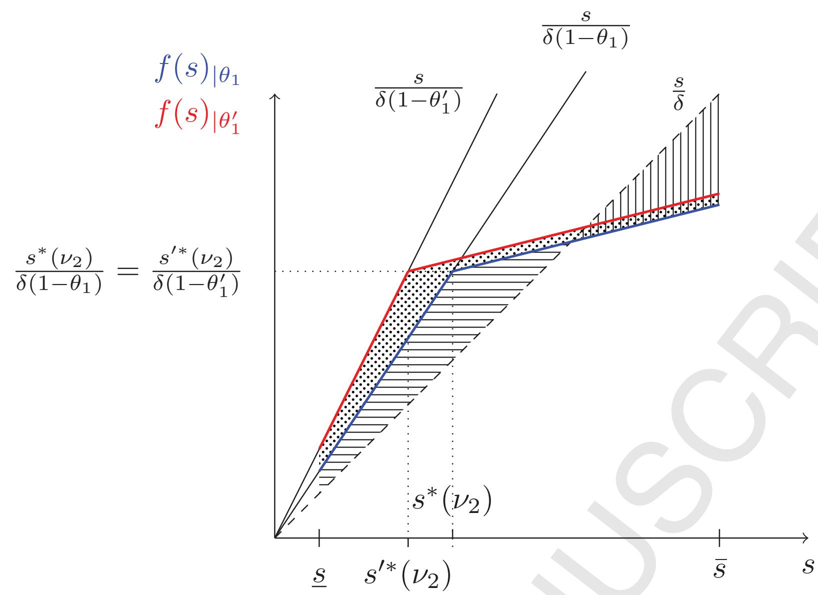

Figure 2: Influence of counterparty quality, with $\theta_1' > \theta_1$ ($v(x) = \delta x$)

A higher counterparty quality θ_1 decreases haircuts since the maximal pledgeable income $p_2(s)/(1-\theta_1)$ increases. Intuitively, a better counterparty has a higher ability to honor debt, which reduces the required down payment. Figure 2 illustrates the effect of an increase in counterparty quality to $\theta_1' > \theta_1$, with the new repo contract depicted in red. The solid line from the origin representing the maximal income that can be pledged rotates counterclockwise, as this income increases when θ_1 increases. We see the value of the repurchase price is also higher in the states $s \geq s^*(\nu_2)$ where investor 1 is unconstrained. Both these changes lead to a decrease of the haircut, as shown by the dotted area in Figure 2.

With regard to the liquidity premium $\mathcal{L}$, counterparty quality θ_1 has instead an ambiguous effect. Indeed, raising θ_1 improves the borrowing capacity of investors 1 in states $s < s^*(\nu_2)$. This reduces the wedge $u'(c_2^2(s)) - v'(c_2^1(s))$ and lowers $s^*(\nu_2)$.[27] This effect is similar to the one we found for an increase in the outstanding amount of the asset,

[27] Observe that θ_1 also affects the spot market price $p_2(s)$ in period 2 for $s \leq s^*(\nu_2)$. Hence, to assess the effect of θ_1 on the borrowing capacity $aMp_2(s)/(1-\theta_1)$, one must also consider this effect. Applying the implicit function theorem to the equation $p_2(s)v'(\omega - aMp_2(s)/(1-\theta_1)) - s = 0$ we see that $\partial[Mp_2(s)/(1-\theta_1)]/\partial\theta_1 > 0$, that is the overall effect of θ_1 on the borrowing capacity is positive.

and tends to reduce the liquidity premium. However, there is a second effect: since the pledgeable income is higher in the states $s < s^*(\nu_2)$ where this is most valuable for investors 1, the asset becomes a better borrowing instrument, which raises its price and so its liquidity premium.

Consider now the effects of re-use on the liquidity premium and the haircut. First, the amount of the asset than can be pledged as collateral increases with ν_2, through the collateral multiplier. This relaxes the investors' borrowing constraint and hence tends to decrease the liquidity premium and to increase the haircut, an effect similar to that of an increase in the asset quantity a. However, collateral re-use affects these variables through other channels. When ν_2 increases, every unit of the asset becomes more valuable since investors can re-use a higher fraction of it, which tends to increase the liquidity premium. Hence, the overall effect of re-use on the liquidity premium is ambiguous. To see why re-use also has an ambiguous effect on the haircut, it is useful to consider Figure 1. An increase in ν_2 lowers the threshold $s^*(\nu_2)$ since investors are less constrained. We see from the figure that this tends to decrease the repurchase price and thus to increase the haircut. However, an increase in ν_2 also increases the slope of the equilibrium repo contract for $s \geq s^*(\nu_2)$ because of the hedging motive, thereby decreasing the haircut. Finally, we observe that the effect of re-use on leverage is also ambiguous. We define leverage as the amount of debt that borrowers raise per dollar of the asset, that is q/p_1. Hence leverage is negatively related to the spot price p_1 of the asset and positively related to the repo price q. The ambiguity of the general equilibrium effects of re-use on these prices thus extends to the effect of re-use on leverage.

3.3.2 Asset risk

We can use our model to compare haircuts and liquidity premia for assets with different risk profiles. To this end, we extend the environment by introducing a second asset. For simplicity, we assume that the second asset has a perfectly correlated payoff with the first asset but carries higher risk. Hence, there is no possibility of hedging positions in one asset with an opposite position in the other asset. Therefore the pattern of equilibrium trades as well as the properties of repo contracts are determined by the same principles as before.

The second asset pays a mean preserving spread of the dividend of the first asset dividend,

$$\rho(s) = s + \sigma(s - \mathbb{E}[s]),$$

where $\sigma > 0$. Investor 1 is still endowed with a units of the first asset and also owns b units of the second asset, while investor 2 is not endowed with any of the assets. The set of available contracts consists of all feasible repos using any of the two assets. It is relatively straightforward to extend the equilibrium analysis of Section 3.2 to this new environment. For each asset, the repurchase price of the equilibrium repo contract is equal to the maximal income that can be pledged selling one unit of that asset in a repo in all states where the first best level of consumption cannot be reached and includes the correction to hedge the price risk otherwise. We then establish the following result.

Proposition 3. *The safer asset always has a higher liquidity premium and a lower haircut than the riskier asset.*

The key intuition behind the result is that the mean preserving spread of the dividend induces a misallocation of collateral value across states. While the two assets have the same expected payoff $\mathbb{E}[s]$, the riskier asset pays relatively more in high states (where there is upside risk) and less in low states (downside risk). An asset is particularly valuable as collateral in low states where investor 1 is borrowing constrained. Since the safer asset pays more in these states, it carries a larger liquidity premium. Turning now to the haircut, the riskier asset has a higher dividend in high states, which ensures a higher borrowing capacity in these states compared to the safer asset. However, investor 1 does not benefit by borrowing more in those states where he attains the first best level of consumption. Thus, since a smaller fraction of the asset dividend is pledged in the equilibrium repo for the second asset, the haircut is larger.[28]

Our results are in line with the empirical evidence that safer assets command lower repo haircuts (see e.g. Gorton and Metrick, 2012). Through this repo collateral channel, our model also rationalizes the findings by Krishnamurthy and Vissing-Jorgensen (2012) and others that safe assets command a (higher) liquidity premium. While investors can sell and pledge both safe and risky assets in our model, the safer assets are more useful as collateral and thus bear a higher premium. Also note that in our model, the same

[28]Note that with risk-neutral preferences, there is no hedging motive and the repurchase price is always equal to the maximum pledgeable income. Equation (29) then shows that the haircut would not depend on asset risk.

stochastic discount factors are used to price the assets in equilibrium since investors can trade both assets at the same time. An advantage of our approach is that the comparison effectively controls for market conditions and its implications can be brought to the data in a more meaningful way.[29]

The next section shows that re-use may lead to endogenous intermediation in equilibrium.

4 Collateral Re-use and Intermediation

In practice, cash is intermediated among market participants through chains of repos.[30] For example, as Figure 3 illustrates, a hedge fund borrows cash through a repo from a dealer bank who finances this transaction by tapping a cash pool, say an insurance company, via another repo. This is surprising because platforms such as Direct Repo™ in the US grant hedge funds direct access to cash pools. So why do traders resort to repo intermediation? In this section we show that these chains of repos may arise in equilibrium. A remarkable feature of our analysis is that intermediation arises endogenously: although the hedge fund is free to trade directly with the insurance company, he still prefers to trade instead with a dealer bank. We explain this feature with differences in counterparty quality for the hedge fund and the dealer bank.[31]

In this section we extend the economy introducing a third type of investor labeled B, for dealer Banks. Investor B is endowed with no asset and ω units of the consumption good in periods 1 and 2 and has the following preferences:

$$U^B(c_1, c_2, c_3) = c_1 + \delta_B c_2 + c_3$$

[29] For completeness, we also considered the effect of changing the risk of the asset in a single asset economy. The equilibrium allocation is different for various level of risk, and we find that a mean preserving spread implies a higher haircut while the effect on the liquidity premium is indeterminate and depends on risk aversion.

