An Interbank Network Determined by the Real Economy

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Abstract

As a means of payment, bank liability circulates in a cycle. A fraction of one bank's liability naturally flows out to another, creating a network of interbank connections. We demonstrate how this network is determined by the production technologies, the resources distribution and the Input-Output network of the real economy. We find banks with a smaller outflow fraction see their funding costs less dependent on the interbank interest rate; the heterogeneity in banks' outflow fraction causes lending inefficiency; and the identities of depositors and borrowers matter. These results will not arise if banks are modelled as intermediaries of loanable funds.

Keywords: circulation of bank liability, interbank network, outflow fraction, identities of depositors and borrowers

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1 Introduction

There have been many studies on interbank networks since the 2007-8 financial crisis and they have shed many important insights. These studies take the approach that models banks as intermediaries of loanable funds. This paper takes a different approach: We model banks as creators of means of payment. In reality, bank deposits are widely accepted as a means of payment. When banks lend to the real economy, they do not part sacks of the currency from their vaults, but credit the borrowers' deposit accounts with the values lent. That is, the means of payment that banks lend out is not the currency, but the deposits that they create, as Keynes (1914) has observed.[1] Deposits are banks' liabilities. Altogether, *banks lend to the real economy their liabilities which it uses as a means of payment*. The loanable-funds approach focuses on the real value of the money that banks lend out, but abstracts from its attribute as a means of payment. This abstraction makes a lot of sense. Indeed, is not lending out money equivalent to lending out the real value that this money represents? The answer seems compellingly YES. In fact, Faure and Gersbach (2017) present a case in which the two approaches lead to the same equilibrium allocation. However, in this paper, we demonstrate that when the circulation of money is concerned, the money-creator approach makes a difference and this difference matters for our understanding of interbank networks.

Naturally, as a means of payment, bank liability circulates in a cycle: It is first lent out from the banking system, then used for layers of trading, and finally deposited back to the banking system. Naturally, a fraction of one bank's liability flows out to another, becoming the former's liability to the latter, as observed by Bianchi and Bigio (2017) and Freixas, Parigi and Rochet (2000). This cycle of circulation, hence, naturally creates an interbank liability network.[2] As this lending-trading-depositing cycle is unnoticed by the

[1] See Chapter 2, "BANK-MONEY".

[2] This network is in fluid because the net interbank positions are in a continuous process of clearing, which sometimes involves debtor banks borrowing reserve from a third-party bank, that is, new interbank liabilities being created to clear old ones. In this paper, we focus on the origination stage and abstract from the clearing stage that follows it.

existing studies on banking networks,[3] so is the interbank network that it creates. This is unsurprising. These studies take the loanable-funds approach and funds are invested and returned, but, unlike means of payment, do not circulate. We model the circulation cycle and characterize how the interbank network that it creates is interwoven with the real economy. Moreover, we examine its implications for bank lending.

The model economy is populated by firms, households and banks. Firms own production technologies, households multiple types of resources (e.g. land, labor), which are the inputs for production.[4] Banks play the role of bridging these two parties. To demonstrate the difference that the money-creator approach makes, we also consider a mirror economy that is identical with the model economy except that the bridging role of banks is modelled with the loanable-funds approach. With this approach, banks first take deposits of resources from households and then lend them to firms. In the model economy, bank's role is modelled with the money-creator approach. As in reality they create and lend deposits to real-economy firms, so in the model economy banks create and lend their liabilities to firms. Then, firms use bank liability to trade resources with households. Finally households deposit the sales incomes with their banks, completing the circulation cycle of bank liability. These two economies provide a level-playing field to compare the two approaches.

In the model economy, the circulation cycle of bank liability creates a network of interbank liabilities and this network is entirely determined by the real economy. Firms' production technologies determine the quantity of liability that each bank lends out and the way that it is split as spending between the multiple types of resources. The sub-streams of all bank's liabilities spent on each type of resources merge into its sales revenue. The distribution of resources across households and their association with banks determine how the sales revenue of resources is split as deposits between banks. Thus inter-flows of liability between any pair of banks, and hence the interbank liability network resultant, are entirely determined by the fabric of the real economy. In contrast, in the mirror econ-

[3] Freixas, Parigi and Rochet (2000) consider the inter-flows of deposits between banks, but they do not consider the lending and trading stages of the circulation.

[4] In the extension of the model, the inputs also include the products themselves as intermediate goods, whereby an Input-Output (I-O) network is accommodated.

omy, equilibrium establishes no interbank network, let alone its connection with the real economy. In equilibrium, banks in deficits of funds borrow from those in surpluses, but the identities of the lender banks are not determined.

In reality, interbank networks that passively result from the circulation of banks' liabilities are important. Lights can be shed by empirical studies that look into the flows of bank reserves. Any interbank debt so passively formed will eventually be settled with a flow of reserves. Observe that such flows are in one direction only because the creditor banks need not pay funds back. In contrast, active borrowing of reserves generates flows in both directions (unless the borrower banks default): A flow of funds to the borrower now must be paired with an inverse flow of repayment to the lender in the future. Therefore, the volume of unpaired flows of reserve funds is indicative of the importance of passively formed interbank debts. Furfine (2003), using the Fedwire funds data, finds that during February and March 1998 there are on average about $15,000$ flows of Fed funds per day, but identifies only about $3,000$ paired transactions of overnight borrowing. Typically the majority of active interbank borrowing is overnight. Then each day about $15,000 - 2 \times 3,000 = 9,000$ flows of Fed funds are not due to active interbank borrowing. While there must be a variety of reasons behind these flows, their sheer quantity still suggests the importance of the passively formed interbank exposures.

Besides for interbank exposures, the circulation of bank liability also has implications for bank lending. In the model economy, we characterize how deposits to any bank flow from banks' lending. Therefore, we capture the general equilibrium effect of bank lending on deposits. This effect is important in reality. Given that bank lending has an impact on household incomes and business profits, it naturally impacts on banks' deposits. By capturing this impact, the money-creator approach delivers the following three new insights.

First, the interbank interest rate has a heterogeneous effect on banks' funding costs. Of each unit of liability that a bank lends out, a fraction is deposited back, and the rest flows out to other banks becoming interbank liabilities. The former amounts to the bank being funded by deposits. Only the latter subjects the bank to the expense of interbank interest. Therefore, a bank with a smaller outflow fraction sees its funding cost less dependent on the interbank interest rate. In contrast, if we use the loanable-funds approach, the funding cost of all banks vary one-to-one with the interbank interest rate. With that approach, banks lend out, or borrow in, the marginal unit of funds on the interbank market. The interbank interest rate, therefore, measures the funding cost for *all* banks.

Second, heterogeneity in the outflow fraction is a source of inefficiency of bank lending. As said above, banks with a smaller outflow fraction rely more on deposits, less on interbank borrowing, and hence have a lower funding cost.[5] They charge a lower lending rate, giving their borrower firms a financial advantage. Thereby these firms obtain too much of the resources. Hence the inefficiency. In contrast, if we use the loanable-funds approach, the interbank interest rate has a homogeneous effect on banks' funding cost and therefore this source of inefficiency does not obtain.

Third, if we use the loanable-funds approach, other things equal, the identities of depositors and borrowers do not matter for banks. A deposit of one thousand dollars from a butcher is equivalent to that from a programmer, a loan of half-million dollars to a pig-farmer equivalent to that to a software developer. In contrast, if we use the money-creator approach and track the circulation of bank liability, we find the identities of depositors and borrowers matter. For example, if a bank mainly lends to pig farmers, then deposits from butchers benefit it more than those from programmers because the former increases the chance that the liability lent out by the bank flows back to itself, thereby reducing its funding cost. Similarly, if the bank's main depositors are butchers, then a loan of half-million dollars to a pig-farmer benefits the bank more than that to a software developer.

This paper contributes to the growing literature on financial networks; for a survey see Allen and Babus (2009), Bougheas and Kirman (2014), Cabrales et al (2015), and Glasserman and Young (2015). While most of the studies in the literature consider an exogenous network, exceptions include Freixas et. al. (2000) and recently Acemoglu et al (2014), Allen et al (2012), Babus (2016), Farboodi (2015) and Zawadowski (2013). These studies have shed many important insights. Freixas et. al. (2000) show the vulnerability of the banking

[5] In reality, if a bank has more extensive branches, then the outflow fraction tends to be smaller, because more likely will the money that it lends out be deposited back.

network to mis-coordinated withdrawals in the manner of Diamond and Dybvig (1983). Allen et. al. (2012) show that systemic risks critically depend on the funding maturity of the banks. Both Acemoglu et al (2014) and Farboodi (2015) underline that an interbank link can bring about both an opportunity of investment and a chance of contagion.[6] Both Acemoglu et al (2014) and Zawadowski (2013) demonstrate that inefficiency is caused by financial-network externalities, namely that a bank fails to internalize the implication of its decision for banks with which it is not directly linked. Babus (2016), based on Allen and Gale (2000), shows that the mutual insurance network bears a small or even nil systemic risk.

The existing studies all model banks as intermediaries of loanable funds. This paper takes the approach that models banks as creators of means of payment. It makes a difference. First, we consider an interbank network that *passively* results from the lending-trading-depositing cycle of circulation of bank liability. This cycle of circulation is unconsidered by the existing studies. Apart from Freixas et al (2000), they all consider interbank networks created by banks *actively* trading funds between themselves. Freixas et al (2000) consider an interbank network formed by deposits moving around, which, with the lending and trading stages absent, does not form the full circulation cycle. As a result, unlike us, they do not consider the role of production technologies or I-O networks in structuring the interbank network, not do they capture the general equilibrium effect of bank lending on deposits or derive the three new insights expounded above.

The money-creator approach of this paper is used by Wang (2019), who shows that a purely nominal monetary policy can relax banks' borrowing constraint. Other recent studies that consider the circulation of bank liability as a means of payment in general equilibrium[7] include Bianchi and Bigio (2017), Donaldson et al (2018), Faure and Gersbach (2016, 2017), Jakab and Kumhof (2015) and Parlour et al (2017). In particular, as the

[6] This trade-off, in a reduced form, is also studied by Blume et al (2013) and Erol and Vehra (2014). Moreover, Glasserman and Young (2015) survey the studies on a similar trade-off, between the benefit of diversification and the cost of possible contagion.

[7] Earlier literature has examined the circulation of private liabilities as a means of payment using search-matching frameworks, e.g. Cavalcanti et. al. (1999) and Williamson (1999).

present paper, Faure and Gersbach (2017) and Jakab and Kumhof (2015) compare the money-creator approach to the loanable-funds approach. The former presents a case in which these two approaches lead to the same equilibrium allocation, the latter showing that the two approaches differ substantially in their quantitative implications. These studies, however, are not concerned with how the interbank network is determined by the real economy.

