The Timing of Takeovers in Growing and Declining Markets*

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February 2, 2010

Abstract

Empirical studies have found that takeover activity is positively related to the absolute size of industry-level shocks. We develop a dynamic framework with competing bidders which explains this pattern. We model both hostile and agreed bids, determining the timing and terms of takeover in each case, and find that takeover occurs only when shocks are sufficiently large in either direction. We examine implications for the efficiency of the market for corporate control, finding that the direction of inefficiency depends on the form of takeover. We also analyze the dependence of takeover activity on the degree of uncertainty about industry conditions.

Keywords: Takeovers, Mergers, Acquisitions, Real options, Timing games.

JEL classification: D44, D81, G34.

*We are grateful to seminar and conference participants at Cambridge, Essex, the Olin School of Business, Southampton, Toronto, the UK Competition Commission, and the Daiwa workshop on Financial Engineering in Tokyo. We thank in particular Dirk Hackbarth, Glenn MacDonald, Robert Pindyck, Juuso Välimäki, and V. Bhaskar for helpful comments. Any errors remain the responsibility of the authors. Robin Mason gratefully acknowledges financial support from the ESRC Research Fellowship Award R000271265.

1 Introduction

When do takeovers occur? Empirical research, summarized by Andrade, Mitchell and Stafford (2001), highlights two consistent features of the pattern of takeover activity. First, takeovers occur in waves, peaking with the business cycle and stock market valuations. Secondly, within a wave, takeover activity clusters strongly by industry; however, there is negligible correlation between the industries involved in different waves. These findings suggest that takeover activity is due in large part to industry-level factors, with takeover representing one form of industry restructuring. Examination of takeover activity at industry level reveals another pattern: that takeover activity is positively related to the *absolute magnitude* of industry shocks. In other words, takeovers occur in industries experiencing large positive *or negative* shocks to industry-specific conditions. Papers providing evidence of this relationship are surveyed below; in summary, these find a 'U'-shaped relationship between takeover activity and industry shocks.

In this paper, we develop a theoretical framework to explain this industry-level phenomenon. We show how takeovers do not occur when industry-specific shocks are moderate but only when they are sufficiently large, in either direction. Our results fit the empirical pattern, explaining both sides of the 'U' within a single model. We go on to consider a number of further questions. How does takeover activity depend on the degree of uncertainty? Is the market for corporate control efficient, in the sense that the timing of takeover is appropriate? How do these conclusions depend on the form of takeover, i.e., the mechanism by which takeover comes about? When the timing of takeover is beyond its direct control, what other actions might the target take to influence the timing of takeover?

We remain broad-minded about the mechanism by which takeover comes about, employing two distinct models. First we model takeover as a competitive bidding process, its timing and terms being determined by the potential acquirers in competition with one another. This situation can be thought of as a hostile takeover, in which the target firm is essentially passive. Subsequently we present a model of agreed takeover, in which the target chooses when to open negotiations with one of the acquirers, and the terms of takeover result from bargaining between the parties. Note that we use the term 'takeover' quite generically: this may be a merger combining the resources of the two firms, or an acquisition of assets where the target's shareholders cease to be owners of the firm. The distinction that we draw concerns the mechanism by which takeover occurs, rather than its format. This difference has important implications for the timing, terms, and efficiency of takeover.

Our modeling approach has two key components. There are two types of acquirer who can make takeover offers, distinguished by their comparative advantages in running the target firm. The values that these firms place on the target are driven by a stochastic industry variable, which may be thought of as industry demand, that varies randomly over time. The first type, the 'growth-acquirer', has the higher valuation when the state is high (e.g., because its synergies generate greater profit when demand is large). Conversely, the second type, the 'decline-acquirer', has the higher valuation when the state is low (e.g., because its fixed costs savings yield greater returns than synergies when demand is low). The second component of our modeling is that the takeover market is competitive: there are two potential acquirers, who compete for the target either directly in a bidding war (the first model), or indirectly via the bargaining process (the second model).

These two components lead to our main result: there is *delay in equilibrium*. Takeover by the growth-acquirer occurs only when the state is sufficiently high, while takeover by the decline-acquirer occurs only when the state is sufficiently low. This result is driven by 'real options' inherent in the takeover decision, which stem from three key features of the situation. First, although it may be delayed, takeover is irreversible: once the target is acquired by one firm, it cannot be reallocated to the other (or the original owners). Secondly, takeover decisions are made in an environment of uncertainty: valuations are driven by a stochastic state variable (industry demand). Third, *competition makes payoffs convex*: with bidding competition, each bidder's payoff function is convex since if it loses it receives a payoff of zero; in agreed takeover, the target can choose between two potential acquirers, giving it a convex payoff function. On their own, irreversibility and uncertainty are not sufficient to cause delay—we show that competition is also required for delay to occur.

For both takeover models, we show that the *extent of delay* increases with the degree of uncertainty. In agreed takeover, convexity of the target's payoff function generates a benefit to waiting for more extreme values of the state variable, and this benefit increases with uncertainty. This is the usual real options effect. When takeover results from bidding competition, convexity of the bidders' payoff functions produces a similar effect, but there is also a second factor which has the opposite implication. The likelihood of being pre-empted, i.e., of the state variable changing by a large amount so that the rival firm acquires the target, also increases with uncertainty. In deciding when to make an offer competing bidders balance these two factors; our result shows that the first effect dominates so that delay increases with uncertainty.

Thirdly, we examine the *efficiency of the market for corporate control* in each case by comparing the timing of takeover with the efficient solution. The efficient solution does involve delay: the target should be taken over by one of the acquirers only when that firm's valuation is sufficiently greater than its rival's. But neither form of takeover generates the efficient degree of delay, and the direction of inefficiency differs between the two cases. With competitive bidding delay is excessive, because competition creates *negative externalities* between bidders, causing them to delay bidding too long. For agreed takeover, by contrast, there is insufficient delay, because the bargaining process creates a *positive externality* between the outside bidder's valuation and the payoff of the target firm, causing it to open negotiations too soon. As a corollary, the analysis implies that takeover will be more delayed when bids are hostile than when they require the target's agreement.

We consider some further implications and extensions of the competitive bidding model. Even when takeover results from competitive bidding, and hence the target lacks direct control, it may be able to influence the timing of takeover through its exposure to industry shocks (e.g., by its choice of investment projects). We show that the target prefers competitive bidding to take place immediately—it suffers from the delay in competitive bids—and so will choose the lowest possible exposure to uncertainty in order to reduce delay. Finally, we consider extensions of the model to more than two bidders.

In summary, our results have the following implications for takeover activity and the market for corporate control. First, they imply that even when there is competition between acquirers, the target may not be taken over immediately. Secondly, takeover activity will be highest during 'booms' and 'busts', i.e., when there extreme shocks to an industry, with less activity in between—fitting the empirical pattern. Finally, the market for corporate control is inefficient: takeover is too delayed when it occurs through competitive bidding, and too rapid when it takes place by agreement.

The empirical literature on takeovers generally is large; see Weston, Chung and Hoag (1990) for a review. The empirical studies of particular relevance for our analysis are four papers which find a positive relationship between takeovers and industry shocks. Mitchell and Mulherin (1996) find that takeover activity in the 1980s was greatest in those industries exposed to the largest shocks. Using two measures of shocks—the absolute value of the difference between a particular industry's sales growth and the average sales growth across all 51 industries in their sample, and the corresponding measure for employment—the authors relate these absolute differences to a number of measures of takeover activity. They find that shocks to sales growth and employment are positively and significantly related to takeover attempts and actual takeovers. In contrast, industry sales growth has no explanatory power for industry variation in takeover activity. Similarly, Andrade and Stafford (2004) find a positive relationship between merger activity and the absolute deviation of sales growth from its long-term mean within an industry. Harford (2005) observes that absolute changes to a number of variables, including profitability, employee growth and sales growth, are all abnormally high prior to increases in takeover activity. Finally, focusing on the effect of demand shocks, Bernile, Lyandres and Zhdanov (2007) also find a significant 'U'-shaped relationship between mergers and shocks.

Among the theoretical literature, relatively few papers deal with the timing of takeovers. A number of papers study the relationship between aggregate shocks and takeovers; for example, Shleifer and Vishny (2003) consider the implications of stock market misvaluation for takeover activity. Fewer papers, however, consider the role of industry-level shocks, and most of these can explain only one side of the 'U'-shaped relationship between takeovers and industry shocks found in the empirical literature. In Morellec and Zhdanov (2005), two firms compete to acquire a target. The authors determine when takeover occurs in equilibrium and the terms of the takeover (i.e., the division of surplus between acquirer and target). In their model, takeovers occur only in growing markets, when the acquirer's relative cash flows are sufficiently large, and competition between bidders speeds up the takeover process. Our results are quite different: takeovers occur in both growing and declining markets, and competition causes delay.

Lambrecht (2004) argues (like us) that merger synergies, resulting from economies of scale, are an increasing function of product market demand. Combining this with fixed merger costs and stochastic demand, merger has call option-like features. Consequently, firms have an incentive to merge only in periods of economic expansion; hence, takeovers are pro-cyclical. In contrast, Lambrecht and Myers (2007) analyze takeovers in declining markets. In their model, the managers of a declining firm are reluctant to shut it down; takeovers are permitted by shareholders as a means to solve this agency problem. The authors characterize how the timing of takeover depends on the identity of the acquirer (for example, whether it is a management buy-out or a raider), and assess the efficiency of the resulting timing of closure.

Bernile, Lyandres and Zhdanov (2007) seek, like us, to provide a theoretical explanation of the 'U'-shaped empirical relationship between takeovers and industry shocks (as well as providing further empirical verification of it). In their theoretical model, incumbent firms have a static incentive to merge in order to increase their market power. But by merging, the incumbents may create an added incentive for an outside firm to enter their industry. The entry decision is unaffected by merger at extreme values of industry profitability: at low levels, entry will not occur, and at high levels, entry will occur regardless of the incumbents' behavior. Hence, merger occurs at these extreme values. For moderate levels of industry profitability, an entrant will enter only if there is insufficient competition in the market, i.e., only if the incumbents have merged. Hence, at these profitability levels, the incumbents choose not to merge in order to deter entry. Clearly, this theory is quite different from ours, though the resulting pattern of takeover activity is broadly similar.

The theoretical papers that we have discussed share with ours a common modeling approach: firms face uncertainty and irreversibility when making their decisions, with uncertainty modeled by a continuous time stochastic process. These papers therefore build on, and contribute to, the growing number of papers analysing strategic interactions in real options settings. See, for example, Smets (1991), Grenadier (1996), Hoppe (2000), Weeds (2002), Lambrecht and Perraudin (2003), Novy-Marx (2007), and Mason and Weeds (2009).

