Information Inertia

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ABSTRACT

We show that aversion to risk and ambiguity leads to information inertia when investors process public news about assets. Optimal portfolios do not always depend on news that is worse than expected; hence, the equilibrium stock price does not reflect this bad news. This informational inefficiency is more severe when there is more risk and ambiguity but disappears when investors are risk neutral or the news is about idiosyncratic risk. Information inertia leads to news momentum (e.g. after earnings announcements) and is consistent with low trading activity of households. An ambiguity premium helps explain the macro and earnings announcement premium.

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Fama's efficient markets hypothesis ignited a lot of empirical and theoretical research on the informational role of asset prices. More recent empirical evidence points to the importance of this role since most of the expected excess return is earned around times when important information is released—the macro and earnings announcements premium. During these times, prices underreact to news, thus, failing to efficiently incorporate this information (Savor (2012)) and leading to news momentum; one of the oldest and most robust manifestations of which is the post-earnings announcement drift or PEAD.

This paper provides a novel economic mechanism which, in the absence of transaction costs, information processing costs, or any other market frictions, leads to optimal portfolios that do not always react to new information and, hence, information is not always reflected in the equilibrium stock price—a phenomenon we refer to as information inertia. The economic mechanism that leads to information inertia relies on the trade-off between over- and underestimating the informativeness of news that is difficult to link to future asset payoffs. Ambiguity averse investors who learn from this news do not want to respond to it for fear of overestimating its informativeness and, thus, underestimating the residual risk. On the other hand, investors do not want to ignore news that predicts a drop in the future value of an asset in this case fearing to underestimate its informativeness. We show that these two effects exactly offset each other, leading to no reaction at all for a range of signals conveying news that is worse than expected in the workhorse learning models in finance—joint normal distribution of signal and asset values/returns combined with CARA/CRRA risk preferences and the multiple prior preference model that exhibits aversion to ambiguity.[1]

[1]The multiple prior or "max-min" formulation of preferences is axiomatized in Gilboa and Schmeidler

To be more concrete about the economic mechanism that leads to information inertia, consider an investor who uses the dividend-price ratio to predict future excess stock market returns. The investor does not know the correlation ρ between the excess return and the predictor or signal and, thus, there is ambiguity about the economic significance β (which is linear in ρ) and the explanatory power ρ^2. Hence, this signal leads to ambiguity about the posterior mean because investors do not know how much weight to put on it and it leads to ambiguity about the residual variance because they do not know how much they can learn from it. Being averse to this ambiguity, the investor considers a family of linear regression models described by the interval $[\rho_a, \rho_b]$ and evaluates the outcome of an investment decision under the model that yields the lowest expected utility, that is, the lowest conditional Sharpe ratio of the asset since the investor has mean-variance preferences. There is a range of signals conveying bad news for which the correlation that minimizes the Sharpe ratio changes with the signal, that is, any decrease in the signal within this range increases the worst case scenario correlation, thus lowering the posterior mean and the residual variance. The resulting decrease in the asset's Sharpe ratio is exactly offset by the decrease in the asset's volatility leaving the optimal stock market investment unchanged; hence, exhibiting information inertia. This stock investment is identical to that of a standard expected utility maximizer who thinks the signal is uninformative ($\rho = 0$), which is surprising, since we rule out such signals ($\rho_a > 0$). The likelihood of investors not reacting to bad news about future excess stock market returns (a dividend-price ratio that is lower than expected) is between

(1989) and it is a commonly used representation of decision-making under ambiguity in financial markets to exploit qualitative differences from standard expected utility models as discussed in Epstein and Schneider (2010).

5% and 20% when we use confidence intervals to proxy for ambiguity as in Garlappi, Uppal, and Wang (2007).

Information inertia in portfolio demand leads to equilibrium stock prices that fail to incorporate some publicly available information. Specifically, there is a range of signals that are worse than expected for which demand for the stock does not depend on the signal and, thus, the stock price does not reflect this information in equilibrium. This informational inefficiency is more severe when the volatility of cash flows and investors' aversion to risk and ambiguity is high but it disappears when investors are risk neutral or if the signal is about the idiosyncratic component of an asset's cash flow. Moreover, ambiguity averse investors require an ex-ante premium for news about an asset's cash flow which, in addition to the premium for systemic risk, helps explain the high macro and earnings announcement premium in the data. If we deviate from the workhorse models by considering (i) different joint distributions of asset payoffs and signals, (ii) economies populated with ambiguity averse and standard expected utility maximizers, and (iii) other preference models that allow for a distinction between ambiguity and ambiguity aversion, then the trade-off between over- and underestimating the informativeness of ambiguous news does not exactly offset. In this case the resulting demand for the asset and its equilibrium price shows reaction to news that is much lower than without ambiguity.

How does our information inertia result fit into the large literature on news momentum? We have identified the new economic effect that learning under ambiguity about the link between information and asset payoffs leads to underreaction to news. Hence, an econometrician regressing excess stock returns on news will estimate a positive coefficient concluding the existence of news momentum. While she would also conclude there is news momentum in a model with expected utility maximizers who underestimate the informativeness of the signal, our model gives additional testable predictions. Specifically, the slope coefficient is higher and, thus, there is more news momentum if there is (i) more risk, (ii) more ambiguity, and (iii) if the signal is more informative about the systematic rather than the idiosyncratic cash flow component of the asset. We find support for all three predictions in the literature that we discuss below.

While our results explain that stock market returns in the US and abroad tend to underreact to macroeconomic news (Wang (2015)), they also shed new light on one of the most robust instances of news momentum—earnings momentum. This is the tendency for a stock's risk-adjusted return to drift in the direction of an earnings news surprise for a certain period of time for news in the form of an earnings announcement, which is the post-earnings announcement drift or PEAD,[2] or news in the form of revisions of analysts' earnings forecasts, which is the post-earnings forecast revision drift or PFRD.[3] Specifically, we show that

[2]The PEAD which is also called the standardized unexpected earnings (SUE) effect was initially proposed by the information content study of Ball and Brown (1968) and verified by Foster, Olsen, and Shevlin (1984), Bernard and Thomas (1989) and Bernard and Thomas (1990) who provide a comprehensive summary of the early work on the PEAD. Comparable and often even stronger results have been reported for analyst forecasts by Doyle, Lundholm, and Soliman (2006), Livnat and Mendenhall (2006), and DellaVigna and Pollet (2009).

[3]Evidence on the existence of the PFRD dates back to Givoly and Lakonishok (1980) and Stickel (1991) who focus largely on individual analyst revision while Chan, Jegadeesh, and Lakonishok (1996) confirm the existence of the PFRD for revision of the consensus analyst forecast. More recent studies that offer new insights into the PFRD are Gleason and Lee (2003), Zhang (2006), and Hui and Yeung (2013). For a recent survey see Kothari, So, and Verdi (2016).

there is a positive drift for excess stock returns that prevails when we adjust excess stock returns for market risk if the earnings news has a firm specific and systematic news component. This prediction is supported by recent empirical studies in the accounting literature: Hui and Yeung (2013) show that the PFRD is driven by under-reaction to industry-wide earnings news and there is no drift for idiosyncratic news and Kovacs (2016) presents similar evidence for the PEAD. Moreover, the positive drift is bigger if there is more risk and ambiguity which is supported by many authors in the literature who use different proxies for risk and ambiguity. Specifically, the PFRD is larger for stocks with lower analyst coverage (Gleason and Lee (2003), Zhang (2006)), greater unexpected delay in processing analyst forecast revisions (Akbas, Markov, Subasi, and Weisbrod (2018)), less well-known analysts (Gleason and Lee (2003)), and lower firm age and size, or by greater analyst forecast dispersion, return volatility, and cash flow volatility (Zhang (2006)). Francis, Lafond, Olsson, and Schipper (2007) find that the PEAD is greater for US stocks with lower earnings quality while Hung, Li, and Wang (2014) find a greater PEAD for financial markets that have lower financial reporting quality. Moreover, Hou, Peng, and Xiong (2009) show that PEAD profits are greater for low volume stocks and during recessions, which are periods where ambiguity and risk is higher.

We show that investors who anticipate news such as a macro announcement or an earnings announcement require an uncertainty premium that consists of a risk and an ambiguity premium to hold stocks. This uncertainty premium which is increasing in risk and ambiguity helps explain the high average excess stock market returns observed in the data before a macroeconomic news announcement about inflation and unemployment (Savor and Wilson

(2013)) or interest rates (Savor and Wilson (2013), Lucca and Moench (2015)) and it helps explain the high excess stock returns before an earnings announcement (Savor and Wilson (2016)). Ai and Bansal (2018) show that deviations from expected utility—e.g. ambiguity aversion—are even necessary to explain the announcement premium. Moreover, Zhou (2015) documents the existence of an ambiguity premium for macroeconomics news and Liu, Chan, and Faff (2018) show similar evidence for the earnings announcement premium.

Our results are also helpful in understanding why households follow simple portfolio rules and do not trade as much as traditional models would predict even after accounting for transaction or information processing costs.[4] Specifically, we would expect to have less trade in markets with more ambiguity averse traders since they always trade in response to changes in the price but not always in response to news.[5] While it is difficult to distinguish trade due to price change from trade due to news, and thus provide direct evidence for our mechanism in the data, the references below show lower trading activity for household characteristics that are typically associated with more ambiguity aversion. Specifically, less financially literate households (Bianchi and Tallon (2018)), less wealthy households (Calvet, Campbell, and Sodini (2009)), and less educated households (Calvet, Campbell, and Sodini (2009), and Bilias, Georgarakos, and Haliassos (2010)) show lower trading activity.

This paper complements recent work on optimal portfolios and equilibrium asset prices

[4]See Ameriks and Zeldes (2004), Bianchi and Tallon (2018), Bodie, Detemple, and Rindisbacher (2009), Calvet, Campbell, and Sodini (2009), Campbell (2006), and Bilias, Georgarakos, and Haliassos (2010) and the references therein for a review of the household portfolio choice literature.

[5]Ambiguity averse investors may trade a lot in response to changes in the price (Bianchi and Tallon (2018)).

when investors process public signals. Epstein and Schneider (2008) show that investors react more to bad signals than to good signals when there is ambiguity about the precision of these signals. Illeditsch (2011) shows that this ambiguity leads to risky portfolios that are insensitive to changes in the stock price—a phenomenon referred to as portfolio inertia. We are the first to show that ambiguity aversion leads to information inertia for risky portfolios and equilibrium prices without relying on information processing costs or other market frictions. Moreover, the economic mechanism that leads to information inertia is novel because it does not occur at the kink in investors' utility in contrast to the portfolio inertia results in Illeditsch (2011).[6]

Our work is also related to a large literature that studies the informational efficiency of prices when there is asymmetric information. For instance, prices do not fully reveal private information in equilibrium, (i) if it is costly to acquire information (Grossman (1976) and Grossman and Stiglitz (1976)), (ii) if there are noise traders (Grossman and Stiglitz (1980)), (iii) if informed investors anticipate how their trades will impact prices (Kyle (1985) and Back, Cao, and Willard (2000)), (iv) if there is ambiguity (Caskey (2009) and Condie and Ganguli (2017)). What is striking in this paper is that a costless informative public signal is not always incorporated in the price (the price is not a sufficient statistic for the signal) when an investor is averse to ambiguity in the workhorse learning models in finance.

There is a growing literature in macroeconomics that imposes an exogenous constraint or cost on the ability of investors to process information in order to explain why macroeconomic

[6] We are not aware of any work with multiple prior preferences that leads to qualitatively different results than standard expected utility that are not due to the kink in utility.

variables exhibit inertia (see Sims (2010) and the references therein). These ideas have also been used in finance to explain information inertia of portfolios (Abel, Eberly, and Panageas (2007)), excess correlation (Peng and Xiong (2006)), financial contagion (Mondria (2010) and (Mondria and Quintana-Domeque 2013)), and portfolio under-diversification (Nieuwerburgh and Veldkamp 2010), among others.[7] In contrast, we derive low sensitivity of portfolios and asset prices to news from a rational choice model with multiple prior utility without imposing exogenous cost or constraints.

The paper proceeds as follows. In Section I, we introduce the information structure and the preferences of investors. In Section II, we solve for optimal demand of ambiguity averse investors and explain why risky and the risk-free portfolio do not always respond to news. In Section III, we solve for the equilibrium prices of individual stocks and the market portfolio and show that the equilibrium stock price fails to incorporate some publicly available information which manifests itself in news momentum. In Section IV, we show the robustness of our information inertia result. Section V concludes. All proofs are relegated to the Appendix and more details are discussed in the Internet Appendix.

I. Information and Preferences

We present in this section a CARA-normal framework in which an ambiguity averse investor in the sense of Gilboa and Schmeidler (1989) learns from an ambiguous signal about the future value of an asset before she makes an investment decision.

[7]See Veldkamp (2011) and the reference therein for an overview of this literature.

There are two dates 0 and 1. Investors can invest in a risk-free asset and a risky asset. Let p denote the price of the risky asset, $\tilde{d}$ the future value or dividend of the risky asset, and θ the number of shares invested in the risky asset. There is no consumption at date zero. The risk-free asset is used as numeraire, so the risk-free rate is zero. The budget constraint is therefore

$$\tilde{w} = w_0 + \left(\tilde{d} - p\right)\theta, \tag{1}$$

in which w_0 denotes initial and $\tilde{w}$ future wealth.

A. *Ambiguity Aversion — Multiple Priors*

Suppose investors receive a signal $\tilde{s}$ about the future asset value $\tilde{d}$. Investors do not know the model m that links the signal to the asset value, that is, they do not know the joint distribution of $\tilde{d}$ and $\tilde{s}$. Moreover, they are averse to ambiguity or model uncertainty in the sense of Gilboa and Schmeidler (1989) and, hence, they consider the set of models $\mathcal{M}$ when making portfolio decisions. Specifically, the ambiguity averse investor chooses a portfolio θ to maximize

$$U(\theta) \equiv \min_{m\in\mathcal{M}} \mathrm{E}_m\left[u\left(w_0 + \left(\tilde{d} - p\right)\theta\right) \mid \tilde{s} = s\right], \tag{2}$$

where $u(\cdot)$ denotes the Bernoulli utility function of the investor and $\mathrm{E}_m\left[\cdot\right]$ the expectation w.r.t. the belief generated by the model m. In the remainder of this paper we will refer to ambiguity averse investors whose preferences under uncertainty are represented with multiple priors/models as MEU investors and to ambiguity neutral investors whose preferences under uncertainty are represented with a single prior/model as standard expected utility maximizers, in short, SEU. For MEU investors, the set $\mathcal{M}$ represents their beliefs about the asset payoff and it is a measure of both the asset's ambiguity and the investor's ambiguity aversion, that is, there is no distinction between ambiguity and ambiguity aversion in the Gilboa and Schmeidler (1989) ambiguity model.[8] To be consistent with models that allow for a distinction between ambiguity and ambiguity aversion that we discuss in Robustness Section IV, we refer to the size of the set $\mathcal{M}$ as the amount of ambiguity that an MEU investor faces when contemplating an investment in an asset after receiving ambiguous information. For both MEU and SEU investors, the curvature of the utility function $u(\cdot)$ determines the investor's risk aversion.

B. Normal Distributions and CARA-Expected Utility

We consider an information structure where the joint distribution of $\tilde{d}$ and $\tilde{s}$ is normal.[9] Specifically,

$$\begin{pmatrix} \tilde{d} \\ \tilde{s} \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} \bar{d} \\ \bar{s} \end{pmatrix}, \begin{pmatrix} \sigma_d^2 & \rho\sigma_d\sigma_s \\ \rho\sigma_d\sigma_s & \sigma_s^2 \end{pmatrix} \right). \tag{3}$$

Hence, a model is described by five parameters, that is, $m = (\bar{d}, \sigma_d, \bar{s}, \sigma_s, \rho)$, where the strictly positive correlation parameter ρ links the normally distributed signal $\tilde{s}$ to the normally distributed asset value $\tilde{d}$. Conditional on knowing the model m, the posterior distribution

[8]MEU or multiple prior preferences imply behavior that is consistent with the experimental evidence in Ellsberg (1961) and more recent portfolio choice experiments discussed by Ahn, Choi, Gale, and Kariv (2014) and Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010).

[9]We discuss other distributions in Robustness Section IV.

of the asset value $\tilde{d}$ is also normal:

$$\tilde{d} \mid \tilde{s} = s \ \sim \mathrm{N}_m \left(\mu(s,m), \sigma^2(m)\right), \tag{4}$$

where $\mu(s,m) = \bar{d} + \rho\sigma_d/\sigma_s(s - \bar{s})$ denotes the conditional mean and $\sigma(m) = \sigma_d\sqrt{1-\rho^2}$ the conditional volatility of $\tilde{d}$ given s.

Suppose investors have CARA utility over future wealth $\tilde{w}$, that is, $u(\tilde{w}) = -e^{-\gamma\tilde{w}}$ with $\gamma > 0$. A standard expected utility or SEU investor who considers the model m chooses a portfolio θ to maximize

$$\bar{U}(\theta) \equiv \ \mathrm{E}_m \left[u\left(w_0 + \left(\tilde{d} - p\right)\theta\right) \mid \tilde{s} = s\right] = u\left(\overline{\mathrm{CE}}(\theta, m)\right), \tag{5}$$

where $\overline{\mathrm{CE}}(\theta, m)$ denotes her certainty equivalent:

$$\overline{\mathrm{CE}}(\theta, m) = \mathrm{E}_m\left[\tilde{w} \mid \tilde{s} = s\right] - \frac{1}{2}\gamma \mathrm{Var}_m\left[\tilde{w} \mid \tilde{s} = s\right] = w_0 + \left(\mu(s,m) - p\right)\theta - \frac{1}{2}\gamma\theta^2\sigma(m)^2. \tag{6}$$

Hence, SEU investors have mean-variance preferences over the asset value $\tilde{d}$ conditional on observing the signal $\tilde{s}$.

C. CARA-Normal Model of Learning under Ambiguity

To determine the utility of an ambiguity averse investor given in equation (2) we need to determine the set of posterior asset distributions, that is, we need to update the preferences of an ambiguity averse investor. We consider the CARA-normal model described in the previous section and follow Gilboa and Schmeidler (1993) to determine the family of posterior asset distributions by applying Bayes rule to each model $m \in \mathcal{M}$. Hence, the utility of an MEU investor with CARA utility $u(\cdot)$ who holds θ shares of the risky asset is

$$U(\theta) = \min_{m \in \mathcal{M}} \mathrm{E}_m \left[u \left(w_0 + \left(\tilde{d} - p \right) \theta \right) \mid \tilde{s} = s \right] = u \left(\mathrm{CE}(\theta) \right), \tag{7}$$

where $\mathrm{CE}(\theta)$ denotes the certainty equivalent of the MEU investor. Specifically,

$$\mathrm{CE}(\theta) = \min_{m \in \mathcal{M}} \overline{\mathrm{CE}}(\theta, m). \tag{8}$$

We focus on ambiguity about the link between information and asset values and, thus, we assume that there is no ambiguity about the marginal distribution of the asset value $\tilde{d}$ and the signal $\tilde{s}$. We also standardize the latter marginal, that is, $\bar{s} = 0$ and $\sigma_s = 1$. Ambiguity about the correlation between the signal and the asset value is described by the interval $[\rho_a, \rho_b]$ with $\rho_a > 0$ and $\rho_b < 1$. Hence, an MEU investor chooses a portfolio θ to maximize her certainty equivalent[10]

$$\mathrm{CE}(\theta) = \min_{\rho \in [\rho_a, \rho_b]} \overline{\mathrm{CE}}(\theta, \rho) = w_0 + \min_{\rho \in [\rho_a, \rho_b]} \left\{ \left(\mu(s, \rho) - p \right) \theta - \frac{1}{2} \gamma \theta^2 \sigma(\rho)^2 \right\}. \tag{9}$$

Learning under ambiguity about the correlation ρ has two important implications for an MEU investor with mean-variance preferences. First, ambiguity is not resolved due to learn-

[10]For a derivation of the certainty equivalent see Proposition 5 in the Appendix.

ing and hence does not disappear.[11] Second, learning from an ambiguous signal leads to ambiguity not only about how much weight to put on the signal when predicting the future asset value ($\mu(s, \rho)$) but also to ambiguity about the amount of residual risk ($\sigma(\rho)$). In other words, the worst case scenario correlation depends on the asset position θ and the signal s and, thus, the worst-case correlation does not always lower posterior means and it does not always increase residual variances. This is distinctly different from a model without learning from an ambiguous signal where the worst case scenario means and variances are chosen independently and thus always move in opposite direction. We discuss the effects of ambiguous news on optimal portfolios in Section II and on equilibrium asset prices in Section III.

II. Portfolio Choice

How do optimal portfolios react to ambiguous news? To answer this question we fix the price p for the remainder of this section and determine the optimal portfolio for the risky asset as a function of the signal.

BENCHMARK (SEU – Portfolio Choice): *The optimal portfolio of an ambiguity neutral investor with belief ρ and risk aversion γ who maximizes her utility given in equation (5)*

[11] Epstein and Schneider (2007) provide thought experiments and a general framework for learning under ambiguity and show that ambiguity does not disappear if there is ambiguity about the data generating mechanism.

after receiving the signal s is

$$\bar{\theta}(s,\rho) \equiv \frac{\mathrm{E}_\rho\left[\tilde{d} \mid \tilde{s}=s\right]-p}{\gamma \mathrm{Var}_\rho\left[\tilde{d} \mid \tilde{s}=s\right]} = \frac{\mu(s,\rho)-p}{\gamma\sigma(\rho)^2}. \tag{10}$$

The posterior asset mean strictly increases in the signal and the residual variance is unaffected by the signal. Hence, the optimal portfolio for an SEU investor always depends on the signal except for the knife edge case of an uninformative signal, that is, $\rho = 0$. The next theorem shows that this is no longer true for optimal portfolios of ambiguity averse investors. These portfolios do not always depend on the signal even if there is no ambiguity about the fact that the correlation between the signal and the future asset value is positive, that is, $\rho_a > 0$.

THEOREM 1 (MEU – Portfolio Choice): *The optimal portfolio of an ambiguity averse investor with risk aversion γ who maximizes her utility given in equation (7) after receiving the signal s is*

$$\theta(s) = \begin{cases} \bar{\theta}(s,\rho_a) & \qquad s \geq s_1 \equiv -\rho_a \max(\lambda_d,0) - \frac{1}{\rho_a}\min(\lambda_d,0) \\ \max\left(\frac{\bar{d}-p}{\gamma\sigma_d^2},0\right) & s_1 > s \geq s_2 \equiv -\rho_b \max(\lambda_d,0) - \frac{1}{\rho_b}\min(\lambda_d,0) \\ \bar{\theta}(s,\rho_b) & s_2 > s \geq s_3 \equiv -\frac{1}{\rho_b}\max(\lambda_d,0) - \rho_b\min(\lambda_d,0) \\ \min\left(\frac{\bar{d}-p}{\gamma\sigma_d^2},0\right) & s_3 > s \geq s_4 \equiv -\frac{1}{\rho_a}\max(\lambda_d,0) - \rho_a\min(\lambda_d,0) \\ \bar{\theta}(s,\rho_a) & \qquad s < s_4. \end{cases} \tag{11}$$

Ambiguity is described by the interval $[\rho_a, \rho_b]$ with $\rho_a > 0$ and $\lambda_d = \frac{\mathrm{E}[\tilde{d}]-p}{\sqrt{\mathrm{Var}[\tilde{d}]}} = \frac{\bar{d}-p}{\sigma_d}$ denotes the unconditional Sharpe ratio of the risky asset.

Suppose the unconditional risk premium of the asset is positive ($\bar{d} - p > 0$). Figure 1 shows that the optimal portfolio does not always depend on signals that convey news that is worse than expected. Specifically, there is a range of bad signals for which an investor's long position in the risky asset does not depend on the signal and there is another range of bad signals for which investors do not hold the risky asset.[12] We briefly discuss the intuition for information inertia of the risk-free portfolio next and then focus on the intuition for information inertia of risky portfolios for the remainder of this section.

[Figure 1 about here.]

