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ESSAYS ON LABOUR SUPPLY AND GROWTH: WITH EMPHASIS ON SKILLS AND INPUT-OUTPUT LINKAGES

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To minha família and moja rodzina.

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Summary

The first chapter incorporates imperfect substitution between two human-capital types and skill-specific technology to Bils and Klenow's (2000) model. Their model relies on the assumptions that (i) different types of workers are perfect substitutes, and (ii) technical changes are neutral. These premises contradict a vast literature showing that (i) the elasticity of substitution between skilled and unskilled workers is much less than infinity, and (ii) that recent technical changes have been favouring skilled rather than unskilled labour. The analysis of the chapter shows that altering the human capital specification leads to diverging conclusions regarding the effect of schooling on human capital and technology.

The second chapter evaluates the impact of the A8 immigration into the UK on the output of its industries. An input-output general equilibrium model featuring three factors of production —capital, low-skill and high-skill labour— and industry-specific factor intensities is developed and calibrated for the country. I show that domestic commerce of intermediate inputs allows for a relay of the supply shock across industries. A model without this feature misestimates industrial output changes the most for industries more connected within the production network.

The third chapter proposes an input-output model with international trade to evaluate the relevance of the production network in the effects on industry output of exogenous macroeconomic shocks. I compute the domestic connectivity of WIOD countries to select two distinct economies: one with a sparse production network —Ireland— and another with a dense one —Korea. I calibrate the model for these countries and apply it to empirically assess the propagation of a labour supply shock and an import price shock through the economies. I find that ignoring input-output linkages results in misestimations of the industry output changes more than three times larger for the dense economy in comparison to the sparse one.

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Chapter 1

Skilled and unskilled labour: Are they worth their weight in growth?

1 Introduction

Despite how intuitive it may seem that investments in human capital foster economic growth, the pendulum of consensus regarding its relative importance has been swinging from enthusiasm to neglect. For the past 20 years, Bils and Klenow (2000) has arguably been accredited as a landmark for the case against education.[1] My study indicates that their results are dependent on the specification of the model, in particular, of the human capital aggregator. This chapter aims to incorporate recent empirical evidence on wages and skill supplies to stress the relevance in the macroeconomic analysis of a framework already widely established in the micro literature.

In the late 1980s' and early 1990s' wave of studies applying neoclassical growth models to then-new cross-country data,[2] Barro (1991) stands out by attesting a positive and strong correlation between schooling and economic growth.[3] Confronting Barro (1991), Bils and Klenow (2000) build a model of human capital accumulation and find that the causal impact of schooling on growth represents less than one-third

[1]See Rossi (2020) for a recent comprehensive review of the literature on the role of human capital on macroeconomic development.

[2]Although the theoretical models, such as Solow (1956), Cass (1965) and Koopmans (1965), where known for decades, the Summers-Heston data set, later coined the Penn World Table, had just been created in 1988.

[3]For a sample of 95 countries, the article estimates that the initial level of human capital, measured as school enrolment rates in 1960, presented a simple correlation of 0.43 with the accumulated per capita GDP growth in the subsequent period until 1985. Controlling for the initial level of GDP per capita in 1960, the partial correlation of school enrolment and posterior growth rises to 0.73.

of the reported correlation, the other two thirds coming from the reverse causality of growth on education and other factors, including demographics.

There are three aspects of Bils and Klenow (2000) that are relevant for my analysis: (i) the use of a Cobb-Douglas production function with labour-augmenting technical change; (ii) the linear human capital aggregation, which relies on the efficiency-units assumption and treats different levels of human capital as perfect substitutes; and (iii) a macro Mincer specification for the human capital accumulation. While the latter constitutes a richer way of measuring human capital for a cross-section of countries based on data on quantity of schooling and age of their cohorts, the former two are in contradiction with the labour market literature.

Several empirical studies documented a continued rise in the relative wages of skilled workers alongside an increase in their labour shares since the 1970s in the US. This stylised fact challenges the notion of neutral technical changes implied by (i), which would result in a ratio of marginal products inversely related to the ratio of quantities. In fact, a vast labour literature reports a skill-biased technical change (SBTC) occurring in the past decades, thus also refuting assumption (ii). Observing skilled workers to be imperfect substitutes for unskilled workers, this work documents their gains from technical innovations in the form of a increasing skill premium.

Those points are brought to the macroeconomic framework by Caselli and Coleman (2002, 2006). Compiling and advancing this work, Caselli (2017) applies a three-level nested constant elasticity of substitution (CES) production function and empirically shows that biased technical changes have been observed world-wide during the past decades. Concerning the unskilled and skilled human capital, he shows that skill-abundant countries tend to use skills more efficiently, i.e. they present a skill biased technical progress. Regarding specifically the imperfect substitutability issue, Jones (2014) proposes a human capital aggregation that embraces relative scarcities among different labour types. When incorporating such “generalized” aggregator in the traditional development accounting, the findings refute the result that physical and human capital stock variations account for a small fraction of income variation across countries.[4]

[4]The human capital aggregator used by Jones (2014) is later criticised by Caselli and Ciccone (2019) for omitting skill-specific technology terms. For applications of the CES considering the skill bias to development accounting, see Caselli and Ciccone (2013) and Jones (2019).

In this chapter, I propose four alternative human capital specifications to reproduce an initial result of Bils and Klenow (2000) consisting of the regressions of human capital and the technology term, together and separately, on schooling. The first specification replicate Bils and Klenow's (2000), the second mimics Jones's (2014), the third is a hybrid of Bils and Klenow's (2000) and Caselli (2017) while the fourth is analogous to Caselli's (2017) human capital aggregator for low and high-skilled labour. The dataset is constructed by employing similar methodology to the benchmark paper. Likewise, the estimations are run over the same period, i.e. I consider only the years of 1960 and 1990. For each year, aggregate or per capita measures are constructed as well as their respective accumulated growth in the 30-year period. Finally, regressions similar to those presented in Bils and Klenow's (2000) Table 2 are run and the results compared to the baseline.

The outline of the chapter is as follows. In Section 2, I present the theory and main assumptions behind the analysis. Section 3 introduces the data and 4 provides a practical guide for the construction of the dataset, calibration and estimation procedures. In 5, I show the results of the analysis and Section 6 concludes. Additionally, Appendix 1.A contains supplementary figures.

2 Conceptual framework

Output is produced assuming the usual constant-returns-to-scale (CRS) Cobb-Douglas with labour-augmenting technology:

$$y_t = {k_t}^{\alpha}(A_t h_t)^{1-\alpha} \tag{1.1}$$

where y is the output flow at time t, k is the stock of physical capital, A is the labour-augmenting technology, and h is the human capital stock. Both stock variables and the output are measured per worker.[5] The parameter α refers to the share of physical capital in the aggregate income and is assumed to equal 1/3.

The main focus of this chapter is to evaluate how different specifications of the component $A_t h_t$ affect the relevance of education on output. The mainstream literature

[5]Following the literature, the labour force is equivalent to the working-age population (WAP), which is required to make the human capital aggregation of age-group values consistent with the national accounting variables. In this way, the analysis abstracts from participation and unemployment issues.

on growth usually considers these two variables separately and h_t as a linear function of lower levels or individual human capital.

This is the case in Bils and Klenow (2000), where the human capital aggregation represents a simple summation of the individual human capitals, which is equivalent to imposing perfect substitutability under the efficiency units assumption.

$$h_t = \sum_a h_{t,a} \times \frac{L_{t,a}}{L_t} \tag{BK}$$

where $h_{t,a}$ is the average human capital of workers in cohort a at time t and $L_{t,a}$ stands for its respective number of workers.[6] Thus, the units here are taken at the cohort level instead of at the individual level.

The adoption of the CES function instead usually considers the technology parameters independently for each level of human capital. Following the SBTC literature and Caselli (2017), the full-fledged CES human capital aggregation reads (time subscripts omitted for clarity):

$$Ah = \left[(A_U h_U)^{\frac{\sigma-1}{\sigma}} + (A_S h_S)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{CES}$$

The quantities h_U and h_S stand for the aggregate supply of unskilled (or low-skill) and skilled (or high-skill) labour, respectively, measured in efficiency units. The parameters A_U and A_S are labour-augmenting technologies specific to each type of human capital.[7] In other words, A_U and A_S are aggregate measures of efficiency or productivity of each category of worker (unskilled/skilled) and relate to many economic factors, including the technological blueprint and the quality of education.

The parameter σ represents the elasticity of substitution between unskilled and skilled labour. In the skill-biased technical change (SBTC) literature surveyed by Acemoglu and Autor (2011), this parameter is estimated to lie between 1 and 2, suggesting that unskilled labour is a gross substitute for skilled labour. Most authors report 1.4 as the most accepted value, after the estimations of Ciccone and Peri (2005), grouping skilled workers as those with completed secondary education or more. This standard value is used throughout the main calculations of this chapter while a range

[6] Note that $\sum_a L_{t,a} = L_t$ represents the labour force.

[7] More broadly, the CES would also include a share parameter which would specifically graduate the representativity of each labour input in the economy. More recently, however, the literature has been dropping this parameter and accepting A_U and A_S to bear both quantitative and qualitative traits.

of values is used to test the sensitivity of the results to this parameter in Section 5.2.

Taking on Bils and Klenow's (2000) approach, the individual human capital is assumed to depend not only on years of schooling but also on the potential years of experience. Therefore, the expression for individual human in this chapter follows the canonical Mincer equation (also called "human capital wage function"):[8]

$$\ln(h_{t,a}) = \theta s_{t,a} + \gamma_1[a - s_{t,a} - 6] + \gamma_2[a - s_{t,a} - 6]^2 \tag{1.2}$$

Here a is also used as a variable, representing the average age of each cohort at time t and $s_{t,a}$ represents their respective average years of schooling. This specification incorporates the micro data on private returns to schooling θ and experience γ_1 and γ_2, respectively.[9]

A final theoretical element is required for the estimation of the technology parameter: the standard equilibrium assumption that inputs are paid their marginal products. Letting $i = U, S$ represents the different skill levels of human capital, the equilibrium wages read:

$$W_i = \frac{\partial y}{\partial h_i} = \frac{\partial y}{\partial h}\frac{\partial h}{\partial h_i} \tag{1.3}$$

In relative terms, this solution implies that the marginal rate of technical substitution equals the wage premium in equilibrium:

$$\frac{W_S}{W_U} = \frac{\partial y/\partial h_S}{\partial y/\partial h_U} \tag{1.4}$$

3 Data

Cross-country information from three different sources and parameter values estimated by related literature are collected. The former is presented in this section while the latter is introduced with the estimation procedure in the next section. The characters in parentheses after each variable described below corresponds to the name used for them in the formulae of this chapter.

[8]This specification is built on the competitive markets hypothesis which grants that wages reflect the returns on human capital increments.

[9]For simplicity, the part of Bils and Klenow's (2000) specification which refers to past cohorts' human capital and the potential issue of diminishing returns in years of schooling is excluded. This omission does not compromise the comparison of the results to Bils and Klenow's (2000), since the authors also report estimations for specifications without these features.

Data on school enrolment comes from Lee and Lee (2016), hereafter Lee-Lee, which covers 111 countries in five-year intervals, from 1820 to 2010. The version 1.0 used in this chapter comes from the most recent update available. The sample selected is restricted to the male population to comply with the Mincerian returns used. It is also the selection used in Bils and Klenow (2000). The variables selected are: Primary (pri); Secondary (sec); and Tertiary (ter) adjusted enrolment ratio in percentage.

The bulk of the data on schooling comes from Barro and Lee (2001), hereafter Barro-Lee, presenting educational data for 146 countries, ranging from 1950 to 2010. The version used is the 2.1. Once more, only the male population subsample was selected. This dataset is the main one used to construct the individual (cohort) human capital since it contains schooling information for each five-year age-group a for the population ranging from 15 to over 75 years old as well as its headcount $L_{a,t}$. The age range selected is a five-year trimmed working-age population (WAP) to make the results comparable to Bils and Klenow (2000), i.e. people from 20 to 60 years old. The description and notation of the variables used regarding school attainment are: Percentage of secondary education (ls); Percentage of secondary complete (lsc); Percentage of tertiary education (lh); Years of Primary Schooling (yr_sch_pri); Years of Secondary Schooling (yr_sch_sec); Years of Tertiary Schooling (yr_sch_ter).

Additionally, information on the theoretical duration of educational levels was retrieved from the World Bank Indicators platform (The World Bank, 2017). The data are provided by the United Nations Educational, Scientific, and Cultural Organization (UNESCO) Institute for Statistics is available yearly for 247 countries since 1970. I use the variable for the number of grades (years) in secondary education $(Duration)$.

Finally, the national accounting variables are collected from version (9.0) of the Penn World Table (PWT) (Feenstra, Inklaar, and Timmer, 2015). The dataset contains the main national accounting information yearly for a range of 182 countries, covering the period of 1950 to 2014. The variables and their respective notation in this chapter are (i) Real GDP at constant 2011 national prices and (ii) Capital stock at constant 2011 national prices.

4 Quantitative analysis

4.1 Constructing the dataset

Individual human capital

I construct the unskilled $h_{U,a}$ and skilled $h_{S,a}$ stocks of human capital at the cohort level via the canonical log-linear Mincerian Equation (1.2), repeated for clarity:

$$\ln(h_{t,a}) = \theta s_{t,a} + \gamma_1[a - s_{t,a} - 6] + \gamma_2[a - s_{t,a} - 6]^2 \tag{1.2}$$

The chosen cut-off for a skilled worker is 'complete secondary education or more' to allow this study to be readily compared with previous literature. Each category comprises of the following levels of education:

- unskilled group: no education, complete primary, some secondary education
- skilled group: complete secondary, some or complete tertiary education

The measures of years of schooling for unskilled and skilled labour, s_U and s_S, are constructed separately using the variables from Barro and Lee (2001) described in Section 3 for each age-group a and time t (omitted for clarity).

$$s_U = yr\_sch\_pri + YR\_sch\_sec\_INC \tag{1.5}$$

$$s_S = yr\_sch\_pri + yr\_sch\_sec + yr\_sch\_ter \tag{1.6}$$

The computation of the average years of schooling of the skilled worker is given simply by the total years of schooling of each cohort. However, in the calculation of the years of schooling of unskilled worker, it is necessary to include those with incomplete secondary education as well as primary education.

Variable $YR\_sch\_sec\_INC$ represent incomplete years of secondary education. It is constructed by combining UNESCO's duration of the secondary education with Barro-Lee's data on years of schooling and shares of population per level of educational attainment, as follows:

$$YR\_sch\_sec\_INC = yr\_sch\_sec - Duration * (lsc + lh)/100 \tag{1.7}$$

While the first term (yr_sch_sec) conveys directly the average *years* of schooling of those with some but not necessarily completed secondary education, variables lsc

and lh represent the *percentage* of people with completed secondary and completed tertiary education, respectively. To translate the latter into years of schooling, I multiply the sum of these shares by the average duration of secondary education given by the variable $Duration$.[10]

The coefficients of the Mincerian equations θ, γ_1 and γ_2 are the cross-country averages as reported and used by Bils and Klenow (2000). The values equal 9.9 per cent for the return to schooling, and 0.0512 and -0.00071 per cent for the experience-earnings profile level and squared, respectively.

The use of the standard Mincer equation which includes potential experience and as well as adopting a unique value for the Mincerian returns for every country is demonstrated by Growiec and Groth (2015) to be the best functional form to construct aggregate measures of human capital based on individual/cohort data on schooling and age. In the context of this study, it could also be justified by the fact that it is not desirable to introduce price differences across countries in the measures of skill supply since it is directly incorporated in the construction of the skill premium. Nevertheless, having access to a larger dataset of Mincerian coefficients could potentially allow for the incorporation of varying measures over time and across levels of education in the future.

Human capital by skill level

Each category of cohort's human capital is aggregated assuming perfect substitution within each group at the cohort level. In that sense, the stock of skilled human capital h_S is defined as a weighted average of that human capital at the cohort level, where the weights are the percentage of people in each cohort by skill level. And analogous for the stock of unskilled human capital h_U. These variables then read:

$$h_U = \sum_a h_{U,a} \times \frac{L_{U,a}}{L} = \frac{1}{L} \sum_a h_{U,a} \times (1 - \tau_a) \times L_a \tag{1.8}$$

$$h_S = \sum_a h_{S,a} \times \frac{L_{S,a}}{L} = \frac{1}{L} \sum_a h_{S,a} \times (\tau_a) \times L_a \tag{1.9}$$

[10]Ideally, each cohort would be combined with the information on duration referent to the year they would approximately have completed the secondary education. But this is not possible for the cohort 1960 since data for this year was not available; data referring to 1970 was used instead. To keep things simple and analogue, cohorts of 1990 were combined with the duration of the corresponding year. The expectation is that most of the increase in the span of this level of education over the past century is being captured in this way.

Variables $h_{U,a}$ and $h_{S,a}$ are constructed using Equation (1.2) while $L_{U,a}$ and $L_{S,a}$ represent the headcount of people in each category of skills. These variables are constructed indirectly, via the share of people in each age group with completed secondary education or more, which is given by τ_a defined as follows:

$$\tau_a = lsc_a + lh_a \tag{1.10}$$

Variables lsc_a and lh_a represent, respectively, the percentages of people with complete secondary and some or complete tertiary education, as provided by Barro-Lee. Defining $L_U = \sum_a L_{U,a} = \sum_a (1 - \tau_a) \times L_a$ and $L_S = \sum_a (\tau_a) \times L_a$, the share of skilled workers across the eight cohorts reads:[11]

$$\tau = \frac{L_S}{L} = \frac{(\sum_a \tau_a \times L_a)}{L} \tag{1.11}$$

Human Capital Aggregators

This section presents the different human capital aggregators evaluated in this study. The first group of specifications considers the technology factor and the human capital function separately, as the Bils and Klenow's (2000) human capital presented in equation (BK). The equation I use for the estimations, however, is slightly modified to allow for an unambiguous comparison with the other specifications, while still adhering to the perfect substitutability hypothesis. Instead of summing all cohorts in each time and country, I consider the summation of the unskilled and skilled human capital, as calculated previously. I label this equation (BK*):

$$h = h_U + h_S \tag{BK*}$$

The alternative Bils and Klenow's (2000) human capital aggregator can be directly compared to Benjamin Jones's (2014), which happens to be a CES specification without technology or share terms. I label this specification (BJ):

$$h = \left[h_U{}^{\frac{\sigma-1}{\sigma}} + h_S{}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{BJ}$$

[11] Note that $L = L_U + L_S$ technically represents the whole WAP, although in the calculations L stands for the five-year trimmed working-age male population, i.e. men aged between 20 and 60 years old.

The second group of specifications considers skill-specific technology terms within the aggregations. For that reason, just like in Caselli and Coleman (2006), the specifications below replace both the human capital h and the labour-augmenting technology term A in the Cobb-Douglas output function presented in Equation (1.1). The first aggregator of this group, labelled (BK-A), is a linear specification given by:

$$Ah = A_U h_U + A_S h_S \tag{BK-A}$$

Specification (BK-A) is contrasted with the standard (CES) specification, reproduced here for easiness of comparison:

$$Ah = \left[(A_U h_U)^{\frac{\sigma-1}{\sigma}} + (A_S h_S)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{CES}$$

Technology terms

For the first group of specifications, (BK*) and (BJ), the technology parameter is the standard residual of the output function, given by Equation (1.12) below. It is fully recovered from the values for output y and physical capital k and the constructed value for h.

$$A = y^{1/(1-\alpha)} k^{-\alpha/(1-\alpha)} h^{-1} \tag{1.12}$$

For the second group, the inference of the technology parameters is less straightforward. Following the literature, especially Caselli (2017), the first step is to isolate one of the technology terms (usually A_U) and rewrite equations (BK-A) and (CES) as:

$$Ah = A_U \left(h_U + \frac{A_S}{A_U} h_S \right) \tag{BK-A*}$$

$$Ah = A_U \left[h_U^{\frac{\sigma-1}{\sigma}} + \left(\frac{A_S}{A_U} h_S \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{CES*}$$

The values of the skill bias[12] A_S/A_U are inferred assuming labour are paid its marginal product, as derived in Equation (1.4). This assumption provides a relationship between skill premium and skill bias. In the case of specification (BK-A), this relation-

[12] The term reflects the fact that increases in A_S/A_U represent skill-biased technical changes.

ship is uncomplicated and reads:

$$W_S/W_U = A_S/A_U \tag{WP BK-A*}$$

For specification (CES), the relationship depends on the relative supply of skills as developed by the SBTC literature. It reads:

$$\frac{W_S}{W_U} = \left(\frac{A_S}{A_U}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{h_S}{h_U}\right)^{\frac{-1}{\sigma}} \tag{WP CES*}$$

The cross-country series for the skill premium in each years is then computed using the constructed relative wages, or the respective wage premium W_S/W_U (as described in the next section) and the relative supplies of skills (given by the ratio of the human capital stocks of skilled and skilled labour constructed as described in the previous section).

Finally, I solve Equations (BK-A*) and (CES*) for A_U separately to find the respective values of each specification. The value for Ah comes straight from the data, via Equation (1.1), and is invariant across the human capital aggregators.

Skill premia

To construct the skill premium for each country c in each period, I first reverse-engineer the Mincer equation to compute their respective wage premia. The aggregate measures of wages for unskilled and skilled labour are inferred based on country-specific Mincerian estimates θ_c and their respective average years of schooling per educational level, $\bar{s}_{U,c}$ and $\bar{s}_{S,c}$.

$$\ln(W_{U,c}) = \theta_c \bar{s}_{U,c} \tag{1.13}$$

$$\ln(W_{S,c}) = \theta_c \bar{s}_{S,c} \tag{1.14}$$

The measures of schooling are calculated from Barro-Lee dataset as the average years of schooling of the male working-age population (aged from 15 to 64). The Mincerian coefficient θ comes from Montenegro and Patrinos (2014), which present the most recent set of comparable estimates for a sample of 139 countries.[13] The variables referring to potential years of experience are omitted following the literature, in

[13]The estimates for the Mincerian returns used in this study were kindly sent by the authors upon request.

particular Caselli (2017). Since the goal is to calculate relative wages or skill premium, the variables related to age would cancel out in the cross-country ratios.[14]

Ideally, one would want to use different coefficients of returns to education per level and for each year since that is how the data on schooling is available. Unfortunately, prior to 1970, the estimates for the returns on education are restricted to a handful of countries. Even for 1990, the sample of countries is quite limited. Moreover, several countries of international relevance do not present estimates for all the levels of education.[15] The present specification implies that the wages of the average skilled worker are higher than the average skilled worker in each country-year strictly due to the additional years of schooling that the more skilled worker possesses.

Enrolment

Since the main exercise of this study is to evaluate how the choice of a linear human capital specification affected the results presented in Bils and Klenow (2000), it suffices to use as the sole regressor of the estimations. The variable of choice is their constructed measure of enrolment, based on shares of enrolled children in school-age.

Their measure of schooling —henceforth BK's measure of schooling— combines the enrolment rates in each of the three main levels of education in the initial period of analysis, i.e. 1960, in each country (variables as described in Section 3):

$$bkschool = 6 * pri/100 + 6 * sec/100 + 5 * ter/100 \tag{1.15}$$

According to the authors, this measure aims at representing the "steady-state average years of schooling" implied by the enrolment rates. They consider the World Bank conventions for the durations of primary, secondary, and tertiary education.

[14]Moreover, the coefficient on level of experience would have its sign reversed. To see these changes, suppose I included the omitted variables in Equations (1.13) and (1.14). The skill premium would read:

$$\frac{W_{S,c}}{W_{U,c}} = \frac{\exp(\theta_c \bar{s}_{S,c} + \gamma_{1,c}[\overline{age} - \bar{s}_{S,c} - 6] + \gamma_{2,c}[\overline{age} - \bar{s}_{S,c} - 6]^2)}{\exp(\theta_c \bar{s}_{U,c} + \gamma_{1,c}[\overline{age} - \bar{s}_{U,c} - 6] + \gamma_{2,c}[\overline{age} - \bar{s}_{U,c} - 6]^2)} = \frac{\exp(\theta_c \bar{s}_{S,c} - \gamma_{1,c}\bar{s}_{S,c} + \gamma_{2,c}[\bar{s}_{S,c}]^2)}{\exp(\theta_c \bar{s}_{U,c} - \gamma_{1,c}\bar{s}_{U,c} + \gamma_{2,c}[\bar{s}_{U,c}]^2)}$$

This alternative specification would have the same two last terms as an analogous calculation for $W_{S,c}$.

[15]I refer here to Montenegro and Patrinos (2014). These missing countries include, among others: Canada; China; Denmark; Egypt; Finland; Vietnam; Germany; Hong Kong; Iceland; Iran; Ireland; Israel; Japan; Kazakhstan; Korea; Luxembourg; Monaco; Netherlands; New Zealand; Romania; Russia; Sweden, Switzerland; and United Kingdom.

4.2 Descriptive analysis

I construct the variables per country for years 1960 and 1990 using the formulae and calibration presented previously. Although each original dataset is relatively large, the intersection of only 74 countries determines the sample size of the dataset. The regressands of the estimations, as in Bils and Klenow (2000), are in terms of the average annual growth rates over the period studied.[16]

Table 1.1 presents the descriptive statistics of the variables of interest in this study. The first set, comprising of real GDP per worker, physical capital and the combined human capital and technology,[17] is common to all specifications. The estimated growth of the latter, at 1.68 per cent, is very close to the one reported by Bils and Klenow (2000, p. 1169), which indicates that the empirical strategy is valid in supporting the proposed comparisons with their estimations.[18]

The next set of variables concerns the measures of schooling. As expected, there is an increase in the participation of skilled workers in the working-age population of 18.2 percentage points (p.p.) on average across countries. This reflects both people studying for longer and the increase in the duration of some levels of education in some countries of the sample over the period.

Regarding the constructed labour market variables, the sample presents on average a small annual decrease of the unskilled aggregate human capital, while the skilled aggregate increases on average 5.9 per cent per year, pushing up the growth of the relative skill supplies h_S/h_U. Meanwhile, the average of the estimated relative wages of the sample increased by 0.45 per cent per year in the period. This calculation is in line with Katz and Murphy (1992), who show that the US wages increased moderately together with higher relative skill supplies between 1963 and 1987.

The set of variables related to the pair of human capital specifications with skill-neutral technology shows striking results. The average aggregate human capital growth varies from 0.61% per year in the modified Bils and Klenow's (2000) specification (BK*) to 1.78% in specification BJ. Logically, the reverse is seen in the estimated growth of the human-capital augmenting technology. The estimations using the original Bils

[16]For a variable x, the average annual growth rate is given by: $(x_{1990}/x_{1960})^{1/30} \times 100 - 100$.
[17]Given the Cobb-Douglas formulation for the output function, $g_h + g_A = \frac{1}{1-\alpha}(g_y - \alpha g_k)$.
[18]This is especially favourable given that my sample surpasses theirs, which has 52 countries.

and Klenow's (2000) specification (BK) is presented here to show its similarity with the modified version.[19]

Table 1.1: Descriptive statistics

Variable	Mean	Std. Dev.	Min.	Max.	N
GDP per worker growth g_y (%)	1.897	1.51	-1.805	5.72	74
Capital per worker growth g_k (%)	2.329	2.122	-2.439	10.888	74
Technology & human capital $g_A + g_h$ (%)	1.681	1.567	-2.174	5.213	74
BK's schooling in 1960 (years)	5.793	2.682	0.403	12.598	74
Change in share of skilled (p.p.)	18.241	11.439	1.539	54.503	74
Change in y.o.s. of unskilled (years)	1.479	0.96	-1.081	3.824	74
Change in y.o.s. of skilled (years)	2.834	1.096	-0.204	5.794	74
Unskilled human capital growth (%)	-0.431	0.833	-3.155	0.528	74
Skilled human capital growth (%)	5.868	2.293	0.577	12.781	74
Relative supplies growth (%)	6.328	2.228	0.951	12.456	74
Skill premium growth (%)	0.451	0.309	0.034	1.559	74
BK human capital growth g_h (%)	0.777	0.326	-0.142	1.83	74
BK* human capital growth g_h (%)	0.611	0.302	-0.268	1.524	74
BJ human capital growth g_h (%)	1.777	0.699	0.09	3.628	74
BK technology growth g_A (%)	0.904	1.513	-2.767	4.738	74
BK* technology growth g_A (%)	1.071	1.531	-2.599	4.881	74
BJ technology growth g_A (%)	-0.083	1.706	-3.495	3.508	74
BK-A human capital growth g_h (%)	0.874	0.544	-0.252	2.984	74
CES human capital growth g_h (%)	4.283	3.869	0.271	18.337	74
BK-A unskilled tech. growth g_{A_U} (%)	0.808	1.492	-2.692	4.847	74
CES unskilled tech. growth g_{A_U} (%)	-2.379	3.327	-13.421	3.964	74
BK-A skilled tech. growth g_{A_S} (%)	1.263	1.558	-2.389	5.103	74
CES skilled tech. growth g_{A_S} (%)	15.662	6.564	4.014	34.347	74
CES skill bias growth (%)	18.541	6.586	3.472	39.004	74

Note: percentage growth represents the average annual growth rate between 1960 and 1990.

In the second group of specifications, the skill bias A_S/A_U presents a cross-country average growth of 18.5% when calculated using (WP CES*). This value is almost 50 times larger than the modest growth of average skill premia[20], pushing upwards the human capital growth measured by (CES) to 4.3% versus 0.87% measured by (BK-A).

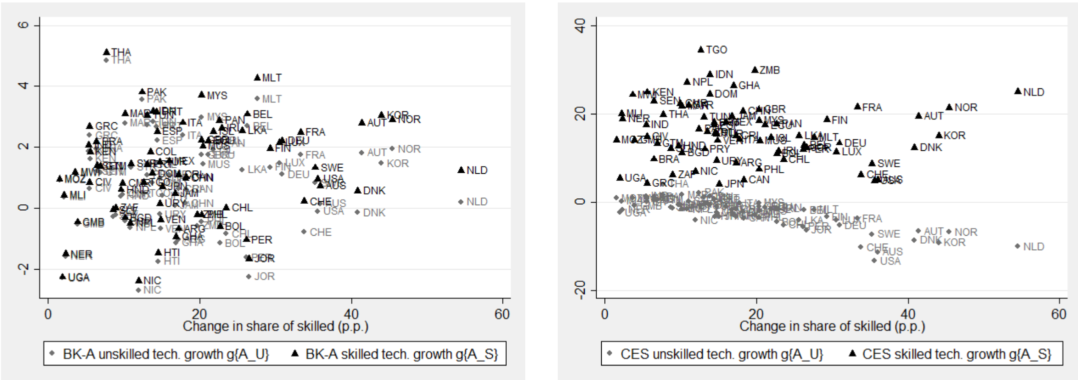
Figure 1.1 plots the growth of the estimated technology terms against the changes in the share of skilled population (τ) over the period. With specification (BK-A), there is no clear distinction between the two coefficients whereas there is a sharp diver-

[19]Notice that in Bils and Klenow (2000) all workers are taken as potentially fully educated whereas Equation (BK*) separates workers according to the shares of educational attainment.

[20]Recall that with specification (BK-A), $A_S/A_U = W_S/W_U$.

gence under the (CES) specification. The growth of the unskilled-specific technology clearly correlates negatively with increases in the share of skilled population while the growth of the skilled-specific technology is rather more heterogeneous and disperse.

The results are in line with the literature reporting that the skill bias is positively correlated with income per worker. Caselli (2017) finds that countries abundant in skilled worker have A_S proportionally higher than A_U, meaning that they tend to use skilled labour relatively more efficiently than unskilled labour.



(a) BK-A (linear) specification

(b) CES specification

Figure 1.1: Technology factors growth (%) over skilled-labour shares changes (p.p.)

5 Results

5.1 Regressions

The analysis concludes with the regressions of the different compositions of technology and human capital growth on BK's measure of schooling. For a generic variable X the estimations comprise of linear regressions of the annual rate of growth g_X over the 30-year period on BK's measure of schooling —given by Equation (1.15)— which is expressed in years.

$$g_X = \beta_0 + \beta_1 bkschool + \epsilon \tag{1.16}$$

The interpretation of the coefficients β_1 that are reported below for each regression is then uncomplicated. They state how much higher, in percentage points (p.p.), the annual growth rate of variable X would have been if the BK's measure of schooling were one year larger for the countries on the sample on average.

Neither I nor Bils and Klenow (2000) argues that this simplistic form is meaningful in explaining the role of schooling on human capital or technology growth. However, these estimations reported in their Table 2, play a key role in the paper's research strategy while being concise enough for a straightforward replication and clear appraisal of Bils and Klenow's (2000) choice of human capital specification.

