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Interval Type-2 Beta Fuzzy Near Sets Approach to Content Based Image Retrieval

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***Abstract*—In computer-based search systems, similarity plays a key role in replicating the human search process which underlies many natural abilities, such as image recovery, language comprehension, decision making, or pattern recognition. The search for images consists in establishing a correspondence between the available images and those sought by the user, by measuring the similarity between the images. In fact, image search per content is generally based on the similarity between the visual characteristics of the images. The distance function used to evaluate the similarity between images depends notonly on the criteria of the search but also on the representation of the characteristics of the image. This is the main idea of a content-based image retrieval (CBIR) system. In this article, we first constructed type-2 beta fuzzy membership of descriptor vectors to help manage inaccuracy and uncertainty of the characteristics extracted from the feature of images. Subsequently, the retrieved images are ranked according to the novel similarity measure, which is noted type-2 fuzzy nearness measure (IT2FNM). By analogy to Type-2 Fuzzy Logic, and motivated by a near sets theory, we advanced a new fuzzy similarity measure (FSM) noted as interval type-2 fuzzy nearness measure (IT-2 FNM). Then, we proposed three new IT-2 FSMs and provided mathematical justification to demonstrate that the proposed FSMs satisfy proximity properties (i.e. reflexivity, transitivity, symmetry, and overlapping). The experimental results generated using three image databases show consistent and significant results.**

***Index Terms*—Interval-Type-2 Fuzzy Sets, Near Sets, Function Beta, Fuzzy similarity measure, CBIR.**

I. Introduction

CONTENT-based image retrieval (CBIR) is one of the basic research challenges that have been studied in depth by the multimedia community for decades, due to its wide range of applications in the search for information and "computer vision", "Database Management", "Man-Machine interface" [1]. Over the years, CBIR systems have been used to efficiently retrieve relevant images from a large set of databases. CBIR techniques recover images visually similar to a given query image. Therefore, several CBIR systems have been developed for image retrieval that represent promising solutions, such as the QBIC image and video content system [2], VisualSEEk [3] and SIMPLIcity [4], etc. Today, CBIR systems continue to face the challenge of the semantic divide in the accuracy of the relevant image and the subjectivity of human perception of visual content due to the incorrect selection of feature extraction methods and the measure of similarity. The effectiveness of such CBIR systems, therefore, depends, among other things, on a better match between the machine and humans, in the mode of representation and description of the information contained in the images. These methods are intended to reduce the notion of visual similarity between images to a simple notion of proximity between descriptors. Besides, the degree of similarity between the images is measured according to the characteristic descriptors which describe the visual content of an image. These features are coded to improve recovery performance using vector descriptors. On the other hand, a review of the most difficult problems of the CBIR has been proposed in [1], [5]–[9]. Although the number of CBIR methods is very large, the functionalities that are designed by the man based on these engineering skills and domain expertise, are difficult to be accurately described by the machine. The search for images similar to a request image is equivalent to searching for neighbors closest to the descriptor of the request image in the description space. In this context, Peters in [10] presented an approach using near sets and tolerance classes. This method is developed within the framework of perceptual systems, where each image or part of an image is considered as a perceptual object [11]. The central idea of the near sets is the research of the similarity between sets of disjoint objects. Thus, the recovery task can be considered as the similarity between sets of objects based on their descriptions. A tolerance measure of nearness (similarity) TNM was introduced in [12], which was applied to problems of image analysis and CBIR [13]–[17]. It was proven that TNM may perform as better than other measures in many real-world applications. In recent years, there have been countless similarity studies in different image processing applications. A summary of several measures is detailed in [18]–[23]. One of the solutions allowing to approach more faithfully the human way of thinking, which is generally characterized by imprecision and ambiguity, is the use of fuzzy logic, which makes it possible to model these characteristics and reduce ambiguity. Fuzzy logic allows

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a gradual transition between the equality of descriptions. Therefore, the integration of near sets with fuzzy sets provides a consistent balanced mix for the development of an efficient soft computing strategy in computer vision. The motivation behind the fuzzy near sets is to define the fuzzy similarity between sets of objects based on their fuzzy description. In fact, according to Peters in [24]a Fuzzy Near set was proposed to represent the similarity between two images. The practical outcomes of the used fuzzy near sets approach in our work [14], show the improvements that can be achieved compared to the results obtained by near sets, but still insufficient compared to the current work. Due to the complexity of the image visual structures, it was found that ambiguity might not be taken with type-1 fuzzy sets, so, the use of type-2 fuzzy logic would be necessary to manage the uncertainty that exists in real-world problems [25]. As type-2 fuzzy sets provide us with more design degrees of freedom , according to Mendel. These Type-2 fuzzy sets are very useful when it is difficult to determine an exact membership function as in Type-1 fuzzy sets [26]. However, when there is no membership uncertainty, the set is automatically reduced to a Type-1 fuzzy set while Type-2 fuzzy sets are now well established and are gaining more and more popularity. Therefore, the use of type-2 fuzzy near sets seems suitable for the paper context. Actually, the image retrieval by using Interval Type-2 Fuzzy Logic has been proven to be a great success in a large variety of applications, such as [27]–[29], among the works can find some: Xing and al. in [30] have been proposed an interval type-2 fuzzy clustering method based on neighborhood information to improve the classification accuracy of remote sensing images with complex land cover. For cancer diagnosis and prognosis, Singh et al. proposed in [31] a robust feature extraction approach based on the principal component analysis with interval type-2 fuzzy membership functions (IT-2PCA). Moreover, in [32], researchers proposed a novel way using the concept of footprint of uncertainty in interval type-2 fuzzy sets. This method can handle the veracity characteristic issue of the big data and reduce the instances to a manageable extent. With the edge-detection method, [33], [34], Patricia used the morphological gradient technique and generalized type-2 fuzzy logic, while Gonzalez applied the interval type-2 fuzzy sets on fuzzy images. In fact, previous literature suggested that interval type-2 fuzzy sets (IT2FS) can offer an alternative that can handle vagueness and uncertainty. The paper of Mittal et al. [35] offers a comprehensive review of the most relevant work in the framework of type-2 fuzzy logic, with its theoretical and practical implications. Castillo in [25] presented a literature review of recent applications using type-2 fuzzy systems based on image processing. One of the most interesting topics in the FS theory is the definition of the measure of similarity between FSs, to indicate their degree of closeness. In this context, several type 2 fuzzy similarity measures (T-2 FSMs) were introduced where Wu and Mendel in [36] presented a comprehensive overview of existing similarity measures for general type-2 FSs and proposed the Jaccard similarity measure for these sets. A comparative study between type 1 and type 2 FSM can be found in [37]. Three new Interval Type-2 Fuzzy Similarity measures (IT-2 FSMs) between Intuitionistic Fuzzy Sets were exposed by Cherif et al in [38]. In this paper, we propose a new Interval Type-2 Beta Fuzzy Near sets approach to Content based Image Retrieval. We first proceed with the feature vectors extraction of all images in the database. Then, the fuzzification of descriptor vectors applied by using the interval type-2 beta fuzzy membership function which deals with the uncertainty of feature extraction characteristics from images. Near sets theory, aims at grouping the images according to some common features, we compare the feature vector of the query image to another image to find the nearest objects. By analogy to Type-2 Fuzzy Logic and being motivated by Henry' study in [16], we advanced then three new interval type-2 fuzzy similarity measures. Since a relevant similarity measure has to fulfill the properties of reflexivity, transitivity, symmetry and overlapping, mathematical demonstrations were provided for our proposed similarity measures. Experimental results on four image databases; SIMPLIcity, Corel-1k, Caltech-101, and Image-Net have been ensured. Within this approach our main contributions are summarized as follows:

1) The development of a new description vector for the images by using an interval type-2 beta TFS (IT2 BFS), which enable gradual transition descriptions.
2) The new use of the near-fuzzy set theory that tolerates the equality of descriptions when comparing elements based on their descriptions.
3) Three new similarity measures between IT-2 fuzzy sets are advanced to obtain a superior approximation and similarity estimation among images.

