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Inflation and Deflationary Biases in Inflation Expectations*

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Abstract

We explore the consequences of losing confidence in the price-stability objective of central banks by quantifying the inflation and deflationary biases in inflation expectations. In a model with an occasionally binding zero-lower-bound constraint, we show that both inflation bias and deflationary bias can exist as a steady-state outcome. We assess the predictions of this model using unique individual-level inflation expectations data across nine countries that allow for a direct identification of these biases. Both inflation and deflationary biases are present and sizable, but different across countries. Even among the euro-area countries, perceptions of the European Central Bank's objectives are very distinct.

Keywords: inflation bias, deflationary bias, confidence in central banks, trust, effective lower bound, inflation expectations, microdata.

JEL classification: E31, E37, E58, D84.

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1. Introduction

The potential existence of inflation bias has structurally shaped central bank policies for the past three decades. While traditionally policy makers focused on preventing high inflation, more recently, the deflationary bias has also attracted a lot of attention, due to the occasionally binding effective lower bound on the interest rate. If economic agents lose confidence in the monetary authority pursuing its price stability objective, their inflation expectations may start deviating from the announced medium-term goals, making it more challenging for a central bank to fulfill its mandate. For the case of higher inflation expectations (inflation bias) this was formalized by Barro and Gordon (1983a, 1983b) and Kydland and Prescott (1977) and for the case of lower inflation expectations, when the economy operates at the zero lower bound (ZLB), (Krugman, 1998; Eggertsson, 2006). Nakov (2008) and Nakata and Schmidt (2018) show that deflationary bias can also occur away from the ZLB in the presence of an occasionally binding ZLB constraint.

In this paper we develop a model that can accommodate both inflation and deflationary biases and empirically test the existence of these biases using a unique individual level data. In the model both biases can occur when the monetary authority is subject to an occasionally binding ZLB constraint and when the perceptions of the monetary authority's objective functions—in particular regarding the cyclical position of the economy—do not coincide with the optimal policy. We show that the deflationary bias exists as long as economic agents attach a positive probability to either a negative demand or a supply shock, pushing the economy to the ZLB. One particularly appealing interpretation of such a shock is a trust shock that acts as a demand shock (Bursian and Faia, 2018) but, in general, could be any adverse demand or supply shock. In this simple environment, we formally prove that, as long as households attach a positive probability that such a shock materializes, deflationary bias exists—that is, inflation expectations are below the target level. Conversely, when the perception of the target level for output gap is positive, inflation expectations are above the target level.[1] The model also allows us to explore whether conservative (Rogoff, 1985) or inflation-targeting central banks reduce the magnitude of inflation and deflationary biases.

We test the predictions of the model, by utilizing a novel individual level data in nine different countries with approximately 85,000 observations. We show that both inflation and deflationary biases are present and sizable in households' inflation expectations. Previous research either shows indirect evidence of inflation bias (among others, Romer, 1993, Ireland, 1999) or identifies a bias in inflation expectations, but cannot claim that it is due to losing confidence in the central bank's price objective (among others, Ehrmann et al., 2017 or Souleles, 2004). In contrast, our survey allows us to directly observe the confidence of agents in the central bank achieving the objective of price stability. Furthermore, instead of considering one-year-ahead expectations only, our survey contains five-year-ahead inflation expectations, mapping closer the medium-run horizon centrals banks have in mind. In our model losing confidence is either a product of believing that the economy could be pushed to the ZLB or that the central bank is pursuing a positive output gap target. As a consequence, inflation expectations deviate from the inflation target of the central bank resulting in

[1] Alternatively, they could perceive asymmetric preferences of the central bank over the business cycle. The effect would be isomorphic. See Ruge-Murcia (2003), Gerlach (2003), and Cukierman and Gerlach (2003).

inflation or deflationary bias. Hence, we estimate the implications of losing confidence in achieving the price objective on inflation expectations.

Our estimates show that losing confidence in the price objective increases medium-run inflation expectations by on average 1 percentage point and short-run expectations by about 1/3 percentage point. Previous empirical evidence on inflation bias relied on testing various implications of the Barro and Gordon (1983a, 1983b) and Kydland and Prescott (1977) models using either the U.S. time series data or a cross-country panel. Romer (1993) tests whether more open economies have lower average inflation, as unexpected monetary expansion leads to real exchange depreciation that mitigates more of the effects of monetary expansion. He finds a strong evidence of the link between openness and inflation and, therefore, concludes in support of the time inconsistency models. Ireland (1999) shows that there exists a cointegrating relationship between unemployment and inflation in the United States: The magnitude of inflation bias becomes higher when the natural rate of unemployment rises.[2]

We also provide evidence of the existence of a deflationary bias. Our estimate of the deflationary bias is on average -2/3 percentage point for medium-run expectations and -1/3 percentage point for short-run expectations. For a comparison, Hills et al. (2016) quantifies the deflationary bias within a calibrated DSGE model that matches some of the key features of the U.S. economy up to 45 basis points. In addition, we do not find evidence that inflation and deflationary biases are mitigated in inflation targeting countries. The deflationary bias may be even larger when the central bank pursues inflation targeting and price stability is the primary objective. However, we observe that countries who pursue inflation targeting experience lower inflation expectations and lower dispersion of inflation expectations at the same time. Mertens and Williams (2018) study the effects of the ZLB on the distribution of inflation expectations and interest rates using options data. They find that the decrease in the natural rate of interest leads to a model-consistent effects on forecast densities for interest rates, but the effect on the densities of inflation expectations is more modest.

For the effective conduct of monetary policy within the monetary union member states should share the same perceptions about the central bank objectives. However, there has been evidence that there is a heterogeneity in the perception of the European Central Bank's (ECB) policy across member states.[3] Using our data and based on the identification from the model, we can infer the average differences across euro-area countries' perceptions regarding the ECB's objective function. The empirical results suggest that Germany, Austria, and France have both inflation and deflationary biases, Spain has only the deflationary bias, and Italy has only the inflation bias. Our model suggests that these differences can be either due to different perceptions of the target level of the output gap or a different relative weight attached to output and inflation in the perception of the

[2] Also using the U.S. time series data, Ruge-Murcia (2003) empirically compares different models that result in an inflation bias. His empirical test suggests that the data prefer the restrictions from the model with asymmetric preferences over those arising from the standard Barro-Gordon model. Ruge-Murcia (2004) performs a cross-country evaluation of the asymmetric preference models by evaluating whether the bias is proportional to the conditional variance of the unemployment rate, and finds supporting evidence for the United States and France. Finally, Surico (2008) presents evidence that the inflation bias was positive and significant in the 1960s and '70s, while it was not significant in the subsequent period.

[3] This is highlighted in, e.g., Goldberg and Klein (2011) as well as in several speeches by central bank officials (see e.g., Cœuré, 2019).

ECB's loss function. Specifically, the model implies that when you increase the target level of output, the deflationary bias decreases and the inflation bias increases. However, when you increase the weight on output relative to inflation in the loss function, both inflation and deflationary bias increase. We find that the perception of the target level of the output gap is the highest in Italy. The differences among other countries are driven by the different relative weights attached to output and inflation. The perception of the weight on the output gap in the loss function is highest in Germany and Austria, and is the lowest in Spain. Finally, our results allow us to indirectly observe the perceived inflation target in these countries, where most of them range between 2 and 3 percent in the medium term. This alludes that after 20 years the ECB still faces challenges in convincing households of their objectives.

Although the question in the survey that we use is specifically on confidence in central banks to achieve its medium-run price objective, our analysis also relates to the literature on trust in central banks more generally. The relevance of trust for economic development and particularly for central banks has long been established, but there are only a few papers that address this topic. Ehrmann et al. (2013) and Hayo and Neumeier (2017) investigate the determinants of trust in the ECB. The relationship between trust and inflation expectations has attracted attention only very recently. Ehrmann et al. (2013) rightly noted that, "If low public trust in central banks is associated with higher household inflation expectations, then swings in public trust in the ECB also directly affect its ability to deliver on its mandate, although the empirical relevance of this proposition has yet to be tested.

Bursian and Faia (2018) explore the interaction between trust and macroeconomic outcomes by endogenizing the level of trust within a DSGE model. They test the validity of their model in a vector autoregression setting using aggregate data and conclude that trust affects inflation expectations. Christelis et al. (2016) use one wave from the Dutch CentERpanel to examine the effect of trust on inflation expectations and show that trust increases inflation expectations on average. Mellina and Schmidt (2018) study the interplay between trust in the ECB, public knowledge about the ECB's objectives, and inflation expectations using the German data. More generally, several papers indirectly study trust in central banks in terms of anchored expectations. If economic agents trust central banks, their inflation expectations should be anchored and not deviate from the announced target or to transitory inflation shocks. See, for instance, Dräger and Lamla (2018).

Our paper is structured as follows: Section 2 presents the model and discusses the analytical results. Section 3 details the empirical strategy and hypotheses and section 4 describes the data. Results are presented in section 5, while section 6 contains several robustness checks. Section 7 concludes.

2. Stylized model

This section presents the stylized model and the policy problem of the central bank. After defining the equilibrium, we present some analytical results that serve as a basis for the hypotheses that we outline in the next section.

2.1. Private Sector

The private sector of the economy has the standard New Keynesian structure formulated, as presented in detail in Woodford (2003) and Galí (2008). A representative, infinitely-living household supplies labor in a perfectly competitive labor market and consumes a basket of differentiated goods produced by firms. Firms maximize profits subject to nominal friction, as in Calvo (1983). To derive closed-form results, we put all model equations except the ZLB constraint in semi-loglinear form. In the case where the central bank has no incentives to deviate from the zero output gap objective, our specification is similar to Nakata and Schmidt (2018). The equilibrium conditions of the private sector are represented by the New Keynesian Phillips curve eq. (1) and the consumption Euler equation eq. (2):

$$\pi_t = \kappa y_t + \beta E_t \pi_{t+1}, \tag{1}$$

and

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r^*) + \tau_t, \tag{2}$$

where π_t is the inflation rate in t, y_t denotes the output gap, and i_t is the level of the nominal interest rate. τ_t is an exogenous shock that is detailed in the following text. The natural real rate of interest, r_t, equals $r^* + \frac{1}{\sigma}\tau_t$. $\sigma > 0$ is the intertemporal elasticity of substitution in consumption, $\beta \in (0, 1)$ is the subjective discount factor, and the deterministic steady state of the natural real rate, r^*, is $\frac{1}{\beta} - 1$. The slope of the New Keynesian Phillips curve, κ, equals:

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\eta\theta)}(\sigma^{-1} + \eta), \tag{3}$$

where $\alpha \in (0, 1)$ is the share of firms in a given period that cannot re-optimize their price, $\theta > 1$ denotes the price elasticity of demand for differentiated goods, and $\eta > 0$ is the inverse Frisch elasticity of labor supply.

In principle, any demand shock, like τ_t, or any supply shock that would push the equilibrium to the ZLB would be sufficient for our results to survive.[4] Thus, the agents in the economy can have potentially different perceptions of which shock causes the central bank to be pushed to the ZLB. One alternative formulation of the shock that delivers the existence of deflationary bias, is the trust shock that materializes when there is a lack of trust between monetary authority and economic agents.[5,6]

[4] In the remainder of the paper, we focus on a demand shock. The equilibrium in the case of supply shock is available upon request.

[5] Note that it is important to clarify the difference between credibility, reputation, confidence, and trust in the context of this discussion. See appendix A for exact definitions.

[6] Trust between households and the central bank can be introduced in the model following Bursian and Faia (2018). In their model trust emerges endogenously as an equilibrium of a strategic interaction game featuring moral hazard and uncertainty on policy actions. In this game, households are betrayal averse, and policy makers have incentives to deviate. We can implement a simplified version of this mechanism via an exogenous demand shock, where the main channel through which trust affects macroeconomic dynamics is preserved. We refer to the appendix where we derive the full version of the model with endogenous trust, but our results are not dependent on the exact definition of the shock.

We implement the demand shock, τ_t, as a two-state Markov process. These processes are commonly used in the effective lower bound literature—for example, Eggertsson and Woodford (2003) and Nakata and Schmidt (2018)—to intuitively describe the underlying mechanisms and transmission processes of the shocks. We assume that τ_t takes the value of either τ_H or τ_L where, for simplicity, we refer to $\tau_H > -\sigma r^*$ as the high state and $\tau_L < -\sigma r^*$ as the low state. The transition probabilities are given by

$$Prob(\tau_{t+1} = \tau_L | \tau_t = \tau_H) = p_H, \tag{4}$$

and

$$Prob(\tau_{t+1} = \tau_L | \tau_t = \tau_L) = p_L. \tag{5}$$

p_H represents the likelihood of switching to the low state in the next period when the economy is in a current high state and will be referred to as the *frequency* of the low state. p_L denotes the likelihood of staying in the low state when the economy is in a current low state and will be referred to as the *persistence* of the low state.

2.2. Society's Objective and the Central Bank's Problem

Society's welfare at time t is represented by the expected discounted sum of future utility flows,

$$V_t = u(\pi_t, y_t) + \beta E_t V_{t+1}, \tag{6}$$

where society's contemporaneous utility function, $u(\pi_t, y_t)$, is given by the standard quadratic function of inflation and the output gap augmented for the possibility that the central bank may be inclined to push the output gap below the natural level,

$$u(\pi, y) = -\frac{1}{2}(\pi^2 + \bar{\lambda}(y)^2). \tag{7}$$

As shown by Woodford (2003), this objective function can be derived using a second-order approximation to the household's preferences. In this case we can further set the $\bar{\lambda}$ to be equal to $\frac{\kappa}{\theta}$.

The form of the central bank's objective function is similar to society's but potentially has important differences—as advocated by the time inconsistency and rules versus discretion debates initiated by Barro and Gordon (1983a, 1983b) and Kydland and Prescott (1977). In this paper, we focus on two potential departures,

$$V_t^{CB} = u^{CB}(\pi_t, y_t) + \beta E_t V_{t+1}^{CB}, \tag{8}$$

where the central bank's contemporaneous utility, $u^{CB}(\pi_t, y_t)$, is of the following form:

$$u^{CB}(\pi, y) = -\frac{1}{2}(\pi^2 + \lambda(y - y^*)^2) \tag{9}$$

Although the central bank's objective function resembles the private sector's function, there are potentially two differences. First, $y^* \geq 0$ represents the central bank's desired level of the output gap, which, if positive, can lead to inflation bias, as proven in proposition 2.[7] Second, the relative weight that the central bank assigns to the stabilization of the output gap, $\lambda > 0$, may potentially differ from $\bar{\lambda}$. Also, the central bank is subject to the ZLB constraint,[8]

$$i_t \geq 0. \tag{10}$$

We assume that the central bank behaves discretionary and the commitment option is not available. The central bank chooses the output gap, inflation rate, and the nominal interest rate in each period t to maximize its objective function, subject to the behavioral constraints of the private sector, while considering the policy functions at time $t+1$ taken as a given. Therefore, the central bank is maximizing the following objective:

$$V_t^{CB}(d_t) = \max_{\pi_t, y_t, i_t} u^{CB}(\pi_t, y_t) + \beta E_t V_{t+1}^{CB}(d_{t+1}). \tag{11}$$

subject to the ZLB constrain in eq. (10) and the private-sector equilibrium conditions detailed in eqs. (1) and (2).

We define the Markov perfect equilibrium as a set of time-invariant value and policy functions $\{V_t^{CB}(\cdot), y(\cdot), \pi(\cdot), i(\cdot)\}$ that solves the central bank's problem described in the preceding text, together with society's value function $V(\cdot)$ that is consistent with $y(\cdot)$ and $\pi(\cdot)$. Armenter (2017), Nakata (2018), and Nakata and Schmidt (2018) point out that there are potentially four Markov perfect equilibria in this economy. The equilibrium that is the most relevant to our current study is the *standard* Markov perfect equilibrium. The *standard* Markov perfect equilibrium fluctuates around a positive nominal interest rate and zero inflation and output. The other potentially interesting equilibrium is the *deflationary* Markov perfect equilibrium that fluctuates around a zero nominal interest rate and negative inflation and output.

The *standard* Markov perfect equilibrium is given by a vector $y_H, \pi_H, i_H, y_L, \pi_L, i_L$ that solves the following system of linear equations:

$$y_H = [(1-p_H)y_H + p_H y_L] + \sigma\left[(1-p_H)\pi_H + p_H\pi_L - i_H + r^*\right] + \tau_H, \tag{12}$$

$$\pi_H = \kappa y_H + \beta\left[(1-p_H)\pi_H + p_H\pi_L\right], \tag{13}$$

$$0 = \lambda(y_H - y^*) + \kappa\pi_H, \tag{14}$$

$$y_L = [(1-p_L)y_H + p_L y_L] + \sigma\left[(1-p_L)\pi_H + p_L\pi_L - i_L + r^*\right] + \tau_L, \tag{15}$$

$$\pi_L = \kappa y_L + \beta\left[(1-p_L)\pi_H + p_L\pi_L\right], \tag{16}$$

[7] There are alternatives, and possibly empirically more plausible formulations of inflation bias that build on asymmetric preferences of the central bank or recession aversion. See, for example, Gerlach (2003), Cukierman and Gerlach (2003), and Ruge-Murcia (2003).

[8] For simplicity we consider a ZLB instead of an effective lower bound that is lower than zero. Results in this paper remain unchanged if we would consider a lower bound $i_t < 0$.

and

$$i_L = 0, \tag{17}$$

and satisfies the non-negativity of the nominal interest rate in the high state and non-positivity of the Lagrange multiplier on the ZLB constraint in the low state:

$$i_H > 0, \tag{18}$$

and

$$\lambda y_L + \kappa \pi_L < 0, \tag{19}$$

x_k denotes the value of variable x in the k state where $k \in \{H, L\}$.