[30] In their guide to the repo market, Baklanova, Copeland, and McCaughrin (2015) state that "*dealers operate as intermediaries between those who lend cash collateralized by securities, and those who seek funding*".

[31] In practice, the transaction between the dealer bank and the insurance company could take place using a Tri-Party agent as a custodian. We abstract from modeling the services provided by the Tri-Party agent. See Federal Reserve Bank of New York (2010) for a discussion of this segment of the repo market. We thus focus on the intermediation provided by the dealer bank to the hedge fund and the insurance company.

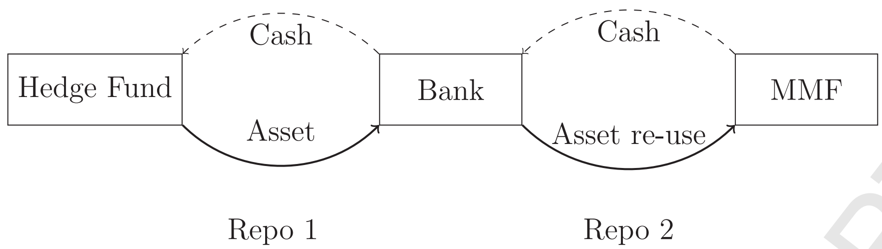

Figure 3: Intermediation with Repo

For simplicity, as a special case of our general specification, we assume here that investor 1 has linear preferences too, that is $v(x) = \delta x$ or:

$$U^1(c_1, c_{2,}c_3) = c_1 + \delta c_2 + c_3$$

We posit $\delta \leq \delta_B < u'(\omega)$. This implies that investor B would also like to borrow from investor 2 in the first period but has no asset to use as collateral, and has weakly lower gains from trade than investor 1. We assume $\theta_B > \theta_1$ so investor B is more creditworthy than investor 1. His greater borrowing capacity will explain why investor B can play a role as an intermediary. All investors are free to participate in the spot market and engage in repo trades with any type of counterparty. We will say that there is intermediation when investor 1 sells his asset to B and B re-sells it to investor 2. We show that intermediation indeed arises in equilibrium. It may take place via a spot or a repo sale from investor 1 to B depending on the relative values of δ and δ_B. Thus our notion of intermediation encompasses more than just chains of repos and we derive below the conditions for each pattern of intermediation to arise. In what follows, it is useful to refer sometimes to agent 1 as the natural borrower, to agent 2 as the natural lender, and to agent B as the intermediary.

4.1 Intermediation via spot trades

We assume first that the natural borrower and the intermediary have the same preferences, that is $\delta = \delta_B$ and only differ in their creditworthiness. We show that in equilibrium intermediation takes place via a spot sale from 1 to B. Note that in this case, there are no direct gains from trade between 1 and B. Hence, the trades between these investors are only driven by the intermediation role played by B.

Proposition 4. ***Intermediation Equilibrium****. Let $\delta = \delta_B$ and $\theta_1 < \theta_B$. When $s^*(\nu_2) > \underline{s}$ (the first best allocation cannot be achieved in the equilibrium with re-use of Proposition 1), in equilibrium, investor 1 sells his asset spot to investor B, who then re-sells it in a repo to investor 2.*

Since it is very similar to the proof of Proposition 5 below, we relegate the proof of this Proposition to Online Appendix D.3. The striking feature in Proposition 4 is that investor 1, who is endowed with the asset, no longer sells it in a repo contract to investor 2, the natural lender. Instead, in equilibrium, investor 1 sells the asset spot to B. Once investor B gains possession of the asset, he finds himself in the same position as investor 1 in the last section vis a vis investor 2. He then engages in an infinite sequence of repo sales and spot purchases of the re-usable collateral with 2. The equilibrium repo contract f_{B2} is specified as in (26), replacing θ_1 with θ_B.

If investor B were not present, we saw in the previous sections that investor 1 would borrow in a repo from investor 2. However, since $\theta_B > \theta_1$, investor B can borrow more than 1 from investor 2 for each unit of the asset. Thus investor B values the asset more and bids up the spot market price. As a result, investor 1 prefers to sell his asset in the spot market, as if he were delegating borrowing to a more creditworthy investor.

Intermediation takes place via a spot sale from investors 1 to B and not via a repo sale. To understand this, observe that, since 1 and B have the same preferences, they cannot benefit from a redistribution of income among them between periods 1 and 2. With a repo, investor 1 would in fact be able to obtain from B more income to be spent in period 1 as compared to a spot sale. However, investor 1's benefit equals what he must pay to B for the transfer. In addition, trading a repo entails a cost because a fraction $1 - \nu_B$ of every unit of the asset transferred to B could not be used to borrow from 2. Hence, investor B would pay a lower price to acquire the asset through a repo purchase, which implies the preference for a spot transaction.

Finally, investor B could be inactive in equilibrium if investor 1 is endowed with a sufficiently high quantity of the asset (that is, $s^*(\nu_2) < \underline{s}$). In the case, investor 1 can attain the first best allocation by trading directly with investor 2 in spite of his lower creditworthiness. An interesting implication of our result is thus that intermediation should be more important precisely when collateral is scarce.

4.2 Chain of repos

We show next that when $\delta < \delta_B$ intermediation may occur via a chain of repos. We call *intermediation equilibrium with a chain of repos* an equilibrium where the following pattern of trades is observed: investor 1 sells the asset in a repo to investor B, who re-uses the asset to sell it in a repo to an investor 2. Since $\delta < \delta_B$, there are now direct gains from trade between 1 and B. However, since $\delta_B < u'(\omega)$, these gains are still smaller than those between 1 and 2. Hence, trades between 1 and B must still be at least partially driven by the intermediation role of B.

It is useful to compare first the chain of repos with alternative patterns of trades. This discussion will shed some light on the conditions stated in the repo chain equilibrium of Proposition 5. When $\delta < \delta_B$ a redistribution of income from period 2 to period 1 in favor of investor 1 is beneficial. It follows from the discussion in the previous section that investor 1 could capture these benefits by using a repo, instead of a spot sale, at the cost of immobilizing collateral. Thus a trade-off emerges now. For investors 1 and B to prefer a repo sale over a spot sale, the direct gains from trade between 1 and B, given by $\delta_B - \delta$, must be sufficiently large relative to the fraction of collateral segregated $1 - \nu_B$. At the same time, the direct gains from trade between B and 2 must be sufficiently large for B to be willing to re-use the asset he acquires from 1 in a repo trade with 2. Otherwise, he will use all the asset in trades with investor 1. This imposes an upper bound on $\delta_B - \delta$. Finally observe that, unlike with a spot sale from investor 1 to B, intermediation with a repo chain involves collateral segregation. Hence, intermediation is preferred to direct trade between investors 1 and 2 if the difference in counterparty quality $\theta_B - \theta_1$ between B and 1 offsets the cost of segregation $1 - \nu_B$.

Investor 2's ability to re-use collateral does not affect qualitatively any of the trade-offs described above so for clarity we set $\nu_2 = 0$ in what follows.[32] We can now state the exact conditions under which a chain of repo arises in equilibrium.

[32]In Online Appendix D.4, we show that an analogous result holds when ν_2 is positive but sufficiently smaller than ν_B.

Proposition 5. ***Chain of Repos***. *Let $\nu_2 = 0$. There exists $\bar{\delta}_B > \underline{\delta_B} > \delta$ such that the equilibrium features intermediation with a chain of repos if and only if $\delta_B \in [\underline{\delta_B}, \bar{\delta}_B]$ and*

$$\frac{\nu_B}{1-\theta_B} \geq \frac{1}{1-\theta_1} \tag{30}$$

Investors 1 sells all the asset in a repo f_{1B} to B with

$$f_{1B}(s) = \frac{s}{1-\theta_1} \qquad \forall s \in [\underline{s}, \bar{s}] \tag{31}$$

Investor B sells part of the asset in a repo f_{B2} to 2 with

$$f_{B2}(s) = \begin{cases} \frac{p_2(s)}{1-\theta_B} & \text{if } s < s^*_{B2} \\ \frac{p_2(s^*_{B2})}{1-\theta_B} & \text{if } s \geq s^*_{B2} \end{cases}$$

for some $s^*_{B2} \in [\underline{s}, \bar{s}]$ and the remaining part in a spot sale to investor 1.