The rest of this paper is organized as follows: Section II presents a brief overview of the previous work employing type-2 fuzzy sets and near sets to the fields of image-based image retrieval. Section II displays the theoretical foundations of fuzzy sets and near sets. While section four, is devoted to the presentation of the sought objectives pursued and to the adopted research methodology. The experiments and results are presented in section five. Finally, a conclusion is presented.

II. Preliminaries

This section presents the basic theoretical concept behind the methodologies used in this research.

A. Near Sets basis

Near sets gather disjoint sets each other [39]. Whenever there are observable similarities between the objects in the sets. The similarity is determined by comparing lists of object feature values which represent an object. A probe function is a real assessed function that represents these characteristics (features). Tolerance relationship gives an intransitive idea about the world [12]. Tolerance near sets is identified by a tolerance relationship based on descriptions. A perceptional system is a group of perceived

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items associated with a group of probe functions. And a perceptual item in a conceptual system can be described as follows. Let O represent a set of perceptual objects, and let B denote a set of real-valued functions, denoted probe functions, representing object features, and let $\phi(x) \in B$, where $\phi_i(x) : \underline{O} \longrightarrow \Re$. In combination, the functions representing object features provide a vector containing measurements (returned values) for an object description, associated with each functional value $\phi_i(x)$ for $x \in X$, where $|\phi| = l$; i.e. the description length is l.

Object Description:
$\phi_B(x) = (\phi_1(x), \phi_2(x), ..., \phi_i(x), \phi_{\mathit{1}}(x))$.
As fellows, the relationship between objects is determined by the probe functions in B. Our senses are defined to probe functions. In the tolerance space, a specific tolerance relation defined in [12] is given by:

Definition:
Let $\langle O, F\rangle$ be a perceptual system and let $\epsilon \in \Re_0^+$ (real). For every $B \subseteq F$ the weak tolerance relation $\cong_{B,\epsilon}$ is defined as:

$$\cong_{B,\epsilon}= \{ (x,y) \in O \times O \,|\, \exists \phi_i \in B \cdot |\phi_i(x) - \phi_i(y)\|_2 \leq \epsilon \} \tag{1}$$

where $\|.\|_2$ is L^2norm.
Notice, the relation $\cong_{B,\epsilon}$ is symmetric and reflexive but not transitive. This relation is very important in finding near sets, since it characterizes tolerance classes within a threshold ϵ.
Finally, the concept of near sets is established on the propositions requiring neighbourhoods and tolerance classes. The separated classes that incorporate similar items are said to be neighbors. The idea behind the nearness measure of Henry and Peters has sought the level of similarity between two variables by omitting the existing correlation between the set of variables, called the tolerance classes. The similarity measures can be grouped into equivalence classes of measures.

Definition : The tolerance nearness measure
Let $\langle O, F\rangle$ be a perceptual system, with $\epsilon \in \Re_0^+$ and $B \subset F$. Moreover, let X, Y be two disjoint sets and $Z = X \cup Y$. A tolerance nearness measure between two sets X and Y is determined by:

$$tNM_{\cong_{B,\epsilon}}(X, Y) = 1 - (\underbrace{\sum}_{C \in H_{B,\epsilon}(Z)} |C|)^{-1} \cdot \underbrace{\sum}_{C \in H_{B,\epsilon}(Z)} |C| \frac{min(|C \cap X|, |C \cap Y|)}{max(|C \cap X|, |C \cap Y|)}) \tag{2}$$

where $H_{\cong_{B,\epsilon}}(Z)$ is the set of tolerance classes.

B. Interval Type-2 Beta Fuzzy basis

In the previous section, it was shown how tolerance relation can be used in modeling the existing imprecision in human visual perception as well as the fuzzy approach. The most important part of the fuzzy logic theory is the modification of the membership values using various fuzzy techniques. The main importance of the Beta functions lies essentially in its capacity to approximate many usual functions (triangular, trapezoidal, or gaussian shapes) [40]. The concept of a type-2 fuzzy sets (T2 FSs) was an extension of the concept of an T1 FS. T2 FSs are now well established and are gaining more and more in popularity. In this work, the type-2 beta function is chosen for modeling of the fuzzy sets.

Definition: A type-2 fuzzy set, denoted $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x, u)$, where $x \in X$ and $u \in J_x \subset [0, 1]$.

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u)/(x, u) J_x \subset [0, 1] \tag{3}$$

where $0 \leq \mu_{\tilde{A}}(x, u) \leq 1$ and J_x is the closure of $\mu_{\tilde{A}}(x, u) > 0$. For any given $x \in X$,

$$\mu_{\tilde{A}}(x) = \int_{u \in J_x} \mu_{\tilde{A}}(x, u)/u \tag{4}$$

This is a second membership function, that is clearly a type-1 fuzzy set. The Uncertainty in the primary MF consists of the union of all MFs. This Uncertainty represents a bounded region that we call the Footprint of Uncertainty (FOU). This region represents a complete description of an IT2 FS. An IT2 is delimited by two Mfs noted the Upper MF (UMF), which is denoted $\bar{\mu}_{\tilde{A}}(x)$ and the Lower MF (LMF), which is denoted $\mu_{\underline{A}}(x)$, i.e.,

$$FOU(\tilde{A}) = \bigcup_{x \in X} u \in J_x; J_x = \left[\bar{\mu}_{\tilde{A}}(x), \underline{\mu}_{\tilde{A}}(x)\right]; \forall x \in X \tag{5}$$

$$\bar{\mu}_{\tilde{A}}(x) \equiv \bar{F}OU(\tilde{A}); \underline{\mu}_{\tilde{A}}(x) \equiv \underline{F}OU(\tilde{A}); \forall x \in X \tag{6}$$

As the Beta primary MF has four variables, we used the Interval type-2 Beta MF with uncertain center $c \in [c_1, c_2]$, a fixed width σ, p and q, and as is expressed in [41]:

$$\beta(x) = (1 + \frac{(p + q)(x - c)}{\sigma p})^p (1 - \frac{(p + q)(c - x)}{\sigma q})^q \tag{7}$$

However, the upper and lower membership functions can be expressed by respectively:

$$\bar{\mu}_{\tilde{\beta}}(x) = \begin{cases} \beta(x; c_1, \sigma, p, q) & x < c_1 \\ 1 & c_1 < x < c_2 \\ \beta(x; c_2, \sigma, p, q) & x > c_2 \end{cases} \tag{8}$$

$$\bar{\mu}_{\tilde{\beta}}(x) = \begin{cases} \beta(x; c_1, \sigma, p, q) & x \leq (c_1 + c_2)/2 \\ \beta(x; c_2, \sigma, p, q) & x > (c_1 + c_2)/2 \end{cases} \tag{9}$$

C. Fuzzy Near Sets

As was mentioned by Peters in [42], [43]: A fuzzy set X is a near set relative to a set Y if the grade of membership of the objects in sets X, Y is assigned to each object by the same membership function ϕ and there is a least one pair of objects $x, y \in X \times Y$ such that $\|\phi(x) - \phi(y)\|_2 \leq \epsilon$, i.e., the description of x is similar to the description y within some ϵ.