The money-creator approach is based on the fact that banks' liabilities are widely accepted as a means of payment, whereas real-economy firms' are not. Explanations for this fact have been offered by studies unconcerned with banking networks. It is attributed to the stronger commitment power of banks by Kiyotaki and Moore (2001), or to their better warehousing technology by Donaldson et al (2018). It is derived by Cavalcanti et al (1999) and Gu et al (2003) using a money-search framework. The last two papers build on the seminal work of Kiyotaki and Wright (1989) on fiat money. However, while our paper is concerned with banks creating means of payment, it has nothing to do with fiat money. Indeed, in Europe, banks had been creating means of payment, by lending out their liabilities in forms of banknotes or bills of exchange, hundreds of years anterior to the existence of fiat money.

The rest of the paper is organized as follows. The model is set up in Section 2. Before it is analyzed in Section 4, we examine the mirror economy in Section 3. The model is extended in Section 5 to accommodate the I-O network. Finally, Section 6 concludes.

2 The Model

The economy lasts for two dates, $t = 0$ for contracting and production, and $t = 1$ for yielding and consumption. We present first the real side of the economy and then the role of banks.

On the real side, there are N sectors, each consisting of the continuum $[0, 1]$ of firms which are managed by their owners. Firms use J types of resources as well as the human capital ξ of the managers to produce the consumption good, corn. The production technology of sector $i \in \mathbf{N} := \{1, 2, ..., N\}$ is:

$$y_i = A_i \xi^{1-\alpha_i} \left(\prod_{j=1}^{J} x_{ij}^{\beta_{ij}} \right)^{\alpha_i},$$

where $\alpha_i \in (0,1)$, $\beta_{ij} \geq 0$ and $\sum_{j=1}^{J} \beta_{ij} = 1$. Without loss of generality, we normalize $\xi = 1$. All resources are owned by households: Household $h \in \mathbf{H} := \{1, 2, ...H\}$ owns z_{hj} units of type j resources, of which the aggregate supply is hence:

$$X_j := \sum_{h \in \mathbf{H}} z_{hj},$$

for $j \in \mathbf{J} := \{1, 2, ..., J\}$. Both N and H are large numbers.

The economy also has N banks. Banks matter for the allocation of resources for a twofold reason. First is the following friction:

Assumption 1 (the Friction): Firms cannot borrow resources directly from households, but banks can.

Second, firms can borrow from banks. On banks' asset side, we assume that there exists bank lending specialization: Firms of sector i can only borrow from Bank i for any $i \in \mathbf{N}$. Bank lending specialization is well documented by empirical studies.[8]

On banks' liability side, we abstract from the process of banks' competition for deposits from households and model its consequence outright. First, the competition leads to a net deposit rate of $r_d \geq 0$. We assume that banks' competition for deposits is imperfect, for which there is strong empirical evidence.[9] As a result, the deposit rate r_d is below the interbank interest rate r_b by a positive gap $\rho > 0$. Second, this competition leads to a network of association of households with banks: Household $h \in \mathbf{H}$ deposits a fraction

[8]See among others Daniels and Ramirez (2008), Jonghe et. al. (2016), Liu and Pogach (2016), Ongena and Yu (2017), and Paravisini et. al. (2014).

[9]See Kopecky and Van Hoose (2012) and Matutes and Vives (2000) and the empirical studies cited therein.

$\zeta_{hi} \geq 0$ of what it owns with Bank i, where $\sum_{i \in \mathbf{N}} \zeta_{hi} = 1$ for any $h \in \mathbf{H}$. Therefore, of any type $j \in \mathbf{J}$ resources, a fraction d_{ji} is deposited with Bank i, where

$$d_{ji} := \frac{1}{X_j} \sum_{h \in \mathbf{H}} z_{hj} \zeta_{hi}.$$

Obviously, $d_{ji} \geq 0$ and $\sum_{i \in \mathbf{N}} d_{ji} = \sum_{i \in \mathbf{N}} \sum_{h \in \mathbf{H}} \frac{z_{hj}}{X_j} \zeta_{hi} = \sum_{h \in \mathbf{H}} \frac{z_{hj}}{X_j} \sum_{i \in \mathbf{N}} \zeta_{hi} = 1.$

Matrix $\{d_{ji}\}_{j \in \mathbf{J}, i \in \mathbf{N}}$ hence represents the distribution of resources between banks, profile $\{A_i, \alpha_i, \beta_{ij}\}_{i \in \mathbf{N}, j \in \mathbf{J}}$ the production technologies. Together, $\{A_i, \alpha_i, \beta_{ij}, d_{ji}\}_{i \in \mathbf{N}, j \in \mathbf{J}}$ represents the real economy.

Due to the Friction, the route for firms to obtain resources from households has to be bridged by banks. There are two approaches to model the bridging role of banks. One is the *loanable-funds approach*. With this approach, banks lend out real goods – typically referred to as funds – and are modelled as *intermediaries of loanable funds*: They first borrow funds from depositors and then lend funds to firms. Using this approach, the role of banks is then modelled as follows.

Approach LF (role of banks): At $t = 0$, banks first take deposits of resources from households and then lend the resources to firms.

In this paper, we use a different approach, which is based on an alternative way of modelling the same friction. Observe that borrowing is a transaction in which the lender gives up her resources in exchange for a promise of the borrower to pay her back. Borrowing can be done if and only if the lender accepts the borrower's promise to pay for the exchange. Therefore, the Friction, that firms cannot borrow resources from households but banks can, is equivalent to the following assumption:

Assumption 1′ (the Friction): Households shun firms' promise to pay corn at $t = 1$, but accept banks', in exchange for their resources.

This assumption captures the fact that banks' liabilities (in the form of deposits) are widely accepted as a means of payment, whereas real-economy firms' are not. By this assumption, rather than first borrow resources from households and then lend the resources to firms, the role of banks is modelled as follows.

Approach MC (role of banks): At $t = 0$, banks lend their promise to pay to firms and firms then use it to exchange resources from households.

Hence this approach models banks as *creators of means of payment*, which is their promise to pay, namely their liabilities. We define *one unit of liability* as a promise to pay one unit of corn at $t = 1$. At $t = 0$, to acquire a bank's promise to pay, a firm enters a loan contract (m, R): Presently the firm receives m units of the bank's liability; and at $t = 1$, the firm will repay mR units of corn to the bank (which will then redeem its liability using the corn paid in). Thus R is the gross lending rate. After borrowing, firms use borrowed bank liability to exchange resources from households. Let the price of type $j \in \mathbf{J}$ resources be p_j. That is, one unit of them is exchanged with p_j units of bank liability. Households deposit all the sales revenue with banks, earning a deposit rate of r_d. We have seen that a fraction d_{ji} of the sales revenue of type $j \in \mathbf{J}$ resources is deposited into Bank $i \in \mathbf{N}$.

At $t = 0$, banks' liabilities thus circulate in a cycle through three stages: Lending, trading, and depositing. This cycle of circulation is illustrated in Figure 1.

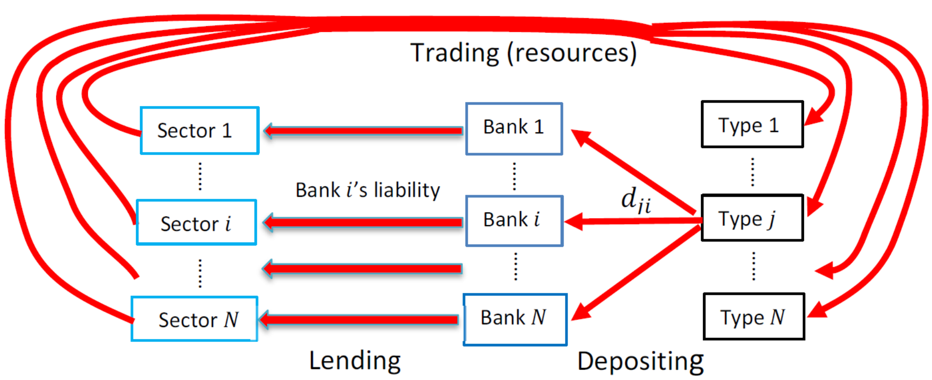

Figure 1: The circulation of banks' liabilities at $t = 0$: They are first lent out to firms, then split as spending between the multiple types of resources, and finally deposited back into the banking system.

The circulation of bank liability generates interbank liability links. In this economy, there is no risk of bank default and each bank's promise to pay one unit of corn at $t = 1$ is worth one unit of corn at $t = 0$. Therefore, banks' liabilities are one-to-one exchangeable at $t = 0$. Hence, when F units of Bank n's liability is deposited with Bank i, what Bank i does is as follows. It adds F units of credit to the depositor's account on the liability side. On the asset side, it now holds Bank n's promise to pay F units of corn, that is, Bank n owes a debt of F to Bank i. Given that the net interbank interest rate is $r_d + \rho$, the deposit changes Bank i's balance sheet as follows.

Assets	Liabilities
Old assets: X	Old liabilities: X
Debt owed by Bank n: $F \times (1 + r_d + \rho)$	The account of the depositor: $F\,(1 + r_d)$
	Gain to the equity: $F \times \rho$

Table 1: The balance sheet of Bank i with a deposit of F units of Bank n's liability

We will track the flows of liability between each pair (i, n) of banks and thus determine the net position Υ_{ni} between them. These net positions together form an network of interbank connections $\{\Upsilon_{ni}\}_{(n,i)\in\mathbf{N}\times\mathbf{N}}$ at $t = 0$.

At $t = 1$, firms produce corn. A firm that borrowed m units of bank liability at $t = 0$ repays the bank with mR units of corn to clear the debt. Banks then use the corn of loan repayment to settle their liabilities to other banks and the depositors. Lastly, households, firms' manager-owners and bank shareholders consume the corn that they have obtained.

Remarks on the model:

This paper aims to investigate the interbank network that results from the circulation of bank liability as a means of payment. We intend to use a model that contains only the elements necessary for this investigation. Hence, we have abstracted from several real-life facts.

First, in reality, when banks issue loans by creating deposits, they are typically subject to constraints regarding the reserve rate and the capital adequacy rate. We abstract from these constraints; incorporating them will not qualitatively change the paper's results.

Second, in reality, banks issue nominal liability, that is, a promise to pay fiat money, whereas in the paper banks issue promises to pay corn, a real good. This abstraction is intended to show that banks' creation of means of payment is unrelated to fiat money. It also gives us the convenience of using a finite-period model for the investigation. The abstraction is harmless. After all, the way in which bank liability circulates is unchanged if banks' liabilities are nominal.

Third, we have abstracted from the process of banks' competition for deposits, because it is an independent issue to our investigation. Suppose our setting is extended to explicitly model this competition. Then in equilibrium, there will be a specific deposit rate r_d and a specific distribution $\{d_{ji}\}_{j\in\mathbf{J},i\in\mathbf{N}}$ of resources. The part of the equilibrium that concerns the circulation of bank liability and the resultant interbank network will coincide with the equilibrium of our model with the parameters r_d and $\{d_{ji}\}_{j\in\mathbf{J},i\in\mathbf{N}}$ taking these specific values. Any particular way of modelling the competition does not matter, so long as it is imperfect and thus leaves a gap $\rho > 0$ between the interbank interest rate r_b and banks' deposit rate r_d. This gap certainly exists in reality.

