Low-Complexity Expectation Propagation Detection for Uplink MIMO-SCMA Systems

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Abstract—We consider uplink sparse code multiple access (SCMA) systems associated with multiple input multiple output (MIMO), where the transmitters and the receiver are equipped with multiple antennas, for enhanced reliability (diversity gain) or improved data rate (multiplexing gain). For each diversity or multiplexing based MIMO scheme combined with SCMA, we develop low-complexity iterative detection algorithms based on the message passing algorithm (MPA) and the expectation propagation algorithm (EPA). We show that the MIMO-SCMA under EPA enjoys the salient advantage of linear complexity (in comparison to the MPA counterpart with exponential complexity) as well as enhanced error rate performances due to the MIMO transmission. We also show that the performance of EPA depends on the codebook size and the number of antennas.

Index Terms—SCMA, diversity, multiplexing, message passing algorithm (MPA), expectation propagation algorithm (EPA).

I. INTRODUCTION

SCMA is an emerging code domain non-orthogonal multiple access (NOMA) paradigm [1] which has attracted increasing research attention over the past years. In principle, coded bits in SCMA are mapped to multi-dimensional sparse codewords which naturally combines the bit-to-constellation modulation with the multi-user spreading. For enhanced diversity gain and/or multiplexing gain, a recent trend is to integrate SCMA with MIMO technology where multiple transmit and receive antennas are deployed. For multiple access channels (MAC), the receive antennas installed at the base-station give rise to larger diversity order and hence may be used for supporting more users with multiple access gain. The diversity-multiplexing tradeoff for MAC was studied in [2]. [3] studied a downlink MIMO-SCMA scheme, where the multiple antennas employed at the transmitter are used for user multiplexing. A diversity scheme based on Alamouti encoding [4] was proposed in [5] for downlink MIMO-SCMA associated with two transmit antennas and two receive antennas. In their scheme, the combiner and the multi-user detector are performed separately at the receiver. Moreover, a multiplexing scheme was proposed for uplink MIMO-SCMA systems by relying on two transmit antennas and four receive antennas, which resulted in better user connectivity than both orthogonal frequency-division multiplexing (OFDM) and single-input single-output (SISO) SCMA. In this work, we consider diversity and multiplexing based MIMO schemes for

uplink MIMO-SCMA systems. More specifically, we investigate MIMO-SCMA associated with space diversity, Alamouti encoding and multiplexing techniques. For each scheme, the equivalent factor graph is constructed and the complexity of MPA over the equivalent factor graph is analyzed. Unlike the Alamouti-based SCMA scheme proposed in [5] for downlink, the combiner and the multi-user detector may not be separated in uplink, thus a joint detector using message passing on the equivalent factor graph is performed. Furthermore, we propose an expectation propagation algorithm (EPA) based detector with linear complexity. To the best of our knowledge, our work is the first which proposes a low-complexity joint detector, based on EPA, for uplink Alamouti-based SCMA scheme. The performance of our EPA detector is analyzed for each MIMO scheme mentioned in the above with different number of receive antennas and the codebook size. Finally, we present the uncoded BER performance for all schemes, under the constraint of a fixed data rate.

Notations: We use x, \mathbf{x} and \mathbf{X} to represent a scalar, a vector and a matrix, respectively. The i^{th} element of \mathbf{x} is denoted by x_i and X_{ij} is the element in the i^{th} row and j^{th} column of \mathbf{X} . diag(\mathbf{x}) returns a diagonal matrix, where its i^{th} diagonal element is x_i . The modulo operator mod(a,m) returns the remainder after division of a by m. |x| returns the absolute value of x if x is a real number and the modulus if x is complex. \mathbf{x}^T denotes the transpose of \mathbf{x} and \mathbf{x}^* returns the complex conjugate of the elements of \mathbf{x} .

II. UPLINK SISO SPARSE CODE MULTIPLE ACCESS

A. System model

We consider the uplink SCMA system, where J single-antenna users $u_{j\in\mathcal{J}}$ share K resources $r_{k\in\mathcal{K}}$ (OFDM subcarriers) to communicate to a single-antenna receiver, where \mathcal{J} and \mathcal{K} is the sets of user and resource indices respectively. The overloading factor is defined as $\lambda = J/K$.

