Congestion-Balanced and Welfare-Maximized Charging Strategies for Electric Vehicles

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Abstract—With the increase of the number of electric vehicles (EVs), it is of vital importance to develop the efficient and effective charging scheduling schemes for all the EVs. In this article, we aim to maximize the social welfare of all the EVs, charging stations (CSs) and power plant (PP), by taking into account the changing demand of each EV, the changing price, the capacity and the congestion balance between different CSs. To this end, two efficient scheduling algorithms, i.e., Centralized Charging Strategy (CCS) and Distributed Charging Strategy (DCS) are proposed. CCS has a slightly better performance than the DCS, as it takes all the information and make the decision in the central control unit. On the other hand, DCS dose not require the private information from EVs and can make decentralized decision. Extensive simulation are conducted to verify the effectiveness of the proposed algorithms, in terms of the performance, congestion balance, and computing complexity.

Index Terms—Social welfare maximization, congestion balance, charging strategy, electric vehicle

1 Introduction

With the increase of greenhouse effect, many countries have set policies and developed several projects to improve the penetration of EVs in their daily lives. In the past ten years, the global stock of battery electric vehicles (BEVs) has passed more than 5 million, with the growth rate 63 percent from the previous year [1]. It is foreseen that the number of EVs will break through 200 million in 2030. It is therefore of vital importance to design the effective and efficient scheduling algorithm for EVs to find the suitable charging station, meanwhile increase their satisfaction and reduce the congestion.

Let us first consider the charging scenario in Fig. 1, where several EVs need to be charged at the same time, but there are only 4 charging stations (CSs) available. We assume there are a central unit (CU) to make the scheduling decision and one power plant (PP) to generate electricity for all the charging stations. One can see that if the charging decision is not made properly, congestion will happen between different CSs and result in the following situations:

- Unbalanced service time of charging stations: In general, the CSs with the heavy charging load cost more time for charging all the queued EVs compared to the CSs with less charging load.
- Wasting of resources: Unbalanced service time causes some CSs overloaded and others underutilized in the long term, which wastes the charging resources for all the EVs.
- Additional investment: Unbalanced charging load among CSs may result in the administrative department to build more CSs or expand the capacity of existing CSs to avoid congestion.

Recently, although direct current (DC) fast charging technology can help complete the charging demand in 30 minutes [3], which decreases charging time for the EVs, it cannot address the unbalanced charging congestions among the CSs. It is therefore of great importance to design the effective changing strategy to balance charging demand and maximize the overall utility function of EVs, CSs and PP, by taking into consideration the changing demand of each EV, the charging price, the capacity and congestion balance between different CSs. The main contributions of this paper are:

- We first define the congestion equation for each CS, and then give the utility functions of all the EVs, CSs and PP. Next, the social welfare maximization are proposed, by taking into consideration of changing demand of each EV, the price, the capacity and congestion balance between different CSs. Then, we present the centralized charging strategy (CCS) and the distributed charging strategy (DCS) to address the proposed problem.
- In CCS, the optimization problem is divided into two parts. In the first part, EVs are distributed to the CSs in a centralized way, which can balance the congestions among different CSs and meanwhile minimize the driving cost between each EV and CS. In the second part, all the charging capacities and power supply are optimized in closed-form by using Lagrangian dual method.

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Manuscript received 6 June 2018; revised 4 June 2020; accepted 13 June 2020. Date of publication 0 . 0000; date of current version 0 . 0000. (Corresponding authors: Kezhi Wang and Kun Yang.) Recommended for acceptance by jianfeng Zhan. Digital Object Identifier no. 10.1109/TPDS.2020.3003270

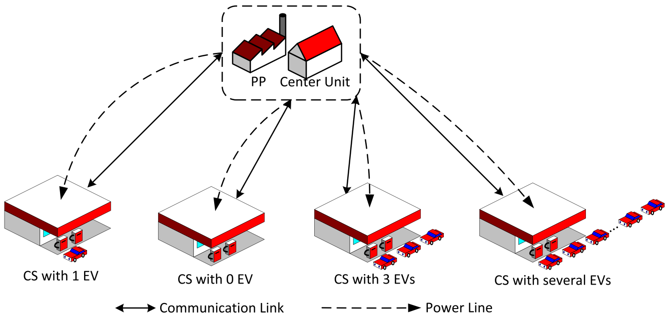

Fig. 1. Unbalanced charging problem.

- In DCS, two stages are proposed. In the first stage, Each EV obtains the updated information from the CSs in real-time, and then selects the best CS, which can not only balance congestions among the CSs, but also minimize the driving distance. In the second stage, a distributed method is proposed to optimize the charging demands, charging capacities and power supply from PP.
- We compare our proposed solutions with the benchmark schemes, including exhaustive search strategy, cross entropy method and multi-agent game solution. We show the advantages of our proposed CCS and DCS from several aspects, i.e., performance, congestion balance and execution time.

The remaining of this paper is organized as follows. Related work is reviewed in Section 2. In Section 3, system model and optimization problem are introduced. In Section 4, the centralized strategy, i.e., CCS is presented. In Section 5, the distributed strategy, i.e., DCS is proposed. Section 6 shows the performance evaluation, followed by Section 7, where we summarize the whole work.

2 Related Work

For EV charging scheme, some research applied charging load to adjust the operation status of power grid, where several indexes of power grid can be optimized, such as smooth load curve [4], power grid frequency, power grid voltage as well as improving uncertainty of power grid operation [5]. However, the above work mainly focused on the time dimension for scheduling EVs, instead of the spatial dimension. From the perspective of EVs, it is important for them to decide and choose the best CSs to charge. There are some other research which studied the scheduling problem in spatial dimension.

In [6], a mechanism was proposed by Bayram *et al.* to schedule the charging behaviours of EVs to avoid the congested CSs. The Stackelberg game was applied to balance the charging requirements. In [7], an intention-aware routing system was presented by Weerdt *et al.* to predict the queuing time in order to reduce the expected journey time for the EVs. In [8], a dynamic pricing strategy was proposed by Xu *et al.* to reduce the queuing delay of EVs at the CSs, but no specific optimization model was put forwarded in this paper. In [9], a strategy was introduced by Malik *et al.* to minimize the queuing time of EVs at the CS, but the congestion balance among different CSs were not considered. In [10], Moghaddam *et al.* presented a smart charging strategy for multiple options for the EV to minimize the charging time, travel time and charging cost. Cao *et al.* in [11] predicted the charging availability of CSs, and EVs' charging requests can be reserved at the specific CS recommended by the controller. Laha *et al.* proposed a game theory method for EVs to select the CSs with the consideration of locations [12], and by selecting the CS with appropriate price and distance, the charging cost of EVs can be minimized. Liu *et al.* in [13] studied a deep reinforcement learning based solution to scheduling the charging behaviours of EVs. The total overhead of EVs including time and charging fares was optimized. In [14], a smart energy management framework was proposed by Zhou *et al.* to reduce the charging cost and improve the quality of service of EV charging. In [15], Tang *et al.* proposed a smart charging strategy to minimize the average charging time of CSs with the assistance of discharging some EVs. Ammous *et al.* in [16] proposed a charging route optimization scheme, which jointly minimized the transit time and charging cost of the EVs. Li *et al.* in [17] proposed a charging navigation routing strategy based on V2V. By optimizing the route and staying position of charge and discharge EVs, the pairing of charge and discharge EVs can be formed.

In addition to optimizing the utility function for EVs, researchers also proposed to optimize the activities for CSs. In [18], stochastic queuing models were employed by Wong *et al.* for the network of public CSs, and with the introduction of some appropriate charging fees, the charging network can achieve the socially optimal congestion balance. Mohsenian-Rad *et al.* in [19] formulated the stochastic optimization problem of time-of-use pricing to study how uncertain departure time can affect the charging schedule of EVs. Lee *et al.* in [20] studied a price competition problem among the CSs with renewable power generators by using the game theory. Zhang *et al.* in [21] proposed a dynamic programming framework to obtain an optimal charging strategy for the EVs at the parking-lots with consideration of the stochastic arrival and departure time of the EVs. In order to maximize the utility of CSs, an online and model-free reinforcement learning method was proposed by Wang *et al.* in [22], which makes the profit of CSs achieve 138 percent higher than the benchmark algorithms. In [23], to maximize the CSs' profit, a tandem queuing network model was proposed by Wang *et al.* to optimize EVs' admission control, pricing and charging scheduling for CSs. In [24], Faridimehr *et al.* proposed a two-stage stochastic programming model to optimize the charging network of CSs, and also a sample average approximation method was adopted to solve this problem for large-scale instances. In [25], an optimal pricing scheme was studied by Zhang *et al.* to minimize the dropping rate of the charging service, where the CSs were modelled as queuing network with multiple servers and heterogeneous service rates.

