Regulatory risk constraints and investment decisions: unintended consequences for the financial system.

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Table of contents

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Abstract

This thesis contributes to the extant research on the impact of regulatory constraints on financial markets, by presenting a collection of three intertwined essays.

The first essay examines the relation between regulatory constraints on market risk and the fluctuation of the financial market, often depicted in the literature but not empirically proven. I find that market volatility is significantly dependent on (and Granger-caused by) the relative market risk exposure of Italian banks, measured as the ratio of value at risk and banks' market risk limit.

In the second paper, I explore the channel for the results obtained in the first paper. The theoretical framework is based on a risk-constrained mean-variance framework. In such a framework, if the constraint binds, the portion of the portfolio invested in risky assets (alpha) is lower than in the unconstrained scenario, as expected. Furthermore, alpha is inversely related to the relative market risk exposure (as above, given by the ratio between value at risk and market risk limit). Empirical tests confirm that this constrained mean-variance framework is more accurate in forecasting investment behaviour in risky assets of the Italian banks than the ordinary mean-variance framework.

In the third essay, I investigate the role of uncertainty, whose relevance has been deeply investigated by several papers. By adding uncertainty to the constrained mean-variance framework built in the previous chapter, I find that an increase in uncertainty determines a decrease of the portion of portfolio invested in risky assets, in line with the literature. I perform empirical tests which confirm the theoretical result, especially in high-volatility periods when constraints bind tighter.

Chapter 1. Introduction[1]

In 2006, Ben Bernanke, Chairman of the Federal Reserve Bank (the Fed) at the time, highlighted the importance of modern risk management as a central element of good supervisory practice, and encouraged the industry to push forward the risk management frontier.

Just a few years later, in 2010, after the beginning of the crisis, Janet Yellen, Chair of the Fed for 2014 to 2018, said that "methods of modern risk management may have intensified the cycle…because of their reliance on metrics such as value at risk [VaR] that are highly sensitive to recent performance, especially volatility. In good times, volatility declines, and value at risk along with it. This pattern generated a pro-cyclical willingness to take on risk and leverage, amplifying and propagating the boom and bust cycle. The vicious cycle of a collapse of confidence, asset fire sales, evaporation of liquidity, and a deleveraging free fall was the mirror image of the manic mortgage market that preceded it" (Yellen 2010).

Still in 2010, the then Chairman of the Fed, Ben Bernanke, said that "during the worst phase of the financial crisis, many economic actors metaphorically threw up their hands and admitted that, given the extreme and, in some ways, unprecedented nature of the crisis, they did not know

[1] *The views expressed in this thesis are of the author, and they do not reflect those of the institution to which he is affiliated.*

what they did not know...The profound uncertainty associated with the ‘unknown unknowns’ during the crisis resulted in panicky selling by investors” (Bernanke 2010).

It is remarkable how the opinion on risk management techniques, and, specifically, on value at risk radically changed in the four years from the first speech to the latest one: from being initially described as a crucial method to be used by banking and financial industries, value at risk became a quasi-evil mechanism which, along with uncertainty, exacerbated the financial crisis.

It is worth noting that before 2007, value at risk was a sort of golden standard among the measures of market risk. Since 1994, when a technical document of JP Morgan-Riskmetrics was released, it has become the most used method to measure the downside risk of banks and investment firms. In fact, VaR is the maximum amount expected to be lost over a given time horizon at a pre-defined confidence level, hence it gives investors a measure of possible expected losses for the following few days and helps them to manage their risk exposures. In 1996, an additional boost to the diffusion of VaR as a risk measure came from the Bank of International Settlements (BIS) which considered value at risk methodologies as acceptable from a regulatory point of view, also as a risk constraint. Since then, VaR-type measures have gained even more favour among financial intermediaries; today, despite criticism, they are still largely used, even by very small banks, to manage and limit exposure to market risk, and they are still a central element of good supervisory practice for defining market risk constraints.

However, as demonstrated by the financial crisis of 2007-08, value at risk cannot adequately capture credit risk inherent in trading exposures, given that it focuses on general market risk and not on the risk of default of counterparties, which can become material in crises. Furthermore, it does not effectively consider market illiquidity. In addition, VaR expresses only maximum losses at a given confidence level (on a certain time horizon); hence it does not discriminate among losses occurring beyond the predetermined level of confidence, thus incentivizing banks to take on tail risk. In particular, illiquidity is one of the major flaws of VaR which emerged in the crisis and the BIS states that when several banks hold exposures of the same traded asset, the market of that asset may rapidly turn illiquid in case of the banking system stress. In fact, at the height of the crisis banks were unable to exit or hedge positions in certain asset markets, suddenly illiquid, thus recording substantial mark-to-market losses. More generally, some of the limits of the VaR were dealt with by the BIS in recent years (Basel Committee 2016b) by complementing and amending the market risk regulation (with the introduction of the expected shortfall measure, see section 5.2). VaR, used as a risk measure for individual financial intermediaries, may have further negative effects on the whole financial system: in periods of financial turmoil market volatility increases and consequently VaR goes up potentially triggering fire sales which in turn increase volatility.

The use of value at risk has been criticized from the beginning, when Danielsson et al. (2001) highlighted that VaR would have induced crashes when they would not have otherwise occurred. However, only the financial crisis of 2007-2009 determined the radical change of opinions reported at the beginning of the section. In fact, the crisis expressly highlighted the inherent problems of VaR, evidencing its drawbacks. In detail, in the immediate aftermath of the crisis, VaR was accused of contributing to the intensification of the cycle, by amplifying and propagating the boom and bust cycle as it homogenizes behaviours of the market participants.

In general, the literature was concordant about the amplification effect caused by VaR: Adrian and Shin (2010, 2013) and Danielsson et al. (2010) found that VaR constraints may have a pro-cyclical effect by amplifying the impact of shocks and increasing market volatility. However empirical evidence in support of such claims is extremely limited.

After the crisis, papers on pro-cyclicality and on the relation between single investors' behaviour and system's reaction have proliferated, as described in the following sections, though the results of such theoretical analyses have been mixed. Danielsson et al. (2004, 2010, 2012) claim that when traders operate under value at risk constraints, market fluctuations are amplified and risk regulation may have the effect of exacerbating price fluctuations; on a different line of reasoning, Adrian and Boyarchenko (2012) state that tightening intermediaries' risk constraints affects the systemic risk-return trade-off by lowering the likelihood of systemic crises.

VaR also affects portfolio allocation decisions, although the results coming from literature are mixed. Alexander and Baptista (2002) and Basak and Shapiro (2001), in different frameworks, show that regulation leads financial institutions to take higher exposure in risky assets in their portfolio allocation strategy. In contrast with these results, Cuoco and Isaenko (2008) and Yiu (2004) find that VaR may reduce allocation to risky assets.

Finally, as reported by Bernanke in the address delivered in 2010 at Princeton University, uncertainty along with VaR has been blamed for worsening the crisis. Several studies show that uncertainty is negatively related to investments in risky assets (e.g. Guetlein 2016). From the empirical side, all the papers on the topics, which are not abundant, are focused on the analysis of behaviour of specific actors of the system in peculiar market situations (e.g. Cont and Wagalath 2014) on the impact on market variance of distressed selling).

Hence, comprehensive evidence, both theoretical and empirical, on the impact of VaR (and possibly of uncertainty) on financial market and portfolio allocation does not exist.

Therefore, the existence of mixed theoretical results, limited empirical support and the lack of comprehensive evidence suggests the need for further theoretical and empirical investigation into the effects of VaR constraints to understand if limitations on risk may really decrease risk exposures of individual banks, while reducing the fluctuations of the whole system.

Hence, the overall goal of this thesis is to answer questions on the still unproven existence of an empirical relation between (regulatory) market risk-constrained investors' decisions and system fluctuations; on the theoretical basis on which such a relation is grounded; and on the impact of risk limits and investment decisions taking into account the effects not only of risk but also of uncertainty.

I limit my analysis geographically to the Italian banking and financial markets which are large enough to be explored, but not internationalized enough (Observatoire de l'Epargne Europeene 2013) to have such relations significantly affected by non-domestic banks. In fact, in more internationalized financial markets, the role of non-domestic banks is relevant; therefore, in such markets, the reaction of financial market risk to banks' behaviour cannot be empirically tested just by looking at domestic bank data. On the contrary, for Italy, which is less internationalized, the relationship between investors' decisions and financial markets is generally stronger and an analysis of this relation is more representative of the effects of investors' behaviour on financial markets.

Before highlighting the main contributions of this thesis, the next section provides some background to the context by reviewing the relevant literature.

1.1 Value at risk and risk exposure

The streams of literature relevant for this thesis concern the theoretical and empirical impact of risk-constraints on investor behaviour and on fluctuations of the system; literature on uncertainty is relevant for chapter 4.

From the theoretical side, the shortcomings of VaR have been repeatedly underlined in the literature. A large number of the papers on this topic, written before the 2007-09 crisis, focus more on the potential impact of risk limits, while the ones published after the crisis focus on the mechanisms enacted by the crisis.

The studies on the potential effects of VaR date back to the first decade of the century and examine possible distortions that such constraints cause to investment decisions or to the investment results. This stream of research gives no conclusive evidence on distortions; conclusions vary from having higher or lower exposure in risky assets to lower returns or to the inexistence of the impact of VaR limits.

Basak and Shapiro (2001) analysed the optimal, dynamic portfolio and the wealth and consumption policies of investors who maximized utility and manage market-risk exposure by using value at risk. They found that VaR risk managers optimally choose a larger exposure to risky assets than non-risk managers, and consequently incur larger losses when losses occur; furthermore, in a general-equilibrium analysis they find that the presence of VaR risk managers amplifies the stock-market volatility at times of down markets and attenuates volatility at times of up markets. In the same line, Sentana (2001) focused on the mean-variance allocation with VaR constraint in a world with one riskless asset and a finite number of risky assets. He looked at three building blocks (mean-variance portfolio frontiers, mean-variance indifference curves, iso-VaRs) to find the best portfolio allocation with a VaR cap. He concludes that the existence of a VaR constraint is a cost for a fund manager in terms of lower return (but also lower risk) and lower Sharpe ratio. Unlike the above, Campbell et al. (2001) show that when expected returns are assumed to be normally distributed, a model with a VaR limit provides almost identical results to the mean-variance approach. Yiu (2004) looked at the optimal portfolio allocation under a VaR-constrained utility maximization problem in a continuous time setting. Yiu used numerical methods to find that when portfolio value increases the VaR constraint becomes active and reduces the allocation of investments to risky assets. Furthermore, Cuoco and Isaenko (2008) found that when VaR is re-evaluated dynamically, the risk exposure of a trader subject to a VaR limit is always lower than that of an unconstrained trader and that the probability of extreme losses are also lower.

From the empirical side there is relatively little research which deals contemporaneously with portfolio optimization and value at risk constraints. Among the most relevant, Puelz (2001) presents four model frameworks that apply VaR to portfolio decisions. One of the frameworks considered is the standard mean-variance. He concludes, also by using empirical data, that VaR-optimal portfolios are penalized in case of loss: in particular, he states that VaR optimal portfolios are more likely to incur large losses when losses occur. In addition, Campbell et al. (2001) found that the higher the confidence level of VaR, the lower the portion of the portfolio invested in risky assets. Cont and Wagalath (2014), in the stream of literature regarding price dynamics, show that distressed selling (due, for instance, to capital requirements set by regulators) has an impact on market variances and covariances. They apply the model to a three-month period immediately after the collapse of Lehman, showing that they cannot refute the hypothesis of no liquidation of assets (fire sales) and find an impact on the variance-covariance matrix. On the basis of data from five major US investment banks, Adrian and Shin (2010) show that financial intermediaries manage their balance sheets actively in a way that causes leverage to be high during booms and low during busts. They conclude that leverage of financial intermediaries is pro-cyclical as a consequence of the active management of balance sheets to respond to changes in prices and measured risk.

1.2 VaR-based regulation and systemic risk

A new wave of studies on risk management have emerged since the beginning of the 2007-09 crisis (and some prior years). Most of them underline the unintended consequences on systemic risk of VaR constraint (e.g. pro-cyclicality, amplification effect) and the possible distortions (or change in behaviour) caused by the constraints on allocation decisions.

Danielsson et al. (2004, 2010, 2012) demonstrate that when risk neutral traders operate under value at risk constraints, market conditions exhibit signs of amplification of shocks through feedback effects: although traders are risk-neutral, the VaR constraint makes them act as if they were risk-averse. Furthermore, the authors build a model where VaR constraint has a role in the amplification effect of deleveraging on volatility, and they also find that when risk is regulated, prices are lower and volatility is higher; hence, risk regulation may have the effect of exacerbating price fluctuations. Shin (2010) looks at the balance sheets of investors to find a pro-cyclical effect of the VaR constraint. In particular, he shows that when VaR is less binding (and investors' equity is larger than necessary), investors use the slack in the balance sheet to purchase additional risky securities thus causing an amplified response to improvements in fundamentals. A few authors put some trading behaviours (e.g. fire sales due to binding constraints such as risk limits) at the centre of their analysis on pro-cyclicality. In this field, Cont and Wagalath (2013) modelled the impact of fire sales on volatility and correlations. They found that the more widespread a security is among different portfolios of various financial agents, the higher the cost of imposing common behaviour via regulatory constraints. Jang and Park (2016) integrate a VaR constraint to fund manager's wealth and ambiguity functions showing that a fund manager using VaR-based risk management is exposed to large losses in bad states.

Adrian and Boyarchenko (2012), in a model where agents are risk-constrained, find a relation between supervisory requirements, cost of risk and systemic risk in the sense that tighter capital requirements shift the term structure of systemic risk downward at the cost of an increased price of risk. Kaplanski and Levy (2015) reached the conclusion that with VaR regulation, institutions face a new regulated capital market line, which induces resource allocation distortion in the economy; only when a riskless asset is available does VaR regulation induce an institution to reduce risk, otherwise the regulation may determine both higher risk and asset allocation distortion. Examining possible distortion or change in behaviour of investors, Alexander and Baptista (2006) find that under certain circumstances, regulation may increase the standard deviation and the probability of extreme losses. More generally, they show that when VaR constraint is imposed it is plausible that certain banks will select riskier portfolios than they would have chosen in the absence of the constraints.

1.3 The interplay of risk and uncertainty

In general, uncertainty is associated with Knight (1921) who separated the notion of risk, as a measurable uncertainty, from the non-measurable, known as Knightian uncertainty. Knightian uncertainty is unobservable, though some proxies can be used to assess its changes over time. Non-Knightian uncertainty refers to the uncertainty of a variable for which the probability distribution of ex-ante realizations can be defined, but the values are not defined.

Literature offers mixed conclusions about the impact of uncertainty on the demand for risky assets. Guetlein (2016) claims that in the standard expected utility framework an increase in risk aversion reduces the demand for risky assets, whereas with ambiguity aversion this in not necessarily the case. Pinar (2014) finds that under certain circumstances ambiguity aversion leads to giving less weight to a fund consisting of risky assets. Illeditsch (2011) builds on the work by Epstein and Schneider (2008), who examined the effect of ambiguous information on stock prices to argue that the interaction between risk and uncertainty can cause drastic changes in the stock prices. Such an interaction may explain the large increase in volatility after unexpected events. Maccheroni et al. (2013) built a mean-variance framework with a risk-free asset, a risky asset, and an ambiguous asset. Using this framework, they found that ambiguity has a negative impact on the fraction of wealth invested in non-free risk assets.

1.4 Main goal of the thesis and chapter preview

The literature on VaR reported above shows that VaR amplifies market shocks, thus causing an increase of volatility, and affects the portfolio decisions of investors. However, no articles provide empirical evidence on the amplification effect. In addition literature does not give a clear view on the direction of the impact of risk limits on portfolio allocation, since not all research has found that imposing risk limits reduces investments in risky assets, as it could be expected. Finally, although much of the literature points towards an inverse impact of uncertainty on investment in risky assets, there is no theoretical evidence of such relation under VaR constraints in a mean-variance framework. Empirical evidence of such an inverse relation is also lacking in literature.

In this thesis I provide supporting empirical evidence of the amplification effect of VaR on market volatility. I find that VaR constraints reduces investments in risky assets, as expected by regulators, and I provide empirical evidence of these effects. Lastly, I confirm the inverse relation between uncertainty and risky investments and empirically demonstrate that this relation is, significantly, especially valid in turbulent periods.

The thesis is structured as follows: chapter 2 contributes to the strand of literature concerning the unintended consequences on the financial system of imposing VaR limits. As mentioned in the literature section, when risk is regulated, volatility is higher; hence, risk regulation may have the effect of exacerbating price fluctuations. Various theoretical papers have found a relation between constrained investors and the system via value at risk. This relation has been extensively discussed but not empirically proven. The original contribution of the chapter is the empirical analysis of the relation between risk constraint and financial system fluctuations, which proves that market volatility is significantly dependant on value at risk of banks and Granger-causes market volatility. I empirically prove this relation for the Italian market, where the impact of the behaviour of Italian banks on the local stock exchange is more direct and visible than in other, more internationalized, financial systems. The result is in line with various papers which theoretically modelled this effect and complements the evidence of other empirical research.

Chapter 3 investigates the theoretical background of the results of chapter 2 and contributes to the strand of literature about the impact of risk limits on investors' decisions. In this chapter, a mean-variance framework is used to determine the optimal percentage alpha of portfolio invested in risky assets, both in a risk-constrained and in an unconstrained setting. The literature on the impact of the constraint on risky investment is mixed. I find that, if the constraint binds, constrained alpha is lower than the unconstrained, giving support to papers showing that risk constraints can reduce risk exposure of individual banks. The empirical part of chapter 3 supports that constrained alpha forecasts the actual risky investments of banks better than the traditional mean-variance framework, thus supporting the assumption that banks that are regulated use the risk-constrained setting to take decisions about risky investments. From the empirical analysis, some additional results give support to literature on fire sales, providing some possible clues on the timing of fire-sales, which may occur while passing from the unconstrained to the constrained framework, in transitions from stable to turmoiled markets.

Lastly, chapter 4 contributes to literature on financial uncertainty and complements the results obtained in chapter 3 about the different investment periods (stable periods or turmoiled periods). This chapter builds on the theoretical results reached in chapter 3 about the constrained alpha to give additional information on the impact of uncertainty, the crucial role of which has been largely described in literature since the financial crisis. The theoretical results are in line with literature, showing that increasing uncertainty decreases the portion of portfolio invested in risky assets, thus providing new evidence since no articles found a closed formula under VaR constraints in a mean-variance framework. As for chapter 3, some empirical tests confirm the statistical significance of market data for constrained alpha opposed to the non-significance of the unconstrained alpha, thus providing new supporting empirical evidence in favour of the inverse relation. Furthermore, I find that in turbulent periods, for constrained-alpha the impact of risk and uncertainty are significant but expected returns are not. Interestingly, in low-volatility periods investments are not driven by risk and uncertainty (not significant) but by expected returns, which are significant and with the expected sign. These results, new in the empirical literature, confirm the results of theoretical papers about the amplification of fluctuations of financial markets: in fact, the constrained investment in risky assets is significantly dependent (with a negative sign) on expected risk and expected uncertainty in turbulent periods but not in stable periods; hence, in turmoil, risky assets held by banks decrease thus amplifying the effect on volatility and uncertainty of the financial market.

The remainder of the thesis is organized as follows. Chapter 2 empirically examines, using the time series techniques, the relation between risk-constraints and market volatility. The conjecture is that risk-constraint Granger-causes market volatility fluctuations.

Chapter 3 investigates the mechanism on which the relation from risk-constraint to market volatility is grounded by theoretically relating risk constraints to investors' choices. In addition, with forecasting techniques based on mixed frequency regressions (MIDAS), the chapter empirically analyses if the banks behave as constrained investors using the model obtained in the theoretical part of the chapter.

Having ascertained in chapter 3 that banks behave like constrained investors, chapter 4 investigates if their investment decisions are also affected by uncertainty, in addition to risk, finding a positive answer from theoretical formulas. Empirical tests, with ordinary least squares (OLS) regressions, confirm that banks invest on the basis of the constrained investors' framework and their investment choices are affected by the existence of uncertainty.

Finally, chapter 5 briefly summarizes the key findings of the thesis and offers directions for future research areas.

Chapter 2. Fuelling fire sales? Prudential regulation and crises: evidence from the Italian market

2.1 Introduction

In 2006, the Chairman of the Federal Reserve Bank (Fed), Ben Bernanke, highlighted the importance of modern risk management as a central element of good supervisory practice and encouraged the industry to push forward the risk management frontier.

Just a few years later, in 2010, after the beginning of the crisis, Janet Yellen, Chair of the Fed from 2014, said that "methods of modern risk management may have intensified the cycle…because of their reliance on metrics such as value at risk that are highly sensitive to recent performance, especially volatility. In good times, volatility declines, and value at risk along with it. This pattern generated a pro-cyclical willingness to take on risk and leverage, amplifying and propagating the boom and bust cycle. The vicious cycle of a collapse of confidence, asset fire sales, evaporation of liquidity, and a deleveraging free fall was the mirror image of the manic mortgage market that preceded it" (Yellen 2010).

In the four years between Bernanke's speech and Yellen's one, the opinion on risk management techniques changed radically: from crucial methods for economic stability and the main contributors to the decline of volatility (Panetta et al. 2006), they became a quasi-evil mechanism of depression which contributed to the worsening and deepening of the crisis

(Bernanke 2008, Financial Stability Forum 2008, Senior Supervisors Group 2008). The mentioned risk management techniques encompass, beyond the ability of the banks to distribute their credit risk by selling it to the market via complex financial products (originate-to-distribute model), the use of risk measures, such as value at risk (VaR), to estimate the potential loss of investments in a predetermined period of time.

Much of the literature underlines some drawbacks of VaR and finds that VaR constraints may have a pro-cyclical effect by amplifying the impact of shocks and affecting market volatility (Adrian and Shin 2010, 2013, Danielsson 2010). Furthermore, Danielsson et al. (2001) claim that the use of value at risk could have induced crashes when they would not have otherwise occurred. Danielsson et al. (2004) found that the main channel of transmission of the amplification effect, in a VaR-constrained framework, is the adjustments of the expected returns and covariances of the investors and the related increase to risk aversion caused by the VaR constraint. The empirical evidence of the above-mentioned effects is very limited and is in favour of the existence of the amplification effect (Adrian and Shin 2010).

This chapter shows empirically that the increasing tightness of the value at risk constraint may amplify the instability of the financial market in crises. To the best of my knowledge, no empirical proof of this relation has been given so far; to measure the tightness of the VaR constraint I use a unique dataset of daily VaR and VaR limits of a sample of Italian banks taken from supervisory reporting.

From the point of view of individual banks or financial agents, value at risk may be seen as the gold standard among the measures of market risk. Since 1994, when a technical document of JP Morgan-Riskmetrics was released, it has become a standard method to measure downside risk of banks and investment firms. In 1996, an additional boost to the diffusion of VaR as a risk measure came from the Bank of International Settlements (BIS) which considered value at risk methodologies as acceptable from a regulatory point of view (Basel Committee 1996). Since then, VaR-type measures have gained even more favour among financial intermediaries and today they are also used by small banks to manage their exposure to market risk. Value at risk is often preferred to other measures because it is easy to understand (it is measured in price units, such as dollars or euros, or as a percentage of portfolio value), it can be used to compute and compare risk of different types of assets and various portfolios, and it can be used to allocate capital to different units, even if it is not easily additive. Therefore, from the point of view of financial intermediaries (individual view), VaR is a good instrument to measure and compare market risk of their investments.

However, from a systemic point of view (systemic view), VaR and other risk management measures may homogenize the behaviours of financial agents, thus amplifying the cycle. This criticism dates back to 2001, when it was raised by the former President of the European Central Bank (Trichet 2001) and by the Financial Market Group of the London School of Economics (Danielsson et al. 2001). After the beginning of the crisis, criticisms of VaR gained so much consensus that even the BIS (Basel Committee 2016a and 2016b), which adopted VaR for regulatory purposes first, mentioned various drawbacks of the method (e.g. inability to adequately capture credit risk inherent in trading exposures; incentives for banks to take on tail risk; inability to capture the risk of market illiquidity) and proposed new risk measures. In particular, illiquidity is one of the major flaws of VaR which emerged in the crisis, as the BIS states that when several banks hold exposures of the same traded asset, the market of that asset may rapidly turn illiquid in case of banking system stress. In fact, at the height of the crisis, banks were unable to exit or hedge positions in certain asset markets, which were suddenly illiquid, thus recording substantial mark-to-market losses. VaR, used as a risk measure for individual financial intermediaries, may have further negative effects on the whole financial system: in periods of financial turmoil, market volatility increases and consequently VaR goes up. The increase of VaR for banks and other financial agents which use VaR to measure their risks increases their exposure to a higher level of market risk. If the risk level is too high banks sell (or even fire sell) some of their financial assets to reduce the risk. Such sales may cause further oscillations (increase of volatility) of the prices of listed securities and, consequently, create an additional rise of VaR, so creating the start of a vicious circle. The circle could be particularly intense if the number of banks using VaR and holding securities in common is high.

In this chapter, the existence of this vicious circle has been tested. The data used is the daily VaR of some Italian banks and their internal limits of VaR, the daily volatility of the Italian financial market and the spread between the yield of the 10-year reference Italian government bond and the German bond (measure of sovereign risk).

The ratio between VaR and the internal limit of VaR (VaR ratio) is used to measure the tightness of the constraint. To take into account any potential endogeneity of the data (see section 2.3), a vector autoregression model (VAR) has been employed. The main results of the empirical tests are that market volatility has a significant, positive relation with lagged VaR ratio and that the VaR ratio does Granger-cause market volatility changes. Moreover, the impact of the value at risk ratio seems to have a kind of overshooting effect since the VaR ratio with longer lags has a negative impact on market volatility.

Despite the huge amount of theoretical literature on risk management and pro-cyclicality, this is the first empirical investigation which directly takes into account data from bank and financial markets to prove the existence of the macro-impacts of VaR.

The results provide additional keys to interpret crisis development, further suggestions for studies on systemic risk and on the unintended consequences of individual regulatory instruments, and evidence of the mechanism that takes place when a measure used to control and contain the risk of individual financial agents has an impact on the whole system and on market variables (such as prices, returns and volatility), via the homogenization of behaviours.

2.2 Theoretical background

A few years ago, several researchers highlighted that using VaR and other volatility-based measures as risk limits might have some drawbacks. On the theoretical side, critiques are based on the idea that market volatility (seen as a market risk measure) is endogenous in the sense that it depends on the behaviour of market players and on the interaction among them. In particular, in bad times, the use of common risk management techniques, such as VaR, increases the similarity of behaviours of different banks so causing unintended and unpredictable effects on the system as a whole. Along with endogeneity, the assumption that the use of VaR and, more generally, of risk limits may amplify ordinary fluctuations of the economic cycle and of financial variables (pro-cyclicality) has been a strong criticism of the use of VaR.