The SCMA encoder for user $u_{j \in \mathcal{J}}$ maps every $\log_2 M$ coded bits to a length-K vector of complex symbols $\mathbf{x}_j = [x_{1j}, x_{2j}, \cdots, x_{Kj}]^T$. The vector \mathbf{x}_j is called SCMA codeword which belongs to a finite set of M codewords \mathcal{C}_j known as SCMA codebook. The SCMA codewords are sparse, with N < K non-zero values. The SCMA structure can be represented by a factor graph \mathcal{G}_j , as depicted in Figure 1.

The circles denote variable nodes, while the squares represent resource nodes. Each variable node represents a codeword sent by a user. A variable node corresponding to a codeword \mathbf{x}_j is connected to a resource node r_k by an edge iif $x_{kj} \neq 0$.

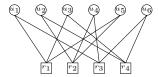


Figure 1. Factor graph representation of a regular SCMA system with 6 users and 4 resources. The overloading factor is $\lambda=150\%$.

The received signal in the uplink can be expressed as $\mathbf{y} = \sum_{j=1}^{J} \operatorname{diag}(\mathbf{h}_j) \mathbf{x}_j + \mathbf{n}$, where $\mathbf{h}_j = (h_1^j, \cdots, h_K^j)^T$ is the channel vector between the jth user and the receiver, $h_k^j \sim \mathcal{CN}(0,1)$ and $\mathbf{n} = (n_1, \cdots, n_K)^T$ is the complex additive white Gaussian noise (AWGN), $n_k \sim \mathcal{CN}(0, N_0)$.

B. Multi-user detection at the receiver

Message passing: The MPA iteratively computes an approximation of the MAP value for each variable node in order to retrieve the estimate $\hat{\mathbf{X}}$. Let $m_{v_j \to r_k}^{(t)}(\mathbf{x})$ (resp. $m_{r_k \to v_j}^{(t)}(\mathbf{x})$) denotes the message associated with the codeword \mathbf{x} , which is transmitted by (to) the variable node v_j to (by) the resource node r_k at the tth iteration of the MPA [1]:

$$\begin{split} m_{r_k \rightarrow v_j}^{(t)}(\mathbf{x}) &= \sum_{\substack{\mathbf{x}_j = \mathbf{x} \\ i \in \mathcal{I}_r(k) \setminus \{j\} \\ \mathbf{x}_i \in C_i}} \frac{1}{\pi N_0} e^{-\frac{\left|y_k - \sum_{i \in \mathcal{I}_r(k)} h_k^i x_{ki}\right|^2}{N_0}} \\ & \cdot \prod_{i \in \mathcal{I}_r(k) \setminus \{j\}} m_{v_i \rightarrow r_k}^{(t-1)}(\mathbf{x}_i), \end{split} \tag{I}$$

and

$$m_{v_j \to r_k}^{(t)}(\mathbf{x}) = \alpha_j \cdot \prod_{\ell \in \mathcal{I}_v(j) \setminus \{k\}} m_{r_\ell \to v_j}^{(t)}(\mathbf{x}),$$
 (2)

where α_j is a normalization factor, $\mathcal{I}_r(k)$ and $\mathcal{I}_v(j)$ are the set of variable node indices connected to resource node r_k and the set of resource node indices connected to variable node v_j , respectively. The complexity of MP detector is $\mathcal{O}(KM^{d_r})$.

Expectation propagation: In [6], an EPA was proposed for uplink SCMA systems, where only the means and variances of the messages are tracked during iterative detection on the factor graph. Furthermore, the EPA has a low detection complexity $\mathcal{O}(KMd_r)$, which scales linearly with the values of M and d_r . The steps performed by the EPA at iteration t are as follows.

1) For each variable node $v_{j\in\mathcal{J}}$, the posterior belief approximated is computed for all $\mathbf{x}_j\in\mathcal{C}_j$ as follows:

$$q^{(t)}(\mathbf{x}_{j}|\mathbf{y}) = \alpha_{j} \cdot p^{(0)}(\mathbf{x}_{j}) \cdot \prod_{k \in \mathcal{I}_{v}(j)} m_{r_{k} \to v_{j}}^{(t-1)}(x_{kj}), \tag{3}$$

where α_j is a normalization factor, $p^{(0)}(\mathbf{x}_j)$ represents the a-priori probability of codeword \mathbf{x}_j .