A. *Information Inertia for the Risk-Free Portfolio*

Why does the risk-free portfolio exhibit information inertia? To answer this question consider first an SEU investor with belief ρ. This investor would buy the asset if the conditional risk premium is positive ($\mu(s, \rho) - p > 0$) and sell short the asset if the conditional risk premium is negative ($\mu(s, \rho) - p < 0$). There is only one signal realization ($s = -\lambda_d/\rho$) for which the conditional risk premium is zero and, thus, an SEU investor would refrain from holding the asset. In contrast, MEU investors buy the asset if there is no ambiguity that the conditional risk premium is positive and sell short the asset if there is no ambiguity that the conditional risk premium is negative. There is a range of bad signals ($-\frac{\lambda_d}{\rho_a} < s < -\frac{\lambda_d}{\rho_b}$) for which the conditional risk premium is positive for some ρ and negative for others and, thus, the

[12]If the unconditional risk premium is negative, then there is a range of good news for which the risk-free portfolio and a portfolio consisting of a short position in the asset exhibits information inertia.

risk-free portfolio exhibits information inertia.[13]

B. Information Inertia for Risky Portfolios

The risk-free portfolio is the only portfolio that perfectly hedges against ambiguity by making utility independent of the unknown parameter ρ. All other portfolios are exposed to ambiguity. The next proposition shows that all these portfolios can be determined by evaluating the optimal portfolio of an SEU investor at the belief ρ that minimizes her utility. Put differently, an MEU investor behaves distinctly different from an SEU investor at a kink of her utility which in this model only occurs at the risk-free portfolio.[14] We focus on the case where the unconditional risk premium and, thus, the unconditional Sharpe ratio is positive ($\lambda_d > 0$) for the remainder of this section.

PROPOSITION 1 (MEU - Characterization of Optimal Portfolios): *Let* $\lambda_d > 0$ *and* $\rho^*(s) = \mathit{argmin}_{\rho \in [\rho_a, \rho_b]} \overline{CE}(\theta(s), \rho)$. *Then*

$$\theta(s) = \begin{cases} 0 & \mathit{if} \quad -\frac{\lambda_d}{\rho_a} \leq s \leq -\frac{\lambda_d}{\rho_b} \\ \bar{\theta}(s, \rho^*(s)) & \mathit{otherwise.} \end{cases} \tag{12}$$

An ambiguity averse investor with mean variance preferences holds portfolios with the

[13]This form of information inertia at the risk-free portfolio also arises in the models of Condie and Ganguli (2011) and Illeditsch (2011).

[14]Illeditsch (2011) considers a model where kinks also occur away from certainty.

highest possible Sharpe ratio that is robust to changes in the correlation ρ. Specifically, let

$$\lambda(s,\rho) = \frac{\mu(s,\rho) - p}{\sigma(\rho)} = \frac{\lambda_d + \rho s}{\sqrt{1-\rho^2}}, \tag{13}$$

denote the conditional Sharpe ratio when the correlation between the signal and the future asset value is ρ. An MEU investor with a long position considers the belief ρ that minimizes the conditional Sharpe ratio $\lambda(s,\rho)$ and an MEU investor with a short position considers the belief ρ that maximizes $\lambda(s,\rho)$. Hence, prior knowledge of the optimal portfolio $\theta(s)$ (other than its sign) is not required when computing the worst case scenario belief $\rho^*(s)$ of an MEU investor as the next proposition shows.

PROPOSITION 2 (MEU - Robust Sharpe Ratio): *Let $\lambda_d > 0$. If $s > -\frac{\lambda_d}{\rho_b}$, then $\bar{\theta}(s,\rho) > 0$ for all $\rho \in [\rho_a, \rho_b]$ and*

$$\rho^*(s) = \underset{\rho\in[\rho_a,\rho_b]}{argmin}\, \lambda(s,\rho) = \begin{cases} \rho_a & if \quad s > -\rho_a\lambda_d \\ -\frac{s}{\lambda_d} & if \quad -\rho_b\lambda_d < s \le -\rho_a\lambda_d \\ \rho_b & if \quad -\frac{\lambda_d}{\rho_b} < s \le -\rho_b\lambda_d. \end{cases} \tag{14}$$

If $s < -\frac{\lambda_d}{\rho_a}$, then $\bar{\theta}(s,\rho) < 0$ for all $\rho \in [\rho_a, \rho_b]$ and $\rho^*(s) = argmax_{\rho\in[\rho_a,\rho_b]}\, \lambda(s,\rho) = \rho_a$.

The robust conditional Sharpe ratio is plotted in the left graph of Figure 2 as a function of the signal. To gain intuition suppose there is no ambiguity that the conditional Sharpe ratio is positive ($s > -\frac{\lambda_d}{\rho_b} = -1.25$) and, thus, the MEU investor has a long position in the asset. If the signal conveys good news ($s > 0$), then an increase in the correlation ρ always increases the conditional Sharpe ratio because a more informative signal raises the conditional mean and reduces the conditional volatility. Hence, the MEU investor behaves like an SEU investor with belief ρ_a (blue dashed circle line in Figure 1). However, if the signal conveys bad news ($-\frac{\lambda_d}{\rho_b} < s < 0$), then an increase in the correlation ρ decreases the conditional mean and volatility and, thus, the effects on the conditional Sharpe ratio are unclear. For some bad news ($-\rho_a\lambda_d = -0.5 \leq s < 0$), the volatility effect dominates and the MEU investor behaves like an SEU investor with belief ρ_a (blue dashed circle line in Figure 1), whereas for other bad news ($-\frac{\lambda_d}{\rho_b} = -1.25 < s \leq -\rho_b\lambda_d = -1$) the mean effect dominates and, thus, the MEU investor behaves like an SEU investor with belief ρ_b (red chain-dotted square line in Figure 1).[15] In both cases demand depends on the signal.

[Figure 2 about here.]

There is a range of bad signals ($-\rho_b\lambda_d = -1 < s < -\rho_a\lambda_d = -0.5$) for which neither the conditional mean nor volatility dominates and the conditional Sharpe ratio is minimized in the interior, that is, $\frac{\partial\lambda(s,\rho)}{\partial\rho} = 0$. In this case, small changes in the correlation change both the mean and volatility but leave the Sharpe ratio unchanged. The worst-case scenario for the conditional Sharpe ratio is the lower envelope given in the left graph of Figure 2. The interior minimizer depends on the signal, that is, $\rho^* = -\frac{s}{\lambda_d}$, because a change in the signal affects only the mean directly, and, thus, the correlation changes in order to have a counterbalancing indirect effect on volatility. We know from Propositions 1 & 2 that in this case the MEU investor behaves like an SEU investor with belief, $\rho^* = -\frac{s}{\lambda_d}$, but the resulting

[15] If $s < -\frac{\lambda_d}{\rho_a} = -2.5$, then there is no ambiguity about the fact that the conditional Sharpe ratio is negative and, thus, the MEU investor has a short position in the asset. In this case the worst case scenario is always a low correlation.

optimal portfolio does not depend on the signal and, hence, coincides with the portfolio of an SEU investor who thinks the correlation between the signal and the asset is zero (green dotted cross line in Figure 1).[16]

B.1. Local Information Inertia — The Intuition

Why are there risky portfolios that do not depend on the signal even though there is no ambiguity about the fact that the signal is informative ($\rho_a > 0$)? To answer this question suppose, $s = -0.75$, in which case the optimal portfolio of the MEU investor and an SEU investor with belief, $\hat{\rho} = (\rho_a + \rho_b)/2$, coincide (black solid and purple dotted plus line in Figure 1). An increase in the signal raises the Sharpe ratio perceived by the SEU and MEU investor and, thus, makes the asset more attractive. The perceived risk for the SEU investor does not change, so her demand for the asset increases (purple dotted plus line). However, an increase in the signal also increases the risk perceived by the MEU investor because in this case the correlation between the asset and the signal decreases and, thus, less risk is resolved by the signal. The increase in the Sharpe ratio is exactly offset by the increase in the volatility, so the MEU investor does not change her portfolio (black solid line). Formally,

$$d\ln\theta(s) = d\ln\lambda\left(s, \rho^*(s)\right) - d\ln\sigma\left(\rho^*(s)\right) = 0, \qquad \forall s \in (-\rho_b\lambda_d, -\rho_a\lambda_d). \tag{15}$$

The left and right graph of Figure 2 show the (log) of the conditional Sharpe ratio and

[16]Even though the demand of an SEU investor with belief $\rho = 0$ and an MEU investor is identical in the critical signal region $(-\rho_b\lambda_d, -\rho_a\lambda_d)$, the utility of the MEU investor is lower than the utility of the SEU investor.

volatility, respectively, when the signal conveys bad news. There is a range of signals for which both the Sharpe ratio and volatility strictly increase in the signal. Specifically, the Sharpe ratio increases with the signal at exactly the same rate as the volatility increases with the signal and, hence, any change of the portfolio due to changes in the Sharpe ratio is exactly offset by a change in risk.

B.2. Risk and Ambiguity Effects

How do changes in risk and ambiguity affect the unresponsiveness of risky portfolios to information? To answer this question, let λ_d measure the amount of risk compensation and $\Delta\rho = \rho_b - \rho_a$ measure the level of ambiguity. Moreover, define with $\Pi(\lambda_d, \Delta\rho)$ the probability of investors exhibiting information inertia for risky asset positions conditional on bad news; determined in the next proposition.

PROPOSITION 3 (Information Inertia – Risk and Ambiguity Effects): *The size of the signal region for which risky portfolios do not react to news is* $\lambda_d(\rho_b - \rho_a) = \lambda_d\Delta\rho$. *The probability of investors exhibiting information inertia for risky asset positions conditional on bad news is*

$$\Pi(\lambda_d, \Delta\rho) = 2\left(\Phi\left(\lambda_d\rho_b\right) - \Phi\left(\lambda_d\rho_a\right)\right), \tag{16}$$

where $\Phi(\cdot)$ *denotes the cumulative distribution function of a standard normal distributed random variable. Moreover, we have the following two comparative static results: (i)* $\frac{\partial\Pi(\Delta\rho,\cdot)}{\partial\Delta\rho} \geq 0$ *and (ii) there exists a Sharpe ratio* $\hat{\lambda}_d$ *that uniquely maximizes* $\Pi(\lambda_d, \cdot)$ *with* $\Pi(\lambda_d = 0, \cdot) = 0$ *and* $\Pi(\lambda_d = \infty, \cdot) = 0$. *The inequality in part (i) is strict if* $\lambda_d > 0$.

There is more information inertia when investors exhibit more ambiguity and there is no information inertia if investors are ambiguity neutral. The severity of information inertia is non-monotonic in the ex-ante risk compensation for the asset and it disappears for small and large λ_d because in both cases the MEU investor behaves like an SEU investor with belief ρ_a.

B.3. Quantitative Importance of Information Inertia

We calibrate an asset return predictability model to determine the quantitative significance of the information inertia results and to sharpen the predictions for the effects of changes in risk for our information results. Suppose MEU investors have mean-variance preferences over the stock market return in excess of the risk-free rate and the signal is a predictor of the future excess stock market return; e.g. the dividend-price ratio. We use the size of the confidence interval for the correlation $\hat{\rho} = (\rho_a + \rho_b)/2$ as a proxy for the level of ambiguity $\Delta\rho = \rho_b - \rho_a$. This is standard in the literature (e.g. Garlappi, Uppal, and Wang (2007)). Let $\hat{a}$ denote the significance level of the confidence interval for $\hat{\rho}$ and T the size of the data sample. The size of the interval strictly decreases in the significance level $\hat{a}$ and hence $\alpha = 1 - \hat{a}$ can be interpreted as a measure for the ambiguity $\Delta\rho$.

We consider $T = 84$ observations of excess returns and dividend-price ratios and three different values for the point estimate of the correlation, that is, $\hat{\rho} \in \{30\%, 40\%, 50\%\}$. These estimates correspond to R^2s in predictive regressions ranging from 0.09 to 0.25 which are similar to the reported predictability results in Koijen and Nieuwerburgh (2011) who use 84 years of data. We focus on predictors for excess returns that are statistically significant to determine the probability of information inertia and, hence, $\rho_a > 0$.

Table I shows the probability of information inertia conditional on signals that convey bad news for long stock positions. We consider five different values for the unconditional Sharpe ratio, that is, $\lambda_d \in \{0.25, 0.3, 0.35, 0.4, 0.5\}$ and ambiguity decreases from $\alpha = 0.99$ to $\alpha = 0.75$. Table I confirms the results of Proposition 3 that the likelihood of information inertia strictly increases in ambiguity α and it shows that the likelihood of information inertia strictly increases in the riskiness of the stock. Moreover, the table demonstrates that information inertia is economically meaningful for reasonable levels of risk. For instance, if $\alpha = 0.99$, then the probability of risky portfolios being unresponsive to bad news is between 8% and 20%.

[Table I about here.]

III. Equilibrium Asset Prices

In this section, we discuss the effects of ambiguous information on asset prices in an exchange economy with a representative agent.[17] We have shown in the previous section that there is a range of ambiguous signals that convey bad news for which a risk and ambiguity averse investor does not change her long position in the asset. Hence, if this signal is about the future value of the market portfolio, then the equilibrium price of the market portfolio does not

[17]We discuss an equilibrium model with multiple agents in the Internet Appendix.

react to a range of signals that conveys bad news leading to informational inefficiencies.[18] We will provide a rigorous analysis of this claim below. However, this raises the question of how ambiguous news about the market portfolio affect the price of an individual stock. Moreover, it raises the question of whether ambiguous news about a firm are always incorporated in the stock price. In order to address these questions we consider an asset pricing model with a large number of assets and distinguish between ambiguous news about the market portfolio and ambiguous news about a firm. We also interested in whether there is a premium for investing in ambiguous news, so we derive ex-ante (before receiving news) and ex-post (after receiving news) asset prices in equilibrium.

A. Two Period Model with Multiple Assets

Suppose there are three dates, 0, 1, and 2. There is representative investor (RI) with CARA utility and aversion to ambiguity in the sense of Gilboa and Schmeidler (1989). The RI consumes at date 2 and can invest in a risk-free one period bond in zero-net-supply and N risky assets in positive-net-supply at date 1 after observing the signal and at date 0 before observing the signal. For $i = 1, \ldots, N$; let $\tilde{d}_i$ denote the liquidating dividend, p_i the date 1 or ex-post price, and p_{i0} the date 0 or ex-ante price of risky asset i. There is no interim consumption and the risk-free one period bond is used as numeraire, so the one period

[18] There is also range of signal values for which investors do not hold the market portfolio or do not change their short position in it. However, these demands do not clear the market.

risk-free rate is always zero. The RI's budget constraints are

$$\tilde{w}_2 = w_1 + \sum_{i=1}^{N} \theta_i \left(\tilde{d}_i - p_i\right) \quad \text{and} \quad \tilde{w}_1 = w_0 + \sum_{i=1}^{N} \theta_{0i} \left(\tilde{p}_i - p_{0i}\right), \tag{17}$$

where θ_i and θ_{0i} denote the number of shares invested in risky asset i at date 1 and 0, respectively. At date 1, the RI receives an ambiguous signal $\tilde{s}$ that is either about the market portfolio (discussed in Section B) or about a single asset (discussed in Section C). We consider the learning model under ambiguity described in Section I and use the recursive approach developed in Epstein and Schneider (2003) to ensure dynamic consistency.[19] Specifically, the date 1 and date 0 value function of an ambiguity averse investor with initial wealth w_0 are

$$V_1(w_1, s) = \max_{(\theta_1,\ldots,\theta_N)\in\mathcal{R}^N} \min_{\rho\in[\rho_a,\rho_b]} \mathrm{E}_\rho \left[u\left(\tilde{w}_2\right) \mid \tilde{s} = s\right], \tag{18}$$

$$V_0(w_0) = \max_{(\theta_{01},\ldots,\theta_{0N})\in\mathcal{R}^N} \mathrm{E}\left[V_1\left(\tilde{w}_1, \tilde{s}\right)\right], \tag{19}$$

respectively. The marginal distribution of the signal is known and, thus, at date 0 there is no ambiguity about the distribution of the value function at date 1.

The liquidating dividend of risky asset i consists of an aggregate and a firm specific component. Specifically, $\tilde{d}_i = \beta_i \, \tilde{d} + \tilde{\varepsilon}_i$, with normally distributed idiosyncratic risk, $\tilde{\varepsilon}_i \sim \mathrm{N}\left(0, v_i\right)$. The supply of each asset is $1/N$, the cash flow betas, β_i, average to one, that

[19]Our approach is similar to Epstein and Schneider (2007) and Epstein and Schneider (2008) who also consider dynamic models with learning under ambiguity. We refer to the Internet Appendix for a detailed discussion.

is, $\frac{1}{N}\sum_{i=1}^{N}\beta_i = 1$, and the idiosyncratic variances $\{v_i\}_{i=1}^{N}$ are uniformly bounded. Hence, by the strong law of large numbers, adding up the dividends paid out by all firms leads to the aggregate dividend $\tilde{d}$, that is, $\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\tilde{d}_i = \tilde{d}$, almost surely. We present results for the case of infinitely many firms ($N = \infty$) in this section. Moreover, in equilibrium the RI consumes the liquidating aggregate dividend $\tilde{d}$, so idiosyncratic risk $\tilde{\varepsilon}_i$ is not part of the market portfolio.

B. *Ambiguous News about the Market Portfolio*

Suppose the signal $\tilde{s}$ is about the aggregate dividend $\tilde{d}$ and does not tell us anything about firm specific cash flows, that is, $\tilde{s}$ and $\tilde{\varepsilon}_i$ are independent for all i. The RI is averse to ambiguity about the correlation between $\tilde{s}$ and $\tilde{d}$ and hence entertains the interval of correlations $[\rho_a, \rho_b]$ when making optimal consumption and investment decisions. The information structure is discussed in detail in Section I and the ex-post and ex-ante value function of the ambiguity averse RI are given in equations (18) and (19), respectively.

BENCHMARK (SEU – Systematic News and Equilibrium Prices): *Consider an economy with infinitely many risky assets and an SEU-RI with risk aversion γ and belief ρ who receives the signal $\tilde{s}$ about the aggregate dividend $\tilde{d}$. The ex-post and ex-ante price of the market portfolio—defined as a claim on $\tilde{d}$—in equilibrium are $\bar{p}_m(s) = \mu(\rho, s) - \gamma\sigma^2(\rho)$ and $\bar{p}_{0,m} = \bar{d} - \gamma\sigma_d^2$, respectively. Moreover, the CAPM holds, and the ex-post and ex-ante equilibrium price of each stock is $\bar{p}_i(s) = \beta_i\bar{p}_m(s)$ and $\bar{p}_{0,i} = \beta_i\bar{p}_{0,m}$, respectively.*

If the stock is exposed to aggregate risk, that is, $\beta_i \neq 0$, then its equilibrium price is monotone in the signal and, hence, the price fully incorporates all available information about aggregate cash flows. This is no longer true when investor are averse to ambiguity as the next theorem shows.

THEOREM 2 (MEU – Systematic News and Equilibrium Prices): *Consider an economy with infinitely many risky assets and an MEU-RI with risk aversion* γ *who receives the signal* $\tilde{s}$ *about the aggregate dividend* $\tilde{d}$. *Ambiguity is described by the interval* $[\rho_a, \rho_b]$ *with* $\rho_a > 0$. *There is a unique equilibrium. The ex-post price of the market portfolio—defined as a claim on the aggregate dividend* $\tilde{d}$*—is*

$$p_m(s) = \begin{cases} \mathrm{E}_{\rho_a}\left[\tilde{d} \mid \tilde{s} = s\right] - \gamma \mathrm{Var}_{\rho_a}\left[\tilde{d} \mid \tilde{s} = s\right] & \text{if } \; s > -\gamma\sigma_d\rho_a \\ \mathrm{E}\left[\tilde{d}\right] - \gamma \mathrm{Var}\left[\tilde{d}\right] & \text{if } \; -\gamma\sigma_d\rho_b \leq s \leq -\gamma\sigma_d\rho_a \\ \mathrm{E}_{\rho_b}\left[\tilde{d} \mid \tilde{s} = s\right] - \gamma \mathrm{Var}_{\rho_b}\left[\tilde{d} \mid \tilde{s} = s\right] & \text{if } \; s < -\gamma\sigma_d\rho_b. \end{cases} \tag{20}$$

The ex-ante price of the market portfolio is

$$p_{0,m} = \mathrm{E}\left[\tilde{d}\right] - \gamma \mathrm{Var}\left[\tilde{d}\right] - \frac{1}{\sqrt{2\pi}} \frac{\sigma_d(\rho_b - \rho_a)}{1 + \frac{\gamma\sigma_d}{\sqrt{2\pi}}(\rho_b - \rho_a)}. \tag{21}$$

Moreover, the CAPM holds and, thus, the ex-post and ex-ante price of each stock is $p_i(s) = \beta_i p_m(s)$ *and* $p_{0,i} = \beta_i p_{0,m}$, *respectively.*

Stocks do not fully incorporate all publicly available information about aggregate cash flows. While good systematic news is always incorporated into the price for stocks that are exposed to aggregate risk ($\beta \neq 0$), bad news is not.[20] Hence, there is an informational

[20] Bad systematic news is good news for stocks with negative cash flow betas.

inefficiency because the price is not a sufficient statistic for the signal. Moreover, investors get compensated for risk and ambiguity when holding an asset that is exposed to ambiguous systematic news ($\beta \neq 0$). We discuss the ambiguity premium and the informational inefficiency below, and since each stock inherits the properties of the market portfolio (the CAPM holds), we drop any asset specific subscripts and refer to p_0 and $p(s)$, more generally, as the ex-ante and ex-post asset price, respectively.

B.1. Ex-Ante Stock Price and the Ambiguity Premium.

The uncertainty premium (UP) for the market portfolio, defined as the unconditional mean of the aggregate dividend minus the ex-ante price of the market portfolio (given in equation (21)), is

$$\mathrm{UP} \equiv \mathrm{E}\left[\tilde{d}\right] - p_0 = \gamma\sigma_d^2 + \frac{1}{\sqrt{2\pi}} \frac{\sigma_d(\rho_b - \rho_a)}{1 + \frac{\gamma\sigma_d}{\sqrt{2\pi}}(\rho_b - \rho_a)}. \tag{22}$$

The uncertainty premium strictly increases in risk aversion γ, the unconditional dividend volatility σ_d, and the amount of ambiguity $\Delta\rho = \rho_b - \rho_a$. Moreover, the UP can be decomposed in the unconditional risk premium, $\gamma\sigma_d^2$, and an ambiguity premium. The ambiguity premium represents the additional compensation for an ambiguity averse investor and it is the reason why the uncertainty premium strictly increases in the amount of ambiguity. However, it strictly decreases in risk aversion and vanishes when risk aversion is sufficiently high. While this seems counterintuitive at first glance, it directly follows from the fact that a very risk averse investor is more worried about the posterior variance than the posterior mean and, thus, she will always behave like an SEU investor with belief, ρ_a, after receiving the signal. Hence, there is no ambiguity about how to react to the signal, so there is no ambiguity premium. Moreover, the ambiguity premium and the risk premium are, as expected, strictly increasing in the ex-ante dividend volatility.

B.2. Ex-post Stock Price and Informational Inefficiencies

Figure 3 shows the equilibrium asset price as a function of the signal. There is a range of signals that convey bad news for which the price does not depend on the signal. To gain intuition consider a two standard deviation bad news surprise ($s = -2$). In this case the equilibrium price is $p = 75$ when there is ambiguity aversion (MEU – black solid line) and when there is no ambiguity aversion and the investor correctly estimates the informativeness of the signal (SEU – dotted purple plus line). If the signal decreases, then the SEU representative investor requires a lower price as compensation for the lower posterior mean in order to hold the market portfolio. In contrast to the SEU investor, the MEU representative investor revises the worst case scenario belief about ρ upwards if the signal drops. The price does not change because the lower posterior mean that would require a drop in the equilibrium price is exactly offset by the lower risk premium that would require an increase in the price.

[Figure 3 about here.]

How do changes in risk and ambiguity affect the informational inefficiency of prices? Similar to the effects of risk and ambiguity on the likelihood of information inertia for risky portfolios, the probability of having an informationally inefficient stock price conditional on bad news strictly increases in ambiguity $\Delta\rho$ but is non-monotonic in risk $\gamma\sigma_d^2$.

B.3. Quantitative Importance of Informational Inefficiencies

The purpose of this subsection is twofold. First, we determine the quantitative significance of the informational inefficiency result. Second, we sharpen the predictions of the effects of changes in risk for the likelihood of obtaining an informational inefficiency. In order to make quantitative assessments, we consider an ambiguity averse representative investor with constant relative risk aversion (CRRA) and one risky asset—the market portfolio—which is a claim on a log normally distributed dividend. Hence, dividend growth rates are normally distributed and the signal is a predictor of the dividend growth rate; e.g. the dividend-price ratio. In this case, there is also a range of signals for which the equilibrium price does not react to news. The results are summarized in the next proposition.