The lower bound $\underline{\delta_B}$ on δ_B ensures investor 1 prefers to sell the asset in a repo rather than spot to investor B. The upper bound $\bar{\delta}_B$ ensures that the direct gains from trade between investors B and 2 are sufficiently large that B prefers to re-use part of the asset to trade with 2. We actually show that, in equilibrium, investor B must be indifferent at the margin between re-selling collateral spot to investor 1 and selling it in a repo to investor 2. For instance, suppose to the contrary that investor B strictly prefers to re-pledge the collateral to investor 2. Then we show that, at the margin, investors 1 and B would rather engage in a spot trade than in a repo. Intuitively, a marginal switch from a repo sale to a spot sale from 1 to B is beneficial since it frees up some of the segregated collateral, allowing B to borrow more from 2.

Condition (30) ensures that intermediation dominates direct trade between investors 1 and 2. It states that $\frac{1}{1-\theta_1}$, the borrowing capacity of investor 1 per unit of asset, is lower than $\frac{\nu_B}{1-\theta_B}$, the borrowing capacity of investor B with one unit of asset acquired in a repo. Since only a fraction ν_B can be re-used by investor B, his higher creditworthiness must compensate for the cost of segregation.

Finally, observe that the repo contract f_{1B} between investors 1 and B does not reflect any hedging motive since both investors are risk neutral. For investors B and 2, the repo contract is instead essentially the same as in Proposition 1 with $\nu_2 = 0$.

To sum up, intermediation via a chain of repos will arise in equilibrium if a third party is more creditworthy than the natural borrower and more efficient at re-deploying collateral than the natural lender. Our analysis thus shows that repo intermediation arises endogenously out of fundamental heterogeneity between traders. Existing models of repo intermediation typically take the chain of possible trades as exogenous. Our approach is helpful to rationalize several features of the repo market. First, we can explain why intermediating repos is still popular despite the emergence of direct trading platforms. Second, in exogenous intermediation models, dealers typically gain and collect fees by charging higher haircuts to borrowers. In our model, the haircut paid by the borrower to the bank may very well be smaller than the one paid by the bank to the lender. Using data from the Australian repo market, Issa and Jarnecic (2016) show that this is indeed the case in most transactions.

5 Implementation

Ou model captures some key features of the repo contracts traded in financial markets, namely the sale of the collateral which allows lenders to re-use the asset and the recourse nature of the loan. However, the equilibrium contract derived in Section 3 does not exactly correspond to repos traded in practice because the repurchase price varies with the future price $p_2(s)$ of the collateral. In a typical repo, the seller commits to repurchase the asset at a fixed price. In fact, market participants quote the fixed interest rate of the repo, which implicitly defines a constant repurchase price. In this section, we propose a two-step implementation of the equilibrium contract in Proposition 1. First, we show that it can be exactly implemented with a combination of a debt-like contract and a hedging contract. Next, we show that the debt-like contract payoff is very similar to that of fixed repo with default in equilibrium.

5.1 Exact Implementation

We first propose an implementation of the equilibrium contract in Proposition 1 with a combination of the two following contracts

$$d(s) = \begin{cases} \frac{p_2(s)}{1-\theta_1} & \text{if } s \leq s^*(\nu_2) \\ \frac{p_2(s^*(\nu_2))}{1-\theta_1} & \text{if } s > s^*(\nu_2) \end{cases}, \qquad h(s) = \nu_2 \max\left\{p_2(s) - p_2(s^*(\nu_2)), 0\right\} \tag{32}$$

Essentially, the original contract f is split into two contracts d and h. The first contract d is similar to a collateralized debt contract with a fixed repayment in high states and a state contingent payment equal to the maximal pledgeable income in low states. Hence, contract d provides the fixed payoff unless the collateral value is too low. The second contract h is a hedging contract against an increase in the price of the collateral above $p_2(s^*(\nu_2))$. Contract h compensates for the price risk faced by the lender who re-uses collateral.

Suppose that both contracts are sold by investor 1 to investor 2 and backed by the same unit of collateral. By construction, the combination of these two contracts has the same promised payoff as the original contract in Proposition 1. Because the collateral requirement is the same and the punishment for default is linear in the payoffs, neither the borrower defaults, nor the lender fails on contracts d and h. Hence, the combination of the debt-like contract d and the hedging contract h exactly implements the original contract.

5.2 Fixed repurchase price

We now establish the connection between the debt-like contract d and a repo with a constant repurchase price and default in equilibrium. Consider the repo with repurchase price

$$\bar{f} = \frac{p_2(s^*(\nu_2))}{1-\theta_1}$$

This fixed promised payoff of contract $\bar{f}$ is the payoff the debt contract d promises only in states $s \geq s^*(\nu_2)$. When $s \leq s^*(\nu_2)$, the promised payoff $\bar{f}$ exceeds the maximal pledgeable income $p_2(s)/(1-\theta_1)$ and hence the borrower defaults. When the borrower defaults, the lender liquidates the collateral and recovers a fraction α of the shortfall for a total payment of $p_2(s) + \alpha(\bar{f} - (1-\kappa)p_2(s))$, as shown by equation (3). This payoff is represented by the upward-sloping solid line on Figure 4. Observe that although it is also increasing in $p_2(s)$, this payoff is different from the payoff of the debt-like contract d, equal to $p_2(s)/(1-\theta_1)$, represented by the dashed line since there are default costs. These default costs also explain the wedge between the lender's payoff and the cost paid by the borrower in these states. This cost is equal to $p_2(s) + (\alpha + \pi_1)(\bar{f} - (1-\kappa)p_2(s))$ since the borrower loses the collateral and incurs costs proportional to the shortfall. This total cost to the borrower is represented by the dashed and dotted line on Figure 4. The wedge between the lender's payoff and the borrower's cost reflects the deadweight loss from the asset liquidation and the non-pecuniary punishment.

Hence, for $s < s^*(\nu_2)$, the implementation of the equilibrium contract with a repo with a constant repurchase price is only approximate. Still, our argument highlights the feature that limited commitment by the borrower implies default in the low states, thus reducing the payoff of the lender, which becomes state-contingent. With a repo with a constant repurchase price, state contingency arises as a consequence of default. Instead, the debt contract d embeds this state contingency in the contract design ex-ante.[33]

6 Conclusion

We analyzed a simple model of repurchase agreements with limited commitment and price risk. Unlike a combination of a sale and future repurchase in the spot market, a repo contract provides insurance against price fluctuations. Due to the recourse nature of repos as well as the ability to re-use collateral, repos expand the borrowing capacity of investors through a collateral multiplier effect. We showed that the repo haircut is an increasing function of counterparty risk and of the asset inherent risk. Safe assets also command a higher liquidity premium than risky ones. We model repos as recourse loans and allow investors to re-use collateral, thus capturing the distinctive aspects of repos from standard collateralized loans. In addition, our model can explain intermediation whereby creditworthy investors borrow on behalf of riskier counterparties.

[33] In a previous version of the paper, we analyzed the model when investors could only trade contracts with non-state contingent repayment terms. We showed that under some conditions, investors were willing to trade fixed repo contracts that induced default and that gains from re-using collateral were still present.

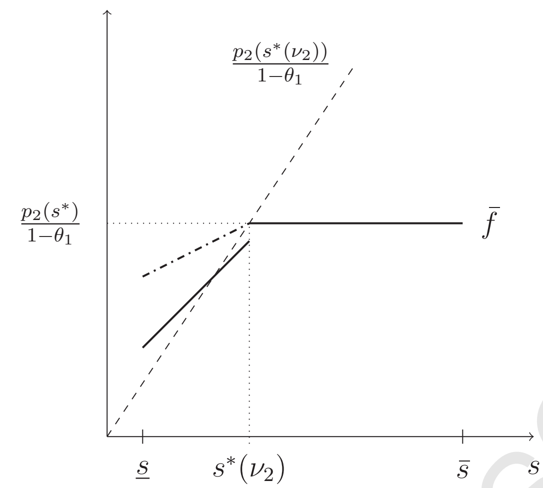

Figure 4: Repo contract with fixed repayment $\bar{f}$

Solid line: lender's payoff. Dashed and dotted line: borrower's cost. Default threshold: $s^*(\nu_2)$.