Specifications with skill-neutral technologies

The first column of Table 1.2 presents the estimates of the regression of the combined growth of human capital and technology, common to all specifications. The coefficient of 0.231 p.p. (standard deviation, s.d., 0.050) is remarkably close to Bils and Klenow's (2000) estimate of 0.238 p.p. (s.d. 0.054) which again validates the analysis proposed here. The following columns show the regressions of human capital and technology growth when applying the first group of specifications, BK* and BJ. The coefficient of the linear human capital aggregator BK* of 0.035 p.p. (s.d. 0.015) is reasonably close to Bils and Klenow's (2000) value of 0.048 p.p. (s.d. 0.009).[21]

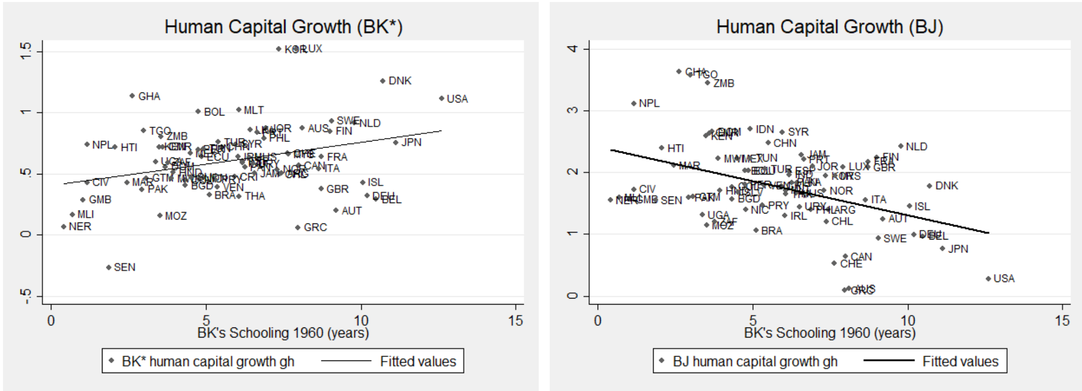
Table 1.2: Human Capital and Technology Growth regressed on Enrolment (BK*, BJ)

		BK*		BJ	
	$g_A + g_h$	g_h	g_A	g_h	g_A
BK's Schooling 1960	0.231***	0.0353*	0.196***	-0.111***	0.337***
	(0.0502)	(0.0146)	(0.0528)	(0.0310)	(0.0526)
Constant	0.341	0.406***	-0.0626	2.418***	-2.036***
	(0.358)	(0.0858)	(0.372)	(0.203)	(0.376)
Observations	74	74	74	74	74
Adjusted R^2	0.145	0.085	0.105	0.169	0.271

Standard errors in parentheses. * $p < 0.05$ ** $p < 0.01$ *** $p < 0.001$

The comparison of the results deriving from BK* and BJ specifications makes it clear that the choice of the human capital specification profoundly affects the results. Even though all the estimates are significant, they are actually significantly different, as shown by their respective standard deviations. Moreover, it seems that applying a (CES) specification without skill-specific technology parameters leads to counter-intuitive empirical results.

[21] Their adjusted R-squared values are also as low as those reported here, reinforcing the low explanatory power of these regression models. The constant, however, does not appear in any of Bils and Klenow's (2000) results.



(a) BK* specification (b) BJ specification

Figure 1.2: Human capital growth regressed on schooling

The imperfect substitutability alone would overestimate the impact of a decrease in any type of human capital, even if compensated by an increase in the other type. As Figure 1.3 shows, this results in a negative relationship between (BJ)'s human capital and schooling, driven mostly by industrialised countries which experience a large increase in educational achievement between 1960 and 1990. That is the case of the USA, where unskilled human capital fell 3.13% in the period while skilled human capital rose 3.0%. While the country's human capital growth of 1.11%,is among the highest with the linear aggregation (BK*) [Figure 1.3a], using Equation (BJ), the human capital growth of only 0.28% is the third smallest among all countries [Figure 1.2b].

In this setting, the residual technology acts as a mere buffer, compensating for the variability found in the human capital aggregation. Since the combined growth of technology and human capital is set, the residual total factor productivity does not play a role in translating Mincer-like quantities of human capital into their respective technological value. In summary, simply assuming imperfect substitutability without incorporating the possibility of skill-biased technical changes can be seen as a deficient approach.

Specifications with skill-biased technical terms

In the estimations of the second group of specifications, the terms between parentheses after A_U on the right-hand side of equations (BK-A*) and (CES*) is the aggregator used to calculate the human capital growth. The analogy with the standard labour-augmenting specification Ah is that A_U is the minimum efficiency that every worker has, while the skilled human capital h_S has its value adjusted by the skill bias within the human capital aggregator. Because it includes the skill-bias A_S/A_U, this aggregator may be seen as not directly comparable to the human capital growth computed using the first group of specifications, even though both this is also a form of "unskilled-adjusted" human capital. Nevertheless, this aggregator is positively a valid measure for comparisons between the pair of specifications (BK-A) and (CES).

The results presented in Table 1.3 show striking differences between the regressions. The slope of 0.115 estimated when considering the linear specification (BK-A) is nearly one order of magnitude smaller than the slope of 0.99 found when using specification (CES). Naturally, the estimates for the effect on the unskilled-specific technology growth are also remarkably different.

Table 1.3: Human Capital and Technology Growth on Enrolment (BK-A, CES)

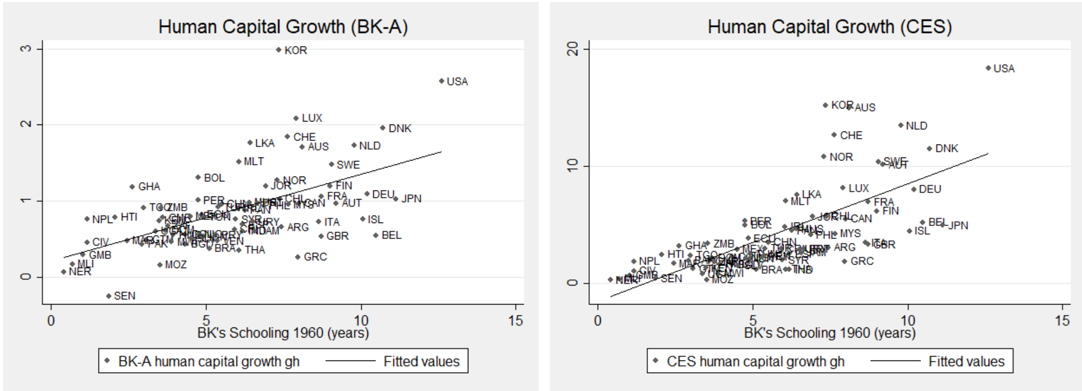
	BK-A		CES	
	g_h	g_{A_U}	g_h	g_{A_U}
BK's Schooling 1960	0.115***	0.116*	0.999***	-0.676***
	(0.0221)	(0.0567)	(0.150)	(0.141)
Constant	0.210	0.133	-1.505*	1.539*
	(0.111)	(0.387)	(0.689)	(0.727)
Observations	74	74	74	74
Adjusted R^2	0.310	0.031	0.473	0.287

Standard errors in parentheses. * $p < 0.05$ ** $p < 0.01$ *** $p < 0.001$

The human capital growth regressed in column 1 is affected by three factors: (i) unskilled human capital; (ii) skilled human capital; and (iii) skill premium. Naturally, the first decreases with education. Non-trivially, the second also correlates negatively with BK's measure of schooling reflecting an initial-condition bias where countries with relatively lower levels of enrolment in 1960 and therefore with very low initial levels of skilled human capital tend to present the highest rates of growth of this category. Skill premium, on the other hand, is positively correlated with BK's measure of schooling which is then the main reason for the slope resulting from specification (BK-A) to be higher than that found with (BK*).[22]

Figure 1.3 plots the annual human capital growth rate across countries computed

[22] Figure 1.A.5 in the Appendix presents the scatter plots and fit lines of the key variables and parameters comprising the regressions of column 1 and 2 of Table 1.3.



(a) BK-A specification

(b) CES specification

Figure 1.3: Human capital growth regressed on schooling

using aggregations (BK-A) and (CES). The values of the latter are much larger and yet they are less disperse than those of the former, i.e. they present smaller standard deviation in relation to the slope coefficient. Thus, the growth of the human capital which accounts for the skill-biased technical changes occurring in the period is strongly correlated with BK's measure of schooling while the corresponding growth of the technology factor A_U mirrors this relationship.[23]

The differences happen because, in the computation of the regressand of column 3, the skilled human capital is adjusted for the skill bias (A_S/A_U) given by Equation (WP CES*) and not only by the skill premium.[24] While, as in the (BK-A) specification, the unskilled human capital h_U falls, the growth of the second term in the aggregation $(h_S A_S/A_U)$ more than compensates it, especially when each term is treated as imperfect substitutes. Even though the calculated skill bias growth is scattered and not directly correlated to BK's measure of schooling on average, country by country the technical changes have boosted skilled labour particularly more where this type of worker became more abundant.[25]

5.2 Sensitivity analysis

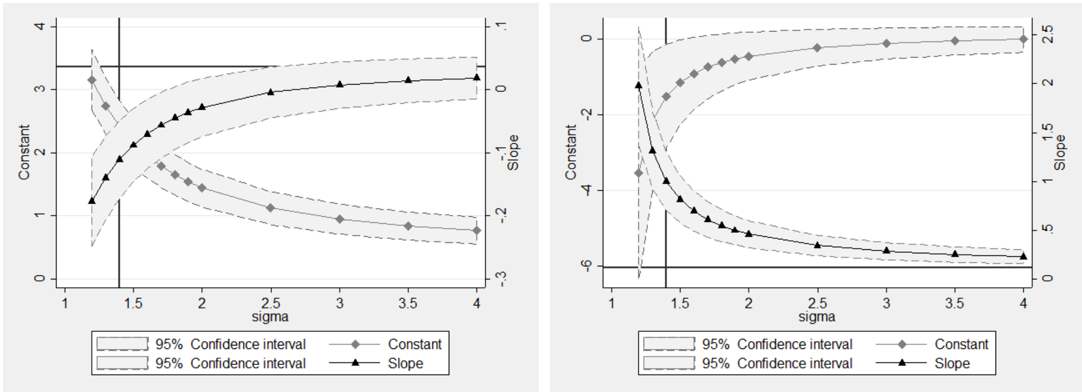
The calculated values for the skill premia across countries and over time are clearly dependent on the value of the elasticity of substitution between unskilled and skilled

[23] Perhaps it is worth pointing out that, for each specification, the sum of the slopes of the regressions of human capital growth g_h and technology growth, either A or A_U, always sum to 0.212 p.p., the slope of the regression of $g_A + g_H$ as presented in column 1 of Table 1.2.

[24] For sigma=1.4, the following simplification aids the interpretation: $A_S/A_U = (W_S/W_U)^3(h_S/h_U)^2$.

[25] Figure 1.A.6 in the Appendix presents the scatter plots and fit lines of the key variables and parameters comprising the regressions of column 3 and 4 of Table 1.3.

labour σ. Recalling that lower values reinforce imperfect substitutability ($\sigma = 0$ implying perfect complementarity), the sensitivity analysis must assess how the choice of 1.4 for this parameter affects the estimations as well as how fast the results converge to those obtained by the linear specification.



(a) BJ specification (b) CES specification

Note: horizontal reference lines refer to the values of the slope (right axis) of each respective linear specification. It is of 0.035 (BK*'s) in Figure 1.4a and of 0.115 (CES's) in Figure 1.4b.

Figure 1.4: Estimates of regressions BJ and CES with varying values of sigma

Figure 1.4a plots the coefficients of the regressions where the human capital growth is calculated using specification (BJ) for values of σ ranging between 1.2 and 4; the vertical reference line is at the benchmark of 1.4 adopted in the analysis. It shows that the estimates for the constant (left axis) and for the slope (right axis) converge reasonably fast towards the values of 0.406 and 0.035, respectively, computed with the linear human capital specification (BK*).[26] For $\sigma = 2$ —arguably the upper bound in the SBTC literature— the slope estimate already reaches -0.03, or one quarter of the -0.11 found with the benchmark for the elasticity of substitution.[27]

A similar convergence pattern is seen in Figure 1.4a, which depicts the estimates for regressions for different values of σ using the human capital computed using specification (CES). The estimates for the constant (left axis) and for the slope (right axis) also converge reasonably fast towards the values of 0.21 and 0.115, respectively, computed with the linear human capital specification (BK-A).[28] For $\sigma = 2$, the slope estimate is quickly debated in more than half, to 0.46 from the value of 0.999 estimated

[26]The results of model BK* are reported in the second column of Table 1.2. Recall that σ tends to infinity in the linear human capital specification.

[27]See column 4 of Table 1.2.

[28]The results of model BK-A are reported in the first column of Table 1.3. Once more, $\sigma \to \infty$ corresponds to the linear human capital specification.

with the benchmark value of the elasticity parameter.[29]

6 Conclusions

This chapter shows that the choice of the human capital specification plays a considerable role in the study of the importance of education to economic growth. I demonstrate that altering the way human capital is perceived in one of the building blocks of Bils and Klenow's (2000) analysis produces critically different results.

Two of the assumptions behind their specifications are analysed separately. On one hand, the premise that unskilled and skilled labour are perfect substitutes is confronted by replacing the linear aggregator by a CES form following Jones's (2014), which only differ from Bils and Klenow's (2000) by treating workers with different levels of skills as imperfect substitutes of each other. On the other hand, I consider skill-specific technology factors following Caselli (2017) in order to incorporate the skill-biased technical changes observed in the studied period of 1960 to 1990.

In the comparison of a linear specification with a CES with a skill-neutral human-capital-augmenting technology term, I find that the relationship between human capital growth and schooling inverts sing, from positive to negative. The negative correlation found goes much beyond possible initial-value bias, since this issue is also present in the linear specification. The problem seems to be that the imperfect substitutability in the non-linear form is punishing more than proportionally decreases in one type of human capital—typically, the unskilled— even when accompanied by increases in the other category of labour. This is the case of the United States, Australia and Greece, for which the CES specification without skill-specific technology terms finds near-zero growth rates of human capital in the period.

In the case of human capital specifications with skill-specific technology terms, the estimations show striking differences. The CES specification produces a relationship between human capital growth and schooling almost ten times larger than the linear form. In this pair of specifications, the human capital aggregator includes the skill bias, i.e. the ratio of skilled over unskilled technology factor. By construction, however, while in the CES specification the term comprises of the skill premium and the

[29]See column 3 of Table 1.3 for the coefficients of the CES model using $\sigma = 1.4$.

relative supplies of skills, in the linear form the skill bias is given solely by the former. The fact that the relative skill supply —which derive from measures of education— boosts the skill bias helps to explain the strong relationship between the human capital growth produced by the CES and schooling. Arguably, when technology is considered as affecting skilled and unskilled workers differently, and these workers are imperfect substitutes of each other, the rewards for education are intensified at the macroeconomic level via the development of technologies that allows the more expensive factor to be used more efficiently.

There is an ongoing debate in the literature (Caselli and Ciccone, 2019; Jones, 2019) regarding the inclusion or not of skill-specific technology terms in the CES human capital aggregation applied to cross-country development analysis. Caselli and Ciccone (2019) argue that the drawback of the omission of these terms lies in the fact that it does not account for factors beyond workers' characteristics to affect the human capital. Jones (2019) disagrees, arguing that the computation of the different levels of human capital using the macro-Mincer specification already attributes to each category of worker its economic value.

This chapter adds to this discussion by contrasting both CES specifications and estimating their diverging relationships with schooling. Moreover, I contribute to growth theory in general by demonstrating that incorporating the conclusive findings of the empirical microeconomic literature is crucial for any subsequent cross-country analysis on the role of human capital on growth.

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1.A Supplementary figures

I present here additional figures depicting relationships between key constructed variables and education. Some repetition of plots serves the purpose of easiness of exposition and comparisons.

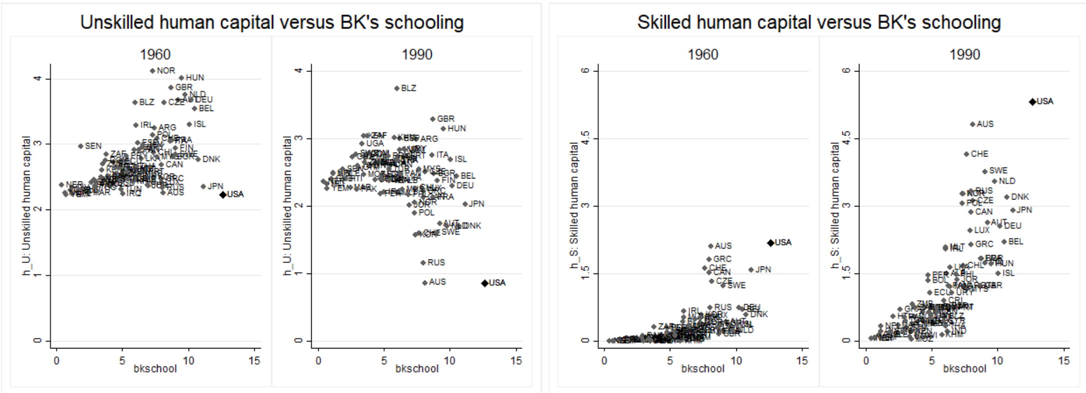


Figure 1.A.1: Unskilled and skilled human capital measures

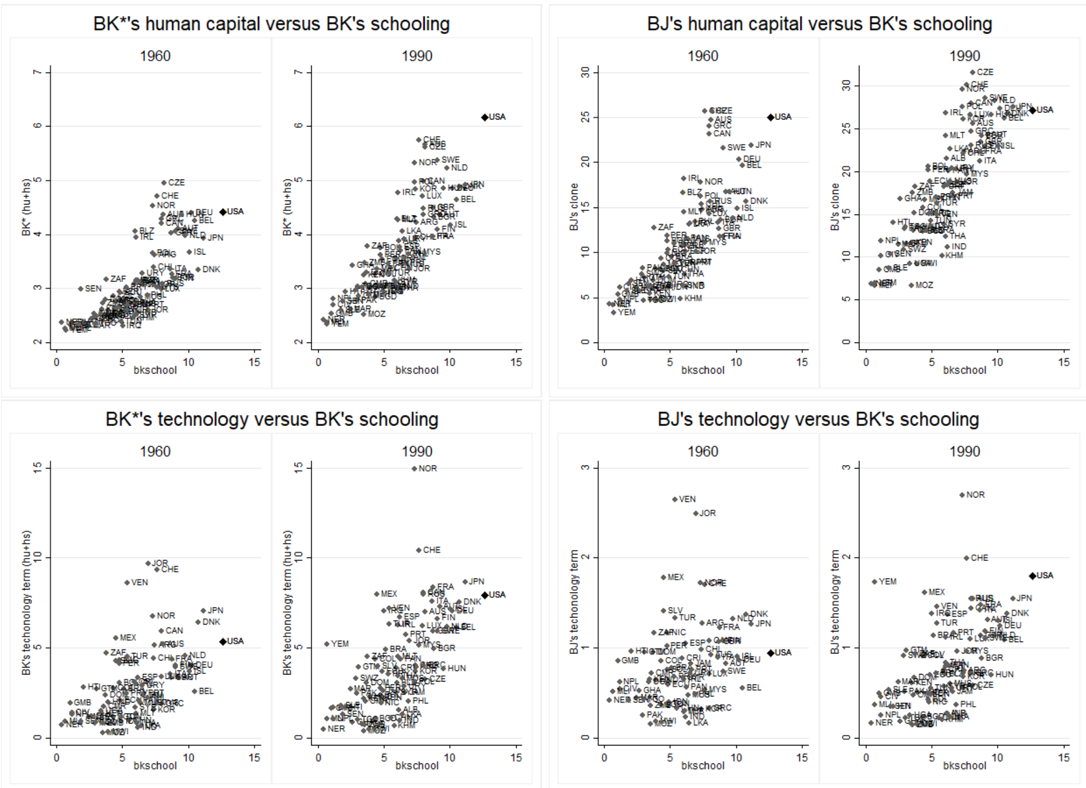


Figure 1.A.2: Comparison of BK*'s and BJ's human capital and technology measures

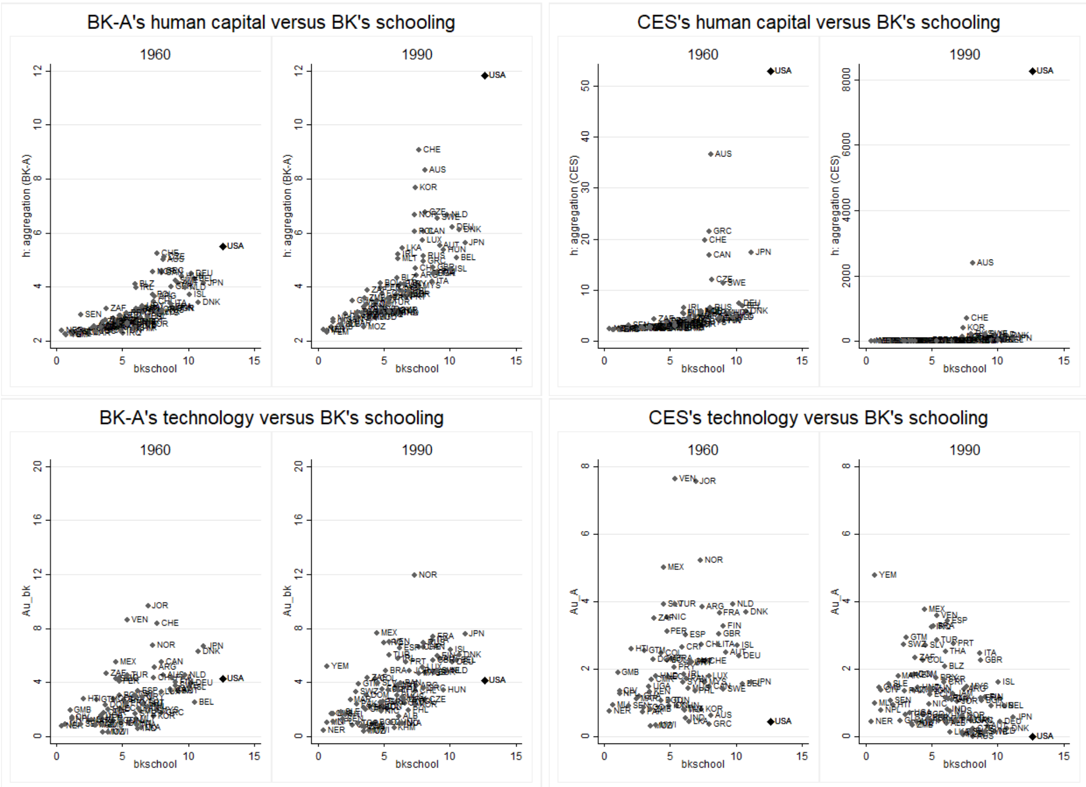


Figure 1.A.3: Comparison of BK-A's and CES's human capital and technology

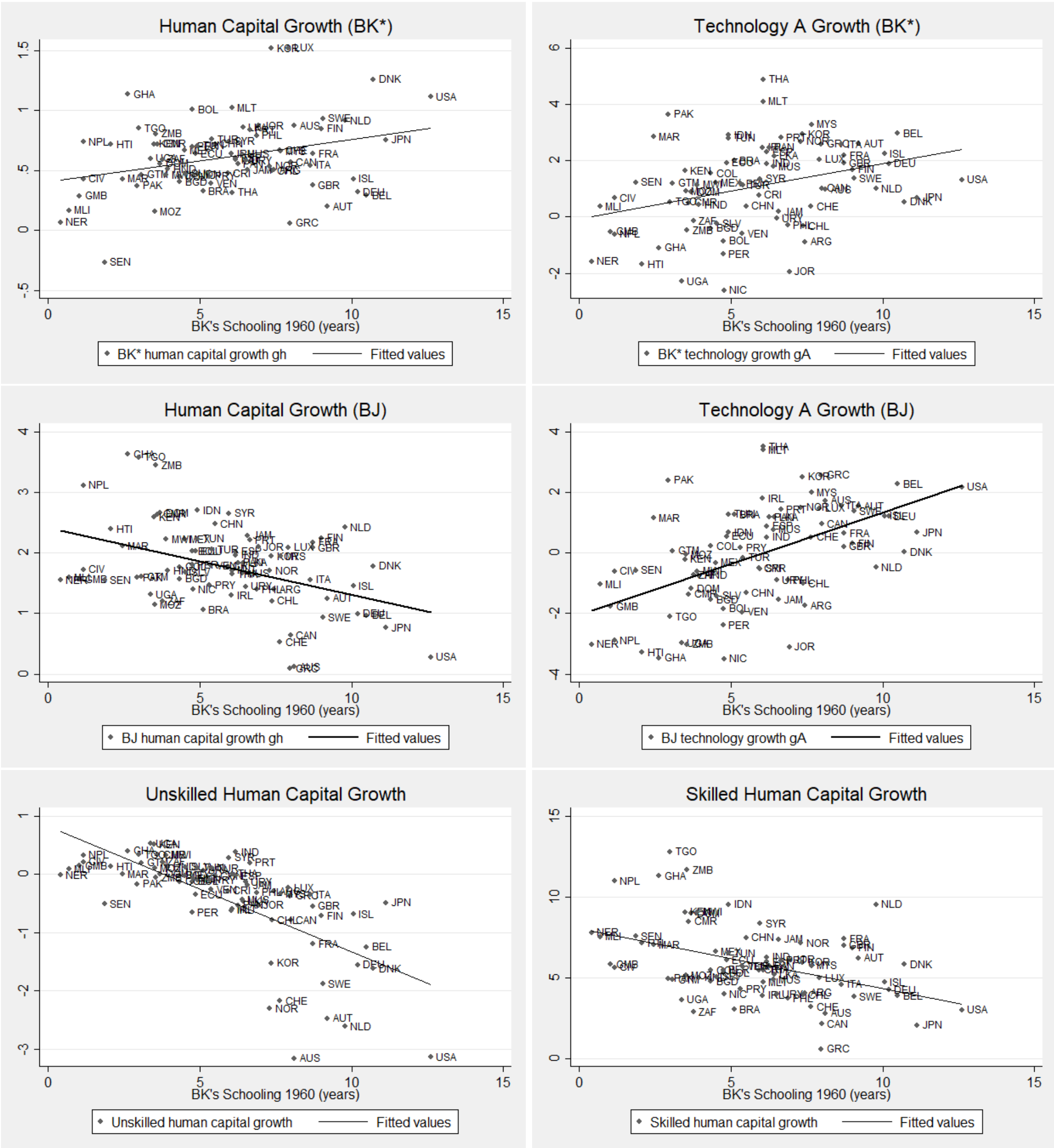


Figure 1.A.4: Specifications BK* and BJ: scatter plots of key variables and parameters on BK's schooling

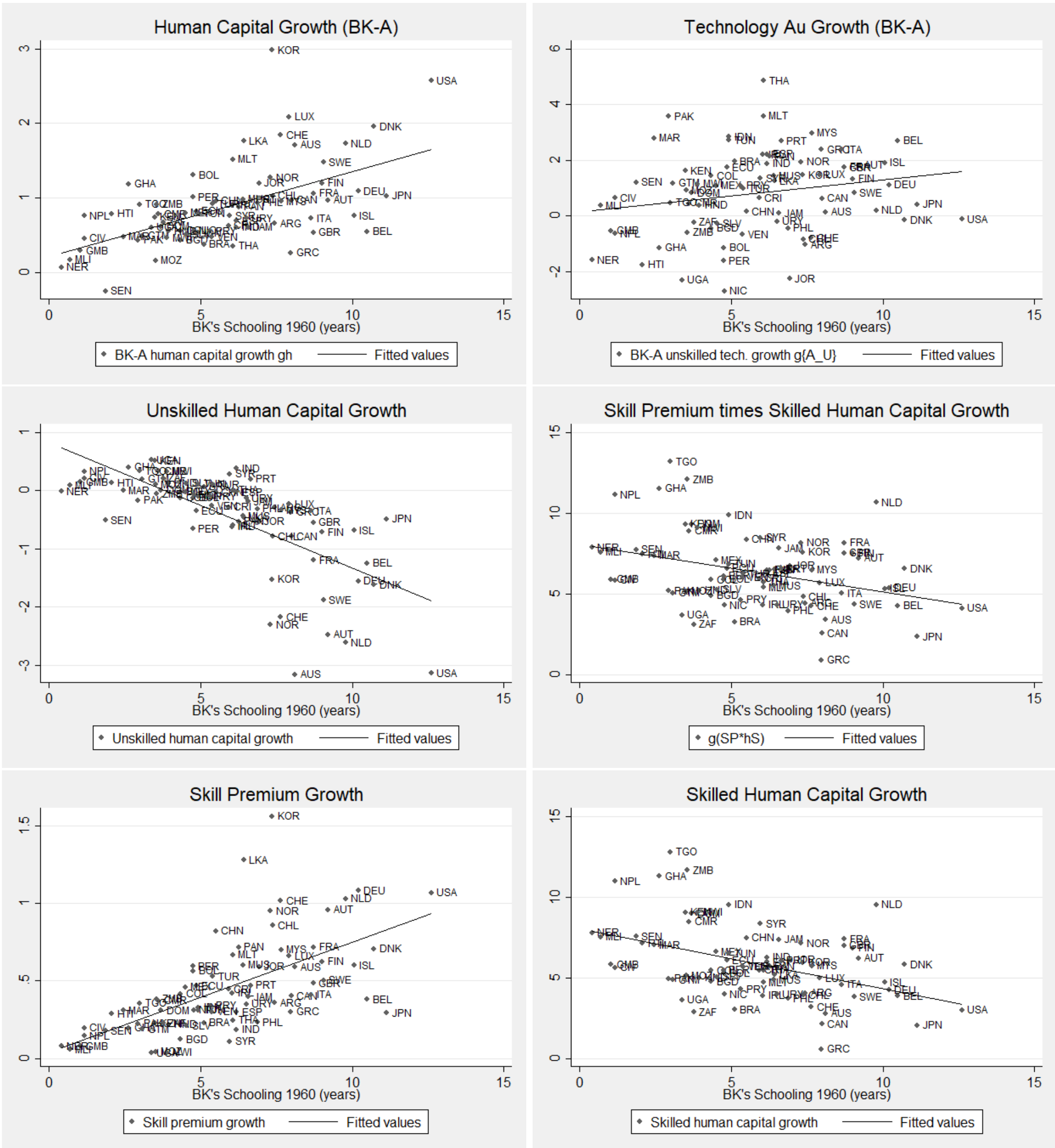


Figure 1.A.5: Specification BK-A: scatter plots of key variables and parameters on BK's schooling

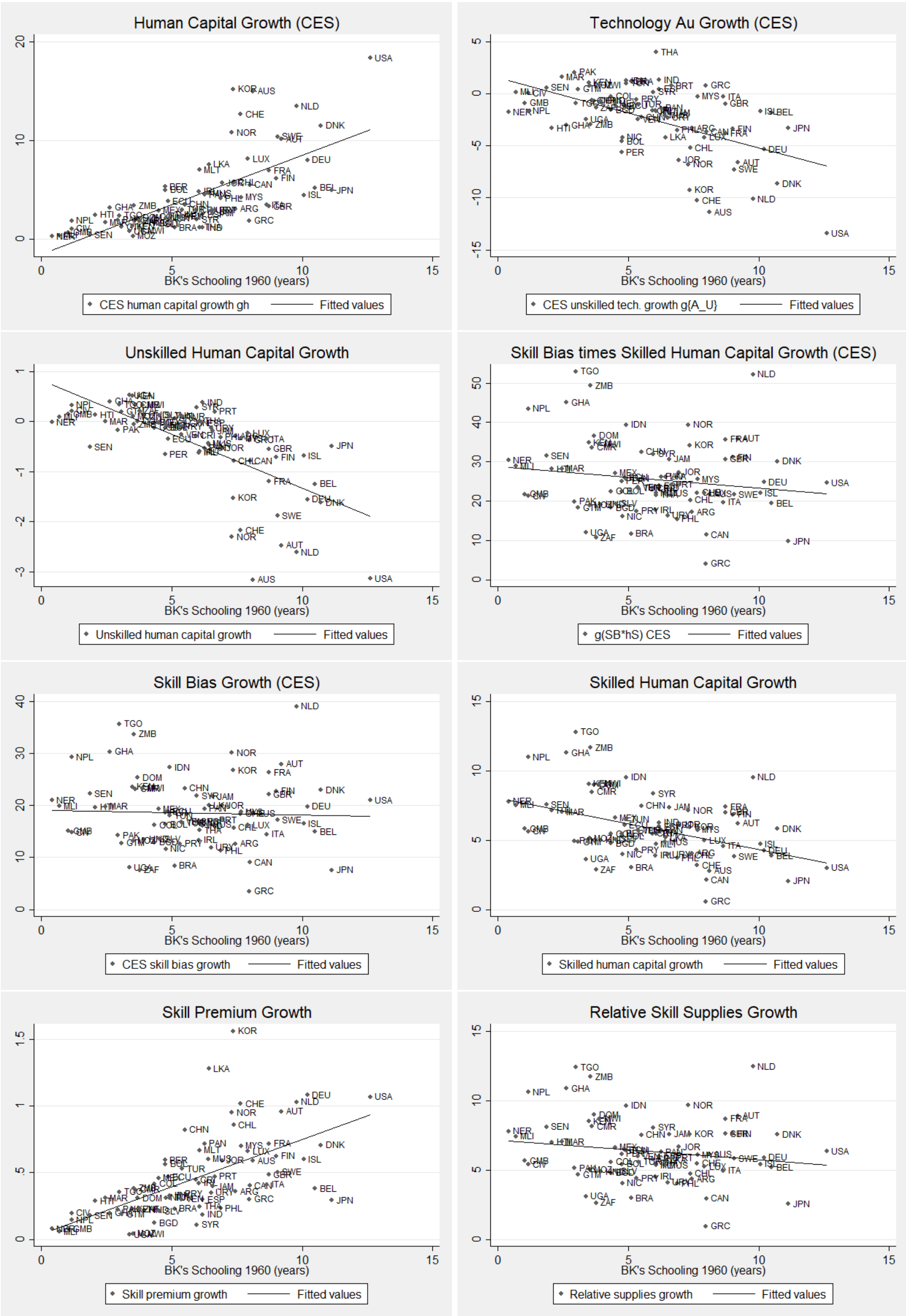


Figure 1.A.6: Specification CES: scatter plots of key variables and parameters on BK's schooling

Chapter 2

A8 skills in the UK: who benefits? Input-output linkages and the transmission of a skill supply shock across industries

1 Introduction

> *Modern economies involve very sophisticated input-output structures. Goods like electricity, financial services, transportation, information technology and healthcare are both inputs and outputs. [...] Despite our intuitive recognition of this point, standard models of macroeconomics and economic growth typically ignore intermediate goods.*
>
> —Charles I. Jones (2013, p. 9).

