Essays on labour market segmentation in Japan

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"Done is better than perfect"

Mark Zuckerberg, Facebook CEO

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Summary

Properly assessing labour market segmentation and its outcomes are essential for the debate of the policy related to atypical work. This thesis document the labour market segmentation in Japan, construct the model, and do counterfactual simulation to contribute to the policy making.

The first chapter documented a rise in atypical jobs and labour market segmentation in Japan. The main findings are the follows. Firstly, there is labour market segmentation between atypical and standard workers in Japan. Secondly, the institutional setting largely affects labour market outcomes. Thirdly, there is a small relationship between educational attainment. The model constructed in the second and the third chapter reflect the analyses of the first chapter.

The second chapter constructs a labour market matching model with heterogeneous jobs and workers that incorporates on-the-job search and different turnover rates. It extends the model of Dolado et al. (2009) by adding different turnover rates to heterogeneous jobs. In line with the paper of Dolado, we prove that the introduction of OTJ search enhances the likelihood of having

an equilibrium with cross-skill matching even if the model contains different turnover rates to heterogeneous jobs. The quantitative implications give the prediction that the unemployment rate of low-productivity workers increases, inequality grows, and welfare decreases as skill-biased technical change proceeds.

The third chapter constructs a search matching model, which incorporates endogenous labour market participation into the segmented labour market by combining the models of Albrecht and Vroman (2002) and Garibaldi and Wasmer (2005) to investigate how to raise the labour force participation rate in the segmented labour market. The quantitative implications predict that the decrease of the value of home production, such as the rapid spread of autonomous robotic vacuum cleaners and cheap housekeeping services, leads an increase in employment.

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Chapter 1

Assessing labour market segmentation: Evidence from Japan

Abstract

Properly assessing labour market segmentation and its outcomes are essential for the debate of the policy related to atypical work[1]. This paper documented a rise in atypical jobs and labour market segmentation in Japan. The main findings are the follows. Firstly, there is labour market segmentation between atypical and standard workers in Japan. Secondly, the institutional setting largely affects labour market outcomes. Thirdly, there is a small relationship between educational attainment and employment types from the

[1]Atypical work refers to employment relationships that do not conform to the standard or "typical" model of full-time, direct employment, and permanent contract.

institutional reason in Japan.

Key Words: Segmented Labor Markets; Japanese Economy; Machine Learning

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1.1 Introduction

1.1.1 Motivation and Summary

The proportion of atypical workers among all workers has been increasing steadily in OECD countries. The combination of strict legal constraints on the termination of regular workers and weak constraints on the creation of atypical works is a likely cause of this increase (Cahuc et al., 2016). Theoretically, high employment protection of permanent contracts will increase labour-market segmentation by encouraging employers to resort to less-secure employment forms where possible (OECD, 2013). Compared with standard work, atypical work has a higher turnover rate due to its character, so atypical workers play an important role in labour market flexibility (Booth, 2002). However, instead of enhancing labour market flexibility, it seems that much concern remain about atypical works.

Properly assessing labour market segmentation and its outcomes are essential for the debate of the policy related to atypical work. Different institutional settings in different countries may lead to different labour markets and outcomes. There has been little researches on this except in regards to the European labour market. Hence, this paper will document the rise of atypical jobs and labour market segmentation in Japan. To do so, we will use data on stock, flow and some estimations. The model constructed in the second and the third chapter reflect the analyses of the first chapter. Japanese data are used, as Japan has the fastest pace of declining population, and the labour force is decreasing. Accumulating knowledge about labour market segmentation in this setting will be useful for discussion.

The main findings are the follows. Firstly, there is labour market segmentation between atypical and standard workers in Japan. The transition between atypical workers and standard workers is low in the short run and the long run. Secondly, the institutional setting largely affects labour market outcomes. Firms and workers correspond to Employment Protection Legislation (EPL) in Japan. This may explain the difference in duration of work and wages. Thirdly, there is a small relationship between educational attainment and employment types from the institutional reason in Japan.

The structure of this chapter is as follows: Section 1.1.2 provides the related literature. Section 1.2 documents the institutional background and Section 1.3 states the data sources. Section 1.4 is the main part of this paper and analyses the Japanese labour market. Section 1.5 provides a conclusion of this chapter.

1.1.2 Related Literature

Research on labour market segmentation and penetration rate of transition has been conducted via theoretical and empirical perspectives. Research by Stoikov et. al in 1972 is important for its theory of the two-tier labour market. They observed a difference between the primary labour market and the secondary labour market, and the main differences are (a) the hourly wage of the secondary market is lower than the primary market, (b) the human capital accumulation of the secondary market is lower than the primary market, and (c) minorities such as blacks and females are particularly distributed in the secondary market.

However, the theory of labour market segmentation and the empirical analysis lost their sense of direction. The straying of this theory comes from the fact that the theory of Doeringer and Piore does not align with the neoclassical economics of free competition and the unity of the market.

Also, it is not an easy task to show labour market segmentation empirically. Dickens and Lang (1985) used a switching model, showing results consistent with the theory of Doeringer and Piore. The primary labour market gives workers a return on educational attainment and experience, but the secondary labour market does not. Moreover, the probability that white people are assigned to the primary labour market is significantly high. However, Sakamoto and Chen (1991), in a similar analysis, found different results. There are a lot of papers which summarise the characteristics of non-regular workers and non-regular work. For example, Cazes and Laiglesia (2015) assessed labour market segmentation in some OECD countries, although they did not include Japan's case.

In Japan, the first paper which empirically showed the two-tier labour market in Japan is by Ishikawa and Dejima (1994). They used a switching model and showed that the existence of multiple tracks is most likely even if they have the same ability and motivation to work. Also, a lot of research summarised the characteristics of standard and atypical work and analyses the difficulty of the transition between standard workers and atypical workers (Ishida, 2005; Sato, 2008; Shikata, 2011).

The contribution of this research is empirical research concerning the above theory of Doeringer and Piore in using Japanese data. We assessed the labour market segmentation from various points. Little research has assessed the labour market segmentation of Japan in a comprehensive way. In particular, using a new method, clustering analysis, we indicate the existence of a segmented labour market in Japan.

1.2 Institutional Background

This section briefly explains the institutions of Japan related with standard jobs and atypical jobs. All elements in the world influence each other. Institutions influence the behaviour of workers and firms, while the behaviour of workers and firms sometimes leads to change in institutions. Economists usually regard institutions as customarily identified with the 'rules of the game' (e.g., Schout and North, 1991). That is, workers and firms behave to maximise their utility given the institutions of a society. In this chapter, I analyse the labour market with standard and atypical workers. Institutions should largely influence this labour market. The following describes the types of employment in Japan, the strictness of Employment Protection Legislation (EPL) compared with the other countries, and the current and past situation of EPL in Japan.

Japanese workers are mainly classified into two types: standard workers and atypical workers. We call standard workers as regular workers and atypical workers as non-regular workers. In the followings, we use the word "regular workers" and "non-regular workers", respectively. Non-regular workers are mainly classified into three types: TAWs[2], contract workers and part-time workers. The classification used by the Japanese statistics are based on how the workers are called at work. As a result, the classification sometimes leaves ambiguity. Regular workers also generally have three characteristics: full-time, direct employment, and permanent contract in general. Direct employment means a worker is directly hired by a firm. Non-regular workers do not have all of these characteristics. A temporary agency worker is not employed directly. He/She has a contract with an agency but works for a firm temporarily. A contract worker has a fixed-term contract and direct employment. A part-time worker usually works part-time.

[2]TAWs refer to temporary agency workers who are employed by temporary work agency, and then hired out to perform his/her work at the user company.

The Employment Protection Indicators published by OECD are used to describe how strict the EPL of Japan is among the developed countries. Figure 1.1 shows the indicators of the strictness of EPL in some OECD countries. These data are from OECD statistics. The left figure shows the strictness of the individual dismissals among regular workers. This measures the procedures and costs involved in dismissing individuals or groups of workers with regular contracts. The right figure shows the strictness of regulations of the use of non-regular workers. This measures the procedures involved in hiring workers on fixed-term or temporary work agency contracts. In the vertical axis, a higher score means stricter regulation. This means France and Spain find it difficult to dismiss regular workers, but it is also difficult to use non-regular workers. From these figures, we can see the EPL in Japan is stricter than in the UK and US, and weaker than France and Spain. In practice, EPL induces labour market segmentation.

The outstanding feature of Japanese EPL for regular workers which is worth mentioning is the definition of justified or unfair dismissal. It is regarded as unfair if firms dismiss workers without 'reasonable cause'. Redundancy dismissals require business reasons for reducing the number of staff, efforts to avoid dismissal, reasonableness of selection criteria and procedures. This implies that firms find it easy to fire non-regular workers in recession.

The Civil Code and the Labor Standards Act of Japan only provide that firms can freely dismiss workers if they provide at least 30 days' advance notice. However, the court historically has judged that a dismissal without reasonable cause is invalid (Employment Dismissal Regulations of Japan). In 2007, the amended Labor Contract Act clearly stipulated this Employment Dismissal Regulations of Japan. This reflects the indicator of Japan having dropped in 2007 in the left figure.

The main features of Japanese EPL for non-regular workers is the allowance of the usage and the definition of justified or unfair dismissal. Historically, to use temporary agency workers were not allowed until 1985. This was in order to prevent intermediate exploitation because of the ambiguities concerning the responsibilities of the employer. In spite of the prohibition, since the 1970s, the number of temporary agency workers has increased. In order to abide this by law, government has decided to allow the use of TAWs in 1986 to regulate the use of TAWs properly (Recruit Works Institutes, 2015). Since then, the system of TAW has consistently moved in the direction of the relaxation of regulations. In 1996 and 1999, the coverage of temporary agency works was largely extended. On the other hand, the use of contract jobs and part-time jobs are never banned to use before.

In terms of dismissals, firms do not need to extend fixed-term contracts at the end of the term of a contract if they do not want to. While it is basically illegal to fire fixed-term workers during the term of a contract, there is no criminal penalty or administrative sanctions.

1.3 Data Sources

Different data sources which complement each other are used to characterise the segmented labour market in this chapter. The panel survey with a large number of samples is not available in Japan, so it is necessary to use multiple data sources. I use four data sources: the Employment Status Survey (ESS), Labour Force Survey (LFS), Longitudinal Survey of Adults in the 21st Century, and General Survey on Diversified Types of Employment.

The main data source is the ESS. The ESS was created by the Statistics Bureau in the Ministry of Internal Affairs and Communications (MIAC), provided by the National Statistics Center. This is cross-sectional data. It started in 1956 and has been conducted every five years. In this paper, I use the individual data from 1992, 1997 and 2002 only, which was provided by the National Statistics Center. The questions about current and past work-related activities such as labour force status, employment type, current and previous employment duration and educational attainment are asked in detail. The sample of the survey is approximately one million persons aged 15 years or over in approximately 0.5 million families. This survey captures one-thirteenth in the Japanese population. The reliability of these statistics is high, because these are national fundamental statistics based on the Statistics Act (Act No.53, 2007). Participants must respond to the national fundamental statistics. There is a penalty for refusal or false information. Areas for taking sample families are randomly selected from about thirty thousand areas which the country is divided into. Then, from these areas, about one million persons in 0.5 million families are selected randomly. Questionnaires are directly distributed to and collected from their houses by investigators.

The LFS has been conducted by the Statistics Bureau in the MIAC every month since July 1947. About 40 thousand households in about 2,900 enumeration districts are surveyed. Questions about current work-related activities are briefly posed to the members aged 15 years old and over (about 1 million persons in total) in those households, which is about one out of thirteen of the Japanese population. The enumerator visits all the households in the sample to ask them to fill out the questionnaires and collects it a few weeks later. These are national fundamental statistics. The survey is conducted as of the last day of each month, in the same two months in two successive years, which we utilize for the flow analysis in my research.

In addition to the above, I use the aggregation data of the Longitudinal Survey of Adults in the 21st Century and the aggregation data of the General Survey on Diversified Types of Employment. The Longitudinal Survey of Adults in the 21st Century are from MHLW. People aged 20-34 in 2002 have been followed every year to investigate their social, demographic and economic situations. The General Survey on Diversified Types of Employment in 2010 is conducted by MHLW. About 17 thousand offices are asked questions about employment-related activities and consciousness of all types of employment.

These data sources compensate for each other. The Employment Status Survey is the micro data with the large sample. It is suitable to use for in estimation, but it is difficult to study long-term trends because the survey is conducted every five years. The Labour Force Survey is monthly data. It is good for studying trend and monthly transitions of employment status. The Longitudinal Survey of Adults in the 21st Century is aggregated panel data. This has many question items, including information on school dropouts. The General Survey on Diversified Types of Employment is the aggregate cross-sectional survey for firms.

1.4 Analysis of the Japanese labour market

1.4.1 A rise in non-regular jobs

This part is the main part of this chapter and presents the analysis of the labour market in Japan. The following results are divided into five parts: a rise in non-regular jobs, the existence of labour market segmentation, the characteristics of non-regular workers and regular workers and the reason to work for a non-regular job, the reason to hire non-regular workers, and the characteristics of regular jobs and non-regular jobs.

Regarding a rise in non-regular jobs, the percentage of non-regular employees among total employees has been increasing in many developed countries such as certain European countries (France, Italy, the Netherlands, Portugal and Spain). It has been repeatedly argued that the rapid increase in non-regular workers results from the combination of stringent legal constraints on the termination of permanent jobs and of weak constraints on the creation of temporary jobs (Cahuc, et al., 2016). The number of non-regular workers has also been increasing steadily in Japan. The percentage of non-regular employees among total employees has increased from 14.4% in 1984 to 35.3% in 2016 according to the Labour Force Survey.

We overview the structural change in the Japanese labour market in Figure 1.2. This graph shows the number of employees, the self-employed, the unemployed, and people not in the labour force. The number of regular workers has been almost stable for these thirty years. It decreased in the late 1990s and the early 2000s, the so-called 'lost decade'. However, it has been increasing in recent years because of the shortage of labour given the shrinking of the Japanese labour force.

The number of each type of employment in non-regular employment is growing. The number of the self-employed is steadily decreasing. The Ministry of Health, Labour and Welfare (2015) points out that the self-employed decreased in number and that non-regular employment offers new jobs to people who were self-employed. The number of temporary agency workers also shows a sharp decrease by about 40% after the Great Recession. This is because firms preferred to fire temporary agency workers during the recession.

1.4.2 Existence of labour market segmentation

According to CTI Reviews (2016), a labour market is seen as segmented if it consists of various sub-groups with little or no crossover capability. The following three sections show there are sub-groups in the Japanese labour market, and the sub-groups are rather separate in the short run and the long run. First, cluster analysis is applied to investigate what kind of sub-groups the labour market is divided into. Then, the existence of labour market segmentation is indicated from a macro perspective. The transfer between regular employment and non-regular employment is observed in the short run and in the long run. Finally, a probit analysis of the determinant of transferring from non-regular employment to regular employment is shown.

Theoretical models explain labour market segmentation as a result of labour market policies, labour market institutions or the payment of efficiency wages in the formal sector (Fields, 2005). When EPL for regular employment is strong, dismissal becomes difficult once a firm employs regular workers. Thus, there is incentive for firms to use a non-regular employment contract with workers. At the same time, the incentive for firms to shift from non-regular employment to regular employment is low. Thus, the transition between from non-regular jobs to regular jobs becomes small.

OECD (2002) shows that, in European countries when EPL indicators for regular jobs are strong, the proportion of non-regular jobs among all jobs is high and the transition between non-regular jobs and regular jobs is low. According to OECD (2002), EPL for regular jobs are relatively strong and

EPL for non-regular jobs are relatively weak in Japan, so it is possible that the transition between non-regular jobs and regular jobs is low.

The following section analyses whether the data are consistent with the theory.

Classification of the labour market in Japan

Cluster analysis is used to examine what kind of sub-groups the Japanese labour market is divided into. Cluster analysis is an analysis in which a collection of data is automatically divided into several groups without external criteria according to the similarity of data. This partition of the observed units into clusters is a faster, more intuitive procedure compared to the examination of a list of regression parameters (Porro et. al, 2004). The basic approaches are hierarchical clustering and k-means clustering (Řezanková, 2014). Hierarchical clustering is often portrayed as the better-quality clustering approach, but it is limited because of its quadratic time complexity. In contrast, K-means and its variants have a time complexity that is linear in the number of documents, but are thought to produce inferior clusters (Steinbach, 2000). As a general conclusion, a k-means algorithm is good for large datasets and hierarchical is good for small datasets (Kaur and Kaur, 2013). One type of hierarchical clustering, agglomerative hierarchical clustering, is applied here. Agglomerative hierarchical clustering is probably the most applied method in economics wherein each observation starts in its own cluster, and pairs of clusters are merged as one moves up the hierarchy to build a hierarchy of clusters. Ward's method is used for measuring the distances between clusters, and a simple matching similarity coefficient is used for measuring the distances between samples. Ward's method is an agglomerative hierarchical clustering procedure, where the criterion for choosing the pair of clusters to merge at each step is based on the optimal value of an objective function of the error sum of squares, and is less susceptible to noise and outliers. Simple matching similarity coefficient works with binary data and is the proportion of matches between the two samples.

The samples used are workers who are 15-54 year old and are not studying per the individual data of Employment Status Survey in 2002 by Ministry of Internal Affairs and Communications. It should be pointed out that older workers who are 55-year-old are not included in the sample. It is expected that older worker are regular workers who are almost retired or non-regular workers who have just retired from regular work. They are excluded to obtain a clearer result of cluster analysis. The variables in the samples for clustering are the binary variables of gender, age, education, occupation, place of living, marital status, work amount, and employment type. The binary variable of gender takes 0 if the sample is female, and it takes 1 if it is male. The binary variable of age takes 0 if the sample is 34 years old or less, and it takes 1 if it is 35 years old or more. The binary variable of education takes 0 if the educational background of the sample is high school graduate or less, and it is 2-year vocational/technical college or more. The binary variable of occupation takes 0 if their work is office work or customer service work, and it takes 1 if their work is manufacturing process work. The binary variable of place of living takes 0 if he/she does not live in the three major metropolitan areas, and it takes 1 if he/she lives in the three major metropolitan areas. The binary variable of marital status takes 0 if he/she is unmarried, and it takes 1 if he/she is married. The binary variable of work amount takes 0 if he/she works part-time, and it takes 1 if he/she works full-time. The binary variable of employment type takes 0 if he/she is a non-regular worker, and it takes 1 if he/she is a regular worker. The number of clusters in cluster analysis is five, since five clusters seem to be the most proper after changing the number of clusters many times and checking the outcomes.

Table 1.1 shows the result of an average of the values of the binary variables of the samples in each cluster. Figure 1.3 is the dendrogram of the result. This analysis reveals that workers in the Japanese labour market are divided into three groups. The first group mainly consists of regular workers and clusters no.1 - no.3 correspond to this group. The second group mainly consists of non-regular workers and cluster no.5 corresponds to this group. The third group includes non-regular workers and regular workers and cluster no.4 corresponds to this group. The details of each cluster follow. The cluster no.1 mainly consists workers with the characteristics such as old, relatively high education, office and service work, married and full-time. The cluster no.2 mainly consists of workers with characteristics such as male, low education and full-time. The cluster no.3 mainly consists of workers with characteristics such as male, low education, manufacturing and full-time. The cluster no.4

consists of regular workers and non-regular workers. It mainly consists of regular and non-regular workers with characteristics such as young, relatively high education, office and service and full-time. The cluster no.5 consists of non-regular workers. It mainly consists of non-regular workers with characteristics such as non-regular workers, old, low education, married, part-time and females. They seem to be a secondary earner, doing housework and child-rearing at the same time.

Segmented labour market in the short run

This section analyses the transfer between non-regular workers and regular workers in the short run. Table 1.2 is the rate to transfer from one employment type to another employment type. The value of each month is calculated as (the flow from the state A to the state B)/(the stock of the previous month in the state A). The average rate to transfer is the average of those values between January 2015 and March 2017.

Over 90% of people in regular work, part-time work, and non-labour force stayed in the same employment type while about 80% of people in temporary agency works and contract works remained in the same employment type. In addition, 2-4% of part-time workers and TAWs move to regular work every month while 7% of contract workers find regular works every month. Thus, regular workers stay regular workers and non-regular workers remain non-regular workers. The limitation of this analysis is that we cannot see the change of job within the same employment types, but the rate of change of job for TAWs and contract workers is still high. Contract workers move to regular work more than the other employment types of non-regular workers. This implies that the possibility that firms make contract works play a role in screening.

Segmented labour market in the long run

This section analyses the transfer between regular work and non-regular work in the long run. We illustrate the relationship between first job and current job by current age-group. The limitation of this analysis is that we cannot grasp whether the person whose first job and current job is non-regular works are always non-regular workers or goes back and forth between non-regular work and regular work. We only know the relationship between first job and current job. This is because the panel data with a large sample are not available in Japan.

Figure 1.4 suggests that employment type of first job greatly affects the employment type of current job. The percentage of persons whose current job is regular work among persons whose first job is regular work is about 80% in the 25-34 age group and about 70% in the 35-54 age group. The percentage of persons whose current job is non-regular work among persons whose first job is regular work is about 10% in the 25-34 age group and about 20% in the 35-54 age group. On the other hand, the percentage of persons whose current job is non-regular work among persons whose first job is non-regular work is about 70% in the 25-44 age group and about 80% in the 45-54 age group. The percentage of persons whose current job is regular work among persons whose first job is non-regular work is about 30% in the 25-44 age group and about 10% in the 45-54 age group.

The macroeconomic analysis mentioned above reveals that the labour market in Japan is segmented in the short run, while the opportunity to transfer from non-regular work to regular work every month is not necessarily low. However, employment type of the first job for most people is the same as that of the current job.

According to the Employment Status Survey of 2012, about 70% of males and about 40% of females changed jobs once or never in the age group from the 30s to the mid-50s. It seems to be relatively easy to move to regular employment in the short run, but a certain number of workers may move back and forth between regular work and non-regular work. To sum up, it can be speculated that non-regular work in the Japanese labour market are 'dead-end' jobs, rather than 'stepping-stone' jobs.

Probit Analysis of the determinant of transferring

The previous results suggest the possibility that non-regular work is not a stepping stone to regular work but a dead end from a macro viewpoint. For further information, the ease of transfer from non-regular work to regular work should have a different rate from a micro viewpoint. We examine who find it easy to move to regular work and who tends to stay in non-regular work for the next step. When the explained variable changes qualitatively, it is typical to use not a linear model but a non-linear model which consider an explained variable to be distributed with particular probability distribution. One of the methods is probit analysis. We estimate the probit model to investigate what factors determine regular workers as people who were working as non-regular workers.

In the following, we summarise the marginal effect of the explaining variables, what percentage the probability of transition from a non-regular work to a regular work increases when the explaining variable changes from 0 to 1, and the z-value in Table 1.3. The sample includes those individuals who are 15-54 years old, and whose previous employment type was non-regular workers, and the data are from the Employment Status Survey of 2002. The value of dependent variable is 1 if the previous employment type was a non-regular job and the current one is a regular job, and the value is 0 if the previous employment type was a non-regular job and the current one is not a regular job. The explanatory variables are the gender dummy, the age dummy, the marriage dummy, the area dummy, the previous company size dummy, and the education dummy. The probability of becoming a regular worker from a non-regular worker is high if the individual is a man, previous job is a contract job, is aged 30-44, lives in rural area, is unmarried, and/or has a higher level of educational attainment.

The result is that males are more likely to move than females, and the difference is large. In terms of employment types, previous contract workers are most likely to move, the next is TAWs, and part-time workers comes last.

In terms of age, the chances of moving to a regular work decrease as they are older if they are over 25 years old. In terms of region, the probability of transition is higher for those who do not live in the three major metropolitan areas. Although it is not significant, non-regular workers in the small company have a higher probability to be a regular worker in terms of company size. There is a possibility that a region or company which is short of hands has more incentive to keep a labour contract with workers permanently. The person who is not married has the higher possibility of transition to regular work. In terms of education, the higher the educational attainment, the more likely they transfer to regular work.

Section 1.4.2 showed that the labour market is divided into subgroups, and the transitions between the groups are disturbed. This is consistent with the theory mentioned in the section. Then, we revealed the characteristics which cause people to move from non-regular work to regular work more easily.

1.4.3 Characteristics of non-regular workers and regular workers and the reason to work in a non-regular job

We look at a number of characteristics of non-regular workers and regular workers in this chapter. First, the characteristics of persons with regular work and non-regular work is illustrated. Age-group, gender, and educational attainment by employment type which show the remarkable characteristics are shown. Then, the reason to work for a non-regular work is analysed by age group and gender in order to understand the characteristics of non-regular workers.

Composition of employment type by gender and age group

The characteristics of employment types by age group and gender for workers in Japan are reported in this section. Figure 1.5 shows the composition of employment types by age group and gender. The aggregate data from Employment Status Survey of 2012 are used. The sample is males and females who are 15 years old or over.

This figure indicates that regular workers include a large number of prime-age males. About 70% of prime-age males work in regular work. On the other hand, about 30-40% of prime-age females work in regular work. Non-regular workers mostly consist of females, old people and young people.

Reasons for being a non-regular worker

Next, the reason to work for a non-regular work is analysed by age group and gender in order to understand the characteristics of non-regular workers. The figure shows the main reasons for being a non-regular worker by gender, age, and employment type. The data are the aggregate data from the Labour Force Survey in 2016. The sample is males and females of 15 years or over in non-regular work. The reason could be chosen from three categories: unable to find a regular job, did not want to be a regular worker, or other reasons.