Equilibrium is defined as follows, where $\mathbf{P} := \{p_j\}_{j\in\mathbf{J}}$ denotes the vector of resource prices. Given there is a large number of banks, each bank is too small to affect the resource prices and hence takes them as given.

Definition 1 *A profile* $\left(\{m_i, R_i\}_{i\in\mathbf{N}}, \mathbf{P}\right)$ *forms an equilibrium, if*

(a) given $(R_i, \mathbf{P})$, m_i *is the optimal demand of Bank* i*'s liability by firms of sector* i;

(b) given $\mathbf{P}$, $\{R_n\}_{n\in\mathbf{N}/\{i\}}$ *and the demand function* $m_i(R_i;\mathbf{P})$, R_i *is the optimal interest rate that the Bank* i *charges; and*

(c) for each $j \in \mathbf{J}$, p_j *clears the market for type* j *resources.*

We will demonstrate how in equilibrium the interbank liability network $\{\Upsilon_{in}\}_{(i,n)\in\mathbf{N}\times\mathbf{N}}$ is determined by the real economy $\left\{A_i, \alpha_i, \beta_{ij}, d_{ji}\right\}_{i\in\mathbf{N},j\in\mathbf{J}}$. Passing on to that, however, we consider a mirror economy that is the identical with the model economy except that banks' role is modelled with Approach LF instead of Approach MC above. This provides a level-playing field to compared the two approaches.

3 The Mirror Economy Where Banks Are Modelled with Approach LF

In the mirror economy, the real side is still $\{A_i, \alpha_i, \beta_{ij}, d_{ji}\}_{i\in\mathbf{N},j\in\mathbf{J}}$ and firms of sector i still borrow only from Bank i for any $\in \mathbf{N}$. However, banks' role is modelled with Approach LF: At $t = 0$, they first take deposits of resources in and then lend them out. Specifically, Bank i first receives deposits of d_{ji} fraction of type j resources for all $j \in \mathbf{J}$ and then lend them to sector i firms, while also borrowing resources from, or lending them to, other banks. A difficulty immediately arises. There is no obvious way to insert the market where resources are traded and their prices $\mathbf{P}$ are determined. Indeed, if firms have obtained all the resources via borrowing from banks, this market seems redundant. However, let us not dwell on this issue; let us suppose that with certain modelling techniques, this difficultly can be overcome and the resource prices $\mathbf{P}$ in units of corn can be determined in a manner of market clearing.

Consider firms' borrowing decision. If a firm of sector i borrows a profile $\{x_{ij}\}_{j\in\mathbf{J}}$ of resources from Bank i, given $\mathbf{P}$, the value m_i of its borrowing is

$$m_i = \sum_{j\in\mathbf{J}} p_j x_{ij}. \tag{1}$$

Given the bank's gross lending rate R, at $t = 1$ the firm repays mR units of corn to settle the loan. Hence, the firm's decision problem is

$$\max_{m_i} A_i \left(\prod_{j\in\mathbf{J}} x_{ij}^{\beta_{ij}}\right)^{\alpha_i} - m_i R, \text{ s.t. } (1). \tag{2}$$

The firm's demand of Bank i's funds is as follows.

$$m_i = \left(\frac{\alpha_i A_i}{R}\right)^{\frac{1}{1-\alpha_i}} \left(\prod_{j\in\mathbf{J}} \left(\frac{\beta_{ij}}{p_j}\right)^{\beta_{ij}}\right)^{\frac{\alpha_i}{1-\alpha_i}} := m_i\left(R;\mathbf{P}\right). \tag{3}$$

Now consider banks' lending decisions. On the liability side, Bank i is deposited with a fraction d_{ji} of type j resources for all $j \in \mathbf{J}$. The total value of its deposits is hence

$$D_i = \sum_{j\in\mathbf{J}} p_j X_j d_{ji}, \tag{4}$$

for which the bank pays a gross deposit rate of $1 + r_d$. On the asset side, if Bank i charges a gross lending rate of R_i, then the demand of its funds is $M_i = m_i(R_i; \mathbf{P})$, from which it earns revenue $M_i R_i$. The surplus (or deficit) $D_i - M_i$ of funds is lent (or borrowed) on the interbank market, earning a gross interest rate of $1 + r_d + \rho$. Therefore, Bank i's value is

$$\begin{aligned}\Pi_i &= M_i R_i + (D_i - M_i)(1 + r_d + \rho) - D_i (1 + r_d) \\ &= M_i (R_i - (1 + r_d)) + (D_i - M_i)\rho. \end{aligned} \tag{5}$$

Formula (5) is intuitive: The profit margin of lending is $R_i - (1 + r_d)$ and the interbank credit position $D_i - M_i$ bears a net return rate of ρ, the gap between the interbank interest rate r_b and the deposit rate r_d. The bank's problem is

$$\max_{R_i} M_i (R_i - (1 + r_d)) + (D_i - M_i)\rho, \text{ s.t. } M_i = m_i(R_i; \mathbf{P}). \tag{6}$$

The optimal lending rate of Bank i is:

$$R_i^* = \frac{1}{\alpha_i}(1 + r_d + \rho). \tag{7}$$

Of this formula, the term $1/\alpha_i$ is the mark-up factor due to the monopolistic power that Bank i has over the firms of sector i, and the term in the brackets is the marginal cost of lending of the bank, denoted by c_i^{LF}. Because $1 + r_d + \rho = 1 + r_b$,

$$c_i^{LF} = 1 + r_b. \tag{8}$$

This equation is intuitive. The opportunity cost of each unit of funds lent out to firms is the interbank rate $1 + r_b$ because if it is not lent to firms, it can be put on the interbank market earning returns at rate $1 + r_b$.

We will make four observation regarding the mirror economy. First, from Problem (6), for the lending decision of Bank i, only the total quantity of deposits D_i and that of loans M_i matter, but the identities of the depositors and the borrowers do not. That is, for banks, other things equal, a deposit of one thousand dollars from a butcher is the same as that from a programmer and a loan of half-million dollars to a pig-farmer is the same as that to a software developer.

Second, $\partial c_i^{LF}/\partial r_b = 1$ for all $i \in \mathbf{N}$, that is, the marginal lending cost for *all* banks varies one-to-one with the interbank interest rate r_b. That is intuitive. In the mirror economy, when making the lending decision, banks take the quantity of funds deposited in as given, and the opportunity cost of funds is the interbank interest rate, homogeneous for all banks.

Third, the equilibrium establishes no network of interbank liabilities. Bank i's net interbank liability position is $\Upsilon_i = M_i - D_i$. In the generic case, $\Upsilon_i \neq 0$ in equilibrium. However, the identities of the counterparties of its interbank positions are indeterminate.

Fourth, due to the Cobb-Douglas form of the production technologies, firms of sector $n \in \mathbf{N}$ spend a fraction β_{nj} of funds they borrow on type j resources. Suppose that with the loanable-funds approach, somehow the equilibrium resource prices $\mathbf{P}^*$ are still determined by the market clearing conditions:

$$\sum_{n \in \mathbf{N}} m_n \left(R_n^*; \mathbf{P}^*\right) \beta_{nj} = p_j^* X_j. \tag{9}$$

Then,

Proposition 1 *If $\alpha_i = \alpha$ for any $i \in \mathbf{N}$, the equilibrium attains the First-Best allocation. That is, the only source of lending inefficiency in the mirror economy is the heterogeneity in banks' monopolistic power.*

Proof. See Appendix. ■

In the mirror economy (as well as in the model economy), resources are allocated between firms, and firms are all subject to the monopolistic power of their banks. Therefore, the monopolistic power per se is not necessarily a source of inefficiency. By the proposition, if all the firms are subject to the monopolistic power of the same strength – i.e. all $\alpha_i = \alpha$ – then its effects for the resource allocation are cancelled out. However, if banks' monopolistic power is heterogeneous, this heterogeneity is a source, and the only source, of lending inefficiency.

We summarize the above four observations in the following proposition.

Proposition 2 *In the mirror economy, which is the identical with the model economy except that banks' role is modelled with Approach LF, the following claims are true.*

(1) The equilibrium establishes no network of interbank liabilities, let alone its connection with the real economy.

(2) Banks' funding cost varies one-to-one with the interbank interest rate.

(3) The identities of depositors and borrowers do not matter for banks.

(4) the heterogeneity in α_i is the only source of lending inefficiency.

These claims are not true if banks are modelled with Approach MC, as is shown in the next section.

4 The Interbank Liability Network Determined by the Real Economy

We go back to the model economy in which the Friction is represented by Assumption $1'$ and banks are modelled as creators of means of payment. In this economy, firms' problem is represented in the same way as in the mirror economy. Because banks do not default, a unit of bank liability (i.e. a bank's promise to pay one unit of corn) is worth one unit of corn. The prices in units of corn in the mirror economy are thus equivalent to the prices in units of bank liability. If at $t = 0$ a sector i firm acquires m_i units of Bank i's liability, then the firm's budget constraint is

$$\sum_{j \in \mathbf{J}} p_j x_{ij} = m_i,$$

exactly the same as (1). Given the gross lending rate R of the bank, at $t = 1$ the firm repays $m_i R$ units of corn to settle the debt, as before. Hence, the firm's decision problem is the same as that given by (2), its demand function for Bank i's liability $m_i(R; \mathbf{P})$ the same as that given by (3). For borrowers, indeed, money is equivalent to the real value that it represents, and the money-creator approach makes no difference.

Now consider banks' decision problem. If Bank i charges lending rate R_i, then it lends $M_i = m_i(R_i; \mathbf{P})$ units of its liability to sector i firms at $t = 0$ and will receive $M_i R_i$

units of corn in return at $t = 1$. Out of these M_i units of liability, suppose D_{own} units are deposited back with Bank i, $M_i - D_{own}$ units flowing out to other banks becoming interbank liabilities. For the inverse flows, let D_{other} denote the quantity of other banks' liabilities that are deposited with Bank i. Given that the gross deposit rate is $1 + r_d$ and the gross interbank rate is $1 + r_d + \rho$, at the end of $t = 0$, Bank i's balance sheet is as follows.

Assets	Liabilities
Loans to firms $(M_i R_i)$	Depositors of its own liability $(D_{own}(1+r_d))$
	Depositors of other banks' liability $(D_{other}(1+r_d))$
Credit to other banks whose liabilities are deposited with Bank i $(D_{other}(1+r_d+\rho))$	Debt owed to the banks with which Bank i's liability is deposited $((M_i - D_{own})(1+r_d+\rho))$
	Equity (Π_i)

Table 2: Bank i's balance sheet at the end of $t = 0$

The bank's deposits sum up to $D_i = D_{own} + D_{other}$. The bank's net liability position to other banks $\Upsilon_i = (M_i - D_{own}) - D_{other} = M_i - D_i$, the same as what have seen with the loanable-funds approach. The bank's value $\Pi_i = M_i R_i + D_{other}(1 + r_d + \rho) - [D_i (1 + r_d) + (M_i - D_{own}) (1 + r_d + \rho)]$. With a little rearrangement,

$$\Pi_i = M_i (R_i - (1 + r_d)) + (D_i - M_i) \rho, \tag{10}$$

which is the same as formula (5) of the bank's value in the mirror economy. That is intuitive. When accounting is concerned, money is equivalent to the value it represents, otherwise, banks create value out of thin air, which no one can do.