In the event of a shock in the supply of a factor, the 'conventional wisdom' —as Charles I. Jones puts it— would be content in stating that the group of industries more intense in its use would be the most affected. Some few could add that the shock would also be felt by other industries, to the extent that they employ that factor. Little explored, however, is how industries that make no use of it would be affected by the shock via intermediate inputs use.

This reasoning equally permeates the media, as seen in the discussion surrounding the withdrawal of the United Kingdom (UK) from the European Union (EU). Information technology (IT) and creative industries fear for the supply of highly skilled workers,[1] while services worry about the large percentages of workers becoming illegible to work in the UK[2] and low-skilled industries foresee a tougher prospect in finding readily available cheap labour.[3] Consistently, there is hardly any consideration about how these supposed supply shortages would affect industries which do not employ these skills.

Migration and investment in education are examples of shocks in the supply of labour or, more specifically, skills. Understanding how these shocks affect the economy in general and the various industries, in particular, is essential for the design of labour market policies. As industries directly affected by a skill supply shock sell their goods as intermediate goods to other industries, the effects of the shock are not limited to the former but include other industries connected through input transactions. The usual impression that the supply of a certain set of skills is of the sole interest of industries using these skills needs to be supplemented by the notion that industrial interconnections propagate labour supply shocks throughout the economy.

To investigate this premise empirically, I explore the large influx of immigrants to the UK coming from one of the so-called A8 accession countries from the EU 2004's enlargement (Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia and Slovenia). The goal is to understand how the A8 immigration affects the real output of UK industries via input-output linkages. To do so, I design counterfactuals removing the stock of low-skilled (3.15%) and high-skilled (2.22%) A8 workers from the universe of employed people in 2014.[4] The combined effect on GDP is of a drop of 1.81% compared to the observed data, 1.17 percentage points (p.p.) from the removal of low-skilled and 0.64 p.p. from the high-skilled A8 workers.

The cross-industry findings challenge the idea that a skill supply shock affects the industries proportionally to their use of that skill —as it would be the case in an economy without linkages. Figure 2.1 plots the low-skill and high-skill nominal shares

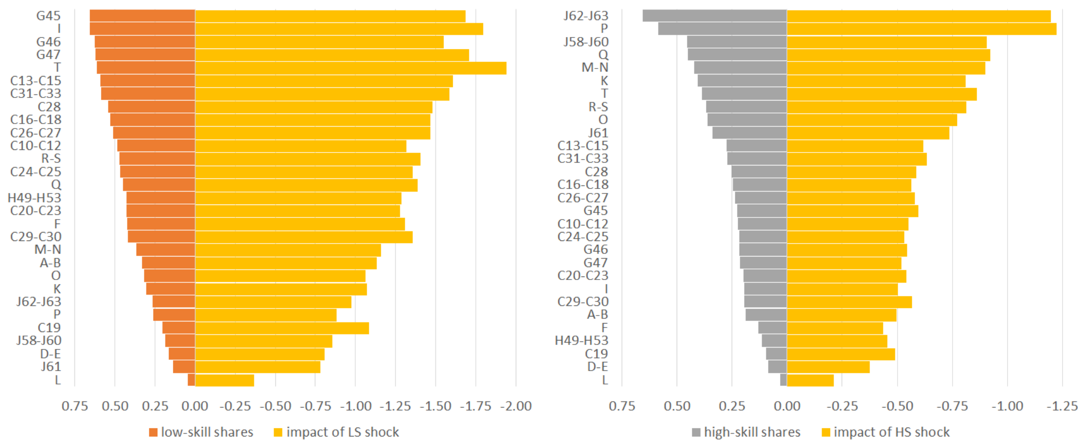
[1] Bethan Staton, "Migration controls threaten job shock to Bristol," *Financial Times*, 10 October 2019, https://www.ft.com/content/9ce7802c-eaa8-11e9-85f4-d00e5018f061 (accessed 21 October 2020)

[2] "Britain's post-Brexit immigration rules worry business," *The Economist*, 20 February 2020, https://www.economist.com/britain/2020/02/20/britains-post-brexit-immigration-rules-worry-business (accessed 21 October 2020)

[3] Judith Evans, "UK farmers warn time running out to find labour for 2021 harvest," *Financial Times*, 28 October 2020, https://www.ft.com/content/78f78ff9-1884-4cc4-8fa0-1608a1452a02 (accessed 04 November 2020)

[4] The total immigrant (non-citizens) employment share in 2004 was less than 10%. More details in section 4 and appendix 2.B.1.

vis-à-vis the output change produced by the removal of A8 workers —as given by the model developed in this chapter. The impacts are not only (i) still large in industries using very little of each skill but also (ii) the relationship between skill usage and output effect is not a monotone one, as shown by the non-smooth decay of the yellow bars on the right of each subfigure. These results indicate that the shock reaches industries less intense in the use of the affected skills via channels other than skill use.



Note: author's calculations. Codes according to ISIC Rev. 4 (Table 2.1).

Figure 2.1: Rankings of skill use and immigration impact per skill level

1.1 Contributions

In this chapter, I analyse how input-output linkages play a role in the transmission of a skill supply shock across industries. My first contribution is to develop a multi-sector closed-economy computable general equilibrium model with industry-specific skill-intensities and input-output linkages. It draws from the family of models stemming from Long and Plosser (1983), in particular Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and Fadinger, Ghiglino, and Teteryatnikova (2016). Like the latter pair, the present model is a static one, abstracting from dynamic aspects; unlike them, I include two different levels of skills and industry-specific factor intensities. Closed-form solutions for the equilibrium values of all endogenous variables are derived, in particular of the GDP [Proposition 2.1] and industry output [Proposition 2.2].

My second contribution is to apply the model to the data, particularly to study the impact of the A8 immigration on the UK economy. This is, to the best of my knowledge, the first calibrated input-output model for the UK economy which matches the factor shares across industries. Notably, I make use of the convenient decomposition of labour into skills categories provided by EU KLEMS. To be as close as possible to the World Input-Output Database (WIOD), which gives the interindustry linkages key to this chapter, the model also includes capital usage varying across industries. By doing so, I present a comprehensive view of the UK production network combined with an elucidative study of the importance of A8 workers to UK industries. I construct the dataset and implement the model on MATLAB, which allows me to contrast the model's first order results with non-linear solutions provided by the counterfactual exercises. Moreover, working on MATLAB endows the research with enough flexibility to test some of the model's hypothesis via robustness checks based on alternative specifications.

My third and most significant contribution is to build a framework to investigate the mechanism through which input-output linkages spread skill shocks across industries and to evaluate its relevance to any given economy. This endeavour unfolds into a number of deeds.

I first build an alternative model lacking the input-output linkages akin to Jones (1965),[5] henceforth the *no-IO model*. Contrasting it with the main model —hence the *IO model*— reveals how much ignoring industrial interconnections can lead to predictions far wide of the mark. I show that in the case of the A8 counterfactual, the output change of less-intense industries can be underestimated up to 0.56 p.p. (for industries using zero of the affected factor) while the impact on more-intense industries is overestimated by a remarkable 42 per cent on average.

I am then able to get to the core of the chapter: to assess how the no-IO model misestimation relates not only (i) to the direct relationship between each pair of industries —trivially given by the input shares— but also (ii) to the indirect transactions —conveyed ultimately by the input-output matrix. By doing so, I can fully explore and understand the interindustry connections of the economy.

Intuitively, the reason for industries more intense in the use of the shocked factor to have output changes smaller than those predicted by the no-IO model lies on the notion that those industries might buy intermediate inputs from the industries less

[5]The neoclassical multi-sector models cited include intermediate goods. Jones (1965) applies an activity analysis model instead. On activity analysis models, see Norton and Scandizzo (1981).

intense in the use of that factor and therefore less affected by the shock. The reverse applies to industries less intense in the shocked factor. The weighted average of the factor intensity of the intermediate input basket of the industries [constructed as per **Definition 2.5**] shows that this phenomenon indeed happens, but does not explain all the differences between the models' predictions.

In reality, the whole chain of intermediate input use matters in the shock transmission. This array is represented by a matrix defined in this chapter [**Definition 2.3**] as the *Leontief-inverse transposed*, since it is an analogue of the static solution matrix in Leontief (1986). Unlike the original one, the matrix studied here has the perspective of input use, i.e. the series of intermediate good purchases from each buying producer to every successive selling industry.

In the IO model, the effect of a factor shock on industry output combines input shares with the affected-factor shares of all industries [**Corollary 2.2**]. By contrast, in the no-IO model, each industry's isolated impact of the labour supply shock is simply given by its respective labour share [**Corollary 2.3**]. In that way, the difference between the two models in gauging the impact of the shock across industries results from a combination of the Leontief-inverse transposed and the total factor shares of every industry. These elements are defined as *linkage weights* [**Definition 2.4**] since they sum to one to each purchasing industry and summarise all upstream input connections relevant to the shock transmission.

One of the key findings of this chapter is given by **Theorem 2.1**, which establishes that the output change of each industry is an average of the labour shares of all industries calibrated by the linkage weights. This means that the industries are not affected solely by their own labour shares but also by those of the industries with which it transacts directly or indirectly throughout the production network. Moreover, the linkage weights determine that the higher the overall influence of an input supplier in the production technology of an industry, the more its isolated impact will be felt by that industry. These results fully explain how the transmission of a labour shock works to average out the impacts across the economy, reducing the output change in more intense and augmenting it in less intense industries.

I explore the equivalence of the Leontief-inverse transposed to an infinite sum of powers of the transpose of the input-output matrix to investigate the relevance of indirect input transactions, i.e. linkages between a pair of industries occurring through at least one other industry. More specifically, the low- and high-skill labour shock themselves are estimated using a truncated sum of the transpose of the input-output matrix in each corresponding order of approximation. The ratios of each order of truncation over the total shock effect are used to assess how fast is the convergence of these sequential approximations for each industry. Slow convergence is observed for industries having more intricate upstream connections, i.e. relying on inputs of industries that also rely considerably on other suppliers.

This endeavour is paramount in explaining differences in output changes among industries with similar first-order interconnections. I find that indeed they happen due to higher-order linkages, coming from the gravity of other more or less connected industries within the chain of upstream transactions. For example, 'real states activities' has the third smallest total input share, at just 23%, but is the one for which the model ignoring input-output linkages performs the worst. Indeed, the few direct suppliers of this industry, in particular financial and insurance activities and construction, are significantly upstream connected within the network.

1.2 Literature review

There has been a thriving revival in the field of input-output economics, motivated both by technical advances in neighbouring fields —e.g. network theory— and from the availability of unprecedentedly refined datasets with hundreds of industries[6] and millions of firms (Carvalho and Tahbaz-Salehi, 2019). Prominent examples of this flourishing literature are Gabaix (2011), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and Baqaee and Farhi (2019).

Chiefly, this chapter is inspired by these articles but pivots them in an original way. While these articles —as their stem, the seminal input-output real business-cycle model of Long and Plosser (1983)— investigate productivity shocks from micro to macro, the focus of the present study is the transmission of aggregate labour/skill shocks in the reverse direction: towards microeconomic entities. Thus, this chapter's

[6]In terms of terminology, the words "industry" and "sector" are often used interchangeably. Historically, the seminal work of Leontief (1941) covered a group of 10 industries while the works such as Uzawa (1961) and Uzawa (1963) extended the Solow-Swan growth model to two sectors, separating consumption to investment goods, which later gave way to the multi-sector growth (MSG) models. I choose 'industry' as it is the term adopted by the data used in this chapter.

contribution to this literature lies in assessing the role of intermediate inputs linkages in transmitting factor supply shocks across industries in a closed-economy model.

The investigation of shifts in industrial composition caused by changes in factor supplies is usually performed by studies in the field of international trade. Placed within the Heckscher-Ohlin theory of international trade —even though it only required a closed-economy model— Rybczynski (1955) applies the then-recently developed production version of the Edgeworth box (Stolper and Samuelson, 1941)[7] to prove what was later known as the Rybczynski theorem: "The maintenance of the same rates of substitution in production after the quantity of one factor has increased must lead to an absolute expansion in production of the commodity using relatively much of that factor, and to an absolute curtailment of production of the commodity using relatively little of the same factor" (Rybczynski, 1955, p.337-338). Allowing for the rates of substitution to adjust, the paper shows that there must be a deterioration in the terms of trade of the good more intense in the increased factor whereas boosted income will guarantee higher production of both goods, relatively more for the one whose relative price lowered.

The initial models of the Heckscher-Ohlin theory, however, did not include intermediate goods. Kemp (1969) and Schweinberger (1975) extended them and prove that the main theorems of the theory would still hold. My contribution to this literature is to evaluate how the input-output linkages would alter the exact product-mix predictions of the models.

Studies of skill shocks in the field of labour economics traditionally focus on their labour market outcomes, overlooking the perspective of the industries. Notable examples are Katz and Murphy (1992) and Goldin and Katz (2007) investigating the changes in the relative supply of skilled workers in the United States and Card (1990), Altonji and Card (1991) and Borjas, Freeman, and Katz (1992) on the effects of immigration. This body of research typically have partial equilibrium models of supply and demand for labour in the background (Ottaviano and Peri, 2013), but there are renowned exemplars of general equilibrium analysis, such as Heckman, Lochner, and Taber (1998).

More recently, however, labour literature has moved to the investigation of the re-

[7] See Humphrey (1996) for a chronicle of the development of the Edgeworth or diagram box.

sponse of firms and local industries, especially within cities.[8] Examples of studies on the impact of immigration on firms' outcomes include Olney (2013), Mitaritonna, Orefice, and Peri (2017) and Dustmann and Glitz (2015). However, these are empirical papers and take a different approach than mine, resorting to reduced form relationships between the supply of labour and firms' outcomes; they do not account for input-output linkages among either firms or industries.[9]

Within both international trade and labour economics, several studies cover the EU expansion (Caliendo, Opromolla, Parro, and Sforza, 2017; Cardoso, 2020) and UK's exit (Van Reenen, 2016; Dhingra, Huang, Ottaviano, Pessoa, Sampson, and Van Reenen, 2017). Nearly none of them study the changes in the industry composition. The one exception, and the work most closely related to this chapter, is Bratsberg, Moxnes, Raaum, and Ulltveit-moe (2019), who develop a factor proportions general equilibrium model to study the effect of 2004's and 2007's EU enlargements on Norway labour market and industry outcomes. Their model considers native and immigrants as having potentially different sets of skills —which includes language— and sorting into occupations that are used with varying proportions across industries. Consistent with the literature finding that firms adjust their production technology to accommodate large inflows of workers,[10] Bratsberg, Moxnes, Raaum, and Ulltveit-moe (2019) depicts a mechanism in which industries not only increase employment in occupations intensive in the skills abundant in the immigrants —responding to their relatively cheaper costs— but also by altering the skill mix and capital-labour mix of their production. Still, however, the paper does not include intermediate good, therefore also ignoring input-output linkages.

The neoclassical model of this chapter stands on the shoulders of Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Jones (2013) and Fadinger, Ghiglino, and Teteryatnikova (2016), being particularly closer to the latter.[11] Vis-à-vis their models, this chapter adds to the literature by splitting labour into two skill levels and adding

[8]See Ottaviano and Peri (2013) for a survey of this literature. According to them, this shift was motivated by the puzzle of unchanged relative wages between incoming immigrants and incumbent workers found in previous work, notably Lewis (2003) and Card and Lewis (2005). The new studies assess whether immigration altered intercity trade, correcting skill imbalances.

[9]Analysing firm-level data, the results obtained regarding the industries are due to clustering firms within the same line of business, which generates a coarser classification than the one provided by the industry-level data.

[10]On this topic, see the comprehensive survey by Lewis (2013).

[11]All cited models are static variations of the 'dynamic multi-sector' models of Long and Plosser (1983), Horvath (1998) and Dupor (1999).

industry-specific factor intensities, as well as having a different focus for the exercises. In this family of neoclassical input-output models, Horvath (1998, 2000) — and more recently vom Lehn and Winberry (2019)— also include technology parameters varying across industries, but do not consider different categories of skills.[12] Notwithstanding, my model is tractable and suitable to be applied to the data, providing a robust tool for input-output analysis.

In terms of application, to the best of my knowledge, this is the first study to consider the impact of EU immigration on UK industries considering their direct and indirect interconnections. Such contribution might prove helpful for aiding policies on immigration, skill supply management and industrial organization.

Most remarkably, insofar as this research topic has not been previously pursued, the framework developed here to the analysis of the transmission mechanism of a factor supply shock across industries is entirely novel. The depiction of the output changes of each industry being a weighted average of other industries' characteristics and the characterisation of the higher-order interconnections among industries allows for a thorough understanding of the role of input-output linkages and are, therefore, the main contributions of this chapter.

1.3 Outline

The structure of this chapter is as follows. Section 2 presents the theoretical framework, which includes the main input-output model with its solutions, the alternative no-IO model, and the key definitions and results supporting the analysis. Section 3 introduces the data, outlines the calibration of the model and gives the rationale for opting for the non-linear solutions produced by the MATLAB implementation. Section 4 turns to the application side, where I show how the model can fortuitously elucidate how the A8 immigration affected UK industries making use of the comprehensive framework developed; in this section, I also present a robustness test for the counterfactual design. Section 5 concludes. Algebraic derivations and proofs, data and computation details, and supplementary results are available in the Appendices 2.A, 2.B and 2.C, respectively.

[12] Horvath (2000) even abstracts from labour altogether.

2 Theoretical analysis

2.1 The input-output model

The main model is an extension of Fadinger, Ghiglino, and Teteryatnikova (2016)'s, to which I incorporate industry-specific factor intensities and decompose the labour stock into two groups: *low-skilled* and *high-skilled*. Workers are identical within each category and can move freely across industries, responding to the demand for skills.[13]

Each industry is viewed as a single representative firm producing a homogeneous good. There are n industries in the economy. The output q_i of each industry i is given by the following constant returns to scale (CRS) Cobb-Douglas technology:[14]

$$q_i = A_i\left(k_i^{\alpha_i} l_i^{\delta_i} h_i^{1-\alpha_i-\delta_i}\right)^{1-\gamma_i} \prod_{j=1}^{n} d_{ji}^{\gamma_{ji}} \tag{2.1}$$

Where the endogenous variables, besides q_i, are:

- k_i: capital
- l_i: low-skilled labour
- h_i: high-skilled labour
- d_{ji}: output of domestic industry j used as input in the production of i[15]

Regarding the technology parameters:[16]

- A_i: industrial total factor productivity
- α_i: factor income share of capital in the production technology of industry i
- δ_i: factor income share of low-skilled labour for industry i
- $1 - \alpha_i - \delta_i$: factor income share of high-skilled labour for industry i
- γ_{ji}: the share of good j in the production technology of firms in industry i, where $\gamma_i = \sum_{j=1}^{n} \gamma_{ji}$

[13]Unlike the literature on immigration, I make no distinction in the model between incomers and native workers. In other words, I assume they are perfect substitutes for each other.

[14]Given the complexity of the model, the adoption of a constant-returns-to-scale (CRS) Cobb-Douglas specification for the industries' output functions is convenient in terms of producing analytical solutions, which are favoured for shocks of greater magnitude. Since the main object of this chapter is of first-order, i.e. the changes in real output, non-linearity issues as discussed by Baqaee and Farhi (2019) do not weigh on the chapter's results. Moreover, the counterfactual design as a shock in one given year matches well with fixed parameters. Altogether, the results should be seen as short-run responses to factor supplies for given technologies.

[15]These elements are also called "materials" or "material inputs" in related literature on trade. See Caliendo, Dvorkin, and Parro (2019).

[16]The Cobb-Douglas technology in (2.1) coupled with competitive factor markets, result in the factor parameters corresponding to their nominal shares in equilibrium. Likewise, the parameters γ_{ji} correspond to the entries of the IO matrix, measuring the value of spending on input j per dollar of production of good i.

Output of industry i is used for final consumption, y_i, or as intermediate good, such that the market clearing condition for each good i holds, i.e.:

$$y_i + \sum_{j=1}^{n} d_{ij} = q_i \tag{2.2}$$

The consumer side is synthesised by a final good aggregation. This can be interpreted as an entity which combines all the y_i left-overs from the industries and transforms them into the aggregate GDP. Consumer preferences are represented by industry-specific final demand parameters $\beta_i \geqslant 0$, $\forall i$, where $\sum_{i=1}^{n} \beta_i = 1$.

$$Y = \prod_{i=1}^{n} y_i^{\beta_i} \tag{2.3}$$

Real GDP (Y) equals nominal GDP (PY) since the price deflator is chosen as the numeraire and its value is normalised to one $(P = 1)$. Production side equals expenditure side GDP, so that final output is equivalent to household's consumption:

$$Y = C \tag{2.4}$$

Households supply the primary factors of production inelastically, receiving competitive market wages w_L and w_H —for low- and high-skilled labour, respectively— and rental on capital w_K. Thus, households' income comprises of the aggregate value-added and is fully used to finance consumption:

$$C = w_K K + w_L L + w_H H \tag{2.5}$$

Finally, factor markets are also assumed to clear, i.e. the sum of each factor employed in every industry equals the exogenous aggregate levels of physical capital K, low-skilled L and high-skilled labour H:

$$K = \sum_{i=1}^{n} k_i \tag{2.6}$$

$$L = \sum_{i=1}^{n} l_i \tag{2.7}$$

$$H = \sum_{i=1}^{n} h_i \tag{2.8}$$

2.2 General equilibrium

The competitive general equilibrium of this economy is trivially given by the profit maximisation of industries and final good aggregator taking prices as given. The formal definition of the equilibrium allocation follows:

Definition 2.1 (Equilibrium) *A competitive equilibrium consists of quantities regarding aggregate output Y and consumption C, the industrial output $\{q_i\}$, intermediate input choices $\{d_{ji}\}$, final good demands $\{y_i\}$ and industry demand for factors $\{k_i\}$, $\{l_i\}$ and $\{h_i\}$, as well as prices of factors w_K, w_L, w_H and goods $\{p_i\}$ for every industry i, j in the economy such that:*

1. **Industry Problem:** *Given good prices $\{p_i\}$ and factor prices w_K, w_L and w_H the representative firm of each industry chooses factors, inputs and output to maximise its profit subject to the technology, such that:*

$$\max_{\{q_i,k_i,l_i,h_i,d_{ji}\}} p_i q_i - w_K k_i - w_L l_i - w_H h_i - \sum_{j=1}^{n} p_j d_{ji}$$

s.t. Equation (2.1).

2. **Final Good Problem:** *Given $\{p_i\}$ and P, the final good aggregator (entity) maximises profit subject to the available technology, such that:*

$$\max_{\{y_i,Y\}} PY - \sum_{i=1}^{n} p_i y_i$$

s.t. Equation (2.3).

3. *setting the the final good price as the numeraire, i.e. $P = 1$, prices clear the markets for all goods and factors, such that:*
   - *industry goods: Equation (2.2)*
   - *final good: Equation (2.4)*
   - *capital: Equation (2.6)*
   - *low-skilled labour: Equation (2.7)*
   - *high-skilled labour: Equation (2.8)*

First order conditions

The Cobb-Douglas combined with efficient markets yields the standard first order conditions (FOCs) for the equilibrium in each industry i in terms of the nominal shares of capital (2.9), low-skilled (2.10) and high-skilled labour (2.11), and the intermediate inputs share (2.12) in the total costs (sales) of an industry:

$$\frac{w_K k_i}{p_i q_i} = \alpha_i(1-\gamma_i) \tag{2.9}$$

$$\frac{w_L l_i}{p_i q_i} = \delta_i(1-\gamma_i) \tag{2.10}$$

$$\frac{w_H h_i}{p_i q_i} = (1-\alpha_i-\delta_i)(1-\gamma_i) \tag{2.11}$$

$$\frac{p_j d_{ji}}{p_i q_i} = \gamma_{ji} \tag{2.12}$$

The necessary equilibrium condition for the final good maximisation renders β_i as each industry's nominal sales share on GDP:

$$\beta_i = \frac{p_i y_i}{Y} \tag{2.13}$$

2.3 Special notes

Industry factor shares

The nominal factor shares as given by Equations (2.9), (2.10) and (2.11) appear often in the subsequent analysis of this chapter. For this reason, it is worth understanding their meaning precisely.

Consider each industry i value added, i.e. its total payment to factors, as given by:

$$va_i = w_K k_i + w_L l_i + w_H h_i \tag{2.14}$$

Then, in equilibrium, having total costs equating total sales, every industry in the IO model must satisfy:

$$p_i q_i = va_i + \sum_{j=1}^{n} p_j d_{ji} \tag{2.15}$$

With these references, I define the 'broad' as given by Equations (2.9), (2.10) and (2.11) and the 'narrow' factor shares as below. The main difference between them is that while the former considers an industry's total costs, i.e. including the costs of the intermediate inputs, the latter are solely in terms of the total costs of factors.

Definition 2.2 (Narrow factor shares) *The narrow nominal shares for capital, low-skilled and high-skilled labour for each industry i are respectively given by the equations below:*

$$\alpha_i = \frac{w_K k_i}{v_i} \tag{2.16}$$

$$\delta_i = \frac{w_L l_i}{v_i} \tag{2.17}$$

$$(1 - \alpha_i - \delta_i) = \frac{w_H h_i}{v_i} \tag{2.18}$$

Aggregate factor shares

To simplify the analysis, I use α and δ to represent aggregate factor shares in the value added given by Equation (2.5), such that:

$$\frac{w_K K}{C} = \alpha \quad (2.19) \qquad \frac{w_L L}{C} = \delta \quad (2.20) \qquad \frac{w_H H}{C} = (1 - \alpha - \delta) \quad (2.21)$$

IO Matrix

The input-output (IO) matrix $\mathbf{\Gamma} = [\gamma_{ji}]$ is the representation of the inter-industry commerce. Each row contains the values of intermediate good j consumed by each industry i per dollar of production. This layout adheres to the way input-output data is usually available, having origins in the rows and destinations in the columns.[17]

$$\mathbf{\Gamma} = [\gamma_{ji}] = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nn} \end{bmatrix}$$

Under this notation, the sum of elements in the i^{th} column of matrix Γ is equal to the total intermediate share of the respective industry: γ_i. These are also called the

[17] It is important to stress here that prominent works more theoretically driven, have opted to express the IO matrix in the reversed direction, i.e. having consuming industries on the rows and producers along the columns. See Baqaee and Farhi (2019) and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012).

"weighted in-degrees".[18] However, since each γ_i includes industry i's own consumption γ_{ii} —and these values tend to be significant—, it is not as much a measure of upstream connectedness as it is a measure of overall reliance on intermediate inputs.

Sales shares

An important parameter not present in the main formulae of the model aids significantly in the derivations and analysis. It is industry i's sales share on GDP, or Domar[19] weight, defined as i's nominal sales $p_i q_i$ over aggregate output.

$$\mu_i = \frac{p_i q_i}{Y} \tag{2.22}$$

Combining the market clearing condition in Equation (2.2) with the equilibrium conditions given by equations (2.12) and (2.13), it is clear that μ_i is a constant parameter and not a variable in the model: i.e. $\mu_i = \sum_{j=1}^{n} \gamma_{ij}\mu_j + \beta_i$.[20] Or in matrix form:

$$\boldsymbol{\mu} = [\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1}\boldsymbol{\beta} \tag{2.23}$$

Matrix $[\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1}$ is known as the "Leontief inverse" and combines the whole chain of gross output production per unit of consumption. If all input shares γ_{ji} are non-zero, positive and smaller that one, the matrix is non-singular.[21]

Industry output versus final good

Before presenting the main results of the model, it is worth explaining why the focus of the analysis is on the industry output q_i and not on the final good y_i. As showed by Equations (2.2) and (2.3), part of the total production of industry i is used by other industries and the remainder is transformed in final consumption. Since intermediate inputs are only transfers between sectors which are cancelled out in the total summation, they are irrelevant in aggregate terms in this model.[22]

[18] Likewise, the "weighted out-degrees" of an industry j are defined as $\gamma_j^{out} = \sum_{i=1}^{n} \gamma_{ji}$. This measure corresponds to the relative importance of industry j's output as an intermediate input in the production network. It is equal to the sum of elements in the j^{th} row of matrix Γ.

[19] Domar (1961).

[20] Moreover, because of intermediate goods, $\sum_{i=1}^{n} \mu_i > 1$ in general.

[21] See Carvalho and Tahbaz-Salehi (2019) for a full characterisation of the Leontief inverse.

[22] Appendix 2.A.1 details the algebra of this property. It shows how aggregate income equals final nominal demand in this model, such that produced and consumed intermediate inputs cancel out.

As such, both final good y_i and its aggregation Y do not capture the variables which are the novel and central part of the analysis. Hence, to assess how each industry is particularly affected by a labour shock, one must study its effect on its total physical production, as given by the real output q_i.

2.4 Analytical results and tools

Equilibrium solutions

Despite the richness of the model, closed-form solutions are derivable for all endogenous variables. I present here the two most relevant for the subsequent analysis.[23]

Proposition 2.1 (Real GDP) *The equilibrium aggregate output is given by:*

$$\begin{aligned}\log Y \quad = \quad & \alpha(\log K - \log \alpha) + \delta(\log L - \log \delta) + (1-\alpha-\delta)(\log H - \log(1-\alpha-\delta)) \\ & + \textstyle\sum_{i=1}^{n} \mu_i(1-\gamma_i)\left[\alpha_i \log \alpha_i + \delta_i \log \delta_i + (1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i)\right] \\ & + \textstyle\sum_{i=1}^{n} \mu_i(1-\gamma_i)\log(1-\gamma_i) + \sum_{i=1}^{n} \mu_i \log A_i \\ & + \textstyle\sum_{i=1}^{n} \mu_i \sum_{j=1}^{n} \gamma_{ji} \log \gamma_{ji} + \sum_{i=1}^{n} \beta_i \log \beta_i \end{aligned} \tag{2.24}$$

The solution clearly shows how the exogenous factor supplies K, L and H relate to the GDP through their respective nominal shares. Furthermore, it confirms that Hulten (1978)'s theorem holds, as the model comprises of CRS technologies and efficient markets. In other words, the industrial multiplier,[24] or the elasticity of aggregate output Y to the productivity of industry i, equals μ_i, i.e. industry i's sales share on GDP.

Proposition 2.2 (Industry output) *The output of each industry i in equilibrium reads:*

$$\begin{aligned}\log q_i \quad = \quad & (1-\gamma_i)\alpha_i(\log K - \log \alpha + \log \alpha_i) \\ & +(1-\gamma_i)\delta_i(\log L - \log \delta + \log \delta_i) \\ & +(1-\gamma_i)(1-\alpha_i-\delta_i)(\log H - \log(1-\alpha-\delta)) \\ & +(1-\gamma_i)(\log(1-\alpha_i-\delta_i) + \log(1-\gamma_i)) + \log A_i + \log \mu_i \\ & + \textstyle\sum_{j=1}^{n} \gamma_{ji} \log \gamma_{ji} - \sum_{j=1}^{n} \gamma_{ji} \log \mu_j + \sum_{j=1}^{n} \gamma_{ji} \log q_j \end{aligned} \tag{2.25}$$

[23] The remaining derivations are available in Appendix 2.A .

[24] The literature —see Chipman (1950)— usually adopts the concept of the industrial or sectoral multiplier to represent the effect on aggregate output Y of a one-per cent change to the productivity of industry i: $\mathrm{d}\log Y/\mathrm{d}\log A_i$.

The appearance of the term $\sum_{j=1}^{n}\gamma_{ji}\log q_j$ on the right-hand side of this equation reveals how each industry's output is recursively related to all other's and makes explicit the model's supply-side characteristic. In this framework, goods are produced not responding to the demand but to the availability of resources: other industry's output j matters to the extend that it is used as an input by industry i.