Other reasons include, for example, a short commuting time.

People who choose TAWs and contract work tend to choose this employment type involuntarily (Figure 1.6). Particularly, it applies to young and middle-aged males. About 60% of them could not find a regular job. Part-time jobs have a different tendency from other employment types. Many of them choose this employment type voluntarily. Particularly, it applies to females. Conditional on the current social system of Japan, many females want to work flexibly as a second earner along with doing the house work and raising a child. The increase of labour participation ratio for females in recent years could reflect that their preference to work flexibly as non-regular workers. Old people relatively prefer to work flexibly. They want to earn money, but they do not want to work heavily as regular workers.

Education of non-regular workers

We examined the educational attainment of non-regular workers regarding education among predetermined characteristics. Figure 1.7 shows the educational attainment by employment type. The sample is males and females aged 15-54 who are not studying according to the Employment Status Survey of 2007.

Part-time workers tend to have a lower educational attainment, and regular workers tend to have a higher educational attainment. About 30, 40, 40 and 50% of part-time workers, contract workers, TAWs, and regular workers have the educational attainment of two-year college graduate or more, respectively. However, there is not a very big difference. Even if we control age and gender, the tendency is the same.

The figure below shows employment type of first job for 2011 graduates and 1983 graduates by educational attainment. Clearly, the percentage of the individual whose first job is a non-regular job increased in all educational attainments. In 1983, it was a few percent among high school graduates or more. In 2013, it was about 30%. The percent is not very different among high school graduates, two-year college graduates, and four-year university graduates.

Thus, educational attainment and job types are not related very much in Japan, as it was suggested in educational sociology. The reasons could be (a) employment placement service in high school, (b) ease of enrolling university and (c) good salary of non-regular jobs at a young age. Firstly, there is a recommendation-based recruitment system specific to Japan for regular work every year. Secondly, according to the School Basic Survey from the Ministry of Education, Culture, Sports, Science and Technology, the university entrance rate was 25% in 1990, and it increased to 50% in 2013. A person who wants to graduate from university can do so if they are not picky in choosing a university, and it result in a decline in the average quality of university graduates. Thirdly, the salary at a young age is almost the same among all employment types as explained in section 1.4.5.

A more detailed classification of educational attainment is applied to the result of labour outcome. Figure 1.8 has a classification of not only graduates but also dropouts. The figures above show the percentage of non-regular workers among workers. The figures below show the percentage of unemployment rate. The left figure is male, and the right figure is female. The sample is male and females aged 20-29, and the data used are the aggregated data of the Longitudinal Survey of Middle-aged and Elderly Persons of 2012. The first three groups include high school graduates, two-year college graduates, and four-year university graduates. The latter four groups include high school dropouts, two-year college dropouts, and four-year university dropouts, and junior high school graduates. In Japan, the later four are the four orange groups occupying about only 10% of the population. The percentage of non-regular workers among the first three groups is clearly high. In addition, the unemployment rates in the later four groups are clearly high. Once they drop out from education, they suffer from low labour outcomes.

1.4.4 The reason for firms to hire non-regular workers

The reasons that firms hire TAWs and contract workers are explored below. The data used are from the General Survey on Diversified Types of Employment in 2010 by MHWL. The target of this survey was 16,866 business offices which hired more than five non-regular workers, and the number of valid responses are from 10,411 business offices, for a valid response rate of 61.7%.

Figure 1.9 presents the answers to the question 'Why do you hire TAWs/contract workers/part-time workers?' These reasons imply there are EPL and firms corresponding to EPL in Japan and create non-regular jobs and regular jobs. Then, workers choose to work non-regular jobs voluntarily and involuntarily. Their reasons for hiring non-regular workers is mainly to save labour costs, including wages, adjustment cost, recruitment cost, and other labour costs. We can see the different reasons by employment type. For example, part-time workers are mainly hired for cost reductions in wages, TAWs are mainly hired for reducing adjustment cost and recruitment cost, and contract workers tend to be hired for screening.

1.4.5 The characteristics of non-regular jobs

This section explores the characteristics of non-regular work. Temporary agency work, part-time work, and contract work are compared with regular work. Firstly, the outstanding characteristics are identified to illustrate the difference of employment types. Then, we present the survival functions of employment duration by employment type. Finally, earnings of each employment type are shown by controlling for the characteristics of workers and work step by step.

Occupation, industry, and training of non-regular jobs

The relationship of occupation, industry, and training of workers and employment types are described. The data used are from the Employment Status Survey of 2007, and the sample is males and females who are more than 15 years old and not studying.

The results clearly indicate that the different types of employment have different roles in the labour market. Many males work for a manufacturing process/elementary job across all employment types (Table 1.4). Many females work as clerical workers and/or in the service industry. Regular workers tend to do relatively specialised or administrative jobs. Contract workers are similar to regular workers. Many male TAWs work for manufacturing process/elementary workers. Many part-time workers work in relatively unskilled jobs such as the hospitality industry. Approximately 40% of part-time workers are working in wholesale trade, retail trade and restaurants.

Then, the proportion of the proportion of taking training offered by a firm by the types of employment is presented. The data used are the aggregated data of the Employment Status Survey in 2012, because the Employment Status Surveys of 1992, 1997 and 2002 do not have the question about training. The data of 2012 consist of people aged 15 or older. That is, these data include persons who are over 60 years of age and are in the middle of studying. However, most of them are not in the labour force or work as part-time workers, so the influence is small. The result is 40% of regular workers receive training and 30% of contract workers receive training, while only 15% of TAWs and part-time workers receive training. Contract workers are more closely treated as regular workers in terms of training, while only 15% of TAWs and part-time workers receive training.

Unemployment risk of non-regular jobs

Unemployment risk is different among employment types. The figure below shows the survival functions of employment duration by employment type. I used the Kaplan-Meier method, a non-parametric statistic used to estimate the survival function from lifetime data. It is also used to measure the length of time people remain employed by employment type.

We can see that the employment duration of TAWs is much shorter than other employment type (Figure 1.10). The survival functions of contract workers and part-time workers are almost same. Regular workers have the longest employment duration. Employment duration and job stability are affected by duration of contract and laws related with employment duration. This is one of the determinant elements in firms. TAWs and contract workers play a role in corresponding to the demand of firms. A contract worker has a role in screening contract workers to choose regular workers within firms. Regular workers play a core role in the firm. Part-time workers play a core role in simple tasks within the firm.

Earnings of non-regular jobs

This section explains earnings. Figure 1.11 shows the hourly wages by employment type by using kernel density estimation. The data used are from the individual data of Employment Status Survey in 2002 by the Ministry of Internal Affairs and Communications, and the samples are workers who are 15-54 years old and are not studying. The variables used for calculation are annual income, hours of work per week, and annual working days. The calculation follows the approach of Ito et al. (2012).

The result is that regular workers earn the most while part-time workers earn the least. Approximately 75% of part-time workers earn 500–1000 yen hourly. All the modes of each employment type are 500–1000 yen, and all distributions are right-skewed. The shapes of the hourly wage distribution for TAWs and contract workers are very similar. Without controlling for any characteristics, we find that non-regular workers earn less than regular workers. This wage gap is widely reported in the empirical literature in many countries.

Next, Figure 1.12 shows the relationship of mean hourly wage and age group by employment type. The data used are the same as in the previous analysis. When workers are young, mean hourly wages are not very different between employment types, especially for TAWs, contract workers, and regular workers in middle/small firms. Moreover, the working environment of small and middle-sized companies is sometimes not good. They often infringe on basic labour rights. These facts may explain why a certain number of young people choose to work in a non-regular job. They do not have a strong incentive to join regular workers in middle/small firms in the short run.

However, as age increases, the wage gap between a regular worker and a non-regular worker expands. The proportion of receiving training is low for non-regular workers. It is more difficult for non-regular workers to accumulate human capital than for regular workers. This seems to reflect the difference of wage in this figure.

Then, the wage functions are estimated to investigate how the hourly wage is decided (Table 1.5). The sample is the same as the one used for calculating the above hourly wage. This is a simple OLS regression controlling for worker and firm characteristics. The dependent variable is log hourly wage. The explanatory variables are employment type dummy, employment duration, square of employment duration, age, square of age, education dummy, firm size dummy, occupation dummy and industry dummy.

The table shows the results of the estimated wage function. Wage differential mainly comes from a worker's characteristics and employer's characteristics. We find a temporary agency workers earn less than a regular worker and more than a part-time worker, holding worker and firm characteristics constant. There is no significant difference between TAWs and contract workers. These differences of log hourly wage by employment type could come from unobserved characteristics. For example, even if they are in the same occupation, the tasks of a regular worker might need a higher skill set. Or it could be just wage discrimination between employment types even if the tasks are totally the same, although the Japanese government promotes a policy of equal pay for equal work between regular workers and non-regular workers.

This seems to be consistent with findings that non-regular workers have traditionally performed different tasks compared with regular workers and are not considered as equals within a firm in Japan (Esteban-Pretel, 2011). Also, the results reveal the characteristics of a higher log hourly wage are higher level of education attainment, longer duration of current employment, higher age and working at bigger firms. The difference of occupations and industries leads to a difference in log hourly wage.

1.5 Conclusion

This paper documented a rise in non-regular jobs and labour market segmentation in Japan. To do so, we used the data concerning stock, flow and some estimations. The main findings are as follows. The EPL in Japan is stricter than in the UK and the US, but weaker than in France and Spain. As the theoretical model predicts the consequences of a combination of stringent legal constraints on the termination of permanent jobs and of weak constraints on the creation of temporary jobs, the number of non-regular workers are increasing, and the transition between non-regular jobs and regular jobs is low. There is labour market segmentation between regular workers and non-regular workers.

Non-regular workers mainly consist of females, young people, and old people. Breadwinners such as prime-age male involuntarily work non-regular jobs, while females and the old who supplementarily work work voluntarily. There is little relationship between educational attainment and employment types from the institutional reason in Japan. Firms correspond to EPL in Japan and create non-regular jobs and regular jobs. Non-regular workers are hired mainly to save labour costs. Then, workers choose to work non-regular jobs voluntarily and involuntarily.

It is more difficult for non-regular workers to accumulate human capital than for regular workers. Such a situation offers one reason for the difference in wages. Characteristics of work regarding employment types differ. Contract jobs are similar to regular jobs, and firms tend to use them for screening. Based on these findings, in the next chapter, I will build an on-the-job search matching model with heterogeneous workers and heterogeneous jobs.

Figures and tables

Figure 1.1: The indicators of the strictness of Employment Protection Legislation in some OECD countries

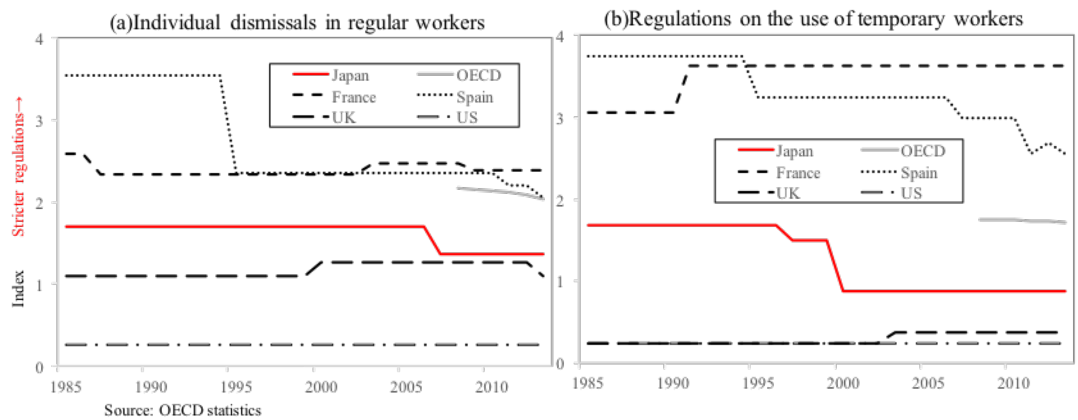


Figure 1.2: Number of workers and unemployed people

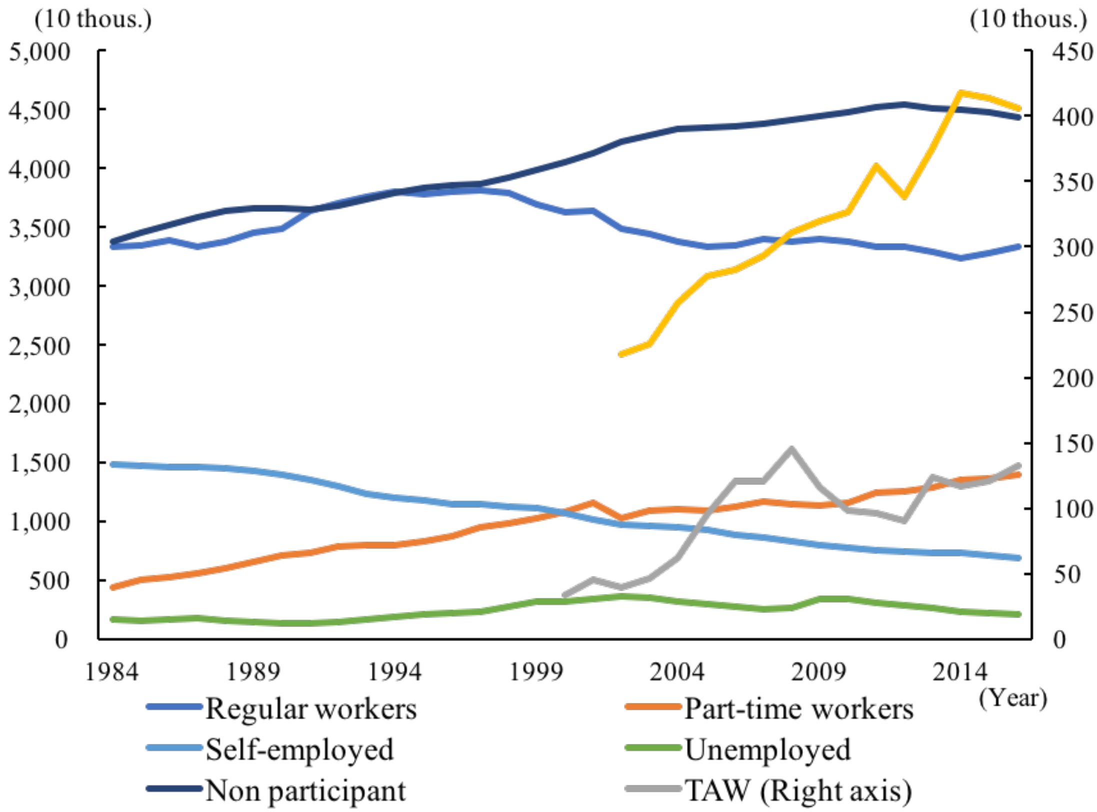


Figure 1.3: Cluster analysis of workers

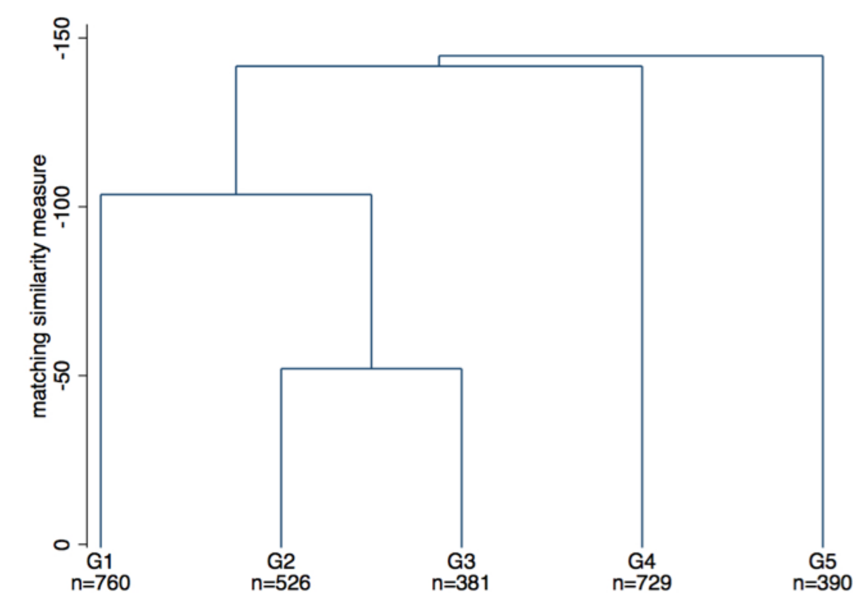
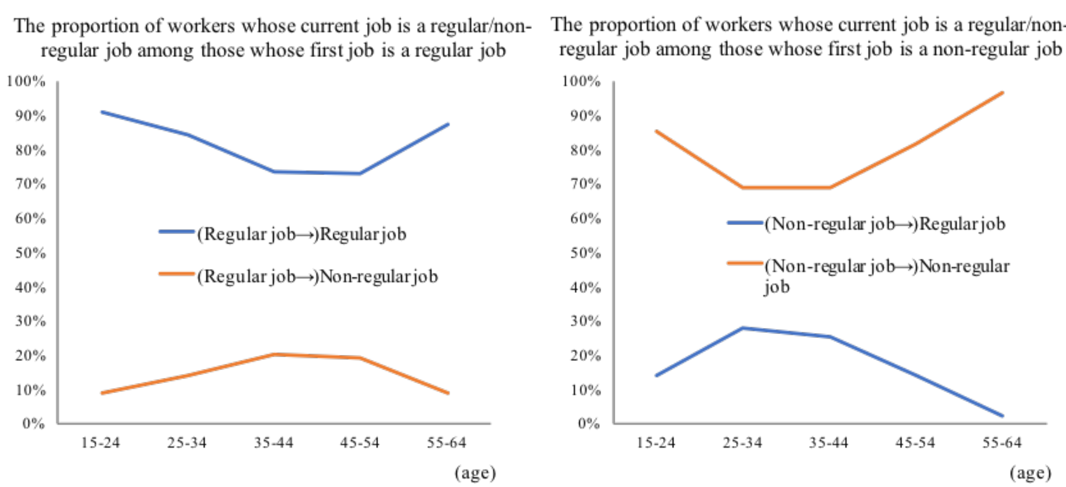
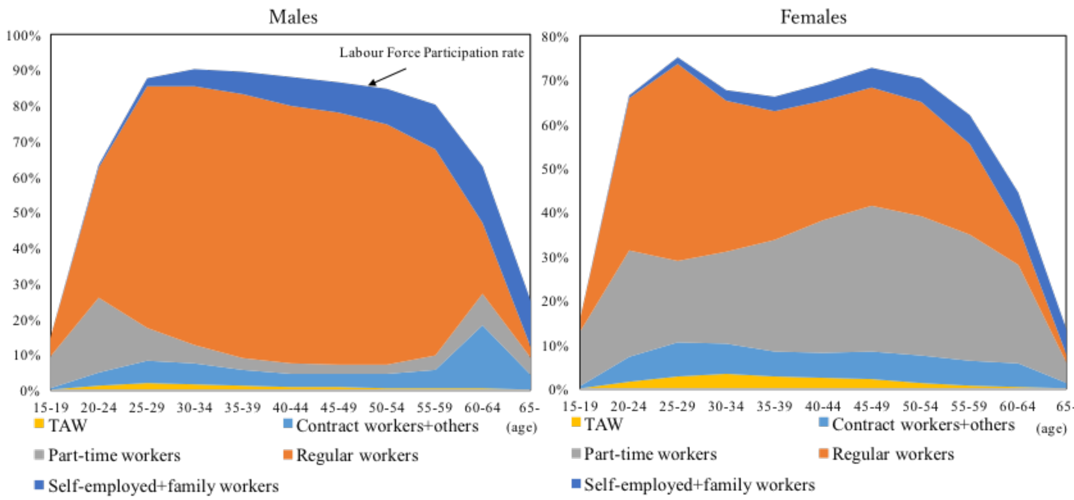


Figure 1.4: The Relationship of First Job and Current job



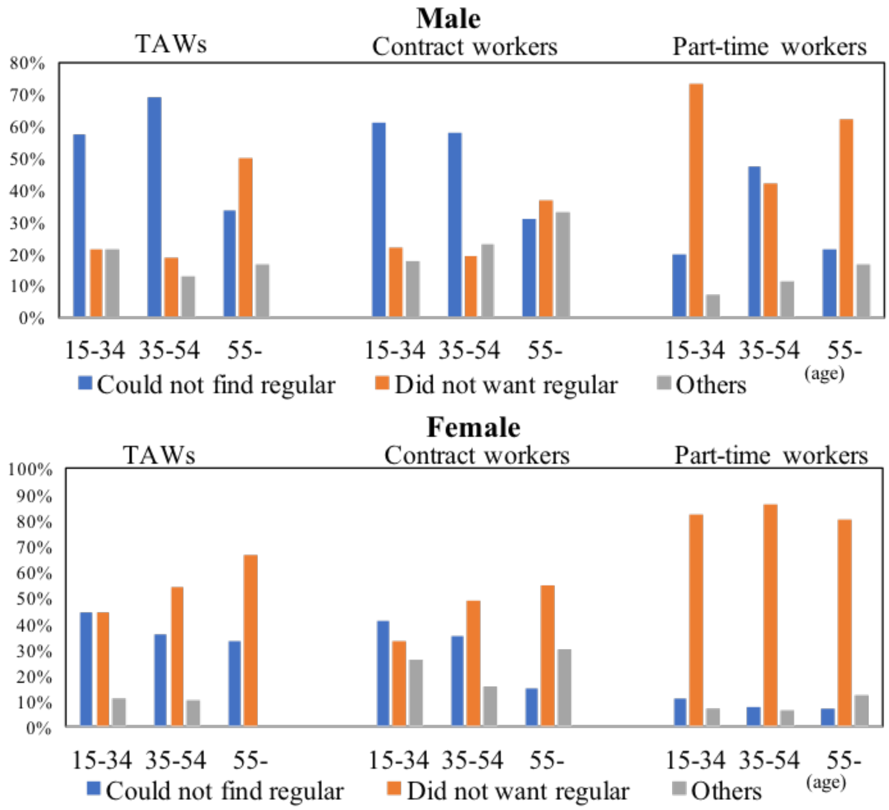
Source: Employment Status Survey (2012) Note: The data of others are omitted for simplicity.