2) Compute the posterior mean $\mu_{kj}^{(t)}$ and variance $\zeta_{kj}^{(t)}$ for each variable node $v_{j\in\mathcal{J}}$ and resource node $r_{k\in\mathcal{I}_v(j)}$ as follows: $\mu_{kj}^{(t)} = \sum_{\mathbf{x}_j\in\mathcal{C}_j} q^{(t)}(\mathbf{x}_j|\mathbf{y}) \cdot x_{kj}$ and $\zeta_{kj}^{(t)} = \sum_{\mathbf{x}_j\in\mathcal{C}_j} q^{(t)}(\mathbf{x}_j|\mathbf{y}) \cdot |x_{kj} - \mu_{kj}^{(t)}|^2$.

3) Calculate the means $\mu_{v_j \to r_k}^{(t)}$ and the variances $\zeta_{v_j \to r_k}^{(t)}$ of the messages $m_{v_j \to r_k}^{(t)} \sim \mathcal{CN}(\mu_{v_j \to r_k}^{(t)}, \zeta_{v_j \to r_k}^{(t)})$:

$$\zeta_{v_{j} \rightarrow r_{k}}^{(t)} = \left(\frac{1}{\zeta_{kj}^{(t)}} - \frac{1}{\zeta_{r_{k} \rightarrow v_{j}}^{(t-1)}}\right)^{-1}, \mu_{v_{j} \rightarrow r_{k}}^{(t)} = \zeta_{v_{j} \rightarrow r_{k}}^{(t)} \left(\frac{\mu_{kj}^{(t)}}{\zeta_{kj}^{(t)}} - \frac{\mu_{r_{k} \rightarrow v_{j}}^{(t-1)}}{\zeta_{r_{k} \rightarrow v_{j}}^{(t-1)}}\right). \quad (4)$$

4) Determine the means $\mu_{r_k \to v_j}^{(t)}$ and the variances $\zeta_{r_k \to v_j}^{(t)}$ of the messages $m_{r_k \to v_j}^{(t)} \sim \mathcal{CN}(\mu_{r_k \to v_j}^{(t)}, \zeta_{r_k \to v_j}^{(t)})$:

$$\begin{split} &\mu_{rk \to v_j}^{(t)} = \frac{1}{h_k^j} \bigg(y_k - \sum_{i \in \mathcal{I}_T(k)} h_k^i \mu_{v_i \to r_k}^{(t)} \bigg), \\ &\zeta_{r_k \to v_j}^{(t)} = \frac{1}{|h_k^j|^2} \bigg(N_0 + \sum_{\substack{i \in \mathcal{I}_T(k) \\ i \neq j}} |h_k^i|^2 \zeta_{v_i \to r_k}^{(t)} \bigg). \end{split} \tag{5}$$

The posterior LLRs are computed using the posterior beliefs approximated in (3). The EPA is initialized with $\mu_{r_k \to v_j}^{(t)} = 0$ and $\zeta_{r_k \to v_j}^{(t)} = \text{INF}$, where INF is a large positive constant.

III. MIMO-SCMA: MULTIPLEXING VERSUS DIVERSITY

Consider a MIMO-SCMA system, with $N_t=2$ transmit antennas and N_r receive antennas. All users are multiplexed over blocks of OFDM subcarriers in the uplink. We study the following schemes combined with SCMA in uplink: space diversity (SD) [7], orthogonal space-time block code (OSTBC) [4] and multiplexing (MUX) [7].

Space diversity: In the context of SD-SCMA, the codeword transmitted by each user is repeated over the N_t transmit antennas. The signal received by the n_r th antenna is $\mathbf{y}^{n_r} = \sum_{n_t=1}^{N_t} \sum_{j=1}^{J} \operatorname{diag}(\mathbf{h}_j^{n_t,n_r}) \cdot \mathbf{x}_j + \mathbf{n}^{n_r}$, where $n_t \in \{1,\cdots,N_t\}$ and $n_r \in \{1,\cdots,N_r\}$ are the transmit and receive antenna indices respectively. $\mathbf{h}_j^{n_t,n_r} = (h_1^{n_t,n_r,j},\cdots,h_K^{n_t,n_r,j})^T$ is the channel vector between the n_t th transmit antenna of user j and the n_r th receive antenna, given that $h_k^{n_t,n_r,j} \sim \mathcal{CN}(0,1)$, while $\mathbf{n}^{n_r} = (n_1^{n_r},\cdots,n_K^{n_r})^T$ is the complex AWGN at the n_t th receive antenna, $n_k^{n_r} \sim \mathcal{CN}(0,N_0)$.

