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Abstract

We consider a world in which individuals have private endowments and trade in markets, while their utility is sensitive to the consumption of their neighbors. Our interest is in understanding how social structure of comparisons, taken together with the familiar fundamentals of the economy – endowments, technology and preferences – shapes equilibrium prices, allocations and welfare.

We find that equilibrium prices and allocations depend on average individual centrality in the social network. As we add links to a social network, the centralities rise and this pushes up prices of the socially sensitive good. Newly linked agents demand more of the socially sensitive good, while the reverse happens with regard to the standard good. We derive a formula to compute the *critical link*, i.e., the new link which maximizes price increase.

We then turn to a model with heterogenous endowments, and find that inequality in network centrality and in wealth inequality reinforce each other. Thus a transfer of resources from less to more central agents raises prices of the socially sensitive good and alters allocations and utilities of all agents. We show by example that poor individuals lose utility while rich individuals gain utility as society moves from segregation to integration.

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1 Introduction

Production, consumption and trade take place at the intersection of society and markets. Traditionally, economists have concentrated on understanding the functioning of markets; in recent years, there has been a resurgence of interest in social interaction. This work has generated an array of models to study economic questions; for the most part, however, these models focus on a setting with pure social interaction and almost entirely abstract from prices and market competition.[1] As the field of social networks matures, we believe it is important to integrate them with traditional models of competition and exchange in markets. This general view informs the approach we take in the present paper to study the effects of relative consumption concerns.

One of the recurrent themes in the study of individual well being is that, in addition to own consumption, it appears also to depend on the consumption of others with whom we interact and compare ourselves. But the happiness and therefore the consumption of these "close by" others in turn depends on the consumption of their friends and so on. Individual decisions on consumption are therefore shaped by the overall pattern of connections which obtain in the society. In order to understand consumption and welfare we therefore need a framework which takes into account of social structure, along with the familiar fundamentals of the economy – endowments, technology and preferences.

In this paper, we propose a simple model which extends the classical model of general equilibrium to allow for social interaction. There is a finite group of individuals who all have identical initial endowments and are price takers. Each of the individuals is also directly affected by the consumption of one good by a *subset* of the other individuals, whom we shall refer to as her *neighbours*. The structure of individual neighbourhoods is modeled as a network. For a fixed set of prices, individual demand depends on consumption of neighbours, whose demand in turn depends on the demand of their neighbours and so on. This suggests that a change in the demand of agent i will have an effect on the demand of her neighbours which will in turn affect the demand of their neighbours, and these changes will in turn feedback on agent i. Thus a study of individual consumption requires us to take into account paths of different lengths via which individual agents are linked in the network.

Our first observation is that the direct and indirect effects of social interaction are captured by eigen vector centrality of the adjacency matrix describing the network.[2] Indeed, in equilibrium, individual consumption can be expressed as a function of her

[1] An important and early exception to this is Montgomery (1991), for recent work which seeks to integrate social structure and markets, see Cassella and Hanaki (2006) and Galeotti (2007). Goyal (2007) provides an overview of the recent research on social networks in economics.

[2] For an early discussion of such centrality measures in the social sciences, see Katz (1953) and Bonacich (1987).

centrality in a social network, while the price is proportional to the average network centrality of agents in the network. As we add links to a social network, there will be more paths between agents, the sum of centralities will rise and this pushes up prices of the socially sensitive good. We also derive a formula to compute the *critical link*, i.e., the new link which maximizes price increase. Roughly speaking, this formula says that, when social comparison effects are small, the critical link is one which connects the two least linked agents. With regard to allocations, we show that, as we add links to a network, the consumption of the socially sensitive good by the connected agents increases, while the reverse happens to non-connected agents.

These results are obtained in a benchmark model in which all agents have the same endowments. We then extend our model to allow for resource heterogeneity. Our main result shows that wealth and network heterogeneity reinforce each other: a transfer of resources from a poorly connected to a well connected agent raises prices and alters allocations and utility across the economy. The model with heterogeneity allows us to explore an interesting theme in the recent happiness literature: the role of shifting social interactions. We illustrate through examples, how poorly endowed individuals lose, while well endowed individuals gain, as we move from a society which is segregated along class lines to an integrated society.

These observations on effects of social networks on welfare naturally lead us to examine the stability of different social networks. We illustrate through an example that different structures ranging from the empty network to the complete network may be stable for given economic fundamentals. We interpret this finding as saying that a variety of different social norms with regard to conspicuous consumption may obtain across societies, which are otherwise identical.[3]

We finally discuss alternative specifications of consumption externalities, e.g., where the externalities depend solely on the average of neighbours, or the maximum of neighbours, or where externalities arise in the consumption of both goods. We find that in such economies there exists equilibria which are insensitive to patterns of social interaction.

There is a vast literature in economics, as well as in other disciplines, on the importance of relative consumption for individual well being. Perhaps, the best known early work is Veblen's (1897) critique of conspicuous consumption. In recent years, relative consumption concerns have been presented as the natural explanation to account for the Easterlin puzzle: the observation that happiness is positively related to incomes in a society at any point in time, but that increases in income in the society over time, appear to have little effect on happiness. Recent papers by Kuhn et al (2008) and Luttmer (2005) present clear empirical evidence in support of the role of social effects

[3]For an interesting recent treatment of endogenous groups in a model of social interactions, see Zanella (2007).

on personal consumption and happiness. Kuhn et al (2008) find that an increase in the incomes of neighbors have significant effects on individual consumption patterns, and moreover that these effects are stronger for immediate neighbors as compared to general neighborhood effects. Luttmer's (2005) work suggests that changes in the incomes of neighbors have effects on self-reported levels of individual happiness and moreover the magnitude of these effects depends on the frequency of interaction with the neighbors.

Over the years, a number of models on relative consumption concerns – both at a personal level (across different selves of an individual, over time) as well as at a social level (across different individuals) – have been proposed. See, e.g., Abel (1990), Arrow and Dasgupta (2007), Easterlin (1974), Cres, Ghiglino and Tvede (1997), Frank (1985), Frey and Stutzer (2002), Hopkins and Kornienko (2004), Layard (2005), Blanchflower and Oswald (2004), de Tella, MacCulloch and Oswald (2003), Veblen (1899) and Dussenbery (1949). While these models differ in many ways, they share one common feature: they suppose that individual utility or well being depends on own consumption and *average* social consumption. However, the motivation for relative consumption effects in this literature typically arises at a local level, i.e., when we compare our consumption with the consumption of friends, colleagues and relatives. This formulation of average consumption is thus restrictive on two counts: one, it precludes the study of size of effects which empirical work suggests are important. Two, the average formulation precludes the study of changing patterns of social interaction, a subject which has been the topic of public debate (see e.g. the discussion in Layard (2005)). These considerations motivate our attempt at developing a framework in which local consumption effects can be studied systematically.[4]

Our paper builds on two earlier papers, Ballester, Calvo-Armengol and Zenou (2006) and Tan (2006). Tan (2006) studies the effect of social networks in a general equilibrium model. There are two main differences between the papers. One, he looks at specific networks – such as star and regular networks – while we allow for arbitrary networks. Our study of general equilibrium in arbitrary networks is made possible due to our key result that the social network effects are summarized in a simple measure of network centrality. We are also able to provide a general set of results on the effects of endowment transfers in a setting with social interaction; to the best of our knowledge these results are new.

Ballester, Calvo-Armengol and Zenou (2006) study a game of social interaction and derive a relation between Nash equilibrium actions and network centralities. While our work uses similar measures of network centrality, the motivation of our paper and the principal findings are quite different from their work. We are interested

[4]There is also an interesting line of research which examines the effects of trading restrictions – modeled in terms of networks – on equilibrium outcomes. See e.g., Gale and Kariv (2007), Kranton and Minehart (2001), and Kakade, Kearns, and Ortiz (2005).

in the ways in which general equilibrium prices, individual consumptions and well being, are jointly shaped by social interaction and competitive markets. This leads us to focus on the ways in which social interactions alter prices, which in turn alter allocations and utilities. Moreover, in our paper, a key issue is how social structure and endowment heterogeneity complement each other in defining prices and utilities. On a technical note, we also note that our characterization of equilibrium obtains for all levels of social consumption effects. This is in contrast to Ballester et. al. (2006) result, as well as most of the literature that follows this paper which requires the social effects to be small.[5]

The rest of the paper is organized as follows: section 2 sets out the basic model of network based consumption externality and section 3 solves this model. Section 4 takes up the model with endowment heterogeneity. Section 5 presents some preliminary findings on stable social networks. Section 6 discusses some alternative formulations, while section 7 contains concluding remarks.

2 Model

We consider a pure exchange economy populated with N consumers, $i = 1, ..., N$. Let $N(i)$ be the set of neighbours of consumer i and let $n_i = |N(i)|$. There are two consumption goods. We denote by p the price of good 2, good 1 being the numeraire. Consumers care about their own consumption of good 1 and good 2. We note x_i the consumption of good 1 by agent i and y_i the consumption of good 2 by agent i. Consumers also care about the consumption of good 2 by their direct neighbours, i.e. consumer i cares also about $\{y_j\}_{j \in N(i)}$. Consumer i is endowed with a bundle of the two goods (ω_i, ν_i), where $\omega_i > 0$ and $\nu_i > 0$. We will represent the pattern of neighbourhoods by G, which is a $n \times n$ matrix of 1's and 0's. An $\{i, j\}$ square in this matrix takes value 1 if and only if i and j are direct neighbours. We will assume that in this matrix the diagonal terms are all set equal 0.

[5]After we had written our paper, we became aware of three new papers which study related ideas Bramoulle, Kranton and D'Amours (2008), Bloch and Querou (2008) and Mookherjee, Napel and Ray (2008). We briefly discuss the latter two papers, as they relate to social interactions and markets. Bloch and Querou (2008) study optimal discriminatory pricing by firms to consumers who experience consumption externalities with respect to their neighbors. They obtain a number of results which relate network centrality of individuals to optimal prices. Mookherjee, Napel and Ray (2008) study a model where families invest in the skills of children and these children then earn wages. The incentives to invest in human capital is related to the average wage in the local social environment. Their main results pertain to the existence of equilibrium in which communities with low and high human capital emerge due to due to the complementarities in the acquisition of human capital across created by the comparison of wages. So, while these papers deal with related themes the economic contexts they study are quite different. So we will not provide a detailed discussion of the results.

In order to model interpersonal comparisons in consumption we let the utility depends on own consumption in the two goods as well as the consumption of the neighbours in the second good. We will assume that:

$$U_i(x_i, y_i, y_{-i}) = u_i(x_i, \Phi(y_i, y_{-i})) \tag{1}$$

where $\Phi : R \times R^{n_i} \to R$ and $-i$ is the set of neighbours to agent i. The function Φ is increasing in y_i and decreasing in each element of the vector y_{-i}. It is also natural to assume that when all neighbours consume y_i, that is each component of y_{-i} is equal to y_i, then the effect of the neighbours vanishes, i.e. $\Phi(y_i, y_i) = y_i$. As we are concerned with interpersonal considerations a minimal specification is

$$\Phi(y_i, y_{-i}) = y_i - \alpha \sum_{j \in N(i)} (y_j - y_i) \tag{2}$$

where α is a real number, which may be interpreted as a measure of the strength of comparison (or externality). The case where $\alpha = 0$ corresponds to the benchmark no externality model. When $\alpha > 0$ individuals are negatively affected by the consumption of their neighbors. By contrast, $\alpha < 0$ corresponds to a positive externality. In our analysis we will focus on the case $\alpha > 0$.

Finally, we assume that u_i has the familiar form

$$u_i(x, y) = x^{\sigma} y^{1-\sigma} \tag{3}$$

with $0 < \sigma < 1$. The utility function can then be written as

$$u_i(x_i, y_i, y_{-i}) = u_i(x_i, y_i, \{y_j\}_{j \in N(i)}) = x_i^{\sigma} \left(y_i[1 + \alpha n_i] - \alpha \left[\sum_{j \in N(i)} y_j \right] \right)^{1-\sigma} \tag{4}$$

The specific form of the utility function is a consequence of the introduction of interpersonal comparisons. The function can be written in a way that allows for a more intuitive interpretation in which the effect of departures from the average is weighted by the number of neighbours, i.e. the social pressure. Indeed, we can write

$$u_i(x_i, y_i, y_{-i}) = u_i(x_i, y_i, \{y_j\}_{j \in N(i)}) = x_i^{\sigma} \left(y_i + \alpha n_i \left[y_i - \frac{1}{n_i} \sum_{j \in N(i)} y_j \right] \right)^{1-\sigma} \tag{5}$$ [6]

Remark: Our utility function may be derived from a more general specification when variations in consumption across individuals are small and individuals have

[6]For the use of a similar utility function in a model of growth and happiness, see Cooper, Garcia-Penalosa and Funk (2001).

identical characteristics. Indeed, subject to this condition we may linearize Φ with respect to y_{-i} near y_i and keep only the first order terms. We then have

$$\Phi(y_i, y_{-i}) \simeq \Phi(y_i, y_i) + \sum_{j \in N(i)} \partial_j \Phi(y_i, y_i)(y_j - y_i) \tag{6}$$

where $\partial_j \Phi(y_i, y_i)$ stands for the partial derivative of Φ in respect to jth neighbour's consumption, evaluated at the point where all consumptions are y_i, i.e. $y_{-i} = y_i$. As all agents are identical we may assume that $\partial_j \Phi(y_i, y_{-i}) = -\alpha$ where $-\alpha$ is the common value of all the partial derivatives of Φ with respect to $-i$.