The structure of the paper is as follows. In section 2 we describe the model of takeover with competitive bids and characterize equilibrium, focusing in particular on the *timing* of takeover. In section 3, we derive the efficient timing of takeover, comparing this with the outcome of competitive bidding. In section 4 we present the alternative model of agreed takeover, solving for the timing of takeover and comparing with the efficient solution. Section 6 outlines some further implications and extensions of the competitive bidding model: the preferences of the target firm and implications for its project choice, and extension of the model to more than two bidders. Section 7 concludes.

2 The Timing of Takeover with Competitive Bids

In this section we model takeover as a competitive bidding process, its timing and terms being determined by offers made by two potential acquirers in competition with one another. The target firm is entirely passive, being acquired by the highest bidder at a time of the acquirer's choosing. The situation is effectively that of a hostile takeover; such an approach to takeovers is common in the US and UK.[1]

2.1 The model

Two risk-neutral firms can each bid to acquire a target firm. When one firm decides to make an offer a bidding war results, with the target sold to the highest bidder.[2] The decisions to bid and acquire can be delayed indefinitely, but once the target has been acquired by one bidder no further actions are considered. Thus, takeover is irreversible, limiting the analysis to one 'cycle' of acquisition.[3] Time is continuous and the time horizon is infinite with $t \in [0, +\infty)$. There is no sunk cost of making an offer (though this could be added).

The potential acquirers are of different types, distinguished by their comparative advantages in running the target firm. The first type, the 'growth-acquirer', exploits synergies such as economies of scale to lower marginal costs and expand production. The second type, the 'decline-acquirer', looks to consolidate and cut fixed costs of production, for example by shedding capacity.[4] The values that the two firms place on the target are driven by a stochastic industry variable that varies randomly over time; this can be thought of as the level of industry demand. The growth-acquirer has the higher valuation when the state variable is high, e.g., because its economies of scale generate greater profit when demand is large. Conversely, the decline-acquirer has the higher valuation when the state variable is low, e.g., because fixed costs savings yield greater returns than scale economies when demand is low.[5]

[1]A hostile bid is defined as one in which the target board's initial reaction is to recommend target shareholders to reject the offer. This accounts for 20–25% of takeovers in the UK and the US; see Schwert (2000).

[2]This is a plausible assumption: there are often multiple suitors for specific targets, and this competition reduces bidders' returns. See Bradley, Desai and Kim (1988) and De, Fedenia and Triantis (1996).

[3]One justification for this assumption is that post-merger integration prevents subsequent divestiture or acquisition of the original business unit.

[4]See Andrade and Stafford (2004) for empirical work on this distinction.

[5]An alternative possibility arises when the bidders are located in different countries. In this case, the

Formally, we normalize the value of the target under current ownership to zero. The valuations of the potential acquirers are represented as $s_i(\theta) : \mathbb{R}_+ \to \mathbb{R}_+, i \in \{G, D\}$, where G denotes the growth-acquirer and D the decline-acquirer. The valuations are functions of an exogenous stochastic variable $\theta \in \mathbb{R}_+$ (industry demand) which evolves according to a geometric Brownian motion (GBM) without drift:

$$d\theta_t = \sigma\theta_t dW_t \tag{1}$$

where $\sigma \in [0, +\infty)$ is the instantaneous standard deviation or volatility parameter, and dW_t is the increment of a standard Wiener process $\{W_t\}_{t\geq 0}$, so that $dW_t \sim N(0, dt)$. The continuous-time discount rate is $r > 0$. The parameters σ and r are common knowledge and constant over time.

The following assumption is made about the acquirers' valuations:

$$s_G(\theta) = \lambda\theta, \lambda > 0; \qquad s_D(\theta) = \eta,\ \eta > 0. \tag{2}$$

The fact that the firms' valuations are non-negative means that we focus on the case in which it is efficient to sell the target; moreover, with two credible bidders at every value of θ, bidding occurs competitively in equilibrium. A crucial feature for our analysis is that the relative valuation $\delta(\theta) \equiv s_G(\theta) - s_D(\theta)$ is upwards single-crossing in θ: firm G's valuation of the target is greater than firm D's when θ is high—above the critical threshold

$$\theta^* \equiv \eta/\lambda.$$

Conversely, firm D has the higher valuation when θ is low—below θ^*.

These linear functional forms are chosen for their analytical convenience. One informal story to support them is that firm G can exploit synergies to lower the unit cost of production of the target after acquisition. When industry demand is θ, firm G can generate an extra profit of the form $\lambda\theta$, where λ is related to the cost saving. Meanwhile, firm D is a consolidator who can reduce the fixed costs of the target by η.

The competitive bidding game has two stages. The first stage is a timing game in which the potential acquirers decide when (i.e., at what level of the state variable) to bid.

state variable is the exchange rate between the countries. If firm 1 is in the same country as the target, it has the higher valuation when the exchange rate is strong; when the exchange rate is weak, the foreign firm 2 has the higher valuation. We are grateful to Pauli Murto for suggesting this example.

Once one or more of the firms decides to make an offer, the game then enters a second, bidding stage in which the firms both submit bids (if they wish). In the timing game, we assume that firms use stationary Markovian strategies; a formal definition of these and of Markov perfect equilibrium is provided in Appendix A. In the bidding stage, firm i acquires the target if and only if its bid b_i exceeds that of its rival b_{-i} and is greater than zero. If it bids successfully, firm i receives the payoff $s_i(\theta)$ while paying its bid to acquire the target. If firm i bids unsuccessfully (or not at all), it receives a payoff of zero. In the event of a non-zero tie ($b_G = b_D > 0$), the target is allocated randomly between the firms. Hence the inter-temporal payoff of firm i is

$$V_i(T, b_G, b_D; \theta_t) = \begin{cases} \mathbb{E}_t\left[(s_i(\theta_T) - b_i(\theta_T))e^{-r(T-t)}\right] & \text{if} b_i > \max[0, b_{-i}], \\ 0 & \text{if} b_i < \max[0, b_{-i}], \\ \frac{1}{2}\mathbb{E}_t\left[(s_i(\theta_T) - b_i(\theta_T))e^{-r(T-t)}\right] & \text{if} b_i = b_{-i} > 0, \\ 0 & \text{if} b_i = b_{-i} = 0, \end{cases} \tag{3}$$

where $T \equiv \min[T_G, T_D]$ and T_i is the (random) first time at which firm i makes a bid; the operator $\mathbb{E}_t$ denotes expectations conditional on information available at time t.

2.2 Takeover with a single acquirer

Suppose there is a single acquirer, of either type G or type D. In our model it is straightforward to determine when a single firm, not faced with a rival bidder, will make its offer. Each firm's valuation of the target s_i is linear in the state variable θ. Consequently, in either case, there is no option value to the firm and hence no incentive to delay bidding. (The firm would have an incentive to delay only if its valuation function were sufficiently convex in θ.) As we shall see, this is in marked contrast to the competitive bidding situation: in the game between the two competing acquirers, equilibrium always involves delay.

2.3 Competitive bidding equilibrium

We look for equilibria with the following two properties regarding behavior in, respectively, the timing and bidding stages of the game.

- **Property 1: Trigger Bids**. The timing of bids is represented by a trigger threshold of the state variable θ for each player. Firm G bids at the first instant that θ hits the interval (or 'stopping region')$[\theta_G, +\infty)$, and firm D bids at the first instant that θ hits the interval $[0, \theta_D]$ (values of the trigger points are to be determined).

- **Property 2: Competitive Cautious Bidding**. Firms' bidding behavior has the following features: (i) if one firm bids, then so does the other; (ii) the firms epsilon-outbid each other; (iii) the losing firm bids cautiously, i.e., so as to be indifferent between winning and not winning at the equilibrium bids. In terms of equilibrium strategies, for $i \in \{G, D\}$,

$$b_i(\theta) = \begin{cases} b_{-i}(\theta) + \epsilon & \text{if} b_{-i} < s_i, \\ s_i & \text{if} b_{-i} \geq s_i, \end{cases}$$

  where $\epsilon > 0$ is arbitrarily small (hereafter, ϵ is generally ignored).

We can now define the equilibrium concept that we shall use to derive the solution to the competitive bidding game.

Definition 1 (Competitive, Cautious, Trigger equilibrium) *Any equilibrium that satisfies properties 1 and 2 is called a Competitive, Cautious, Trigger (CCT) equilibrium.*

Trigger strategies seem the natural ones to consider given the single-crossing property of the relative valuation function $\delta(\theta)$. Restricting attention to competitive bidding means that we can focus on the *timing* (rather than the level) of bids as our primary interest. We concentrate on cautious equilibria to rule out arbitrary outcomes in which the losing bidder effectively uses weakly dominated strategies.[6] A CCT equilibrium is characterized, therefore, by the two trigger points θ_G and θ_D. (In Appendix A we demonstrate formally the existence of a CCT equilibrium.)

To determine the equilibrium trigger points, the game is solved backwards as follows. In the bidding stage, competitive cautious bidding (property 2) ensures that each firm bids at most its actual valuation of the target, s_i. Combined with the single-crossing property of the relative valuation $\delta(\theta)$, this implies that (were bidding to take place) Goutbids D for $\theta > \theta^*$ but is outbid by its rival for $\theta < \theta^*$. A firm that is outbid receives a payoff of zero; we can therefore conclude that each firm bids only in the interval where it is not the losing bidder, i.e., that $\theta_G \geq \theta^* \geq \theta_D$.

[6] The same equilibrium notion is used in the equilibrium analysis of Bergemann and Välimäki (1996) and Felli and Harris (1996).