Proposition: Fuzzy sets (X, ϕ) , (Y, ϕ) are near sets if, and only if there exists at least one tolerance class $x/ \cong_{\phi,\epsilon}$

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in (X, ϕ) and $y/\cong_{\phi,\epsilon}$ in (Y, ϕ) such that $x/\cong_{\phi,\epsilon} \rhd\lhd_{\phi,\epsilon} y/\cong_{\phi,\epsilon}$ [43]. This model of near fuzzy sets was used in our paper in [14].

D. Interval type-2 Fuzzy similarity measure IT2FSM

A similarity measure between two sets (objects) is usually measured using by quantification of objects' attributes. Various similarity measures e.g. Jaccard defined in [37], S1, S2, proposed in [38] measures are applied between interval type-2 fuzzy sets to define the similarity score. In this section, we briefly present expressions of these similarity measures between type-2 fuzzy sets.

Jaccard's Type-2 Similarity Measure:

Let $\tilde{A}$ and $\tilde{B}$ be interval type-2 fuzzy sets (IT-2 FSs). A Jaccard Similarity measure [37] between two IT-2 FSs denoted as IT2FSMj (A, B) is presented as follows:

$$FSM_J(\tilde{A}, \tilde{B}) = \frac{p(\tilde{A} \cap \tilde{B})}{p(\tilde{A} \cup \tilde{B})} \tag{10}$$

$$= \frac{\int_x min(\bar{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{B}}(x))\mathrm{d}x + \int_x min(\underline{\mu}_{\tilde{A}}(x), \underline{\mu}_{\tilde{B}}(x))\mathrm{d}x}{\int_x max(\underline{\mu}_{\tilde{A}}(x), \underline{\mu}_{\tilde{B}}(x))\mathrm{d}x + \int_x max(\underline{\mu}_{\tilde{A}}(x), \underline{\mu}_{\tilde{B}}(x))\mathrm{d}x} \tag{11}$$

where $p(\tilde{A} \cap \tilde{B})$ and $p(\tilde{A} \cup \tilde{B})$ are the cardinalities of $\tilde{A} \cap \tilde{B}$ and $\tilde{A} \cup \tilde{B}$, respectively. The Jaccard type-2 similarity measure, satisfies the four similarity properties; reflexivity, symmetry, transitivity and overlapping.

Cherif's Interval Type-2 Fuzzy Similarity Measures:

Cherif et al. in [38] have generated new IT2FSM as an extension of the Baccour's distance proposed in [44] and named as IT2FSM1 and IT2FSM2.

$$IT2FSM_1(\tilde{A}, \tilde{B}) = 1 - \frac{(\bar{s} + \underline{s})}{2} \tag{12}$$

with

$$\bar{s} = \frac{1}{2n}\sum_{i=1}^{n}\left(\frac{|\bar{\mu}_{\tilde{A}}(x_i) - \bar{\mu}_{\tilde{B}}(x_i)|}{\bar{\mu}_{\tilde{A}}(x_i) + \bar{\mu}_{\tilde{B}}(x_i)} + \frac{|\bar{\mu}_{\tilde{A}}(x_i) - \bar{\mu}_{\tilde{B}}(x_i)|}{2 - \bar{\mu}_{\tilde{A}}(x_i) - \bar{\mu}_{\tilde{B}}(x_i)}\right) \tag{13}$$

and

$$\underline{s} = \frac{1}{2n}\sum_{i=1}^{n}\left(\frac{|\underline{\mu}_{\tilde{A}}(x_i) - \underline{\mu}_{\tilde{B}}(x_i)|}{\underline{\mu}_{\tilde{A}}(x_i) + \underline{\mu}_{\tilde{B}}(x_i)} + \frac{|\underline{\mu}_{\tilde{A}}(x_i) - \underline{\mu}_{\tilde{B}}(x_i)|}{2 - \underline{\mu}_{\tilde{A}}(x_i) - \underline{\mu}_{\tilde{B}}(x_i)}\right) \tag{14}$$

IT2FSM1 measure satisfies the four analogy properties.

$$IT2FSM_2(\tilde{A}, \tilde{B}) = 1 - \frac{(\bar{s} + \underline{s})}{2} \tag{15}$$

with

$$\bar{s} = \frac{1}{2n}\sum_{i=1}^{n}\left(\frac{|\bar{\mu}_{\tilde{A}}(x_i) - \bar{\mu}_{\tilde{B}}(x_i)|}{\bar{\mu}_{\tilde{A}}(x_i) + \bar{\mu}_{\tilde{B}}(x_i)} + \frac{|\bar{\mu}_{\tilde{B}}(x_i) - \bar{\mu}_{\tilde{A}}(x_i)|}{\bar{\mu}_{\tilde{A}}(x_i) + \bar{\mu}_{\tilde{B}}(x_i)}\right) \tag{16}$$

and

$$\underline{s} = \frac{1}{2n}\sum_{i=1}^{n}\left(\frac{|\underline{\mu}_{\tilde{A}}(x_i) - \underline{\mu}_{\tilde{B}}(x_i)|}{\underline{\mu}_{\tilde{A}}(x_i) + \underline{\mu}_{\tilde{B}}(x_i)} + \frac{|\underline{\mu}_{\tilde{B}}(x_i) - \underline{\mu}_{\tilde{A}}(x_i)|}{\underline{\mu}_{\tilde{A}}(x_i) + \underline{\mu}_{\tilde{B}}(x_i)}\right) \tag{17}$$

IT2FSM2 measure satisfies only reflexivity, transitivity, and overlapping properties.

Subsequently, based on the verified properties, IT2FSM1 and IT2FSM2 are Fuzzy Similarity Measures.

Fuzzy Similarity Measure Properties:

Definition: The properties of a fuzzy similarity measure between three sets A, B and C of $FS(X)$ are proposed in [38], [37], [44], as follows:

- P1. Reflexivity: If $S(A, B) = 1$ then $A = B$
- P2. Symmetry: $S(A, B) = S(B, A)$
- P3. Transivity: If $A < B < C$ then $S(A, B) > S(A, C)$ and $S(B, C) < S(A, C)$
- P4. Overlapping: If $A \cap B = \varnothing$ then $S(A, B) > 0$ otherwise $S(A, B) = 0$

A similarity value can vary from -1 to 1. For two fuzzy sets A and B, a unit similarity measure means that the two sets are similar; and when the similarity value is -1 that means that the two sets are exactly opposite.

III. A Proposed Interval-Type-2 Beta Fuzzy Near Method (IT2BFNM) in Image Retrieval System

This section describes the content-based image retrieval system used. The proposed work can be well defined by four main steps. Fig. 1 shows a block diagram. As soon as images are collected and associated and the feature vectors are extracted, the fuzzification process is applied. Then, the resemblance between feature vectors will be computed using the matching mechanism according to the near sets theory. Finally, similarity measures will be calculated to retrieve the nearest images. Those steps are described by the following algorithm:

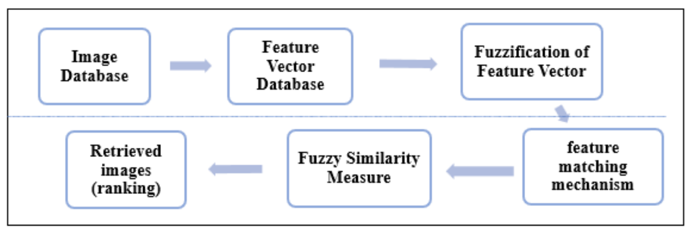

Fig. 1. The strategy of the methodology of work

Proposed algorithm

1) For each image of database, extract feature vector
2) For each image, apply fuzzification of feature vector
3) For each feature vector, combine the feature vectors of the query and any feature vector of database
4) Apply the feature matching mechanism of the combination of two feature vectors (see [14])
5) Measure the Fuzzy Nearness Similarity
6) Repeat steps 3-5 for each image with all images of database
7) Rank images with Fuzzy similarity Measure

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A. Pre-processing database

1) Partitioning Image: This research is based on a set-theoretic approach to image analysis where each image is viewed as a set of visual elements (describable objects). Each visual element is a part of the image that can be visually perceived and described. However, we decided to decompose the images of our base into blocks of a fixed size where each block is approximately assimilated to a sub-image. To control this point, a study has been undertaken in [14].