We emphasize that in the above formulation both the individual consumption as well as the size of the total consumption of neighbors matters. This is consistent with some recent empirical work by Luttmer (2005). In this paper, the author emphasizes the role of neighbourhoods in shaping individual welfare. In particular, Table 8 in his paper shows that increasing the number of meetings with neighbours matters for the magnitude of the effect of neighbours. This empirical work does not distinguish between more meetings with the same neighbors and a larger number of neighbors. This suggests that it is the overall magnitude of difference between personal consumption and consumption of neighbors which may be more appropriate.

While we believe that our formulation is consistent with empirical work, we are aware that there is no consensus on the form of interpersonal comparisons of consumption. We therefore feel that it is important to examine the effects of alternative formulations of interpersonal comparisons. Section 6 presents our analysis of a number of alternative formulations.

We will suppose that good x is the numeraire good and that the price of good y is p. A consumer i's optimization program reads

$$\begin{array}{cl} \max_{(x_i, y_i)} & u_i(x_i, y_i, \{y_j\}_{j \in N(i)}) \\ s.t. & x_i + p y_i = \omega_i + p \nu_i \end{array} \tag{7}$$

Let $(\widehat{x}_i, \widehat{y}_i)$ solve this problem.

A general equilibrium is a strictly positive price p and a vector of allocations $(\widehat{x}_i, \widehat{y}_i)_{i \in N}$ such that

1. Markets clear: $\sum_{i \in N} \widehat{x}_i = \sum_{i \in N} \omega_i$, $\sum_{i \in N} \widehat{y}_i = \sum_{i \in N} \nu_i$.
2. For each $i \in N$, $(\widehat{x}_i, \widehat{y}_i)$ solves the optimization problem outlined above.

We will study equilibrium in which $x_i > 0, y_i > 0$ as well as $y_i + \alpha n_i \left[y_i - \frac{1}{n_i} \sum_{j \in N(i)} y_j \right] > 0$ for all i.

The first order conditions associated with the optimization problem are

$$\begin{aligned} 0 &= \sigma x_i^{\sigma-1}\left(y_i[1+\alpha n_i] - \alpha\left[\sum_{j\in N(i)} y_j\right]\right)^{1-\sigma} - \lambda \\ 0 &= x_i^{\sigma}(1-\sigma)[1+\alpha n_i]\left(y_i[1+\alpha n_i] - \alpha\left[\sum_{j\in N(i)} y_j\right]\right)^{-\sigma} - \lambda p \\ 0 &= \omega_i + p\nu_i - x_i - py_i \end{aligned} \tag{8}$$

For fixed prices, how does a neighbor's consumption of the socially sensitive good affect the marginal returns on own consumption of y? We note from the second line in (8) that for fixed n_i, marginal utility to y_i is clearly increasing in consumption of y by a neighbor. Next we consider the impact of an additional neighbor. It is easily seen that so long as consumption by the new neighbor is equal or larger than current own consumption, the marginal utility from consumption of y_i will increase. But, with a bit of algebra, we can also see that if the consumption of the neighbor is significantly smaller than current own and current neighbors consumption then the marginal utility from y_i may actually fall upon the addition of a new neighbor.

We observe that in our model the first order conditions are necessary for an interior optimum. We will restrict attention to interior solutions of the maximization problem faced by individuals.[7] The demands for goods 1 and 2 are given by:

$$f_i^1(p, \{y_j\}_{j\in N(i)}) = \sigma\left(\omega_i + p\nu_i - \frac{\alpha}{1+\alpha n_i} p \sum_{j\in N(i)} y_j\right) \tag{9}$$

$$f_i^2(p, \{y_j\}_{j\in N(i)}) = \frac{1-\sigma}{p}\left(\omega_i + p\nu_i + \frac{\sigma}{1-\sigma}\frac{\alpha}{1+\alpha n_i} p \sum_{j\in N(i)} y_j\right) \tag{10}$$

We can use equation (10) to obtain for each i:

$$y_i - \frac{\alpha\sigma}{1+\alpha n_i}\sum_{j\in N(i)} y_j - \frac{1-\sigma}{p}(\omega_i + p\nu_i) = 0 \tag{11}$$

This can be rewritten as

$$y_i - \frac{\alpha\sigma}{1+\alpha n_i} G_i \cdot Y - \frac{1-\sigma}{p}(\omega_i + p\nu_i) = 0 \tag{12}$$

[7] We note that the optimum will be interior if endowments are positive, prices are positive and relative consumption effects are sufficiently small.

where Y is the N-dimensional vector of good 2 consumption, G_i is the ith row of the $N \times N$ matrix of connections, i.e. the adjacency matrix.

The demands y_i for all consumers may be expressed in matrix form as

$$\left[I - \alpha\sigma G^N\right] Y - \frac{1-\sigma}{p} W = 0 \tag{13}$$

where I is the identity matrix and G^N is the $N \times N$ matrix of connections in which every row is normalized so that the sum of the elements add to $\frac{n_i}{1+\alpha n_i}$, and W is the N-dimensional vector of individual wealth.

Whenever $\left[I - \alpha\sigma G^N\right]$ is invertible we can write the demand for good y as:

$$Y = \frac{1-\sigma}{p} \left[I - \alpha\sigma G^N\right]^{-1} W \tag{14}$$

3 Homogeneous agents

In this section we will study the case where all individuals have the same preferences and the same endowments. In this benchmark case, with no social interaction, the equilibrium is unique and characterized by no trade and the price is simply given by a ratio of endowments and the relative importance assigned by individual preferences to the two goods. How does social interaction affect equilibrium prices and allocations? Our first finding is that general equilibrium prices can be expressed as a function of the average centrality of the network, and that an individual's consumption is proportional to her network centrality. We then examine the effects of changing networks. We show that adding a link to a network always raises price of the socially sensitive good and a link between i and j alters their equilibrium consumption of the goods in proportion to their centrality in the initial network. We illustrate the quantitative significance of these effects with the help of numerical examples.

Since endowments are identical, the wealth of an agent is given by

$$W_i = \omega_i + p\nu_i = \omega + p\nu \tag{15}$$

which means that

$$W = (\omega + p\nu)J \tag{16}$$

where J is the N-dimensional vector of ones. We can now use equation 14 to obtain:

$$Y = \frac{1-\sigma}{p} \left[I - \alpha\sigma G^N\right]^{-1} J(\omega + p\nu) \tag{17}$$

We now introduce our concept of network centrality.

Definition 1 *Let G^N be the $N \times N$ adjacency matrix in which a row i is normalized by $\frac{1}{1+\alpha n_i}$, where n_i is the degree of agent i. Then we define the centrality vector B by*

$$B = \left[I - \alpha\sigma G^N\right]^{-1} J \tag{18}$$

where J is the N dimensional vector of ones.

When $\alpha\sigma$ is smaller than the inverse of the modulo of the largest eigenvalue of G^N, the inverse $\left[I - \alpha\sigma G^N\right]^{-1}$ can be expressed as a power series

$$\left[I - \alpha\sigma G^N\right]^{-1} = \sum_{s=0}^{\infty} \left(\alpha\sigma G^N\right)^s \tag{19}$$

In our model, observe that the condition for convergence is always met. This is because, from the Perron-Frobinius Theorem we know that an eigen value is less than the maximum sum across all rows. In our case, it can be checked that the maximum across all rows is indeed smaller than σ. By assumption $\sigma < 1$ and so the series converges. This is an important point: in Ballester, Calvo-Armengol and Zenou (2006), convergence has required that local effects be small, i.e., α be sufficiently small. In our framework, due to the normalization, implicit in the definition of the matrix G^N, we do not require any assumptions on the magnitude of the local effect.

The element (i, j) of this matrix can be written as

$$\left\{\left[I - \alpha\sigma G^N\right]^{-1}\right\}_{(i,j)} = \sum_{s=0}^{\infty} (\alpha\sigma)^s \{(G^N)^s\}_{(i,j)} \tag{20}$$

where $\{(G^N)^s\}_{(i,j)}$ counts the number of paths starting in j and ending at i of length s, weighted by the factors defined in Definition 1. This expression provides a nice interpretation of centrality in terms of interactions with neighbours of increasing distance. In fact the centrality of an individual B_i, reflects the weighted sum of paths of all the different possible lengths.

We now have all the notation and concepts needed to state our first result which characterizes the relation between market equilibrium and social interaction. Let $\overline{B} = \frac{1}{N}\sum_{k=1}^{N} B_k$.

Proposition 1 *There exists an interior equilibrium. In this equilibrium, the allocations of goods for individual i are*

$$x_i = \omega \frac{1 - (1-\sigma)B_i}{1 - (1-\sigma)\overline{B}} \tag{21}$$

$$y_i = B_i \frac{1-\sigma}{p}(\omega + p\nu) = \frac{B_i}{\overline{B}}\nu \tag{22}$$

while the price is given by

$$p = \frac{\omega}{\nu}\left[\frac{1}{1-\sigma}\frac{1}{\overline{B}} - 1\right]^{-1} \tag{23}$$

Proof: The proof of existence is constructive. From equations (17) and (18) it follows that individual allocations are collinear in B:

$$Y = B\frac{1-\sigma}{p}(\omega + p\nu) \tag{24}$$

In order to find the price p that equates supply and demand we solve the market clearing equation:

$$\sum_{i=1}^{N} y_i = \frac{1-\sigma}{p}(\omega + p\nu)\sum_{i=1}^{N} B_i = N\nu \tag{25}$$

so that

$$\frac{1-\sigma}{p}(\omega + p\nu) = \frac{N\nu}{\sum_{k=1}^{N} B_k} \tag{26}$$

Therefore

$$Y = \frac{B}{\sum_{k=1}^{N} B_k} N\nu = \frac{B}{\overline{B}}\nu \tag{27}$$

with $\overline{B} = \frac{1}{N}\sum_{i=1}^{N} B_i$. Equivalently we have

$$y_i = \frac{B_i}{\sum_{k=1}^{N} B_k} N\nu = \frac{B_i}{\overline{B}}\nu \tag{28}$$

Finally, from equation 26 note that the equilibrium price is given by the equation

$$(1-\sigma)\frac{\omega}{p} = \frac{N\nu}{\sum_{i=1}^{N} B_i} - (1-\sigma)\nu \tag{29}$$

leading to

$$p = \frac{\omega}{\nu}\left[\frac{1}{1-\sigma}\frac{N}{\sum_{i=1}^{N} B_i} - 1\right]^{-1} \tag{30}$$

From

$$x_i = \omega + p\nu - py_i \tag{31}$$

we get

$$
\begin{aligned}
x_i &= \omega + \frac{\omega}{\nu}\left[\frac{1}{1-\sigma}\frac{N}{\sum_{k=1}^{N} B_k} - 1\right]^{-1}\left[\nu - \frac{B_i}{\sum_{k=1}^{N} B_k} N\nu\right] \\
&= \omega\frac{N-(1-\sigma)NB_i}{N-(1-\sigma)\sum_{k=1}^{N} B_k} = \omega\frac{1-(1-\sigma)B_i}{1-(1-\sigma)\overline{B}}
\end{aligned} \tag{32}
$$

At this stage it is necessary to check that this allocation is indeed an equilibrium. Under our hypothesis on $\alpha\sigma$, $B_i > 0$ and $\overline{B} > 0$. We need to verify that $p > 0, x_i > 0, y_i > 0$ and $y_i(1+\alpha n_i) - \alpha\sum_{j\in N(i)} y_j > 0$ for all i. These inequalities are verified in the Appendix.