PROPOSITION 4 (Informational Inefficiencies in a CRRA Ambiguity Model): *Consider a one-period model with one risky asset that is a claim on a log normally distributed liquidating dividend $e^{\tilde{d}}$ and an MEU-RI with CRRA coefficient γ who receives the signal $\tilde{s}$ about $\tilde{d}$. Ambiguity about the joint-normal distribution of $\tilde{s}$ and $\tilde{d}$ is described by the interval $[\rho_a, \rho_b]$ with $\rho_a > 0$. Then the unique equilibrium price is*

$$p(s) = \begin{cases} e^{-\gamma\sigma_d^2(1-\rho_a^2)} \mathrm{E}_{\rho_a}\left[e^{\tilde{d}} \mid \tilde{s} = s\right] & \text{if} \quad s > -\gamma\sigma_d\rho_a \\ e^{-\gamma\sigma_d^2} \mathrm{E}\left[e^{\tilde{d}}\right] & \text{if} \quad -\gamma\sigma_d\rho_b \leq s \leq -\gamma\sigma_d\rho_a \\ e^{-\gamma\sigma_d^2(1-\rho_b^2)} \mathrm{E}_{\rho_b}\left[e^{\tilde{d}} \mid \tilde{s} = s\right] & \text{if} \quad s < -\gamma\sigma_d\rho_b, \end{cases} \tag{23}$$

where σ_d denotes the volatility of dividend growth $\tilde{d}$. The size of the signal region for which the equilibrium price does not react to news is $\gamma\sigma_d(\rho_b - \rho_a) = \gamma\sigma_d\Delta\rho$ and the probability of *an informational inefficiency conditional on bad news is*

$$\Pi_p(\gamma\sigma_d, \Delta\rho) = 2\left(\Phi\left(-\gamma\rho_a\sigma_d\right) - \Phi\left(-\gamma\rho_b\sigma_d\right)\right), \tag{24}$$

where $\Phi(\cdot)$ *denotes the cumulative distribution function of a standard normal distributed variable. Moreover, we have the following two comparative static results: (i)* $\frac{\partial \Pi_p(\cdot,\Delta\rho)}{\partial \Delta\rho} \geq 0$ *and (ii) there exists a* $\widehat{\gamma\sigma}_d$ *that maximizes* $\Pi_p(\gamma\sigma_d, \cdot)$ *and* $\Pi_p(\gamma\sigma_d = 0, \cdot) = 0$ *and* $\Pi_p(\gamma\sigma_d = \infty, \cdot) = 0$*. If* $\Delta\rho > 0$ *and* $\gamma > 0$*, then the inequality in part (i) is strict and the maximum in part (ii) is unique.*

The qualitative effects of risk and ambiguity on the equilibrium price in the CRRA-log normal model are the same as in the CARA-normal model, so we focus on the quantitative effects. Specifically, we consider $T = 84$ observations of dividend growth rates and dividend-price ratios, as well, as three different values for the point estimate of the correlation between the dividend growth rate and the dividend price ratio, that is, $\hat{\rho} = (\rho_a + \rho_b)/2 \in \{30\%, 40\%, 50\%\}$. These estimates correspond to R^2s in predictive regressions ranging from 0.09 to 0.25 which are similar to the reported predictability results in Koijen and Nieuwerburgh (2011) who use 84 years of data. We use $\alpha = 1 - \hat{a}$ to measure the level of ambiguity $\Delta\rho = \rho_b - \rho_a$, where $\hat{a}$ denotes the significance level for the confidence interval of $\hat{\rho}$.

Figure 4 shows the equilibrium stock price as a function of the signal (standardized PD-ratio) for different values of the unconditional expected excess return, that is, $\gamma\sigma_d^2 \in \{2.5\%, 5\%, 7.5\%, 10\%\}$ when $\hat{\rho}^2 = 16\%$ and $\alpha = 0.99$. There is a range of PD-ratios that convey news that is worse than expected for which the equilibrium price does not react, so the price is not efficiently incorporating the information conveyed by the PD-ratio about dividend growth into the equilibrium price. The size of the inaction region increases with risk but at the same time the realizations of the PD-ratio for which there is no price reaction become less likely with an increase in risk. Hence, the probability of having no price reaction is not monotone in risk as shown in Proposition 4. Table 2 shows that the likelihood of having no price reaction to bad news about dividends strictly increases for reasonable levels of the unconditional asset risk premium. This likelihood is also increasing in ambiguity $\alpha \in \{0.9, 0.95, 0.99\}$. Moreover, Table 2 demonstrates that the probability of having an equilibrium price that does not react to bad news is economically meaningful. For instance, if $\alpha = 0.99$ then this probability is between 7% and 33%.

[Figure 4 & Table II about here.]

C. *Ambiguous News about a Firm*

We consider the CARA-normal model and assume that investors receive ambiguous news about a single asset and, thus, w.l.o.g. the signal $\tilde{s}$ is about the first asset with liquidating dividend $\tilde{d}_1 = \beta_1 \tilde{d} + \tilde{\varepsilon}_1$ and $\beta_1 > 0$. The representative investor (RI) is averse to ambiguity about the correlation between $\tilde{s}$ and $\tilde{d}_1$, and, hence, entertains the interval of positive correlations $[\rho_a, \rho_b]$ when making optimal consumption and investment decisions. The signal is informative about the aggregate dividend $\tilde{d}$ and the idiosyncratic cash flow component $\tilde{\varepsilon}_1$ but it does not tell us anything about the idiosyncratic risk of all other assets and, thus, $\tilde{s}$ is independent of $\tilde{\varepsilon}_i$, $\forall i \neq 1$. Moreover, the parameter $\omega = \frac{\beta_1 \mathrm{Cov}(\tilde{s}, \tilde{d})}{\mathrm{Cov}(\tilde{s}, \tilde{d}_1)}$ measures how much of the positive correlation between the first asset's dividend and the signal is due to the systematic cash flow component and $1 - \omega = \frac{\mathrm{Cov}(\tilde{s},\tilde{\varepsilon}_1)}{\mathrm{Cov}(\tilde{s},\tilde{d}_1)}$ measures the amount that is due to the idiosyncratic cash flow component.

BENCHMARK (SEU – News about a Stock and Equilibrium Prices): *Consider an economy with infinitely many risky assets and an SEU-RI with risk aversion* γ *and belief* ρ *who receives the signal* $\tilde{s}$ *about the first asset's dividend* $\tilde{d}_1$. *The first asset has a positive cashflow beta, that is,* $\beta_1 > 0$. *The ex-post price of the first asset is*

$$\bar{p}_1(s, \rho) = \mathrm{E}_\rho \left[\tilde{d}_1 \mid \tilde{s} = s \right] - \gamma \mathrm{Cov}_\rho \left[\tilde{d}_1, \tilde{d} \mid \tilde{s} = s \right]. \tag{25}$$

The ex-post price of the market portfolio—defined as a claim on the aggregate dividend $\tilde{d}$*—is*

$$\bar{p}_m(s, \rho) = \mathrm{E}_\rho \left[\tilde{d} \mid \tilde{s} = s \right] - \gamma \mathrm{Var}_\rho \left[\tilde{d} \mid \tilde{s} = s \right]. \tag{26}$$

The ex-post price of every other individual asset is $\bar{p}_i(s, \rho) = \beta_i \bar{p}_m(s, \rho)$, $\forall i \neq 1$. *The ex-ante price of the market portfolio is* $p_{0,m} = \bar{d} - \gamma \sigma_d^2$. *Moreover, the CAPM holds ex-ante for all assets and, thus,* $p_{0,i} = \beta_i p_{0,m}$, $\forall i$.

The information about the first asset is fully incorporated into its price because the posterior mean strictly increases in the signal. If the signal is correlated with the aggregate dividend, that is, $\omega > 0$, then the posterior mean of the market portfolio strictly increases in the signal and, thus, the information about the market portfolio is fully incorporated into its price too. This is no longer true when investor are averse to ambiguity as the next theorem shows.

THEOREM 3 (MEU – News about a Stock and Equilibrium Prices): *Consider an economy with infinitely many risky assets and an MEU-RI with risk aversion* γ *who receives the signal* $\tilde{s}$ *about the first asset's dividend* $\tilde{d}_1$. *Ambiguity is described by the interval* $[\rho_a, \rho_b]$ *with* $\rho_a > 0$ *and the first asset has a positive cashflow beta, that is,* $\beta_1 > 0$. *There is a unique equilibrium. The ex-post price of the first asset is*

$$p_1(s) = \begin{cases} \bar{p}_1(s, \rho_a) & \text{if } s > \hat{s}_a \equiv -\gamma \frac{\omega}{\beta_1} \sigma_1 \rho_a \\ \beta_1 \left(\bar{d} - \gamma \sigma_d^2 \right) & \text{if } \hat{s}_b \leq s \leq \hat{s}_a \\ \bar{p}_1(s, \rho_b) & \text{if } s < \hat{s}_b \equiv -\gamma \frac{\omega}{\beta_1} \sigma_1 \rho_b, \end{cases} \tag{27}$$

where $\sigma_1^2 = \beta_1^2 \sigma_d^2 + v_1$ *denotes the unconditional variance of the first asset and* $\bar{p}_1(s, \cdot)$ *is given in equation (25). The ex-post price of the market portfolio—defined as a claim on the aggregate dividend* $\tilde{d}$*—is*

$$p_m(s) = \begin{cases} \bar{p}_m(s, \rho_a) & \text{if } s > \hat{s}_a \\ \bar{d} - \gamma \sigma_d^2 & \text{if } \hat{s}_b \leq s \leq \hat{s}_a \\ \bar{p}_m(s, \rho_b) & \text{if } s < \hat{s}_b, \end{cases} \tag{28}$$

where $\bar{p}_m(s, \rho)$ *is given in equation (26). The ex-post price for all other assets is* $p_i(s) =$

$\beta_i p_m(s)$, $\forall i \neq 1$. The ex-ante price of the first asset and the market portfolio are

$$p_{0,1} = \beta_1(\bar{d} - \gamma\sigma_d^2) - \frac{\sigma_1(\rho_b - \rho_a)}{\sqrt{2\pi} + \gamma\frac{\omega}{\beta_1}\sigma_1(\rho_b - \rho_a)} \tag{29}$$

$$p_{0,m} = \bar{d} - \gamma\sigma_d^2 - \frac{\frac{\omega}{\beta_1}\sigma_1(\rho_b - \rho_a)}{\sqrt{2\pi} + \gamma\frac{\omega}{\beta_1}\sigma_1(\rho_b - \rho_a)}, \tag{30}$$

respectively. For all other assets we have that $p_{0,i} = \beta_i p_{0,m}$, $\forall i \neq 1$.

If the signal about the first asset's dividend is correlated with the aggregate dividend, that is, $\omega > 0$ then there is a range of signals for which the price of the market portfolio and each individual asset with exposure to the market ($\beta_i \neq 0$) does not react to news that is worse than expected. Hence, there is informational inefficiency because equilibrium prices are no longer sufficient statistics for the signal. The intuition for the price inaction regions when investors process ambiguous news about an asset is the same as when the process ambiguous news about the market portfolio. Specifically, any ambiguous signal that is correlated with the aggregate dividend affects the utility of the MEU representative investor. If the signal conveys news that is worse than expected, then the MEU-RI is worried about a high ρ that would lead to a low posterior mean of the asset and a low ρ because that would lead to a high residual variance. Hence, there is a range of signals for which the worst-case scenario correlation depends on the signal, so the price of every asset exposed to the market does not depend on the signal. The effects of an increase in the systematic component of the signal measured by ω on the inaction region are similar to that of an increase in risk aversion γ, that is, the size of the inaction region increases with ω but at the same time the critical signal values $\hat{s}_a$ and $\hat{s}_b$ become less likely. Hence, the probability of having an informational inefficiency is not always monotone in the signal.[21] Importantly, if $\omega = 0$ and, thus, the signal is about the idiosyncratic component of the asset, then the utility of the investor is not affected by the signal, so there is no inaction region in equilibrium. Hence, as long as the signal is correlated with the aggregate dividend there is a tradeoff between the affects of the correlation on the posterior mean and the residual variance and therefore, the information is not efficiently incorporated into equilibrium asset prices.

D. News Momentum

We show in this section that aversion to risk and ambiguity leads to news momentum when investors process i) news about the stock market (e.g. macroeconomic news) and (ii) news about a firm (e.g. earnings news). This news momentum is more severe if the signal is on average more informative and if there is more risk and ambiguity. Moreover, it disappears when the news is only about the idiosyncratic asset component. To determine the post-news drift for qualitatively reasonable risk premia, we consider in Section D.1 the CRRA-log normal model.

D.1. Macroeconomic News

Suppose the signal is about the market portfolio which is a claim on a liquidating aggregate dividend; e.g macroeconomic news. Moreover, consider the CRRA-log normal ambiguous

[21] In contrast to risk aversion which is unbounded, the systematic component of the signal ω is bounded by one and, thus, the probability of having an informational inefficiency does not go to zero with an increase in ω.

information model described in Section III.B.3 and denote with $\hat{\rho}$ the correlation between the log dividend growth rate, $\tilde{d}$, and the predictor, $\tilde{s}$, that generates the data. We simulate a long time series of $\tilde{d}$, $\tilde{s}$, and equilibrium prices $p(\tilde{s})$ from our model (see Proposition 4) and run the following regression:

$$\tilde{d} - \ln\left(p(\tilde{s})\right) = \text{constant} + \text{slope} \times \tilde{s} + \text{noise}. \tag{31}$$

Figure 5 shows the slope of this regression as a function of the unconditional risk premium of the asset for six economies that differ with respect to ambiguity measured by, α, and the average informativeness of the signal measured by, $\hat{\rho}$. We standardize the LHS to make the slope coefficients comparable across economies.

[Figure 5 about here.]

The (log) of the equilibrium price is a strictly increasing linear function of the signal in a SEU-RI economy and, thus, the equilibrium price is a sufficient statistic. If the SEU-RI has the correct belief (black dotted line in Figure 5), that is, $\rho = \hat{\rho}$, then the information is correctly incorporated into the price and the slope coefficient is zero. On the other hand, if the SEU-RI belief is different than that of the econometrician, then the information is fully but incorrectly incorporated into the price and the slope coefficient is different from zero. Specifically, it is negative if the SEU-RI overreacts to news and it is positive if the SEU-RI underreacts to news. However, in both cases the slope coefficient would depend neither on the risk in the economy nor the average informativeness of the signal.

If investors are averse to ambiguity, then the (log) of the equilibrium price is a piecewise linear function of the signal that is constant for a range of bad signals. Hence, prices are no longer sufficient statistics and, thus, they are not informationally efficient. Moreover, this local information inertia—an extreme form of underreaction—and the underreaction to good news leads to news momentum. The economic significance of the news momentum regression—the slope—depends on risk, ambiguity, and the average informativeness of the signal. Specifically, Figure 5 shows that the slope strictly increases in risk unless there is no ambiguity (black dotted line). Moreover, the blue dashed, red chain-dotted, and green star lines show that the slope increases in ambiguity, α, for fixed amount of risk and an average signal informativeness of, $\hat{\rho} = 0.4$. Similarly, the black solid, green star, and purple plus lines show that the slope increases in the average signal informativeness, $\hat{\rho}$, for a fixed amount of risk and ambiguity $\alpha = 0.99$.

Why is there more news momentum when there is more risk, more ambiguity, or if the signal is on average more informative? The reason is that the probability of the equilibrium price overreacting to news strictly decreases in risk $\gamma\sigma_d$, ambiguity $\Delta\rho = \rho_b - \rho_a$, and the average signal informativeness $\hat{\rho} = \frac{\rho_a+\rho_b}{2}$, because in all three cases the critical signal value $s_b = -\gamma\sigma_d\rho_b$ and, thus, the probability of having signal realizations below it decreases. Hence, the economic significance of news momentum increases in the average signal informativeness, risk, and ambiguity.

D.2. Earnings News

Suppose the signal is about a firm which is a claim on a liquidating dividend that consist of an idiosyncratic component and a systematic component; e.g an earnings news surprise or a revision of an analyst's forecast. The dividend exposure to the systematic component is measured by the positive cash-flow beta β_1. Moreover, consider the CARA-normal ambiguous information model described in Section C and denote with $\hat{\rho}$ the correlation between the dividend, $\tilde{d}_1$, and the signal, $\tilde{s}$, that generates the data. We simulate a long time series of liquidating dividends $\tilde{d}_1$ and $\tilde{d}$, signals $\tilde{s}$, and equilibrium prices $p_1(\tilde{s})$ and $p_m(\tilde{s})$ from our model (see Theorem 3). If we run news momentum regressions similar to the previous section for the stock and the market portfolio, then we get positive slope coefficients if the signal is correlated with fundamentals, that is, if $\omega > 0$. The results are similar to the previous case, and, thus omitted. If $\omega = 0$, then the slope is zero for the market portfolio because the signal does not contain any information about aggregate fundamentals and it is zero for the firm because investors do not care about the variance of idiosyncratic risk.

We have established in the previous section that ambiguous macroeconomic news lead to news momentum which is more pronounced if there is more risk, ambiguity, and if the average informativeness of the signal is higher. We get the same results for ambiguous news about the earnings of a firm in the form of an earnings news surprise or a revision of analysts' forecasts in this section. However, the empirical evidence for PEAD and PFRD is for risk-adjusted returns and not returns in excess of the risk-free rate. Hence, to show that our model also leads to PEAD and PFRD we run the following regression:

$$\left(\tilde{d}_1 - p_1(\tilde{s})\right) - \beta_1 \left(\tilde{d}_m - p_m(\tilde{s})\right) = \text{constant} + \text{slope} \times \tilde{s} + \text{noise}. \tag{32}$$

The right graph of Figure 5 shows the slope of this regression as a function of the unconditional stock market risk premium for five economies that differ with respect to ω, which measures how much we can learn from the signal about the systematic asset component. We standardize the LHS to make the slope coefficients comparable across economies and use the same parameter values as in the previous section. Moreover, we assume that 25% of the variation in the dividend of the first asset is due to its market exposure and the cash flow beta is, $\beta_1 = 1.5$. The black solid line shows that there is no earnings momentum if the signal is either only about the idiosyncratic component ($\omega = 0$) or only about the systematic component ($\omega = 1$). The first result follows directly from the fact that there is neither news momentum for excess stock returns nor for excess stock market returns. The second result is at first glance surprising because there is news momentum for the stock and the market portfolio if the signal is only about the aggregate dividend. The reason for the zero slope is that in this case the CAPM holds for the stock and, thus, after adjusting for risk there is no reaction to the signal at all, so there is no news momentum. If the signal contains information about both the systematic and idiosyncratic component of the asset, then the CAPM does not hold for the stock and, thus, there is news momentum. Hence, the slope coefficient is non-monotonic in ω and it is zero for $\omega = 0$ or $\omega = 1$ as the black solid line, the red dotted-square line, the blue dashed circle line, and the black chain dotted line in the right graph of Figure 5 confirm.

IV. Robustness

Before we discuss the robustness of our information inertia results, we re-emphasize the economic mechanism that leads to information inertia. Specifically, in our ambiguous information model investors are ambiguous about the correlation between a signal and the future value of an asset. Learning from this ambiguous signal leads to ambiguity about the posterior mean because investors don't know how much weight to put on the signal and it leads to ambiguity about the residual variance because they do not know how much they learn from observing the signal. Hence, models where investors are only ambiguous about the mean or the variance are inconsistent with our learning model. Moreover, models where investors choose the mean and variance independently to compute the worst case scenario are also inconsistent with our learning model. For example, suppose an investor contemplates a long position in the asset and receives a bad signal. Choosing the worst case scenario for the mean leads to putting a lot of weight on the signal, but at the same time, this means a low residual variance because investors learn a lot from this signal; thus being the best case scenario for the variance.

Combining the insights from this learning model under ambiguity with the workhorse learning models in Finance—joint normal distribution of signal and asset values/returns combined with CARA/CRRA risk preferences and the multiple prior preference model that exhibits aversion to ambiguity—leads to the stark results that there is a range of signal values for which neither demand nor equilibrium price depend on the signal. Hence, there is information inertia. The information inertia result is particularly surprising because there is no ambiguity about the fact that the correlation is positive, that is, $\rho_a > 0$. In what follows, we discuss deviations from the workhorse model that do not lead to information inertia but instead lead to strong underreaction of the equilibrium asset price to a range of signals that are worse than expected. This underreaction leads to news momentum as discussed in the previous section.

We consider three different model specifications all of which are discussed in detail in the Internet Appendix: (i) an heterogenous agent economy with MEU and SEU investors, (ii) a model in which the representative investor has preferences that allow for a distinction between ambiguity and ambiguity aversion, that is, the GHTV model introduced in Gajdos, Hayashi, Tallon, and Vergnaud (2008), and (iii) a model where the representative investor has CRRA preferences and the dividend is either positive or negative skew log normally distributed. The parameter α measures the fraction of MEU investors in part (i), it measures the degree of ambiguity aversion in part (ii), and the size of the ambiguity interval in part (iii). The left graph of Figure 6 shows the equilibrium price as a function of the signal in all three case and the right graph of Figure 6 shows the slope of a regression of excess returns on a constant and the signal. For comparison, we also show the equilibrium price and the economic significance of news momentum for the CRRA-log normal model which is also used to set the benchmark parameters in the CARA/normal model. The parameters in all other models are calibrated to the equilibrium price function obtained in the CRRA-log normal model. The blue-dashed circle line and black solid line verify that there is a range of signals for which the equilibrium price does not react in the CRRA-log normal and the CARA-normal model, respectively. Deviating from these two models does not lead to inaction but to a lower reaction to news or a more severe informational inefficiency. Specifically, the price in the CRRA-negative skew log-normal model is not a sufficient statistic for a bigger range of signals than in the two benchmark models. The right graph of Figure 6 shows that deviating from the CARA and CRRA models by considering (i) different joint distributions of asset payoffs and signals, (ii) economies populated with ambiguity averse and standard expected utility maximizers, and (iii) preference models that allow for a distinction between ambiguity and ambiguity aversion leads to similar slopes of the news momentum regressions. Hence, the implications of our information inertia models for news momentum are robust.

[Figure 6 about here.]

V. Conclusion

We introduce in this paper a novel economic mechanism which, in the absence of transaction costs, information processing costs, or any other market frictions, leads to optimal portfolios that do not always react to public information and, hence, this information is not reflected in the equilibrium stock price—a phenomenon we refer to as information inertia. The economic mechanism that leads to information inertia relies on the trade-off between over- and underestimating the informativeness of news that ambiguity averse investors face when processing it. Information inertia for optimal portfolios and equilibrium asset prices is a local phenomenon that occurs for news that is worse than expected in the workhorse learning models in Finance—joint normal distribution of signal and asset values/returns combined with CARA/CRRA risk preferences and the multiple prior preference model that exhibits aversion to ambiguity. If we deviate from these models by considering (i) different joint distributions of asset payoffs and signals, (ii) economies populated with ambiguity averse and standard expected utility maximizers, and (iii) other preference models that allow for a distinction between ambiguity and ambiguity aversion, then there is no information inertia but the demand for the asset and its equilibrium price shows reaction to news that is much lower than without ambiguity.

Our new economic mechanism helps explain why households follow simple portfolio rules and do not trade as much as traditional models would predict even after accounting for transaction or information processing costs. Moreover, it sheds new light on one of the most robust instances of underreaction to news, that is, the post-earnings announcement drift (PEAD) and the post-forecast revision drift (PFRD). Specifically, we show that the slope (drift) when regressing risk-adjusted returns on a constant and a signal strictly increases in risk and ambiguity. Moreover, it vanishes if the news is only about the idiosyncratic component of the asset. We find support for all three predictions in the literature. Our ambiguity model provides additional predictions that, as far as we know, have not been tested yet. For instance, the post-news drift is bigger if the news is on average more informative and it varies depending on (i) risk, (ii) ambiguity, (iii) and the average informativeness if one conditions on the size of the negative news surprise.

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Appendix. Proofs

We first determine the certainty equivalent of the MEU investor in Proposition 5 before we prove Proposition 1, Proposition 2, and Theorem 1.