Our simple model delivers rich implications about the repo market but leaves many venues for future research. We argued that counterparty risk is a fundamental determinant of the terms of trade in repo contracts. In Europe, over the past few years, bilateral repo transactions between banks moved increasingly to the centrally cleared segment of the market (see Mancini, Ranaldo, and Wrampelmeyer, 2015). In this case, clearing implies novation of trades by the central counterparties. Novation bears some similarities with intermediation although terms of trades cannot be adjusted and risk may end up being concentrated on a single agent. We believe our model could be extended to account for this evolution. When it comes to re-use, besides the limit on the amount of collateral that can be re-deployed, we assumed a frictionless process. Traders establish and settle positions smoothly although many rounds of re-use may be involved. Although we did not investigate this aspect in the present work, we believe that in the presence of frictions such as bilateral trading, collateral re-use may contribute to market fragility. This extension would complement the recent literature, such as Biais, Heider, and Hoerova (2018), who have shown the negative impact of spot market fire sales on secured lending markets.

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Appendix A: Default procedure in Master Agreements

This Appendix describes the default procedure as stated in paragraph 10 of the General Master Repo Agreement (GMRA) of the Securities Industry and Financial Markets Association (SIFMA) and the International Capital Market Association (ICMA). The GMRA, governed by English law is the most widely used Agreement. According to Baklanova, Copeland, and McCaughrin (2015), most U.S. repo dealers would rather use SIFMA's Master Repurchase Agreement (MRA), governed by New York state law, for domestic U.S. counterparties but the provisions for default stated in paragraph 11 of the MRA are essentially the same as those in the GRMA.

The default procedure is initiated by one of the parties sending a default notice to the other party, after an event of default occurred. Events of default include, for example, the borrower failing to repay the lender at the agreed date, the lender failing to deliver equivalent securities on the repurchase date, either party failing to transfer margins when margins are due, or either party being in a general state of insolvency with respect to other obligations triggering bankruptcy.

Within 20 days following an event of default, if the event is continuing, the non-defaulting party may specify an Early Termination Date (ETD) for all outstanding obligations pertaining to the contract. These obligations and their values are determined by the non-defaulting party. The latter has to present a statement to the defaulting party detailing the amount due under each obligation, as soon as reasonably practicable. The amounts due in relation to the repo contract are netted against each other. The party with a positive balance is liable to pay the other party the day after the statement is provided and interests shall apply between the ETD and the effective payment date following usual market conventions. A delay between the ETD and the statement date may occur, for instance, as the non-defaulting counterparty needs to value the collateral and possibly other securities used as margins in the transaction, as explained below.

To determine the market value of the securities used in the repo, the non-defaulting party can[34] (1) choose the realized price when (s)he has traded these securities on or

[34] According to the ICMA's summary of the Master Agreement, this value should be a measure of the securities' fair market value, calculated using whatever pricing sources and methods the non-defaulting party deems appropriate in his reasonable opinion. Sources can include, without limitation, securities with similar maturities, terms and credit characteristics. In effect, the calculation of the Net Value is marking-to-model (calculating a theoretical fundamental price).

about the early termination date, (2) use the arithmetic means of price quotes by two or more market dealers, (3) determine a value for the securities whenever s(he) deems it not reasonable to trade the securities at the quotes provided by the market dealers, e.g. if the securities are illiquid. In any of these scenarios all reasonable transaction costs, realized or expected, related to the purchase or sale of these securities (including commissions, fees, any mark-up, mark-down or premium paid for guaranteed delivery) can be deducted from the securities' value to obtain the default market value of the securities.

The defaulting party shall also be liable for the amount of all reasonable legal and other professional expenses incurred by the non-defaulting party in connection with, or as a consequence of, an event of default. In addition, the non-defaulting party can claim any loss or expense s(he) incurred in replacing transactions or in hedging its exposure in connection with the termination of the repo contract. However, no party may claim any amount related to consequential losses or damages due to an event of default. The default market value of the securities and the other expenses specified above play a key role in the determination of the balance due.

Note that the GMRA distinguishes between an outright default by the lender and a settlement fail whereby the borrower expects the lender to return the collateral but with some delay. The latter event is not often treated as an event of default (see paragraph 10(a)(ii)), because delay may result from internal miscommunication on the type of securities to be transferred, or genuine difficulties for the lender to find these securities (see Fleming and Garbade, 2005). Still, settlement fails disrupt the functioning of the repo market and the Treasury Market Practices Group (TPMG) introduced a fails charge for U.S. Treasuries in 2009, while the European Union is planning on introducing similar penalties.

Finally, while it is not explicitly stated in the GMRA it is worth pointing out that, in case a party is insolvent, any unsecured claim resulting from the procedure described above will be treated as a junior claim in the bankruptcy proceedings.

Appendix B: Repos without collateral segregation.

In the model, the lender segregates a fraction $1 - \nu_2$ of the collateral. This is optimal because he is only able re-trade a fraction ν_2 of the asset. In most repos however, no collateral is segregated. This means that lenders can potentially re-use all the asset although they do not always exert this right. A small modification to our setup allows us to rationalize this observation. Assume that the lender can only re-trade with probability ν_2. Instead of being deterministic, the ability to re-trade collateral is now stochastic.[35]

This small difference between the two models has important implications for incentives and collateral segregation. Suppose indeed that investors want to achieve a re-use rate of collateral of ν_2. With a deterministic access to the spot market, lenders only need to be able to re-use a fraction ν_2 of the asset and can segregate the remaining fraction $1 - \nu_2$. With a probabilistic access, borrowers must now grant full re-use rights of collateral, which means that the lender does not segregate any asset. With full re-use rights, the lender is more likely to default since he can abscond with all the collateral. The no-default constraint indeed writes

$$f(s) \geq p_2(s) \tag{33}$$

This condition is tighter than the no-default constraint (6) in our baseline model where the re-usable fraction is ν_2.

In the model with probabilistic access to the spot market, a trade-off thus arises between collateral re-use and the incentives of the lender. Observe indeed that (6) may be violated by the equilibrium contract in Proposition 1. Investors may then consider segregating a fraction $\xi > 0$ of the collateral. This relaxes the no-default constraint of the lender (33), that becomes $f(s) \geq (1 - \xi)p_2(s)$. However, segregating collateral also reduces the re-use rate which falls to $\nu_2(1 - \epsilon)$. This trade-off is moot in the baseline model since then, the no-default constraint of the lender does not bind when investors only segregate the collateral that cannot be re-used.

Under some simple condition though, the equilibrium allocation of the probabilistic model is the same than in the main text. It can only be the case if investors segregate no collateral since otherwise the re-use rate would be strictly lower than ν_2. We are left to derive the condition under which the no-default constraint (33) holds for the equilibrium contract in Proposition 1. The condition is tighter in the highest state $\bar{s}$ where it writes

$$\frac{p_2(s^*(\nu_2))}{1 - \theta_1} + \nu_2(p_2(\bar{s}) - p_2(s^*(\nu_2)) \geq p_2(\bar{s})$$

[35]To preserve the tractability of the new model, one could assume that each investor is made of a continuum of small traders who face idosyncratic access shocks to the spot market. By, the law of large number, investor would be able on average to re-use at most a fraction ν_2 of collateral without segregation.

It can be shown that this condition simplifies to

$$c^2_{2,*} - \omega > a \frac{\bar{s}}{v'(c^1_{2,*})} \tag{34}$$

Condition (34) states that the net promised payoff (the left hand side) exceeds the total value of the collateral pledged in the highest state $\bar{s}$ (the right hand side). If (34) is satisfied, the equilibrium is then identical except that collateral needs not be segregated.

Appendix C: Proofs

C.1 Proof of Proposition 1

We show here that a competitive equilibrium exists where investor 1 only sells one contract, does not sell the asset spot, and investor 2 does not sell a repo contract to investor 1. The proof has several steps. In step 1, we derive the first order conditions for the individual problem (10). In step 2, we determine the conditions under which investors do not wish to trade other repos. This allows us to characterize the equilibrium repo contract and the spot market price in period 2 (Step 3). We finish the proof with a series of claims that we prove in Online Appendix D.2. Claim 3 states that investors do not trade other repo contracts. Claim 4 establishes that, when condition (8) holds, investors do not trade repos inducing default. Finally, Claim 5 states that the equilibrium allocation is unique. To streamline the presentation, we also relegate the proof of some intermediate results to the Online Appendix.

Step 1: First order conditions for the individual choice problem (10).

