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• LETTER •

Output Feedback Control for Mobile Robot Systems with significant External Disturbances

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Dear Editor,
The control of nonholonomic systems, like those describing mobile robots, has attracted considerable research attention in recent decades due to its theoretical and practical importance. Brockett's theorem states that nonholonomic systems cannot be stabilized to an equilibrium point by any source of smooth or continuous state feedback. Additionally, the required accuracy of measuring the system states is often unachievable in practice. These factors introduce considerable difficulty to the problem of controlling mobile robot and other nonholonomic systems.

Several studies have addressed the output feedback stabilization of uncertain nonholonomic systems[1-3]. However, these studies require the first subsystem of the considered system to be linear and known, and the virtual control directions also need to be known. Some researchers have attempted apply output feedback control when the first subsystem has uncertain parameters. One example uses a constructive observer design to control a mobile robot model[1]. Recently, a novel adaptive output feedback controller for global stabilization of nonholonomic systems in chained form was provided for a system with unknown virtual control directions[4]. The effects of non-vanishing external disturbances were not considered in the design of these controllers, although most practical systems are affected by significant external disturbances.

Several methods are available for solving the disturbance problem[5-7]. In general, to reject the external disturbances, the extended state observer (ESO) [5] is used in active disturbance rejection control. Additionally, the disturbance observer-based control technique provides a promising approach to handle the system disturbances[6]. For the mobile robot systems, good results for stabilization and tracking control have been reported when the nonholonomic constraints are precisely known[8]. However, the presence of external disturbances often introduces dynamic constraints because of parameter uncertainties. To the best of our knowledge, very few reports have proposed output feedback for use in mobile robot systems or other nonholonomic systems that require anti-interference control.

Motivated by the above observations, this letter proposes a robust output feedback controller for use in mobile robot systems that suffer parametric uncertainties, deviation in the measured angles, and non-vanishing external disturbances. The proposed controller is designed to handle two challenges in a nonholonomic system: the unknown system parameters and the presence of non-

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vanishing external disturbance. First, these challenges are overcome by generalizing the disturbance as an extended state. Second, an ESO is constructed with the gain from an off-line time-varying Riccati matrix differential equation to estimate the unmeasurable states. These two strategies yield a controller that applies time-varying output feedback. The proposed robust controller drives the system states to the origin asymptotically.

In the presence of parametric uncertainties and non-vanishing external disturbances, the dynamics of a unicycle-type mobile robot with angle measurement errors can be described as

$$\begin{aligned} \dot{x}_c &= p_1^* v cos(\theta+\varepsilon) + c_0 x_c, \\ \dot{y}_c &= p_1^* v sin(\theta+\varepsilon), \\ \dot{\theta} &= p_2^* \omega + d(t) \end{aligned} \tag{1}$$

where $c_0 x_c$ is the horizontal drift, p_1^* and p_2^* are bounded unknown positive parameters determined by the radius of the rear wheels, $d(t)$ is the time-varying external disturbance, and ε is a small bias orientation. This letter assumes that $d(t)$ can be decomposed into $d(t) = d_1(t)d_2(t)$, where $d_1(t) \neq 0$ is known and bounded, and $d_2(t)$ is unknown but $\dot{d}_2(t) \in l_2$, i.e., $d(t)$ is a time-varying disturbance that is relatively fast. The control problem is steering the robot toward the origin while not knowing the values of unknown parameters, the angle measurement error, or the direction and magnitude of an external disturbance. Indeed, if the external interference, angle deviation, and nonlinear drift are ignored, and all state variables are measurable, the model (1) is a classical nonholonomic system. The second-order approximation of system (1) near $\theta = 0$ is given by[1] as follows

$$\begin{aligned} \dot{x}_l &= p_1^* \left(1 - \frac{\varepsilon^2}{2}\right) v + c_0 x_l, \\ \dot{y}_l &= p_1^*(\theta_l v + \varepsilon v), \\ \dot{\theta}_l &= p_2^* \omega + d(t). \end{aligned} \tag{2}$$