Proposition 1. *The standard Markov perfect equilibrium exists if and only if*

$$p_L \leq p_L^*(\Theta_{(-p_L)})$$

and

$$p_H \leq p_H^*(\Theta_{(-p_H)})$$

where i) for any parameter x, $\Theta_{(-x)}$ denotes the set of parameter values excluding x, and ii) the cutoff values $p_L^*(\Theta_{(-p_L)})$ and $p_H^*(\Theta_{(-p_H)})$ are given in appendix B.1.

Proof. See appendix B.1. □

The two conditions guarantee the non-positivity of the Lagrange multiplier in the low state and the non-negativity of the nominal interest rate in the high state. When the frequency of the low state, p_H, is high, the central bank reduces the nominal interest rate aggressively to mitigate the deflationary bias. Thus, for the policy rate to be positive in the high state, p_H must be sufficiently low. With $p_L > p_L^*(\Theta_{(-p_L)})$, inflation and output in the low state are positive when they satisfy the consumption Euler equation and the Phillips curve: When the persistence of the low state, p_L, is high, inflation and output in a current low state are largely dependent on private-sector expectations of output and inflation in the next period's low state. Thus, positive inflation and output in the low state can be self fulfilling. However, such positive inflation and output cannot be an equilibrium because the central bank would have incentives to raise the nominal interest rate from zero in the low state. This incentive manifests itself in the positive Lagrange multiplier in the low state when inflation and output are positive.

The other three possible Markov perfect equilibria in this framework are: the *deflationary* Markov perfect equilibrium, which is briefly discussed in the preceding text; the *ZLB-free* Markov perfect equilibrium, where the ZLB constraint does not bind in either state, and the *topsy-turvy* Markov perfect equilibrium, where the ZLB binds in the high state but not in the low state. The latter two equilibria are less likely to occur. For the purpose of this paper, we do not consider these three equilibria. We focus on the standard Markov perfect equilibrium, as this one seems more relevant for the set of countries that we consider in the empirical part of this paper. All these economies have positive long-run inflation expectations and most surveys indicate that economic agents expect the central bank to eventually raise interest rates. Furthermore, all countries have lowered the interest rate after the end of 2015, which marks the end of the time sample for our empirical analysis. Mertens and Williams (2018) also find empirical support in the United States for this type of equilibrium over the deflationary (liquidity trap) equilibrium.

2.3. Analytical results

When the conditions for the existence of the equilibrium hold, it is possible to show that depending on the values of y^*, we can observe either inflation or deflationary bias. Conditional on y^*, the signs of the endogenous variables can be determined.

Proposition 2. *When the conditions for the existence of the equilibrium hold, we can observe either inflation or deflationary bias depending on the values of y^*.*

(a) For any $\lambda \geq 0$ and $y^* \geq 0$:[9]

- $\pi_H \leq 0$ *iff* $y^* \leq \widetilde{y^*}$ *and* $\pi_H > 0$ *otherwise, where* $\widetilde{y^*} \equiv -\beta p_H r_L (\kappa C)^{-1}$
- $\pi_L \leq 0$ *iff* $y^* \leq \widehat{y^*}$ *and* $\pi_L > 0$ *otherwise, where* $\widehat{y^*} \equiv B r_L (D\kappa\lambda)^{-1}$
- $y_H > 0$
- $y_L \leq 0$ *iff* $y^* \leq \overline{y^*}$ *and* $y_L > 0$ *otherwise, where* $\overline{y^*} \equiv (1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda(-\lambda\left[(1-\beta)C + (1-\beta p_L)\right]\kappa)^{-1} r_L$

(b) For any $\lambda \geq 0$ and $y^* = 0$: $\pi_H \leq 0$, $y_H > 0$, $i_H < r_H$, $\pi_L < 0$, and $y_L < 0$.

(c) With $\lambda = 0$: $\pi_H = 0$.

Proof. See appendix B.2. □

In this proposition we can observe the interaction between the shock and the perceived target for the output gap, y^*. As long as $y^* < min\{\widehat{y^*}, \overline{y^*}\}$, we observe that output and inflation are below target values in the low state, as the ZLB constraint is binding and monetary policy cannot offset the shock. However, higher y^* reduces these effects, and if $y^* \geq \widehat{y^*}$, inflation becomes positive. In the high state, firms lower prices because of a positive probability of τ_L (low state) that leads to a reduction in the expected marginal costs of production. This raises the expected real interest rate that incentivizes households to postpone their consumption plans. These anticipation effects are mitigated by the central banks' lowering of nominal rates. In the literature, this effect is usually referred to as deflationary bias and alone causes inflation in the high state to be negative. With $y^* > 0$ we have an additional effect that, in equilibrium, raises inflation and leads to inflation bias if $y^* \geq \widetilde{y^*}$. As the central bank would like to stabilize the output around y^*, this raises inflation

[9] We use the following definitions in this proposition: $B = \kappa^2 + \lambda(1-\beta(1-p_H))$, $C = \frac{(1-p_L)}{\sigma\kappa}(1-\beta p_L + \beta p_H) - p_L$, and $D = -1 - C$.

because of a tradeoff between inflation and output gap stabilization. Output in the high state is positive, irrespective of the value of y^*. To be precise, both effects of y^* and r_L lead to higher output (see propositions 4 and 5). These mechanisms are consistent with those described in the inflation bias and deflationary bias literature. As described in the last part of proposition 2, when setting $\lambda = 0$ or in other words appointing a conservative central banker (Rogoff, 1985) there are no inflation or deflationary biases.

We now further establish several results on how the degree of conservatism affects endogenous variables in both states.

Proposition 3. *How the degree of conservatism affects endogenous variables depends on the values of y^*. Higher conservatism (lower λ) reduces the absolute distance of inflation from 0 irrespective of y^*.*

(a) For any $\lambda \geq 0$ and $y^* \leq \widetilde{y^*}$: $\frac{\partial \pi_H}{\partial \lambda} \leq 0$, $\frac{\partial \pi_L}{\partial \lambda} \leq 0$, $\frac{\partial y_L}{\partial \lambda} \leq 0$, and $\frac{\partial y_H}{\partial \lambda} < 0$.

(b) For any $\lambda \geq 0$ and $y^* > \widetilde{y^*}$: $\frac{\partial \pi_H}{\partial \lambda} > 0$, $\frac{\partial \pi_L}{\partial \lambda} > 0$, $\frac{\partial y_L}{\partial \lambda} > 0$, and $\frac{\partial y_H}{\partial \lambda} > 0$.

Proof. See appendix B.3. □

This proposition states that, as the central bank cares relatively more about inflation, both inflation and deflationary biases that can occur in the high state will be lower, and inflation will move closer to zero in the high state. When deflationary bias prevails ($y^* \leq \widetilde{y^*}$) and inflation in the high state is negative, then a lower value of λ increases inflation. However, when inflation bias prevails ($y^* \geq \widetilde{y^*}$) and inflation in the high state is positive, then a lower value of λ decreases inflation in this state.

Proposition 4 details the effect of the y^* on expectations:

Proposition 4. *For any $y^* \geq 0$: $\frac{\partial \pi_H}{\partial y^*} > 0$, $\frac{\partial \pi_L}{\partial y^*} > 0$, $\frac{\partial y_H}{\partial y^*} > 0$, and $\frac{\partial y_L}{\partial y^*} > 0$.*

Proof. See appendix B.4. □

Higher y^* increases expectations of inflation in both states. This is a standard result in the inflation bias literature. Note that, in the case where deflationary bias is present, an increase in y^* means that the deflationary bias is reduced or even that expectations are now in the region of inflation bias.

Proposition 5. *For any τ_L: $\frac{\partial \pi_H}{\partial \tau_L} > 0$, $\frac{\partial \pi_L}{\partial \tau_L} > 0$, $\frac{\partial y_H}{\partial \tau_L} < 0$, and $\frac{\partial y_L}{\partial \tau_L} > 0$.*

Proof. See appendix B.5. □

The effects of the size of the shock, τ_L, on expectations is described in proposition 5. The increase in τ_L would increase inflation expectations in both states, while it would decrease output in the high state and increase the output in the low state.

Next, we turn out attention to p_H. In the next proposition, we focus on the effect on π_H, which is the main object of the empirical part of our study.

Proposition 6. *For any p_H that satisfies conditions for existence of equilibrium $\frac{\partial \pi_H}{\partial p_H} < 0$ and for any p_L that satisfies the existence conditions $\frac{\partial \pi_H}{\partial p_L} < 0$.*

Proof. See appendix B.6 □

As we can see in proposition 6, the effect of p_H on inflation expectations is always negative. Thus, when there is a higher probability of the shock, which would push the economy to the ZLB, the inflation expectations are lower and the deflationary bias is higher.

We show that the effects of p_L and p_H on π_H are very similar, as the effect of p_L is negative for π_H. A higher value of p_L will increase deflationary bias (or decrease inflation bias) in both the high and low state. As discussed in the preceding text, even in the high state, firms are aware that there is a possibility of having a bad shock and falling into a low state. When firms know that the low state will last even longer, they are likely to further lower prices, given the expected fall in marginal costs of production will persist for a longer duration. The central bank will then have to mitigate the anticipation effects by further lowering the nominal interest rate. This would lead to more deflationary bias. In the low state, an increase in the persistence will require a stronger response from the central bank to stabilize the economy. We would expect lower nominal interest rates and would expect inflation to fall in the low state. Thus, increasing the persistence of the low state would lead to further deflationary bias.

Corollary 1 summarizes the results for p_H:

Corollary 1. *We can observe the following effects on π_H depending on the level of y^*:*

(a) For any $y^* \leq \widetilde{y^*}$: $\pi_H \leq 0$ and $\frac{\partial \pi_H}{\partial p_H} < 0$ and $\frac{\partial \pi_H}{\partial p_L} < 0$ and $\frac{\partial \pi_H}{\partial \lambda} \leq 0$ and $\frac{\partial \pi_H}{\partial y^*} > 0$.

(b) For any $y^* > \widetilde{y^*}$: $\pi_H > 0$ and $\frac{\partial \pi_H}{\partial p_H} < 0$ and $\frac{\partial \pi_H}{\partial p_L} < 0$ and $\frac{\partial \pi_H}{\partial \lambda} > 0$ and $\frac{\partial \pi_H}{\partial y^*} > 0$.

Proof. See proofs of the propositions in the preceding text. □

This corollary states that there are two regions of y^*. In the first region, when $y^* \leq \widetilde{y^*}$, π_H is nonpositive. Collorary 1 shows that this will also be coupled with a negative effect of p_H. Thus, in this case, we have a deflationary bias and higher p_H that further increases the deflationary bias. For this case, we also have that higher conservatism and higher y^* decreases the inflation bias, as formulated in propositions 3 and 4. When y^* is larger than $\widetilde{y^*}$, we have the case where inflation bias exists and a higher p_H and higher degree of conservatism decrease the amount of inflation bias, while higher y^* increases the inflation bias.

3. Hypotheses and Empirical Strategy

This section details the hypotheses that will be tested using survey data and explains the empirical strategy. The first hypothesis states that both inflation and deflationary biases can occur when economic agents assign either a positive probability of hitting the ZLB constraint (deflationary bias) or when they perceive y^* to be positive (inflation bias). As monetary authorities in all countries in the sample decreased the interest rates after the end of the sample, we consider that our sample is best described by a *standard* Markov perfect equilibrium in a high state, given that, in the low state, the equilibrium features interest rates at the effective lower bound. We assume that households that take part in this survey use the above model to forecast inflation and that they know the correct structural parameters, but they form their own perceptions of the four non-structural parameters that influence inflation expectations and could lead to lower confidence in the central bank achieving its inflation target. The four non-structural parameters are the (perceived) probability of entering the low steady state (p_H), the probability of staying in the low steady state (p_L), the degree of conservatism (λ), and the output gap target level (y^*). This will guarantee that the heterogeneity of their responses can be explained using differences in perceptions of these parameters.

The survey asks "How confident, if at all, are you that the central bank is currently pursuing the correct policies in order to meet its target of price stability (that is, inflation around [target]) over the medium term (that is, the next 3 - 5 years)?" We assume that there are two possibilities, within the model, that lead the economic agents to reply that they are not confident that the central bank is pursuing the right policies to meet its inflation target. The first possibility is due to the perception of y^* being positive. In cases where economic agents believe that $y^* > 0$, they would expect higher than target inflation, which actually leads to higher inflation. Thus, the central bank would not be pursuing the right policies to achieve the inflation objective. The second possibility is when the agents perceive a positive probability of the shock, (p_H), that could push the economy to the ZLB. As we saw in the theoretical model, this leads to inflation expectations that are lower than the inflation target, which causes the actual inflation to be below the target.[10] Once again, in this case, the central bank is not pursuing the right policies to meet its inflation objective—inflation is too low—and thus they should answer that they are not confident that the central bank will achieve its inflation objective.[11] The exact size of these effects will depend on the perceptions of λ. For simplicity of the analysis in the empirical part of this paper, we will assume that p_L and p_H are fixed and known and that all consumers will have the correct perceptions of the probability and average duration at the ZLB, as it is not possible to distinguish the effects between all non-structural parameters. Thus, we will explain the heterogeneity of inflation expectations with λ and y^*. This choice is supported by the fact that we control for the macroeconomic conditions in respective countries, which at least partially controls for p_L and p_H.[12]

Hypothesis 1. *Depending on y^*, for $\lambda, p_H > 0$ there exists either deflationary or inflation bias, as described in proposition 2.*

Testing this hypothesis entails examining whether households with low confidence have inflation expectations either lower or higher than their inflation objective, depending on their perceptions of

[10] In practice and in the model that we outlined, it could also be the combination of both factors. In this case the resulting inflation expectations will exhibit the net effect.

[11] There were several proposals in the literature and policy circles on how to alleviate this concern. Among others, one proposal suggested that (temporary) price level targeting could alleviate these concerns. In fact, a simple way to achieve the target would be to set $\lambda = 0$. See Nakata and Schmidt (2018).

[12] Note that in a more realistic model, p_H would depend on the distance of interest rate from the ZLB (see Nakata, 2017), so it is reasonable to expect that this probability is state(time)-dependent, although our model assumes that it is a constant.

y^*. As the structural parameters have differential effects on π_H, we can furthermore test the effects of y^* and λ.

The Hypotheses 2 and 3 test the difference of inflation expectations within the monetary union, and study the differential effects of λ and y^*. Households within the European Monetary Union have been experiencing common monetary policy since the establishment of the ECB in 1999. There are two potential sources of the difference in inflation expectations within a monetary union: λ and y^*. The model suggests a clear distinction between these two effects, as they imply a differential effect on deflationary bias when the level of confidence decreases.

Hypothesis 2. *Within a monetary union, differences in inflation expectations, could be explained by the perception of* y^*. *Within a monetary union, the perception of* λ *is the same across all countries.*

A cross-country comparison of inflation expectations, given the level of confidence, implies that an increase in λ reduces both inflation and deflation bias, while an increase in y^* reduces deflationary bias and increases the inflation bias. Therefore, when a country is compared to a reference country, a country with lower inflation bias does not necessarily also have lower deflation bias.

Hypothesis 3. *Level of confidence varies across EMU countries: Countries with higher confidence have lower variance of inflation expectations.*

Testing this hypothesis involves testing whether unconditional variance of inflation expectations (disagreement of inflation expectations around the target level) is lower in countries, where the level of confidence is higher (lower p_H). This examines the relevance of inflation and deflationary bias for the distribution of inflation expectations.

4. Data

Our main dataset consists of individual level data across nine countries from 11 survey waves from the end of 2013 to the end of 2015 with a total of 84,735 observations.[13] To analyze the data, we combine countries and survey waves to form a panel of data across individuals, countries, and time. The microdata we use in this paper is collected by YouGov, an online research center focusing on the perceptions and opinions of individuals across the world.[14]

The survey has information on inflation expectations, confidence in central bank, trust in the government, and general characteristics of the individual. To measure inflation expectations, the

[13] More specifically, our country sample includes individuals living in Austria, France, Germany, Hong Kong, Italy, Singapore, Spain, Switzerland, and the United Kingdom. Surveys have been conducted beginning of each quarter.

[14] YouGov conducts surveys using Active Sampling: It predetermines who is allowed to participate in the survey in order to maximize the representativeness of the sample. Each survey is anonymous and takes under 10 minutes to complete and YouGov provides a monetary incentive for completing the survey. After surveys are conducted, the data is statistically weighted to correspond to the national population profile of all adults over the age of 18. These weights are calculated based on Census data, large-scale random probability surveys, election results, and national statistic agencies. YouGov, specifically, weights based on age, gender, social class, region, party identity, and the readership of individual newspapers. YouGov's results have been shown to be comparable in accuracy to other major polling entities and have a high predictive accuracy for actual outcomes in national and regional elections. YouGov operates in France, Germany, and the United Kingdom. They use a partner to conduct surveys in Austria, Hong Kong, Italy, Singapore, Spain, and Switzerland. YouGov's public opinion research is conducted according to the Market Research Society guidelines.

survey asks the participants to provide their short-run inflation expectations,(what they expect inflation to be 12 months from the date of the survey) and their medium-run inflation expectations, (what they expect inflation to be five years from the date of the survey). To measure central bank confidence, the survey asks the following question: "How confident, if at all, are you that the central bank is currently pursuing the correct policies in order to meet its target of price stability (i.e., inflation around [target]) over the medium term (i.e., the next 3 - 5 years)?" The individual then chooses between "Not at all," "Not very," "Fairly confident," or "Very confident." To measure trust in the government, the survey asks the individual the following question: "To what extent do you agree or disagree with the following statement? 'I think that the government is currently following the right economic policies for [Country]'." The participants then choose between the following answers: "Strongly disagree," "Tend to Disagree," "Neither agree nor disagree," "Tend to Agree," and "Strongly agree." General characteristics that YouGov surveys include general characteristics, including the participants gender, age, and region in which they are currently living.[15]

The macroeconomic variables we use are annualized CPI inflation rate, short-run interest rate, real GDP growth, and the unemployment rate. We collect these variables from the ECB for all European Union (EU) countries and from the central bank websites for all countries outside of the European Union except Singapore, whose data come from the Singapore Government Department of Statistics. We classify a country as inflation targeting based on the central bank mission, as stated on the country's central bank website. Those countries who state a specific number as their inflation target are labeled as inflation targeting in our sample.[16]

4.1. Summary Statistics

In the following text, we introduce our confidence measure and provide first evidence on the link between confidence and inflation expectation.