On these points, Danielsson et al. (2010) demonstrate that when risk neutral traders operate under value at risk constraints, market conditions exhibit signs of amplification of shocks through feedback effects. This effect complements other amplifications of shocks highlighted in literature (for instance, very recently by Kokas et al. (2019)). Similarly, but in a different framework, Adrian and Boyarchenko (2012) described a model where agents are risk-constrained, and they studied the impact of prudential policies on the trade-off between system-wide distress and risk pricing. They uncovered a relation between supervisory requirements, cost of risk and systemic risk. In particular, they found that tighter capital requirements shift the term structure of systemic risk downward at the cost of an increased price of risk.

Discussing pro-cyclicality, Panetta et al. (2009) show that many variables (e.g. capital regulation, accounting standards and managers' incentives) can have a pro-cyclical impact. Adrian and Shin (2006) highlight that the mechanism of targeting the leverage level by market agents can foster pro-cyclicality. They also suggest that some micro-behaviours which have macro-impact may have the same vicious effect (e.g. the use of the VaR model to determine internal capital allocation). Again in 2010, Adrian and Shin said that some accounting rules also contribute to increased turmoil: in particular, the mark-to-market principle applied to balance sheets of financial intermediaries along with VaR constraints can foster pro-cyclicality and have an impact on market liquidity and on volatility measures. In 2013, the same authors (Adrian and Shin 2013) studied the balance sheets of five major investment banks and showed that pro-cyclicality can be due to the fact that banks actively manage their balance sheets in order to keep constant the ratio between their value at risk and equity.

Furthermore, some authors have shown that prudential regulation has a greater impact on pro-cyclicality than accounting standards (Amel-Zadeh et al. 2014, Brousseau et al. 2014, Jones 2015), and that such prudential regulation, when oriented more to individual banks than to the financial system, may not give the right relevance to the possible systematic impact (also on financial markets) of banks' risks (Fiordelisi and Marqués-Ibañez 2013).

All the previously mentioned problems (pro-cyclicality, leverage, market liquidity, risk management) have been put together from a stream of literature which examine panic behaviours; in this framework, panic happens when the circular relationship between market risk and asset price level causes shifts in risk, mainly generated by self-fulfilling behaviours (Bacchetta et al. 2012).

A few of the above-mentioned criticisms of VaR have recently been addressed by the Basel committee, which have implemented some ad hoc advancements to the framework of risk measures (Basel Committee 2012, 2013, 2014, 2016b). However, some important drawbacks persist, as the Committee itself admits (Basel Committee 2016a).

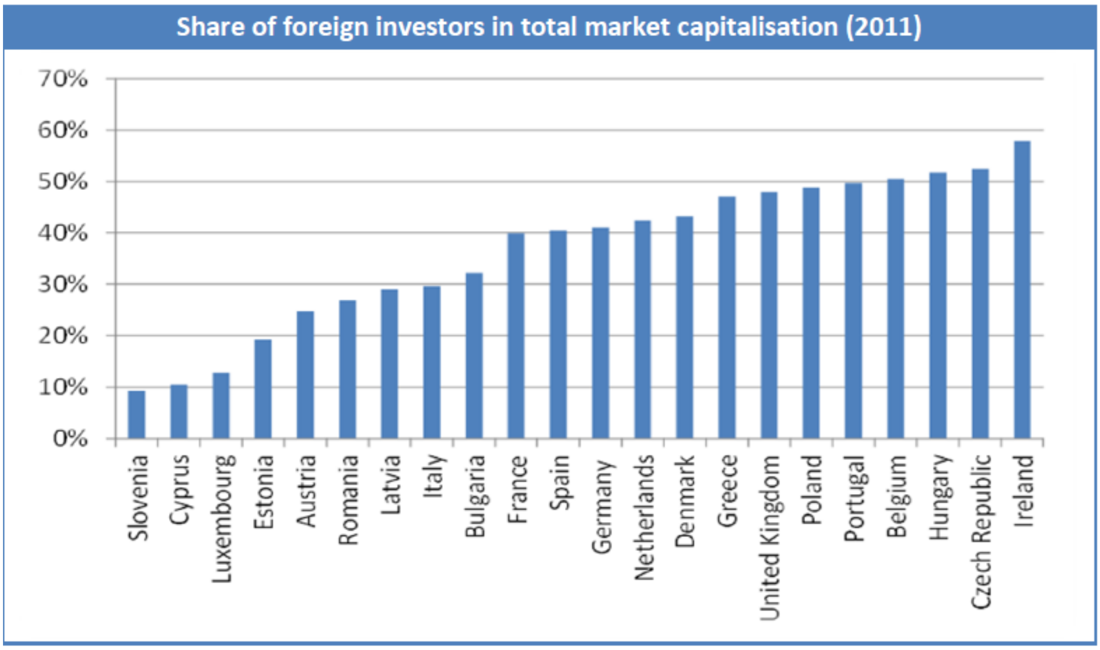
The above-mentioned papers are mainly theoretical. On the empirical side the literature on the vicious circle between market risk measures and financial markets is not extensive. In 2010, in the aftermath of the financial crisis, the need for empirical research on ways in which regulations can be designed to make bank capital less pro-cyclical was underlined (Wilson et al. 2010).

Cont and Wagalath (2014), in the stream of literature regarding price dynamics, showed that distressed selling (due, for instance, to capital requirements set by regulators) can have an impact on market variances and covariances. They applied the model to a three-month period immediately after the collapse of Lehman, showing that they could not refute the hypothesis of no liquidation of assets (fire sales), and found an impact of it on the variance-covariance matrix. Adrian and Shin (2010), on the basis of data from five major US investment banks, showed that financial intermediaries manage their balance sheets actively in a way that causes leverage to be high during booms and low during busts. They concluded that leverage of financial intermediaries is pro-cyclical as a consequence of the active management of balance sheets to respond to changes in prices and measured risk. In their regressions they found a significant negative relation between lagged change in VaR and change in leverage. The result was based on quarterly data for five investment banks for a sample period of 15 years ending at the first quarter of 2008. Furthermore, they explored the nexus between deleveraging of these banks and volatility of the market by looking at the repo market (weekly data, from 1990 to 2008, from all primary dealers), finding that the growth rate of repos on dealers' balance sheets significantly forecast innovations in the market volatility index (VIX).

In this chapter, the relation between the tightness of the VaR constraint (VaR ratio) and market volatility is directly tested. Daily data on value at risk and financial markets are used (450 observations, one-and-a-half year window) and, on the basis of time series techniques, the existence of Granger causality between the VaR ratio and market volatility is also verified. Granger causality does not imply a true causality relation among variables but indicates whether one time series is useful in forecasting another and the direction of such type of causality. This feature of Granger causality is extremely useful for the chapter goal, which is to verify the existence of a flow of information and its direction between time series. Hence, the methodology is different from the one used by Adrian and Shin (2010) which was based on a longer time-span but on two different panel regressions and two different samples with different data frequencies. in detail, they studied the correlations firstly between VaR and leverage and secondly between volatility and repos. An additional significant difference from other empirical papers is related to the geographical perimeter of the variables: Adrian and Shin (2010) and Cont and Wagalath (2014) used US data, whereas the focus of this work is on Italian data. This choice is based on the idea that Italian market is not as international as other European (or

US) markets. This feature is particularly important for this type of analysis: to perform an empirical analysis of the relation between national financial intermediaries and the corresponding national financial market requires a focus on markets where domestic financial intermediaries may have a relevant impact on the local financial market. In fact, the Italian market is one of the least internationalized among the largest European nations with regards to both stocks and government bonds (the two main components of the portfolio of securities for Italian banks). For stocks, a report to the European Commission (Observatoire de l'Epargne Europeene 2013) shows that in 2011 (a period immediately before the time window of the empirical analysis of the present chapter), the Italian Stock Exchange was less internationalized than other large European countries (Spain, France, United Kingdom, Germany) in terms of the share of foreign investment to total market capitalization of the Stock Exchange (see Fig. 2.1).

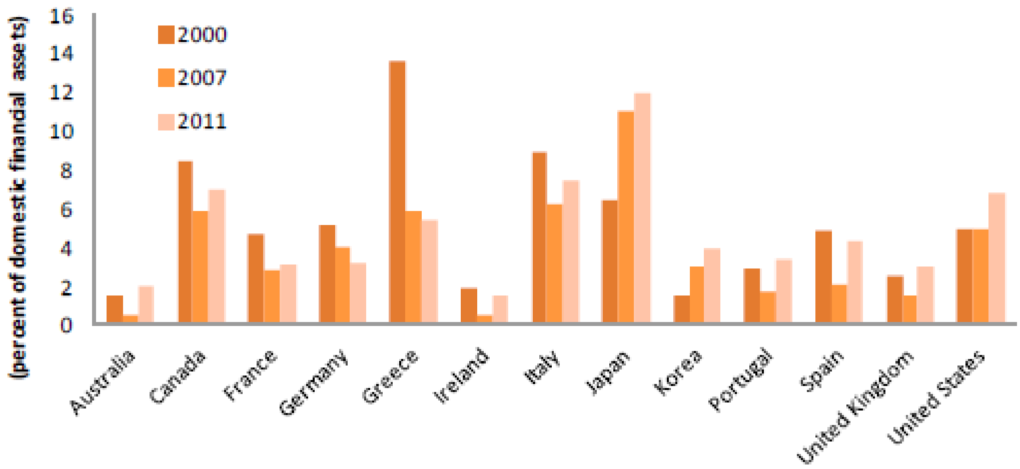
Fig. 2.1. Foreign investments to total market capitalization



Share of foreign investment investors, as a percentage of total market capitalization per EU country, as of 31 December 2011. Foreign investors are defined as any investors whose residence differs from the registration country of the company whose shares they hold. Foreign investors can be European (other than national) or non-European. Source: Observatoire de l'Epargne Europeene - OEE (Paris, France) (2013).

With regards to government bonds, an IMF working paper (Andritzky 2012) shows that in 2011 the share of government securities held domestically was higher in Italy than in other large European countries (see Fig. 2.2).

Fig. 2.2. Government securities held by domestic investors



Sources: Country authorities, OECD.
1/ First observation refers to 2002 for Korea and 2001 for Ireland (estimated).

Portion of domestically held government securities as per cent of total domestic financial assets.
Source of the graph: R. Andritzky (2012).

Furthermore, the relation between Italian Stock Exchange and Italian banks, which is a relevant information for this chapter, is stronger than in other countries. On the basis of the same European report cited before (Observatoire de l'Epargne Europeene 2013), Italian banks hold a higher share of listed companies relative to other large European countries (9%; against less than 1% for the UK, 5% for Germany and 4% for France).

Against this background, the analysis of the relationships between domestic banking data and national financial data for Italy is more significant than for the other above-mentioned countries, where national financial data may also be influenced by the behaviour of non-domestic banks.

2.3 Methodology

The test of the relation between market volatility and the VaR ratio (VaR of banks divided by their internal limits of VaR) is based on two major steps. The first step is to compute the volatility of the market; the second step is to examine market volatility in relation with the VaR ratio and possibly with other relevant variables. There is no obvious dependant-independent relation between the VaR ratio and market volatility. In fact, by definition, VaR depends on market volatility. Conversely, for the research question of the chapter, VaR may impact volatility.

The endogeneity of the variables is supported by the literature. In fact Adrian and Shin (2006, 2010, 2013) show how the use of risk limits measured in terms of VaR may determine fire sales, which have an impact on market volatility and, in turn, on VaR (hence on risk limit); Cont and Wagalath (2014) show that fire sales may have an impact on market data. This double-way interaction between market volatility and VaR exactly expresses the endogeneity idea where it is not known ex-ante which variable drives the other, and which one can be considered exogenous.

To deal with such endogeneity and with the possible reverse causality problem, I opted to use the vector-autoregression technique (VAR), where current values of variables are put in relation only with lagged ones (Brooks 2007), without any contemporaneous terms. It is worth noting that, based on the public information published by the major Italian banks, the value at risk model used by banks in the period under analysis was based on historical simulation not on the parametric model. Hence, the VaR definition was based on the worst 1% loss level (in a 10-day period) and not on a measure of market volatility; the variables used in the regressions are thus neatly distinct. Nevertheless, since they refer to similar concepts, in section 2.5.2 I perform some additional tests to further check the robustness of the results obtained in the baseline regressions. In the limited number of empirical research articles on the impact of value at risk, the VAR approach has not been used. I opted for this approach since it helps to deal with endogeneity and to test the lead-lag relations among interconnected variables. Furthermore, it let me exploit the unique daily dataset available on VaR and internal risk limits. In the chapter, some specific robustness tests are performed to control for possible concerns related with the use of the method (lag order selection, reverse causality).

The VAR methodology is widely used in literature to test lead-lag relations among variables; Lafuente-Luengo (2009) uses it to find evidence of the intraday lead-lag relationship between futures market volatility and spot market volatility; Bec and Gollier (2009) use it to show that VaR is influenced by the state of the financial market cycle and Chomicz-Grabowska and Orlowski (2020) examine the dynamic interactions between financial market risk (VIX) and some key macroeconomic stability variables with this technique. However, given the uniqueness of the data used in this chapter (VaR and VaR limits for individual banks), VAR has not been used in literature for testing the relation between the VaR constraint and market volatility.

Finally, in the VAR framework used in this chapter, a variable regarding Italian government bonds is also considered since these bonds greatly affected the Italian financial market in the period examined.

2.3.1 Volatility computation

The first step of the analysis is the computation of market volatility. In the baseline empirical tests I use the annualized 10-day volatility of the returns of the market index, in line with the regulatory requirements for VaR (Basel Committee on Banking Supervision 1996).

As mentioned in the previous section, the use of this volatility variable may cause some endogeneity doubts to arise. In addition, autocorrelation of returns may affect independence of returns, impacting on both the VaR and the 10-day volatility measures. To control for these possible concerns, in the robustness checks I reperform the vector autoregression by substituting the 10-day volatility variable with volatility coming from the ARMA-GARCH (autoregressive moving average model for the mean with generalized autoregressive conditional heteroscedasticity model for the variance) approach, thus cleaned of the autocorrelation effects. To compute the GARCH volatility, I iteratively perform the following steps (Box Jenkins approach):

- assess the stationarity of the series of returns by looking at the sample autocorrelation and partial autocorrelation functions;
- select the stationary model for conditional mean (autoregressive AR, moving average MA, or ARMA model) on the basis of the autocorrelation and partial autocorrelation functions of the dependent time series;
- estimate the coefficients that best fit the selected ARMA model;
- check the non-correlation and homoscedasticity of the residuals; in this case, estimation of GARCH models for volatility and a further check of the partial autocorrelation function of squared residuals.

The ARMA(p,q) model is described by the following equation:

$$Y_t = \phi_0 + \sum_{i=1}^{p} \phi_i Y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \epsilon_t \qquad (2.1)$$

where:

p refers to the number of autoregressive terms,

q refers to the number of lagged error terms,

ϕ refers to the coefficients of the autoregressive terms and the constant,

θ refers to the coefficient of the moving average terms.

Seasonal adjustments can be added to (2.1) in order to best capture seasonal variations in the data. For instance, a seasonal moving average (SMA) can be included when a seasonal moving average term (with lags) captures some economic regularities of the data. The resulting MA lag structure would be obtained from the product of the lag polynomial specified by the MA terms and the one specified by any SMA terms. For instance, if the third addendum of (2.1) is a standard second-order MA process without seasonality:

$$Y_t = \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} \tag{2.2}$$

which can be written as:

$$Y_t = (1 + \theta_1 L + \theta_2 L^2)\epsilon_t \tag{2.3}$$

where L is the lag operator such as $L^k Y_t = Y_{t-k}$.

With the inclusion of a seasonality factor at lag 3, the polynomial becomes

$$Y_t = (1 + \theta_1 L + \theta_2 L^2)(1 + \omega_3 L^3)\epsilon_t \tag{2.4}$$

As explained at the beginning of the section, in the last step of the Box-Jenkins approach, if the time series plots of residuals and of squared residuals of the ARMA model show some clusters of volatility (typical of financial series) the model should be improved, by adding an ARCH/GARCH models for volatility.

ARCH(p) models, where p refers to the order of the lagged autoregressive terms of previous innovations, have the following form:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 , \quad \omega > 0, i > 0, \alpha_i \geq 0 \qquad (2.5)$$

$\epsilon_t = \sigma_t z_t$ where z_t is white noise.

Relative to the ARCH effects, the GARCH model assumes that the conditional variances of innovations follow an ARMA model. In that case, GARCH (p, q) refers to the following representation:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 ,$$

$$\omega > 0, i > 0, \alpha_i \geq 0, \beta_i \geq 0 \qquad (2.6)$$

$\epsilon_t = \sigma_t z_t$ where z_t is white noise and $\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i < 1$ to have stationarity.

2.3.2 Vector autoregression

With the second step of the analysis, the relation between market volatility and the VaR ratio of the banks is examined by using a Vector Autoregression technique. The vector autoregression (VAR) is an econometric model used to capture the linear interdependencies among multiple time series. VAR models generalize the univariate autoregressive model (AR model) by allowing for more than one evolving variable. Each variable has an equation explaining its evolution based on its own lags and the lags of the other model variables.

In mathematical terms, a VAR model describes the evolution of a set of k variables over the same period (t = 1, ..., T) as a linear function of only their past values. The variables are collected in a k × 1 vector y_t, where the i-th element, $y_{i,t}$, is the observation at time "t" of the i-th variable.

The representation of a VAR is as follows:

$$\mathbf{y_t} = \beta_0 + \sum_{i=1}^{p} \beta_i \mathbf{y_{t-i}} + \mu_t \qquad (2.7)$$

where $\mathbf{y_t}$ is a $k \times 1$ vector of variables determined by p lags of all k variables in the system, μ_t is a $k \times 1$ vector of error terms, β_0 is a $k \times 1$ vector of constant term coefficients and β_i are $k \times p$ matrices of coefficients of the ith lag of y_t.

Since only lagged values of the variables appear on the right-hand side of the equations, simultaneity is not an issue and OLS yields consistent estimates. To choose the maximum lag p in the VAR model, specific tests are used.

All variables have to be of the same order of integration; in detail, to run a VAR in levels all variables have to be I(0) (stationary).

In this chapter, as reported in the empirical section (2.5.1), all variables are stationary or reduced to stationarity hence the VECM (vector error correction model) is not used, but an unrestricted VAR in the reduced form. In fact, the goal of the chapter is not to study the long-run equilibrium and the related adjustment process, for which VECM would be one of the possible useful approaches, but to examine the lead-lag relation among variables, and to test if the risk constraint (measured with the VaR ratio) Granger-causes the market volatility variable.

2.4 Data

2.4.1 Main variables

In the present chapter three types of data are used: financial market data; government bond data; measures of exposure to market risk of some banks.

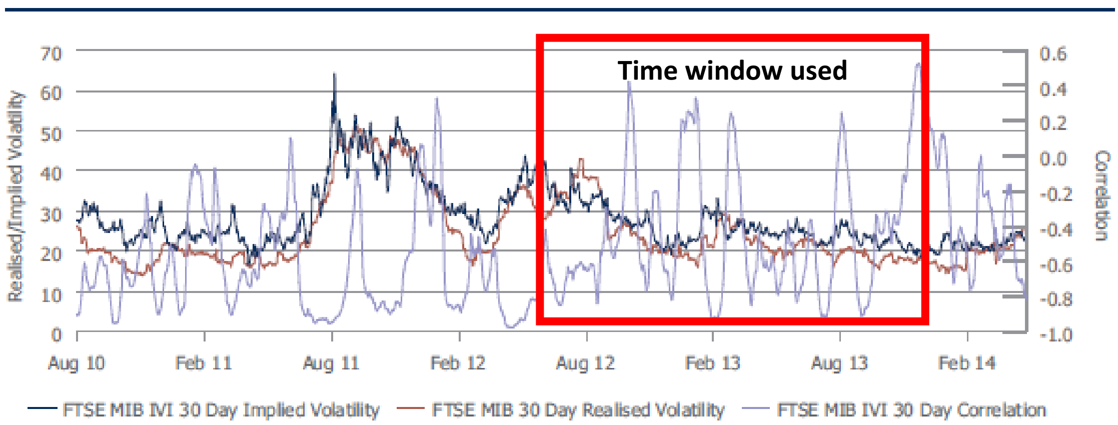
The main financial market data used come from Datastream and are the daily closing prices of the Italian Stock Exchange Index (FTSE MIB). The Italian Index FTSE MIB is a weighted average of the quotes of its components (40 shares). The weights are available on the site of the London Stock Exchange group, which Italian Stock Exchange is a member of. The index accounts for around 80% of the capitalization of the Stock Exchange.

The government bond data come from Bloomberg and are the daily quotes for the 10-year Italian government bond and the 10-year German government bond. The Italian government bond yield has become particularly important since 2011, when the sovereign bond crisis occurred. During that period the spread between the yield of the Italian and the German government bonds became a crucial reference for financial analysts and was used as a measure of the credit (or sovereign) risk of the country.

Data about banks comes from supervisory databases and measures daily market risk exposure of Italian banks (i.e. value at risk) which have a validated internal model to measure market risk exposure. To have a validated internal model means that banks have been authorized to measure their market risk exposure by using their own value at risk model instead of the standard regulatory one. In the dataset, beyond the level of value at risk there are also data on the internal limits of VaR set by these banks to manage their market risk. In this chapter, the ratio between value at risk level and the internal limit of VaR is used as the measure of the risk constraint impact, with the assumption that the higher the value of this ratio, the higher the risk incurred relative to the maximum acceptable risk (where the maximum acceptable risk level is the internal limit). Moreover, the higher this ratio, the more likely that the bank would sell some financial assets to reduce its risk. Further data on banks (e.g. the securities bought and sold by banks in a certain period), used in this chapter for descriptive purposes, are from the central bank databases, too. It is important to note that regulatory market risk exposure concerns only portfolios held with trading intent; therefore, our VaR measure basically considers the portfolio of banks held for trade (HFT).

For all the data mentioned, the time window of the analysis starts from the beginning of April 2012 and ends at the end of December 2013 (see Fig. 2.3).

Fig. 2.3. Volatility in the time window used



Left scale: volatility; right scale: correlation; blue line - realized 30-day volatility; red line - Implied 30-day volatility (implied in options, IVI30, as computed by FTSE). grey line - right scale, correlation between realized and implied volatility.
Source: FTSE (2014).

I have chosen this time span for four reasons. Firstly, for this period I have a representative sample of banks in the VaR dataset. Secondly, it is a period of high volatility; this higher-than-usual volatility is a good starting point to test the research idea of pro-cyclicality in turmoil periods caused by the use of regulatory measures. Thirdly, volatility shows both increases and decreases in the period; this is useful in order to test our hypothesis not only applicable in periods of sharp increase of volatility. Lastly, this period does not include the peak of the second half of 2011, which was more related to a specific (sovereign risk crisis, i.e. issuer risk) than to general market risk as the analysis of specific market risk is beyond the objective of this chapter.

2.4.2 Summary statistics

By following the methodology described in section 2.3.1 the returns on the closing price of the FTSE MIB index have been preliminarily calculated in order to compute volatility. In the period examined, the index returns have a mean close to zero and a daily standard deviation of 1.6% (see table 2.1, column a), which is equivalent to an annual volatility of almost 25%.

Table 2.1. Summary statistics – Italian stock exchange index, value at risk and government bonds.

Statistics	FTSE MIB index – returns (a)	VaR / internal limit of VaR (b)	Yield Gov't bond (BTP)(%) (c)	Spread Italy-Germany (basis points) (d)
Mean	0.00	0.51	4.79	324
Median	0.00	0.50	4.54	302
Maximum	0.06	0.74	6.49	523
Minimum	-0.05	0.35	3.80	220
Std. Dev.	0.02	0.08	0.67	76
Number of observations	456	428	440	440

Time window from 01 April 2012 to 31 December 2013. Daily observations. Index returns are computed as log variations of daily closing prices (source Datastream). The ratio between VaR and internal limits is based on supervisory reporting data; the source of government bond data is Bloomberg. BTPs (Buoni poliennali del Tesoro) are Italy's government bonds.

Public information on risks released by every bank (retrievable from the website of each bank) show that in the sample period the largest part of market risk was generated by both stocks and Italian government bonds held. In fact, Italian government bonds (BTP, Buoni Poliennali del Tesoro) with fixed rate and medium-long term maturity average around 15% of the HFT portfolio and 35% of trading activity in the period. As mentioned before, in the period examined government bonds had a large impact on value at risk because the sovereign risk market crisis started in the second half of 2011.

As in the empirical analysis I use the sovereign risk variable measured by the spread between Italy and Germany's bonds. In table 2.1 I also show this variable (column d).

Data on the reference Italian bond show an average yield of 4.8% with a standard deviation of 0.67% (table 2.1, column c). In the period, the spread with Germany's 10-year bond, which was a proxy of the credit (sovereign) risk measure for financial investors, has been on average equal to 324 basis points (table 2.1, column d). The average exposure of the Italian banks to market risk relative to their own internal limit (VaR ratio) has been around 50%, with a maximum of 74%, and a high range of variability (table 2.1, column b).

The sample of banks used in this study is representative of the system: the trading activity of these banks accounts for more than 40% of the negotiations on the components of the stock exchange index and more than 50% of the held-for-trade portfolio of the Italian banking system.

Since the empirical part is based on data on VaR of Italian banks and volatility of the Italian financial market, it is important that securities traded by domestic banks do represent a not-immaterial portion of the whole trading activity in the financial market. From the data collected, in the sampled period Italian banks traded around 20% of the value of negotiations of the first five securities of the index (shares bought and sold by banks divided by market turnover by value), whose weight accounts for more than 50% of the Italian index. To be coherent with the data used in the estimations (see previous section), the cited incidence on market turnover is computed only on proprietary trading and on securities coming from the held for trading (HFT) portfolio of banks, which is the portfolio used by banks to measure market risk (i.e. the perimeter of VaR calculations). If I added the turnover of other portfolios (e.g. available-for-sale portfolio) which are also traded by banks, the weight would be far higher.

2.5 Results

As mentioned in the previous section, I tested the relation between the Italian market volatility and the VaR ratio. This ratio is computed as the simple average of the VaR ratios (level of VaR divided by the internal limit) of Italian banks, with the assumption that if a bank, whatever its dimension, is close to the VaR limit, it will start to fire sell thus triggering the amplification effect on market volatility.

Given that VaR concerns expected future losses, to measure market volatility I used both the historical standard deviation of the market (the annualized 10-day volatility of the returns of the market index, as described in section 2.3.1), which assume that history will repeat itself, and, in the robustness tests, the best possible estimations of volatility calculated on the basis of an ARMA-GARCH model, which takes into account all peculiarities of the financial series of returns.