Some researchers also studied the optimization of the social utility (also known as social welfare), which focused on maximizing the utility of all the participating entities. In [26], Tucker *et al.* proposed an online pricing mechanism to reserve park and charge spot for the EVs. Alinia *et al.* in [27] studied the charging scenario with limited charging capacity of the CSs and uncertainty of the arrivals of the EVs. The social welfare maximization problem was formulated and

TABLE 1
Notations

Symbol	Description
N, M	the number of EVs and CSs, respectively.
S_N, S_M	the sets of $\{1, 2, \ldots, N\}$ and $\{1, 2, \ldots, M\}$, respectively.
r_i	the charging satisfaction weight of EV$_i$.
m	the parameter of congestion weight of CS.
β	average energy consumption β kWh for 1 km.
p_{last}	last charging price.
x_i	the charging demand of EV$_i$.
s_{ij}	the decision variable of EV$_i$ selecting CS$_j$.
d_{ij}	Manhattan distance from EV$_i$ to CS$_j$.
L^{pp}	power generation capacity of PP.
p^{pp}	electricity price of PP for 1 kWh.
a, b, c	parameters of electricity generation cost.
L_j^{cs}	the charging load capacity of CS$_j$.
p_j^{cs}	the charging price of CS$_j$ for 1 kWh.
N_j^{pile}	the number of charging piles of CS$_j$.
N_j	the number of EVs selecting CS$_j$.
$\mathbf{L^{cs}}$	the vector of $\left\{L_j^{cs} \vert j \in S_M\right\}$.
$\mathbf{x}$	the vector of $\{x_i \vert i \in S_N\}$.
$\mathbf{s}$	the $M \times N$ matrix contains all the variables s_{ij}.
$\mathbf{p^{cs}}$	the vector of $\left\{p_j^{cs} \vert j \in S_M\right\}$.

solved approximately. In [28], Wang *et al.* researched a four-stage charging game of EVs in a smart community and all the parties in the energy network can obtain their optimal strategies. In [29], the charging network consisting of public CSs with different service options were proposed by Moradipari *et al.* By assigning EVs to the best CSs, the social welfare objective function was optimized.

There are also other related issues were considered, such as safety and V2G (vehicle to grid) networks, etc. In [30], Zhou *et al.* proposed a secure V2G energy trading framework, and based on which, the EVs' charging scheme was implemented. In [31], Zhou *et al.* researched the demand response mechanism for EVs' networks, where the energy trading among EVs was kept safe by a consortium blockchain. Additionally, in [32], Yu *et al.* studied the energy network of EVs, which used V2G technology to supply power to multiple districts and showed that the mobility of the symmetrical EVs' energy network can balance the power demand of different districts.

By analysing the above research work, we did not find the studies that considered the social warfare utility of all the EVs, CSs as well as PP, and meanwhile taking the congestion between CSs into account, to design the scheduling schemes for all the EVs. Next, we will show the proposed problem and the solutions.

3 System Model

In this section, we introduce the system model. We first summarize the main notations in Table 1.

3.1 Congestion Definition

We assume one charging pile can only charge one EV at a time. Then, the congestion rate of the charging station for the jth CS can be defined as:

$$con_j = \frac{N_j - N_j^{pile}}{N_j}, \tag{1}$$

where the above equation can be explained as: 1) when $N_j \geq N_j^{pile}$, the con_j is the probability of congestion, and 2) when $N_j < N_j^{pile}$, the con_j is a value that indicating the degree of no queuing chance. One can also see that $con_j \in (-\infty, +1)$ with N_j varying from 0 to $+\infty$.

Since the congestion or queuing affects the waiting time of EVs, we can define the weighting coefficient ρ_j as follows:

$$\rho_j = \frac{2}{1 + \frac{N}{\sum_{j=1}^{M} N_j^{pile}}\left(1 - con_j\right)} - 1, \tag{2}$$

where $\frac{N}{\sum_{j=1}^{M} N_j^{pile}}$ is a constant and shows the average number of EVs on each charging pile. According to (1), one can have $1 > con_j \geq -\infty$, then one can also have $\rho_j \in (-1, +1)$. After some manipulation, one can further obtain:

$$\rho_j = \frac{\frac{\sum_{i=1}^{N} s_{ij}}{N_j^{pile}} - \frac{N}{\sum_{j=1}^{M} N_j^{pile}}}{\frac{\sum_{i=1}^{N} s_{ij}}{N_j^{pile}} + \frac{N}{\sum_{j=1}^{M} N_j^{pile}}}, \tag{3}$$

where $\sum_{i=1}^{N} s_{ij}$ is the number of EVs that is charged on the jth charging station.

3.2 Utility Functions

The utility function of PP can be defined as:

$$U^{pp} = p^{pp} \sum_{j=1}^{M} L_j^{cs} - \left(a(L^{pp})^2 + bL^{pp} + c\right), \tag{4}$$

where $p^{pp} \sum_{j=1}^{M} L_j^{cs}$ is the revenue from selling $\sum_{j=1}^{M} L_j^{cs}$ kWh electricity at the price p^{pp}; $a(L^{pp})^2 + bL^{pp} + c$ is the cost function of power generation, which is widely used in the literatures [33], [34].

Then, the utility function of jth CS can be defined as:

$$U_j^{cs} = p_j^{cs} \sum_{i=1}^{N} s_{ij} x_i - p^{pp} L_j^{cs}, \tag{5}$$

where $p_j^{cs} \sum_{i=1}^{N} s_{ij} x_i$ is the revenue from charging $\sum_{i=1}^{N} s_{ij} x_i$ kWh at the charge rate of p_j^{cs}; $p^{pp} L_j^{cs}$ is the cost of purchasing L_j^{cs}kWh from the power plant at the price p^{pp}.

Then, the utility function of ith EV can be defined as:

$$U_i^{ev} = \sum_{j=1}^{M} \left(s_{ij}\left((m - \rho_j) r_i \ln x_i - p_j^{cs} x_i - p_{last} d_{ij} \beta\right)\right). \tag{6}$$

The above U_i^{ev} includes three parts, which can be explained as follows:

Part 1: $(m - \rho_j) r_i \ln x_i$ is a weighted charging satisfaction function. The term $\ln x_i$ has been widely used in the literatures [5], [35]. Although it is different from some literature like [33], $\ln x_i$ satisfies the two properties in [33]: 1) utility function is non-decreasing; 2) marginal profit is non-increasing. Also, r_i represents the weight of each user regarding the charging satisfaction. Different users may have different weights because they have different views on charging. In addition, $(m-\rho_j)$ represents the congestion weight of the jth charging station. Larger con_j also leads to bigger ρ_j, and then result in the smaller congestion weight as well as the lower charging satisfaction. One can see that the above definition is also consistent with the real situation. This is because if the number of EVs in the charging station is larger, the waiting time for charging is longer, thus may decrease the charging satisfaction. Also, parameter m represents the congestion weight dilution factor. The larger m may lead to smaller difference of charging satisfaction caused by congestion.

Part 2: $p_j^{cs} x_i$ is the cost of charging x_ikWh at price p_j^{cs}.

Part 3: $p_{last} d_{ij} \beta$ is the cost of driving distance of d_{ij}; β is the power consumption per distance unit and p_{last} is the charging price of last time, which is set fixed for all the EVs for simplicity.

3.3 Problem Formulation

We formulate the social welfare maximization problem as follows:

$$
\begin{aligned}
\mathbf{P1} \quad & \max_{L^{pp}, p^{pp}, \mathbf{L^{cs}}, \mathbf{s}, \mathbf{x}, \mathbf{p^{cs}}} \quad U^{pp} + \sum_{j=1}^{M} U_j^{cs} + \sum_{i=1}^{N} U_i^{ev} \\
s.t. \quad & \mathrm{C1}: \quad L^{pp} \geq \sum_{j=1}^{M} L_j^{cs}, \\
& \mathrm{C2}: \quad L_j^{cs} \geq \sum_{i=1}^{N} s_{ij} x_i, \ j \in S_M \\
& \mathrm{C3}: \quad \sum_{j=1}^{M} s_{ij} = 1, \ i \in S_N \\
& \mathrm{C4}: \quad s_{ij} \in \{0, \ 1\}, \ j \in S_M; \ i \in S_N \\
& \mathrm{C5}: \quad x_i^{max} \geq x_i \geq x_i^{min}, \ i \in S_N \\
& \mathrm{C6}: \quad p_j^{cs} \geq 0, \ p^{pp} \geq 0, \ j \in S_M,
\end{aligned}
$$

where C1 means the total capacity of power plant L^{pp} should be larger than the total requirement of all the CSs; C2 denotes that the total charging capacity of the jth charging station of L_j^{cs} should be larger than all the charging demands of EVs choosing this charging station; C3 means that each EV selects only one CS for charging; C4 shows that the charging decision variable s_{ij} is the binary variable; C5 is the constraint for charging demand of each EV; C6 means that the charging price p_j^{cs} and the electricity price p^{pp} should be non-negative.