A difference is made when we comes to money circulation. Funds, as a real good, are invested and returned, but they do not circulate. In contrast, banks' liabilities, as a means of payment, naturally circulate. Consider the circulation of any Bank n's liability. First, M_n units of it are lent to sector n firms. Then, according to the Cobb-Douglas technology of the firms, a β_{nj} fraction of the bank's liability is spent on type j resources. It follows that the total sales revenue of this type of resources is

$$E_j = \sum_{n \in \mathbf{N}} M_n \beta_{nj}. \tag{11}$$

A fraction d_{ji} of this revenue is deposited with Bank i. Hence, the bank's total deposit is $D_i = \sum_{j \in \mathbf{J}, n \in \mathbf{N}} M_n \beta_{nj} d_{ji}$. With some rearrangement,

$$D_i = M_i \times f_{ii} + \sum_{n \in \mathbf{N}/\{i\}} M_n \times f_{ni}, \tag{12}$$

where for any $(n, i) \in \mathbf{N} \times \mathbf{N}$,

$$f_{ni} := \sum_{j \in \mathbf{J}} \beta_{nj} d_{ji} \tag{13}$$

is the fraction of bank n's liability flowing into Bank i: Out of one unit of liability that Bank n lends out, fraction β_{nj} is spent on each type $j \in \mathbf{J}$ of resources and out of this spending fraction of d_{ji} is deposited into Bank i, hence in total a fraction f_{ni} of Bank n's liability flowing into Bank i.

Considering that $M_n f_{ni}$ units of Bank n's liability flow to Bank i and inversely $M_i f_{in}$ units of Bank i's liability flow to Bank n, the net liability position of Bank i to Bank n is hence $\Upsilon_{in} = M_i f_{in} - M_n f_{ni}$. Therefore, the circulation of banks' liabilities naturally gives rise to an interbank liability network $\boldsymbol{\Upsilon} = \{\Upsilon_{in}\}_{(i,n) \in \mathbf{N} \times \mathbf{N}}$. In contrast, no such a network arises in the mirror economy.

Moreover, according to equation (12), for any Bank i, each Bank $n \in \mathbf{N}$ sees fraction f_{ni} of the means of payment M_n that it lends out flows to Bank i and all these flows merges together to form Bank i's deposits D_i. We thus capture the general equilibrium effect of lending by the banking system on banks' deposits. Substitute (12) for D_i in equation (10) and Bank i's value is

$$\Pi_i(R_i) = M_i [R_i - (1 + r_d) - (1 - f_{ii}) \rho] + \rho \sum_{n \in \mathbf{N}/\{i\}} M_n \times f_{ni}.$$

The bank's problem is hence:

$$\max_{R_i} \Pi_i(R_i), \quad s.t. \quad M_i = m_i(R; \mathbf{P}).$$

The optimal lending rate of Bank i is thus:

$$R_i^* = \frac{1}{\alpha_i} \left[1 + r_d + (1 - f_{ii}) \rho \right]. \tag{14}$$

As in the mirror economy, the term $1/\alpha$ is the mark-up factor due to the monopolistic power of Bank i and the term in the square brackets is the bank's marginal cost of lending, denoted by c_i^{MC}. Because $\rho = (1 + r_b) - (1 + r_d)$,

$$c_i^{MC} = f_{ii} (1 + r_d) + (1 - f_{ii}) (1 + r_b). \tag{15}$$

For intuition of equation (15), observe that out of each unit of liability that Bank i lends out, f_{ii} unit circulates back to the bank and the rest $1 - f_{ii}$ unit flows out to other banks, becoming interbank liabilities. For the former, the bank pays the deposit rate $1 + r_d$, and for the latter, the interbank rate $1 + r_b$. Hence equation (15).

Finally, with the money-creator approach, at the trading stage of the circulation, naturally markets open where firms use bank liability to exchange resources with households and the resource prices $\mathbf{P}$ are determined. We have found the aggregate spending on type j resources E_j in (11). With bank n charges interest rate R_n^*, the size of its lending is $M_n = m_n (R_n^*; \mathbf{P})$. The aggregate demand for type j resources is E_j/p_j, while the aggregate supply is X_j. The market clearing condition for each type $j \in \mathbf{J}$ resources is thus:

$$\frac{1}{p_j} \sum_{n \in \mathbf{N}} m_n (R_n^*; \mathbf{P}) \beta_{nj} = X_j. \tag{16}$$

These market clearing conditions together determine the equilibrium resource prices $\mathbf{P}^*$.

We prove the existence of a unique equilibrium in the following proposition.

Proposition 3 *There is a unique equilibrium. The equilibrium establishes a unique interbank liability network $\mathbf{\Upsilon} = \{\Upsilon_{in}\}_{(i,n) \in \mathbf{N} \times \mathbf{N}}$, of which the entry Υ_{in} is the net liability that Bank i owes to Bank n and*

$$\Upsilon_{in} = m_i (R_i^*; \mathbf{P}^*) f_{in} - m_n (R_n^*; \mathbf{P}^*) f_{ni},$$

where function $m_i (R; \mathbf{P})$ is given by (3), R_i^* by (14), and f_{ni} by (13).

Proof. See Appendix. ■

Thus far, we have considered the two economies that are identical except the way in which the role of banks is modelled. It is modelled with the loanable-funds approach in the mirror economy and the money-creator approach in the model economy. The two economies are compared in the subsection below, so as to elicit the new insights that the money-creator approach sheds.

4.1 The new insights

1. *The Interbank network* $\boldsymbol{\Upsilon}$ *is determined by the real economy* $\left\{A_i, \alpha_i, \beta_{ij}, d_{ji}\right\}_{i\in\mathbf{N}, j\in\mathbf{J}}$.

In the model economy, the equilibrium establishes a unique interbank liability network

$$\boldsymbol{\Upsilon} = \left\{m_i\left(R_i^*; \mathbf{P}\right) f_{in} - m_n\left(R_n^*; \mathbf{P}\right) f_{ni}\right\}_{(i,n)\in\mathbf{N}\times\mathbf{N}}.$$

Of this matrix, the demand functions $m_i\left(R; \mathbf{P}\right)$, given by (3), are determined by the technologies $\left\{A_i, \alpha_i, \left\{\beta_{ij}\right\}_{j\in\mathbf{J}}\right\}_{i\in\mathbf{N}}$; and the inter-flow fractions $f_{in} = \sum_{j\in\mathbf{J}} \beta_{ij} d_{jn}$ are determined both by the technologies and the resource distribution $\left\{d_{ji}\right\}_{j\in\mathbf{J}, i\in\mathbf{N}}$. The interbank liability network $\boldsymbol{\Upsilon}$, in its entirety, is determined by and interwoven with the real economy $\left\{A_i, \alpha_i, \beta_{ij}, d_{ji}\right\}_{i\in\mathbf{N}, j\in\mathbf{J}}$. In contrast, if we follow the loanable-funds approach, by Proposition 2, the equilibrium establishes no network of interbank liabilities, let alone its connection with the real economy. The difference arises because in the model economy, banks pump out their liabilities, which, as a means of payment, naturally circulate in a cycle (as illustrated in Figure 1), and the inter-flows of banks' liabilities create an interbank liability network. In contrast, in the mirror economy, banks lend out funds, and while banks do run a surplus or deficit of funds, the identities of their counterparties are indeterminate without further assumptions.

2. *The interbank interest rate* r_b *has a heterogeneous effect on banks' lending cost.*

In the model economy, by (15), $\partial c_i^{MC}/\partial r_b = 1 - f_{ii}$. That is, a unit increase in the interbank rate r_b raises Bank i's lending cost by $1 - f_{ii}$ unit, where $1 - f_{ii}$ is the outflow fraction of the bank. In contrast, if we follow the loanable-funds approach, all banks' lending cost varies one to one with the interbank rate r_b. The difference arises because in the model economy, out of each unit of liability that Bank i lends out, fraction f_{ii} unit circulates back, only the rest $1 - f_{ii}$ unit flowing out to other banks and subjecting the bank to the expense of interbank interest. In contrast, this effect of lending on deposits is unconsidered with the loanable-funds approach. Instead, when making lending, banks take the size of their deposits as given. They hence lend out, or borrow in, the marginal unit of funds on the interbank market. The interbank interest rate, therefore, measures the funding cost for *all* banks.

3. *The identities of depositors and borrowers matter.*

The higher the flowed-back fraction f_{ii}, the lower is the lending cost of Bank i because the cost of the flowed-back liability is lower than that of the flowed-out by a gap $\rho > 0$. By (13), $f_{ii} = \sum_{j \in \mathbf{J}} \beta_{ij} d_{ji}$. The value of f_{ii} is raised by a positive sorting of $\left\{\beta_{ij}\right\}_{j \in \mathbf{J}}$ with $\{d_{ji}\}_{j \in \mathbf{J}}$. Intuitively, if the borrowers of a bank heavily use a type j resources (i.e. β_{ij} is high), then a deposit of the sales revenue of this type of resources channels more of the money that the bank lends out to flow back to itself, thus benefiting it more, than a deposit of another type's, even if the two deposits are of the identical size in equilibrium. Similarly, although in the model economy, the borrowers of a bank use the same production technology, in general, if a bank is deposited with a great fraction of the sales revenue of a type j resources (i.e. d_{ji} is high), then a borrower that uses more of this type of resources benefits the bank more than one that uses less of it. For example, a deposit of ten-thousand dollars from a pig-farmer benefits the bank more than that from a programmer if the bank mainly lends to sausage producers; a loan of half a million dollars to a sausage producer is more profitable than that to a software developer if the bank receive deposits mainly from pig-farmers. That is, the identities of depositors and borrowers matter, whereas they do not if we follow the loanable-funds approach according to Proposition 2.

4. *Heterogeneity in the outflow fraction is a source of lending inefficiency.*

If we follow the loanable-funds approach, by Proposition 2, the heterogeneity in α_i is the only source of lending inefficiency. In contrast, if we follow the money-creator approach, we identify a new source of lending inefficiency: Heterogeneity in the outflow fraction $1-f_{ii}$ of banks. Intuitively, from (15) Bank i's lending cost is $c_i^{MC} = 1 + r_d + (1 - f_{ii})\rho$. Hence, the smaller the outflow fraction $1 - f_{ii}$, the lower the lending cost of Bank i, and hence the lower the lending rate of the bank, which gives Sector i firms an advantage that is irrelevant to their production technology. As a result, they obtain too many resources relative to the First-Best allocation.