Finally, I controlled for the impact of the spread (hereinafter the spread) between Italian government bonds and the German bonds. In fact, this spread is a measure of sovereign risk which is not of interest for the goal of the research. However, in the period considered, for Italy this risk (measured by the spread) may have affected the market volatility variable used to measure general market risk. The changes of the spread are funnelled towards the market volatility through two channels. One of the transmission mechanisms of a shock is via the price of the shares of the banks listed in the market: an increase of the spread causes a decrease of the value of the portfolio held by Italian banks (a relevant part of their portfolio consists of BTPs) and, for listed bank, of the value of their shares. In turn, this has an impact on market volatility of the market index which in Italy is strongly dependant on banks' shares (in December 2013, almost one third of the market capitalization of the index was related to banks). The second transmission mechanism is via the risk management techniques (VaR). Since a not-irrelevant portion of the portfolio held by banks consists of BTP, a negative shock on BTP price due to the increase of the spread has an impact on VaR and the VaR ratio (since the internal risk limit does not immediately change); consequently, banks may decide to sell risky financial assets to reduce their market risk exposure, thus fuelling the market instability.

2.5.1. Main results

Before estimating the VAR model, I assessed the stationarity of the variables by running the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests. The results of these tests show that all the variables, except the GARCH volatility, were not stationary (at a 5% level of significance) unless purged from the trend effect (Brooks 2007). Hence, I detrend the non-stationary variables in order to have all stationary variables in the unrestricted vector autoregressive model (VAR) (see table 2.2, columns g and h). In line with literature (Brooks 2007, Lutkepohl 2007, Chomicz-Grabowska et al. 2020, Ozcicek and McMillin 1999), to calculate the right VAR dimensions (i.e. the best lag order), which is two, I use the information criteria, in particular the Schwarz information criterion (SIC). In fact, Lutkepohl (2007) suggests using such criterion if consistency and not forecasting ability is the goal of the econometric test to be run. Since in this chapter I do not use VAR for forecasting, I opted for the SIC as the main criterion to determine the number of lags. For the sake of completeness for the regressions performed, the Akaike information criterion also suggests using a VAR order equal to two for the regressions reported in both table 2.3 and table 2.4 (see table 2.2). The resulting model is stable and removes most of the autocorrelation (Lutkepohl 2007). To control for the robustness of the lag order choice, some additional checks are performed in section 2.5.2.

Table 2.2. Lag order selection and stationarity tests

Lags	Lag order selection			
	VAR table 2.3		VAR table 2.4	
	SIC (a)	**AIC** *(b)*	**SIC** *(c)*	**AIC** *(d)*
Lag 1	-21.93	-22.05	-33.23	-33.35
Lag 2	-21.94§	-22.16§	-34.14§	-34.35§
Lag 3	-21.75	-22.06	-33.95	-34.27
Lag 4	-21.65	-22.07	-33.82	-34.24
Lag 5	-21.51	-22.05	-33.68	-34.22
Lag 6	-21.37	-22.02	-33.52	-34.17

Variable	*Stationarity checks*			
	Unit root test (ADF) *(e)*	**Unit root test (PP)** *(f)*	**Unit root test on detrended variable (ADF)** *(g)*	**Unit root test on detrended variable (PP)** *(h)*
Volatility (10-day returns)	-1.49	-1.69*	-4.20***	-5.30***
Spread ITA-GER	-0.78	-0.82	-3.76***	-3.78***
VaR Ratio	-1.12	-1.40	-4.52***	-4.37***
VaR ratio on HFT	-0.67	-0.66	-12.02***	-12.79***
Volatility (GARCH)	-2.52**	-2.42**	NA	Na
Volatility of the BTP yield	-1.19	-1.68*	-3.51***	-4.62***

Lag order selection: AIC stands for Akaike information criterion, SIC stands for Schwarz information criterion. I limit the analysis to the sixth lag to consider the weekend effect (see section 2.5.2); the result does not change up to 10 lags.
*Stationarity checks: null: the variable has a unit root. Columns (e) and (f): tests on the variables; columns (g) and (h): tests on the detrended, stationary variables (Brooks 2007). ADF (augmented Dickey Fuller) test performed with the automatic lag length selection (based on SIC criterion): PP (Phillips Perron) test based on Bartlett kernel spectral estimation method and the automatic selection (Newey West) for the bandwidth. GARCH has not been detrended ("NA", columns (g) and (h)), given that the unit root null is rejected at 5% (columns (e) and (f)). § indicates lag order selected by the criterion. *** - significant at 0.01 level, ** - significant at 0.05 level, * - significant at 0.1 level. Number of observations reported in table 2.3 and 2.4.*

The results of the regressions (2.7) reported in table 2.3 (column a) show that the coefficient between the VaR ratio with one lag and market-volatility is significant with the expected positive sign, as often underlined in theoretical literature. The coefficient of the two-lag VaR ratio is significant but with a negative sign, giving empirical evidence also of the overshooting effects (or spirals) highlighted in literature.

Table 2.3. Vector autoregression – volatility, VaR ratio and spread (SIC lag order)

Variables	Volatility (a)	VaR ratio (b)	Spread ITA-GER (c)
Volatility (-1)	0.954***	0.149	-0.349
	(0.05)	(0.36)	(0.28)
Volatility (-2)	-0.049	-0.010	0.529*
	(0.05)	(0.36)	(0.29)
VaR ratio (-1)	0.016**	0.830***	0.074*
	(0.006)	(0.05)	(0.04)
VaR ratio (-2)	-0.018**	0.09*	-0.05
	(0.006)	(0.05)	(0.04)
Spread ITA-GER (-1)	0.002	-0.035	1.026**
	(0.009)	(0.06)	(0.05)
Spread ITA-GER (-2)	0.004	0.011	-0.089*
	(0.008)	(0.06)	(0.05)
Constant	-0.00	-0.00	0.00
	(0.00)	(0.00)	(0.00)

*Vector autoregression among the simple average of the VaR ratio (VaR divided by internal limit), the volatility of the returns of the index and the spread between reference government bonds of Italy and Germany. The (-1) and (-2) labels after a variable name stand for lag 1 and lag 2 respectively. Daily data for Italian banks and Italian financial market from April 2012 to December 2013. *** - significant at 0.01 level, ** - significant at 0.05 level, * - significant at 0.1 level. Number of observations: 383.*

An additional result reported in table 2.3, in addition to the autocorrelation of variables typical of financial series, is that the spread is positively related to the VaR ratio with one lag (though at a lower level of significance, see column c), thus confirming a possible transmission channel through the sale of government bonds (an increase of VaR for bonds held in portfolio may determine the sale of such bonds, the decrease of their price and the increase of their yield and of the Italy-Germany spread).

Furthermore, the VaR ratio and the spread cause (in terms of Granger causality) volatility of the market (at 5% significance) whereas the market volatility and the spread do not cause (in terms of Granger causality) the VaR ratio, thus showing that the information flows from the VaR ratio and the spread towards market volatility.

As is known, Granger causality is not a measure of causality but a demonstration that past values of VaR ratio contain information that helps to predict future volatility. Since the VaR ratio is based on expected volatility, the result of the empirical test, with a lagged VaR ratio impacting on current volatility, may not be an answer to the research question (i.e. if the regulatory risk limits affect market volatility) but may simply be evidence of the obvious fact that since VaR ratio is based on expected volatility it is the best way to predict future volatility. To control for this concern and to further test for the possible endogeneity and reverse causality of the VaR and volatility variables (see section 2.3), I ran the same vector autoregression by substituting the VaR ratio variable with a variable which is not related to regulatory limits, such as VaR divided by the trading portfolio of the bank. If the results obtained in table 2.3 were simply dependant on the fact that today's VaR contains expectations on future volatility or if the endogeneity concerns had affected the results, this new regression should give similar results to before; if the previous results were also dependant on the VaR limit (i.e. the denominator of the VaR ratio used in table 2.3), the new variable (VaR on HFT) would not be significant.

The vector autoregression with the best lags (there are two) selected on the basis of the SIC criterion (see table 2.2), shows no significant correlation between the VaR variable and volatility (table 2.4, column a) nor any Granger causality between the two. Since the only difference between the VaR-variable used in table 2.3 (VaR divided by the VaR limit) and the one used in table 2.4 (VaR divided by the value of the HFT portfolio) is the denominator, I can conclude that it was exactly the VaR limit to determine the significance reported in the column a of table 2.3.

Table 2.4. Vector autoregression – volatility, VaR on HFT and spread (SIC lag order)

Variables	Volatility (a)	VaR on HFT (b)	Spread ITA-GER (c)
Volatility (-1)	0.949***	0.000	-0.362
	(0.05)	(0.001)	(0.29)
Volatility (-2)	-0.045	-0.000	0.557**
	(0.05)	(0.001)	(0.29)
VaR on HFT (-1)	-0.086	0.915***	-2.204
	(1.96)	(0.03)	(11.63)
VaR on HFT (-2)	-1.43	-0.001	-1.152
	(1.62)	(0.03)	(9.57)
Spread ITA-GER (-1)	-0.001	-0.000	1.024***
	(0.009)	(0.00)	(0.05)
Spread ITA-GER (-2)	0.005	0.00	-0.093*
	(0.009)	(0.00)	(0.05)
Constant	-0.00	0.00	0.00
	(0.00)	(0.00)	(0.00)

*Vector autoregression among the simple average of the VaR on HFT (VaR divided by the value of the HFT portfolio), the volatility of the returns of the index and the spread between reference government bonds of Italy and Germany. The (-1) and (-2) labels after a variable name stand for lag 1 and lag 2 respectively. Daily data on Italian banks and Italian financial market from April 2012 to December 2013. *** - significant at 0.01 level, ** - significant at 0.05 level, * - significant at 0.1 level. Number of observations: 383.*

2.5.2 *Robustness tests*

The regressions performed so far are based on three main assumptions: the number of lags suggested by the criteria is correct; the expected volatility is measurable using the latest volatility data; and the risk of the investment in government bonds is correctly measured by the spread. The first assumption is particularly relevant, given the sensitivity of VAR models to the choice of the number of lags. The others may cause inconsistent estimations due to the use of inaccurate measures of the phenomenon under analysis if inappropriate variables have been chosen.

Therefore, in this section I test if the results hold also with different lags of the variables and with different measures of the risk related to the government bonds and market volatility.

One of the most debated topics in VAR literature is the lag order selection. Although information criteria are generally used (Brooks 2007, Lutkepohl 2007) to select the lag order of VARs, Lutkepohl suggests a criterion based on the likelihood ratio statistic (sequential modified likelihood ratio). On the basis of this criterion the best lag order at 5% significance is five (the chosen starting maximum lag is six, consistent with the evidence that the data shows a weekend effect, as reported below; see table 2.7).

The results of the regression (2.7) with five lags are reported in table 2.5 and show that, notwithstanding the higher order and the reduced efficiency of the model (in terms of information criteria, see the lag order selection part of table 2.2), the core results do not change: the VaR ratio with one lag is significant (at 5%) with the expected positive sign (column a); the VaR ratio with two lags is negative and significant at 5% (column a) thus confirming the overshooting effect. The model is stable and there is no autocorrelation (at 1%) up to lag 10.

Table 2.5. Vector autoregression – volatility, VaR ratio and spread

(LR lag order)

Variables	Volatility (a)	VaR ratio (b)	Spread ITA-GER (c)
Volatility (-1)	0.922***	0.079	-0.394
	(0.054)	*(0.403)*	*(0.325)*
Volatility (-2)	-0.009	0.173	0.208
	(0.073)	*(0.547)*	*(0.442)*
Volatility (-3)	0.068	0.310	0.268
	(0.072)	*(0.540)*	*(0.436)*
Volatility (-4)	0.022	-0.794	0.197
	(0.070)	*(0.524)*	*(0.423)*
Volatility (-5)	-0.156***	0.306	-0.011
	(0.050)	*(0.379)*	*(0.306)*
VaR ratio (-1)	0.016**	0.816***	0.093**
	(0.007)	*(0.055)*	*(0.044)*
VaR ratio (-2)	-0.020**	0.025	-0.058
	(0.009)	*(0.071)*	*(0.057)*
VaR ratio (-3)	-0.003	-0.070	-0.007
	(0.010)	*(0.072)*	*(0.058)*
VaR ratio (-4)	0.007	0.183**	0.001
	(0.010)	*(0.071)*	*(0.057)*
VaR ratio (-5)	-0.002	-0.019	0.006
	(0.007)	*(0.055)*	*(0.044)*
Spread ITA-GER (-1)	0.005	-0.027	0.994***
	(0.009)	*(0.067)*	*(0.054)*
Spread ITA-GER (-2)	0.003	0.072	-0.033
	(0.013)	*(0.095)*	*(0.077)*
Spread ITA-GER (-3)	-0.032***	-0.043	-0.079
	(0.013)	*(0.096)*	*(0.077)*
Spread ITA-GER (-4)	0.029***	-0.165*	-0.078
	(0.013)	*(0.099)*	*(0.080)*
Spread ITA-GER (-5)	0.004	0.149**	0.132**
	(0.009)	*(0.069)*	*(0.056)*
Constant	-0.000	-0.000	0.0004
	(0.000)	*(0.001)*	*(0.001)*

*Lag order selected on the basis of the Lutkepohl's LR. Variables: simple average of the VaR ratio (VaR divided the value of the internal limit), volatility of the returns of the index; the spread between reference government bonds of Italy and Germany. The (-1) … (-5) labels after a variable name stand for lag 1 … lag 5 respectively. Daily data on Italian banks and Italian financial market from April 2012 to December 2013. *** - significant at 0.01 level, ** - significant at 0.05 level, * - significant at 0.1 level. Number of observations: 346.*

As in the previous section, if I change the VaR ratio to the VaR on HFT ratio, the lag order suggested by the LR statistics is six and the significance of the VaR variable at lag one fades away (Table 2.6, column a), although the model remains stable and with no autocorrelation (at 1%) up to lag 10.

For market volatility, the variable used is based on the past realization of actual volatility. However, VaR is based on expected volatility, hence it is useful to check if the relation and the Granger causality would change if a different measure of the expected future volatility is used. It is worth noting that the market volatility variable based on past results does contain some know regularities (autocorrelation, ARCH effect) typical of several financial time series which may affect the results of the regressions (see section 2.3 and 2.3.1 about endogeneity and reverse causality); therefore the substitution of the 10-day volatility with the GARCH estimated volatility helps also to control for the effect of autocorrelations.

Table 2.6. Vector autoregression – volatility, VaR on HFT and spread

Variables	Volatility (a)	VaR on HFT (b)	Spread ITA-GER (c)
Volatility (-1)	0.894***	-0.001	-0.359
	(0.055)	*(0.001)*	*(0.333)*
Volatility (-2)	-0.009	0.001	0.277
	(0.074)	*(0.001)*	*(0.447)*
Volatility (-3)	0.087	0.000	0.197
	(0.073)	*(0.001)*	*(0.442)*
Volatility (-4)	0.031	-0.001	0.315
	(0.072)	*(0.001)*	*(0.436)*
Volatility (-5)	-0.096	0.002	-0.774*
	(0.070)	*(0.001)*	*(0.426)*
Volatility (-6)	-0.079	-0.002*	0.719**
	(0.052)	*(0.001)*	*(0.314)*
VaR on HFT (-1)	2.370	0.825***	12.959
	(3.326)	*(0.056)*	*(20.016)*
VaR on HFT (-2)	-7.390*	0.049	-4.633
	(4.297)	*(0.072)*	*(25.857)*
VaR on HFT (-3)	4.916	-0.035	-22.407
	(4.306)	*(0.072)*	*(25.910)*
VaR on HFT (-4)	1.218	0.097	3.063
	(4.357)	*(0.073)*	*(26.215)*
VaR on HFT (-5)	-0.708	0.013	19.588
	(3.684)	*(0.062)*	*(22.168)*
VaR on HFT (-6)	-2.111	-0.017	-9.178
	(1.653)	*(0.028)*	*(9.950)*
Spread ITA-GER (-1)	0.003	-0.0001	0.989***
	(0.009)	*(0.0001)*	*(0.055)*
Spread ITA-GER (-2)	0.003	0.0001	-0.010
	(0.013)	*(0.0002)*	*(0.077)*
Spread ITA-GER (-3)	-0.026***	-0.000	-0.111
	(0.013)	*(0.0002)*	*(0.078)*
Spread ITA-GER (-4)	0.026***	-0.0004*	-0.068
	(0.013)	*(0.0002)*	*(0.080)*
Spread ITA-GER (-5)	-0.010	0.0001	0.082
	(0.013)	*(0.0002)*	*(0.080)*
Spread ITA-GER (-6)	0.012	0.0002	0.046
	(0.009)	*(0.0002)*	*(0.057)*
Constant	-0.000	0.000	0.000
	(0.000)	*(0.000)*	*(0.000)*

*Note. Vector autoregression among the simple average of the VaR divided by the HFT portfolio, the volatility of the index returns and the spread between government bonds of Italy and Germany. The (-1) … (-6) labels stand for lag 1 … lag 6 respectively. Daily data on Italian banks and Italian financial market from April 2012 to December 2013. *** - significant at 0.01 level, ** - significant at 0.05 level, * - significant at 0.1 level. Number of observations: 334.*

Therefore, I computed a different measure for volatility using the ARMA-GARCH model, thus cleaning the autocorrelation effects from the time series. To estimate the ARMA I initially performed the usual checks on the series of the returns of the Italian stock index. This series has no unit roots; hence it can be used without applying any procedures to have stationarity, and shows autocorrelation for various lags (Ljung-Box statistics reject the null of absence of autocorrelation). On the basis of the information criteria (AIC, SIC), the model which takes into account autocorrelation of the data used is an ARMA(1,1), with a seasonal moving average at the 5th lag (see table 2.7).

Table 2.7. Returns of the Italian market index (ARMA model)

Variables	Coefficients
C	0.00
	(0.00)
AR(1)	0.59**
	(0.24)
MA(1)	-0.64***
	(0.23)
Seasonal-MA(5)	-0.13***
	(0.05)

Dependent variable: daily returns of the index. AR(1) is the variable for the autoregressive component of order 1; MA(1) is the variable for the moving average component of order 1, Seasonal-MA(5) is the variable for the seasonal moving average of order 5.
*Level of significance: * - significant at 10%, ** - significant at 5%, *** - significant at 1%*
Number of observations: 455.

The seasonal variable helps the model to incorporate the weekend effect, typical of financial market series. Table 2.8 show the average returns for weekdays (column a), with an evident difference between returns for Fridays and Mondays.

Table 2.8. Returns of the index for weekdays. Summary statistics

Weekday	Mean (a)	Maximum (b)	Minimum (c)	Std. Dev. (d)	Observations (e)
Monday	-0.001	0.0300	-0.046	0.0151	91
Tuesday	0.001	0.0361	-0.051	0.0160	92
Wednesday	-0.00	0.0373	-0.037	0.0150	91
Thursday	0.002	0.0547	-0.047	0.0159	91
Friday	0.001	0.063	-0.044	0.0159	91
Sample	**0.000**	**0.064**	**-0.051**	**0.0156**	**456**

Daily returns of the index computed on the closing price of the Italian index (FTSE MIB). Time window from 1 April 2012 to 31 December 2013. Source: Datastream.

The model so structured solves the problem of autocorrelations of residuals, is stationary and invertible. However, the squared residuals show strong autocorrelation at least up to one month, which is a sign of conditional heteroscedasticity. In fact, the heteroscedasticity ARCH-LM (autoregressive conditional heteroscedasticity, Lagrange multiplier) test refutes the null of absence of ARCH in the residuals. Hence I complemented the model with an ARCH framework; the one which fits the dynamics of the data and has the lowest value of the usual information criteria is the GARCH(1,1), with no asymmetry and a t-student distribution of residuals (table 2.9).

Table 2.9. GARCH model – variance equation

Variables	Coefficients
C	0.00
	(0.00)
RESIDUAL(-1)^2	-0.02**
	(0.01)
GARCH(-1)	1.01***
	(0.01)

Variance equation: Dependent variable: variance of the daily returns. RESIDUAL(-1)^2 is the first term of a GARCH(1,1) model (error squared, lagged 1); GARCH(-1) is the second term of a standard GARCH(1,1) model (variance lagged 1).
*Level of significance: * - significant at 10%, ** - significant at 5%, *** - significant at 1%*
Number of observations included for the regressions: 455.

With respect to other models (GARCH, Threshold-ARCH, Exponential-GARCH, Power-ARCH) and other distributions of residuals (normal or generalized error distribution) this model has two important advantages: (i) it has the lowest value of the Akaike information criterion among those with significant coefficients for the variance equation; (ii) it cleans the correlogram of squared residuals (non-reject of the null of absence of correlation) and passes the ARCH-LM test (the null of absence of further ARCH effects is not rejected). The result is in line with other studies which found that models with t-student distribution of residuals have better statistical features given the non-normality (fat tails) of the financial series (Talpsepp and Rieger 2010).

The VAR framework of the regression (2.7), where the volatility is measured by GARCH and the number of optimal lags is one (SIC criterion), confirms the existence of a relation flowing from the VaR ratio towards market volatility (Granger causality at 10%). The results of the regression reported in table 2.10 show the significance, even at a lower level, of the VaR ratio at lag one in the regression having market volatility (GARCH) as a dependent variable (column a). In addition, the relation between spread and the VaR ratio is confirmed (column c, significance at 5%).

Table 2.10. Vector autoregression – market volatility (GARCH), VaR ratio and spread

Variables	Volatility (GARCH) (a)	VaR ratio (on internal limit) (b)	Spread ITA-GER (c)
Volatility GARCH(-1)	0.992***	0.044	0.167
	(0.004)	*(0.194)*	*(0.15)*
VaR ratio (-1)	0.001*	0.907***	0.031**
	(0.0004)	*(0.02)*	*(0.016)*
Spread ITA-GER (-1)	-0.000	-0.014	0.944***
	(-0.00)	*(-0.02)*	*(0.02)*
Constant	0.000	-0.001	-0.002
	(0.00)	*(0.003)*	*(0.002)*

*Vector autoregression among the simple average of the VaR ratio (VaR divided by internal limit), the volatility of the returns of the index estimated with a GARCH model, and the spread between reference government bonds of Italy and Germany. The (-1) label after a variable name stands for lag 1. Daily data on Italian banks and Italian financial market from April 2012 to December 2013. *** - significant at 0.01 level, ** - significant at 0.05 level, * - significant at 0.1 level*
Number of observations: 401.

If I substitute the VaR ratio with the VaR divided by the amount of the HFT portfolio, the VaR variable (with volatility as dependent variable) becomes not significant (table 2.11, column a) and Granger causality disappears, thus confirming that it was due to the regulatory limit .

Table 2.11. Vector autoregression – market volatility (GARCH), VaR on HFT and spread

Variables	**Volatility (a)**	**VaR on HFT (b)**	**Spread ITA-GER (c)**
Volatility (-1)	1.038***	-0.002	-1.62
	(0.05)	*(0.006)*	*(2.00)*
Volatility (-2)	-0.043	0.002	1.83
	(0.05)	*(0.00)*	*(1.99)*
VaR on HFT (-1)	0.125	0.916***	-1.68
	(0.29)	*(0.03)*	*(11.6)*
VaR on HFT (-2)	0.30	-0.001	-1.60
	(0.25)	*(0.03)*	*(9.58)*
Spread ITA-GER (-1)	-0.000	-0.000	1.03***
	(0.00)	*(0.00)*	*(0.05)*
Spread ITA-GER (-2)	0.000	0.000	-0.09**
	(0.00)	*(0.00)*	*(0.05)*
Constant	0.000	0.000	-0.003
	(0.00)	*(0.00)*	*(0.002)*

*Vector autoregression among the simple average of the VaR on HFT (VaR divided by the value of the held for trade portfolio), the volatility of the returns of the index, and the spread between reference government bonds of Italy and Germany. The (-1) and (-2) labels after a variable name stand for lag 1 and lag 2 respectively. Daily data on Italian banks and Italian financial market from April 2012 to December 2013. *** - significant at 0.1 level, ** - significant at 0.05 level, * - significant at 0.1 level.*
Number of observations: 387.

For the second robustness test, I tested the sensitivity of the result to the spread variable. Hence, I reperformed the VAR tests by substituting the variable spread, with a variable more similar to the one used in value at risk model, that is the standard deviation of the volatility of returns of the Italian government bonds. I calculate the change of the daily yield of the BTP and computed the ten-day standard deviation (that is the regulatory time horizon for value at risk). The time series has no unit root, when detrended.

The significance of the VaR ratio with the right positive sign at lag one and the overshooting effect (negative sign at lag two) are confirmed (table 2.12, column a); the yield volatility variable is strongly significant at lag two with the expected positive sign, showing that the increased volatility at time t-2 has an impact on the VaR ratio at time t-1 and affects volatility at time t. Granger causality is confirmed, too.

Table 2.12. Vector autoregression – market volatility, VaR ratio and volatility of the spread

Variables	Market volatility (a)	VaR ratio (on internal limit) (b)	Volatility of the yield of the reference Italian government bond (BTP) (c)
Market volatility (-1)	0.942***	0.149	-0.083
	(0.05)	(0.36)	(0.07)
Market volatility (-2)	-0.035	-0.053	0.033
	(0.05)	(0.36)	(0.07)
VaR ratio (-1)	0.016**	0.835***	-0.016*
	(0.01)	(0.05)	(0.01)
VaR ratio (-2)	-0.016**	0.09*	0.010
	(0.01)	(0.05)	(0.01)
BTP Yield Volatility (-1)	-0.067*	0.220	0.968***
	(0.03)	(0.26)	(0.05)
BTP Yield Volatility (-2)	0.099***	-0.233	-0.045
	(0.03)	(0.26)	(0.05)
Constant	0.000	-0.000	0.000
	(0.00)	(0.00)	(0.00)

*Vector autoregression among the simple average of the VaR Ratio (VaR divided by internal limit), the volatility of the returns of the index, and the volatility of the yield of the reference government bonds of Italy. The (-1) and (-2) labels after a variable name stand for lag 1 and lag 2 respectively. Daily data on Italian banks and Italian financial market from April 2012 to December 2013. *** - significant at 0.1 level, ** - significant at 0.05 level, * - significant at 0.1 level.*
Number of observations: 383.

2.6 Conclusions

The financial crisis of 2007-2009 is a good starting point to discuss the impact of micro-regulation (regulation of single financial intermediaries) on macro behaviour or macro variables (e.g. financial stability, liquidity of the system).