Roughly 60 percent of the survey population is not confident with respect to the central bank in their country meeting the inflation close to their target level.[17] This is not unexpected, as this period is dominated by very low inflation rates in Europe. Furthermore, the values are comparable to the EU Eurobarometer survey. The Eurobarometer survey asks "For each of the following institutions, please tell me if you tend to trust it or tend not to trust it or don't know?". During that period roughly 30 percent of the respondents trusted the ECB which is very close to the average of 28 percent we report. Interestingly, we can observe that we have some country heterogeneity. While in most of the EMU countries, the majority is not confident, in countries like United Kingdom and Switzerland, the majority is confident that the central bank will meet their objective.

For both short- and medium-run inflation expectations, the means differ across the range of confidence in the central bank. Using a t-test to compare the mean inflation rate and a Kruskal-Wallis equality of populations rank test to compare the median inflation rate across confidence level,

[15] With respect to the ordering of the questionnaire, the survey starts with the questions on inflation expectations, followed by the question on the confidence in the central bank and the question on the government's economic policy.

[16] We also classify the ECB as an inflation targeting central bank.

[17] See Table A.2 for details. It contains the shares of consumers that are confident for overall and specific characteristics (age, gender, etc.), countries and over time.

we find that, for each confidence level, the mean and median inflation expectation in both short- and medium-run are statistically significant at the 99 percent confidence level when compared to the remaining levels of confidence.[18] We furthermore observe that individuals that are not confident have, on average, a 1 percent higher short-run inflation expectation and almost a 1.5 percent higher medium-run inflation expectation. From these observations, so far, we can only infer that the majority tend to have higher inflation expectations when confidence is lost. However, our model predicts that losing confidence, under certain assumptions, may also lead to lower inflation expectations.

To gain some insight into this potential pattern, we present two graphs. First, we look at the share of "not confident" individuals across the spectrum of inflation expectations, and second, we compare how the distribution of inflation expectations changes depending on whether people are confident or not.

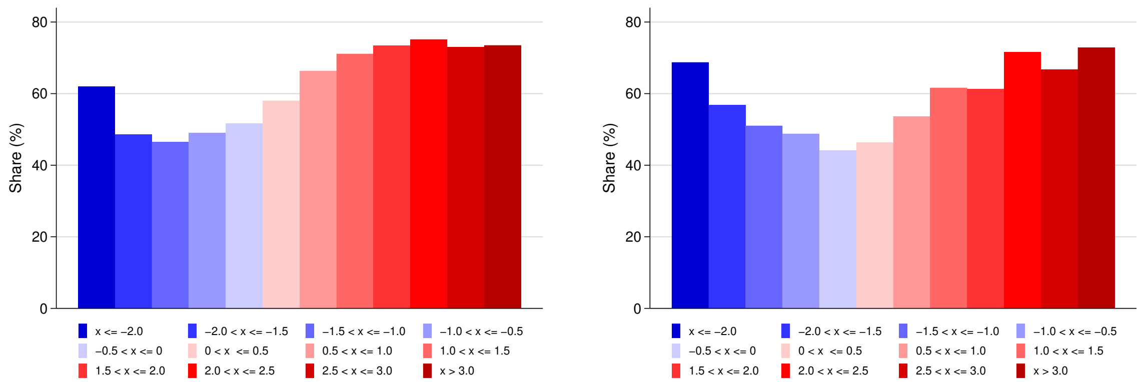
For the first approach, we calculate the share of people that are confident for intervals of inflation expectations. If we plot the share of confidence across different bins of inflation expectations and there is no deflation bias, we should expect a rising share of people being not confident with rising inflation expectations. However, if a deflation bias exists, we should expect a u-shape relationship. Close to the target inflation rate, the confident shares should be very high. If we move away from this area there should be fewer people who are confident. Most people who have no confidence could expect either very high inflation or very low inflation. Figure 1 shows the resulting distribution for the share of people who are not confident across different levels of inflation expectations for short- and medium-run inflation expectations. We can clearly observe the u-shaped pattern. Most people that are confident have inflation expectations around 1.0 percent to 2.0 percent. Hence losing confidence leads to having high expectations >2.5 percent or very low expectations <0 percent which clearly indicates the potential of generating both inflation as well as deflation bias.

Figure 2 shows the distribution of expectations of confident and not confident individuals for short- and medium-run expectations. Not-confident individuals are presented in pink columns. We can observe that losing confidence moves the distribution substantially to the right and causes the well-documented inflation bias. However, on closer inspection, we can observe that there is a movement to lower inflation expectations as well. This movement is clearly visible for medium-run inflation expectations at the bracket 0-1 percent.

In both graphs we observe that there is a substantial inflation bias of "not-confident" individuals as there is a higher amount of people with higher expectations. However, of particular interest and in line with our model predictions, we also observe an increasing amount of individuals who have lower inflation expectations as a response to losing confidence and hence have a deflationary bias. This deflationary bias is particularly strong for medium-term inflation expectations.

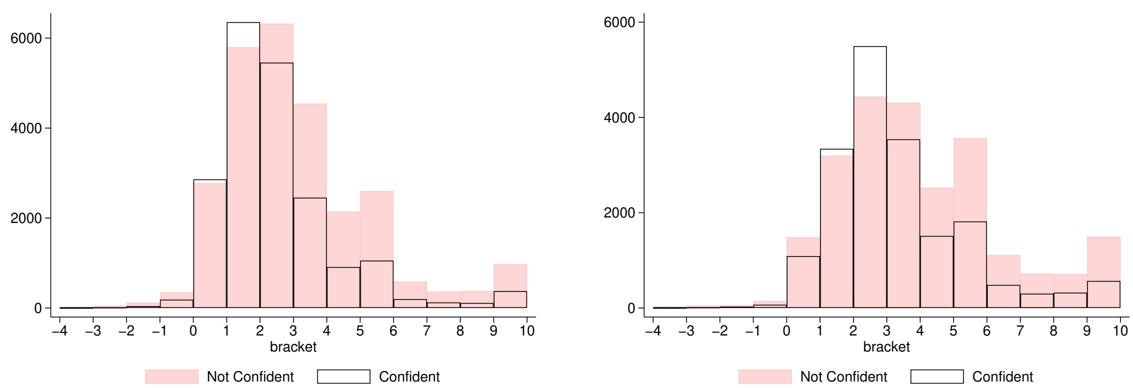
[18] In Table A.1, we compare the means for short-run inflation expectations and medium-run inflation expectations by the range of confidence in the central bank and compare these means across political orientation, inflation targeting countries, trust in government, countries, gender, age, and survey wave.

Figure 1. Confidence Level Shares By Distance from Inflation Target For Not Confident Sample



(a) Short Run (b) Medium Run

Figure 2. Distribution of Short- and Medium-run Inflation Expectations of Confident and Non-Confident Individuals



(a) Short Run (b) Medium Run

Notes: The black outlined clear bars show the distribution of confident individuals' inflation expectations, while the pink bars show the distribution of expectations of not-confident individuals.

5. Results

In this section we use econometric means to test our hypotheses derived from our theoretical model. As already indicated in the previous section there seems to be support for the conjecture that lack of confidence is linked to both, an inflation as well as a deflationary bias.

To test hypotheses 1 and 2 we use the following equation and adjust the set of explanatory variables as necessary:

$$\pi^e_{i,j,t} = \alpha + \beta NC_{i,j,t} + \delta_x \pi^{e,x}_{i,j,t} + \Phi_x \pi^{e,x}_{i,j,t} NC_{i,j,t} + \Gamma Z_{i,j,t} + \mu_j + \nu_t + \varepsilon_{i,j,t}; x \in H, L;$$

where the subscripts i, j, t denote individual i, country j, and time t. π^e represents medium-term expectations (3-5 years ahead) or short-term expectations (1-year ahead) in some of the robustness checks. NC (not confident) captures whether the individual is confident in the central bank achieving inflation close to 2 percent. $\pi^{e,H}_{i,j,t}$ and $\pi^{e,L}_{i,j,t}$ represent the above and below thresholds that are explained in the following text. The vector Z contains several control variables including individual characteristics as well as macroeconomic control variables. μ and ν are region and year fixed effects and ε is the i.i.d error term. For the errors we use two-way clustering in region and time.

We control for socioeconomic characteristics, such as gender and age; the macroeconomic situation in the country proxied by the short-term interest rate; economic growth; and the inflation rate. Literature on survey inflation expectations commonly identifies the bias in inflation expectations. Sociodemographic characteristics and macroeconomic conditions explain part of this bias (see, for example Ehrmann et al., 2017 or Souleles, 2004). Finally, we control for trust in the government. As there might be a lack of trust in institutions in general, we need to make sure that we only identify the lack of confidence in central banks and do not confuse it with attitudes toward other policy making bodies.

As one-year-ahead expectations may be strongly influenced by transitory shocks, we opt to focus on medium-run expectations that are not prone to such short-term movements and may reflect longer-run effects such as changes in policy strategy or objectives. Results for short-run expectations are presented in the robustness section.

In table 1, we test hypothesis 1. Hypothesis 1 is tested by adding the not-confident indicator dummy and by interacting this indicator with thresholds for inflation expectations below 1.5 percent and above 2.5 percent. Therefore, we consider expectations between 1.5 percent and 2.5 percent, which are centered around the official target for inflation targeting central banks, to be the reference level region to which we will compare expectations in the other two regions. In effect, we calculate the average effects of not confident in three regions of inflation expectations. Notably this is an ad-hoc threshold indicating a moderate deviation of expectations from the inflation target. We report a number of robustness checks and alternative definitions of these thresholds, among others, we re-estimate the same equation with thresholds of 1 percent and 3 percent and where we calculate the country-specific thresholds that may not be necessarily centered around the official target.[19] As our model predicts the possibility of both inflation and deflationary biases, we expect the above interaction term to be positive (adding to the inflation bias) and the below interaction term to be negative (adding to the deflationary bias). This hypothesis would be rejected if the below interaction term is positive or insignificant indicating that a loss in confidence results in an inflation bias independent the probability of the low state. In such a case, this result would confirm the standard inflation bias prediction. Notably, being able to identify and quantify the inflation bias as such would still be a substantial contribution. Similarly, if the above interaction term is not significant or negative this outcome would imply a rejection of the hypothesis about inflation bias.

In column (1) of table 1, we include, aside from the control variables and the time and region fixed effects, the main variable of interest, but we do not include the interaction terms for different

[19]Results are reported in the robustness section in table 4.

Table 1: Medium-Run Expectations

	(1)	(2)	(3)	(4)
Not Confident	1.107***	1.102***	-0.401***	-0.526***
	(0.0519)	(0.0518)	(0.0345)	(0.135)
Gov. Mistrust	0.231***	0.274***	0.157***	0.156***
	(0.0534)	(0.0538)	(0.0484)	(0.0485)
Inflation Target		-1.781***	-0.425**	0.152
		(0.192)	(0.193)	(0.197)
Below			-1.357***	-1.175***
			(0.0270)	(0.0667)
Below_nc			-0.120***	-0.122***
			(0.0352)	(0.0351)
Above			3.456***	4.119***
			(0.0847)	(0.0866)
Above_nc			1.500***	1.533***
			(0.0764)	(0.0782)
Nc_it				0.131
				(0.142)
Below_it				-0.202***
				(0.0688)
Above_it				-0.744***
				(0.105)
Observations	46,785	46,785	46,785	46,785
R-squared	0.073	0.074	0.278	0.278
MacroVar	Yes	Yes	Yes	Yes
Socio	Yes	Yes	Yes	Yes
Region FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes

Two-way (region time) clustered standard errors in parentheses. "Above" represents dummy variable denoting a threshold of 2.5 percent and "Below" a threshold of 1.5 percent respectively. Medium-run expectations are 3-5 years ahead. All regressions include regional and year fixed effects. Errors are two-way clustered over time and region. "Below_nc" and "Above_nc" represent interaction terms between "Not Confident" and the "Above" or "Below" threshold. "Nc_it" denotes the interaction term between "Inflation Target" and "Not Confident". "Above_it" and "Below_it" are the interaction terms between "Inflation Targeting" and the "Above" and "Below" threshold variables. "FE" denotes fixed effects. "Socio" stands for control variables on socioeconomic characteristics and "MacroVar" for macroeconomic control variables.

*** $p<0.01$

** $p<0.05$

* $p<0.1$

regions of distribution. As such, these results represent the average results for the whole distribution of inflation expectations. We can show that the coefficient estimate of being not confident in the central bank is statistically highly significant and has a positive sign. Losing confidence in a central bank on average generates the classical result of an inflation bias. Notably, we not only provide empirical evidence of this result, but are now in a position to exactly quantify inflation bias as resulting in a 1.1 percent higher medium-run inflation expectations. Given that inflation targets are around 2 percent in our country sample this number reflects a substantial bias. However, our analysis below confirms that there is a significant heterogeneity in inflation expectations and that not all households exhibit inflation bias.

In column (2) we add a dummy variable for inflation-targeting countries. Our main variable of interest remains highly significant. Looking at the relevance of inflation targeting, the corresponding coefficient estimate is negative implying that inflation targeting reduces medium-run expectations by 1.8 percent. Hence, we can re-confirm the effect of inflation targeting of reducing inflation expectations and bringing them closer to the target level.

In column (3) we dig deeper and test hypothesis 1 that is, the simultaneous existence of an inflation and deflationary bias. Therefore, as described above, we add the dummy variables "Above" and "Below" and the corresponding interaction terms of above and below with being not confident (denoted as Above_nc and Below_nc). Those interaction terms are of particular interest here.

The first important observation regarding this estimation setup is the sharp increase in the explanatory power of the specification. While in the previous columns the R^2 was approximately 0.07, when we account for deflationary and inflation bias, the R^2 increases to 0.33. Hence, accounting for this non-linearity improves the quality of this estimation substantially.

Looking at the estimation results (for 1.5 percent and 2.5 percent as thresholds), we observe that the coefficient estimates of above_nc and below_nc are statistically significant and have opposite signs. The estimate for above_nc is positive, implying that having inflation expectations above 2.5 percent and being not confident leads to an inflation bias for medium-term expectations of 1.5-0.4=1.1 percent. To the contrary, the coefficient estimate of below_nc is negative implying that individuals having inflation expectations below 1.5 percent, losing confidence leads to even lower inflation expectations and a deflationary bias (for medium-term expectations: -0.12-0.40=-0.52 percent). With these results, we can provide first-time evidence of the simultaneous existence of inflation and deflationary bias and quantify it.

In column (4), we explore how controlling for inflation targeting affects the results for inflation and deflationary biases. For this purpose we interact inflation targeting with our threshold variables. As inflation targeting should reduce the variability of inflation expectations, we expect that the interaction term with "Above" should be negative and the interaction term with "Below" should be positive.

We can observe that there is little change in the coefficient estimates of our main variables of interest. This estimation leads to very similar results for both inflation and deflationary biases: 1 percent and 2/3 percent, respectively. We observe that inflation targeting shifts the whole distribution of inflation expectations to the left, as both coefficient estimates of "Above_it" and "Below_it"

Table 2: EMU Medium-Run Expectations

	(1) EMU	(2) Germany	(3) Austria	(4) France	(5) Spain	(6) Italy
Not Confident	-0.150***	-0.266***	-0.0245	0.0115	-0.0397	-0.0891
	(0.0355)	(0.0615)	(0.0768)	(0.0626)	(0.0812)	(0.0823)
Gov. Mistrust	-0.0156	-0.286***	0.0229	-0.0268	0.0459	0.129**
	(0.0297)	(0.0616)	(0.103)	(0.0529)	(0.0544)	(0.0598)
Below	-1.285***	-0.940***	-1.149***	-1.062***	-1.204***	-1.264***
	(0.0313)	(0.0510)	(0.129)	(0.0538)	(0.0668)	(0.0519)
Below_nc	-0.232***	-0.334***	-0.607***	-0.384***	-0.192**	0.00548
	(0.0397)	(0.0760)	(0.190)	(0.0723)	(0.0839)	(0.0746)
Above	4.202***	3.356***	3.396***	4.805***	5.413***	4.367***
	(0.104)	(0.160)	(0.228)	(0.284)	(0.218)	(0.189)
Above_nc	0.919***	1.078***	0.916***	0.804**	-0.360	1.577***
	(0.113)	(0.187)	(0.247)	(0.315)	(0.259)	(0.228)
Observations	26,194	6,534	2,820	4,208	6,205	6,427
R-squared	0.292	0.238	0.235	0.349	0.332	0.308
Macro	Yes	Yes	Yes	Yes	Yes	Yes
Socio	Yes	Yes	Yes	Yes	Yes	Yes
Region FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes	Yes

Two-way (region-time) clustered standard errors are in parentheses. “Above” represents a dummy variable denoting a threshold of 2.5 percent and “Below” a threshold of 1.5 percent respectively. Medium-run expectations are 3-5 years ahead. All regressions include region and time fixed effects. “Below_nc” and “Above_nc” represent interaction terms between “Not Confident” and the “Above” or “Below” threshold. “FE” denotes fixed effects. “Socio” stands for control variables on socioeconomic characteristics and “MacroVar” for macroeconomic control variables.
*** p<0.01
** p<0.05
* p<0.1

are negative and significant. Especially, the coefficient on the interaction with above is quite large. This means that the dispersion of inflation expectations is lower in the inflation targeting countries. Here we have to note that our sample has only two central banks that are not pursuing inflation targeting, so these results may be affected by the selection of countries in our sample.

Hypothesis 2 is tested running the same regression for hypothesis 1 column (3) but for the EMU countries only. Furthermore, to explore the different perception of y^*, we estimate the specification of each country separately. With that setup, we can analyze country specific inflation and deflation bias effects of losing confident.