To exmine this point in a simple manner, we consider a special case in which all the sectors have the same production technology:

$$\left(A_i, \alpha_i, \beta_{ij}\right) = \left(A, \alpha, \beta_j\right)$$

for any $i \in \mathbf{N}$ and $j \in \mathbf{J}$. Because $\alpha_i = \alpha$ for all $i \in \mathbf{N}$, the equilibrium would attain the first-best allocation if we follow the loanble-funds approach. Given that all the sectors have the same technology, in the first-best allocation $\left\{x_{ij}^{FB}\right\}_{i\in\mathbf{N},j\in\mathbf{J}}$, firms of different sectors should obtain an equal quantity of resources; that is,

$$\frac{x_{ij}^{FB}}{x_{nj}^{FB}} = 1 \tag{17}$$

for any $(i, n, j) \in \mathbf{N} \times \mathbf{N} \times \mathbf{J}$. Now look at the equilibrium allocation $\left\{x_{ij}^{*}\right\}_{i\in\mathbf{N},j\in\mathbf{J}}$ of the model economy. All the firms spend a fraction β_j of the budget on type $j \in \mathbf{J}$ resources. Hence, for any $(i, n, j) \in \mathbf{N} \times \mathbf{N} \times \mathbf{J}$,

$$\frac{x_{ij}^{*}}{x_{nj}^{*}} = \frac{m_i}{m_n}.$$

By (3), m_i is in proportion to $(1/R_i)^{\frac{1}{1-\alpha}}$, while the gross lending rate R_i, by (14), is in proportion to the marginal cost $1 + r_d + (1 - f_{ii})\rho$. Therefore,

$$\frac{x_{ij}^{*}}{x_{nj}^{*}} = \frac{\left(\frac{1}{1+r_d+(1-f_{ii})\rho}\right)^{\frac{1}{1-\alpha}}}{\left(\frac{1}{1+r_d+(1-f_{nn})\rho}\right)^{\frac{1}{1-\alpha}}}, \tag{18}$$

which is different to the socially optimal allocation given in (17) if banks have a heterogeneous outflow fraction $1 - f_{ii}$. To characterize which sectors obtain too much of resources relative to the first allocation, and which too little, we define the average marginal lending cost across all banks by

$$c^e := \left(\frac{1}{N} \sum_{i \in \mathbf{N}} \left(\frac{1}{1 + r_d + (1 - f_{ii})\,\rho} \right)^{\frac{1}{1-\alpha}} \right)^{-(1-\alpha)}.$$

Then, we have the following proposition.

Proposition 4 *Assume* $(A_i, \alpha_i, \beta_{ij}) = (A, \alpha, \beta_j)$ *for any* $i \in \mathbf{N}$ *and* $j \in \mathbf{J}$. *A sector* $i \in \mathbf{N}$ *obtains too much of resources if Bank i's lending cost* $c_i^{MC} < c^e$ *and too little if* $c_i^{MC} > c^e$. *Sectors associated with banks that have the minimum outflow fraction obtain the greatest quantity of resources, those with the maximum outflow fraction the smallest. Moreover, the higher the gap* ρ, *the more the former sectors obtain and the less the latter.*

Proof. See Appendix. ■

Thus far, we have examined how the interbank network resulting from money circulation is structured by the production technologies and the resource distribution of the real economy. In the next section, the model is extended to incorporate an Input-Output (I-O) network, where, therefore, the economy has multiple consumption goods. This multiplicity imposes two technical difficulties. One is that we need an ex-post market to determine the (relative) prices of the multiple goods and this market need open before the loan repayment. The other is the following. Thus far, the model economy has only one consumption good – corn – and it is convenient to allow firms to use corn for the settlement of their loans. It is no longer so in the extension where there are multiple consumptions, otherwise, banks each end up with multiple goods and a new goods market need open to exchange them after the loan settlement. Both difficulties can be overcome by requiring firms to settle the loans with bank liability instead of their product. First, this requirement commands that a market open before the loan settlement where firms sell their product for bank liability, a market where the relative price of multiple goods will be determined if they exist. This helps overcome the first difficulty is overcome. Second, as will be shown, this requirement also leads banks to issue new liability to buy firms' product on this market. This helps overcome the second difficulty. Therefore, the present model is thus modified to accommodate the requirement in the following subsection, before we pass on to the extension. This modification certainly draws the model closer to reality as well. As the net deposit rate r_d plays no role whatsoever, hereafter we assume $r_d = 0$.

4.2 A Modification of the Present Model

In this subsection, firms are required to repay loans with money – i.e. bank liability – instead of their product. Hence, at $t = 1$, a market opens before the loan settlement where firms sell their product for bank liability. On this market, among buyers are naturally households, which have sold their resources to firms and are holding bank liability as deposits of their sales revenue. Banks are buyers too; in equilibrium banks will issue new liability to buy corn on the market. Otherwise, the aggregate demand of money by firms, which is equal to the aggregate loan obligations $\sum_{i\in\mathbf{N}} M_i R_i$, is greater than the aggregate supply, namely the aggregate of deposits $\sum_{i\in\mathbf{N}} D_i$. That is because all the deposits, in the model economy, flow from the money loaned out by banks in the first place and hence $\sum_{i\in\mathbf{N}} D_i = \sum_{i\in\mathbf{N}} M_i$ and $R_i > 1$ for any $i \in \mathbf{N}$. The clearing of the corn market, therefore, commands that banks issue new liability to buy corn at $t = 1$; indeed, that is how they obtain their real profit. The timing of events at $t = 1$ is accordingly changed as follows.

1. Firms produce corn.

2. Each Bank $i \in \mathbf{N}$ issues T_i units of new liability (i.e. promise to pay corn later within date 1). Then, the corn market opens, where banks use the newly issued liabilities and households use bank liability that they have deposited at $t = 0$ to buy corn from firms.

3. Firms use the money from the sale of corn to clear their debts. In particular, sector i firms each pay back $M_i R_i$ units of bank liability to Bank i. Observe that part of these $M_i R_i$ units of bank liability might be the liability of a bank $j \neq i$. Hence, new interbank liabilities are formed as a result of loan settlement.

4. All the interbank liabilities are netted and then settled with a payment of corn. Note that the interbank liabilities formed previously at $t = 0$ bear interest at rate ρ, while those

newly formed bear none as they are almost immediately cleared.

5. Corn is consumed.

At stage 3, firms can use any banks' liabilities to settle their debts. Therefore, at stage 2 the prices of all banks' liabilities in the unit of corn are the same, denoted by p. As we can see, $p = 1$, that is, a unit of bank liability is equivalent to one unit of corn. Therefore, no part of the previous analysis needs to change.

Proposition 5 *$p = 1$ and the aggregate new issuance is equal to the aggregate bank profit in equilibrium.*

Proof. We construct the aggregate demand $D(p)$ for bank liability and the aggregate supply $S(p)$ and then show the unique price equating the demand to the supply is $p = 1$. Regarding the demand, sector i firms want to obtain $M_i R_i$ units of bank liability as long as it can afford it, namely, $M_i R_i p \leq Y_i$, where Y_i is the quantity of their output, otherwise, the firms default and demand Y_i/p units of money. Hence, the demand of sector i firms is $\min(M_i R_i, Y_i/p)$. Observe that given any price of bank liability p at $t = 1$, at $t = 0$ no firms will borrow such a quantity of it that they will default at $t = 1$. Taking into account firms' borrowing decision at $t = 0$, hence, we are not concerned with the part of p where default happens at $t = 1$. Therefore, the aggregate demand is $D(p) = L := \sum_{i \in \mathbf{N}} M_i R_i$, the aggregate loan repayments.

Now consider the supply. Bank i has issued M_i units of liability at date 0, which will be used by households to buy corn on the market at $t = 1$. If it newly issues T_i units of liability, it obtains $T_i p$ units of corn and there are $M_i + T_i$ units of its liability on the market. All of them will return back to the banking system because no households or firms want bank liability at the end of $t = 1$. Bank i receives $M_i R_i$ units of liability (its own or other banks') from the loan repayments. Hence, its net interbank liability is thus $M_i + T_i - M_i R_i = T_i - M_i(R_i - 1)$, which the bank clears using corn of as amount. Altogether, with the new issuance, the bank's value is $\Pi_i = T_i p - [T_i - M_i(R_i - 1)] + (D_i - M_i)\rho = M_i(R_i - 1) + (D_i - M_i)\rho + T_i(p - 1)$. The profit margin of the new issuance is hence $p-1$. This is intuitive: One more liability newly issued can be used to buy p units of corn, but it adds one unit of interbank liability which needs one unit of corn to clear. Therefore, the aggregate of newly issued liabilities is

$$T(p) = \left\{ \begin{array}{c} \infty \text{ if } p > 1 \\ [0, \infty] \text{ if } p = 1 \\ 0 \text{ if } p < 1 \end{array} \right\}$$

and the aggregate supply of bank liability on the corn market $S(p) = T(p) + M$, where $M := \sum_{i \in \mathbf{N}} M_i$ is the aggregate quantity issued at $t = 0$ or the aggregate deposits. Observe that $M < L$. The aggregate demand and supply functions are thus illustrated by Figure 2 below.

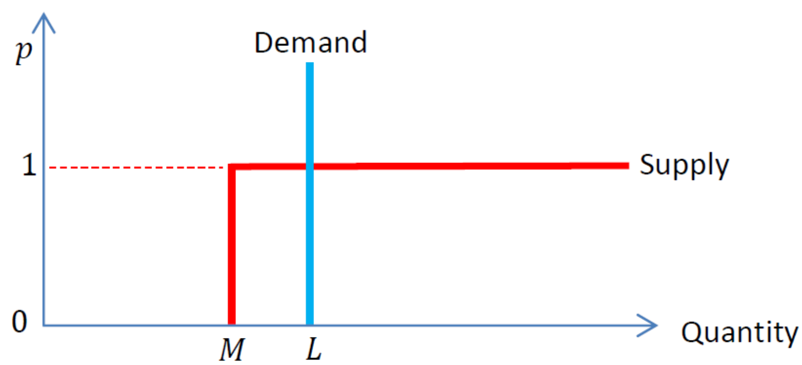

Figure 2: The aggregate demand and supply of bank liability on the corn market at stage 2 of $t = 1$

Obviously, the market clearing price $p = 1$, at which $T(p) + \sum_{i \in \mathbf{N}} M_i = \sum_{i \in \mathbf{N}} M_i R_i$, or equivalently $T(p) = \sum_{i \in \mathbf{N}} M_i (R_i - 1)$, that is, the aggregate new issuance equals the aggregate bank profit. ■

While the aggregate new issuance is pinned down in equilibrium, individual banks' is not. At $p = 1$, banks are indifferent between issuing new liability to buy corn on the market and obtaining corn from its interbank credit positions. This indeterminacy leads to multiple equilibria. In what follows we focus on one in which Bank i newly issues $T_i = \Pi_i = M_i (R_i^* - 1) - \Upsilon_i \rho$ for any $i \in \mathbf{N}$, that is, each bank obtains all its profit from the corn market, none from the interbank clearance. This equilibrium will be referred to as the *convenient equilibrium.* It is called so because of the convenience that in this equilibrium, interbank liabilities are wholly cleared by netting, no payment of corn needed.