■

This result explains how markets and social structures jointly shape prices, allocations and welfare. We first illustrate the effect of social structure on equilibrium outcomes and welfare. We note that, in our model, the addition or deletion of a neighbor alters the utility function of the individual, making utility comparisons difficult. We will therefore restrict ourselves to comparing utilities of agents whose neighborhood, and hence preferences, remain unchanged.

Example 1 *Social embeddedness and economic outcomes*

In this economy all agents has an endowment of 10 units of either good and $\alpha = \sigma = 0.5$. Figure 1 presents four standard networks: empty, star, ring, and core-periphery. Figure 2 summarizes information on market equilibrium prices, allocations and utilities. We would like to bring out two points: one, social structure has significant effects on prices and allocations. A move from the empty network to the ring raises prices by a 100%, but has no effects on allocations. On the other hand, moving from the ring to the star lowers prices by almost 12% and leads to 16% increase in consumption of the socially sensitive good and a 29% decrease in consumption of standard good by the central agent. Our second point is that social structure has substantial effects on welfare: the utility of the peripheral agents in the star and the core-periphery network (with one neighbor each) are very different.

△

We now discuss the role of market interaction in shaping equilibrium outcomes.

Example 2 *No man is an island: General equilibrium effects*

In this economy all agents have an endowment of 10 units of either good and $\alpha = \sigma = 0.5$. Figure 3 presents four networks with progressively increasing links but they all share the feature that agent 8 is isolated. Figure 4 summarizes information on market equilibrium prices, allocations and utilities. We would like to bring out two points: one, as the connected part of society gets more densely linked, the price of the socially sensitive good steadily increases while its consumption steadily moves away from the isolated agent and toward the more connected individuals. Two, we observe that the utility of the isolated agent actually increases from 10.44 in the initial network all the way to 11.36 when the core social group constitutes a complete hexagon.

△

First, we observe that equilibrium price depends on the average centrality: networks with higher average centrality will exhibit higher prices for the socially sensitive good. We also observe that prices are not affected by the distribution of centralities. This is an aspect of the linear structure of the model. Moreover, equilibrium price is increasing in the importance of the socially sensitive good, which is reflected in the value of $1 - \sigma$.

Second, we note that the consumption of socially sensitive good y_i is proportional and increasing in the centrality of agent i. Correspondingly, the consumption of the standard good x decreases with the centrality. We now comment on the relation between degree and consumption. One may expect that a higher degree individual will be led to compare himself with more neighbors and this will push him toward higher consumption of the socially sensitive good. While this is clearly true, it is also the case that the consumption of the neighbors is in turn affected by their degree and so forth. The centrality measure captures this indirect effect of the structure of interaction. These considerations are reflected in the following recursive formulation of the centrality measure:

$$B_k = 1 + \alpha\sigma \frac{1}{1 + \alpha n_k} \sum_{j \in N(k)} B_j \tag{33}$$

The consumption y_i of the socially sensitive good by agent i is increasing in the number of direct neighbours and also positively affected by the centrality of these neighbors. We observe that individual consumption of good 1 is larger than initial endowment if and only if centrality is below the average, i.e. $\sum_{k=1}^{N} B_k > NB_i$ or $\frac{1}{N}\sum_{k=1}^{N} B_k > B_i$. The reverse is true for good 2. The intuition is that the more central an agent is, the more she consumes of good 2 in order to cope with the larger negative externality.

Changing networks: Our characterization of prices in Proposition 1 tells us prices are an increasing function of average network centrality. Adding links in a network raises paths between players and this raises average centralities, which in turn raises raises prices. The following result summarizes these ideas.

Proposition 2 *For small α, starting from any network $G \neq G^c$, an addition of a link raises p; it therefore follows that this price is minimized in the empty network and maximized in the complete network.*

Adding links clearly increases number of paths between any two agents, and this pushes towards greater centrality for all individuals. In our setting, it also alters the weights of the paths as the number of neighbors also appears in the denominator of the interaction matrix G^N. This presents some technical problems which complicate the argument significantly and we are obliged to restrict attention to small values of α. The details of the computations are provided in the appendix.

We now examine the nature of the critical link: this is the link which has maximum impact on price of socially sensitive good. From Proposition 1 and the discussions above, this is the link which increases the sum (or average) centralities in the network. we have been unable to obtain a general result for arbitrary levels of social spillover, but we are able to say something for small α.

Proposition 3 *Fix a network $g \neq g^c$. For sufficiently small α, the critical link g_{ij} is the solution to the following problem:*

$$\max_{ij} \frac{\alpha\sigma}{(1+\alpha(n_i+1))\,(1+\alpha n_i)} + \frac{\alpha\sigma}{(1+\alpha(n_j+1))\,(1+\alpha n_j)} \tag{34}$$

This result says that for instance in any network g with two or more isolated agents, a critical link would be between a pair of isolated agents. This result is somewhat counter-intuitive, as the first guess as the location of a critical link would be that it should connected agents with high degree. There are two reasons underlying our result: one, we focus on sufficiently small α, which allows us to ignore second and higher order effects of a link. two, in our model the increment to centrality of two connected agents varies inversely with their current degree, due to the normalization involved in the construction of the adjacency matric G^N.

We now examine how new links affect individual demands. To gain some intuition for the forces at work, consider the case where individuals are located around a circle and a link is added between two individuals i and j. In the initial network all individuals are in a symmetric situation and so the first effect of the additional link is that it increases the marginal return from increasing consumption of good 2 for both i and j. Such an increase in turn leads creates pressure on the demands for good 2 from the neighbors of i and j and the effects may be rather large depending on the centrality of these individuals. However, when α is very small, these second order or indirect social effects on the neighbors of i and j are relatively small and the first order direct effects prevail. The following result summarizes our analysis of the effects of additional links on equilibrium.

Proposition 4 *Suppose a new link is added between two agents i and j. There is an $\hat{\alpha} > 0$ such that for $\alpha < \hat{\alpha}$, there is an increase in the consumption of the socially sensitive good and a decrease in the consumption of the standard good by both i and*

j. Correspondingly, the consumption of the socially sensitive good decreases while the consumption of the standard good increases, for all agents $h \neq i, j$

The following example illustrates the effects of an additional link on equilibrium prices and allocations.

Example 3 *The critical link.*

In Figure 5 we illustrate two ways of adding links, in one case we add a link between two central agents, while in the second case we add a link between two spoke players. The parameter values and effects of link are given in Figure 6: we find that adding a link between the hub agents has a lower impact on prices as compared to adding link between the spoke players. Similar relative ranking also obtains under different values of α.

$\triangle$

4 Wealth heterogeneity

In the previous section we studied the effects of social interaction in a setting with identical agents. This section explores the ways in which heterogeneity in endowments is mediated via the social network and how they together shape market equilibrium and allocations. The principle insight of this analysis is that heterogeneities in endowments and network centrality are complementary in their effects, i.e., the wealth of an individual affects equilibrium prices and allocations in proportion to her centrality! We exploit this property to show that redistributions of wealth across individuals with different network centralities have significant price and allocation effects. By contrast, in the absence of social interaction, such redistributions have no effect on equilibrium prices.

Recall that the demand for the socially sensitive good is given by:

$$Y = \frac{1-\sigma}{p}\left[I - \alpha\sigma G^N\right]^{-1} W \tag{35}$$

Let the (column) vector of ω_i be denoted Ω and let Ψ denote the (column) vector of ν_i. Then, we have

$$W = \Omega + p\Psi \tag{36}$$

If then we note $M = \left[I - \alpha\sigma G^N\right]^{-1}$ we get

$$Y = \frac{1-\sigma}{p} M(\Omega + p\Psi) \tag{37}$$

$$= \frac{1-\sigma}{p} M\Omega + (1-\sigma)M\Psi \tag{38}$$

Note that $M\Omega$ is the network centrality weighted by the endowments of good ω and we shall refer to it as $\omega-$ centrality and denote it by B_ω. Similarly $M\Psi$ is the $\nu-$centrality and we denote it by B_ν. Note that we recover the homogeneous case by setting $M\Omega = MJ\omega = B\omega$ and $M\Psi = MJ\nu = B\nu$ where B is as in Definition 1 above. The equilibrium price vector can be obtained from the market clearing condition

$$\sum_{i=1}^{N} y_i = \sum_{i=1}^{N} \nu_i \Longleftrightarrow Y'J = \Psi'J \tag{39}$$

We get

$$\begin{aligned}\Psi'J &= \frac{1-\sigma}{p}\left[M(\Omega + p\Psi)\right]'J \\ &= \frac{1-\sigma}{p}\left[M\Omega\right]'J + (1-\sigma)\left[M\Psi\right]'J\end{aligned}$$

so that

$$p = \frac{(1-\sigma)\left[M\Omega\right]'J}{\Psi'J - (1-\sigma)\left[M\Psi\right]'J} \tag{40}$$

Finally, the equilibrium allocation of the socially sensitive good is

$$Y = \frac{\Psi'J - (1-\sigma)\left[M\Psi\right]'J}{\left[M\Omega\right]'J} M\Omega + (1-\sigma)M\Psi \tag{41}$$

while the allocation in the non-socially sensitive good can be obtained from

$$X = W - pY \tag{42}$$

Let R_ν denote the total endowment of the socially sensitive good and let R_ω denote the total endowment of the other good. We summarize our discussion in the following result.

Proposition 5 *In an interior equilibrium the allocation of individual i is given by:*[8]

$$y_i = \frac{R_\nu - (1-\sigma)\sum_{i\in N} B_{\nu,i}}{\sum_{i\in N} B_{\omega,i}} B_{\omega,i} + (1-\sigma)B_{\nu,i}. \tag{43}$$

$$x_i = \omega_i + \frac{(1-\sigma)B'_\omega J}{R_\nu - (1-\sigma)B'_\nu J} v_i) - \frac{(1-\sigma)B'_\omega J}{R_\nu - (1-\sigma)^2 B'_\nu J}[B_{v,i} + B_{w,i}) \tag{44}$$

Furthermore, the equilibrium price is given by

$$p = \frac{(1-\sigma)B'_\omega J}{R_\nu - (1-\sigma)B'_\nu J} \tag{45}$$

[8] As in the basic model, the inverse of the adjacency matrix is well defined; an equilibrium exists when the 'net' consumption of the socially sensitive good and prices are positive.

We observe that the sum of $\omega-$centralities can be expressed as $\sum_i \sum_j M_{ij}\omega_j$. The above result says that the price of the socially sensitive good is increasing in the sum of $\omega-$centralities. This is intuitive: this increase may be caused by an increase in endowments or an an increase in the network centrality per se. An increase in endowment of the standard good raises incomes and leads to an increase in demand for the socially sensitive good; the price of the socially sensitive good has to increase to offset this increased demand. Similarly, keeping endowments of the standard good fixed, an increase in centralities of agents increases demand for the socially sensitive good and this necessitates an increase in equilibrium prices (as endowments are constant). The effects of changes in $\nu-$ centrality are more complicated. It can be checked that an increase in $\nu-$ centralities caused by pure network changes will raise the price of the socially sensitive good. However, an increase in endowment of some agents will lower price. The magnitude of this effect on price will depend on the centrality of the agents whose endowments have been altered: in particular, the fall in price is smaller the larger the centrality of the agents who are given more endowments. Similarly, we observe that the equilibrium allocations are related to the weighted centralities of agents. In particular, the consumption of the socially sensitive good is increasing in the ω-centrality as well as the ν-centrality of an agent, while the converse is true for the equilibrium allocation of the standard good, *ceteris paribus*.

We next consider the effect of redistributions in endowments. In our model, with *no* consumption externalities a redistribution leaves the price unchanged. We can deduce this from the formula for prices in Proposition 5 and noting that B_w and B_v will simply be equal to the aggregate endowments of these two goods, respectively, when $\alpha = 0$. When $\alpha > 0$, social interaction has significant effects on prices: a transfer from a less central to a more central agent will raise prices, the transfer in the reverse direction will lower prices. It is useful to define $\tilde{M}_i = \sum_j M_{ji}$, as the weighted sum of all paths of all lengths from all agents to agent i. The following result summarizes these observations regarding the effects of a redistribution in endowments.

Proposition 6 *Suppose $\tilde{M}_q > \tilde{M}_{q'}$. The price of a socially sensitive good is increasing in transfers from agent q' to q and decreasing in transfers from q to q'. There are no price effects of a transfer from q' to q, if $\tilde{M}_q = \tilde{M}_{q'}$.*

Proof: Recall from equation (45) that prices are given by

$$p = \frac{(1-\sigma)B'_\omega J}{R_\nu - (1-\sigma)B'_\nu J} \tag{46}$$

where $B'_\omega J = \sum_{i=1}^N \sum_{j=1}^N M_{ij}\omega_j$, and $B'_\nu J = \sum_{i=1}^N \sum_{j=1}^N M_{ij}\nu_j$. Let us consider a transfer Δ of the socially sensitive good from q' to q. We will focus on the term $B'_\nu J$, as all other terms remain unchanged.