Figure 1.5: The Composition of Employment Types by Age Group (2012)



Source: Employment Status Survey (2012)

Figure 1.6: The Main Reason for Being a Non-Regular Worker by Gender, Age, and Employment Type



Source: LFS (2016)

Figure 1.7: Education and employment type

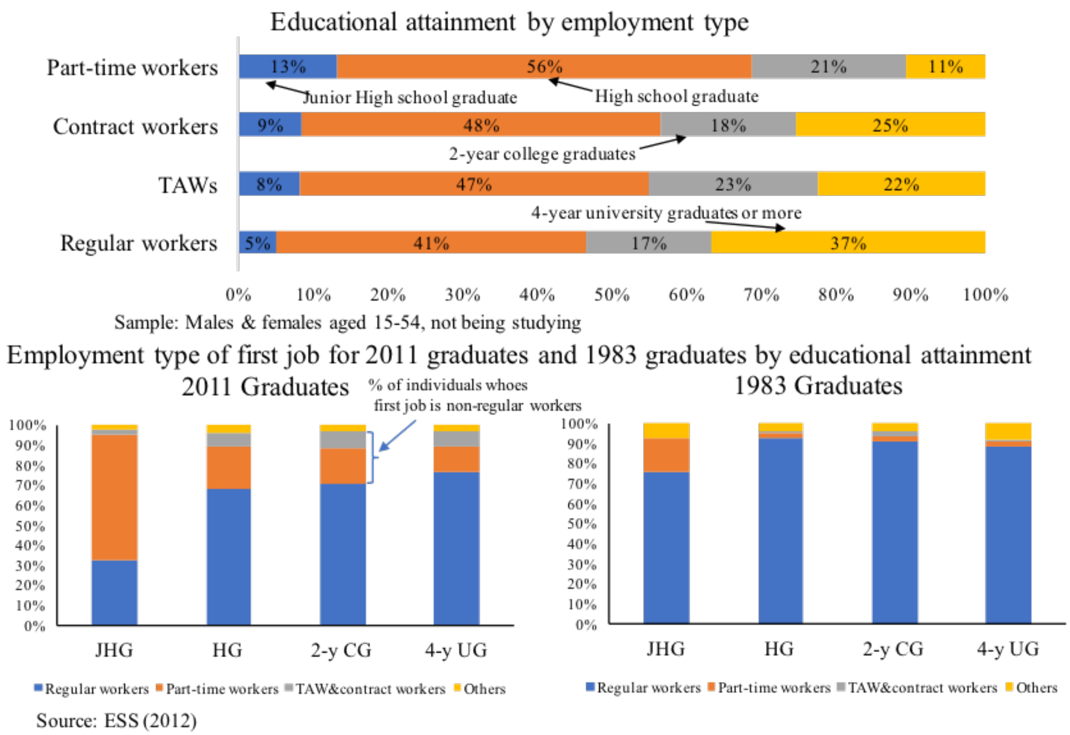
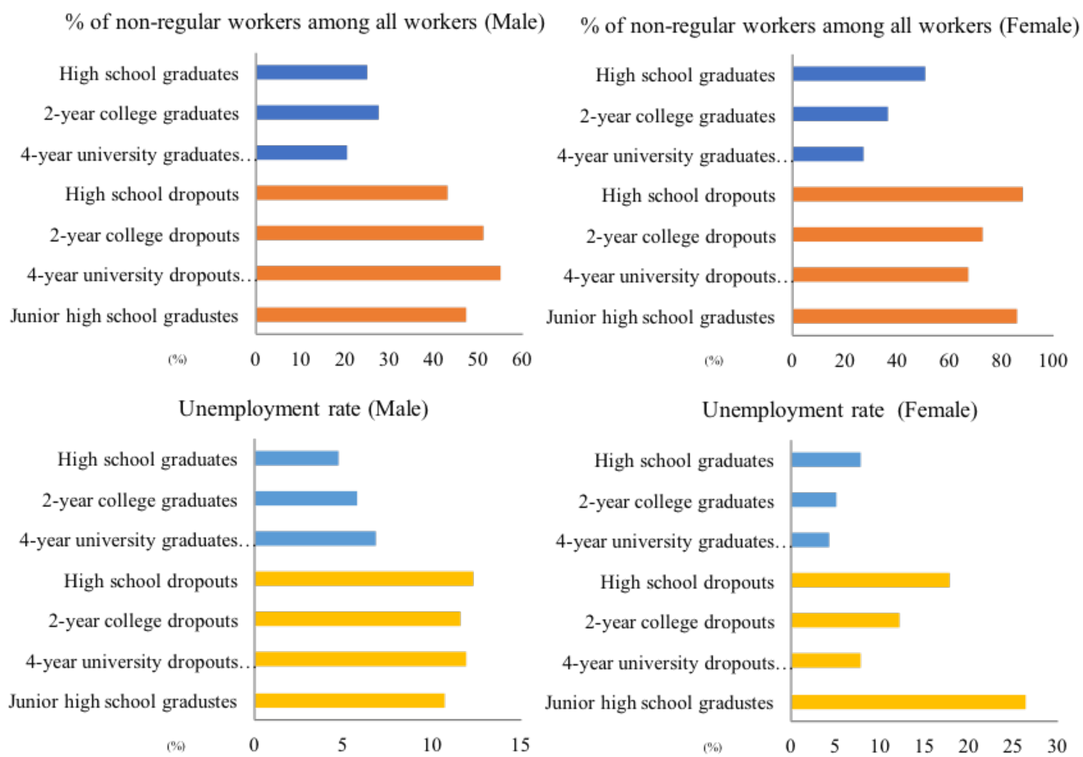


Figure 1.8: Labour market outcomes of dropouts



Sample: males&females aged 20-29, Source: The Longitudinal Survey of Middle-aged and Elderly Persons (2012)

Figure 1.9: Reason to hire non-regular workers

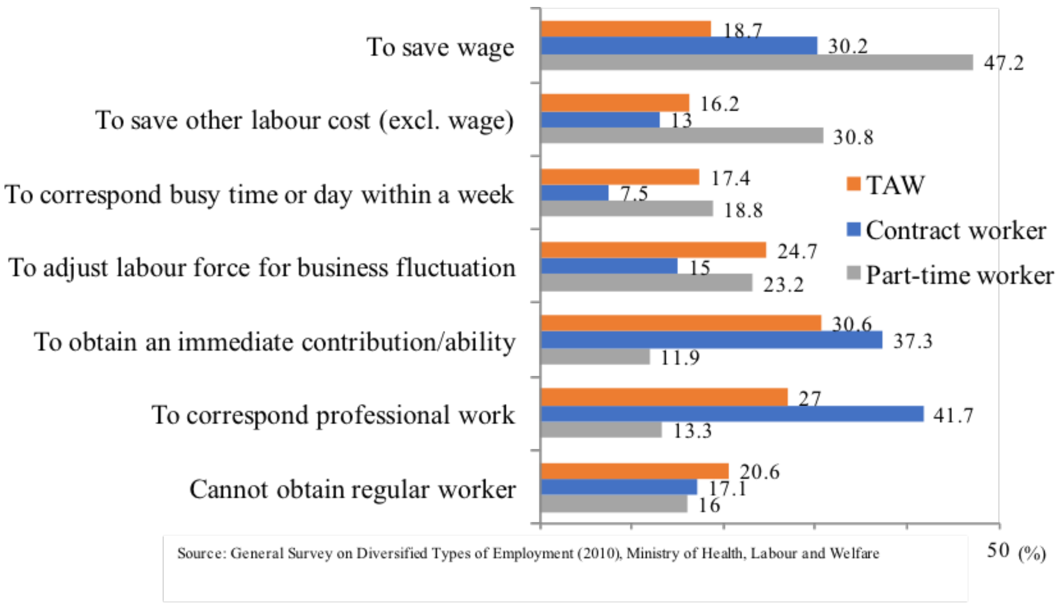
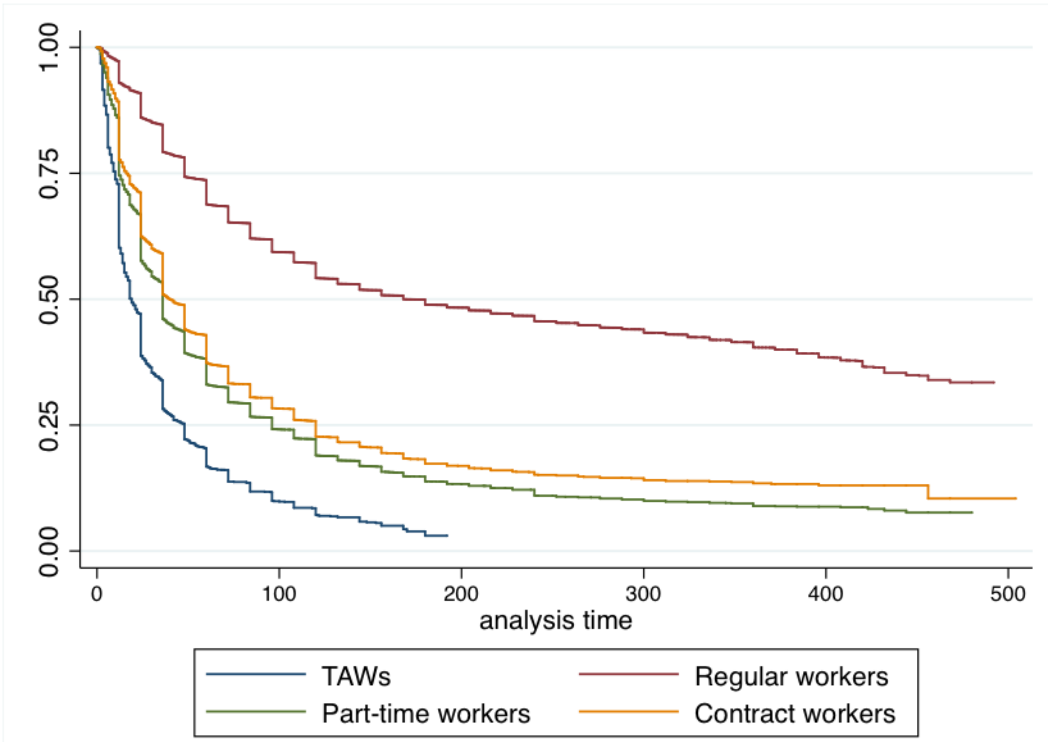
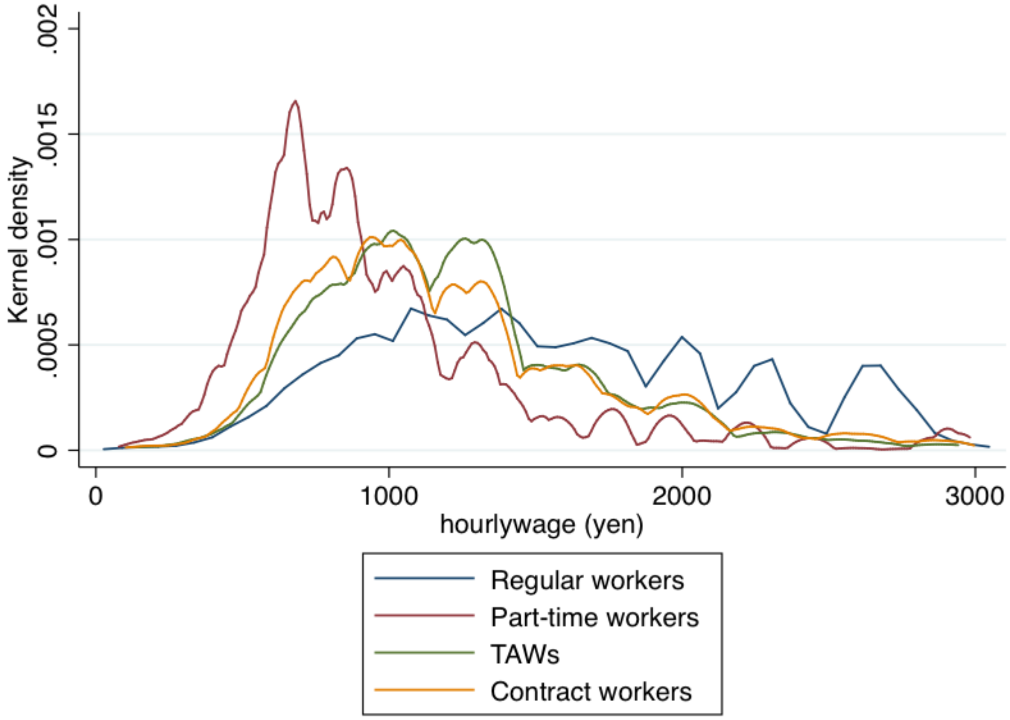


Figure 1.10: Survival functions of employment duration for 1 firm by employment type



Source: Employment Status Survey, 2002, Sample: Males & Females aged 15-54 who are not working

Figure 1.11: Hourly wage by employment type



Sample: Males & females aged 15-54 who are not studying
Source: Employment Status Survey, 2002

Figure 1.12: Relationship of mean hourly wage and age-group

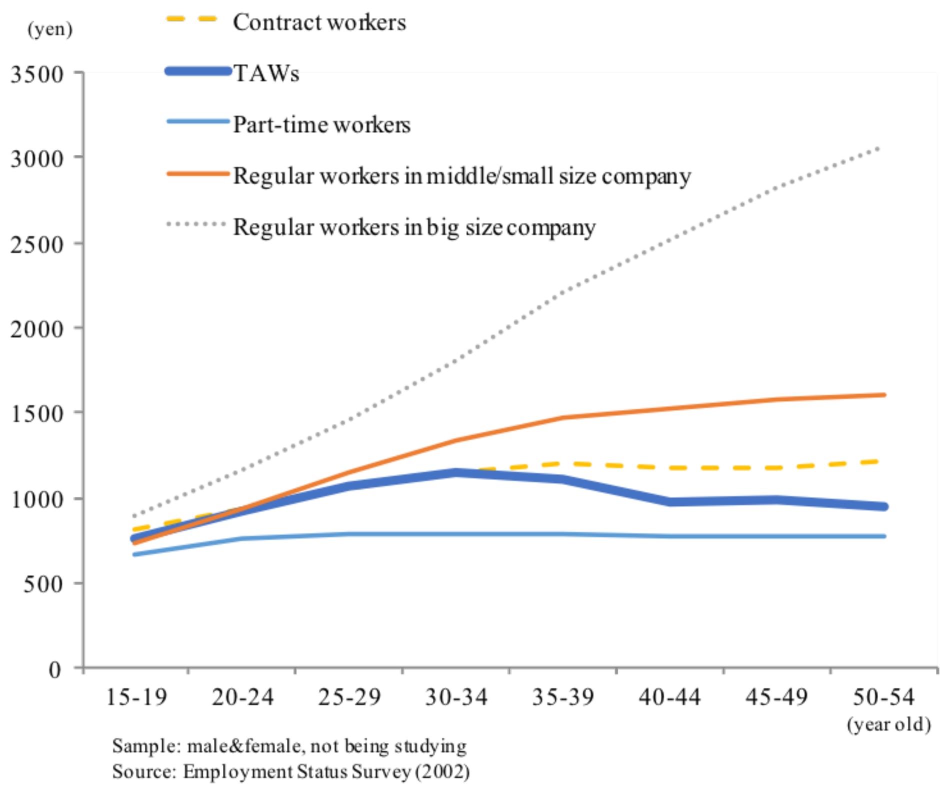


Table 1.1: Individuals with Each Characteristic in Each Cluster

	1	2	3	4	5
Regular work	97%	95%	96%	70%	0%
Males	57%	87%	93%	41%	11%
Middle-age (35-54)	100%	44%	99%	10%	94%
High education	52%	13%	18%	67%	16%
Manufacturing	5%	63%	100%	19%	41%
Living urban area	27%	29%	30%	30%	28%
Married	90%	34%	99%	27%	87%
Full-time	100%	71%	100%	88%	25%
N	760	526	381	729	390

Sample: Non-regular workers, who are 15-54 year-old, not being studying
Source: Employment Status Survey (2002)
Note: 1% of sample were chosen randomly because of the functional restriction of the Stata

Table 1.2: Average Monthly Transition Rate Based on the Stock of the Previous Month

Previous＼Current	Regular	Part-time	TAW	Contract	Unemployed	Non-participant
Regular worker	96.4%	0.7%	0.1%	0.6%	0.2%	0.2%
Part-time worker	2.2%	91.9%	0.4%	1.1%	0.6%	1.5%
TAW	3.7%	5.3%	79.7%	6.5%	1.7%	1.3%
Contract worker	7.4%	5.6%	3.0%	78.9%	0.7%	0.6%
Unemployed	2.8%	5.4%	1.2%	1.0%	76.2%	10.9%
Non-participant	0.2%	0.5%	0.0%	0.0%	0.5%	97.9%

Source: Labour Force Survey 2015-2017, Sample: Aged 15 and over males and females

Table 1.3: Probit Analysis of Transition from Previous Non-Regular Workers

VARIABLES (1:the previous type was NR and current is R)	Probit		
Gender<female>			
Male	0.244	(0.005)	***
Previous Status <Contract worker>			
Part-time worker	-0.048	(0.007)	***
TAW	-0.111	(0.012)	***
Age < 20-24 year-old>			
15-19 year-old	-0.140	(0.019)	***
25-29 year-old	0.054	(0.008)	***
30-34 year-old	0.037	(0.009)	***
35-39 year-old	0.032	(0.009)	***
40-44 year-old	0.021	(0.009)	*
45-49 year-old	0.021	(0.009)	*
50-54 year-old	0.026	(0.009)	**
Area <Not living three largest metropolitan regions>			
Living three largest metropolitan regions	-0.054	(0.005)	***
Marriage status <Not being Marriage>			
Being Marriage	-0.039	(0.005)	***
Previous Company Size <small companies (-99 worker)>			
Middle companies (100-999 worker)	-0.013	(0.006)	*
Large companies (1000- worker or public office)	-0.008	(0.007)	
Education <High school graduate>			
Junior high school graduate	-0.068	(0.007)	***
2-year vocational/technical college	0.056	(0.006)	***
4-year university graduate or more	0.113	(0.008)	***
Observations		37056	

Sample: Males & females aged 15-54 who are not studying
Source: Employment Status Survey, 2002

Table 1.4: The Percentage Distribution of Occupation Groups and Industry Groups by Gender, and the Proportion of Taking Training Offered

	Regular worker	Contract worker	TAW	Part-time worker
Male				
Occupation (n=162,484)	(100%)	(100%)	(100%)	(100%)
Specialist professionals	13.1	11.2	8.6	3.1
Clerical workers	17.1	11.6	10.5	5.9
Sales workers	14.8	10.9	3.9	14.0
Service workers	3.5	6.5	2.8	15.8
Manufacturing process/elementary	38.1	45.2	69.4	49.2
Industry (n=164,268)	(100%)	(100%)	(100%)	(100%)
Construction	13.1	11.5	3.3	16.1
Manufacture	26.5	22.2	60.2	14.3
Transport and communications	10.2	11.8	7.2	9.7
Whole sale trade, retail trade and	15.1	13.3	3.8	31.7
Service industry	20.9	31.8	15.8	21.9
Female				
Occupation (n=130,974)	(100%)	(100%)	(100%)	(100%)
Specialist professionals	26.6	19.2	3.7	7.0
Clerical workers	36.8	37.1	58.4	24.5
Sales workers	9.3	12.0	7.7	14.8
Service workers	11.0	15.8	7.0	19.7
Manufacturing process/elementary	15.1	14.2	22.6	32.3
Industry (n=132,399)	(100%)	(100%)	(100%)	(100%)
Construction	4.0	1.9	2.6	1.7
Manufacture	19.1	11.7	26.0	18.2
Whole sale trade, retail trade and restaurant	16.2	17.2	11.8	41.2
Finance and insurance	5.4	5.1	12.6	2.1
Service industry	46.7	47.5	26.3	28.6
Training				
% of taking training at firm	39.3	26.6	14.5	16.3

Source: Employment Status Survey (2002, 2012)
Sample: Males and females aged 15-54, who is not studying
Note: The items with large values in each section are only picked up, so that the sum of each items in each section is less than 100%.
Note2: The data of training is only from Employment Status Survey (2012)

Table 1.5: OLS Estimation Results for the Wage Equation

Log hourly wage	Coefficient	Standard Error	
Gender <Female>			
Male	0.325	(0.00)	***
Employment duration	0.002	(0.00)	***
Employment duration square	0.000	(0.00)	***
Age	0.038	(0.00)	***
Age square	0.000	(0.00)	***
Emplyment status <TAW>			
Regular worker	0.167	(0.01)	***
Part-time worker	-0.190	(0.01)	***
Contract worker	0.001	(0.01)	
Education <High school graduate>			
2-year vocational/technical college	0.071	(0.00)	***
4-year university graduate or more	0.186	(0.00)	***
Company Size <small companies (-99 worker)>			
Middle companies (100-999 worker)	0.150	(0.00)	***
Large companies (1000- worker or public office)	0.300	(0.00)	***
Occupation<Specialist professionals>			
Administrative and managerial workers	0.136	(0.01)	***
Clerical workers	-0.095	(0.00)	***
Sales workers	-0.166	(0.00)	***
Service workers	-0.202	(0.00)	***
Security workers	-0.380	(0.01)	***
Agriculture and forestry workers	-0.268	(0.02)	***
Transport and communication workers	-0.249	(0.01)	***
Manufacturing process workers	-0.230	(0.00)	***
Industry dummy	○		
Constant term	5.770	(0.02)	***
Observations	213234		

Source: Employment Status Survey (2002), Ministry of Internal Affairs and Communications
Sample: Males & Females aged 15-54 who are not studying
Note: Robust version doesn't change the result.

Chapter 2

On-The-Job Search in a Matching Model with Temporary Jobs and Permanent Jobs

Abstract

The objective of this chapter is to construct a labour market matching model with heterogeneous workers and jobs that incorporates OTJ search and different turnover rates. This research extends the model of Dolado et al. (2009) by incorporating different turnover rates to heterogeneous jobs. In line with the paper of Dolado et al. (2009), we show that when we introduce OTJ search into the model, the likelihood of having an equilibrium with cross-

skill matching is enhanced even if the model contains different turnover rates to heterogeneous jobs. The quantitative implications give the prediction that the unemployment rate of low-productivity workers increases, inequality grows, and welfare decreases as skill-biased technical change proceeds. The simulation also shows that the effect to add the OTJ search to this economy is not affected largely.

Key Words: Segmented Labor Markets; Job Search

JEL Codes: J4; J6

2.1 Introduction

The explosion of information and communication technologies profoundly affects the labour market in the world. The effect of development of information technology on the labour market has a range of topical issues. One of the issues is that these technologies increase the relative productivity of skilled labour which is called as skill-biased technological change. The other issue is that the internet dramatically increase efficiency when we look for a job. However, little is known about how it affect to the outcome of the Japanese segmented labour market. There is no quantitative simulation for the Japanese labor market, and it is very important to understand the effects of these issues for policy making.

My first chapter assessed the labour market segmentation in Japan. In the second chapter, we construct the model based on the analysis of the first chapter. The model gives the simulation for the debate of the political direction of non-regular jobs to consider the effect of the development of the information and communication technologies. The objective of this chapter is to construct a labour market matching model with heterogeneous workers and jobs that incorporates OTJ search and different turnover rates. We use this theoretical model to investigate counterfactual scenarios and welfare effects in Japanese labour market.

Many papers build a search model with non-regular jobs and regular jobs, but there are few models which include the OTJ search as far as I know. The similar model to us is Bentolila et al. (2012). They construct the model in which a non-regular worker has the possibility to move to a regular work if the asset value of a regular work is higher than that of unemployed when the contract terminates with a certain probability.

In Japan, Nosaka (2011) and Ariga and Okazawa (2011) build and calibrate a search model with non-regular jobs and regular jobs. However, their interests and models are quite different from our model. Nosaka (2011) builds a search matching model and analyzes the impact of decreasing Japan's productivity on employment by contract types, however there is only one type of worker in the model. Ariga and Okazawa (2011) builds and calibrates a model of competitive search focusing the role of the cost of training and work experience. The model can reproduce a set of stylized facts concerning major impacts of the decade long stagnation and subsequent changes in the Japanese labor market.

There are two papers which are closely related to my research. Albrecht and Vroman (2002) extend a random matching model by adding heterogeneous workers and heterogeneous jobs. They build a model to find the role of skill in the labour market. Dolado et al. (2009) add the OTJ search to the model of Albrecht and Vroman. They investigate the implication of OTJ search to the labour market outcomes systematically. They also show that the likelihood of having an equilibrium with cross-skill matching is enhanced by introducing the OTJ search.

This research extends the model of Dolado et al. (2009) by incorporating different turnover rates to heterogeneous jobs to describe the segmented labour market more precisely. This research contributes to provide the counterfactual simulation for the debate of the political direction of two-tier labour market in Japan, because the number of studies for Japan has been limited. In line with the paper of Dolado et al. (2009), we show that the likelihood of having an equilibrium with cross-skill matching is enhanced by introducing OTJ search into the model even if the model contains different turnover rates to heterogeneous jobs. The quantitative implications give the prediction that the unemployment rate of low-productivity workers increases, inequality grows, and welfare decreases as skill-biased technical change proceeds. The simulation also shows that the effect to add the OTJ search to this economy is not affected largely.

The structure of this paper is as follows: Section 2.2 provide a brief look at the evidence of the model setting. Section 2.3 explains the model. Section 2.4 shows the equilibrium of this model. Section 2.5 gives the quantitative implications. Finally, section 2.6 provides a conclusion to this work.

2.2 A Brief Look at the Evidence of the Model Setting

In this section, we describe the evidence of the model.

Firstly, a large percentage of dropouts get non-regular work as their first job, and a large percentage of graduates get regular work as their first job. They tend to stay there for the rest of their professional career. The evidence indicates that non-regular jobs are a large percentage of the first job of dropouts just after leaving school. The data which was conducted in Tokyo, the capital of Japan, shows about 60-70% of dropouts start their career in non-regular work, in contrast to about 10-40% of graduates (Table 2.1). On the other hand, the first job largely affects their subsequent job in Japan (ex. Ishida 2005, Sato 2011). My first chapter also analyses the Employment Status Survey in 2012. This indicated that about 70% of non-regular workers whose first job is a non-regular job stay at non-regular jobs, and about 70% of regular workers whose first job is a regular job stay at a regular job.

Secondly, a large percentage of dropouts work for non-regular work, although a large percentage of graduates work for regular work and some percentage of graduates work for non-regular work. The data which was conducted in Tokyo shows that the rate of regular workers are about 10% for graduates and about 50-80 % for dropouts.

Thirdly, the labour market outcome of graduates are clearly better than that of dropouts. We note the data of the labour market outcome for graduates and dropouts in the Table 2.2. The table indicates the difference of labour market outcome such as hourly wage and unemployment rate between dropouts and graduates. It compares the outcome between dropouts and graduates by types of school.

We construct a model based on these data in the next section, although the available data is limited. The model has two types of jobs and two types of workers. Two types of jobs are; non-regular work and regular work. The two types of workers are; low-productivity and high-productivity. We regard dropouts as low-productivity workers and graduates as high-productivity workers. High-productivity workers stay in regular work, non-regular work, or unemployed. Low-productivity workers stay in non-regular work or unemployed.

2.3 The Model

2.3.1 Basic Assumptions

We consider a continuous time economy in steady state. It is easier to build and calculate the continuous-time model than the discrete-time model. There is a unit mass of infinitely lived workers and a mass of infinitely lived firms. We can believe the assumption that individuals are infinitely lived because maybe many people plan for their dynasty. This assumption makes it easier to solve the problem. Both agents are risk neutral, have a time-separable intertemporal utility function, and discount the future at rate $r > 0$. Workers and firms are interested in maximizing expected discounted income. There is on-the-job search by matched workers. A mismatched worker can move to a better job without an intervening spell of unemployment. This means a high-productivity worker in a non-regular job can move to a regular job. We assume that the distribution of productivities across workers is exogenous. More specifically, there are two types of workers: a fraction μ of the workers in the population has the low-productivity level, y(n), a fraction $1 - \mu$ has the high-productivity level, y(s).