By applying (4), (5) and (6) into **P1**, one can have:

$$
\begin{aligned}
\mathbf{P2} \quad & \min_{L^{pp}, \mathbf{s}, \mathbf{x}} \quad \left(a (L^{pp})^2 + b L^{pp} + c \right) \\
& - \sum_{i=1}^{N} \left(\sum_{j=1}^{M} \left(s_{ij} \left((m - \rho_j) r_i \ln x_i - p_{last} d_{ij} \beta \right) \right) \right) \\
s.t. \quad & \mathrm{C3}, \ \mathrm{C4}, \ \mathrm{C5}, \ \mathrm{C7} \ ,
\end{aligned}
$$

where the constraint C6 is removed. As the variable L_j^{cs} is not shown in the objective function, one can apply constraint C7 according to C1 and C2 as follows:

$$
L^{pp} \geq \sum_{j=1}^{M} \sum_{i=1}^{N} s_{ij} x_i. \tag{7}
$$

One can see that **P2** is a MINLP and is difficult to tackle in general. Although some literatures have proposed standard methods [36], such as branch and bound method, these algorithms have very high complexity, especially in the large-scale scenario. Next, we will introduce centralized solution, i.e, CCS as well as decentralized solutions, i.e., DCS.

4 Centralized Charging Strategy

4.1 Charging Station Selection

In this subsection, we first tackle the integer variables $\{s_{ij} | i \in S_N, j \in S_M\}$ in **P2**. We define two functions to express the objective of **P2** as follows:

$$
f_1(s_{ij}) = \sum_{i=1}^{N} \sum_{j=1}^{M} \left(s_{ij} (m - \rho_j) r_i \ln x_i \right), \tag{8}
$$

and

$$
f_2(s_{ij}) = -\sum_{i=1}^{N} \sum_{j=1}^{M} \left(s_{ij} p_{last} d_{ij} \beta \right). \tag{9}
$$

Next, we introduce methods to maximize $f_1(s_{ij})$ and $f_2(s_{ij})$ respectively. Note that we use f_1 and f_2 to denote $f_1(s_{ij})$ and $f_2(s_{ij})$ for simplicity, respectively.

4.1.1 Maximization of f_1

We first analyse the properties of f_1 as follows.

Lemma 1. *In order to maximize f_1, every EV should select the CS which can maximize contribution to the increase of f_1.*

Proof. It is obvious that EV$_i$ should select the CS, which can produce an maximal increment $\Delta f_1^{(i)}$ of f_1. Then, one can have the maximum value of f_1 as $f_1^{max} = \sum_{i=1}^{N} \max \Delta f_1^{(i)}$. □

Lemma 2. *In order to maximize the $\Delta f_1^{(i)}$, the EV$_i$ should select the CS with the minimal N_j / N_j^{pile}.*

Proof. Assume the i-th EV selects the j-th CS, where it already has N_j EVs here. Then the parameter ρ_j can be given as:

$$
\rho_j = \left(1 - \frac{\frac{2N}{\sum_{j=1}^{M} N_j^{pile}}}{\frac{N_j}{N_j^{pile}} + \frac{N}{\sum_{j=1}^{M} N_j^{pile}}} \right). \tag{10}
$$

According to (8), the i-th EV should select the CS with the minimal ρ_j to maximize $\Delta f_1^{(i)}$. Then, one can see that we can select CS with the minimal N_j / N_j^{pile} to get the maximal $\Delta f_1^{(i)}$. □

Lemma 3. *In the CS selection process, the difference between the maximum and minimum value of $\left\{ N_j / N_j^{pile} | j \in S_M \right\}$ should be less than $1 / N_{jmin}^{pile}$, where N_{jmin}^{pile} is the minimal number of charging piles among all the CSs.*

Proof. Without loss of generality, we sort the numbers of charging piles as $N_1^{pile} \leq N_2^{pile} \leq \ldots \leq N_M^{pile}$. Then we have $N_{jmin}^{pile} = N_1^{pile}$.

If the first EV selects the CS, it will select the M-th CS according to **Lemma 2**. Then we have the order as:

$$\frac{0}{N_1^{pile}} \leq \frac{0}{N_2^{pile}} \leq \ldots \leq \frac{1}{N_M^{pile}}. \tag{11}$$

One can find

$$\frac{1}{N_M^{pile}} - \frac{0}{N_1^{pile}} \leq \frac{1}{N_{jmin}^{pile}}. \tag{12}$$

If the second EV selects the CS, one can get the same conclusion as above.

Assume that the k-th EV selects the CS and then we have the order as:

$$\frac{N_1}{N_1^{pile}} \leq \frac{N_2}{N_2^{pile}} \leq \ldots \leq \frac{N_M}{N_M^{pile}}, \tag{13}$$

and we also have

$$\frac{N_M}{N_M^{pile}} - \frac{N_1}{N_1^{pile}} \leq \frac{1}{N_{jmin}^{pile}}. \tag{14}$$

Then if the $(k+1)$th EV selects the CS, there exist two situations:

(a) if the condition:

$$\frac{N_1}{N_1^{pile}} + \frac{1}{N_1^{pile}} > \frac{N_M}{N_M^{pile}}, \tag{15}$$

is satisfied, the sorting is changed and one can have:

$$\frac{N_1}{N_1^{pile}} + \frac{1}{N_1^{pile}} - \frac{N_2}{N_2^{pile}} < \frac{1}{N_{jmin}^{pile}}. \tag{16}$$

(b) if the condition:

$$\frac{N_1}{N_1^{pile}} + \frac{1}{N_1^{pile}} \leq \frac{N_M}{N_M^{pile}}, \tag{17}$$

is satisfied, one can also have:

$$\frac{N_M}{N_M^{pile}} - \frac{N_2}{N_2^{pile}} < \frac{1}{N_{jmin}^{pile}}. \tag{18}$$

Therefore, the **Lemma 3** is proved. □

Theorem 1. *If $\left\{s_{ij}|i \in S_N, j \in S_M\right\}$ is determined, the values of $\left\{N_j/N_j^{pile}|j \in S_M\right\}$ is bounded by:*

$$T = \left[\frac{N}{\sum_{j=1}^M N_j^{pile}} - \frac{1}{N_{jmin}^{pile}}, \frac{N}{\sum_{j=1}^M N_j^{pile}} + \frac{1}{N_{jmin}^{pile}}\right], \tag{19}$$

and the optimal value of N_j/N_j^{pile} is close to $\frac{N}{\sum_{j=1}^M N_j^{pile}}$.

Proof. Without loss of generality, we assume the order of N_j/N_j^{pile} as:

$$\frac{N_1}{N_1^{pile}} \leq \frac{N_2}{N_2^{pile}} \leq \ldots \leq \frac{N_M}{N_M^{pile}}, \tag{20}$$

according to which, one can also get the following inequality:

$$\frac{N_1}{N_1^{pile}} \leq \frac{\sum_{j=1}^M N_j}{\sum_{j=1}^M N_j^{pile}} \leq \frac{N_M}{N_M^{pile}}. \tag{21}$$

In the **Lemma 3**, we have:

$$\frac{N_1}{N_1^{pile}} - \frac{N_M}{N_M^{pile}} \leq \frac{1}{N_{jmin}^{pile},} \tag{22}$$

then one can further get:

$$\frac{N}{\sum_{j=1}^M N_j^{pile}} - \frac{1}{N_{jmin}^{pile}} \leq \frac{N_1}{N_1^{pile}}, \tag{23}$$

and

$$\frac{N}{\sum_{j=1}^M N_j^{pile}} + \frac{1}{N_{jmin}^{pile}} \geq \frac{N_M}{N_M^{pile}}, \tag{24}$$

where $N = \sum_{j=1}^M N_j$. Then we can prove that N_j/N_j^{pile} is bounded by (19). In addition, when N_{jmin}^{pile} and the number of EVs N is large, one can have:

$$\left(\frac{N}{\sum_{j=1}^M N_j^{pile}} - \frac{1}{N_{jmin}^{pile}}\right) \approx \left(\frac{N}{\sum_{j=1}^M N_j^{pile}} + \frac{1}{N_{jmin}^{pile}}\right), \tag{25}$$

which means the values of $\left\{N_j/N_j^{pile}|j \in S_M\right\}$ equal to each other, and is close to $\frac{N}{\sum_{j=1}^M N_j^{pile}}$ for all the CSs. In this case, it is also found that the congestion con_j between each CS can be also balanced, which is consistent to the real-world scenario. □

4.1.2 Maximization of f_2

To maximize f_2, one can see that all the EVs should select the nearest CSs to reduce the travelling distance to the CSs. However, if all the EVs select the nearest CSs, the maximal value of f_1 is affected. In order to maximize both f_1 and f_2, the following heuristic algorithm is proposed.