2) Feature Vector Database: In Near Set theory, an element visual is those that represent something in the physical world, and hence they can be perceived and described. Describing the element is possible through a set of characteristics (features). A visual element is a sub-image (as described in the previous section) that can be perceived and described by a probe function (color, shape, or texture). However, in this step, a visual descriptor has to be generated automatically. The choice of the probe function is often detailed in [14].

3) Fuzzification of feature vector: In this step, each sub-image is associated with a fuzzy function that assigns a value (between 0 and 1) to each feature of vector. The building or choosing a proper membership function Interval Type-2 Beta Fuzzy function is described in the previous section.

B. Feature matching mechanism

The next step consisting to compare the features of each image to any image feature in the database using a tolerance fuzzy relation to obtain a satisfactory matching in near sets sens. Amir in [45] defined a tolerance fuzzy relation $\cong_{\phi,\epsilon}$ between feature vectors. A fuzzy relation $\hat{R}$ is a 'fuzzy set' defined as follows where the membership function represents the degree of membership of each pair of elements in the relation. Furthermore, a tolerance fuzzy relation is a relation that is reflexive, symmetric, and transitive.

Definition: Let O a set of describable objects, B a set of probe functions and Φ_B is the set of feature vectors. Suppose $\|.\|_2$ is a distance function on (Φ_B, d). Let $\epsilon < \epsilon\prime \in \Re$. A perceptual fuzzy tolerance relationship $\hat{\cong}_{B,\epsilon} : O \times O \longrightarrow [0,1]$ is defined as follows:

$$
\begin{array}{ll}
\hat{\cong}_{B,\epsilon} & \\
= 1 & if \left\|\left(\Phi_B\left(x\right), \Phi_B\left(y\right)\right\|_2 < \epsilon \\
= \frac{\epsilon\prime - \|(\Phi_B(x),\Phi_B(y)\|_2}{\epsilon\prime-\epsilon} & if \epsilon < \left\|\left(\Phi_B\left(x\right), \Phi_B\left(y\right)\right\|_2 < \epsilon\prime \\
= 0 & otherwise
\end{array}
\tag{18}
$$

The focus of the descriptive near set theory is to assess similarity in terms of the descriptions of objects within the sets. For each visual element x_0 in the union of all sub-images ($x_0 \in X \cup Y$), where X and Y are two objects (images), find the tolerance classes with respect to the tolerance relation $\cong_{\phi,\epsilon}$. Tolerance classes are composed of the query points of successive neighborhoods, and then all the tolerance classes containing $x \in X$ are subsets of the neighborhood of x [14]. Finding tolerance classes is based on the Maximal Clique Enumeration (MCE) approach. This concept consists of using a tree structure to find all the maximal cliques through a depth-first search, where each call to Clique Enumerate creates a new child node. The general idea is to find maximal cliques through a Depth-First Search where the branches are formed based on candidate cliques and the backtracking occurs once a maximal clique has been discovered. This algorithm finds all the tolerance classes. The main idea behind using tolerance classes is the conjecture that when we look at two images, we tend to group image elements based on similarity to the element of interest at the point of gaze.

C. Fuzzy Similarity Measure

In this section, we proposed three new fuzzy similarity measures between IT-2 FSs. The notion of a similarity measure can be formalized through quantifying the differences between two objects.

1) Interval Type-2 Fuzzy Nearness Measures 1:

In a tolerance space view to image correspondence, nearness between sets of describable objects X, Y is defined by comparing the tolerance classes of almost similar objects in a tolerance space that covers both images. The idea behind the nearness measure of Henry and Peters has sought the level of similarity between two variables by omitting the existing correlation between the set of variables, called the tolerance classes. The similarity measures can be grouped into equivalence classes of measures. The Tolerance Fuzzy Nearness Measure between two fuzzy sets $X; Y$ is based on the concept that equivalent classes formed from objects in the union $Z = X \cup Y$ should be uniformly divided between X and Y if these sets are similar. A Tolerance Fuzzy Nearness Measure (TFNM) is proposed in this paper. By analogy to Type-2 Fuzzy Logic [26], the intersection of any FS will be replaced by minimum. Then based on the Equation 2, TFNM will be defined by the following definition.

Definition: Let $\langle O, F\rangle$ be a perceptual system, with $\epsilon \in \Re_0^+$ and $B \subset F$. Let X and Y be two disjoint sets. A tolerance fuzzy nearness measure (IT2FNM1) between two sets X and Y is determined by:

$$IT2FNM1 = 1 - \frac{(\bar{s} + \underline{s})}{2} \tag{19}$$

where

$$\bar{s}_{\cong_{B,\epsilon}}(\tilde{X}, \tilde{Y}) = \underbrace{\sum}_{\bar{\mu}_{\tilde{C}} \in H_{B,\epsilon}(Z)} |\bar{\mu}_{\tilde{C}}|)^{-1} \underbrace{\sum}_{\bar{\mu}_{\tilde{C}} \in H_{B,\epsilon}(Z)} |\bar{\mu}_{\tilde{C}}| \frac{min(|\bar{\mu}_{\tilde{C}} \cap \bar{\mu}_{\tilde{X}}|, |\bar{\mu}_{\tilde{C}} \cap \bar{\mu}_{\tilde{Y}}|)}{max(|\bar{\mu}_{\tilde{C}} \cap \bar{\mu}_{\tilde{X}}|, |\bar{\mu}_{\tilde{C}} \cap \bar{\mu}_{\tilde{Y}}|)} \tag{20}$$

and

$$\underline{s}_{\cong_{B,\epsilon}}(\tilde{X},\tilde{Y}) = \underbrace{\sum}_{\underline{\mu}_{\tilde{C}}\in H_{B,\epsilon}(Z)} |\underline{\mu}_{\tilde{C}}|)^{-1}\cdot \underbrace{\sum}_{\underline{\mu}_{\tilde{C}}\in H_{B,\epsilon}(Z)} |\underline{\mu}_{\tilde{C}}|\cdot \frac{min(|\underline{\mu}_{\tilde{C}}\cap \underline{\mu}_{\tilde{X}}|,|\underline{\mu}_{\tilde{C}}\cap \underline{\mu}_{\tilde{Y}}|)}{max(|\underline{\mu}_{\tilde{C}}\cap \underline{\mu}_{\tilde{X}}|,|\underline{\mu}_{\tilde{C}}\cap \underline{\mu}_{\tilde{Y}}|)} \tag{21}$$

where $H_{\cong_{B,\epsilon}}(Z)$ is the set of fuzzy tolerance classes. Note that X, Y are pairs of images and X, Y represent sets of describable objects (visual elements) corresponding to images X, Y. When the cardinality of a fuzzy set is defined as the sum of the membership values of all the elements in a set (as defined in [44]). The Interval Type-2 Fuzzy Nearness Measure is the mean of Tolerance Type-2 Fuzzy-upper Nearness Measure and Interval Type-2 Fuzzy-lower Nearness Measure.