PROPOSITION 5 (Preferences): *Let* $\hat{\theta}_a \equiv -s/(\gamma\rho_a\sigma_d)$ *and* $\hat{\theta}_b \equiv -s/(\gamma\rho_b\sigma_d)$*. The certainty equivalent of an investor with risk aversion* γ *and ambiguity described by* $[\rho_a, \rho_b]$ *who has the received signal* s *is*

$$CE(\theta) = \begin{cases} \mathrm{E}_{\rho_a}\left[\tilde{w} \mid \tilde{s} = s\right] - \frac{1}{2}\gamma \mathrm{Var}_{\rho_a}\left[\tilde{w} \mid \tilde{s} = s\right] & \text{if } \; \theta \leq \min\left(\hat{\theta}_a, 0\right) \text{ or } \; \theta > \max\left(\hat{\theta}_a, 0\right) \\ \mathrm{E}\left[\tilde{w}\right] - \frac{1}{2}\gamma \mathrm{Var}\left[\tilde{w}\right] - \frac{s^2}{2\gamma} & \text{if } \; \min\left(\hat{\theta}_a, 0\right) < \theta \leq \min\left(\hat{\theta}_b, 0\right) \\ \mathrm{E}_{\rho_b}\left[\tilde{w} \mid \tilde{s} = s\right] - \frac{1}{2}\gamma \mathrm{Var}_{\rho_b}\left[\tilde{w} \mid \tilde{s} = s\right] & \text{if } \; \min\left(\hat{\theta}_b, 0\right) < \theta \leq \max\left(\hat{\theta}_b, 0\right) \\ \mathrm{E}\left[\tilde{w}\right] - \frac{1}{2}\gamma \mathrm{Var}\left[\tilde{w}\right] - \frac{s^2}{2\gamma} & \text{if } \; \max\left(\hat{\theta}_b, 0\right) < \theta \leq \max\left(\hat{\theta}_a, 0\right). \end{cases} \tag{A1}$$

The certainty equivalent $CE(\theta)$ *is a continuous and strictly concave function of the stock demand* θ*. Moreover, it is continuously differentiable except for the portfolio* $\theta = 0$ *if* $s \neq 0$*.*

Proof of Proposition 5. The certainty equivalent $\mathrm{CE}(\theta)$ of the ambiguity averse MEU investor satisfies

$$\mathrm{CE}(\theta) = \min_{\rho \in [\rho_a, \rho_b]} \overline{\mathrm{CE}}(\theta, \rho).$$

Note that

$$\frac{\partial \overline{\mathrm{CE}}(\theta, \rho)}{\partial \rho} = \theta\sigma_d s + \gamma\theta^2\sigma_d^2\rho.$$

Consider three cases, (i) $s = 0$, (ii) $s > 0$, and (iii) $s < 0$. Case (i): $s = 0 \Leftrightarrow \hat{\theta}_a = \hat{\theta}_b = 0$. Then $\frac{\partial \overline{\mathrm{CE}}(\theta,\rho)}{\partial \rho} > 0$ for all $\rho \in [\rho_a, \rho_b]$. Thus the minimum of $\overline{\mathrm{CE}}(\theta, \rho)$ is attained at ρ_a and hence,

$$\text{CE}(\theta) = \min_{\rho\in[\rho_a,\rho_b]} \overline{\text{CE}}(\theta,\rho) = \overline{\text{CE}}(\theta,\rho_a) \text{ for all } \theta \in \rho.$$

$\overline{\text{CE}}(\theta,\rho_a)$ is continuously differentiable and strictly concave in θ for all $\theta \in \mathbb{R}$ and thus so is $\text{CE}(\theta)$. Case (ii): $s > 0 \Leftrightarrow \hat{\theta}_a < \hat{\theta}_b < 0$. Suppose $\theta < \hat{\theta}_a < 0$ or $\theta > 0$. Then $\frac{\partial\overline{\text{CE}}(\theta,\rho)}{\partial\rho} > 0$ for all $\rho \in [\rho_a,\rho_b]$. Thus, the minimum of $\overline{\text{CE}}(\theta,\rho)$ is attained at ρ_a. Suppose $\hat{\theta}_b < \theta < 0$. Then $\frac{\partial\overline{\text{CE}}(\theta,\rho)}{\partial\rho} < 0$ for all $\rho \in [\rho_a,\rho_b]$. Thus, the minimum of $\overline{\text{CE}}(\theta,\rho)$ is attained at ρ_b. Suppose $\hat{\theta}_a \leq \theta \leq \hat{\theta}_b$. Then, since $\frac{\partial^2\overline{\text{CE}}(\theta,\rho)}{\partial\rho^2} > 0$, the minimum is attained when $\frac{\partial\overline{\text{CE}}(\theta,\rho)}{\partial\rho} = 0$, i.e. $\rho^*(\theta) \equiv \underset{\rho\in[\rho_a,\rho_b]}{\text{argmin}}\ \overline{\text{CE}}(\theta,\rho) = \frac{-s}{\gamma\sigma_d\theta}$. Note that $\rho^* \in [\rho_a,\rho_b]$ when $\hat{\theta}_a \leq \theta \leq \hat{\theta}_b < 0$ and that

$$\overline{\text{CE}}(\theta,\rho^*) = \text{E}[\tilde{w}] - \frac{1}{2}\gamma\text{Var}[\tilde{w}] - \frac{s^2}{2\gamma} = \overline{\text{CE}}(\theta,0) - \frac{s^2}{2\gamma}.$$

Using the above, we get

$$\text{CE}(\theta) = \begin{cases} \overline{\text{CE}}(\theta,\rho_a) & \text{if } \theta \leq \hat{\theta}_a \text{ or } 0 < \theta \\ \overline{\text{CE}}(\theta,0) - \frac{s^2}{2\gamma} & \text{if } \hat{\theta}_a < \theta \leq \hat{\theta}_b \\ \overline{\text{CE}}(\theta,\rho_b) & \text{if } \hat{\theta}_b < \theta \leq 0 \end{cases}$$

as desired. $\text{CE}(\theta)$ is continuous for all $\theta \in \mathbb{R}$ and $\rho \in [\rho_a,\rho_b]$ and $\overline{\text{CE}}(0,\rho_a) = \overline{\text{CE}}(0,\rho_b)$. $\overline{\text{CE}}(\theta,\rho)$ is continuously differentiable for all $\theta \in \mathbb{R}$ and $\rho \in [\rho_a,\rho_b]$ and the $\frac{\partial^2\overline{\text{CE}}(\theta,\rho)}{\partial\theta^2} \leq 0$ for all $\theta \in \mathbb{R}$ and $\rho \in [\rho_a,\rho_b]$. Thus, for any $\theta \neq 0$ there is an open neighborhood for such $\text{CE}(\theta)$ is continuously differentiable and $\frac{\partial^2\text{CE}(\theta)}{\partial\theta^2}$ exists and is non-positive. To verify concavity and non-differentiability of $\text{CE}(\theta)$ at $\theta = 0$, we calculate the left derivative $\text{CE}'^{-}(\theta)$ and the right derivative $\text{CE}'^{+}(\theta)$ at $\theta = 0$.

$$\text{CE}'^{-}(0) \equiv \lim_{\theta \uparrow 0} \frac{\partial \text{CE}(\theta)}{\partial \theta} = \bar{d} + \rho_b \sigma_d s - p$$

$$\text{CE}'^{+}(0) \equiv \lim_{\theta \downarrow 0} \frac{\partial \text{CE}(\theta)}{\partial \theta} = \bar{d} + \rho_a \sigma_d s - p$$

Thus, $\text{CE}'^{-}(0) > \text{CE}'^{+}(0)$, so $\text{CE}(\theta)$ is strictly concave for all $\theta \in \mathbb{R}$, not differentiable at $\theta = 0$, and continuously differentiable at all $\theta \neq 0$. Case (iii): $s < 0 \Leftrightarrow \hat{\theta} > \hat{\theta}_b > 0$. Using reasoning similar to that for the above case, we get

$$\text{CE}(\theta) = \begin{cases} \overline{\text{CE}}(\theta, \rho_a) & \text{if } \theta \leq 0 \text{ or } \hat{\theta}_a < \theta \\ \overline{\text{CE}}(\theta, \rho_b) & \text{if } 0 < \theta \leq \hat{\theta}_b \\ \overline{\text{CE}}(\theta, 0) - \frac{s^2}{2\gamma} & \text{if } \hat{\theta}_b < \theta \leq \hat{\theta}_a \end{cases}$$

and that $\text{CE}(\theta)$ is continuous and strictly concave in $\theta \in \mathbb{R}$. Moreover, $\text{CE}(\theta)$ is continuously differentiable at all $\theta \neq 0$. Finally, combining the above cases provides the desired expression and properties for $\text{CE}(\theta)$. □

Proof of Proposition 1. If we find a demand that maximizes the MEU's certainty equivalent, then we know it is the unique optimal demand because the MEU's certainty equivalent is a continuous and strictly concave function of demand. Suppose $\lambda_d > 0$ and consider five different cases for the signal: (i) $s \leq s_4 \equiv -\frac{\lambda_d}{\rho_a}$, (ii) $s_4 < s \leq s_3 \equiv -\frac{\lambda_d}{\rho_b}$, (iii) $s_3 < s \leq s_2 \equiv -\rho_b \lambda_d$, (iv) $s_2 < s \leq s_1 \equiv -\rho_a \lambda_d$, and (v) $s > s_1$. The condition $s \leq s_4$ implies that $\bar{\theta}(s, \rho_a) = \frac{\lambda_d + \rho_a s}{\gamma \sigma_d (1 - \rho_a^2)} \leq 0$. Hence, $\text{CE}(\bar{\theta}(s, \rho_a)) = \overline{\text{CE}}(\bar{\theta}(s, \rho_a), \rho_a)$, and the correlation that minimizes $\overline{\text{CE}}(\bar{\theta}(s,\rho)$ is ρ_a. Moreover, $\theta(s) = \bar{\theta}(s,\rho_a)$ is the optimal demand. The condition $s_3 < s \leq s_2$ implies that $0 < \bar{\theta}(s,\rho_b) = \frac{\lambda_d+\rho_b s}{\gamma\sigma_d(1-\rho_b^2)} \leq \hat{\theta}_b = -\frac{s}{\gamma\rho_b\sigma_d}$. Hence, $\text{CE}(\bar{\theta}(s,\rho_b)) = \overline{\text{CE}}(\bar{\theta}(s,\rho_b),\rho_b)$, and the correlation that minimizes $\overline{\text{CE}}(\bar{\theta}(s,\rho)$ is ρ_b. Moreover, $\theta(s) = \bar{\theta}(s,\rho_b)$. The condition $s_2 < s \leq s_1$ implies that $\hat{\theta}_b = -\frac{s}{\gamma\rho_b\sigma_d} < \frac{\lambda_d}{\gamma\sigma_d} \leq \hat{\theta}_a = -\frac{s}{\gamma\rho_a\sigma_d}$. Hence, $\text{CE}(\bar{\theta}(s,0)) = \overline{\text{CE}}(\bar{\theta}(s,\rho^*),\rho^*)$, and the correlation that minimizes $\overline{\text{CE}}(\bar{\theta}(s,\rho)$ is ρ^*. Moreover, $\theta(s) = \bar{\theta}(s,0)$. The condition $s > s_1$ implies that $\bar{\theta}(s,\rho_a) = \frac{\lambda_d+\rho_a s}{\gamma\sigma_d(1-\rho_a^2)} > \hat{\theta}_a = -\frac{s}{\gamma\rho_a\sigma_d}$. Hence, $\text{CE}(\bar{\theta}(s,\rho_a)) = \overline{\text{CE}}(\bar{\theta}(s,\rho_a),\rho_a)$, and the correlation that minimizes $\overline{\text{CE}}(\bar{\theta}(s,\rho))$ is ρ_a. Moreover, $\theta(s) = \bar{\theta}(s,\rho_a)$ is the optimal demand. Putting these four cases together leads to $\theta(s) = \bar{\theta}(s,\rho^*(s))$ with $\rho^*(s) = \text{argmin}_{\rho\in[\rho_a,\rho_b]}\overline{\text{CE}}(\theta(s),\rho)$. The condition $s_4 < s \leq s_3$ implies that $\bar{\theta}(s,\rho_b) \leq 0 < \bar{\theta}(s,\rho_a)$. In this case, choosing $\theta(s) > 0$ does not satisfy $\overline{\text{CE}}(\theta(s),\rho)\forall\rho \in [\rho_a,\rho_b]$. Similarly, choosing $\theta(s) < 0$ does not satisfy $\overline{\text{CE}}(\theta(s),\rho)\forall\rho \in [\rho_a,\rho_b]$. Hence, $\theta(s) = 0$ and the MEU's certainty equivalent does not depend on ρ. □

Proof of Proposition 2. Let $s > -\frac{\lambda_d}{\rho_b}$. Then, $s > -\frac{\lambda_d}{\rho}$ for all $\rho \in [\rho_a,\rho_b]$ since $-\frac{\lambda_d}{\rho}$ is increasing in ρ. From this it follows that for all $\rho \in [\rho_a,\rho_b]$, $\frac{\bar{d}+\rho\sigma_d s-p}{\sigma(\rho)} = \lambda(s,\rho) > 0$ and hence $\overline{\theta}(s,\rho) = \frac{\lambda(s,\rho)}{\gamma\sigma(\rho)} > 0$. Moreover, $\frac{d^2}{d\rho^2}\lambda(s,\rho) > 0$ for all $\rho \in [\rho_a,\rho_b]$, so $\lambda(s,\rho)$ is strictly convex in ρ. Letting ρ' denote the unique minimizer of $\lambda(s,\rho)$ over $[\rho_a,\rho_b]$, the first-order (Kuhn-Tucker) condition for the constrained minimization yields

$$0 \leq \frac{s+\lambda_d\rho'}{(1-\rho'^2)}$$

with equality holding if $\rho_a < \rho' < \rho_b$. This yields the result that $\rho^*(s)$ is the unique minimizer of $\lambda(s,\rho)$ over $[\rho_a, \rho_b]$. An analogous argument shows that if $s < -\frac{\lambda_d}{\rho_a}$, then for all $\rho \in [\rho_a, \rho_b]$, $\lambda(s,\rho) < 0$ and hence $\overline{\theta}(s,\rho) < 0$. Moreover, in this case, $\frac{d^2}{d\rho^2}\lambda(s,\rho) < 0$ for all $\rho \in [\rho_a, \rho_b]$, so $\lambda(s,\rho)$ is strictly concave in ρ and the first order condition yields the result that $\rho^*(s) = \rho_a$ is the unique maximizer of $\lambda(s,\rho)$ over $[\rho_a, \rho_b]$. □

Proof of Theorem 1. If $\lambda_d > 0$, then optimal demand $\theta(s)$ is given in equation (12) of Proposition 1. Plugging in for $\rho^*(s)$ given in equation (14) of Proposition 2 leads to optimal demand given in equation (11) of Theorem 1. The proof for the case $\lambda_d \leq 0$ is similar to the case of $\lambda_d > 0$ and, thus, to save space we provide it in the Internet Appendix. □

Proof of Proposition 3. The expression for MEU investor optimal portfolio given in equation (11) yields that MEU investors are long the stock, but demand does not react to bad news, that is, $\theta(s) = \overline{\theta}(s, 0) > 0$ if and only if $\lambda_d > 0$ and $s_1 > s \geq s_2$, where $s_1 = -\rho_a\lambda_d$ and $s_2 = -\rho_b\lambda_d$. Hence, the size of the signal region for which risky portfolios do not react to news is $(\rho_b - \rho_a)\lambda_d$. Moreover, $\tilde{s} \sim \mathrm{N}(0, 1)$, so it follows that the probability of investors exhibiting information inertia for a risky asset position conditional on $\tilde{s} \leq 0$ (bad news) is as given in (16). Clearly, $\lim_{\lambda_d \to \infty} \Pi(\lambda_d, \cdot, \cdot) = \Pi(\lambda_d = 0, \cdot, \cdot) = 0$. Moreover, this probability is a concave function of λ_d with $\frac{d\Pi(\lambda_d,\cdot,\cdot)}{d\lambda_d} = (\phi(\lambda_d\rho_b)\rho_b - \phi(\lambda_d\rho_a)\rho_a)$. Hence, $\Pi(\lambda_d, \cdot, \cdot)$ is maximized at $\hat{\lambda}_d = \sqrt{2\frac{\ln(\rho_a/\rho_b)}{\rho_a^2 - \rho_b^2}}$. Moreover, we have that $\frac{d\Pi(\cdot,\Delta\rho,\mathrm{cot})}{d\Delta\rho} = 2\lambda_d\left(\phi(\lambda_d\rho_b) + \phi(\lambda_d\rho_b)\right) \geq 0$, with strict inequality if $\lambda_d > 0, \Delta\rho > 0$. □

Proof of Theorem 2. We determine equilibrium asset prices by backward induction using the ex-post stochastic discount factor (SDF)

$$M_{\tilde{d}}(s) = \frac{u'\left(\tilde{d}\right)}{\mathrm{E}_{\rho^*(s)}\left[u'\left(\tilde{d}\right) \mid \tilde{s} = s\right]}, \qquad \text{where}, \qquad u'(d) = \gamma e^{-\gamma d},$$

and $\rho^*(s)$ minimizes utility from consuming the aggregate dividend $\tilde{d}$ in equilibrium,

$$\rho^*(s) = \underset{\rho\in[\rho_a,\rho_b]}{\operatorname{argmin}} \mathrm{E}_\rho \left[u\left(\tilde{d}\right) \mid \tilde{s} = s\right] = \begin{cases} \rho_a & \text{if } s > -\gamma\sigma_d\rho_a \equiv s_a \\ -\frac{s}{\gamma\sigma_d} & \text{if } s_b \leq s \leq s_a \\ \rho_b & \text{if } s < s_b = -\gamma\sigma_d\rho_b. \end{cases}$$

The ex-post price of the market portfolio is

$$p_m\left(s\right) = \mathrm{E}_{\rho^*(s)}\left[\tilde{d}\, M_{\tilde{d}}(s) \mid s\right] = \bar{p}_m\left(\rho_a, s > s_a\right) + \bar{p}_m\left(0, s_b \leq s \leq s_a\right) + \bar{p}_m\left(\rho_b, s < s_b\right).$$

The marginal value of wealth is

$$\begin{aligned} V'\left(s\right) = u'\left(\mu\left(s, \rho^*(s)\right) - \frac{1}{2}\gamma\sigma^2\left(\rho^*(s)\right)\right) &= u'\left(\mu_a(s) - \frac{1}{2}\gamma\sigma_a^2\right) 1_{\{s>s_a\}} \\ + u'\left(\mu_b(s) - \frac{1}{2}\gamma\sigma_b^2\right) 1_{\{s<s_b\}} &+ u'\left(\bar{d} - \frac{1}{2}\gamma\sigma_d^2 - \frac{1}{2}\frac{s^2}{\gamma}\right) 1_{\{s_b\leq s\leq s_a\}}. \end{aligned}$$

Ex-ante there is no ambiguity about the distribution for $\tilde{s}$ and thus the SDF is

$$M_{\tilde{s}} = \frac{V'\left(\tilde{s}\right)}{\mathrm{E}\left[V'\left(\tilde{s}\right)\right]} = \frac{u'\left(\mu\left(s, \rho^*(s)\right) - \frac{1}{2}\gamma\sigma^2\left(\rho^*(s)\right)\right)}{u'\left(\bar{d} - \frac{1}{2}\gamma\sigma_d^2\right)\left(1 + \frac{\gamma\sigma_d}{\sqrt{2\pi}}(\rho_b - \rho_a)\right)}.$$

The ex-ante market portfolio price is therefore

$$p_{0,m} = \mathrm{E}\left[p_m\left(s\right)\ M_{\tilde{s}}\right] = \bar{d} - \gamma\sigma_d^2 - \frac{1}{\sqrt{2\pi}}\frac{\sigma_d(\rho_b - \rho_a)}{1 + \frac{\gamma\sigma_d}{\sqrt{2\pi}}(\rho_b - \rho_a)}.$$

The ex-post and ex-ante equilibrium price of stock i is

$$\begin{aligned} p_i\left(s\right) &= \mathrm{E}\left[\tilde{d}_i\ M_{\tilde{d}}(s) \mid \tilde{s} = s\right] = \mathrm{E}\left[\left(\beta_i\tilde{d} + \tilde{\varepsilon}_i\right)\ M_{\tilde{d}}(s) \mid \tilde{s} = s\right] = \beta_i p_m(s) \\ p_{0,i} &= \mathrm{E}\left[p_i\left(s\right)\ M_{\tilde{s}}\right] = \beta_i\,\mathrm{E}\left[p_m\left(s\right)\ M_{\tilde{s}}\right] = \beta_i p_{0,m}, \end{aligned}$$

respectively, and, thus, the CAPM holds. □

Proof of Proposition 4. We determine the equilibrium asset price of the market portfolio in an economy with an MEU representative investor with CRRA risk preferences who receives an ambiguous signal about the dividend growth rate $\tilde{d}$. Ambiguity about the joint-normal distribution of $\tilde{s}$ and $\tilde{d}$ is given by the interval $[\rho_a, \rho_b]$. Moreover, for any $\rho \in [\rho_a, \rho_b]$ the residual variance of the dividend growth is $\sigma_d^2(1-\rho^2)$ and the posterior mean of the aggregate dividend, $e^{\tilde{d}}$, is $e^{\bar{d}+\rho\sigma_d s}$. The ex-post stochastic discount factor (SDF) is

$$M_{\tilde{d}}(s) = \frac{u'\left(e^{\tilde{d}}\right)}{\mathrm{E}_{\rho^*(s)}\left[u'\left(e^{\tilde{d}}\right) \mid \tilde{s} = s\right]}, \qquad \text{where,} \qquad u'(w) = w^{-\gamma},\ \gamma > 0,$$

and $\rho^*(s)$ minimizes utility from consuming the aggregate dividend $e^{\tilde{d}}$ in equilibrium,

$$\rho^*(s) = \underset{\rho \in [\rho_a, \rho_b]}{\operatorname{argmin}} \mathrm{E}_\rho \left[u\left(e^{\tilde{d}}\right) \mid \tilde{s} = s \right] = \begin{cases} \rho_a & \text{if } s > -\gamma\sigma_d\rho_a \\ -\frac{s}{\gamma\sigma_d} & \text{if } -\gamma\sigma_d\rho_b \leq s \leq -\gamma\sigma_d\rho_a \\ \rho_b & \text{if } s < -\gamma\sigma_d\rho_b. \end{cases}$$

The ex-post price of the market portfolio is

$$p(s) = e^{-\gamma\sigma_d^2(1-(\rho^*(s))^2)} \mathrm{E}_{\rho^*(s)} \left[e^{\tilde{d}} \mid \tilde{s} = s \right].$$

Hence, plugging in for $\rho^*(s)$ lead to the expression in (23) for the equilibrium price. Moreover, the equilibrium price does not depend on news because

$$p(s) = e^{-\gamma\sigma_d^2} \mathrm{E}\left[e^{\tilde{d}} \right] \qquad \forall\, s \in [-\gamma\sigma_d\rho_b, -\gamma\sigma_d\rho_a].$$

Hence, the size of the signal region for which the stock price does not react to news is $\gamma\sigma_d(\rho_b - \rho_a)$. Moreover, $\tilde{s} \sim \mathrm{N}(0,1)$, so the probability of the equilibrium price exhibiting information inertia conditional on $\tilde{s} \leq 0$ is as given in (24). To prove the first comparative static results we show that the first derivative of Π_p w.r.t. $\Delta\rho$ is strictly positive if $\Delta\rho > 0$ and it vanishes if $\Delta\rho = 0$. Specifically, $\frac{\partial \Pi_p(\cdot,\cdot,\Delta\rho)}{\partial \Delta\rho} = 2\gamma\sigma_d \left[\phi(-\gamma\sigma_d\rho_a) + \phi(-\gamma\sigma_d\rho_b)\right] \geq 0$. To prove the second comparative static result, we compute the first and second derivative of Π_p w.r.t. $\gamma\sigma_d$. Specifically, $\frac{\partial \Pi_p(\gamma\sigma_d,\cdot,\cdot)}{\partial \gamma\sigma_d} = 2\left(\phi(-\gamma\sigma_d\rho_b)\rho_b - \phi(-\gamma\sigma_d\rho_a)\rho_a\right)$ and $\frac{d^2\Pi(\gamma\sigma_d,\cdot,\cdot)}{d(\gamma\sigma_d)^2} = 2\gamma\sigma_d\left(\phi(-\gamma\sigma_d\rho_a)\rho_a^3 - \phi(-\gamma\sigma_d\rho_b)\rho_b^3\right)$. If $\gamma\sigma_d > 0$, then the second derivative is always negative and, the first derivative vanishes if $\widehat{\gamma\sigma}_d = \sqrt{\frac{1}{2}\ln\left(\frac{2\ln(\rho_a/\rho_b)}{(\rho_a^2-\rho_b^2)}\right)}$. The probability Π_p is zero if $\gamma\sigma_d = 0$ because in this case there is no inaction region and it is zero if $\gamma\sigma_d$ approaches infinity because $\lim_{x\to-\infty}\Phi(x) = 0$. Hence, $\widehat{\gamma\sigma_d}$ is the unique maximizer of Π_p.