4.1.3 Heuristic Algorithm

To maximize the f_1 and f_2, the values of $\left\{N_j/N_j^{pile}|j \in S_M\right\}$ should comply with the **Theorem 1**, and the sum of all the travelling distances for all the EVs should also be minimized. Then, the heuristic algorithm is presented in **Algorithm 1**.

In line 1 of **Algorithm 1**, all the distances are sorted by ascending order, and the value of $N/\sum_{j=1}^M N_j^{pile}$ is calculated. From line 2 to line 8, according to the order of distance, each EV is assigned to a suitable CS. For each CS, its N_j should be less than the $Nearestinteger(Ave \cdot N_j^{pile})$, where the $Nearestinteger(z)$ is a function to get the nearest integer of z. From line 9 to line 21, for the EVs which do not select any CS will be assigned to the suitable CSs in the end.

4.2 Optimal Charging Demand

When the integer variable s is determined, **P2** becomes the convex problem w.r.t L^{pp} and x_i as

$$\begin{aligned}\textbf{P3}\quad &\min_{L^{pp},\mathbf{x}}\ \Big(a(L^{pp})^2+bL^{pp}+c\Big)\\&-\sum_{i=1}^{N}\left(\sum_{j=1}^{M}\big(s_{ij}\big((m-\rho_j)r_i\ln x_i-p_{last}d_{ij}\beta\big)\big)\right)\\&s.t.\ \ \text{C5, C7}\end{aligned}$$

P3 can be solved by applying the interior point method with the help of CVX toolbox [37]. However, the complexity of the above method may be high. Next, we obtain the closed-form solution by applying the Lagrangian method as follows

$$\begin{aligned}&\mathcal{L}_{x_i^{min}\le x_i\le x_i^{max}}(L^{pp},x_i,\lambda)=\Big(a(L^{pp})^2+bL^{pp}+c\Big)-\\&\sum_{i=1}^{N}\left(\sum_{j=1}^{M}\big(s_{ij}\big((m-\rho_j)r_i\ln x_i-p_{last}d_{ij}\beta\big)\big)\right)+\\&\lambda\left(\sum_{j=1}^{M}\sum_{i=1}^{N}s_{ij}x_i-L^{pp}\right).\end{aligned}\tag{26}$$

One can see that solving **P3** is equivalent to minimizing $\mathcal{L}(L^{pp},x_i,\lambda)$. By taking the first derivative with respect to L^{pp} and x_i, and equate the results to zero, one can get

$$L^{pp*}=\frac{\lambda-b}{2a}.\tag{27}$$

and

$$x_i^*=\begin{cases}x_i^{min},\ if\ \frac{(m-\rho_j)r_i}{\lambda}<\ x_i^{min}\\ \frac{(m-\rho_j)r_i}{\lambda},\ if\ \frac{(m-\rho_j)r_i}{\lambda}\in\big[x_i^{min},\ x_i^{max}\big]\\ x_i^{max},\ if\ \frac{(m-\rho_j)r_i}{\lambda}>\ x_i^{max}\end{cases}.\tag{28}$$

We put the EVs whose charging demands are the boundary values such as x_i^{min} or x_i^{max} into the set $S_{N'}$. Then, we further put (27) and (28) into (26), and the dual function of $\mathcal{L}(L^{pp},x_i,\lambda)$ can be obtained as

$$\begin{aligned}&g(\lambda)=\left(\frac{-\lambda^2+2\lambda b-b^2}{4a}+c\right)-\\&\sum_{i=1,i\notin S_{N'}}^{N}\left(\sum_{j=1}^{M}\Big(s_{ij}\Big(u\ln\frac{u}{\lambda}-p_{last}d_{ij}\beta\Big)\Big)\right)-\\&\sum_{i=1,i\in S_{N'}}^{N}\left(\sum_{j=1}^{M}\big(s_{ij}\big(u\ln x_i-p_{last}d_{ij}\beta\big)\big)\right)+\\&\left(\lambda\sum_{j=1}^{M}\sum_{i=1,i\in S_{N'}}^{N}s_{ij}x_i+\sum_{j=1}^{M}\sum_{i=1,i\notin S_{N'}}^{N}s_{ij}u\right),\end{aligned}\tag{29}$$

where $u=(m-\rho_j)r_i$ and $g(\lambda)$ is a concave function. By taking the first derivative of the above equation and equate the result to zero, one can get

$$\begin{aligned}\frac{\partial g(\lambda)}{\partial\lambda}&=\lambda^2-\lambda b-2\lambda a\left(\sum_{j=1}^{M}\sum_{i=1,i\in S_{N'}}^{N}s_{ij}x_i\right)\\&-2a\sum_{i=1,i\notin S_{N'}}^{N}\sum_{j=1}^{M}\big(s_{ij}(m-\rho_j)r_i\big)=0.\end{aligned}\tag{30}$$

Then, we have:

$$\lambda^*=\frac{-h\pm\sqrt{h^2-4k}}{2},\tag{31}$$

where h is:

$$h=-b-2a\left(\sum_{j=1}^{M}\sum_{i=1,i\in S_{N'}}^{N}s_{ij}x_i\right),\tag{32}$$

and k is

$$k=-2a\sum_{i=1,i\notin S_{N'}}^{N}\left(\sum_{j=1}^{M}\big(s_{ij}(m-\rho_j)r_i\big)\right).\tag{33}$$

According to the function $\frac{\partial g(\lambda)}{\partial\lambda}$, one can see that if $\lambda=0$, $\frac{\partial g(\lambda)}{\partial\lambda}\le 0$, which means the optimal value of λ is non-positive. This violates the range constraint of Lagrange multipliers. Then the optimal value of λ can be obtained as

$$\lambda^*=\frac{-h+\sqrt{h^2-4k}}{2}.\tag{34}$$

Based on the above analysis, we propose the algorithm to solve **P3** as **Algorithm 2**.

Algorithm 1. CS Selection Algorithm

Input: $DIS=\{d_{ij}|i\in S_N,j\in S_M\}$, $\{N_j^{pile}|j\in S_M\}$
Output: s
sort DIS by ascending and let $Ave\leftarrow N/\sum_{j=1}^{M}N_j^{pile}$;
for $k=1$ to $N\cdot M$ **do**
  get EV id i and CS id j of $DIS(k)$;
  if $\sum_{i=1}^{N}s_{ij}$ < *Nearest integer*$(Ave\cdot N_j^{pile})$ **then**
    $s_{ij}\leftarrow 1, s_{ip}\leftarrow 0,\forall p\in S_M,p\neq j;$
    break;
  end
end
if $\sum_{j=1}^{M}\sum_{i=1}^{N}s_{ij}<N$ **then**
  for $i=1$ *to* N **do**
    if $\sum_{j=1}^{M}s_{ij}=0$ **then**
      for $k=1$ *to* $N\cdot M$ **do**
        get EV id u and CS id v of $DIS(k)$;
        if $u=i$ *and* $\sum_{i=1}^{N}s_{iv}<Ave\cdot N_v^{pile}$ **then**
          $s_{iv}\leftarrow 1, s_{ip}\leftarrow 0,\forall p\in S_M,p\neq v;$
          break;
        end
      end
    end
  end
end
return s.

Algorithm 2. Optimal Charging Demand and Capacity

Input: $\mathbf{s}, a, b, c, m, r_i, p_{last}, d_{ij}, \beta, x_i^{max}, x_i^{min}$
Output: L^{pp*}, x_i^*
Initialize all L^{pp}, x_i;
repeat
  calculate optimal L^{pp} by calling (27);
  calculate optimal x_i by calling (28);
  calculate optimal λ by calling (34);
  calculate $\mathcal{L}(L^{pp}, x_i, \lambda)$;
until *all charging demand x_i are not changed*
return L^{pp*}, x_i^*.

Then, the overall algorithm of CCS is as **Algorithm 3**.

Algorithm 3. Overall Algorithm for CCS

Input: $a, b, c, m, r_i, p_{last}, d_{ij}, \beta, x_i^{max}, x_i^{min}, DIS = \{d_{ij}|i \in S_N, j \in S_M\}, \{N_j^{pile}|j \in S_M\}$
Output: $\mathbf{s}, L^{pp*}, x_i^*$
calling **Algorithm 1** to get the optimal $\mathbf{s}$;
calling **Algorithm 2** to get the optimal L^{pp} and x_i;
return $\mathbf{s}, L^{pp*}, x_i^*$.