Besides there are two types of jobs; non-regular jobs and regular jobs, and jobs are either filled or vacant. Filled non-regular jobs and regular jobs break up at the exogenous rate σ_n and σ_s, respectively. We assume the rate at which non-regular jobs break up, σ_n, is larger than the rate at which regular jobs break up, σ_s as with the real world. A non-regular job and regular job have the minimum productivity required of a worker hired in the jobs. A minimum productivity requirement of a non-regular job is the low-productivity level, y(n), that is, a non-regular job can be filled by either a low-productivity worker or a high-productivity worker. In contrast, a requirement of a worker's productivity for a regular job is the high-productivity level, y(s). Therefore, a regular job can only be filled by high-productivity workers. The output produced is given by y(n) when a job is filled with a non-regular job by a low-productivity worker or a high-productivity worker. The output produced is given by 0 when a job is filled with a regular job by a low-productivity worker. The output produced is given by y(s) when a job is filled with a regular job by a high-productivity worker.

This model is constructed based on the evidence in the Section 2.2. Firstly, dropouts tend to drift non-regular jobs. Most of graduates tend to work for regular jobs, while some of them work for non-regular jobs. Secondly, the labour market outcome of graduates are clearly better than that of dropouts. A firm pay w(i,j) to a worker when a job is filled. A firm pay the instantaneous cost of a vacancy c when a job is unfilled. A firm choose the type of a vacant job to maximize the value of the vacancy when a vacant job is created.

2.3.2 Matching

Through an imperfect matching technology vacant jobs and job seekers meet randomly and are matched. It is assumed that our matching process is undirected as with Albrecht and Vroman (2002). In addition, OTJ search by mismatched workers is allowed as with Dolado et al. (2009). Therefore a mismatched worker can move to a more suitable job by a job-to-job transition. Job seekers and firms are assumed to meet each other randomly according to an usual constant return to scale matching function:

$$m[v(s)+v(n),u(h)+u(l)+\lambda e(n,h)]$$

where v(i) is the mass of vacancies of type i, u(j) is the mass of unemployed workers of type j, e(n,h) is the mass of mismatched workers, and $\lambda \in [0,\infty)$ is the relative search intensity of mismatched workers. When $\lambda = 1$, it is interpreted that the arrival rate of job offers is independent of the employment status of the job seeker. When $\lambda = 0$, there is no OTJ search. Lastly we define $\theta = [v(s)+v(n)]/[u(h)+u(l)+\lambda e(n,h)]$ as the effective labour market tightness and we assume that the meeting function is strictly increasing in both arguments. Therefore we can write the contact rate of a job seeker is equivalent to $f(\theta) = \theta p(\theta)$ during unemployment and $\lambda f(\theta) = \lambda\theta p(\theta)$ during employment which is smaller than $f(\theta)$ and the contact rate of a firm as $p(\theta) = m(1, 1/\theta)$. The properties of the matching function are that $f'(\theta) > 0$ and $p'(\theta) < 0$. We assume that $\lim_{\theta\to\infty} f(\theta) = \lim_{\theta\to 0} p(\theta) = \infty$ and $\lim_{\theta\to 0} f(\theta) = \lim_{\theta\to\infty} p(\theta) = 0$.

2.3.3 Wage Determination

We focus on one important aspect of OTJ search in our analysis. We know that workers sometimes move to the same type of job to obtain a higher wage through OTJ search. Allowing for this feature would complicate our model greatly. Thus, we only pursuit of a better match for mismatched worker. We have two strong assumptions about wage determination which follows Pissarides (1994). Firstly, the wage can continually revised without any cost.

This means that we rule out the long-term contracts. Secondly, under a linear surplus-splitting rule which entitles workers to a fraction $\beta \in (0,1)$ of the flow rents, wages are decided.

These assumptions result in a wage-setting rule which looks the same as the typical solution of Nash bargaining. Formally, we let V(i) the asset value of a vacant job of type i and U(j) denote the asset value of unemployment for a worker of type j. Likewise, we let J(i,j) and W(i,j) be the asset value of the worker and the firm obtained from a match that combines a worker of type j with a job of type i. We define the surplus as $S(i,j) \equiv J(i,j) + W(i,j) - U(j) - V(i)$. In any match with a positive match surplus, the wage w(i,j) is constant and satisfies the following rule of sharing:

$$\beta[J(i,j) - V(i)] = (1-\beta)[W(i,j) - U(j)] \tag{2.1}$$

The equation (2.1) indicates that there are no wage differences among identical workers in the same type of job. Lastly, it is assumed when mismatched workers find a regular job, they resign their jobs in the rest of the analysis.

2.3.4 Asset Values

We describe the asset value equations of workers and firms in this section. The value function of a high-productivity unemployed worker, U(h), is:

$$rU(h) = b + f(\theta)\{\zeta \max[W(n,h) - U(h), 0] + (1-\zeta)[W(s,h) - U(h)]\} \tag{2.2}$$

where the share of non-regular job vacancies is denoted as $\zeta = v(n)/[v(n) + v(s)]$. The high-productivity unemployed worker will accept a non-regular job if his/her lifetime income will be improved by this. The asset value of a high-productivity worker in a non-regular job W(n,h) is given by:

$$rW(n,h) = w(n,h) + \delta_n[U(h) - W(n,h)] + \lambda f(\theta)(1-\zeta)[W(s,h) - W(n,h)] \tag{2.3}$$

The last term on the RHS of (2.3) means the expected gain if they successfully move to a regular job through OTJ search.

The asset value equations of a low-productivity worker during unemployment, a low-productivity worker in a non-regular job, and a high-productivity worker in a regular job are usual :

$$rU(l) = b + f(\theta)\zeta[W(n,l) - U(l)] \tag{2.4}$$

$$rW(n,l) = w(n,l) + \delta_n[U(l) - W(n,l)] \tag{2.5}$$

$$rW(s,h) = w(s,h) + \delta_s[U(h) - W(s,h)] \tag{2.6}$$

The value equations of vacant jobs are defined. we let $\psi = [u(h)+u(l)]/[u(h)+u(l)+\lambda e(n,h)]$ denote the share of unemployed job seekers. In a similar way, the share of low-productivity workers in the pool of unemployed is denoted by $\phi = u(l)/[u(l) + u(h)]$. Then, the value functions for a non-regular job vacancy, V(n), is given by:

$$rV(n) = -c + \psi p(\theta)\{\phi[J(n,l) - V(n)] + (1-\phi)\max[J(n,h) - V(n), 0]\} \quad (2.7)$$

Similarly, we can write the asset value equation for a regular job vacancy, V(s), by:

$$rV(s) = -c + (1 - \psi\phi)p(\theta)[J(s,h) - V(s)] \quad (2.8)$$

The asset value equations of filled jobs are given by:

$$rJ(s,h) = y(s) - w(s,h) + \delta_s[V(s) - J(s,h)] \quad (2.9)$$

$$rJ(n,l) = y(n) - w(n,l) + \delta_n[V(n) - J(n,l)] \quad (2.10)$$

$$rJ(n,h) = y(n) - w(n,h) + [\delta_n + \lambda f(\theta)(1-\zeta)][V(n) - J(n,h)] \quad (2.11)$$

The separation rate $\delta_n + \lambda f(\theta)(1 - \zeta)$ in (2.11) is greater than δ_n. This represents that mismatched workers have chance to leave their employer and move to a regular job when they find a regular job. How this feature impacts the choices of workers and firms in equilibrium is analysed in the next parts.

2.4 Equilibrium

2.4.1 Definition of Equilibrium

We define the set of equilibrium in this section. We are primarily interested in the equilibria with cross-skill matching and on-the-job search, therefore we consider the case first. In this case, the surplus of filling a non-regular job with a high-productivity worker is non-negative, $S(n, h) \geq 0$. Another case is ex-post segmentation, $S(n, h) < 0$, which is presented later.

We begin by deriving the equilibrium surplus expressions. From the free-entry condition for non-regular jobs, namely V(n)=0, the equation (2.5) and (2.10), S(n,l) satisfies:

$$(r + \delta_n)S(n, l) = y(n) - rU(l) \tag{2.12}$$

Along with (2.1), this implies that the wage of low-productivity workers, w(n,l), is denoted as:

$$w(n, l) = rU(l) + \beta[y(n) - rU(l)] \tag{2.13}$$

Similarly, from (2.1), (2.6), (2.9) plus the free-entry condition for regular jobs, V(s)=0, the corresponding expressions of high-productivity workers in regular jobs are:

$$(r+\delta_s)S(s,h) = y(s) - rU(h) \tag{2.14}$$

$$w(s,h) = rU(h) + \beta[y(s) - rU(h)] \tag{2.15}$$

Likewise, the following is the corresponding expressions for mismatched workers:

$$\begin{aligned} &[r+\delta_n+\lambda f(\theta)(1-\zeta)]S(n,h) \\ &= y(n) - rU(h) + f(\theta)\lambda(1-\zeta)\beta S(s,h) \end{aligned} \tag{2.16}$$

$$\begin{aligned} w(n,h) &= rU(h) + \beta[y(n) - rU(h)] \\ &-f(\theta)\lambda(1-\zeta)\beta(1-\beta)S(s,h) \end{aligned} \tag{2.17}$$

We compare the equation (2.16) and (2.12). There are two important differences. The first difference is that the output gotten by a mismatched worker is discounted at a higher rate than the one of a low-productivity worker. The second difference is that the value of S(n,h) contains the expected gains from on-the-job search which amount to $f(\theta)\lambda(1-\zeta)\beta S(s,h)$. Mismatched workers compensate their employers by accepting a wage reduction given by $f(\theta)\lambda(1-\zeta)\beta(1-\beta)S(s,h)$ because the actual gains from on-the-job search occur not to the firm but to the worker.

We proceed to the reservation values of low-productivity workers and high-productivity workers. From (2.2) and (2.4), we obtain:

$$rU(l) = b + f(\theta)\zeta\beta S(n,l) \tag{2.18}$$

$$rU(h) = b + f(\theta)\beta[\zeta S(n,h) + (1-\zeta)S(s,h)] \tag{2.19}$$

This yields the following expressions using (2.12), (2.14) and (2.16):

$$rU(l) = \frac{(r+\delta_n)b + f(\theta)\beta\zeta y(n)}{r+\delta_n + f(\theta)\beta\zeta} \tag{2.20}$$

$$rU(h) = \frac{(r+\delta_s)\alpha_{1n}b + f(\theta)\beta[\zeta(r+\delta_s)y(n) + (1-\zeta)\alpha_{2n}y(s)]}{\alpha_{2n}\alpha_{3s} + (r+\delta_s)f(\theta)\beta\zeta(1-\lambda)} \tag{2.21}$$

We define $\alpha_{1n} = r + \delta_n + f(\theta)\lambda(1-\zeta)$, $\alpha_{2n} = r + \delta_n + f(\theta)\lambda(1-\zeta+\beta\zeta)$ and $\alpha_{3s} = r + \delta_s + f(\theta)(1-\zeta)\beta$ which are discount factors.

Lastly, we insert the previous expressions for rU(j) and S(i,j) plus the wage rule (2.1) into both (2.7) and (2.8), then the two zero-profit conditions V(s)=0 and V(n)=0 can be written as:

$$\begin{aligned} \frac{c}{(1-\beta)p(\theta)\psi} &= \frac{\phi[y(n)-b]}{r+\delta_n+f(\theta)\zeta\beta} \\ &+(1-\phi)\frac{(r+\delta_s+\lambda f(\theta)(1-\zeta)\beta)\{y(n)-rU(h)\}}{(r+\delta_s)(r+\delta_n+\lambda f(\theta)(1-\zeta))} \end{aligned} \tag{2.22}$$

$$\frac{c}{(1-\beta)p(\theta)(1-\psi\phi)} = \frac{\alpha_{1n}[y(s)-b] + f(\theta)\beta\zeta[y(s)-y(n)]}{\alpha_{2n}\alpha_{3s} + (r+\delta_s)f(\theta)\beta\zeta(1-\lambda)} \tag{2.23}$$

Here, rU(h) is from Equation (2.21). We can see that (2.22) and (2.23) constitute the first two equilibrium relationships of the model. The steady state flow conditions for u(h), u(l) and e(n,h) give us the remaining ones. We denote the total mass of unemployed workers as $u \equiv u(l) + u(h)$. Then, these conditions are expressed as follows:

$$\zeta f(\theta)\phi u = \delta_n(\mu - \phi u) \tag{2.24}$$

$$f(\theta)(1-\phi)u = \delta_n e(n,h) + \delta_s e(s,h) \tag{2.25}$$

$$\zeta f(\theta)(1-\phi)u = [\delta_n + f(\theta)\lambda(1-\zeta)]e(n,h) \tag{2.26}$$

Here we know $e(n,h)+e(s,h) = (1-\mu) - u(1-\phi)$. We define an equilibrium with cross-skill matching when OTJ search exists from these five conditions.

Definition 1.

A steady-state equilibrium with cross-skill matching and OTJ search consists of a set of value functions for S(i,j), W(i,j), U(j), J(i,j) and V(i) that satisfy (2.2)–(2.11), (2.12), (2.14) and (2.16) plus a vector $\{u, \theta, \phi, \zeta, \psi\}$ *such that*

1. *All matches produce a non-negative surplus for the equilibrium values of* $\{\theta, \zeta\}$

2. *The vector* $\{u, \theta, \phi, \zeta, \psi\}$ *solves the steady state conditions (2.24) to (2.26) and the free entry conditions (2.22) and (2.23)*

Before moving to another case, we mention about the corner solution. Our equilibrium in the cross-skill matching case has a corner solution for some parameter combinations. In this case, only non-regular jobs vacancies are created.

Then, we describe the necessary conditions which define an ex-post segmentation equilibrium. An equilibrium with ex post segmentation arises in Albrecht and Vroman's model ($\lambda = 0$) when the productivity gap between regular jobs and non-regular jobs is relatively large and/or when high-productivity workers occupy a relatively large share of the population. These conditions still hold in this model with OTJ search and the different rate of breaking up when λ is relatively small but positive.

Formally, when high-productivity workers do not want to work in non-regular jobs ($S(n,h) < 0$), the expression for rU(h) in (2.21) goes to:

$$rU(h) = \frac{(r+\delta_s)b + f(\theta)(1-\zeta)\beta y(s)}{r+\delta_s+f(\theta)(1-\zeta)\beta}$$

while the solution for rU(l) is same as (2.20). We replace these solutions in (2.12) and (2.14), and find:

$$S(n,l) = \frac{y(n)-b}{r+\delta_n+f(\theta)\zeta\beta}$$

$$S(s,h) = \frac{y(s)-b}{r+\delta_s+f(\theta)(1-\zeta)\beta}$$

As a consequence, we can define an ex post segmentation equilibrium as follows:

Definition 2.

A steady state equilibrium with ex post segmentation can be described by a vector $\{\theta, \phi, \zeta, u\}$ *which generates an value for high-productivity workers* $rU(h) > y(n)$ *and solves these four equilibrium conditions (2.27)-(2.30):*

$$\frac{c}{(1-\beta)p(\theta)} = \frac{\phi[y(n)-b]}{r+\delta_n+f(\theta)\beta\zeta} \tag{2.27}$$

$$\frac{c}{(1-\beta)p(\theta)} = \frac{(1-\phi)[y(s)-b]}{r+\delta_s+f(\theta)\beta(1-\zeta)} \tag{2.28}$$

$$f(\theta)\zeta\phi u = \delta_n(\mu-\phi u) \tag{2.29}$$

$$f(\theta)(1-\zeta)(1-\phi)u = \delta_s[1-\mu-(1-\phi)u] \tag{2.30}$$

There is no job-to-job in the ex post segmentation equilibrium because there are no mismatched workers.

2.4.2 Equilibrium Configurations

In this section, we will prove that the likelihood of having an equilibrium with cross-skill matching are enhanced when we introduce OTJ search into the model ($0 < \lambda \leq 1$). For the first step, we recall that the existence of an equilibrium with cross-skill matching is guaranteed under two conditions:

(I) firms must be willing to provide both regular jobs and non-regular jobs

(II) the high-productivity workers must be willing to accept employment in non-regular jobs.

It is sufficient to exclude the corner solution where firms exclusively offer non-regular jobs to guarantee condition I. The following result indicates that this requirement places a lower-bound on the share of high-productivity workers and on the difference in productivity between regular and non-regular jobs:

Proposition 1.

We let θ^* as the labour market tightness associated with a single job-type distribution. If the inequality (2.31)

$$\frac{\mu(r+\delta_n)}{(1-\mu)[r+\delta_n+f(\theta^*)\beta]} < \frac{y(s)-y(n)}{y(n)-b} \tag{2.31}$$

is satisfied, then firms offer both regular and non-regular jobs.

Proof. See Appendix.

Here, θ^* denote the unique value of the labour market tightness that solves (A.1) where rU is given by (A.2) in the Appendix. Then, we need to ensure that firms with non-regular jobs and high-productivity workers are willing to match, i.e., $S(n,h) > 0$ in order to guarantee condition II. It is shown this leads to the following condition in the appendix:

Proposition 2.

A necessary condition for the existence of an equilibrium with cross-skill *matching when firms offer both regular jobs and non-regular jobs is that,*

$$\frac{r + \delta_s + f(\theta)\beta\lambda(1-\zeta)}{f(\theta)\beta(1-\zeta)(1-\lambda)} > \frac{y(s) - y(n)}{y(n) - b} \tag{2.32}$$

for the equilibrium values of ζ and θ in this type of equilibrium.

Proof. See Appendix

Generally, we can only verify the above inequality (2.32) once the values of θ and ζ have been determined at the equilibrium. The exception is that the condition (2.32) is always verified in an economy where unemployed and employed job seekers face equal contact rates, i.e., $\lambda = 1$. This is because the RHS of (2.32) approaches infinity λ tends to 1. Thus, the following result holds:

Corollary 1.

There always exists a cross-matching equilibrium with the same search intensity for all job seekers ($\lambda = 1$) when the condition (2.31) is satisfied.

If the inequality (2.31) $\frac{y(s)-y(n)}{y(n)-b} > \frac{\mu(r+\delta_n)}{(1-\mu)[r+\delta_n+f(\theta^*)\beta]}$ is satisfied, then firms offer both regular and non-regular jobs. At the same time, here $\lambda = 1$. This means the inequality (2.32) $\frac{y(s)-y(n)}{y(n)-b} < \frac{r+\delta_s+f(\theta)\beta\lambda(1-\zeta)}{f(\theta)\beta(1-\zeta)(1-\lambda)}$ is satisfied. To sum up, an equilibrium with cross-skill matching exists.

This is the benchmark result, and we can get some intuition for this by considering the expression for $S(n,h)$ which is obtained after replacing $U(h)$ in (2.16) by (2.19):

$$S(n,h) = \frac{y(n) - [b + f(\theta)(1-\lambda)(1-\zeta)\beta S(s,h)]}{r + \delta_n + f(\theta)[\lambda(1-\zeta) + \beta\zeta]} \tag{2.33}$$

When y(n) is larger than the bracketed term that means the opportunity cost of a high-productivity worker who accepts a non-regular job, $S(n,h) > 0$ from the numerator of (2.33). When we rule out the OTJ search ($\lambda = 0$), the opportunity cost is the expected income of a high-productivity worker under ex post segmentation. If we allow OTJ search, this makes the opportunity cost reduce. That is, the opportunity cost is b when $\lambda = 1$ because unemployed and mismatched job seekers face the same contact rates. This implies that $S(n,h) > 0$ immediately, given our assumption that $y(n) > b$.

Proposition 2 provide the necessary condition for the existence of an equilibrium with cross-skill matching when firms offer both regular jobs and non-regular jobs. The next Proposition 3 goes to provide a sufficient condition in terms of λ that exclude ex post segmentation. Again, the idea that a rise in λ lowers the opportunity cost of mismatch for high-productivity workers is used to prove it:

Proposition 3.

For any economy that satisfies the inequality (2.31) there exists a value

$\lambda \in (0, 1)$ *such that the equilibrium is always cross-skill matching for any* $\lambda \in (\bar{\lambda}, 1]$.

Proof. See Appendix.

For $\lambda > \bar{\lambda}$, the equilibrium is always cross-skill matching in this economy. The threshold value $\bar{\lambda}$ is defined the lowest value of λ at which a firm and a high-productivity worker with a non-regular job can deviate from an ex post segmentation equilibrium without incurring a loss.

However, at lower values than $\bar{\lambda}$, notice that the above argument does not exclude the existence of a cross-skill matching equilibrium. If they deviate from an ex post segmentation equilibrium for any $\lambda < \bar{\lambda}$, an individual worker-firm pair will incur a loss by definition. Yet, firms would react by increasing the proportion of non-regular jobs in the economy if all high-productivity workers were to collectively start to accept non-regular jobs. This would make regular jobs more scarce and, hence, for the same value of λ, all high-productivity workers may now find it optimal to accept non-regular jobs. Thus, for some intermediate values of λ the equilibrium may exhibit either ex post segmentation or cross-skill matching. For future purposes, let us denote the lower-bound of this interval as $\underline{\lambda}$. Then, multiple equilibria are possible in the range $[\underline{\lambda},\bar{\lambda}]$.

The last step is that we show that the introduction of OTJ search never leads to the destruction of an cross-skill matching equilibrium, using similar arguments:

Proposition 4.

The same economy will have an equilibrium with cross-skill matching with OTJ search for any $\lambda > 0$*, if an economy that generates an equilibrium with cross-skill matching when* $\lambda = 0$*.*

Proof. See Appendix.

If an economy has a cross-skill matching equilibrium at $\lambda = 0$, then the same economy has a cross-skill matching equilibrium at $\lambda > 0$.

To sum up, the results in Propositions 1. to 4. indicates that OTJ search apparently narrows the scope for ex post segmentation equilibria. This leads to the two alternative equilibrium configurations 1) and 2).

1) While it may exhibit both types of equilibria for the intermediate range of values $\lambda \in [\underline{\lambda}, \bar{\lambda}]$, the economy exhibits cross-skill matching for any $\lambda > \bar{\lambda}$ and ex post segmentation for $\lambda < \underline{\lambda}$.

2) The economy always exhibits cross-skill matching.

In line with the the paper of Dolado et al. (2009), we can prove this even if the model contains different turnover rates to heterogeneous jobs.

2.5 Quantitative implications

2.5.1 Calibration

Finally, in this section, we calibrate our model to match the following statistics for the Japanese labor market (Table 2.3).

Calibration is a method which derive parameter values from actual data, and those parameters characterize the behavior of the economy. Using this model, we investigate about counterfactual scenarios for the effect of skill-biased technical change and the existence of on-the-job search of Japanese labour market. Set the time period to a quarterly. Let μ to be 0.152 reflect the statistics of a fraction of the workers in the population who has the low-productivity level. We set δ_s and δ_n to be 0.0121 and 0.0409 to match the statistics of the quarterly transfer rate from regular job to unemployment and the quarterly transfer rate from non-regular job to unemployment. The discount factor r is set to be 0.0053 because the real 10 years' government bond yield was 2.1 on average from 1990 to 2013, which we take from Matsuoka (2015). The matching function is assumed to be Cobb-Douglas, $f(\theta) = M\theta^a$ with $M = 0.362, a = 0.542$ which is estimated in Cabinet Office (2016). We set instantaneous value of leisure b to be 0.4, which we take from Esteban-Pretel et al. (2010) that estimates the search model in the Japanese labour market. We normalize $y(n) = 1$. The bargaining power is set to be 0.5 as usual. Let λ (relative search intensity captured by the parameter) to be 0.9153 reflect the ratio of quarterly transfer rate from non-regular job

(0.1459) to regular job to quarterly transfer rate from unemployment to regular job (0.1594).

The remaining parameters are $y(s)$ (the productivity of high-productivity worker) and c (the instantaneous cost of a vacancy) which we cannot easily guess from the data and the previous literature related with Japan. We set the two parameters to match the following two calibration targets (Table 2.4): (1) an average hourly wage with low-productivity level/an average hourly wage with high-productivity level (0.8481) and (2) an average hourly wage with non-regular job/an average hourly wage with regular job (0.8568). The sample of (1) and (2) is young workers aged 20-29, not being studying, and is from Longitudinal Survey of Adults in the 21st Century (2012 Cohort) and Employment Status Survey (2002), respectively. To calculate the corresponding model statistics, we solve our model for the stationary equilibrium characterized above. The parameters are estimated by minimizing the sum of squared distances between the simulated moments and the data moments. The results of $y(s)$ and c become 1.096 and 0.183, respectively.

The Table 2.5 shows the result in the benchmark model solved computationally. This table is the labour market outcomes in the benchmark model. Overall, the value of labour market outcomes matches the Japanese economy well. Unemployment rate is usually 3-4% and the value of the outcome matches well. In terms of labour market tightness θ, the statistics of vacancies over unemployed are usually around 1, and the the outcome value of θ is a little big. The other outcomes basically seem to be appropriate as well.

The parameter reflects the Japanese labour market well.

2.5.2 Comparative Statistics

Our next objective is to show that our model delivers predictions about the relationship between some parameters and labour market outcomes. We consider two counterfactual scenarios in the Japanese labour market. One is skill-biased technological change occurs. The other is the use of the internet for job search increases.

The Figure 2.1 shows the result of comparative statistics of $y(s)$ to consider skill-biased technological change. Note that there is a cross-productivity equilibrium during calculation by changing the parameters. This tells as the effect of skill-biased technical change. Skill-biased technical change means high-productivity workers are only affected by technological change. The baseline parameter of $y(s)$ is 1.1. The baseline parameter of $y(n)$ is 1. The value of $y(s)$ is moved between 1 and 2. As the value of $y(s)$ becomes larger, the relative value of regular jobs increase, and the unemployment rate of low-productivity workers increase, inequality grows, and welfare decreases.