Corollary 2.1 (Vector of industry output) *The complete solution for the industry output can only be obtained in matrix form. Let* $\mathfrak{q}_i = \log q_i$ *and the* i^{th} *row of vector* $\boldsymbol{V}$ *be composed of all the constant terms of* (2.25) *—those prior to* $\sum_{j=1}^{n}\gamma_{ji}\log q_j$*— such that vector* $\mathfrak{q}$ *reads:*

$$\mathfrak{q} = \left[\boldsymbol{I} - \boldsymbol{\Gamma}'\right]^{-1}\boldsymbol{V} \tag{2.26}$$

Equation (2.26) is key in the toolkit developed in this chapter and is discussed in extension further on.

Comparative statics

The IO model's prediction for the effects on the output of an industry are given by the first derivative of its equilibrium solution with respect to the shocked variable. It follows from Corollary 2.1 that this differentiation is rather straightforward.

For clarity, I present below the effects of a low-skilled, a high-skilled and a total labour supply shock. For the latter, I consider $E = L + H$, i.e. the sum of the low- and high-skilled labour supply.

Corollary 2.2 (Output effect) *The vectors of the effect of a low-skilled, high-skilled and total labour supply shock on the real output of the industries in the IO model are given by the product of the Leontief-inverse transposed and a vector comprised of the 'broad' low-skill, high-skilled and total labour shares of each industry* i*, i.e. the nominal shares* δ_i*,* $(1-\alpha_i-\delta_i)$ *and* $(1-\alpha_i)$*, respectively, times the total factor share* $(1-\gamma_i)$*.*

$$\begin{bmatrix} \frac{\mathrm{d}\log q_1}{\mathrm{d}\log L} \\ \frac{\mathrm{d}\log q_2}{\mathrm{d}\log L} \\ \vdots \\ \frac{\mathrm{d}\log q_n}{\mathrm{d}\log L} \end{bmatrix} = \left[\boldsymbol{I} - \boldsymbol{\Gamma}'\right]^{-1} \cdot \begin{bmatrix} (1-\gamma_1)\delta_1 \\ (1-\gamma_2)\delta_2 \\ \vdots \\ (1-\gamma_n)\delta_n \end{bmatrix} \tag{IO$_{LS}$}$$

$$\begin{bmatrix} \frac{\mathrm{d}\log q_1}{\mathrm{d}\log H} \\ \frac{\mathrm{d}\log q_2}{\mathrm{d}\log H} \\ \vdots \\ \frac{\mathrm{d}\log q_n}{\mathrm{d}\log H} \end{bmatrix} = \left[\boldsymbol{I} - \boldsymbol{\Gamma}'\right]^{-1} \cdot \begin{bmatrix} (1-\gamma_1)(1-\alpha_1-\delta_1) \\ (1-\gamma_2)(1-\alpha_2-\delta_2) \\ \vdots \\ (1-\gamma_n)(1-\alpha_n-\delta_n) \end{bmatrix} \tag{IO$_{HS}$}$$

$$\begin{bmatrix} \frac{\mathrm{d}\log q_1}{\mathrm{d}\log E} \\ \frac{\mathrm{d}\log q_2}{\mathrm{d}\log E} \\ \vdots \\ \frac{\mathrm{d}\log q_n}{\mathrm{d}\log E} \end{bmatrix} = \left[\boldsymbol{I} - \boldsymbol{\Gamma}'\right]^{-1} \cdot \begin{bmatrix} (1-\gamma_1)(1-\alpha_1) \\ (1-\gamma_2)(1-\alpha_2) \\ \vdots \\ (1-\gamma_n)(1-\alpha_n) \end{bmatrix} \tag{IO$_{ES}$}$$

Noticeably, the three results are analogue of each other. They all have the Leontief-inverse transposed premultiplying a "vector shock" comprised of the 'broad' nominal shares of the affected factor for all industries. These results will be explored in depth in section 2.5.[25]

Contrast with a model without input-output linkages

The model without linkages is a specialisation of the full model where industries do not use other industries' output as intermediate inputs, i.e. all the intermediate inputs γ_{ji} equal zero. The formulae is the same as before, except by this restriction. For clarity, Equations (2.1) and (2.2) are replaced with the following, respectively:

$$q_i = A_i\left(k_i^{\alpha_i} l_i^{\delta_i} h_i^{1-\alpha_i-\delta_i}\right) \tag{2.27}$$

$$q_i = y_i \tag{2.28}$$

Likewise, Equations (IO$_{LS}$), (IO$_{HS}$) and (IO$_{ES}$) regarding the impact of low-skilled, high-skilled and total labour supply, respectively, are also adjusted accordingly.

Corollary 2.3 (Output effect no-IO model) *In the no-IO model, the effect of a low-skilled, high-skilled and total labour supply shock on the real output of the industries*

[25] These results are closely related to Baqaee (2015)'s "network-adjusted labour intensity". Being supply-side shocks, they propagate downstream and, as such, they require an upstream measure to assess how much each industry relies on the output of the affected industries.

is proportional to their affected 'narrow' factor shares, such that:

$$\frac{\mathrm{d}\log q_i}{\mathrm{d}\log L} = \delta_i \tag{no-IO$_{LS}$}$$

$$\frac{\mathrm{d}\log q_i}{\mathrm{d}\log H} = (1 - \alpha_i - \delta_i) \tag{no-IO$_{HS}$}$$

$$\frac{\mathrm{d}\log q_i}{\mathrm{d}\log E} = (1 - \alpha_i) \tag{no-IO$_{ES}$}$$

Unlike the IO model, the no-IO model predicts that the impact of the shocks on each industry is fully determined by its own usage of the affected factor. As expected, the other industries' output and parameters play no role in it.

Leontief-inverse transposed

The Leontief inverse in Equation (2.23) takes into account all direct and indirect effects at work along the *downstream* chains, i.e. from the seller's perspective.[26] Having the input-output transpose matrix Γ' instead captures the *upstream* or buyers' chains, as it is the case in equations (IO$_{LS}$), (IO$_{HS}$) and (IO$_{ES}$).[27] For this reason, I coin matrix $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$ the *Leontief-inverse transposed* matrix.

Definition 2.3 (Leontief-inverse transposed) *Matrix $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$ is defined as the Leontief-inverse transposed. Each element υ_{ij} of it represents the overall relevance of the input of industry j in the production technology of industry i.*

In the Leontief-inverse transposed, the elements portray how much each industry relies on every other industry's inputs, while each element of the Leontief inverse represents how important each industry is as a supplier to every other industry. In that way, υ_{ij} captures all the direct and indirect transactions of each input j over the supply chain of each good i.

[26] A simple way of demonstrating this result, as done by Fadinger, Ghiglino, and Teteryatnikova (2016) is to represent the Leontief-inverse as an infinite geometric series post-multiplied by a vector of ones and truncated at its first-order term. This leads to the vector of out-degrees $\boldsymbol{\gamma}^{\boldsymbol{out}}$:

$$\begin{aligned} [I - \Gamma]^{-1}\mathbf{1} &= \left(\textstyle\sum_{k=0}^{\infty}\Gamma^k\right)\mathbf{1} = \left(I + \Gamma + \Gamma^2 + ...\right)\mathbf{1} \\ &\approx [I + \Gamma]\mathbf{1} = \mathbf{1} + \boldsymbol{\gamma}^{\boldsymbol{out}} \end{aligned}$$

The approximation is rather coarse and does not hold numerically, but it is helpful in the visualisation of the direction of the effects captured by this measure.

[27] Algebraically, the Leontief-inverse transposed is trivially the transpose of the Leontief-inverse, i.e. $[I - \Gamma^T]^{-1} = [[I - \Gamma]^{-1}]^T$.

Nonetheless, the direction of the effect must not be understood rigidly. Because there are loops in the network, a supposed downstream effect may also include some transactions happening upstream, in the sense that an initially selling sector may eventually be a buyer in a chain.[28] In essence, all that is relevant in this interpretation is the starting point, which is either a view from the sellers' or from the buyers' perspective.

Linkage weights

Remarkably, all the common portions of Equations (IO_{LS}), (IO_{HS}) and (IO_{ES}) are derived from the input shares γ_{ji}. For each industry i, these terms come from the product of each element υ_{ij} of the row i of the Leontief-inverse transposed with the elements of the column vector of total factor shares. In fact, since each row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$ is a linear combination of the vector $\mathbf{1} - \boldsymbol{\gamma}$,[29] the result of this product is the vector of ones. I define the elements of this dot product as *linkage weights*.

Definition 2.4 (Linkage weights) *The linkage weights ρ_{ij} of each industry i with respect to industry j are defined as:*

$$\rho_{ij} = \upsilon_{ij}(1 - \gamma_j) \tag{2.29}$$

In essence, they represent the importance of every industry j in spreading the factor shock to each industry i. Moreover, these terms premultiply the isolated shock experienced by each industry —given by the shares of the affected factor— and therefore play a key role in the transmission of the shocks. In fact, these linkage weights render the output impact in the IO model as a weighted average of the isolated output impact predicted by the no-IO model.

Theorem 2.1 (Shock dispersion via linkages) *The effect of a labour (skill) shock on the real output of an industry i is given by the average of the labour (skill) nominal shares of the input-supplying industries j weighted by the linkage weights ρ_{ij}. Equa-*

[28] For example, matrix Γ^2 considers all the two-step transactions, such that $1 \rightarrow 2 \rightarrow 1$ would be a chain of industry 1 selling to 2 and then buying something back from industry 2.

[29] Recall $\gamma_i = \sum_{j=1}^{n} \gamma_{ji}$, i.e. the sum of row i of $\boldsymbol{\Gamma}'$ or, equivalently, the sum of column i of $\boldsymbol{\Gamma}$.

tions (IO_{LS}), (IO_{HS}) *and* (IO_{ES}) *can be simplified accordingly to read:*

$$\frac{\mathrm{d}\log q_i}{\mathrm{d}\log L} = \sum_j \rho_{ij}\delta_j \tag{2.30}$$

$$\frac{\mathrm{d}\log q_i}{\mathrm{d}\log H} = \sum_j \rho_{ij}(1-\alpha_j-\delta_j) \tag{2.31}$$

$$\frac{\mathrm{d}\log q_i}{\mathrm{d}\log E} = \sum_j \rho_{ij}(1-\alpha_j) \tag{2.32}$$

In other words, the total effect of the labour shock is a combination of the labour shares of all industries from which a producer buys, graduated by their relative importance in the whole upstream chain of transactions. They are therefore the precise way of expressing how the shocks are average out across industries in the IO model.

2.5 Investigating the transmission mechanism

The main takeaway of the theoretical analysis presented so far is that in the IO model the effect of a labour or skill shock on the output of an industry comprises of terms deriving from the input shares and the nominal shares of the affected factor for all industries. By contrast, the no-IO model predicts that each industry's isolated impact of the labour supply shock comprises solely of its respective labour share.

I explore two ways of understanding the role of input-output linkages in the transmission of the shocks across industries: (i) the direct input purchases of each industry; and (ii) the transactions of higher orders along the input supply. I focus exclusively on the two skill categories, leaving aside the combined shock for the sake of better readability of the analysis.

First round IO effects: Skill content of input purchases

A key idea of this chapter is that a model ignoring interindustry connections tend to underestimate the output impact of a labour shock to more labour-intensive industries and overestimates it for less intensive ones. I explore the intuitive idea that the averaging out of the impacts across industries may result from their direct transactions by constructing the weighted average of the factor intensity of the intermediate input basket of the industries.

The intermediate input bundle of an industry is composed of goods with different factor intensity. By skill level, I calculate the weighted average of the labour content of the input basket of a buying industry i by taking each producer j's skill intensity (either δ_j for low-skill or $(1-\alpha_j-\delta_j)$ for high-skill) and weighting it by the relative amount of good j used by i $(p_j d_{ji}/\sum_j p_j d_{ji} = \gamma_{ji}/\gamma_i)$.

Definition 2.5 (Skill content) *The low-skill and high-skill content of the input purchases of industry i are respectively defined as:*

$$LSC_i = \sum_j \delta_j \left(\frac{\gamma_{ji}}{\gamma_i}\right) \tag{LSC}$$

$$HSC = \sum_j (1-\alpha_j-\delta_j) \left(\frac{\gamma_{ji}}{\gamma_i}\right) \tag{HSC}$$

Additionally, I discount an industry's own skill content to gauge whether it purchases goods relatively more or less intensive than its own production. I then investigate if these differences could explain the averaging out of the skill shocks by comparing these values with the ratios of the predictions of the no-IO over the IO model. I present these results in section 4.2, which shows that these transactions fall short of explaining all input-output interconnections.

Higher IO effects: Speed of convergence of the shocks' approximations

Formally, the upstream array of intermediate input use and its role in transmitting the labour shocks are embedded in the Leontief-inverse transposed $[I-\Gamma']^{-1}$. As shown by Corollary 2.2, in the IO model the effect on output of industry i is a combination of two components:

- a common *shock vector*, consisting of each industry-specific labour shares times the correspondent total factor share $(1-\gamma_j)$; and
- the respective row of matrix $[I-\Gamma']^{-1}$, which graduates the importance of the shock vector. This is the key component, since it is the varying part across industries and determines how much of the shock in every industry in the economy will be transmitted to each industry.

The equivalence of matrix $[\boldsymbol{I}-\boldsymbol{\Gamma}']^{-1}$ to an infinite sum of powers of matrix $\boldsymbol{\Gamma}'$ can be explored to investigate the relevance of the initial orders of the sum in explaining the final matrix. In other words, depending on how fast the power series converges to its limit, more or less important are the higher orders of the approximation.

$$\left[\boldsymbol{I}-\boldsymbol{\Gamma}'\right]^{-1}=\sum_{k=0}^{\infty}\boldsymbol{\Gamma}'^{k}=\boldsymbol{I}+\boldsymbol{\Gamma}'+\boldsymbol{\Gamma}'^{2}+\dots \tag{2.33}$$

Each order of approximation of the Leontief-inverse transposed can be interpreted as the transactions occurring at the correspondent number of steps in the chain, i.e. from no intermediate input purchases (zeroth order), passing by direct transactions with input suppliers (first order), to those links more indirectly relating a pair of industries (higher orders). For instance, the second order of approximation includes all cases in which an industry g sells to j which then sell to an industry i.

To aid the analysis, I calculate the approximations of the shocks themselves on the output of the industries. Taking a low-skill labour supply shock of magnitude as an example, the equations below illustrate the first three orders of approximation:

- Zeroth-order approximation

$$\begin{bmatrix}\frac{\mathrm{d}\log q_1}{\mathrm{d}\log L}\\ \frac{\mathrm{d}\log q_2}{\mathrm{d}\log L}\\ \vdots\\ \frac{\mathrm{d}\log q_n}{\mathrm{d}\log L}\end{bmatrix}=\begin{bmatrix}(1-\gamma_1)\delta_1\\ (1-\gamma_2)\delta_2\\ \vdots\\ (1-\gamma_n)\delta_n\end{bmatrix} \tag{IO$_{LS}$0}$$

- First-order approximation

$$\begin{bmatrix}\frac{\mathrm{d}\log q_1}{\mathrm{d}\log L}\\ \frac{\mathrm{d}\log q_2}{\mathrm{d}\log L}\\ \vdots\\ \frac{\mathrm{d}\log q_n}{\mathrm{d}\log L}\end{bmatrix}=\left[I+\Gamma'\right]\cdot\begin{bmatrix}(1-\gamma_1)\delta_1\\ (1-\gamma_2)\delta_2\\ \vdots\\ (1-\gamma_n)\delta_n\end{bmatrix} \tag{IO$_{LS}$1}$$

- Second-order approximation

$$\begin{bmatrix}\frac{\mathrm{d}\log q_1}{\mathrm{d}\log L}\\ \frac{\mathrm{d}\log q_2}{\mathrm{d}\log L}\\ \vdots\\ \frac{\mathrm{d}\log q_n}{\mathrm{d}\log L}\end{bmatrix}=\left[I+\Gamma'+\Gamma'^{2}\right]\cdot\begin{bmatrix}(1-\gamma_1)\delta_1\\ (1-\gamma_2)\delta_2\\ \vdots\\ (1-\gamma_n)\delta_n\end{bmatrix} \tag{IO$_{LS}$2}$$

Although not identical, the zeroth order of approximation can be interpreted as an analogue to the impact predicted by the no-IO model; it differs by the total share of factors $(1 - \gamma_i)$ multiplying each industry's labour share. The first-order approximation includes the direct transactions only, from an industry to its suppliers. Each subsequent order includes a set of transactions one step farther than the preceding. Moreover, since the approximations are for the actual impact on output, the results will vary even though the matrices are the same in both cases. This happens because the industries have distinct shares of high- and low-skill labour and they will be affecting other industries along the chain of transactions accordingly.

In section 4.2, I recast matrix $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$ into the sum of powers of matrix $\boldsymbol{\Gamma}'$ times the respective shock vectors to investigate the relative importance to an industry of its indirect input transactions. The results confirm that industries' higher-order connections matter in explaining the input-output linkages, especially for industries which has few first-order transactions.

3 Quantitative analysis

3.1 Dataset

The developed models are applied in this chapter to the study of the United Kingdom. The national input-output tables (NIOT) are extracted from the World Input-Output Dataset (WIOD), Release 2016 (Timmer, Dietzenbacher, Los, Stehrer, and de Vries, 2015). The remainder —including the industry-level series on payments to labour and employment by skill level— comes from EU KLEMS (Jäger, 2017).[30] The analysis is focused on 2014, the most recent year in both databases.[31]

The constructed dataset is attuned to some simplifying assumptions, particularly that factor markets are efficient so that competition wields unique factor prices. This leads to the merging of some industries, which are aggregated in 29 groups, as listed in Table 2.1 together with their number and codes.[32]

[30] Full description of the data is available in Appendix 2.B.1.

[31] Using 2003 —the year before the EU expansion that included the A8 countries— as the base year would unintentionally incorporate some parametric changes in the calculations. In Appendix 2.B.3, I discuss this alternative computation in details.

[32] Appendix 2.B.1 describes this process.

Table 2.1: Dataset's industry codes and description

No	Code	Description
1	A-B	Agriculture, forestry and fishing; Mining and quarrying
2	C10-C12	Manufacture of food products, beverages and tobacco products
3	C13-C15	Manufacture of textiles, wearing apparel and leather products
4	C16-C18	Manufacture of wood etc. except furniture; Manufacture of paper etc.; Printing and reproduction of media
5	C19	Manufacture of coke and refined petroleum products
6	C20-C23	Chemicals and chemical products; Rubber and plastics products; Other non-metallic mineral products
7	C24-C25	Basic metals and fabricated metal products, except machinery and equipment
8	C26-C27	Electrical and optical equipment
9	C28	Machinery and equipment n.e.c.
10	C29-C30	Transport equipment
11	C31-C33	Other manufacturing; repair and installation of machinery and equipment
12	D-E	Electricity, gas and water supply
13	F	Construction
14	G45	Wholesale and retail trade and repair of motor vehicles and motorcycles
15	G46	Wholesale trade, except of motor vehicles and motorcycles
16	G47	Retail trade, except of motor vehicles and motorcycles
17	H49-H53	Transport and storage; Postal and courier activities
18	I	Accommodation and food service activities
19	J58-J60	Publishing, audiovisual and broadcasting activities
20	J61	Telecommunications
21	J62-J63	IT and other information services
22	K	Financial and insurance activities
23	L	Real estate activities

No	Code	Description
24	M-N	Professional, scientific, technical, administrative and support service activities
25	O	Public administration and defence; Compulsory social security
26	P	Education
27	Q	Health and social work
28	R-S	Arts, entertainment, recreation; Other service activities
29	T	Activities of households as employers; Activities of households for own use

3.2 Calibration

To be as anchored as possible to the data and given that the research question of this chapter focuses on factor changes, aggregate (K, L and H) and industry level (k_i, l_i and h_i) factor quantities are taken directly from the data. Factor prices w_K, w_L and w_H are extracted from data values for aggregate factor compensations under the perfect competition assumption which asserts that factors are paid the same fare by all industries.

In monetary terms, industry gross output values are constructed as the sum of total input use and value added, while the (left-over) consumption good equals gross output minus the total sales of intermediate good.[33] Data for nominal and real value added across industries are used to calculate good prices. Aggregate price P is normalised to equal one and good prices p_i are adjusted accordingly. Industry output q_i, consumption good y_i and input-output variables d_{ji} are then extracted from their data nominal values. In possession of these quantities, industry level parameters α_i, δ_i and γ_{ji} are computed using Equations (2.9), (2.10) and (2.12).

Leaving out data on international trade, aggregate output is set to equal domestic consumption as given by Equation (2.5). This allows for β_i to be computed via Equation (2.13). Likewise, aggregate parameters α and δ are determined by Equations (2.19) and (2.20), respectively.

[33] Further details in Appendix 2.B.1.

For the no-IO model, domestic input-output data is also left out, so that both q_i and y_i equals industry value added va_i in real terms (divided by p_i); and μ_i equals β_i following Equations (2.13) and (2.22). Expectedly, the estimations for these shares vary significantly across the models.[34] More importantly, since total payments to factors[35] is unchanged, *factor shares* α_i *and* δ_i *are the same in both models.* Good and factor prices in the no-IO are also identical to those in the IO model, as well as all aggregate variables.

The dataset is constructed in MATLAB, where the models are solved. The productivity parameters A_i are calculated as the level required to match the output computed by the models to data values in each industry.

Finally, I classify the industries as either or not low-skill and high-skill intensive by comparing their skill intensity [δ_i and $(1 - \alpha_i - \delta_i)$] with the respective the national average [δ and $(1 - \alpha - \delta)$]. Since there are three factors in the model, an industry can be neither low- nor high-skill intensive —in which case it is capital-intensive— or it can be both, i.e. labour-intensive.

Factor and input shares

Figure 2.2 illustrates the main industry parameters computed as described in Section 3.2. Industry skill intensities (L: low and H: high) are labelled below their codes. Nominal shares of capital α_i, low-skill δ_i, high-skill $(1 - \alpha_i - \delta_i)$ and total intermediate inputs γ_i are respectively labelled as 'k_shares', 'l_shares', 'h_shares' and 'gammai'. It seems to be the case that high-skill intensive industries have somewhat lower input shares than capital-intensive ones.

[34] I present these differences in Appendix 2.A.5.
[35] Or, equivalently, value added va_i as given by Equation (3.14).

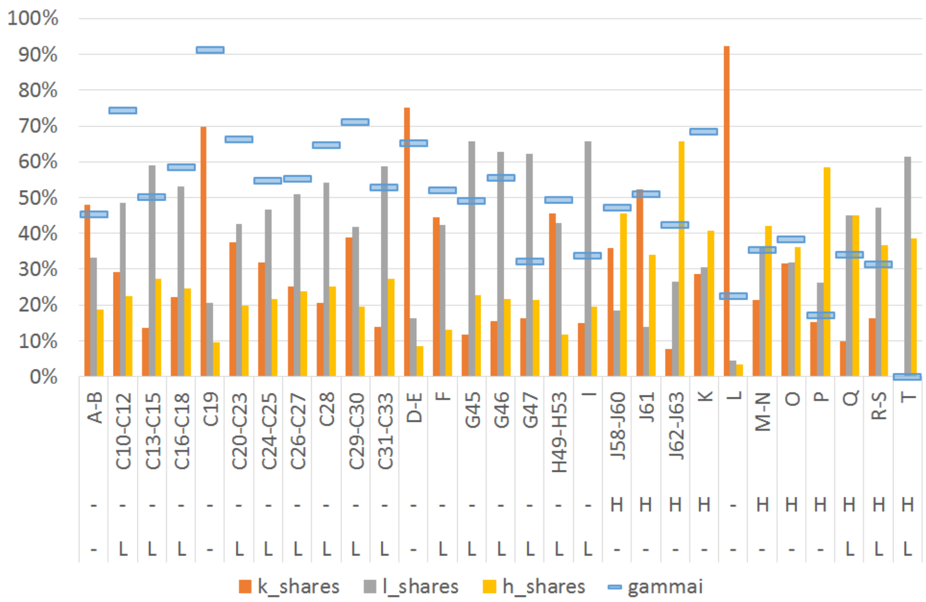


Figure 2.2: Nominal shares of factors and total usage of intermediate inputs (2014)

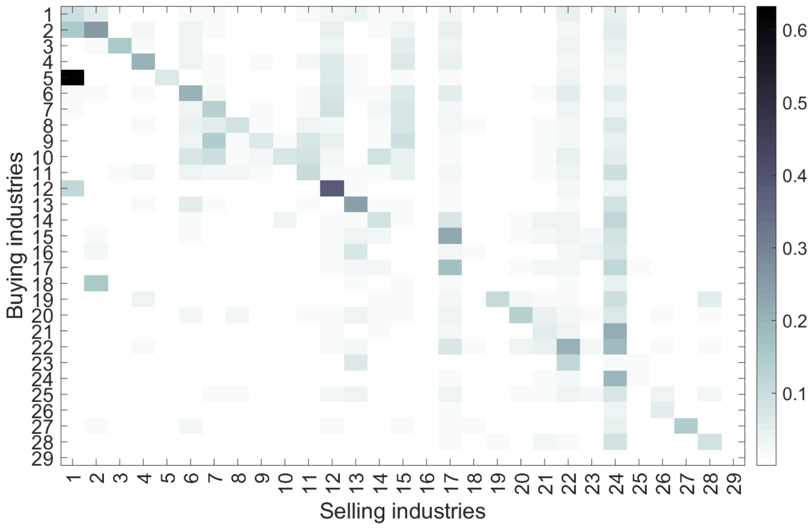
Input-output linkages

Figure 2.3 plots three different representations of the input-output linkages: (a) the IO matrix Γ transposed; (b) the Leontief-inverse transposed $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$; and (c) the matrix of the linkage weights. The IO matrix presented in plot 2.3a is transposed to have the same perspective as the other plots, i.e. each industry's use of other industries' goods are shown along the rows of the plots.

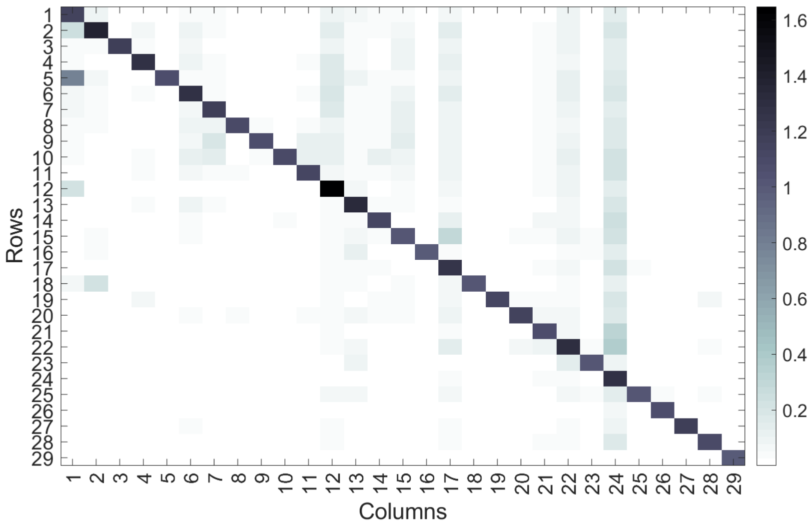
Even though the diagonal elements of matrix Γ are not particularly large, these values of both Leontief-inverse transposed and the matrix of linkage weights are prominent. This reflects the fact that each industry's own parameters tend to be more relevant in transmitting any exogenous shock than those of other industries. Whilst each row of the Leontief-inverse transposed spins from a buying industry i, the linkage weights include other industries' values of $(1 - \gamma_{ji})$. This may affects the ranking of the relevance of selling industries.[36] Both representations are helpful for the analysis.

Matrix $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$ summarises all the upstream transactions occurring to infinity. Notwithstanding, the full transmission of the shock relay of all industries to each particular industry is entirely subsumed by the linkage weights. Moreover, the latter is useful in depicting the relevance of the impact on every industry j to each industry i in a standardised range between zero and one.

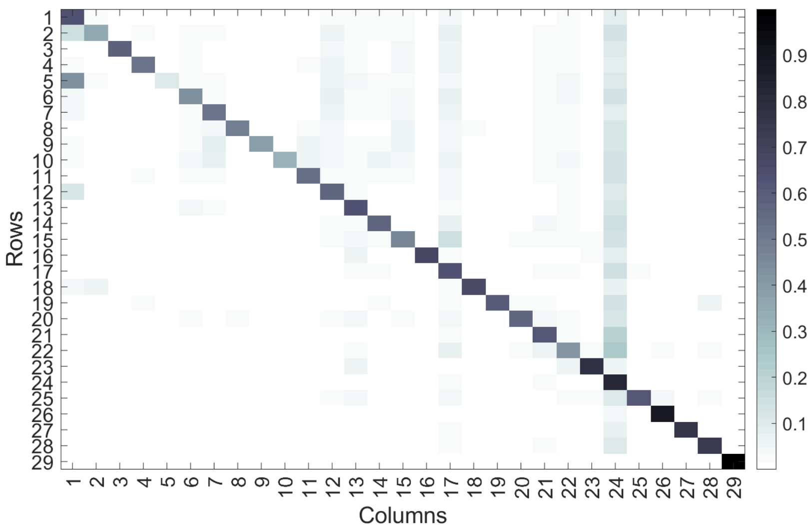
[36]Industry C19 is an example of such a case. I study this industry in detail in section 4.3.



(a) IO matrix transposed ($\mathbf{\Gamma}'$)



(b) Matrix $[\boldsymbol{I} - \mathbf{\Gamma}']^{-1}$



(c) Matrix of Weights

Figure 2.3: Representations of input-output linkages

3.3 Assessing the accuracy of first order approximations

The rich dataset conceived as a closed system allows for the full implementation of the general equilibrium model and computation of the counterfactual directly on MATLAB. Very few adjustments are required to make the data consistent with the model. The most significant relates to the single factor prices assumption, contradicting the data which has those varying across industries.

The exact numerical solutions are executed for both models in all the scenarios and presented in the results. Not relying on the model's first-order approximations is particularly important when the magnitude of the shocks differ significantly from the one-percent benchmark.

The examples below illustrate the divergence on the predictions of the effect on GDP of labour supply shocks of 1%, 10% and 100%. First-order approximation performs quite well for relatively small counterfactual changes (up to 10%) but not so well for larger ones. Recalling E stands for total labour supply, underscore-'obs' refers to values calibrated as explained in Section 3.2 while underscore-'cf' represents the values computed in the counterfactual scenarios.

- Counterfactual 1: 1% increase in E
  - Increasing the total supply of labour by one per cent: $E_{cf} = 1.01 \cdot E_{obs}$
  - Model's predictions: $\frac{\mathrm{d}\log Y}{\mathrm{d}\log E} = (1-\alpha) = 0.6563\%$
  - Results: $\frac{Y_{cf}-Y_{obs}}{Y_{obs}} = 0.6552\%$
- Counterfactual 2: 10% increase in E
  - Increasing the total supply of labour by one per cent: $E_{cf} = 1.1 \cdot E_{obs}$
  - Model's predictions: $\frac{\mathrm{d}\log Y}{\mathrm{d}\log E} = (1-\alpha) \times 10 = 6.563\%$
  - Results: $\frac{Y_{cf}-Y_{obs}}{Y_{obs}} = 6.455\%$
- Counterfactual 3: 100% increase in E
  - Increasing the total supply of labour by one per cent: $E_{cf} = 2 \cdot E_{obs}$
  - Model's predictions: $\frac{\mathrm{d}\log Y}{\mathrm{d}\log E} = (1-\alpha) \times 100 = 65.63\%$
  - Results: $\frac{Y_{cf}-Y_{obs}}{Y_{obs}} = 57.61\%$

Opting for the non-linear solutions

For the application to the A8 immigration that follows, I opt to use the non-linear solutions since they are the exact values and easily computable in MATLAB after the models are solved there. In fact, given the relatively small magnitude of the shocks — 3.15% for low-skilled and 2.22% for high-skilled workers— the first-order approximations are nearly the same as the non-linear results. I present a comparison of the main results produced by both approaches in Appendix 2.B.4.

4 Application to A8 Immigration in the UK

In May 2004, the European Union expanded from 15 to 25 member states. The ten newcomers included eight low-income countries, the so-called A8 group: Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia and Slovenia. The inflow of A8 workers to the UK was relatively high. According to UK Annual Population Survey (APS) Household Dataset, the stock of A8 workers in 2014 represented 2.76% of the UK employed labour force in 2014. This seemingly low share is actually substantial since it makes up about one quarter of all non-British nationals.[37] By skill group, the shares were of 3.15% and 2.22% of low-skilled and high-skilled workers, respectively.

To evaluate this inflow of workers, it would be ideal to have a UK economy without those immigrants as the base of comparison. To construct this counterfactual, however, I would have to determine what would have been the respective parameters and level of capital of this economy. The alternative approach adopted here is to simply remove the A8 stock in a given year and see how the output of the industries respond to this counterfactual shock. Therefore, I consider the removal of these workers in each group of labour as the counterfactual shock, using these percentages as the values for each numerical exercise. In section 4.4, I present a robustness check using the year before the EU expansion, i.e. 2003, as the base year in an attempt to test the other extreme case in which all parametric changes are associated with the

[37] This refers to the nationality declared by the employed people. The vast majority of the universe (over 90%) is of British nationals, which certainly include immigrants who already acquired British citizenship. Although the application process only required five years of legal residency, non-EU nationals would have much more incentive than EU nationals to engage in the request for British citizenship. Full data description in the Appendix 2.B.1.