□

Proof of Theorem 3. The proof proceeds similarly to that of Theorem 2 and is provided in the Internet Appendix for completeness. □

Table I
Information Inertia of Optimal Portfolios.

This table shows the probability of information inertia for risky long positions conditional on a bad news surprise for different ambiguity (α), explanatory power of the PD-ratio ($\hat{\rho}^2$), and unconditional Sharpe ratios (λ_d). The size of the data sample is fixed at $T = 84$. The PD-ratio is always statistically significant because the confidence interval that represents ambiguity never contains zero and, thus, $\rho_a > 0$ for all $\alpha \in \{0.75, 0.9, 0.95, 0.99\}$.

	Sharpe Ratio (λ_d)					**Sharpe Ratio (λ_d)**				
	0.25	0.30	0.35	0.40	0.50	0.25	0.30	0.35	0.40	0.50
$\boldsymbol{\hat{\rho}^2}$	($\alpha = 0.99$)					($\alpha = 0.95$)				
9%	10.15%	12.16%	14.16%	16.15%	20.09%	7.79%	9.34%	10.87%	12.40%	15.43%
16%	9.40%	11.26%	13.09%	14.91%	18.49%	7.20%	8.62%	10.03%	11.42%	14.17%
25%	8.43%	10.08%	11.71%	13.32%	16.45%	6.44%	7.70%	8.94%	10.17%	12.57%
$\boldsymbol{\hat{\rho}^2}$	($\alpha = 0.90$)					($\alpha = 0.75$)				
9%	6.56%	7.86%	9.16%	10.45%	13.00%	4.61%	5.52%	6.43%	7.34%	9.14%
16%	6.06%	7.25%	8.44%	9.61%	11.93%	4.25%	5.09%	5.92%	6.74%	8.37%
25%	5.41%	6.47%	7.52%	8.55%	10.56%	3.79%	4.53%	5.26%	5.99%	7.40%

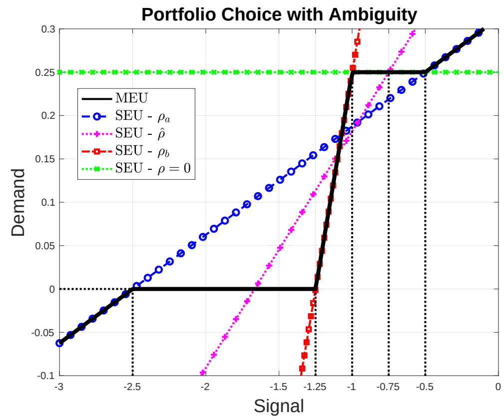


Figure 1. Information Inertia for the Risk-free and Risky Portfolio. The graph shows the optimal asset demand as a function of the signal for an MEU investor (black solid line), for an SEU investor who underestimates the informativeness of the signal (ρ_a, blue dashed circle line), for an SEU investor who correctly estimates the informativeness of the signal ($\hat{\rho}$, purple dotted plus line), for an SEU investor who overestimates the informativeness of the signal (ρ_b, red chain-dotted square line), and for an SEU investor who does not receive the signal (green dotted cross line). There is a range of signals for which a risky and the risk-free portfolio do not react to signals that convey news that is worse than expected. The parameters are $\bar{d} = 100$, $p = 95$, $\sigma_d^2 = 20$, $\rho_a \sigma_d = 2$, $\hat{\rho}\sigma_d = 3$, $\rho_b \sigma_d = 4$, and $\gamma = 1$.

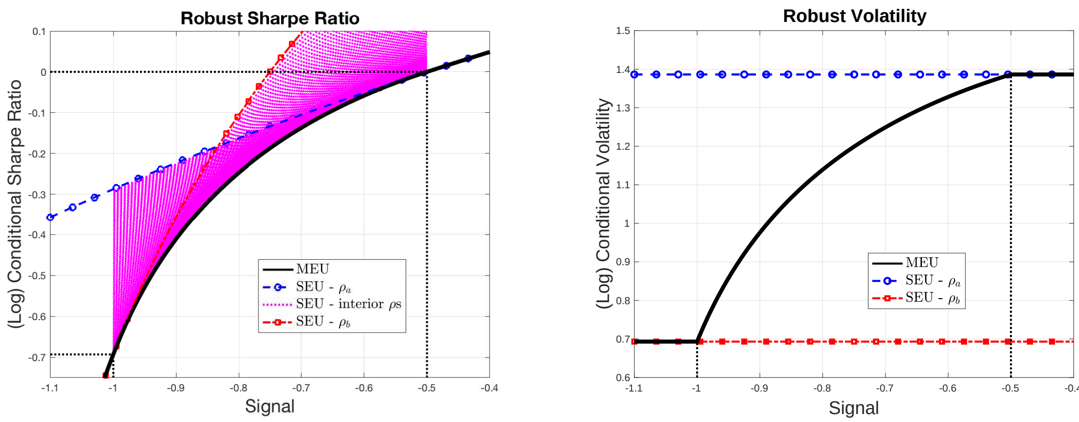


Figure 2. Information Inertia for the Risky Portfolio – the Mechanism. The left graph shows the investors' perceived (log of the) conditional Sharpe ratio and the right graph shows the investors' perceived (log of the) conditional volatility as a function of the signal. The blue dashed circle line represents an SEU investor who underestimates the informativeness of the signal (ρ_a), the dotted purple lines represent SEU investors with different beliefs $\rho \in (\rho_a, \rho_b)$, the red chain-dotted square line represents an SEU investor who overestimates the informativeness of the signal (ρ_b), and the black solid line represents the MEU investor. There is a range of signals for which the Sharpe rate and the volatility change at the same rate leading to no change in the demand for the asset and thus local information inertia for risky portfolios. The parameters are $\bar{d} = 100$, $p = 95$, $\sigma_d^2 = 20$, $\rho_a \sigma_d = 2$, $\rho_b \sigma_d = 4$, and $\gamma = 1$.

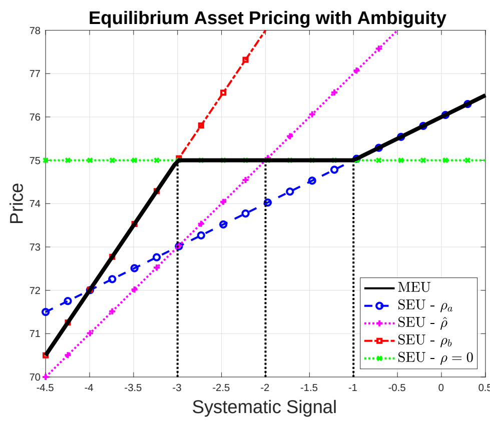
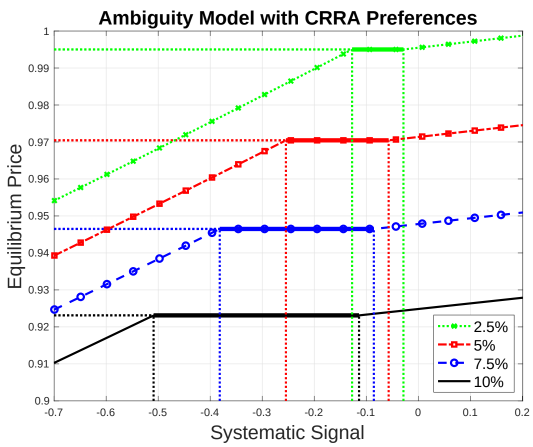


Figure 3. Informational Inefficiency. The graph shows the equilibrium price of the market portfolio as a function of a signal about the future aggregate dividend. The blue dashed circle line represents an economy with an SEU-RI who underestimates the informativeness of the signal, the dotted purple plus line represents an economy with a SEU-RI who correctly estimates the informativeness of the signal, the red chain-dotted square line represents an economy with an SEU-RI who overestimates the informativeness of the signal, and the black solid line represents an economy with an MEU-RI. There is a range of signals for which the equilibrium price does not react (black solid line) to changes in the signal leading to an informational inefficiency. The parameters are $\bar{d} = 100$, $\sigma_d = 5$, $\rho_a = 0.2$, $\hat{\rho} = (\rho_a + \rho_b)/2 = 0.4$, $\rho_b = 0.6$, and $\gamma = 1$.



	Risk Premium ($\gamma\sigma_d^2$)			
	2.5%	5%	7.5%	10%
$\hat{\boldsymbol{\rho}}^2$	($\alpha = 0.99$)			
9%	8.47%	16.81%	24.92%	32.68%
16%	7.85%	15.52%	22.83%	29.64%
25%	7.04%	13.85%	20.19%	25.87%
$\hat{\boldsymbol{\rho}}^2$	($\alpha = 0.95$)			
9%	6.50%	12.91%	19.15%	25.15%
16%	6.01%	11.89%	17.50%	22.75%
25%	5.38%	10.58%	15.43%	19.78%
$\hat{\boldsymbol{\rho}}^2$	($\alpha = 0.90$)			
9%	5.47%	10.88%	16.14%	21.20%
16%	5.06%	10.00%	14.73%	19.15%
25%	4.52%	8.89%	12.97%	16.64%

Figure 4. Informational Inefficiency – Quantitative Importance. The figure shows the equilibrium price as a function of a signal with an average informativeness about dividend growth of $\hat{\rho}^2 = 16\%$ and ambiguity $\alpha = 0.99$. The unconditional risk premium is ranging from 2.5% to 10%. There is a range of PD-ratios for which the equilibrium price does not depend on the PD-ratio, so the dividend information conveyed by the PD-ratio is not efficiently incorporated into the price. The table shows the probability of obtaining an informational inefficiency conditional on a bad news surprise for different ambiguity (α), average informativeness ($\hat{\rho}^2$), and unconditional risk premiums ($\gamma\sigma_d^2$). The severity of this informational inefficiency increases in ambiguity and risk. The economy consists of an ambiguity averse RI with CRRA utility and the expected rate of dividend growth is 2% and the dividend growth volatility is 12%. The PD-ratio is always statistically significant because the confidence interval that represents ambiguity never contains zero, so $\rho_a > 0$ for all $\alpha \in \{0.9, 0.95, 0.99\}$.

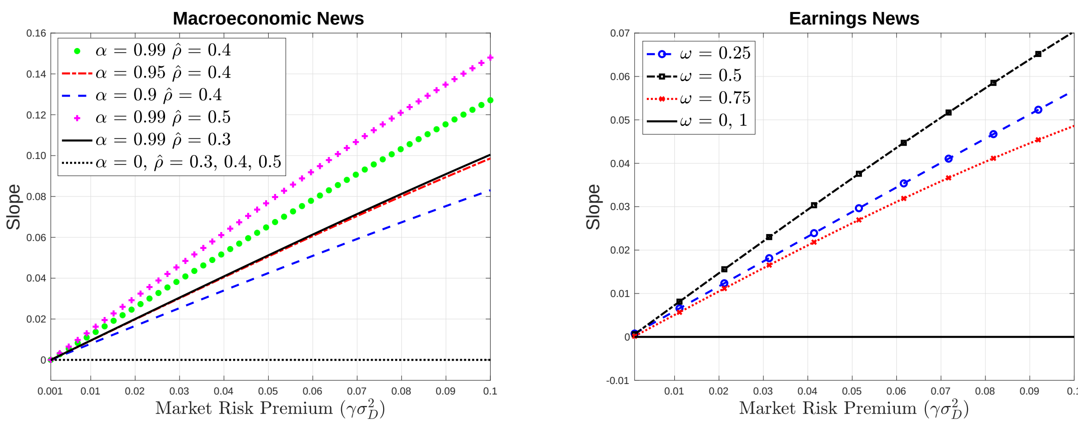


Figure 5. News Momentum. Both graphs show the slope of a news momentum regression as a function of the stock market risk premium. The left graph shows that excess stock market returns underreact to news about the market portfolio (e.g. macroeconomic news) and the resulting positive slope coefficient increases in the stock market risk premium measured by $\gamma\sigma_d^2$, ambiguity measured by, α, and the average informativeness of the signal measured by, $\hat{\rho}$. The right graph shows that risk adjusted stock returns underreact to news about a firm (e.g. earnings news) and the resulting positive slope coefficient increases in the stock market risk premium measured by $\gamma\sigma_d^2$ and is non-monotonic in ω which measures how much of the positive correlation between the firm's dividend and the signal is due to the systematic cash flow component. The cash flow beta of the asset shown in the right graph is $\beta_1 = 1.5$ and the market explains 25% of the variation in the assets return. Similar to the left graph, the slope on the right graph would also increase in ambiguity and the average informativeness of the signal.

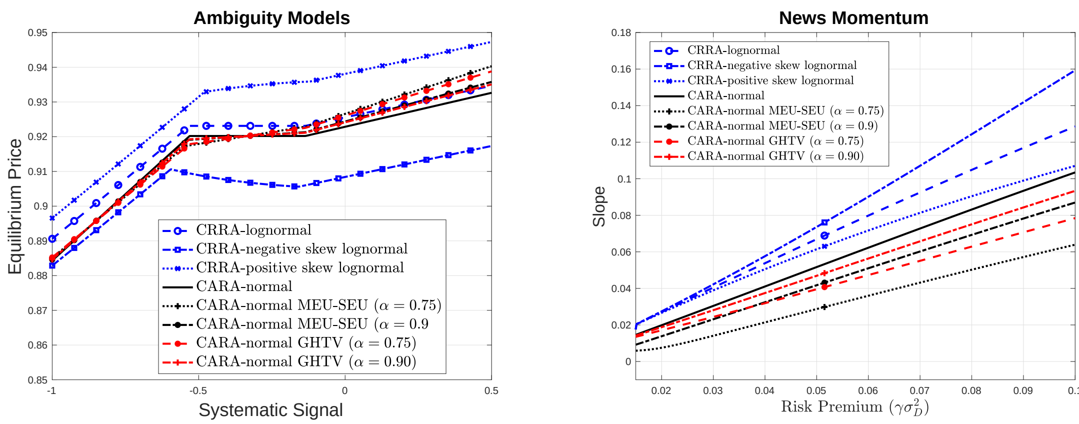


Figure 6. Robustness. The left graph shows the equilibrium price as a function of the signal and the right graph shows the slope coefficients from a regression of excess returns on a constant and the signal. The blue-dashed circle line and black solid line verify that there is a range of signals for which the equilibrium price does not react in the CRRA/log-normal and the CARA/normal model, respectively. Deviating from these models by considering (i) different joint distributions of asset payoffs and signals, (ii) economies populated with ambiguity averse and standard expected utility maximizers, and (iii) other preference models that allow for a distinction between ambiguity and ambiguity aversion leads to low sensitivity to news instead of information inertia. The economic significance of the news momentum drift is of similar magnitude in all cases.

Internet Appendix for

"Information Inertia"

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ABSTRACT

This Internet Appendix serves as a companion to the paper 'Information Inertia'. It provides proofs and details not included in the main paper. We present these in the order that they appear in the main paper.

We present proofs and details not included in the main paper in the following order.

(i) In Section I, we provide details for the optimal demand of an MEU investor. Specifically, we state and prove a result of independent interest (Theorem IA.1) which determines the MEU optimal demand as a function of price. We also provide a proof for the case of a negative Sharpe ratio in Theorem 1 (main paper), which is analogous to the case of a positive Sharpe ratio and, thus, was omitted from the main paper to save space.

(ii) In Section II, we discuss the recursive approach developed in Epstein and Schneider (2003) to ensure dynamic consistency of our two-period learning model under ambiguity that we use in Section III (main paper) to derive equilibrium asset prices before and after receiving a signal about future asset payoffs.

(iii) In Section III, we determine the price of the market portfolio in equilibrium when the economy is populated by investors who all receive the same signal about fundamentals but differ with respect to risk aversion and ambiguity. In this case, there does not exist a representative investor. Moreover, we also prove the existence of a representative investor if investors have different risk aversion but the same ambiguity.

(iv) In Section IV, we provide the proof for Theorem 3 (main paper) where we derive equilibrium asset prices when an investor receives ambiguous news about the future payoff of a firm. This model is more general because it allows the signal to be correlated with the systematic and idiosyncratic component of an asset but the proof is similar to that of Theorem 2 (main paper), and hence was omitted to save space.

(v) In Section V we discuss the implications on portfolio choice and equilibrium prices of two leading models that distinguish between ambiguity and ambiguity aversion — (i) the non-smooth Gajdos, Hayashi, Tallon, and Vergnaud (2008) model (Section A) and the smooth Klibanoff, Marinacci, and Mukerji (2005) model (Section B).

(vi) In Section VI, we discuss the skew normal distribution, which is used in the Section IV (Robustness) of the main paper to demonstrate the robustness of our main results.

I. Optimal Demand

We first state and prove a result (Theorem IA.1) showing the MEU optimal portfolio as a function of price and then provide a proof for Theorem 1 (main paper) which covers all three cases of a positive, negative, and zero Sharpe ratio.

THEOREM IA.1 (Optimal Demand)**.** *Optimal demand at price p for an investor with risk aversion γ and ambiguity described by $[\rho_a, \rho_b]$, who has recieved signal s is*

$$\theta(s,p) = \begin{cases} \overline{\theta}(s,\rho_a,p) & p \le p_1(s) \equiv \mu(s,\rho_a) - \gamma\sigma^2(\rho_a)\max\left(\hat{\theta}_a, 0\right) \\ \max\left(\overline{\theta}(s,0,p),0\right) & p_1(s) < p \le p_2(s) \equiv \mu(s,\rho_b) - \gamma\sigma^2(\rho_b)\max\left(\hat{\theta}_b, 0\right) \\ \overline{\theta}(s,\rho_b,p) & p_2(s) < p \le p_3(s) \equiv \mu(s,\rho_b) - \gamma\sigma^2(\rho_b)\min\left(\hat{\theta}_b, 0\right) \\ \min\left(\overline{\theta}(s,0,p),0\right) & p_3(s) < p \le p_4(s) \equiv \mu(s,\rho_a) - \gamma\sigma^2(\rho_a)\min\left(\hat{\theta}_a, 0\right) \\ \overline{\theta}(s,\rho_a,p) & p > p_4(s), \end{cases} \tag{IA.1}$$

where $\mu(s,\rho) = \bar{d} + \rho\sigma_d s$ and $\sigma^2(\rho) \equiv \sigma_d^2(1-\rho^2)$.

Proof of Theorem IA.1. Consider three cases: (i) $s = 0$, (ii) $s > 0$, and $s < 0$. For expositional clarity, we make the dependence on p and s explicit.

(i) $s = 0 \Leftrightarrow \hat{\theta}_a = \hat{\theta}_b = 0$, so it follows from Proposition 5 (main paper) that $\theta(s,p) = \overline{\theta}(s,\rho_a,p)$ for all $p \in \mathbb{R}$.

(ii) $s > 0 \Leftrightarrow \hat{\theta}_a < \hat{\theta}_b < 0$. Consider five sub-cases: (a) $p \le p_1 = \mu(s,\rho_a)$, (b) $p_1 < p \le p_2 = \mu(s,\rho_b)$, (c) $p_2 < p \le p_3 = \mu(s,\rho_b) - \gamma\sigma^2(\rho_b)\hat{\theta}_b$, (d) $p_3 < p \le p_4 = \mu(s,\rho_a) - \gamma\sigma^2(\rho_a)\hat{\theta}_a$, and (e) $p_4 < p$.

(ii)(a) Suppose $p \leq p_1$. We show that $\theta(s,p) = \overline{\theta}(s,\rho_a,p)$. First, note that

$$\overline{\theta}(s,\rho_a,p) = \frac{\mu(s,\rho_a) - p}{\gamma\sigma^2(\rho_a)} \geq \frac{\mu(s,\rho_a) - p_1}{\gamma\sigma^2(\rho_a)} = 0. \tag{IA.2}$$

Moreover, for any $\theta > 0$, $\mathrm{CE}(\theta) = \overline{\mathrm{CE}}(\theta,\rho_a)$ from Proposition 5 (main paper) Thus, since $\mathrm{CE}(\theta)$ is strictly concave, $\overline{\theta}(s,\rho_a,p)$ is the local and global maximizer of $\mathrm{CE}(\theta)$ for all $p \leq p_1$.

(ii)(b) Suppose $p_1 < p \leq p_2$. We show that $\theta(s,p) = 0$. First, note that since $\rho_a\sigma_d > 0$,

$$\overline{\theta}(s,0,p) = \frac{\bar{d} - p}{\gamma\sigma_d^2} < \frac{\bar{d} - p_1}{\gamma\sigma_d^2} \leq \frac{\mu(s,\rho_a) - p_1}{\gamma\sigma_d^2} = 0. \tag{IA.3}$$

Since $\mathrm{CE}(\theta)$ is strictly concave, it suffices to show that $\theta = 0$ is a local maximizer. Given Proposition 5 (main paper) there exists $\epsilon > 0$ such that

$$\mathrm{CE}(\theta) = \begin{cases} \overline{\mathrm{CE}}(\theta,\rho_b) & \text{if } -\epsilon < \theta \leq 0 \\ \\ \overline{\mathrm{CE}}(\theta,\rho_a) & \text{if } 0 \leq \theta < \epsilon. \end{cases} \tag{IA.4}$$

For $-\epsilon < \theta \leq 0$,

$$\mathrm{CE}(0) - \overline{\mathrm{CE}}(\theta,\rho_b) = \theta\left(p - \bar{d} - \rho_b\sigma_d s\right) + \frac{1}{2}\gamma(\sigma_d^2(1-\rho_b^2)) \geq 0 \tag{IA.5}$$

when $p \leq p_2$.

For $0 \leq \theta < \epsilon$,

$$\mathrm{CE}(0) - \overline{\mathrm{CE}}(\theta, \rho_a) = \theta \left(p - \bar{d} - \rho_a \sigma_d s\right) + \frac{1}{2}\gamma(\sigma_d^2(1-\rho_a^2)) \geq 0 \qquad \text{(IA.6)}$$

when $p_1 \leq p$. Combining the above, shows that $\theta = 0$ is a local and hence global maximizer of $\mathrm{CE}(\theta)$ for $p_1 < p \leq p_2$.

(ii)(c) Suppose $p_2 < p \leq p_3$. We show that $\theta(s, p) = \overline{\theta}(s, \rho_b, p)$. First, note that

$$\overline{\theta}(s, \rho_b, p) = \frac{\mu(s, \rho_b) - p}{\gamma\sigma^2(\rho_b)} < \frac{\mu(s, \rho_b) - p_2}{\gamma\sigma^2(\rho_b)} = 0 \qquad \text{(IA.7)}$$

when $p_2 < p$ and that

$$\overline{\theta}(s, \rho_b, p) = \frac{\mu(s, \rho_b) - p}{\gamma\sigma^2(\rho_b)} \geq \frac{\mu(s, \rho_b) - p_3}{\gamma\sigma^2(\rho_b)} = \hat{\theta}_b \qquad \text{(IA.8)}$$

when $p \leq p_3$.

From Proposition 5 (main paper) , $\mathrm{CE}(\theta) = \overline{\mathrm{CE}}(\theta, \rho_b)$ when $\hat{\theta}_b < \theta \leq 0$. Thus, given strict concavity of $\mathrm{CE}(\theta)$, $\overline{\theta}(s, \rho_b, p)$ is a local and hence global maximizer of $\mathrm{CE}(\theta)$ when $p_2 < p \leq p_3$.

(ii)(d) Suppose $p_3 < p \leq p_4$. We show that $\theta(s, p) = \overline{\theta}(s, 0, p)$. First, note that since $\rho_a \sigma_d > 0$,

$$\overline{\theta}(s, 0, p) = \frac{\bar{d} - p}{\gamma\sigma_d^2} < \frac{\bar{d} - p_3}{\gamma\sigma_d^2} < \frac{\bar{d} - p_2}{\gamma\sigma_d^2} \leq 0. \qquad \text{(IA.9)}$$

Also, $p_3 = \mu(s, \rho_b) - \gamma\sigma^2(\rho_b)\hat{\theta}_b = \bar{d} - \gamma\sigma_d^2\hat{\theta}_b$ and $p_4 = \mu(s, \rho_a) - \gamma\sigma^2(\rho_a)\hat{\theta}_a =$

$\bar{d} - \gamma\sigma_d^2\hat{\theta}_a$. Hence,

$$\hat{\theta}_a \leq \overline{\theta}(s,0) < \hat{\theta}_b \tag{IA.10}$$

when $p_3 < p \leq p_4$.