Various theoretical studies have found a relation between micro and macro via value at risk (VaR), a measure of market risk used by several banks and also included in some regulatory documents, by finding an amplification effect of shocks (on market volatility) of VaR constraints imposed on individual investors (banks). Large amounts of theoretical (Danielsson et al. 2004) and empirical (Adrian and Shin 2010) literature support the existence of an amplification effect of shocks due to VaR constraints. On the basis of Danielsson et al. (2004) the main channel of transmission of the amplification effect, in a theoretical framework with a VaR constraint, is the adjustments of the investors' expected returns and covariances and the related increase of risk aversion caused by the constraint. This chapter provides new empirical evidence of the fact that a measure of the tightness of VaR ratio for Italian banks Granger-causes the variation of the volatility of the index of the Italian financial market, giving support to the theoretical results obtained by the relevant literature, and complementing the existing empirical evidence based on US data. I also found a significant positive relation (and Granger causality), although at a lower significance level, between the VaR ratio and the expected volatility measured by the GARCH model, seen as a measure of volatility expected by investors, thus providing empirical evidence of the "expectation channel" described by Danielsson (2004) as the way used by the VaR constraint to amplify shocks. The above mentioned relation has been found for the Italian market, where the impact of behaviours of Italian banks on the local stock exchange is more directly visible than in other, more internationalized, financial system such as the US, for which literature has found similar results but in a more indirect way. In fact, for the US market Adrian and Shin (2010) show the existence of an indirect nexus between VaR and VIX in two steps, by using data related only to five investment banks and by running two different regressions based on two different datasets. More specifically, they show that the growth of leverage is negatively related to the lagged growth of VaR and positively related to the growth of repos (quarterly data). In an additional regression, they show that the variation of implied volatility is negatively related to the lagged growth of repo (weekly data). Therefore, from the research of Adrian and Shin (2010), an indirect, positive relation between change of VaR and VIX appears to exist.

The results obtained in this chapter confirm the assumptions about the existence of the amplification effect on the volatility of the market caused by the existence of risk limits, in line with the results presented in several theoretical models (Danielsson et al. 2010, Adrian and Shin 2006, Cont and Wagalath 2014). Furthermore, the main findings complement the existing empirical literature (Adrian and Shin 2010).

In addition, I also got some empirical confirmation about the positive relation between the increase of other, market-related risks (namely, sovereign risk) and market volatility and I found some evidence of the overshooting effects on market volatility of the application of risk limits.

As stated in the introduction, Janet Yellen pointed out the effect that methods of modern risk management and value at risk may have had on the intensification of the cycle (Danielsson et al. 2010). On the basis of the empirical relation found in this chapter, some doubts about unintended consequences of VaR constraints are confirmed: VaR measures used to control individual behaviour and to reduce risk to banks may have an amplification effect on market volatility thus affecting the results of the banks which could be induced, by the interaction of the increased volatility and tightening risk limit, to fire sell some assets.

Therefore, from a policy point of view, setting limits on market risk in order to contain risks (and losses) of individual banks, if not supplemented with decisions concerning the increase of risks at a macro level, produces the unintended effect of increasing market volatility, thus potentially triggering fire sales and increasing losses of banks. Any possible changes to market risk regulation should therefore try to break such relation between individual constraints and market volatility by using risk limits which are less directly related to market volatility or by adding some additional rules which can limit or monitor the amplification effect.

From the research side, this result draws attention to the matter and requires further analysis to define a theoretical framework to better understand the theoretical channels and dynamics on which this empirical evidence is grounded and to offer additional evidence about the direction to follow in order to improve regulation and risk management practices.

Chapter 3. Risk limits and portfolio allocation in a mean-variance world

3.1 Introduction

As mentioned in chapter 2, in most crises when macroeconomic conditions worsen, the risk appetite of financial agents declines and investors start to cut their risk exposures by selling assets thus impacting the rest of the financial system and causing the deterioration of the whole market.

The consequent fall in prices and increase of standard risk measures (such as volatility) determines an additional reduction of the exposure to risky assets, thus feeding the vicious circle.

The financial crisis of ten years ago reheated debates on endogeneity of risk, pro-cyclicality of regulation and the impact on systemic risk of prudential rules and risk management techniques.

In 2010, the former chairman of the Fed, Ben Bernanke, argued that the financial crisis was a failure of economic engineering (i.e. risk-management techniques of financial institutions and financial regulatory system) and economic management (i.e. management of complex financial institutions and day-to-day supervision of these institutions).

During the period, risk management techniques had become widespread. In particular, value at risk (VaR) has become the standard measure of market risk since it summarizes in a single, easy-to-understand number the downside risk arising out of financial market variability (Jorion 2007). Given its useful characteristics (i.e. it provides a common measure of risk across different positions and risk factors and enables aggregation of risks of positions taking into account the ways in which risk factors correlate with each other), VaR is still widely used to monitor market risk among practitioners. In addition, the Basel Accords chose it as a measure of risk limit and as the basis for regulatory capital calculation.

However, during the crisis, risk management and, especially, risk limits and VaR, appear to have amplified the shock of the financial crisis (see chapter 2), as underlined by Danielsson et al. (2010) and Adrian and Shin (2010). Although much of the literature agrees on the amplification effect due to risk limits, the relationship between such risk limits and investment decisions, which can cause such amplification, is still unclear. The existence of an amplification effect of shocks may be explained by an excess of investment in risky assets under VaR constraints, which is then corrected (even with fire sales) in periods of shocks (Basak and Shapiro 2001); however, the fact that risk constraint causes an excess of investment in risky assets is counterintuitive and not in line with the goals of regulators which impose risk limits to reduce the exposure of individual investors to risky assets. Hence, understanding the impact of risk limits on investor's decisions is not straightforward, given the existence of the amplification effect. The data on the topic in literature are contradictory too.

Alexander and Baptista (2002), in a mean-value at risk (mean-VaR) framework, found that risk-averse agents may end up selecting portfolios with larger standard deviation (thus riskier) if they switch from variance to VaR as a measure of risk. Along the same line, Basak and Shapiro (2001) show that regulation leads financial institutions to take on higher exposure to risky assets and that the presence of VaR risk management amplifies stock market volatility.

In contrast with previous results, Cuoco and Isaenko (2008) found that the risk exposure of a trader subject to a VaR limit is always lower than that of an unconstrained trader. Yiu (2004) also founds that VaR may reduce allocation to risky assets.

None of the above-mentioned articles searched for empirical support of the theoretical results found.

In this chapter I find that, in a theoretical mean-variance framework when VaR constraint binds, investments in risky assets decrease. Specifically, I obtain a new closed formula for the allocation of wealth between risky and risk-free assets for VaR-constrained investors, and I provide new empirical evidence in support of the formula obtained. The theoretical result supports the literature claiming that VaR constraint reduces risky investments, as expected by regulators, and it is a step toward understanding both the theoretical framework standing behind the behaviour of VaR-constrained investors and the impact of imposing risk limits on investors.

The approach of analysing the impact of a VaR limit on investors' decisions is not completely new in literature; however, previous articles are not focused on portfolio allocations (e.g. Basak and Shapiro (2001) focus on the terminal wealth of a VaR agent). Other researchers solve the problem by using numerical methods (e.g. Yiu 2004) or perform a graphical analysis of the effect of the constraint (e.g. Sentana 2001), without searching for a closed formula. To the best of my knowledge, no previous paper found a closed formula for portfolio allocation between riskless and risky assets in a mean-variance framework. In addition, unlike several papers on the topic I performed some empirical tests, limited to Italy, which confirm that the constrained optimization solution found in the theoretical part of the chapter is more accurate in forecasting bank's risky investments than the unconstrained one. The empirical test was conducted using mixed-data sampling (MIDAS) regressions in order to exploit the information content of higher frequency data. I limited the empirical analysis geographically to the Italian banking and financial markets which are large enough to be explored, but not internationalized enough to have the relations between the domestic financial market and national banks significantly affected by non-domestic banks. In fact, in 2011, one of the first years included in the time window used in the empirical analysis, the share of stocks held by foreign investors (in terms of market capitalization) was lower in Italy than in other large European countries (Spain, France, United Kingdom, Germany) (Observatoire de l'Epargne Europeene 2013). In the same year, the share of government securities held domestically was higher in Italy than in other large European countries (Andritzky 2012).

This chapter is structured as follows: section 3.2 describes the relevant literature, section 3.3 introduces the theoretical framework used, and section 3.4 describes the empirical tests, while section 3.5 concludes.

3.2 The effects of risk limits: relevant literature

In the Markowitz model, risk is the variability of returns of the investors' portfolio. As such it is measured by standard deviation or variance. However, variance is not necessarily sufficient for capturing risk since two distributions with different shapes and different downside risk can have the same variance (Rosenberg and Schuermann 2004). In such cases measures such as skewness and kurtosis may complement variance to quantify risks.

Another way to overcome the drawbacks of variance is to examine the percentiles of the distribution, as in the value at risk (VaR) approach. Jorion (2007) argues that the greatest advantage of VaR is that it summarizes in a single, easy-to-understand number, the downside risk arising out of financial market variability for any institution. Furthermore, VaR provides a common measure of risk across different positions and risk factors, enables aggregating the risks of positions taking into account the ways in which risk factors correlate with each other, and takes full account of all driving risk factors (other traditional measures, such as the Greeks, look at one risk factor at a time). Such additional characteristics give VaR an edge over traditional risk assessment methods; therefore, VaR is widely used among practitioners to monitor market risk. Furthermore, the Basel Accords acknowledge the important role of VaR by choosing it as a risk limit measure and as the basis for regulatory capital calculation.

3.2.1 Theoretical approaches

The impact of VaR-based constraints on investment decisions has been studied in literature with mixed results.

Alexander and Baptista (2002) compared mean-VaR and mean-variance frontiers and found that the standard deviation (hence, the risk level) of the optimal portfolio of a risk-averse agent may increase if the investor decides to use VaR as the relevant measure of risk. In their article they examined the effects of VaR by substituting value at risk to standard deviation in the standard mean-variance framework, thus describing a new mean-value at risk setting. Using the same line of reasoning, Tsao (2010) suggested that the traditional mean-variance framework for portfolio selection should be revised when the investor's concern was the VaR instead of the standard deviation, incorporated VaR in the portfolio selection process, and proposed a mean-VaR efficient frontier. He concluded that risk-averse investors might allocate wealth inefficiently if decisions were based on the mean-variance framework instead of the mean-VaR setting, where inefficiency is measured using the loss of return given a level of risk. More recently, Tsafack and Tchana (2019), in a Markowitz setup, found that VaR restrictions affected manager performance even more negatively than other restrictions such as short selling constraints, especially in a more volatile market.

Therefore, for these studies the use of VaR instead of volatility or variance as a measure of risk, significantly affects portfolio allocation choices and portfolio performances. For portfolio allocation, the use of VaR causes an increase in portfolio risk (Alexander and Baptista 2002, Tsao 2010).

A second strand of literature examines investment decisions under VaR constraints, with mixed results. Basak and Shapiro (2001) analysed the effects of VaR-based risk management on optimal wealth, consumption choices and portfolio decisions. In their model, risk managers are utility maximizers (with utility derived from wealth) in an ongoing economic setting and must comply with a VaR constraint. This requires wealth to decrease below a given floor only with a pre-specified probability. Their model shows that regulation may lead financial institutions to accept higher exposure to risky assets, supporting also the idea that, in a regulated framework, losses are larger when they occur. Furthermore, Basak and Shapiro showed that the presence of VaR risk management amplifies stock market volatility at times of down markets and attenuates the volatility at times of up markets. Along the same line, and in the same period, Sentana (2001) focuses on mean-variance allocation with VaR constraints. He considered a world with one riskless asset and a finite number of risky assets. In this world, Sentana introduced three basic building blocks – mean-variance portfolio frontiers, mean-variance indifference curves, and iso-VaRs – to find the best portfolio allocation with a VaR cap. Specifically, he starts from a portfolio selected with the mean-variance approach, then he determines the degree of leverage for the chosen position and some iso-VaRs, which are graphical lines of portfolios which share the same VaR (for a fixed probability level) in an expected excess return-standard deviation plan. Finally, he concludes that the existence of a VaR constraint is a cost for a fund manager in terms of lower return (but also lower risk) and a lower Sharpe ratio.

Additional evidence with results differing from Basak and Shapiro's (2001), comes from the analysis of Yiu (2004), who formulate a constrained utility maximization problem in a continuous time setting to look at the optimal portfolio when value at risk is imposed. Using numerical methods, Yiu found that when portfolio value increases, the VaR is active and the allocation to risky assets is reduced. Cuoco and Isaenko (2008) present a framework where expected utility of the terminal value of the trading portfolio is maximized, subject to the constraint that the VaR of the portfolio is not larger than some pre-specified level. In this framework, based on numerical computations founded on assumed values of the parameters, they find that, when VaR is re-evaluated dynamically, the risk exposure of a trader subject to a VaR limit is always lower than the one of an unconstrained trader.

Alexander and Baptista (2002), in a mean-variance framework with a VaR constraint, find that under specific circumstances (i.e. highly risk averse banks whose unconstrained optimal portfolio lies on the efficient frontier above the minimum variance portfolio but below the VaR-constrained portfolio), regulation may cause an increase of the standard deviation of the optimal portfolio and of the probability of extreme losses; more generally, they show that when a VaR constraint is imposed it is plausible that certain banks will end up selecting riskier portfolios.

Lastly, a third wave of studies on risk management have emerged from the beginning of the 2007-09 crisis (and some years before). These papers mainly focus on some unintended consequences of systemic risk of the VaR constraint (e.g. pro-cyclicality, amplification effect) and some possible distortions (or changes in behaviour) determined on allocation choices. In the strand of literature concerning the systemic effect of the constraint, Shin (2010) looks at the balance sheets of investors to find a pro-cyclical effect of the VaR constraint. Specifically, he shows that when VaR is less binding (and investors' equity is larger than necessary), investors use the slack in the balance sheet to purchase additional risky securities thus causing an amplified response (overshooting) to improvements in fundamentals. Cont and Wagalath (2013) model the impact of fire sales on volatility and correlations to find that the more widespread a security is in portfolios of different financial agents, the higher the amplification effect and the cost of imposing common behaviour via regulatory constraints. Danielsson et al. (2012) set a model where VaR constraints have a role in the amplification effect on volatility caused by deleveraging and found also that when risk is regulated, prices are lower and volatility is higher; hence, price fluctuation is higher. Kaplanski and Levy (2015) reach the conclusion that with VaR regulation institutions face a new regulated capital market line which induces resource allocation distortion in the economy. Only when a riskless asset is available will VaR regulation induce an institution to reduce risk otherwise regulation may both cause risk to increase and asset allocation to be distorted. More recently, Vasileiou and Samitas (2020) examined the data of five European market indexes, to confirm that VaR models based on historical data contribute to pro-cyclicality and overreaction in the stock market.

In summary, on the basis of the contributions of the above-mentioned articles, VaR constraint seems to have a negative impact on the returns of the investors and seems to determine larger losses when a loss occurs. With regards to portfolio allocation, the literature provides no concordant evidence that VaR constraints cause a decrease of risky investments.

In this chapter I investigate the impact of VaR constraint on portfolio allocation by assuming that investors take decisions on the basis of the mean-variance framework where VaR is a constraint.

The approach of analysing the impact of VaR limit on investors' decisions is not completely new in literature; however, previous articles are not focused on portfolio allocations (e.g. Basak and Shapiro (2001), who examined the terminal wealth of a VaR agent). Other researchers (Yiu 2004) analyse the problem in a continuous-time setting and solve it by using numerical methods, without looking for a closed formula. Sentana (2001) bases his solution on a graphical analysis of the problem with Iso-VaR lines. Therefore, no previous papers have found a closed formula for portfolio allocation between riskless and risky assets in a mean-variance framework.

The mean-variance framework is a cornerstone of the investment analysis, and related literature often uses it as a benchmark or as a starting point of several studies, some mentioned in this section. Furthermore, mean and variance, the coordinates to measure return and risk, are still widely used by practitioners around the world. In fact, as documented by Eun and Lee (2010), one of the effects of the increased financial integration among developed countries has been the significant convergence of the risk-return characteristics of 17 developed stock markets (Italy included), as it happens when common variables and models are widely used by the practitioners. Therefore, in the empirical part of the chapter I use the mean-variance framework to compare the results obtained for the risk-constrained scenario.

From the theoretical framework I find that under a mean-variance setup, and if constraint binds, risky investments depend solely on risk variables (and not on returns) and are lower than in the unconstrained scenario, as expected. In addition, differing from several papers on the topic, I perform empirically tests on the result obtained in the theoretical part.

3.2.2 Empirical literature

By moving the focus to empirical articles there is relatively little research which deals contemporaneously with portfolio optimization and value at risk constraints. Among the most relevant, Puelz (2001) presents four models with VaR limits applied to portfolio decisions; one of the frameworks is the standard mean-variance. By using monthly returns of six national indices (for 1984-1999) and simulated scenarios, he concludes that VaR-optimal portfolios are more likely to incur large losses when losses occur. Campbell et al. (2001), in the empirical part of their study, recognize that VaR is successful in containing exposure to risky assets since they find that the higher the confidence level of VaR the lower the portion of the portfolio invested in risky assets.

However, to the best of my knowledge, no study has directly tested the relation between the theoretical result of a risk-constrained portfolio allocation problem and the actual allocation choices of investors. In this chapter, based on data of financial markets and data coming from the supervisory reporting, after having found a solution to the constrained optimization problem, I find that this is more accurate in forecasting banks' behaviour than the standard, unconstrained mean-variance setup.

3.3 Theoretical framework: risk limits in the mean variance framework

Risk management, based on variance-type measures of risk, is inherently incorporated in the Markowitz mean-variance framework; hence, it is almost natural to add a risk limit to such a framework in order to study the impact of risk limits on the optimal solution.

In this section I solve the problem of maximizing mean-variance utility of the investors, under a VaR constraint, to find that the portion of wealth invested in risky assets is lower for VaR-constrained investors than for unconstrained ones.

3.3.1 Standard mean-variance framework

The starting point of the exercise is the mean-variance framework used by Bacchetta et al. (2012) where investors maximize mean-variance utility over their portfolio return as follows:

$$\max_{\alpha_t} E_t\left(R^p_{t+1}\right) - 0.5\gamma\left[\sigma^2_t\left(R^p_{t+1}\right)\right] \qquad (3.1)$$

where γ measures risk aversion, $\sigma^2_t\left(R^p_{t+1}\right)$ is the variance of portfolio returns at t+1 expected at time t, R^p_{t+1} is the portfolio return at time t+1:

$$R^p_{t+1} = \alpha_t \, E_t\,(R^k_{t+1}) + (1-\alpha_t)\,R \qquad (3.2)$$

therefore also $\sigma^2_t\left(R^p_{t+1}\right)$ depends on α_t.

In (3.2), α_t stands for the share of the portfolio invested in equity, R is the gross return of free-risk bonds and $E_t\,(R^k_{t+1})$ is the return on the equity at t+1. The equity return is computed as follows:

$$R^k_{t+1} = \frac{A_{t+1}+Q_{t+1}}{Q_t} \qquad (3.3)$$

where Q_{t+1} is the tomorrow's equity price, A_t represents the dividends and $A_t = \bar{A} + m\, S_t$ with S as an exogenous state variable that follows a stochastic process and Ā is the constant dividend when m=0.

From the maximization condition, Bacchetta et al. (2012) find that the portion invested in equity is equal to:

$$\alpha_t = \frac{E_t\,(R_{t+1}^k) - R}{\gamma\, \sigma_t^2\; (R_{t+1}^k)} \tag{3.4}$$

PROPOSITION 3.1: *In an overlapping generation setup where investors have an endowment W and purchase risk-free bonds (yielding an exogenous constant return R) and risky equity (yielding a gross return of R_{t+1}^k) and maximize mean-variance utility, the portion of wealth invested in risky assets (unconstrained alpha) is positively related to the excess return of the risky asset and inversely related to the expected variance of the risky assets:*

Therefore, the portion of wealth invested in risky assets increases if the excess returns of equity with respect to bonds increases or if the variance of the expected returns decreases. The alpha obtained in (3.4) is the solution to the unconstrained maximization problem stated at (3.1).

3.3.2 Constrained mean-variance framework

I now introduce a general risk-limit constraint in the following form:

$$\max_{\alpha_t} E_t\left(R^p_{t+1}\right) - 0.5\gamma\left[\sigma^2_t\left(R^p_{t+1}\right)\right]$$

$$\text{subject to: } f\left[\sigma_t\left(R^p_{t+1}\right)\right] \leq Risk\ Limit \qquad (3.5)$$

where *f* is positive linear in portfolio volatility; risk limit (hereafter RL) is a positive number and $\sigma_t\left(R^p_{t+1}\right)$ is positive in alpha.

The model represents the maximization problem faced by an investor subject to a market risk limit; the variable subject to the constraint is a measure of risk, such as a function of volatility of portfolio returns.

From the constrained maximization procedure I find that when the constraint binds, investments in risky assets are represented as follows:

$$\mathrm{f}^{-1}(\sigma_t(\mathrm{R}_{p,t+1}))=\mathrm{RL} \qquad (3.6)$$

where $\mathrm{f}^{-1}(\sigma_t(\mathrm{R}_{p,t+1}))$ is the inverse of the risk function included in the constraint. Hence, the proportion of wealth invested in risky assets in the case of binding constraint is related only to the function used to measure risk and to the risk limit.

As expected, when the constraint does not bind, the proportion of wealth invested in risky assets is equal to (3.4); therefore, it is directly related to the excess return and inversely to the variance of risky assets.

From the optimization procedure, I also find that:

PROPOSITION 3.2: *For any risk-constrained maximization of the form (3.5) either investment in risky assets is the same as in the unconstrained case, if constraint is not binding, or risky investment is strictly lower, if constraint binds.*

Proof is reported in appendix 3.A.

3.3.3 VaR-based risk limit

In this section I adapt the general results to the specific risk limit I am analyzing, which is the value at risk. VaR is a probabilistic metric of market risk used by banks and other investors to monitor risk exposure of their trading portfolios. Value at risk indicates the maximum loss expected on an investment, at a certain confidence level and over a given time horizon.

VaR has become so widespread as a measure of risk, that in literature it is not unusual to assess investments in a mean-VaR mapping instead of the traditional mean-variance one (see, among others, Alexander and

Baptista 2006). However, this approach does not take into account that investment decisions (based on returns and volatility) and monitoring of the investment by the application of regulatory constraints (based on VaR) are two different steps of the investment process with the investment step preceding the monitoring phase. VaR affects the first step (i.e. the investment decisions) only as a constraint since investors cannot buy or sell some securities if such operations will cause a breach of VaR limit. Therefore, unlike the above-mentioned stream of literature, I assume that in ordinary investment activity the investment criteria are first based on the traditional risk-return (mean-variance) framework and, only in addition are VaR limits taken into account as a constraint. Hence, the VaR limits are more a constraint to the decision mechanisms than a variable of the risk-return mapping.

Following Kaplanski and Levy (2015) and Alexander and Baptista (2002, 2006), I express value at risk as a positive number measured as $z\sigma_t\left(R_{t+1}^p\right)$ where σ_t is the volatility of portfolio returns at t+1 expected at time t, and z is the parameter corresponding to the chosen confidence level. If the returns are not assumed to be zero, VaR becomes $z\sigma_t\left(R_{t+1}^p\right) - E_t\left(R_{t+1}^p\right)$. In a VaR-limited investment process, investors would not buy a portfolio of securities with VaR higher than a risk limit V. In line with the general framework (3.5), the VaR-constrained maximization becomes:

$$\max_{\alpha_t} E_t\left(R_{t+1}^p\right) - 0.5\gamma\left[\sigma_t^2\left(R_{t+1}^p\right)\right]$$

$$\text{subject to: } z\sigma_t\left(R_{t+1}^p\right) - E_t\left(R_{t+1}^p\right) \leq V \qquad (3.7)$$

PROPOSITION 3.3: *In the framework described in proposition 3.1, where investors maximize mean-variance utility over their portfolio return under a VaR limit constraint V and the constraint binds, the portion of wealth invested in risky assets (constrained alpha) is positively related to the level of the limit and to the excess of the expected return of the risky asset over the riskless one, and inversely related to the expected volatility of the risky asset:*

$$\alpha_t^C = \frac{V+R}{\mathrm{z}\,\sigma_t\,(R_{t+1}^k) - (\mathrm{E_t}\,(R_{t+1}^k) - \mathrm{R})} \qquad (3.8)$$

The proof, in line with the maximization procedure reported in the appendix 3.A for the general case, is reported in appendix 3.B.

The analysis of the VaR-constrained alpha (3.8) highlights that when the pre-determined risk limit V increases, the portion of wealth invested in risky assets increases, hence, the looser the risk limit, the higher the portion of the portfolio invested in risky assets is (in line with Campbell et al. 2001). Furthermore, the higher the VaR of the portfolio (denominator), the lower the alpha; more generally, when the expected volatility (sigma) increases relative to the risk limit V, then alpha decreases. The relation of alpha with the expected returns of risky assets is positive, as expected.

In the framework set in proposition 3.3, when the constraint does not bind I get the same unconstrained alpha as in (3.4), hence the unconstrained alpha is positively related to the excess of the expected return of the risky asset over the riskless one, and inversely related to the expected volatility of the risky asset.

PROPOSITION 3.4: *In the framework set in proposition 3.1, if the constraint binds the investment in risky assets (constrained alpha) is lower than the risky investment in an unconstrained scenario; if the constraint does not bind the value of the portion invested in risky assets is equal to that obtained in the ordinary (unconstrained) mean-variance setting.*

Proof is reported in appendix 3.B.

When the constraint binds, the bank reduces alpha to a level that is lower than the unconstrained alpha. This would cause an increase of asset sales which have an impact on market volatility. This result is in line with literature (e.g. Danielsson et al. 2012) which highlights that VaR can be pro-cyclical and with that some literature (Cuoco and Isaenko 2008, Yiu 2004) which shows that risky investments are lower for VaR-constrained investors than for unconstrained ones.

3.4 Empirical tests: methodology and evidence

After having found the closed-form solutions for risky investments (for constrained and unconstrained investors), in this section I perform some empirical tests on the above stated alphas (equations (3.4) and (3.8)).

In order to verify if the VaR-constrained formula adequately represents investors' behaviour, I test its accuracy of forecasting the investment behaviour of Italian banks. I measure the forecasting ability of the formula for constrained investors by comparing it with the standard mean-variance framework (for unconstrained investors) used as a benchmark. This comparison is also useful to better understand which of the following banks' investment process is actually used by banks: (i) an investment approach driven by the standard mean-variance framework, followed by a phase, not included in the investment process, when banks regularly check the compliance with risk limits; (ii) an investment approach that incorporates risk-limit assessment from the beginning, thus taking constrained investment decisions.

Specifically, I estimate the coefficients of (3.4) and (3.8) on a sub-sample of the available data, and then I forecast the portion of the portfolio invested in risky assets by Italian banks for the residual part of the sub-sample, both for the constrained and the unconstrained alpha.

The final aim is to evaluate if the constrained alpha equation has a better performance in forecasting the real alpha (i.e., the observed alpha, which is the portion of trading portfolio invested in risky assets by Italian banks), than the unconstrained one. For the empirical tests, I use daily data from the Italian financial market to estimate the expected volatility, and monthly data of the balance sheets of Italian banks (supervisory reporting) for the calculation of alpha.

Since data used have different frequencies, the empirical exercise is mainly based on the MIDAS approach, which gives the opportunity to deal with data of different frequency without losing the intra-monthly information content of the daily data.