Table 2 contains the estimation results. In the first column we replicate the specification of Table 1 column (3) for the EMU countries in our sample. The other columns replicate the same specification for the individual member country.

Column (1), which replicates previous results for the EMU sub-sample, confirms the simultaneous existence of inflation and deflationary biases for the EMU countries. The coefficients of above_nc and below_nc are positive and negative respectively and confirm together with the "‘not confident"' coefficient estimate the existence of inflation and deflationary biases (-0.15+0.92=0.77 and -0.14-0.23=-0.37). These estimates are a touch smaller than for our full sample of countries. In columns (2)-(6), we replicate the same specification for each EMU member state individually. We observe that Germany, France, and Austria have inflation bias as well as deflationary bias. Spain only has a deflationary bias, while Italy only has an inflation bias. In terms of size, Italy has the strongest response of losing confidence in the price stability objective resulting in a high inflation bias followed by Germany, Austria, and France. Regarding the deflationary bias Germany and Austria have the highest coefficient estimate, followed by France and Spain. Overall, it is remarkable how different the responses to losing confidence in terms of size and propensity for inflation and deflationary biases are across the member countries of the monetary union, despite having the same experience with the ECB. That said, our model allows us to identify the drivers of this observed heterogeneity.

From our model, given the form of the objective function in eq. (9), the differences in the perceptions can be either due to λ or y^*. Our theoretical framework allows us to compare country pairs and determine if the differences in the perceptions are due to λ or y^*, as the two have different effects on inflation and deflationary biases. Most strikingly, we can say that perceptions of y^* in Italy are significantly higher than in any other EMU country in the sample. Thus, in Italy, deflationary bias is not present, while there is a high inflation bias due to perceptions that the ECB is targeting a positive output gap. Differences in perceptions among other countries are mostly guided by different perceptions of the weight that is associated to the output gap in the ECB's objective function (λ). The perceptions of λ are highest in Austria and Germany, followed by France, and the lowest in Spain, where inflation bias is not significant. Thus, we can argue that, in Austria and Germany, households are worried most about the ECB not pursuing a clear hierarchical mandate, where inflation objective is the primary goal.

There is one additional possibility regarding differences in the perceptions of the ECB's objective function that is not explicitly modeled in eq. (9): differences with respect to the perceptions of the inflation target. Although the ECB has clearly stated that the objective is to keep inflation "close, but below 2 percent inflation", Paloviita et al. (2017) has shown that in practice, this means the inflation is around 1.7 percent. To see how different the perception is across the member states in our sample, we use the following approach to investigate whether the perceived inflation target is within the 1.5 percent and 2.5 percent range that we specify: We use our main regression, as in table 2, with one small adjustment that introduces an additional dummy variable for expectations above 5 percent to investigate which range for inflation target maximizes the fit of the model. We run a grid search with a constant one-percentage-point spread between the lower bound and the upper bound that maximizes the overall fit of the model, for each country separately. We find that for the medium-run expectations the best fit for all countries is roughly a range between 2 percent and 3 percent (see figure A.1). Thus, it is considerably above the ECB's inflation objective, although the absolute differences in fit are particularly small for Spain and France, while they are larger in particular for Austria and Germany. This has a clear implication regarding anchoring of inflation expectations, suggesting that ECB still faces a challenge convincing households of the medium-run

Table 3: Confidence and Dispersion

	(1) Short-Run	(2) Medium-Run	(3) Short-Run IQR	(4) Medium-Run IQR	(5) Short-Run Region	(6) Medium-Run Region
Not Confident	2.931***	2.746***	0.663*	1.939***	2.095***	1.828***
	(0.648)	(0.658)	(0.364)	(0.559)	(0.441)	(0.421)
Constant	2.443***	3.391***	1.440***	2.289***	2.839***	3.685***
	(0.442)	(0.457)	(0.248)	(0.351)	(0.286)	(0.276)
Observations	97	97	97	97	1,025	1,021
R-squared	0.189	0.201	0.033	0.114	0.027	0.022

Robust standard errors are in parenthesis. The dependent variable is the variance of inflation expectations within a country per time. "Not_confident" is the share of population within a country that indicated being not confident.
*** p<0.01
** p<0.05
* p<0.1

inflation objective.[20] These results thus complement results in the previous paragraph, where we observed that the perception in Germany and Austria are in line with the ECB not putting enough weight on inflation developments (relative to output gap) in its utility function. In Germany and Austria it is believed that the inflation target that ECB is pursuing is slightly larger, both in the short-run and in the medium-run, compared to the publicly stated objective.

Hypothesis 3 can be tested by comparing the variance and dispersion of inflation expectations against the share of people that are confident across countries and time. Our model predicts that with more people being not confident, inflation and deflationary biases are increasing, implying a positive correlation. Table 3 shows a simple bi-variate ordinary least squares regression between the variance of short- and medium-run inflation expectations (across countries and time) against the share of people being not confident. We provide evidence of that for both horizons of expectations of a higher share of the not confident population increases the dispersion of beliefs. Again the medium-run effect is stronger and exerts a higher level of statistical significance. This result is robust to using alternative robust measures of dispersion such as the interquartile range (that is, the difference between the 25th and 75th quartiles), as we can see in columns (3) and (4). We further check the validity of these conclusions by computing the variance by region instead of by country. These results are reported in columns (5) and (6) and confirm our baseline estimation results using country-level variance.

6. Robustness

In this section we conduct several types of robustness checks to solidify our results. Specifically, we check the robustness of the specification in table 1 column (3). Table 4 contains all the different

[20]The more striking difference among countries actually emerges if we repeat the same exercise using short-run expectations. In that case, the fit for Spain, Italy, and France is maximized for the 1.5 percent to 2.5 percent range–in line with ECB's objective. While for Germany and Austria, it is maximized in the 2 percent to 3 percent range, similar to the medium-run expectations.

robustness exercises we executed. For ease of reading, column (1) of table 4 is our benchmark estimation result of column (3) of table 1.

6.1. Short-run Effects

Our main results focus on medium-run expectations. Nevertheless, it is interesting to check what the short-run (one-year ahead) effect of losing confidence is and whether we can already observe the same asymmetry in the short-run. One would expect that, in the short run, the deflationary and inflation biases are smaller. Column (2) of table 4 contains the estimation results for this time horizon. Overall, we observe qualitatively identical results. We provide evidence for inflation and deflationary biases. In terms of size, the effects are lower than in the medium run, as we would have expected. Losing confidence increases one year ahead expectations by approximately 0.65 percent compared to 1.1 percent for the medium run. This is not unexpected as one year ahead is a shorter horizon were the effect of losing confidence should not fully materialize.

6.2. Government Trust

One could argue that consumers may state that they are not confident in the central bank—not necessarily because of inflation alone but because of a general lack of trust of government policy. Usually this is hard to tackle, as one must compare the opinion toward other government bodies or entities with the opinion regarding the central bank. To account for this possibility we decided to exclude all respondents that distrust the government. With this, we exclude everybody that distrusts the government and also has no confidence in the central bank, thereby we account for a potential general negative attitude toward public bodies. Estimation results are presented in column (3) of table 4. Again, our results hold, and we confirm, even in this substantially reduced sample, the inflation and deflationary biases. While the inflation bias is about 0.8 percent, the deflationary bias is about 0.4 percent.

6.3. Higher Order Fixed Effects

While we include a set of variables to control for macroeconomic events one might argue that we might miss some relevant variation in the data. To control for that we estimate region times time fixed effects accounting for any variation at one quarter within one region (most countries in the sample are comprised of several regions) that could drive our results. Estimation results are presented in column (4) of table 4 and again are qualitatively identical.

6.4. Bootstrap Standard Errors

Our standard errors are clustered at time times regional level and therefore account for joint variation of regions at one point in time. But of course, one could argue that we are not accounting for the possibility that the errors follow a different pattern. To capture this concern we decided to bootstrap our error at this level of our clustered standard errors. Results are presented in column (5) of table 4. Again our main results remain unaffected.

Table 4: Robustness Checks

	(1) Main	(2) Short-Run	(3) Gov. Trust	(4) Higher Order FE	(5) Bootstrap	(6) Heckman	(7) 1%-3%	(8) Country Specific	(9) Full Measure
Not Confident	-0.401***	-0.230***	-0.328***	-0.411***	-0.401***	-0.054***	-0.354***	-0.334***	
	(0.03)	(0.05)	(0.04)	(0.04)	(0.03)	(0.01)	(0.03)	(0.04)	
Gov. Mistrust	0.157**	-0.077		0.149**	0.157**	-0.014	0.134**	0.111*	0.078
	(0.05)	(0.04)		(0.05)	(0.05)	(0.02)	(0.05)	(0.05)	(0.05)
Inflation Target	-0.425*	-0.136	-0.216	0.470	-0.425	-0.108	-0.182	-0.310	-0.442*
	(0.19)	(0.19)	(0.24)	(0.25)	(0.24)	(0.07)	(0.19)	(0.19)	(0.19)
Below	-1.357***	-0.834***	-1.362***	-1.372***	-1.357***	-2.775***	-1.615***	-1.035***	-1.413***
	(0.03)	(0.04)	(0.03)	(0.03)	(0.03)	(0.14)	(0.03)	(0.03)	(0.02)
Below_nc	-0.120***	-0.157**	-0.055	-0.115**	-0.120***	-1.543***	-0.161***	-0.285***	-0.049***
	(0.04)	(0.06)	(0.05)	(0.04)	(0.03)	(0.20)	(0.04)	(0.03)	(0.01)
Above	3.456***	1.895***	3.516***	3.462***	3.456***	1.169***	3.962***	3.466***	4.146***
	(0.08)	(0.07)	(0.09)	(0.09)	(0.09)	(0.02)	(0.10)	(0.09)	(0.06)
Above_nc	1.500***	0.880***	1.116***	1.483***	1.500***	0.250***	1.379***	1.427***	0.643***
	(0.08)	(0.08)	(0.10)	(0.08)	(0.08)	(0.02)	(0.08)	(0.08)	(0.03)
Not ConfidentF									-0.146***
									(0.01)
R-squared	0.278	0.164	0.276	0.292	0.278		0.308	0.300	0.279
Observations	46785	50074	25764	46747	46785	73355	46785	46785	46785
Region FE	Yes	Yes	Yes		Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes		Yes	Yes	Yes	Yes	Yes
Macrovar	Yes	Yes	Yes		Yes	Yes	Yes	Yes	Yes
Socio	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Region×Time FE				Yes					

If not otherwise specified in the column: Two-way (region time) clustered standard errors in parentheses. “Above” represents dummy variable denoting a threshold of 2.5 percent and “Below” a threshold of 1.5 percent in all columns but column (8) respectively. See footnote 20 for country-specific thresholds. Medium-run expectations are 3-5 years ahead. All regressions include region and time fixed effects. “Below_nc” and “Above_nc” represent interaction terms between “Not Confident” and the “Above” or “Below” threshold. “FE” denotes fixed effects. “Socio” stands for control variables on socioeconomic characteristics and “MacroVar” for macroeconomic control variables. “Not ConfidentF” refers to the measuring confidence on a scale between -2 to 2 while “Not Confident” is a 0/1 variable.

*** $p<0.01$

** $p<0.05$

* $p<0.1$

6.5. Heckman Selection Bias

Another issue to check is a potential selection bias. To account for that we use a Heckman selection approach. In the selection equation we use the same set of variables plus two additional variables one capturing the fear of inflation and the second inferring expected income change. Results are presented in column (6) of table 4. Again our results remain qualitatively identical. Results suggest that the size of the inflation and deflationary biases may be partially influenced by the selection bias, as the size of deflationary bias increases and the size of inflation bias decreases. This may be because responders who are more likely to report inflation bias more often do not answer the questions regarding long-run expectations.

6.6. Alternative Threshold of 1 Percent to 3 Percent

As already mentioned in the main text the threshold we chose was ad hoc, and results might hinge upon that. To counter this argument, we re-estimated the main table for an alternative threshold of 1 percent to 3 percent. Results are presented in column (7) of table 4. Comparing this table to the main table we can observe that we have qualitative identical results as well as quantitatively very similar results. The overall bias is 1.107 in Table 1 and has exactly the same value in column (7) of table 4. The above_nc and below_nc coefficients are both slightly smaller. Hence, changing the threshold does not affect our results.

An additional check for the thresholds is to utilize another question in the survey that asks to the tendency to agree with the following statement: "Rising inflation is giving me and my family cause for concern at the moment." As for the not confident questions, the answers range from strongly agree to strongly disagree. Table 5 shows the categories of being concerned about rising inflation against the corresponding average short and medium run inflation expectations in this category. As can be seen, the average responses of those who agree and disagree are broadly in line with the thresholds that we considered that is, 1.5 percent and 2.5 percent.

Table 5: Inflation Concern and Short and Medium-Run Inflation Expectations

Concern	π_s^e	π_m^e
Strongly disagree	1	2
Tend to disagree	1.5	2.4
Neither agree nor disagree	2	3
Tend to agree	2.5	3.5
Strongly agree	3	4

π_s^e denotes averaged short-run expectations while π_m^e represents averaged medium-run expectations in the corresponding 'concern' category.

6.7. Alternative Country-Specific Thresholds

To provide yet another robustness test on the thresholds, we compute country-specific thresholds. Our goal is to find a county-specific thresholds, where the reference group will have no inflation and no deflation biases. As you can see in column (2) of table 6, there is a deflationary bias in the reference group, as the coefficient on not confident is significantly negative. That means that at least in some countries the reference group thresholds are potentially too low. To study this we implement a country-specific regressions with a fixed reference group width (difference between above and below). We find that at least for Singapore and Honk Kong the reference group is potentially set too low.[21] Results are presented in column (8) of table 4. Comparing this table to the main table, we can observe that we have qualitative identical results. As one would expect, by more correctly identifying the region where deflation bias, the estimate of inflation bias slightly increases, by about 0.1 percent, while the estimate of inflation bias is virtually unchanged.

6.8. Full Measure of Confidence

For ease of exposition we decided to work with a 0/1 measure of confidence. Our survey, however, captures confidence in an ordinal scale. To see whether this simplification might affect our results we re-estimate the main table with this full (ordinal) measure. Results are reported in column (9) of table 4. Again, the results remain qualitatively identical. Given that this variable is not 0 and 1 but ranges between -2 and 2, we cannot directly compare the coefficient estimates.

6.9. Inflation Targeting and Inflation/Deflation Bias

In this section we test the implications of central bank design on inflation expectations. One reason for the introduction of inflation targeting was to control inflation also via the inflation expectations. Our sample consists of both inflation and non-inflation targeting countries, although only two out of nine countries are not inflation targeting countries. We have already shown that the dispersion of inflation expectations is lower in inflation targeting countries than in other countries in our sample. Here we also test implications to the inflation and deflationary biases in these two groups of countries.

We test the difference in sizes of these biases by implementing a triple interaction term between above (below), non-confident and inflation targeting. As inflation targeting, according to our model, should reduce both biases, we expect a negative coefficient for the above interaction and a positive coefficient for the below interaction term.

Table 6 presents the results. Column (1) provides the estimation of our main table for comparison. In column (2) we observe that the results for our standard thresholds are the same in all countries. These results suggest that there is no statistical difference in inflation and deflationary biases among inflation and non-inflation targeting central banks, although we confirm the results

[21] In specific, we set the reference group width to 3-4 percent for Hong Kong; 2-3 percent for Singapore; 1-2 percent for Switzerland; 2-3 percent for Germany, UK, and Italy; and 1.5-2.5 percent for the remaining countries.

Table 6: Inflation Targeting and Confidence

	(1)	(2)	(3) Country Specific
Gov. Mistrust	0.231***	0.156***	0.111**
	(0.0534)	(0.0485)	(0.0473)
Not Confident	1.107***	-0.389***	-0.377***
	(0.0519)	(0.0707)	(0.0914)
Above		4.188***	3.983***
		(0.110)	(0.128)
Not Confident×above		1.374***	1.266***
		(0.160)	(0.162)
Inflation Target		0.213	0.0551
		(0.192)	(0.189)
Not Confident×Inflation Target		-0.0153	0.0522
		(0.0777)	(0.0965)
above×Inflation Target		-0.822***	-0.611***
		(0.144)	(0.161)
Not Confident×above×Inflation Target		0.175	0.220
		(0.181)	(0.184)
Below		-1.224***	-1.165***
		(0.0752)	(0.0695)
Not Confident×below		-0.0309	0.0529
		(0.143)	(0.0893)
Below×Inflation Target		-0.152*	0.149**
		(0.0795)	(0.0737)
Not Confident×below×Inflation Target		-0.0923	-0.375***
		(0.147)	(0.0947)
Observations	46,785	46,785	46,785
R-squared	0.073	0.278	0.300
MacroVar	Yes	Yes	Yes
Socio	Yes	Yes	Yes
Region FE	Yes	Yes	Yes
Time FE	Yes	Yes	Yes

Robust standard errors in parentheses. “Above” represents dummy variable denoting a threshold of 2.5 percent and “Below” a threshold of 1.5 percent respectively for columns (1) and (3). See footnote 20 for country-specific thresholds. Medium-run expectations are 3-5 years ahead. All regressions include region and time fixed effects. “FE” denotes fixed effects. “Socio” stands for control variables on socioeconomic characteristics and “MacroVar” for macroeconomic control variables.

*** $p<0.01$

** $p<0.05$

* $p<0.1$

from Table 1 that the dispersion of inflation expectations is smaller in inflation targeting countries in our sample.

However, in column (3) of table 6 we perform an additional check—that is also potentially an interesting robustness check for our results overall—where we assign a country-specific threshold for the reference group, as in section 6.7. After adjusting the reference group in column (3) of table 6, we do not find a significant difference in the inflation bias among these two groups of countries (about 1.25 percent), however the deflationary bias is twice as large among inflation targeting countries, 0.75 percent, compared to other countries in our sample.