Accordingly, each firm's value function has three components, holding over different ranges of θ. For $\theta \geq \theta_G$, firm G bids immediately, acquiring the target with a net payoff of $\lambda\theta - \eta$ (ignoring ϵ, which is arbitrarily small), while D receives a payoff of zero. For $\theta \leq \theta_D$, firm D bids immediately, acquiring the target with a net payoff of $\eta - \lambda\theta$, while G receives a payoff of zero. In the continuation region before either firm has bid, i.e., for $\theta \in (\theta_D, \theta_G)$, in any short time interval dt starting at time t firm i receives a flow payoff of 0 and experiences a capital gain or loss dV_i. The Bellman equation for the value of the bidding opportunity to firm i is therefore

$$V_i = \exp(-rdt)\mathbb{E}_t\left[V_i + dV_i\right].$$

Itô's lemma and the GBM equation (1) gives the ordinary differential equation (ODE)

$$\frac{1}{2}\sigma^2\theta^2 V_i''(\theta) - rV_i(\theta) = 0.$$

The general solution of this homogeneous ODE is $V_i = A_i\theta^{\alpha} + B_i\theta^{\alpha+1}$, where A_i and B_i are constants determined by boundary conditions (discussed below), and $\alpha > 0$ is the positive root of the quadratic equation $\mathcal{Q}(z) = \frac{1}{2}\sigma^2 z(z+1) - r$:

$$\alpha = \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8r}{\sigma^2}}\right) > 0. \tag{4}$$

Putting the three parts together, firm G's value function is

$$V_G(\theta) = \begin{cases} 0 & \theta \leq \theta_D, \\ A_G\theta^{-\alpha} + B_G\theta^{\alpha+1} & \theta_D < \theta < \theta_G, \\ \lambda\theta - \eta & \theta \geq \theta_G. \end{cases} \tag{5}$$

$B_G\theta^{\alpha+1}$ is an option term anticipating G's bid for the target; $A_G\theta^{-\alpha}$ is an option-like term anticipating firm D's bid. Similarly, D's value function is

$$V_D(\theta) = \begin{cases} \eta - \lambda\theta & \theta \leq \theta_D, \\ A_D\theta^{-\alpha} + B_D\theta^{\alpha+1} & \theta_D < \theta < \theta_G, \\ 0 & \theta \geq \theta_G. \end{cases} \tag{6}$$

Equilibrium is derived from a number of boundary conditions, which together determine the two trigger points and four constants. By arbitrage, the trigger threshold θ_G for firm G must satisfy a value-matching condition. Optimality requires a second, smooth-pasting condition to be satisfied;[7] this condition requires the components of firm G's value function to meet smoothly at θ_G. θ_D is not chosen optimally by firm G; hence the smooth-pasting optimality condition does not apply here for firm G. Value functions are forward-looking, however, and so a value-matching condition applies at θ_D. Hence there are three boundary conditions for firm G—value-matching and smooth-pasting at θ_G, and value-matching at θ_D:

$$\begin{aligned} A_G\theta_G^{-\alpha} + B_G\theta_G^{\alpha+1} &= \lambda\theta_G - \eta, \\ -\alpha A_G\theta_G^{-\alpha-1} + (\alpha+1)B_G\theta_G^{\alpha} &= \lambda, \\ A_G\theta_D^{-\alpha} + B_G\theta_D^{\alpha+1} &= 0. \end{aligned}$$

These three equations can be combined to yield

$$\left(\frac{\theta_G}{\theta_D}\right)^{2\alpha+1} = \frac{(\alpha+1)\theta_G - \alpha\theta^*}{-\alpha\theta_G + (\alpha+1)\theta^*}. \tag{7}$$

Equation (7) gives implicitly the best-response correspondence of firm G, which we denote $\theta_G(\theta_D)$.

A similar story holds for firm D. Value-matching and smooth-pasting hold at θ_D, and value-matching alone at θ_G:

$$\begin{aligned} A_D\theta_D^{-\alpha} + B_D\theta_D^{\alpha+1} &= \eta - \lambda\theta_D, \\ -\alpha A_D\theta_D^{-\alpha-1} + (\alpha+1)B_D\theta_D^{\alpha} &= -\lambda, \\ A_D\theta_G^{-\alpha} + B_D\theta_G^{\alpha+1} &= 0. \end{aligned}$$

These three equations can be combined to give

$$\left(\frac{\theta_G}{\theta_D}\right)^{2\alpha+1} = \frac{-\alpha\theta_D + (\alpha+1)\theta^*}{(\alpha+1)\theta_D - \alpha\theta^*},$$

which gives implicitly the best-response correspondence of firm D, which we denote $\theta_D(\theta_G)$.

[7] See Dixit and Pindyck (1994) for an explanation.

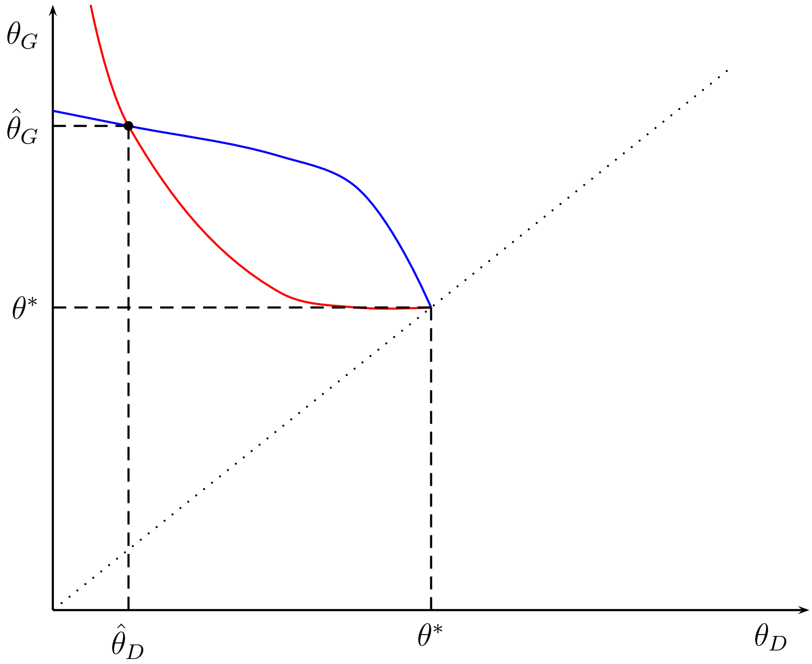

Figure 1: Reaction functions

Note that the firms' best responses are well-defined and non-empty for all $\theta_G, \theta_D \in (\underline{\theta}, \bar{\theta})$, where

$$\bar{\theta} \equiv \left(\frac{\alpha+1}{\alpha}\right)\theta^* > \theta^*, \qquad \underline{\theta} \equiv \left(\frac{\alpha}{\alpha+1}\right)\theta^* < \theta^*. \tag{8}$$

A CCT equilibrium exists if there is a solution to the simultaneous equations (7) and (8) with $\bar{\theta} > \theta_G \geq \theta^* \geq \theta_D > \underline{\theta}$. The following proposition establishes the existence of such a solution.

Proposition 1 (CCT equilibrium) *There exists a unique solution to equations (7) and (8) with* $\bar{\theta} > \theta_G > \theta^* > \theta_D > \underline{\theta}$.

Proof. See Appendix B. □

The CCT equilibrium is illustrated in figure 1. Note from the figure (see also the proof of the proposition) that the reaction functions also intersect at $\theta_G = \theta_D = \theta^*$. The following corollary, which follows immediately from the relative slopes of the reaction functions around the point $\theta_G = \theta_D = \theta^*$, means that this solution can be ignored in the remainder of the analysis.

Corollary 1 *The solution $\theta_G = \theta_D = \theta^*$ to equations (7) and (8) is not stable under the best-response dynamic (when strategies are restricted to be CCT).*

Proposition 1 and corollary 1 imply that takeovers occur only when shocks to the industry variable θ are sufficiently large: when θ rises above θ_G or falls below θ_D . For moderate shocks such that θ lies in the interval (θ_D, θ_G), the firms delay making offers. Our model can, therefore, explain the empirical findings of Mitchell and Mulherin (1996) and others, surveyed in the Introduction, that takeovers occur in industries experiencing large negative and positive shocks.

Two features of our model are crucial for generating this result. The first is that the firms' valuations of the target are imperfectly correlated, with the relative valuation function $\delta(\theta) = \lambda\theta - \eta$ being upward-sloping in θ. We interpret this feature as capturing two distinct types of bidder: one whose plans for the target are based on a growing market (firm G), the other whose plans are based on a declining market (firm D). The second feature is that the takeover market is competitive. As pointed out in section 2.2, with a single acquirer there is no delay—the target is acquired immediately. When there is a rival bidder, however, each firm's payoff function becomes convex: it is bounded below by 0 when the firm loses, and is equal to the difference in the firms' valuations when the firm wins. This convexity creates an incentive to delay bidding: rather than bidding immediately, or at the first instant when it has the larger valuation, each firm waits until the difference in valuations is sufficiently large. This feature—that competition makes payoffs convex—is quite general; we therefore expect to see bidder delay in more general environments than the one that we consider in this paper.

2.4 The effect of uncertainty

An important comparative static involves the effect of an increase in uncertainty (i.e., the parameter σ) on the trigger points θ_G and θ_D. A standard property of single-firm real option models is that when uncertainty increases, irreversible actions are more delayed (i.e., occur at a higher level of the state variable for firm G, and at a lower level for firm D). The reason for this is that delay allows for the possibility that the random process (1) might change; if it goes in an adverse direction (down for firm G, up for firm D), then the firm need not act. The greater the variance of the process, the more valuable is the option created by this asymmetric situation, hence more delay occurs.

The situation is complicated in the multiple firm case by the threat of pre-emption. If firm G delays, then it may lose its option altogether should firm D act in the meantime. This consideration can be seen in the value function in equation (5), for example. The term $B_G\theta^{\alpha+1} > 0$ is firm G's valuation of the option to delay due to the single-firm real options effect; the option-like term $A_G\theta^{-\alpha} < 0$ is the decrease in the valuation due to the possibility that firm D, not firm G, acquires the target—i.e., the pre-emption effect.

There are, therefore, two factors pulling in opposite directions when uncertainty increases; the comparative statics of θ_G and θ_D with respect to σ are determined by the balance between these two factors. Proposition 2 shows the net effect of the two.

Proposition 2 (Uncertainty and competitive bidding equilibrium) $\partial\theta_G/\partial\sigma > 0$ *and* $\partial\theta_D/\partial\sigma < 0$ *for all* $\sigma \geq 0$.

Proof. See Appendix B. □

The proposition shows that the standard option effect (which acts to increase θ_G and lower θ_D as σ increases) outweighs the pre-emption effect (which acts in the opposite direction). This need not always be the case: when the random process in equation (1) has a strictly positive drift μ,[8] there are cases when the pre-emption effect dominates. We have shown analytically (in a proof available on request) that when $\mu > 0$, in the limit as $\sigma \to +\infty$, the comparative statics are reversed, i.e., $\partial\theta_G/\partial\sigma < 0$ and $\partial\theta_D/\partial\sigma > 0$. Numerical analysis suggests that this outcome occurs when the drift parameter is large (close to, but below, the interest rate r) and there is a substantial degree of uncertainty. When μ is large, the opportunity cost of holding the option to bid is small, and so the option value is large. When σ is large, the option value is large. Hence both conditions (μ and σ large) ensure that the firms' options to bid are very valuable. But when σ is large, each firm assesses a high probability that the state variable hits the other firm's trigger point. The optimal response of each firm to an increase in σ is then to decrease delay, so that θ_G falls and θ_D rises, to limit the probability of pre-emption and so preserve its large option value.[9]

In a sense, therefore, the result in proposition 2 is not entirely general, though it holds in the model we have specified: the comparative static with respect to σ is unambiguous

[8] i.e., $d\theta_t = \mu\theta_t dt + \sigma\theta_t dW_t$ where $\mu \in (0, r)$ is the drift parameter. The restriction $\mu < r$ ensures that there is a positive opportunity cost to holding the option to bid to acquire the target. This means that each firm bids at some finite value of the state variable, rather than holding the option in perpetuity.

[9] See Mason and Weeds (2009) for a more general analysis of when the standard real options comparative static is reversed because of pre-emption.

only when $\mu = 0$. But the numerical analysis suggests that, even with a large drift, the extent of delay increases with σ for almost all values of the volatility parameter.