I focus the analysis only on the Italian banks and the Italian stock market since they are large enough to be explored, but not internationalized enough to have such relations significantly affected by non-domestic banks. In fact, in 2011, one of the first years included in the time window used in the empirical analysis, the share of stocks held by foreign investors (in terms of market capitalization) was lower in Italy than in other large European countries (Spain, France, United Kingdom, Germany) (Observatoire de l'Epargne Europeene 2013). In the same year the share of government securities held domestically was higher in Italy than in other large European countries (Andritzky 2012).

In more internationalized financial markets the role of non-domestic investors and foreign banks is relevant; thus, in such markets, the reaction of financial market risk to banks' behaviour cannot be empirically tested just by looking at domestic banks data. On the contrary, for Italy, given its lower level of internationalization, the relationship between investors' decisions and financial markets is generally stronger and an analysis of this relation is more representative of the effects of investors' behaviour on financial markets.

3.4.1 Methodology

Typical regression models relate variables sampled at the same frequency. In the presence of variables with different frequencies researchers often aggregate the higher-frequency observations to the lowest available frequency. This technique has some drawbacks since it causes the loss of potential useful information (Foroni and Marcellino 2013). Direct modelling of mixed frequency data may help to solve this problem. Some of the most used techniques to direct model mixed frequency variables are bridge equations (Baffigi et al. 2004), mixed frequency VAR and the MIDAS approach.

Bridge equations link low-frequency variables and time-aggregated indicators. Forecasts of the high-frequency indicators are provided by specific high-frequency time series models, then the forecasted values are aggregated and plugged into the bridge equations for the analysis of the low frequency variable. One of the drawbacks of the model is the two-step approach which requires a forecast of high-frequency variables and the following aggregation to obtain forecasts of the low-frequency variable.

Mixed frequency VAR (MF VAR) aims to examine co-movements of the mixed frequency series. In the classical framework (Mariano and Murasawa 2010), the state-space representation is used, treating low-frequency series as high-frequency series with missing observations.

Borrowing from existing literature about distributed lag models, Ghysels et al. (2004), Ghysels et al. (2007) and Andreou et al. (2013) proposed the MIDAS class of models, which allow dependent and independent variables to be sampled at heterogeneous frequencies. The approach has been thoroughly described in several publications (Ghysels et al. 2005, 2006, 2007, Armesto et al. 2010).

In this approach, lagged explanatory variables are weighted by coefficients that come from deterministic specifications (e.g. Almon lags, beta polynomials, step functions). In literature, the MIDAS models are usually used for forecasting exercises. Further technical details on MIDAS models are reported in appendix 3.C.

Foroni and Marcellino (2014a) compared the different approaches to macroeconomic aggregates, finding that MIDAS and bridge equations appear to forecast better than the MF-VAR. Schumacher (2016), still using macroeconomic variables, does not find a clear preference between MIDAS and bridge equations in terms of forecasting performance.

In this chapter I opt for the MIDAS approach which overcomes the drawbacks of the traditional methods that solve the frequency disparity by aggregating the variables at the lowest frequency thus losing valuable information. In addition, MIDAS shows better forecast performance than the MF VAR (Foroni and Marcellino 2014a). Furthermore, while bridge equations are mostly used in macroeconomics, MIDAS is mainly used in banking and finance literature. For instance, Ghysels et al. (2006) use MIDAS to predict volatility on the basis of returns sampled at different frequencies. Furthermore, in a GARCH context, MIDAS is used also to predict volatility of commodities (Pan et al. 2017). Recently Audrino et al. (2019) use the MIDAS approach to predict bank failures.

Given its capacity to take account of the most recent high-frequency data MIDAS analysis is extremely powerful for forecasting exercises. In this chapter, I use it to determine if the constrained formula is more appropriate than the unconstrained one to forecast the portion of the portfolio invested in risky assets by Italian banks. My expectation is that, given that banks are constrained in their behaviour by market risk regulation, the constrained alpha formula has the best forecast accuracy. I will test several types of weighting schemes (Almon, Beta, Step), and a more traditional aggregation scheme, which is the simple average of the high frequency data for each low frequency point.

Against this methodological background, I estimate the regressions for unconstrained alpha and constrained alpha. Starting from (3.4) and (3.8), for the regressions I use the inverse of alpha as the dependent variable and the inverse of the right-hand side ratios as regressors.

Hence, for the unconstrained alpha the regression is represented as follows:

$$A_t = \beta_0 + \beta_1 \frac{\gamma \ \sigma^2{}_t(\mathrm{R}_{k,t+1})}{[\mathrm{E_t}\,(\mathrm{R_{k,t+1}}) - \mathrm{R}]} + \epsilon_t \tag{3.9}$$

While for the constrained alpha, the regression is based on the following model:

$$\mathrm{A_t} = \beta_0 + \frac{\beta_1 z\, \sigma_{\,t}\,(\mathrm{R_{k,t+1}}) - \beta_2(\mathrm{E_t}\,(\mathrm{R_{k,t+1}}) - \mathrm{R})}{\mathrm{V+R}} + \epsilon_t \tag{3.10}$$

3.4.2 Data

The data comes from the supervisory reporting, has a monthly periodicity and covers a time span of nine years (June 2010 – May 2019).

For the estimation, the expected yield of free risk assets (R) is reasonably set equal to zero. To compute alpha, I use the ratio between the value of unencumbered listed shares held by the Italian banking system and the total held for trading (HFT) portfolio. I chose listed shares, since they are the risky asset class most impacted by VaR. I limited the analysis to the unencumbered shares since they are freely disposable in case the VaR constraint starts to bind; hence they are the portion of the portfolio which is freely impacted by investment decisions. The portfolio chosen is the HFT because it is the one relevant from a regulatory point of view (i.e. the portfolio the VaR regulation is applied to); hence it is the closest one to the concept of a VaR-constrained investment portfolio expressed in the previous section.

With reference to the other variables used for the regressions: to measure market yield, I compute the annualized daily return from the closing value of the Italian market index; for the expected volatility (and variance) I picked the daily volatility (variance) implied in the at-the-money index options as reported by the volatility index for the Italian market index (which is the equivalent of the VIX). In particular, I use the end-of-day values of the IV-MIB (implied volatility of the Italian market index MIB) index, which measures the annualized 30-day volatility implied in some selected options listed in the national derivative market (IDEM) (i.e. near-term, out-of-the money options with non-zero bid and ask prices[2]). The

[22] For greater details about the calculation methodology used by FTSE MIB, see the https://www.borsaitaliana.it/borsa/indici/indici-di-volatilita/dettaglio.html?indexCode=IVMIB30&lang=it and https://research.ftserussell.com/products/downloads/FTSE_Implied_Volatility_Index_Series.pdf

realized volatility variable is the annualized 30-day volatility computed on the daily return of the closing values of the FTSE MIB market index.

The time frame of the available data is around nine years. In line with literature on forecasting I used the first larger part of the sample (seven years from June 2010 to May 2017) for the estimation of the model and I used the last part of the observations (two years, from June 2017 to May 2019) to check the quality of the forecast. In the robustness test, I also performed some checks on different periods.

For the forecasting exercise, I used all the main weighting schemes (Almon, Beta, Step) of the MIDAS model in addition to the unrestricted MIDAS (U-MIDAS) and the traditional method of averaging out the high-frequency data.

3.4.3 Results

The expected results of the tests are that investment behaviour of Italian banks is always affected by regulatory VaR constraints; hence, the forecast based on the constrained alpha formula (3.8) is expected to be closer to the actual data than the unconstrained alpha one.

This expectation does not change under the different weighting schemes which I use to assess if the results are crucially dependant on some specific weighting scheme.

3.4.3.1 Goodness of fit of the regressions

In order to decide on the best forecasting model, I used the usual model selection criteria (Gujarati 2003), which measure the goodness of fit on the basis of the variance explained by the model (r-squared-type indicators) or the likelihood function (information criteria). The criteria presented in this section are the following: R-squared, adjusted R-squared, Akaike information criterion (AIC) and Schwarz information criterion (SIC).

In table 3.1, I firstly run the regressions for the MIDAS models and the averaged high-frequency data model, for the entire period examined (June 2010 – May 2019). This shows that most MIDAS regression have a moderate to high level of r-squared for the constrained formula (in column a, excluding regression average, most of the row values range from 0.50 to 0.77); for the unconstrained scenario, r-squared is often lower.

The models of both scenarios, especially for MIDAS regressions, are penalized if the number of regressors are taken into account. In fact, the adjusted r-squared (columns (b) and (g)) are much lower than the r-squared values. The model that best fits the data, having a higher adjusted r-squared, is the beta model for both the constrained and the unconstrained solutions. Looking at the lowest information criteria, beta is generally the preferred model (only in the constrained scenario and for the Schwarz criterion it is the second best model) as highlighted in bold in table 3.1.

Table 3.1. Goodness of fit for the 2010-2019 period

	R squared	Adjusted R squared	AIC	SIC
	(a)	**(b)**	**(c)**	**(d)**
		Constrained		
Almon 2	0.37	0.02	5.38	5.54
Almon 3	0.50	0.19	5.22	5.44
Beta	0.54	**0.26**	**5.12**	5.34
Regression average	0.21	0.19	5.14	**5.22**
Step	0.61	0.21	5.33	5.99
U-MIDAS	0.77	0.24	5.35	6.64
		Unconstrained		
	(f)	**(g)**	**(h)**	**(i)**
Almon 2	0.38	0.09	5.24	5.35
Almon 3	0.44	0.15	5.22	5.34
Beta	0.48	**0.20**	**5.16**	**5.28**
Regression average	0.01	-0.01	5.36	5.41
Step	0.33	-0.01	5.38	5.48
U-MIDAS	0.52	0.03	5.53	6.19

Comparison of various indexes measuring the goodness of fit.
Estimation period: June 2010 – May 2019.
AIC stands for Akaike information criterion; SIC for Schwarz information criterion
Almon 2 (or 3) stands for the PDL (polynomial distributed lag)/Almon weighting scheme with 2 (or 3) polynomial degrees; the number of lags has been chosen by using the automatic best lag selection provided by Eviews; Beta stands for the beta step weighting (20 lags). Step stands for the step weighting scheme (2 steps); the number of lags has been chosen on the basis of the automatic best lag algorithm provided by Eviews; U-MIDAS stands for the unrestricted MIDAS (20 lags). Regression average means that the forecast has been run starting from the regression of the monthly variable on the monthly average of the daily variables.
In bold is the best model suggested by the criteria (the highest adjusted r-squared, the lowest AIC, the lowest SIC), for the constrained scenario and for the unconstrained scenario.

If I limit the analysis only to the estimation period (June 2010 – May 2017) and I perform an in-sample goodness-of-fit analysis, I get mixed results.

In fact, as reported in table 3.2, for the constrained scenario the model that best fits the data, having a higher adjusted r-squared, is the step model. The one with the lowest information criteria, is the Almon with two polynomial degrees. In the unconstrained scenario, the preferred model is the Almon with three degrees, which has the highest adjusted r-squared and the lowest information criteria.

Therefore, the selection of the model based on the goodness-of-fit analysis gives no straightforward results. Looking at the entire sampling period the suggested model appears to be the MIDAS model with a beta weighting function (table 3.1). However, by limiting the analysis to the estimation period, there are two suggested models (for the constrained formula: MIDAS model with Almon weighting function and two polynomial degrees; for the unconstrained formula: MIDAS model with Almon weighting function and three polynomial degrees). None of these is the beta model suggested for the entire period. The conclusions on model selections which can be drawn from the analysis performed in table 3.1 and 3.2 are thus extremely mixed.

Table 3.2. Goodness of fit – estimation period 2010-2017

	R squared	Adjusted R squared	AIC	SIC
	(a)	**(b)**	**(c)**	**(d)**
		Constrained		
Almon 2	0.53	0.26	**5.08**	**5.26**
Almon 3	0.53	0.24	5.14	5.39
Beta	0.51	0.21	5.18	5.43
Regression average	0.13	0.11	5.22	5.31
Step	0.70	**0.33**	5.18	5.92
U-MIDAS	0.79	-0.01	5.50	6.95
		Unconstrained		
	(f)	**(g)**	**(h)**	**(i)**
Almon 2	0.39	0.09	5.25	5.35
Almon 3	0.53	**0.28**	**5.03**	**5.17**
Beta	0.48	0.22	5.12	5.26
Regression average	0.01	-0.01	5.33	5.39
Step	0.38	0.09	5.26	5.36
U-MIDAS	0.61	0.15	5.41	6.15

Comparison of various indexes measuring the goodness of fit.
Estimation period: June 2010 – May 2017.
AIC stands for Akaike information criterion; SIC for Schwarz information criterion
Almon 2 (or 3) stands for the PDL (polynomial distributed lag)/Almon weighting scheme with 2 (or 3) polynomial degrees; the number of lags has been chosen by using the automatic best lag selection provided by Eviews; Beta stands for the beta step weighting (20 lags). Step stands for the step weighting scheme (2 steps); the number of lags has been chosen on the basis of the automatic best lag algorithm provided by Eviews; U-MIDAS stands for the unrestricted MIDAS (20 lags). Regression average means that the forecast has been run starting from the regression of the monthly variable on the monthly average of the daily variables.
In bold the best model suggested by the criteria (the highest adjusted r-squared, the lowest AIC, the lowest SIC), for the constrained scenario and for the unconstrained scenario.

Against this background, it is worth noting that the model is used for forecasting purposes in this chapter. As reported in the literature, the forecasting ability and goodness of fit may not always have the same results, since a well-fitted model can be poor at forecasting. More specifically, the relevant literature (Wooldridge 2003, Gujarati 2003, Brooks 2002, Alexander 2008) highlights that the application of the usual model selection criteria, when the model is used for forecasting, may favour the choice of models which are overfitted on the current data and do not have good forecasting performance. Therefore, given the forecasting purposes of the present chapter, I chose the best model on the basis of the out-of-sample forecasting ability, as suggested by Wooldrige, Gujarati and Alexander. The results of the out-of-sample forecasting accuracy are reported in the following section.

3.4.3.2 Baseline results

The results of the analysis of the accuracy of the out-of-sample forecasts, listed in table 3.3, show that constrained alpha is far better in forecasting the investment decisions in risky assets of Italian banks than the unconstrained alpha. In the table, for every weighting scheme used for the high-frequency variable (reported in the rows), the indicators of forecasting accuracy (RMSE, MAE, MAPE, SMAPE, Theil's u2 as defined in the note of table 3.3) are much lower in the constrained scenario (columns (a) through (e)) than in the unconstrained (columns (f) through (j)). Hence, forecasting is more accurate with the constrained formula than with the unconstrained one.

Among the constrained alphas, the best are the regression of alpha on the monthly average of the daily data for the variable (for RMSE, MAE; MAPE SMAPE) and MIDAS with the beta weighting scheme (for Theil's u2), highlighted in bold in the table. The results suggest that using daily data on the basis of the MIDAS approach does not supply additional information to the forecast since the monthly average appears to have better forecasting performance.

Table 3.3. Forecast evaluation for alpha

Comparison of actual values with static forecast regressions

	No obs.	RMSE (a)	MAE (b)	MAPE (c)	SMAPE (d)	Theil - u2 (e)
		Constrained				
Almon 2	73	3.36	2.75	32.20	25.10	0.91
Almon 3	73	3.33	2.79	32.30	25.38	0.95
Beta	73	3.03	2.52	29.18	23.73	**0.76**
Regression average	107	**2.86**	**2.46**	**27.68**	**23.32**	1.16
Step	73	3.69	3.24	35.30	28.86	1.03
U-MIDAS	73	3.70	3.14	31.40	27.23	1.17
		Unconstrained				
		(f)	**(g)**	**(h)**	**(i)**	**(j)**
Almon 2	59	4.03	3.16	39.07	28.30	1.08
Almon 3	74	5.04	3.97	44.84	36.22	1.25
Beta	74	4.05	3.16	39.20	28.31	1.08
Regression average	108	4.02	3.23	39.34	28.91	1.63
Step	74	4.14	3.30	39.55	28.96	1.07
U-MIDAS	74	6.65	5.05	55.30	37.40	1.28

Comparison of various indexes measuring the quality of forecast.
Estimation period: June 2010 – May 2017; forecast period: June 2017 – May 2019.
RMSE (root mean squared error), MAE (mean absolute error), MAPE (mean absolute percentage error), SMAPE (symmetric mean absolute percentage error) and Theil (U2) are the usual measures for forecasting evaluation.
Almon 2 (or 3) stands for the PDL/Almon weighting scheme with 2 (or 3) polynomial degrees; the number of lags has been chosen by using the automatic best lag selection provided by Eviews; Beta stands for the beta step weighting (20 lags). Step stands for the step weighting scheme (2 steps); the number of lags has been chosen on the basis of automatic best lag algorithm provided by Eviews. U-MIDAS stands for the unrestricted MIDAS (20 lags). Regression average means that the forecast has been run starting from the regression of the monthly variable on the monthly average of the daily variables. In bold the best forecasting results.

However, given that the indicators are not fully concordant, to better assess if one of the two methods is clearly better than the other, I carried out the one-step Diebold Mariano test. On the basis of this test, the two forecasts under analysis (Beta-MIDAS and monthly average) are not significantly different.

Hence I report the regressions regarding these two methods. As stated in table 3.4 and table 3.5, for both regressions the expected volatility variable (based on implied volatility measured by Italian VIX) has a significant impact, with the expected positive sign on the inverse of constrained alpha (hence with a negative sign on constrained alpha); the level of significance is 1%.

Table 3.4. Impact of monthly average of daily data

Dependent variable: inverse constrained alpha

Variables	Coefficients
Implied volatility	23.14***
	(5.77)
Yield	0.18
	(0.51)
C	7.30***
	(1.62)

*OLS estimates for model (3.10). Dependent variable (monthly): inverse constrained alpha; regressors (monthly): implied volatility of the Italian market index (to measure the expected volatility in (3.10)) and the returns (measured by the monthly average of daily returns of the index). Standard errors in parenthesis. - *** - significant at 1% level, ** - significant at 5% level; * - significant at 10% level. Number of observations: 107.*

In contrast, the yield variables, relevant for the unconstrained alpha, are not significant.

For the MIDAS regression (table 3.5), only the slope is examined since it has a direct impact on the dependent variable without being directly affected by the weighting scheme. The results show that implied volatility is highly significant with the expected sign (slope 20.70), while yield is not significant. Hence, the results confirm that banks' investment behaviour is highly dependent on expected risk, while investment decisions appear not to be influenced by the yield. Such results, in line with literature and expectations, seem reasonable since the analysis is limited to investment decisions in risky assets in a risk-constrained environment.

Table 3.5. MIDAS regression with beta weighting

Dependent variable: inverse constrained alpha

Variables	Coefficients
C	7.97***
	(1.97)
Implied volatility (-1)	
Slope	20.70***
	(7.12)
Beta-01	0.02
	(35.30)
Beta-02	0.03
	(35.30)
Yield (-1)	
Slope	0.10
	(0.17)
Beta-01	-0.03
	(34.66)
Beta-02	-0.02
	(34.66)

*MIDAS estimates for model (3.10). Dependent variable (monthly): inverse alpha (inverse of the ratio between listed unencumbered shares and HFT portfolio); regressors (daily): implied volatility and returns of the Italian market index Standard errors in parenthesis. - *** - significant at 1% level, ** - significant at 5% level; * - significant at 10% level. Number of observations: 73.*

3.4.3.3 Robustness

Given that the time span of data used is so long that it includes completely different evolutions of risk, the results about the prevalence of constrained alpha on unconstrained alpha can be affected by the time windows used to estimate the regression and to assess the forecast. To check if the forecast accuracy is conditioned by the time windows used for estimation and forecast, I recomputed the indicators, for constrained alpha and unconstrained alpha, by estimating the models on the first quartile of the observations (from June 2010 up to May 2012) and by forecasting alpha for the residual part of the sampled period (from June 2012 up to May 2019). I did not use the U-MIDAS and the step weighting scheme since they have convergence problems due to the limited number of observations.

The goodness-of-fit indicators for the new estimation period (June 2010 – May 2012) have very low, and even negative, values, given the few observations. For the unconstrained scenario, the best model is the regression on the averaged high-frequency data; for the constrained one, the selection criteria do not suggest a unique model. The model with the highest adjusted r-squares and the lowest Akaike information criterion is the MIDAS with Almon weighting scheme (three degree-polynomial). For the Schwarz criterion the suggested model is the MIDAS with beta weighting scheme.

Table 3.6. Goodness of fit – estimation period 2010-2012.

	R squared	Adjusted R squared	AIC	SIC
	(a)	**(b)**	**(c)**	**(d)**
		Constrained		
Almon 2	0.39	0.01	5.45	0.39
Almon 3	0.79	**0.59**	**4.61**	0.79
Beta	0.28	-0.05	5.12	**0.28**
Regression average	0.46	0.41	4.83	0.46
		Unconstrained		
	(f)	**(g)**	**(h)**	**(i)**
Almon 2	0.16	-0.21	5.57	5.72
Almon 3	0.16	-0.29	5.67	5.87
Beta	0.20	-0.20	5.60	5.80
Regression average	0.04	**0.00**	**5.32**	**5.41**

Comparison of various indexes measuring the goodness of fit.
Estimation period: June 2010 – May 2012.
AIC stands for Akaike information criterion; SIC for Schwarz information criterion
Almon 2 (or 3) stands for the PDL/Almon weighting scheme with 2 (or 3) polynomial degrees; the number of lags has been chosen by using the automatic best lag selection provided by Eviews; Beta stands for the beta step weighting (20 lags). Regression average means that the forecast has been run starting from the regression of the monthly variable on the monthly average of the daily variables.
In bold the best model suggested by the criteria (the highest adjusted r-squared, the lowest AIC, the lowest SIC), for the constrained and unconstrained scenarios.

As reported in section 3.4.3.1, I therefore selected the model on the basis of the out-of-sample forecasting accuracy, in line with relevant literature (see table 3.7).

For the out-of-sample forecasting, even under such a limited estimation time window the constrained alpha (table 3.7, columns (a) through (e)) has better forecasting accuracy than the unconstrained alpha (table 3.7, columns (f) through (j)), although the difference is smaller than with the previous test (see table 3.3). The best model is confirmed to be the MIDAS with beta weighting scheme.

Table 3.7. Forecast evaluation for alpha

Comparison of actual values with static forecast regressions

	No obs.	RMSE	MAE	MAPE	SMAPE	Theil - u2
		(a)	(b)	(c)	(d)	(e)
			Constrained			
Almon 2	19	3.70	3.06	24.00	25.14	1.18
Almon 3	19	4.26	3.46	26.07	29.12	1.40
Beta	19	**1.41**	**0.50**	**6.10**	**4.75**	**0.38**
Regression average	24	3.65	3.04	24.09	24.88	1.19
			Unconstrained			
		(f)	(g)	(h)	(i)	(j)
Almon 2	19	3.60	2.94	24.45	23.67	1.11
Almon 3	19	3.95	3.14	26.05	26.33	1.23
Beta	19	1.62	0.58	7.10	5.31	0.47
Regression average	24	4.40	1.23	14.97	10.02	2.25

Estimation period: May 2010 – May 2012; forecast period from June 2012 to May 2019.
RMSE (root mean squared error), MAE (mean absolute error), MAPE (mean absolute percentage error) and SMAPE (symmetric mean absolute percentage error) and Theil (U2) are the usual measures for forecasting evaluation.
Almon 2 (or 3) stands for the PDL/Almon weighting scheme with 2 (or 3) polynomial degrees; the number of lags has been chosen by using the automatic best lag selection provided by Eviews; Beta stands for the beta step weighting (20 lags). Step stands for the step weighting scheme (2 steps); the number of lags has been chosen on the basis of automatic best lag algorithm provided by Eviews. U-MIDAS stands for the unrestricted MIDAS (20 lags). Regression average means the forecast has been run starting from the regression of the monthly variable on the monthly average of the daily variables. In bold the best forecast indicators.

To check if the result is affected by the choice of volatility variable (implied volatility), in the following robustness test I change it and use the volatility estimated by the best fitted GARCH model, for the best forecasting model (Beta-MIDAS). For the time span used, the best ARMA-GARCH model, on the basis of the usual information criteria, is the one-lag autoregressive model with one-lag moving average (ARMA(1,1)) for yield, and E-GARCH, with one asymmetric order and t-distribution of residuals for variance (in line with literature on financial series modelling on non-normal conditional error distributions, for instance see Brooks (2007). I run the beta weighting scheme without endpoint or shape constraints for the period June 2010 – May 2017, to forecast the period June 2017 – May 2019. Forecast indicators with ARMA-GARCH are even better than those stated in table 3.3 for beta-MIDAS constrained alpha, hence its accuracy in forecasting is better than one of the other models examined in table 3.3.

Examining the estimation coefficients reported in table 3.8, the slope of volatility (equal to 24.80) is significant and with the expected sign, in line with the results reported in table 3.5.

Table 3.8. MIDAS regression with beta weighting (*constrained alpha)*

Variables	Coefficients
C	7.64***
	(1.45)
GARCH Volatility – Lags: 20	
Slope	24.80***
	(6.16)
Beta-1	3.25
	(5.85)
Beta-2	3.73
	(8.28)
Beta-3	-0.08
	(0.04)
Return – Lags: 20	
Slope	0.60
	(0.69)
Beta-1	0.52
	(61.57)
Beta-2	0.52
	(61.57)
Beta-3	-0.73
	(1.96)

GARCH Volatility – Beta shape

Lag	Coefficient	Distribution
0	3.24	*
1	3.08	*
2	2.62	*
3	1.93	*
4	1.15	*
5	0.38	*
6	-0.27	*
7	-0.76	*
8	-1.02	*
9	-1.04	*
10	-0.83	*
11	-0.41	*
12	0.15	*
13	0.81	*
14	1.50	*
15	2.13	*
16	2.65	*
17	3.02	*
18	3.20	*
19	3.24	*

*MIDAS estimates for model (3.10). Dependent variable (monthly): inverse of alpha (inverse of the listed unencumbered shares divided by the HFT); regressors (daily): volatility (estimated with a GARCH model) and returns of the Italian market index standard errors in parenthesis. - *** - significant at 1% level, ** - significant at 5% level; * - significant at 10% level. Number of observations: 84. The forecast evaluation indicators for the model are the following: RMSE: 2.46; MAE 2.11; MAPE 22.2; SMAPE: 19.5; Theil U2: 0.89.*

Besides helping in studying the relation among variables, the MIDAS approach also contributes to the analysis of the evolution of the relations between variables for different lags. From table 3.8, the first two beta coefficients are significantly different from 1 thus showing a peculiar U-shaped beta. This shape may be an expression of the overshooting (and pro-cyclical) effect reported in literature in case of risk-constrained investment decisions and highlighted in the previous chapter.