5 Extension: The I-O Network and the Interbank Network

In order to incorporate the I-O network, certain changes to the previous model in section 2 need to be made. First, in the previous model, all sectors produce one and the same good. Now they produce different goods. Sector $i \in \mathbf{N}$ produces good i. These goods can be used both for consumption and as an intermediate good for production. More specifically, in sector $i \in \mathbf{N}$, an firm uses a bundle of resources $\{x_{ij}\}_{j \in \mathbf{J}}$ and a bundle of intermediate goods $\{y_{in}\}_{n \in \mathbf{N}}$ to produce good i, according to the following production function:

$$y_i = A_i \left(\prod_{j \in \mathbf{J}} x_{ij}^{\beta_{ij}} \right)^{\alpha_i \gamma_i} \left(\prod_{n \in \mathbf{N}} y_{in}^{w_{in}} \right)^{\alpha_i (1-\gamma_i)},$$

where $\alpha_i \in (0,1)$, all the other power coefficients are non-negative, $\sum_{j \in \mathbf{J}} \beta_{ij} = 1$ and $\sum_{n \in \mathbf{N}} w_{in} = 1$ for any $i \in \mathbf{N}$. γ_i represents the importance of resources and w_{in} the relative importance of intermediate good n in the production of good i.

Second, in the baseline model, for each sector $i \in \mathbf{N}$, there is only one bank – that is Bank i – that monopolizes lending to all firms of the sector. Keeping this assumption would imply a situation where Bank i's lending decision, by affecting all the producers of good i, would have a non-negligible effect on the price of the good. This effect seems unrealistic. To avoid it, in the extension, we assume that in each sector there is a large number B of symmetric banks, each of which monopolizes lending to fraction $1/B$ of firms in the sector. As a result, a single bank still has only negligible influence on the price of any good or any resource, and is thus a price taker. To simplify exposition, in our analysis below, for each sector $i \in \mathbf{N}$ we group all the B banks of the sector into one bank and refer to it as Bank i.

Third, given there are multiple goods, the utility function of households, firm manager-owners and bank shareholders is now assumed to be:

$$u(c_1, c_2, ..., c_N) = \prod_{n \in \mathbf{N}} c_n^{\theta_n}, \tag{19}$$

where $\theta_n \geq 0$ for any $n \in \mathbf{N}$ and $\sum_{n \in \mathbf{N}} \theta_n = 1$. If $\theta_i = 0$, then good i is not a consumption good, but an intermediate good solely. The Cobb-Douglas form of the utility function again means that the aggregate consumption spending on good n is fraction θ_n of the aggregate income.

We pick good 1 as the numeraire. As such, bank liability takes the form of a promise to pay good 1. As in the previous model, one unit of liability is defined as a promise to pay one unit of good 1.

Lastly, as in the previous model, we model banks' role with the money-creator approach. In parallel to Assumption 1', the friction of the economy is represented by the following assumption.

Households shun firms' promise to pay corn at $t = 1$, but accept banks', in exchange for their resources.

Assumption 2: Firms' promise to pay is not accepted, but banks' is, to exchange for intermediate goods or resources.

At $t = 0$, the firm receives m units of the bank's liability and at $t = 1$, the firm pays mR units of corn to the bank, where R is the gross interest rate of lending. After borrowing, firms use borrowed bank liability to exchange resources from households. Let the price of type $j \in \mathbf{J}$ resources be p_j. That is, one unit of these resources is exchanged for p_j units of bank liability. Households deposit all the sales revenue with banks, earning a deposit rate of r_d. We have seen that household $h \in \mathbf{H}$ deposits a fraction ζ_{hi} of its income with Bank $i \in \mathbf{N}$ and as a result a fraction d_{ji} of the sales revenue of type $j \in \mathbf{J}$ resources is deposited into Bank $i \in \mathbf{N}$.

In particular, to obtain intermediate goods at $t = 1$, firms have to place orders at $t = 0$ with the full payment in the form of bank liability. The timing of events at $t = 0$ is then as follows.

1. Bank $i \in \mathbf{N}$ posts the gross interest rate of lending R_i.
2. Firms of sector $i \in \mathbf{N}$ borrow m_i units of Bank i's liability and use it as a means of payment to buy type j resources at price p_j for $j \in \mathbf{J}$ and and to order intermediate

good n at price q_n for $n \in \mathbf{N}/\{i\}$. In equilibrium, this price will be the same as the good's price at $t = 1$. Hence, each firm of sector i obtains revenue $v_i = q_i \sum_{n \in \mathbf{N}/\{i\}} y_{ni}$ from the pre-sale order of its product. To simplify exposition, we assume that the debt that firms owe to their banks is *non-callable*, that is, they cannot use the pre-sale revenue to partly clear their debt at $t = 0$. As a result, firms of sector i, while presently depositing v_i with Bank i, are still obligated to pay back $m_i R_i$ to the bank at $t = 1$.

3. Households deposit their sales incomes, with fraction d_{ji} of the sales revenue of type j resources deposited with Bank i for $j \in \mathbf{J}, i \in \mathbf{N}$.

4. Interbank liabilities are bilaterally netted, whereby we find the net liability position Υ_{in} between any pair of Banks i and $n \in \mathbf{N}/\{i\}$.

As was said in subsection 4.2, in the extension, firms settle their loans using bank liability. Hence, the timing at $t = 1$ follows that in subsection 4.2 and is as follows.

1. Firms produce the goods. In this process, all the intermediary goods that have been ordered at $t = 0$ are delivered.

2. Each Bank $i \in \mathbf{N}$ issues T_i units of new liability. Then, the goods markets open, where banks use the newly issued liabilities and households use deposits to buy goods from firms. Firms use the sales revenue plus that from pre-sale orders – of value v_i for sector i firms – to buy goods that they do not produce by themselves.

3. Firms use bank liability that remains with them to settle their loans. As a result of this debt settlement, new interbank liabilities are formed.

4. Interbank liabilities are cleared, first with netting and then with good 1.

5. Agents consume the goods that they have obtained.

As was shown in subsection 4.2, on the good markets at stage 2 of $t = 1$, bank liability is valued at par, that is, one unit of bank liability is worth one unit of good 1. We focus on the convenient equilibrium with Bank i's new issuance $T_i = \Pi_i$ for any $i \in \mathbf{N}$ and interbank liabilities are wholly cleared with netting. As before, $\mathbf{P} = (p_1, p_2, ..., p_J)$ denotes the vector of resource prices. Let $\mathbf{Q} := (q_1, q_2, ..., q_N)$ denote the vector of the good prices. Given good 1 being the numeraire., $q_1 = 1$.

As before, we start with the analysis of firms' demand for bank liability. Suppose at

$t = 0$, a sector i firm borrows m_i units of bank liability to buy a bundle of factors of production $\{x_{ij}\}_{j\in\mathbf{J}}$ and order a bundle of intermediate goods $\{y_{in}\}_{n\in\mathbf{N}/\{i\}}$. Then, the budget constraint is

$$\sum_{j\in\mathbf{J}} p_j x_{ij} + \sum_{n\in\mathbf{N}/\{i\}} q_n y_{in} = m_i. \tag{20}$$

Meanwhile, the firm receives a revenue of

$$v_i = q_i \sum_{n\in\mathbf{N}/\{i\}} y_{ni} \tag{21}$$

from the orders of its own product and deposits this revenue with Bank i. At $t = 1$, out of the total product y_i, y_{ii} has been used for its own production and $\sum_{n\in\mathbf{N}/\{i\}} y_{ni}$ has been pre-sold to other sectors. Hence, at $t = 1$, the sales revenue is $q_i\left(y_i - \sum_{n\in\mathbf{N}} y_{ni}\right)$, out of which the firm pays back the bank $m_i R$. The firm's objective function is thus

$$\begin{aligned} & q_i \times \left[A_i\left(\prod_{j=1}^{J} x_{ij}^{\beta_{ij}}\right)^{\alpha_i\gamma_i}\left(\prod_{n=1}^{N} y_{in}^{w_{in}}\right)^{\alpha_i(1-\gamma_i)} - \sum_{n\in\mathbf{N}} y_{ni}\right] - m_i R + v_i \\ = \; & q_i \times \left[A_i\left(\prod_{j=1}^{J} x_{ij}^{\beta_{ij}}\right)^{\alpha_i\gamma_i}\left(\prod_{n=1}^{N} y_{in}^{w_{in}}\right)^{\alpha_i(1-\gamma_i)} - y_{ii}\right] - m_i R, \end{aligned} \tag{22}$$

where the first term represents *the gross revenue of its production*, denoted by s_i – that is, $s_i := q_i\left(y_i - y_{ii}\right)$ – and the second the total cost. This gross revenue s_i needs to be tracked because it will be used for the market -clearing of good i. At the optimum, the spending on each input that depends on bank finance – that is, any type $j \in \mathbf{J}$ of resources and any intermediate good $n \in \mathbf{N}/\{i\}$ – is a fixed fraction of the budget:

$$p_j x_{ij} = \frac{\gamma_i \beta_{ij}}{1 - (1-\gamma_i) w_{ii}} m_i; \tag{23}$$

$$q_n y_{in} = \frac{(1-\gamma_i) w_{in}}{1 - (1-\gamma_i) w_{ii}} m_i. \tag{24}$$

To understand these fractions, note that in the firm's production, its own product takes weight

$$\tau_i := \alpha_i (1-\gamma_i) w_{ii}. \tag{25}$$

Thus, the rest of the outputs, those that depend on bank finance, altogether take weight $\alpha_i - \tau_i$ (recall that $1 - \alpha_i$ is the weight for the human capital of the firm's manager-owner). Hence, the fraction of spending on type j resources is thus $\alpha_i\gamma_i\beta_{ij}/(\alpha_i - \tau_i) =$

$\gamma_i\beta_{ij}/\left(1-\left(1-\gamma_i\right)w_{ii}\right)$ and that on intermediate good n is thus $\alpha\left(1-\gamma_i\right)w_{in}/\left(\alpha_i-\tau_i\right)=\left(1-\gamma_i\right)w_{in}/\left(1-\left(1-\gamma_i\right)w_{ii}\right)$. At the optimum, the sector i firm's demand for bank liability is

$$m_i=\left(\frac{1}{R}\right)^{\frac{1-\tau_i}{1-\alpha_i}}\left(\frac{\alpha_i-\tau_i}{1-\tau_i}\xi_i\prod_{j\in\mathbf{J}}p_j^{-\frac{\alpha_i\gamma_i\beta_{ij}}{1-\tau_i}}\prod_{n\in\mathbf{N}}q_n^{-\frac{\alpha_i(1-\gamma_i)w_{in}}{1-\tau_i}}q_i^{\frac{1}{1-\tau_i}}\right)^{\frac{1-\tau_i}{1-\alpha_i}}:=m_i\left(R;\mathbf{P},\mathbf{Q}\right) \tag{26}$$

and its gross revenue is

$$s_i=\left[\frac{1}{R}\right]^{\frac{\alpha_i-\tau_i}{1-\alpha_i}}\left[\left(\frac{\alpha_i-\tau_i}{1-\tau_i}\right)^{\frac{\alpha_i-\tau_i}{1-\tau_i}}\xi_i\prod_{j=1}^{N}p_j^{-\frac{\alpha_i\gamma_i\beta_{ij}}{1-\tau_i}}\prod_{n\in\mathbf{N}}q_n^{-\frac{\alpha_i(1-\gamma_i)w_{in}}{1-\tau_i}}q_i^{\frac{1}{1-\tau_i}}\right]^{\frac{1-\tau_i}{1-\alpha_i}}:=s_i\left(R;\mathbf{P},\mathbf{Q}\right), \tag{27}$$

where ξ_i is a constant:

$$\xi_i:=\frac{1-\tau_i}{\tau_i}\left[A_i\tau_i\right]^{\frac{1}{1-\tau_i}}\prod_{j=1}^{N}\left(\frac{\left(1-\gamma_i\right)w_{in}}{1-\left(1-\gamma_i\right)w_{ii}}\right)^{\frac{\alpha_i\gamma_i\beta_{ij}}{1-\tau_i}}\prod_{n\in\mathbf{N}/\{i\}}\left(\frac{\left(1-\gamma_i\right)w_{in}}{1-\left(1-\gamma_i\right)w_{ii}}\right)^{\frac{\alpha_i(1-\gamma_i)w_{in}}{1-\tau_i}}. \tag{28}$$

Now we move to consider banks' decision problem. If Bank $i\in\mathbf{N}$ charges R_i, it lends out $M_i=m_i\left(R_i;\mathbf{P},\mathbf{Q}\right)$ units of its liability. Then, similar to Table 2 of the preceding section, the bank's balance sheet is as follows.