Overall, while we find evidence that inflation targeting reduces inflation bias, we provide evidence that it increases deflationary bias. Notably, we face the same empirical limitations other studies have in analyzing the implications of inflation targeting. We have only a few countries that are not inflation targeting countries in our sample and consequently they might not be seen as an optimal comparison group.

7. Conclusion

In this paper, we investigate implications of losing confidence in the price objective of the central bank for households' inflation expectations. We first build a model with an occasionally binding zero-lower-bound constraint and a central bank's loss function that is potentially different from the optimal one. We show that in this environment inflation expectations can either have inflation or deflationary bias. Second, we test several hypotheses derived from the model using individual-level data on inflation expectations and confidence in a central bank's ability to achieve the specified inflation objective for nine countries. This novel survey design allows us to directly test for inflation and deflationary biases.

Our results suggest that both inflation and deflationary biases are present in the survey data. They are also sizable: Losing confidence implies an inflation bias of about 1 percentage point and a deflationary bias of -2/3 percentage point for the medium-run inflation expectations. Inflation and deflationary biases are somewhat smaller for the short-run inflation expectations, but still significant and relevant for policy makers. This result shows that communication strategies (management of expectations) are critical for central banks to achieve the inflation objective. For our sample of countries, we also find that countries who pursue inflation targeting have lower inflation expectations and lower dispersion of inflation expectations as well. However, no support is found that inflation and deflationary biases are mitigated: Our results indicate that in our sample the deflationary bias may even become larger under an inflation targeting regime.

Furthermore, our model allows us to identify the average differences in the perceptions of European Central Bank's objective function across euro-area countries in our sample. This is particularly interesting as the EMU countries share the exact same experience in terms of the ECB's monetary policy. The empirical results show quite remarkable differences: Germany, Austria, and France have both inflation and deflationary biases, Spain has a deflationary bias only, and Italy has only an inflation bias. Our model allows us to dissect these differences and interpret them as either a result of different perceptions of the target level of the output gap or different relative weights attached to output and inflation in the perception of the ECB's loss function. We can show that the perception of the target level of the output gap is the highest in Italy among these countries. The differences among other countries are mostly driven by the different relative weights attached to output and inflation. These results indicate that the ECB faces an ongoing challenge in convincing households of their objectives.

Overall, our results highlight the importance of communication and the relevance of a sound central bank reputation, particularly when the economy is operating close to the zero lower bound and using nonstandard monetary policy tools.

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Appendix A. Model with Endogenous Trust

When discussing inflation or deflationary bias often several descriptions of the relationship between the central bank and economic agents are put forward. Note that it is important to clarify the difference between credibility, reputation, confidence, and trust in the context of this discussion. While all these concepts are inherently related, the game theoretic literature distinguishes between them. To clarify the objective of the question asked in the survey, it relates to the public confidence that the central bank is currently pursuing appropriate policies to achieve price stability over the medium term. The main difference among credibility (reputation), trust, and confidence is that credibility (reputation) consists of the characteristics of the institution or individual (one-sided), while trust and confidence are inherently two-sided relationships, as they are characterized by the preferences of both agents involved in this *game* (relationship).[22] In economic terms, trust can be defined as "the belief or perception by one party (for example, a principal) that the other party (for example, an agent) to a particular transaction will not cheat" (Knack, 2001). It is more difficult to disentangle the difference between trust and confidence as they are strongly related. Potentially, trust could be a broader concept than confidence, as one could argue that confidence is based on trust, or that it is a perception of trust. In a game theoretic setup, a trust game embeds moral hazard due to uncertainty of which action will be implemented, while a reputation game is characterized by asymmetric information on the type of agent.

A.1. The Marginal Utility with Respect to Consumption

$$U^A(C_t, \tau_t, \phi_t) = \tilde{\alpha_1}(\tau_t, \phi_t) - \tilde{\alpha_2}(\tau_t)\, e^{-\delta C_t}$$

$$\text{where } \tilde{\alpha_1}(\tau_t, \phi_t) = \alpha_1 + \alpha_2\tau_t - \alpha_3(\phi_t)\,\tau_t^2$$

$$\text{and } \quad \tilde{\alpha_2}(\tau_t) = \alpha_1 + \alpha_2\tau_t$$

So $U^A(C_t, \tau_t, \phi_t)$ becomes

$$U^A(C_t, \tau_t, \phi_t) = \alpha_1 + \alpha_2\tau_t - \alpha_3(\phi_t)\,\tau_t^2 - (\alpha_1 + \alpha_2\tau_t)\, e^{-\delta C_t}$$

$$\text{where } \tau_t = \psi_{1,t}\left(1 - e^{-\delta C_t}\right) + \varepsilon_t^{\tau}$$

Taking the derivative of the utility function with respect to consumption we have:

[22] Credibility and reputation depend solely on the actions and characteristics of the institution or individual. Reputation involves learning based on past experience; in other words, repeated credible actions and achieved targets lead to a certain reputation and uncertainty regarding the *type* of agents slowly dissipates. Generally, trust in institutions and policy making is a wider concept than reputation, as it is the nexus of both preferences of the trustee and the trustor (see, Bursian and Faia, 2018).

$$\frac{U^A}{\partial C_t} = \alpha_2 \frac{\tau_t}{\partial C_t} - 2\alpha_3\left(\phi_t\right)\tau_t \frac{\tau_t}{\partial C_t} + \alpha_1 \delta e^{-\delta C_t} + \alpha_2\left(-\tau_t \delta e^{-\delta C_t} + e^{-\delta C_t}\frac{\tau_t}{\partial C_t}\right)$$
$$\text{where } \frac{\tau_t}{\partial C_t} = \delta \psi_{1,t} e^{-\delta C_t}$$

Plugging in for $\frac{\tau_t}{\partial C_t}$:

$$\begin{aligned}\frac{U^A}{\partial C_t} &= \alpha_2 \psi_{1,t}\delta e^{-\delta C_t} - 2\alpha_3\left(\phi_t\right)\psi_{1,t}^2 \delta e^{-\delta C_t} + 2\alpha_3\left(\phi_t\right)\psi_{1,t}^2\delta e^{-2\delta C_t} - 2\alpha_3\left(\phi_t\right)\psi_{1,t}\varepsilon_t^\tau \delta e^{-\delta C_t}\\ &+ \alpha_1\delta e^{-\delta C_t} - \alpha_2\psi_{1,t}\delta e^{-\delta C_t} + \alpha_2\delta e^{-2\delta C_t} - \alpha_2\varepsilon_t^\tau\delta e^{-\delta C_t} + \alpha_2\psi_{1,t}\delta e^{-2\delta C_t}\end{aligned}$$

Which simplifies to:

$$\begin{aligned}U_{C,t}^A &= \left[-2\alpha_3\left(\phi_t\right)\psi_{1,t} - \alpha_2\right]\varepsilon_t^{\tau_t}\delta e^{-\delta C_t} + \left[\alpha_1 - 2\alpha_3\left(\phi_t\right)\psi_{1,t}^2\right]\delta e^{-\delta C_t}\\ &+ \left[2\alpha_3\left(\phi_t\right)\psi_{1,t} + \alpha_2\right]\psi_{1,t}\delta e^{-2\delta C_t} + \alpha_2\delta e^{-2\delta C_t}\end{aligned}$$

The steady state for the marginal utility with respect to consumption is:

$$\begin{aligned}U_C^A &= \left[-2\alpha_3\left(\phi\right)\psi_1 - \alpha_2\right]\varepsilon^\tau\delta e^{-\delta C} + \left[\alpha_1 - 2\alpha_3\left(\phi\right)\psi_1^2\right]\delta e^{-\delta C}\\ &+ \left[2\alpha_3\left(\phi\right)\psi_1 + \alpha_2\right]\psi_1\delta e^{-2\delta C} + \alpha_2\delta e^{-2\delta C}\end{aligned}$$

A.2. The Log-Linearized Marginal Utility with Respect to Consumption

$$\begin{aligned}U_C^A\hat{U_{C,t}^A} &= \left[-2\alpha_3(\phi)\varepsilon^\tau\delta e^{-\delta C} - 4\alpha_3(\phi)\psi_1\delta e^{-\delta C} + 4\alpha_3(\phi)\psi_1\delta e^{-2\delta C} + \alpha_2\delta e^{-2\delta C}\right]\psi\hat{\psi_{1,t}}\\ &+ [-\left(-2\alpha_3(\phi)\psi_1 - \alpha_2\right)\varepsilon^\tau\delta^2 e^{-\delta C} - \left(\alpha_1 - 2\alpha_3(\phi)\psi_1^2\right)\delta^2 e^{-\delta C}\\ &- 2\left(2\alpha_3(\phi)\psi_1 + \alpha_2\right)\psi_1\delta^2 e^{-2\delta C} - 2\alpha_2\delta^2 e^{-2\delta C}]C\hat{C}_t\\ &+ [\left(-2\alpha_3(\phi)\psi_1 - \alpha_2\right)\delta e^{-\delta C}]\varepsilon^\tau\hat{\varepsilon_t^\tau}\end{aligned}$$

Cleaning up the log-linearization:

$$\begin{aligned}U_C^A\hat{U_{C,t}^A} &= [-2\alpha_3(\phi)\varepsilon^\tau\psi_1\delta e^{-\delta C} + \alpha_2\varepsilon^\tau\delta e^{-\delta C} - \alpha_2\varepsilon^\tau\delta e^{-\delta C} + \alpha_1\delta e^{-\delta C} - \alpha_1\delta e^{-\delta C}\\ &- 2\alpha_3(\phi)\psi_1^2\delta e^{-\delta C} - 2\alpha_3(\phi)\psi_1^2\delta e^{-\delta C} + 2\alpha_3(\phi)\psi_1^2\delta e^{-2\delta C} + 2\alpha_3(\phi)\psi_1^2\delta e^{-2\delta C}\\ &+ \alpha_2\delta e^{-2\delta C} - \alpha_2\delta e^{-2\delta C} + \alpha_2\psi_1\delta e^{-2\delta C}]\hat{\psi_{1,t}}\\ &- \delta[\left(-2\alpha_3(\phi)\psi_1 - \alpha_2\right)\varepsilon^\tau\delta e^{-\delta C} + \left(\alpha_1 - 2\alpha_3(\phi)\psi_1^2\right)\delta e^{-\delta C} + \left(2\alpha_3(\phi)\psi_1 + \alpha_2\right)\psi_1\delta^2 e^{-2\delta C}\\ &+ \left(2\alpha_3(\phi)\psi_1 + \alpha_2\right)\psi_1\delta^2 e^{-2\delta C} + \alpha_2\delta^2 e^{-2\delta C} + \alpha_2\delta^2 e^{-2\delta C}]C\hat{C}_t\\ &+ [\left(-2\alpha_3(\phi)\psi_1 - \alpha_2\right)\delta e^{-\delta C}]\varepsilon^\tau\hat{\varepsilon_t^\tau}\end{aligned}$$

Using the definition of the steady state of marginal utility with respect to consumption:

$$\begin{aligned}
\hat{U^A_{C,t}} = &\left[\frac{U^A_C - \delta\left[\left(\alpha_2\varepsilon^\tau + \alpha_1 + 2\alpha_3(\phi)\psi_1^2\right)\delta e^{-\delta C} + \left(\alpha_2 - 2\alpha_3(\phi)\psi_1\right)e^{-2\delta C}\right]}{U^A_C}\right]\hat{\psi_{1,t}} \\
&- \delta C\left[\frac{U^A_C + \delta\left[\left(2\alpha_3(\phi)\psi_1 + \alpha_2\right)\psi_1 + 2\alpha_2\right]e^{-2\delta C}}{U^A_C}\right]\hat{C}_t \\
&- \left[\frac{\left[2\alpha_3(\phi)\psi_1 + \alpha_2\right]\varepsilon^\tau \delta e^{-\delta C}}{U^A_C}\right]\hat{\varepsilon^\tau_t}
\end{aligned}$$

$$\begin{aligned}
\hat{U^A_{C,t}} = &\left[1 - \frac{\delta\left[\left(\alpha_2\varepsilon^\tau + \alpha_1 + 2\alpha_3(\phi)\psi_1^2\right)\delta e^{-\delta C} + \left(\alpha_2 - 2\alpha_3(\phi)\psi_1\right)e^{-2\delta C}\right]}{U^A_C}\right]\hat{\psi_{1,t}} \\
&- \delta C\left[1 + \frac{\delta\left[\left(2\alpha_3(\phi)\psi_1 + \alpha_2\right)\psi_1 + 2\alpha_2\right]e^{-2\delta C}}{U^A_C}\right]\hat{C}_t \\
&- \left[\frac{\left[2\alpha_3(\phi)\psi_1 + \alpha_2\right]\varepsilon^\tau \delta e^{-\delta C}}{U^A_C}\right]\hat{\varepsilon^\tau_t}
\end{aligned}$$

A.3. Log-Linearizing the Euler Equation

$$U^A_{C,t} = \beta E_t U^A_{C,t+1}\frac{1+i_t}{\pi_{t+1}}$$

When we log-linearize we get:

$$U^A_C \hat{U^A_{C,t}} = \frac{\beta U^A_C}{\pi} i\hat{i_t} + \frac{\beta U^A_C(1+i)}{\pi^2}\pi\hat{\pi_{t+1}} + \frac{\beta(1+i)}{pi}U^A_C \hat{U^A_{C,t+1}}$$

This simplifies to:

$$\hat{U^A_{C,t}} = \frac{\beta i}{\pi}\hat{i_t} + \frac{\beta(1+i)}{\pi}\hat{\pi_{t+1}} + \frac{\beta(1+i)}{\pi}\hat{U^A_{C,t+1}}$$

Appendix B. Proof of Propositions

B.1. Proof of Proposition 1

The *standard* Markov perfect equilibrium is given by a vector $y_H, \pi_H, i_H, y_L, \pi_L, i_L$ that solves the following system of linear equations:

$$y_H = \left[(1-p_H)y_H + p_H y_L\right] + \sigma\left[(1-p_H)\pi_H + p_H\pi_L - i_H + r^*\right] + \tau_H, \tag{20}$$

$$\pi_H = \kappa y_H + \beta\left[(1-p_H)\pi_H + p_H\pi_L\right], \tag{21}$$

$$0 = \lambda(y_H - y^*) + \kappa\pi_H, \tag{22}$$

$$y_L = \left[(1-p_L)y_H + p_L y_L\right] + \sigma\left[(1-p_L)\pi_H + p_L\pi_L - i_L + r^*\right] + \tau_L, \tag{23}$$

$$\pi_L = \kappa y_L + \beta\left[(1-p_L)\pi_H + p_L\pi_L\right], \tag{24}$$

and

$$i_L = 0, \tag{25}$$

and satisfies the non-negativity of the nominal interest rate in the high state

$$i_H > 0, \tag{26}$$

ϕ_L denotes the Lagrange multiplier on the ZLB constraint in the low state:

$$\phi_L := \lambda y_L + \kappa \pi_L. \tag{27}$$

We will prove the four preliminary propositions (proposition 1.A-1.D), and use these propositions to prove the main proposition (proposition 1) on the necessary and sufficient conditions for the existence of the standard Markov perfect equilibrium.

Let

$$A(\lambda) := -\beta\lambda p_H, \tag{28}$$
$$B(\lambda) := \kappa^2 + \lambda(1-\beta(1-p_H)), \tag{29}$$
$$C := \frac{(1-p_L)}{\sigma\kappa}(1-\beta p_L + \beta p_H) - p_L, \tag{30}$$
$$D := -\frac{(1-p_L)}{\sigma\kappa}(1-\beta p_L + \beta p_H) - (1-p_L) = -1 - C, \tag{31}$$

and

$$E(\lambda) := A(\lambda)D - B(\lambda)C. \tag{32}$$

Assumption 1.A: $E(\lambda) \neq 0$.

Throughout the rest of this proof, we will assume that Assumption 1.A holds.

Proposition 1.A: There exists a vector $y_H, \pi_H, i_H, y_L, \pi_L, i_L$ that solves (20)-(25).