From Proposition 5 (main paper) , $\text{CE}(\theta) = \overline{\text{CE}}(\theta, 0) - \frac{s^2}{2\gamma}$ when $\hat{\theta}_a < \theta \leq \hat{\theta}_b < 0$. Thus, since $\text{CE}(\theta)$ is strictly concave, $\overline{\theta}(s,0,p)$ is the local and global maximizer of $\text{CE}(\theta)$ for $p_3 < p \leq p_4$.

(ii)(e) Suppose $p_4 < p$. We show that $\theta(s,p) = \overline{\theta}(s,\rho_a,p)$. First, note that

$$\overline{\theta}(s,\rho_a,p) = \frac{\mu(s,\rho_a) - p}{\gamma\sigma^2(\rho_a)} < \frac{\mu(s,\rho_a) - p_4}{\gamma\sigma^2(\rho_a)} \leq \hat{\theta}_a = 0. \tag{IA.11}$$

Moreover, for any $\theta < \hat{\theta}_a$, $\text{CE}(\theta) = \overline{\text{CE}}(\theta,\rho_a)$ from Proposition 5 (main paper) Thus, since $\text{CE}(\theta)$ is concave, $\overline{\theta}(s,\rho_a,p)$ is the local and global maximizer of $\text{CE}(\theta)$ for all $p > p_4$.

Using the above, we get

$$\theta(s,p) = \begin{cases} \overline{\theta}(s,\rho_a,p) & \text{if } p \leq p_1 \\ 0 & \text{if } p_1 < p \leq p_2 \\ \overline{\theta}(s,\rho_b,p) & \text{if } p_2 < p \leq p_3 \\ \overline{\theta}(s,0,p) & \text{if } p_3 < p \leq p_4 \\ \overline{\theta}(s,\rho_a,p) & \text{if } p_4 < p. \end{cases} \tag{IA.12}$$

as desired.

(iii) When $s < 0 \Leftrightarrow \hat{\theta}_a > \hat{\theta}_b > 0$, then it follows from Proposition 5 (main paper) that $p_1 = \mu(s, \rho_a) - \gamma\sigma^2(\rho_a)\hat{\theta}_a$, $p_2 = \mu(s, \rho_b) - \gamma\sigma^2(\rho_b)\hat{\theta}_b$, $p_3 = \mu(s, \rho_b)$, and (d) $p_4 = \mu(s, \rho_a)$. Thus, using similar reasoning as above, we get

$$\theta(s,p) = \begin{cases} \overline{\theta}(s, \rho_a, p) & \text{if } p \leq p_1 \\ \overline{\theta}(s, 0, p) & \text{if } p_1 < p \leq p_2 \\ \overline{\theta}(s, \rho_b, p) & \text{if } p_2 < p \leq p_3 \\ 0 & \text{if } p_3 < p \leq p_4 \\ \overline{\theta}(s, \rho_a, p) & \text{if } p_4 < p. \end{cases} \tag{IA.13}$$

as desired.

Combining the three cases above provides the desired expression for $\theta(s, p)$. □

Proof of Theorem 1 (main paper). Consider three cases: (i) $\lambda = 0$, (ii) $\lambda > 0$, and (iii) $\lambda < 0$ and the expression for demand in Theorem IA.1. We omit the dependence on p for expositional ease.

(i) Suppose $\lambda = 0$. Then $s_1 = s_2 = s_3 = s_4 = 0$ and from Theorem IA.1, $\theta(s) = \overline{\theta}(s, \rho_a)$ if $p \leq p_1 \Leftrightarrow s \geq 0$ and if $p > p_4 \Leftrightarrow s < 0$.

(ii) $\lambda > 0$. Then $s_1 = -\rho_a \lambda > s_2 = -\rho_b \lambda > s_3 = -\frac{\lambda}{\rho_b} > s_4 = -\frac{\lambda}{\rho_a}$.

Then from Theorem IA.1 the following holds.

$$\theta(s) = \begin{cases} \overline{\theta}(s, \rho_a) & \text{if } p \leq p_1 \Leftrightarrow s \geq s_1 \\ \overline{\theta}(s, 0) & \text{if } p_1 < p \leq p_2 \Leftrightarrow s_1 > s \geq s_2 \\ \overline{\theta}(s, \rho_b) & \text{if } p_2 < p \leq p_3 \Leftrightarrow s_2 > s \geq s_3 \\ 0 & \text{if } p_3 < p \leq p_4 \Leftrightarrow s_4 \leq s < s_3 \\ \overline{\theta}(s, \rho_a) & \text{if } p > p_4 \Leftrightarrow s < s_4. \end{cases} \tag{IA.14}$$

(iii) $\lambda < 0$. Then $s_1 = -\frac{\lambda}{\rho_a} > s_2 = -\frac{\lambda}{\rho_b} > s_3 = -\rho_b \lambda > s_4 = -\rho_a \lambda$.

Then from Theorem IA.1 the following holds.

$$\theta(s) = \begin{cases} \overline{\theta}(s, \rho_a) & \text{if } p \leq p_1 \Leftrightarrow s \geq s_1 \\ 0 & \text{if } p_1 < p \leq p_2 \Leftrightarrow s_1 > s \geq s_2 \\ \overline{\theta}(s, \rho_b) & \text{if } p_2 < p \leq p_3 \Leftrightarrow s_2 > s \geq s_3 \\ \theta(s, 0) & \text{if } p_3 < p \leq p_4 \Leftrightarrow s_4 \leq s < s_3 \\ \overline{\theta}(s, \rho_a) & \text{if } p > p_4 \Leftrightarrow s < s_4. \end{cases} \tag{IA.15}$$

Combining the above cases provides the desired expression. □

II. Dynamic Consistency

Epstein and Schneider (2003) axiomatize an intertemporal version of multiple-priors utility that is dynamically consistent. In order to do that they modify the preference model at date zero such that the (i) conditional preference at each decision node satisfy the Gilboa and Schmeidler (1989) axioms[1] and (ii) the process of conditional preferences is dynamically consistent. Hence, instead of choosing a parameter from the ambiguity interval (the set of parameters that describe the decisions makers aversion to ambiguity) to minimize conditional one-step ahead expected utility at future decision nodes the decision maker chooses a function for each future decision node that maps the possible realizations at the decision node into the ambiguity interval and minimizes the conditional one-step ahead expected utility at this decision node. Therefore, date zero preferences are defined over all signal and dividend realizations and thus have to satisfy the so called rectangularity condition of Epstein and Schneider (2003).[2] In a nutshell, if the backward induction solution does not coincide with the forward planning solution (and thus the model is not dynamically consistent), then date zero-preferences are modified such that the forward planning solution coincides with the backward induction solution. Hence, the backward induction ambiguity model represents investors preferences and the modified forward planning model is another way to represent

[1]Beliefs at each decision node are updated prior by prior using Bayes rule to determine the conditional one-step ahead expected utility at each decision node.

[2]The dimension of the set of beliefs that describes the aversion to ambiguity at date zero is equal to the number of future decision nodes.

them.[3]

Our information inertia results do not rely on dynamics and can be expressed clearly in the model described in Sections II-III (main paper). Hence we do not formulate a dynamic model there. However, if we were to consider a dynamic setting and include a trading date 0 before the signal is received at date 1, then we can take the backward induction solution and write down the forward planning model to match it. The marginal distribution of the signal is normal with zero mean and unit variance and the conditional distribution of the dividend given the signal is normal with mean $\bar{d} + \sigma_d \rho s$ and variance $\sigma_d^2(1-\rho^2)$. There is no ambiguity about the marginal distribution of the signal and thus we only need to account for ambiguity after observing the signal. Hence, in the modified forward planning model, the decision maker at date zero uses the marginal distribution of the signal and chooses a function $\rho(s)$ from all functions $\rho : \mathcal{R} \to [\rho_a, \rho_b]$ that describes the set of conditional distributions of the dividend given the signal s.[4]

III. Investor Heterogeneity

In this section, we determine the price of the market portfolio in equilibrium when the economy is populated by investors who all receive the same signal about fundamentals but

[3]As noted by Epstein and Schneider (2003) on page 16: "there is an important conceptual distinction between the set of probability laws that the decision maker views as possible ... and the set of priors ... that is part of the representation of preference. Only the latter includes elements of reasoning or processing, backward induction for example, on the part of the decision-maker."

[4]The analysis is very similar to the dynamic extension of the Epstein and Schneider (2008) model which is described in Back (2010) on page 430 and 431.

may differ with respect to risk and ambiguity aversion. We show that there is a range of bad signals for which investors' long position in the market portfolio does not depend on the signal and thus the equilibrium price of the market portfolio does not depend on these signals.

Suppose there are H investors who all receive the signal $\tilde{s}$ about the future value of the dividend $\tilde{d}$. Investors may differ with respect to their initial wealth, and their aversion to risk and ambiguity. Let w_{0h} denote investor h's initial wealth, $\gamma_h > 0$ her risk aversion coefficient, and $[\rho_{ah}, \rho_{bh}]$ the interval that represents her ambiguity and ambiguity aversion with $0 < \rho_{ah} \leq \rho_{bh} < 1 \, \forall \, h \in \{1, \ldots, H\}$. As in the main paper, we refer to the interval as representing the investor's ambiguity.

An equilibrium in this economy is defined as follows:

DEFINITION IA.1 (Equilibrium)**.** *The signal-to-price map $p(s)$ is an equilibrium if and only if (i) each investor chooses a portfolio θ_h to maximize*

$$\min_{\rho_h \in [\rho_{ah}, \rho_{bh}]} \mathrm{E}_{\rho_h} \left[u_h \left(w_{0h} + \left(\tilde{d} - p(s) \right) \theta_h \right) \mid \tilde{s} = s \right], \qquad \forall s \in \mathbb{R} \tag{IA.16}$$

and (ii) markets clear, that is, $\sum_{h=1}^{H} \theta_h = 1$ and investors consume the liquidating dividend $\tilde{d}$ at date 1*.*

A. Homogeneous Ambiguity Aversion

We know that if all investors are standard expected utility maximizers, then there exists a representative investor (SEU-RI) with these preferences.[5] We show in the next proposition that this is still true when all investors have the same ambiguity and we determine the utility of the ambiguity averse representative investor (MEU-RI) in equilibrium.[6]

PROPOSITION IA.1 (MEU-RI and Equilibrium Utility)**.** *Assume that all investors have the same ambiguity* $[\rho_a, \rho_b]$*. Then there exists a representative investor with initial wealth* $w_0 = \sum_{h=1}^{H} w_{0h}$ *and aggregate risk tolerance* $1/\gamma \equiv \sum_{h=1}^{H} 1/\gamma_h$*. Moreover, the utility of the MEU-RI in equilibrium is*

$$\min_{\rho \in [\rho_a, \rho_b]} \mathrm{E}_\rho \left[u\left(\tilde{d}\right) \mid \tilde{s} = s \right] = u\left(\mu\left(s, \rho^*(s)\right) - \frac{1}{2} \gamma \sigma^2 \left(\rho^*(s)\right) \right), \tag{IA.17}$$

where

$$\rho^*(s) = \begin{cases} \rho_a & \text{if} \quad s \geq -\gamma \sigma_d \rho_a \\ -\frac{s}{\gamma \sigma_d} & \text{if} \quad -\gamma \sigma_d \rho_b < s < -\gamma \sigma_d \rho_a \\ \rho_b & \text{if} \quad s \leq -\gamma \sigma_d \rho_b. \end{cases} \tag{IA.18}$$

For the remainder of this subsection we consider a representative investor (MEU-RI)

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[5]See Chapter 7 in Back (2010).

[6]Wakai (2007) and Illeditsch (2011) show that there exists a representative investor when investors have the same ambiguity but differ w.r.t. their CARA coefficient.

with initial wealth w_0, risk aversion γ, and ambiguity $[\rho_a, \rho_b]$.[7]

Her equilibrium utility is determined by minimizing the equilibrium utility of an SEU-RI over her belief ρ. The utility of the SEU-RI is strictly increasing in the posterior mean of the dividend and strictly decreasing in the residual variance of the dividend. Hence, the belief ρ that minimizes the SEU-RI's utility depends on the nature of the news.

Suppose the signal conveys bad news ($s < 0$), then the worst case for the posterior mean $\mu(s, \rho)$ is a high correlation because in this case the investor significantly revises the value of the dividend downwards whereas the worst case for the residual variance $\sigma^2(\rho)$ is a low correlation because in this case there is less risk resolved by the signal. If the signal conveys very bad news ($s \leq -\gamma\sigma_d\rho_b$), then the mean dominates and the MEU-RI investor behaves like an SEU-RI investor with belief ρ_b. If the signal conveys moderately bad or good news ($s \geq -\gamma\sigma_d\rho_a$), then the MEU-RI investor behaves like an SEU-RI investor with belief ρ_a. There is a range of bad signal values ($-\gamma\sigma_d\rho_b < s < -\gamma\sigma_d\rho_a$) for which neither the posterior mean nor the residual variance dominates and utility is minimized in the interior.

The equilibrium price when the representative investor is an SEU investor with belief ρ is

$$\bar{p}(s, \rho) = \mathrm{E}_\rho\left[\tilde{d} \mid \tilde{s} = s\right] - \gamma \mathrm{Var}_\rho\left[\tilde{d} \mid \tilde{s} = s\right] = \mu(s, \rho) - \gamma\sigma^2(\rho). \tag{IA.19}$$

The price of the asset is strictly increasing in the signal and hence it fully incorporates all available information. This is no longer true when the representative investor is averse

[7] We discuss the properties of the equilibrium price when investors have different ambiguity in the next subsection.

to ambiguity as the next theorem shows.

THEOREM IA.2 (Equilibrium Price)**.** *Consider an economy with an MEU-RI with risk aversion γ and ambiguity $[\rho_a, \rho_b]$ who receives the signal s. There is a unique equilibrium price,*

$$p(s) = \begin{cases} \mathrm{E}_{\rho_a}\left[\tilde{d} \mid \tilde{s} = s\right] - \gamma \mathrm{Var}_{\rho_a}\left[\tilde{d} \mid \tilde{s} = s\right] & \text{if} \quad s > -\gamma\sigma_d\rho_a \\ \mathrm{E}\left[\tilde{d}\right] - \gamma \mathrm{Var}\left[\tilde{d}\right] & \text{if} \quad -\gamma\sigma_d\rho_b \leq s \leq -\gamma\sigma_d\rho_a \\ \mathrm{E}_{\rho_b}\left[\tilde{d} \mid \tilde{s} = s\right] - \gamma \mathrm{Var}_{\rho_b}\left[\tilde{d} \mid \tilde{s} = s\right] & \text{if} \quad s < -\gamma\sigma_d\rho_b. \end{cases} \tag{IA.20}$$

Moreover, $p(s) = \bar{p}(s, \rho^*(s))$ where $\bar{p}(\cdot)$ is given in equation (IA.19) and $\rho^*(s)$ is given in equation (IA.18).

The left graph of Figure IA.1 shows the equilibrium price as a function of the signal. There is a range of signals that convey bad news for which the price does not depend on the signal even though the utility of the RI is sensitive to changes in these signals.

Why does the price not always incorporate signals that convey bad news? We know from Theorem IA.2 that the equilibrium price $p(s)$ coincides with the equilibrium price $\bar{p}(s, \rho^*(s))$ in an economy with an SEU-RI whose belief about the correlation minimizes her utility from consuming the dividend.[8] Consider a two standard deviation bad news surprise ($s = -2$). In this case the equilibrium price is $p = 75$ when there is ambiguity aversion and when there is no ambiguity aversion $\beta_m = 2$ (see left graph of Figure IA.1). If the signal decreases, then the SEU-RI requires a lower price as compensation for the lower posterior mean in order

[8]We know from the previous section that an MEU investor behaves differently from an SEU investor only if she holds the risk-free portfolio which is not an equilibrium allocation.

to hold the market portfolio. However, the MEU-RI revises the worst case scenario belief about ρ upwards if the signal drops. The price does not change because the lower posterior mean that would require a drop in the equilibrium price is exactly offset by the lower risk premium that would require an increase in the price.

Formally,

$$dp(s) = d\mu(s, \rho^*(s)) - \gamma d\sigma^2(\rho^*(s)) = 0, \qquad \forall\, s \in (-\gamma\sigma_d\rho_b, -\gamma\sigma_d\rho_a). \tag{IA.21}$$

The right graph of Figure IA.1 shows the posterior mean and residual variance perceived by the MEU-RI as a function of the signal. The graph shows that there is a range of signals for which both the mean and variance are strictly increasing in the signal. Moreover, the posterior mean increases at the same rate as the residual variance increases in this signal range. Hence, any change in the price due to changes in the posterior mean is exactly offset by a change in the residual variance.

B. Heterogeneous Ambiguity Aversion

We show in the next proposition that equilibrium prices still fail to incorporate all available public information when investors are heterogeneous in their ambiguity.[9]

PROPOSITION IA.2 (Information Inertia). *Let* $1/\gamma \equiv \sum_{h=1}^{H} 1/\gamma_h$ *denote aggregate risk*

[9]We do not report the equilibrium price outside of the inaction region but provide numerical examples in Figure IA.2.

tolerance and let $[\rho_a, \rho_b] \equiv \bigcap_{h=1}^{H} [\rho_{ah}, \rho_{bh}] \neq \emptyset$. *Then the equilibrium price is*

$$p(s) = \mathrm{E}\left[\tilde{d}\right] - \gamma \mathrm{Var}\left[\tilde{d}\right] \qquad \forall\, s \in \left[-\gamma\sigma_d\rho_b, -\gamma\sigma_d\rho_a\right]. \tag{IA.22}$$

To gain intuition consider an economy populated by two MEU investors with ambiguity aversion $[\rho_{a1}, \rho_{b1}] = [0.1, 0.4]$ and $[\rho_{a2}, \rho_{b2}] = [0.2, 0.6]$, respectively. The left graph of Figure IA.2 shows the equilibrium price and the middle graph shows their equilibrium portfolios as a function of the signal. Consider the five different signal regions (i) $(-\infty, -3.15]$, (ii) $[-3.15, -2]$, (iii) $[-2, -1]$, (iv) $[-1, -0.5]$, and (v) $[-0.5, \infty)$. Both MEU investors behave like SEU investors with beliefs $\rho_{b1} = 0.4$ and $\rho_{b2} = 0.6$ in the first signal range and thus the equilibrium price depends on these signals. The equilibrium portfolio of the second MEU investor (red dashed line) is increasing in the signal because her worst case scenario belief ($\rho_{b2} = 0.6$) is larger than the worst case scenario belief of the second MEU investor (blue chain-dotted line) and thus she puts more weight on the signal. The analysis is similar for the fifth signal range because with good news the worst case scenario for both investors is a low ρ.

For the other three ranges of signals there is at least one investor who ignores the signal and uses her prior when choosing her optimal portfolio. In other words, there is at least one investor who behaves as if the signal is uninformative even though her utility is negatively affected by it. Consider the second signal range. The first MEU investor still behaves like an SEU investor with belief $\rho_{b1} = 0.4$ but the second MEU investor does not rely on the signal. Hence her demand, which is increasing for the first range of signals, is now decreasing because neither mean nor variance depends on the signal and the equilibrium price increases with it. The equilibrium price still depends on the signals in the second region because of the first investor but not as much as for the first range of signals. Both investors do not rely on the signals in the third region and hence the equilibrium price does not depend on these signals. The intuition for the fourth signal range is similar to the second. In this case the first investor does not rely on the signal when choosing her optimal portfolio and hence in equilibrium her asset demand decreases with the signal.

There is no information inertia in optimal portfolios of SEU investors and hence the equilibrium price always depends on the signal in their presence. But how much do SEU investors move the price? To answer this question, we consider a unit mass of investors where α denotes the fraction of MEU investors and $1 - \alpha$ denotes the fraction of SEU investors. The third graph of Figure IA.2 shows that when the fraction of MEU investors is sufficiently large, then there are signal regions for which the equilibrium price shows lower sensitivity to news than the price in an economy where the SEU-RI has the belief ρ_a.

C. *Proofs of Results*

Proof of Proposition IA.1. Using $\theta_h(s,p)$ to denote the demand at price p for investor h who recieves signal s, it follows from Theorem IA.1 that

$$\theta_h(s,p) = \begin{cases} \frac{\mu(s,\rho_a)-p}{\gamma_h \sigma^2(\rho_a)} & p \le p_1 \\ \max\left(\frac{\bar{d}-p}{\gamma_h \sigma_d^2}, 0\right) & p_1 < p \le p_2 \\ \frac{\mu(s,\rho_b)-p}{\gamma_h \sigma^2(\rho_b)} & p_2 < p \le p_3 \\ \min\left(\frac{\bar{d}-p}{\gamma_h \sigma_d^2}, 0\right) & p_3 < p \le p_4 \\ \frac{\mu(s,\rho_a)-p}{\gamma_h \sigma^2(\rho_a)} & p > p_4, \end{cases} \tag{IA.23}$$

where p_1, p_2, p_3, p_4 are as in Theorem IA.1 due to homogeneous ambiguity aversion $[\rho_a, \rho_b]$ across investors.

Aggregating individual demands leads to aggregate demand,

$$\theta(s,p) = \sum_{h=1}^{H} \theta_h(s,p) \begin{cases} \frac{\mu(s,\rho_a)-p}{\gamma \sigma^2(\rho_a)} & p \le p_1 \\ \max\left(\frac{\bar{d}-p}{\gamma \sigma_d^2}, 0\right) & p_1 < p \le p_2 \\ \frac{\mu(s,\rho_b)-p}{\gamma \sigma^2(\rho_b)} & p_2 < p \le p_3 \\ \min\left(\frac{\bar{d}-p}{\gamma_h \sigma_d^2}, 0\right) & p_3 < p \le p_4 \\ \frac{\mu(s,\rho_a)-p}{\gamma \sigma^2(\rho_a)} & p > p_4, \end{cases} \tag{IA.24}$$

with $\frac{1}{\gamma} = \sum_{h=1}^{H} \frac{1}{\gamma_h}$ risk tolerance, wealth $w_0 = \sum_{h=1}^{H} w_0 h$ and ambiguity described by $[\rho_a, \rho_b]$.

The representative investor holds the risky asset in equilibrium and consumes the dividend. Thus, using CE(θ) and Proposition 1 (main paper) with $\theta(s,p) = 1$ for the RI yields

(IA.17) as the representative investor utility in equilibrium. Since $u' > 0$, the equilibrium utility can be computed by solving

$$\min_{\rho \in [\rho_a, \rho_b]} \mu(s,\rho) - \frac{1}{2}\gamma\sigma^2(\rho). \tag{IA.25}$$

Since $\mu(s,\rho) - \frac{1}{2}\gamma\sigma^2(\rho)$ is strictly convex in ρ over $[\rho_a, \rho_b]$, the following first order (Kuhn-Tucker) condition is necessary and sufficient for the solution to the constrained minimization problem.

$$0 \leq \rho(s + \sigma_s\gamma) \tag{IA.26}$$

with equality if $\rho \in (\rho_a, \rho_b)$. Solving this yields (IA.18) as desired. □

Proof of Theorem IA.2. We make the dependence of demand on price p explicit for expositional clarity. Market clearing requires that $\theta(s,p) = 1$ since there is one unit of the risky asset in aggregate.

Consider three cases: (i) $s > -\gamma\rho_a\sigma_d$, (ii) $-\gamma\rho_b\sigma_d \leq s \leq -\gamma\rho_a\sigma_d$, and (iii) $s < -\gamma\rho_b\sigma_d$.

(i) Suppose $s > -\gamma\rho_a\sigma_d$. Then $\hat{\theta}_a < 1$. We need to verify that markets clear when $p(s) = \mu(s,\rho_a) - \gamma\sigma^2(\rho_a)$. From Theorem 1 (main paper), it follows that

$$\theta(s,p) = \overline{\theta}(s,\rho_a,p) = \frac{\mu(s,\rho_a) - p(s)}{\gamma\sigma^2(\rho_a)} = 1 \tag{IA.27}$$

if and only if

$$p(s) = \mu(s, \rho_a) - \gamma\sigma^2(\rho_a) \leq p_1 = \mu(s, \rho_a) - \gamma\sigma^2(\rho_a) \max\left\{\hat{\theta}_a, 0\right\} \tag{IA.28}$$

or

$$p(s) = \mu(s, \rho_a) - \gamma\sigma^2(\rho_a) > p_4 = \mu(s, \rho_a) - \gamma\sigma^2(\rho_a) \min\left\{\hat{\theta}_a, 0\right\}. \tag{IA.29}$$

Since $\hat{\theta}_a < 1$, $p(s) \leq p_1$ and the result follows.