A8 immigration.

The evaluation of the A8 is performed separately for each skill level and combined as a whole labour supply shock on the reference year of 2014 —the most recent for which there is data available. They are labelled *LS CF*, *HS CF* and *ES CF* for the low-skilled, high-skill and total labour supply counterfactuals, respectively. I compare the impact of the removal of those immigrants from the labour stock on the industrial gross output q_i produced under the framework of the models with and without input-output linkages.

The impact of these shocks on the main aggregate variables is plotted in Figure 2.4. The values refer to both models since by construction intermediate inputs affect only the distribution of the shocks but not the aggregate values. The removal of A8 workers causes a reduction in the GDP and in rental on capital of the same magnitude also due to the models' specifications. In the combined case, it reaches 1.81 per cent. The effects on low-skill and high-skill wages vary depending on the exercise being implemented. Since the stock of A8 low-skilled workers is larger than the stock of A8 high-skilled workers, its removal causes the largest impacts.

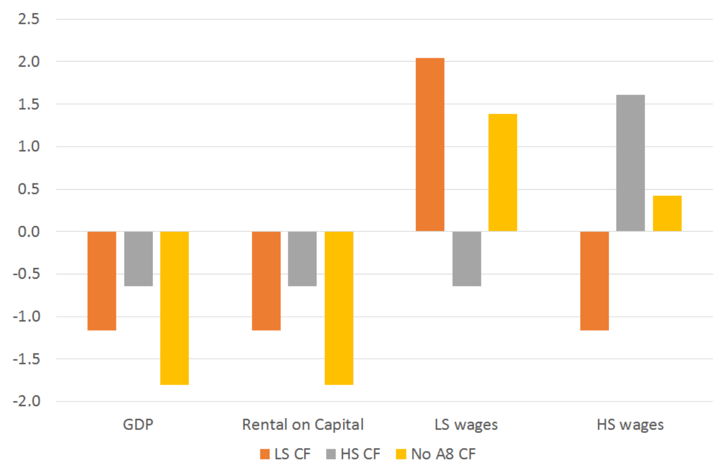


Figure 2.4: Counterfactual effects on aggregates measures (percentage change)

4.1 Results

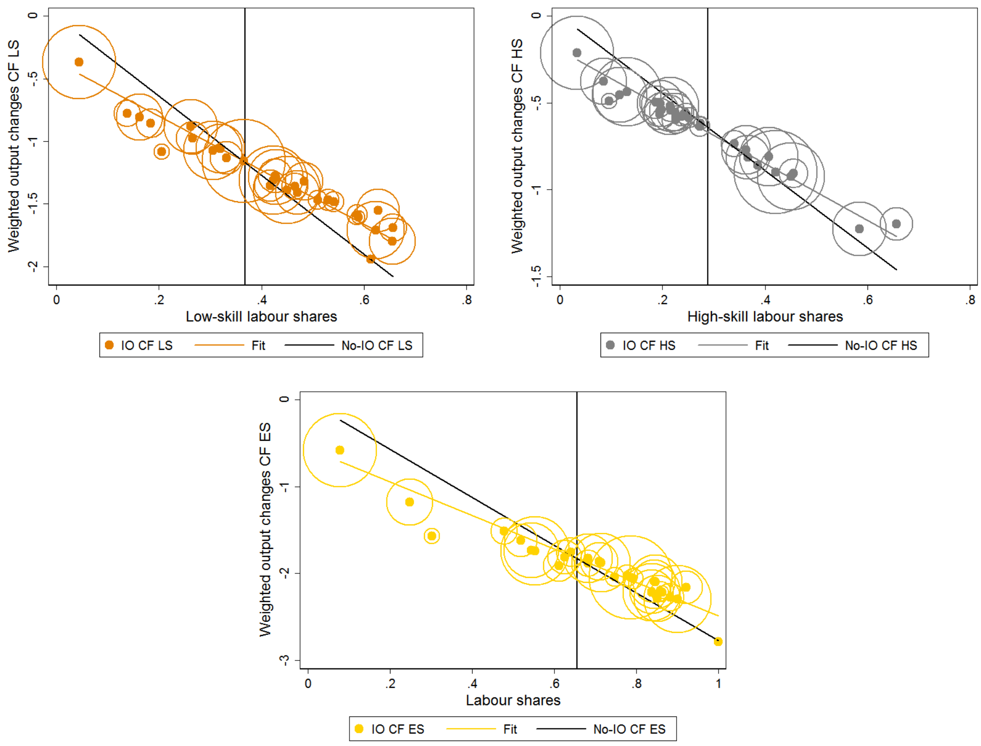
As showed analytically in Section 2.4, at the industry level, the no-IO model misestimates the output impact of the shocks, when compared to the full model. I quantitatively investigate these effects concerning the A8 immigration to the UK. In Section 4.1, I contrast both models predictions to industry output in the counterfactual scenarios to make explicit how the no-IO model's misestimation relates to the intensity of each industry in the shocked factor. Section 4.1 presents the glaring overall misestimation of the no-IO model for the output effect of the industries according to their shocked-factor intensity. In Section 4.1, I introduce a statistic key to posterior analysis: the ratios of the models' predictions for the industry output changes. Finally, Section 4.1 address the question of which industries, in particular, are the most affected by the A8 shock in contrast to the which have their output effect the most and the least misestimated by the no-IO model.

Output changes

Figure 2.5 shows the output changes produced by each pair of models resulting from the removal of the low-skill (CF LS), high-skill (CF HS) and all (CF ES) A8 workers. The downward sloping lines represent the predictions of the no-IO model, which amount to the magnitude of the shocks themselves times each industry's factor shares.[38] The colourful lines represent the linear regressions of the counterfactual changes calculated with the main model. The regression is weighted by each industry representativity in the total gross output of the economy, i.e. the size of each industry in terms of nominal gross output $p_i q_i$, as depicted by the hollow circles.

In each plot of Figure 2.5, industries on the left-hand side are defined as less intensive whereas those on the right-hand side are more intensive in the use of the factor than the national average. Unequivocally, the former group have their counterfactual output changes more negative while the latter present less negative output changes than those predicted by a model without linkages. Graphically, the fit lines for the counterfactual changes are less steep under the full than under the no-IO model.

[38] In the case of a one-percent shock, the locus of the predictions of the no-IO model over the respective factor shares would simply be the 45-degree line; a different magnitude of the counterfactual shock merely rotates this line.



Note: vertical reference lines represent the aggregate low-skilled ($\delta = 0.3677$) and high-skilled labour ($1 - \alpha - \delta = 0.2887$) shares.

Figure 2.5: Output changes over labour shares (weighted by output shares)

The labour supply shocks produce a milder impact on industries more intensive in the use of the affected factor than that predicted by a model without input-output linkages. In contrast, industries less intensive in the use of each factor are more affected in the IO model. The results suggest that there is a sort of transfer of the impact of the shock from more intensive to less intensive industries via intermediate inputs. Output q_i increases more than the no-IO for industries on the left and vice-versa for those on the right-hand side of the plot.

Overall no-IO misestimation

One way of assessing the global error of the model without input-output transactions is to sum the differences between the predictions of the model for each industry; graphically, summing the distances between the each dot and the black line in Figure 2.5. In the case of the total labour supply shock (CF ES), the sum of all the underestimation produced by no-IO model reaches 2.57 percentage points for the capital-intensive industries while the total overestimation is of 2.76 pp; overall, the no-IO overestimate the negative effect on the output of the industries by 0.19 pp.[39]

The coefficients of the regression lines plotted in Figure 2.5 can also give a glimpse of the overall misestimation produced by a model that ignores input-output transactions. Since the no-IO model predicts an impact in each industry equivalent to its affected factor share times the magnitude of the shock, the slopes of its fit lines are given by the latter with the intercepts nearly at the origin. The comparison of these lines with the fit lines of the IO model shows how much the no-IO model underestimate (overestimate) the magnitude of the shock for industries less (more) intensive in the affected factor.

Table 2.2 shows the values of those regression lines. As expected, the no-IO coefficients are such that the constants are roughly zero and the slopes are equivalent to the counterfactual shocks. The constants of the I-O model regressions indicate how much, on average, an industry using zero of the affected factor would have its output reduced solely due to I-O linkages. In the case of the total shock CF ES, this fall would be of 0.56%, comprising of 0.37% from the low-skill and 0.20% from the high-skill shock. The slopes indicate how much more the output of an industry would be reduced by each unit of labour share it has. On average, the factor share coefficients of the no-IO model are 42% larger than those of the I-O model.

In summary, the underestimation of the no-IO model for the output changes hits 0.56 percentage point (pp) in the limit,[40] whilst the incremental overestimation is of 42% on average.

[39] Using the nominal gross output to weight each industry's error results in much smaller differences, since it turns out that the most misestimated industries have low participation in the economy's gross output. Capital-intensive are underestimated by 0.09pp whilst labour-intensive industries are overestimated by 0.07pp.

[40] In the data, the lowest share of labour if of 0.08% for industry L, which presented an output decrease of 0.58% under the IO model and of 0.22% under the no-IO (a difference of 0.36 percentage points) whereas the biggest underestimation of the no-IO model (0.70 pp) regards industry C19, which has the third lowest labour share (30%).

Table 2.2: Fit lines for counterfactual output changes on factor shares

	ES	No-IO ES	LS	No-IO LS	HS	No-IO HS
e_shares	-1.93***	-2.75***				
	(0.14)	(0.10)				
l_shares			-2.16***	-3.16***		
			(0.13)	(0.00)		
h_shares					-1.63***	-2.22***
					(0.07)	(0.00)
constant	-0.56***	-0.02	-0.37***	-0.01***	-0.20***	-0.00***
	(0.10)	(0.07)	(0.06)	(0.00)	(0.02)	(0.00)
Observations	29	29	29	29	29	29
Adjusted R^2	0.963	0.967	0.969	1.000	0.980	1.000

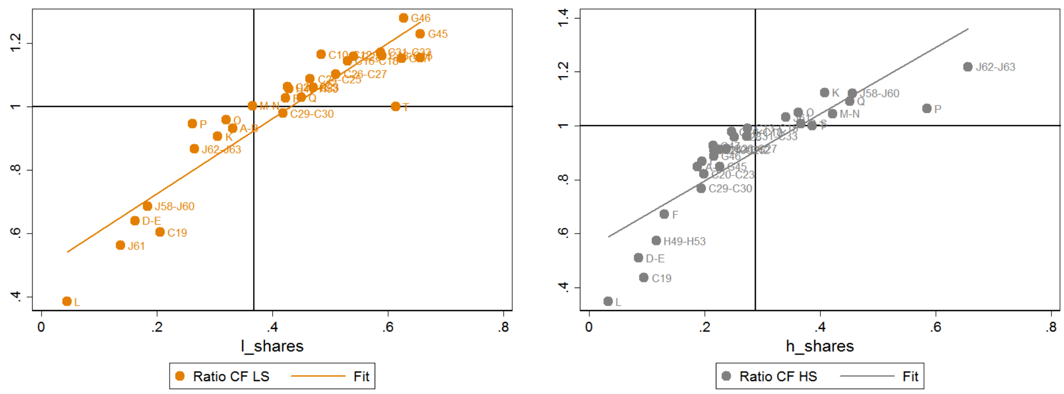
Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: Stata's weighting option *pweights* denotes the inverse of the probability of each observation.

Ratios

It is clearly the case that the industries are dissimilarly affected by the labour shock. The ratio of the prediction of the no-IO model over that of the IO model gives an idea of how much the former misestimates the impact on the output of an industry. Indirectly, it is a measure of how much the input-output linkages affect each industry's outcome beyond the expected influence of its own labour shares.

In the A8 counterfactual exercises, all shocks (removal of A8 workers from labour force) and calculated impact (counterfactual output growth) are negative. Thus, the calculated ratios (impact estimated by the no-IO over impact of IO model) are all positive. The no-IO model underestimates the impact of a counterfactual shock if its calculated values are less negative than in the IO model, i.e. the ratio of the impacts calculated by each model is below one, while the reverse applies for an overestimation. This corresponds to the points being below or above the horizontal reference line in Figure 2.6.



Note: vertical reference lines represent the national shares of low-skilled ($\delta = 0.3677$) and high-skilled labour ($1 - \alpha - \delta = 0.2887$).

Figure 2.6: Ratio of immigration effects (no-IO / IO)

The results are in line with the expected predictions of the models, with most ratios lying on quadrants I or III of the plots. The impacts of the CF LS (left plot of Figure 2.6) tend to be underestimated for industries less intensive in low-skill labour (left-hand side of the vertical line) and overestimated for the low-skill intensive industries. Similar pattern occurs with the ratios of the CF HS (plot on the right-hand side of Figure 2.6) regarding more and less high-skill intensive industries.

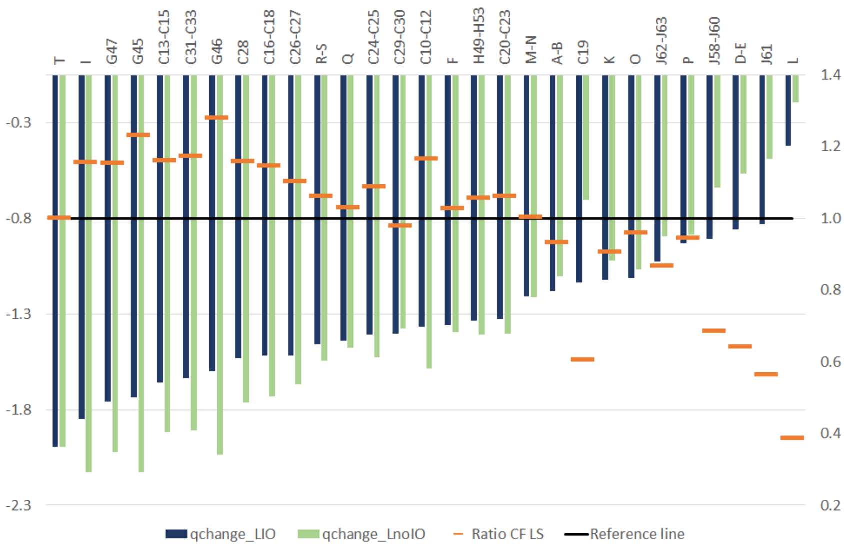
Most and least affected industries

The disparities between the models on the predicted output effect of the A8 immigration over the industries are substantial. The no-IO model portrays the impact of a labour shock as simply proportional to each industry's labour share. In reality, the outcomes are a combination of the labour shares and the input-output transactions of all industries. Therefore, the industry most misestimated by the no-IO model does not necessarily present extreme values of output changes or labour shares.

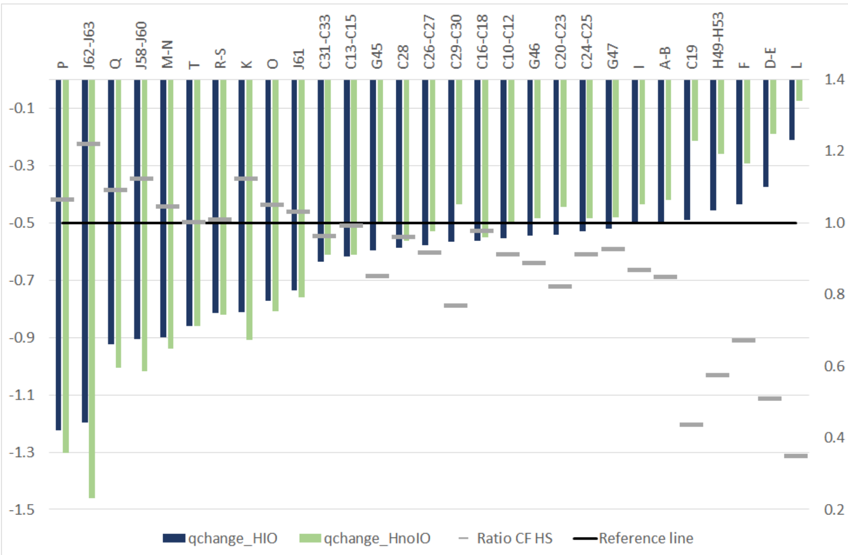
Figure 2.7 plots the models' predictions for the industry output changes alongside their ratios.[41] Notice that the counterfactual impacts on industry T ("Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use") are nearly the same in both models, i.e. the ratios are equal to one. This is a consequence of its close to zero use of intermediate inputs (γ_{ij}). Hereafter, this industry is ignored in the comparisons but remains in the plots

[41] The values plotted are a combination of those in Figures 2.1, 2.5 and 2.6. See Appendix 2.C.1 for numerical comparisons.

as a point of reference for the analysis.



(a) LSCF most least



(b) HSCF most least

Figure 2.7: IO and no-IO output changes (left) and ratios (right)

In the case of the CF LS (Figure 2.7a), the largest output decline is suffered by industry I "Accommodation and Food Service Activities" (-1.80 per cent), while industry G45 "Wholesale and retail trade and repair of motor vehicles and motorcycles" has the largest no-IO output drop (-2.08%). Notwithstanding, the most overestimated output fall is for industry G46 "Wholesale trade, except of motor vehicles and motorcycles". On the opposite end, the smallest output decline predicted by models IO and no-IO are both for industry L "Real estate activities" (-0.37% and -0.14%, respectively), which also has the most underestimated output change. Finally, in the group of those industries getting similar predictions by both models there are industries M-N “Professional, scientific, technical, administrative and support service activities” and C29-C30 “Transport equipment”, with rather similar low-skill shares (42% and 37%, respectively). The latter is an interesting case, since it is the only low-skill intensive industry (slightly) underestimated by the no-IO model.

In the case of the CF HS (Figure 2.7b), the largest output decline is suffered by industry P “Education” (-1.23 per cent), while industry J62-J63 “IT and other information services” has the largest no-IO output drop (-1.46%). The latter also happens to be the most overestimated by the no-IO model. Industry L “Real estate activities” has once more the smallest output decline predicted by the models (IO -0.21% and no-IO -0.07%) and is the most underestimated. The industries barely misestimated by the no-IO model are R-S “Arts, entertainment, recreation and other service activities” and C13-C15 “Manufacture of textiles, wearing apparel and leather products” which have fairly different high-skill shares (38% and 29%, respectively).

In both counterfactuals, industries L “Real estate activities”, D-E “Electricity, Gas and Water Supply” and C19 “Manufacture of coke and refined petroleum products” have their output contractions consistently underestimated by the no-IO model. Being capital-intensive industries, they are relatively less intensive in both low- and high-skilled labour, suggesting that the transmission of the labour supply shocks via input-output linkages is especially relevant for those industries.

4.2 The role of input-output linkages

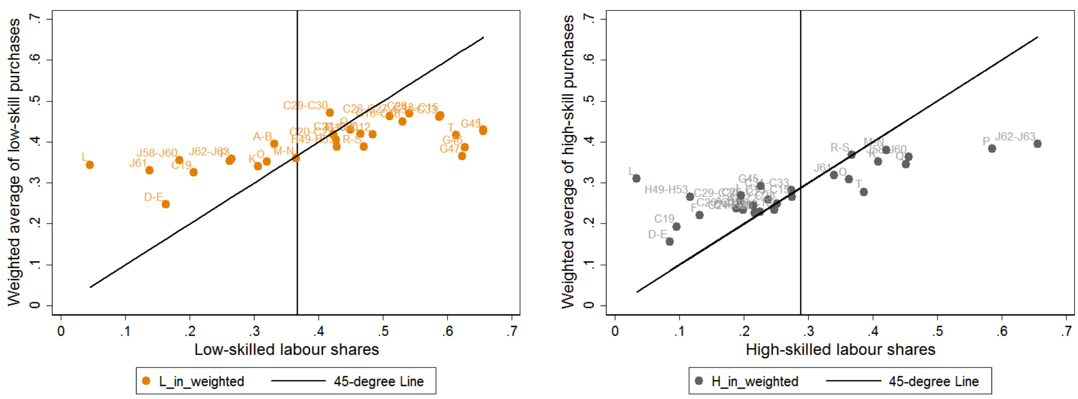
The no-IO model misestimation of the output impact of the shocks occurs because it ignores the transactions of intermediate inputs across industries, depicting the impact of a shock on an industry solely as its affected-factor share. In reality, the IO linkages counterbalance the impact of a factor supply shock in a specific industry, as industries intensive in the altered factor transact with others both more and less intensive in that factor.

In Section 4.2, I show that the more intensive an industry is in a factor, the more likely it is to buy from relatively less intensive industries and, therefore, to have part of its impact reduced. The reverse is also true, so less intensive industries have their impact enlarged by IO linkages. When compared to one industry's own shocked-factor share, the results confirm the expectations that more (less) intensive industries which are the most over(under)estimated by the no-IO model tend to use inputs less (more) intensive in the shocked factor.

The relationship between the differences in the models predictions and the shocked-factor intensity of input use is not a one-to-one, however. Higher order input transactions also play a role. In Section 4.2, I present the quantitative results for the approximations of the shocks to confirm that industries most misestimated by the no-IO model tend to be the most upstream connected into the production network.

Skill content of input purchases

The weighted average of the sellers' skill intensity tend to be smaller than the buyer's for industries intensive in the use of that factor and reversely for an industry in the other side of the skill spectrum. Figure 2.8 plots the weighted averages given by Equations (LSC) and (HSC) over skill intensity. It shows that this pattern is true with very few exceptions, with only industry C29-C30 in Figure 2.8a worthy of note. Within categories, however, it is also true that industries tend to buy moderately more inputs from their own skill-intensity peers.

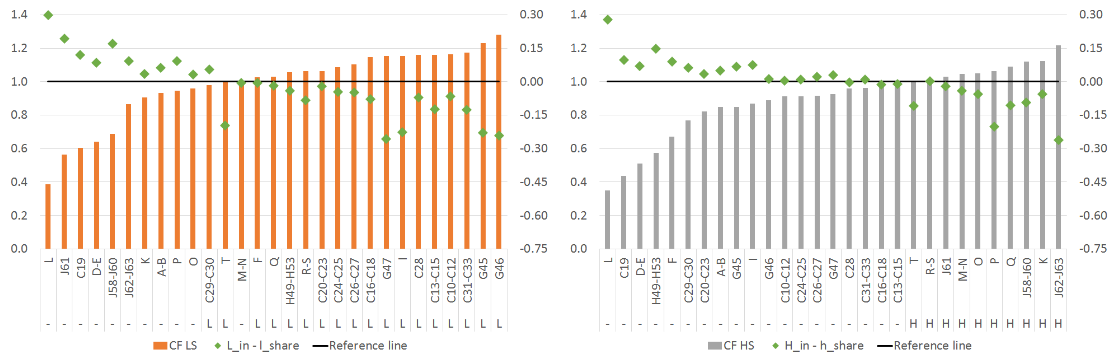


(a) Low-skill content of input use

(b) High-skill content of input use

Note: low- and high-skill content of the industry input basket calculated via Equations (LSC) and (HSC), respectively.

Figure 2.8: Skill content of input bundle over skill intensity



(a) Ratios of models' output impact (left); relative low-skill intensity of input use (right)

(b) Ratios of models' output impact (left); relative high-skill intensity of input use (right)

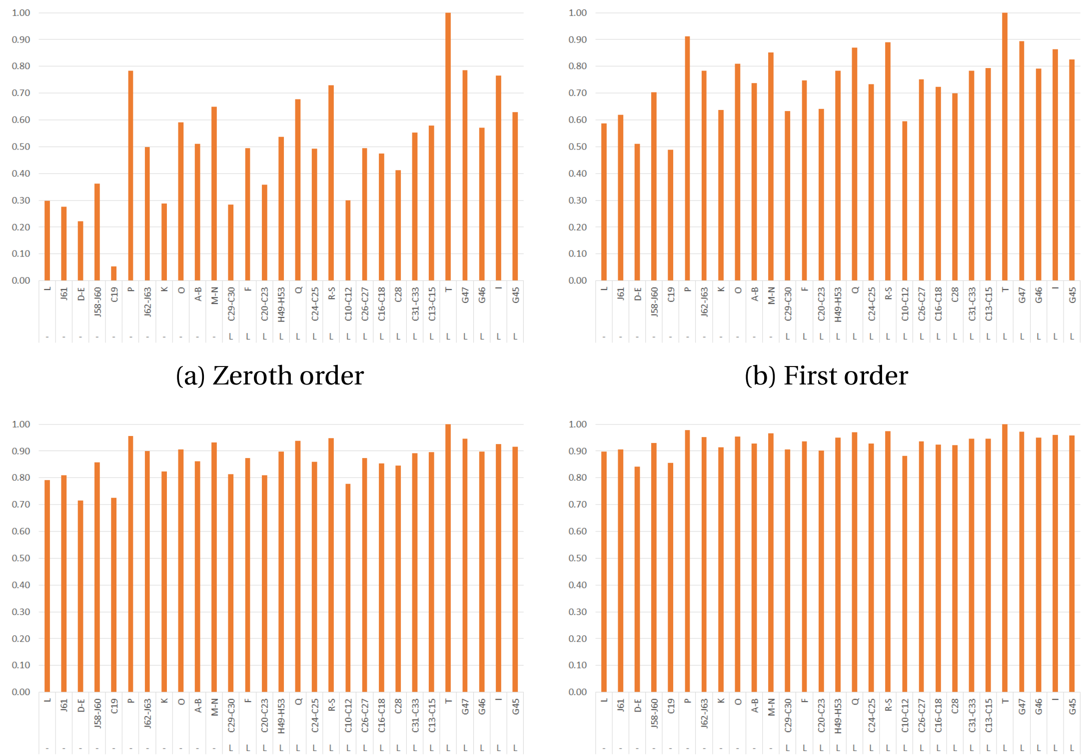
Figure 2.9: No-IO's misestimation and relative skill content of input bundle

Figure 2.9 plots the industries' relative skill intensity of the input basket as the green diamonds (right axis) and the bars of misestimation ratios (left axis) ranked by the latter. The plots for LS CF and HS CF show a negative relationship between the former and the latter. These patterns partially confirm the expectations, i.e. the further the skill intensity of an industry's input bundle is in relation to its own, the more the model without input-output linkages misestimates the impact of A8 immigration.

This is not an infallible rule, however. As the jumps in the green diamonds show, the skill content of each industry's intermediate input use compared to its own do not fully explain the differences in the models' predictions. I explore the higher order of input-output connections in the next section.

Speed of convergence of the shocks' approximations

Figures 2.10 and 2.11 plot the ratio of each order of approximation to the correspondent total impact on the industries. In the the low-skill labour supply shock example, this corresponds to each order ($IO_{LS}0$), ($IO_{LS}1$) etc. divided by (IO_{LS}) for each industry. These ratios give the percentage of the total impact explained by each order of approximation in each industry and thus gauge the relevance of the indirect intermediate input transactions. The faster the fraction explained converges to 100% the less relevant the transactions farther along the chain are for an industry. Reversely, the industries showing the lowest ratios by the third order are assumed to be those whose transactions are more entangled into the production network for the goods intensive in the affected factor.



(a) Zeroth order

(b) First order

(c) Second order

(d) Third order

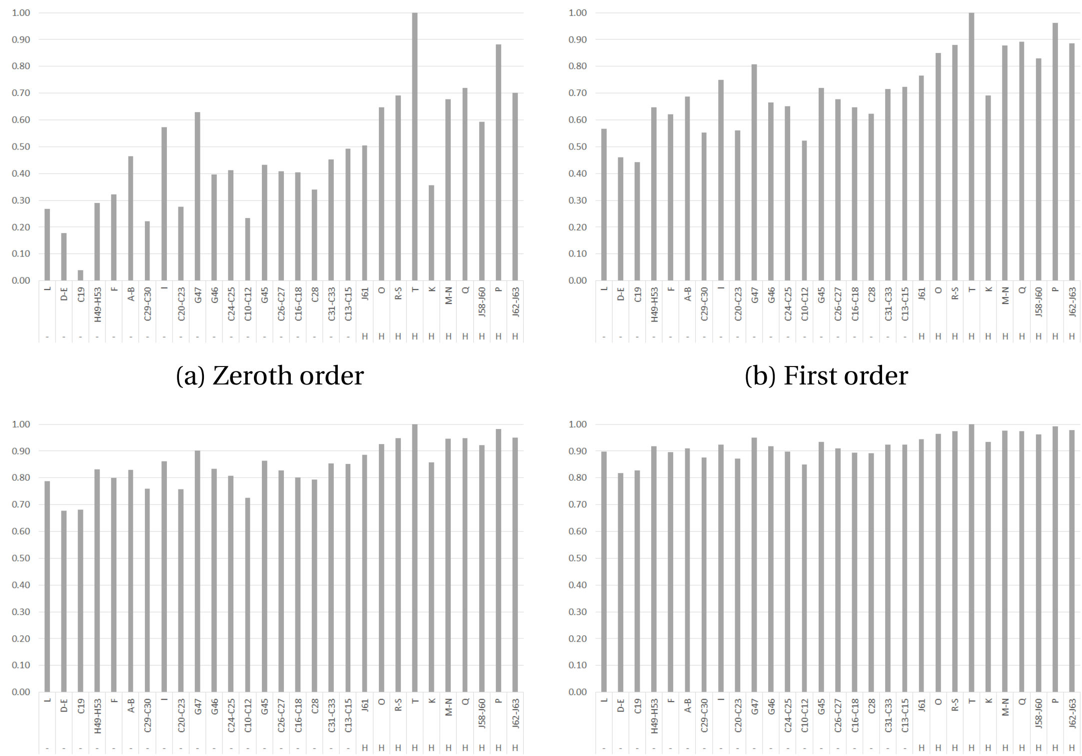
Figure 2.10: LS CF ratios of approximation (sorted by low-skill shares)

In the calculations of these ratios, I assume one-percent reductions in the stock of low-skilled and high-skilled labour. I do this for simplicity, since the magnitude of the shocks vanishes when computing the quotients.

Overall, the first-order impact as given by the ratio of the zeroth order approximation is on average of about just 50% of the total effect, showing the importance of higher order IO effects.[42] Moreover, the average first order effect is of just 73% of the total across all industries, pointing that a quarter of the effect is still unaccounted for. Only by the third order of approximation that all industries have their output impact more than 80% appraised.

The three industries showing the lowest ratios are the same in both counterfactuals: D-E, C19, C10-C12, L, C20-C23 and C29-C30. Interestingly, two industries present very low speed of convergence for the CF LS but relatively fast for CF HS: J61 and K; and other two have ratios converging relatively fast in the CF HS but slow in the CF LS: F and A-B.

[42]Reminding that industry T “Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use” is ignored in this comparison since it has no capital nor intermediate input use. Thus, the no-IO model estimates exactly same impact as IO model in both counterfactual exercises. It stays in the plots as a reference point.



(a) Zeroth order

(b) First order

(c) Second order

(d) Third order

Figure 2.11: HS CF ratios of approximation (sorted by high-skill shares)

As expected, the slow speed of convergence and therefore the upstream connectedness of the industries help to explain the discrepancies in the negative relationship between the direct input purchases and the models' differences plotted in Figure 2.9. To name a couple of examples, industries C19 and D-E are both significantly underestimated by the no-IO model, even though they do not present input use significantly more intensive in the use of the shocked factor in comparison to their own, in both counterfactuals.

In the group of fastest convergence, there are industries Q and R-S, labour-intensive services not much connected within the production network.[43] After these two, the five industries presenting the highest ratios for the HS CF are all high-skill intensive (J58-J60, J62-J63, M-N, O and P) while for the LS CF the group is comprised of a mix of low- and high-skill intensive industries (G45, G47, I, M-N and P). The result for the low-skill shock (LS CF) is surprising because the zeroth-order ratio itself is expected to be higher for industries directly affected.

In this section, I showed that higher order input-output effects are important for

[43] Figures 2.C.54 and 2.C.56 in the Appendix show the input-output linkages of these industries.

transmitting supply shocks, with effects very heterogeneous across industries depending on their connection to the network. In the next section, I move on to case studies in order to explore this further, and explain key linkages in the empirical UK network.

4.3 Specific industries

The analytical framework I presented is especially useful in explaining the output impact on industries that have non-trivial combinations of parameters. I select three industries for an in-depth analysis based on their particularities, namely:

1. L "Real estate activities" is the most underestimated by the no-IO model even though it has the third lowest total input share;
2. C19 "Manufacture of coke and refined petroleum products" has the highest input share which is heavily skewed towards one supplier and notwithstanding it is highly connected in the input-output network;
3. J61 'Telecommunications" presents opposing transmissions for the low-skill vis-à-vis the high-skill shock.

These case studies demonstrate the proficiency of the model. I show that the resources provided by the developed framework can successfully extricate complex input-output linkages to explain ambiguous industry-level results while providing a comprehensive understanding of a country's industrial interconnections.