Assets	Liabilities
Loans to firms (M_iR_i)	Deposit of its own liability by households (D_{own})
	Deposit of other banks' liability by households (D_{other})
	Deposit of other banks's liability by sector i firms (v_i)
Credit to other banks whose liabilities are deposited with $Banki$ $((D_{other}+v_i)(1+\rho))$	Debt owed to the banks with which $Banki$'s liability is deposited $((M_i-D_{own})(1+\rho))$
	Equity (Π_i)

Table 3: a bank's balance sheet at the end of $t=0$

The bank's total deposits $D_i = D_{own} + D_{other} + v_i$, of which $D_i^H := D_{own} + D_{other}$ is from households, v_i from sector i firms. We can find its net interbank liability position $\Upsilon_i = M_i - D_i$, the same as (**??**), and its value

$$\Pi_i = M_i \left(R_i - 1 - \rho\right) - D_i \rho. \tag{29}$$

To calculate D_i, we first find D_i^H, the deposits from households of the sales revenue of resources, and then v_i, the deposits from the sector i firms. Regarding the former, similar to the preceding section, by (23), the aggregate spending E_j on type j resources is

$$E_j = \sum_{n \in \mathbf{N}} M_n \frac{\gamma_n \beta_{nj}}{1 - (1 - \gamma_n) w_{nn}}. \tag{30}$$

Of this spending, fraction d_{ji} is deposited into Bank i. Hence, the deposits into the bank from households are in total equal to

$$D_i^H = M_i \times f_{ii}^H + \sum_{n \in \mathbf{N}/\{i\}} M_n \times f_{ni}^H,$$

where for any Bank $n \in \mathbf{N}$,

$$f_{ni}^H := \frac{\gamma_n}{1 - (1 - \gamma_n) w_{nn}} \sum_{j \in \mathbf{J}} \beta_{nj} d_{ji} \tag{31}$$

is the fraction of its liability deposited into Bank i by households. Observe that f_{ni}^H equals the fraction in the preceding section (given by 13) multiplied by $\gamma_n / \left[1 - (1 - \gamma_n) w_{nn}\right]$, because here not all but only fraction $\gamma_n / \left[1 - (1 - \gamma_n) w_{nn}\right]$ of Bank n's liability is spent on resources.

The deposit from the firms with Bank i is the pre-sale revenue v_i of their product. By (24) the spending $q_i y_{ni}$ of sector n on intermediate good i, for $n \neq i$, is $M_n f_{ni}^E$, where

$$f_{ni}^E := \frac{(1 - \gamma_n) w_{ni}}{1 - (1 - \gamma_n) w_{nn}} \tag{32}$$

denote the fraction of Bank n's liability that flows into Bank i due to the sector i firms' deposits. In total, this channel generates the following quantity of deposits for Bank i:

$$v_i = \sum_{n \in \mathbf{N}/\{i\}} M_n f_{ni}^E. \tag{33}$$

Put together, the total deposit of Bank i is thus

$$\begin{aligned} D_i &= D_i^H + v_i \\ &= M_i \times f_{ii}^H + \sum_{n\in\mathbf{N}/\{i\}} M_n f_{ni}, \end{aligned} \tag{34}$$

where for $n \in \mathbf{N}/\{i\}$,

$$\begin{aligned} f_{ni} &= f_{ni}^H + f_{ni}^E \\ &= \frac{1}{1-(1-\gamma_n)w_{nn}}\left(\gamma_n \sum_{j\in\mathbf{J}} \beta_{nj} d_{ji} + (1-\gamma_n)w_{ni}\right), \end{aligned} \tag{35}$$

is the total fraction of Bank n's liability that flows into to Bank i. Observe from (34) that while other banks' liabilities are brought into Bank i via depositing by both households and firms (hence $f_{ni}^H + f_{ni}^E$), only household depositing can bring the money that Bank i lends out back to itself, because its borrower firms need no bank finance to obtain their own product.

By (29), Bank i's value is the following:

$$\Pi_i\left(M_i; \{M_n\}_{n\in\mathbf{N}/\{i\}}\right) := M_i\left[R_i - 1 - \left(1-f_{ii}^H\right)\rho\right] + \rho \sum_{n\in\mathbf{N}/\{i\}} M_n f_{ni}.$$

The bank's problem is thus

$$\max_{M_i, R_i} \Pi_i\left(M_i; \{M_n\}_{n\in\mathbf{N}/\{i\}}\right), s.t. M_i = m_i\left(R_i; \mathbf{P}, \mathbf{Q}\right),$$

where the demand function $m_i(R; \mathbf{P}, \mathbf{Q})$ is given by (26). As was said, a single bank has negligible effects on prices $(\mathbf{P}, \mathbf{Q})$. Hence, the bank takes $(\mathbf{P}, \mathbf{Q})$ as given in solving the above decision problem. The optimum lending rate of Bank i, denoted by R_i^*, is then:

$$R_i^* = \frac{1-\tau_i}{\alpha_i - \tau_i}\left[1 + \left(1-f_{ii}^H\right)\rho\right]. \tag{36}$$

Similar to formula (14) for the optimal lending rate in the preceding section, the first term is the mark-up factor due to the bank's monopolistic power over the firms; and the term in the square parentheses is the marginal cost of lending. Here the mark-up factor is no longer $1/\alpha_i$ because the input of intermediate good i is self financed. The lending size of Bank i as function of prices is thus

$$M_i(\mathbf{P}, \mathbf{Q}) = m_i(R_i^*; \mathbf{P}, \mathbf{Q}), \tag{37}$$

and the gross revenue of a firm in sector i is thus

$$s_i(\mathbf{P}, \mathbf{Q}) = s_i(R_i^*; \mathbf{P}, \mathbf{Q}),$$

where functions $m_i(R; \mathbf{P}, \mathbf{Q})$ and $s_i(R; \mathbf{P}, \mathbf{Q})$ are respectively given by (26) and (27).

Now we determine prices $(\mathbf{P}, \mathbf{Q})$ using market clearing conditions. The total spending E_j on type j resources is given by (30) and their total supply in value is $p_j X_j$. The market clearing condition for each type $j \in \mathbf{J}$ resources is thus:

$$\sum_{n\in\mathbf{N}} M_n(\mathbf{P}, \mathbf{Q}) \frac{\gamma_n \beta_{nj}}{1 - (1 - \gamma_n) w_{nn}} = p_j X_j. \tag{38}$$

We have $q_1 = 1$. For any good i, it is either used for consumption or as an intermediate good. Thus,

$$c_i + \sum_{n\in\mathbf{N}} y_{ni} = y_i.$$

Recall that the gross revenue of a firm producing good i is $s_i = q_i(y_i - y_{ii})$ and the total value of the good i used as an intermediate good for other sectors is $v_i = q_i \sum_{n\in\mathbf{N}/\{i\}} y_{ni}$. Hence, the market clearing for good i is

$$q_i c_i + v_i = s_i. \tag{39}$$

Alternatively, the equation says that the firm obtains revenue by selling good i either to consumers or to producers. We have found $s_i = s_i(\mathbf{P}, \mathbf{Q})$. By (33),

$$v_i = \sum_{n\in\mathbf{N}/\{i\}} M_n(\mathbf{P}, \mathbf{Q}) f_{ni}^E := v_i(\mathbf{P}, \mathbf{Q}).$$

Because of the the Cobb-Douglas form of the utility function in (19), the aggregate consumption spending on good i – that is $q_i c_i$ – is θ_i fraction of the aggregate income, $\sum_{n\in\mathbf{N}} (s_n - v_n)$. Hence, for any good $i \in \mathbf{N}$, the market clearing condition is

$$\theta_i \sum_{n\in\mathbf{N}} (s_n(\mathbf{P}, \mathbf{Q}) - v_n(\mathbf{P}, \mathbf{Q})) = s_i(\mathbf{P}, \mathbf{Q}) - v_i(\mathbf{P}, \mathbf{Q}). \tag{40}$$

Only $N-1$ of these N equations are independent.[10] Pick any $N-1$ of them and these $N-1$ equations and the J resource-market clearing equations (given by 38) determine all the other $N+J-1$ prices than q_1 (which is 1). Thus price profile $(\mathbf{P}, \mathbf{Q})$ is determined.

The whole equilibrium profile is thus determined. In the same way in which Proposition 3 is proved, we can prove that there is a unique equilibrium in the extension. Hence the following proposition. $\left\{A_i, \alpha_i, \beta_{ij}\right\}_{i \in \mathbf{N}, j \in \mathbf{J}}$

Proposition 6 *As a result of money circulation, the technologies* $\left\{A_i, \alpha_i, \beta_{ij}\right\}_{i \in \mathbf{N}, j \in \mathbf{J}}$, *the resource distribution* $\{d_{ji}\}_{(j,i) \in \mathbf{J} \times \mathbf{N}}$ *and the Input-Output network* $\{w_{in}\}_{(i,n) \in \mathbf{N} \times \mathbf{N}}$ *of the real economy determines a unique interbank liability network* $\{\Upsilon_{in}\}_{(i,n) \in \mathbf{N} \times \mathbf{N}}$, *of which the entry* Υ_{in} *is the net liability that Bank* i *owes to Bank* n *and*

$$\Upsilon_{in} = M_i(\mathbf{P}, \mathbf{Q}) f_{in} - M_n(\mathbf{P}, \mathbf{Q}) f_{ni},$$

where $M_i(\mathbf{P}, \mathbf{Q})$, *given by (37), is Bank* i*'s lending size, and* f_{in}, *to be found with (35), is the fraction of money lent out by Bank* i *flowing into Bank* n.