We can rearrange the system of equations (20)-(25) and eliminate y_H and y_L.
Using (22) we have:

$$y_H = y^* - \frac{\kappa}{\lambda}\pi_H$$

We substitute this value for y_H into equation (21):

$$\begin{aligned}
\pi_H &= \kappa y_H + \beta[(1-p_H)\pi_H + p_H\pi_L] \\
&= \kappa[y^* - \frac{\kappa}{\lambda}\pi_H] + \beta[(1-p_H)\pi_H + p_H\pi_L] \\
\beta p_H \pi_L + \kappa y^* &= \pi_H + \frac{\kappa^2}{\lambda}\pi_H - \beta(1-p_H)\pi_H
\end{aligned}$$

if we multiply this expression by λ:

$$[\kappa^2 + \lambda(1-\beta(1-p_H))]\pi_H - \beta\lambda p_H \pi_L = \kappa\lambda y^* \tag{33}$$

When we solve for y_H in equation 21 and y_L in equation 24 we have:

$$y_H = \frac{1}{\kappa}\pi_H - \frac{1}{\kappa}\beta[(1-p_H)\pi_H + p_H\pi_L] \tag{34}$$
$$y_L = \frac{1}{\kappa}\pi_L - \frac{1}{\kappa}\beta[(1-p_L)\pi_H + p_L\pi_L] \tag{35}$$

We substitute these values for y_H and y_L into equation 23:

$$\begin{aligned}
(1-p_L)\left[\frac{1}{\kappa}\pi_L - \frac{1}{\kappa}\beta[(1-p_L)\pi_H + p_L\pi_L]\right] &= (1-p_L)\left[\frac{1}{\kappa}\pi_H - \frac{1}{\kappa}\beta[(1-p_H)\pi_H + p_H\pi_L]\right] \\
&\quad + \sigma[(1-p_L)\pi_H + p_L\pi_L + r^*] + \tau_L
\end{aligned}$$

$$\left[\frac{(1-p_L)}{\kappa}(1-\beta p_L + \beta p_H) - \sigma p_L\right]\pi_L - \left[\frac{(1-p_L)}{\kappa}(1+\beta p_H - \beta p_L) + \sigma(1-p_L)\right]\pi_H = \sigma r^* + \tau_L \tag{36}$$

Therefore we have two unknowns, π_H and π_L, and two equations:

$$\begin{bmatrix} A(\lambda) & B(\lambda) \\ C & D \end{bmatrix} \begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \begin{bmatrix} \kappa\lambda y^* \\ r_L \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \frac{1}{A(\lambda)D - B(\lambda)C} \begin{bmatrix} D & -B(\lambda) \\ -C & A(\lambda) \end{bmatrix} \begin{bmatrix} \kappa\lambda y^* \\ r_L \end{bmatrix} \tag{37}$$

where $r_L = r^* + \frac{1}{\sigma}\tau_L$.
Therefore, we have:

$$\pi_H = \frac{A(\lambda)}{E(\lambda)} r_L - \frac{C}{E(\lambda)} \kappa\lambda y^* \tag{38}$$

and

$$\pi_L = \frac{-B(\lambda)}{E(\lambda)} r_L + \frac{D}{E(\lambda)} \kappa\lambda y^* \tag{39}$$

This gives us the following Phillips curves in both states:

$$\begin{aligned} y_H &= y^* - \frac{\kappa}{\lambda}\pi_H \\ y_H &= y^* - \frac{\kappa}{\lambda}\left[\frac{A(\lambda)}{E(\lambda)} r_L - \frac{C}{E(\lambda)} \kappa\lambda y^*\right] \\ y_H &= \frac{\beta\kappa p_H}{E(\lambda)} r_L + \left(1 + \frac{C}{E(\lambda)}\kappa^2\right) y^* \end{aligned} \tag{40}$$

$$\begin{aligned} y_L &= \frac{1}{\kappa}\left[\pi_L - \beta\left[(1-p_L)\pi_H + p_L\pi_L\right]\right] \\ y_L &= -\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda}{\kappa E(\lambda)} r_L - \lambda\left[(1-\beta)C + (1-\beta p_L)\right]\frac{y^*}{E(\lambda)} \end{aligned} \tag{41}$$

Proposition 1.B: Suppose (20)-(25) are satisfied. Then $\phi_L < 0$ if and only if $E(\lambda) < 0$
Proof: Notice that

$$\begin{aligned} \phi_L &= \lambda y_L + \kappa\pi_L \\ &= \lambda\left[-\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda}{\kappa E(\lambda)} r_L\right] \\ &- \lambda\left[(1-\beta)C + (1-\beta p_L)\right]\frac{\lambda y^*}{E(\lambda)} \\ &+ \kappa\left[\frac{-B(\lambda)}{E(\lambda)} r_L + \frac{D}{E(\lambda)}\kappa\lambda y^*\right] \end{aligned}$$

Group terms:

$$\begin{aligned} \phi_L = &-\left[\frac{\lambda}{\kappa}\left[(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda\right] + \kappa B(\lambda)\right]\frac{r_L}{E(\lambda)} \\ &+ \left[\kappa^2 D - \lambda\left[(1-\beta)C + (1-\beta p_L)\right]\right]\frac{\lambda y^*}{E(\lambda)} \end{aligned}$$

Now simplify the y^* term:

$$
\begin{aligned}
&= \left[\kappa^2 D - \lambda\left[(1-\beta)C + (1-\beta p_L)\right]\right] \frac{\lambda y^*}{E(\lambda)} \\
&= \left[-\left[\kappa^2 + \lambda(1-\beta)\right] \frac{(1-\beta p_L + \beta p_H)}{\sigma\kappa}(1-p_L) - [\kappa^2+\lambda](1-p_L)\right] \frac{\lambda y^*}{E(\lambda)}
\end{aligned}
\tag{42}
$$

Putting all the terms together we have:

$$
\begin{aligned}
\phi_L = &-\left[\frac{\lambda}{\kappa}\left[(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda\right] + \kappa B(\lambda)\right] \frac{r_L}{E(\lambda)} \\
&-(1-p_L)\left[\left[\kappa^2 + \lambda(1-\beta)\right] \frac{(1-\beta p_L + \beta p_H)}{\sigma\kappa} + [\kappa^2+\lambda]\right] \frac{\lambda y^*}{E(\lambda)}
\end{aligned}
\tag{43}
$$

Thus $(1-p_L)\left[\frac{(\lambda-\lambda\beta+\kappa^2)}{\sigma\kappa}(1-\beta p_L + \beta p_H) + [\kappa^2+\lambda]\right] \leq 0$ if $p_L \leq 1$.

We have that $\tau_L < 0, (1-\beta p_L)\kappa^2 > 0, (1-\beta)(1+\beta p_H - \beta p_L)\lambda \geq 0$, and $\kappa B(\lambda) \geq 0$. Also, if $E(\lambda) < 0$, then $\phi_L < 0$.

However, if $y^* > 0$, given that:

$$
-\left[\frac{\lambda}{\kappa}\left[(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda\right] + \kappa B(\lambda)\right] r_L > (p_L - 1)\left[\frac{(\lambda-\lambda\beta+\kappa^2)}{\sigma\kappa}(1-\beta p_L + \beta p_H) + (\kappa^2+\lambda)\right] \lambda y^*
$$

then we come to the same conclusion as above that if $\phi_L < 0$, then $E(\lambda) < 0$ and also if $E(\lambda) < 0$, then $\phi_L < 0$.

Proposition 1.C: $E(\lambda) < 0$ if and only if $p_L^* < (\Theta_{-p_L})$
Proof: Let $E(\cdot)$ be a function of p_H and p_L for this purpose.

$$
\begin{aligned}
E(p_H, p_L) =& A(p_H, p_L)D - B(p_H, p_L)C \qquad (44) \\
=& -\beta p_H \lambda(-1-C) - \left[\kappa^2 + \lambda\left(1 - \beta(1-p_H)\right)\right] C \\
=& \beta p_H \lambda - \left[\kappa^2 + \lambda(1-\beta)\right]\left[\frac{(1-p_L)}{\sigma\kappa}(1-\beta p_L + \beta p_H) - p_L\right]
\end{aligned}
\tag{45}
$$

Let $\Gamma = \kappa^2 + \lambda(1-\beta)$.

$$
\begin{aligned}
E(p_H, p_L) =& \\
=& \beta p_H \lambda - \Gamma\left[\frac{(1-p_L)}{\sigma\kappa}(1-\beta p_L + \beta p_H) - p_L\right] \\
=& -\Gamma\beta\frac{1}{\sigma\kappa}p_L^2 + \Gamma\left[\frac{1}{\sigma\kappa}(1+\beta+\beta p_H) + 1\right] p_L + \beta\lambda p_H - \Gamma\frac{1}{\sigma\kappa}(1+\beta p_H) \\
:=& q_2 p_L^2 + q_1 p_L + q_0
\end{aligned}
\tag{46}
$$

Where we have that:

$$q_0 := \beta\lambda p_H - \Gamma\frac{1}{\sigma\kappa}(1+\beta p_H) \tag{47}$$

$$q_1 := \Gamma\left[\frac{1}{\sigma\kappa}(1+\beta+\beta p_H)+1\right] \tag{48}$$

$$q_2 := -\Gamma\beta\frac{1}{\sigma\kappa} \tag{49}$$

The function, $E(\cdot,\cdot)$, has the following two properties:
Property 1: $E(p_H, 1) > 0$ **for any** $0 \leq p_H \leq 1$**.**

$$\begin{aligned} E(p_H, p_L) =& -\Gamma\beta\frac{1}{\sigma\kappa} + \Gamma\left[\frac{1}{\sigma\kappa}(1+\beta+\beta p_H)+1\right] + \beta\lambda p_H - \Gamma\frac{1}{\sigma\kappa}(1+\beta p_H) \\ =& \Gamma + \beta\lambda p_H > 0 \end{aligned} \tag{50}$$

Property 2: $E(p_H, p_L)$ **is maximized at** $p_L > 1$ **for any** $0 \leq p_H \leq 1$**.**

$$\begin{aligned} \frac{\partial E(p_H,p_L)}{\partial p_L} =& 2q_2 p_L^* + q_1 = 0 \\ \leftrightarrow p_L^* =& -\frac{q_1}{2q_2} \\ =& \frac{\Gamma\left[\frac{1}{\sigma\kappa}(1+\beta+\beta p_H)+1\right]}{2\Gamma\beta\frac{1}{\sigma\kappa}} \\ =& \frac{\left[\frac{1}{\sigma\kappa}(2\beta+(1-\beta)+\beta p_H)+1\right]}{2\beta\frac{1}{\sigma\kappa}} > 1 \end{aligned} \tag{51}$$

$$\tag{52}$$

Property 1 and property 2 imply together that i) one root of $E(\cdot, p_L)$ is below 1 and ii) $E(\cdot, p_L) < 0$ below this root. We will call this root $p_L^*(\Theta_{-p_L})$

$$p_L^*(\Theta^1_{-p_L}) := \frac{-q_1 - \sqrt{q_1^2 - 4q_2 q_0}}{2q_2}. \tag{53}$$

Based on the properties outlined above, if $E(\lambda) < 0$, then $p_L < p_L^*(\Theta_{-p_L})$. Likewise, if $p_L < p_L^*(\Theta_{-p_L})$, then $E(\lambda) < 0$. This completes the proof of proposition 1.C. Proposition 1.C. holds regardless of whether the system of linear equations (20)-(25) is satisfied or not.

Proposition 1.D: Suppose (20)-(25) are satisfied and $E(\lambda) < 0$**. Then** $i_H > 0$ **if and only if** $p_H < p_H^*(\Theta_{-p_H})$**.**

Proof: i_H comes from rearranging y_H

$$y_H = [(1-p_H)y_H + p_H y_L] + \sigma[(1-p_H)\pi_H + p_H\pi_L - i_H + r^*] + \tau_H \tag{54}$$

We multiply by $\frac{1}{\sigma}$:

$$\begin{aligned} i_H &= \frac{1}{\sigma}[(1-p_H)y_H + p_H y_L] + (1-p_H)\pi_H + p_H\pi_L + r^* + \frac{1}{\sigma}\tau_H - \frac{1}{\sigma}y_H \\ &= \frac{1}{\sigma}[-p_H y_H + p_H y_L] + (1-p_H)\pi_H + p_H\pi_L + r_H \end{aligned}$$

where $r_H = r^* + \frac{1}{\sigma}\tau_H$.
We have that i_H is equal to the following:

$$i_H = \frac{1}{\sigma}\left[-p_H y_H + p_H y_L\right] + (1-p_H)\pi_H + p_H\pi_L + r_H$$

where $r_H = r^* + \frac{1}{\sigma}\tau_H$.

Now we plug in for y_H, y_L, π_H, and π_L:

$$\begin{aligned}
i_H = &\frac{-p_H}{\sigma}\left[\frac{\beta\kappa p_H}{E(\lambda)}r_L + \left(1 + \frac{C}{E(\lambda)}\kappa^2\right)y^*\right] \\
&+ \frac{p_H}{\sigma}\left[-\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda}{\kappa E(\lambda)}r_L - \left[(1-\beta)C + (1-\beta p_L)\right]\frac{\lambda y^*}{E(\lambda)}\right] \\
&+ (1-p_H)\left[\frac{A(\lambda)}{E(\lambda)}r_L - \frac{C}{E(\lambda)}\kappa\lambda y^*\right] \\
&+ p_H\left[\frac{-B(\lambda)}{E(\lambda)}r_L + \frac{D}{E(\lambda)}\kappa\lambda y^*\right] + r_H
\end{aligned} \tag{55}$$

Now we group the r_L and y^* terms:

$$\begin{aligned}
i_H = &\left[-p_H\left(\left[\left(1+\kappa^2\frac{C}{E(\lambda)}\right) + \left[(1-\beta p_L) + (1-\beta)C\right]\frac{\lambda}{E(\lambda)}\right]\frac{1}{\sigma} - \frac{C\lambda\kappa}{E(\lambda)} - \frac{D\lambda\kappa}{E(\lambda)}\right) - \frac{C\lambda\kappa}{E(\lambda)}\right]y^* \\
&+ \left[-p_H\left(\left[\kappa\beta p_H + \frac{(1-\beta p_L)\kappa^2 + \lambda(1-\beta)(1-\beta p_L + \beta p_H)}{\kappa}\right]\frac{1}{\sigma} + A(\lambda) + B(\lambda)\right) + A(\lambda)\right]\frac{r_L}{E(\lambda)} + r_H
\end{aligned}$$

Now we look at the y^* term and simplify it:
First we pull out $\frac{1}{E(\lambda)}$ of the expression and group with the y^* term:

$$\left[-p_H\left(\left[\left(E(\lambda) + \kappa^2 C\right) + \left[(1-\beta p_L) + (1-\beta)C\right]\lambda\right]\frac{1}{\sigma} - C\lambda\kappa - D\lambda\kappa\right) - C\lambda\kappa\right]\frac{y^*}{E(\lambda)}$$

First, we will look at the term multiplied by $\frac{1}{\sigma}$:

$$\begin{aligned}
&\left[\left(E(\lambda) + \kappa^2 C\right) + \left[(1-\beta p_L) + (1-\beta)C\right]\lambda\right]\frac{1}{\sigma} \\
&\left[-\beta\lambda p_H D - \left[\kappa^2 + \lambda(1-\beta(1-p_H))\right]C + \kappa^2 C + \left[(1-\beta p_L) + (1-\beta)C\right]\lambda\right]\frac{1}{\sigma} \\
&\left[\beta\lambda p_H + \lambda(1-\beta p_L)\right]\frac{1}{\sigma}
\end{aligned} \tag{56}$$

Then we look at the term multiplied by $-p_H$

$$\begin{aligned}
&-p_H\left(\left[\beta\lambda p_H + \lambda(1-\beta p_L)\right]\frac{1}{\sigma} - C\lambda\kappa - D\lambda\kappa\right) \\
&-p_H\left(\left[\beta\lambda p_H + \lambda(1-\beta p_L)\right]\frac{1}{\sigma} + \lambda\kappa\right)
\end{aligned} \tag{57}$$

So our y^* term is:

$$\left[-p_H\left([\beta\lambda p_H+\lambda(1-\beta p_L)]\frac{1}{\sigma}+\lambda\kappa\right)-C\lambda\kappa\right]\frac{y^*}{E(\lambda)}$$
$$\left[\left(-\beta\lambda p_H^2-\lambda p_H+\lambda\beta p_L p_H-(1-p_L)\lambda(1-\beta p_L)-(1-p_L)\lambda\beta p_H\right)\frac{1}{\sigma}-p_H\lambda\kappa+p_L\lambda\kappa\right]\frac{y^*}{E(\lambda)} \tag{58}$$

Now we group our y^* term by power of p_H:

$$\left[\frac{-\beta\lambda}{\sigma}p_H^2+\left(\frac{1}{\sigma}[\beta p_L\lambda-\lambda-\lambda(1-p_L)\beta]-\lambda\kappa\right)p_H-\frac{\lambda(1-p_L)(1-\beta p_L)}{\sigma}+p_L\lambda\kappa\right]\frac{y^*}{E(\lambda)} \tag{59}$$

So our full simplified y^* term is:

$$\left[\frac{-\beta\lambda}{\sigma}p_H^2-\lambda\left(\frac{1+\beta(1-2p_L)}{\sigma}+\kappa\right)p_H-\lambda\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma}-p_L\kappa\right)\right]\frac{y^*}{E(\lambda)}$$

Next, we group the terms of r_L terms and simplify the expression:

$$\left[-p_H\left(\left[\kappa\beta p_H+\frac{(1-\beta p_L)\kappa^2+\lambda(1-\beta)(1-\beta p_L+\beta p_H)}{\kappa}\right]\frac{1}{\sigma}+A(\lambda)+B(\lambda)\right)+A(\lambda)\right]\frac{r_L}{E(\lambda)}$$
$$\left[-p_H\left(\frac{[\kappa^2+\lambda(1-\beta)][\beta p_H+(1-\beta p_L)]}{\kappa\sigma}+\kappa^2+\lambda(1-\beta)]\right)-\beta\lambda p_H\right]\frac{r_L}{E(\lambda)} \tag{60}$$

We know that $\Gamma=\kappa^2+\lambda(1-\beta)$, so we have:

$$\left[-p_H\frac{\Gamma[\beta p_H+(1-\beta p_L)]}{\kappa\sigma}-p_H\kappa^2-p_H\lambda+p_H\lambda\beta)-\beta\lambda p_H\right]\frac{r_L}{E(\lambda)}$$
$$\left[-p_H\frac{\Gamma[\beta p_H+(1-\beta p_L)]}{\kappa\sigma}-p_H\kappa^2-p_H\lambda\right]\frac{r_L}{E(\lambda)} \tag{61}$$

Now we group our r_L term by the power of p_H:

$$\left[\frac{-\Gamma\beta}{\kappa\sigma}p_H^2-\left(\frac{\Gamma(1-\beta p_L)}{\kappa\sigma}+\kappa^2+\lambda\right)p_H\right]\frac{r_L}{E(\lambda)} \tag{62}$$

With the simplification of the y^* term and r_L term, our full expression for i_H becomes:

$$\begin{aligned}i_H=&\left[\frac{-\beta\lambda}{\sigma}p_H^2-\lambda\left(\frac{1+\beta(1-2p_L)}{\sigma}+\kappa\right)p_H-\lambda\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma}-p_L\kappa\right)\right]\frac{y^*}{E(\lambda)}\\&+\left[\frac{-\Gamma\beta}{\kappa\sigma}p_H^2-\left(\frac{\Gamma(1-\beta p_L)}{\kappa\sigma}+\kappa^2+\lambda\right)p_H\right]\frac{r_L}{E(\lambda)}+r_H\end{aligned} \tag{63}$$