Proposition 2 can be contrasted with the findings in Bernile, Lyandres and Zhdanov (2007). Their model, like ours, has an upper and lower trigger such that merger occurs if the state variable rises above (falls below) the upper (lower) trigger. They find, however, that both of their triggers increase with the degree of uncertainty. Consequently, in their model, the zone in which no takeover activity is observed may actually decrease as uncertainty increases. In contrast, in our model, this zone increases with uncertainty. This offers the prospect of distinguishing empirically between the two models.

3 Efficient Timing of Takeover

Efficiency requires that the target be acquired by the firm with the higher valuation. The identity of that firm is stochastic: when $\theta < \theta^*$, it is firm D; when $\theta > \theta^*$, it is firm G. The timing of acquisition must also be efficient. We denote the efficient trigger points by θ_L for the lower threshold and θ_H for the upper one. The efficient allocation rule then takes the form "award the target to firm D immediately if $\theta \in [0, \theta_L]$, to firm G immediately if $\theta \in [\theta_H, \infty)$, otherwise wait".

The social value function is as follows. Acquisition by firm i has a social value equal to its full valuation $s_i(\theta)$; any payment for the target is merely a transfer and so is not subtracted from the social value. Between the efficient trigger points, i.e., for $\theta \in (\theta_L, \theta_H)$, there is a social option value which takes the familiar form. Putting these components together, the social value function is

$$W(\theta) = \begin{cases} \eta & \theta \leq \theta_L, \\ A_E \theta^{-\alpha} + B_E \theta^{\alpha+1} & \theta \in (\theta_L, \theta_H), \\ \lambda\theta & \theta \geq \theta_H. \end{cases} \tag{9}$$

The efficient triggers θ_L and θ_H are determined optimally, thus value-matching and smooth-pasting conditions apply at both points. These four equations yield, after elimination of the option value coefficients A_E and B_E,

$$\theta_H = \left(\frac{\alpha+1}{\alpha}\right)^{\frac{1}{2\alpha+1}} \theta^* > \theta^*, \tag{10}$$

$$\theta_L = \left(\frac{\alpha}{\alpha+1}\right)^{\frac{1}{2\alpha+1}} \theta^* < \theta^*. \tag{11}$$

(The symmetry of the equations implies that $\theta_H = \theta^{*2}/\theta_L$.)

Since pre-emption is not an issue in the efficient solution, the triggers θ_H and θ_L should have the the usual property of delay for situations of irreversible action under uncertainty; that is, θ_H should be increasing and θ_L decreasing in σ. The next proposition confirms this conjecture.

Proposition 3 (Uncertainty and efficient triggers) *1.*

(i) $\lim_{\sigma\to 0} \theta_H/\theta_L = 1$*;*

(ii) $\lim_{\sigma\to\infty} \theta_H/\theta_L = \infty$*;*

(iii) $\partial\theta_H/\partial\sigma > 0, \partial\theta_L/\partial\sigma < 0$*.*

Proof. Equations (10) and (11) imply that

$$\frac{\theta_H}{\theta_L} = \left(\frac{\alpha+1}{\alpha}\right)^{\frac{2}{2\alpha+1}}.$$

Parts (i) and (ii) of the proposition follow immediately from observing that as $\sigma \to 0$, $\alpha \to \infty$, and as $\sigma \to \infty$, $\alpha \to 0$, respectively. Part (iii) of the proposition will be proved for θ_H; an equivalent argument holds for θ_L. Since α is a decreasing function of σ,

$$\frac{\partial}{\partial\sigma}\left(\frac{\alpha+1}{\alpha}\right) > 0 \text{ and } \frac{\partial}{\partial\sigma}\left(\frac{1}{2\alpha+1}\right) > 0.$$

This implies immediately that θ_H is increasing in σ. □

In the next proposition we compare directly the competitive bidding equilibrium and efficient triggers.

Proposition 4 (Inefficiency of competitive bidding equilibrium) $\theta_G \geq \theta_H$ *and* $\theta_D \leq \theta_L$*.*

Proof. See Appendix B. $\square$

Proposition 4 tells us that there is too much delay in the competitive bidding equilibrium. A comparison of payoffs reveals why this is so. The efficient payoff is $\pi_E \equiv \max\{\lambda\theta, \eta\}$; in the competitive bidding model the payoff of firm G is $\pi_G \equiv \max\{\lambda\theta - \eta, 0\}$ and that of firm D is $\pi_D \equiv \max\{\eta - \lambda\theta, 0\}$. Competitive payoffs are lower because the acquirer must outbid its rival, paying the amount of latter's valuation to the owner of the target firm. Hence, *competition creates negative externalities*: $\pi_i = \pi_E - s_{-i} < \pi_E$ for $i \in \{G, D\}$. Notice that the competitive payoff of a firm is lower than the efficient payoff both when it wins the target and when it loses. The reduction in payoff on winning leads the firm to bid later (relative to the efficient trigger); the reduction in payoff on losing leads to an earlier bid. When an firm chooses its trigger point optimally the former effect dominates, so that negative externalities lead to more delay in equilibrium.

To understand why, consider first the choice of trigger point by firm G. Firm G's competitive payoff π_G is lower than the efficient payoff π_E by an amount η (the valuation of firm D), both when it wins the target and when it loses to firm D. Note, however, that firm G will be the first to bid only when $\theta \geq \theta^*$, when it has the higher valuation. In determining G's trigger point, the payoff reduction on winning is more important than the reduction on losing, because the former occurs immediately while the latter occurs in the future (when θ falls below θ^*) and so is discounted. Although the undiscounted payoff reductions are equal, the reduction on winning has greater weight. So, for a given lower trigger point (θ_L, say), the upper trigger in competitive equilibrium is greater than the efficient upper trigger. The shape of the reaction functions (established in proposition 1) then ensures that $\theta_G \geq \theta_H \geq \theta_L \geq \theta_D$.

This argument for firm G relied on firm D's valuation of the target being a constant, η. A more general intuitive argument can be made. Over the interval $\theta \in (\theta_D, \theta_G)$, firm G's value function (given in equation (5)) has two components. The first, $OL_G(\theta) \equiv A_G\theta^{-\alpha} < 0$, is an option-like term anticipating firm D's successful bid; the second $O_G(\theta) \equiv B_G\theta^{\alpha+1} > 0$ is an option term relating to its own successful bid. At firm G's optimally-chosen trigger point θ_G, value-matching and smooth-pasting conditions imply that $O_G(\theta_G) > -OL_G(\theta_G) > 0$ and $O'_G(\theta_G) > -OL'_G(\theta_G) > 0$ (where the prime denotes the derivative with respect to θ). Hence the option term is of greater magnitude in terms of both level and slope.[10] In short, the value from winning is more important than the

[10] Equivalently for firm D: its value function in equation 6 has two components, the option term

value from losing, hence firm G's trigger point lies above the efficient level. This argument suggests that the result in proposition 4 should generalize beyond linear valuation functions, although we have not shown this analytically.[11]

More generally, a second effect may also arise: the negative externalities may not only shift equilibrium payoffs, but also change the degree of convexity, relative to the efficient payoff. If the externalities create more (less) convexity, then they will tend to lead to more (less) delay in equilibrium, other things equal. Firm G's equilibrium payoff function is (weakly) more convex than the efficient payoff function iff

$$\frac{s''_G(\theta)}{s'_G(\theta)} \geq \frac{s''_D(\theta)}{s'_D(\theta)} \forall\ \theta,$$

assuming non-zero first derivatives. With linear valuation functions this condition is satisfied with equality (since $s''_G = s''_D = 0$), hence this effect does not arise in our analysis.

4 The Timing of Agreed Takeover

In several (especially continental European and south-east Asian) countries, hostile takeovers are a rarity:[12] either the company is controlled by a majority shareholder, or incumbent management can block unwanted bids through a variety of measures. Instead, then, takeover can occur only by means of an agreement reached between the target and acquirer. In this section we present an alternative model, of agreed takeover, in which the target chooses when to open negotiations with one of the acquirers, and the terms of takeover result from bargaining between the parties.

4.1 The model

In our setting with a single target and two potential acquirers, agreed takeover is modeled as follows. The target firm chooses when to open negotiations with one of two possible

$O_D(\theta) \equiv A_D\theta^{-\alpha} > 0$ relating to its own successful bid, and the option-like term $OL_D(\theta) \equiv B_D\theta^{\alpha+1}$ anticipating firm G's successful bid. Value-matching and smooth-pasting at firm D's trigger point θ_D imply that $O_D(\theta_D) > -OL_D(\theta_D) > 0$ and $-O'_D(\theta_D) > -OL'_D(\theta_D) > 0$. Again, the option term is of greater magnitude in level and slope.

[11] Numerical investigation supports this optimism.

[12] Schneper and Guillén (2004) report that between 1988 and 1998, 431 hostile takeovers attempts were announced in the US and 220 in Britain, while only 20 were announced in France, 12 in Sweden, five in Germany, three in Malaysia, two in Thailand, and just one each in Chile and Japan.

acquirers, $i \in \{G, D\}$, though not with both simultaneously.[13] If negotiations break down without agreement, the chosen firm goes away forever; the target can then negotiate with the other acquirer. Valuation functions of the two acquirers, $s_G(\theta)$ and $s_D(\theta)$, are as described by (2), and the stochastic state variable θ (industry demand) evolves according to equation (1). Entering into negotiations is costless (a negotiation cost could be added) and can be delayed indefinitely, but once the target has been acquired no further actions are considered. Time is continuous and the discount rate is $r > 0$.

The game has two stages. The first is a timing stage in which the target chooses when (i.e., at what level of the state variable) to open negotiations with one of the potential acquirers. The second stage is a bargaining game between the target and the chosen acquirer; we assume that the surplus from merger is divided according to the Nash bargaining solution.[14] In bargaining with the first firm, the target has the outside option of opening negotiations with the second; this possibility is also incorporated into the analysis.

As before the game is solved backwards, starting with the bargaining stage. Suppose that the target chooses to open negotiations with firm i. With risk-neutral players, the Nash bargaining solution (NBS) is the division of surplus that solves the following maximization problem

$$\max_{t_i} (t_i - d_T)(s_i - t_i - d_i),$$

where t_i is the target's share of the surplus from takeover by i (i.e., of i's valuation s_i), and d_T, d_i are the disagreement payoffs of the target and potential acquirer respectively. If no agreement is reached, firm i walks away empty-handed; thus $d_i = 0$. The target, by contrast, has the outside option of negotiating with the other firm $-i$; since both acquirers' valuations are strictly positive for all $\theta > 0$, this is preferable to remaining independent. In its negotiations with i, therefore, the target's disagreement payoff is its gain from subsequently negotiating with $-i$. Thus, to solve the bargaining problem, we must first ascertain the value of the target's outside option.