6 Conclusion

The existing studies on banking networks have shed many important insights using the loanable-funds approach. This approach focuses on the real value of the money that banks lend out, but abstracts from its attribute as a means of payment. In this paper, we highlight this attribute: Banks lend to the real economy their liabilities which it uses as a means of payment. Using this approach, we shed four new insights that have eluded the existing studies.

First, the circulation of bank liability as a means of payment creates an interbank liability network and this network is structured by the real economy. As a means of payment, banks' liabilities naturally circulate following the real economic activity, and one bank's liability naturally flows into another. These inter-flows of liability create a network of

[10] As is well known, if the markets for $N-1$ goods clear, then the market for the remained good clears too. A straight way to see this is to note that summing up both sides of (40) over $i \in \mathbf{N}$ reaches an identity.

interbank exposures. Because these flows are directed by the real economic activity, the structure of this network is entirely determined by the fabric of the real economy.

Second, the interbank interest rate has a heterogeneous effect on banks' funding cost. Out of each unit of liability that a bank lends out, only a fraction flows out to other banks subjecting the bank to the cost of interbank interest. A unit increase in the interbank interest rate raises the bank's funding cost by this outflow fraction. This fraction, determined by the fabric of the real economy, differs across banks in the generic case.

Third, the heterogeneity in the outflow fraction across banks is a source of lending inefficiency. A bank with a smaller outflow fraction has a lower funding cost and thus charges a lower lending rate. This gives its real-economy borrowers an advantage irrelevant to their production technologies. As a result, these borrows muster more resources than they should according to their technologies.

Fourth, not only the sizes of deposits and loans matter, so do the identities of depositors and borrowers. A deposit of ten-thousand dollars from a programmer benefits the bank more than that from a pig-farmer if it mainly lends to software developers; a loan of half-million dollars to a software developer is more profitable than that to a sausage producer if the bank receives deposits mainly from programmers.

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Appendix

Proof of Proposition 1:

By the First Welfare Theorem, the First-Best allocation is attained in the competitive equilibrium in the absence of the Essential Friction, that is, if firms can obtain resources directly from households. In this frictionless case, the problem of Sector i firm's is

$$\max_{\{x_{ij}\}} A_i \left(\prod_{j \in \mathbf{J}} x_{ij}^{\beta_{ij}} \right)^{\alpha_i} - \sum_{j \in \mathbf{J}} p_j^{FB} x_{ij}.$$

This problem is a special case of problem (2) above in which $R = 1$; intuitively the case in which firms do not need banks to obtain real resources is equivalent to the case in which firms pay no costs for banks' services, that is, $R = 1$. The First-Best demand of Sector i for type j resources is thus:

$$x_{ij}^{FB} = \frac{1}{p_j^{FB}} m_i \left(1; \mathbf{P}^{FB}\right),$$

where the First-Best competitve equilibrium prices satisfy:

$$\sum_{n \in \mathbf{N}} m_n \left(1; \mathbf{P}^{FB}\right) \beta_{nj} = p_j^{FB} X_j \tag{41}$$

for any $j \in \mathbf{J}$. From (3), it follows that

$$\begin{aligned} m_i (R; \mathbf{P}) &= R^{\frac{-1}{1-\alpha_i}} m_i (1; \mathbf{P}) \\ m_i (R; \lambda \mathbf{P}) &= \left(\lambda^{-1}\right)^{\frac{\alpha_i}{1-\alpha_i}} m_i (R; \mathbf{P}). \end{aligned}$$

Hene, if $\alpha_i = \alpha$ and hence $R_i^* = R^*$ for any $i \in \mathbf{N}$, then

$$\begin{aligned} m_n (R_n^*; \mathbf{P}^*) &= R^{*\frac{-1}{1-\alpha}} m_n (1; \mathbf{P}^*) \\ &= R^{*\frac{-1}{1-\alpha}} \lambda^{\frac{\alpha}{1-\alpha}} m_n (1; \lambda \mathbf{P}^*). \end{aligned}$$

Substitute it into (9) and we have

$$\begin{aligned} R^{*\frac{-1}{1-\alpha}} \lambda^{\frac{\alpha}{1-\alpha}} \sum_{n \in \mathbf{N}} m_n (1; \lambda \mathbf{P}^*) \beta_{nj} &= p_j^* X_j \Leftrightarrow \\ \sum_{n \in \mathbf{N}} m_n (1; \lambda \mathbf{P}^*) \beta_{nj} &= R^{*\frac{1}{1-\alpha}} \lambda^{\frac{-\alpha}{1-\alpha}} p_j^* X_j. \end{aligned}$$

Hence, if we let λ satisfy

$$R^{*\frac{1}{1-\alpha}}\lambda^{\frac{-\alpha}{1-\alpha}} = \lambda \Leftrightarrow \lambda = R^*,$$

then

$$\sum_{n\in\mathbf{N}} m_n\left(1; \lambda\mathbf{P}^*\right)\beta_{nj} = \lambda p_j^* X_j,$$

that is, $\lambda\mathbf{P}^*$ is a solution for the market clearing conditions (41) in the First-Best allocation. Given that these simultaneous equations have a unique solution (as the technologies are all strictly concave),

$$\lambda\mathbf{P}^* = \mathbf{P}^{FB}.$$

Hence, the first allocation is identical with the equilibrium allocation, because

$$\begin{aligned}
x_{ij}^{FB} &= \frac{1}{p_j^{FB}} m_i\left(1; \mathbf{P}^{FB}\right) \\
&= \frac{1}{\lambda p_j^*} m_i\left(1; \lambda\mathbf{P}^*\right) \\
&= \frac{R^{*\frac{1}{1-\alpha}}}{\lambda^{\frac{1}{1-\alpha}} p_j^*} m_i\left(R^*; \mathbf{P}^*\right) \\
|_{\lambda=R^*} &= \frac{1}{p_j^*} m_i\left(R^*; \mathbf{P}^*\right) \\
&= x_{ij}^*.
\end{aligned}$$

Q.E.D.

Proof of Proposition 3:

The objective function in the firm's problem (2) is equal to

$$\left[A_i\left(\prod_{j\in\mathbf{J}} x_{ij}^{\beta_{ij}}\right)^{\alpha_i} - \sum_{j\in\mathbf{J}} p_j x_{ij}\right] - m\left(R-1\right),$$

where the term in the square brackets is the firm's profit if it faces no friction of payment and could use its own IOU as a means of payment, and the second term $m\left(R-1\right)$ represents the cost of this friction to the firm; indeed, as was said above, $m\left(R-1\right)$ is the cost that this firm pays for renting the bank's commitment power. As a result, if $R_i = 1$ for any

$i \in \mathbf{N}$, then the equilibrium is reduced to the (perfect) competitive equilibrium. By (14), R_i^* is independent of $\mathbf{P}$. By (3), if we define

$$\widehat{A}_i := \frac{A_i}{R_i^*},$$

then the demand function $m_i\left(R_i^*;\mathbf{P}\right)$ is the same as $m_i\left(1,\mathbf{P};\widehat{A}_i\right)$, that is, the same as the firm's total demand for resources under the (perfect) competitive equilibrium. Considering that the demand for each type $j \in \mathbf{J}$ of resources equals $\frac{1}{p_j}\sum_{i\in\mathbf{N}} m_i\beta_{ij}$, then the aggregate demand for each type of resources in the model economy is equal to that in the competitive equilibrium with $A_i = \widehat{A}_i$. Therefore, the equilibrium allocation $\{x_{ij}\}_{i\in\mathbf{N},j\in\mathbf{J}}$ is the same as that of the competitive equilibrium with $A_i = \widehat{A}_i$ for any $i \in \mathbf{N}$. According to Welfare Theorem 1, then, the equilibrium allocation $\{x_{ij}\}_{i\in\mathbf{N},j\in\mathbf{J}}$ is the one that maximizes the aggregate product, that is, it is the solution to the following social planner's problem:

$$\max_{\{x_{ij}\}_{i\in\mathbf{N},j\in\mathbf{J}}} \sum_{i\in\mathbf{N}} \widehat{A}_i \left(\prod_{j\in\mathbf{J}} x_{ij}^{\beta_{ij}}\right)^{\alpha_i}$$
$$s.t. \sum_{i\in\mathbf{N}} x_{ij} = X_j \text{ for each } j \in \mathbf{J}.$$

The objective function is strictly concave. Hence, there exists a unique solution to the maximisation problem. Hence, a unique equilibrium exists.

Q.E.D.

Proof of Proposition 4:

We have shown that the quantity of any type of resources that Sector i firms obtain is in proportion to $\left(1/\left[1+r_d+\left(1-f_{ii}\right)\rho\right]\right)^{\frac{1}{1-\alpha}}$ and therefore takes γ_i fraction of the aggregate supply, where

$$\begin{aligned}
\gamma_i \quad : \quad &= \frac{\left(\frac{1}{1+r_d+(1-f_{ii})\rho}\right)^{\frac{1}{1-\alpha}}}{\sum_{n\in\mathbf{N}}\left(\frac{1}{1+r_d+(1-f_{nn})\rho}\right)^{\frac{1}{1-\alpha}}} \\
&= \frac{\left(\frac{1}{1+r_d+(1-f_{ii})\rho}\right)^{\frac{1}{1-\alpha}}}{N\left(\frac{1}{c^e(\rho)}\right)^{\frac{1}{1-\alpha}}} \\
&= \frac{1}{N}\times\left(\frac{c^e(\rho)}{1+r_d+(1-f_{ii})\rho}\right)^{\frac{1}{1-\alpha}}.
\end{aligned}$$

Recall that in the first best allocation $\gamma_i^* = 1/N$. Hence, $\gamma_i > \gamma_i^*$ if and only if $1 + r_d + (1 - f_{ii})\rho < c^e(\rho) \Leftrightarrow c_i^{MC} < c^e(\rho)$. Moreover, $\gamma_i = \max\{\gamma_n | \in \mathbf{N}\}$ if $f_{ii} = \max\{f_{nn}|n \in \mathbf{N}\}$, $\gamma_i = \min\{\gamma_n | \in \mathbf{N}\}$ if $f_{ii} = \min\{f_{nn}|n \in \mathbf{N}\}$. Hence the first part of the proposition is proved. For the second part, note that

$$\gamma_i = \frac{1}{\sum_{n\in\mathbf{N}}\left(\frac{1+(1-f_{ii})\rho}{1+(1-f_{nn})\rho}\right)^{\frac{1}{1-\alpha}}}$$

and $\frac{1+(1-f_{ii})\rho}{1+(1-f_{nn})\rho}$ is decreasing (increasing) with ρ if $f_{ii} > f_{nn}$ ($f_{ii} < f_{nn}$). Therefore, if $f_{ii} = \max\{f_{nn}|n \in \mathbf{N}\}$, then γ_i is increasing with ρ and if $f_{ii} = \min\{f_{nn}|n \in \mathbf{N}\}$ then γ_i is decreasing with ρ.

Q.E.D.