OSTBC: In the OSTBC scheme, two data symbols s_1 and s_2 are transmitted by two transmit antennas ($N_t=2$) over two channel use periods (T=2). It can be characterized by its encoding matrix of size $N_t \times T$:

$$\mathbf{S} = \begin{bmatrix} -s_2^* & s_1 \\ s_1^* & s_2 \end{bmatrix},\tag{6}$$

OSTBC-SCMA is equivalent to replacing s_1 and s_2 in (6) by SCMA codewords for each user. The received signal by the n_r th antenna at the time slot $t \in \{t_0, t_0 + 1\}$, is given by:

$$\mathbf{y}^{n_r}(t) = \sum_{n_t=1}^{N_t} \sum_{j=1}^{J} \operatorname{diag}(\mathbf{h}_j^{n_t, n_r}) \cdot \mathbf{x}_j(n_t, t) + \mathbf{n}^{n_r}, \tag{7}$$

where $\mathbf{x}_j(n_t, t)$ is the codeword transmitted by user u_j from the n_t th transmit antenna at time t. The following holds:

$$\mathbf{x}_{j}(n_{t}, t_{0} + 1) = \begin{cases} -\mathbf{x}_{j}^{*}(n_{t} + 1, t_{0}) \text{ if } n_{t} = 1\\ \mathbf{x}_{j}^{*}(n_{t} - 1, t_{0}) \text{ if } n_{t} = 2. \end{cases}$$
(8)

OSTBC combiner and multi-user detection can not be decoupled in the uplink due to the fact that the SCMA codewords are not multiplied by the same fading channel. Hence, a joint detection of all codewords sent by users during a space-time block should be considered.

Spatial multiplexing: In order to achieve an improved multiplexing gain, we consider MUX-SCMA where the N_t transmit antennas at each user are used for transmitting N_t independent SCMA codewords. The received signal at the n_r th antenna reads $\mathbf{y}^{n_r} = \sum_{n_t=1}^{N_t} \sum_{j=1}^{J} \mathrm{diag}(\mathbf{h}_j^{n_t,n_r}) \cdot \mathbf{x}_j(n_t) + \mathbf{n}^{n_r}$.

IV. REDUCED COMPLEXITY DETECTION

A. Space diversity

In this scheme, the factor graph $\mathcal{G}_{\mathrm{SD}}$ is composed of J variable nodes and KN_r resource nodes r_k . Each variable node corresponds to a user u_j . The received signal $y_k^{n_r}$ by the n_r th receive antenna at the kth subcarrier is mapped to the resource $r_{(n_r-1)K+k}$ of the factor graph. For the SCMA system given in Fig. 1 the factor graph of the SD scheme having $N_r=2$ is given in Fig. 2(a). Both the MPA and EPA presented in Section II-B can be directly applied to $\mathcal{G}_{\mathrm{SD}}$ after replacing $h_{(n_r-1)K+k}^i$ by $\sum_{n_t=1}^{N_t} h_k^{n_t,n_r,i}$, where i is the variable node index. The complexities of MP and EP algorithms are $\mathcal{O}(KN_rM^{d_r})$ and $\mathcal{O}(KN_rMd_r)$, respectively.

B. OSTBC

In OSTBC-SCMA, the factor graph \mathcal{G}_{OSTBC} is composed of JN_c variable nodes v_i and TKN_r resource nodes r_k , where $N_c = 2$ is the number of SCMA codewords transmitted during a space-time block and T = 2 is the number of channel use time slots. Fig. 2(b) shows the factor graph of the OSTBC-SCMA scheme for the SCMA system given by the factor graph in Fig. 1. The factor graph \mathcal{G}_{OSTBC} consists of two groups of variable nodes v_j , where $j \in \{1, \dots, JN_c\}$. The first group V_0 corresponds to the first codeword x sent by each user and the second group V_1 corresponds to the second codeword $\tilde{\mathbf{x}}$. There are also two groups of resource nodes r_k , where $k \in \{1, \dots, TKN_r\}$. The first group \mathcal{R}_0 corresponds to the subcarriers at the first channel use time slot t_0 , while the second group \mathcal{R}_1 corresponds to those received in $t_0 + 1$. The complexity orders of the MP and EP algorithms are $\mathcal{O}(2KN_rM^{2d_r})$ and $\mathcal{O}(4KN_rMd_r)$, respectively. In order to apply the MP algorithm in Section II-B on the factor graph $\mathcal{G}_{\text{OSTBC}}$, the message from the resource node to the variable node given by (1) should be updated by incorporating the expressions of the received signal in OSTBC scheme given in (7) and (8). Indeed, while the same message updating given by (1) is used by the resource nodes belonging to \mathcal{R}_0 , equation (1) should be modified for the resource nodes