In its negotiations with the remaining acquirer, were these to take place, the target has no (further) outside option; if bargaining failed, the payoffs of both target and firm $-i$ would be zero. Applying the NBS with $d_T = d_{-i} = 0$, the target's gain from agreeing

[13] In effect, we assume that the target can commit to sell at the time of its choosing, and not at any other time. An alternative assumption might be that both parties must agree to open negotiations; bargaining would then take place at the first θ that is in the stopping regions of both parties.

[14] For details of Nash's bargaining approach see Osborne and Rubinstein (1990), chapter 2.

takeover with $-i$ (after failing to reach agreement with i) is $\frac{1}{2}s_{-i}$.[15] This amount forms the target's disagreement payoff in its negotiations with firm i. Applying the NBS to this problem, the target's payoff from agreeing takeover with i is

$$t_i(\theta) = \frac{1}{2}s_i(\theta) + \frac{1}{4}s_{-i}(\theta). \tag{12}$$

To solve the timing stage, consider the target's payoff from agreeing takeover with each of the potential acquirers. From (12), the target prefers takeover by firm G (respectively, D) for $\theta >$ (respectively, $<$) θ^*. The target's payoff function from agreeing takeover with its chosen acquirer is

$$t(\theta) = \begin{cases} \frac{1}{2}\eta + \frac{1}{4}\lambda\theta & \theta \le \theta^*, \\ \frac{1}{2}\lambda\theta + \frac{1}{4}\eta & \theta \ge \theta^*. \end{cases}$$

This function is increasing and convex in θ (with a kink at θ^*), implying that the target has an option value of delaying negotiations. Its optimal strategy is represented by a pair of trigger thresholds $\{\theta_B, \theta_U\}$, with $\theta_B < \theta^* < \theta_U$, such that the target opens negotiations with firm D for $\theta \le \theta_B$ and with firm G for $\theta \ge \theta_U$.

The target's value function is given by

$$V_T = \begin{cases} \frac{1}{2}\eta + \frac{1}{4}\lambda\theta & \theta \le \theta_B, \\ A_T\theta^{-\alpha} + B_T\theta^{\alpha+1} & \theta_B < \theta < \theta_U, \\ \frac{1}{2}\lambda\theta + \frac{1}{4}\eta & \theta \ge \theta_U. \end{cases} \tag{13}$$

The trigger points θ_B and θ_U are determined by the following value-matching and smooth-pasting conditions

$$\begin{aligned} A_T\theta_B^{-\alpha} + B_T\theta_B^{\alpha+1} &= \frac{1}{2}\eta + \frac{1}{4}\lambda\theta_B, \\ -\alpha A_T\theta_B^{-\alpha-1} + (\alpha+1)B_T\theta_B^{\alpha} &= \frac{1}{4}\lambda, \\ A_T\theta_U^{-\alpha} + B_T\theta_U^{\alpha+1} &= \frac{1}{2}\lambda\theta_U + \frac{1}{4}\eta, \\ -\alpha A_T\theta_U^{-\alpha-1} + (\alpha+1)B_T\theta_U^{\alpha} &= \frac{1}{2}\lambda. \end{aligned}$$

[15] Note that, given the functional forms assumed for s_G and s_D, the target's payoff from takeover by $-i$ is linear in θ. Hence it gains no option value by delaying agreement with the remaining bidder: this second round of bargaining would therefore take place immediately.

These equations can be combined to give

$$\left(\frac{\theta_U}{\theta_B}\right)^{\alpha} = \frac{\alpha\theta_B + 2(\alpha+1)\theta^*}{2\alpha\theta_U + (\alpha+1)\theta^*}, \tag{14}$$

$$\left(\frac{\theta_U}{\theta_B}\right)^{\alpha} = \frac{2(\alpha+1)\theta_U + \alpha\theta^*}{(\alpha+1)\theta_U + 2\alpha\theta^*\frac{\theta_U}{\theta_B}}. \tag{15}$$

We are looking for a solution to the simultaneous equations (14) and (15) with $\theta_U > \theta^* > \theta_B$. The next proposition demonstrates existence and uniqueness of such a solution.

Proposition 5 (Agreed takeover triggers) *There exists a unique solution to equations (14) and (15) with $\theta_U > \theta^* > \theta_B$.*

Proof. See Appendix B. □

4.2 The effect of uncertainty

Again, an important comparative static involves the effect of an increase in uncertainty (i.e., the parameter σ) on the trigger points θ_U and θ_B. With the timing of negotiations chosen by the target, pre-emption is not an issue for agreed takeover. Thus, the triggers should have the the usual property of delay for situations of irreversible action under uncertainty; i.e., θ_U should be increasing and θ_B decreasing in σ. Proposition 6 confirms this conjecture.

Proposition 6 (Uncertainty and agreed takeover) $\partial\theta_U/\partial\sigma > 0$ *and* $\partial\theta_B/\partial\sigma < 0$ *for all* $\sigma \geq 0$.

Proof. See Appendix B. □

4.3 Comparison with the efficient solution

The trigger points for agreed takeover can be compared with the efficient triggers θ_H and θ_L. The next proposition ranks these trigger points.

Proposition 7 (Inefficiency of agreed takeover) $\theta_U < \theta_H$ *and* $\theta_B > \theta_L$.

Proof. See Appendix B. □

Proposition 7 tells us that agreed takeover takes place too soon, compared with the efficient solution. A comparison of payoffs in the two cases reveals why this is so. The efficient payoff is $\pi_E \equiv \max\{\lambda\theta, \eta\}$; with agreed takeover the target's payoff is $\pi_T = \max\{\frac{1}{2}\lambda\theta + \frac{1}{4}\eta, \frac{1}{2}\eta + \frac{1}{4}\lambda\theta\}$. If, instead, the target's payoff were $\max\{\frac{1}{2}\lambda\theta, \frac{1}{2}\eta\}$ (which would be the case in the absence of the outside option), it would be a simple rescaling of the efficient payoff ($\frac{1}{2}\pi_E$), resulting in the identical trigger points $\{\theta_H, \theta_L\}$. Relative to this, the target's payoff when it agrees takeover with bidder i is increased by one-quarter of the valuation of the outside bidder $-i$: i.e., $\pi_T^i = \frac{1}{2}\pi_E + \frac{1}{4}s_{-i}$ for $i \in \{G, D\}$. Hence the target's payoff is less convex, implying less delay.

This feature arises from the target's outside option in the bargaining game with the acquirer, which allows it to gain an additional share of the takeover surplus related to the valuation of the outside bidder. In effect, the bargaining process creates a *positive externality* between the valuation of the outside bidder $-i$ and the target's payoff from agreeing takeover with i, which reduces delay.

As a corollary of proposition 7, we can also rank the triggers for agreed takeover against those for the competitive bidding model. Taken together with proposition 4, the findings imply that $\theta_U < \theta_G$ and $\theta_B > \theta_D$: agreed takeovers occur with less delay than competitive bids.

5 Summary of Results

Summarizing, our analytical results are as follows.

1. Competitive bidding: θ_G is greater than θ^* and increasing in σ; θ_D is less than θ^* and decreasing in σ (propositions 1 and 2);

2. Efficient solution: θ_H is greater than θ^* and increasing in σ; θ_L is less than θ^* and decreasing in σ (equations (10) and (11), and proposition 3);

3. Agreed takeover: θ_U is greater than θ^* and increasing in σ; θ_B is less than θ^* and decreasing in σ (propositions 5 and 6);

4. Ranking of trigger points: $\theta_G \geq \theta_H > \theta_U$ and $\theta_D \leq \theta_L < \theta_B$ (propositions 4 and 7).

These results are illustrated by the numerical analysis shown in figure 2, which plots the various trigger points against different values of σ. In this numerical analysis, we set

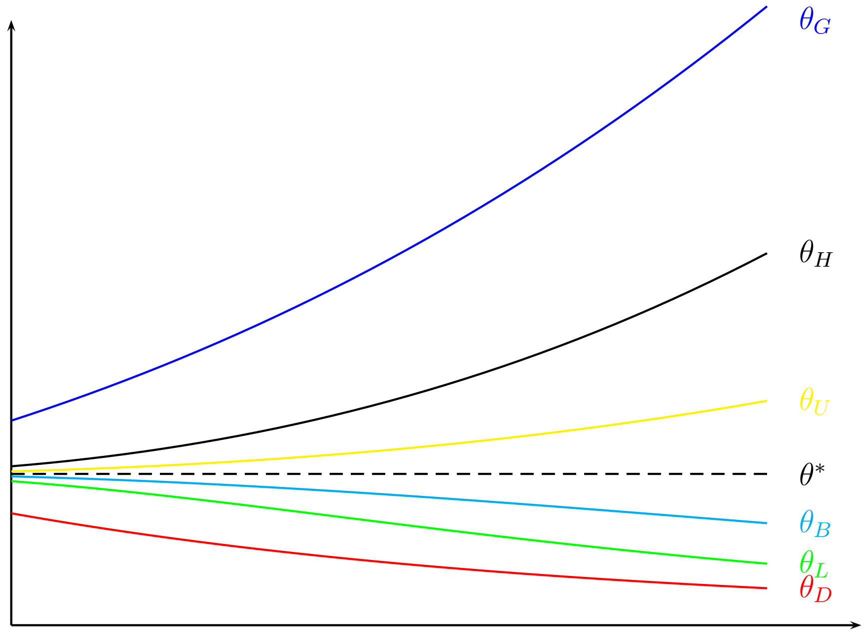

Figure 2: Triggers against degree of uncertainty σ

the interest rate r to 5%, firm D's value η to 10 and the slope of firm G's value λ to 1. With these values, $\theta^* = 10$.

6 Extensions

6.1 Target preferences and project choice

In section 2, the timing of takeover is determined by competing bidders, with the target as a passive player. Nonetheless, in this setting it is possible that the target may be able to influence the timing of takeover through its exposure to industry shocks, say by its choice of investment projects. Here we assess the target firm's preferences regarding the timing of takeover in the competitive bidding model. Then, assuming that the target can influence the volatility parameter σ through its choice of projects, we briefly consider the target's incentives regarding project choice.

When acquisition occurs, the owner of the target firm receives the valuation of the losing bidder. Its payoff is therefore

$$u(\theta) = \begin{cases} \lambda\theta & \theta \leq \theta_D, \\ \eta & \theta \geq \theta_G. \end{cases}$$

For $\theta \in (\theta_D, \theta_G)$ the target's value function consists of two option-like terms anticipating takeover by one or other of the acquirers; thus, the target's value function is

$$U(\theta) = \begin{cases} \lambda\theta & \theta \leq \theta_D, \\ A_U\theta^{-\alpha} + B_U\theta^{\alpha+1} & \theta_D < \theta < \theta_G, \\ \eta & \theta \geq \theta_G. \end{cases} \tag{16}$$

The coefficients A_U and B_U are determined by value-matching conditions at the bidders' trigger points θ_G and θ_D; there are no corresponding smooth-pasting conditions as the triggers involve no optimality on the part of the target.