Let γ^i_1 denote the Lagrange multiplier of the collateral constraint (14) in problem (10), for investor $i = 1, 2$. The variable $\gamma^i_2(s)$ denotes the Lagrange multiplier on the no short sale constraint (17) in period 2 and state s for investor $i = 1, 2$. As we wrote in the main text, it is convenient here to simply write f for the contract f_{12} sold in equilibrium by investor 1 to 2 and $q = q_{12}(f)$ for its price. The first order conditions of problem (10) with respect to $a_1^i, a_2^i(s)$ for $i = 1, 2$ and b^{12}, l^{21} are:

$$-p_1 + \mathbb{E}[p_2(s)v'(c_2^1(s))] + \gamma_1^1 = 0, \tag{C.1}$$

$$q - \mathbb{E}\left[f(s)v'(c_2^1(s))\right] - \gamma_1^1 = 0, \tag{C.2}$$

$$-p_1 + \mathbb{E}\left[p_2(s)u'(c_2^2(s))\right] + \gamma_1^2 = 0, \tag{C.3}$$

$$-q + \mathbb{E}\left[f(s)u'(c_2^2(s))\right] + \nu_2\gamma_1^2 = 0, \tag{C.4}$$

$$-p_2(s)v'(c_2^1(s)) + \gamma_2^1(s) + s = 0, \tag{C.5}$$

$$-p_2(s)u'(c_2^2(s)) + \gamma_2^2(s) + \beta s = 0. \tag{C.6}$$

We will verify that investor 2 does not sell a repo contract to investor 1. Hence the first order conditions with respect to l^{12}, b^{21} for a contract $f_{21} \in \mathcal{F}_{21}(\mathbf{p}_2)$ are not reported above.

Step 2: Conditions on the equilibrium contract f

We determine then the conditions f must satisfy to ensure investors do not trade other repo contracts. Consider an arbitrary repo contract $\tilde{f}_{12} \in \mathcal{F}_{12}(\mathbf{p}_2)$ different from f. For f to be the only traded contract, the price $q_{12}(\tilde{f}_{12})$ must be such that investor 1 does not wish to sell $\tilde{f}_{12}$ and investor 2 does not wish to buy it. Observe that investor 1 prefers not to sell $\tilde{f}_{12}$ as long as its price is lower than $\mathbb{E}\left[\tilde{f}_{12}(s)v'(c_2^1(s))\right] + \gamma_1^1$, where we used the marginal rate of substitution of investor 1, evaluated at the equilibrium allocation, to determine his marginal willingness to sell $\tilde{f}_{12}$. Similarly, investor 2 prefers not to buy this contract if the price is higher than $\mathbb{E}\left[\tilde{f}_{12}(s)u'(c_2^2(s))\right] + \nu_2\gamma_1^2$. Hence, it is possible to find a price $q_{12}(\tilde{f}_{12})$ such that there is no trade in equilibrium of repo contract $\tilde{f}_{12}$ iff the following condition holds:

$$\mathbb{E}\left[\tilde{f}_{12}(s)v'(c_2^1(s))\right] + \gamma_1^1 \geq \mathbb{E}\left[\tilde{f}_{12}(s)u'(c_2^2(s))\right] + \nu_2\gamma_1^2 \tag{C.7}$$

The above inequality can be rewritten, using equations (C.2) -(C.4) above to substitute for γ_1^1 and γ_1^2, as:

$$\mathbb{E}\left[\left(f(s) - \tilde{f}_{12}(s)\right)\left(u'(c_2^2(s)) - v'(c_2^1(s))\right)\right] \geq 0 \tag{C.8}$$

which must hold for all $\tilde{f}_{12} \in \mathcal{F}_{12}(\mathbf{p}_2)$. Similarly there is no trade for repo contract

$\tilde{f}_{21} \in \mathcal{F}_{21}(\mathbf{p}_2)$ sold by investor 2 to investor 1 if:

$$\mathbb{E}\left[\tilde{f}_{21}(s)u'(c_2^2(s))\right] + \gamma_1^2 \geq \mathbb{E}\left[\tilde{f}_{21}(s)v'(c_2^1(s))\right] + \nu_1\gamma_1^1 \tag{C.9}$$

Substituting for γ_1^2 and γ_1^1 using equations (C.2)-(C.4), the condition becomes:

$$\begin{aligned}\mathbb{E}\left[\tilde{f}_{21}(s)\left(u'(c_2^2(s)) - v'(c_2^1(s))\right)\right] \geq & \frac{\nu_1}{1-\nu_2}\mathbb{E}\left[(f_{12}(s) - \nu_2 p_2(s))\left(u'(c_2^2(s)) - v'(c_2^1(s))\right)\right] \\ & - \frac{1}{1-\nu_2}\mathbb{E}\left[(f_{12}(s) - p_2(s))\left(u'(c_2^2(s)) - v'(c_2^1(s))\right)\right]\end{aligned} \tag{C.10}$$

Step 3: Equilibrium contract f and spot market price $\mathbf{p}_2$

We first prove that if (C.8) and (C.10) hold, then we must have $u'(c_2^2(s)) \geq v'(c_2^1(s))$ for all s of any subset of $[\underline{s}, \bar{s}]$ of positive measure.[36] This means that investor 1 never over-borrows in period 1. Suppose this were not the case on a subset $\mathcal{S}_0$ of $[\underline{s}, \bar{s}]$. To establish a contradiction we need to consider two cases.

Suppose first there exists a subset $\mathcal{S}_1 \subseteq \mathcal{S}_0$ of positive measure such that $\epsilon_1 := \min_{s\in\mathcal{S}_1} f(s) > 0$. Consider a repo contract with payoff $\tilde{f}_{12}(s) = f(s) - \epsilon_1$ for $s \in \mathcal{S}_1$ and $\tilde{f}_{12}(s) = f(s)$ otherwise. Condition (C.8) for such contract is clearly incompatible with $u'(c_2^2(.)) < v'(c_2^1(.))$ on $\mathcal{S}_0$.

If no such subset exists, this implies that $f = 0$ almost surely on $\mathcal{S}_0$. Using investor 1 budget constraint (12) in period 2 and states $s \in S_0$ with $f = 0$, we obtain:

$$c_2^1(s) = \omega + p_2(s)(a_1^1 - a_2^1(s)) \geq \omega + p_2(s)(a - a_2^1(s)) = \omega + p_2(s)a_2^2(s) \tag{C.11}$$

The inequality follows from the claimed property that investor 1 does not sell the asset in the spot market in period 1 and thus that $a_1^1 \geq a_0^1 = a$. To derive the third equality, we used the spot market clearing condition in period 2. The investors' short sale constraint (17) in period 2 implies that $a_2^2(s) \geq 0$. Hence, $c_2^1(s) \geq \omega$ for all $s \in S_0$, which together with the assumption that $u'(\omega) > v'(\omega)$ implies that $u'(c_2^2(s)) > v'(c_2^1(s))$. This again contradicts the claim $u'(c_2^1(s)) < v'(c_2^2(s))$ for all $s \in S_0$.

We next show that investor 2 does not hold the asset after period 2, that is $a_2^2(s) = 0$.

[36] All similar assertions in this proof require the qualification "on any subset of $[\underline{s}, \bar{s}]$ of positive measure", though we sometimes omit the statement in what follows.

Using equations (C.5)-(C.6), we have

$$\gamma_2^2(s) = \gamma_2^1(s) + p_2(s)\left[u'(c_2^2(s)) - v'(c_2^1(s))\right] + (1-\beta)s \tag{C.12}$$

Since $\beta < 1$, the Lagrange multiplier $\gamma_2^1(s)$ is non negative and $u'(c_2^2(s)) \geq v'(c_2^1(s))$, it follows that $\gamma_2^2(s) > 0$ in any state s. Since $\gamma_2^2(s)$ is the Lagrange multiplier on the no short sale constraint (17) of agent 2 in period 2 and state s, we have that $a_2^2(s) = 0$. By market clearing, $a_2^1(s) = a_1^1 - a_2^2(s) \geq a$ since $a_1^1 \geq a$ because investor 1 is assumed not to sell the asset spot in period 1 and hence $\gamma_2^1(s) = 0$. Using the budget constraint (12) of investor 1 in period 2, state s and the property that investors do not trade spot in period 2, we get:

$$c_2^1(s) = \omega - b^{12}f(s) + (a_1^1 - a_2^1(s))p_2(s) = \omega - b^{12}f(s) + (a_1^1 - a)p_2(s) \tag{C.13}$$

Plugging this expression in equation (C.5), we obtain:

$$\forall s, \quad p_2(s)v'(\omega - b^{12}f(s) + (a_1^1 - a)p_2(s)) = s \tag{C.14}$$

We now distinguish the cases where the collateral constraint does not bind from the case where it binds. In the first case, the following claim holds, for which we relegate the proof to the Online ppendix.