The final expression for i_H grouped by the power of p_H:

$$\begin{aligned} i_H =& \left[\frac{-\beta\lambda}{\sigma}\frac{y^*}{E(\lambda)} - \frac{\Gamma\beta}{\kappa\sigma}\frac{r_L}{E(\lambda)}\right] p_H^2 \\ &- \left[\lambda\left(\frac{1+\beta(1-2p_L)}{\sigma} + \kappa\right)\frac{y^*}{E(\lambda)} + \left(\frac{\Gamma(1-\beta p_L)}{\kappa\sigma} + \kappa^2 + \lambda\right)\frac{r_L}{E(\lambda)}\right] p_H \\ &- \lambda\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L\kappa\right)\frac{y^*}{E(\lambda)} + r_H \end{aligned}$$

We want to show that $i_H > 0$ when $E(\lambda) < 0$:
We will multiply the expression by $-E(\lambda)$:

$$\left[\frac{\beta\lambda}{\sigma}y^* + \frac{\Gamma\beta}{\kappa\sigma}r_L\right]p_H^2 + \left[\lambda\left(\frac{1+\beta(1-2p_L)}{\sigma} + \kappa\right)y^* + \left(\frac{\Gamma(1-\beta p_L)}{\kappa\sigma} + \kappa^2 + \lambda\right)r_L\right]p_H + \lambda\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L\kappa\right)y^* - E(\lambda)r_H > 0$$

$$\begin{aligned} &\left[\frac{\beta\lambda}{\sigma}y^* + \frac{\Gamma\beta}{\kappa\sigma}r_L\right]p_H^2 + \left[\lambda\left(\frac{1+\beta(1-2p_L)}{\sigma} + \kappa\right)y^* + \left(\frac{\Gamma(1-\beta p_L)}{\kappa\sigma} + \kappa^2 + \lambda\right)r_L - r_H\beta\lambda + r_H\Gamma\beta\frac{1-p_L}{\kappa\sigma}\right]p_H \\ &+ \lambda\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L\kappa\right)y^* + r_H\Gamma\left(\frac{(1-p_L)}{\kappa\sigma}(1-\beta p_L) - p_L\right) > 0 \end{aligned}$$

Now we divide by Γ and $-r_L$:

$$\begin{aligned} &\left[\frac{-\beta\lambda}{\sigma\Gamma r_L}y^* - \frac{\beta}{\kappa\sigma}\right]p_H^2 - \left[\lambda\left(\frac{1+\beta(1-2p_L)}{\sigma} + \kappa\right)\frac{y^*}{\Gamma r_L} + \frac{(1-\beta p_L) + (1-p_L)\beta\frac{r_H}{r_L}}{\kappa\sigma} + \frac{\kappa^2 + \lambda(1-\beta\frac{r_H}{r_L})}{\Gamma}\right]p_H \\ &- \frac{\lambda}{\Gamma r_L}\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L\kappa\right)y^* - \frac{r_H}{r_L}\left(\frac{(1-p_L)}{\kappa\sigma}(1-\beta p_L) - p_L\right) > 0 \end{aligned}$$

Let

$$P(p_H) = \phi_2 p_H^2 + \phi_1 p_H + \phi_0 \tag{64}$$

where

$$\phi_0 := -\frac{\lambda}{\Gamma r_L}\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L\kappa\right)y^* - \frac{r_H}{r_L}\left(\frac{(1-p_L)}{\kappa\sigma}(1-\beta p_L) - p_L\right) \tag{65}$$

$$\phi_1 := \left[\lambda\left(\frac{1+\beta(1-2p_L)}{\sigma} + \kappa\right)\frac{y^*}{\Gamma r_L} + \frac{(1-\beta p_L) + (1-p_L)\beta\frac{r_H}{r_L}}{\kappa\sigma} + \frac{\kappa^2 + \lambda(1-\beta\frac{r_H}{r_L})}{\Gamma}\right] \tag{66}$$

$$\phi_2 := \left[\frac{-\beta\lambda}{\sigma\Gamma r_L}y^* - \frac{\beta}{\kappa\sigma}\right] \tag{67}$$

$$\tag{68}$$

Property 1: $\phi_0 > 0$

$$\begin{aligned} i_H =& \left[\frac{-\beta\lambda}{\sigma}\frac{y^*}{E(\lambda)} - \frac{\Gamma\beta}{\kappa\sigma}\frac{r_L}{E(\lambda)}\right] p_H^2 \\ &- \left[\lambda\left(\frac{1+\beta(1-2p_L)}{\sigma} + \kappa\right)\frac{y^*}{E(\lambda)} + \left(\frac{\Gamma(1-\beta p_L)}{\kappa\sigma} + \kappa^2 + \lambda\right)\frac{r_L}{E(\lambda)}\right] p_H \\ &- \lambda\left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L\kappa\right)\frac{y^*}{E(\lambda)} + r_H \end{aligned}$$

If we have that $p_H = 0$, then the expression reduces to:

$$i_H = -\lambda \left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L \kappa \right) \frac{y^*}{E(\lambda)} + r_H \tag{69}$$

When $p_H = 0$ we have $i_H = -\lambda \left(\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L \kappa \right) \frac{y^*}{E(\lambda)} + r_H$. $\frac{(1-p_L)(1-\beta p_L)}{\sigma} - p_L \kappa > 0$, when $p_L < p_L^*(\Theta^2_{-p_L}) := \frac{\beta\kappa\sigma+1-\sqrt{(\beta\kappa\sigma+1)^2-4\beta}}{2\beta}$.

This completes the proof of property 1.

Property 2: $\phi_2 < 0$

In order for $\phi_2 < 0$, we must have that:

$$\left[\frac{-\beta\lambda}{\sigma\Gamma r_L} y^* - \frac{\beta}{\kappa\sigma} \right] < 0$$
$$\frac{-\beta\lambda}{\sigma\Gamma r_L} y^* < \frac{\beta}{\kappa\sigma}$$

We will multiply by $-r_L$ since we have by assumption that $r_L < 0$

$$\frac{\beta\lambda}{\sigma\Gamma} y^* < -r_L \frac{\beta}{\kappa\sigma}$$
$$\frac{\lambda}{\Gamma} y^* < -r_L \frac{1}{\kappa}$$
$$y^* < -r_L \frac{\Gamma}{\kappa\lambda}$$
$$y^* < -r_L \frac{\kappa^2 + \lambda(1-\beta)}{\kappa\lambda} \qquad r_L < -y^* \frac{\kappa\lambda}{\kappa^2 + \lambda(1-\beta)} \tag{70}$$

As long as $r_L < -y^* \frac{\kappa\lambda}{\kappa^2+\lambda(1-\beta)}$, then $\phi_2 < 0$. This completes the proof of Property 2.

Property 1 and property 2, $\phi_0 > 0$ and $\phi_2 < 0$, imply that one root of (64) is non-negative and $i_H > 0$ if and only if p_H is below this non-negative root, given by

$$p_H^*(\Theta_{-p_H}) := \frac{-\phi_1 - \sqrt{\phi_1^2 - 4\phi_0\phi_2}}{2\phi_2}. \tag{71}$$

This completes the proof of proposition 1.D.

With these four preliminary propositions (1.A -1.D), we have what we need to prove proposition 1.

Proposition 1: There exists a vector $\{y_H, \pi_H, ß_H, y_L, \pi_L, i_L\}$ that solves the system of linear equations (20)-(25) and satisfies $\phi_L < 0$ and $i_H > 0$ if and only if $p_L < p_L^*(\Theta_{-p_L})$ and $p_H < p_H^*(\Theta_{-p_H})$, where $p_L^*(\Theta_{-p_L}) = min\{p_L^*(\Theta^1_{-p_L}), p_L^*(\Theta^2_{-p_L})\}$.[23]

Proof of "if" part: Suppose that $p_L < p_L^*(\Theta_{-p_L})$ and $p_H < p_H^*(\Theta_{-p_H})$. According to proposition 1.A there exists a vector $\{y_H, \pi_H, ß_H, y_L, \pi_L, i_L\}$ that solves the system of linear equations (20)-(25). According to proposition 1.B and 1.C, $E(\lambda) < 0$ and $\phi_L < 0$. According to proposition 1.D and the fact that $E(\lambda) < 0$, $i_H > 0$. This completes the "if" part of the proof.

Proof of "only if" part: Suppose that $\phi_L < 0$ and $i_H > 0$. According to proposition 1.A there exists a vector $\{y_H, \pi_H, ß_H, y_L, \pi_L, i_L\}$ that solves the system of linear equations (20)-(25). According to proposition 1.B and 1.C, $E(\lambda) < 0$ and $p_L < p_L^*(\Theta_{-p_L})$. According to proposition 1.D and the fact that $E(\lambda) < 0$, $p_H < p_H^*(\Theta_{-p_H})$. This completes the "only if" part of the proof.

[23] It is straightforward to show that the sufficient (but not necessary) condition for $\Theta^1_{-p_L}) < p_L^*(\Theta^2_{-p_L})$ is that $\kappa\sigma < 2/\beta$.

B.2. Proof of proposition 2

We characterize the sign of inflation and output depending on whether it is low or high state. We will use the restriction regarding r_L that guarantees us the existence and $E(\lambda) < 0$, the inequalities on $A(\lambda)$, $A(\lambda) < 0$; $B(\lambda)$, $B(\lambda) > 0$; C and D. Namely that when $E(\lambda) < 0$, $C > 0$ and $D < 0$.

When $y^* = 0$, we get, as in Nakata and Schmidt (2018), that $\pi_H \leq 0$, $\pi_L < 0$, $y_H > 0$ and $y_L < 0$.

However, when y^* does not equal zero, our equations are augmented.

$$\pi_H = \frac{A(\lambda)}{E(\lambda)} r_L - \frac{C}{E(\lambda)} \kappa \lambda y^*$$

Given that $-\frac{C}{E(\lambda)}\kappa\lambda y^*$ is a positive number, it is possible under certain conditions for π_H to be positive. Whenever, $y^* > \frac{A(\lambda) r_L}{C\kappa\lambda}$ then $\pi_H > 0$. Under the assumption that restrictions for the existence of equilibrium are satisfied (proposition 1) we can conclude that:

$$\pi_H = \begin{cases} \frac{A(\lambda)}{E(\lambda)} r_L - \frac{C}{E(\lambda)} \kappa\lambda y^* \leq 0, & \text{iff } y^* \leq -\frac{\beta p_H}{\kappa C} r_L \\ \frac{A(\lambda)}{E(\lambda)} r_L - \frac{C}{E(\lambda)} \kappa\lambda y^* > 0, & \text{iff otherwise} \end{cases}$$

Note that $-\frac{\beta p_H}{\kappa C} r_L = \frac{A(\lambda) r_L}{C\kappa\lambda}$.

$$\pi_L = \frac{-B(\lambda)}{E(\lambda)} r_L + \frac{D}{E(\lambda)} \kappa\lambda y^*$$

Given that $\frac{D}{E(\lambda)}\kappa\lambda y^* > 0$, it is possible under certain conditions for π_L to be positive. Whenever $y^* > \frac{B(\lambda) r_L}{D\kappa\lambda}$, then $\pi_H > 0$.

$$\pi_L = \begin{cases} \frac{-B(\lambda)}{E(\lambda)} r_L + \frac{D}{E(\lambda)} \kappa\lambda y^* < 0, & \text{iff } y^* < \frac{B(\lambda) r_L}{D\kappa\lambda} \\ \frac{-B(\lambda)}{E(\lambda)} r_L + \frac{D}{E(\lambda)} \kappa\lambda y^* \geq 0, & \text{iff otherwise} \end{cases}$$

$$y_H = \frac{\beta\kappa p_H}{E(\lambda)} r_L + \left(1 + \frac{C}{E(\lambda)}\kappa^2\right) y^*$$

$y_H > 0$ as long as $\frac{\beta\lambda p_H}{E(\lambda)} r_L + \left(1 + \frac{C}{E(\lambda)}\kappa^2\right) y^* > 0$
We will multiply by $-E(\lambda)$:

$$\begin{aligned} &-\beta\kappa p_H r_L + \left(-E(\lambda) - C\kappa^2\right) y^* > 0 \\ &-\beta\kappa p_H r_L > \lambda\left(\beta p_H - (1-\beta)C\right) y^* \end{aligned} \tag{72}$$

If $(\beta p_H - (1-\beta)C) < 0$, then we have:

$$\frac{-\beta\kappa p_H}{\lambda\left(\beta p_H - (1-\beta)C\right)} r_L < y^* \tag{73}$$

Since both the numerator and denominator are negative, and given that $r_L < 0$, $\frac{-\beta\kappa p_H}{\lambda(\beta p_H - (1-\beta)C)} r_L < 0$. $y^* \geq 0$, therefore, this equality will always hold. Thus, $y_H > 0$ when $\beta p_H - (1-\beta)C < 0$. Solving this inequality leads us to the condition $p_L < p_L^*(\Theta^3_{-p_L})$. Now we can straightforwardly show that $p_L^*(\Theta^3_{-p_L}) > p_L^*(\Theta^1_{-p_L})$ from the proposition 1 (existence of the equilibria), and thus for all values of p_L that satisfy the existence conditions we have that $y_H > 0$.[24]

$$y_L = -\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta p_H - \beta p_L)\lambda}{\kappa E(\lambda)} r_L - \lambda\left[(1-\beta)C + (1-\beta p_L)\right]\frac{y^*}{E(\lambda)}$$

[24] By comparing $p_L^*(\Theta^3_{-p_L})$ and $p_L^*(\Theta^1_{-p_L})$, one can easily show that $p_L^*(\Theta^3_{-p_L}) > p_L^*(\Theta^1_{-p_L})$ as long as $-\kappa^2\beta p_H < 0$, which is always true.

Given that $-\lambda\left[(1-\beta)C+(1-\beta p_L)\right]\frac{y^*}{E(\lambda)} > 0$, it is possible under certain conditions for y_L to be positive. If $y^* > \frac{(1-\beta p_L)\kappa^2+(1-\beta)(1+\beta p_H-\beta p_L)\lambda}{-\lambda[(1-\beta)C+(1-\beta p_L)]\kappa} r_L$, then y_L is positive.

$$y_L = \begin{cases} -\frac{(1-\beta p_L)\kappa^2+(1-\beta)(1+\beta p_H-\beta p_L)\lambda}{\kappa E(\lambda)} r_L - \lambda\left[(1-\beta)C+(1-\beta p_L)\right]\frac{y^*}{E(\lambda)} < 0, & \text{iff } y^* < \frac{(1-\beta p_L)\kappa^2+(1-\beta)(1+\beta p_H-\beta p_L)\lambda}{-\lambda[(1-\beta)C+(1-\beta p_L)]\kappa} r_L \\ -\frac{(1-\beta p_L)\kappa^2+(1-\beta)(1+\beta p_H-\beta p_L)\lambda}{\kappa E(\lambda)} r_L - \lambda\left[(1-\beta)C+(1-\beta p_L)\right]\frac{y^*}{E(\lambda)} \geq 0, & \text{iff otherwise} \end{cases}$$

B.3. Proof of Proposition 3

In this proposition we characterize how λ affects inflation and output in the low and high state. We will use the restriction that $E(\lambda) < 0$, $r_L < 0$ and the inequalities on $A(\lambda)$, $A(\lambda) < 0$; $B(\lambda)$, $B(\lambda) > 0$; C and D. Namely, when $E(\lambda) < 0$, $C > 0$ and $D < 0$.

$$\begin{aligned}
\frac{\partial \pi_H}{\partial \lambda} =& \frac{A'(\lambda)E(\lambda) - A(\lambda)E'(\lambda)}{E(\lambda)^2} r_L - \frac{E(\lambda) - \lambda E'(\lambda)}{E(\lambda)^2} C\kappa y^* \\
=& \frac{A'(\lambda)\left[-A(\lambda) - A(\lambda)C - B(\lambda)C\right] - A(\lambda)\left[-A'(\lambda) - A'(\lambda)C - B'(\lambda)C\right]}{E(\lambda)^2} r_L \\
&- \frac{\left[-A(\lambda) - A(\lambda)C - B(\lambda)C\right] - \lambda\left[-A'(\lambda) - A'(\lambda)C - B'(\lambda)C\right]}{E(\lambda)^2} C\kappa y^* \\
=& \frac{-\beta p_H \lambda\left(1-\beta+\beta p_H\right) + \beta p_H\left(\kappa^2 + \left(1-\beta+\beta p_H\right)\lambda\right)}{E(\lambda)^2} C r_L \\
&- \frac{\left[\beta\lambda p_H + \beta\lambda p_H C - \left[\kappa^2 + \lambda\left(1-\beta(1-p_H)\right)\right]C\right] - \lambda\left[\beta p_H + \beta p_H C - \left(1-\beta(1-p_H)\right)C\right]}{E(\lambda)^2} C\kappa y^* \\
=& \frac{\beta p_H \kappa^2}{E(\lambda)^2} C r_L + \frac{\kappa^2 C}{E(\lambda)^2} C\kappa y^* \qquad (74) \\
& \qquad (75)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pi_L}{\partial \lambda} =& \frac{-B'(\lambda)E(\lambda) + E'(\lambda)B(\lambda)}{E(\lambda)^2} r_L + \frac{E(\lambda) - \lambda E'(\lambda)}{E(\lambda)^2} D\kappa y^* \\
=& \frac{A'(\lambda)B(\lambda) - A(\lambda)B'(\lambda)}{E(\lambda)^2} D r_L - \frac{\kappa^2 C}{E(\lambda)^2} D\kappa y^* \\
=& -\frac{\beta p_H \kappa^2}{E(\lambda)^2} D r_L - \frac{\kappa^2 C}{E(\lambda)^2} D\kappa y^* \qquad (76)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial y_H}{\partial \lambda} =& \frac{-E(\lambda)}{E(\lambda)^2} \beta\kappa p_H r_L + \frac{-E(\lambda)}{E(\lambda)^2} C\kappa^2 y^* \\
=& \frac{-\left[\beta p_H - (1-\beta)C\right]}{E(\lambda)^2} \beta\kappa p_H r_L + \frac{-\left[\beta p_H - (1-\beta)C\right]}{E(\lambda)^2} C\kappa^2 y^* \\
=& \frac{-\left[\beta p_H - (1-\beta)C\right]}{E(\lambda)^2} \left[\beta\kappa p_H r_L + C\kappa^2 y^*\right]
\end{aligned}$$