Define initial endowment distribution as (ω, ν), and the new endowment distribution as (ω, ν'). Note that $\nu_i = \nu'_i$ for all agents except q and q', where $\nu'_q = \nu_q + \Delta$ while $\nu'_{q'} = \nu_{q'} - \Delta$. $B'_\nu J$ can be written as

$$\sum_{i=1}^{N}\sum_{j=1}^{N} M_{ij}\nu_j = \sum_{i\neq q,q'}\sum_{j\neq q,q'} M_{ij}\nu_j + \sum_{j} M_{qj}\nu_j + \sum_{j} M_{q'j}\nu_j + \sum_{i\neq q,q'} M_{iq}\nu_i + \sum_{i\neq q,q'} M_{iq'}\nu_i \tag{47}$$

In the same way, we can write the post transfer weighted centrality $B'_{\nu'}J$ as:

$$\sum_{i=1}^{N}\sum_{j=1}^{N} M_{ij}\nu'_j = \sum_{i\neq q,q'}\sum_{j\neq q,q'} M_{ij}\nu'_j + \sum_{j} M_{qj}\nu'_j + \sum_{j} M_{q'j}\nu'_j + \sum_{i\neq q,q'} M_{iq}\nu'_q + \sum_{i\neq q,q'} M_{iq'}\nu'_{q'} \tag{48}$$

However, note that $\nu_i = \nu'_i$, for all $i \neq q, q'$. So we can rewrite (48) as follows:

$$\begin{aligned}\sum_{i=1}^{N}\sum_{j=1}^{N} M_{ij}\nu'_j \;=\;& \sum_{i\neq q,q'}\sum_{j\neq q,q'} M_{ij}\nu_j + \sum_{j\neq q,q'} M_{qj}\nu_j + M_{qq}\nu'_q + M_{qq'}\nu'_{q'} + \\ & \sum_{j\neq q,q'} M_{q'j}\nu_j + M_{q'q'}\nu'_{q'} + M_{qq'}\nu'_q + \sum_{i\neq q,q'} M_{iq}\nu'_q + \sum_{i\neq q,q'} M_{iq'}\nu'_{q'}\end{aligned}$$

Comparing equations (47) and the previous equation, we can say that $B'_{\nu'}J > B'_{\nu}J$ if and only if

$$\begin{aligned}& M_{qq}\nu'_q + M_{qq'}\nu'_{q'} + M_{q'q}\nu'_q + M_{q'q'}\nu'_{q'} + \sum_{i\neq q,q'} M_{iq}\nu'_q + \sum_{i\neq q,q'} M_{iq'}\nu'_q \\ > \;& M_{qq}\nu_q + M_{qq'}\nu_{q'} + M_{q'q}\nu_q + M_{q'q'}\nu_{q'} + \sum_{i\neq q,q'} M_{iq}\nu_q + \sum_{i\neq q,q'} M_{iq'}\nu_{q'}\end{aligned} \tag{49}$$

This inequality holds if and only if $\tilde{M}_q = \sum_i M_{iq} > \sum_i M_{iq'} = \tilde{M}_{q'}$.

■

The above result allows us to make a number of observations. First observe that the arguments in the proof tell us that the magnitude of change in sum of weighted centralities is proportional to the relative centralities of the agents directly involved in the transfer of endowments: indeed it is easy to see that $B'_{\nu'}J - B'_{\nu}J = \Delta[M_q - M_{q'}]$, so that the larger the difference in centralities, the larger the effect on price. Moreover, a transfer between two agents i and j with the same centrality has no effect on prices. Since the endowments of all agents other than i and j are unaffected by this transfer, and the prices remain unchanged, it follows that the equilibrium allocations of all these agents are unaffected as well. Matters are significantly more complicated when the centralities of agents are different as there are two effects to take into account: the change in aggregate weighted centrality as well as the change in the weighted centrality of specific individuals.

We now study numerical examples to quantify the magnitude of the price effects and also to determine the utility implications of wealth and network differences across agents.

Example 4 *Wealth and centrality are complementary.*

Let $\alpha = 0.5$ and $\sigma = 0.1$. We suppose that the network consists of two distinct components, each with 4 agents forming a ring. We assume also that there is a link between agents 1 and 5 as represented in network 2 of Figure 7. We suppose that 2 of the agents are rich, they each have 3 units of each goods. The remaining 6 agents have each 1 unit of each good. We compare the situation in which the 2 rich are the least central, agents 3 and 7, respectively, to the situation in which the rich are very central, agents 1 and 5, respectively. We can interpret this as an instance of transfer of resources from less central to more central agents. The results are reported in Figure 8. Then Proposition 6 tells us that price of the socially sensitive good will increase; the example shows that the price change and the consumption patterns is substantial. The price increases by approximately 30%. Finally, we observe that the resource transfer has large utility effects: wealth differences are reinforced by centrality, so that rich agents have a higher utility when they are more central, while poor agents, such as 2 and 6, are worse off when the rich agents are more central!

$\triangle$

Example 5 *Integrated and segregated societies.*

Let $\alpha = 0.5$ and $\sigma = 0.5$. Again we consider a network with two distinct components, each with 4 agents. These networks are given in Figure 9. Suppose are 4 rich players with endowments of 10 units of the standard good and 2 units of the socially sensitive good. The 4 poor players have 5 units and 1 unit of the two goods, respectively. We consider two types of societies, segregated (in which the poor and rich live in different components) and integrated (in which the rich and poor live are mixed in the same component). The results are reported in Figure 10. The main insight is that the utility of the poor agents is significantly lower in integrated communities as compared to segregated communities; the converse is true for rich agents!

$\triangle$

The recent literature on happiness highlights the role of changing social comparisons in understanding relatively low increases in happiness over time, see e.g., Layard (2005). The previous example shows how changing neighbors does have interesting and powerful effects on utility levels: in particular, substituting a poor neighbor by a rich neighbor unambiguously lowers the utility of a person.

The findings on utility of rich and poor under segregation and integration suggest that rich agents will desire links with the poor, who will in turn try and avoid the rich! In a world where link formation requires consent on the part of both agents, this suggests that a society segregated by economic class may well be stable. We briefly explore this idea in section below.

5 Choosing the right neighbours

Our results show that social structure has an impact on prices, allocations and welfare. In this section we examine the incentives of agents to form and sustain social relations. The idea we explore is a simple one: a social network is not incentive compatible if an agent can improve his lot by severing a link or if a pair of agents can improve their welfare by forming a tie. We emphasize that we are implicitly assuming that a link reflects a social relation and is therefore bilateral in nature and requires mutual agreement. Following Jackson and Wolinsky (1996), we say that a network g is *pairwise stable* if no agent wishes to delete any link and if no pair of agents gains by forming a link.

In our framework, welfare comparisons associated with the addition or deletion of a link are delicate because they the set of neighbors changes, which in turn alters the utility function. One way to move on this would be to follow Maccheroni, Marinacci and Rustichini (2008), who consider inter-dependent preferences in which the composition of the neighborhood changes. We would then have to show that our utility function is a representation of their preference order. We decided that this will take us too far afield; so here we will simply assume that the choice is determined by the value of the utility as described in equations (4) and (5). Keeping this qualification in mind, we are able to state the following preliminary result.

Proposition 7 *For small α, the empty network is pairwise stable.*

Proof: In the empty network the equilibrium is characterized by no trade: so $x_i = \omega$ and $y_i = \nu$, while the utility is given by $\omega^\sigma \nu^{1-\sigma}$. Denote by $p(g^e)$ the equilibrium price under the empty network. Let us examine the incentives to form a link for an agent i: if i were to form a link with some agent j, then we know from Proposition 2 that, for small α, in network $g = g^e + g_{ij}$ the equilibrium price $p(g)$ will be higher. At the same time, the centrality of i will increase relative to the others, and so Proposition 1 tells us his consumption (x_i, y_i) will satisfy $x_i < \omega$ and $y_i > \nu$. Note that by symmetry, $x_j = x_i$ and $y_j = y_i$. So the utility of agent i in network g is $x_i^\sigma y_i^{1-\sigma}$. Observe next that the new allocation lies on the new budget, which is strictly under the old budget line, for all bundles north-west of the initial endowments (ω, ν). It then follows that there exists a bundle $(\hat{x}, \hat{y})$ with $\hat{x} > x_i$, and $\hat{y} > y_i$, such that $\hat{x} + \hat{y}p(g^e) < \omega + \nu p(g^e)$. The bundle $(\hat{x}, \hat{y})$ lies in the interior of the budget set with price $p(g^e)$, but agent i chooses (ω, ν), so it follows that $\omega^\sigma \nu^{1-\sigma} > \hat{x}^\sigma \hat{y}^{1-\sigma}$. Thus we have shown that:

$$x_i^\sigma y_i^{1-\sigma} < \hat{x}^\sigma \hat{y}^{1-\sigma} < \omega^\sigma \nu^{1-\sigma}. \tag{50}$$

In other words, agent i strictly loses by forming a link. Thus the empty network is pairwise stable.

■

We have been unable to characterize all stable networks. The complication we face is that when two agents form a link their utility functions change and this makes a comparison of pre and post link situations difficult for general networks. We have examined a few numerical examples. These are presented in Figures 11 and 12. Figure 11 illustrates that the complete and empty network are stable, while Figure 12 makes the point that players in a complete component have an incentive to delete a link if there is an isolated player. This example provides a simple illustration of how general equilibrium forces can shape the formation of social networks.

6 Discussion

In the paper so far, we have taken the view that individuals compare their consumption of good 2 to the consumption of good 2 by others in their neighborhood, and that they care about the total deviation in consumptions. In this section we will explore a number of alternative possible specifications with a view to clarifying the scope of our results.

6.1 All goods are positional

First we consider the simple case where consumption externalities are relevant in a symmetric way for both goods. In that case, the utility is given by

$$\hat{u}_i(x_i, x_{-i}, y_i, y_{-i}) = \left(x_i[1+\alpha n_i] - \alpha \left[\sum_{j \in N(i)} x_j \right] \right)^{\sigma} \left(y_i[1+\alpha n_i] - \alpha \left[\sum_{j \in N(i)} y_j \right] \right)^{1-\sigma} \tag{51}$$

It is possible to show that there exists an equilibrium which is identical in prices and allocations to an equilibrium in the economy with no consumption externalities. This is analogous to a point made in a recent paper by Arrow and Dasgupta (2007). They consider a dynamic model of work, leisure and savings, and find that if consumption and leisure are equally susceptible to consumption externalities then there is no distortion in equilibrium.

6.2 Comparisons with the average

We now turn to the examination of a Suppose that individuals care about their consumption relative to the average consumption of their neighbours. In this case,

$$u(x_i, y_i, \{y_j\}_{j\in N(i)}) = x_i^{\sigma} \left(y_i + \alpha \left[y_i - \frac{1}{n_i} \sum\nolimits_{j \in n_i(g)} y_j \right] \right)^{1-\sigma} \tag{52}$$

In the homogeneous case, this average formulation suggests a natural outcome: every consumer chooses the same bundle of goods. The demand function for good 2 becomes

$$f_i^2(p, \{y_j\}_{j\in N(i)}) = \frac{1-\sigma}{p} \left(\omega_i + p\nu_i + \frac{\sigma}{1-\sigma} \frac{\alpha}{1+\alpha} p \frac{1}{n_i} \sum_{j \in N(i)} y_j \right). \tag{53}$$

Now we see that the analysis done in the previous sections can be extended in a straightforward way to cover the case of average consumption effects. Indeed, the matrix G^N can now be defined as the $N \times N$ matrix of connections in which every row is normalized so that the sum of the elements add to $\frac{1}{1+\alpha}$. It is possible to show that there is an equilibrium in which agents choose bundles which are identical to their endowments and that this constitutes an equilibrium; in other words, network centrality plays no role in shaping prices and allocations. However, the network does play a role when agents are heterogeneous in their endowments as Figure 13 shows.

6.3 Direct negative externalities

Suppose that the consumption of others exerts a direct negative effect on neighbours. In other words, suppose the consumption of good 2 by agent i constitutes a bad for his neighbors, and directly lowers their net consumption of good 2. The utility of individuals is then given by:

$$u_i(x_i, y_i, y_{-i}) = u_i(x_i, y_i, \{y_j\}_{j\in N(i)}) = x_i^{\sigma} \left(y_i - \alpha \sum_{j \in N(i)} y_j \right)^{1-\sigma} \tag{54}$$

In this case, the methods we have developed can be applied in a simpler form, and the equilibrium price and allocations are proportional to the Katz-Bonacich centrality as before and we can establish slightly stronger versions of the results on effects of adding links on allocations and prices.