If the coordinates are transformed as follows, $x_0 = x_l,\ x_1 = y_l,\ x_2 = (\theta_l + \varepsilon),\ u = \omega,\ u_0 = v$, the system can be rewritten as

$$\begin{aligned} \dot{x}_0 &= p_1^* \left(1 - \frac{\varepsilon^2}{2}\right) u_0 + c_0 x_0, \\ \dot{x}_1 &= p_1^* x_2 u_0, \\ \dot{x}_2 &= p_2^* u + d(t). \end{aligned} \tag{3}$$

Evidently, this system is a third-order nonholonomic system in chained form, and if the perturbation ε is unknown, the state x_2 is not measurable.

To handle the unknown parameters p_1^* and c_0, we can take u_0 for the first subsystem of (3) as

$$\dot{u}_0 = -k_1 sgn(g_0) x_0 - k_2 u_0, \tag{4}$$

where $g_0 = p_1^* \left(1 - \varepsilon^2/2\right)$ and the gains k_i's are assigned such that the equation

$$s^2 + (k_2 - c_0)s + k_1|g_0| - k_2 c_0 = 0, \tag{5}$$

has two negative real roots. As g_0 and c_0 are bounded, values of k_1 and k_2 that satisfy equation (5) are determined easily (if c_0 is negative, we can simply choose $k_1 = 0$ and $k_2 > 0$, i.e., neither the sign of g_0 nor the bounds of g_0 and c_0 need to be known) The aforementioned design guarantees that x_0 and u_0 are convergent to zero exponentially and $\dot{u}_0/u_0$ is measurable and bounded for small perturbations ε.

To construct the Kalman filter observer for subsystem x, we introduce another transformation $\zeta_1 = (1/p_1^* p_2^*) x_1$, $\zeta_2 = (1/p_2^*) x_2$ and find that

$$\begin{aligned} \dot{\zeta}_1 &= \zeta_2 u_0, \\ \dot{\zeta}_2 &= u + (1/p_2^*) d(t). \end{aligned} \tag{6}$$

Using the change of variables for a nonholonomic system $z_1 = \zeta_1/u_0,\ z_2 = \zeta_2$ and further letting $z_3 = \bar{d}(t) = (1/p_2^*) d_2(t)$, we have the following dynamics

$$\begin{aligned} \dot{z}_1 &= -\frac{\dot{u}_0}{u_0} z_1 + z_2, \\ \dot{z}_2 &= u + d_1(t) z_3, \\ \dot{z}_3 &= \dot{\bar{d}}(t) = h(t). \end{aligned} \tag{7}$$

Notbly z_i is not available in the design of controller because the parameters p_1^* and p_2^* are unknown. Conversely, $p_1^* p_2^* z_1$ is an available signal in the controller design. Let $z = [z_1, z_2, z_3]^T$, the dynamics can be rewritten into the following compact form

$$\dot{z} = A(t) z + C_2 u + C_3 h(t), \tag{8}$$

where $A = [-\dot{u}_0/u_0, 1, 0; 0, 0, d_1(t); 0, 0, 0]$, $C_2 = [0, 1, 0]^T$, $C_3 = [0, 0, 1]^T$. The extended state Kalman observer can be given as follows[9]

$$\dot{\widehat{z}} = A(t)\widehat{z} + C_2 u - P C_1 C_1^T \widehat{z}, \tag{9}$$

where the observer gain $P(t)$ is updated by the following time-varying Riccati differential equation

$$\begin{cases} \dot{P} = P A^T(t) + A(t) P - P C_1 C_1^T P + I, \\ P(0) = P_0 > 0, \quad C_1 = [1, 0, 0]^T. \end{cases} \tag{10}$$