Observe that the target's payoff function $u(\theta)$ is concave. Thus greater delay, i.e., a widening of the trigger points θ_G and θ_D, is harmful to the target. If the target firm's owner were able to determine the timing of acquisition, it would choose to sell immediately. Note that this finding does not require a functional form assumption such linearity of the valuation functions. It depends only on the assumption that the difference in the valuations is (weakly) increasing in θ, and that bidding is competitive so that the target receives the lower of the two bidders' valuations.

The target cannot directly control the timing of takeover—bid timing is determined by the acquirers—but it may be able to influence this indirectly through the volatility parameter σ. Following e.g., Sung (1995), Cadenillas, Cvitanić and Zapatero (2004, 2007) and Décamps, Mariotti and Villeneuve (2006), we assume that the target can influence the volatility parameter σ through its choice of which project(s) to pursue. We now consider briefly the target's incentives regarding project choice.

Note that in our model, there is an alternative interpretation of volatility. The stochastic process θ drives the gap between the valuations of the target firm and acquirer G, with this difference being scaled by λ. Thus, the volatility σ may be interpreted as the degree of correlation between the value of these assets when controlled by the target and under the alternative ownership of firm G, where a higher value of σ represents a reduction in the correlation between these valuations. The degree of correlation between the valuations of the target and acquirer D, with a fixed cost reduction, is unaffected. However, since D must outbid G to win the target, the payoff functions of both bidders (net of bid payment) depend on θ and the correlations of both are affected by the target's choice of σ.

Numerical analysis suggests that an increase in σ reduces the target's pre-bid value (the middle segment of the target's value function in equation (16)). This is illustrated in figure 3, in which σ increases from 0.2 to 0.4. From proposition 2, we know that the acquiring firms' triggers widen when the degree of uncertainty σ increases. Hence in the figure, θ_D shifts to the left and θ_G to the right. The result is a downward shift in the target's value function. Hence in this numerical example, the target prefers the lower value of σ. While we have not been able to establish this analytically, numerical examples suggests that the result holds more generally.

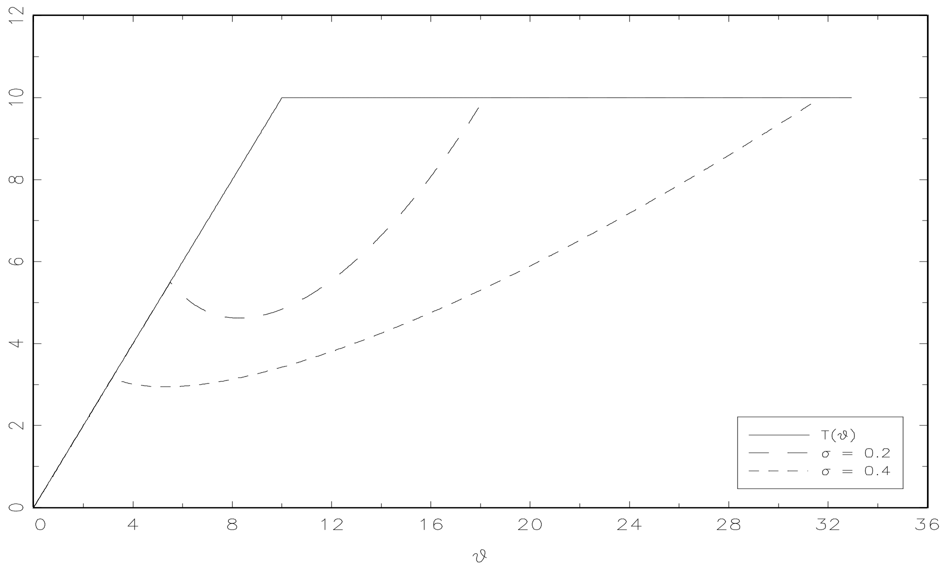

Figure 3: The target's value function against σ

6.2 More than two bidders

Finally, we consider how the modeling and results in section 2 might be affected if there were more than two bidders for the target firm.

We have fixed exogenously the number of bidders to equal two.[16] Whatever the number of bidders, the critical assumption for our analysis is that the bidders' valuations can be ordered with respect to a single state variable. With just two bidders, this means that the relative valuations of the bidders must be single-crossing. There is then a single value of the state variable so that the identity of the highest valuation bidder is determined by whether the state variable is above or below that level. With more than two bidders, the same basic assumption is required: that the set of values of a single state variable can be sectioned into disjoint and contiguous intervals. A particular bidder then has the highest valuation only if the level of the state variable is within a particular interval.

Subject to this assumption, a number of possibilities arise. Suppose first that there are two broad types of bidders: growth-acquirers (G-bidders) and decline-acquirers (D-bidders). For simplicity, suppose that there is a single D-bidder, whose valuation of the target is $\eta > 0$. But there are two G-bidders: one with a valuation of the target of $\lambda_1\theta$ (we label this bidder as the G_1-bidder), the other (the G_2-bidder) with a valuation $\lambda_2\theta$, where $0 < \lambda_2 < \lambda_1$. The G_2-bidder never acquires the target in equilibrium, but it acts as a constraint on the G_1-bidder when the latter sets its bid. When $\theta \geq \theta_1^* \equiv \eta/\lambda_1$, the G_1-bidder has the highest valuation. Within this interval, the identity of the second highest valuation bidder depends on $\theta_2^* \equiv \eta/\lambda_2 > \theta_1^*$. For $\theta \in [\theta_1^*, \theta_2^*)$, the second highest valuation bidder is the D-bidder, and so the G_1 -bidder must pay η in order to acquire the target. When $\theta > \theta_2^*$, the G_2-bidder has the second highest valuation and the G_1-

[16] The empirical evidence (which is not extensive) provides some support for this feature. In De, Fedenia and Triantis (1996), out of 660 contests over the period 1962–1988, 460 involve just one bidder, and 200 involve more than one bidder. The authors do not report how many bidders are involved in contests in which there are multiple bidders, but in those 200 contests, only 279 bidders were involved. This indicates that, where there are multiple bidders, there are relatively few bidders; and that some bidders are involved in more than one contest. A similar story emerges from Boone and Mulherin (2002), who study a sample of 50 US firms acquired by private auction over the period 1989–1998. The mean (respectively, median) number of firms initially contacted by the target was 63.2 (respectively, 50). Of the firms initially contacted, an average of 28.7 firms (median of 18) indicated interest by signing confidentiality agreements. An average of 6.3 firms submitted preliminary proposals. An average of 2.6 potential bidders (median of 2) submitted binding written offers. The authors contrast these statistics with the number of competing public bids for the firms in the auction sample. On average, there were 1.3 competing public bids; the median was 1. Of course, little can be said about the presence of potential bidders, who do not submit bids but affect the bidding behavior of firms that do actually bid. Nevertheless, these findings provide informal support for our modeling decision to limit the number of bidders to two.

bidder must pay $\lambda_2\theta > \eta$ to acquire the target. In consequence, the payoff function of the G_1-bidder were it to bid at θ becomes

$$G_1(\theta) = \begin{cases} 0 & \theta \leq \theta_1^*, \\ \lambda_1\theta - \eta & \theta \in (\theta_1^*, \theta_2^*) \\ (\lambda_1 - \lambda_2)\,\theta & \theta \geq \theta_2^*. \end{cases}$$

Compared with the two-bidder case, there is a kink at θ_2^* : after this point the payoff function flattens, with a slope of $(\lambda_1 - \lambda_2)$ rather than λ_1.

If λ_2 is low, such that G_1's unconstrained trigger (in the absence of the G_2-bidder) $\theta_G < \theta_2^*$, then the presence of the G_2-bidder has no effect on the equilibrium and efficient solutions. But for higher values of λ_2 such that the unconstrained $\theta_G > \theta_2^*$, the equilibrium behavior of the G_1-bidder is constrained by the G_2-bidder. Note that the G_1-bidder's payoff function is *concave* over the interval (θ_1^*, ∞) ; with θ_D unchanged, the G_1-bidder would therefore bid at the kink θ_2^* itself. The optimal behavior of the D-bidder will also change: since triggers are strategic complements, θ_D will be higher. In other words, the increase in competition among G-bidders reduces the extent of equilibrium delay, but does not eliminate it. Hence our main result is robust to this type of extension, as long as there are asymmetries between bidders of each type.

The picture is more complicated when there are more than two possible winning bidders. To illustrate the issues, suppose that there are three possible winning bidders, labeled D, M, and G. The D-bidder has a valuation of the target of $\overline{\eta}$, the M-bidder has a target valuation of $\underline{\eta} + \underline{\lambda}\theta$, and the G-bidder values the target at $\overline{\lambda}\theta$, where $0 < \underline{\eta} < \overline{\eta}$ and $0 < \underline{\lambda} < \overline{\lambda}$. (The M-bidder can be viewed as having a combined strategy: some asset stripping and some exploitation of synergies.) The bidders' valuations as functions of θ are shown in figure 4. With these valuation functions there are three critical values of the state variable: $\underline{\theta}^* \equiv \left(\overline{\eta} - \underline{\eta}\right)/\underline{\lambda}$; $\theta^* \equiv \overline{\eta}/\overline{\lambda}$ (the critical threshold in the two-bidder case), and $\bar{\theta}^* \equiv \underline{\eta}/\left(\overline{\lambda} - \underline{\lambda}\right)$, where $\underline{\theta}^* < \theta^* < \bar{\theta}^*$. Standard arguments suggest that the D-bidder bids in the interval $[0, \theta_D]$, where $\theta_D < \underline{\theta}^*$; the G-bidder bids in the interval $[\theta_G, +\infty)$, where $\theta_G > \bar{\theta}^*$; and the M-bidder bids in the interval $[\underline{\theta}_M, \bar{\theta}_M]$, where $\underline{\theta}^* < \underline{\theta}_M < \bar{\theta}_M < \bar{\theta}^*$. The bidding intervals are illustrated in figure 4, in bold along the horizontal axis. In this case, therefore, takeovers occur at moderate values of the state variable as well as at extreme (large or small) values. Nevertheless, the general feature remains: given an initial value

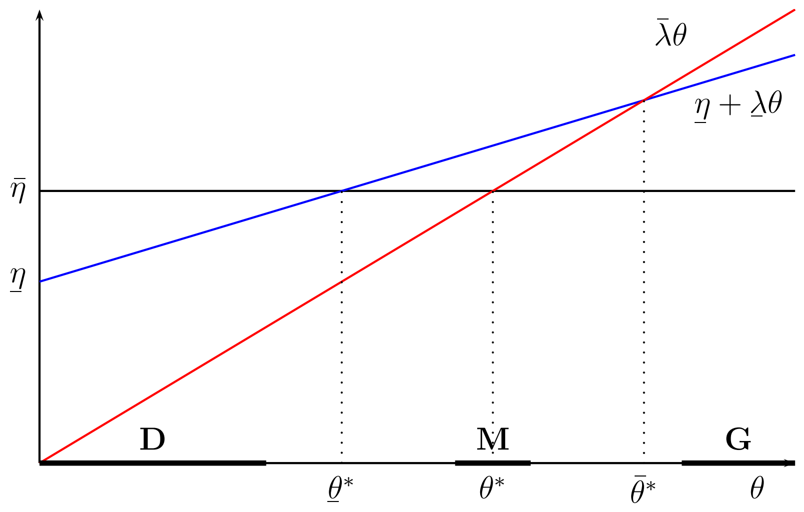

Figure 4: Valuations with 3 bidders

of the state at which no takeover occurs, a sufficiently large positive or negative shock to the state can lead to a takeover occurring.