$$\mathcal{R}_0$$
, equation (1) should be modified for the resource nodes belonging to \mathcal{R}_1 as follows, according to (8): $m_{r_k \to v_j}^{(t)}(\mathbf{x}) = \sum_{\substack{\mathbf{x}_j = \mathbf{x} \\ i \in \mathcal{I}_r(k)}} \frac{1}{\pi^{N_0}} \cdot e^{-\frac{|y_k - \sum_{v_i \in \mathcal{V}_0} h_k^i x_{k_i}^* + \sum_{v_i \in \mathcal{V}_1} h_k^i x_{k_i}^*|^2}{N_0}}$.

 $\prod_{u\in\mathcal{I}_r(k)\setminus\{j\}} m_{v_u\to r_k}^{(t-1)}(\mathbf{x}_u)$. To apply the EPA, only step 4) in the algorithm given in Section II-B should be modified after updating the expressions of means and variances in (4) as follows. Let r_k be a resource node corresponding to the n_r th receive antenna and v_j is a variable node corresponding to a user u_ℓ belonging to the group \mathcal{V}_p , where $p\in\{0,1\}$ and $\ell=\mathrm{mod}(j-1,J)+1\in\mathcal{J}$.

Result 1: The expressions of means and variances from resource nodes to variable nodes in OSTBC-SCMA are given by (9) and (10) respectively, where $n_t = p+1, \ n_t' = p'+1, \ p' \in \{0,1\}, \ \ell' = \operatorname{mod}(i-1,J)+1, \ \bar{n}_t = 2-p \ \operatorname{and} \ \bar{n}_t' = 2-p'.$

$$\boldsymbol{\zeta}_{r_k \rightarrow v_j}^{(t)} = \left\{ \begin{array}{l} \frac{1}{|\boldsymbol{h}_k^{n_t, n_r, \ell}|^2} \binom{N_0 + \sum_{i \in \mathcal{I}_r(k)} |\boldsymbol{h}_k^{n_t', n_r, \ell'}|^2 \boldsymbol{\zeta}_{v_i \rightarrow r_k}^{(t)} \right) & \text{if } r_k \in \mathcal{R}_0 \\ & (v_i \in \mathcal{V}_{p'}) \\ \frac{1}{|\boldsymbol{h}_k^{\overline{n}_t, n_r, \ell}|^2} \binom{N_0 + \sum_{i \in \mathcal{I}_r(k)} |\boldsymbol{h}_k^{\overline{n}_t', n_r, \ell'}|^2 \boldsymbol{\zeta}_{v_i \rightarrow r_k}^{(t)} \right) & \text{if } r_k \in \mathcal{R}_1 \\ & (v_i \in \mathcal{V}_{p'}) \end{array}$$

Result 1 is formulated using equations (7) and (8).

C. MUX-SCMA

The factor graph \mathcal{G}_{MUX} is composed of N_tJ variable nodes v_j and KN_r resource nodes r_k , where $N_t=2$. The factor graph of MUX-SCMA scheme with $N_r=1$ is depicted in Fig. 2(c), for the SCMA system in Fig. 1. \mathcal{G}_{MUX} consists of two groups of variable nodes corresponding to two codewords sent by two antennas. The complexities of the MPA and EPA are increased to $\mathcal{O}(KN_rM^{2d_r})$ and $\mathcal{O}(2KN_rMd_r)$, respectively, compared to SD-SCMA scheme. It can be observed that an MUX-SCMA scheme is equivalent to a SISO-SCMA system having N_tJ users and an equivalent number of resources. Thus, both the MPA and EPA demonstrated in Section II-B can be directly applied to \mathcal{G}_{MUX} .