It is worth noting that some variables in **P1** are not addressed, such as L_j^{cs}, p_j^{cs} and p^{pp}. These variables are not included in the objective function of **P2**, and we can define their values arbitrarily in their value ranges. For example, we can make $L_j^{cs*} = \sum_{i=1}^N s_{ij} x_i$, which does not effect the optimal value of **P1**.

Algorithm 4. CS Selection Algorithm (EV Part)

Input: $\{\rho_j|j \in S_M\}$, $Ave \leftarrow N/\sum_{j=1}^M N_j^{pile}$, $\{N_j^{pile}|j \in S_M\}$, $DIS_i = \{d_{ij}|j \in S_M\}$
Output: $\{s_{ij}|j \in S_M\}$
1 receive the updated N_j in real time;
2 sort DIS_i in ascending order;
3 **for** $k = 1$ *to* M **do**
4   get $DIS_i(k)$, store CS id into j;
5   **if** $N_j < Nearest\ integer(Ave \cdot N_j^{pile})$ **then**
6     $s_{ij} \leftarrow 1, s_{ip} \leftarrow 0, \forall p \in S_M, p \neq j$;
7   **break**;
8   **end**
9 **end**
10 **if** $\sum_{u=1}^M s_{iu} = 0$ **then**
11   **for** $k = 1$ *to* M **do**
12   get $DIS_i(k)$, store CS id into j;
13   **if** $N_j < Ave \cdot N_j^{pile}$ **then**
14       $s_{ij} \leftarrow 1, s_{ip} \leftarrow 0, \forall p \in S_M, p \neq j$;
15   **break**;
16   **end**
17   **end**
18 **end**
19 send CS selection $\{s_{ij}|j \in S_M\}$ to CSs;
20 **return** $\{s_{ij}|j \in S_M\}$.

In CCS, one can see that all the parameters of EV and CS should be sent and known by central control centre. But in real life, some private information is not provided such as the charging satisfaction weight r_i for the privacy concerns. Therefore, next, we will introduce a distributed charging strategy, i.e., DCS.

5 Distributed Charging Strategy

For the DCS, two stages are introduced. In the first stage (i.e., Stage-I), the EVs select the suitable charging stations separately according to **Theorem 1**, in order to determine $\{s_{ij}|i \in S_N, j \in S_M\}$. In the second stage (i.e., Stage-II), other continuous variables are addressed.

5.1 Stage-I of DCS

According to **Theorem 1**, each EV selects charging station based on that the value N_j/N_j^{pile} of each CS, which approximately equals to $\frac{N}{\sum_{j=1}^M N_j^{pile}}$. We propose two algorithms for EV and CS in **Algorithm 4** and **Algorithm 5**, respectively.

In **Algorithm 4**, there are mainly two parts. The first part (from lines 3 to 9) is similar to the part from lines 2 to 8 in **Algorithm 1**. The second part (from lines 10 to 18) is similar to the part from lines 9 to 21 in **Algorithm 1**.

Algorithm 5. CS Selection Algorithm (CS Part)

Input: $Ave \leftarrow N/\sum_{j=1}^M N_j^{pile}$, synchronized timer
Output: N_j, ρ_j
initialize the ρ_j and N_j^{pile};
repeat
  receive the $\{s_{ij}|j \in S_M\}$ from EV_i;
  update and broadcast the number of EVs currently select this CS N_j;
until *synchronized timer expires*
calculate the ρ_j;
return N_j, ρ_j.

Also, in **Algorithm 5**, the CSs wait the selection decision from each EV, and then update the number of EVs as N_j.

5.2 Stage-II of DCS

After we decide the variable $\{s_{ij}|i \in S_N, j \in S_M\}$, the rest of the problem is given as

$$\begin{aligned} \mathbf{P4} \quad & \min_{L^{pp}, \mathbf{L^{cs}}, \mathbf{x}} \quad \left(a(L^{pp})^2 + bL^{pp} + c\right) - \\ & \sum_{i=1}^N \left(\sum_{j=1}^M \left(s_{ij} \left((m - \rho_j) r_i \ln x_i - p_{last} d_{ij} \beta \right) \right) \right) \\ & s.t. \;\; \text{C1, C2, C5.} \end{aligned}$$

As the price parameters p_j^{cs} and p^{pp} are not in the constraints, they can be removed. Then, to solve **P4**, we first write the Lagrange function as

$$\begin{aligned} \mathcal{L}(L^{pp}, \mathbf{L^{cs}}, \mathbf{x}, \lambda_1, \boldsymbol{\lambda_2}) = & \left(a(L^{pp})^2 + bL^{pp} + c\right) - \\ & \sum_{i=1}^N \left(\sum_{j=1}^M \left(s_{ij} \left((m - \rho_j) r_i \ln x_i - p_{last} d_{ij} \beta \right) \right) \right) \\ & + \lambda_1 \left(\sum_{j=1}^M L_j^{cs} - L^{pp} \right) \\ & + \sum_{j=1}^M \left(\lambda_{2j} \left(\sum_{i=1}^N s_{ij} x_i - L_j^{cs} \right) \right), \end{aligned} \tag{35}$$

where $\boldsymbol{\lambda_2} = \{\lambda_{2j}|j \in S_M\}$, $\mathbf{L^{cs}} = \{L_j^{cs}|j \in S_M\}$, $\mathbf{x} = \{x_i|i \in S_N\}$, and $x_i^{max} \geq x_i \geq x_i^{min}, \forall i \in S_N$.

Then, the dual function is as

$$\begin{aligned}\mathcal{D}(\lambda_1, \lambda_2) &= \min_{L^{pp}, \mathbf{L^{cs}}, \mathbf{x}} \mathcal{L}(L^{pp}, \mathbf{L^{cs}}, \mathbf{x}, \lambda_1, \lambda_2) \\ &= P(\lambda_1) + \sum_{j=1}^{M} C_j(\lambda_1, \lambda_{2j}) + \sum_{i=1}^{N} E_i(\lambda_{2j}),\end{aligned} \tag{36}$$

where

$$P(\lambda_1) = \min_{L^{pp}} \left(a(L^{pp})^2 + bL^{pp} + c\right) - \lambda_1 L^{pp} \tag{37}$$

$$C_j(\lambda_1, \lambda_{2j}) = \min_{\mathbf{L^{cs}}} \ \lambda_1 L_j^{cs} - \lambda_{2j} L_j^{cs}, \ j \in S_M \tag{38}$$

$$\begin{aligned}E_i(\lambda_{2j}) = \min_{\mathbf{x}} &\sum_{j=1}^{M} \left(s_{ij}(p_{last} d_{ij}\beta + \lambda_{2j} x_i)\right) \\ &- \sum_{j=1}^{M} \left(s_{ij}((m-\rho_j) r_i \ln x_i)\right), \ i \in S_N.\end{aligned} \tag{39}$$

According to $P(\lambda_1)$, one can get the optimal solution of L^{pp} as

$$L^{pp*} = \frac{\lambda_1 - b}{2a}. \tag{40}$$

Then, the optimal solution of x_i can be obtained according to (39) as

$$x_i^* = \begin{cases} x_i^{\max}, \ if \ \frac{(m-\rho_j)r_i}{\lambda_{2j}} > x_i^{\max} \\ \frac{(m-\rho_j)r_i}{\lambda_{2j}}, \ if \ \frac{(m-\rho_j)r_i}{\lambda_{2j}} \in \left[x_i^{\min}, \ x_i^{\max}\right]. \\ x_i^{\min}, \ if \ \frac{(m-\rho_j)r_i}{\lambda_{2j}} < x_i^{\min} \end{cases} \tag{41}$$

The optimal Lagrange multipliers can be obtained by maximizing the dual problem as

$$\mathbf{P5} \quad \max_{\lambda_1 \geq 0, \lambda_2 \geq \mathbf{0}} \mathcal{D}(\lambda_1, \lambda_2).$$

In order to solve **P5**, first we put (40) into $\mathcal{D}(\lambda_1, \lambda_{2j})$, and let $\frac{\partial D(\lambda_1, \lambda_{2j})}{\partial \lambda_1} = 0$, then the optimal value of λ_1 is given as

$$\lambda_1^* = 2a \sum_{j=1}^{M} L_j^{cs} + b. \tag{42}$$

We can also find the optimal solution of λ_{2j} in the same way. However, as the optimal value of λ_{2j} requires the private information from EVs like r_i, we employ the gradient descent method to get the optimal value of λ_{2j} as

$$\lambda_{2j}^{(t+1)} = \left[\lambda_{2j}^{(t)} - \gamma\left(\sum_{i=1}^{N} s_{ij} x_i - L_j^{cs}\right)\right]^+, \ \forall j \in S_M, \tag{43}$$

where $\sum_{i=1}^{N} s_{ij} x_i$ is charging demand from EVs that select the jth CS and L_j^{cs} is the charging capacity distributed from PP according to the following equation

$$L_j^{cs} = L^{pp*}\left(\sum_{i=1}^{N} s_{ij} x_i \Big/ \sum_{j=1}^{M} \sum_{i=1}^{N} s_{ij} x_i\right), \tag{44}$$