The Figure 2.2 shows the result of comparative statistics of λ. We give thought to the increase of the use of the internet for job search. If the use of internet increase when we look for a job, people who have a job may benefit more from that. A number of people who use internet are growing and more ads are posted on the website of public employment service. We consider the change of the increase of the use of internet for job search as the change of the value of the search intensity λ. Note that there is a cross-productivity equilibrium during calculation by changing the parameters. The baseline parameter of λ is 0.93. We moved the value of the search intensity λ between 0 and 1. As the value of λ becomes larger, a high productivity worker becomes happier to work for a non-regular job. The unemployment rate of low-productivity workers increase, inequality grows, and welfare decreases. However, compared with the previous figures, we can see the interval of the vertical axis is mush small in this figure. Even if $\lambda = 0$ or $\lambda = 1$, the effect to add the OTJ search to the outcome of this economy is not affected largely.

2.6 Conclusion

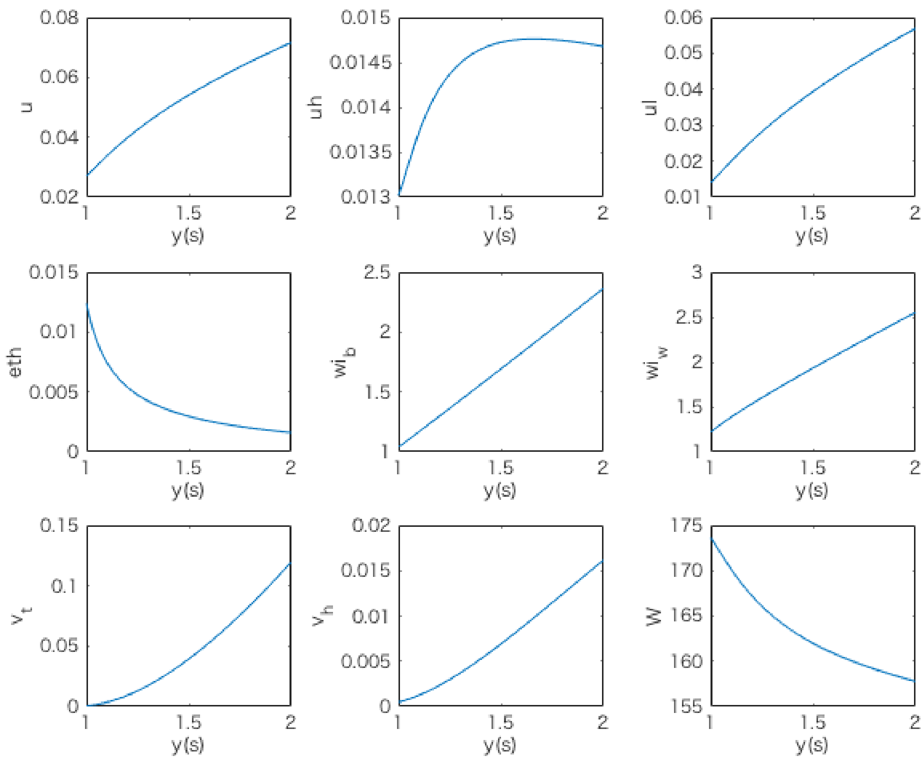
The objective of this chapter is to construct a labour market matching model with heterogeneous jobs and workers that incorporates on-the-job search and different turnover rates. This paper extends the model of Dolado by incorporating different turnover rates to heterogeneous jobs. The number of studies for Japan has been limited. We use this theoretical model to investigate about counterfactual scenarios and welfare effects in Japanese labour market.

In line with the paper of Dolado et al. (2009), we prove that the introduction of OTJ search enhances the likelihood of having an equilibrium with cross-skill matching even if the model contains different turnover rates to heterogeneous jobs. The quantitative implications give the prediction that the unemployment rate of low-productivity workers increases, inequality grows, and welfare decreases as skill-biased technical change proceeds. The simulation of the model also shows that the effect to add the OTJ search to this economy is not affected largely.

In the future, it may be possible that the model incorporates the endogeneity of job destruction. This allows to examine the impact of exogenous macro shocks on labour market outcomes through not only job creation but also job destruction, although this may reduce the simplicity and clarity of the model. Also, we can extend the model slightly to consider various policy effects. I would like to consider for the future study.

Figures and tables

Figure 2.1: $u, \theta, \phi, \zeta, \psi$ as a function of $y(s)$

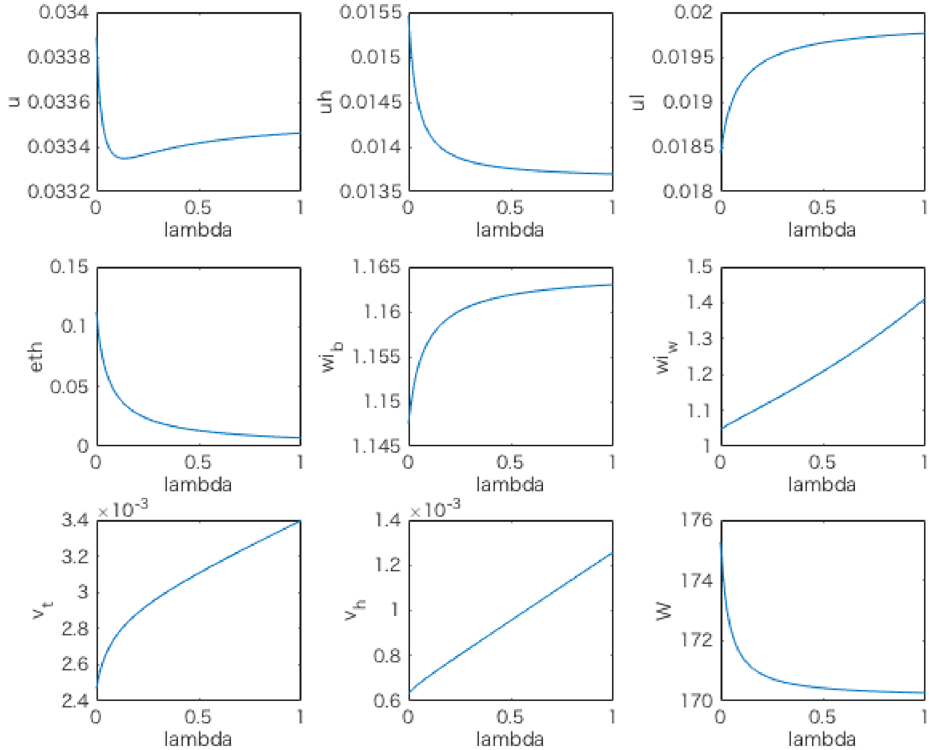


Note: The baseline parameter of y(s) is 1.1.

W_{ib} : Between group wage inequality, W_{iW} : Within group wage inequality

V_t : Wage variance for all, V_h : Wage variance for high productivity worker

Figure 2.2: $u, \theta, \phi, \zeta, \psi$ as a function of λ



Note: The baseline parameter of λ is 0.93.

W_{ib} : Between group wage inequality, W_{iW} : Within group wage inequality

V_t : Wage variance for all, V_h : Wage variance for high productivity worker

Table 2.1: The first job just after leaving school by educational attainment in Tokyo

	% of regualr workers	% of non-regular workers	Unemployment rate
High school graduates	45.1	41.7	7.1
2-year college graduates	61.5	30.0	5.9
4-year university graduates	76.2	14.9	6.1
High school dropouts and junior high school graduates	7.8	58.9	27.8
2-year college dropouts and 4-year university dropouts	7.5	70.9	15

Data: The Work Style Survey of Tokyo (2011)

Sample: males and females aged 20-29.

Note: The survey was conducted in only Tokyo, so the distribution of characteristics may be different from those of Japan.

"% of regular workers" means the proportion of non-regular workers among population.

Table 2.2: The difference of labour market outcomes between dropouts and graduates

	Hourly wage	Unemployent rate
Dropouts/Graduates: High school	90	2.4
Dropouts/Graduates: 2-year college	86	2.2
Dropouts/Graduates: 4-year university or more	74	1.9

Data: Longitudinal Survey of Adults in the 21st Century (2012 Cohort)

Sample: males and females aged 20-29.

Note: Hourly wage are calculated only among employed workers of each group.

Table 2.3: Summary of statistics for moments (except for $y(s)$ and c)

Statistics	Statistics for the moments	Source	Related parameters
0.1525	A fraction of the low-productivity workers in the population	LSA (2012 Cohort)	μ
0.0121	Quarterly transfer rate from regular job to unemployment	LFS	δs
0.0409	Quarterly transfer rate from non-regular job to unemployment	LFS	δn
0.9288	Quarterly transfer rate from non-regular job to regular job/that from unemployment to regular job	LFS	λ
0.362	Estimated parameters of matching function	Cabinet Office (2016)	M
0.542	Estimated parameters of matching function	Cabinet Office(2016)	a
0.0053	Discount factor	Matsuoka (2015)	r
0.4	Instantaneous value of leisure	Esteban-pretel et al. (2011)	b
0.5	Bargaining power	Hornstein et al. (2005)	β

Table 2.4: Parameter values for $y(s)$, c

Statistics	Statistics for the moments	Source
0.848	An average hourly wage with low-productivity level/an average hourly wage with high-productivity level	LSA(2012 Cohort)
0.857	An average hourly wage with temporary job/an average hourly wage with regular job	ESS(2002)

Parameter	Statistics for the moments	Value
y(s)	The output from skilled job	1.096
c	The instantaneous cost of a vacancy cost	0.183

Table 2.5: Labour Market Outcomes

Labour market tightness θ	2.333
Share of temporary vacancies ζ	0.359
Share of less-productivity unemployed ϕ	0.590
Share of unemployed job seekers ψ	0.826
Unemployment rate u	0.033
Mass of mismatched workers e(t, h)	0.008
Unemployment rate highly-productivity workers u(h)	0.014
Unemployment rate less-productivity workers u(l)	0.020
Wage of less-productivity workers w(t, l)	0.922
Wage of highly-productivity workers in temporary jobs w(t, h)	0.779
Wages of highly-productivity worker in regular job w(r, h)	1.075
JTJ total separations	0.261
Wage Inequality	
between groups	1.163
Within groups	1.380
Variance of wages	
Total	0.003
Highly-productivity workers	0.001

Appendix

Proof of proposition 1

Firstly, we consider an equilibrium in which firms offer exclusively non-regular jobs ($\zeta = 1$). Then, the value functions of vacancies are given by:

$$rV(n) = -c + p(\theta)(1-\beta)\frac{y(n) - rU}{r + \delta_n} = 0 \tag{A.1}$$

where

$$rU = b + f(\theta)\beta\frac{y(n) - rU}{r + \delta_n} \tag{A.2}$$

is the identical outside-option value of high-productivity and low-productivity workers. We let θ^* denote the unique value of the labour market tightness that solves (A.1) where rU is given by (A.2). It must hold to exclude an equilibrium of this type:

$$rV(s) = -c + p(\theta^*)(1-\beta)(1-\mu)\frac{y(s) - rU}{r + \delta_s} > 0 \tag{A.3}$$

that is, a deviant firm can make positive profits by opening a regular job. Then, we can get the condition (A.4) by using (A.1) and (A.3):

$$y(n) - rU < (1-\mu)[y(s) - rU] \tag{A.4}$$

Eventually, we solve for in (A.2) and replace it into (A.4). Then, we can get (2.31).

Proof of proposition 2

In an equilibrium with cross-skill matching, a positive surplus is needed to be generated for all three possible types of matches. First, to show that $S(n,l)$ and $S(s,h)$ is positive, we substitute (2.20) into the RHS of (2.12) yields:

$$S(n,l) = \frac{y(n) - b}{r + \delta_n + f(\theta)\zeta\beta} \qquad (A.5)$$

$$S(s,h) = \frac{\alpha_{1n}[y(s) - b] + f(\theta)\beta\zeta[y(s) - y(n)]}{\alpha_{2n}\alpha_{3s} + (r + \delta_s)f(\theta)\beta\zeta(1 - \lambda)} \qquad (A.6)$$

where $S(n,l)$ and $S(s,h)$ are positive because $y(s) > y(n) > b$.

Then, we obtain $S(n,h)$ from (2.16) by using (2.14) and replacing $S(s,h)$ by (A.6). It becomes after some algebraic manipulation:

$$S(n,h) = \frac{\alpha_{3s}[y(n) - b] - f(\theta)\beta(1 - \zeta)(1 - \lambda)[y(s) - b]}{\alpha_{2n}\alpha_{3s} + (r + \delta_s)f(\theta)\beta\zeta(1 - \lambda)} \qquad (A.7)$$

We finish to prove as we notice that condition (2.32) is equal to $S(n,h) > 0$, that is, $\alpha_{3s}[y(n) - b] > f(\theta)\beta(1 - \zeta)(1 - \lambda)[y(s) - b]$.

Proof of proposition 3

We consider an economy with a unique ex post segmentation equilibrium when $\lambda = 0$. Here, the associated equilibrium values for θ and ζ are denoted by (θ^e, ζ^e). The expected value of an unemployed high-productivity worker in this economy is:

$$rU(h) = b + f(\theta^e)\beta(1-\zeta^e)S(s,h) \qquad (A.8)$$

where $S(s,h) = \frac{y(s)-b}{r+\delta_s+f(\theta^e)(1-\zeta^e)\beta}$. It must be that because high-productivity workers refuse to work in non-regular jobs in this case. This means that it requires that $rU(h) > y(n)$. Then, we consider the same economy when $\lambda > 0$. The minimum wage at which a high-productivity worker would be willing to accept a non-regular job can be derived in this case. We define this minimum wage as $\underline{w}$. A firm with a non-regular job and a high-productivity worker can both obtain a gain if they accept to match and fix some wage $w \in (\underline{w}, y(n))$ if $\underline{w} < y(n)$. The ex post segmentation equilibrium would cease to exist in such a case. In the below, it is shown there always exists some value of $\lambda < 1$ for which this is the case.

We denote $W_n(w)$ as the lifetime income of a deviant high-productivity worker who accepts an arbitrary wage w to work in a non-regular job. As mismatched workers will quit a non-regular job when they find a regular job, the value equation for $W_n(w)$ satisfies:

$$rW_n(w) = w + \delta[U(h) - W_n(w)] + \lambda f(\theta^e)(1-\zeta^e)\beta S(s,h) \qquad (A.9)$$

Here, (A.9) is strictly increasing in w. We combine (A.8) and (A.9) and find that:

$$W_n(w) - U(h) = (w - b - f(\theta^e)(1-\lambda)(1-\zeta^e)\beta S(s,h))/(r+\delta_n) \quad (A.10)$$

Similarly, the asset value of a firm with a non-regular job that offers a high-productivity worker a wage w satisfies:

$$J_n(w) = \frac{y(n) - w}{r + \delta_n + \lambda f(\theta^e)(1 - \zeta^e)} \qquad (A.11)$$

$\underline{w}$ can be defined implicitly by the following condition:

$$W_n(\underline{w}) = U(h) \qquad (A.12)$$

It follows that the solution is given by from (A.10):

$$\underline{w} = b + f(\theta^e)(1 - \lambda)(1 - \zeta^e)\beta S(s, h) \qquad (A.13)$$

that is, firms need to pay high-productivity workers at least their opportunity cost. What is more, (A.11) implies that a firm with a non-regular job would like to offer this minimum acceptable wage as long as $y(n) - \underline{w} > 0$. We note that for $\lambda = 0, \underline{w} = rU(h)$ and $J_n(w) < 0$ while for $\lambda = 1, \underline{w} = b$ and $J_n(w) > 0$. If we put differently, if workers cannot perform OTJ search, the ex post segmentation equilibrium is well defined because a firm with a non-regular job would make negative profits if it were to pay a high-productivity worker his/her opportunity cost $\underline{w} = rU(h)$ .On the contrary, an unemployed job seeker and a mismatched worker have the same chances to match with a regular job when $\lambda = 1$. Hence, the worker would like to accept this job when $w \geq b$. Because $y(s) > b$, a firm with a non-regular job can thus make a high-productivity worker an offer $w \in (b, y(n))$ such that the worker and the firm are strictly better off when they deviate. From here, it means that there exists a $\bar{\lambda} \in (0,1)$ such that $J_n(w) > 0$ for any $\lambda > \bar{\lambda}$, for any pair $(\theta^e, \zeta^e) \in (0, \infty) \times (0,1)$. The RHS of (A.13) defines ζ as a continuously decreasing function of λ that maps $[0,1]$ onto $[b, rU(h)]$ for given values of θ and ζ. Therefore, as $rU(h) > y(n) > b$, there exists a unique value $\lambda \in (0,1)$ which is denoted by $\bar{\lambda}$, such that $y(n) - \underline{w} = 0$ whereas $J_n(w) > 0$ for all $\lambda > \bar{\lambda}$.

Proof of proposition 4

Again, $\underline{w} = rU(h)$ for $\lambda = 0$ holds but this time we have that $rU(h) < y(n)$ because we think the case that the equilibrium exhibits cross-skill matching. Thus, it must be $y(n) - w > 0$ for any $\lambda > 0$ because $\underline{w}$ is decreasing in λ. Accordingly, a firm with a non-regular job and a high- productivity worker incur a loss if they deviate from the equilibrium in this case.

Chapter 3

A search matching model with heterogeneous jobs and workers, and endogenous labour market participation

Abstract

We construct a search matching model, which incorporates endogenous labour market participation into the segmented labour market by combining the models of Albrecht and Vroman (2002) and Garibaldi and Wasmer (2005) to investigate how to raise the labour force participation rate in the segmented labour market. The quantitative implications predicts that the decrease of the value of home production, such as the rapid spread of autonomous robotic

vacuum cleaners and cheap housekeeping services, leads an increase in employment.

Key Words: Segmented Labor Markets; Job Search; Household Production

JEL Codes: J4; J6

3.1 Introduction

There are significant gaps between the labour-market participation of men and women of working age, owing to the unequal distribution of household tasks and career breaks in an increasingly volatile world of work and production. In addition, a significant percentage of older persons continue working despite being older than the established retirement age. The labour market is sometimes segmented in a way that a large percentage of prime-age males work in regular work, and non-regular workers mostly consist of females, old people and young people, as we see the Japanese case in the Chapter 1.

The sustainability of many welfare-state programs requires high employment and high labour force participation in order to strengthen the tax base and avoid distortions. It is very important to keep high labour force participation especially for the government of countries where the number of children and labour force population are decreasing. However, we know little about the interactions between workers' participation decisions and firms' incentives to create jobs in the segmented labour market.

We construct a search matching model that incorporate endogenous labour market participation in the segmented labour market in this chapter. We use this model and do a counterfactual simulation by changing the parameters of the model. We consider how to raise the labour force participation rate in the segmented labour market, what increases employment in regular jobs and non-regular jobs and what decreases the unemployment rate of low-skilled workers and high-skilled workers. This model concentrate on understanding endogenous labour market participation in the segmented labour market and do not consider immigrants. There are not many immigrants in Japan. The data of Japan is suitable to use to simulate this model. The quantitative implications predicts that a decrease in the value obtained from home production leads to an increase in employment. We think the rapid spread of autonomous robotic vacuum cleaners and cheap housekeeping services decrease the value obtained from home production. The data of Japan is applied to do the counterfactual simulation.

The two closest papers to my research are Albrecht and Vroman (2002) and Garibaldi and Wasmer (2005). The model of Albrecht and Vroman (2002) is certainly the amendable and concise one. It assumed two worker types, low-skill and middle-skill. The number of workers in each type were decided exogenously in a given market. There were frictions in the process by which vacancies and unemployed workers contacted one another. The surplus generated by a match was given to a worker and a firm according to the Nash bargaining solution. The flow output of a match depended on the skill level of the worker and on the job type, which was decided by the firm when it created the vacancy. The model of Garibaldi and Wasmer (2005) incorporate the endogenous labour market participation features into standard models of search. In their world, a worker's decision to participate was determined by an entry decision and an exit decision. They included several categories of individuals: attached employed workers, unattached employed workers, marginally attached non-employed workers and true non-participants, and the solution was characterised using only three equations.

There are a few papers related to my research. For example, the model of Satchi and Temple (2008) was about a matching model with an endogenous labour force participation decision in segmented labour markets. They developed a general equilibrium model with matching frictions in the urban labour market, the possibility of self-employment in the informal sector, and a scope for rural-urban migration. It featured an urban manufacturing sector, an urban informal sector, and rural agriculture. Underemployment arose because of matching frictions in the formal sector labour market. In equilibrium, workers not employed in the formal sector could choose between self-employment in the informal sector and working in the agricultural sector. This is a matching model with an endogenous labour force participation decision, but the main conceptual difference in my model is that the value of the home production is endogenous, rather than simply an exogenous value of leisure, and this will vary with the extent of agricultural employment.

Henriksen (2017) set up a general equilibrium model with ex ante homogeneous firms and heterogeneous workers in a two-homogeneous country setting. The workers and firms matched in a frictional labour market, with the workers endogenously determining their search intensity. This model focused on frictions in the labour market and international trade and not on the segmented labour market, but they also included the concept of non-participation in the model.

Boeri and Garibaldi (2005) investigated the border between formal employment, shadow employment, and unemployment in an equilibrium model of the labour market with market frictions. Their model had two sectors in the labour market, heterogeneous workers, and endogeneity of the choice of both workers and firms to go idle in an equilibrium model of the labour market with market frictions. They considered the state of unemployed, but did not consider the state of non-participation.

Barnichon and Figura (2013) were interested in estimating a matching function and constructed a generalised matching function that explicitly allowed for worker heterogeneity and segmentation in the labour market, because they were concerned the matching function implied that the job finding rate depended only on labour market tightness.

As far as I know, there are no papers that construct a search matching model that incorporates endogenous labour market participation in the segmented labour market. We constructed the model by combining the models of Albrecht and Vroman (2002) and Garibaldi and Wasmer (2005) to investigate how to raise the labour force participation rate in the segmented labour market, what increases employment in regular jobs and non-regular jobs, and what decreases the unemployment rate of low-skilled workers and high-skilled workers. Also, the number of studies for Japan are limited. This research is useful to do the counterfactual simulation for the debate of the labour force participation rate for women and old people in Japan.

The structure of this paper is as follows: Section 3.2 explains the model. Section 3.3 shows the equilibrium of the model. Section 3.4 gives the quantitative implications. Finally, section 3.5 provides a conclusion to this work.

3.2 The Model

3.2.1 Basic Assumptions

The model in this chapter is different from the model in the second chapter in that it includes the state of full-time home production.

We consider a continuous time economy in a steady state. There is a unit mass of infinitely lived workers and a mass of infinitely lived firms. Both agents are risk neutral, have a time-separable intertemporal utility function, and discount the future at rate $r > 0$. Workers and firms are interested in maximising their expected discounted income. There is not on-the-job search by matched workers for the simplicity.

The distribution of productivities across workers is exogenous. Specifically, we assume a two-point distribution: a fraction μ of the workers in the population have the low-productivity level, $y(n)$, a fraction $1-\mu$ have the high-productivity level, $y(s)$. The measure of workers is normalised to one.

Likewise, there are two types of jobs, non-regular jobs and regular jobs. Jobs are either vacant or filled. Filled non-regular jobs and regular jobs break up at the exogenous rate δ_n and δ_s, respectively. We assume the rate at which non-regular jobs break up, δ_n, is larger than the rate at which regular jobs break up, δ_s. A non-regular job and regular job have the minimum productivity required of a worker hired for the jobs. A minimum productivity requirement of a non-regular job is the low-productivity level, $y(n)$, that is, a non-regular job can be filled by either type of worker. In contrast, a requirement of a worker's productivity for a regular job is the high-productivity level, $y(s)$. Thus, a regular job can only be filled by high-productivity workers. The technology is such that when a job is filled with a non-regular job by a low-productivity worker or a high-productivity worker, the output produced is given by $y(n)$. When a job was filled with a regular job by a low-productivity worker, the output produced is given by 0. When a job is filled with a regular job by a high-productivity worker, the output produced was given by $y(s)$.

When a job is filled, a firm pay $w_2(i, j, x)$ to a worker. When a job is unfilled, a firm pay the instantaneous cost of a vacancy c. When a vacant job is created, its skill requirement is chosen to maximise the value of the vacancy. In addition to that, the state of full-time home production is included in this model. Workers wanting to participate in the labour market undertake a time-consuming search. The time allocation problem of the worker is defined as follows: h_w is the number of hours actually worked, h_s is the search intensity necessary to obtain a job, and h_h is the choice of hours spent in leisure/home production. The time constraint is thus

$$1 = h_w + h_s + h_h \text{ with } h_w \in \{0, e_i\} \text{ and } h_s \in \{0, b_2\}$$

where e_i is the inelastic number of hours worked for a job in type i, and b_2 is the inelastic number of hours spent looking for a job. There is no on-the-job search, so the job search and employment are mutually exclusive activities. It follows that in the three states W, U, and H (where W is employment, U is unemployment, and H is full-time home production), the flow utility of agents is given by

$$\begin{aligned} w(i, j, x) &= (1 - e_i)x + w_2(i, j, x) \\ b &= (1 - b_2)x \\ d &= x \end{aligned}$$

where x is home productivity and $w_2(i, j, x)$ is the total wage received for the e hours worked. Throughout this section, we assumed $1 \geq e_r \geq e_n \geq b_2$. We assume the number of hours worked for a regular job is not less than the number of hours worked for a non-regular job. In addition, in line with the paper of Garibaldi and Wasmer (2005), we further assume a fully indivisible labor supply as one in which entering the labor market involves a sacrifice of home production regardless of the employment status and avoid unimportant constant terms but without implication for the results, so that we assume $e_i = b_2 = 1$ for a normalization. It is important to note that, following Becker (1965), home production or leisure consumption are formally expressed in the same way (raising an individual's utility). We assume that there is some heterogeneity in the valuation of non-market activities. Concretely, home productivity x is heterogeneous and stochastic, and its value changed according to a Poisson process at rate π. Conditional on the arrival rate of a shock, the value of home productivity takes a value from a continuous distribution $f(x)$ and $c.d.f F(x)$, defined over the support $x \in [xmin, xmax]$. For a non-employed individual, the participation decision is whether to spend 0 or b_2 hours in the labour market, whereas the participation decision for an employed worker is whether to work e_i or 0 hours: our model is an extensive margin model and we ignore such issues as the intertemporal elasticity of substitution, bargaining over hours, and work sharing.