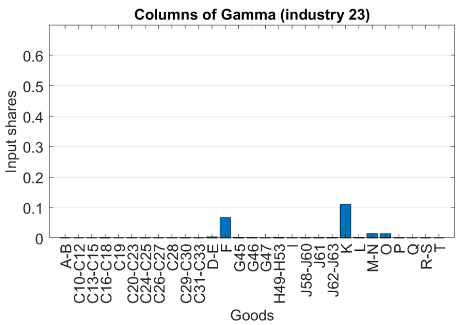
L: "Real estate activities"

Being the most capital-intensive industry, with a capital share of 92% (Figure 2.2), industry L "Real estate activities" is the least affected in terms of output fall according to both models in both counterfactuals. Given their own small shares of low- (4%) and high-skill (3%) labour, the discounted skill content of their input basket is by far the largest, which helps to explain the shock transmission. As Figure 2.9 shows, L's output drop is the most underestimated by the no-IO model.

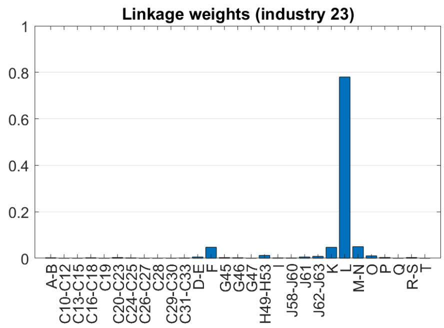
This indicates that the linkage effects are quite relevant even though its total usage of intermediate inputs (Figure 2.2) is the third lowest at 23%. As shown by Figure 2.12a, this is comprised of very few goods: mostly from industries K "financial and insurance activities" (11%) and F "construction" (7%); to a lesser extent from industries M-N "Professional, scientific, technical, administrative and support service activities" and O "Public administration and defence; compulsory social security" (both around 1%).

Despite the low direct purchases, indirectly industry L seem to be well connected in the production network via industries K, F, and M-N. Interestingly, transactions of highest orders also play a role here, as industries K and F also rely significantly[44] on inputs from industry M-N.

These indirect linkage effects clearly show up in the shock approximations, as industry L presents a rather slow convergence compared to other industries (Figures 2.10 and 2.11). It figures within the third and the sixth lowest ratios depending on the counterfactual and order of approximation. Interestingly, its first order approximation is not as low as that of industry C19 which reflects the highest degree of relevance of its own factors (Figure 2.12b).



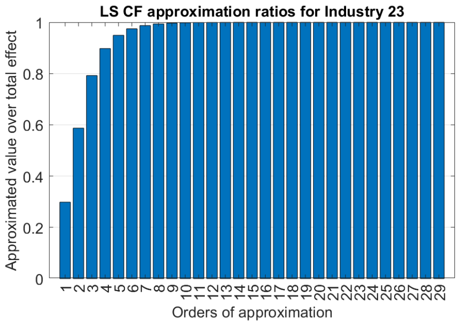
(a) Nominal shares of purchases (γ_{ji})



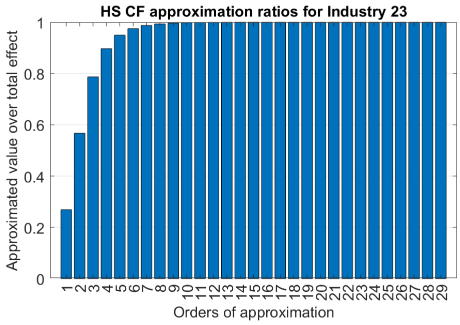
(b) Weights

Figure 2.12: Industry L's linkage effects

[44]Figures 2.C.44 and 2.C.26 in the Appendix.



(a) Approximations LS CF shock



(b) Approximations HS CF shock

Figure 2.13: Industry L's approximations for the shocks

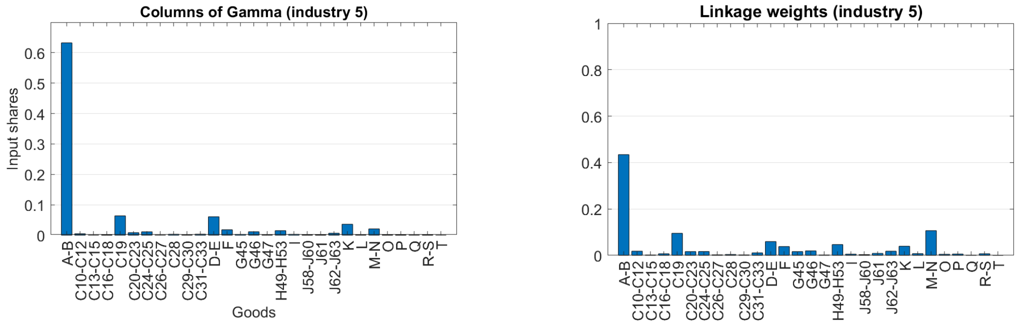
C19: "Manufacture of coke and refined petroleum products"

Similarly to L, industry C19 "Manufacture of coke and refined petroleum products" is heavily capital-intensive (third largest capital share at 70%, Figure 2.2) and significantly underestimated by the no-IO model in both counterfactuals (Figure 2.9). Unlike industry L, C19 presents the highest use of intermediate inputs at 91% (Figure 2.2), most of it from industry A-B "Agriculture, Forestry and Fishing & Mining and quarrying" with a share of 63% (Figure 2.14a).

These two facts combined make industry C19 the only one presenting another industry almost as relevant as itself in its Leontief-inverse transposed row (Figure 2.3b) and, more curiously, not having its own linkage weight higher than all others (Figure 2.14b).[45] Unsurprisingly, even the direct input-output effects are quite relevant for this industry, reflected in the lowest ratio of the zeroth order of approximation and largest gap to the first order approximation among all industries (Figures 2.10a and 2.11a).

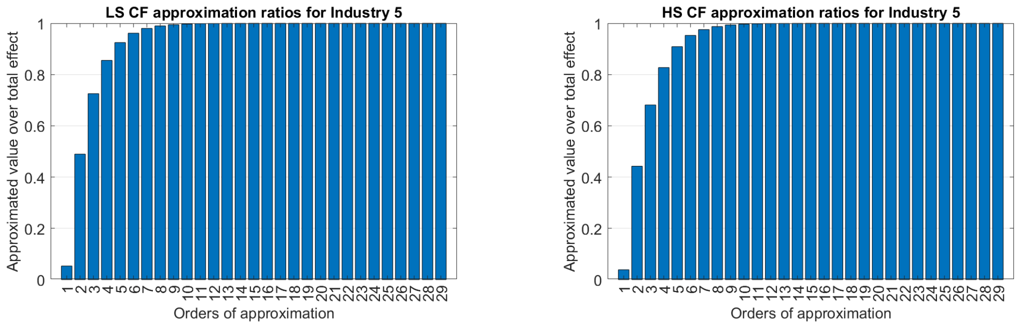
But once again, there are more than one tier of transactions at play, as industry A-B is not only relevant as a supplier to C19 but also to industry D-E "Electricity, Gas and Water Supply" (Figure 2.C.24a), another significant supplier to C19. All in all, industry C19 seems to be well connected in the production network, presenting relatively low ratios throughout all initial orders (Figures 2.10 and 2.11).

[45]Note that the linkage weights combine the rows of the Leontief-inverse transposed with each industry's total factor share $(1-\gamma_i)$, the latter being a very low value for industry C19 given its very high total input use.



(a) Nominal shares of purchases (γ_{ji}) (b) Weights

Figure 2.14: Industry C19's linkage effects



(a) Approximations LS CF shock (b) Approximations HS CF shock

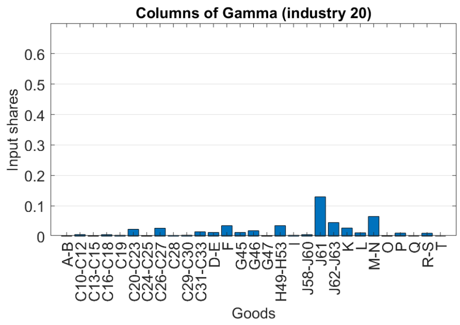
Figure 2.15: Industry C19's approximations for the shocks

J61: "Telecommunications"

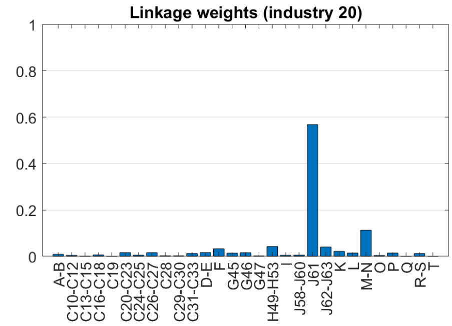
This is a case of an industry presenting significantly different transmissions for the low-skilled compared to the high-skilled labour shocks. J61: "Telecommunications" illustrates that even though the linkage weights are unique for each industry, independently of the shock, the factor shares of the supplying industries compared to one's own are what give the colours of the final results.

Industry J61 is the second most underestimated by the no-IO model in the LS CF, partially reflecting the much larger than its own low-skill intensity of its input purchases (Figure 2.9). It has an average overall input share at 51% (Figure 2.2), relatively well spread among several industries. Having the second smallest low-skilled labour share (14%), nearly all industries J61 buys from transmit the shock in a way not captured by the no-IO model. This explains the rather slow rate of convergence for the LS CF shock, in particular the low ratio for the zeroth order approximation.

Regarding the HS CF, the picture is quite different. Having a high-skilled labour share just slightly above the national average, its input purchases are intensive in the use of that factor nearly as much as itself. This explains why the no-IO model only mildly overestimates the output decrease of this industry in the HS CF. Moreover, most of its direct input purchases comes from high-skill intensive industries, which explains its relatively larger ratios of approximation from the first order upwards.

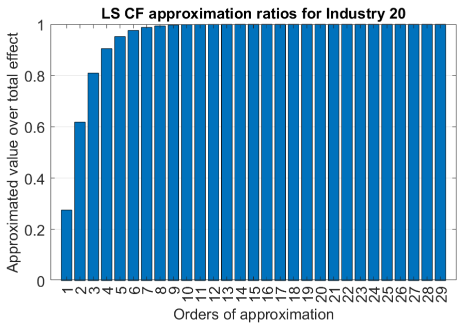


(a) Nominal shares of purchases (γ_{ji})

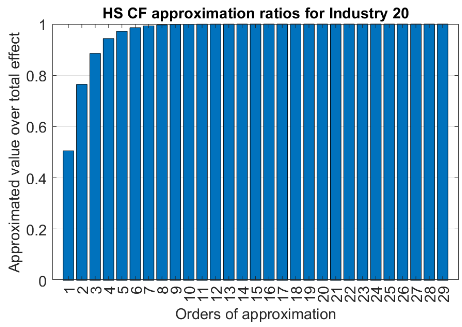


(b) Weights

Figure 2.16: Industry J61's linkage effects



(a) Approximations LS CF shock



(b) Approximations HS CF shock

Figure 2.17: Industry J61's approximations for the shocks

4.4 Robustness check

In the counterfactual exercises performed, I implicitly assumed that the labour supply had no effect on the parameters. Of course, this is an extreme simplification when, in reality, it is reasonable to expect that the industries may adjust their production technology to the relatively higher supply and lower prices of factors and goods. Dustmann and Glitz (2015) explicitly investigate the response of industries to the relative abundance of a given skill group and find that most of the adjustments occur in the form of changes in the production technology, namely the relative skill intensities.

Motivated by this finding, I investigate the other extreme assumption: to suppose that all the observed changes in the parameters from 2003 to 2014 were a result of the A8 immigration. One of the advantages of working with a numerical implementation in MATLAB is to be able to alter some constituents of the economy when performing counterfactuals. I present here the results of an alternative counterfactual where the UK's economy in 2014 has not only the labour supply —as in the main counterfactual exercises— but also all the technology parameters of 2003.[46] The idea is that, in reversal, removing the stock of A8 workers would also bring back the production patterns used before the EU enlargement.

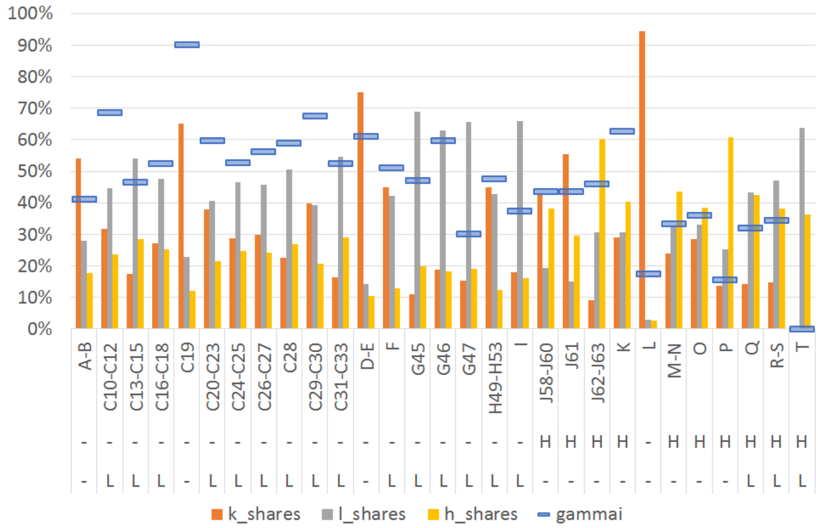


Figure 2.18: Nominal shares of factors and total usage of intermediate inputs (2003)

A visual comparison of parameters can be done by contrasting Figure 2.18 with Figure 2.2, which display the values regarding 2014. It shows some but not much change over this span of 10 years. This is also true for the IO matrices, plotted in Figure 2.19 for both comparison years.

Regarding the output changes across the industries, the results are only moderately affected. The overall underestimation (slope) is now smaller, at 32% compared to 42%, while the intercept takes the value of -0.42 in contrast to -0.56 (Figure 2.20 and Table 2.3).

[46]Due to a data limitation, the values of the skill shares of the industries, which comes from KLEMS, correspond to year 2008, the earliest available.

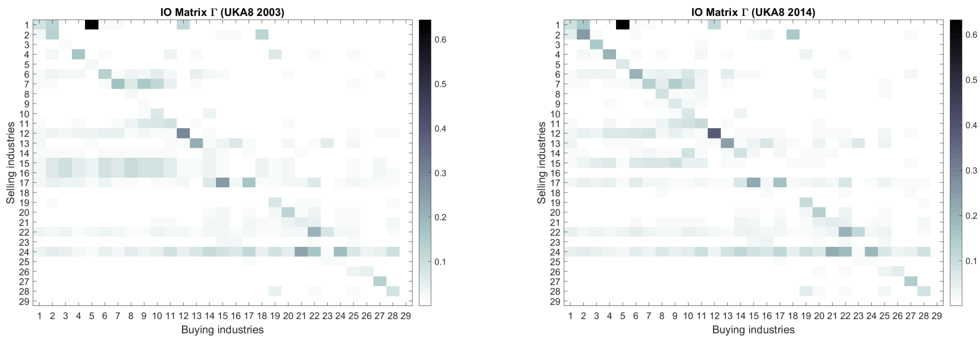


Figure 2.19: IO Matrix (2003 vs 2014)

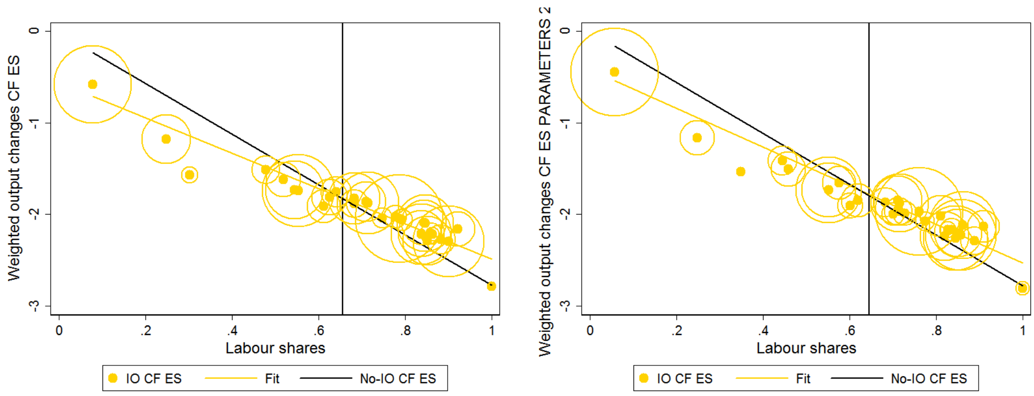


Figure 2.20: Output changes with parameters from 2014 (top) vs 2003 (bottom)

Table 2.3: Fit lines for output changes on factor shares PARAMETERS 2003

	ES	No-IO ES
e_shares	-2.11***	-2.77***
	(0.12)	(0.09)
_cons	-0.42***	-0.01
	(0.09)	(0.07)
N	29	29
adj. R^2	0.97	0.97

Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: Stata's weighting option *pweights* denotes the inverse of the probability of each observation.

5 Conclusions

In this chapter, I developed a theoretical framework to study the role of input-output linkages in the transmission of a skill supply shock. I showed that ignoring these interconnections can lead to misestimations of about half of the impact of the shock on specific industries.

I made three main contributions. First, I constructed a rich yet tractable input-output model with two levels of skills, for which I derived closed-form solutions for all endogenous variables. Then, I brought the model to the data, applying it to the A8 expansion to the UK, which brought low-skilled and high-skilled workers representing about one quarter of the country's immigrant labour force. Finally, a comprehensive set of analytical results were produced to fully understand the transmission of a supply shock through the production network.

Remarkably, I demonstrate that, unlike in its twin model without input-output, in the main model the effect of an increase in labour supply on the output of an industry is not solely dependant on its share of labour use. There is a transfer of output gains from more intensive to less intensive industries reflecting their input use. The fact that more (less) intensive industries tend to purchase intermediate goods from less (more) intensive industries gives the right intuition for the relay of the skill shocks along the intensity spectrum. However, it considers only the direct purchases among producers represented by the input shares and thus disregards the whole upstream chain of transactions.

To fully capture the misestimations resulting from disregarding input-output connections it is crucial to study also the indirect or higher orders of interactions. I established that the elements defined here as the linkage weights calibrate the importance of each supplying industry while summarising all the upstream transactions relevant to each producing industry. In fact, they are the true way in which each industry receives the isolated impact of all other industries and, in that way, how the shock spread out throughout the production network. Moreover, a decomposition of the Leontief-inverse transposed into infinite terms of a summation paved the way for a thorough characterisation of the higher order of input transactions affecting each industry.

The extensive analytical framework allowed for an elucidative understanding of the impact of the A8 immigration on the UK industries. In the case of high-skilled workers, the most affected industry is not 'IT and other information services' as predicted by the no-IO model, but 'Education' while for low-skill immigration the largest impact was felt by 'Accommodation and Food Service Activities' and not 'Wholesale and retail trade and repair of motor vehicles and motorcycles'. In both cases, the misestimated industries are well connected within the production network which worked to mitigate the impact on their output.

5.1 Limitations

Naturally, this chapter has a few limitations. Most of them resulting from modelling choices that offer the benefit of parsimony at the expense of completeness.

First, regarding the characteristics of the immigrant versus the native worker, to presume that they are homogeneous in every economically relevant dimension is to blatantly ignore the whole literature on immigration. This is certainly a limitation of the model regarding any application on immigration issues but does not compromise the study of other skill supply or labour shocks.

Secondly, the adoption of a Cobb-Douglas production function implies a unitary elasticity of substitution between each pairwise combination of factors and intermediate inputs. Although including two levels of skills separately is better than adding them together —which would imply perfect substitutability, i.e. infinite elasticity—, the labour literature is unanimous in estimating the elasticity of substitution between skilled (college-educated) and unskilled workers in the range of 1.2 to 1.4 (Acemoglu and Autor, 2011). Likewise, the studies consistently suggest the existence of complementarity (elasticity of substitution smaller than one) between capital and skilled-labour (Krusell, Ohanian, Ríos-Rull, and Violante, 2000; Lewis, 2011). Knowingly this does not speak to the literature on skill-biased technical change, polarization or automation.[47] Clearly, if a set of skills, or labour in general, becomes dearer it would be more likely substituted by another factor or even a good. It is not trivial to predict how much these simplifications affect the analysis, but an extension of this chapter adopting a constant elasticity of substitution (CES) specification would certainly be a fruitful road of research. The downside would be the abandonment of closed-form solutions to the sole reliance on first-order approximations.

Furthermore, the focus of this paper on industry output quantities (q_i) can be considered less relevant from the producer's point of view than the industry output val-

[47]See Goldin and Katz (2007), Autor, Katz, and Kearney (2006), Acemoglu and Autor (2011) and Acemoglu and Restrepo (2020).

ues ($p_i q_i$). Indeed, a labour shock will affect both the equilibrium physical quantity and the price of a good in the model. However, given the Cobb-Douglas structure for industry demands, my model contains the well-known result that the industry-level sales are simply denoted by their Domar weights in equilibrium, i.e. $p_i q_i = \mu_i Y$. This means that the change in sales at the industry level following a change in labour supply is the same for all industries since they are all equivalent to the change in the GDP. Besides, there is a very interesting IO structure that I am able to focus on, namely the Leontief-inverse transposed, which gives me clean results on how changes in q(i) depend on the IO structure. Hence, focusing on industry sales, rather than physical quantity, is not interesting in this chapter. However, one might rightly be concerned that industry sales, or even employment, might be more important in reality. Doing so requires altering the structure of the model so that the novel IO effects that I identify for physical quantities can spill over to other variables. This is an interesting avenue for potential future work.

Additionally, a related limitation of the model lies in having the technology parameters exogenously given. This simplification is at odds with the profusion of studies documenting that the technical progress in the past few decades has been interacting with the relative supply of skills Beaudry, Doms, and Lewis (2006); Voigtländer (2014) indicating that there is a within-industry adjustment mechanism to skill imbalances as well as the between-industry one studied here (Lewis, 2013; Dustmann and Glitz, 2015). Moreover, the model does not include international trade, which also plays a role in the technological choices of the industries. Partial equilibrium models, such as Caselli and Coleman (2006), suggest alternative formulae to endogenise the technology parameters at the expense of making wages exogenous variables. This chapter, on the other hand, considers both labour demand per industry and aggregate wages as endogenous while making use of the exact values for factors and input shares provided by the dataset in each year of analysis. Insofar as questions remain regarding the validity of the results, robustness check exercises testing alternative parameters can be performed. In the case of this chapter's application, the robustness test showed very little influence of the change of the parameters on the results in the period studied.

Finally, other simplifications, such as assuming perfect competition —and therefore same equilibrium factor prices across industries— and closing the international dimension of the economy, may also compromise the analysis and results it produced. To address the former would require a departure from this family of neoclassical input-output models, which is not currently desired given that there is still much to be explored within this framework. The latter limitation, however, is being overcome in my next chapter, where I extend the main model to incorporate international trade.

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2.A Model's derivations

2.A.1 Irrelevance of intermediate inputs in aggregate terms

Ignoring measurement errors, nominal value-added GDP (total income) should equal final nominal demand (expenditure), i.e. $GDP = w_K K + w_L L + w_H H = C$.

Value added GDP:

$$w_K K + w_L L + w_H H = \sum_{i=1}^{n} p_i q_i - \sum_{i=1}^{n}\sum_{j=1}^{n} p_j d_{ji} \tag{2.34}$$

Expenditure GDP:

$$Y = C = \sum_{i=1}^{n} p_i y_i = \sum_{i=1}^{n} p_i q_i - \sum_{i=1}^{n}\sum_{j=1}^{n} p_i d_{ij} \tag{2.35}$$

Since $C = w_K K + w_L L + w_H H$, it must be true that:

$$\underbrace{\sum_{i=1}^{n} \underbrace{\sum_{j=1}^{n} p_j d_{ji}}_{\text{total usage of inputs by } i}}_{\text{summing across destinations } i} = \underbrace{\sum_{i=1}^{n} \underbrace{\sum_{j=1}^{n} p_i d_{ij}}_{\text{total inputs produced by } i}}_{\text{summing across origins } i} \tag{2.36}$$

Equivalently, the total amount of intermediate inputs used equals the quantity produced:

$$\underbrace{\sum_{i=1}^{n} \underbrace{\sum_{j=1}^{n} \gamma_{ji} p_i q_i}_{\text{summing across origins } j}}_{\text{summing across destinations } i} = \underbrace{\sum_{i=1}^{n} \underbrace{\sum_{j=1}^{n} \gamma_{ij} p_j q_j}_{\text{summing across destinations } j}}_{\text{summing across origins } i} \tag{2.37}$$

QED.

2.A.2 Proofs

Proof of Proposition 2.1. Real GDP [48]

In logs, the FOCs for the factors and inputs in each industry given by Equations (2.9),

[48] Derivations adapted from Fadinger, Ghiglino, and Teteryatnikova (2016).

(2.10), (2.11) and (2.12) read:

$$\log k_i = \log \alpha_i + \log(1-\gamma_i) + \log p_i + \log q_i - \log w_K \tag{2.38}$$

$$\log l_i = \log \delta_i + \log(1-\gamma_i) + \log p_i + \log q_i - \log w_L \tag{2.39}$$

$$\log h_i = \log(1-\alpha_i-\delta_i) + \log(1-\gamma_i) + \log p_i + \log q_i - \log w_H \tag{2.40}$$

$$\log d_{ji} = \log \gamma_{ji} + \log p_i + \log q_i - \log p_j \tag{2.41}$$

Substituting them into Equation (2.1) in logs gives:

$$\begin{aligned}
\log q_i &= \log A_i + (1-\gamma_i)\left[\alpha_i \log k_i + \delta_i \log l_i + (1-\alpha_i-\delta_i)\log h_i\right] + \\
&\quad + \textstyle\sum_{j=1}^n \gamma_{ji} \log d_{ji} \\
&= \log A_i + \\
&\quad +(1-\gamma_i)\alpha_i[\log \alpha_i + \log(1-\gamma_i) + \log p_i + \log q_i - \log w_K] + \\
&\quad +(1-\gamma_i)\delta_i[\log \delta_i + \log(1-\gamma_i) + \log p_i + \log q_i - \log w_L] + \\
&\quad +(1-\gamma_i)(1-\alpha_i-\delta_i)[\log(1-\alpha_i-\delta_i) + \log(1-\gamma_i) + \log p_i + \log q_i - \log w_H] + \\
&\quad + \textstyle\sum_{j=1}^n \gamma_{ji}(\log \gamma_{ji} + \log p_i + \log q_i - \log p_j) + \\
0 &= \log A_i + \log p_i + (1-\gamma_i)\log(1-\gamma_i) + \\
&\quad +(1-\gamma_i)\alpha_i[\log \alpha_i - \log w_K] + \\
&\quad +(1-\gamma_i)\delta_i[\log \delta_i - \log w_L] + \\
&\quad +(1-\gamma_i)(1-\alpha_i-\delta_i)[\log(1-\alpha_i-\delta_i) - \log w_H] + \\
&\quad + \textstyle\sum_{j=1}^n \gamma_{ji}(\log \gamma_{ji} - \log p_j) + \\
0 &= \log A_i + \log p_i + (1-\gamma_i)\log(1-\gamma_i) + \\
&\quad +(1-\gamma_i)\alpha_i \log \alpha_i - (1-\gamma_i)\alpha_i \log w_K + \\
&\quad +(1-\gamma_i)\delta_i \log \delta_i - (1-\gamma_i)\delta_i \log w_L + \\
&\quad +(1-\gamma_i)(1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i) - (1-\gamma_i)(1-\alpha_i-\delta_i)\log w_H + \\
&\quad + \textstyle\sum_{j=1}^n \gamma_{ji}\log \gamma_{ji} - \sum_{j=1}^n \gamma_{ji}\log p_j
\end{aligned} \tag{2.42}$$

Solving for $\delta_i \log w_L$:

$$\begin{aligned}
\delta_i \log w_L &= \alpha_i \log \alpha_i + \delta_i \log \delta_i + (1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i) + \log(1-\gamma_i) \\
&\quad -\alpha_i \log w_K - (1-\alpha_i-\delta_i)\log w_H \\
&\quad + \tfrac{1}{1-\gamma_i}\left[\log A_i + \log p_i + \textstyle\sum_{j=1}^n \gamma_{ji}\log \gamma_{ji} - \sum_{j=1}^n \gamma_{ji}\log p_j\right]
\end{aligned} \tag{2.43}$$

Substituting for w_K and w_H using (2.19) and (2.21) in logs:

$$\begin{aligned}
\delta_i \log w_L &= \alpha_i \log \alpha_i + \delta_i \log \delta_i + (1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i) + \log(1-\gamma_i) \\
&\quad -\alpha_i\left(\log\alpha + \log C - \log K\right) \\
&\quad -(1-\alpha_i-\delta_i)\left(\log(1-\alpha-\delta) + \log C - \log H\right) \\
&\quad +\tfrac{1}{1-\gamma_i}\left[\log A_i + \log p_i + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^n \gamma_{ji}\log p_j\right] \\
&= \alpha_i \log \alpha_i + \delta_i \log \delta_i + (1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i) + \log(1-\gamma_i) \\
&\quad -\alpha_i\log\alpha - \alpha_i\log C + \alpha_i\log K \\
&\quad -(1-\alpha_i-\delta_i)\log(1-\alpha-\delta) - (1-\alpha_i-\delta_i)\log C + (1-\alpha_i-\delta_i)\log H \\
&\quad +\tfrac{1}{1-\gamma_i}\left[\log A_i + \log p_i + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^n \gamma_{ji}\log p_j\right] \\
&= \alpha_i \log \alpha_i + \delta_i \log \delta_i + (1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i) + \log(1-\gamma_i) \\
&\quad -\alpha_i\log\alpha - (1-\alpha_i-\delta_i)\log(1-\alpha-\delta) \\
&\quad +\alpha_i\log K + (1-\alpha_i-\delta_i)\log H - (1-\delta_i)\log C \\
&\quad +\tfrac{1}{1-\gamma_i}\left[\log A_i + \log p_i + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^n \gamma_{ji}\log p_j\right]
\end{aligned} \tag{2.44}$$

Now, use $C = w_L L/\delta$ [Equation (2.20)]:

$$\begin{aligned}
\delta_i \log w_L &= \alpha_i \log \alpha_i + \delta_i \log \delta_i + (1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i) + \log(1-\gamma_i) \\
&\quad -\alpha_i\log\alpha - (1-\alpha_i-\delta_i)\log(1-\alpha-\delta) \\
&\quad +\alpha_i\log K + (1-\alpha_i-\delta_i)\log H - (1-\delta_i)\log w_L - (1-\delta_i)\log L + (1-\delta_i)\log\delta \\
&\quad +\tfrac{1}{1-\gamma_i}\left[\log A_i + \log p_i + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^n \gamma_{ji}\log p_j\right]
\end{aligned} \tag{2.45}$$

To get:

$$\begin{aligned}
\log w_L &= \alpha_i \log \alpha_i + \delta_i \log \delta_i + (1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i) + \log(1-\gamma_i) \\
&\quad -\alpha_i\log\alpha - (1-\alpha_i-\delta_i)\log(1-\alpha-\delta) + (1-\delta_i)\log\delta \\
&\quad +\alpha_i\log K + (1-\alpha_i-\delta_i)\log H - (1-\delta_i)\log L \\
&\quad +\tfrac{1}{1-\gamma_i}\left[\log A_i + \log p_i + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^n \gamma_{ji}\log p_j\right]
\end{aligned} \tag{2.46}$$

Now, consider the vector $\boldsymbol{\mu}'\boldsymbol{Z}$ where $\boldsymbol{\mu}$ is the $(nX1)$ vector of multipliers and $\boldsymbol{Z}$ is a diagonal matrix with $Z_{ii} = 1-\gamma_i$ such that $\boldsymbol{\mu}'\boldsymbol{Z} = \boldsymbol{\beta}'[\boldsymbol{I}-\boldsymbol{\Gamma}']^{-1}\cdot\boldsymbol{Z}$. Take the i^{th} element of this matrix and multiply by the equation above.

$$\begin{aligned}\mu_i(1-\gamma_i)\log w_L &= \mu_i(1-\gamma_i)\left[\alpha_i\log\alpha_i+\delta_i\log\delta_i+(1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i)\right]\\ &\quad+\mu_i(1-\gamma_i)\left[\log(1-\gamma_i)-\alpha_i\log\alpha\right]\\ &\quad+\mu_i(1-\gamma_i)\left[(1-\delta_i)\log\delta-(1-\alpha_i-\delta_i)\log\left(1-\alpha-\delta\right)\right]\\ &\quad+\mu_i(1-\gamma_i)\left[\alpha_i\log K+(1-\alpha_i-\delta_i)\log H-(1-\delta_i)\log L\right]\\ &\quad+\mu_i\log A_i+\mu_i\log p_i+\mu_i\textstyle\sum_{j=1}^n\gamma_{ji}\log\gamma_{ji}-\mu_i\sum_{j=1}^n\gamma_{ji}\log p_j\end{aligned} \tag{2.47}$$