Proof and justification: See Appendix Proof of IT2FNM1 properties.

2) Interval Type-2 Fuzzy Nearness Measures 2:

As noted in Section 2, for 2 IT-2 FSs A and B, Jaccard's similarity is represented by the intersection between A and B divided by the maximum of their union. According to Henry's metric-free description-based nearness measure [16], so it is a measure of similarity since

$$Similarity = 1 - Distance \tag{22}$$

according to [37]. Therefore, this measure is equivalent to that of the Jaccard similarity measure. By analogy to fuzzy Logic [26] and according to set operations, the absolute difference of (X and Y) and the algebraic sum of X and Y reduce to the union and the intersection between A and B respectively. Thus, based on the equation of the nearness measure 2 and as the false membership function will be replaced by 1ʇ(x), the equation (12) will be transformed as:

$$IT2FNM2 = 1 - \frac{(\bar{s}+\underline{s})}{2} \tag{23}$$

where

$$\bar{s}_{\cong_{B,\epsilon}}(X,Y) = (2\times \underbrace{\sum}_{\bar{\mu}_C\in H_{B,\epsilon}(Z)} |\bar{\mu}_C|)^{-1}\cdot \underbrace{\sum}_{\bar{\mu}_C\in H_{B,\epsilon}(Z)} |\bar{\mu}_C| (\frac{|min(|\bar{\mu}_C\cap\bar{\mu}_X|,|\bar{\mu}_C\cap\bar{\mu}_Y|)|}{max(|\bar{\mu}_C\cap\bar{\mu}_X|,|\bar{\mu}_C\cap\bar{\mu}_Y|)}+ \frac{|min(|\bar{\mu}_C\cap\bar{\mu}_X|,|\bar{\mu}_C\cap\bar{\mu}_Y|)|}{2-max(|\bar{\mu}_C\cap\bar{\mu}_X|,|\bar{\mu}_C\cap\bar{\mu}_Y|)}) \tag{24}$$

and

$$\underline{s}_{\cong_{B,\epsilon}}(X,Y) = (2\times \underbrace{\sum}_{\underline{\mu}_C\in H_{B,\epsilon}(Z)} |\underline{\mu}_C|)^{-1}\cdot \underbrace{\sum}_{\underline{\mu}_C\in H_{B,\epsilon}(Z)} |\underline{\mu}_C| (\frac{|min(|\underline{\mu}_C\cap\underline{\mu}_X|,|\underline{\mu}_C\cap\underline{\mu}_Y|)|}{max(|\underline{\mu}_C\cap\underline{\mu}_X|,|\underline{\mu}_C\cap\underline{\mu}_Y|)}+ \frac{|min(|\underline{\mu}_C\cap\underline{\mu}_X|,|\underline{\mu}_C\cap\underline{\mu}_Y|)|}{2-max(|\underline{\mu}_C\cap\underline{\mu}_X|,|\underline{\mu}_C\cap\underline{\mu}_Y|)}) \tag{25}$$

where $\bar{s}$ and $\underline{s}$ defined the upper and lower values respectively.

Proof and justification: See Appendix Proof of IT2FNM2 properties.

3) Interval Type-2 Fuzzy Nearness Measures 3: By the same principle, the Interval Type-2 Fuzzy similarity Measure S3 is defined by the generalization from the similarity measure presented in Equation 15 between IFS. The latter are computing the mean of lower and upper values. For two IT-2 FSs X and Y, the following formula is hence obtained:

$$IT2FNM3 = 1 - \frac{(\bar{s}+\underline{s})}{2} \tag{26}$$

where

$$\bar{s}_{\cong_{B,\epsilon}}(X,Y) = (2\times \underbrace{\sum}_{\bar{\mu}_C\in H_{B,\epsilon}(Z)} |\bar{\mu}_C|)^{-1}\cdot \underbrace{\sum}_{\bar{\mu}_C\in H_{B,\epsilon}(Z)} |\bar{\mu}_C| (\frac{|min(|\bar{\mu}_C\cap\bar{\mu}_X|,|\bar{\mu}_C\cap\bar{\mu}_Y|)|}{max(|\bar{\mu}_C\cap\bar{\mu}_X|,|\bar{\mu}_C\cap\bar{\mu}_Y|)}+ \frac{|min(|\bar{\mu}_C\cap\bar{\mu}_Y|,|\bar{\mu}_C\cap\bar{\mu}_X|)|}{max(|\bar{\mu}_C\cap\bar{\mu}_Y|,|\bar{\mu}_C\cap\bar{\mu}_X|)}) \tag{27}$$

and

$$\underline{s}_{\cong_{B,\epsilon}}(X,Y) = (2\times \underbrace{\sum}_{\underline{\mu}_C\in H_{B,\epsilon}(Z)} |\underline{\mu}_C|)^{-1}\cdot \underbrace{\sum}_{\underline{\mu}_C\in H_{B,\epsilon}(Z)} |\underline{\mu}_C| (\frac{|min(|\underline{\mu}_C\cap\underline{\mu}_X|,|\underline{\mu}_C\cap\underline{\mu}_Y|)|}{max(|\underline{\mu}_C\cap\underline{\mu}_X|,|\underline{\mu}_C\cap\underline{\mu}_Y|)}+ \frac{|min(|\underline{\mu}_C\cap\underline{\mu}_Y|,|\underline{\mu}_C\cap\underline{\mu}_X|)|}{max(|\underline{\mu}_C\cap\underline{\mu}_Y|,|\underline{\mu}_C\cap\underline{\mu}_X|)}) \tag{28}$$

where $\bar{s}$ and $\underline{s}$ defined the upper and lower values respectively.

Proof and justification: See Appendix Proof of IT2FNM3 properties.

The best similarity measure provides the largest number of relevant images. The measure of similarity between images is assimilated to a calculation of the distance between the descriptor vector of the query image and that of an image of the base. Both the distance is small as the two images are similar.

D. Returning query results

The system returns the result of the search as a list of ordered images according to the similarity between their descriptors and the descriptor of the query image. The effectiveness of the search is evaluated according to the number of images relevant and irrelevant to the query, found in a database: a search making it possible to find, in an image database, all the images relevant to the request, and no irrelevant image, is perfectly effective.

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IV. EXPERIMENTS AND RESULTS

In this section, we describe the image data sets used. Then, we deal with the experimental results of the proposed method. Finally, we discussed the comparison of these results with the results of other CBIR systems.

A. DataBase

The performance is evaluated on the Corel-1K, SIMPLIcity, Caltech-101 and ImageNet image repository. The Corel-1K image repository contains 1000 images in the form of 10 semantic categories and each semantic category contains 100 images with resolution sizes of 128 × 192 or 192 × 128. The SIMPLIcity database contains 1000 natural color images available for download from [46]. This database is numbered between 0 to 999 and divided into 10 conceptually different categories (named here as target sets C0 to C9). Images are 384 × 256 pixels. The original CALTECH database consists of images of 101 categories. Each category contains 40 to 800 image samples. Each image is roughly 300 × 200 in size. The Image-Net [47] is a large-scale image database. It is used for indexing and retrieval complex and multi-category images. It contains more than 100k datasets (i.e. called synsets). In this study, we used only the 20 most popular synsets including 10000 images: airplane, minivan, train, car, fire engine, tank, army, musket, tiger, quail, tortoise, snake, ladle, box, pot, towel, dam, mountain, fisherman and volcano. Fig. 2 displays sample images from Corel-1k, SIMPLIcity, Caltech-101 and ImageNet databases. Any image from the database can be selected as a query image and compared to all images in the database using either the fuzzy nearness approaches mentioned above. The experimentation consists of calculating the similarity measures between each query image and all images. Subsequently, the images will be sorted based on their similarity to the query image. The experimentation is performed using each one of the proposed similarity measures in previous chapters. The experimentation was generated using $\epsilon = 0,3$ as a value produced the best results in [14].