3.2.2 Matching

Job seekers and firms with vacant jobs are matched together in pairs through an imperfect matching technology. Like Albrecht and Vroman (2002), we assume that the matching process is undirected, and we only allow for job search during unemployment.

The total flow of random contacts between a job seeker and a firm is determined by a standard CRS meeting function:

$$m[v(n) + v(s), u(l) + u(h)]$$

where $u(j)$ is the mass of unemployed workers in type j and $v(i)$ was the mass of vacancies in type i. Finally, we assume that m[.,.] is strictly increasing in both arguments, and we define the effective labour market tightness by $\theta = [v(n) + v(s)]/[u(l) + u(h)]$. Accordingly, we can write the contact rate of a firm as $p(\theta) = m(1, 1/\theta)$, while the contact rate of a job seeker is equal to $f(\theta) = \theta p(\theta)$ during unemployment. The properties of m[.,.] guarantees that $p'(\theta) < 0$, $f'(\theta) > 0$, and we assume that $\lim_{\theta \to 0} p(\theta) = \lim_{\theta \to \infty} f(\theta) = \infty$ and $\lim_{\theta \to \infty} p(\theta) = \lim_{\theta \to 0} f(\theta) = 0$. Formally, let $U(j, x)$ denote the value of unemployment for a worker in type j and type x, who produce x units of utility per unit of time if they are engaged in home production, and $V(i)$ denote the value of a vacant job in type i. Similarly, let $J(i, j, x)$ and $W(i, j, x)$ denote the proceeds for the firm and the worker from a match that combine a job in type i, a worker in type j and a type of home production, x. In any match with a positive match surplus $S(i, j, x) \equiv W(i, j, x) + J(i, j, x) - V(i) - \max\{U(j, x), H(j, x)\}$, the constant wage $w(i, j, x)$ then satisfies the following sharing rule based on the typical Nash bargaining solution:

$$(1 - \beta)[W(i, j, x) - \max\{U(j, x), H(j, x)\}] = \beta[J(i, j, x) - V(i)] \qquad (3.1)$$

Condition (3.1) rules out any wage differences among identical workers in the same type of job and the same type of home productivity.

3.2.3 Asset Values

We are now able to define the asset value equations of workers and firms. Let $\zeta = v(n)/[v(n) + v(s)]$ denote the share of non-regular job vacancies. Then the asset value equation of a high productivity unemployed worker was given by:

$$\begin{aligned} rU(h,x) &= b + f(\theta)[\zeta[\max\{W(n,h,x), U(h,x)\} - U(h,x)] \\ &+ (1-\zeta)[\max\{W(s,h,x), U(h,x)\} - U(h,x)]] \\ &+ \pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x')\} dF(x') \end{aligned} \tag{3.2}$$

The equation (3.2) states that the value of a high productivity unemployed worker is the sum of the utility flow, the capital gain of being an employed worker who works for a non-regular work with the arrival rate $f(\theta)\zeta$, and the capital gain of being an employed worker who works for a regular work with the arrival rate $f(\theta)(1-\zeta)$, and the capital gain (or loss) from a home production shock, after which workers reoptimize (they decide whether to keep looking for the job U' or leave activity H').

$$\begin{aligned} rU(l,x) &= b + f(\theta)\zeta[\max\{W(n,l,x), U(l,x)\} - U(l,x)] + \\ &\pi \int_{xmin}^{xmax} \max\{U(l,x'), H(l,x')\} dF(x') \end{aligned} \tag{3.3}$$

The equation (3.3) states that the value of a low productivity unemployed worker is the sum of the utility flow, the capital gain of being an employed worker who worked for a non-regular work with the arrival rate $f(\theta)\zeta$, and the capital gain (or loss) from a home production shock, after which workers reoptimize (they decide whether to keep looking for the job U' or leave activity H').

$$rW(n,h,x) = w(n,h,x) + \delta_n[\max\{(U(h,x) - W(n,h,x)), (H(h,x) - W(n,h,x))\}]$$
$$+\pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x'), W(n,h,x')\} dF(x') \tag{3.4}$$

$$rW(n,l,x) = w(n,l,x) + \delta_n[\max\{(U(l,x) - W(n,l,x)), (H(l,x) - W(n,l,x))\}]$$
$$+\pi \int_{xmin}^{xmax} \max\{U(l,x'), H(l,x'), W(n,l,x')\} dF(x') \tag{3.5}$$

$$rW(s,h,x) = w(s,h,x) + \delta_s[\max\{(U(h,x) - W(s,h,x)), (H(h,x) - W(s,h,x))\}]$$
$$+\pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x'), W(s,h,x')\} dF(x') \tag{3.6}$$

The equation (3.4) states that the value of a high productivity non-regular worker is the sum of the wage, the capital loss of being an unemployed worker with the probability δ_n, and the capital gain (or loss) from a home production shock, after which workers reoptimize (they decide whether to hold onto the job $W(n,h,x')$, look for another job $U(h,x')$, or leave activity $H(h,x')$). The equation (3.5) and the equation (3.6) is the value of a low productivity non-regular worker and the value of a high productivity regular worker, respectively.

$$rH(l,x) = d + \pi \int_{xmin}^{xmax} \max\{U(l,x'), H(l,x')\} dF(x') \quad (3.7)$$

$$rH(h,x) = d + \pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x')\} dF(x') \quad (3.8)$$

The equation (3.7) is the value of a worker engaging in full-time home production. This is the sum of the utility flow and the capital gain (or loss) from a home production shock, after which workers reoptimize their behaviour. Next, to define the asset value equations of vacant jobs, we let $\phi = u(l)/[u(l)+u(h)]$ denote the share of low productivity workers in the pool of unemployed. Accordingly, we can write the asset value equation for a non-regular job vacancy, V(n), as:

$$rV(n) = -c + p(\theta)\{\phi[J_{nl}^e - V(n)] + (1-\phi)\max\{(J_{nh}^e - V(n)), 0\}\} \quad (3.9)$$

$$J_{nl}^e = \frac{\int_{xmin}^{x_v^l} J(n,l,x')dF(x')}{F(x_v^l)} \quad (3.10)$$

$$J_{nh}^e = \frac{\int_{xmin}^{x_v^h} J(n,h,x')dF(x')}{F(x_v^h)} \quad (3.11)$$

where J_{nl}^e and J_{nh}^e are the expected value of the job given wage bargaining, and x_v^l and x_v^h are the cutoff points which are described in 3.2.4. The term J_{nl}^e and J_{nh}^e takes into account the density of workers actively looking for a job in the market. Workers in the interval $[xmin, x_v^l]$ and $[xmin, x_v^h]$ are actively looking for a job.

Similarly, the corresponding expression for a regular job vacancy, V(s), satisfies:

$$rV(s) = -c + (1-\phi)p(\theta)[J^e_{sh} - V(s)] \tag{3.12}$$

$$J^e_{sh} = \frac{\int_{xmin}^{x_v^h} J(s,h,x')dF(x')}{F(x_v^h)} \tag{3.13}$$

The equations (3.9) and (3.12) of the asset value equation for a vacancy of a regular job and a non-regular job do not depend on x.

Finally, the asset values of filled jobs verify:

$$\begin{aligned} rJ(s,h,x) = y(s) - w(s,h,x) + \delta_s[V(s) - J(s,h,x)] \\ +\pi \int_{xmin}^{xmax} \max\{J(s,h,x'), V(s)\}dF(x') \end{aligned} \tag{3.14}$$

$$\begin{aligned} rJ(n,l,x) = y(n) - w(n,l,x) + \delta_n[V(n) - J(n,l,x)] \\ +\pi \int_{xmin}^{xmax} \max\{J(n,l,x'), V(n)\}dF(x') \end{aligned} \tag{3.15}$$

$$\begin{aligned} rJ(n,h,x) = y(n) - w(n,h,x) + \delta_n[V(n) - J(n,h,x)] \\ +\pi \int_{xmin}^{xmax} \max\{J(n,h,x'), V(s)\}dF(x') \end{aligned} \tag{3.16}$$

The equation (3.14) states that the value of filled job with a high productivity regular worker is the sum of utility flow, the capital loss when an employer leaves with the probability δ_s, and the capital gain after a change in workers' characteristics—possibly leading to job destruction if the worker quits.

3.2.4 Reservation Strategies

This section explains the characteristics of the value functions and the workers' strategies based on the value of x.

Firstly, the value functions $W(i, j, x)$, $U(j, x)$, and $H(j, x)$ are monotonically non-decreasing functions of x. This is obvious because the worker with $W(n, h, x_1)$ is more advantageous in all aspects than the worker with $W(n, h, x_2)$ when $x_1 \geq x_2$, for example. The detail is in Appendix A.

Also, the value functions $W(i, j, x)$, $U(j, x)$, and $H(j, x)$ are piecewise linear functions of x. Between the cut-off points, using the wages derived in section 3.2.5, we can rewrite the Bellman equations of the employed (attached and unattached), unemployed and those not in the labour force. This is the simultaneous linear equation, so we can get the constant value of the slopes.

Next, we introduce the cut-off points x_v^l, x_q^l, x_q^{hn}, x_q^{hs}, and x_v^h defined by

$$
\begin{aligned}
U(l, x_v^l) &= H(l, x_v^l) \\
W(n, l, x_q^l) &= H(l, x_q^l) \\
W(n, h, x_q^{hn}) &= H(h, x_q^{hn}) \\
W(s, h, x_q^{hs}) &= H(h, x_q^{hs}) \\
U(h, x_v^h) &= H(h, x_v^h)
\end{aligned}
$$

In the following, we assume that the interval between $xmax$ and $xmin$ is enough large to consider the positional relations of the value functions. The

Figure 3.1 describes the following discussion by a figure.

Low-productivity workers behave in the same way as workers in Garibaldi's (2005) paper. In this case, it is always $W(n, l, x) > U(l, x)$. Also, above x_q^l one finds only workers engaged in full-time home production, that is non-participants. Between the two cut-off points x_v^l and x_q^l, one finds two categories of workers, some of them are non-participants but do not search for a job and those that are employed workers. Finally, below x_v^l one finds both unemployed job seekers and employed workers.

High-productivity workers behave differently because we must think two types of jobs. When x is such that $W(s, h, x) \leq H(h, x)$, that is, above x_q^{hs}, one finds only workers engaged in full-time home production or non-participants. Here, we think the case when x is such that $W(s, h, x) > H(h, x)$. In this case, when x is such that $W(n, h, x) < U(h, x)$, $W(s, h, x) > U(h, x)$. When x is such that $W(n, h, x) > U(h, x)$, $W(s, h, x) > W(n, h, x)$. (Appendix B) The rest we needed to think is the positional relation of $W(n, h, x)$ and $U(h, x)$ when x is such that $W(s, h, x) > H(h, x)$. From appendix C, we know that the value of $W(n, h, x) - U(h, x)$ is independent of x. This means the parameters decide which of $W(n, h, x)$ and $U(h, x)$ is higher, when $W(s, h, x) > H(h, x)$. Thus, when x is such that $W(s, h, x) > H(h, x)$, $W(s, h, x)$ is the highest and $H(h, x)$ is the lowest among the value functions for high-productivity workers, and the positional relationship of $W(n, h, x)$ and $U(h, x)$ is dependent on the parameters of the model.

Our interest is the case of a cross-skill matching equilibrium, because the real world is so. We derive a sufficient condition for a cross-skill matching equilibrium in Appendix D.

Proposition 1

A sufficient condition for a cross-skill matching equilibrium ($W(n,h,x) > U(h,x)$) to exist is

$$\beta[y(n) + \pi V(n) - rV(n)] - \beta[b + f(\theta)\frac{1-\zeta}{r+\delta_s}\beta[y(s) - y(n)]] \geq 0$$

3.2.5 Wage Determination

By substituting the value functions (3.2)-(3.8) and (3.14)-(3.16) into the equation (3.1), the expression for wages, the equation (3.17)-(3.22), came easily. The detail is in Appendix E.

$$\begin{aligned} w^a(n,l,x) &= \beta[y(n) + \pi V(n) - rV(n)] \\ &+(1-\beta)[b + f(\theta)\zeta[W(n,l,x) - U(l,x)]] \end{aligned} \tag{3.17}$$

$$\begin{aligned} w^a(s,h,x) &= \beta[y(s) + \pi V(s) - rV(s)] \\ +(1-\beta)[b + f(\theta)\{\zeta[\max\{W(n,h,x), U(h,x)\} &- U(h,x)] \\ +(1-\zeta)[\max\{W(s,h,x), U(h,x)\} &- U(h,x)]\}] \end{aligned} \tag{3.18}$$

$$\begin{aligned} w^a(n,h,x) &= \beta[y(n) + \pi V(n) - rV(n)] \\ +(1-\beta)[b + f(\theta)\{\zeta[\max\{W(n,h,x), U(h,x)\} &- U(h,x)] \\ +(1-\zeta)[\max\{W(s,h,x), U(h,x)\} &- U(h,x)]\}] \end{aligned} \tag{3.19}$$

$$w^{na}(n,l,x) = \beta[y(n) + \pi V(n) - rV(n)] + (1-\beta)d \tag{3.20}$$

$$w^{na}(s,h,x) = \beta[y(s) + \pi V(s) - rV(s)] + (1-\beta)d \tag{3.21}$$

$$w^{na}(n,h,x) = \beta[y(n) + \pi V(n) - rV(n)] + (1-\beta)d \tag{3.22}$$

where a refers to attached workers ($U \geq H$) and na refers to unattached workers ($H \geq U$). The attached workers mean they will not leave the labour market after a job destruction shock. The unattached workers mean they will leave the labour market after a job destruction shock.

3.2.6 Steady State conditions

In this section, we explain the equilibrium stock of workers in different states. The detail of the description of the flow rates between the stocks and the simultaneous linear equations in ten unknowns are in Appendix F. The flow rates are expressed by the parameters and the endogenous variables. The stocks of workers are obtained by solving the linear equations, which we can not solve manually but computationally. Thus, the stocks of workers are expressed by the parameters and the endogenous variables.

3.3 General Equilibrium

We define the general equilibrium in this section. We consider the case where high-productivity workers can work non-regular jobs. There are many high-productivity workers working non-regular jobs in the real world. We think the case of Proposition 1 in section 3.2.4. In this case, the parameter in this model satisfies $W(n,h,x) > U(h,x)$.

The following is the definition of general equilibrium.

Definition 1.

A steady-state equilibrium with cross-skill matching consists of a set of value functions for $W(i,j,x)$, $J(i,j,x)$, $V(i)$, $U(j,x)$ and $H(j,x)$ that satisfy the equations of entry margin, job creation, and quit margin, wage rules (3.17)-(3.22) plus a vector $\{\zeta, \theta, \phi, S, x_v^l, x_q^l, x_q^{hn}, x_q^{hs}, x_v^h\}$ such that (1) All matches produce a non-negative surplus for the equilibrium values (2) The vector $\{\zeta, \theta, \phi, S, x_v^l, x_q^l, x_q^{hn}, x_q^{hs}, x_v^h\}$ solves the free entry conditions ($V(i) = 0$) and the steady state conditions.

The equations are as follows:

(Entry margin)

$$\frac{x_v^l}{f(\theta)\zeta} = \frac{x_q^l - x_v^l}{r + \delta_n}\frac{1}{\beta} \tag{3.23}$$

$$\frac{x_v^h}{f(\theta)} = \frac{\zeta}{\beta}\frac{x_q^{hn} - x_v^h}{r + \delta_n} + \frac{1-\zeta}{\beta}\frac{x_q^{hs} - x_v^h}{r + \delta_s} \tag{3.24}$$

(Job creation)

$$\frac{c}{(1-\phi)p(\theta)} = \frac{x_q^{hs} - x_v^h}{r + \delta_s}\frac{1}{1-\beta} \tag{3.25}$$

$$c = p(\theta)\{\phi\frac{x_q^l - x_v^l}{r + \delta_n}\frac{1}{1-\beta} + (1-\phi)\frac{x_q^{hn} - x_v^h}{r + \delta_n}\frac{1}{1-\beta}\} \tag{3.26}$$

(Quit margin)

$$y(n) - x_q^l + \pi \frac{1}{r+\delta_n} \int_{x_v^l}^{x_q^l} F(x)dx = 0 \tag{3.27}$$

$$y(s) - x_q^{hs} + \pi \frac{1}{r+\delta_s} \int_{x_v^h}^{x_q^{hs}} F(x)dx = 0 \tag{3.28}$$

$$y(n) - x_q^{hn} + \pi \frac{1}{r+\delta_n} \int_{x_v^h}^{x_q^{hn}} F(x)dx = 0 \tag{3.29}$$

The equations of Entry margin are derived from using $U(l, x_v^l) = H(l, x_v^l)$ and $U(h, x_v^h) = H(h, x_v^h)$. The equations of Quit margin are derived from $W(n, l, x_q^l) = H(l, x_q^l)$, $W(n, h, x_q^{hn}) = H(h, x_q^{hn})$ and $W(s, h, x_q^{hs}) = H(h, x_q^{hs})$. The equations of Job creation are obtained from the asset value equations of labour demand. Here, S is the vector of the stock of workers, [E_a^{hs}, E_{na}^{hs}, E_a^{hn}, E_{na}^{hn}, E_a^{ln}, E_{na}^{ln}, N_U^h, N_U^l, N^h, N^l]. The steady state conditions are in Appendix F.

3.4 Quantitative implications

3.4.1 Numerical Example

Finally, in this section, we show the numerical example that illustrates the Japanese labour market (Table 3.1). The baseline parameter values are chosen with the following criteria in mind. First, the parameter should be chosen to illustrate the Japanese labour market. Second, the parameter values themselves should be reasonable. Finally, the values of the endogenous variables that follow from these parameter values should also be reasonable. Using this model, we investigate counterfactual scenarios for the effect of the technical change on home production of Japanese labour market.

Set the time period to quarterly. Let μ to be 0.658 to reflect the statistics of the fraction of the workers in the population who have the low-productivity level. We set δ_s and δ_n to be 0.0121 and 0.0409 to match the statistics of the quarterly transfer rate from a regular job to unemployment, and the quarterly transfer rate from a non-regular job to unemployment. The discount factor r is set to be 0.05, which we take from Albrecht and Vroman (2002). The matching function is assumed to be Cobb-Douglas, $f(\theta) = M\theta^a$ with $M = 0.362, a = 0.542$, which is estimated by the Cabinet Office (2016). We set the instantaneous value of leisure b to be 0.4 from Esteban-Pretel et al. (2010), which estimated the search model in the Japanese labour market. We normalise $y(n) = 1$. $y(s) = 1.103$ and $c = 0.256$ are taken from chapter 2, respectively. The bargaining power is set to be 0.5 as usual. Let π to be 0.0409 and Λ to be 1 because we assume that the extent of the value of π and Λ can be similar to the value of a non-regular job and low-productivity worker. The distribution of home productivity is exponential with the parameter $\Lambda = 1$.

The Table 3.2 shows the result in the benchmark model, solved computationally. This figure shows the labour market outcomes in the benchmark model. Overall, the value of labour market outcomes seems to match the Japanese economy to some extent. The unemployment rate is usually 3–4%. The parameter reflects the Japanese labour market well. In terms of labour market tightness θ, the statistics of vacancies over unemployed are usually around 1, so the result of the θ is somewhat high. The other outcomes seem to be basically appropriate. Note that the equations converged and that there is a cross-productivity equilibrium. The value of outcome is all positive, and the size relationship of the outcome is appropriate ($x_v^l < x_q^l$ and $x_v^h < x_q^{hn} < x_q^{hs}$).

3.4.2 Comparative Statistics

Our next objective is to show that our model delivers predictions about the relationship between a parameter and labour outcomes. We consider one counterfactual scenario in the Japanese labour market, which is the average of home production (Λ^{-1}) change occurs. Recently, autonomous robotic vacuum cleaners and cheap housekeeping services are rapidly becoming popular. This seems to have led to a decrease in the outcome of home production. The substitution between manual cleaners and "autonomous robotic vacuum cleaners and cheap housekeeping services" is not strong in the Japanese context.

Figure 3.2 shows the result of the comparative statistics of Λ. Note that there is a cross-productivity equilibrium during calculation, the value of outcome is all positive and the size relationship of the outcome is appropriate ($x_v^l < x_q^l$ and $x_v^h < x_q^{hn} < x_q^{hs}$) during changing the parameters. It tells as the effect of home production technological change. The baseline parameter of Λ is 1. The value of Λ moved between 0.8 and 1.2. As the value of Λ becomes larger, the non-participants and the unemployed people increase. This is because the relative value to work and being unemployed increases. This result shows the decrease of the value of home production leads an increase in employment.

3.5 Conclusion

We construct a search matching model, which incorporate endogenous labour market participation into the segmented labour market. Our model allows for a rather precise description of the labour market. It included several categories of individuals: attached regular and non-regular employed workers, unattached regular and non-regular employed workers, unemployed workers, marginally attached non-employed workers and true non-participants.

We use this model and do the counterfactual simulation by changing the parameters of the model. We consider how to raise the labour force participation rate in the segmented labour market, what increased employment in regular jobs and non-regular jobs, and what decreases the unemployment rate of low-skilled workers and high-skilled workers. I apply the data of Japan to do the counterfactual simulation. The quantitative implication predicts that the decrease of the value of home production, such as the rapid spread of autonomous robotic vacuum cleaners and cheap housekeeping services, leads an increase in employment.

We can extend the model slightly to consider various policy effects. I would like to consider this as the future study.