This Riccati differential equation is solvable. The observation error variables are defined as $e_i = z_i - \widehat{z}_i,\ i = 1, 2, 3$, so the error dynamics satisfies

$$\dot{e} = \left(A(t) - P C_1 C_1^T\right) e + P C_1 z_1 + C_3 h(t). \tag{11}$$

Let $\sigma_1 = \widehat{z}_2 - \alpha_1$, we get

$$\dot{z}_1 = -\frac{\dot{u}_0}{u_0}z_1 + z_2$$
$$-\frac{\dot{u}_0}{u_0}z_1 + e_2 + \sigma_1 + \alpha_1, \quad (12)$$
$$\dot{\widehat{z}}_2 = d_1(t)\widehat{z}_3 - p_{21}(t)\widehat{z}_2 + u, \quad (13)$$
$$\dot{\widehat{z}}_3 = -p_{31}(t)\widehat{z}_3, \quad (14)$$

where $p_{21}(t)$ and $p_{31}(t)$ are the corresponding elements of the matrix $P(t)$.

Although the term z_1 is unknown, the term $p_1^*p_2^*z_1$ is measurable. Applying the backstepping method to the equations (12)–(14), we determined that there exist proper (large) positive constants K, L_1 and L_2 such that the controller defined by

$$\alpha_1 = -Kp_1^*p_2^*z_1, \quad (15)$$
$$u = -d_1(t)\widehat{z}_3 + p_{21}(t)\widehat{z}_2 + Kp_1^*p_2^*\frac{\dot{u}_0}{u_0}z_1$$
$$-L_1 sgn(\sigma_1)K|p_1^*p_2^*z_1| - L_2\sigma_1. \quad (16)$$

can drive the states z_1, e, σ_1 to zero and z_3 and $\widehat{z}_3$ are bounded. Furthermore, it follows that the states ζ_1, ζ_2 and x_1, x_2 converge to zero. This convergence also ensures that the (x_l, y_l, θ_l) converges to the equilibrium $(0, 0, -\varepsilon)$.

Theorem 1. For system (2), if the non-vanishing disturbance $d(t)$ satisfies $d(t) = d_1(t)d_2(t)$, where $d_1(t) \neq 0$ is known and bounded, $d_2(t)$ is unknown, but $\dot{d}_2(t) \in l_2$, the unknown parameters p_1^* and p_2^* are positive and bounded, and the controller is set as (15)–(16), then all signals in the closed-loop system are bounded and (x_l, y_l, θ_l) converges to the equilibrium $(0, 0, -\varepsilon)$.

Numerical results. We assumer that $p_i^* \in [0.9, 1]$, $\varepsilon \in [0, 1]$ and $d(t) = 0.5e^{-t} - 1$. To illustrate the response of the closed-loop system, Figure 1 shows the results of a simulation with the system and design parameters set to $p_i^* = 1$, $\varepsilon = 0.1$, $c_0 = -0.2$, $k_1 = 0$, $k_2 = 1$, $K = 3$, $L_1 = 4$, $L_2 = 5$ and the initial conditions as $(x_l(0), y_l(0),\ \theta_l(0) + \varepsilon)$=$(1, 1, -1)$, $\widehat{z}(0) = [0, -1, 0]^T$, $P(0) = diag\{0.1, 0.2, 0.3\}$,

Notably the non-vanishing external disturbance $d(t) = 0.5e^{-t} - 1$ will make exiting control methods inapplicable to this mobile robot system even if all state variables are measurable. However, the aforementioned controller is effective with our combination of the ESO and dynamic observer techniques. The simulation results show the necessity of considering non-vanishing external disturbances in the control design as well as the effectiveness of the proposed method.

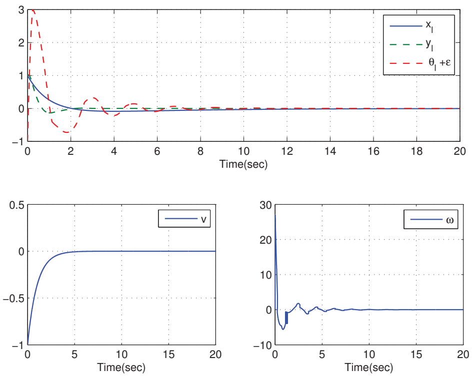

Figure 1 Responses of the closed-loop system.

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