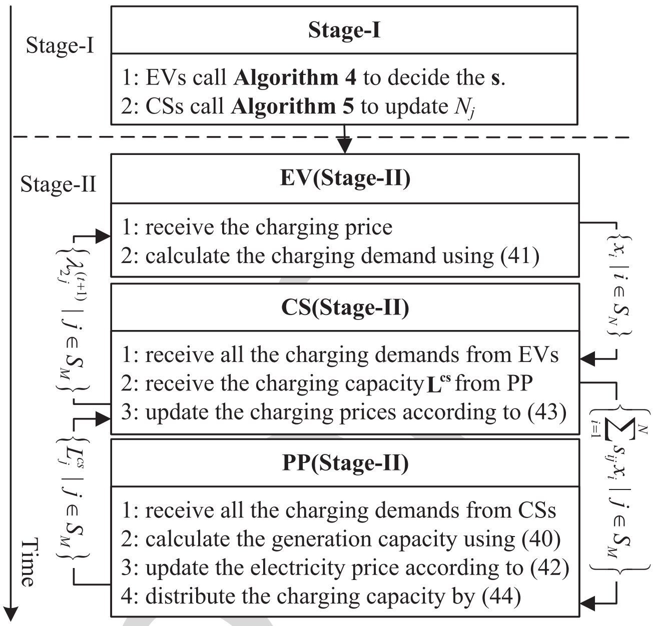

Fig. 2. The flowchart of DCS

where (44) guarantees $L^{pp} = \sum_{j=1}^{M} L_j^{cs}$. This can be seen as one of the necessary conditions for **P4** to minimize its objective function.

One can also see that the expressions of (37), (38) and (39) are similar to that of (4), (5) and (6). Then, the Lagrange multipliers λ_1 and λ_{2j} can be the prices p^{pp} and p_j^{cs} respectively. Therefore, the price variables are solved.

We also present the flow chart of DCS in Fig. 2.

The process of DCS can be explained as follows.

Step 1: DCS starts the first stage with a timer $T1$. Each EV calls **Algorithm 4** to select the CS independently. Each CS calls **Algorithm 5** in response to EV's selection.

Step 2: When $T1$ expires or all the EVs complete the CS selection, they can enter into stage 2. Then, all the EVs, CSs and PP initialize their variables.

Step 3: Each EV receives the charging price from the CSs and calculate its charging demands according to (41), and then sends their demands to the CS which they select.

Step 4: All the CSs receive the demands from EVs, and then forward them to PP.

Step 5: PP receives all the charging demands from CSs and then checks that if all the demands converge. If so, it can end and exit. Otherwise, PP calculates the optimal generation capacity using (40) and then updates the electricity price according to (42) and distributes the generation capacity to each CS according to (44). Finally, PP sends the updated charging capacities to all the CSs.

Step 6: CSs receive the charging capacities from the PP, and then update the charging prices according to (43) and send the prices to all the EVs. After that, it can go to *Step 2*.

6 Performance Evaluation

6.1 Parameters Setting

Assume there are N EVs and M CSs randomly distributed in a 50 km × 50 km square area, and 1 PP supplies electricity to all the CSs. Also assume there are 5 types of EVs with the battery capacities *cap* and shown in Table 2 [39].

TABLE 2
Parameters for EV Models

EV Model	Battery Capacity (kWh)	Assumed Market Share
Tesla Model X	90	35%
Nissan Leaf	30	25%
BMW i3	33	15%
Chevy Bolt	60	15%
Kia Soul EV	27	10%

TABLE 3
Simulation Parameters Setting

Parameter	Value	Parameter	Value
p_{last}	1.0\$/kWh	β	0.2kWh/km
a	10^{-5}\$/kW^{2}h	b	0.1\$/kWh
c	10\$	m	1.0
r_i	$[10, 50]$	γ	10^{-7}
$\lambda_1^{(0)}$	1.0\$/kWh	$\lambda_{2j}^{(0)}, j \in S_M$	1.0\$/kWh

The maximal charging demand x_i^{max} is randomly generated in the interval $[0.8 \times cap, cap]$, and the minimum charging demand x_i^{min} is randomly generated from $[0.1 \times cap, 0.3 \times cap]$. Similar to [40], we set the number of charging piles in each CS randomly from $[3, 8]$.

Also, the distance from ith EV to jth CS is denoted by the Manhattan distance as $\left|x_i^{ev} - x_j^{cs}\right| + \left|y_i^{ev} - y_j^{cs}\right|$, $i \in S_N$; $j \in S_M$, where x_i^{ev}, x_j^{cs} are the horizontal coordinates, and y_i^{ev}, y_j^{cs} are the vertical coordinates.

In Table 3, we set the initial value of λ_1 as $\lambda_1^{(0)}$; and set the initial value of λ_{2j} as $\lambda_{2j}^{(0)}, j \in S_M$. In addition, β is set according to [41]. The value of a and b are set according to [42]. For the range of a, b and c, we analyse them as follows. In order to ensure that $a(L^{pp})^2 + bL^{pp} + c$ always increase to the positive axis, b should be no less than 0. Also, the parameter c represents the fixed costs such as maintenance and therefore we set $c > 0$. a is also given as $a > 0$.

Normally the charging cost of the EV is larger than the distance cost, i.e., $p_j^{cs} x_i \gg p_{last} d_{ij} \beta, \forall i \in S_N$. In the simulation, we set the charging cost 10 times the same as the distance cost. We also obtain an upper bound for p_{last}: $p_{last} < \sum_{i=1}^{N} \sum_{j=1}^{M} s_{ij} p_j^{cs} x_i / (10\beta \sum_{i=1}^{N} \sum_{j=1}^{M} s_{ij} d_{ij})$, where p_j^{cs} is close to 1 in the optimization process. we also set the upper bound of p_{last} as 1.0. Also, after several tests, we can determine the appropriate p_{last} value according to the above inequality.

The simulation is performed on MATLAB R2016b installed in the computer equipped with Intel Core i5-7500 3.4 GHz processor with 4 cores and 8 GB memory.

6.2 Comparison Strategies

6.2.1 Nearest Distance Charging Strategy

In the Nearest Distance Charging Strategy (NDCS), each EV selects its nearest CS. Then other continuous variables are solved by applying DCS.

6.2.2 Random Selection Charging Strategy

In the Random Selection Charging Strategy (RSCS), each EV randomly selects the CS, and other continuous variables are solved by applying DCS.

6.2.3 Exhaustive Strategy

In the Exhaustive Strategy (ES), all the possible combinations of the selection variables of s are checked. After the decision is determined, **P3** is solved by using the method in CCS.

6.2.4 Cross Entropy Method Strategy

The Cross Entropy Method Strategy (CEMS) is an intelligent optimization algorithm, which has the state transition probability matrix storing the probabilities of selection decisions. The parameters are set according to [43], where the rarity parameter is 0.03; the smoothing parameter is 0.9; the stopping constant is 10 and the number of samples per iteration is 100. In each iteration, the EVs select the CSs according to the transition probability matrix first, and then the other variables are optimized by using the method in CCS.

6.2.5 Multi-Agent Game Strategy

Here, we further put forward the game theory based strategy i.e., Multi-Agent Game Strategy (MAGS), where all the EVs, CSs and PP are denoted as agents. We optimize their variables independently and then exchange the information among them until convergence. For i-th EV which selects the jth CS, its optimal charging demand is as

$$x_i^* = \max\Big\{\min\Big\{(m - \rho_j) r_i / p_j^{cs},\ x_i^{\max}\Big\}, x_i^{\min}\Big\}. \tag{45}$$

Also, the charging price p_j^{cs} is updated by:

$$p_j^{cs,(t+1)} = p_j^{cs,(t)} - \gamma \left[L_j^{cs} - \sum_{i=1}^{N} s_{ij} x_i \right]. \tag{46}$$

The capacity of PP is:

$$L^{pp*} = (p^{pp} - b)/(2a), \tag{47}$$

and its price is:

$$p^{pp*} = 2a \sum_{j=1}^{M} \sum_{i=1}^{N} s_{ij} x_i + b. \tag{48}$$

Also, charging capacity of CS is distributed by:

$$L_j^{cs} = L^{pp*} \left(\sum_{i=1}^{N} s_{ij} x_i \Big/ \sum_{j=1}^{M} \sum_{i=1}^{N} s_{ij} x_i \right). \tag{49}$$

A flow chart of MAGS is proposed in Fig. 3, which has the similar execution process as the stage-II in DCS.