We can also model the negative externality as taking place at the average level. In this case,

$$u(x_i, y_i, \{y_j\}_{j\in N(i)}) = x_i^{\sigma} (y_i - \frac{\alpha}{n_i} \sum_{j \in n_i(g)} y_j)^{1-\sigma} \tag{55}$$

The average formulation suggests a natural outcome: every consumer chooses the same bundle of goods. Clearly then the externality faced by every consumer is the same and so the optimal consumption bundle under a common set of prices will be the same as well. The market clearing equations are then $nx_i = n\omega$ and $ny_y = n\nu$.

The equilibrium allocation is then (ω, ν) . The equilibrium price can be obtained from eq. (3)

$$\omega = \sigma \left(\omega + p\nu + \alpha \frac{1}{n_i} p \sum_{j \in N(i)} \nu \right) = \sigma \left(\omega + p\nu + \alpha p\nu \right) \tag{56}$$

giving

$$\omega(1 - \sigma) = p(\sigma\nu + \sigma\alpha\nu) = p\sigma(1 + \alpha) \tag{57}$$

so that

$$p = \frac{1 - \sigma}{\sigma(1 + \alpha)} \tag{58}$$

Note that it is independent of the network, as is to be expected since demands are independent of the network connections. However, note that the equilibrium price is related to the magnitude of the externality, and is indeed decreasing in the value of α. We expect that network structure will matter, even if individuals care about average consumptions if endowments are heterogeneous, as in the previous section.

7 Concluding remarks

Relative consumption concerns appear to be important in day to day life and economists have been studying them for at least a hundred years, following the work of Veblen (1989). This interest has spawned a large theoretical literature which examines the implications of relative consumption concerns; for the most part, this work assumes that individuals care about their own consumption as well as the average consumption of society at large.

Introspection as well as recent empirical work suggest that we care about the relative consumption of those *close to us*, i.e., our neighbors, friends, relatives, acquaintances and colleagues. This motivates us to incorporate social networks into a market economy with a view to understanding how relative consumption concerns shape economic exchange and well being.

We start with a characterization of equilibrium prices and allocations as a function of (eigen vector) centralities in the social network. Indeed, in equilibrium, individual consumption can be expressed as a function of her centrality in a social network, while the price is proportional to the average network centrality of agents in the network. We show that adding links to a social network leads to higher centralities and this in turn pushes up the price of the socially sensitive good. We also derive a formula to compute the *critical link*, i.e., the new link which maximizes price increase. Roughly speaking, this formula says that, when social comparison effects are small, the critical link is one which connects the two least linked agents. With regard to allocations, we show that, as we add links to a network, the consumption of the socially sensitive good by the connected agents increases, while the reverse happens to non-connected agents.

We then turn to a model with heterogenous endowments, and find that inequality in network centrality and in wealth inequality reinforce each other. Thus a transfer of resources from less to more central agents raises prices of the socially sensitive good and alters allocations and utilities of all agents.

The large effects of networks on equilibrium prices, allocations and welfare naturally raise the question: where do these networks come from and which networks are likely to be stable? Our findings in Example 5 about the welfare effects of integration *vs* segregation suggest that rich agents will desire links with their poor cohort, while the opposite pressures will work on the poor. In a world where link formation requires consent on the part of both agents, this suggests that stable communities should consist of agents with similar endowments; in other words, a society segregated by economic class. We have reported some preliminary findings in this paper; we leave more systematic analysis of this problem to another paper.

8 Appendix

Proof of Proposition 1: the verification of inequalities Consider an arbitrary coordinate k

$$\begin{aligned}
B_k &= 1+\alpha\sigma\{G^N J\}_k+\alpha^2\sigma^2\{(G^N)^2 J\}_k+\alpha^3\sigma^3\{(G^N)^3 J\}_k+... \\
&= 1+\alpha\sigma\sum_{h=1}^{N} g^N_{kh}+\alpha^2\sigma^2\sum_{q=1}^{N}\sum_{p=1}^{N} g^N_{kp}g^N_{pq}+ \\
&\quad \alpha^3\sigma^3\sum_{q=1}^{N}\sum_{p=1}^{N}\sum_{h=1}^{N} g^N_{kp}g^N_{ph}g^N_{hq}+... \\
&= 1+\alpha\sigma\sum_{j\in N(k)}\frac{1}{1+\alpha n_k}+\alpha^2\sigma^2\sum_{p=1}^{N} g^N_{kp}\sum_{q=1}^{N} g^N_{pq}+ \\
&\quad +\alpha^3\sigma^3\sum_{p=1}^{N} g^N_{kp}\sum_{h=1}^{N} g^N_{ph}\sum_{q=1}^{N} g^N_{hq}+... \\
&= 1+\alpha\sigma\frac{n_k}{1+\alpha n_k}+\alpha^2\sigma^2\sum_{p=1}^{N} g^N_{kp}\frac{n_p}{1+\alpha n_p}+ \\
&\quad +\alpha^3\sigma^3\sum_{p=1}^{N} g^N_{kp}\sum_{h=1}^{N} g^N_{ph}\frac{n_h}{1+\alpha n_h}+... \\
&= 1+\alpha\sigma\frac{n_k}{1+\alpha n_k}+\alpha^2\sigma^2\frac{1}{1+\alpha n_k}\sum_{p\in N(k)}\frac{n_p}{1+\alpha n_p}+ \\
&\quad +\alpha^3\sigma^3\frac{1}{1+\alpha n_k}\sum_{p\in N(k)}\frac{1}{1+\alpha n_p}\sum_{h\in N(p)}\frac{n_h}{1+\alpha n_h}+... \qquad (59)
\end{aligned}$$

The first term is the degree n_k of the agent k scaled down by $1+\alpha n_k$. The quantity $\frac{\alpha n_k}{1+\alpha n_k}$ is increasing in n_k and bounded by 0 and 1. The second term is the sum over the neighbours $N(k)$ of the degrees of the neighbours (also scaled down). This decomposition also allows to define B recursively

$$B_k = 1 + \alpha\sigma \frac{1}{1+\alpha n_k} \sum_{j \in N(k)} B_j \tag{60}$$

Now, from the series decomposition and since $\frac{\alpha n_k}{1+\alpha n_k} < 1$, we have that

$$B_k < 1 + \left[\sum_{s=1}^{\infty} \sigma^s \right] = 1 + \frac{\sigma}{1-\sigma} \tag{61}$$

We then get the bounds

$$1 < B_k < \frac{1}{1-\sigma} \tag{62}$$

The condition for the price to be positive is

$$\frac{1}{1-\sigma} \frac{N}{\sum_{i=1}^{N} B_i} > 1 \tag{63}$$

which clearly holds since $\frac{1}{1-\sigma} > B_i$ for all i. The condition $x_i > 0$ is satisfied as both $(1-\sigma)B_i < 1$ and $(1-\sigma)\overline{B} < 1$ hold. The condition $y_i > 0$ is automatically satisfied as $B_k > 1$. Finally, we consider the condition $y_i(1+\alpha n_i) - \alpha \sum_{j \in N(i)} y_j > 0$ which may be rewritten as

$$y_i - \alpha \frac{1}{(1+\alpha n_i)} \sum_{j \in N(i)} y_j > 0 \tag{64}$$

or

$$B_i - \alpha \frac{1}{(1+\alpha n_i)} \sum_{j \in N(i)} B_j > 0 \tag{65}$$

Using the recursive formulation for B, we see that the condition indeed holds

$$1 + \alpha\sigma \frac{1}{1+\alpha n_i} \sum_{j \in N(i)} B_j - \alpha \frac{1}{(1+\alpha n_i)} \sum_{j \in N(i)} B_j \tag{66}$$

$$= \quad 1 - (1-\sigma) \frac{\alpha}{1+\alpha n_i} \sum_{j \in N(i)} B_j \tag{67}$$

$$> \quad 1 - (1-\sigma) \frac{\alpha n_i}{1+\alpha n_i} \frac{1}{1-\sigma} \tag{68}$$

$$> \quad 1 - 1 \tag{69}$$

$$> \quad 0 \tag{70}$$

■

Proof of Proposition 2: If a link is added between i and j, the matrix G is modified to $\widetilde{G}=G+\Delta_{ij}+\Delta_{ji}$ where Δ_{ij} is the matrix defined by $\Delta_{ij}=\{t_{pq}\}_{p,q=1}^{N}$ with $t_{ij}=1$ and $t_{pq}=0$ otherwise. In order to evaluate how the vector B is modified by the addition of the link it is useful to note that

$$G^{N}=\left(G^{T}\Lambda\right)^{T} \tag{71}$$

where $(.)^{T}$ means transpose and Λ is defined as

$$\Lambda=\begin{bmatrix}\frac{1}{1+\alpha n_1} & 0 & 0 & 0\\ 0 & \frac{1}{1+\alpha n_2} & 0 & 0\\ 0 & 0 & \ldots & 0\\ 0 & 0 & 0 & \frac{1}{1+\alpha n_n}\end{bmatrix} \tag{72}$$

Then the addition of a link between i and j implies that G^{N} becomes $\widetilde{G}^{N}$ and

$$\begin{aligned}\widetilde{G}^{N} &= \left(\widetilde{G}^{T}\widetilde{\Lambda}\right)^{T}\\ &= \left(\left(G+\Delta_{ij}+\Delta_{ji}\right)^{T}\widetilde{\Lambda}\right)^{T}\end{aligned}$$

with $\widetilde{\Lambda}$ defined as Λ but with the elements at position i and j modified to become $\frac{1}{1+\alpha(n_i+1)}$ and $\frac{1}{1+\alpha(n_j+1)}$.

Therefore, introducing a link between i and j modifies B as follows

$$\widetilde{B} = \left[\sum_{s=0}^{\infty}\alpha^{s}\sigma^{s}(\widetilde{G}^{N})^{s}\right]J \tag{73}$$

$$= \left[\sum_{s=0}^{\infty}\alpha^{s}\sigma^{s}\left[\left(\left(G+\Delta_{ij}+\Delta_{ji}\right)^{T}\widetilde{\Lambda}\right)^{T}\right]^{s}\right]J \tag{74}$$

Clearly, introducing a connection between i and j affects the value of all the elements of the vector B. However, this effect is vanishing with the distance and as $\alpha\sigma \longrightarrow 0$. It is then useful to look at the first terms in the expression for B noting that all terms are positive. We consider an arbitrary coordinate k. We get for the first three terms

$$\begin{aligned}B_k &= 1+\alpha\sigma\frac{n_k}{1+\alpha n_k}+\alpha^{2}\sigma^{2}\sum\nolimits_{p=1}^{N} g_{kp}^{N}\frac{n_p}{1+\alpha n_p}+\\ &\quad +\alpha^{3}\sigma^{3}\sum\nolimits_{p=1}^{N} g_{kp}^{N}\sum\nolimits_{h=1}^{N} g_{ph}^{N}\frac{n_h}{1+\alpha n_h}+\ldots\\ &= 1+\alpha\sigma\frac{n_k}{1+\alpha n_k}+\alpha^{2}\sigma^{2}\frac{1}{1+\alpha n_k}\sum\nolimits_{p\in N(k)}\frac{n_p}{1+\alpha n_p}+\\ &\quad +\alpha^{3}\sigma^{3}\frac{1}{1+\alpha n_k}\sum\nolimits_{p\in N(k)}\frac{1}{1+\alpha n_p}\sum\nolimits_{h\in N(p)}\frac{n_h}{1+\alpha n_h}+\ldots\end{aligned} \tag{75}$$

The expansion for B_k allows us to evaluate the effect of the introduction of a link between i and j. Denote this change as ΔB_k. Using the series expansion we then define $\Delta B_k = \sum_{s=1}^{\infty} \Delta B_k^s$ where the suffix indicates the power of $\alpha\sigma$. For the first term in the series we have

$$\begin{aligned}
\Delta B_i^1 &= \alpha\sigma \left[\frac{n_i + 1}{1 + \alpha\{n_i + 1\}} - \frac{n_i}{1 + \alpha n_i} \right] \\
&= \frac{\alpha\sigma}{[1 + \alpha\{n_i + 1\}]\,[1 + \alpha n_i]} \qquad (76)
\end{aligned}$$