3.5 Conclusion

The financial crisis proved that the modern risk management techniques cannot prevent crises from happening. In addition, various authors have underlined that some of these risk management techniques, such as value at risk for market risk, may deepen the recession and, more generally, may have a pro-cyclical effect by affecting investors' behaviour.

In literature, research about the impact of value at risk on investment decisions deliver mixed results. Alexander and Baptista (2002), in a mean-VaR framework, found that risk-averse agents may end up selecting riskier portfolios if they use VaR as a risk measure. Basak and Shapiro (2001) also found that regulation leads financial institutions to accept higher exposure to risky assets.

In contrast with these results, Cuoco and Isaenko (2008) find that exposure to risky assets is lower for a trader subject to a VaR limit than for an unconstrained investor. Yiu (2004) also finds that VaR may reduce allocation to risky assets. Such theoretical results are not supported by the empirical analysis.

In this chapter, I solved a constrained maximization problem in a theoretical mean-variance framework, and I found that when VaR constraint binds, investments in risky assets are lower than in an unconstrained environment, in line with some of the literature. In addition, I found that when the constraint does not bind, the portion of wealth invested in risky assets is in line with the standard mean-variance result, as expected.

Unlike several papers on the topic, I also performed some empirical tests, limited to Italy, finding that the VaR-constrained solution discussed in the theoretical part of the chapter is more accurate in forecasting banks' investments in risky assets than the unconstrained one, thus giving empirical support to the result obtained in the theoretical part.

In terms of policy, the results obtained appear to back the regulators' decision to impose risk limits on individual banks. In fact, the application of VaR causes a decrease of investments in risky assets, thus potentially reducing the exposure to market risk, in line with regulators' goal. The result found that risky investments are lower under the constrained-VaR scenario than in the unconstrained one, which paves the way for additional analyses on the amplification effect, whose existence has been empirically proved in chapter 2. In fact, on the basis of the result of this chapter the shift from an unconstrained scenario to a constrained one (or from a low-volatility period to a high-volatility period) causes a reduction of investment in risky assets and therefore may explain the above-mentioned amplification effect. This additional effect of the model, which can further explain the conclusions reached in chapter 2, will be tested in chapter 4 where the model defined in the present chapter will be complemented with a measure of uncertainty.

Appendix 3.A

The maximization problem with a general risk constraint reported in the chapter is represented as follows:

$$\max_{\alpha_t} E_t\left(R_{P,t+1}\right) - 0.5\gamma\left[\sigma_t^2\left(R_{P,t+1}\right)\right]$$

$$\text{subject to: } f\left[\sigma_t\left(R_{P,t+1}\right)\right] \leq \mathit{Risk\ Limit} \qquad (3.5)$$

The related Lagrangian expression is

$$\mathcal{L} = \alpha_t \mathrm{E_t}\left(\mathrm{R_{k,t+1}}\right) + (1-\alpha)\mathrm{R} - 0.5\,\gamma\,\alpha^2\sigma_t^2\left(R_{k,t+1}\right) +$$

$$\lambda\left[\mathrm{RL} - \mathrm{f}\left(\sigma_t\left(\mathrm{R}_{p,t+1}\right)\right)\right] \qquad (3.A.1)$$

Since it is a one-variable optimization problem with one inequality constraint, I verify the conditions reported in Baldani et al. (1996) for the binding constraint ($\lambda > 0$) and the non-binding constraint ($\lambda = 0$) scenarios.

Non-binding constraint scenario.

When the constraint does not bind ($\lambda = 0$), conditions are the following:

a) $\lambda = 0$

b) $\frac{\partial \mathcal{L}}{\partial \alpha} = 0$

c) $\frac{\partial \mathcal{L}}{\partial \lambda} \geq 0$

d) $\lambda \frac{\partial \mathcal{L}}{\partial \lambda} = 0$, which holds given that $\lambda = 0$ (see a)

Condition b) can be written as follows:

$$\frac{\partial \mathcal{L}}{\partial \alpha} = [\mathrm{E_t}\,(\,\mathrm{R_{k,t+1}}) - \mathrm{R}] \;-\; \gamma\,\alpha\,\sigma^2{}_{\mathrm{t}}(\,\mathrm{R}_{k,t+1}) \;-\; \lambda\left[\frac{\partial\, f\left[\sigma_t\left(\mathrm{R}_{P,t+1}\right)\right]}{\partial \alpha}\right] = 0$$

(3.A.2)

where lambda is equal to zero (non-binding constraint). From 3.A.2 I obtain:

$$\alpha_t = \frac{[\mathrm{E_t}\,(\,\mathrm{R_{k,t+1}}) - \mathrm{R}]}{\gamma\;\sigma^2{}_{\mathrm{t}}(\,\mathrm{R}_{k,t+1})} \qquad (3.A.3)$$

which is the usual identity for alpha in the unconstrained scenario.

Hence, in the case of the non-binding constraint, the portion of wealth invested in risky assets is given by (3.A.3) and is directly related to the excess return and inversely related to the variance of risky assets.

Finally, from c) I obtain:

$$\mathrm{RL} - \mathrm{f}(\sigma_t(\mathrm{R}_{p,t+1})) \geq 0 \qquad (3.A.4)$$

then

$$\mathrm{f}^{-1}(\sigma_t(\mathrm{R}_{p,t+1})) \leq \mathrm{RL} \qquad (3.A.5)$$

Hence, when the constraint does not bind, the portion of wealth invested in risky assets is at a level below the maximum possible, where the maximum is the one that determines that $\mathrm{f}(\sigma_t(\mathrm{R}_{p,t+1}))$ is equal to the risk limit.

Binding constraint scenario

When the constraint binds ($\lambda > 0$), conditions are the following:

a) $\lambda > 0$

b) $\frac{\partial \mathcal{L}}{\partial \alpha} = 0$

c) $\frac{\partial \mathcal{L}}{\partial \lambda} \geq 0$

d) $\lambda \frac{\partial \mathcal{L}}{\partial \lambda} = 0$ i.e. $\frac{\partial \mathcal{L}}{\partial \lambda} = 0$ (given that $\lambda > 0$, as in a))

From b) I get the following expression:

$$\alpha_t = \frac{[E_t(R_{k,t+1}) - R]}{\gamma\ \sigma^2{}_t(R_{k,t+1})} - \lambda\,\frac{[\frac{\partial f[\sigma_t(R_{P,t+1})]}{\partial\alpha}]}{\gamma\ \sigma^2{}_t(R_{k,t+1})} \qquad (3.A.6)$$

which can be written as follows:

$$\alpha_C = \alpha_U - \lambda\,\frac{[\frac{\partial f[\sigma_t(R_{P,t+1})]}{\partial\alpha}]}{\gamma\ \sigma^2{}_t(R_{k,t+1})} \qquad (3.A.7)$$

Where, for the sake of simplicity, α_U is the unconstrained alpha (see (3.A.3)), whereas $\alpha_C$is the VaR-constrained alpha.

Regarding the second term of the difference reported in the right-hand side, in the scenario examined lambda is strictly positive (see a), the numerator of the ratio is positive (since sigma is positive in alpha by assumption), and the denominator is positive. As a consequence, the entire second term of the right-hand side is negative. Therefore, when the constraint binds, the constrained alpha is lower than the unconstrained one, as is reasonable.

From c) and d) I get:

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0$$

Hence

$$\mathrm{RL} - \mathrm{f}(\sigma_t(\mathrm{R}_{p,t+1}))=0 \qquad (3.A.8)$$

which can be written, in terms of alpha, as:

$$\mathrm{f}^{-1}(\sigma_t(\mathrm{R}_{p,t+1}))=\mathrm{RL} \qquad (3.A.9)$$

Hence the proportion of wealth invested in risky assets, in case of binding constraints, is given by (3.A.7) and is related only to the function used to measure risk and to the risk limit.

Appendix 3.B

The maximization problem with a VaR risk constraint reported in this chapter is represented as follows:

$$\max_{\alpha_t} E_t\left(R^p_{t+1}\right) - 0.5\gamma\left[\sigma^2_t\left(R^p_{t+1}\right)\right]$$

$$\text{subject to: } z\sigma_t\left(R^p_{t+1}\right) - E_t\left(R^p_{t+1}\right) \leq V \qquad (3.B.1)$$

The Lagrangian expression associated to the constrained maximization is the following:

$$\mathcal{L} = \mathrm{E_t}\left(\mathrm{R_{p,t+1}}\right) - 0.5\gamma\sigma^2{}_\mathrm{t}\left(\mathrm{R}_{p,t+1}\right) + \lambda\left[\mathrm{V} - \mathrm{z}\,\sigma_t\left(\mathrm{R}_{p,t+1}\right) + \mathrm{E}_t\left(\mathrm{R}_{p,t+1}\right)\right]$$

which is written also as follows:

$$\mathcal{L} = \left[\alpha_t \mathrm{E_t}\left(\mathrm{R_{k,t+1}}\right) + (1-\alpha_t)\mathrm{R}\right] - 0.5\,\gamma\,\sigma^2{}_\mathrm{t}\left(\alpha_t\,\mathrm{R}_{k,t+1}\right) +$$

$$+\lambda\left[\mathrm{V} - \mathrm{z}\,\sigma_t\left(\alpha\,\mathrm{R}_{k,t+1}\right) + \left(\alpha_t \mathrm{E_t}\left(\mathrm{R_{k,t+1}}\right) + (1-\alpha_t)\mathrm{R}\right)\right]$$

given that risk (sigma) for the free risk asset is equal to zero by definition.

Non-binding scenario

Following Baldani et al. (1996), I verify the following conditions for the non-binding constraint ($\lambda = 0$) scenario:

a) $\lambda = 0$

b) $\frac{\partial \mathcal{L}}{\partial \alpha} = 0$

c) $\frac{\partial \mathcal{L}}{\partial \lambda} \geq 0$

d) $\lambda \frac{\partial \mathcal{L}}{\partial \lambda} = 0$, which holds given that $\lambda = 0$ (see a)

Condition b) can be written as follows:

$$\frac{\partial \mathcal{L}}{\partial \alpha} = [\mathrm{E_t}(\mathrm{R_{k,t+1}}) - \mathrm{R}] - \gamma\, \alpha\, \sigma^2{}_{\mathrm{t}}(\mathrm{R}_{k,t+1}) +$$

$$+\lambda\, [-\mathrm{z}\, \sigma_t \left(\mathrm{R}_{k,t+1}\right) + (\mathrm{E_t}(\mathrm{R_{k,t+1}}) - \mathrm{R})] = 0 \quad (3.B.2)$$

from which I get:

$$\alpha_t = \frac{[\mathrm{E_t}(\mathrm{R_{k,t+1}}) - \mathrm{R}]}{\gamma\ \sigma^2{}_{\mathrm{t}}(\mathrm{R}_{k,t+1})} - \lambda\, \frac{[\mathrm{z}\, \sigma_t\, (\mathrm{R}_{k,t+1}) - (\mathrm{E_t}(\mathrm{R_{k,t+1}}) - \mathrm{R})]}{\gamma\ \sigma^2{}_{\mathrm{t}}(\mathrm{R}_{k,t+1})} \quad (3.B.3)$$

when lambda is equal to zero, (3.B.3) becomes:

$$\alpha_t = \frac{[\mathrm{E_t}\,(\,\mathrm{R_{k,t+1}}) - \mathrm{R}]}{\gamma\ \sigma^2{}_\mathrm{t}(\,\mathrm{R}_{k,t+1})} \tag{3.B.4}$$

which is the usual identity for alpha in the unconstrained scenario.

It is worth noting that c) implies

$$\mathrm{V} - \mathrm{z}\,\sigma_t\left(\mathrm{R}_{p,t+1}\right) + \mathrm{E}_t\left(\mathrm{R}_{p,t+1}\,\right) \geq 0$$

Hence

$$\mathrm{V} - \mathrm{z}\,\sigma_t\left(\alpha\,\mathrm{R}_{k,t+1}\right) + (\alpha_t \mathrm{E_t}\,(\,\mathrm{R_{k,t+1}}) + (1 - \alpha_t)\mathrm{R}) \geq 0$$

This means that:

$$\alpha_t \leq \frac{V+R}{\mathrm{z}\,\sigma_t\,(\,\mathrm{R}_{k,t+1}) + (\mathrm{E_t}\,(\,\mathrm{R_{k,t+1}}) - \mathrm{R})} \tag{3.B.5}$$

Hence, the portion of wealth invested in risky assets under a non-binding constraint is lower than the expression reported at the right-hand side of (3.8.5), which is the value of alpha under a binding constraint (see (3.B.8)).

Binding constraint

When the constraint binds ($\lambda > 0$), the conditions are the following:

a) $\lambda > 0$

b) $\frac{\partial \mathcal{L}}{\partial \alpha} = 0$

c) $\frac{\partial \mathcal{L}}{\partial \lambda} \geq 0$

d) $\lambda \frac{\partial \mathcal{L}}{\partial \lambda} = 0$ i.e. $\frac{\partial \mathcal{L}}{\partial \lambda} = 0$ (given that $\lambda > 0$, as in a))

From b) I get the (3.B.3) expression which can be written as follows:

$$\alpha_C = \alpha_U - \lambda \frac{[z\, \sigma_t\, (R_{k,t+1}) - (E_t\, (R_{k,t+1}) - R)]}{\gamma\, \sigma^2{}_t(R_{k,t+1})} \qquad (3.B.6)$$

Where, for the sake of simplicity, α_U is the unconstrained alpha (3.B.4) whereas α_C is the VaR-constrained alpha.

With reference to the second term of the difference reported in the right-hand side, in the scenario examined, lambda is strictly positive (see a)), the numerator of the ratio is positive, as detailed below, and the denominator is positive. As a consequence, the entire second term of the right-hand side is negative. Therefore, the constrained alpha is lower than the unconstrained one, as is reasonable.

As mentioned above, the numerator of the ratio is positive. In fact, from the constraint I know that:

$$\text{VaR} = z\,[\sigma_t\,(R_{p,t+1})] - E_t\,(R_{p,t+1})$$

that is:

$$\text{VaR} = z\,\sigma_t\left(\alpha_t\, R_{k,t+1}\right) - (\alpha_t E_t\,(R_{k,t+1}) + (1-\alpha_t)R)$$

from which I get:

$$\frac{VaR+R}{\alpha_t} = z\,\sigma_t\left(R_{k,t+1}\right) - (E_t\,(R_{k,t+1}) - R) \qquad (3.B.7)$$

On the basis of the assumptions of the model, VaR and alpha are positive, and the return R of the risk-free asset is close to zero. Hence, the right-hand side of (3.B.7) (which is the numerator of the second term of (3.B.6)) is positive, too.

From (3.B.6) I get:

$$\alpha_t = \frac{V+R}{\mathrm{z}\,\sigma_t\,(\mathrm{R}_{k,t+1}) - (\mathrm{E_t}\,(\,\mathrm{R_{k,t+1}}) - \mathrm{R})} \qquad (3.B.8)$$

which is in line with condition d). In fact, from d) I get:

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -\mathrm{z}\,\sigma_t\left(\alpha\,\mathrm{R}_{k,t+1}\right) + (\alpha \mathrm{E_t}\,(\,\mathrm{R_{k,t+1}}) + (1-\alpha)\mathrm{R}) + \mathrm{V} = 0$$

$$(3.B.9)$$

which, in turn, satisfies also condition c). From (3.B.8) I obtain the result for constrained alpha, which is the expression (3.B.8).

For the sake of completeness, from (3.B.3) the value of lambda for the Lagrangian equation is the following:

$$\lambda = \frac{[\mathrm{E_t}\,(\,\mathrm{R_{k,t+1}}) - \mathrm{R}]}{\mathrm{z}\,\sigma_t\,(\mathrm{R}_{k,t+1})\, - (\mathrm{E_t}\,(\,\mathrm{R_{k,t+1}}) - \mathrm{R})} - \alpha^c\,\frac{\gamma\;\sigma^2{}_{\mathrm{t}}\left(\,\mathrm{R}_{k,t+1}\right)}{\mathrm{z}\,\sigma_t\,(\mathrm{R}_{k,t+1})\, - (\mathrm{E_t}\,(\,\mathrm{R_{k,t+1}}) - \mathrm{R})}$$

Hence lambda is positive when $[\mathrm{E_t}\,(\mathrm{R_{k,t+1}}) - \mathrm{R}] > \alpha^c \gamma\ \sigma^2{}_{\mathrm{t}}(\mathrm{R}_{k,t+1})$; therefore if, and only if, $\alpha^u > \alpha^c$, which is true in the constrained scenario under analysis.

Appendix 3.C

For the MIDAS model, the starting point could be considered the traditional regression used for forecasting:

$$y_t = \beta_0 + \beta_1 X_t + \epsilon_t \qquad (3.C.1)$$

where h denotes the forecast horizon and both y_{t+h} and X_t are sampled at a low frequency (e.g. in this chapter, monthly). In this approach X_t is an aggregate of high-frequency series (e.g. daily series). The aggregation scheme of high-frequency data (e.g. equal weight for simple averages of high-frequency variables) may be represented as follows:

$$y_t = \beta_0 + \beta_1 \left(\sum_{\tau=0}^{S-1} X_{\frac{t-\tau}{S}} \right) \lambda \; + \epsilon_t \qquad (3.C.2)$$

where $X_{(t-\tau)/S}$ are the data at the τ high frequency periods prior to t, with S values for each low frequency data point. The aggregation approach may be thought of as one in which the component higher frequency lags all enter the low frequency regression with a common coefficient, λ.

MIDAS regressions assume that the aggregation of high frequency data is captured by a known weight function; hence the mixed frequency model may be represented as follows:

$$y_t = \beta_0 + \beta_1 f(X^H_{\frac{t}{S}}, \theta, \lambda) + \epsilon_t \qquad (3.C.3)$$

where y_t is the dependent variable, sampled at a low frequency; $X^H_{t/S}$ is a set of regressors sampled at a higher frequency with S values for each low frequency value; θ, λ are vectors of parameters.

MIDAS regressions allow a more flexible weighting structure than traditional low-frequency models and can also be more parsimonious. Moreover, the MIDAS framework can easily accommodate the timely releases of high-frequency data.

As mentioned before, different classes of MIDAS model exist, based on different weighting functions.

Foroni and Marcellino (2014b), Foroni et al. (2015), and Marcellino and Schumacher (2010) referred to the unrestricted MIDAS regression as the one where there is no predefined weighting scheme for the high-frequency data:

$$y_t = \beta_0 + \ \beta_t \sum_{\tau=0}^{S-1} X_{(t-\tau)/S}^{H}{}' + \epsilon_t \qquad (3.C.4)$$

where $X_{(t-\tau)/S}^{H}$ are the data at the τ high frequency periods prior to t. This approach estimates a distinct coefficient for each of the high frequency lag regressors. Hence, U-MIDAS does not alleviate the issue of requiring a large number of coefficients.

Some of the most used weighting schemes which help to reduce the number of coefficients to be estimated are the Almon lag, the step function and the beta function weightings.

In the Almon lag weighting (also called polynomial distributed lag or PDL weighting), for each high frequency lag the regression coefficients are modelled as a lag polynomial in the MIDAS parameters. The resulting restricted regression model can be written as follows:

$$y_t = \beta_0 + \beta_i \sum_{j=0}^{p} Z_{i,t}{}' + \epsilon_t \qquad (3.C.5)$$

$$Z_{i,t} = \sum_{\tau=0}^{k-1} (\tau^i) X_{(t-\tau)/S}^{H} \qquad (3.C.6)$$

where p is the Almon polynomial order, and k the chosen number of lags.

In the step function weighting the regression model can be written as follows:

$$y_t = \beta_0 + \beta_t \ \sum_{\tau=0}^{k-1}(\tau^i)X_{(t-\tau)/S}^{H}{}' + \epsilon_t \qquad (3.C.7)$$

where k is the number of lagged high frequency periods chosen, and the coefficient β_t is the same for every observation included in the step-length time window; hence it restricts consecutive lags to have the same coefficient (Forsberg and Ghysels 2007).

Lastly, Ghysels et al. (2007) considered the beta polynomial as a possible alternative for the weighting function, assuming that the weights are determined by a few hyperparameters θ.

The corresponding regression model is given by:

$$y_t = \beta_0 + \ \beta_t \ \sum_{i=0}^{k} Z_{i,t}{}' + \epsilon_t \qquad (3.C.8)$$

where

$$Z_{i,t} = \left(\frac{\omega_i^{\theta_1-1} - (1-\omega_i)^{\theta_2-1}}{\sum_{j=0}^{k}\omega_j^{\theta_1-1}\left(1-\omega_j\right)^{\theta_2-1}} + \theta_3\right) X_{(t-i)/S}^{H} \qquad (3.C.9)$$

The beta function is extremely flexible and can take many shapes, including gradually increasing or decreasing, flat, humped, or U-shaped, depending on the values of the three MIDAS parameters (θ_1, θ_2, θ_3). In practice the beta function is usually restricted with θ_1=1, θ_3=0. The first restriction implies that the shape of the weight function depends on a single parameter (slow decay if θ_2 >1, slow increase when θ_2 <1); the restriction θ_3=0 implies that there are zero weights at the high frequency lag endpoints (0 and k-1).

Chapter 4. Uncertainty and risk in a VaR – constrained portfolio choice

4.1 Introduction

Chapter 3 shows that the relationship between investors' decisions and regulatory risk limits is theoretical founded and is supported by empirical evidence. More specifically, the theoretical constrained mean-variance framework, presented in the previous chapter, confirms that the adoption of regulatory VaR has a risk-limiting impact on investors' choices. In addition, the theoretical results of chapter 3 pave the way for additional analyses on the amplification effect, since the shift from an unconstrained to a constrained scenario causes a reduction of the investments in risky assets and therefore may explain the amplification effect empirically proved in chapter 2. In this chapter, by using the closed form formula on risky investments found in chapter 3, to which I add an uncertainty variable, I will further test the model to assess its ability to describe also the amplification effect and the impact of uncertainty on investors' decisions.

I enrich the model with an uncertainty variable because, as demonstrated in literature, risk is not the sole parameter taken into account by investors. Especially since the financial crisis, uncertainty has come to light as a relevant variable to explain market behaviour. In 2010, the former Chairman of the Fed, Ben Bernanke, said that "during the worst phase of the financial crisis, many economic actors metaphorically threw up their hands and admitted that, given the extreme and, in some ways, unprecedented nature of the crisis, they did not know what they did not know...The profound uncertainty associated with the 'unknown unknowns' during the crisis resulted in panicky selling by investors" (Bernanke 2010).

In fact, since the financial crisis of the 2007-2008, papers on risk management and uncertainty have proliferated. In general, during a financial crisis, investors start to cut their risk exposure by selling assets, thus impacting the rest of the financial system and causing a deterioration of the whole market. The consequent fall of prices and increase of volatility determine the need to further reduce exposure to risky assets, thus feeding a vicious circle. In such circumstances the reinforcing effect of uncertainty becomes more visible.

In this chapter, building further on the results obtained in chapter 3, I examine the impact of risk limits and uncertainty in a simple framework where agents have mean-variance preferences and choose to allocate their wealth between risk-free bonds and risky assets. In line with other papers, I find that uncertainty is inversely related to the proportion of wealth invested in risky assets, potentially contributing to the vicious circle (increase of risk and uncertainty, increase of fire sales and volatility) typical of crises.

Several other papers have studied uncertainty in the field of finance, also in portfolio allocation problems. Bekaert and Hoerova (2014) find that the variance premium, which is the difference between the squared VIX and an estimate of the conditional variance and is used as a measure of risk aversion, is a significant predictor of stock returns, but the conditional variance, as a measure of uncertainty, mostly is not. However, conditional variance robustly and significantly predicts economic activity, whereas variance premium has no predictive power for future output growth.

Drechsler (2013) indicates that fluctuation in the variance premium strongly reflects changes in the level of Knightian uncertainty and predicts monthly stock returns. In the same line, Bollerslev et al. (2014) find that a measure of variance risk premium, seen as a proxy for aggregate economic uncertainty, predicts aggregate stock market returns. In an empirical analysis of the dynamics of investors' beliefs, Ozoguz (2009) finds a positive relation between uncertainty and volatility.

With regards to portfolio allocation, Maccheroni et al. (2013) set a theoretical mean-variance framework adjusted for ambiguity with a risk-free asset, a risky asset, and an ambiguous asset. In this framework Maccheroni et al. find that ambiguity has an inverse impact on the fraction of wealth invested in non-risk-free assets.

However, to the best of my knowledge, no previous research focused on the portfolio allocation with uncertainty by considering also a VaR constraint in a mean-variance framework. Furthermore, theoretical articles concerning the impact of uncertainty on portfolio allocation do not provide empirical evidence in support of their theories. In addition, among the empirical articles, no researchers empirically investigate how a VaR-constrained portfolio allocation framework (including uncertainty and obtained in a mean-variance setup) concretely works, in two different periods of high and low volatility.

In the empirical part of the chapter I test the results obtained in the theoretical part by using a unique dataset on Italian banks and official data for the Italian stock market, with the idea that the relation between banks' behaviour and the domestic financial market is more visible in Italy than in other big European countries or in the United States where the role of international financial agents is stronger and the relation between national stock exchanges and domestic banks is weaker. The empirical tests with OLS confirm the theoretical dynamics; furthermore, in line with literature, in high-volatility periods the importance of uncertainty is stronger than in low-volatility periods.

The results obtained provide further theoretical and empirical support to the role of uncertainty in portfolio allocation choices, especially in turmoiled periods, and at the same time raise further concerns about the role of uncertainty in more stable times.

Therefore, the contribution of this chapter is twofold. In the first part, I find a new closed formula for portfolio allocation which, unlike theoretical literature, includes both a VaR-constraint and uncertainty in a mean-variance framework; in the second part, I successfully complete some empirical tests of this formula, providing supporting evidence to the formula and finding that banks' behaviour is more influenced by uncertainty in periods with higher volatility. To the best of my knowledge, no empirical test of the impact of both VaR constraint and uncertainty have been previously performed in literature.

4.2 The effects of risk limits and uncertainty: relevant literature

The chapter examines the impact of risk and uncertainty on investment decisions in a regulated environment, from a theoretical and empirical points of view. Hence, it is related to theoretical papers which analyse the impact of risk and regulation on financial agents' behaviour and the effects of uncertainty on investment decisions, and to empirical papers which analyse the role of risk and uncertainty on investments. However, the chapter provides a new theoretical contribution by adding a VaR constraint to the framework and, unlike other papers, empirical evidence in support of the framework.

With respect to the influence of future risk on current investment decisions, Bacchetta et al. (2012) focus on the relationship between risk and price in a mean-variance environment. They assume that asset price depends (negatively) on asset risk, defined as the variance of tomorrow's asset price. In addition, they add a variable ("S") which can be a fundamental variable (e.g. dividends) or a variable extrinsic to the model (a sunspot variable). Since they assume that the variance of tomorrow's asset price (risk) depends on the variable "S", the asset price (which depends on risk) is also related to "S". This relationship between asset risk and asset price level can generate self-fulfilling shifts in risk, coordinated around the variable "S". In general, the relation between risk and investment decisions examined by the authors is classically related to Markowitz's Modern Portfolio Theory.