The convergence condition is set to:

$$CF = \frac{\left| \sum_{i=1}^{N} x_i - L^{pp} \right|}{L^{pp}} \le \sigma, \tag{50}$$

where the parameter σ is set to 0.001.

6.3 Convergence Performance

In this simulation, the number of CSs M is set to 50, and the number of EVs N is set as 1000. The convergence curve is

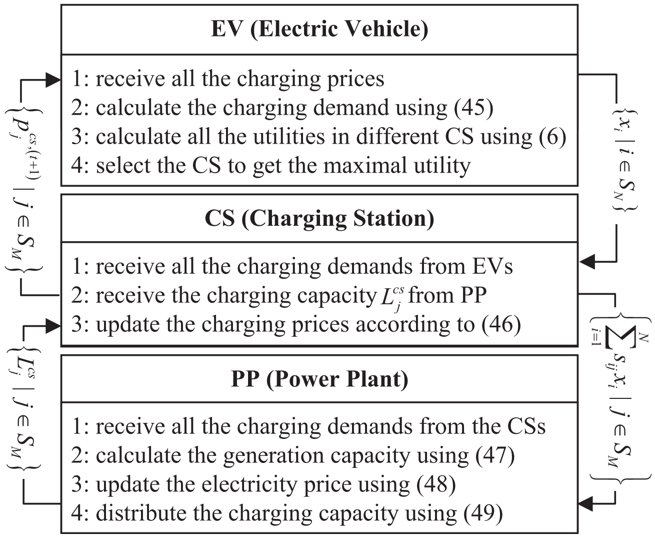

Fig. 3. The flowchart of MAGS.

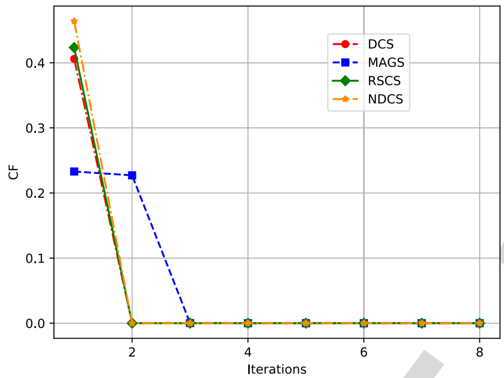

Fig. 4. The convergence performance.

shown in Fig. 4, where one can see that the iterations of DCS, RSCS and NDCS are the same, as they adopt the same process to optimize their continuous variables. Although the iterations of MAGS is 3, its running time is much higher than that of DCS, which can be found in Fig. 5.

In Fig. 5, one can see that the running time of MAGS is the highest among the compared algorithms. The reason for this is that MAGS needs to dynamically exchange information between EV and CS. In addition, when EV selects the CS, it changes the parameter ρ, which may lead to instability of MAGS and then affect the convergence of MAGS.

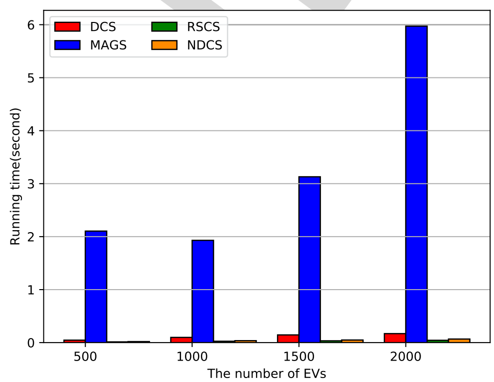

Fig. 5. The performance of running time.

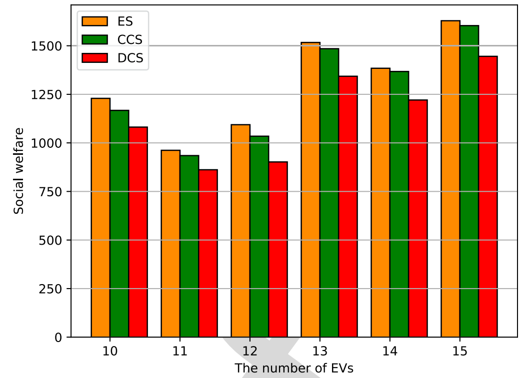

Fig. 6. The optimality of social welfare in small scale.

6.4 Performance in Small Scale Scenario

Considering the high complexity of ES, we only check it in a small scale. First, we define the CS congestion equilibrium (i.e., balance) indexes (CEI) as:

$$CEI = \sum_{j=1}^{M} \left| con_j - \frac{\sum_{j=1}^{M} con_j}{M} \right|. \tag{51}$$

Here, the number of CSs M is set to 3, and we randomly generate the number of charging piles for each CS from [1,3]. The number of EVs increases from 10 to 15. The convergence factor σ of DCS is set to 0.000001. Other parameters are the same as before.

In Fig. 6, one can see that the performance of CCS and DCS are quite similar to ES. The difference between ES and CCS is less than 1.5 percent, when the number of EV equals to 15.

In Fig. 7, we further present the CEI indexes of the three algorithms. One can see that the CEIs of CCS and DCS are smaller than those of ES in most cases. When the number of EVs is above 14, the CEIs of CCS and DCS are close to 0, which indicates that the congestion degree of all the CSs is nearly the same.

6.5 Performance in Large Scale Scenario

In this section, the performance of CCS and DCS is examined, with the comparison to intelligent optimization algorithm, i.e., CEMS. The number of CSs M is set to 50, and the number of EVs increases from 1000 to 2000, with a step of 200. The number of charging piles at the CS is randomly selected from [3, 8]. Convergence parameter σ of DCS is set to 0.000001.

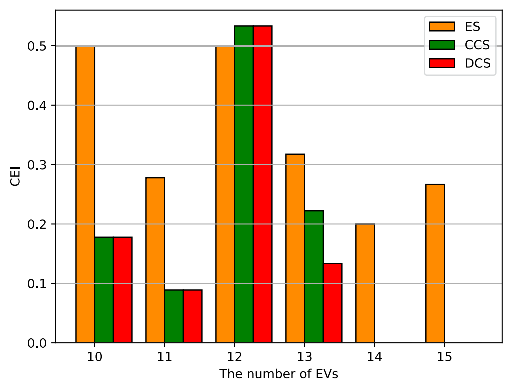

Fig. 7. The congestion balance indexes CEI in small scale.

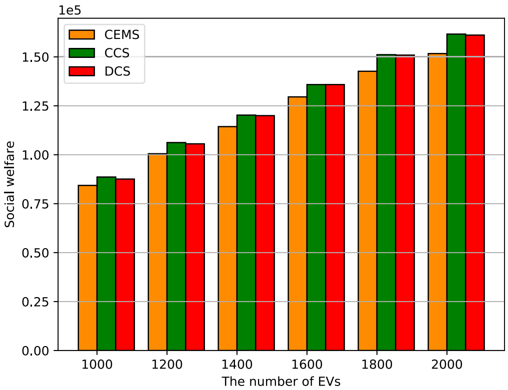

Fig. 8. The optimality of social welfare in large scale.

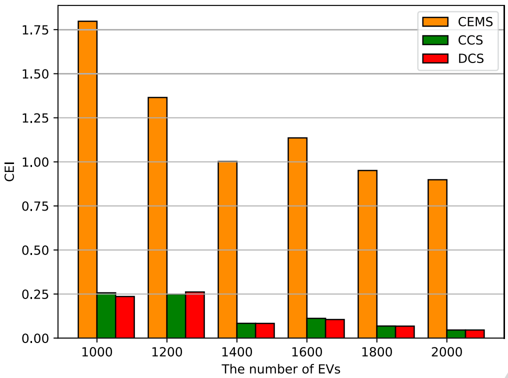

Fig. 9. The congestion balance indexes CEI in large scale.

As shown in Fig. 8, the performance of CEMS is worse than that of CCS and DCS. This is because CEMS is based on the transfer probability matrix when choosing charging stations and by storing better optimization results, the selection of CS may reach to the better results. Also, CEMS may fall into the local optimization due to the parameter setting.

In Fig. 9, one sees that the congestion balance of CEMS is not as good as CCS and DCS, which may lead to insufficient utilization of resources of the charging station.

6.6 Influence of the Number of EVs and CSs

In this section, we set the number of CSs to 20, and the number of EVs increasing from 1000 to 4000 in a step of 200. The convergence parameter of σ is set to 0.001. Other parameters are shown in Table 3.

According to Fig. 10, one can see the performance of CCS is the best among all the compared algorithms. When the number of EVs increases, the performance gap increases as well. The performance of RSCS and NDCS is worse than that of other algorithms.

From Fig. 11, one sees that the CEI indexes of NDCS and RSCS are much larger than those of the other three algorithms in the above sub-figure, which shows the poor congestion balance of NDCS and RSCS. In the below sub-figure of Fig. 11, one can see that the CEI index of MAGS is larger than that of CCS and DCS. In addition, we find that the CEI of MAGS is more than ten times as that of CCS and DCS.