Figures and tables

Figure 3.1: Value functions of home productivity x for high-productivity workers

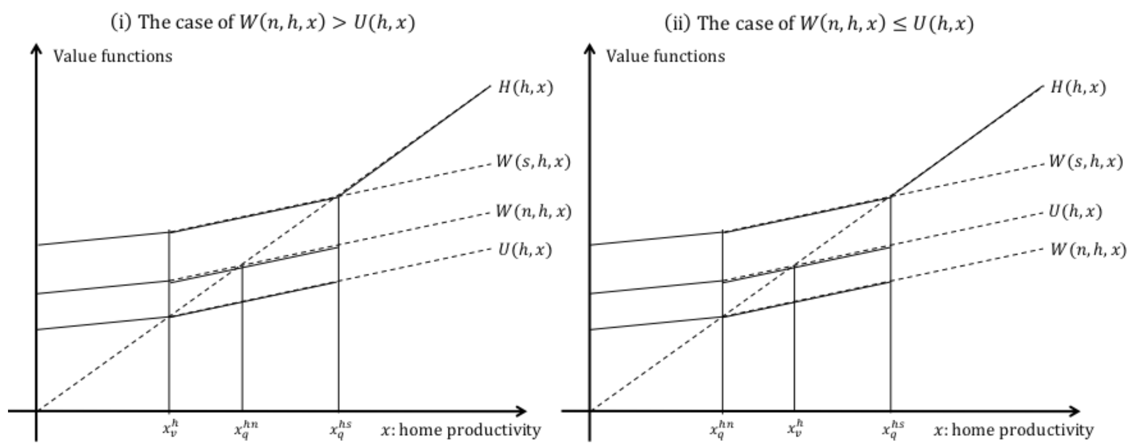


Figure 3.2: ζ, θ, u, n, e as a function of Λ

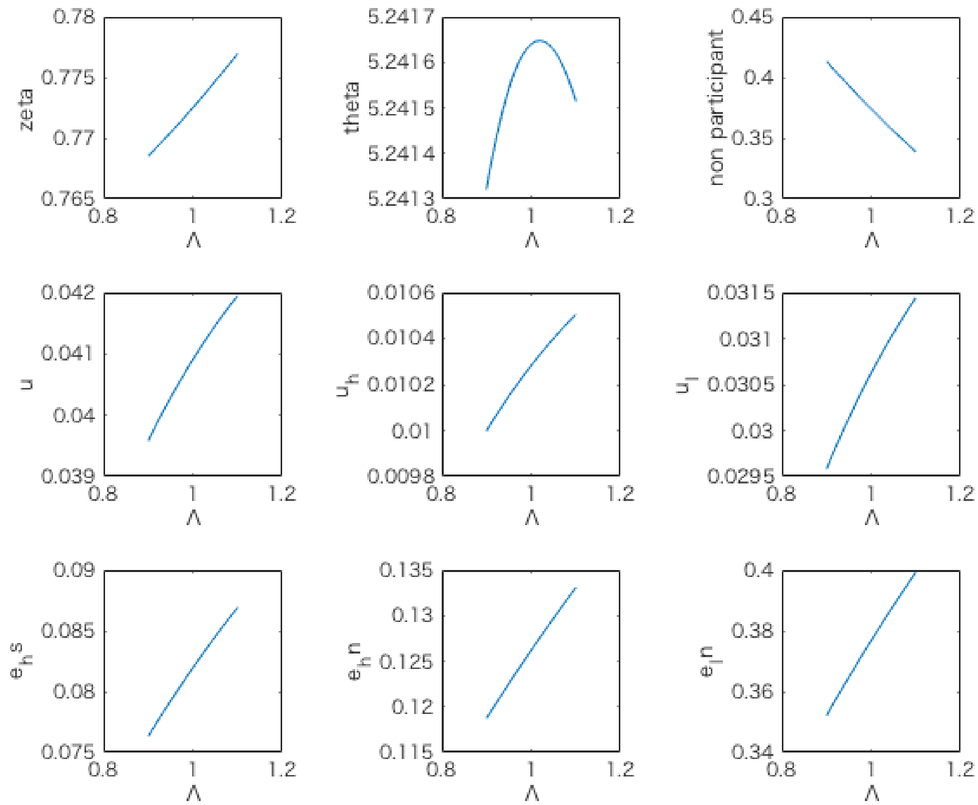


Table 3.1: Summary of the baseline parameters

Parameter	Definition	Statistics	Source
μ	A fraction of the low-productivity workers in the population	0.6580	LSA (2019 Cohort)
y(s)	The output from regular job	1.0960	Chapter 2
y(n)	The output from non-regular job	1	Chapter 2
δs	Quarterly transfer rate from regular job to unemployment	0.0121	LFS
δn	Quarterly transfer rate from temporary job to unemployment	0.0409	LFS
c	The intantaneous cost of a vacancy cost	0.183	Chapter 2
π	The rate that the value of home production productivity changes	0.0409	Author calculation
Λ	The coefficient for the accumulated distribution which the value of home productivity takes a value from	1	Author calculation
β	Bargaining power	0.5	Albrecht and Vroman (2002)
r	Discount factor	0.0053	Albrecht and Vroman (2002)
M	Estimated parameters of matching function	0.362	Cabinet Office (2016)
a	Estimated parameters of matching function	0.542	Cabinet Office(2016)

Table 3.2: Labour market outcomes in the benchmark model

Labour market tightness θ	5.24
Share of non-regular job vacancies ζ	0.77
Share of less-productivity unemployed ϕ	0.75
x_l_v	0.95
x_l_q	1.01
x_hn_q	1.00
x_hs_q	1.13
x_h_v	0.99
Unemployment rate u	0.04
non participant n	0.37
Unemployment rate highly-productivity workers u(h)	0.01
Unemployment rate less-productivity workers u(l)	0.03
Employment rate highly-productivity workers in regular job	0.08
Employment rate highly-productivity workers in non-regular job	0.13
Employment rate less-productivity workers in non-regular job	0.38

Table 3.3: The definition of the stocks of workers

E_a^{hs}	High-skilled employed attached in a regular job	E_{na}^{hs}	High-skilled employed unattached in a regular job
E_a^{hn}	High-skilled employed attached in a non-regular job	E_{na}^{hn}	High-skilled employed unattached in a non-regular job
E_a^{ln}	Low-skilled employed attached in a non-regular job	E_{na}^{ln}	Low-skilled employed unattached in a non-regular job
$N_U^h(=u(h))$	High-skilled unemployed	$N_U^l(=u(l))$	Low-skilled unemployed
N^h	High-skilled nonparticipants (in full-time home production)	N^l	Low-skilled nonparticipants (in full-time home production)

Table 3.4: The flows rate between the stocks

$e_a^{hs}e_{na}^{hs}$	Flows from E_a^{hs} to E_{na}^{hs}	$e_a^{hs}e_{na}^{hs} = \pi(F(x_q^{hs}) - F(x_v^h))$
$e_{na}^{hs}e_a^{hs}$	Flows from E_{na}^{hs} to E_a^{hs}	$e_{na}^{hs}e_a^{hs} = \pi F(x_v^h)$
$e_a^{hn}e_{na}^{hn}$	Flows from E_a^{hn} to E_{na}^{hn}	$e_a^{hn}e_{na}^{hn} = \pi(F(x_q^{hn}) - F(x_v^h))$
$e_{na}^{hn}e_a^{hn}$	Flows from E_{na}^{hn} to E_a^{hn}	$e_{na}^{hn}e_a^{hn} = \pi F(x_v^h)$
$e_a^{hs}u^h$	Flows from E_a^{hs} to N_U^h	$e_a^{hs}u^h = \delta_s$
$u^h e_a^{hs}$	Flows from N_U^h to E_a^{hs}	$u^h e_a^{hs} = f(\theta)(1-\zeta)$
$e_a^{hn}u^h$	Flows from E_a^{hn} to N_U^h	$e_a^{hn}u^h = \delta_n$
$u^h e_a^{hn}$	Flows from N_U^h to E_a^{hn}	$u^h e_a^{hn} = f(\theta)\zeta$
$e_a^{hs}n^h$	Flows from E_a^{hs} to N^h	$e_a^{hs}n^h = \pi(1 - F(x_q^{hs}))$
$e_a^{hn}n^h$	Flows from E_a^{hn} to N^h	$e_a^{hn}n^h = \pi(1 - F(x_q^{hn}))$
$u^h n^h$	Flows from N_U^h to N^h	$u^h n^h = \pi(1 - F(x_v^h))$
$n^h u^h$	Flows from N^h to N_U^h	$n^h u^h = \pi F(x_v^h)$
$e_a^{ln}e_{na}^{ln}$	Flows from E_a^{ln} to E_{na}^{ln}	$e_a^{ln}e_{na}^{ln} = \pi(F(x_q^l) - F(x_v^l))$
$e_{na}^{ln}e_a^{ln}$	Flows from E_{na}^{ln} to E_a^{ln}	$e_{na}^{ln}e_a^{ln} = \pi F(x_v^l)$
$e_a^{ln}u^l$	Flows from E_a^{ln} to N_U^l	$e_a^{ln}u^l = \delta_n$
$u^l e_a^{ln}$	Flows from N_U^l to E_a^{ln}	$u^l e_a^{ln} = f(\theta)\zeta$
$e_a^{ln}n^l$	Flows from E_a^{ln} to N^l	$e_a^{ln}n^l = \pi(1 - F(x_q^l))$
$u^l n^l$	Flows from N_U^l to N^l	$u^l n^l = \pi(1 - F(x_v^l))$
$n^l u^l$	Flows from N^l to N_U^l	$n^l u^l = \pi F(x_v^l)$
$e_{na}^{hs}n^h$	Flows from E_{na}^{hs} to N^h	$e_{na}^{hs}n^h = \delta_s + \pi(1 - F(x_q^{hs}))$
$e_{na}^{hn}n^h$	Flows from E_{na}^{hn} to N^h	$e_{na}^{hn}n^h = \delta_n + \pi(1 - F(x_q^{hn}))$
$e_{na}^{ln}n^l$	Flows from E_{na}^{ln} to N^l	$e_{na}^{ln}n^l = \delta_n + \pi(1 - F(x_q^{ln}))$

Appendix A increasing value functions

(1) Low-skilled workers

In this appendix, we show that the value functions of low skilled workers, $U(l,x)$, $W(n,l,x)$, and $H(l,x)$, are increasing. We substitute x_1 and x_2 into x in the equations (3.3), (3.5), and (3.7). Here, $x_1 < x_2$ and $\Delta = x_2 - x_1$.

$$\begin{aligned} rU(l,x_1) = b + f(\theta)\zeta[\max\{W(n,l,x_1),U(l,x_1)\} - U(l,x_1)] \\ +\pi \int_{xmin}^{xmax} \max\{U(l,x'),H(l,x')\}dF(x') \end{aligned} \tag{A.1}$$

$$\begin{aligned} rW(n,l,x_1) = w(n,l,x_1) + \delta_n[\max\{(U(l,x_1) - W(n,l,x_1)),(H(l,x_1) - W(n,l,x_1))\} \\ +\pi \int_{xmin}^{xmax} \max\{U(l,x'),H(l,x'),W(n,l,x')\}dF(x') \end{aligned} \tag{A.2}$$

$$rH(l,x_1) = d + \pi \int_{xmin}^{xmax} \max\{U(l,x'),H(l,x')\}dF(x') \tag{A.3}$$

$$\begin{aligned} rU(l,x_2) = b + f(\theta)\zeta[\max\{W(n,l,x_2),U(l,x_2)\} - U(l,x_2)] \\ +\pi \int_{xmin}^{xmax} \max\{U(l,x'),H(l,x')\}dF(x') \end{aligned} \tag{A.4}$$

$$rW(n,l,x_2) = w(n,l,x_2) + \delta_n[\max\{(U(l,x_2) - W(n,l,x_2)), (H(l,x_2) - W(n,l,x_2))\}$$
$$+\pi \int_{xmin}^{xmax} \max\{U(l,x'), H(l,x'), W(n,l,x')\} dF(x') \tag{A.5}$$

$$rH(l,x_2) = d + \pi \int_{xmin}^{xmax} \max\{U(l,x'), H(l,x')\} dF(x') \tag{A.6}$$

We subtract (A.3) from (A.6), and we know $H(l,x)$ is increasing function in , because

$$rH(l,x_2) - rH(l,x_1) = x_2 - x_1 > 0 \tag{A.7}$$

We subtract (A.1) from (A.4) and (A.2) from (A.5), and organize the formula by using the fact $W(n,l,x) > U(l,x)$[1].

$$\begin{aligned}(r + f(\theta)\zeta)\{U(l,x_2) - U(l,x_1)\} &= (1 - b_2)(x_2 - x_1) \\ &+ f(\theta)\zeta\{W(n,l,x_2) - W(n,l,x_1)\}\end{aligned} \tag{A.8}$$

$$\begin{aligned}(r + \delta_n)\{W(n,l,x_2) - W(n,l,x_1)\} &= w(n,l,x_2) - w(n,l,x_1) \\ +\delta_n \max\{U(l,x_2), H(l,x_2)\} &- \delta_n \max\{U(l,x_1), H(l,x_1)\}\end{aligned} \tag{A.9}$$

[1] This is easily derived by subtracting (3.3) from (3.5).

Firstly, we think the interval of x which satisfies $U(l,x) \leq H(l,x)$. This means we think the case that x_1 and x_2 exist in $U(l,x) \leq H(l,x)$. We want to know the value functions are increasing or not in this interval.
From the wage determination[2]

$$w^{na}(n,l,x) = \beta[y(n) + \pi V(n) - rV(n)] + (1-\beta)d$$
$$w^{na}(n,l,x_2) - w^{na}(n,l,x_1) = (1-\beta)(x_2 - x_1)$$

Then, (A.9) is

$$(r+\delta_n)\{W(n,l,x_2)-W(n,l,x_1)\} = (1-\beta)(x_2-x_1)+\delta_n\{H(l,x_2)-H(l,x_1)\}$$

and we know when x_1 and x_2 exist in $U(l,x) \leq H(l,x)$, then $W(n,l,x_2) - W(n,l,x_1)$ is always positive, and $W(n,l,x)$ is increasing function. From (A.8), $U(l,x)$ is increasing function.

Second, we think the interval of x which satisfies $U(l,x) \geq H(l,x)$. This means we think the case x_1 and x_2 exist in $U(l,x) \geq H(l,x)$.
From the wage determination,

$$w^{a}(n,l,x_2) - w^{a}(n,l,x_1) = (1-\beta)(x_2 - x_1)$$
$$+(1-\beta)f(\theta)\zeta\{[W(n,l,x_2) - W(n,l,x_1)] - [U(l,x_2) - U(l,x_1)]\}$$

[2]The detail of the wage determination is in the Appendix E.

Then, by using (A.8) and the expression of $w^a(n,l,x_2) - w^a(n,l,x_1)$, (A.9) becomes

$$[(r + \delta_n - (1-\beta)f(\theta)\zeta) - (\delta_n - (1-\beta)f(\theta)\zeta)\frac{f(\theta)\zeta}{r + f(\theta)\zeta}]\{W(n,l,x_2) - W(n,l,x_1)\}$$
$$= (1-\beta)(x_2 - x_1) + (\delta_n - (1-\beta)f(\theta)\zeta)\frac{(1-b_2)(x_2-x_1)}{r + f(\theta)\zeta}$$

The coefficient of $W(n,l,x_2) - W(n,l,x_1)$ is positive, and the RHS of the equation is also positive. Thus, $W(n,l,x)$ is increasing function. From (A-8), $U(l,x)$ is increasing function.

To sum up, $W(n,l,x)$, $U(l,x)$, and $H(l,x)$ is increasing function in x.

(2) High-skilled workers

Next, we show that the value functions of high skilled workers, $U(h,x)$, $W(n,h,x)$, $W(s,h,x)$, and $H(h,x)$, are increasing. We substitute x_1 and x_2 into x in the equations (3.3), (3.5), and (3.7). Here, $x_1 < x_2$ and $\Delta = x_2 - x_1$ is very small.

$$\begin{aligned} r(U(h,x_2) - U(h,x_1)) &= (1-b_2)(x_2 - x_1) \\ +f(\theta)\{\zeta[\max\{W(n,h,x_2), U(h,x_2)\} &- U(h,x_2)] \\ +(1-\zeta)[\max\{W(s,h,x_2), U(h,x_2)\} &- U(h,x_2)]\} \\ -f(\theta)\{\zeta[\max\{W(n,h,x_1), U(h,x_1)\} &- U(h,x_1)] \\ +(1-\zeta)[\max\{W(s,h,x_1), U(h,x_1)\} &- U(h,x_1)]\} \end{aligned} \tag{A.10}$$

$$\begin{aligned} r(W(n,h,x_2)-W(n,h,x_1)) &= w(n,h,x_2)-w(n,h,x_1) \\ +\delta_n[\max\{(U(h,x_2)-W(n,h,x_2)),(H(h,x_2)-W(n,h,x_2))\}] \\ -\delta_n[\max\{(U(h,x_1)-W(n,h,x_1)),(H(h,x_1)-W(n,h,x_1))\}] \end{aligned} \tag{A.11}$$

$$\begin{aligned} r(W(s,h,x_2)-W(s,h,x_1)) &= w(s,h,x_2)-w(s,h,x_1) \\ +\delta_s[\max\{(U(h,x_2)-W(s,h,x_2)),(H(h,x_2)-W(s,h,x_2))\}] \\ -\delta_s[\max\{(U(h,x_1)-W(s,h,x_1)),(H(h,x_1)-W(s,h,x_1))\}] \end{aligned} \tag{A.12}$$

$$r(H(h,x_2)-H(h,x_1)) = x_2 - x_1 \tag{A.11}$$

From (A.13), we know $H(l,x)$ is increasing function in x.

(I)$H(h,x) \geq U(h,x)$

Firstly, we think the interval of x which satisfies $H(h,x) \geq U(h,x)$. This means we think the case x_1 and x_2 exist in $H(h,x) \geq U(h,x)$. We want to know the value functions are increasing or not in this interval.

From the wage determination,

$$w(n,h,x_2)-w(n,h,x_1) = w^{na}(n,h,x_2)-w^{na}(n,h,x_1) = (1-\beta)(x_2-x_1)$$

Then, (A.11) is

$$(r+\delta_n)(W(n,h,x_2)-W(n,h,x_1)) = (1-\beta)(x_2-x_1)+\delta_n[H(h,x_2)-H(h,x_1)]$$

Thus, $W(n,h,x)$ is increasing function in x. Then, $W(s,h,x)$ is also increasing function in x. This leads that $U(h,x)$ is increasing function in x as well.

(II)$H(h,x) \leq U(h,x)$

Secondly, we think the interval of x which satisfies $H(h,x) \leq U(h,x)$. This means we think the case x_1 and x_2 exist in $H(h,x) \leq U(h,x)$. We want to know in this interval, the value functions are increasing or not. When $U(h,x') \geq H(h,x')$, it is always $W(s,h,x) > U(h,x)$ as we show in the Appendix B.

From the wage determination,

$$
\begin{aligned}
w^a(n,h,x_1) - w^a(n,h,x_2) = (1-\beta)(1-b_2)(x_2 - x_1) \\
+(1-\beta)f(\theta)\{\zeta[\max\{W(n,h,x_2), U(h,x_2)\} - U(h,x_2)] \\
+(1-\zeta)[W(s,h,x_2) - U(h,x_2)]\} - (1-\beta)f(\theta)\{\zeta[\max\{W(n,h,x_1), U(h,x_1)\} \\
-U(h,x_1)] + (1-\zeta)[W(s,h,x_1) - U(h,x_1)]\}
\end{aligned}
$$

Then, considering with these facts, we also think two cases of the interval of x, that is $W(n,h,x) \leq U(h,x)$ and $W(n,h,x) \geq U(h,x)$.

(i)$W(n,h,x) \leq U(h,x)$

From the wage determination and (A.10), (A.11), and (A.12), we simplify and get (A.14).

$$W(s,h,x_2) - W(s,h,x_1) = \frac{(1-\beta)(1-b_2)(x_2-x_1) + \frac{\delta_s-(1-\beta)f(\theta)(1-\zeta)}{r+f(\theta)(1-\zeta)}(1-b_2)(x_2-x_1)}{r+\delta_s-(1-\beta)f(\theta)(1-\zeta) - \frac{\delta_s-(1-\beta)f(\theta)(1-\zeta)}{r+f(\theta)(1-\zeta)}f(\theta)(1-\zeta)} \tag{A.14}$$

Then, we simplify the RHS of (A.14), and find $W(s,h,x_2) - W(s,h,x_1)$ is positive, and $W(s,h,x)$ is increasing function. From this, we can easily show that $W(n,h,x)$ and $U(h,x)$ is increasing function as well.

(ii)$W(n,h,x) \geq U(h,x)$

From the wage determination and (A.10), (A.11), and (A.12), we simplify and get these four equations.

$$(r+f(\theta))(U(h,x_2)-U(h,x_1)) = (1-b_2)(x_2-x_1)$$
$$+f(\theta)\zeta[W(n,h,x_2)-W(n,h,x_1)] + f(\theta)(1-\zeta)[W(s,h,x_2)-W(s,h,x_1)] \tag{A.15}$$

$$r(W(n,h,x_2)-W(n,h,x_1)) = w(n,h,x_2)-w(n,h,x_1)$$
$$+\delta_n[U(h,x_2)-W(n,h,x_2)] - \delta_n[U(h,x_1)-W(n,h,x_1)] \tag{A.16}$$

$$r(W(s,h,x_2)-W(s,h,x_1)) = w(s,h,x_2)-w(s,h,x_1)$$
$$+\delta_s[U(h,x_2)-W(s,h,x_2)] - \delta_s[U(h,x_1)-W(s,h,x_1)] \tag{A.17}$$

$$w^a(s,h,x_1) - w^a(s,h,x_2) = w^a(n,h,x_1) - w^a(n,h,x_2)$$
$$= (1-\beta)(1-b_2)(x_2-x_1) + (1-\beta)f(\theta)\{\zeta[W(n,h,x_2)-U(h,x_2)]$$
$$+(1-\zeta)[W(s,h,x_2)-U(h,x_2)]\} - (1-\beta)f(\theta)\{\zeta[W(n,h,x_1)$$
$$-U(h,x_1)] + (1-\zeta)[W(s,h,x_1)-U(h,x_1)]\} \quad \text{(A.18)}$$

We simplify them and get (A.19)

$$(r+\delta_s-(1-\beta)f(\theta)(1-\zeta)-(1-\beta)f(\theta)\zeta\frac{r+\delta_s}{r+\delta_n})(W(s,h,x_2)-W(s,h,x_1))$$
$$=(1-\beta)(1-b_2)(x_2-x_1)+(\delta_s-(1-\beta)f(\theta)+(1-\beta)f(\theta)\zeta\frac{\delta_n-\delta_s}{r+\delta_n})$$
$$[\frac{(1-b_2)(x_2-x_1)}{r+f(\theta)-f(\theta)\zeta\frac{\delta_n-\delta_s}{r+\delta_n}}+\frac{f(\theta)\zeta}{r+f(\theta)-f(\theta)\zeta\frac{(\delta_n-\delta_s)}{(r+\delta_n)}}\frac{(r+\delta_s)}{(r+\delta_n)}[W(s,h,x_2)-W(s,h,x_1)]$$
$$+\frac{f(\theta)(1-\zeta)}{r+f(\theta)-f(\theta)\zeta\frac{\delta_n-\delta_s}{r+\delta_n}}[W(s,h,x_2)-W(s,h,x_1)]] \quad \text{(A.19)}$$

(A.19) is simplified, and then we know the coefficient of $W(s,h,x_2)-W(s,h,x_1)$ is very complicated to be shown to be positive. We multiply $(r+f(\theta)-f(\theta)\zeta\frac{\delta_n-\delta_s}{r+\delta_n})(r+\delta_n)(r+\delta_n)$ to each term and simplify and get

$$\frac{(rr(r+\delta_n)(r+\delta_n)+r(r+\delta_n)(r+\delta_n)\delta_s+r(r+\delta_n)(r+\delta_n)\beta f(\theta)(1-\zeta)+r(r+\delta_n)(r+\delta_s)\beta f(\theta)\zeta)}{(r+f(\theta)-f(\theta)\zeta\frac{\delta_n-\delta_s}{r+\delta_n})(r+\delta_n)(r+\delta_n)}$$

We know this is positive, and then $W(s,h,x)$ is increasing function. From this, we can easily show that $W(n,h,x)$ and $U(h,x)$ is increasing function as well.

To sum up, the value functions of low-skilled workers and high-skilled workers, $W(n,l,x)$, $U(l,x)$, $H(l,x)$, $W(s,h,x)$, $W(n,h,x)$, $U(h,x)$ and $H(h,x)$, are increasing in x.

Appendix B The characteristics of the value functions of high-skilled workers

(i) When $U(h,x) \geq H(h,x)$,

$$\begin{aligned} rU(h,x) = b + f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\} - U(h,x)] \\ +(1-\zeta)[\max\{W(s,h,x),U(h,x)\} - U(h,x)]\} \\ +\pi \int_{xmin}^{xmax} \max\{U(h,x'),H(h,x')\}dF(x') \end{aligned} \tag{B.1}$$

$$\begin{aligned} rW(n,h,x) = w(n,h,x) + \delta_n[U(h,x) - W(n,h,x)] \\ +\pi \int_{xmin}^{xmax} \max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x') \end{aligned} \tag{B.2}$$

$$\begin{aligned} rW(s,h,x) = w(s,h,x) + \delta_s[U(h,x) - W(n,h,x)] \\ +\pi \int_{xmin}^{xmax} \max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x') \end{aligned} \tag{B.3}$$

$$rH(h,x) = d + \pi \int_{xmin}^{xmax} \max\{U(h,x'),H(h,x')\}dF(x') \tag{B.4}$$

From equation (B.1) and (B.3),

$$
\begin{aligned}
r[W(s,h,x) - U(h,x)] = w(s,h,x) - b + \delta_s[U(h,x) - W(s,h,x)] \\
-f(\theta)\{\zeta[\max\{W(n,h,x), U(h,x)\} - U(h,x)] \\
+(1-\zeta)[\max\{W(s,h,x), U(h,x)\} - U(h,x)]\} \\
+\pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x'), W(n,h,x')\} dF(x') \\
-\pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x')\} dF(x')
\end{aligned}
$$

Here, when $W(n,h,x) < U(h,x)$, if we assume $W(s,h,x) \le U(h,x)$, then LHS is less than 0 and RHS is positive as we know that $w(s,h,x)$ is not zero. This is contradiction, and we find when $W(n,h,x) < U(h,x)$, it is $W(s,h,x) > U(h,x)$.

When $W(n,h,x) \ge U(h,x)$, from equation (B.2) and (B.3),

$$
\begin{aligned}
r[W(s,h,x) - W(n,h,x)] = w(s,h,x) - w(n,h,x) \\
+\delta_s[U(h,x) - W(s,h,x)] - \delta_n[U(h,x) - W(n,h,x)] \\
+\pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x'), W(n,h,x')\} dF(x') \\
-\pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x'), W(n,h,x')\} dF(x')
\end{aligned}
$$

$\Leftrightarrow$

$$
\begin{aligned}
r[W(s,h,x) - W(n,h,x)] = w(s,h,x) - w(n,h,x) + \delta_s[U(h,x) - W(s,h,x)] \\
-\delta_n[U(h,x) - W(n,h,x)] - \delta_s W(n,h,x) + \delta_s W(n,h,x)
\end{aligned}
$$

$\Leftrightarrow$

$$
\begin{aligned}
(r+\delta_s)[W(s,h,x)-W(n,h,x)] &= w(s,h,x)-w(n,h,x) \\
&+(\delta_s-\delta_n)[U(h,x)-W(s,h,x)]
\end{aligned}
$$

We know $w(s,h,x) > w(n,h,x)$ and so that RHS is positive, as a result it is $W(s,h,x) > W(n,h,x)$.