(ii) Suppose $-\gamma\rho_b\sigma_d \leq s \leq -\gamma\rho_a\sigma_d$. Then $\hat{\theta}_b \leq 1 \leq \hat{\theta}_a$. We need to verify that markets clear when $p(s) = \bar{d} - \gamma\sigma_d^2$. From Theorem 1 (main paper) it follows that

$$\theta(s, p) = \overline{\theta}(s, 0, p) = \frac{\bar{d} - p(s)}{\gamma\sigma_d^2} = 1 \tag{IA.30}$$

if and only if

$$p(s) = \bar{d} - \gamma\sigma_d^2 > p_1 = \mu(s, \rho_a) - \gamma\sigma^2(\rho_a) \max\left\{\hat{\theta}_a, 0\right\} \tag{IA.31}$$

and

$$p(s) = \bar{d} - \gamma\sigma_d^2 \leq p_2 = \mu(s, \rho_b) - \gamma\sigma^2(\rho_b) \max\left\{\hat{\theta}_b, 0\right\}. \tag{IA.32}$$

Since $\hat{\theta}_a \geq 1$ and $\mu(s, \rho_a) - \gamma\sigma^2(\rho_a)\hat{\theta}_a = \bar{d} - \gamma\sigma_d^2\hat{\theta}_a$, we have $p(s) > p_1$.

If $\hat{\theta}_b \leq 0$, then $s \geq 0$. So, $p(s) = \bar{d} - \gamma\sigma_d^2 \leq \bar{d} + \rho_b\sigma_d s = \mu(s, \rho_b) = p_2$. If $0 < \hat{\theta}_b$, then since $\hat{\theta}_b \leq 1$ we $p(s) = \bar{d} - \gamma\sigma_d^2 \leq \bar{d} - \gamma\sigma_d^2\hat{\theta}_b = \mu(s, \rho_b) - \gamma\sigma^2(\rho_b)\hat{\theta}_b = p_2$. So, $p_1 < p(s) \leq p_2$

(iii) Suppose $s < -\gamma\rho_b\sigma_d$. Then $\hat{\theta}_b > 1$. We need to verify that markets clear when $p(s) = \mu(s, \rho_b) - \gamma\sigma^2(\rho_b)$. From Theorem 1 (main paper), it follows that

$$\theta(s, p) = \overline{\theta}(s, \rho_b, p) = \frac{\mu(s, \rho_b) - p(s)}{\gamma\sigma^2(\rho_b)} = 1 \tag{IA.33}$$

if and only if

$$p(s) = \mu(s, \rho_b) - \gamma\sigma^2(\rho_b) > p_2 = \mu(s, \rho_b) - \gamma\sigma^2(\rho_b) \max\left\{\hat{\theta}_b, 0\right\} \tag{IA.34}$$

and

$$p(s) = \mu(s, \rho_b) - \gamma\sigma^2(\rho_b) \leq p_3 = \mu(s, \rho_b) - \gamma\sigma^2(\rho_b) \min\left\{\hat{\theta}_b, 0\right\}. \tag{IA.35}$$

Since $\hat{\theta}_b > 1$, $p_2 < p(s) \leq p_3$ and the result follows.

Combining the above cases provides the desired result. $\square$

Proof of Proposition IA.2. Using $\theta_h(s)$ to denote the demand for investor h, it follows from

Theorem IA.1 that

$$\theta_h(s,p) = \begin{cases} \frac{\mu(s,\rho_{ah})-p}{\gamma_h\sigma^2(\rho_{ah})} & p \le p_{1h} \equiv \mu(s,\rho_{ah}) - \gamma_h\sigma^2(\rho_{ah})\max\left(\hat{\theta}_{ah},0\right) \\ \max\left(\frac{\bar{d}-p}{\gamma_h\sigma_d^2},0\right) & p_{1h} < p \le p_{2h} \equiv \mu(s,\rho_{bh}) - \gamma_h\sigma^2(\rho_{bh})\max\left(\hat{\theta}_{bh},0\right) \\ \frac{\mu_{bh}-p}{\gamma_h\sigma^2(\rho_{bh})} & p_{2h} < p \le p_{3h} \equiv \mu(s,\rho_{bh}) - \gamma_h\sigma^2(\rho_{bh})\min\left(\hat{\theta}_{bh},0\right) \\ \min\left(\frac{\bar{d}-p}{\gamma_h\sigma_d^2},0\right) & p_{3h} < p \le p_{4h} \equiv \mu(s,\rho_{ah}) - \gamma_h\sigma^2(\rho_{ah})\min\left(\hat{\theta}_{bh},0\right) \\ \frac{\mu(s,\rho_{ah})-p}{\gamma_h\sigma^2(\rho_{ah})} & p > p_{4h}, \end{cases} \tag{IA.36}$$

where $\hat{\theta}_{ah} \equiv -s/(\gamma_h\rho_{ah}\sigma_d)$ and $\hat{\theta}_{bh} \equiv -s/(\gamma_h\rho_{bh}\sigma_d)$.

We first show that there exists an equilibrium. Individual demand given in equation (IA.36) is continuous and non-increasing in p with $\lim_{p\to-\infty}\theta_h(s,p) = \infty$ and $\lim_{p\to\infty}\theta_h(s,p) = -\infty$ for all $h \in \{1,\ldots,H\}$. Hence, aggregate demand $\theta(s,p) = \sum_{h=1}^H \theta_h(s,p)$ is continuous and non-increasing in p with $\lim_{p\to-\infty}\theta(s,p) = \infty$ and $\lim_{p\to\infty}\theta(s,p) = -\infty$. Hence, there exists an equilibrium because the market clearing condition $\theta(s,p) - 1 = 0$ has always a solution.

We next determine the equilibrium stock price $p(s)$ for all $s \in [-\gamma\sigma_d\rho_b, -\gamma\sigma_d\rho_a]$. By assumption we have that $\rho_a = \max\{\rho_{a1},\ldots,\rho_{ah}\}$ and $\rho_b = \min\{\rho_{b1},\ldots,\rho_{bh}\}$. Hence, since $s < 0$,

$$p_1(s) \equiv \max_{h\in\{1,\ldots,H\}} p_{1h}(s) = \max_{h\in\{1,\ldots,H\}} \left\{\bar{d} + \frac{\sigma_d}{\rho_{ah}}s\right\} = \bar{d} + \frac{\sigma_d}{\rho_a}s \tag{IA.37}$$

$$p_2(s) \equiv \min_{h\in\{1,\ldots,H\}} p_{2h}(s) = \min_{h\in\{1,\ldots,H\}} \left\{\bar{d} + \frac{\sigma_d}{\rho_{bh}}s\right\} = \bar{d} + \frac{\sigma_d}{\rho_b}s. \tag{IA.38}$$

We have that $\rho_b \ge \rho_a$ and thus (i) $[-\gamma\sigma_d\rho_b, -\gamma\sigma_d\rho_a] \ne \emptyset$ and (ii) $p_2(s) \ge p_1(s)$ for all $s \in [-\gamma\sigma_d\rho_b, -\gamma\sigma_d\rho_a]$.

It follows from equations (IA.36)-(IA.38) that

$$\theta_h(s,p) = \frac{\bar{d} - p}{\gamma_h \sigma_d^2} \qquad \forall\, p_1(s) \le p \le p_2(s), \quad \text{and} \quad \forall h \in \{1, \dots, H\}. \tag{IA.39}$$

Aggregating over all investors leads to

$$\theta(s,p) = \sum_{h=1}^{H} \theta_h(s,p) = \frac{\bar{d} - p}{\sigma_d^2} \sum_{h=1}^{H} \frac{1}{\gamma_h} = \frac{\bar{d} - p}{\gamma \sigma_d^2} \qquad \forall\, p_1(s) \le p \le p_2(s).$$

Imposing the market clearing condition $\theta(s,p)$=1 leads to the price $p(s) = \bar{d} - \gamma\sigma_d^2$. Finally, the desired result follows from noting that $p_1(s) \le \bar{d} - \gamma\sigma_d^2 \le p_2(s)$ if and only if $-\gamma\sigma_d\rho_b \le s \le -\gamma\sigma_d\rho_a$. □

IV. Equilibrium Prices

In Section IV, we provide the proof for Theorem 3 (main paper) where we derive equilibrium asset prices when an investor receives ambiguous news about the future payoff of a firm. This model is more general because it allows the signal to be correlated with the systematic and idiosyncratic component of an asset but the proof is similar to that of Theorem 2 (main paper), and hence was omitted to save space.

Proof of Theorem 3 (main paper). We solve for the prices by backward induction using the

SDF

$$M_{\tilde{d}}(s) = \frac{u'\left(\tilde{d}\right)}{\mathrm{E}_{\rho^*(s)}\left[u'\left(\tilde{d}\right) \mid \tilde{s} = s\right]}, \tag{IA.40}$$

where $\rho^*(s)$ minimizes ex-post utility from consuming the aggregate dividend in equilibrium. Using reasoning similar to that for Theorem 2 (main paper) yields that

$$\rho^*(s) = \begin{cases} \rho_a & \text{if } s > -\gamma \frac{\omega}{\beta_1} \sigma_1 \rho_a \equiv \hat{s}_a \\ -\frac{s}{\gamma \sigma_1 \frac{\omega}{\beta_1}} & \text{if } \hat{s}_b \leq s \leq \hat{s}_a \\ \rho_b & \text{if } s < -\gamma \frac{\omega}{\beta_1} \sigma_1 \equiv \hat{s}_b \end{cases} \tag{IA.41}$$

Hence, the ex-post price of asset 1 is

$$\begin{aligned} p_1(s) &= \mathrm{E}_{\rho^*(s)}\left[\tilde{d}_1\, M_{\tilde{d}}(s) \mid \tilde{s} = s\right] \\ &= \bar{p}_1(\rho_a, s > s_a) + \bar{p}_1(0, s_b \leq s \leq s_a) + \bar{p}_1(\rho_b, s < s_b) \end{aligned} \tag{IA.42}$$

and the ex-post market portfolio price is

$$\begin{aligned} p_m(s) &= \mathrm{E}_{\rho^*(s)}\left[\tilde{d}\, M_{\tilde{d}}(s) \mid \tilde{s} = s\right] \\ &= \bar{p}_m(\rho_a, s > s_a) + \bar{p}_m(0, s_b \leq s \leq s_a) + \bar{p}_m(\rho_b, s < s_b) \end{aligned} \tag{IA.43}$$

The individual ex-post stock prices for $i \neq 1$ are

$$p_i(s) = \mathrm{E}_{\rho^*(s)}\left[(\beta_i \tilde{d} + \varepsilon_i)\, M_{\tilde{d}}(s) \mid \tilde{s} = s\right] = \beta_i p_m(s). \tag{IA.44}$$

The equilibrium value function is

$$
\begin{aligned}
V(s) &= u\left(p_m(s) + \frac{1}{2}\gamma\sigma_m^2\left(\rho^*(s)\right)\right) \\
&= u\left(\mu_m(s,\rho_a) - \frac{1}{2}\gamma\sigma_m^2(\rho_a)\right)1_{\{s>s_a\}} + u\left(\mu_m(s,\rho_b) - \frac{1}{2}\gamma\sigma_m^2(\rho_b)\right)1_{\{s<s_b\}} \\
&+ u\left(\bar{d} - \frac{1}{2}\gamma\sigma_d^2 - \frac{1}{2}\frac{s^2}{\gamma}\right)1_{\{s_b\le s\le s_a\}}
\end{aligned}
\tag{IA.45}
$$

where $\mu_m(s,\rho) = \bar{d} + \frac{\omega}{\beta_1}\sigma_1\rho$ and $\sigma_m^2(\rho) = \sigma_d^2 - \left(\frac{\omega}{\beta_1}\sigma_1\rho\right)^2$.

Ex-ante there is no ambiguity about the distribution for $\tilde{s}$ and thus the SDF is

$$
M_{\tilde{s}} = \frac{V'(\tilde{s})}{\mathrm{E}\left[V'(\tilde{s})\right]}, \tag{IA.46}
$$

where $V'(s)$ denotes the marginal value of wealth.

The ex-ante market portfolio price is therefore

$$
p_{0,m} = \mathrm{E}\left[p_m(s)\ M_{\tilde{s}}\right] = \bar{d} - \gamma\sigma_d^2 - \frac{1}{\sqrt{2\pi}}\frac{\frac{\omega_1}{\beta_1}\sigma_1(\rho_b-\rho_a)}{1+\frac{\gamma\frac{\omega_1}{\beta_1}\sigma_1}{\sqrt{2\pi}}(\rho_b-\rho_a)} \tag{IA.47}
$$

Similarly,

$$
p_{0,1} = \mathrm{E}\left[p_1(s)\ M_{\tilde{s}}\right] = \beta_1(\bar{d} - \gamma\sigma_d^2) - \frac{1}{\sqrt{2\pi}}\frac{\sigma_1(\rho_b-\rho_a)}{1+\frac{\gamma\frac{\omega}{\beta_1}\sigma_1}{\sqrt{2\pi}}(\rho_b-\rho_a)} \tag{IA.48}
$$

Finally, for all $i \neq 1$,

$$
p_{0,i} = \mathrm{E}\left[p_i(s)\ M_{\tilde{s}}\right] = \beta_i p_{0,m}. \tag{IA.49}
$$

$\square$

V. Separating Ambiguity and Ambiguity Aversion

In this section, we discuss the implications on portfolio choice and equilibrium price of separating ambiguity and ambiguity aversion. We consider two well known models of decision-making: (i) the non-smooth GHTV model (Gajdos, Hayashi, Tallon, and Vergnaud (2008)) and (ii) the smooth KMM (Klibanoff, Marinacci, and Mukerji (2005)) model.

A. The GHTV Model

In this section, we study optimal portfolios and equilibrium stock prices when investors are averse to ambiguity in the sense of Gajdos, Hayashi, Tallon, and Vergnaud (2008). This preferences model, in short the GHTV model, allows for a distinction between ambiguity and ambiguity aversion. We show that if investors are sufficiently ambiguity averse then there is a range of good and bad signals for which risky portfolios is not very sensitive to news. Moreover, there is a range of bad signals for which the price of the market portfolio is not very sensitive to news if the representative investor is sufficiently ambiguity averse.[10]

Suppose there are two dates 0 and 1. Investors can invest in a risk-free asset and a risky asset. Let p denote the price of the risky asset, $\tilde{d}$ the future value or dividend of the risky

[10] If we follow Bianchi and Tallon (2014) and consider a slightly modified GHTV preference model, then all the results in the paper go through if we change the interval $[\rho_a, \rho_b]$ to $[\rho_a^\alpha, \rho_b^\alpha]$, where $\rho_a^\alpha = \alpha\rho_a + (1-\alpha)\rho_S$, $\rho_b^\alpha = \alpha\rho_b + (1-\alpha)\rho_S$, and $\rho_S = (\rho_a + \rho_b)/2$. In this case the amount of ambiguity is measured by the interval $[\rho_a, \rho_b]$ and α measures the degree of ambiguity aversion.

asset, and θ the number of shares invested in the risky asset. There is no consumption at date zero. The risk-free asset is used as numeraire, so the risk-free rate is zero. Hence, future wealth $\tilde{w}$ is given by

$$\tilde{w} = w_0 + \left(\tilde{d} - p\right)\theta, \tag{IA.50}$$

in which w_0 denotes initial wealth.

Suppose investors receive a signal $\tilde{s}$ about the future value $\tilde{d}$ of the asset. The joint distribution of $\tilde{d}$ and $\tilde{s}$ is normal:

$$\begin{pmatrix} \tilde{d} \\ \tilde{s} \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} \bar{d} \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_d^2 & \beta \\ \beta & 1 \end{pmatrix}\right), \tag{IA.51}$$

where $\beta = \rho\sigma_d$. Investors do not know the correlation between $\tilde{d}$ and $\tilde{s}$ and consider a family of joint distributions described by $\rho \in [\rho_a, \rho_b]$ with $\rho_a > 0$ and $\rho_b < 1$ when making decisions.[11]

We follow Gilboa and Schmeidler (1993) and determine the family of conditional dividend distributions given the signal by applying Bayes rule to each correlation. Hence, standard normal-normal updating for each $\rho \in [\rho_a, \rho_b]$ leads to

$$\tilde{d} \mid \tilde{s} = s \;\sim \mathrm{N}_\rho\left(\mu(s,\rho), \sigma^2(\rho)\right), \tag{IA.52}$$

where $\mu(s,\rho) = \bar{d} + \beta s$ denotes the conditional mean and $\sigma(\rho) = \sigma_d\sqrt{1-\rho^2}$ the conditional

[11] There is no ambiguity about the marginal distribution of the signal and hence there is no loss in generality by normalizing the mean and the variance of the signal to zero and one, respectively.

volatility of $\tilde{d}$ given s.

Suppose investors have CARA utility over future wealth $\tilde{w}$ (i.e. $u(\tilde{w}) = -e^{-\gamma\tilde{w}}$ with $\gamma > 0$). The utility of a risk and ambiguity averse investors who holds θ shares of the risky asset is

$$U_{\mathrm{GHTV}}(\theta) = \alpha \min_{\rho\in[\rho_a,\rho_b]} \mathrm{E}_\rho\left[u\left(\tilde{w}\right) \mid \tilde{s} = s\right] + (1-\alpha)\, \mathrm{E}_{\rho_S}\left[u\left(\tilde{w}\right) \mid \tilde{s} = s\right], \tag{IA.53}$$

where $\rho_S = (\rho_a + \rho_b)/2$. There are no closed form solutions for optimal demand and equilibrium price but it is straightforward to compute them numerically. Suppose the unconditional Sharpe ratio of the asset is positive $\lambda_d > 0$. The left graph of Figure IA.3 shows that the optimal portfolio does not react much to news if ambiguity aversion α is sufficiently high. Suppose there is a representative investor who is maximizing utility given in equation (IA.53). In equilibrium, the representative investor holds the stock and consumes the liquidating dividend $\tilde{d}$. The right graph of Figure IA.3 shows the equilibrium stock price as a function of the signal for different aversion to ambiguity α. The price reacts moderately to signals that convey good and moderately bad news and it reacts strongly to signals that convey very bad news. There is a range of signals that convey bad news for which the price does not react much to news if α is sufficiently large.

B. The KMM Model

We study in this section the implications for portfolio choice and asset pricing when investors face Bayesian model or parameter uncertainty and we distinguish between ambiguity and aversion to ambiguity by considering the smooth ambiguity model axiomatized in Klibanoff, Marinacci, and Mukerji (2005). We provide characterisations of the optimal portfolio (Proposition IA.3) and equilibrium price (Proposition IA.4) below and illustrate that there are signal regions for which portfolios and prices show lower sensitivity to news than an SEU investor with belief ρ_a, if aversion to ambiguity is sufficiently high. Consider the model described in Section I (main paper) and assume that the correlation between the signal and the dividend is random. Let $\mathbb{P}$ denote the prior distribution for $\tilde{\rho}$ with support $[\rho_a, \rho_b] \subset (0,1)$.[12] The joint distribution of $\tilde{d}$ and $\tilde{s}$ conditional on knowing the correlation ρ is normal and given in equation (2) (main paper). Hence, standard Bayesian updating leads to

$$\tilde{d} \mid \tilde{s} = s, \tilde{\rho} = \rho \sim \mathrm{N}_\rho \left(\mu(s,\rho), \sigma^2(\rho)\right). \tag{IA.54}$$

The investor does not learn anything about the correlation ρ after observing the signal and hence the prior $\mathbb{P}$ coincides with the posterior.[13] Let $u(\cdot)$ denote the function that measures attitudes toward risk and $\phi(\cdot)$ the function that measures attitudes towards ambiguity. The utility of an ambiguity averse investor in the sense of Klibanoff, Marinacci, and Mukerji (2005) who holds θ shares of the risky asset is therefore

$$\mathrm{E}_{\mathbb{P}}\left[\phi\left(\mathrm{E}_{\tilde{\rho}}\left[u\left(\tilde{w}\right) \mid \tilde{s} = s\right]\right)\right] = \mathrm{E}_{\mathbb{P}}\left[\phi\left(u\left(\overline{\mathrm{CE}}(\theta,\tilde{\rho})\right)\right)\right], \tag{IA.55}$$

[12]The SEU investors of the previous sections have dogmatic priors over the correlation ρ.

[13]Investors can draw inferences about the correlation ρ, if they observe a time series of $\tilde{d}$ and $\tilde{s}$. In this case $\mathbb{P}$ in equation (IA.55) below would be the distribution of $\tilde{\rho}$ conditional on observing the signal $\tilde{s} = s$.

where $\overline{\mathrm{CE}}(\theta,\rho)$ denotes the certainty equivalent of an SEU investor with dogmatic belief ρ. If $\phi(\cdot)$ is linear, then investors are neutral to ambiguity and thus we call them BMU investors (investors who face Bayesian model uncertainty) otherwise we call them KMM investors.

For the remainder of this section we assume that investors have constant absolute risk aversion and constant relative ambiguity aversion, that is, $u(w) = -e^{-\gamma w}$ and $\phi(u) = -\frac{1}{1+\alpha}(-u)^{1+\alpha}$ with γ positive and α nonnegative.[14] Hence, the certainty equivalent $\overline{\mathrm{CE}}(\theta,\rho)$ is given in equation (6) (main paper).

C. *Portfolio Choice*

Let $\theta(s)$ denote the portfolio of a KMM investor that maximizes utility given in equation (IA.55). The properties of $\theta(s)$ are summarized in the next proposition.

PROPOSITION IA.3 (Portfolio Choice)**.** *For every distribution $\mathbb{P}$ with support $[\rho_a,\rho_b]\subset[0,1]$ such that utility given in equation (IA.55) exists, let $\mathbb{Q}(\rho;s,\theta(s))$ denote the risk and ambiguity adjusted distribution of $\tilde{\rho}$ conditional on $\tilde{s}=s$. Specifically,*

$$d\mathbb{Q}(\rho;s,\theta(s)) = \frac{e^{-\gamma(1+\alpha)\left(\sigma_d s\rho\,\theta(s)+\frac{1}{2}\gamma\sigma_d^2\rho^2\theta(s)^2\right)}}{\mathrm{E}_{\mathbb{P}}\left[e^{-\gamma(1+\alpha)\left(\sigma_d s\tilde{\rho}\,\theta(s)+\frac{1}{2}\gamma\sigma_d^2\tilde{\rho}^2\theta(s)^2\right)}\right]}d\mathbb{P}(\rho) \tag{IA.56}$$

[14] We choose constant absolute risk aversion for $u(\cdot)$ so that conditional on knowing ρ investors have mean-variance preferences. The choice of constant relative ambiguity aversion for $\phi(\cdot)$ simplifies the analysis but does not change the qualitative result of this section.

The optimal portfolio is unique and implicitly given by

$$\theta(s) = \frac{\lambda^{\mathbb{Q}}(s, \theta(s))}{\gamma \sigma^{\mathbb{Q}}(s, \theta(s))} \tag{IA.57}$$

where

$$\sigma^{\mathbb{Q}}(s, \theta(s)) = \sigma_d \sqrt{1 - \mathrm{E}_{\mathbb{Q}(\rho; s, \theta(s))}\left[\tilde{\rho}^2 \mid \tilde{s} = s\right]} \tag{IA.58}$$

$$\lambda^{\mathbb{Q}}(s, \theta(s)) = \frac{\bar{d} - p + \sigma_d s \mathrm{E}_{\mathbb{Q}(\rho; s, \theta(s))}\left[\tilde{\rho} \mid \tilde{s} = s\right]}{\sigma^{\mathbb{Q}}(s, \theta(s))}. \tag{IA.59}$$

Why does the Sharpe ratio and volatility depend on the position in the asset? To answer this question consider first the case where $\alpha = 0$. The BMU investor hedges against parameter uncertainty by adjusting its distribution for risk. The risk adjusted probability $\mathbb{Q}$ depends on the position in the asset because the effects of different realizations of $\tilde{\rho}$ on utility depend on the asset position. For instance, suppose an investor who contemplates a long position in the asset receives a signal that conveys bad news. If the long position is very large, then the investor is more concerned about the residual variance and thus the risk adjusted probability of low correlation states is higher than the actual probability of these states. Similarly, for a moderate long position in the asset, the investor is more concerned about a low posterior mean and thus the risk adjusted probability of high correlation states exceeds the actual probability of these states.[15] A KMM investor is also averse to ambiguity and thus puts additional weight on the states of the world for which ρ has adverse effects on utility. An increase in risk aversion would also make a BMU investor more concerned

[15]There is no parameter uncertainty for the risk free portfolio and thus $\mathbb{Q}$ and $\mathbb{P}$ coincide.

about parameter uncertainty. However, an increase in risk aversion has the indirect effect of decreasing the asset position which makes her less concerned about parameter uncertainty.