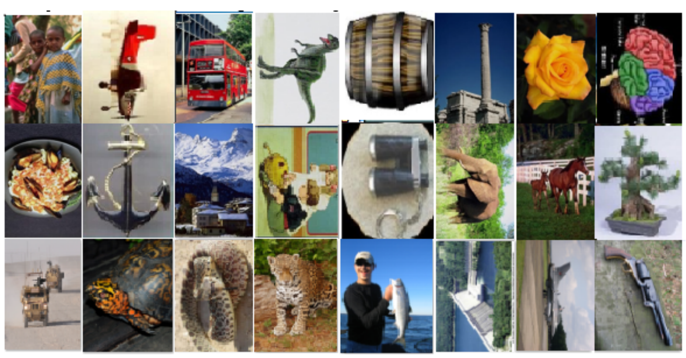

Fig. 2. Sample images from Corel 10k, SIMPLIcity, Caltech-101 and ImageNet databases

Three different methods were used in this experiment to evaluate precision and recall of the image retrieval. In the **first evaluation method**, mean average precision for each category between the proposed measures on the basis of the formula 19, 23 and 26. In a **second method** of retrieval evaluation, both precision and recall were calculated at each number of the 100 most similar images (each category) and the values of precision were plotted against recall. In a **third evaluation method** Comparison of average precision for each category between the proposed measures, results published in [5]–[9] on the SIMPLIcity, corel-1k, caltech-101 and ImageNet databases. The experimental results were provided by different approaches :

- **BFNS** : Our proposal (Beta Fuzzy Near System) is based on the beta fuzzy Near sets approach with fuzzy nearness measure [14].
- **IT2BFNS1** : Our proposal (Interval Type-2 Beta Fuzzy Near System 1) is based on the interval type-2 beta fuzzy Near set approach with fuzzy nearness measure 1.
- **IT2BFNS2** : Our proposal (Interval Type-2 Beta Fuzzy Near System 2) is based on the interval type-2 beta fuzzy Near set approach with fuzzy nearness measure2.
- **IT2BFNS3** : Our proposal (Interval Type-2 Beta Fuzzy Near System 3) is based on the interval type-2 beta fuzzy Near set approach with fuzzy nearness measure 3.

We present some examples of experimental results obtained from the measures studied.

B. Performance Measurement for Similar Image Retrieval

To assess the effectiveness of our system, we interested in calculating the two most commonly used measures. For a given query image, a retrieved image is considered relevant if and only if it belongs to the same image category as the query image. The precision is the number of relevant images retrieved in relation to the total number of images proposed by the search engine for a given query. Precision is calculated using the following formula:

$$precision = \frac{|\{relevant\ images\} \cap \{retrieved\ images\}|}{\{retrieved\ images\}} \tag{29}$$

The recall is the ratio between the number of relevant images in the set of images found and the number of relevant images in the image base. Recall is defined as follows:

$$Recall = \frac{|\{relevant\ images\} \cap \{retrieved\ images\}|}{\{relevant\ images\}} \tag{30}$$

In practice, in order to evaluate the system, we use several queries. It is therefore necessary to calculate the AP (Average Precision) for all the requests corresponding to each level of recall according to the following formula:

$$AP = \frac{\sum_i (precision\ of\ the\ query\ i)}{Number\ of\ requests} \tag{31}$$

This measurement is calculated on all queries and represents the arithmetic mean of the different precision of each

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category. To evaluate the experiments, the AP (Average Precision) has been calculated, the different results will be presented. The Mean Average Precision (MAP) is the mean of all the APs of the different categories. Precision and Recall are interesting for a final evaluation of the best image of one category, however for larger evaluation purposes, we consider the Precision/Recall curve. This curve is the set of all the couples (Precision, Recall) for each number of images returned by the system. The curve always starts from the top left $(1, 0)$ and ends in the bottom right $(0, 1)$. Between these two points, the curve decreases regularly. A good Precision/Recall curve is a curve that decreases slowly, which means that at the same time, the system returns a lot of relevant images and few of them are lost.

C. Results and Analysis

In the first experimentation, the retrieval results of the average precision and average recall are reported in Table I. As seen in this table, IT2BFNS3 outperforms IT2BFNS1 and IT2BFNS2 similarity measures in all categories, and we show that, for the proposed IT2BFNS3, three image classes 'Elephant', 'Flowers' and 'Horses' have attained 100% average precision.

The second experimentation have been performed on Caltech-101 data collection. We selected randomly fifteen categories; the contents include airplanes, ferry, camera, brain, minaret, motorbrikes, etc. To use the Caltech database in a similar manner as for the SIMPLIcity database, 100 images of each class are randomly selected. The average precision and recall rates for 15 categories of the Caltech-101 database are shown in Table II. Similarly, IT2BFNS3 provides better precision in those categories. The recall rates for Caltech-101 are also promising. Most of the categories have high recall rates, while a few have average rates. The Windsor chair and camera have average rates. The mean average precision obtained for this benchmark is 88.7%, and the mean average recall is 50.05%.

The third experimentation is done for evaluating the proposed fuzzy similarity measures on the Corel-1k database. The average precisions and recalls of all categories in this database are reported in Table III.

Results show that IT2BFNS3 has a higher average precision rate in some categories, and a significantly better average recall rate. The recall rate is improved by 19.14% over those from the existing measures. IT2BFNS3 outperforms other measures by at least 5.51% in terms of average precision. The horses, flower and elephant categories are relatively easy to retrieved, and most of the existing methods provide better results in these categories, it reaches a rate of 100%. Table IV, shows a comparison between performance analysis of proposed and existing measures in terms of average precision and average recall on 20 catégories of Image-Net datasets. Therefore, we can draw the following conclusions; first, the experiments performed on Image-Net are a very challenging task. In fact, the average precision rates obtained for each category are around 63% which implies that they achieve an equivalent performance for all proposed measures. On the other hand, the proposed IT2FBNS3 method achieves the highest precision rates in some categories since Image-Net synsets are difficult to categorize. Secondly, The average recall rates obtained for each category are around 37%, where IT2FBNS1 achieves the best performance with 39.49% performance. Then, the image-Net shows average results due to its complex nature of images which falls into the different semantic groups at the same time.