Claim 1. *If $\gamma_1^1 = 0$, investors reach the first best-allocation. We have that $s^*(\nu_2) \leq \underline{s}$. The spot market price at date 1 is given by $p_1 = E[s]$. Finally, there exists ν^* such that the first-best allocation is attained if $\nu_2 \geq \nu^*$ where*

$$\nu^* = \frac{s^*(0) - \underline{s}}{s^*(0) - (1-\theta_1)\underline{s}} \tag{C.15}$$

We can now focus on the case where the collateral constraint binds, that is $\gamma_1^1 > 0$. This means that $b^{12} = a_1^1$. Suppose there exists a subset $\mathcal{S}_0$ of $[\underline{s}, \bar{s}]$ where $u'(c_2^2(s)) > v'(c_2^1(s))$ for $s \in \mathcal{S}_0$. If not, the analysis in the proof of Claim (1) would appliy and the first-best allocation would be attained in all states. Then for (C.8) to hold, for $s \in \mathcal{S}_0$, $f(s)$ must take the maximum possible value in $\mathcal{F}_{12}(\mathbf{p}_2)$, that is $f(s) = p_2(s)/(1-\theta_1)$.

Then, using equations (C.1)-(C.4), we have

$$\gamma_1^2 = \frac{1}{1-\nu_2}\mathbb{E}\left[(f(s) - p_2(s))\left(u'(c_2^2(s)) - v'(c_2^1(s))\right)\right] \tag{C.16}$$

This proves that $\gamma_1^2 > 0$ since either $u'(c_2^2(s)) - v'(c_2^1(s)) = 0$ or $u'(c_2^2(s)) > v'(c_2^1(s))$ and $f(s) > p_2(s)$ for $s \in \mathcal{S}_0$ The collateral constraint of investor 2, equation (14) binds, that is

$$\nu_2 b^{12} + a_1^2 = 0, \quad \nu_2 b^{12} = a_1^1 - a \tag{C.17}$$

Using our previous result that $b^{12} = a_1^1$, we find that $a_1^1 = b^{12} = a/(1-\nu_2)$. Hence, substituting for b^{12} and a_1^1 in equation (C.14), we obtain:

$$c_2^1(s) = \omega - \frac{a}{1-\nu_2}(f(s) - \nu_2 p_2(s)) \tag{C.18}$$

In the states $s \in \mathcal{S}_0$, using that $f(s) = p_2(s)/(1-\theta_1)$, we can rewrite equation (C.14) to obtain equation (23). This proves that $p_2(s)$ is strictly increasing in s. In addition, since $u'(c_2^2(s)) > v'(c_2^1(s))$ by definition of $c_{2,*}^1$ we have $c_2^1(s) = \omega - aM\frac{p_2(s)}{1-\theta_1} > c_{2,*}^1$.

On the other hand, if for some states $s \in [\underline{s}, \bar{s}]$ the inequality $u'(c_2^2(s)) \geq v'(c_2^1(s))$ holds as an equality, we have $c_2^1 = c_{2,*}^1$. This is compatible with (C.18) if and only if

$$f(s) = \nu_2 p_2(s) + \frac{1-\nu_2}{a}(\omega - c_{2,*}^1) = \frac{p_2(s^*(\nu_2))}{1-\theta_1} + \nu_2(p_2(s) - p_2(s^*(\nu_2)) \tag{C.19}$$

where to obtain the second inequality, we used the definition of $s^*(\nu_2)$. The spot price in period 2 is given by $p_2(s)v'(c_{2,*}^1) = s$.

We then show that there exists a threshold $\widehat{s}$ such that $c_2^1(s) > c_{2,*}^1$ for $s < \widehat{s}$ and $c_2^1(s) = c_{2,*}^1$ for $s \geq \widehat{s}$ and that this threshold is equal to $s^*(\nu_2)$.

Claim 2. In equilibrium, we have that $c_2^1(s) > c_{2,*}^1$ for $s < s^*(\nu_2)$ and $c_2^1(s) = c_{2,*}^1$ for $s \geq s^*(\nu_2)$.

Collecting our previous results, we have show that f is given by equation (26) and the spot market price p_2 is given by (25). The first order conditions with respect to spot trades in period 1 hold with

$$p_1 = \mathbb{E}[p_2(s)v'(c_2^1(s)] + \gamma_1^1 = \mathbb{E}[s] + \frac{1}{1-\nu_2}\mathbb{E}[(f(s) - \nu_2 p_2(s))(u'(c_2^2(s)) - v'(c_2^1(s))]$$

We finish the proof with a series of claims that we prove in Online Appendix D.2.

Claim 3. *Investors do not engage in other repo trades.*

In Claim 3, we verify that investor 1 does not use another repo contract to sell the asset and that investor 2 re-uses the asset through a spot sale rather than a repo sale.

Claim 4. *Investor* 1 *does not sell a repo contract inducing default when condition* (8) *holds.*

In the proof of Claim 4, we show that investor 1 never sells a contract leading him to default in some states. Under condition (8), the non-pecuniary cost offsets the benefits from pledging more income through the recourse component of the loan.

Claim 5. *The equilibrium allocation is unique and the pattern of trades is also unique when* $s^*(\nu_2) \geq \underline{s}$.

In the proof of Claim 5, we show that there the pattern of trades we consider comes without loss of generality to prove that the equilibrium is unique. This concludes the proof of Proposition 1.

C.2 Proof of Proposition 2

We first the derive the expression for the liquidity multiplier. The price $\hat{p}_1$ of a (virtual) asset without collateral value is equal to the highest valuation for that asset among investors 1 and 2. We have

$$\hat{p}_1 = \max\left\{\mathbb{E}[p_2(s)v'(c_2^1(s)), \mathbb{E}[p_2(s)u'(c_2^2(s))]\right\} = \mathbb{E}[p_2(s)u'(c_2^2(s))] \tag{C.20}$$

where the second equality follows from the result that $u'(c_2^2(s)) \geq v'(c_2^1(s))$ for all s, that we proved in Proposition 1. We thus obtain

$$\begin{aligned} \mathcal{L} = p_1 - \hat{p}_1 &= \mathbb{E}[p_2(s)u'(c_2^2(s))] + \gamma_1^2 - \mathbb{E}[p_2(s)u'(c_2^2(s))] \\ &= \frac{1}{1-\nu_2}\mathbb{E}[(f(s) - p_2(s))(u'(c_2^2(s)) - v'(c_2^1(s))] \\ &= \frac{\theta_1}{(1-\nu_2)(1-\theta_1)}\int_{\underline{s}}^{s^*(\nu_2)} \big(p_2(s)u'(c_2^2(s)) - s\big)\, \mathrm{d}G(s) \end{aligned} \tag{C.21}$$

The second line justifies expression (28) in the main text. To obtain the third line, we replaced $f(s)$ by its equilibrium value and we used equation (C.5). Differentiating $\mathcal{L}$ with respect to a, we obtain:

$$\frac{\partial \mathcal{L}}{\partial a} = \frac{\theta_1}{(1-\nu_2)(1-\theta_1)} \int_{\underline{s}}^{s^*(\nu_2)} \left[\frac{\partial p_2(s)}{\partial a} u'(c_2^2(s)) + \frac{\partial [a p_2(s)]}{\partial a} \frac{M p_2(s)}{1-\theta_1} u''(c_2^2(s)) \right] \mathrm{d}G(s) \tag{C.22}$$

where we used the relationship $c_2^2(s) = 2\omega - c_2^1(s)$ and equation (22) for the expression of $c_2^1(s)$. Applying the implicit function theorem to equation (25), we obtain that $\frac{\partial p_2(s)}{\partial a} \leq 0$ and $\frac{\partial [a p_2(s)]}{\partial a} > 0$. Since u is strictly concave, this proves that $\frac{\partial \mathcal{L}}{\partial a} < 0$.