The second order term ΔB_i^2 is

$$\begin{aligned}
\Delta B_i^2 &= \alpha^2\sigma^2 \frac{1}{1 + \alpha(n_i + 1)} \left[\sum\nolimits_{p \in N(i)} \frac{n_p}{1 + \alpha n_p} + \frac{n_j + 1}{1 + \alpha n_j + 1} \right] \\
&\quad - \alpha^2\sigma^2 \frac{1}{1 + \alpha n_i} \sum\nolimits_{p \in N(i)} \frac{n_p}{1 + \alpha n_p} \\
&= \alpha^2\sigma^2 \left[\frac{1}{1 + \alpha(n_i + 1)} - \frac{1}{1 + \alpha n_i} \right] \sum\nolimits_{p \in N(i)} \frac{n_p}{1 + \alpha n_p} \\
&\quad + \alpha^2\sigma^2 \frac{1}{1 + \alpha(n_i + 1)} \frac{n_j + 1}{1 + \alpha n_j + 1} \\
&= \alpha^2\sigma^2 \frac{-\alpha}{(1 + \alpha(n_i + 1))\,(1 + \alpha n_i)} \sum\nolimits_{p \in N(i)} \frac{n_p}{1 + \alpha n_p} \\
&\quad + \alpha^2\sigma^2 \frac{1}{1 + \alpha(n_i + 1)} \frac{n_j + 1}{1 + \alpha n_j + 1} \\
&= \alpha\sigma^2 \frac{1}{1 + \alpha(n_i + 1)} \left[\frac{\alpha n_j + 1}{1 + \alpha n_j + 1} - \frac{\alpha}{1 + \alpha n_i} \sum\nolimits_{p \in N(i)} \frac{\alpha n_p}{1 + \alpha n_p} \right] \qquad (77)
\end{aligned}$$

Consider now how the centralities of the neighbours of i and j are affected. Denote by h a generical neighbour of i, i.e. $h \in N(i), h \neq i, j$. Then, the addition of a link between agent i and j has no effect on n_h so that $\Delta B_h^1 = 0$. The second term in the expression of B_h is $\alpha^2\sigma^2 \frac{1}{1+\alpha n_h} \sum_{p \in N(h)} \frac{n_p}{1+\alpha n_p}$ so that

$$\begin{aligned}
\Delta B_h^2 &= \alpha^2\sigma^2 \frac{1}{1 + \alpha n_h} \left[\frac{n_i + 1}{1 + \alpha(n_i + 1)} + \sum\nolimits_{\substack{p \in N(h) \\ p \neq i}} \frac{n_p}{1 + \alpha n_p} \right] \\
&\quad - \alpha^2\sigma^2 \frac{1}{1 + \alpha n_h} \left[\frac{n_i}{1 + \alpha n_i} + \sum\nolimits_{\substack{p \in N(h) \\ p \neq i}} \frac{n_p}{1 + \alpha n_p} \right] \\
&= \alpha^2\sigma^2 \frac{1}{1 + \alpha n_h} \left[\frac{n_i + 1}{1 + \alpha(n_i + 1)} - \frac{n_i}{1 + \alpha n_i} \right] \\
&= \alpha^2\sigma^2 \frac{1}{1 + \alpha n_h} \frac{1}{(1 + \alpha(n_i + 1))\,(1 + \alpha n_i)} \qquad (78)
\end{aligned}$$

For further neighbours, only higher order terms in $\alpha\sigma$ are non-zero.

We are in a position to evaluate how $\sum_{k=1}^{N} B_k$ is affected by the new link. Let $\sum_{k=1}^{N} \widetilde{B}_k$ be the value of the sum after the new link is introduced. If we keep only the first two terms of the expansion we have:

$$\begin{aligned}\Delta \sum_{k=1}^{N} B_k &= \sum_{k=1}^{N} \widetilde{B}_k - \sum_{k=1}^{N} B_k \\ &\simeq \Delta B_i^1 + \Delta B_i^2 + \Delta B_j^1 + \Delta B_j^2 + \sum_{k\in N(i)} \Delta B_k^2 + \sum_{k\in N(j)} \Delta B_k^2 \end{aligned} \tag{79}$$

Now,

$$\begin{aligned}\Delta B_i^1 + \Delta B_j^1 &\simeq \alpha\sigma \left[\frac{n_i+1}{1+\alpha(n_i+1)} + \frac{n_j+1}{1+\alpha(n_j+1)} - \frac{n_i}{1+\alpha n_i} - \frac{n_j}{1+\alpha n_j} \right] \\ &\simeq \frac{\alpha\sigma}{(1+\alpha(n_i+1))\,(1+\alpha n_i)} + \frac{\alpha\sigma}{(1+\alpha(n_j+1))\,(1+\alpha n_j)} \end{aligned} \tag{80}$$

On the other hand,

$$\begin{aligned}\Delta B_i^2 + \Delta B_j^2 \simeq\ & \alpha^2\sigma^2 \frac{-\alpha}{(1+\alpha(n_i+1))\,(1+\alpha n_i)} \sum\nolimits_{h\in N(i)} \frac{n_h}{1+\alpha n_h} \\ & +\alpha\sigma^2 \frac{1}{1+\alpha(n_i+1)} \frac{\alpha n_j}{1+\alpha n_j} \\ & +\alpha^2\sigma^2 \frac{-\alpha}{(1+\alpha(n_j+1))\,(1+\alpha n_j)} \sum\nolimits_{h\in N(j)} \frac{n_h}{1+\alpha n_h} \\ & +\alpha\sigma^2 \frac{1}{1+\alpha(n_j+1)} \frac{\alpha n_i}{1+\alpha n_i} \end{aligned} \tag{81}$$

Finally,

$$\begin{aligned}\sum_{h\in N(i)\cup N(j)} \Delta B_h^2 =\ & \frac{\alpha^2\sigma^2}{(1+\alpha(n_i+1))\,(1+\alpha n_i)} \sum_{h\in N(i)} \frac{1}{1+\alpha n_h} \\ & + \frac{\alpha^2\sigma^2}{(1+\alpha(n_j+1))\,(1+\alpha n_j)} \sum_{h\in N(j)} \frac{1}{1+\alpha n_h} \end{aligned} \tag{82}$$

Therefore,

$$\begin{aligned}
\Delta\sum_{k=1}^{N} B_k &= \frac{\alpha\sigma}{(1+\alpha(n_i+1))\,(1+\alpha n_i)} + \frac{\alpha\sigma}{(1+\alpha(n_j+1))\,(1+\alpha n_j)} \\
&+\alpha\sigma^2 \frac{1}{1+\alpha(n_j+1)}\frac{\alpha n_i}{1+\alpha n_i} \\
&+\alpha\sigma^2 \frac{1}{1+\alpha(n_i+1)}\frac{\alpha n_j}{1+\alpha n_j} \\
&+\alpha^2\sigma^2 \frac{1}{(1+\alpha(n_i+1))\,(1+\alpha n_i)}\,(1-\alpha)\sum\nolimits_{h\in N(i)} \frac{n_h}{1+\alpha n_h} \\
&+\alpha^2\sigma^2 \frac{1}{(1+\alpha(n_j+1))\,(1+\alpha n_j)}\,(1-\alpha)\sum\nolimits_{h\in N(j)} \frac{n_h}{1+\alpha n_h} \qquad (83)
\end{aligned}$$

Finally, recall that the equilibrium price is given by

$$p = \frac{\omega}{\nu}\left[\frac{1}{1-\sigma}\frac{1}{\overline{\overline{B}}} - 1\right]^{-1} \qquad (84)$$

with $\overline{B} = \frac{1}{N}\sum_{k=1}^{N} B_k$. Therefore, as the first two terms increase strictly, and we can ignore the higher order terms, by suitably lowering α, the sum $\overline{B}$ increases when a new link is added. This implies that the equilibrium price also increases.

■

Proof of Proposition 3: Fix a network $g \neq g^c$; the critical link g_{ij}, solves the following problem:

$$\max_{g_{ij}} \bar{B}(g+g_{ij}) - \bar{B}(g). \qquad (85)$$

We note from proof of Proposition 2, that for small enough α this is equivalent to a link which maximizes the first order effects of a change in network, i.e., maximizes $\Delta B_i^1 + \Delta B_j^1$. From equation 80 the first order effects of a link between i and j are given by:

$$\begin{aligned}
\Delta B_i^1 + \Delta B_j^1 &\simeq \alpha\sigma\left[\frac{n_i+1}{1+\alpha(n_i+1)} + \frac{n_j+1}{1+\alpha(n_j+1)} - \frac{n_i}{1+\alpha n_i} - \frac{n_j}{1+\alpha n_j}\right] \\
&\simeq \frac{\alpha\sigma}{(1+\alpha(n_i+1))\,(1+\alpha n_i)} + \frac{\alpha\sigma}{(1+\alpha(n_j+1))\,(1+\alpha n_j)} \qquad (86)
\end{aligned}$$

So the critical link maximizes the expression in 86; the proof now follows.

■

Proof of Proposition 4: First, consider the change in the consumption of the socially sensitive good by an agent h with $h \neq i, j$ when a link is added between i and

j.Let $\widetilde{y}_h$ denote the corrected value of y_h. A similar convention is used for the other variables. We have

$$\widetilde{y}_h = \frac{\widetilde{B}_h}{\sum_{k=1}^N \widetilde{B}_k} N\nu \tag{87}$$

It will turn out that first order terms are sufficient to characterise the bahavior as $\alpha \mapsto 0$. From last section we know that this of order:

$$\Delta\Sigma \equiv \sum_{k=1}^N \widetilde{B}_k - \sum_{k=1}^N B_k = \frac{\alpha\sigma}{(1+\alpha(n_i+1))(1+\alpha n_i)} + \frac{\alpha\sigma}{(1+\alpha(n_j+1))(1+\alpha n_j)} \tag{88}$$

Furthermore, $\Delta B_h^1 = 0$. Then

$$\begin{aligned}
\widetilde{y}_h - y_h &= \frac{B_h}{\Delta\Sigma + \sum_{k=1}^N B_k} N\nu - \frac{B_h}{\sum_{k=1}^N B_k} N\nu \\
&= N\nu \left(\frac{(B_h)\left(\sum_{k=1}^N B_k\right) - (B_h)\left(\Delta\Sigma + \sum_{k=1}^N B_k\right)}{\left(\Delta\Sigma + \sum_{k=1}^N B_k\right)\left(\sum_{k=1}^N B_k\right)} \right) \\
&= N\nu \left(\frac{-B_h \Delta\Sigma}{\left(\Delta\Sigma + \sum_{k=1}^N B_k\right)\left(\sum_{k=1}^N B_k\right)} \right) \\
&\simeq -N\nu\Delta\Sigma \frac{B_h}{\left(\sum_{k=1}^N B_k\right)^2}
\end{aligned} \tag{89}$$

As $\alpha \mapsto 0$ we get

$$\widetilde{y}_h - y_h \simeq -2N\nu\alpha\sigma \frac{B_h}{\left(\sum_{k=1}^N B_k\right)^2} \leq 0 \tag{90}$$

Remark: We only need $\alpha n_i << 1$ so that $(1+\alpha(n_i+1))(1+\alpha n_i) \simeq 1$ for all i.