Other researchers focus their analyses on regulatory constraints, for instance in terms of impact of value at risk (VaR), on investment decisions. VaR is the regulatory standard measure of market risk. The results of these studies are mixed; however, in general they show that, in specific circumstances, the impact of VaR constraints may unintentionally alter the investment allocation process and amplify shocks. Jang and Park (2016) integrated a VaR constraint in the fund manager's wealth function, and found that fund managers using VaR-based risk management are exposed to large losses in turbulent times, as did Vorst (2001) and Meral (2019). Furthermore, Danielsson et al. (2004) show that VaR-constrained operators can cause the cycle to be more pronounced. Brunnermeier and Pedersen (2009) called margin spirals the amplification effect – via feedback – coming from capital constraints.

Still examining the impact of regulatory constraints on investment allocation, Alexander and Baptista (2006) find that, in a mean-variance framework with VaR constraint, regulation may increase the probability of extreme losses. Furthermore, the portfolio chosen in a constrained framework would be even riskier than the one chosen in an unconstrained environment.

Finally, with respect to the impact of uncertainty in the field of finance, it is worth noting that, despite the proliferation of papers on the topics, the definition is not uniform; furthermore, empirical evidence about the impact of uncertainty on investment decisions is not copious.

In general, uncertainty is associated with Knight (1921) who separated the notion of risk, as a measurable uncertainty, from the non-measurable one, known as Knightian uncertainty. Knightian uncertainty is unobservable, though some proxies can be used to assess changes over time. Non-Knightian uncertainty refers to the uncertainty of a variable for which the probability distribution of ex-ante realizations can be defined, but the values are not defined.

In literature, uncertainty has been variously defined and measured. Looking at the classification by Makarova (2014), macroeconomic Knightian uncertainty may be classified in at least two main categories: policy uncertainty (which is related to policy actions) and macro (or financial) uncertainty.

The former has been extensively examined in literature. For this chapter, the results obtained by Vinogradov (2012), who focused on regulatory ambiguity in the market equilibrium framework, are of interest, as he concluded that some negative effects of ambiguity can only be seen in times of high aggregate risk. Additionally, in the policy uncertainty strand, Baker et al. (2016) developed the Economic Policy Uncertainty index (EPU) for US, Canada, China, the Eurozone, France, Germany, India, Italy, Spain and UK. For the US they measured uncertainty on the basis of three components: frequency of newspaper references to economic policy uncertainty, tax provision, and disagreement among professional forecasters.

With regard to macro (or financial) uncertainty, the most influential papers are by Jurado et al. (2015), and by Bloom (2009). Jurado et al. define uncertainty as "the conditional volatility of a disturbance that is unforecastable from the perspective of economic agents". They use a large-scale dynamic factor model with stochastic volatility to extract joint forecastable components from 279 macroeconomic and financial indicators (for US only) allowing for idiosyncratic shocks in each of the indices. This analysis is complemented by the analysis of common variation of uncertainty at firm level by examination of a panel of 155 firms. In Bloom (2009), the VIX (Chicago Board Options Exchange Market Volatility Index) is used as a proxy for of macroeconomic uncertainty. Although VIX was originally designed for measurement of uncertainties related to financial markets only, it has become a widely used measure of uncertainty (e.g.

Haddow et al. (2013), for a critique see Bekaert et al. (2013)). In order to measure uncertainty, some of the literature uses the variance risk premium (Drechsler 2013), which is the difference between the squared VIX and an estimate of the conditional variance (Carr and Wu 2009, Bekaert et al. 2013). The variance premium is nearly always positive and displays substantial time-variation. In addition, Miao et al. (2018) has recently offered an ambiguity-related interpretation of variance premium, finding that most of the variance premium could be attributed to ambiguity aversion. A recent paper (Slim et al. 2019) uses variance risk premium to enhance the accuracy of VaR in measuring market risk.

Uncertainty has also been applied to asset allocation problems, sometimes by adding an ambiguous asset category to the traditional risky/ riskless dichotomy. Investors face risk and ambiguity when they evaluate an investment in an asset because they know neither the future realization of the asset's payoff (risk), nor the probability of it occurring (ambiguity). In this stream of literature, financial investors have a form of aversion not only to risk but also to ambiguity. Guetlein (2016) studied the relation between risk and ambiguity attitude and reached the conclusion that in the standard expected utility framework an increase in risk aversion reduces the demand for risky assets, whereas in a model considering ambiguity and risk aversion an increase in risk aversion does not necessarily determine an decrease of investment in uncertain (meaning risky and ambiguous) assets. Pinar (2014) also examines the impact of ambiguity aversion in a mean-variance setting, by finding, among other results, that under certain circumstances ambiguity aversion leads to giving less weight to a fund consisting of risky assets in a portfolio composed of a riskless asset and the risky asset fund. Illeditsch (2011) builds on the work by Epstein and Schneider (2008), who examine the effect of ambiguous information on stock prices, to argue that the interaction between risk and uncertainty can cause drastic changes in the stock prices. This interaction may explain the large increase in volatility after unexpected events. Gollier (2011) sets a framework where the Arrow-Pratt approximation is exact (i.e. normality of the priors, constancy of absolute risk aversion), and investors' preferences exhibit constant absolute risk aversion (CARA) and constant relative ambiguity aversion. In this framework, he finds that the optimal demand for the uncertain asset is negatively related to both the risk measure (and risk aversion) and ambiguity (and ambiguity aversion). One of the most relevant papers on the topic is by Maccheroni et al. (2013) who identified a new framework by adding model uncertainty and ambiguity aversion variables. Under the standard conditions used for the Arrow-Pratt approximation to be exact (i.e. risk is normally distributed and the utility function is exponential) and using CARA utility functions both for risk and uncertainty, they set a mean-variance framework adjusted for ambiguity with a risk-free asset, a risky asset and an ambiguous asset. Using this framework Maccheroni et al. found that ambiguity has an inverse impact on the fraction of wealth invested in non-risk-free assets.

In contrast to this chapter, no theoretical paper on uncertainty also focuses on the impact of risk limits on decisions of investment in a mean-variance framework.

On the empirical side, there is no concordant way to measure uncertainty. The quantification methods (or combinations of them) used in literature are numerous; among them the implied volatility measured with VIX (Bloom 2009); the disagreement among forecasters (Giordani and Söderlind 2003, Clements and Harvey 2011); the ARCH/GARCH-type models, where conditionally-autoregressive errors are associated with uncertainty (Elder 2004, Kontonikas 2004, Daal et al. 2005 Neanidis and Savva 2011); the uncertainty of the parameters, the variables, the data or the model (Onatski and Williams 2003, Orlik and Veldkamp 2013, Fritsche and Glass 2014); and the distribution of ex-post forecast errors (Jordà et al. 2013, Knüppel 2014).

A more recent measure of uncertainty (Izhakian 2016) built on volatility of probabilities of returns. Additionally, on the empirical side, Brenner and Izhakian (2018) examined the effects of risk, ambiguity and ambiguity attitudes on excess returns. They show that, in case of a high probability of losses, the effect of ambiguity on excess returns is negative, while in the case of a high probability of gains it is positive.

Finally, it is worth noting that the number of papers dealing contemporaneously with portfolio optimization, value at risk constraints, and ambiguity (or uncertainty) is limited. Puelz (2001) uses real data to conclude that VaR optimal portfolios are more likely to incur large losses when losses occur. Campbell et al. (2001) find that the higher the confidence level of VaR, the lower the percentage of risky asset in the portfolio. In the value at risk strand of literature, model uncertainty has been examined by Opschoor et al. (2014), who obtain a more accurate forecast of the left tail of the distribution by combining density forecasts to account for model uncertainty. Along the same line, Peng et al. (2018) also enrich the VaR with uncertainty thus obtaining an increase in accuracy of forecasting the VaR.

In contrast to this chapter, no empirical papers examine the relation among portfolio allocation decisions taken under VaR constraints in a mean-variance framework, and uncertainty.

To sum up, in this chapter, in the theoretical part I extend the mean-variance framework with the risk constraint determined in chapter 3 by adding an ambiguity measure. In line with other papers, such as Maccheroni et al. (2013), I find that risk and uncertainty are inversely related to the portion of wealth invested in risky assets. The result is new, given that literature does not find this result under a VaR constraint in a mean-variance framework. The empirical part, given the limited number of empirical articles on the topic and the use of a unique dataset, produces several new pieces of evidence. More specifically, I examine if the inverse theoretical relation among investments in risky assets, risk and uncertainty holds empirically. Based on a data of the Italian financial market and unencumbered stocks held by Italian banks, for the period June 2010 – May 2019, I confirm this inverse negative relation between the portion of wealth invested in risky assets and risk and uncertainty measures. Furthermore, in line with Vinogradov (2012) and Brenner and Izhakian (2018), the impact is negative and significant in high-volatility periods; in more stable periods the relation between investment in risky assets and the uncertainty measure is not significant. In literature no empirical evidence has been produced on the impact of risk and uncertainty on investment decisions in a VaR-constrained framework.

4.3 Theoretical framework

4.3.1 The unconstrained and the constrained mean-variance framework

As in section 3.3, I start from the standard model of maximization of mean-variance utility over portfolio return proposed by Bacchetta et al. (2012) (unconstrained framework); then, I determine a closed-form solution of the maximization problem under market risk constraint (constrained framework).

I report the main steps of the maximization procedure in this section. Further details on the procedure are in section 3.3.2 and appendix 3.B.

The model designed by Bacchetta et al. (2012) considers an overlapping generation setup where investors are born with wealth (W). Investors have an endowment W and purchase risk-free bonds (exogenous constant return R) and risky equity. They allocate their wealth between a risky equity and a risk-free bond that pays a gross return R. In their model, investors maximize mean-variance utility over their portfolio return:

$$\max_{\alpha_t} E_t\left(R_{t+1}^{p}\right) - 0.5\gamma\left[\sigma_t^2\left(R_{t+1}^{p}\right)\right] \qquad (4.1)$$

where γ measures risk aversion, $\sigma_t^2\left(R_{t+1}^{p}\right)$ is the variance of portfolio returns at t+1 expected at time t, R_{t+1}^{p} is the portfolio return at time t+1:

$$R_{t+1}^{p} = \alpha_t\, E_t\,(R_{t+1}^{k}) + (1-\alpha_t)\, R \qquad (4.2)$$

therefore $\sigma_t^2\left(R_{t+1}^{p}\right)$ also depends on α_t.

In (4.2), α_t stands for the share of the portfolio invested in equity, R is the gross return of a free-risk bond and $E_t\,(R_{t+1}^{k})$ is the return of the equity at t+1. The equity return is computed as follows:

$$R_{t+1}^{k} = \frac{A_{t+1}+Q_{t+1}}{Q_t} \qquad (4.3)$$

where Q_{t+1} is tomorrow's equity price; A_t represents the dividends and $A_t = \bar{A} + m\, S_t$ with S as an exogenous state variable that follows a stochastic process, and Ā is the constant dividend when m=0.

From the maximization condition, Bacchetta et al. (2012) find that the portion invested in equity is equal to:

$$\alpha_t = \frac{E_t\,(R_{t+1}^{k}) - R}{\gamma\, \sigma_t^2\; (R_{t+1}^{k})} \qquad (4.4)$$

Therefore, the portion of wealth invested in risky assets increases if the excess returns of equity with respect to bonds increases or if the variance of the expected returns decreases. The alpha obtained in (4.4) is the solution to the unconstrained maximization problem stated at (4.1).

To move towards a constrained framework from the model (4.1 – 4.4), I assume that investment decisions of professional investors are bound by risk limit constraints imposed by internal risk management or, ultimately, by regulation. Such constraints are not considered in the model described above, therefore I add them to the maximization problem stated in (4.1).

I focus on market risk limits which are more relevant for security investments process. The standard way to measure market risk limits is by computing the value at risk of the investment. Value at risk (VaR) is a probabilistic metric of market risk used by banks and other investors to monitor risk exposure of their trading portfolios. Value at risk indicates the maximum loss expected on an investment, at a certain confidence level, and over a given time horizon.

Following Kaplanski and Levy (2015) and Alexander and Baptista (2002, 2006), I express value at risk as a positive number measured as $z\sigma_t\left(R_{t+1}^p\right)$ where σ_t is the volatility of portfolio returns at t+1 expected at time t, and z is the confidence level parameter corresponding to the chosen confidence level. If the returns are not assumed to be zero, VaR becomes $z\sigma_t\left(R_{t+1}^p\right) - E_t\left(R_{t+1}^p\right)$. In a VaR-limited investment process, investors would not buy a portfolio of securities with VaR higher than a risk limit V. Specifically:

$$\max_{\alpha_t} E_t\left(R_{t+1}^p\right) - 0.5\gamma\left[\sigma_t^2\left(R_{t+1}^p\right)\right]$$

$$\text{subject to: } z\sigma_t\left(R_{t+1}^p\right) - E_t\left(R_{t+1}^p\right) \leq V \qquad (4.5)$$

As reported in section 3.3.2, from this constrained maximization, the portion of wealth invested in risky assets in a mean-variance framework with VaR-constraint is the following:

$$\alpha_t^C = \frac{V+R}{\mathrm{z}\,\sigma_t\,(R_{t+1}^k) - (\mathrm{E_t}\,(R_{t+1}^k) - \mathrm{R})} \tag{4.6}$$

4.3.2 Expected risk and uncertainty (unexpected risk)

The inclusion of uncertainty in the optimal demand for assets has been examined in literature under diverse frameworks and with different assumptions. Gollier (2011) and Maccheroni et al. (2013), taking into account the standard condition of having the Arrow-Pratt approximation as exact, find that the optimal demand for uncertain assets is negatively related to both risk and ambiguity variables. Anderson et al. (2009), in a different context, augment their model to take into account uncertainty.

More specifically, Maccheroni et al. (2013) start from the Arrow-Pratt approximation for the certain equivalent of the uncertain prospect $w + h$:

$$c(w + h, \mathrm{Prob}) \approx w + E_{Prob}\,(h) - \frac{1}{2}\,\lambda_u(w)\sigma_{Prob}^2\,(h)$$

where w is initial wealth, h is the investment, *Prob* is the probabilistic model associated with the approximation, $\sigma^2_{Prob}\ (h)$ is the variance of h with respect to *Prob*, and $\lambda_u(w)$ is the coefficient that links risk premium and variance and is determined by the risk aversion of the investor.

Starting from this approximation, and under the assumptions reported in their article, Maccheroni et al. write the portfolio problem as follows:

$$\max_{\alpha_t} E_{Prob}\left(R^p_{t+1}\right) - \frac{\lambda}{2}\left[\sigma^2_{Prob}\left(R^p_{t+1}\right)\right] - \frac{\theta}{2}\ \sigma^2_{\mu}\ \left(E\left(R^p_{t+1}\right)\right)$$

where $E_{Prob}\left(R^p_{t+1}\right)$ is the expected return of the portfolio under *Prob*, $\sigma^2_{\mu}\ E\left(R^p_{t+1}\right)$ is the variance of portfolio returns under μ (which is the prior of the investor over the possible probability models), and λ and θ represent the investor's attitude toward risk and ambiguity respectively.

Following Maccheroni et al. (2013), I augment the constrained maximization (4.5) for uncertainty as follows:

$$\max_{\alpha_t} E_t\left(R^p_{t+1}\right) - 0.5\gamma\left[\sigma^2_t\left(R^p_{t+1}\right)\right] - 0.5\theta\left[\widetilde{\sigma^2_t}\left(R^p_{t+1}\right)\right]$$

$$\text{subject to: } z[\sigma_t\left(R^p_{t+1}\right) + \theta\left[\widetilde{\sigma^2_t}\left(R^p_{t+1}\right)\right] - E_t\left(R^p_{t+1}\right) \leq V \qquad (4.7)$$

where θ is the uncertainty aversion, $\widetilde{\sigma_t^2}\left(R_{t+1}^p\right)$ is the measure for uncertainty and all the other symbols have the same meaning as in (4.5)

For the constraint, it is worth noting that regulation aims only at limiting risk. However, risk management units at banks are using increasingly accurate measures of the left tails of distributions to monitor. Therefore, in (4.7) although the regulatory limit V of the constraint does not change with respect to (4.5), I included the ambiguity variable in the measure of risk used by the bank (left hand side of the constraint) in line with literature (Opschoor et al. 2014) which shows that if model uncertainty is accounted for, a more accurate forecast of the left tail of the distribution is obtained.

Furthermore, as underlined by Brenner and Izhakian (2018), there is no agreement in literature yet on the exact functional form to represent ambiguity in portfolio preferences. Since in this chapter I will measure ambiguity with VIX (calculated for the Italian financial market, see section 4.4.1), it is natural to assume that it has some analogies with risk, hence:

$$\widetilde{\sigma_t^2}\left(R_{t+1}^p\right) = \alpha_t^2 \, \widetilde{\sigma_t^2}(R_{t+1}^k)$$

Against this background, the unconstrained and the constrained alphas, reported in (4.4) and (4.6) become:

$$\alpha_t^U = \frac{E_t\,(R_{t+1}^k) - R}{\gamma\,\sigma_t^2\,(R_{t+1}^k) + \theta\,\widetilde{\sigma_t^2}(R_{t+1}^k)} \tag{4.8}$$

$$\alpha_t^C = \frac{V+R}{\mathrm{z}[\,\sigma_t\,(R_{t+1}^k) + \widetilde{\sigma_t^2}(R_{t+1}^k)] - (\mathrm{E_t}\,(R_{t+1}^k) - \mathrm{R})} \tag{4.9}$$

where, in line with literature uncertainty is inversely related to the portion of wealth invested in risky asset; however, literature does not consider the VaR constraint,

4.3.3 Hypotheses

The theoretical equations (4.8) and (4.9) may be empirically tested with OLS to assess the impact of every addendum. For the unconstrained alpha, I ran the following regression:

$$A_t^U = \gamma_1 \frac{\gamma\,\sigma_t^2\,(R_{t+1}^k)}{E_t\,(R_{t+1}^k) - R} + \gamma_2\,\frac{\theta\,\widetilde{\sigma_t^2}(R_{t+1}^k)}{E_t\,(R_{t+1}^k) - R} + \varepsilon_t \tag{4.10}$$

where A^U is the inverse of the unconstrained alpha.

The addenda represent, respectively, the expected risk corrected for the expected excess return (which is closely related to the inverse of the

Sharpe ratio and to the inverse of the solution of the Merton's Portfolio problem), and the uncertainty corrected for the expected excess returns.

For the equation (4.9), where V and R are exogenous variables, I run the following regression, to assess the impact of every addendum:

$$A_t^C = \gamma_1 \sigma_t \left(R_{t+1}^k\right) + \gamma_2 \, \widetilde{\sigma}_t\left(R_{t+1}^k\right) + \gamma_3\left(\mathrm{E_t}\left(R_{t+1}^k\right) - \mathrm{R}\right) + \varepsilon_t \quad (4.11)$$

where A_t^C is the inverse of the constrained alpha and γ_i (with i=1…3) are the coefficients of the addenda at the denominator of (4.9) and the uncertainty measure multiplied by z/(V+R).

As reported in previous sections, the first addendum represents the expected risk, the second is a measure of uncertainty, and the third addendum is a measure of the expected excess return.

The main hypothesis is that investments in risky assets are driven by the constrained formula since regulation is applied in every market scenario. Hence, the coefficients of regressors for the constrained alpha, uncertainty included, are expected to be significant. Furthermore, I expect that when volatility is high, and thus the market is more turbulent and VaR constraint hits, the risk and ambiguity variables become more relevant.

4.4 Data, empirical methodology and results.

4.4.1 Data

In this section the equations (4.10) and (4.11) are empirically tested. I use data of the Italian financial market and of Italian banks, the latter coming from supervisory reporting.

More specifically, for alpha, I use the ratio between the amount of unencumbered listed shares held by the Italian banking system, which is based on a unique dataset coming from supervisory reporting, and the total held for trading (HFT) portfolio. I limited the analysis to the unencumbered shares, since they are freely disposable if the VaR constraint becomes tight. Among unencumbered shares, only the listed ones are included because, having an official market value, they are the risky asset class most impacted by VaR. I limited the analysis to the HFT portfolio because it is the one relevant from a regulatory point of view (i.e. it is the portfolio targeted by market risk regulation, which is based on value at risk), hence it is the closest to the idea of a VaR-constrained investment portfolio .

As mentioned above, in this section data on the HFT portfolio of banks, total unencumbered shares, and listed unencumbered shares come from monthly supervisory reporting and cover a time span of nine years (June 2010 – May 2019). The starting date has been chosen for comparability reasons since before June 2010 there is a break in the series reported by supervised entities.

For the other market-related variables (volatilities and returns), daily market data have been used for the same time span (May 2010 – June 2019). In details, for the market yield, I computed the annualized daily return from the closing value of the Italian market index. For the VIX variable, I used the daily end-of-day values of the IV-MIB index (henceforth also Italian VIX), which measures the annualized 30-day volatility implied in some selected options listed in the national derivative market (IDEM) (i.e. near-term, out-of-the money options with non-zero bid and ask prices[33]).

4.4.2 Empirical methodology

In the regression models (4.10) and (4.11), risk is represented as $\sigma_t^2\ (R_{t+1}^k)$ which is the expected variance of future returns of the risky asset while uncertainty is reported as $\widetilde{\sigma_t^2}(R_{t+1}^k)$.

To measure uncertainty, in line with literature (e.g. Bloom 2009), I use the Italian VIX indicator, for the Italian financial market. In fact, as underlined by Haddow et al. (2013), VIX is one of the most widely used

[33] For further details about the calculation methodology used by FTSE, see the https://www.borsaitaliana.it/borsa/indici/indici-di-volatilita/dettaglio.html?indexCode=IVMIB30&lang=it and https://research.ftserussell.com/products/downloads/FTSE_Implied_Volatility_Index_Series.pdf

indicators of uncertainty. In the robustness tests, I check the stability of the results by using different measures of uncertainty and risk.

To measure risk, the expected variance is computed on the basis of the most appropriate GARCH model. More generally, for the expectation-related variables, I assume that market investors use an ARMA-GARCH forecasting model based on daily market data; such models take into account the autocorrelation feature typical of financial series, highlighted by the analysis of correlograms.

The model that best deals with such autocorrelations, in the period under analysis (June 2010 – May 2019) is the one-lag autoregressive model with one-lag moving average (ARMA(1,1)). The LM test shows that ARCH effects are present in the data; modelling volatility with an ARCH-type model is therefore suggested. On the basis of the Akaike information criterion, the best ARCH-type model is a E-GARCH, with one asymmetric order and t-distribution of residuals, in line with literature on financial series modelling (e.g. Brooks 2007) which highlighted that for financial series, the distribution of residuals of non-emerging markets is often non-normal, and asymmetric volatility response (typical of E-GARCH with respect to ordinary GARCH) can capture some peculiarities of the data.

I then use this model to estimate the expected returns (from the ARMA model) and the expected realized volatility (the ARCH part of the model).

The regression is performed by ordinary least squares. To deal with serial correlation among residuals, I use the Cochrane-Orcutt procedure (Gujarati 2003, Brooks 2002), which is a two-step estimation of a linear regression model with first-order serial correlation in the errors. In the first step, the first-order autocorrelation coefficient ρ is estimated on the basis of an AR(1) (autoregression with one lag) estimation on the residuals of the standard OLS. In the second step this estimate is used to rescale the variables. The regression in terms of rescaled variables has no serial correlation in the errors. More formally, a generic standard regression:

$$y_t = \beta_0 + \beta_1 x_t + \epsilon_t$$

becomes:

$$y_t^* = \beta_0^* + \beta_1 x_t^* + \epsilon_t$$

where:

$$y_t^* = y_t + \rho y_{t-1}$$

$$x_t^* = y_t + \rho x_{t-1}$$

$$\beta_0^* = \frac{\beta_0}{1-\rho}$$

4.4.3 Baseline results

In table 4.1, I report some summary statistics related to alpha. The table shows that unencumbered listed shares are on average 8.4% of the HFT (table 4.1, column a, mean) portfolio, with a maximum in period of around 20% (table 4.1, column a, maximum).

Table 4.1. Listed shares on HFT portfolio. Summary statistics.

	Unencumbered listed shares (a)	**Listed shares (b)**
Mean	0.084	0.067
Median	0.075	0.061
Maximum	0.198	0.167
Minimum	0.049	0.025
Standard Deviation	0.030	0.025
Number of observations	108	108

Descriptive statistics for the unencumbered listed shares held by Italian banks and classified as held for trading (HFT). Monthly data for the period June 2010 – May 2019.

Although the percentage seems to be low, it is still relevant as a proxy of risky asset class, since most of the HFT portfolio is composed of government bonds. Robustness tests will deal with this peculiar composition of the portfolio.

The descriptive statistics for the market variables are summarized in table 4.2: expected volatility is on average 23% (table 4.2, column a, mean) and daily returns are on average close to zero (table 4.2, column b, mean).

Table 4.2. Market variables. Summary statistics.

	Expected risk (a)	**Expected return (%) (b)**
Mean	0.227	0.001
Median	0.213	0.000
Standard Deviation	0.080	3.812
Number of observations	2,348	2,348

Descriptive statistics for the expected risk and the expected return of the Italian stock market. Daily data for the period 1 June 2010 – 31 May 2019.

I then run the ordinary least squares regressions for equations (4.10) and (4.11), by using monthly data for the time span from June 2010 to May 2019. To convert the daily market data into monthly data, I use the last observation of each month instead of using different criteria (e.g. the monthly average), since any investment choice is usually based on the most recent data.

For the ordinary least squares regression of (4.10) and (4.11), the Jarque-Bera and the Breusch-Pagan-Godfrey tests, for normality and for homoskedasticity respectively, are passed and correlation between residuals and regressors is not significant. On the basis of the test performed (variance inflation factor) collinearity is not a problem.

However, the analysis of residuals shows some serial correlation (on the basis of the Durbin Watson test, confirmed by the Breusch-Godfrey LM test and correlograms).

Some positive serial correlation among residuals is in fact reasonable, since our investors, which are banks, do not create their portfolio from scratch every month. To clean data from such correlations, I applied the Cochrane-Orcutt procedure and re-perform the regression. To measure the risk variable of (4.10) I used the expected volatility coming from the ARMA-GARCH procedure described in section 4.4.2. For the ambiguity variable, as above mentioned, I used the Italian VIX, in line with Bloom (2009). The results are reported in table 4.3, where the significant variables have the expected sign.

Table 4.3. Constrained scenario estimation – corrected for autocorrelation.