Does a third bidder reduce the size of the shock that is required to induce takeover? There are two conflicting effects. Takeover—by the M-bidder—now occurs for intermediate values of θ between $\underline{\theta}_M$and $\bar{\theta}_M$; with just the D-bidder and G-bidder, no takeover occurs for intermediate θ. But the presence of the M-bidder also changes the equilibrium behavior of the G- and D-bidders, as each now competes directly with the M-bidder rather than with one another. Suppose that the initial value of θ lies in the interval $\left(\theta^*, \bar{\theta}^*\right)$, so that bidding competition occurs between the G- and M-bidders. Assuming an interior solution for $\bar{\theta}_M$,[17] the bidders' value functions resemble those in the two-bidder case, with value-matching and smooth-pasting conditions at the bidder's own trigger and value-matching alone at its rival's. Considering the G-bidder (a corresponding analysis holds for the M-bidder relative to the D-bidder in the two-bidder case), its payoff function is altered: in the two-bidder case the G-bidder's payoff was $\max\left(0, \overline{\lambda}\theta - \overline{\eta}\right)$; when competing against the M-bidder this becomes $\max\left(0, \left(\overline{\lambda} - \underline{\lambda}\right)\theta - \underline{\eta}\right)$. If it were the case that $\overline{\eta}/\overline{\lambda} = \underline{\eta}/\left(\overline{\lambda} - \underline{\lambda}\right)$ (and thus $\theta^* = \bar{\theta}^*$), this would represent a simple rescaling of payoffs and G's trigger point would be unchanged. But with $\bar{\theta}^* > \theta^*$, the increasing segment of G's payoff is shifted upwards relative to this, raising G.[18] A corresponding analysis of bidding competition between the D- and M-bidders over the lower interval of

[17]Otherwise, the M-bidder will act at θ^*, where its payoff has a concave kink.

[18]A corresponding analysis holds for the M-bidder's payoff compared with that of the D-bidder in the two-bidder model, implying that $\bar{\theta}_M$ exceeds θ_D in the two-bidder case.

θ implies that θ_D is shifted downwards compared with the two-bidder case. Thus, with three bidders, the takeover triggers of the G- and D-bidders are widened (i.e., θ_G rises and θ_D falls). Taken together, this analysis implies that takeover continues to be delayed in equilibrium, while the size of the shock required to induce takeover may increase or decrease.[19]

In summary: allowing for additional bidders does not change the general feature that a sufficiently large positive or negative shock to the state variable is needed to stimulate takeover, although the precise pattern of takeover activity is likely to be affected.

7 Conclusions

We have developed a theoretical framework modeling the timing of takeovers, in order to explain the empirical finding that takeovers occur after both positive and negative shocks to industry-specific conditions. Our approach relies on two key assumptions: that there are two types of acquirers distinguished by their comparative advantages in running the acquired firm, and that the takeover market is competitive. We model both hostile takeover, where the timing and terms are determined by the competing bidders, and agreed takeover, where the target chooses when to open negotiations with one of the potential acquirers. In both cases, we show that in equilibrium takeover occurs only when there is a sufficiently large shock to industry conditions. We show that this delay, and hence the market for corporate control, is not efficient. Moreover, the direction of inefficiency depends on the form of takeover: hostile bidding occurs too late, and agreed takeover too early, compared with the efficient solution. We also show that the extent of the delay increases in the degree of uncertainty about industry conditions; this result (which is not found in Bernile, Lyandres and Zhdanov (2007)) suggests an empirical test of our model.

We have briefly examined the incentives and behavior of the target firm when faced with competitive bidding. In future work this line might be further developed, looking at the range of target actions that might influence the timing of (hostile) takeover. For agreed takeover, the impact of different selling mechanisms might be assessed. Finally, the separation of ownership and control implies that managerial, rather than shareholder,

[19]Numerical analysis suggests that the gap between θ_G and $\bar{\theta}_M$ exceeds that between the triggers in the two-bidder case, thus if the initial value of θ is in this range, there might be more rather than less delay.

incentives determine the behavior of the target; in this context an analysis of managers' payoffs becomes relevant.

Appendix A

Formalization of the timing game

To formalize the timing game described in section 2.1, we define two further state variables x and y. At any time $t \geq 0$, let $x = 0$ if firm G has not made a bid for the target at any time $\tau \leq t$ and $x = 1$ otherwise; let $y \in \{0, 1\}$ indicate the same for firm D. The current state of the game is the triple (θ, x, y). At any time $t > 0$, past realizations of the process described by equation 1 and past bid decisions constitute the history of the game. The firms are assumed to use stationary Markovian strategies: actions depend on only the current state and the strategy formulation itself does not vary with time.[20] A (pure) Markovian strategy for firm $i \in \{G, D\}$ (at time t) has two parts: (i) when $x_t = y_t = 0$, a measurable function[21] $m_i(\theta_t) : \mathbb{R}_+ \rightarrow \{0, 1\}$ which takes the value 0 when the firm has not made a bid, and the value 1 when the firm makes a bid; (ii) when $x_t + y_t > 0$, a measurable function $b_i(\theta_t) : \mathbb{R}_+ \rightarrow \mathbb{R}$ describing the bid submitted by the firm.

Markov perfect equilibrium is defined as follows.

Definition 2 ***(Markov Perfect Equilibrium, MPE)***
A pair of strategies $(m_G^*, b_G^*), (m_D^*, b_D^*)$, with $T^* \equiv \min[T_G^*, T_D^*]$ and $T_i^* \equiv \inf\{t | m_i^*(\theta_t) = 1\}$, is a Markov Perfect Equilibrium if and only if, for all θ_t and $i \in \{G, D\}$,

$$V_i(T^*, b_i^*, b_{-i}^*; \theta_t) \geq V_i(T, b_i^*, b_{-i}^*; \theta_t), \forall m_i(\theta_t),$$
$$V_i(T^*, b_i^*, b_{-i}^*; \theta_t) \geq V_i(T^*, b_i, b_{-i}^*; \theta_t), \forall b_i$$

where $T \equiv \min[T_i, T_{-i}^*], T_i \equiv \inf\{t | m_i(\theta_t) = 1\}$.

Note that deviation strategies are not required to be Markovian.

[20]For further explanation, see Maskin and Tirole (1988) and Fudenberg and Tirole (1991). Non-Markovian equilibria may exist. Since we want to analyze how the resolution of uncertainty affects the takeover game, we concentrate on equilibria in Markovian strategies. This allows us to rule out collusive equilibria with continuation strategies that depend on information that is not payoff-relevant.

[21]Measurability is with respect to the filtration $\mathcal{F}$ of the complete probability space $(\Omega, \mathcal{F}, P)$ on which the Wiener process $\{W_t\}_{t \geq 0}$ in equation (1) is defined.

Existence of a CCT equilibrium

To show that a CCT equilibrium exists, consider first the bidding stage when at least one of the firms has made a bid. By the standard 'Bertrand' argument in common knowledge bidding games, when one firm bids competitively and cautiously, then the best response is a competitive, cautious bid. Given this behavior in the bidding stage, now consider the prior stage when the firms must decide when to bid; and suppose that firm D bids at the first time that the state variable hits the interval $[0, \theta_D]$. Firm G's value function V_0 when it is not bidding is

$$V_0 = \begin{cases} 0 & \theta \leq \theta_D, \\ A_G \theta^{-\alpha} + B_G \theta^{\alpha+1} & \theta > \theta_D. \end{cases}$$

Its value function when it makes a bid is given by

$$V_1 = \begin{cases} \lambda\theta - \eta - \epsilon & \text{if } s_G(\theta) > s_D(\theta) \\ 0 & \text{otherwise.} \end{cases}$$

By a standard argument (see Dixit and Pindyck (1994)), the continuation region (i.e., in which no bid is made) for firm G is the half-open interval $[0, \hat{\theta})$, while the stopping region (in which a bid is made) is the interval $[\hat{\theta}, \infty)$, for some $\hat{\theta}$.

An analogous argument holds for firm D's best response when firm G uses a CCT strategy. In summary: the best response to a CCT strategy is itself a CCT strategy. Hence

Proposition 8 *A CCT equilibrium exists, and is given by the solutions to equations (7) and (8).*

Of course, non-CCT equilibria exist—there are Markovian equilibria that are non-CCT (for example, the continuum of equilibria with non-cautious bidding in the bidding stage of the game); and there are non-Markovian equilibria. Our focus on Markovian CCT equilibrium is motivated by our interest in the timing of non-collusive bids.

Appendix B

Proof of Proposition 1

Proposition 1 (CCT equilibrium) There exists a unique solution to equations (7) and (8) with $\bar{\theta} > \theta_G > \theta^* > \theta_D > \underline{\theta}$.

Proof. The symmetry of equations (7) and (8) implies that

$$\theta_G = \frac{\theta^{*2}}{\theta_D}.$$

Let $w \equiv \theta_G/\theta^*$. Equation (7) then implies that

$$w^{4\alpha+2} = \frac{(\alpha+1)w - \alpha}{-\alpha w + \alpha + 1}. \tag{17}$$

This equation defines a polynomial in w. We are interested in roots of this polynomial that satisfy $w \geq 1$. The first root is clearly $w = 1$. There must be (at least) a second root, since the left-hand side of equation (17) is greater than zero and finite for $w \in [\alpha/(\alpha+1), (\alpha+1)/\alpha]$, while the right-hand side is zero at $w = \alpha/(\alpha+1)$ and tends to infinity as $w \to (\alpha+1)/\alpha$.