Moreover, it can be seen from the Fig. 12 that the time consumed by MAGS is much longer than that of other compared algorithms. The reason why the running time of

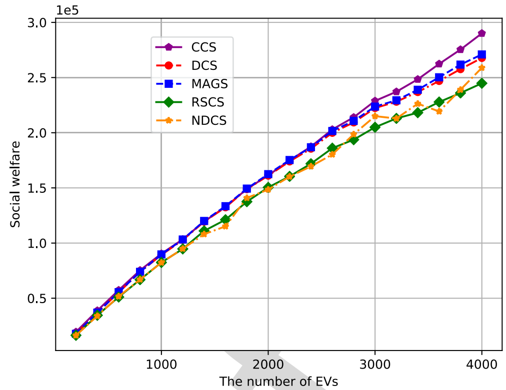

Fig. 10. The maximal social welfare vs the number of EVs.

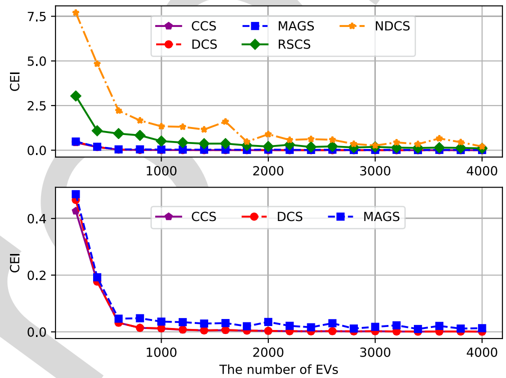

Fig. 11. The CEI vs. the number of EVs.

MAGS dose not increase linearly is that the charging range of all the EVs and the number of charging piles of CSs are randomly generated, and therefore result in different convergence performance.

It is also worth noting that the execution time of CCS is the same as that of DCS, both of which are very small. This is because CCS is not solved by the internal point method or other iterative methods, but based on the closed-form solutions.

To study the influence of the changing number of CSs, we set the number of EVs as 2000, with the number of CSs increases from 10 to 50 and step size setting as 2.

It can be seen from Fig. 13 that when the number of CSs changes, the performance of CCS and DCS is close to that of MAGS, and much better than that of NDCS and RSCS. However, when we compare the running time, MAGS is much longer than other four compared algorithms.

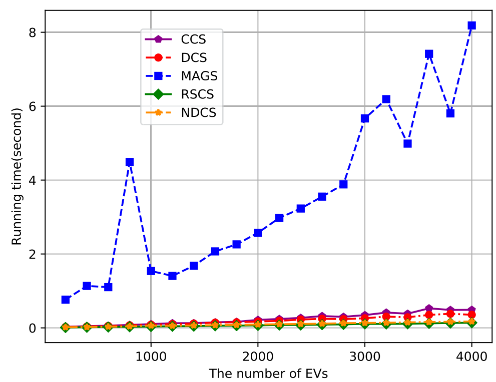

Fig. 12. The running time vs the number of EVs.

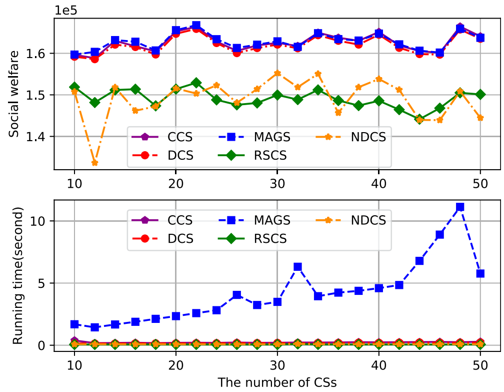

Fig. 13. The social welfare and running time vs. the number of CSs.

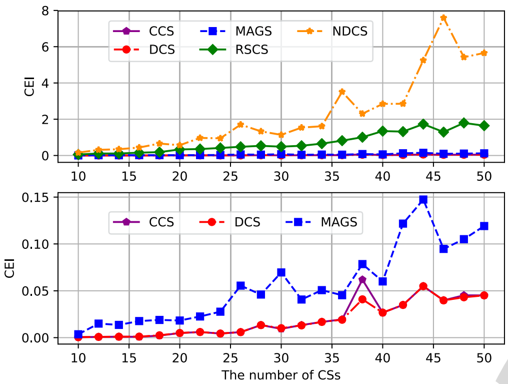

Fig. 14. The CEI vs. the number of CSs.

In Fig. 14, with the increase of the number of CSs, the CEI values of the five comparison algorithms increase as well. The CEI value of NDCS is the largest, followed by that of RSCS. The CEI values of the above two algorithms are much higher than those of the other three comparison algorithms, i.e., CCS, DCS and MAGS.

By further comparing the CEI values of CCS, DCS and MAGS, we find that the CEI of MAGS is higher than those of CCS and DCS, which illustrates the considerable performance of CCS and DCS in terms of the congestion balance.

6.7 The Influence of Parameters m and r_i

In this part, the number of EVs is set to 2000 and the number of CSs is set to 20. Also, m is increased from 1.0 to 4.8 in a

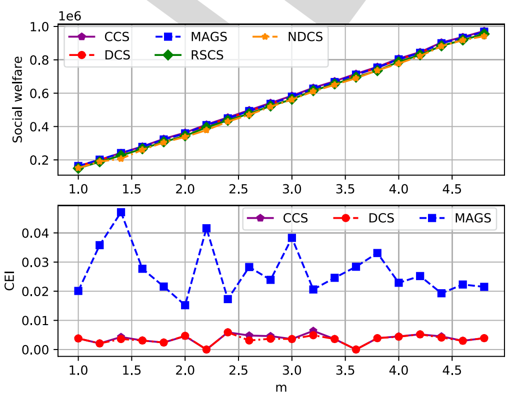

Fig. 15. Social welfare and CEI vs. m.

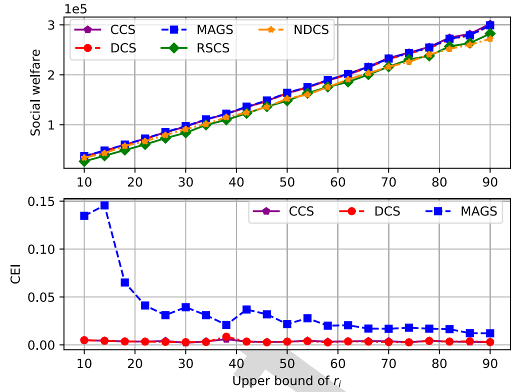

Fig. 16. Social welfare and CEI vs. upper bound of r_i.

step of 0.2. As r_i is set as a random parameter, we set its upper bound increasing from 10 to 90 with the step size of 4.

In Fig. 15, one can see that the performance of the five compared algorithms increases linearly with the increase of m, and the performance of CCS, MAGS and DCS is slightly higher than that of NDCS and RSCS.

Also, as the CEI indexes of NDCS and RSCS are not as good and therefore we do not put their CEI indexes in Fig. 15. It can be seen from Fig. 15 that the CEI index of MAGS is much larger than that of CCS and DCS. Also, with the increase of m, CCS and DCS still have considerable performance in terms of congestion balance.

In Fig. 16, one can see that the CCS, DCS and MAGS perform better than NDCS and RSCS. In addition, one sees that the CEI index of MAGS decreases with the increase of the upper bound of r_i, but it is still has larger value than that of CCS and DCS.

7 Conclusion

In this paper, we have proposed the smart charging scheduling model for electric vehicles considering social welfare maximization and congestion balance between different charging stations. We first presented the utility functions of power plant, charging stations and electric vehicles, and then proposed the social welfare maximization problem, which turns to be a MINLP and difficult to address. We proposed the centralized algorithm, i.e., CCS as well as the distributed algorithm, i.e., DCS to tackle the problem successfully. CCS has better performance than DCS but requires the private information from the EVs, whereas DCS can run decentralized and therefore, users do not upload their personal information to the control centre for resource allocation. We verified both algorithms via simulation in terms of social welfare, congestion balance of charging station and executing time.

For the future work, we aim to further study the charging scheduling algorithm integrated with renewable energy. Additionally, we plan to integrate the computing requirement in the charging algorithm, with the help of the popular mobile edge computing technologies.

Acknowledgements

This work was supported in part by the Natural Science Foundation of China (Grant No. 61620106011 and 61772087), in part by Zhongshan City Team Project (Grant No. 180809 162197874), in part by the Outstanding Youth Project of Hunan Province Education Department (Grant No. 18B162), and in part by the Double First-class International Cooperation and Development Scientific Research Project of Changsha University of Science and Technology (Grant No. 2018IC23).

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