To further the understanding, the second experimentation is done by presenting in fig. 3(a); an example query from the beach category of SIMPLIcity database. Fig. 3(b) shows the first 20 images extracted from the best search of the query are sorted and displayed according to the nearness measure. In this figure, IT2FBNS3 (first row), IT2FBNS2 (second row), IT2FBNS1 (thrid row), and FBNS (four row), are shown. The first row contains only 13 images from the class and 7 irrelevant images of other classes. Then IT2BFNS3 method leaves less mistakes then other measures. Fig. 3(c) shown the precision and recall curve for the query with different images retrieved for this database. Some of the curves have an acute inflection point. These points represent the location where the remaining IT2BFNM values for a particular request become null. It can be seen that images in category "beach" can be retrieved with very high precision with the measure IT2BFNM3. Fig. 4 is another example of an image of Imag-Net database. Fig. 4(b) shows that IT2FBNS3 give more relevant images then other methods. Fig. 4(c) shown the precision and recall curve. It can be seen also that IT2BFNM3 method have a better precision. To demonstrate the effectiveness of the new similarity measure, the nearness measure had compared with three different kinds of similarity measures in [15]; Tolerance Nearness Measure (TNM), Earth Mover's Distance (EMD), and Integrated Region Matching (IRM). Hence, it is experimentally verified that TNM is an effective similarity measure in image retrieval. Table V encompasses the results obtained for different distance metrics and with recent CBIR algorithms [8], [48] using the SIMPLIcity, Corel-10K, and Caltech-101 databases. This, clearly shows that, the Euclidean distance metric delivers the better result than other metrics. It produced 69% precision and 7.5% recall in SIMPLIcity, 36.83% precision and 7.5% recall in Corel-10k, and 65% precision and 7.5% recall in Caltech-101. In the third experimentation, the best results of our proposed similarity measure are evaluated and compared to them in [5]–[9]. The numerical results of the average precision are reported in table VI. As seen in this table, the average precision of our proposed measures on Corel-1k is 88.65%. This is significantly better than others. Table IV-C shows the average precision rate for the best proposed metric and existing research metrics for SIMPLIcity database. The existing methods also report good results in many categories but the proposed method reports the highest mean average precision.

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Categories	BFNS		IT2BFNS1		IT2BFNS2		IT2BFNS3	
	Precision(%)	Recall(%)	Precision(%)	Recall(%)	Precision(%)	Recall(%)	Precision(%)	Recall(%)
Africa	76.3	53	16.51	50.49	21,84	50.49	79.02	55
Beach	72.5	65	28.60	78	22.27	83	7693	91
Building	76.2	61	24.33	68	20.59	75	75.74	87
Bus	92.3	68	25.10	74	23.47	85	91,91	92
Dinosaur	100	91	54.82	91	55.43	97	89.72	100
Elephant	74.8	61	69.05	69	100	75	100	83
Flower	89.2	55	100	65	100	71	100	81
Horse	100	58	100	65	98	76	100	80
Mountain	66.8	53	58.85	61	80	69	90	76
Food	68.7	64	80.35	71	87.05	75	92.02	92
Average	81.18	60	82.35	67	84.76	66	88.2	79

TABLE I

Comparison of the average precision obtained by the proposed similarity measures for SIMPLIcity database

Categories	BFNS		IT2BFNS1		IT2BFNS2		IT2BFNS3	
	Precision(%)	Recall(%)	Precision(%)	Recall(%)	Precision(%)	Recall(%)	Precision(%)	Recall(%)
Airplanes	76	48.02	95	51.1	72	54	94	57.18
Ferry	75	32.05	66	34	66	37.11	88	41
Camera	77	48.2	85	51	72	54.02	84	57.11
Brain	71	44.8	79	51.2	70	54.5	91	56.22
Cougarface	72	45	83	49.9	71	51.05	87	45
Grad piano	76	44.05	90	44.05	73	53	92	56
Dalmation	70	39	73	43	68	46	90	49
Dollar bill	76	37	35	41.78	40	44.21	86	47.11
Starfish	77	32	45	37	74	42.2	88	47.68
Soccer ball	74	36.08	82	41.22	71	43.05	87	51.55
Minaret	74	31.46	66	33.01	79	37.5	88	45.05
Motorbikes	73	35.88	78	37	72	41.78	90	43
Revolver	70	36	73	39.03	71	42	85	46.66
Sunflower	50	35.29	66	38.44	68	43	92	50.05
Windsorchair	51	35.29	74	38.44	73	43	84	50.05
Average	71	35.29	69.5	38.44	72.8	43	88.7	50.05

TABLE II

Comparison of the average precision obtained by the proposed similarity measures on the Caltech-101 database

Categories	BFNS		IT2BFNS1		IT2BFNS2		IT2BFNS3	
	Precision(%)	Recall(%)	Precision(%)	Recall(%)	Precision(%)	Recall(%)	Precision(%)	Recall(%)
Africa	73.3	2.05	66.5	7.49	72	7.95	89.02	20.03
Beach	59.5	12.05	60	4.04	65	9.40	70.5	14.05
Building	67.2	2.15	66.9	4.84	72	7.22	78.14	18.20
Bus	72.3	1.72	76	10.82	85	16.68	92.3	20.50
Dinosaur	99.5	19.93	96.2	18.72	98.8	19.83	98	23.47
Elephant	84.1	9.93	98	9.08	100	12.84	100	14.12
Flower	82.3	17.10	88	13.54	84	17.26	100	23.12
Horse	100	18.64	100	10.19	98	18.85	100	25.78
Mountain	66.8	9.83	58.85	4.52	80	10.51	82	14.95
Food	78.7	2.23	80.35	6.12	80	10.32	80	17.25
Average	78.34	9.65	79.03	8.94	83.48	13.09	88.65	19.14

TABLE III

Comparison of the average precision obtained by the proposed similarity measures for Corel-1k database

V. Conclusions and Future Work

In this paper, an Interval Type-2 Beta Fuzzy Sets approach to Content-Based Image Retrieval was presented. This work has improved the feature visual used in content-based image retrieval application by introducing the Interval Type-2 Beta function. The usage of fuzzy-similarity relations is proposed to ameliorate the performance of similarity for image retrieval. Three Interval Type-2 measures were proposed as an extension of some nearness ones. Mathematical Proves were done to show that the measures advanced are really proximity ones. The proposed measures are working on a list of features, unlike classification based CBIR which requires a large training set. Experimental results show that the proposed approach with IT2FBNS3 measure gives a higher performance that achieves a precision rate of almost 90% over the SIMPLIcity, Corel-1k, and Caltech-101 datasets and a precision rate of almost 65% over the big Image-Net datasets. The CBIR problem is still a challenging problem. One of the biggest challenges in CBIR is that, in many cases, there is little correlation between the low-level image features and high-level semantics. Indeed, current CBIRs still lack the accuracy of the relevant image due to incorrect selection of feature extraction methods. Consequently, implementing more visual features for comparing images and to incorporate more powerful feature extraction/selection methods is

Categories	BFNS		IT2BFNS1		IT2BFNS2		IT2BFNS3	
	Precision(%)	Recall(%)	Precision(%)	Recall(%)	Precision(%)	Recall(%)	Precision(%)	Recall(%)
Airliner	68	34.6	72.3	43	72	44.2	70.8	43.6
minivan	66.3	33.2	62.4	40	51.7	32.3	67.8	32.3
train	78	44.5	77	44.5	76	44.5	77	44.5
car	72.5	34.8	71	34.5	72	34.5	72	34.5
fire-engine	68	44.5	73	44.5	69.8	44.5	77.3	44.5
tank	77.4	38.2	75.6	38.2	87	38.2	78.2	38.2
army	72	44.2	68.2	38.2	70	44.2	72	38.2
musket	56.6	32.5	44.8	32.5	42.6	32.5	55.5	32.5
tiger	66	29.52	66.8	32.5	66	29.52	62.8	44.5
quail	54.3	34.08	60.73	29.8	58.9	32.5	57.8	32.5
tortoise	52	32.5	52	32.5	51.7	37.5	52.8	32.5
snake	49.2	32.5	52	32.5	52	32.5	50.8	32.5
ladle	72	37.5	73	42.6	72	42.6	73	42.6
box	66.7	37.5	66	42.6	56.8	43	68	37.5
pot	44.7	42.6	34.5	37.5	45.6	32.5	42.2	32.5
towel	70	37.5	68.2	32.5	72.8	43	74.5	32.5
dam	66.7	42.6	70	38.4	71	32.5	72.8	38.4
mountain	67	32.5	70	38.4	73	32.5	72	38.4
fisherman	51	35.29	64	38.4	66	38.4	67.7	35.29
volcano	48.2	35.29	54	38.4	52.7	32.5	52	32.5
Average	63.3	36.77	63.77	39.49	63.98	37.19	63.95	36.99