(ii) When $U(h,x) < H(h,x)$,

$$
\begin{aligned}
rU(h,x) = b + f(\theta)\{&\zeta[\max\{W(n,h,x),U(h,x)\}-U(h,x)] \\
&+(1-\zeta)[\max\{W(s,h,x),U(h,x)\}-U(h,x)]\} \\
&+\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x')\}dF(x')
\end{aligned}
\tag{B.5}
$$

$$
\begin{aligned}
rW(n,h,x) &= w(n,h,x)+\delta_n[H(h,x)-W(n,h,x)] \\
+\pi&\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x')
\end{aligned}
\tag{B.6}
$$

$$
\begin{aligned}
rW(s,h,x) &= w(s,h,x)+\delta_s[H(h,x)-W(s,h,x)] \\
+\pi&\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x'),W(s,h,x')\}dF(x')
\end{aligned}
\tag{B.7}
$$

$$rH(h,x) = d + \pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x')\} dF(x') \tag{B.8}$$

From equation (B.6), (B.7), and (B.8)

$$\begin{aligned} r[W(n,h,x) - H(h,x)] = w(n,h,x) - d + \delta_n[H(h,x) - W(n,h,x)] \\ +\pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x'), W(n,h,x')\} dF(x') \\ -\pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x')\} dF(x') \end{aligned}$$

$$\begin{aligned} r[W(s,h,x) - H(h,x)] = w(s,h,x) - d + \delta_s[H(h,x) - W(s,h,x)] \\ +\pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x'), W(s,h,x')\} dF(x') \\ -\pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x')\} dF(x') \end{aligned}$$

$$(r + \delta_n)[W(n,h,x) - H(h,x)] = w(n,h,x) - d + \pi\beta\overline{S}$$

$$(r + \delta_s)[W(s,h,x) - H(h,x)] = w(s,h,x) - d + \pi\beta\overline{S}$$

If we substitute $w^{na}(n,h,x)$ and $w^{na}(s,h,x)$ into these equations and simplify, we find this equation become smaller as x increases.

And also when $W(s,h,x) > H(h,x)$, from equation (B.6) and (B.7),

$$r[W(s,h,x) - W(n,h,x)] = w(s,h,x) - w(n,h,x)$$
$$+\delta_s[H(h,x) - W(s,h,x)] - \delta_n[H(h,x) - W(n,h,x)]$$

$\Leftrightarrow$

$$(r+\delta_n)[W(s,h,x) - W(n,h,x)] = w(s,h,x) - w(n,h,x)$$
$$+(\delta_n - \delta_s)[W(s,h,x) - H(h,x)]$$

Here, we know $w(s,h,x) > w(n,h,x)$, so that it is $W(s,h,x) > W(n,h,x)$.

Appendix C The independency of the value function

We think the positional relation of $W(n,h,x)$ and $U(h,x)$ when x is such that $W(s,h,x) > H(h,x)$.

When $U(h,x) \geq H(h,x)$, we know that $W(s,h,x) > W(n,h,x)$ if $W(n,h,x) \geq U(h,x)$ and $W(s,h,x) > U(h,x)$ if $W(n,h,x) < U(h,x)$. Using these information, we consider the positional relationship of $W(n,h,x)$ and $U(h,x)$ in terms of x.

$$rU(h,x) = b + f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\} - U(h,x)]$$
$$+(1-\zeta)[\max\{W(s,h,x),U(h,x)\} - U(h,x)]\}$$
$$+\pi \int_{xmin}^{xmax} \max\{U(h,x'),H(h,x')\}dF(x')$$

$$rW(n,h,x) = w(n,h,x) + \delta_n[U(h,x) - W(n,h,x)]$$
$$+\pi \int_{xmin}^{xmax} \max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x')$$

and then,

$$r[W(n,h,x) - U(h,x)] = w(n,h,x) - b + \delta_n[U(h,x) - W(n,h,x)]$$
$$-f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\} - U(h,x)]$$
$$+(1-\zeta)[\max\{W(s,h,x),U(h,x)\} - U(h,x)]\}$$
$$+\pi \int_{xmin}^{xmax} \max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x')$$
$$-\pi \int_{xmin}^{xmax} \max\{U(h,x'),H(h,x')\}dF(x')$$

$\Leftrightarrow$

$$
\begin{aligned}
r[W(n,h,x)-U(h,x)] = \beta[y(n)+\pi V(n)-rV(n)] \\
+(1-\beta)[b+f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\}-U(h,x)] \\
+(1-\zeta)[\max\{W(s,h,x),U(h,x)\}-U(h,x)]\}] \\
-b+\delta_n[U(h,x)-W(n,h,x)] \\
-f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\}-U(h,x)] \\
+(1-\zeta)[\max\{W(s,h,x),U(h,x)\}-U(h,x)]\} \\
+\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x') \\
-\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x')\}dF(x')
\end{aligned}
$$

$\Leftrightarrow$

$$
\begin{aligned}
(r+\delta_n)[W(n,h,x)-U(h,x)] = \beta[y(n)+\pi V(n)-rV(n)] \\
-\beta[b+f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\}-U(h,x)] \\
+(1-\zeta)[\max\{W(s,h,x),U(h,x)\}-U(h,x)]\} \\
+\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x') \\
-\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x')\}dF(x')
\end{aligned}
$$

Here, we know $W(s,h,x) > U(h,x)$. Then,

$$\begin{aligned}(r+\delta_n)[W(n,h,x)-U(h,x)] &= \beta[y(n)+\pi V(n)-rV(n)] \\ &-\beta[b+f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\}-U(h,x)] \\ &+(1-\zeta)[W(s,h,x)-U(h,x)]\}] \\ &+\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x') \\ &-\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x')\}dF(x')\end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned}(r+\delta_n+\beta f(\theta)(1-\zeta))[W(n,h,x)-U(h,x)] &= \beta[y(n)+\pi V(n)-rV(n)] \\ &-\beta[b+f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\}-U(h,x)] \\ &+(1-\zeta)[W(s,h,x)-W(n,h,x)]\}] \\ &+\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x') \\ &-\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x')\}dF(x')\end{aligned}$$

Now, we think the case of $W(n,h,x) = U(h,x)$. We substitute $W(n,h,x) = U(h,x)$ into the previous equation.

$$\begin{aligned}
0 &= \beta[y(n) + \pi V(n) - rV(n)] \\
&-\beta[b + f(\theta)(1-\zeta)[W(s,h,x) - W(n,h,x)]] \\
&+\pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x'), W(n,h,x')\} dF(x') \\
&-\pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x')\} dF(x')
\end{aligned}$$

Here, we know

$$\begin{aligned}
(r+\delta_s)[W(s,h,x) - W(n,h,x)] &= w(s,h,x) - w(n,h,x) \\
&+(\delta_s - \delta_n)[U(h,x) - W(n,h,x)]
\end{aligned}$$

Also, $w(s,h,x) - w(n,h,x)$ is obtained from Appendix E.

$$\begin{aligned}
w(s,h,x) - w(n,h,x) &= \beta[y(s) + \pi V(s) - rV(s)] \\
&+(1-\beta)[b + f(\theta)\{\zeta[\max\{W(n,h,x), U(h,x)\} - U(h,x)] \\
&+(1-\zeta)[\max\{W(s,h,x), U(h,x)\} - U(h,x)]\}] \\
&-\beta[y(n) + \pi V(n) - rV(n)] \\
&+(1-\beta)[b + f(\theta)\{\zeta[\max\{W(n,h,x), U(h,x)\} - U(h,x)] \\
&+(1-\zeta)[\max\{W(s,h,x), U(h,x)\} - U(h,x)]\}]]
\end{aligned}$$

$\Leftrightarrow$

$$w(s,h,x) - w(n,h,x) = \beta[y(s) - y(n)]$$

That is,

$$(r+\delta_s)[W(s,h,x) - W(n,h,x)] = \beta[y(s) - y(n)] + (\delta_s - \delta_n)[U(h,x) - W(n,h,x)]$$

We substitute this into the main equation, and we get

$$\begin{aligned} 0 &= \beta[y(n) + \pi V(n) - rV(n)] \\ &-\beta[b + f(\theta)(1-\zeta)\frac{\beta[y(s)-y(n)] + (\delta_s - \delta_n)[U(h,x) - W(n,h,x)]}{r+\delta_s}] \\ &+\pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x'), W(n,h,x')\} dF(x') \\ &-\pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x')\} dF(x') \end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned} 0 &= \beta[y(n) + \pi V(n) - rV(n)] \\ &-\beta[b + f(\theta)(1-\zeta)\frac{\beta[y(s)-y(n)]}{r+\delta_s}] \\ &+\pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x'), W(n,h,x')\} dF(x') \\ &-\pi \int_{xmin}^{xmax} \max\{U(h,x'), H(h,x')\} dF(x') \end{aligned}$$

When we try to solve this equation, we find that given parameters θ and ζ are decided and the value of x is independent on that. This means whether

$W(n,h,x)$ is bigger than $U(h,x)$ or not is depend on the parameters of the model. Once the parameters are given, the positional relationship of $W(n,h,x)$ and $U(h,x)$ are fixed in terms of x.

Appendix D A sufficient condition for a cross-skill matching equilibrium

When $U(h,x') \geq H(h,x')$, we know that $W(s,h,x) > W(n,h,x)$ if $W(n,h,x) \geq U(h,x)$ and $W(s,h,x) > U(h,x)$ if $W(n,h,x) < U(h,x)$. Using these information, we consider the positional relationship of $W(n,h,x)$ and $U(h,x)$.

$$\begin{aligned} rU(h,x) = b + f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\} - U(h,x)] \\ +(1-\zeta)[\max\{W(s,h,x),U(h,x)\} - U(h,x)]\} \\ +\pi\int_{xmin}^{xmax} \max\{U(h,x'),H(h,x')\}dF(x') \end{aligned}$$

$$\begin{aligned} rW(n,h,x) = w(n,h,x) + \delta_n[U(h,x) - W(n,h,x)] \\ +\pi\int_{xmin}^{xmax} \max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x') \end{aligned}$$

and then,

$$r[W(n,h,x)-U(h,x)] = w(n,h,x) - b + \delta_n[U(h,x) - W(n,h,x)]$$
$$-f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\} - U(h,x)]$$
$$+(1-\zeta)[\max\{W(s,h,x),U(h,x)\} - U(h,x)]\}$$
$$+\pi\int_{xmin}^{xmax} \max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x')$$
$$-\pi\int_{xmin}^{xmax} \max\{U(h,x'),H(h,x')\}dF(x')$$

$$r[W(n,h,x)-U(h,x)] = \beta[y(n) + \pi V(n) - rV(n)]$$
$$+(1-\beta)[b + f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\} - U(h,x)]$$
$$+(1-\zeta)[\max\{W(s,h,x),U(h,x)\} - U(h,x)]\}] - b$$
$$+\delta_n[U(h,x) - W(n,h,x)] - f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\} - U(h,x)]+$$
$$(1-\zeta)[\max\{W(s,h,x),U(h,x)\} - U(h,x)]\}$$
$$+\pi\int_{xmin}^{xmax} \max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x')$$
$$-\pi\int_{xmin}^{xmax} \max\{U(h,x'),H(h,x')\}dF(x')$$

$$(r+\delta_n)[W(n,h,x)-U(h,x)] = \beta[y(n) + \pi V(n) - rV(n)]$$
$$-\beta[b + f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\} - U(h,x)]$$
$$+(1-\zeta)[\max\{W(s,h,x),U(h,x)\} - U(h,x)]\}]$$
$$+\pi\int_{xmin}^{xmax} \max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x')$$
$$-\pi\int_{xmin}^{xmax} \max\{U(h,x'),H(h,x')\}dF(x')$$

Here, we know $W(s,h,x) > U(h,x)$ and then,

$$
\begin{aligned}
(r+\delta_n)[W(n,h,x)-U(h,x)] &= \beta[y(n)+\pi V(n)-rV(n)] \\
&-\beta[b+f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\}-U(h,x)] \\
&+(1-\zeta)[W(s,h,x)-U(h,x)]\}] \\
&+\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x') \\
&-\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x')\}dF(x')
\end{aligned}
$$

$$
\begin{aligned}
&(r+\delta_n+\beta f(\theta)(1-\zeta))[W(n,h,x)-U(h,x)] \\
&= \beta[y(n)+\pi V(n)-rV(n)]-\beta[b+f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\}-U(h,x)] \\
&+(1-\zeta)[W(s,h,x)-W(n,h,x)]\}] \\
&+\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x') \\
&-\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x')\}dF(x')
\end{aligned}
$$

Here we know

$$
(r+\delta_s)[W(s,h,x)-W(n,h,x)] = \beta[y(s)-y(n)]+(\delta_s-\delta_n)[U(h,x)-W(n,h,x)]
$$

We substitute this into the main equation,

$$
\begin{aligned}
&(r+\delta_n+\beta f(\theta)(1-\zeta))[W(n,h,x)-U(h,x)] \\
&= \beta[y(n)+\pi V(n)-rV(n)]-\beta[b+f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\}-U(h,x)] \\
&\quad +\frac{1-\zeta}{r+\delta_s}[\beta[y(s)-y(n)]+(\delta_s-\delta_n)[U(h,x)-W(n,h,x)]]\}] \\
&\quad +\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x') \\
&\quad -\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x')\}dF(x')
\end{aligned}
$$

$$
\begin{aligned}
&(r+\delta_n+\beta f(\theta)(1-\zeta)+\beta f(\theta)\frac{(1-\zeta)(\delta_n-\delta_s)}{r+\delta_s}[W(n,h,x)-U(h,x)] \\
&= \beta[y(n)+\pi V(n)-rV(n)]-\beta[b+f(\theta)\{\zeta[\max\{W(n,h,x),U(h,x)\}-U(h,x)] \\
&\quad +\frac{1-\zeta}{r+\delta_s}[\beta[y(s)-y(n)]]\}] \\
&\quad +\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x'),W(n,h,x')\}dF(x') \\
&\quad -\pi\int_{xmin}^{xmax}\max\{U(h,x'),H(h,x')\}dF(x')
\end{aligned}
$$

When

$$\beta[y(n)+\pi V(n)-rV(n)]-\beta[b+f(\theta)\{\frac{1-\zeta}{r+\delta_s}\beta[y(s)-y(n)]\}]\geq 0$$

if we postulate $W(n,h,x)\leq U(h,x)$, then the LHS is less than 0 and the RHS is positive and it is contradiction. So, When

$$\beta[y(n)+\pi V(n)-rV(n)]-\beta[b+f(\theta)\{\frac{1-\zeta}{r+\delta_s}\beta[y(s)-y(n)]\}]\geq 0,$$

we find $W(n,h,x) > U(h,x)$.

Appendix E Wage determination

The proof involves computing the average surplus of workers, firms, and the match. Firstly, we define

$$\overline{S_w} = \beta \overline{S} = \int_{xmin}^{xmax} \max\{W', U', H'\} - \max\{U', H'\} dF(x')$$

and

$$\overline{S_f} = (1-\beta)\overline{S} = \int_{xmin}^{xmax} \max\{J' - V', 0\} dF(x')$$

The following functions are obtained from the value functions of firm side.

$$(r+\delta_s)[J(s,h,x) - V(s)] = y(s) - w(s,h,x) + \pi(1-\beta)\overline{S}(s,h) + \pi V(s) - rV(s)$$

$$(r+\delta_n)[J(n,l,x) - V(n)] = y(s) - w(n,l,x) + \pi(1-\beta)\overline{S}(n,l) + \pi V(n) - rV(n)$$

$$(r+\delta_n)[J(n,h,x) - V(n)] = y(s) - w(n,h,x) + \pi(1-\beta)\overline{S}(n,h) + \pi V(n) - rV(n)$$

When $H > U$, the following functions of workers are obtained if you subtract H from W.

$$(r + \delta_n)[W(n, l, x) - H(l, x)] = w(n, l, x) - d + \pi\beta\overline{S}(n, l)$$

$$(r + \delta_s)[W(s, h, x) - H(h, x)] = w(s, h, x) - d + \pi\beta\overline{S}(s, h)$$

$$(r + \delta_n)[W(n, h, x) - H(h, x)] = w(n, h, x) - d + \pi\beta\overline{S}(n, h)$$

When $H \leq U$, the following functions of workers are obtained if you subtract U from W.

$$(r+\delta_n)[W(n, l, x)-U(l, x)] = w(n, l, x)-b-f(\theta)\zeta[W(n, l, x)-U(l, x)]+\pi\beta\overline{S}(n, l)$$

$$\begin{aligned}(r + \delta_s)[W(s, h, x) - U(h, x)] &= w(s, h, x) - b + \pi\beta\overline{S}(s, h) \\ &-f(\theta)\{\zeta[\max\{W(n, h, x), U(h, x)\} - U(h, x)] \\ &+(1 - \zeta)[\max\{W(s, h, x), U(h, x)\} - U(h, x)]\}\end{aligned}$$

$$\begin{aligned}(r + \delta_n)[W(n, h, x) - U(h, x)] &= w(n, h, x) - b + \pi\beta\overline{S}(n, h) \\ &-f(\theta)\{\zeta[\max\{W(n, h, x), U(h, x)\} - U(h, x)] \\ &+(1 - \zeta)[\max\{W(s, h, x), U(h, x)\} - U(h, x)]\}\end{aligned}$$

Finally, we substitute those functions into equation (3.1) by each job type and worker type. Then we get,

$$w^a(n,l,x) = \beta[y(n) + \pi V(n) - rV(n)] + (1-\beta)[b + f(\theta)\zeta[W(n,l,x) - U(l,x)]]$$

$$w^a(s,h,x) = \beta[y(s) + \pi V(s) - rV(s)]$$
$$+(1-\beta)[b + f(\theta)\{\zeta[\max\{W(n,h,x), U(h,x)\} - U(h,x)]$$
$$+(1-\zeta)[\max\{W(s,h,x), U(h,x)\} - U(h,x)]\}]$$

$$w^a(n,h,x) = \beta[y(n) + \pi V(n) - rV(n)]$$
$$+(1-\beta)[b + f(\theta)\{\zeta[\max\{W(n,h,x), U(h,x)\} - U(h,x)]$$
$$+(1-\zeta)[\max\{W(s,h,x), U(h,x)\} - U(h,x)]\}]$$

Thus,

$$w^{na}(n,l,x) = \beta[y(n) + \pi V(n) - rV(n)] + (1-\beta)d$$

$$w^{na}(s,h,x) = \beta[y(s) + \pi V(s) - rV(s)] + (1-\beta)d$$

$$w^{na}(n,h,x) = \beta[y(n) + \pi V(n) - rV(n)] + (1-\beta)d$$

Appendix F Stock

Let us first establish some notation. We use lowercase letters (e.g. $e_a^{hs} e_{na}^{hs}$) to denote the flows rate between the stocks, while we use upper letters for the stocks of workers (e.g. E_a^{hs}). We define of the stocks of workers in the Table 3.4. The flows rate between those stocks are easily described by the parameters and some endogenous variables as in the Table 3.5.

We have 10 upper letters for the stock of workers to think. One can write the evolution of the stocks of workers in the 10 categories by

(Low-productivity worker)

$$\frac{dE_a^{ln}}{dt} = -(e_a^{ln}u^l + e_a^{ln}n^l + e_a^{ln}e_{na}^{ln})E_a^{ln} + u^l e_a^{ln} N_U^l + e_{na}^{ln} e_a^{ln} E_{na}^{ln} \tag{F.1}$$

$$\frac{dE_{na}^{ln}}{dt} = -(e_{na}^{ln}n^l + e_{na}^{ln}e_a^{ln})E_{na}^{ln} + e_a^{ln} e_{na} E_a^{ln} \tag{F.2}$$

$$\frac{dN_U^l}{dt} = -(u^l e_a^{ln} + u^l n^l)N_U^l + e_a^{ln} u^l E_a^{ln} + n^l u^l N^l \tag{F.3}$$

$$\frac{dN^l}{dt} = -n^l u^l N^l + e_{na}^{ln} n^l E_{na}^{ln} + u^l n^l N_U^l + e_a^{ln} n^l E_a^{ln} \tag{F.4}$$

(High-productivity worker)

$$\frac{dE_a^{hs}}{dt} = -(e_a^{hs}u^h + e_a^{hs}n^h + e_a^{hs}e_{na}^{hs})E_a^{hs} + u^h e_a^{hs} N_U^h + e_{na}^{hs} e_a^{hs} E_{na}^{hs} \tag{F.5}$$

$$\frac{dE_{na}^{hs}}{dt} = -(e_{na}^{hs}n^h + e_{na}^{hs}e_a^{hs})E_{na}^{hs} + e_a^{hs} e_{na}^{hs} E_a^{hs} \tag{F.6}$$

$$\frac{dE_a^{hn}}{dt} = -(e_a^{hn}u^h + e_a^{hn}n^h + e_a^{hn}e_{na}^{hn})E_a^{hn} + u^h e_a^{hn} N_U^h + e_{na}^{hn} e_a^{hn} E_{na}^{hn} \tag{F.7}$$

$$\frac{dE_{na}^{hn}}{dt} = -(e_{na}^{hn}n^h + e_{na}^{hn}e_a^{hn})E_{na}^{hn} + e_a^{hn} e_{na}^{hn} E_a^{hn} \tag{F.8}$$

$$\frac{dN_U^h}{dt} = -(u^h e_a^{hs} + u^h e_a^{hn} + u^h n^h)N_U^h + e_a^{hs} u^h E_a^{hs} + e_a^{hn} u^h E_a^{hn} + n^h u^h N^h \tag{F.9}$$

$$\frac{dN^h}{dt} = -n^h u^h N^h + e_{na}^{hs} n^h E_{na}^{hs} + e_{na}^{hn} n^h E_{na}^{hn} + u^h n^h N_U^h + e_a^{hs} n^h E_a^{hs} + e_a^{hn} n^h E_a^{hn} \tag{F.10}$$

Here, the proportion of low skilled workers is μ.

$$E_a^{ln} + E_{na}^{ln} + N_U^l + N^l = \mu \tag{F.11}$$

Also, the total number of the stocks of workers is 1.

$$E_a^{hs} + E_a^{hn} + E_a^{ln} + N_U^h + N^h + E_{na}^{hs} + E_{na}^{hn} + E_{na}^{ln} + N_U^l + N^l = 1 \tag{F.12}$$

The evolution of the stocks of workers from (F.1) to (F.10) goes to zero in the equilibrium. We will drop (F.4) and (F.10) because any one of 4 equations from (F.1) to (F.4) and any one of 6 equations from (F.5) to (F.10) does not give us any information. We have 10 equations, (F.1)-(F.3), (F.5)-(F.9), (F.11) and (F.12). Now, we have 10 unknown stocks of workers and 10 equations, and can solve simultaneous linear equations in 10 unknowns.

It is difficult to solve this manually, so we use inverse matrix in the computer. The below Table 3.3 is the matrix of simultaneous linear equations in 10 unknowns. We can get these 10 unknowns by solving $AB = C$. where

$$A = \begin{pmatrix} E_a^{hs} \\ E_a^{hn} \\ E_a^{ln} \\ N_U^h \\ N^h \\ E_{na}^{hs} \\ E_{na}^{hn} \\ E_{na}^{ln} \\ N_U^l \\ N^l \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & B_{23} & 0 & 0 & 0 & 0 & B_{28} & B_{29} & 0 \\ 0 & 0 & B_{33} & 0 & 0 & 0 & 0 & B_{38} & 0 & 0 \\ 0 & 0 & B_{43} & 0 & 0 & 0 & 0 & 0 & B_{49} & B_{410} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ B_{61} & 0 & 0 & B_{64} & 0 & B_{66} & 0 & 0 & 0 & 0 \\ B_{71} & 0 & 0 & 0 & 0 & B_{76} & 0 & 0 & 0 & 0 \\ 0 & B_{82} & 0 & B_{84} & 0 & 0 & B_{87} & 0 & 0 & 0 \\ 0 & B_{92} & 0 & 0 & 0 & 0 & B_{97} & 0 & 0 & 0 \\ B_{101} & B_{102} & 0 & B_{104} & B_{105} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \mu \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$B_{23} = -(e_a^{ln}u^l + e_a^{ln}n^l + e_a^{ln}e_{na}^{ln}), B_{28} = e_{na}^{ln}e_a^{ln}, B_{29} = u^l e_a^{ln}$

$B_{33} = e_a^{ln}e_{na}^{ln}, B_{38} = -(e_{na}^{ln}n^l + e_{na}^{ln}e_a^{ln})$

$B_{43} = e_a^{ln}u^l, B_{49} = -(u^l e_a^{ln} + u^l n^l), B_{410} = n^l u^l$

$B_{61} = -(e_a^{hs}u^h + e_a^{hs}n^h + e_a^{hs}e_{na}^{hs}), B_{64} = u^h e_a^{hs}, B_{66} = e_{na}^{hs}e_a^{hs}$

$B_{71} = e_a^{hs}e_{na}^{hs}, B_{76} = -(e_{na}^{hs}n^h + e_{na}^{hs}e_a^{hs})$

$B_{82} = -(e_a^{hn}u^h + e_a^{hn}n^h + e_a^{hn}e_{na}^{hn}), B_{84} = u^h e_a^{hn}, B_{87} = e_{na}^{hn}e_a^{hn}$

$B_{92} = e_a^{hn}e_{na}^{hn}, B_{97} = -(e_{na}^{hn}n^h + e_{na}^{hn}e_a^{hn})$

$B_{101} = e_a^{hs}u^h, B_{102} = e_a^{hn}u^h, B_{104} = -(u^h e_a^{hs} + u^h e_a^{hn} + u^h n^h), B_{105} = n^h u^h$

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