The left graph of Figure IA.4 shows the optimal portfolio as a function of the signal when the unconditional Sharpe ratio is positive and $\tilde{\rho}$ is uniformly distributed on the interval $[\rho_a, \rho_b]$. The black solid line represents an MEU investor, the red solid line represents a BMU investor ($\alpha = 0$), and the other three lines represent KMM investors with different degrees of ambiguity aversion α. There is a range of signals for which risky portfolios become less and less sensitive to news as ambiguity aversion increases. Moreover, the figure shows that there are signal ranges for which asset demand is strictly increasing in α, which is consistent with Gollier (2011) who also finds that an increase in aversion to ambiguity does not always lead to a decrease in asset demand.

Why does the sensitivity to news for some risky portfolios decrease with aversion to ambiguity? The intuition for this result is similar to the intuition for the information inertia result. Asset demand for a KMM investor is increasing in the Sharpe ratio and decreasing in the volatility of the asset. Both the Sharpe ratio and volatility are determined by averaging over $\tilde{\rho}$ using the risk and ambiguity adjusted probability $\mathbb{Q}$. The risk and ambiguity adjustment depends on the signal and thus the conditional volatility and Sharpe ratio depend on the signal. The right graph of Figure IA.4 shows the (log) of the conditional Sharpe ratio and volatility perceived by a KMM investor for different degrees of ambiguity aversion α. If α is sufficiently large, then there is a range of signals for which both the conditional Sharpe ratio and volatility increase at approximately the same rate and thus the portfolio does not react much to these signals.[16]

D. Equilibrium Price

Suppose there exists a representative investor with prior $\mathbb{P}$ over the correlation $\tilde{\rho}$. In equilibrium the representative investor holds the asset ($\theta = 1$) and consumes the liquidating dividend $\tilde{d}$. The properties of the equilibrium price are summarized in the next proposition.

PROPOSITION IA.4. *The unique equilibrium price is*

$$p(s) = \bar{d} - \gamma\sigma_d^2 + s\sigma_d \, \mathrm{E}_{\mathbb{Q}(\rho,s)}\left[\tilde{\rho} \mid \tilde{s} = s\right] + \gamma\sigma_d^2 \mathrm{E}_{\mathbb{Q}(\rho,s)}\left[\tilde{\rho}^2 \mid \tilde{s} = s\right], \tag{IA.60}$$

where $\mathbb{Q}(\rho, s)$ denotes the risk and ambiguity adjusted equilibrium distribution of the correlation $\tilde{\rho}$ conditional on $\tilde{s} = s$. Specifically,

$$d\mathbb{Q}(\rho; s) = \frac{e^{-\gamma(1+\alpha)\left(\sigma_d s\rho + \frac{1}{2}\gamma\sigma_d^2\rho^2\right)}}{\mathrm{E}_{\mathbb{P}}\left[e^{-\gamma(1+\alpha)\left(\sigma_d s\tilde{\rho} + \frac{1}{2}\gamma\sigma_d^2\tilde{\rho}^2\right)}\right]} d\mathbb{P}(\rho). \tag{IA.61}$$

The left graph of Figure IA.5 shows the equilibrium price as a function of the signal when $\tilde{\rho}$ is uniformly distributed on the interval $[\rho_a, \rho_b]$. The black solid line represents an economy with an MEU-RI the red solid line represents an economy with a BMU-RI ($\alpha = 0$), and the other three lines represent economies with a KMM-RI with different degrees of ambiguity aversion α. There is a range of signals for which the equilibrium price becomes less sensitive

[16] There is also a range of signals for which the risk-free portfolio does not react much to news if α is sufficiently large.

to changes in the signal when α increases.

Why does the sensitivity of the equilibrium price to news decrease with aversion to ambiguity? Intuitively, the price increases with the posterior mean and decreases with the posterior variance. The RI hedges against risk and ambiguity and thus both the mean and variance depend on the signal. The right graph of Figure IA.5 shows that there is a range of signal values for which both the risk and ambiguity adjusted mean and variance increase at approximately the same rate and thus the equilibrium price does not react much to these signals.

E. Proofs

Proof of Proposition IA.3. For all $\theta \in \mathbb{R}$, the function $\xi(\theta, s, \rho)$,

$$\xi(\theta, s, \rho) = \frac{e^{-\gamma(1+\alpha)\left(\sigma_d s\rho\,\theta + \frac{1}{2}\gamma\sigma_d^2\rho^2\theta^2\right)}}{\mathrm{E}_{\mathbb{P}}\left[e^{-\gamma(1+\alpha)\left(\sigma_d s\tilde{\rho}\,\theta + \frac{1}{2}\gamma\sigma_d^2\tilde{\rho}^2\theta^2\right)}\right]} \tag{IA.62}$$

is non-negative and $\mathrm{E}_{\mathbb{P}}\left[\xi(\theta, s, \tilde{\rho})\right] = 1$ hence $d\mathbb{Q}\left(s, \rho, \theta\right)$ as defined in (IA.56) is a conditional probability distribution.

The utility $U(\theta)$ of a KMM investor from holding portfolio θ is as given in (IA.55) with $u(w) = -e^{-\gamma w}, \gamma > 0$ and $\phi(u) = -\frac{1}{1+\alpha}\left(-u\right)^{1+\alpha}, \alpha \geq 0$. The first-order condition for this investor's optimal portfolio is

$$0 = U'(\theta) = \mathrm{E}_{\mathbb{P}}\left[\phi'(u(\overline{\mathrm{CE}}(\theta, \tilde{\rho})))u'(\overline{\mathrm{CE}}(\theta, \tilde{\rho}))\left(\lambda(s, \tilde{\rho}) + \gamma\theta\sigma(\tilde{\rho})\right)\sigma(\tilde{\rho})\right]. \tag{IA.63}$$

The second derivative of (IA.55) with respect to θ is

$$U''(\theta) = \mathrm{E}_{\mathbb{P}}\left[\left(\lambda(s,\tilde{\rho}) + \gamma\theta\sigma(\tilde{\rho})\right)^2 \sigma^2(\tilde{\rho}) \left(\phi''(\cdot)u'(\cdot) + \phi'(\cdot)u''(\cdot)\right) - \phi'(\cdot)u'(\cdot)\gamma\sigma^2(\tilde{\rho}) \right] < 0 \tag{IA.64}$$

given $u' > 0$, $u'' < 0$ and $\phi' > 0, \phi'' \leq 0$.

Hence, KMM investor utility is strictly concave in θ and optimal portfolio is unique. Solving for $\theta(s)$ using the first-order condition and $\phi'(u) = (-u)^\alpha$ and $u'(w) = \gamma e^{-\gamma w}$ yields

$$\begin{aligned} \theta(s) &= \frac{\mathrm{E}_{\mathbb{P}}[\phi'(\cdot)u'(\cdot)(\lambda(s,\tilde{\rho})\sigma(\tilde{\rho}))]}{\gamma \mathrm{E}_{\mathbb{P}}[\sigma^2(\tilde{\rho})]} \\ &= \frac{(\bar{d}-p)+\sigma_d s \mathrm{E}_{\mathbb{Q}(\rho;s,\theta(s))}[\tilde{\rho}|\tilde{s}=s]}{\gamma\sigma_d^2(1-\mathrm{E}_{\mathbb{Q}}[(\tilde{\rho}^2)])} \\ &= \frac{\lambda^{\mathbb{Q}(s,\theta(s))}}{\gamma\sigma^{\mathbb{Q}}(s,\theta(s))} \end{aligned} \tag{IA.65}$$

as desired.

□

Proof of Proposition IA.4. Setting $\theta(s) = 1$ in (IA.56) and using $\mathbb{Q}(\rho, s)$ to denote $\mathbb{Q}(\rho; s, 1)$ yields the distribution of $\tilde{\rho}$ condtional on $\tilde{s} = s$ in (IA.61).

The representative investor holds the risky asset and consumes the dividend in equilibrium. Setting $\theta(s) = 1$ in the first-order condition (IA.63) yields,

$$\lambda^{\mathbb{Q}(s,1)} = \gamma\sigma^{\mathbb{Q}}(s,1). \tag{IA.66}$$

Using the expressions for $\lambda^{\mathbb{Q}}(s,\theta(s))$ and $\sigma^{\mathbb{Q}}(s,\theta(s))$ from Proposition IA.3 with $\theta(s) = 1$

and $\mathbb{Q}(\rho, s)$ for $\mathbb{Q}(\rho; s, 1)$, solving for $p(s)$ yields (IA.60) as the unique equilibrium price. $\square$

VI. Different Payoff Distributions

Consider the dividend growth rate $\tilde{d}$ and assume that its mean and variance is $\bar{d}$ and σ_d^2, respectively. Suppose we regress the dividend growth rate, $\tilde{d}$, on a constant and the (standardized) signal $\tilde{s}$. There is ambiguity about the correlation between the signal and the dividend growth rate, ρ, and hence there is ambiguity about the posterior dividend growth rate because the slope coefficient is a linear function of ρ and there is ambiguity about the posterior variance because the regression rsquared is ρ^2. Hence, the residual variance of the dividend growth rate is $\sigma(\rho)^2 = \sigma_d^2(1-\rho^2)$ and the posterior mean of the dividend is $\mathrm{E}[e^{\tilde{d}} \mid \tilde{s} = s] = e^{\bar{d}+\sigma_d \rho s}$.

In contrast to the previous section, we assume that the error term of the regression, $\tilde{\eta}$, is skew normally distributed and has unit variance. Let $G_\eta(t; a, b, c)$ denote the moment generating function of a skew normally distributed random variable with location parameter a, scale parameter b, and shape parameter c. Hence, the mean is $a + bc\sqrt{\frac{2}{\pi}}$, the variance is $b^2\left(1 - \frac{2c^2}{\pi}\right)$, and skewness $\frac{4-\pi}{2}\frac{\left(c\sqrt{2/\pi}\right)^3}{(1-2c^2/\pi)^{\frac{3}{2}}}$. The moment generating function of $\tilde{\eta}$ is

$$G_\eta(t; a, b, c) = 2e^{at+\frac{1}{2}b^2t^2}\Phi(b\lambda t), \qquad \lambda = \frac{c}{\sqrt{1+c^2}}, \tag{IA.67}$$

where the parameters are chosen such that the random variable η has zero mean and unit variance. If $\lambda = 0$, then $\tilde{\eta}$ is normally distributed and if λ is positive (negative), then $\tilde{\eta}$ has a positive (negative) skewness.

Consider an economy with an representative SEU investor with CRRA utility and belief ρ who consumes the liquidating dividend $e^{\tilde{d}}$. The equilibrium utility of the SEU investor and the stochastic discount factor in this economy are

$$\bar{U}(\rho, s) = \mathrm{E}_\rho\left[u\left(e^{\tilde{d}}\right) \mid \tilde{s} = s\right] \qquad \text{and} \qquad M_{\tilde{d}}(s) = \frac{e^{-\gamma\tilde{d}}}{\mathrm{E}_\rho\left[e^{-\gamma\tilde{d}} \mid \tilde{s} = s\right]}, \tag{IA.68}$$

respectively. The risk aversion coefficient γ is strictly positive and, thus, the equilibrium price is

$$\bar{p}(s, \rho) = \mathrm{E}\left[e^{\tilde{d}} M_{\tilde{d}}(s) \mid \tilde{s} = s\right] = \frac{G_\eta\left((1-\gamma)\sigma(\rho); a, b, c\right)}{G_\eta\left(-\gamma\sigma(\rho); a, b, c\right) G_\eta\left(\sigma(\rho); a, b, c\right)} \mathrm{E}\left[e^{\tilde{d}} \mid \tilde{s} = s\right].$$

For instance, if the regression error term is normally distributed, then

$$\bar{p}(s, \rho; \lambda = 0) = e^{-\gamma\sigma_d^2(1-\rho^2)} \mathrm{E}\left[e^{\tilde{d}} \mid \tilde{s} = s\right].$$

The equilibrium utility of an MEU-RI is

$$U(s) = \min_{\rho \in [\rho_a, \rho_b]} \bar{U}(\rho, s) = \bar{U}(\rho^*(s), s). \tag{IA.69}$$

where $\rho^*(s)$ is the worst case scenario correlation and, thus, in general will depend on the signal. The SDF in this economy is therefore

$$M_{\tilde{d}}(s) = \frac{e^{-\gamma\tilde{d}}}{\mathrm{E}_{\rho^*(s)}\left[e^{-\gamma\tilde{d}} \mid \tilde{s} = s\right]},$$

Hence, the equilibrium price is

$$p(s) = \mathrm{E}_{\rho^*(s)}\left[e^{\tilde{d}} M_{\tilde{d}}(s) \mid \tilde{s} = s\right] = \bar{p}(s, \rho^*(s)). \tag{IA.70}$$

For instance, if the regression error term is normally distributed, then

$$p(s) = \begin{cases} e^{-\gamma\sigma_d^2(1-\rho_a^2)}\mathrm{E}_{\rho_a}\left[e^{\tilde{d}} \mid \tilde{s} = s\right] & \text{if} \quad s > -\gamma\sigma_d\rho_a \\ e^{-\gamma\sigma_d^2}\mathrm{E}\left[e^{\tilde{d}}\right] & \text{if} \quad -\gamma\sigma_d\rho_b \le s \le -\gamma\sigma_d\rho_a \\ e^{-\gamma\sigma_d^2(1-\rho_b^2)}\mathrm{E}_{\rho_b}\left[e^{\tilde{d}} \mid \tilde{s} = s\right] & \text{if} \quad s < -\gamma\sigma_d\rho_b. \end{cases} \tag{IA.71}$$

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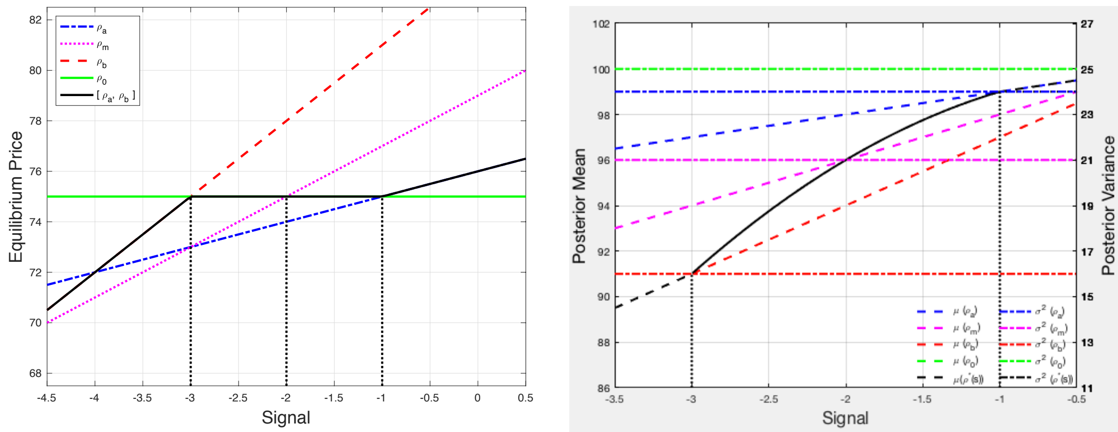

Figure IA.1. Equilibrium Price
The left graph shows the equilibrium price and the right graph shows the RI's perceived posterior mean and variance as a function of the signal. Red lines represent an SEU-RI economy with belief $\beta_b = \rho_b \sigma_d = 3$, purple lines represent an SEU-RI economy with belief $\beta_m = \sigma_d(\rho_a + \rho_b)/2 = 2$, blue lines represent an SEU-RI economy with belief $\beta_a = \rho_a \sigma_d = 1$, green lines represent an SEU-RI economy with belief $\rho_0 = 0$, and black lines represent an MEU-RI economy with ambiguity aversion $[\rho_a, \rho_b]$. In the right graph dashed lines represent the mean and chain-dotted lines the variance. The parameters are $\bar{d} = 100$, $\sigma_d^2 = 25$, and $\gamma = 1$.

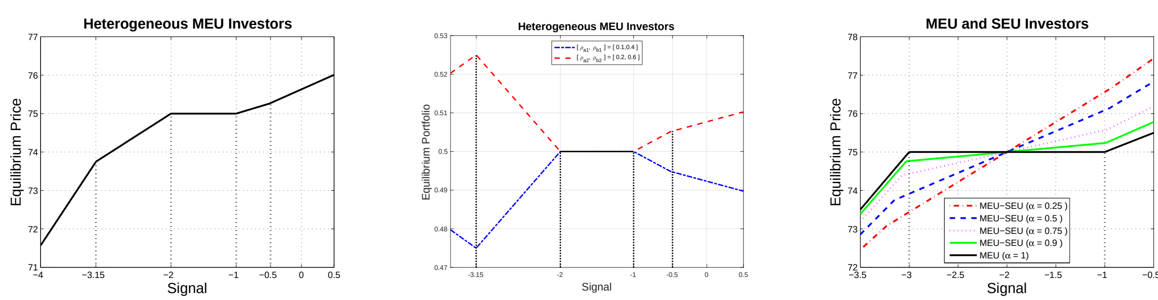

Figure IA.2. Equilibrium Price and Portfolio
The left graph shows the equilibrium price and the middle graph shows the equilibrium portfolios as a function of the signal in an economy populated with heterogenous MEU investors. The blue chain-dotted line shows the optimal portfolio of an MEU investor with ambiguity aversion $[\rho_{a1}, \rho_{b1}] = [0.1, 0.4]$ and the red dashed line shows the optimal portfolio of an MEU investor with ambiguity aversion $[\rho_{a2}, \rho_{b2}] = [0.2, 0.6]$. The right graph shows the equilibrium price as a function of the signal when there is a unit mass of investors where α denotes the fraction of MEU investors and $1 - \alpha$ denotes the fraction of SEU investors. The parameters are $\bar{d} = 100$, $\sigma_d^2 = 25$ and $\gamma_1 = \gamma_2 = 1$.

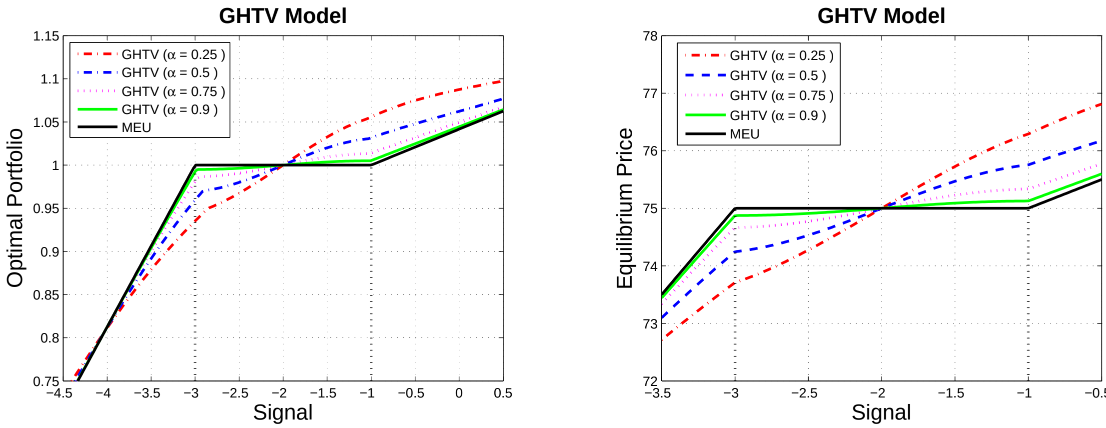

Figure IA.3. Demand and Equilibrium Price in the GHTV Model

The left graph shows the optimal portfolio as a function of the signal when $p = 75$. The right graphs show the equilibrium price of the market portfolio as a function of the signal. The black lines represent an MEU investor and the colored lines represent GHTV investors with different degree of aversion to ambiguity α. The parameters are $\rho_a = 0.2$, $\rho_b = 0.6$, $\bar{d} = 100$, $\sigma_d^2 = 25$, and $\gamma = 1$.

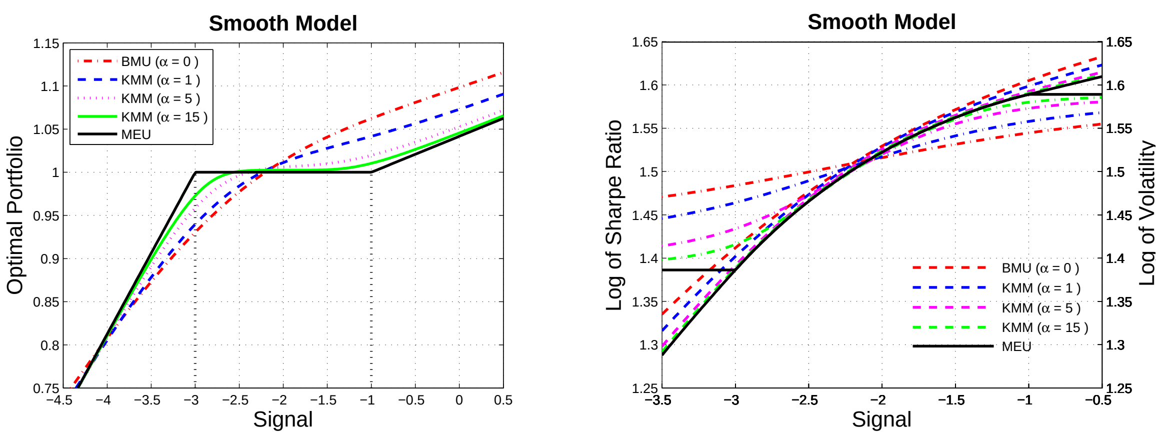

Figure IA.4. Optimal Portfolio

The left graph shows the optimal portfolio and the right graph shows the risk and ambiguity adjusted (log of the) conditional Sharpe ratio and volatility as a function of the signal. The black lines represent an MEU investor, the red lines represent a BMU investor ($\alpha = 0$), and the other lines represent KMM investors with different degrees of ambiguity aversion α. In the right graph dashed lines represent the Sharpe ratio and chain-dotted lines the volatility. The parameter ρ is uniformly distributed on the interval $[\rho_a, \rho_b] = [0.2, 0.6]$ and $\bar{d} = 100$, $p = 75$, $\sigma_d^2 = 25$, and $\gamma = 1$.

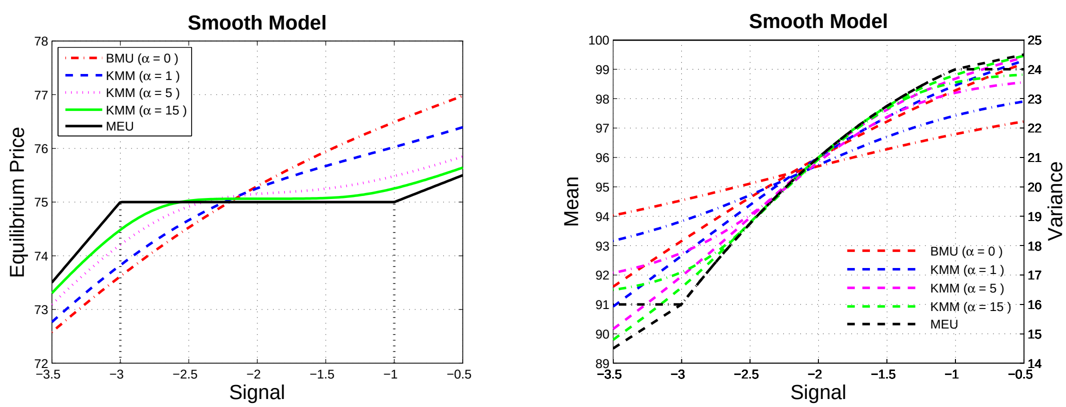

Figure IA.5. Equilibrium Price

The left graph shows the equilibrium price and the right graph shows the risk and ambiguity adjusted conditional mean and variance as a function of the signal. The black lines represent an MEU investor, the red lines represent a BMU investor ($\alpha = 0$), and the other three lines represent KMM investors with different ambiguity aversion α. In the right graph dashed lines represent the mean and chain-dotted lines the variance. The parameter ρ is uniformly distributed on the interval $[\rho_a, \rho_b]$ and $\bar{d} = 100$, $\sigma_d^2 = 25$, and $\gamma = 1$.