We now show that there is no more than 1 root such that $w > 1$. The slope of the left-hand side of equation (17) is $(4\alpha+2)w^{4\alpha+1}$; at an intersection point, this must equal

$$\left(\frac{4\alpha+2}{w}\right)\frac{(\alpha+1)w - \alpha}{-\alpha w + \alpha + 1}.$$

The slope of the right-hand side is

$$\frac{2\alpha+1}{(-\alpha w + \alpha + 1)^2}.$$

At $w = 1$, the slope of the left-hand side is clearly greater than the slope of the right-hand side. Hence at the 'next' intersection point (i.e., the intersection point with the lowest $w > 1$), it must be that the slope of the right-hand side is greater than the slope of the left-hand side. I.e., at this intersection point,

$$\frac{2\alpha+1}{(-\alpha w + \alpha + 1)^2} > \left(\frac{4\alpha+2}{w}\right)\frac{(\alpha+1)w - \alpha}{-\alpha w + \alpha + 1}.$$

Rearranging this inequality gives

$$-\alpha(\alpha+1)(4\alpha+2)w^2 + ((\alpha+1)^2(4\alpha+2) + \alpha^2 - 2\alpha - 1)w - \alpha(\alpha+1) < 0.$$

The left-hand side of this expression is a quadratic in w. Given the signs of the coefficients in the quadratic, if the inequality is satisfy for an $w^* > 1$, it must be satisfied for all $w \geq w^*$. Hence at any intersection point $w > 1$, it must be that the slope of the right-hand side of equation (17) is greater than the slope of the left-hand side. There can be, therefore, only one intersection point $w > 1$. □

Proof of Proposition 2

Proposition 2 (Uncertainty and competitive bidding equilibrium) $\partial\theta_G/\partial\sigma > 0$ and $\partial\theta_D/\partial\sigma < 0$ for all $\sigma \geq 0$.

Proof. The proof of both parts of the proposition concentrates on the reaction function of firm G, equation (7); the symmetry in equations (7) and (8) means that the equivalent result for firm D follows immediately. Equation (7) can be rewritten as

$$\frac{\theta_G}{(\Theta(\theta_G))^{\frac{1}{2\alpha+1}}} = \theta_D$$
$$\text{where}\Theta(\theta_G) \equiv \frac{(\alpha+1)\theta_G - \alpha\theta^*}{-\alpha\theta_G + (\alpha+1)\theta^*}.$$

Consider the denominator of the left-hand side of this reaction function. Taking logs and differentiating with respect to σ gives

$$\frac{\partial}{\partial\sigma}\ln\left((\Theta(\theta))^{\frac{1}{2\alpha+1}}\right) = \frac{2\alpha_\sigma}{(2\alpha+1)^2}\left(-\ln\Theta(\theta) + \frac{1}{2}\left(\frac{\Theta^2(\theta)-1}{\Theta(\theta)}\right)\right)$$

where $\alpha_\sigma \equiv \partial\alpha/\partial\sigma$. Since $\alpha_\sigma < 0$, the sign of this expression is determined by the sign of

$$-\ln\Theta(\theta) + \frac{1}{2}\left(\frac{\Theta^2(\theta)-1}{\Theta(\theta)}\right). \tag{18}$$

Consider the function $\phi(x) \equiv -\ln x + \frac{1}{2}\left(\frac{x-1}{x}\right)(1+x)$. $\phi(1) = 0$; and for $x > 1$, the first derivative is

$$\phi'(x) = \frac{1}{2}\left(\frac{x-1}{x}\right)^2 > 0.$$

Hence $\phi(x) > 0$ for $x > 1$. The expression in equation (18) is therefore positive for all values of σ. So the denominator of the left-hand side of the reaction function of firm G is decreasing in σ; and consequently, the reaction function of firm G shifts upwards when σ increases. With the symmetric argument for firm D's reaction function, the proposition follows. $\square$

Proof of Proposition 4

Proposition 4 (Inefficiency of competitive equilibrium) $\theta_G \geq \theta_H$ and $\theta_D \leq \theta_L$.

Proof. The proof works by showing (through contradiction) that $\theta_G/\theta_D \geq \theta_H/\theta_L$; since $\theta_G = (\theta^*)^2/\theta_D$ and $\theta_H = (\theta^*)^2/\theta_L$, this gives the result immediately. To simplify expressions, let $\tilde{\theta}_k \equiv \theta_k/\theta^*$ for $k \in \{G, D, H, L\}$. From equations (10) and (11),

$$\frac{\tilde{\theta}_H}{\tilde{\theta}_L} = \left(\frac{\alpha+1}{\alpha}\right)^{\frac{2}{2\alpha+1}};$$

from equation (7),

$$\frac{\tilde{\theta}_G}{\tilde{\theta}_D} = \left(\frac{(\alpha+1)\tilde{\theta}_G - \alpha}{-\alpha\tilde{\theta}_G + \alpha + 1}\right)^{\frac{1}{2\alpha+1}}.$$

Suppose that $\tilde{\theta}_H/\tilde{\theta}_L \geq \tilde{\theta}_G/\tilde{\theta}_D$. Manipulation of the above expressions shows that this would mean that

$$\tilde{\theta}_G \leq \frac{\alpha^3 + (\alpha+1)^3}{\alpha(\alpha+1)(2\alpha+1)}.$$

Similarly, from equation (8),

$$\frac{\tilde{\theta}_G}{\tilde{\theta}_D} = \left(\frac{-\alpha\tilde{\theta}_D + \alpha + 1}{(\alpha+1)\tilde{\theta}_D - \alpha}\right)^{\frac{1}{2\alpha+1}}.$$

$\tilde{\theta}_H/\tilde{\theta}_L \geq \tilde{\theta}_G/\tilde{\theta}_D$ then implies that

$$\tilde{\theta}_D \geq \frac{\alpha(\alpha+1)(2\alpha+1)}{\alpha^3 + (\alpha+1)^3}.$$

Hence $\tilde{\theta}_H/\tilde{\theta}_L \geq \tilde{\theta}_G/\tilde{\theta}_D$ implies that $\tilde{\theta}_G/\tilde{\theta}_D \leq 1$—a contradiction (since $\theta_G \geq \theta_D$). Therefore $\tilde{\theta}_H/\tilde{\theta}_L \leq \tilde{\theta}_G/\tilde{\theta}_D$. The proposition follows. $\square$

Proof of Proposition 5

Proposition 5 (Agreed takeover triggers) There exists a unique solution to equations (14) and (15) with $\theta_U > \theta^* > \theta_B$.

Proof. Equations (14) and (15) together imply that

$$\theta_U = \frac{\theta^{*2}}{\theta_B}. \tag{19}$$

Let $y \equiv \theta_U/\theta^*$. Substituting into equation (14) yields

$$y^{2\alpha+1} = \frac{2(\alpha+1)y + \alpha}{2\alpha y + (\alpha+1)}. \tag{20}$$

This equation defines a polynomial in y. We are interested in roots of this polynomial that satisfy $y \geq 1$, demonstrating the existence of a solution $\theta_U \geq \theta^*$.

First we show that there must be (at least) one root $y \geq 1$. At $y = 1$, the left-hand side of this expression is unity while the right-hand side equals $\frac{3\alpha+2}{3\alpha+1} > 1$, thus $y = 1$ is not a root. As $y \to \infty$, the left-hand side of (20) approaches infinity while the right-hand side remains finite, tending towards $\frac{\alpha+1)}{\alpha}$. Hence there must be at least one root $y > 1$.

Next we demonstrate that there is no more than one root such that $y > 1$. Both sides of equation (20) are strictly increasing in y; the left-hand side is strictly convex while the right-hand side is strictly concave. Thus the two cross only once; i.e., there exists a single root $y > 1$.

Let $z \equiv \theta^*/\theta_B$. Substituting this and (19), equation (15) becomes

$$z^{2\alpha+1} = \frac{2(\alpha+1)z + \alpha}{2\alpha z + (\alpha+1)}. \tag{21}$$

This equation takes the same form as (20). Thus, an identical proof demonstrates the existence of a unique root $z \geq 1$, i.e., of a solution $\theta_B < \theta^*$. □

Proof of Proposition 6

Proposition 6 (Uncertainty and agreed takeover) $\partial\theta_U/\partial\sigma > 0$ and $\partial\theta_B/\partial\sigma < 0$ for all $\sigma \geq 0$.

Proof. The proof proceeds with reference to the proof of proposition 5. We start with the first part of the proposition, $\partial\theta_U/\partial\sigma > 0$. We have defined $y \equiv \theta_U/\theta^*$. Since θ^* is strictly positive and independent of σ, $\partial y/\partial\sigma$ has the same sign as $\partial\theta_U/\partial\sigma$. The root α, defined in (4), is strictly decreasing in σ, thus $\partial y/\partial\sigma$ and $\partial y/\partial\alpha$ take opposite signs. So, the proposition can be proven by demonstrating $\partial y/\partial\alpha < 0$.

Equation (20) defines a polynomial in y. Taking logs and differentiating with respect to α, then rearranging, yields

$$\frac{(2y-1)(2y+1)}{(2(\alpha+1)y+\alpha)(2\alpha y+\alpha+1)} + 2\ln(y) = -\left(\frac{\partial y}{\partial\alpha}\right)\frac{\alpha(\alpha+1)(2\alpha+1)(2y+1)^2}{y(2(\alpha+1)y+\alpha)(2\alpha y+\alpha+1)}$$

Since $\theta_U > \theta^*$, $y > 1$ and $\ln(y) > 0$. Thus the equation is satisfied only if $\partial y/\partial\alpha < 0$.

The second part of the proposition, $\partial\theta_B/\partial\sigma < 0$, follows from an identical proof based on equation (21) the corresponding polynomial in $z \equiv \theta^*/\theta_B$, demonstrating that $\partial z/\partial\alpha < 0$. This implies that $\partial\theta_B/\partial\alpha > 0$, i.e., $\partial\theta_B/\partial\sigma < 0$. □

Proof of Proposition 7

Proposition 7 (Inefficiency of agreed takeover) $\theta_U < \theta_H$ and $\theta_B > \theta_L$.

Proof. Equation (20) defines a polynomial in $y \equiv \theta_U/\theta^*$. The proof of proposition 5 demonstrates the existence of a single root $y > 1$, i.e., that there exists a unique solution $\theta_U > \theta^*$. We now wish to compare θ_U with the efficient trigger θ_H. The left-hand side of (20) is strictly increasing and convex in y, and approaches infinity as $y \to \infty$. The right-hand side of (20) is strictly increasing and concave in y; as $y \to \infty$ it remains finite, tending towards $\frac{\alpha+1}{\alpha}$. Thus we can write

$$y^{2\alpha+1} = \frac{2(\alpha+1)y+\alpha}{2\alpha y+(\alpha+1)} < \frac{\alpha+1}{\alpha}.$$

Thus

$$y < \left(\frac{\alpha+1}{\alpha}\right)^{\frac{1}{2\alpha+1}}.$$

Substituting for y and rearranging, we can infer that

$$\theta_U < \left(\frac{\alpha+1}{\alpha}\right)^{\frac{1}{2\alpha+1}}\theta^* = \theta_H.$$

Equation (21), the equivalent polynomial in $z \equiv \theta^*/\theta_B$, generates a corresponding proof for θ_B, yielding

$$\theta_B > \left(\frac{\alpha}{\alpha+1}\right)^{\frac{1}{2\alpha+1}} \theta^* = \theta_L.$$

$\square$

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