Turning then to the haircut, using the equilibrium expression for the repo contract f and the consumption of investor 1 in period 2, we obtain:

$$\begin{aligned} \mathcal{H} = p_1 - q &= \mathbb{E}[(p_2(s) - f(s)) v'(c_2^1(s))] \\ &= -\int_{\underline{s}}^{s^*(\nu_2)} \frac{\theta_1}{1-\theta_1} s \mathrm{d}G(s) + (1-\nu_2) \int_{s^*(\nu_2)}^{\bar{s}} \left(s - \frac{s^*(0)}{1-\theta_1} \right) \mathrm{d}G(s) \end{aligned} \tag{C.23}$$

where, to derive the final expression, we substituted for $f(s)$ thanks to (26), we used equation (C.5) and the definition of $s^*(\nu_2)$ to introduce $s^*(0)$. Observe that $\mathcal{H}$ only depends on a through $s^*(0)$. Hence:

$$\frac{\partial \mathcal{H}}{\partial a} = -\frac{1}{1-\theta_1} \frac{\partial s^*(0)}{\partial a} [1 - G(s^*(\nu_2))] \tag{C.24}$$

This expression is positive because, from equation (24), $s^*(0)$ is decreasing in a. The effect of counterparty quality θ_1 is negative since:

$$\frac{\partial \mathcal{H}}{\partial \theta_1} = -\frac{1}{(1-\theta_1)^2} \left[\int_{\underline{s}}^{s^*(\nu_2)} s \mathrm{d}G(s) + s^*(0)[1 - G(s^*(\nu_2))] \right] < 0 \tag{C.25}$$

C.3 Proof of Proposition 3

In the proof, we refer to the first asset as asset A and to the second asset as asset B, with dividend, respectively $\rho^A(s) = s$ and $\rho^B(s) = s + \sigma(s - E[s])$. The repo equilibrium with two assets is similar to the one asset case. Investor 1 sells his holdings of asset $i = A, B$ in a repo f^i. Let $s^{**}(\nu_2)$ be the minimal state where the first best allocation can be reached, defined by:

$$\omega + M\frac{a\rho^A(s^{**}(\nu_2)) + b\rho^B(s^{**}(\nu_2))}{(1-\theta_1)v'(c^1_{2,*})} = c^2_{2,*}. \tag{C.26}$$

The repurchase price for the equilibrium repo on asset $i \in \{A, B\}$ is:

$$f^i(s) = \begin{cases} \dfrac{p^i_2(s)}{1-\theta_1} & \text{if } s \leq s^{**}(\nu_2) \\ \dfrac{p^i_2(s^{**}(\nu_2))}{(1-\theta_1)} + \nu_2(p^i_2(s) - p^i_2(s^{**}(\nu_2)) & \text{if } s > s^{**}(\nu_2) \end{cases} \tag{C.27}$$

where p^i_2 is the spot market price of asset $i = A, B$ in period 2, given by:

$$\begin{cases} p^i_2(s)v'\left(\omega - M\frac{ap^B_2(s)+bp^B_2(s)}{(1-\theta_1)}\right) - \rho^i(s) = 0 & s \leq s^{**}(\nu_2) \\ p^i_2(s)v'(c^1_{2,*}) = \rho^i(s) & s > s^{**}(\nu_2) \end{cases} \tag{C.28}$$

Using the derivations in the proof of Proposition 2, the liquidity premium for asset $i = A, B$ is then

$$\mathcal{L}^i = \frac{\theta_1}{(1-\nu_2)(1-\theta_1)}\int_{\underline{s}}^{s^{**}(\nu_2)} \rho^i(s)\left[\frac{u'\left(c^2_2(s)\right)}{v'(c^1_2(s))} - 1\right] dG(s) \tag{C.29}$$

Let us define $l(s) := u'\left(c^2_2(s)\right)/v'(c^1_2(s)) - 1$ and denote $k = (1-\nu_2)(1-\theta_1)/\theta_1$. We obtain:

$$\mathcal{L}^A - \mathcal{L}^B = \frac{1}{k}\int_{\underline{s}}^{s^{**}(\nu_2)} (s - \rho^B(s))l(s)dG(s) = -\frac{\sigma}{k}\int_{\underline{s}}^{s^{**}(\nu_2)} (s - \mathbb{E}[s])l(s)dG(s) \tag{C.30}$$

We need to show that the integral in the expression above has a negative sign. Note that $l(s)$ is strictly decreasing in s on $[\underline{s}, s^{**}(\nu_2)]$. This follows from the fact that $c^2_2(s) = \omega + aM\frac{p_2(s)}{1-\theta_1}$ and $p_2(s)$ is increasing in s, so that $c^2_2(s)$ is increasing in s and $c^1_2(s)$ is decreasing in s, while u' and v' are decreasing since u and v are concave functions. This implies that, for all s, $[l(s) - l(\mathbb{E}[s])][s - \mathbb{E}(s)] \leq 0$. We thus obtain

$$\int_{\underline{s}}^{s^{**}(\nu_2)} (s - \mathbb{E}[s])l(s)dG(s) \leq l(\mathbb{E}[s])\int_{\underline{s}}^{s^{**}(\nu_2)} (s - \mathbb{E}[s])dG(s)$$

Since $\int_{\underline{s}}^{s^{**}(\nu_2)}(s - \mathbb{E}[s])dG(s)$ is negative for any value of $s^{**}(\nu_2) \in [\underline{s}, \bar{s}]$, the expression on the right hand side of the inequality above is negative and so $\mathcal{L}^A - \mathcal{L}^B > 0$, which proves our claim.

The haircut for the equilibrium repo on asset $i = A, B$ can be obtained proceeding along similar lines to the argument in the proof of Proposition 2:

$$\mathcal{H}_i = \mathbb{E}[s] - \int_{\underline{s}}^{s^{**}(\nu_2)} \frac{\rho^i(s)}{1-\theta_1} dF(s) - \int_{s^{**}(\nu_2)}^{\bar{s}} \left[\frac{\rho^i(s^{**}(\nu_2))}{1-\theta_1} + \nu_2(\rho^i(s) - \rho^i(s^{**}(\nu_2)) \right] dF(s) \tag{C.31}$$

Hence, we obtain:

$$\mathcal{H}^B - \mathcal{H}^A = \frac{\sigma}{1-\theta_1} \int_{s^{**}(\nu_2)}^{\bar{s}} (s - s^{**}(\nu_2)) dF(s) - \nu_2 \sigma \int_{s^{**}(\nu_2)}^{\bar{s}} (s - s^{**}(\nu_2)) dF(s) > 0$$

where we used $\int_{\underline{s}}^{\bar{s}} (s - \mathbb{E}[s]) dF(s) = 0$ to obtain the expression on the right hand side. The safer asset A always commands a lower haircut than the risky asset. The inequality is strict if $s^{**}(\nu_2) > \underline{s}$, that is investor 1 is borrowing constrained.

C.4 Proof of Proposition 5

We show in what follows that there exists an equilibrium where investors 1 sell all their asset in a repo f_{1B} to investor B, who in turn re-uses the asset acquired as collateral to sell it partly spot (to some investor 1) and partly in a repo f_{B2} to investors 2. The first order condition for investor 1 spot trade in period 1 is given by (C.1) while that with respect to the repo trade of contract f_{B1} is given by (C.32) below. The first order conditions of investor B with respect to spot trades and repo trades of contract f_{B1} and f_{B2} are then given by equations (C.33)-(C.35) below, and those of investor 2 with respect to spot trades and repo trades of contract f_{B2} are given, respectively, by (C.5) with $\nu_2 = 0$ and (C.36):

$$q_{1B} - \delta \mathbb{E}[f_{1B}(s)] - \gamma_1^1 = 0 \tag{C.32}$$

$$-p_1 + \delta_B \mathbb{E}[p_2(s)] + \gamma_1^B = 0 \tag{C.33}$$

$$-q_{1B} + \delta_B \mathbb{E}[f_{1B}(s)] + \nu_B \gamma_1^B = 0 \tag{C.34}$$

$$q_{B2} - \delta_B \mathbb{E}\left[f_{B2}(s)\right] - \gamma_1^B = 0 \tag{C.35}$$

$$-q_{B2} + \mathbb{E}\left[f_{B2}(s) u'(c_2^2(s))\right] = 0 \tag{C.36}$$

By an argument similar to that in the proof of Proposition 1, we can show that only investor 1, who has the greatest marginal utility for consumption in period 3, carries the asset into period 3. Since $v(x) = \delta x$ here, we obtain $p_2(s) = s/\delta$.