We now analyze the effect on the agents who have formed a link. Again consider only the first order terms. We have

$$\widetilde{y}_i \simeq \frac{B_i + \frac{\alpha\sigma}{(1+\alpha(n_i+1))(1+\alpha n_i)}}{\Delta\Sigma + \sum_{k=1}^N B_k} N\nu \tag{91}$$

and

$$\widetilde{y}_j \simeq \frac{B_j + \frac{\alpha\sigma}{(1+\alpha(n_j+1))(1+\alpha n_j)}}{\Delta\Sigma + \sum_{k=1}^N B_k} N\nu \tag{92}$$

Therefore,

$$\widetilde{y}_i - y_i \simeq \frac{B_i + \frac{\alpha\sigma}{(1+\alpha(n_i+1))(1+\alpha n_i)}}{\Delta\Sigma + \sum_{k=1}^{N} B_k} N\nu - \frac{B_i}{\sum_{k=1}^{N} B_k} N\nu \tag{93}$$

Similarly,

$$\widetilde{y}_j - y_j \simeq \frac{B_i + \frac{\alpha\sigma}{(1+\alpha(n_j+1))(1+\alpha n_j)}}{\Delta\Sigma + \sum_{k=1}^{N} B_k} N\nu - \frac{B_j}{\sum_{k=1}^{N} B_k} N\nu \tag{94}$$

So that the aggregate change in consumption of socially sensitive good by i and j is:

$$y_i - y_i + y_j - y_j] = \frac{B_i + B_j + \Delta\Sigma}{\Delta\Sigma + \sum_{k=1}^{n} B_i} - \frac{B_i + B_j}{\sum_{k=1}^{n} B_k} > 0. \tag{95}$$

We now examine the effects on demand of good 1, when a single connection is added to the network between i and j. Assume that $(1 + \alpha(n_i + 1))(1 + \alpha n_i) \simeq 1$ or equivalently that $\alpha n_i << 1$, for all i then. For agent i, we have

$$\begin{aligned} \frac{1}{\omega}\widetilde{x}_i &= \frac{N - (1-\sigma)N\widetilde{B}_i}{N - (1-\sigma)\sum_{k=1}^{N} \widetilde{B}_k} \\ &\simeq \frac{N - (1-\sigma)N(\alpha\sigma + B_i)}{N - (1-\sigma)[2\alpha\sigma + \sum_{k=1}^{N} B_k]} \end{aligned} \tag{96}$$

Therefore

$$\frac{1}{\omega N}\Delta x_i = \frac{1}{\omega N}(\widetilde{x}_i - x_i) \tag{97}$$

$$\begin{aligned} &= \frac{1 - (1-\sigma)(\alpha\sigma + B_i)}{N - (1-\sigma)[2\alpha\sigma + \sum_{k=1}^{N} B_k]} \\ &\quad - \frac{1 - (1-\sigma)B_i}{N - (1-\sigma)\sum_{k=1}^{N} B_k} \end{aligned} \tag{98}$$

After reducing to a common denominator, the numerator becomes

$$\begin{aligned} &Num = [1 - (1-\sigma)(\alpha\sigma + B_i)][N - (1-\sigma)\sum\nolimits_{k=1}^{N} B_k] \\ &-[1 - (1-\sigma)B_i][N - (1-\sigma)[2\alpha\sigma + \sum\nolimits_{k=1}^{N} B_k]] \end{aligned} \tag{99}$$

Let

$$A_1 = 1 - (1-\sigma)B_i \text{ and } A_2 = N - (1-\sigma)\sum\nolimits_{k=1}^{N} B_k \tag{100}$$

Then

$$\begin{aligned} Num &= [A_1 - (1-\sigma)\alpha\sigma)][A_2] - [A_1][A_2 - (1-\sigma)2\alpha\sigma] \\ &= [-(1-\sigma)(\alpha\sigma)]A_2 + A_1(1-\sigma)2\alpha\sigma \\ &= [(1-\sigma)\alpha\sigma][2A_1 - A_2] \end{aligned} \tag{101}$$

Then

$$\begin{aligned} Num &= [(1-\sigma)\alpha\sigma] \\ &\quad [2 - 2(1-\sigma)B_i - N + (1-\sigma)\textstyle\sum_{k=1}^N B_k] \\ &= [(1-\sigma)\alpha\sigma]\left(2 - N + (1-\sigma)\left[\textstyle\sum_{k=1}^N B_k - 2B_i\right]\right) \end{aligned} \tag{102}$$

Finally we get

$$\Delta x_i = \omega N[(1-\sigma)\alpha\sigma]\frac{2 - N + (1-\sigma)[\sum_{k=1}^N B_k - 2B_i]}{\left(N - (1-\sigma)[2\alpha\sigma + \sum_{k=1}^N B_k\right)\left(N - (1-\sigma)\sum_{k=1}^N B_k\right)} \tag{103}$$

Note that the condition for a positive price is $(1-\sigma)\sum_{k=1}^N B_k < N$ and that $B_i > 1$ by construction. Therefore, $\Delta x_i < 0$.

For a node h with $h \neq i, j$, the increase in consumption of first good is computed as follows.

$$\begin{aligned} \frac{1}{\omega}\Delta x_h &= \frac{1}{\omega}(\widetilde{x}_h - x_h) \\ &= \frac{N - (1-\sigma)NB_h}{N - (1-\sigma)[2\alpha\sigma + \sum_{k=1}^N B_k]} \\ &\quad - \frac{N - (1-\sigma)NB_h}{N - (1-\sigma)[\sum_{k=1}^N B_k]} \end{aligned} \tag{104}$$

First observe that $\Delta x_h/\omega > 0$, as $N - (1-\sigma)[\sum_{k=1}^N B_k] > 0$. After reducing to the same denominator, the numerator becomes:

$$\begin{aligned} Num &= [N - (1-\sigma)NB_h] \\ &\quad \left[\left(N - (1-\sigma)[\sum_{k=1}^N B_k]\right) - \left(N - (1-\sigma)[2\alpha\sigma + \sum_{k=1}^N B_k]\right)\right] \\ &= [N - (1-\sigma)NB_h]\,[(1-\sigma)2\alpha\sigma] \\ &= N(1-\sigma)2\alpha\sigma - (1-\sigma)^2 2\alpha\sigma NB_h \\ &= N(1-\sigma)2\alpha\sigma\,[1 - (1-\sigma)B_h] \end{aligned} \tag{105}$$

Therefore the increase in x_h decreases with the (relative) centrality of h; in other words, for fixed $\sum B_i$, an increase in B_h reduces the increase in demand for good 1.

■

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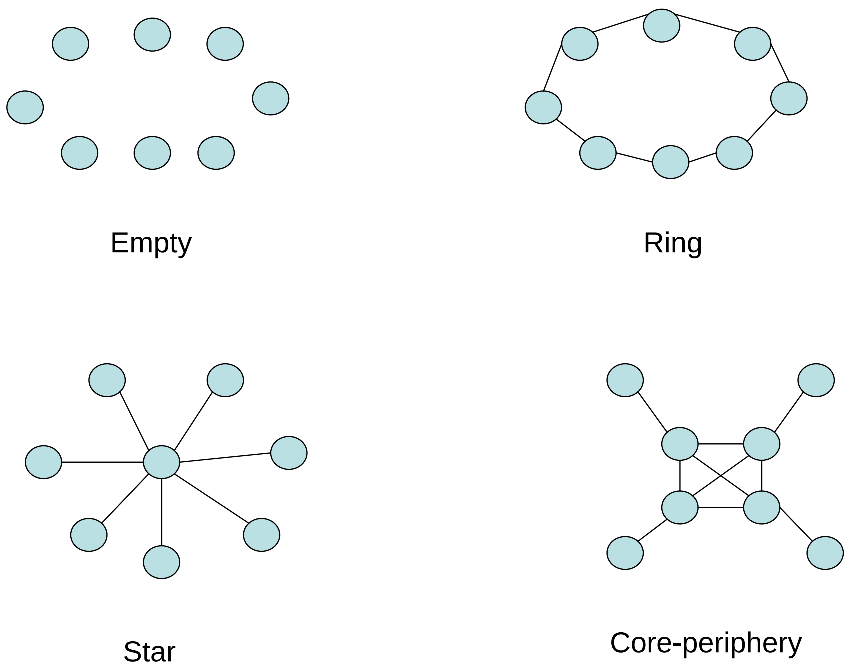

Figure 1: Classical social networks

	Empty	Star	Ring	Core-periphery
x1	10	7.1233	10	8.75
x2	10	10.411	10	8.75
x3	10	10.411	10	8.75
x4	10	10.411	10	8.75
x5	10	10.411	10	11.25
x6	10	10.411	10	11.25
x7	10	10.411	10	11.25
x8	10	10.411	10	11.25
y1	10	11.6279	10	10.6452
y2	10	9.7674	10	10.6452
y3	10	9.7674	10	10.6452
y4	10	9.7674	10	10.6452
y5	10	9.7674	10	9.3548
y6	10	9.7674	10	9.3548
y7	10	9.7674	10	9.3548
y8	10	9.7674	10	9.3548
u1	10	11.3672	10	9.9393
u2	10	9.5919	10	9.9393
u3	10	9.5919	10	9.9393
u4	10	9.5919	10	9.9393
u5	10	9.5919	10	9.8987
u6	10	9.5919	10	9.8987
u7	10	9.5919	10	9.8987
u8	10	9.5919	10	9.8987
Price	1	1.7671	2	1.9375

endow.=10
alpha=0.5
sigma=0.5

Figure 2: Social networks shape general equilibrium

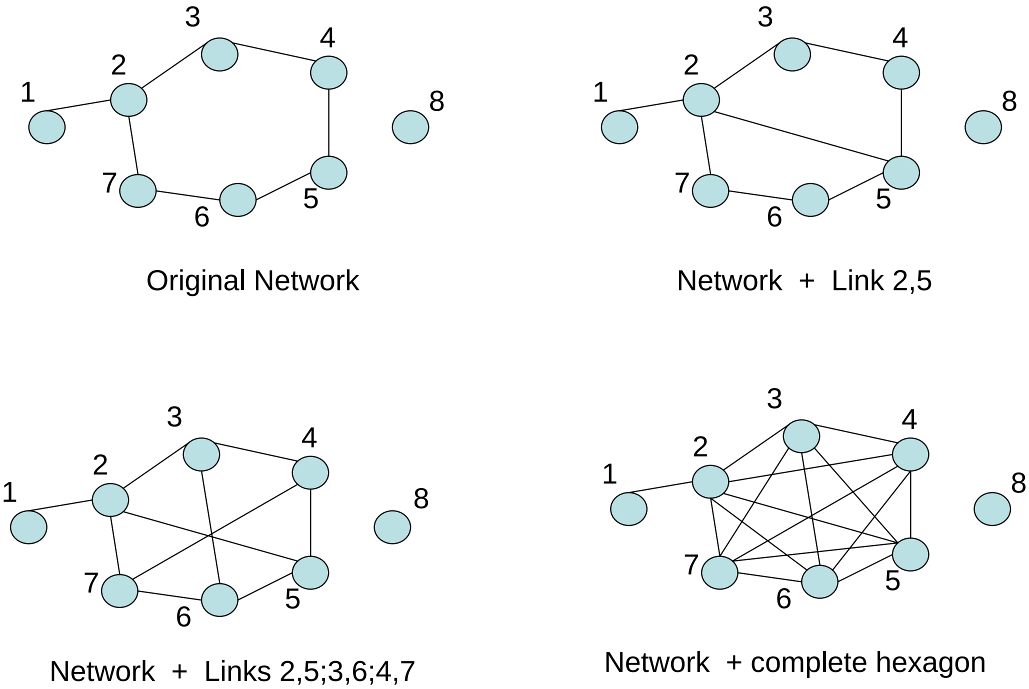

Figure 3: Networks with Isolated Agent

	Ring 2 to 7	Ring+Link 2-5	Ring+2-5,3-6,4-7	Complete hexa
x1	10.7933	11.0183	11.3892	12.4379
x2	8.5529	8.0418	8.2078	8.6765
x3	9.2643	9.4517	8.673	8.2997
x4	9.3545	9.5039	9.6842	8.3345
x5	9.3645	8.5117	8.673	9.1296
x6	9.3545	9.5039	8.7104	8.3345
x7	9.2643	9.4517	9.6307	8.2997
x8	14.0518	14.517	15.0318	16.4877
y1	9.5618	9.465	9.3076	8.9389
y2	10.7994	11.0288	10.8933	10.5761
y3	10.4064	10.2881	10.6614	10.7401
y4	10.3566	10.2606	10.1574	10.7249
y5	10.351	10.7819	10.6614	10.3788
y6	10.3566	10.2606	10.6428	10.7249
y7	10.4064	10.2881	10.1841	10.7401
y8	7.7619	7.6269	7.4921	7.1762
u1	9.8247	9.7813	9.9198	10.0499
u2	10.0508	10.096	9.9846	9.9145
u3	9.7375	9.6886	9.6152	9.484
u4	9.8323	9.7421	9.6569	9.5238
u5	9.8428	9.7549	9.6152	9.5234
u6	9.8323	9.7421	9.6569	9.5238
u7	9.7375	9.6886	9.6152	9.484
u8	10.4436	10.5223	10.7081	10.8775
Price	1.8104	1.9034	2.1133	2.2975