Summing over all industries i:

$$\begin{aligned}\textstyle\sum_{i=1}^n\mu_i(1-\gamma_i)\log w_L &= \textstyle\sum_{i=1}^n\mu_i(1-\gamma_i)\left[\alpha_i\log\alpha_i+\delta_i\log\delta_i+(1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i)\right]\\ &\quad+\textstyle\sum_{i=1}^n\mu_i(1-\gamma_i)\left[\log(1-\gamma_i)-\alpha_i\log\alpha\right]\\ &\quad+\textstyle\sum_{i=1}^n\mu_i(1-\gamma_i)\left[(1-\delta_i)\log\delta-(1-\alpha_i-\delta_i)\log\left(1-\alpha-\delta\right)\right]\\ &\quad+\textstyle\sum_{i=1}^n\mu_i(1-\gamma_i)\left[\alpha_i\log K-(1-\delta_i)\log L+(1-\alpha_i-\delta_i)\log H\right]\\ &\quad+\textstyle\sum_{i=1}^n\mu_i\log A_i+\sum_{i=1}^n\mu_i\sum_{j=1}^n\gamma_{ji}\log\gamma_{ji}\\ &\quad+\textstyle\sum_{i=1}^n\mu_i\log p_i-\sum_{i=1}^n\mu_i\sum_{j=1}^n\gamma_{ji}\log p_j\end{aligned} \tag{2.48}$$

Manipulating the two terms of the last row and then (2nd to 3rd line) using (2.23):

$$\begin{aligned}\textstyle\sum_{i=1}^n\mu_i\log p_i-\sum_{i=1}^n\mu_i\sum_{j=1}^n\gamma_{ji}\log p_j &=\\ \textstyle\sum_{i=1}^n\mu_i\log p_i-\sum_{j=1}^n\log p_j\sum_{i=1}^n\mu_i\gamma_{ji} &=\\ \textstyle\sum_{i=1}^n\mu_i\log p_i-\sum_{j=1}^n\log p_j(\mu_j-\beta_j) &=\\ \textstyle\sum_{i=1}^n\mu_i\log p_i-\sum_{i=1}^n\log p_i(\mu_i-\beta_i) &= \textstyle\sum_{i=1}^n\beta_i\log p_i\end{aligned} \tag{2.49}$$

It must be true that:

$$\begin{aligned}\textstyle\sum_{i=1}^n\mu_i(1-\gamma_i)\log w_L &= \textstyle\sum_{i=1}^n\mu_i(1-\gamma_i)\left[\alpha_i\log\alpha_i+\delta_i\log\delta_i+(1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i)\right]\\ &\quad+\textstyle\sum_{i=1}^n\mu_i(1-\gamma_i)\left[\log(1-\gamma_i)-\alpha_i\log\alpha\right]\\ &\quad+\textstyle\sum_{i=1}^n\mu_i(1-\gamma_i)\left[(1-\delta_i)\log\delta-(1-\alpha_i-\delta_i)\log\left(1-\alpha-\delta\right)\right]\\ &\quad+\textstyle\sum_{i=1}^n\mu_i(1-\gamma_i)\left[\alpha_i\log K-(1-\delta_i)\log L+(1-\alpha_i-\delta_i)\log H\right]\\ &\quad+\textstyle\sum_{i=1}^n\mu_i\log A_i+\sum_{i=1}^n\mu_i\sum_{j=1}^n\gamma_{ji}\log\gamma_{ji}+\sum_{i=1}^n\beta_i\log p_i\end{aligned} \tag{2.50}$$

Finally, I use the FOC of maximisation problem of the final good aggregator, i.e.:

$$\frac{\partial Y}{\partial y_i}=\frac{p_i}{P}=\beta_i\frac{Y}{y_i}\Rightarrow y_i=\beta_i\frac{PY}{p_i} \tag{2.51}$$

Using $P = 1$, substitute into the final product aggregator:

$$Y = \prod_{i=1}^{n} \left(\beta_i \frac{Y}{p_i} \right)^{\beta_i} = Y \prod_{i=1}^{n} \left(\frac{\beta_i}{p_i} \right)^{\beta_i} \tag{2.52}$$

Which implies that:

$$1 = \prod_{i=1}^{n} \left(\frac{\beta_i}{p_i} \right)^{\beta_i} = \frac{\prod_{i=1}^{n} {\beta_i}^{\beta_i}}{\prod_{i=1}^{n} {p_i}^{\beta_i}} \Leftrightarrow \prod_{i=1}^{n} {\beta_i}^{\beta_i} = \prod_{i=1}^{n} {p_i}^{\beta_i} \tag{2.53}$$

In logs:

$$\sum_{i=1}^{n} \beta_i \log \beta_i = \sum_{i=1}^{n} \beta_i \log p_i \tag{2.54}$$

Therefore:

$$\begin{aligned} \textstyle\sum_{i=1}^{n} \mu_i(1-\gamma_i) \log w_L \quad &= \textstyle\sum_{i=1}^{n} \mu_i(1-\gamma_i) \left[\alpha_i \log \alpha_i + \delta_i \log \delta_i + (1-\alpha_i-\delta_i) \log(1-\alpha_i-\delta_i)\right] \\ &+ \textstyle\sum_{i=1}^{n} \mu_i(1-\gamma_i) \left[\log(1-\gamma_i) - \alpha_i \log \alpha\right] \\ &+ \textstyle\sum_{i=1}^{n} \mu_i(1-\gamma_i) \left[(1-\delta_i) \log \delta - (1-\alpha_i-\delta_i) \log (1-\alpha-\delta)\right] \\ &+ \textstyle\sum_{i=1}^{n} \mu_i(1-\gamma_i) \left[\alpha_i \log K - (1-\delta_i) \log L + (1-\alpha_i-\delta_i) \log H\right] \\ &+ \textstyle\sum_{i=1}^{n} \mu_i \log A_i + \sum_{i=1}^{n} \mu_i \sum_{j=1}^{n} \gamma_{ji} \log \gamma_{ji} + \sum_{i=1}^{n} \beta_i \log \beta_i \end{aligned} \tag{2.55}$$

$$\begin{aligned} \log w_L \quad = \quad & (\log K - \log \alpha) \left[\tfrac{\sum_{i=1}^{n} \mu_i(1-\gamma_i)\alpha_i}{\sum_{i=1}^{n} \mu_i(1-\gamma_i)} \right] \\ & + (\log L - \log \delta) \left[\tfrac{\sum_{i=1}^{n} \mu_i(1-\gamma_i)\delta_i}{\sum_{i=1}^{n} \mu_i(1-\gamma_i)} - 1 \right] \\ & + (\log H - \log (1-\alpha-\delta)) \left[\tfrac{\sum_{i=1}^{n} \mu_i(1-\gamma_i)(1-\alpha_i-\delta_i)}{\sum_{i=1}^{n} \mu_i(1-\gamma_i)} \right] \\ & + \tfrac{\sum_{i=1}^{n} \mu_i(1-\gamma_i)[\alpha_i \log \alpha_i + \delta_i \log \delta_i + (1-\alpha_i-\delta_i) \log(1-\alpha_i-\delta_i)]}{\sum_{i=1}^{n} \mu_i(1-\gamma_i)} \\ & + \tfrac{1}{\sum_{i=1}^{n} \mu_i(1-\gamma_i)} \left[\textstyle\sum_{i=1}^{n} \mu_i \log A_i + \sum_{i=1}^{n} \mu_i \sum_{j=1}^{n} \gamma_{ji} \log \gamma_{ji} \right] \\ & + \tfrac{1}{\sum_{i=1}^{n} \mu_i(1-\gamma_i)} \left[\textstyle\sum_{i=1}^{n} \mu_i(1-\gamma_i) \log(1-\gamma_i) + \sum_{i=1}^{n} \beta_i \log \beta_i \right] \end{aligned} \tag{2.56}$$

Now, substitute $p_j d_{ji} = \gamma_{ji} p_i q_i$ and $p_i y_i = \beta_i Y$ into the market clearing condition for good j in nominal terms. Then expand Y using $\beta_i Y = \beta_i (C)$:

$$\begin{aligned} p_j y_j + \textstyle\sum_{i=1}^{n} p_j d_{ji} &= p_j q_j \\ \beta_j Y + \textstyle\sum_{i=1}^{n} \gamma_{ji} p_i q_i &= p_j q_j \\ \beta_j (C) + \textstyle\sum_{i=1}^{n} \gamma_{ji} p_i q_i &= p_j q_j \end{aligned} \tag{2.57}$$

Denote $\text{þ}_j = p_j q_j$. Then, in matrix form: $C\boldsymbol{\beta} + \boldsymbol{\Gamma}\boldsymbol{\text{þ}} = \boldsymbol{\text{þ}}$. Hence,

$$\boldsymbol{\text{þ}} = [\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1} C\boldsymbol{\beta} = C\boldsymbol{\mu}$$

Where $\boldsymbol{\mu} = [\boldsymbol{I} - \boldsymbol{\Gamma}]^{-1}\boldsymbol{\beta}$ is fully determined by parameters. Therefore $\not{p}_i = p_i q_i = C\mu_i$. Moreover, Equation (2.2) in nominal and aggregate terms $\sum_{i=1}^n p_i q_i = \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} p_j q_j + \sum_{i=1}^n p_i y_i$ implies:

$$
\begin{aligned}
Y = \sum_{i=1}^n p_i y_i \quad &= \quad \sum_{i=1}^n p_i q_i - \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} p_j q_j \\
&\quad \sum_{i=1}^n p_i q_i - \sum_{j=1}^n p_j q_j \sum_{i=1}^n \gamma_{ij} \\
&\quad \sum_{i=1}^n p_i q_i - \sum_{j=1}^n p_j q_j \gamma_j \\
&\quad \sum_{i=1}^n p_i q_i - \sum_{i=1}^n \gamma_i p_i q_i
\end{aligned} \tag{2.58}
$$

Combining the formulas for the GDP Y from the expenditure and production sides:

$$
\begin{aligned}
\sum_{i=1}^n p_i q_i - \sum_{i=1}^n \gamma_i p_i q_i &= C \\
\frac{\sum_{i=1}^n p_i q_i}{Y} - \frac{\sum_{i=1}^n \gamma_i p_i q_i}{Y} &= \frac{C}{Y} \\
\sum_{i=1}^n \mu_i - \sum_{i=1}^n \gamma_i \mu_i &= \frac{C}{Y}
\end{aligned} \tag{2.59}
$$

Which allows us to conclude that:

$$
\frac{C}{Y} = \sum_{i=1}^{n} \mu_i (1 - \gamma_i) = 1 \tag{2.60}
$$

By construction, the summation of the Domar shares discounting intermediate input shares equals one. Using Equations (2.20) and (2.60), I get the solution for low-skilled wages:

$$
\begin{aligned}
\log w_L \quad = \quad & \alpha(\log K - \log \alpha) - (1-\delta)(\log L - \log \delta) + (1-\alpha-\delta)(\log H - \log(1-\alpha-\delta)) \\
& + \sum_{i=1}^n \mu_i (1-\gamma_i) \left[\alpha_i \log \alpha_i + \delta_i \log \delta_i + (1-\alpha_i-\delta_i) \log(1-\alpha_i-\delta_i)\right] \\
& + \sum_{i=1}^n \mu_i (1-\gamma_i) \log(1-\gamma_i) + \sum_{i=1}^n \mu_i \log A_i \\
& + \sum_{i=1}^n \mu_i \sum_{j=1}^n \gamma_{ji} \log \gamma_{ji} + \sum_{i=1}^n \beta_i \log \beta_i
\end{aligned} \tag{2.61}
$$

Finally, substituting Equation (2.61) into Equation (2.20) I get Equation (2.24). QED.

Proof of Proposition 2.2. Industry output

From the first order conditions of firm i's profit maximisation and using Equation (2.22) in logs, I get the solutions for factor usage:

$$\log k_i = \log \alpha_i + \log(1-\gamma_i) + \log \mu_i + \log Y - \log w_K \tag{2.62}$$

$$\log l_i = \log \delta_i + \log(1-\gamma_i) + \log \mu_i + \log Y - \log w_L \tag{2.63}$$

$$\log h_i = \log(1-\alpha_i-\delta_i) + \log(1-\gamma_i) + \log \mu_i + \log Y - \log w_H \tag{2.64}$$

Now, consider (2.1) in logs:

$$\log q_i = \log A_i + (1-\gamma_i)\left[\alpha_i \log k_i + \delta_i \log l_i + (1-\alpha_i-\delta_i)\log h_i\right] + \sum_{j=1}^{n} \gamma_{ji} \log d_{ji} \tag{2.65}$$

Simplifying the terms within square brackets using Equations (2.62), (2.63) and (2.64):

$$\begin{aligned}
\alpha_i \log k_i + \delta_i \log l_i + (1-\alpha_i-\delta_i)\log h_i = \\
= \alpha_i\left[\log \alpha_i + \log(1-\gamma_i) + \log \mu_i + \log Y - \log w_K\right] \\
+\delta_i\left[\log \delta_i + \log(1-\gamma_i) + \log \mu_i + \log Y - \log w_L\right] \\
+(1-\alpha_i-\delta_i)\left[\log(1-\alpha_i-\delta_i) + \log(1-\gamma_i) + \log \mu_i + \log Y - \log w_H\right] = \\
= \alpha_i\left[\log \alpha_i - \log w_K\right] + \delta_i\left[\log \delta_i - \log w_L\right] + (1-\alpha_i-\delta_i)\left[\log(1-\alpha_i-\delta_i) - \log w_H\right] \\
+ \log(1-\gamma_i) + \log \mu_i + \log Y
\end{aligned} \tag{2.66}$$

$$\begin{aligned}
\log q_i \quad = \quad & \log A_i + (1-\gamma_i)\left[\log(1-\gamma_i) + \log \mu_i + \log Y + \alpha_i(\log \alpha_i - \log w_K)\right] \\
& +(1-\gamma_i)\left[\delta_i(\log \delta_i - \log w_L) + (1-\alpha_i-\delta_i)(\log(1-\alpha_i-\delta_i) - \log w_H)\right] \\
& + \textstyle\sum_{j=1}^{n} \gamma_{ji} \log d_{ji}
\end{aligned} \tag{2.67}$$

Recalling that d_{ji} refers to the demand of input j (origin) by industry i (destination). From firm's i FOCs:

$$\begin{aligned}
\log d_{ji} \quad &= \quad \log \gamma_{ji} + \log(p_i q_i) - \log p_j \\
&= \quad \log \gamma_{ji} + \log \mu_i + \log Y - \log p_j
\end{aligned} \tag{2.68}$$

Using (2.22) in logs, $\log p_j = \log \mu_j + \log Y - \log q_j$, I can express $\log d_{ji}$ in terms of $\log q_j$:

$$\log d_{ji} = \log \gamma_{ji} + \log \mu_i - \log \mu_j + \log q_j \tag{2.69}$$

Substituting Equation (2.69) into (2.67):

$$
\begin{aligned}
\log q_i &= \log A_i + (1-\gamma_i)\left[\log(1-\gamma_i) + \log\mu_i + \log Y + \alpha_i(\log\alpha_i - \log w_K)\right] \\
&+ (1-\gamma_i)\left[\delta_i(\log\delta_i - \log w_L) + (1-\alpha_i-\delta_i)(\log(1-\alpha_i-\delta_i) - \log w_H)\right] \\
&+ \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} + \gamma_i\log\mu_i - \sum_{j=1}^n \gamma_{ji}\log\mu_j + \sum_{j=1}^n \gamma_{ji}\log q_j
\end{aligned} \tag{2.70}
$$

Combining like terms:

$$
\begin{aligned}
\log q_i &= \log A_i + \log\mu_i + (1-\gamma_i)\log Y + (1-\gamma_i)\alpha_i(\log\alpha_i - \log w_K) \\
&+ (1-\gamma_i)\left[\delta_i(\log\delta_i - \log w_L) + (1-\alpha_i-\delta_i)(\log(1-\alpha_i-\delta_i) - \log w_H)\right] \\
&+ (1-\gamma_i)\log(1-\gamma_i) + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^n \gamma_{ji}\log\mu_j + \sum_{j=1}^n \gamma_{ji}\log q_j
\end{aligned} \tag{2.71}
$$

Substituting for the equilibrium values of Y, w_K, w_K and w_H:

$$
\begin{aligned}
\log q_i &= \log A_i + \log\mu_i \\
&+ (1-\gamma_i)\left[\alpha_i\log\alpha_i + \delta_i\log\delta_i + (1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i)\right] \\
&+ (1-\gamma_i)\log Y \\
&- (1-\gamma_i)\alpha_i\log w_K \\
&- (1-\gamma_i)\delta_i\log w_L \\
&- (1-\gamma_i)(1-\alpha_i-\delta_i)\log w_H \\
&+ (1-\gamma_i)\log(1-\gamma_i) + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^n \gamma_{ji}\log\mu_j + \sum_{j=1}^n \gamma_{ji}\log q_j
\end{aligned} \tag{2.72}
$$

I get:

$$
\begin{aligned}
\log q_i &= \log A_i + \log\mu_i \\
&+ (1-\gamma_i)\left[\alpha_i\log\alpha_i + \delta_i\log\delta_i + (1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i)\right] \\
&+ (1-\gamma_i)\alpha_i\left[(\log K - \log\alpha)\right] \\
&+ (1-\gamma_i)\delta_i\left[(\log L - \log\delta)\right] \\
&+ (1-\gamma_i)(1-\alpha_i-\delta_i)\left[(\log H - \log(1-\alpha-\delta))\right] \\
&+ (1-\gamma_i)\log(1-\gamma_i) + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^n \gamma_{ji}\log\mu_j + \sum_{j=1}^n \gamma_{ji}\log q_j
\end{aligned} \tag{2.73}
$$

Simplifying, I get Equation (2.25). QED.

Proof of Corollary 2.1. Vector of industry output

Expressing Equation (2.25) in matrix form:

$$\begin{bmatrix} \log q_1 \\ \log q_2 \\ \vdots \\ \log q_n \end{bmatrix} = V + \begin{bmatrix} \gamma_{11} & \gamma_{21} & \cdots & \gamma_{n1} \\ \gamma_{12} & \gamma_{22} & \cdots & \gamma_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{1n} & \gamma_{2n} & \cdots & \gamma_{nn} \end{bmatrix} \cdot \begin{bmatrix} \log q_1 \\ \log q_2 \\ \vdots \\ \log q_n \end{bmatrix} \tag{2.74}$$

One can easily solve for the vector of the log of industry output to get Equation (2.26). QED.

Proof of Corollary 2.2. Output effect

It is easy to see that deriving Equation (2.26) with respect to L, H and E leads to Equations (IO_{LS}), (IO_{HS}) and (IO_{ES}), respectively. QED.

Proof of Corollary 2.3. Output effect no-IO model

Letting all γ_{ji} equal zero in Equations (IO_{LS}), (IO_{HS}) and (IO_{ES}), I get respectively Equations (no-IO_{LS}), (no-IO_{HS}) and (no-IO_{ES}). QED.

Proof of Theorem 2.1. Linkage weights

The elements of Equations (IO_{LS}), (IO_{HS}) and (IO_{ES}) composed of input shares γ_{ji} comprises of the Leontief-inverse transposed $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$ and the vector $\mathbf{1} - \boldsymbol{\gamma}$ as given below:

$$\left[I - \begin{pmatrix} \gamma_{11} & \gamma_{21} & \cdots & \gamma_{n1} \\ \gamma_{12} & \gamma_{22} & \cdots & \gamma_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{1n} & \gamma_{2n} & \cdots & \gamma_{nn} \end{pmatrix} \right]^{-1} \cdot \begin{bmatrix} (1-\gamma_1) \\ (1-\gamma_2) \\ \vdots \\ (1-\gamma_n) \end{bmatrix} \tag{2.75}$$

To demonstrate that each scalar product equals one, notice that postmultiplying the Leontief-inverse transposed (not inverted) by a vector of ones leads to the vector of total factor shares, i.e.:

$$\left[I - \begin{pmatrix} \gamma_{11} & \gamma_{21} & \cdots & \gamma_{n1} \\ \gamma_{12} & \gamma_{22} & \cdots & \gamma_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{1n} & \gamma_{2n} & \cdots & \gamma_{nn} \end{pmatrix}\right] \cdot \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} (1-\gamma_1) \\ (1-\gamma_2) \\ \vdots \\ (1-\gamma_n) \end{bmatrix} \tag{2.76}$$

This is true because each $\gamma_i = \sum_{j=1}^{n} \gamma_{ji}$ stands for exactly the sum of the row i of $\mathbf{\Gamma}' = [\gamma_{ij}]$. Now, premultiplying matrix $[\boldsymbol{I} - \mathbf{\Gamma}']^{-1}$ on both sides leads to:

$$\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = \left[I - \begin{pmatrix} \gamma_{11} & \gamma_{21} & \cdots & \gamma_{n1} \\ \gamma_{12} & \gamma_{22} & \cdots & \gamma_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{1n} & \gamma_{2n} & \cdots & \gamma_{nn} \end{pmatrix}\right]^{-1} \cdot \begin{bmatrix} (1-\gamma_1) \\ (1-\gamma_2) \\ \vdots \\ (1-\gamma_n) \end{bmatrix} \tag{2.77}$$

QED.

2.A.3 General equilibrium solutions

For completeness and easiness of consultation, I present all model's solutions.

Equilibrium Y

$$\begin{aligned} \log Y \;=\;& \alpha(\log K - \log\alpha) + \delta(\log L - \log\delta) + (1-\alpha-\delta)(\log H - \log(1-\alpha-\delta)) \\ &+ \textstyle\sum_{i=1}^{n} \mu_i(1-\gamma_i)\left[\alpha_i\log\alpha_i + \delta_i\log\delta_i + (1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i)\right] \\ &+ \textstyle\sum_{i=1}^{n} \mu_i(1-\gamma_i)\log(1-\gamma_i) + \sum_{i=1}^{n}\mu_i\log A_i \\ &+ \textstyle\sum_{i=1}^{n} \mu_i \sum_{j=1}^{n}\gamma_{ji}\log\gamma_{ji} + \sum_{i=1}^{n}\beta_i\log\beta_i \end{aligned} \tag{(2.24)}$$

Equilibrium w_K

$$\begin{aligned} \log w_K \;=\;& (\alpha-1)(\log K - \log\alpha) + \delta(\log L - \log\delta) + (1-\alpha-\delta)(\log H - \log(1-\alpha-\delta)) \\ &+ \textstyle\sum_{i=1}^{n} \mu_i(1-\gamma_i)\left[\alpha_i\log\alpha_i + \delta_i\log\delta_i + (1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i)\right] \\ &+ \textstyle\sum_{i=1}^{n} \mu_i(1-\gamma_i)\log(1-\gamma_i) + \sum_{i=1}^{n}\mu_i\log A_i \\ &+ \textstyle\sum_{i=1}^{n} \mu_i \sum_{j=1}^{n}\gamma_{ji}\log\gamma_{ji} + \sum_{i=1}^{n}\beta_i\log\beta_i \end{aligned} \tag{2.78}$$

Equilibrium w_L

$$\begin{aligned}
\log w_L \quad = \quad & \alpha(\log K - \log\alpha) - (1-\delta)(\log L - \log\delta) + (1-\alpha-\delta)(\log H - \log(1-\alpha-\delta)) \\
& + \textstyle\sum_{i=1}^n \mu_i(1-\gamma_i)\left[\alpha_i \log\alpha_i + \delta_i \log\delta_i + (1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i)\right] \\
& + \textstyle\sum_{i=1}^n \mu_i(1-\gamma_i)\log(1-\gamma_i) + \sum_{i=1}^n \mu_i \log A_i \\
& + \textstyle\sum_{i=1}^n \mu_i \sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} + \sum_{i=1}^n \beta_i \log\beta_i
\end{aligned} \tag{(2.61)}$$

Equilibrium w_H

$$\begin{aligned}
\log w_H \quad = \quad & \alpha(\log K - \log\alpha) + \delta(\log L - \log\delta) + \left[(1-\alpha-\delta) - 1\right](\log H - \log(1-\alpha-\delta)) \\
& + \textstyle\sum_{i=1}^n \mu_i(1-\gamma_i)\left[\alpha_i \log\alpha_i + \delta_i \log\delta_i + (1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i)\right] \\
& + \textstyle\sum_{i=1}^n \mu_i(1-\gamma_i)\log(1-\gamma_i) + \sum_{i=1}^n \mu_i \log A_i \\
& + \textstyle\sum_{i=1}^n \mu_i \sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} + \sum_{i=1}^n \beta_i \log\beta_i
\end{aligned} \tag{2.79}$$

Factors usage

$$\log k_i = \log\alpha_i + \log(1-\gamma_i) + \log\mu_i + \log Y - \log w_K \tag{(2.62)}$$

$$\log l_i = \log\delta_i + \log(1-\gamma_i) + \log\mu_i + \log Y - \log w_L \tag{(2.63)}$$

$$\log h_i = \log(1-\alpha_i-\delta_i) + \log(1-\gamma_i) + \log\mu_i + \log Y - \log w_H \tag{(2.64)}$$

Demand for intermediate inputs

$$\log d_{ji} = \log\gamma_{ji} + \log\mu_i - \log\mu_j + \log q_j \tag{(2.69)}$$

Industry output

$$\begin{aligned}
\log q_i \quad = \quad & (1-\gamma_i)\alpha_i(\log K - \log\alpha + \log\alpha_i) \\
& + (1-\gamma_i)\delta_i(\log L - \log\delta + \log\delta_i) \\
& + (1-\gamma_i)(1-\alpha_i-\delta_i)(\log H - \log(1-\alpha-\delta)) \\
& + (1-\gamma_i)(\log(1-\alpha_i-\delta_i) + \log(1-\gamma_i)) + \log A_i + \log\mu_i \\
& + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^n \gamma_{ji}\log\mu_j + \sum_{j=1}^n \gamma_{ji}\log q_j
\end{aligned} \tag{(2.25)}$$

Prices of goods/inputs

$$\begin{aligned}\log p_i = & -(1-\gamma_i)\alpha_i(\log K - \log\alpha + \log\alpha_i) + (1-\gamma_i)\alpha(\log K - \log\alpha) \\ & -(1-\gamma_i)\delta_i(\log L - \log\delta + \log\delta_i) + (1-\gamma_i)\delta(\log L - \log\delta) \\ & -(1-\gamma_i)(1-\alpha_i-\delta_i)(\log H - \log(1-\alpha-\delta) + \log(1-\alpha_i-\delta_i)) \\ & +(1-\gamma_i)(1-\alpha-\delta)(\log H - \log(1-\alpha-\delta)) \\ & +(1-\gamma_i)\sum_{i=1}^n \mu_i(1-\gamma_i)\alpha_i\log\alpha_i + (1-\gamma_i)\sum_{i=1}^n \mu_i(1-\gamma_i)\delta_i\log\delta_i \\ & +(1-\gamma_i)\sum_{i=1}^n \mu_i(1-\gamma_i)(1-\alpha_i-\delta_i)\log(1-\alpha_i-\delta_i) \\ & -(1-\gamma_i)\log(1-\gamma_i) + (1-\gamma_i)\sum_{i=1}^n \mu_i(1-\gamma_i)\log(1-\gamma_i) \\ & -\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} + (1-\gamma_i)\sum_{i=1}^n \mu_i\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} \\ & -\log A_i + (1-\gamma_i)\sum_{i=1}^n \mu_i\log A_i + (1-\gamma_i)\sum_{i=1}^n \beta_i\log\beta_i + \sum_{j=1}^n \gamma_{ji}\log p_j\end{aligned} \tag{2.80}$$

Let $\mathfrak{p}_i = \log p_i$ and $\boldsymbol{W}$ such that each i^{th} row of vector consists of all the constant terms of (2.80) (those prior to $\sum_{j=1}^n \gamma_{ji}\log p_j$). Solving for $\mathfrak{p}$::

$$\mathfrak{p} = \left[I - \Gamma'\right]^{-1}\boldsymbol{W} \tag{2.81}$$

2.A.4 Comparative Statics

Effect of change in H on Real GDP

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log H} = (1-\alpha-\delta) \tag{2.82}$$

Effect of a change in H on wages

$$\frac{\mathrm{d}\log w_K}{\mathrm{d}\log H} = (1-\alpha-\delta) \tag{2.83}$$

$$\frac{\mathrm{d}\log w_L}{\mathrm{d}\log H} = (1-\alpha-\delta) \tag{2.84}$$

$$\frac{\mathrm{d}\log w_H}{\mathrm{d}\log H} = (-\alpha-\delta) \tag{2.85}$$

Factors usage

$$\frac{\mathrm{d}\log k_i}{\mathrm{d}\log H} = \frac{\mathrm{d}\log l_i}{\mathrm{d}\log H} = 0 \tag{2.86}$$

$$\frac{\mathrm{d}\log h_i}{\mathrm{d}\log H} = 1 \tag{2.87}$$

Inputs usage and industries' final goods

$$\frac{\mathrm{d}\log d_{ij}}{\mathrm{d}\log H} = \frac{\mathrm{d}\log y_i}{\mathrm{d}\log H} = \frac{\mathrm{d}\log q_i}{\mathrm{d}\log H} \tag{2.88}$$

Effect of change in H on $\log q_i$

Taking the derivatives of the solution in matrix form, as given by (2.26) $\mathfrak{q} = [I - \Gamma']^{-1}\boldsymbol{V}$:

$$\begin{bmatrix} \frac{\mathrm{d}\log q_1}{\mathrm{d}\log H} \\ \frac{\mathrm{d}\log q_2}{\mathrm{d}\log H} \\ \vdots \\ \frac{\mathrm{d}\log q_n}{\mathrm{d}\log H} \end{bmatrix} = \begin{bmatrix} I - \begin{pmatrix} \gamma_{11} & \gamma_{21} & \dots & \gamma_{n1} \\ \gamma_{12} & \gamma_{22} & \dots & \gamma_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{1n} & \gamma_{2n} & \dots & \gamma_{nn} \end{pmatrix} \end{bmatrix}^{-1} \cdot \begin{bmatrix} (1-\gamma_1)(1-\alpha_1-\delta_1) \\ (1-\gamma_2)(1-\alpha_2-\delta_2) \\ \vdots \\ (1-\gamma_n)(1-\alpha_n-\delta_n) \end{bmatrix} \tag{2.89}$$

Goods' prices

Taking the derivatives of the solution in matrix form, as given by (2.81) $\mathfrak{p} = [I - \Gamma']^{-1}\boldsymbol{W}$:

$$\begin{bmatrix} \frac{\mathrm{d}\log p_1}{\mathrm{d}\log H} \\ \frac{\mathrm{d}\log p_2}{\mathrm{d}\log H} \\ \vdots \\ \frac{\mathrm{d}\log p_n}{\mathrm{d}\log H} \end{bmatrix} = \begin{bmatrix} I - \begin{pmatrix} \gamma_{11} & \gamma_{21} & \dots & \gamma_{n1} \\ \gamma_{12} & \gamma_{22} & \dots & \gamma_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{1n} & \gamma_{2n} & \dots & \gamma_{nn} \end{pmatrix} \end{bmatrix}^{-1} \cdot \begin{bmatrix} (1-\gamma_1)(1-\alpha-\delta) - (1-\gamma_1)(1-\alpha_1-\delta_1) \\ (1-\gamma_2)(1-\alpha-\delta) - (1-\gamma_2)(1-\alpha_2-\delta_2) \\ \vdots \\ (1-\gamma_n)(1-\alpha-\delta) - (1-\gamma_n)(1-\alpha_n-\delta_n) \end{bmatrix} \tag{2.90}$$

The Cobb-Douglas produces "offsetting" impacts on quantities and prices, in the sense that the total impact nominal quantity $p_i q_i$ must be equal to $(1-\alpha-\delta)$.

2.A.5 Parameters in the model without linkages

Ignoring intermediate inputs means leaving out a large fraction of the data for industry output. By forcing total production to equal final good ($q_i = y_i$), the no-IO model imposes that $\beta_i = \mu_i$, since:

$$\beta_i = \frac{p_i y_i}{Y} = \frac{p_i q_i}{Y} = \mu_i \tag{2.91}$$

The differences between the parameters estimated under the auxiliary (no-IO) and the main (IO) model are plotted in Figure 2.A.1. As expected, the parameters in the no-IO model (yellow bars) are much smaller than the μ_i (dark gray) in the IO model. Industry K, for instance, which has the fourth largest sales share in the main model (which considers all the data) ends up with a mediocre estimative under the no-IO model.