Variables	Coefficients
C	5.73*** *(2.09)*
Risk (expected volatility)	10.48*** *(3.76)*
Uncertainty (Italian VIX)	15.00*** *(5.65)*
Exp. Return	-1.09 *(1.40)*

*OLS estimates for the constrained model (4.11). Dependent variable inverse of alpha (inverse of the listed unencumbered shares divided by the HFT); regressors: expected volatility (estimated with a GARCH model) and uncertainty (estimated with Italian VIX); returns of the Italian market index estimated on the basis of the same ARMA-GARCH model used for the expected volatility. Correction for autocorrelation of residuals with the Cochrane-Orcutt procedure. Standard errors in parenthesis. *** - significant at 1% level, ** - significant at 5% level; * - significant at 10% level. Number of observations: 107.*

It is interesting to note that in table 4.3, the expected return is not significant, though it has the expected sign. In fact, when the regulatory constraint is active, risk measures seem to have a greater impact on the investment decisions. Unlike the constrained alpha, the unconstrained scenario (regression (4.11)) has no significant variable up to a confidence level of 10%, and after the correction for autocorrelation of residuals, for the entire period. The results are reported in table 4.4. Therefore, for the entire sampled period, the constrained alpha framework helps to explain the relationships among the risk, uncertainty and banks' behaviour.

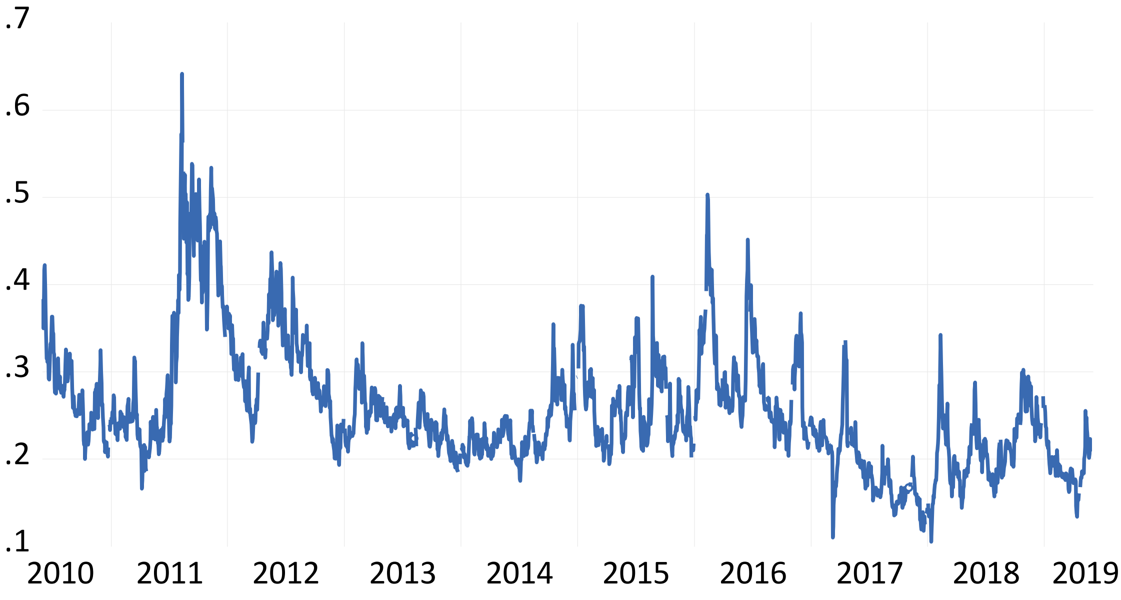
Table 4.4. Unconstrained scenario estimation – corrected for autocorrelation.

Variables	Coefficients
C	12.84*** (0.73)
Risk (expected volatility)	0.12 (0.15)
Uncertainty (Italian VIX)	-0.13 (0.12)

*OLS estimates for the unconstrained model (4.10). Dependent variable inverse of alpha (inverse of the listed unencumbered shares divided by the HFT); regressors: expected volatility (estimated with a GARCH model) and uncertainty (estimated with Italian VIX); returns used in (4.10) are estimated on the basis of the same ARMA-GARCH model used for the expected volatility. Correction for autocorrelation of residuals with the Cochrane-Orcutt procedure. Standard errors in parenthesis. - *** - significant at 1% level, ** - significant at 5% level; * - significant at 10% level. Number of observations: 107.*

From a graphical inspection of implied volatility (see figure 4.1) taken as a proxy of expected volatility, for the period under analysis I see that the first part of the time span has a higher volatility (in fact, it coincides with the period of higher financial turmoil) than the second one.

Fig. 4.1. Implied volatility of the Italian market index FTSE MIB.



Implied volatility for the Italian stock market, as measured by the index FTSE MIB IVI 30 *for the period 1 June 2010 – 31 May 2019; daily data. Source: Bloomberg.*

To identify the existence of a possible break in the series, I used Perron's (1989) unit root break method with innovative outlier (the break occurs gradually without unusual innovation, which differs from the additive outliers where breaks occur immediately) and a break only in the intercept (minimum Dickey-Fuller t-statistics). On the basis of this test, the break in the series is estimated to occur in September 2016. For the regression reported in table 4.3 the Chow breakpoint test also rejects the null of no break in September 2016.

Against this background, I assume that in the pre-break period (high volatility), the constrained formula is the one with the most significant impact of risk and uncertainty on investor decisions, while in the post-break period (less turbulent) I expect risk constraints to be less relevant since the market returns are less volatile. Quite interestingly, the results of the regression (table 4.5) show that in high-volatility periods the constrained alpha is significantly related to volatility-type variables (table 4.5, column a, variables risk and uncertainty). In this time window, uncertainty (significant at 5%) has a larger impact, in terms of magnitude, than expected volatility (significant at 1%). In low-volatility periods, the estimated coefficients for the variables of the constrained formula are always non-significant (table 4.5, columns b and d, variables risk and uncertainty). In addition, in these more stable periods, the uncertainty variable may even change sign (though the variable is not significant), as underlined in literature (table 4.5, columns b and d, variable uncertainty).

Table 4.5 Constrained and unconstrained scenarios, pre and post break

Variable	Constrained scenario		Unconstrained Scenario	
	Pre-break (1) (a)	Post-break (b)	Pre-break (1) (c)	Post-break (1) (d)
C	8.18***	10.56***	14.17***	9.95***
	(1.93)	(3.67)	(0.57)	(1.11)
Risk (expected volatility)	10.32***	2.13	-0.48	-6.93
	(3.90)	(11.65)	(3.03)	(10.18)
Uncertainty (Italian VIX)	12.59**	-4.68	-0.59	-0.30
	(6.10)	(15.95)	(0.61)	(0.53)
Exp. Return	-0.36	-4.57	-	-
	(1.39)	(4.99)	-	-

*OLS estimates for models (4.10) and (4.11). Dependent variable inverse of alpha (inverse of the listed unencumbered shares divided by the HFT); regressors: expected volatility (estimated with a GARCH model) and uncertainty (estimated with Italian VIX); returns of the Italian market index estimated on the basis of the same ARMA-GARCH model used for the expected volatility. Correction for autocorrelation of residuals with the Cochrane-Orcutt procedure. Break in September 2016. Standard errors in parenthesis. - *** - significant at 1% level, ** - significant at 5% level; * - significant at 10% level. (1) Corrected for autocorrelation of residuals. Number of observations: 76 for (a), 30 for (b), 75 for (c), 32 for (d).*

4.4.4 Tests of robustness

4.4.4.1 General framework of the robustness tests

In this section I evaluate the sensitivity of the results to the different choices made in the empirical tests performed in the previous section. More specifically, I substitute the return variable, which has been modelled on the basis of an ARMA model, with the return of the last day of every month, to check if the results obtained in previous sections were highly dependent on the modelling of the returns with the ARMA approach. Furthermore, I add a control variable related to sovereign risk to the regression. In fact, for Italy, in the period being analyzed, this risk (measured by the spread between the yields of Italian and German government bonds) affected the market volatility variable, which I use as the basis to measure general market risk. The variation of this spread affected market volatility through two channels. First, an increase in the spread caused a decrease to the value of the portfolio held by Italian banks (a significant part of it consists of Italian government bonds) and, for listed banks, of the value of their shares. This effect had an impact on market volatility of the market index, which in Italy is strongly dependant on banks' shares (in December 2013, almost one third of the market capitalization of the index was related to banks). An additional transmission of shock of the spread to market volatility was via risk management techniques (VaR). Since a significant portion of the portfolio held by banks consisted of BTP, a negative shock on BTP price, due to an increase of the spread, had an impact on VaR; consequently, in the period under analysis, banks may have decided to sell any type of risky financial assets to reduce their market risk exposure, so fuelling the market instability.

Therefore, to control for the quality of the market volatility variable as a measure of general market risk (net of the impact of sovereign risk), I added a variable measuring the spread between the yield of Italian and German government bonds.

Finally, given Bloom's (2009) criticism of the use of VIX as a measure of uncertainty (see Bekaert et al. 2013) I opted to use different measures of uncertainty. More explicitly, I use the variance risk premium (the difference between squared VIX and conditional volatility) which Drechsler (2013) finds to be related to uncertainty but Bekaert et al. associate to risk aversion. The use of the variance risk premium, interpreted as risk aversion, helps to control the robustness of the results obtained for the uncertainty for the risk aversion variable.

In the tests, I also used a measure of unexpected volatility, more related to the idea of uncertainty expressed by Jurado et al. (2015) and Maccheroni et al. (2013). In line with relevant literature, both theoretical and empirical (Maccheroni 2013, Jang and Park 2016, Puelz 2001, Peng et al. 2018, Bekaert and Hoerova 2014), in the examination of the impact on investment decisions, no idiosyncratic characteristics of investors are considered as control variables. In fact, from an economic point of view, when VaR binds because of the regulatory constraint, the need to sell risky assets appears to be independent from any individual investors' features. Furthermore, the main purpose of the analysis is the examination of the relation between the banking system as a whole and the financial market to assess possible macro-supervision implications, and is not the analysis of specific characteristics of banks which can affect market volatility.

4.4.4.2 Results of the robustness tests

In this section I perform some robustness tests, starting from checking the sensitivity of the results obtained in previous sections to the modelling of the returns. Hence, I substitute the expected return variable, calculated on the basis of the ARMA model, with the return for the last day of the month, where the return is calculated as the annualized daily return of the market index. The idea of using the latest information available to form expectations about the future variations of that variable is in line with the adaptive expectation hypothesis; with this different variable, volatility and uncertainty maintain a good level of significance in the pre-break period (table 4.6, column a, variables volatility and uncertainty).

Table 4.6. Constrained scenario – adaptive expected returns.

Variables	Coefficients	
	Pre-break (1)	**Post-break**
	(a)	**(b)**
C	10.23***	10.63***
	(1.94)	(3.52)
Risk (expected volatility)	9.79**	-5.52
	(4.04)	(9.6)
Uncertainty (Italian VIX)	12.45**	1.66
	(6.10)	(14.45)
Exp. Return	0.05	0.12
	(0.07)	(0.19)

*OLS estimates for model (4.11). Dependent variable inverse of alpha (inverse of the listed unencumbered shares divided by the HFT); regressors: expected volatility (estimated as with a GARCH model) and uncertainty (estimated with Italian VIX); returns of the Italian market index estimated on the basis of return of the last day of the month. Correction for autocorrelation of residuals with the Cochrane-Orcutt procedure. Break in September 2016. Standard errors in parenthesis. - *** - significant at 1% level, ** - significant at 5% level; * - significant at 10% level. (1) corrected for autocorrelation*
Number of observations: 74 for (a), 32 for (b).

As a further robustness test, I add to the regression an additional variable which has a specific impact on the Italian FTSE MIB index. This variable is the spread between the yield of the 10-year German government bonds and the yield of the 10-year Italian government bonds. This variable usually has a strong impact on the evolution of the Italian market index and expresses the sovereign risk included in the HFT portfolio of the banks. Since I am not interested in sovereign risk, but only in the impact of general market risk, I control for the impact of the spread variable. My expectation is that this variable is significant and has a positive effect on alpha (an increase of the spread should decrease the investment in Italian government bonds and, potentially, increase alpha), which in this regression on the inverse of alpha means a coefficient with a negative sign. In fact, table 4.7 shows that the spread is significant for the relations under analysis, and with the expected negative sign (table 4.7, column a, variable spread). Despite the introduction of a new variable, the main conclusions remain unchanged: in fact, volatility and uncertainty still have a significant impact on the investment decisions in the high-volatility period (table 4.7, column a, variables risk and uncertainty). It is interesting to note that in the low-volatility period, expected returns are significant, although at a low-level, thus confirming that in stable periods returns become a relevant variable (table 4.7, column b, variables expected returns).

Table 4.7. Constrained and unconstrained scenarios, pre and post break with sovereign risk

Variable	Constrained scenario		Unconstrained scenario	
	Pre-break (1) (a)	Post-break (b)	Pre-break (1) (c)	Post-break (1) (d)
C	10.10***	13.60**	16.45***	14.15***
	(1.94)	(5.09)	(1.09)	(2.69)
Risk (expected volatility)	11.36***	21.70	-0.54	-4.80
	(3.76)	(12.14)	(2.95)	(9.79)
Uncertainty (Italian VIX)	14.89**	-19.40	-0.50	-0.19
	(5.83)	(15.25)	(0.60)	(0.51)
Spread Ita-Ger gov't	-0.01***	-0.02	-0.01**	-0.03*
	(0.004)	(0.02)	(0.004)	(0.015)
Exp. Return	-0.06	-10.38*	-	-
	(1.33)	(5.20)	-	-

*OLS estimates for models (4.10) and (4.11). Dependent variable inverse of alpha (inverse of the listed unencumbered shares divided by the HFT); regressors: expected volatility (estimated with a GARCH model) and uncertainty (estimated with Italian VIX); returns of the Italian market index estimated on the basis of the same ARMA-GARCH model used for the expected volatility; sovereign risk measured as the spread of the yield of the Italian and German reference government bonds.. Correction for autocorrelation of residuals with the Cochrane-Orcutt procedure. Break in September 2016. Standard errors in parenthesis. - *** - significant at 1% level, ** - significant at 5% level; * - significant at 10% level*
Number of observations: 74 for (a), 32 for (b), 75 for (c), 32 for (d)
(1) Corrected for autocorrelation of residuals.

In addition, the spread variable is also significant in the unconstrained scenario (table 4.7, columns (c) and (d), variable spread). The sign of the regressor is the expected one, since an increase of the spread would cause a decrease of investment in risk-free assets, hence an increase of alpha (and therefore a decrease of the inverse of alpha, which is the dependent variable of the regression).

Finally, starting from the robustness test reported in table 4.7, I also check the stability of my results to other different measures of risk and uncertainty, as described in section 4.4.3.1. The variable representing risk in the model presented in this chapter comes from the representation of value at risk as $z[\sigma_t(R^p_{t+1})]$. A recent article (Slim et al. 2019) empirically tests that, for developed countries (and long trading positions), the incorporation of the variance risk premium into the GARCH model greatly enhances the accuracy of VaR in measuring market risk. Therefore, I enrich the measure of risk used in previous regression, which was based on GARCH, with a measure of variance risk premium that is the difference between implied volatility (as measured by the Italian VIX) and realized (ex post) volatility. In addition, variance risk premium may be interpreted as a risk aversion measure (Bekaert et al. 2013, Bollerslev and Marrone 2014); therefore, adding it to the regression helps to control for risk aversion in the results regarding uncertainty.

For the uncertainty variable, I consider the critiques of Bekaert et al. (2013), who underline that the VIX is a proxy that includes both uncertainty and risk aversion measures. Therefore, as explained in section 4.4.3.1, I use, as a measure of uncertainty, the difference between the realized (ex post) volatility at time *t* and the volatility that I expected (at time *t-1*) for time *t* (on the basis of the GARCH model). This unexpected volatility seems more in line with the Knightian concept of uncertainty and with the model uncertainty variable used by Maccheroni et al. (2013), since it represents the difference between the volatility that investors expected on the basis of their model and the true realization of volatility. The results of this regression, reported in table 4.8, confirm that risk and uncertainty are strongly significant and have the expected sign in the high-volatility period (table 4.8, column a, variables risk and uncertainty). Furthermore, in the stable period, uncertainty is significant, though at a low level, with the sign reversed with respect to the high-volatility period (table 4.8, column b, variable uncertainty). This means that an increase of uncertainty in a calm period may even determine an increase of risky investments, possibly to increase the return of the portfolio. In fact, the variable expected returns (of risky assets) is also significant (at a low level) with the correct sign (table 4.8, column b, variable expected return). These results appear to be in line with that literature (Puelz 2001) which supports the idea that in a low-volatility period there is an accumulation of potential risk, thus determining a greater loss for risk-constrained investors when losses occur.

Table 4.8. Constrained and unconstrained scenarios, pre and post break with sovereign risk. New measures of risk and uncertainty

Variable	Constrained scenario		Unconstrained scenario	
	Pre-break (1) (a)	Post-break (b)	Pre-break (1) (c)	Post-break (1) (d)
C	11.03***	12.85**	16.34***	14.89***
	(1.78)	*(4.67)*	*(1.08)*	*(2.67)*
Risk (expected volatility)	23.01***	0.92	1.18	-0.69
	(6.53)	*(15.80)*	*(4.31)*	*(13.96)*
Variance Risk Premium	18.57**	-5.76	-11.38	-6.72
	(8.78)	*(18.04)*	*(9.81)*	*(20.09)*
Uncertainty (unexp. volatility)	21.92**	-37.3*	-0.66	-10.21
	(8.67)	*(20.86)*	*(11.50)*	*(9.66)*
Spread Ita-Ger gov't	-0.01***	-0.02	-0.01**	-0.03*
	(0.004)	*(0.01)*	*(0.004)*	*(0.015)*
Exp. return	-0.66	-9.77*	-	-
	(1.44)	*(5.09)*	-	-

*OLS estimates for models (4.10) and (4.11). Dependent variable inverse of alpha (inverse of the listed unencumbered shares divided by the HFT); regressors: expected volatility (estimated with a GARCH model), Variance risk premium (computed as the difference between the Italian VIX and the realized volatility), uncertainty (estimated as the difference between the realized volatility at time t and the volatility expected at time t-1 for time t); returns of the Italian market index estimated on the basis of the same ARMA-GARCH model used for the expected volatility; sovereign risk measured as the spread of the yield of the Italian and German reference government bonds. Correction for autocorrelation of residuals with the Cochrane-Orcutt procedure. Break in September 2016. Standard errors in parenthesis. - *** - significant at 1% level, ** - significant at 5% level; * - significant at 10% level (1). Corrected for autocorrelation of residuals.*
Number of observations: 76 for (a), 32 for (b), 75 for (c), 32 for (d).

4.5 Conclusion

The financial crisis proved that modern risk management techniques cannot stop a crisis from happening. In addition, various authors have underlined that some of the risk management techniques, such as value at risk for market risk, may deepen the recession and, more generally, may be pro-cyclical. In the analysis of the crisis, the role of ambiguity and uncertainty has also been extensively discussed.

In this chapter, by assuming that investors use a constrained mean-variance model, augmented for uncertainty, when managing their trading portfolio, I analysed the impact of imposing VaR constraints to limit market risk exposure. I found that uncertainty is inversely related to the portion of wealth invested in risky assets. More explicitly, I leveraged the constrained maximization problem presented in chapter 3, where I found a solution for portfolio allocation of VaR-constrained investors operating under a mean-variance framework. The solution shows that when VaR constraint binds, investments in risky assets is lower than in an unconstrained environment, in line with some of literature. In addition, I found that when the constraint does not bind, the portion of wealth invested in risky assets is in line with the standard mean-variance result, as expected. I enriched this solution by incorporating the impact of ambiguity, as in Maccheroni et al. (2013); the results show that ambiguity is negatively related to risky investments.

Unlike several papers on the topic, I also performed some empirical tests, limited to Italy, and found that the VaR-constrained solution found in the theoretical part of the chapter explains risky investments of Italian banks better than the traditional, unconstrained mean-variance formula. More specifically, empirical tests for the Italian market (security market and Italian banking system) confirm the negative relations between uncertainty and risky investments which were highlighted by the theoretical framework.

Both the theoretical and the empirical results, although in line with relevant literature, are completely new. In fact, the inverse relation between uncertainty and risky investments had already been found in literature, but in a framework different from mean-variance and without the application of the VaR constraint. In addition, to the best of my knowledge, existent literature provides no empirical evidence on the relation between the portion of the portfolio invested in risky assets and several measures of risk and uncertainty. In this respect I used a unique dataset, based on details retrievable from supervisory reporting, to identify which part of the portfolio of the Italian banks is invested in risky assets. Furthermore, no similar analysis has previously been performed for Italy.

An additional new result coming from the empirical tests is that volatility and uncertainty have a strong impact on investment in risky assets exclusively in high-volatility periods. This behaviour does not emerge in low-volatility periods; hence, in relatively calm periods, the increase of volatility or uncertainty appears to have no significant negative impact on the portion of the portfolio invested in risky assets. This result seems consistent with the behaviour, already highlighted in literature, of accumulating risky assets in calm periods and it helps to explain the possible conundrum raised by the joint exam of the results obtained in chapter 2 and chapter 3. In fact, in chapter 2 I found empirical support for the amplification effect of VaR on market volatility. As in Basak and Shapiro (2001), this effect, could have been explained by the fact that regulation leads financial institutions to take on higher exposure to risky assets which causes an amplification of stock market volatility at times of down markets. However, in chapter 3, I found that risk limits decrease risky investments. This result provided no direct support to the amplification mechanism found in chapter 2. The results of chapter 4 connect the results of the previous chapters by finding that, as highlighted in chapter 3, investments in risky assets are lower, especially in high-volatility periods, when the risk limit binds. Hence the possible accumulation of risk in low-volatility periods may be the cause of the amplified effect in high-volatility periods when investors, as shown in chapter 3, have to reduce the portion invested in risky assets below the unconstrained level.

This result is also extremely interesting in terms of policy implications, since it confirms that risk limits accomplish their function of reducing risk exposure for investors when limits are binding, in turmoil, whereas they seem to be less effective in stable periods. Therefore, to avoid unintended amplifications of shocks, regulation should be complemented also by mechanisms of risk limitation to be activated in calm periods, which should go beyond those that were already implemented in the period examined in my analysis.

Chapter 5. Concluding remarks

5.1 Conclusions

Over the last two decades a new stream of literature analysing the effects of the regulatory constraints on the behaviour of market participants, and on the financial and banking system, has proliferated. Despite the significant progress made in this area, many findings are mixed and not supported by empirical evidence.

As a first step, this thesis searches for empirical evidence on the impact of risk limits on market fluctuations. In fact, the existence of such evidence, not proven in literature, is the cornerstone of all additional investigations, especially on further impacts of imposing a risk limit based on value at risk (VaR). Several papers have examined the impact of risk constraints on the market, with limited empirical evidence and mixed theoretical results. Chapter 2 of the thesis gives evidence of the relations between banking and financial systems, showing that market risk limits on banks Granger-causes (hence occurs before) market fluctuations. Such evidence, completely new in literature, provides a substantial contribution to the discussion related to a market (the Italian market) which is not as international as other European (or US) markets where domestic financial intermediaries may have a relevant impact on the local financial market.

However, such empirical evidence does not give information about the theoretical framework supporting the data dynamics examined. To complement the empirical analysis in chapter 2, I focused on the choices of investors in a mean-variance framework, constrained by a VaR risk limit. Such constrained optimization is based on the idea that regulation intervenes as a constraint over investment decisions based on mean-variance analysis. The theoretical result is then supported by empirical evidence which, by exploiting all the available data (using MIDAS regressions), show that behaviour of banks may be better forecasted by the constrained alpha formula than by the unconstrained one.

Finally, I complement the analysis with the possible impact of uncertainty on the model obtained in chapter 3. When uncertainty is also considered, as in chapter 4, evidence that investors behave like risk-constrained agents is confirmed; furthermore, there is evidence that in stable periods returns are significant, while in turbulent periods risk and uncertainty becomes relevant. These results concerning constrained optimization without and with uncertainty are new in literature and relevant to better understand the impact on the system of imposing a constraint on individual participants in the market.

The overall contribution is that the regulatory constraints influence not only the way individual banks invest but also financial markets and some market variables used to measure risk. Furthermore, such constraints may determine different investment behaviour in stable periods compared with turbulent periods. The results obtained are robust also from the empirical side as they are confirmed by the several different econometric techniques used.

5.2 The overall contribution of the thesis and future research.

The results obtained are very relevant for the topic under discussion and contribute to the existing literature, whose results are still mixed. In fact, the results reported in the chapters support, on a theoretical and an empirical side, the existence of unintended consequences of imposing risk limit measures based on market variables (such as value at risk), both on changing investors' behaviour and financial system fluctuations. The latter are extremely important since they may affect financial markets, the banking system and even some variables (e.g. market volatility) used to measure risks.

Hence, the thesis provides further evidence in support of the opportunity to revise the regulatory measures and constraints for market risk given that they may exacerbate financial crises, as suggested by some parts of the literature. In fact, the Basel Committee is moving in the direction of revising the market risk measures and limits. The Basel Committee on Banking Supervision (2019) has recently acknowledged that the design of the VaR metrics created incentives to hold positions that featured significant tail risks but were subject to limited risk in normal conditions. Therefore, to overcome the pre-existing system, the Basel

Committee has proposed using a new model based on the expected shortfall (ES), which should capture the tail risks that are not accounted for in the existing VaR measures. While current VaR calculates the losses at a single cut-off point in the distribution (e.g. 97.5%), ES looks at the average of the losses which exceed such a cut-off point. Hence, the ES seems to rely on a cut-off point, which can be the current VaR, so VaR could stay as a crucial variable of the risk constraining framework.

Such possible, future evolution of the regulation, which maintains the risk-limit approach based on the distribution of empirical data, further confirms the relevance of the topics of the thesis and its results, focused on the unintended effect of risk limits on market variables (e.g. volatility) and on investors' behaviour.

From the results of thesis, the goal of risk limits (i.e. reducing investment in risky assets) seems to have been reached, though at a cost of impacting the market volatility, possibly exacerbating financial crises. Hence, the above-mentioned innovations in regulation, by keeping the risk-limit approach, should still achieve the goal of diminishing risky investments. However, as the thesis show, the goal is better achieved when market volatility is high, while in low-volatility periods the impact does not seem in line with the regulation ambitions.

Therefore, from the results of the research done here, two additional issues seem to still need to be tackled by regulation: the impact of risk limits on financial systems, hence the relations between individual regulation and the financial system; and the effects of such limits in periods of transition from low-volatility to high-volatility periods.

Hence, the results of the thesis provide more solid ground to build further directions of research on.

In particular, further theoretical analysis may be done on the results of chapter 3 to model the passage from constrained to unconstrained alphas. Furthermore, it would be interesting to leverage on the results obtained in chapter 4, where two different investment behaviours emerge for stable or turbulent periods, to investigate the impact on the market of the passage between the two regimes of the market, both on the theoretical and empirical sides

Finally, further theoretical and empirical analysis could be done to investigate the interactions between individual-based regulation and the macro-prudential behaviour of supervision authorities.

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