As shown in proposition 2, $(\beta p_H - (1-\beta)C) < 0$ for any plausible value of p_L that satisfy the existence of equilibria (proposition 1) then $\frac{\partial y_H}{\partial \lambda} > 0$ iff $y^* > \frac{-\beta p_H}{C\kappa} r_L$ and $\frac{\partial y_H}{\partial \lambda} \leq 0$ iff $y^* \leq \frac{-\beta p_H}{C\kappa} r_L$.

$$
\begin{aligned}
\frac{\partial y_L}{\partial \lambda} =& \frac{(1-\beta)(1-\beta p_L+\beta p_H)E(\lambda) - \big((1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L+\beta p_H)\lambda\big)\, E'(\lambda)}{\kappa E(\lambda)^2} r_L \\
&- \frac{E(\lambda)-\lambda E'(\lambda)}{E(\lambda)^2}\left[(1-\beta)C + (1-\beta p_L)\right] y^* \\
=& \left[\frac{(1-\beta)(1-\beta p_L+\beta p_H)\left[A(\lambda)D - B(\lambda)C\right]}{\kappa E(\lambda)^2}\right] r_L \\
&- \left[\frac{\big((1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L+\beta p_H)\lambda\big)\left[A'(\lambda)D - B'(\lambda)C\right]}{\kappa E(\lambda)^2}\right] r_L + \frac{\kappa^2 C}{E(\lambda)^2}\left[(1-\beta)C + (1-\beta p_L)\right] y^* \\
=& \frac{\beta\kappa p_H}{E(\lambda)^2}\left[(1-\beta)C + (1-\beta p_L)\right] r_L + \frac{\kappa^2 C}{E(\lambda)^2}\left[(1-\beta)C + (1-\beta p_L)\right] y^*
\end{aligned} \tag{77}
$$

B.4. Proof of Proposition 4

In this proposition we characterize how y^* affects inflation and output in the low and high state. We will use the restrictions that $E(\lambda) < 0$, $r_L < 0$ and the inequalities on $A(\lambda)$, $A(\lambda) < 0$; $B(\lambda)$, $B(\lambda) > 0$; C and D. Namely, when $E(\lambda) < 0$, $C > 0$ and $D < 0$.

$$
\frac{\partial \pi_H}{\partial y^*} = -\frac{C}{E(\lambda)}\kappa\lambda > 0
$$

$$
\frac{\partial \pi_L}{\partial y^*} = \frac{D}{E(\lambda)}\kappa\lambda > 0
$$

$$
\begin{aligned}
\frac{\partial y_H}{\partial y^*} =& \left(1 + \frac{C}{E(\lambda)}\kappa^2\right) \\
=& -E(\lambda) - C\kappa^2 \\
=& -\big[\beta\lambda p_H - (\kappa^2 + \lambda[1-\beta])C\big] - C\kappa^2 \\
=& -\beta\lambda p_H + \lambda(1-\beta)C \\
=& -\lambda(\beta p_H + (1-\beta)C)
\end{aligned}
$$

Since we've imposed in our earlier propositions that $(\beta p_H + (1-\beta)C) < 0$, then we have that $\frac{\partial y_H}{\partial y^*} > 0$

$$
\frac{\partial y_L}{\partial y^*} = \frac{-\lambda[(1-\beta) + (1-\beta)p_L]}{E(\lambda)} > 0
$$

B.5. Proof Proposition 5

In this proposition we characterize how τ_L affects inflation and output in the low and high state. We will use the restrictions that $E(\lambda) < 0$, $r_L < 0$ and the inequalities on $A(\lambda)$, $A(\lambda) < 0$; $B(\lambda)$, $B(\lambda) > 0$; C and D. Namely, when $E(\lambda) < 0$, $C > 0$ and $D < 0$.

$$
\frac{\partial \pi_H}{\partial \tau_L} = -\frac{A(\lambda)}{E(\lambda)}\frac{1}{\sigma} > 0
$$

$$
\frac{\partial \pi_L}{\partial \tau_L} = \frac{-B(\lambda)}{E(\lambda)}\frac{1}{\sigma} > 0
$$

$$\frac{\partial y_H}{\partial \tau_L} = \frac{\beta\kappa p_H}{E(\lambda)}\frac{1}{\sigma} < 0$$

$$\frac{\partial y_L}{\partial y^*} = -\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1+\beta-\beta p_L)\lambda}{E(\lambda)\kappa} > 0$$

B.6. Proof Proposition 6

In this proposition we first characterize how p_H affects inflation and output in the low and high state. We will use the restriction that $E(\lambda) < 0$, $r_L < 0$ and the inequalities on $A(\lambda)$, $A(\lambda) < 0$; $B(\lambda)$, $B(\lambda) > 0$; C and D. Namely, when $E(\lambda) < 0$, $C > 0$ and $D < 0$.

$$\begin{aligned}
E(\lambda) =& -\beta\lambda p_H\left[-1-C\right] - \left[\kappa^2 + \lambda(1-\beta(1-p_H))\right]C \\
E(\lambda) =& \beta\lambda p_H - \left[\frac{1-p_L}{\sigma\kappa}(1-\beta p_L+\beta p_H) - p_L\right]\left[\kappa^2+\lambda(1-\beta)\right]
\end{aligned}$$

$$\frac{\partial E(\lambda)}{\partial p_H} = \beta\lambda - \frac{1-p_L}{\sigma\kappa}\beta[\kappa^2+\lambda(1-\beta)]\frac{\partial C}{\partial p_H} = \frac{1-p_L}{\sigma\kappa}\beta\frac{\partial A}{\partial p_H} = -\beta\lambda$$

$$\begin{aligned}
\frac{\partial \pi_H}{\partial p_H} =& \frac{\frac{\partial A(\lambda)}{\partial p_H}E(\lambda) - A(\lambda)\frac{\partial E(\lambda)}{\partial p_H}}{E(\lambda)^2}r_L - \frac{\frac{\partial C}{\partial p_H}E(\lambda) - C\frac{\partial E(\lambda)}{\partial p_H}}{E(\lambda)^2}\kappa\lambda y^* \\
\frac{\partial \pi_H}{\partial p_H} =& \left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L) - p_L\right)\beta\lambda\left[\frac{[\kappa^2+\lambda(1-\beta)]}{E(\lambda)^2}r_L - \frac{\lambda\kappa}{E(\lambda)^2}y^*\right] \qquad (78)
\end{aligned}$$

If we want to examine when $\frac{\partial \pi_H}{\partial p_H} < 0$:

$$\frac{\partial \pi_H}{\partial p_H} = \left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L) - p_L\right)\beta\lambda\left[\frac{[\kappa^2+\lambda(1-\beta)]}{E(\lambda)^2}r_L - \frac{\lambda\kappa}{E(\lambda)^2}y^*\right] < 0 \qquad (79)$$

We thus need to show that $\left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L) - p_L\right)\beta\lambda\left[[\kappa^2+\lambda(1-\beta)]r_L - \lambda\kappa y^*\right] < 0$

We know that $\beta\lambda > 0$ and that $\left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L) - p_L\right) > 0$, as the negative root of this equation is equivalent to $p_L^*(\Theta^2_{-p_L})$ that is needed for the existence of the equilibria. Thus, our expression becomes:

$$\begin{aligned}
&\left(\frac{1-p_L}{\sigma\kappa}(1-\beta p_L) - p_L\right)\beta\lambda\left[[\kappa^2+\lambda(1-\beta)]r_L - \lambda\kappa y^*\right] < 0 && (80)\\
&\left[[\kappa^2+\lambda(1-\beta)]r_L - \lambda\kappa y^*\right] < 0 && (81)\\
&\lambda\kappa y^* > [\kappa^2+\lambda(1-\beta)]r_L && (82)
\end{aligned}$$

This expression will always hold since $y^* > 0$ and $r_L < 0$.

In the second part of this proposition we characterize how p_L affects inflation and output in the low and high state. We will use the restriction that $E(\lambda) < 0$, $r_L < 0$ and the inequalities on $A(\lambda)$, $A(\lambda) < 0$; $B(\lambda)$, $B(\lambda) > 0$; C and D. Namely, when $E(\lambda) < 0$, $C > 0$ and $D < 0$.

$$\frac{\partial E(\lambda)}{\partial p_L} = -\left(\frac{1}{\sigma\kappa}(1+\beta(1-2p_L+p_H)) + 1\right)[\kappa^2+\lambda(1-\beta)]$$

$$\frac{\partial C}{\partial p_L} = -(\frac{1}{\sigma\kappa}(1+\beta(1-2p_L+p_H))+1)$$

$$\frac{\partial A}{\partial p_L} = 0$$

$$\frac{\partial \pi_H}{\partial p_L} = \frac{\frac{\partial A(\lambda)}{\partial p_L}E(\lambda) - A(\lambda)\frac{\partial E(\lambda)}{\partial p_L}}{E(\lambda)^2} r_L - \frac{\frac{\partial C}{\partial p_L}E(\lambda) - C\frac{\partial E(\lambda)}{\partial p_L}}{E(\lambda)^2}\kappa\lambda y^*$$

$$\frac{\partial \pi_H}{\partial p_L} = \frac{\beta\lambda p_H\left((\frac{1}{\sigma\kappa}(1+\beta(1-2p_L+p_H))+1)[\kappa^2+\lambda(1-\beta)]\right)}{E(\lambda)^2} r_L \tag{83}$$

$$- \frac{-\left(\frac{1}{\sigma\kappa}(1+\beta(1-2p_L+p_H))+1\right)E(\lambda) + C\left((\frac{1}{\sigma\kappa}(1+\beta(1-2p_L+p_H))+1)[\kappa^2+\lambda(1-\beta)]\right)}{E(\lambda)^2}\kappa\lambda y^* \tag{84}$$

$$\tag{85}$$

Note that $\frac{\partial E(\lambda)}{\partial p_L} < 0$ and $\frac{\partial C}{\partial p_L} < 0$, as $-2p_L + p_H > -\frac{\sigma\kappa+1}{\beta} - 1$ for any $0 < p_L, p_H < 1$. Thus, it is always the case that $\frac{\partial \pi_H}{\partial p_L} < 0$.

Appendix C. Summary Statistics

Table A.1: Summary Statistics for Expected Inflation For Combined Confidence Measure

			Political Orientation		Inflation Targeting		Gov. Trust		Country								
	No. Obs.	Total	Right	Left	No	Yes	Trust	Distrust	Austria	France	Germany	Hong Kong	Italy	Singapore	Spain	Switzerland	UK
Short-Run Inflation																	
Not Confident	29,233	3.84***	3.74***	3.84***	5.08***	3.56***	3.83***	3.85***	3.39***	3.09***	3.53***	$5.37^{**\diamond}$	4.68***	4.59***	$3.65^{\diamond\diamond\diamond}$	3.52***	2.75***
Confident	21,246	2.81	2.75	3.04	4.39	2.51	2.91	2.45	2.84	2.62	2.44	5.06	3.30	4.00	3.69	1.99	1.98
Medium-Run Inflation																	
Not Confident	27,204	5.34***	5.13***	5.08***	7.29***	4.88***	5.24***	5.39***	5.09***	4.66***	5.21***	7.82***	5.41***	6.40***	$4.48^{\diamond\diamond\diamond}$	5.39***	4.29***
Confident	19,928	3.84	4.06	3.65	5.71	3.45	3.92	3.53	3.59	3.62	3.47	6.56	3.69	5.22	4.36	3.33	3.07

		Gender		Age			Survey										
	No. Obs.	Male	Female	Young	Middle	Old	3	4	5	6	7	8	9	10	11	12	13
Short Run Inflation																	
Not Confident	29,233	3.33***	4.47***	4.59***	3.99***	3.37***	4.37***	4.07***	3.99***	4.10***	3.80***	3.84***	3.78***	3.45***	3.57***	3.66***	3.64***
Confident	21,246	2.53	3.31	3.82	3.11	2.26	3.58	3.22	3.28	2.92	3.18	2.80	2.79	2.53	2.39	2.56	2.41
Medium Run Inflation																	
Not Confident	27,204	4.95***	5.82***	6.11***	5.48***	4.84***	5.68***	5.52***	5.64***	5.59***	5.44***	5.18***	5.20***	5.14***	5.11***	5.08***	5.12***
Confident	19,928	3.55	4.35	4.81	4.03	3.32	4.40	4.14	4.19	4.19	3.94	3.79	3.85	3.58	3.61	3.64	3.49

Notes: The mean (median) of each group variable is compares between confident and not confident. For example, when testing Austria we are comparing the mean (median) of Not Confident to the mean (median) of Confident within Austria. $\star$,$\star\star$, $\star\star\star$ denote significance at the 10, 5, and 1 percent, respectively, for the two sample t test. $\diamond$,$\diamond\diamond$, $\diamond\diamond\diamond$ denote significance at the 10, 5, and 1 percent, respectively, for the Kruskal-Wallis equality of populations rank test.

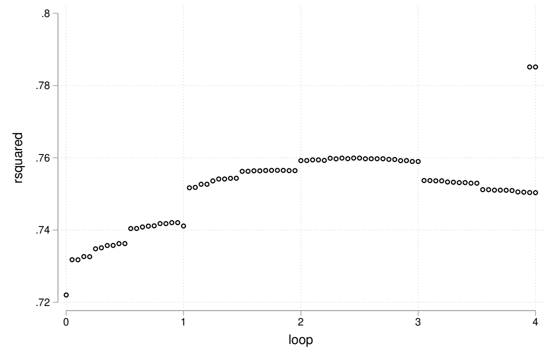
Table A.2: Share of Confidence Combined Confidence Measure

		Political Orientation		Inflation Targeting		Gov. Trust		Country									
	Total	Right	Left	No	Yes	Trust	Distrust	Austria	France	Germany	Italy	Spain	EU	Hong Kong	Singapore	Switzerland	UK
Not Confident	61.98***	57.35***	71.58***	60.78***	62.19***	43.65***	83.26***	73.55***	77.54***	66.54***	65.88***	77.37***	71.77***	72.54***	48.55**	37.02***	40.29***
Confident	38.02	42.65	28.42	39.22	37.81	56.35	16.74	26.45	22.46	33.46	34.12	22.63	28.23	27.46	51.45	62.98	59.71

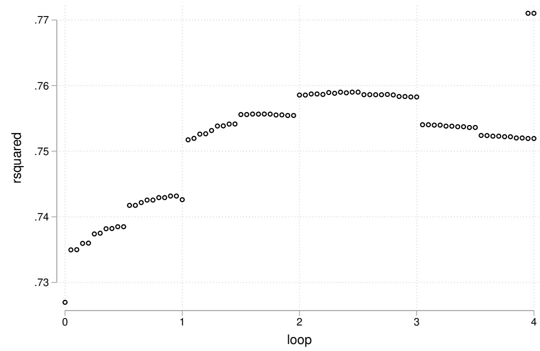
		Gender		Age			Survey										
	Total	Male	Female	Young	Middle	Old	3	4	5	6	7	8	9	10	11	12	13
Not Confident	61.98***	56.86***	67.24***	62.55***	63.11***	61.32***	69.67***	65.28***	69.24***	65.40***	61.62***	61.04***	61.54***	57.71***	58.16***	59.23***	57.17***
Confident	38.02	43.14	32.76	37.45	36.89	38.68	30.33	34.72	30.76	34.60	38.38	38.96	38.46	42.29	41.84	40.77	42.83

*Notes:*The median of each group variable is compares between confident and not confident. For example, when testing Austria we are comparing the median of Not Confident to the median of Confident within Austria. *,**,*** denote significance at the 10, 5, and 1 percent, respectively, for a one-sample test of proportion.

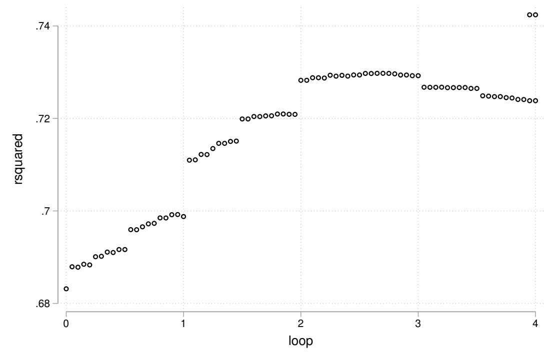
Figure A.1. Perception of ECB's inflation target in different euro-area countries.



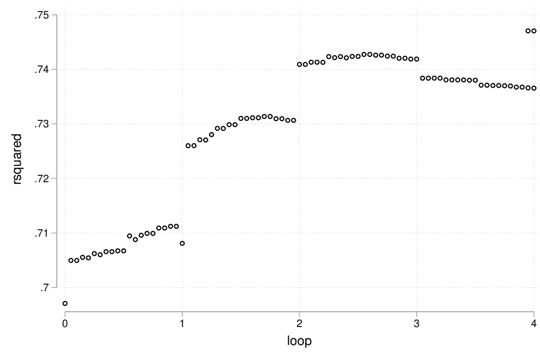
(a) Spain



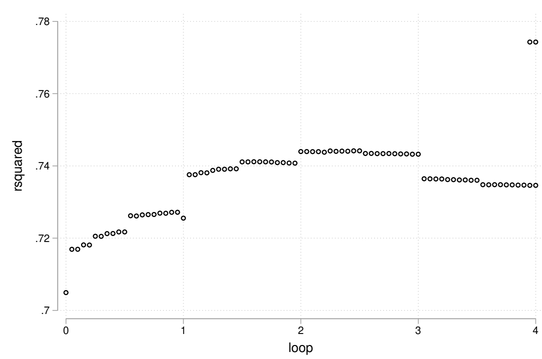
(b) Italy



(c) Germany



(d) Austria



(e) France