Step 1: Equilibrium Repo Contracts

It follows from the analysis in Proposition 1 and the fact that investors 1 and B are both risk neutral, that the repo contract f_{1B} sold by investor 1 to B must be given by:

$$f_{1B}(s) = \frac{p_2(s)}{1-\theta_1}, \quad \forall s. \tag{C.37}$$

We now characterize the repo contract f_{B2} sold by investor B to investor 2. From equations (C.32) to (C.34), we obtain

$$\gamma_1^B = \frac{\delta_B - \delta}{1-\nu_B}\mathbb{E}[f_{1B}(s) - p_2(s)], \qquad \gamma_1^1 = \gamma_1^B + (\delta_B - \delta)\mathbb{E}[p_2(s)] \tag{C.38}$$

Hence, since $f_{1B}(s) > p_2(s)$ for all s, we get $\gamma_1^B > 0$ and thus $\gamma_1^1 > 0$. This implies that the collateral constraints of both investors 1 and B bind or:

$$a_1^1 = b^{1B}, \qquad a_1^B + \nu_B b^{1B} = b^{B2} \tag{C.39}$$

The first equation states that investor 1 sells in a repo the amount of asset he is endowed plus what he buys in the spot market. The second equation states that investor B re-uses all the collateral acquired in the repo with investor 1, that is $\nu_B b^{1B}$, both to sell it in the spot market (since $a_1^B < 0$ in the claimed equilibrium) and to sell it in a repo to investor 2. By spot market clearing in period 1 we have $a = a_1^1 + a_1^B + a_1^2$ and, since in the claimed equilibrium investor 2 does not trade spot, that is $a_1^2 = 0$, we obtain:

$$a = (1-\nu_B)b^{1B} + b^{B2} \tag{C.40}$$

From equation (C.40) it follows that the possible values of b^{B2} compatible with equilibrium are $[0, \nu_B a]$. The highest possible value is obtained by setting $b^{1B} = a$ and corresponds to the situation where investor B does not re-sell spot any of the re-usable collateral bought in repo f_{1B} so that investor 1 may only sell repo his endowment $a_0^1 = a$. For any given value of b^{B2}, the pattern of trades between investors B and 2 is given as in Proposition

1 with $\nu_2 = 0$. Hence, the equilibrium repo contract f_{B2} sold by B to 2 is:

$$f_{\mathrm{B2}}(b^{B2}, s) = \begin{cases} \frac{p_2(s)}{1-\theta_B} & \text{if } s < s^*(b^{B2}) \\ \frac{p_2\left(s^*(b^{B2})\right)}{1-\theta_B} & \text{if } s \geq s^*(b^{B2}) \end{cases}$$

where $s^*(b^{B2})$ is defined by an expression analogous to (24):

$$c_{2,*}^2 = \omega + b^{B2}\frac{p_2(s^*(b^{B2}))}{1-\theta_B} = \omega + b^{B2}\frac{s^*(b^{B2})}{\delta(1-\theta_B)}$$

For $s \geq s^*(b^{B2})$, investor 2 consumption in period 2 equals the first best level $c_{2,*}^2$.

Step 2: Re-use of collateral

We now determine the quantity b^{B2} sold in the repo by investor B to investor 2. This will also pin down the amount b^{1B} sold in the repo by investor 1 to investor B via equation (C.40). From equations (C.37) and (C.38), we obtain

$$\gamma_1^B = \frac{(\delta_B - \delta)\theta_1}{(1-\nu_B)(1-\theta_1)}\mathbb{E}[p_2(s)] \tag{C.41}$$

while from (C.35) and (C.36) we get:

$$\gamma_1^B = \int_{\underline{s}}^{s^*_{B2}(b^{B2})} \left[u'\left(c_2^2(b^{B2}, s)\right) - \delta_B\right] \frac{p_2(s)}{1-\theta_B}\mathrm{d}G(s) \tag{C.42}$$

where $c_2^2(b^{B2}, s) = \omega + b^{B2} f_{B2}(b^{B2}, s)$. Substituting (C.41) above for γ_1^B in (C.42) yields:

$$\int_{\underline{s}}^{s^*_{B2}(b^{B2})} \left[u'\left(c_2^2(b^{B2}, s)\right) - \delta_B\right] \frac{p_2(s)}{1-\theta_B}\mathrm{d}G(s) = \frac{(\delta_B - \delta)\theta_1}{(1-\nu_B)(1-\theta_1)}\mathbb{E}[p_2(s)] \tag{C.43}$$

This relationship proves the property stated below Proposition 5 that investor B is indifferent between re-using the collateral to sell it in a repo to investor 2 (the left hand side) or to sell it spot to investor 1 (the right hand side). Since

$$\int_{\underline{s}}^{s^*_{B2}(b)} \left[u'\left(c_2^2(b, s)\right) - \delta_B\right] \mathrm{p}_2(\mathrm{s})\mathrm{d}G(s)$$

is strictly decreasing in b, there is at most one value of b^{B2} satisfying equation (C.43).

To establish the claimed property of the equilibrium, we have to prove that the solution lies in the feasible range for b^{B2}, which we showed is $[0, \nu_B a]$. The condition that $b^{B2} \geq 0$ yields

$$\frac{u'(\omega) - \delta_B}{1 - \theta_B} \geq \frac{(\delta_B - \delta)\theta_1}{(1 - \nu_B)(1 - \theta_1)} \tag{C.44}$$

or equivalently

$$\delta_B \leq \bar{\delta}_B := \frac{\frac{\theta_1}{(1-\nu_B)(1-\theta_1)}\delta + \frac{u'(\omega)}{1-\theta_B}}{\frac{\theta_1}{(1-\nu_B)(1-\theta_1)} + \frac{1}{1-\theta_B}} \tag{C.45}$$

Observe in particular that $\bar{\delta}_B \leq u(\omega)$. The condition $b^{B2} \leq v^B a$ is equivalent to:

$$\int_{\underline{s}}^{s^*_{B2}(\nu_B a)} \frac{u'\left(\omega + \nu_B a \frac{s}{\delta(1-\theta_B)}\right) - \delta_B}{1 - \theta_B} s \mathrm{d}F(s) \leq \frac{(\delta_B - \delta)\theta_1}{(1 - \nu_B)(1 - \theta_1)} \tag{C.46}$$

or

$$\delta \geq \underline{\delta}_B := \frac{\frac{\theta_1}{(1-\nu_B)(1-\theta_1)}\delta + \int_{\underline{s}}^{s^*_{B2}(\nu_B a)} \frac{u'\left(\omega+\nu_B a \frac{s}{\delta(1-\theta_B)}\right)}{1-\theta_B} s \mathrm{d}F(s)}{\frac{\theta_1}{(1-\nu_B)(1-\theta_1)} + \int_{\underline{s}}^{s^*_{B2}(\nu_B a)} \frac{1}{1-\theta_B} s \mathrm{d}F(s)} \tag{C.47}$$

with $\underline{\delta}_B \geq \delta$. Since $\delta_B \leq \bar{\delta}_B$ and $\delta_B \geq \underline{\delta}_B$ are respectively equivalent to (C.44) and (C.46), it is easy to see from these expressions that $\underline{\delta}_B \leq \bar{\delta}_B$

Step 3: No other profitable trades

We are left to show that investors do not wish to engage in other trades. The argument in the proof of Proposition 1 still applies to show that investor 2 does not wish to sell a repo to any other investor and that investor B does not sell a repo to 1. Hence, we are left to verify that investor 1 does not wish to bypass investor B. In other words, there should be no repo contract that investor 1 desires to sell to investor 2. Hence, for any $\tilde{f}_{12} \in \mathcal{F}_{12}(\boldsymbol{p}_2)$ the following inequality must hold:

$$\delta \mathbb{E}[\tilde{f}_{12}(s)] + \gamma_1^1 \geq \mathbb{E}\left[\tilde{f}_{12}(s) u'(c_2^2(s))\right] \tag{C.48}$$

Using equations (C.1) to (C.36) to substitute for γ_1^1, we obtain:

$$\mathbb{E}\left[\left(\tilde{f}_{12}(s) - p_2(s)\right)\left(u'(c_2^2(s)) - \delta\right)\right] \leq \mathbb{E}\left[\left(f_{B2}(s) - p_2(s)\right)\left(u'(c_2^2(s)) - \delta_B\right)\right] \tag{C.49}$$

This inequality holds for all $\tilde{f}_{12} \in \mathcal{F}_{12}(\boldsymbol{p}_2)$ if it holds for the highest value of the payoff in $\mathcal{F}_{12}(\boldsymbol{p}_2)$, $\frac{p_2(s)}{1-\theta_1}$. Substituting this value into the inequality above and rearranging terms we obtain:

$$
\begin{aligned}
& 0 \leq \mathbb{E}\left[\left(f_{B2}(s) - \frac{p_2(s)}{1-\theta_1}\right)\left(u'(c_2^2(s)) - \delta_B\right)\right] - \mathbb{E}\left[\left(f_{12}(s) - p_2(s)\right)\left(\delta_B - \delta\right)\right] \\
\Leftrightarrow \quad & 0 \leq \left[\frac{1}{1-\theta_B} - \frac{1}{1-\theta_1}\right] \int_{\underline{s}}^{s^*_{B2}(b^{B2})} \left[u'\left(c_2^2(b^{B2}, s)\right) - \delta_B\right] p_2(s) \mathrm{d}G(s) - \frac{\theta_1(\delta_B - \delta)}{1-\theta_1}\mathbb{E}[p_2(s)] \\
\Leftrightarrow \quad & 0 \leq \left[1 - \frac{1-\theta_B}{1-\theta_1}\right]\gamma_1^B - \gamma_1^B(1-\nu_B) \\
\Leftrightarrow \quad & 0 \leq \frac{\nu_B}{1-\theta_B} - \frac{1}{1-\theta_1}
\end{aligned}
$$

where the last inequality is condition (30).