TABLE IV
Comparison of the average precision obtained by the proposed similarity measures on the Image-Net database

database	Performance	Similarity metrics		
		Manhattan	chi-Square	Euclidean
SIMPLIcity	Precision	40	40	69
	Recall	5	5	7.5
Corel-10k	Precision	43.35	46	36.83
	Recall	5.20	5.52	4.42
Caltech-101	Precision	39	29	65
	Recall	5.5	4	7.5

TABLE V
Comparison of average presicion and recall with different similarity metrics for the SIMPLIcity, corel-10k, caltech-101 databases

set	Results reported in [7]	Results reported in [8]	Results reported in [6]	Results reported in [5]	Results reported in [9]	Best result in our work
C0	88.05%	90%	74.80%	82%	90%	89.02%
C1	79.5%	60%	45.85%	60%	60%	70.05%
C2	67.65%	90%	58.00%	67%	90%	78.14%
C3	100%	75%	77.45%	95%	100%	92.3%
C4	100%	100%	99.75%	100%	75%	95%
C5	93.10%	70%	61.60%	95%	100%	100%
C6	100%	90%	80.55%	100%	70%	100%
C7	100%	100%	94.90%	100%	90%	100%
C8	77.75%	70%	45.05%	63%	100%	82%
C9	89.30%	90%	72.80%	71%	70%	80%
average	89.50%	83.5%	71.05%	71.05%	84.5%	88.65%

TABLE VI
Comparison of average presicion of each category between some techniques on the Corel-1k database

challenging. Also, the development a content-based image retrieval system architecture to support querying very large image databases for a wide variety of data sets in different domain will be good solution. Investigation of these problems is part of our future work.

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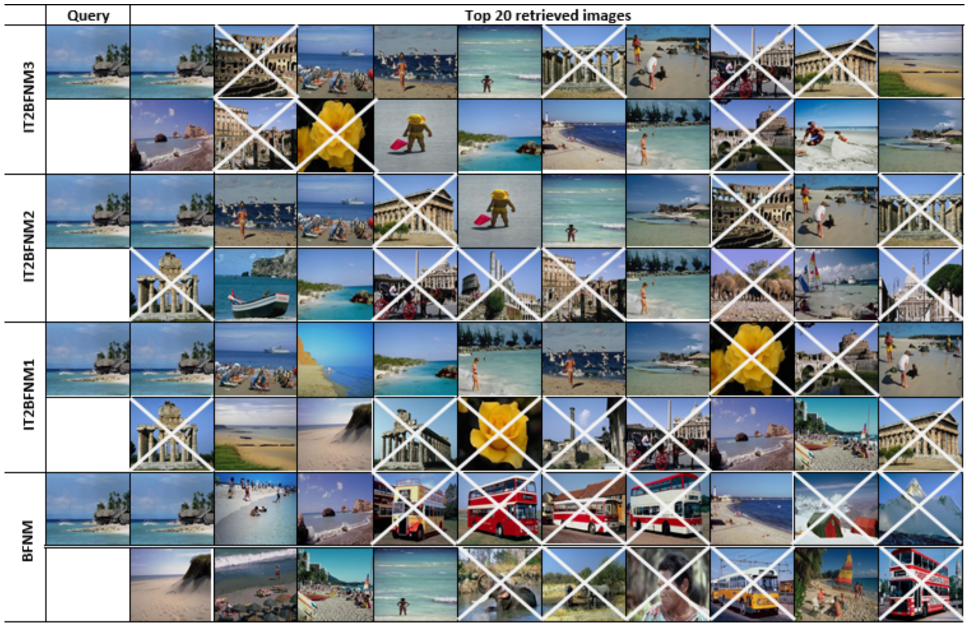
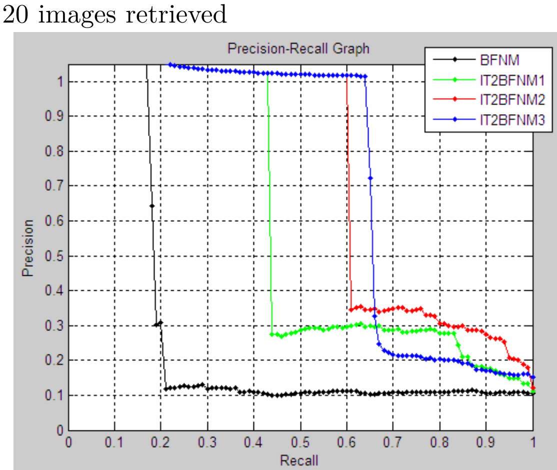


(c) first 20 images retrieved

(a) query image



(b) Precision/Recall curve

Fig. 3. Experiment results of the best image : SIMPLIcity database

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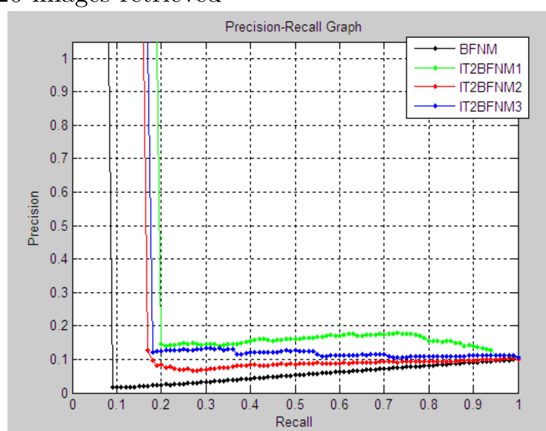
(c) first 20 images retrieved

(a) query image



(b) Precision/Recall curve

Fig. 4. Experiment results of an image : Image-Net database

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set	Results reported in [49]	Results reported in [45]	Results reported in [50]	Results reported in [17]	Best result in our work
C0	76.3%	83.8%	80%	66.35%	79.02%
C1	72.5%	73.6%	94%	52.45%	76.93%
C2	86.2%	85.9%	81%	43.85%	75.74%
C3	92.3%	71.4%	99%	69.85%	91.91%
C4	100%	100%	100%	100%	89.72%
C5	74.8%	69.05%	100%	94.70%	100%
C6	89.2%	100%	100%	83.75%	100%
C7	100%	100%	98%	78.75%	100%
C8	66.8%	58.85%	80%	64.25%	90%
C9	78.7%	80.35%	80%	71.3%	92.02%
average	76%	73%	80.35%	72.33%	88.2%

TABLE VII
COMPARISON OF AVERAGE PRESICION OF EACH CATEGORY BETWEEN SOME TECHNIQUES ON THE SIMPLICITY DATABASE

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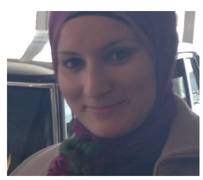

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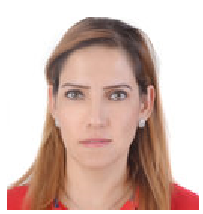

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