V. SIMULATION RESULTS AND DISCUSSION

We consider an SCMA system whose factor graph is given in Fig. 1. For each of the MIMO arrangements presented in Section III, let ρ be the maximum number of bits per user transmitted by two antennas during two channel time slots. For example, MUX-SCMA can transmit 4 codewords, which is equivalent to $\rho=8$ bits/user/2 transmit antennas/2 channel use periods, when a codebook of size M=4 codewords is employed. Furthermore, the bit error rate (BER) performance is studied for a fixed value of ρ in all schemes in order to enable fair comparison. SCMA codebooks are designed according to the method proposed in [8]. We use damping to improve the performance of EPA.

Figure 3(a) depicts the uncoded BER performance curves of the SD, OSTBC and MUX systems, where $N_r = 1$ and $\rho = 4$. The MPA is employed with 6 iterations. It can be observed that a BER gain of nearly 4 dB is obtained using OSTBC-SCMA with respect to MUX-SCMA, at BER = 10^{-5} . Moreover, it can be seen that OSTBC-SCMA outperforms SD-SCMA by 3 dB at BER = 10^{-5} . To further demonstrate the efficiency of OSTBC-SCMA, we also consider a space-time code (STC) scheme based on both space and time diversity, which is obtained after removing the minus sign and the conjugate operators in the encoding matrix (6). Figure 3(a) shows that OSTBC-SCMA exhibits a better BER performance than STC-SCMA at the same complexity of detection. This demonstrates the ability of OSTBC-SCMA for improving the reliability of transmission although the OSTBC combiner and signal decoding are no more separable.

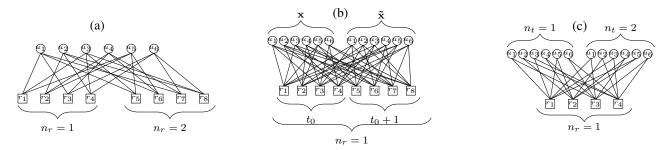


Figure 2. Factor graph representations of (a) SD-SCMA with $N_r = 2$, (b) OSTBC-SCMA with $N_r = 1$, (c) MUX-SCMA with $N_r = 1$.

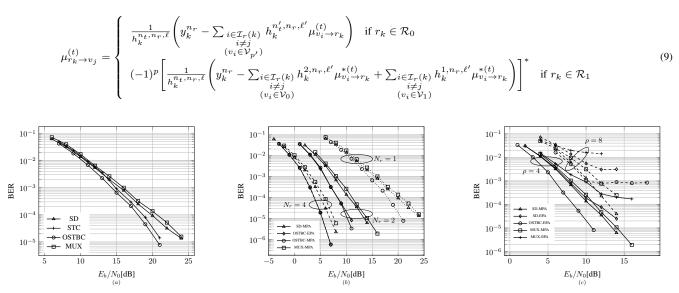


Figure 3. (a) BER performance using MPA. $\rho=4, N_r=1$. (b) Effect of $N_r, \rho=4$. (c) Comparison for different values of ρ . $N_r=2$

Figure 3(b) shows the improvement in the BER performance for the OSTBC, SD and MUX schemes, as the number of receive antennas increases. The BER performance for OSTBC-SCMA scheme is portrayed for both MPA and EPA, except when $N_r=1$, where the EPA suffers from high error floor at BER = 10^{-2} . It can be seen that the performance of EPA is close to that of MP, especially when N_r increases. This indicates that by adding more receive antennas the low complexity EPA detector can be used more efficiently.

Figure 3(c) depicts the BER performance of uncoded MIMO-SCMA, when $N_r=2$, at different values of ρ . The BER performances of SD-SCMA and MUX-SCMA are given for both the MPA and EPA algorithms. However, the BER performance of OSTBC-SCMA is given only for the EPA, since the complexity of MPA increases with the codebook size. It can be observed that EPA may exhibit high error floor depending on the codebook size and on the number of receive antennas. The problem of error floor in EPA may be solved using channel coding.

VI. CONCLUSION

We have studied various uplink MIMO-SCMA schemes and investigated low-complexity detection algorithms based on EPA. We analyzed the performance of the EPA based on the MIMO-SCMA scheme, the codebook size M and the number of receive antennas N_r . Results showed that the performance of EPA improves when M decreases and N_r increases.

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