endow.=10.
alpha=0.5
sigma=0.5

Figure 4: No man is an island

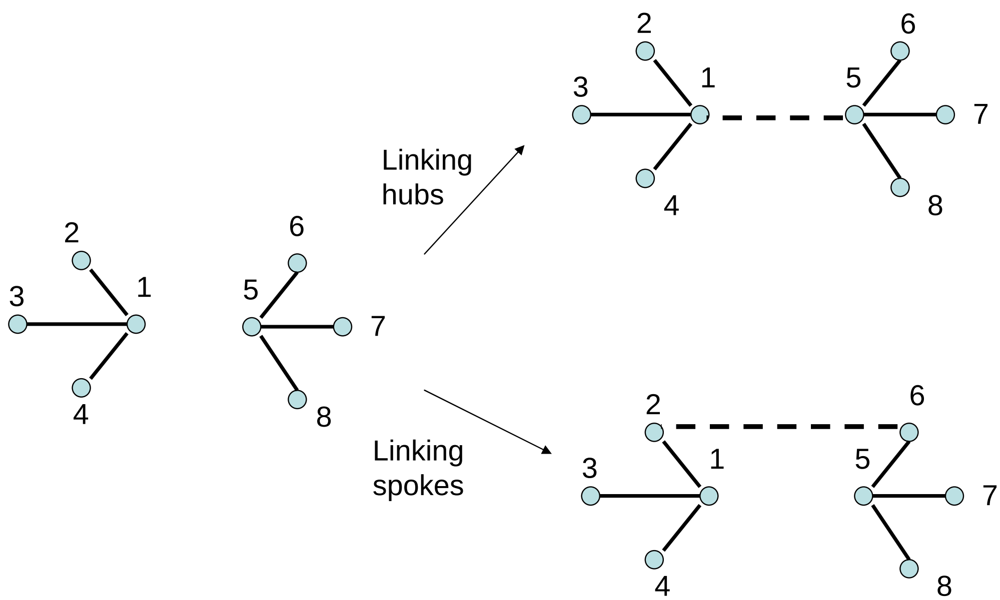

Figure 5: Critical links

	Two Stars	Linked centers	Linked laterals
x1	8.5714	8	8.7943
x2	10.4762	10.6667	9.3617
x3	10.4762	10.6667	10.922
x4	10.4762	10.6667	10.922
x5	8.5714	8	8.7943
x6	10.4762	10.6667	9.3617
x7	10.4762	10.6667	10.922
x8	10.4762	10.6667	10.922
y1	10.8333	11.1111	10.6564
y2	9.7222	9.6296	10.3475
y3	9.7222	9.6296	9.4981
y4	9.7222	9.6296	9.4981
y5	10.8333	11.1111	10.6564
y6	9.7222	9.6296	10.3475
y7	9.7222	9.6296	9.4981
y8	9.7222	9.6296	9.4981
u1	10.351	10.328	10.2596
u2	9.7996	9.7373	9.7685
u3	9.7996	9.7373	9.8698
u4	9.7996	9.7373	9.8698
u5	10.351	10.328	10.2596
u6	9.7996	9.7373	9.7685
u7	9.7996	9.7373	9.8698
u8	9.7996	9.7373	9.8698
Price	1.7143	1.8	1.8369

endow.=10
alpha=0.5
sigma=0.5

Figure 6: Linking hubs and spokes

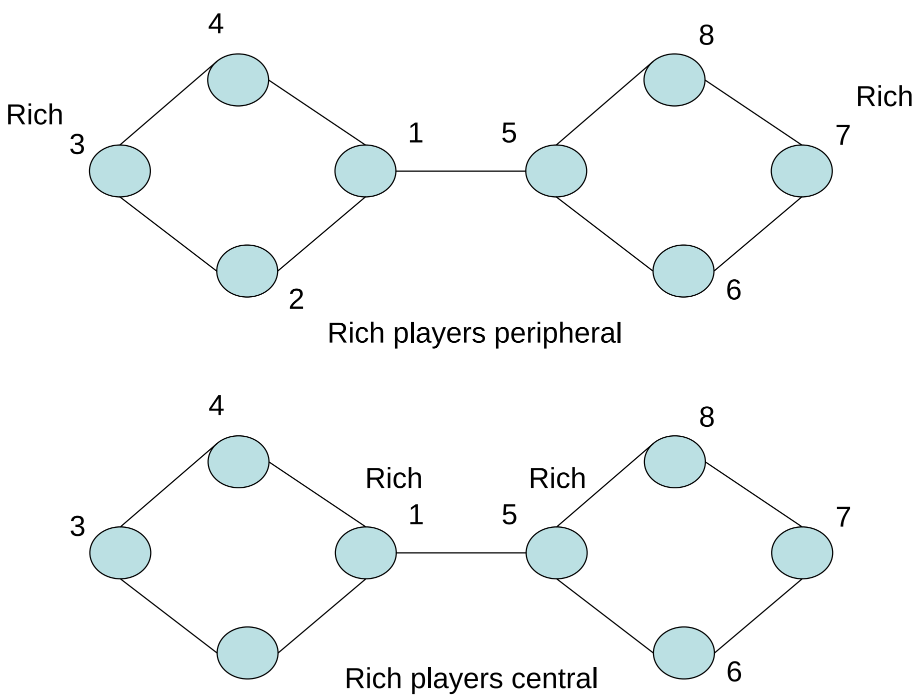

Figure 7: Rich and poor central agents

	Rich+Isolated Endowments	Rich+Isolated Allocations	Rich+Connected Endowments	Rich+Connected Allocations
x1	1	0.806	3	4.5852
x2	1	0.1429	1	0.1429
x3	3	4.9083	1	1.1291
x4	1	0.1429	1	0.1429
x5	1	0.806	3	4.5852
x6	1	0.1429	1	0.1429
x7	3	4.9083	1	1.1291
x8	1	0.1429	1	0.1429
y1	1	1.0104	3	2.9261
y2	1	1.0461	1	1.04
y3	3	2.8974	1	0.994
y4	1	1.0461	1	1.04
y5	1	1.0104	3	2.9261
y6	1	1.0461	1	1.04
y7	3	2.8974	1	0.994
y8	1	1.0461	1	1.04
u1		0.9564		4.7889
u2		0.1387		0.1221
u3		4.7645		0.9647
u4		0.1387		0.1221
u5		0.9564		4.7889
u6		0.1387		0.1221
u7		4.7645		0.9647
u8		0.1387		0.1221
Price		18.6045		21.4389

Alpha=0.5
Sigma=0.1

Figure 8: Wealth and centrality are complementary

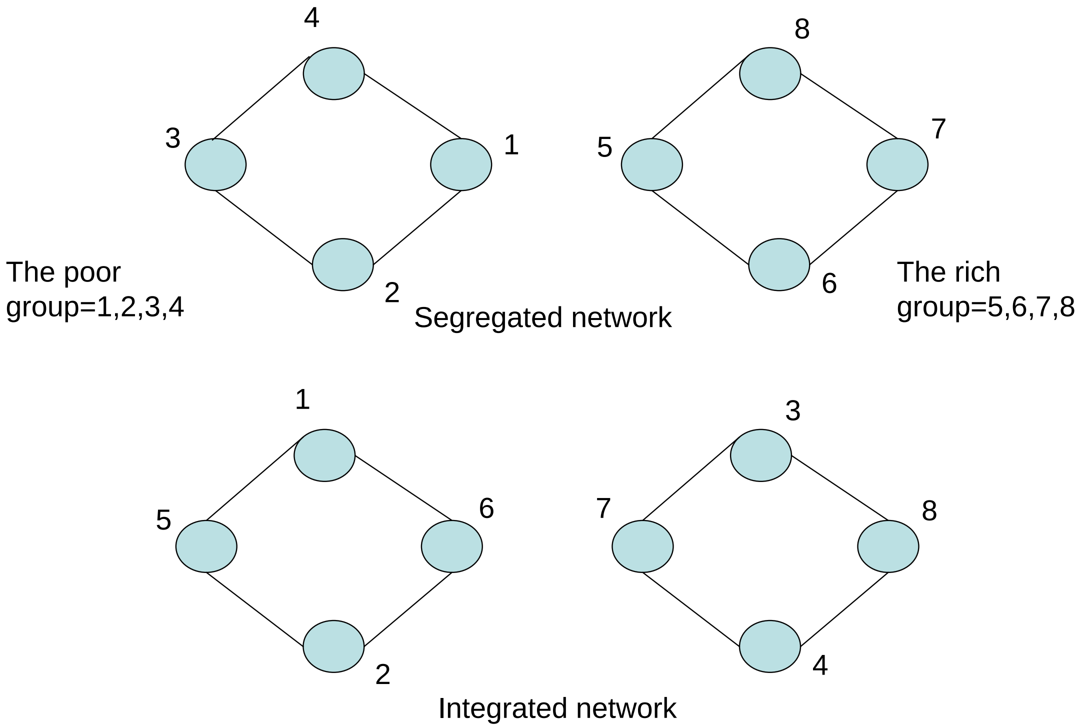

Figure 9: Segregated and integrated societies

	Endowments	Segregated	Integrated
x1	5	5	3
x2	5	5	3
x3	5	5	3
x4	5	5	3
x5	10	10	12
x6	10	10	12
x7	10	10	12
x8	10	10	12
y1	1	1	1.2
y2	1	1	1.2
y3	1	1	1.2
y4	1	1	1.2
y5	2	2	1.8
y6	2	2	1.8
y7	2	2	1.8
y8	2	2	1.8
u1	2.2361	2.2361	1.3416
u2	2.2361	2.2361	1.3416
u3	2.2361	2.2361	1.3416
u4	2.2361	2.2361	1.3416
u5	4.4721	4.4721	5.3666
u6	4.4721	4.4721	5.3666
u7	4.4721	4.4721	5.3666
u8	4.4721	4.4721	5.3666
Price		10	10

alpha=0.5
sigma=0.5

Figure 10: Keeping up with the neighbors

	Complete	Delete 1-2	Empty	Add 1-2
x1	10	10.411	10	8.4211
x2	10	10.411	10	8.4211
x3	10	9.863	10	10.5263
x4	10	9.863	10	10.5263
x5	10	9.863	10	10.5263
x6	10	9.863	10	10.5263
x7	10	9.863	10	10.5263
x8	10	9.863	10	10.5263
y1	10	9.906	10	11.4286
y2	10	9.906	10	11.4286
y3	10	10.0313	10	9.5238
y4	10	10.0313	10	9.5238
y5	10	10.0313	10	9.5238
y6	10	10.0313	10	9.5238
y7	10	10.0313	10	9.5238
y8	10	10.0313	10	9.5238
u1	10	9.9606	10	9.8102
u2	10	9.9606	10	9.8102
u3	10	10.0088	10	10.0125
u4	10	10.0088	10	10.0125
u5	10	10.0088	10	10.0125
u6	10	10.0088	10	10.0125
u7	10	10.0088	10	10.0125
u8	10	10.0088	10	10.0125
Price	4.5	4.3699	1	1.1053

Endow.=10.
alpha=0.5
sigma=0.5

Figure 11: Complete and empty networks are stable

	7 complete+8	Delete 1-2
x1	9.3677	9.4808
x2	9.3677	9.4808
x3	9.3677	9.3624
x4	9.3677	9.3624
x5	9.3677	9.3624
x6	9.3677	9.3624
x7	9.3677	9.3624
x8	14.4262	14.2264
y1	11.0488	10.8941
y2	11.0488	10.8941
y3	11.0488	11.0979
y4	11.0488	11.0979
y5	11.0488	11.0979
y6	11.0488	11.0979
y7	11.0488	11.0979
y8	2.6586	2.722
u1	9.5236	9.5293
u2	9.5236	9.5293
u3	9.5236	9.554
u4	9.5236	9.554
u5	9.5236	9.554
u6	9.5236	9.554
u7	9.5236	9.554
u8	12.1816	12.058
Price	0.6029	0.5807

Endow.=10.
alpha=0.9
sigma=0.9

Figure 12: General equilibrium effects on link stability

	Endowm	Empty	Disconected	Link 1-5
x1	1	2.26	2.3034	2.0967
x2	1	2.26	2.3034	2.3
x3	1	2.26	2.3034	2.3033
x4	1	2.26	2.3034	2.3
x5	5	3.74	3.6966	3.9033
x6	5	3.74	3.6966	3.7
x7	5	3.74	3.6966	3.6967
x8	5	3.74	3.6966	3.7
y1	1	0.9417	0.9598	0.9662
y2	1	0.9417	0.9598	0.9599
y3	1	0.9417	0.9598	0.9598
y4	1	0.9417	0.9598	0.9599
y5	1.5	1.5583	1.5402	1.5338
y6	1.5	1.5583	1.5402	1.5401
y7	1.5	1.5583	1.5402	1.5402
y8	1.5	1.5583	1.5402	1.5401
u1		1.4805	1.0476	0.9536
u2		1.4805	1.0476	1.046
u3		1.4805	1.0476	1.0475
u4		1.4805	1.0476	1.046
u5		2.45	1.6812	1.7752
u6		2.45	1.6812	1.6827
u7		2.45	1.6812	1.6812
u8		2.45	1.6812	1.6827
Price		21.6	32.4	32.4

alpha=0.5
sigma=0.1

Figure 13: Effects of neighbor averages in an unequal society