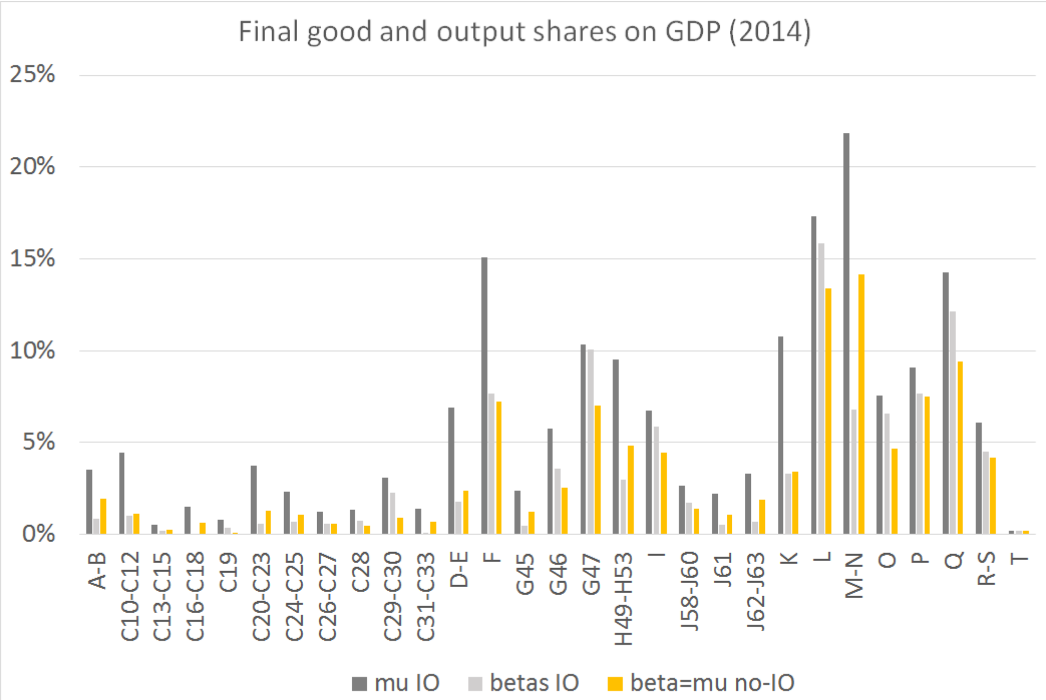


Figure 2.A.1: Comparison of the parameters estimated by the IO and no-IO models

2.B Quantitative Analysis

2.B.1 Dataset

EU KLEMS

The EU KLEMS Growth and Productivity Accounts (Jäger, 2017) provides measures capital (K), labour (L), energy (E), materials (M) and service (S) use of 34 industries (ISIC Rev. 4 industry classification). Release 2017 (revised in July 2018) covers the EU-28 countries, several EU aggregates, and the United States over the period of 1995 to 2015. I extracted the "Basic File" —which contains the main stock and flow variables— and the "Labour Input File" —with the data disaggregated by skill level— for the UK.

The variables extracted from the "Basic File" were:

- VA: Gross value added at current basic prices (in millions of national currency)

- QI: Gross value added, volume (2010 prices)
- EMP: Number of persons engaged (thousands)
- LAB: Labour compensation (in millions of national currency)
- CAP: Capital compensation (in millions of national currency)
- K_GFCF: Nominal capital stock (in millions of national currency)

The values in pounds were converted to dollars using exchange rate provided by WIOD. The good prices were constructed from the ratio of nominal over real value added ($PRI = VA/QI$).

From the "Labour Input File", I constructed the shares of employment type in each industry total employment using the higher education threshold as follows:

- L_shares: Intermediate & No formal qualifications
- H_shares: University graduates (ISCED_5 + 6)

Likewise, the aggregate shares of labour compensation were obtained in order to calculate market wages:

- LAB_L_shares: percentage of labour compensation paid to low-skill workers
- LAB_H_shares: percentage of labour compensation paid to high-skill workers

Observed factor prices were then extracted under the assumption of competitive markets as:

- $w_L = (LAB \cdot LAB\_L\_shares)/(\sum_i L\_shares_i \cdot EMP_i)$: low-skill wages
- $w_H = (LAB \cdot LAB\_H\_shares)/(\sum_i H\_shares_i \cdot EMP_i)$: high-skill wages
- $w_K = CAP/K\_GFCF$: rental on capital

WIOD

The World Input-Output Dataset (Timmer, Dietzenbacher, Los, Stehrer, and de Vries, 2015), Release 2016 covers 43 countries and a model for the rest of the world for the period 2000-2014. Data for 56 sectors are classified according to the International Standard Industrial Classification revision 4 (ISIC Rev. 4). I extracted the National IO tables (NIOT) only for the UK. The input-output values II_{ji} are denoted in millions of US dollars, are represent the nominal sales from industry j to industry i.

In order, the following variables were derived from the dataset, for each industry i:

- Total sum of intermediate inputs used: $II_i^{in} = \sum_{j=1}^{N} II_{ji}$
- Value added: $va_i = w_K \cdot k_i + w_L \cdot l_i + w_H \cdot h_i$

- Gross output of each industry i equals: $p_i q_i = va_i + II_i^{in}$
- Total sum of intermediate inputs produced: $II_i^{out} = \sum_{j=1}^{N} II_{ij}$
- Final sales: $p_i y_i = p_i q_i - II_i^{out}$

Industries in the dataset

Given the competitive markets assumption and the order of the construction of variable as above, some industries end up with negative values of final sales y_i. To overcome this issues, those industries are merged with nearby industries. The process resulted in the following 29 industries:

1. A-B Agriculture, Forestry and Fishing & Mining and quarrying
2. C10-C12 Manufacture of food products, beverages and tobacco products
3. C13-C15 Manufacture of textiles, wearing apparel and leather products
4. C16-C18 Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials & Manufacture of paper and paper products & Printing and reproduction of recorded media
5. C19 Manufacture of coke and refined petroleum products
6. C20-C23 Chemicals and chemical products & Rubber and plastics products, and other non-metallic mineral products
7. C24-C25 Basic metals and fabricated metal products, except machinery and equipment
8. C26-C27 Electrical and optical equipment
9. C28 Machinery and equipment n.e.c.
10. C29-C30 Transport equipment
11. C31-C33 Other manufacturing; repair and installation of machinery and equipment
12. D-E ELECTRICITY, GAS AND WATER SUPPLY
13. F CONSTRUCTION
14. G45 Wholesale and retail trade and repair of motor vehicles and motorcycles
15. G46 Wholesale trade, except of motor vehicles and motorcycles
16. G47 Retail trade, except of motor vehicles and motorcycles
17. H49-H53 Transport and storage & Postal and courier activities
18. I ACCOMMODATION AND FOOD SERVICE ACTIVITIES

19. J58-J60 Publishing, audiovisual and broadcasting activities
20. J61 Telecommunications
21. J62-J63 IT and other information services
22. K FINANCIAL AND INSURANCE ACTIVITIES
23. L REAL ESTATE ACTIVITIES
24. M-N PROFESSIONAL, SCIENTIFIC, TECHNICAL, ADMINISTRATIVE AND SUPPORT SERVICE ACTIVITIES
25. O Public administration and defence; compulsory social security
26. P Education
27. Q Health and social work
28. R-S ARTS, ENTERTAINMENT, RECREATION AND OTHER SERVICE ACTIVITIES
29. T Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use

APS

The Annual Population Survey (APS) household dataset[49] for the year 2014 was retrieved from UK Data Service. I used the ILODEFR variable to select only the individuals “In Employment”.

Regarding the skills, the HIQUL11D values (1) Degree or equivalent; and (2) Higher education are equivalent to NVQ level 4 and above, which on its turn correspond to ISCED 5+, i.e. the high-skill category in EU KLEMS. For immigration, the value (3) of NATOX7_EUL_Sub “European Union EU8” was used to qualify individuals declaring their nationality as of one of the A8 countries. Finally, the data was adjusted for sampling weights using the variable PWTA17.

In numbers, the stock of workers in 2014 was of 30.5 million. Low- and high-skilled workers totalled 17.9 and 12.5 million, respectively. Within this universe, the stock of A8 workers was of 840.2 thousand, composed of 562.7 thousand low-skilled and 277.5 thousand high-skilled workers.

[49] Office for National Statistics, Social Survey Division (2019).

2.B.2 Computation of the counterfactuals

The definition and calculation of the variable of interest q_i, the gross output of industry, differ between the two models.

IO model

In the IO model, from the expenditure point of view, letting $v_i = w_K k_i + w_L l_i + w_H h_i$ represent the value-added of industry i, nominal sales equates total costs:

$$p_i q_i = v_i + \sum_{j=1}^{n} p_j d_{ji} \tag{expenditure}$$

no-IO model

In the no-IO model, the output of an industry i is all used for used final consumption, y_i, which also equates to the industry's value-added. In nominal terms:

$$p_i q_i = p_i y_i = v_i \tag{production=expenditure}$$

Effect of shocks

Using 2014 as the base year,[50] the formula for calculating the effect of the labour supply shocks is identical in the two models. In other words, the impact of the changes in the input-output structure over time (γ_i)[51] does not play a role here.

$$\begin{aligned} \frac{q_{i,2014CF}}{q_{i,2014}} - 1 &= \frac{v_{i,2014CF}}{v_{i,2014}} \frac{p_{i,2014}}{p_{i,2014CF}} \frac{(1-\gamma_{i,2014})}{(1-\gamma_{i,2014CF})} - 1 \\ &= \frac{v_{i,2014CF}}{v_{i,2014}} \frac{p_{i,2014}}{p_{i,2014CF}} - 1 \end{aligned} \tag{IO}$$

The last simplification derive from the fact that a labour supply shock does not affect the parameters in this (Cobb-Douglas) model. Clearly, this is the same as the

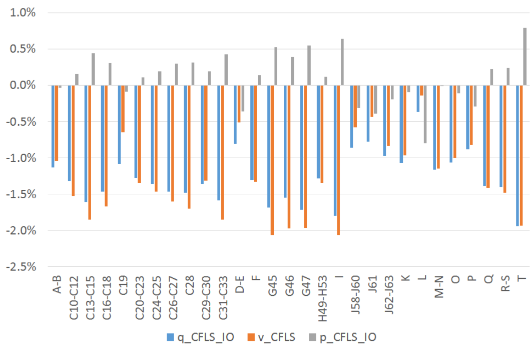
[50] Even though the EU expansion that incorporated the A8 countries took place in 2004, I choose not to use 2003 as the base year as it would yield different formulas. See Appendix **??** for details.

[51] In reality, γ_i is one dimension smaller than the whole IO matrix, reflecting only the total share of intermediate input usage of each industry.

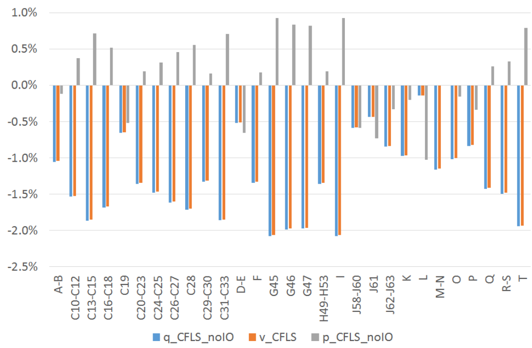
formula for the real industrial output change in the no-IO model:

$$\frac{q_{i,2014CF}}{q_{i,2014}} - 1 = \frac{v_{i,2014CF}}{v_{i,2014}} \frac{p_{i,2014}}{p_{i,2014CF}} - 1 \tag{noIO}$$

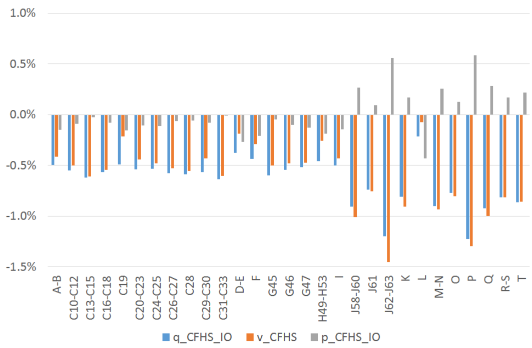
Even though the calculated value-added is the same in both models, the observed and counterfactual values for output and prices differ between the two models (Figure 2.B.1).



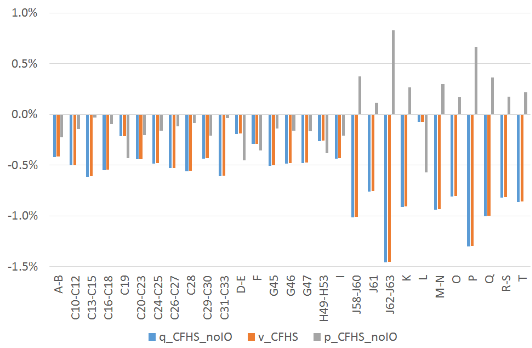
(a) CF LS IO



(b) CF LS no-IO



(c) CF HS IO



(d) CF HS no-IO

Figure 2.B.1: Counterfactual effects on output, value-added and prices

2.B.3 Base year 2003 versus 2014

Comparison of data calculations using two different base years: 2003 and 2014. Using 2003 as the base year, yields the following ratios for the growth rates in each model:

$$\frac{q_{i,2014CF}}{q_{i,2003}} = \frac{v_{i,2014CF}}{v_{i,2003}} \frac{p_{i,2003}}{p_{i,2014CF}} \frac{(1-\gamma_{i,2003})}{(1-\gamma_{i,2014})} \tag{IO}$$

Which has the no-IO counterpart:

$$\frac{q_{i,2014CF}}{q_{i,2003}} = \frac{v_{i,2014CF}}{v_{i,2003}} \frac{p_{i,2003}}{p_{i,2014CF}} \tag{noIO}$$

Changes in gross output in the IO model reflect not only the changes in the value added (factor quantities and prices) good prices over the period (as in the no-IO model), but also the changes in the II purchases (γ_i) from 2003 to 2014.

Impact

The impact of the supply shock is calculated as the difference between the real output growth in the counterfactual scenario minus the observed growth in each model:

$$\begin{aligned} Impact_i &= \tfrac{q_{i,2014CF}}{q_{i,2003}} - \tfrac{q_{i,2014}}{q_{i,2003}} \\ &= \tfrac{(1-\gamma_{i,2003})}{(1-\gamma_{i,2014})} \left[\tfrac{v_{i,2014CF}}{v_{i,2003}} \tfrac{p_{i,2003}}{p_{i,2014CF}} - \tfrac{v_{i,2014}}{v_{i,2003}} \tfrac{p_{i,2003}}{p_{i,2014}} \right] \end{aligned} \tag{IO}$$

$$\begin{aligned} Impact_i &= \tfrac{q_{i,2014CF}}{q_{i,2003}} - \tfrac{q_{i,2014}}{q_{i,2003}} \\ &= \tfrac{v_{i,2014CF}}{v_{i,2003}} \tfrac{p_{i,2003}}{p_{i,2014CF}} - \tfrac{v_{i,2014}}{v_{i,2003}} \tfrac{p_{i,2003}}{p_{i,2014}} \end{aligned} \tag{noIO}$$

While using 2014 as the base year, the impact is simply the effect of on real output:

$$\begin{aligned} Impact_i &= \tfrac{q_{i,2014CF}}{q_{i,2014}} - \tfrac{q_{i,2014}}{q_{i,2014}} \\ &= \tfrac{v_{i,2014CF}}{v_{i,2014}} \tfrac{p_{i,2014}}{p_{i,2014CF}} - 1 \end{aligned} \tag{IO=noIO}$$

Ratio

I plot below the ratios of the impact calculated with the no-IO model over the one with the IO model using each base year. Recall that in the formulae above, only the calculated value-added v_i is the same across models.

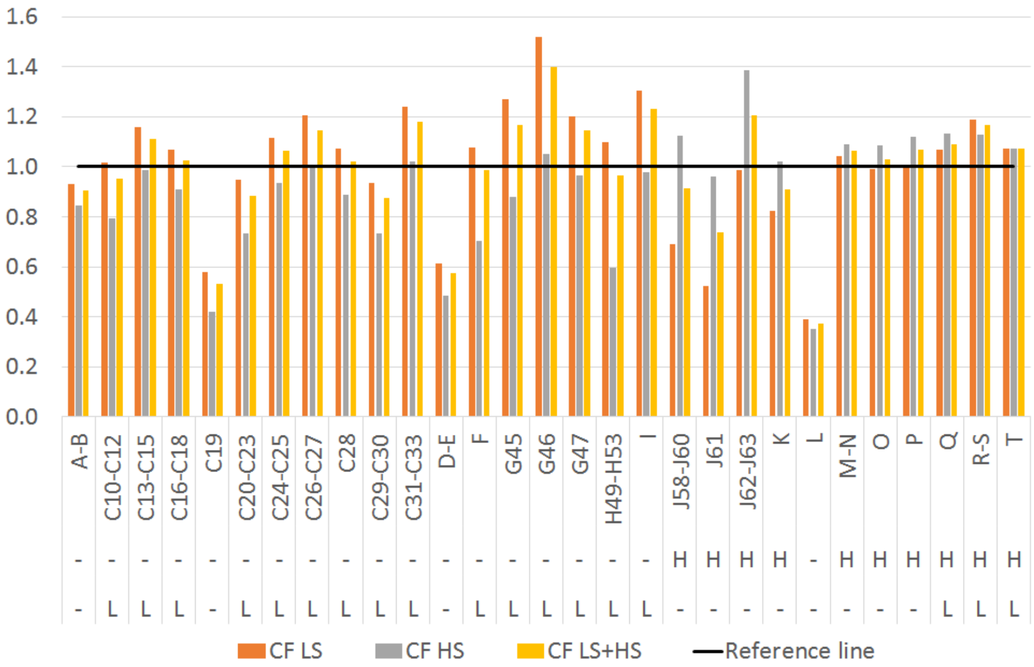


Figure 2.B.2: 2003: Ratio of immigration effects (no-IO / IO)

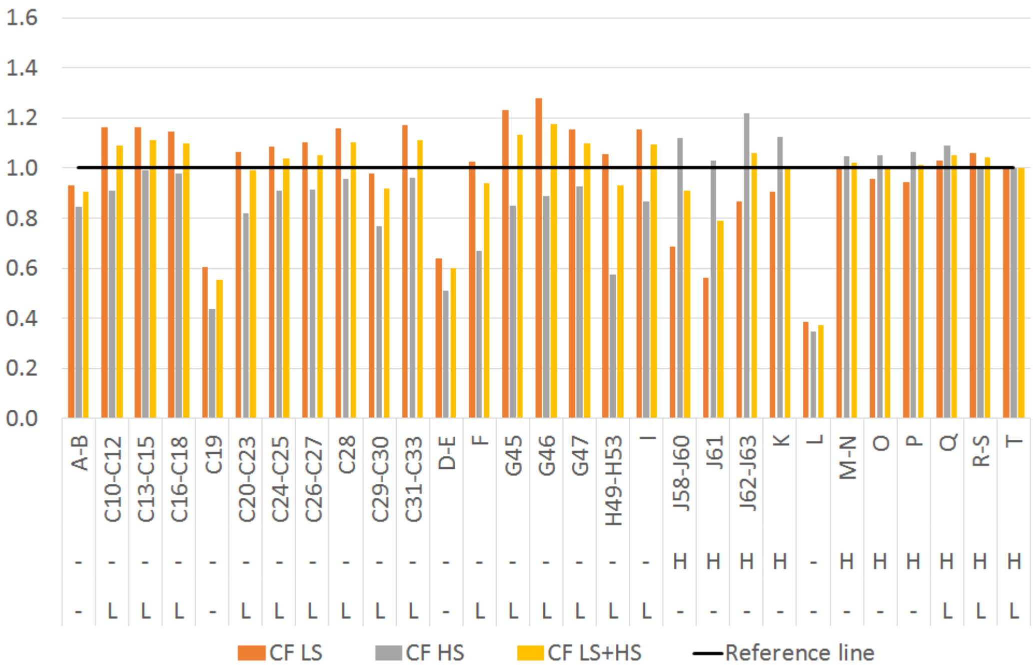


Figure 2.B.3: 2014: Ratio of immigration effects (no-IO / IO)

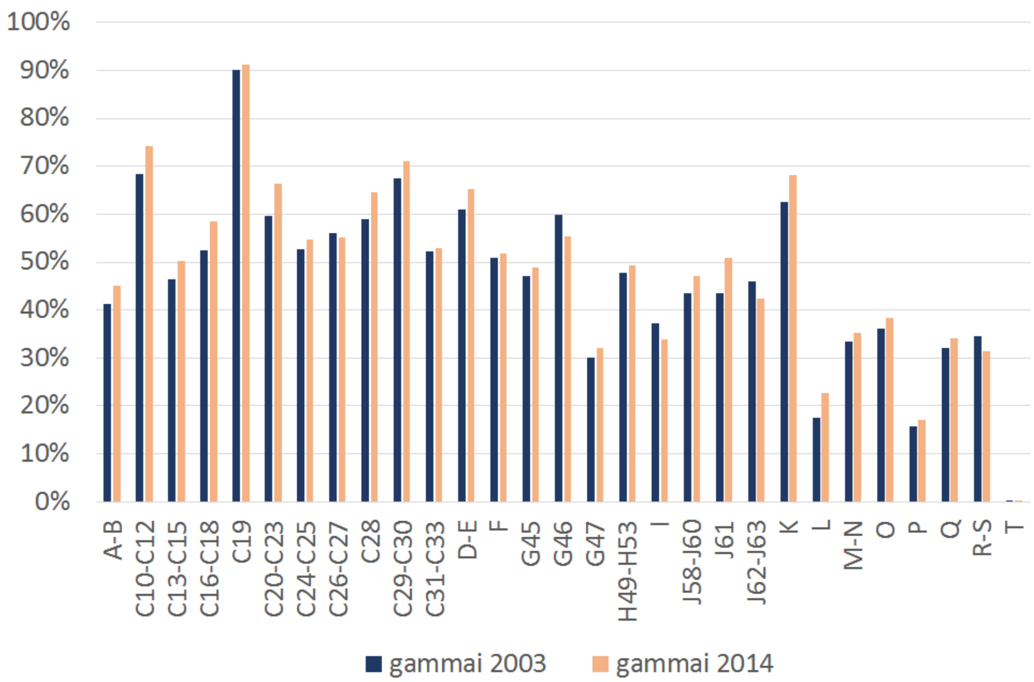


Figure 2.B.4: Intermediate input usage by industry 2003 and 2014 (γ_i)

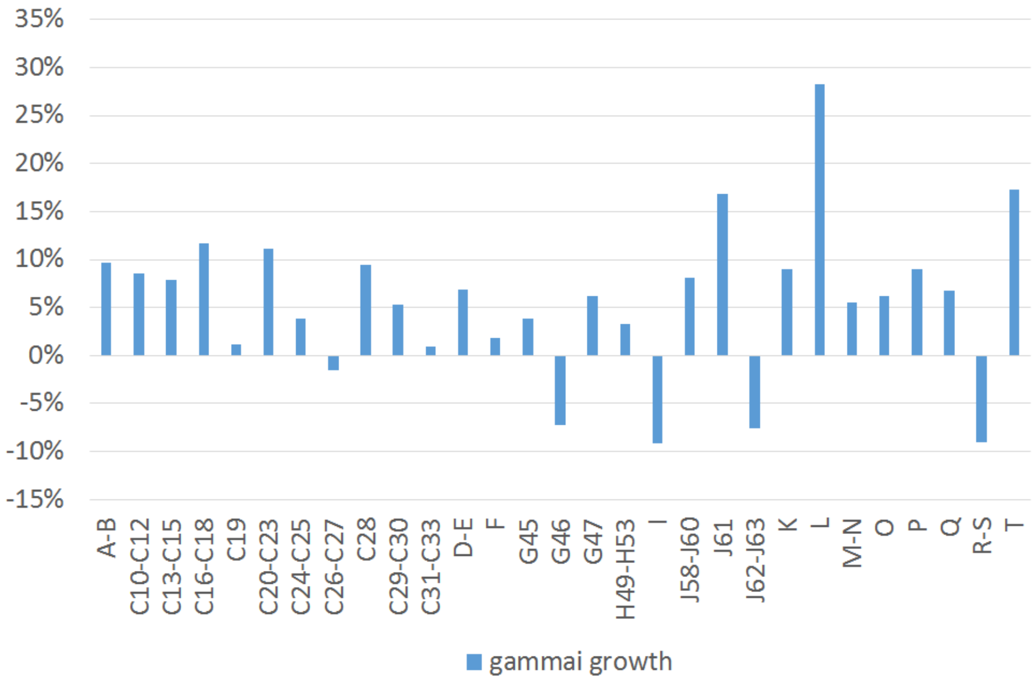
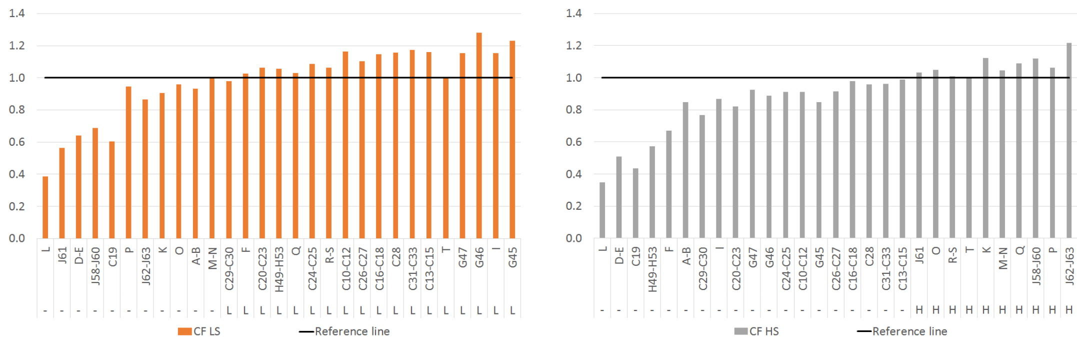


Figure 2.B.5: Intermediate input growth by industry: 2014 over 2003

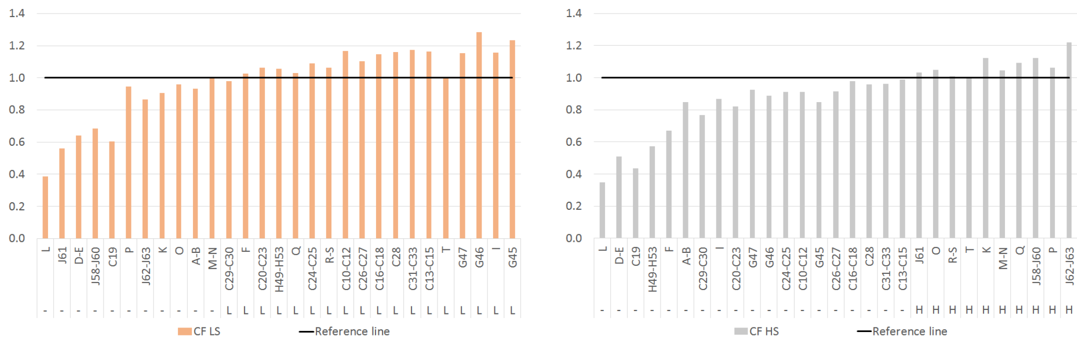
2.B.4 Non-linear solutions versus first order approximations

The results using the non-linear solutions, as given by the constructed counterfactual data, and those predicted by the model's first order approximatios are virtually identical. The figures below portray the estimations presented by this chapter done using the former (labelled 'Data') and the latter (labelled 'Model').



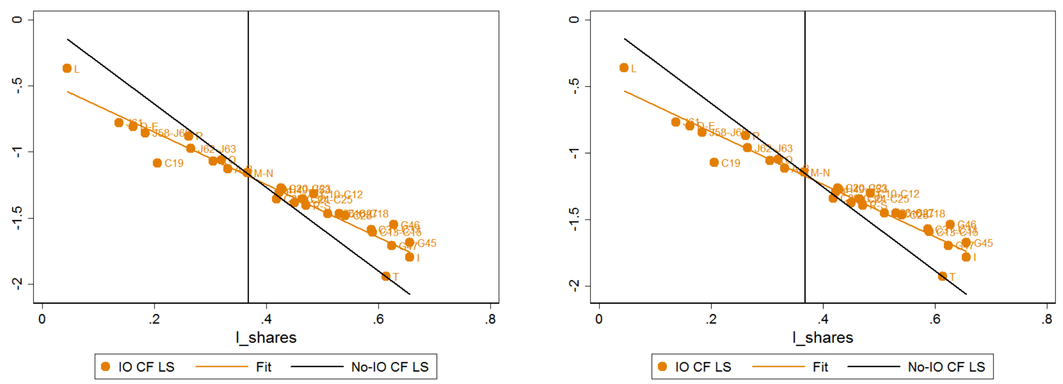
(a) CF LS (ranked by low-skilled shares) (b) CF HS (ranked by high-skilled shares)

Figure 2.B.6: Data: ratio of immigration effects (no-IO / IO)



(a) CF LS (ranked by low-skilled shares) (b) CF HS (ranked by high-skilled shares)

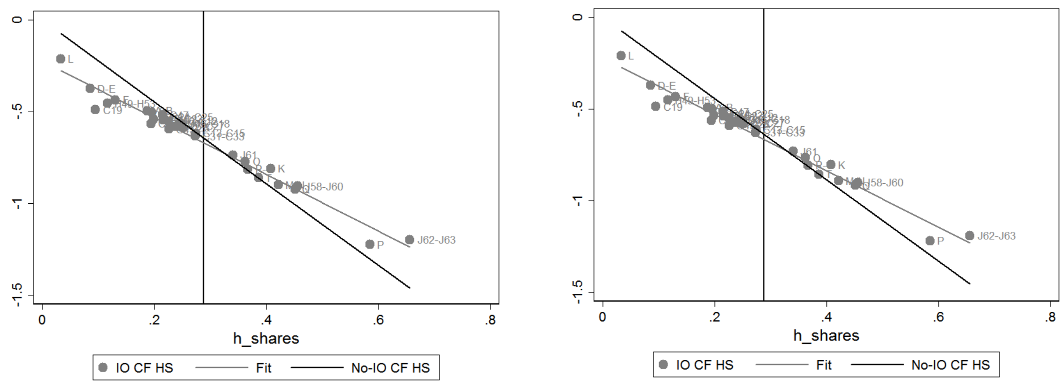
Figure 2.B.7: Model: ratio of immigration effects (no-IO / IO)



(a) Data (b) Model

Note: vertical line represents national average.

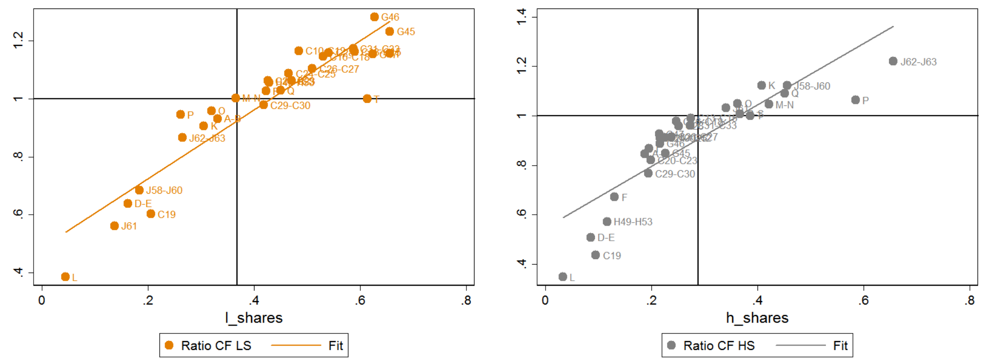
Figure 2.B.8: LS CF: output changes versus low-skill shares



(a) Data (b) Model

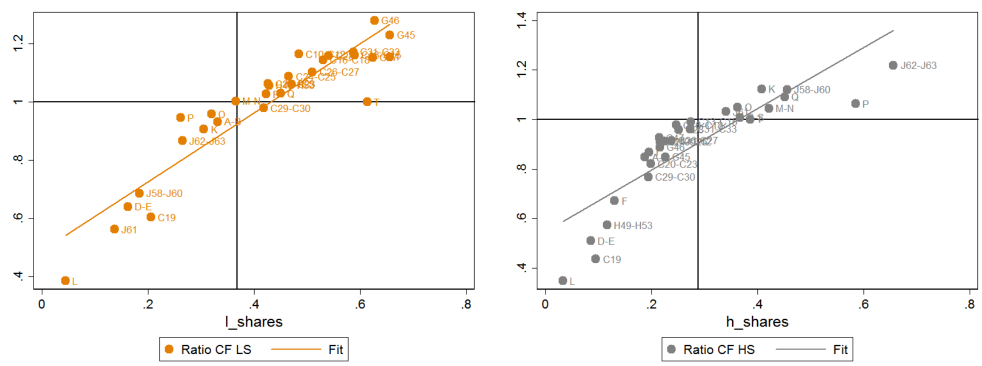
Note: vertical line represents national average.

Figure 2.B.9: HS CF: output changes versus high-skill shares



Note: vertical reference lines correspond to the national share of low-skill (left) and high-skilled (right).

Figure 2.B.10: Model: No-IO model's misestimation of impact of A8 immigration



Note: vertical reference lines correspond to the national share of low-skill (left) and high-skilled (right).

Figure 2.B.11: Data: No-IO model's misestimation of impact of A8 immigration

2.C Supplementary Results

2.C.1 Most and least affected

Output changes reported below after each industry code refer to the IO model and the no-IO model, outside and inside the parenthesis, respectively.

LS CF

Industries most affected by the shock:

1. T -1.94 (-1.94)
2. I -1.80 (-2.08)
3. G47 -1.71 (-1.97)

Industries least affected by the shock:

1. L -0.37 (-0.14)
2. J61 -0.78 (-0.44)
3. D-E -0.81 (-0.52)

Industries most overestimated:

1. G46 -1.55 (-1.99)
2. G45 -1.69 (-2.08)
3. C31-C33 -1.59 (-1.86)

Industries most underestimated:

1. L -0.37 (-0.14)
2. J61 -0.78 (-0.44)
3. C19 -1.09 (-0.66)

Industries least misestimated:

1. T -1.94 (-1.94)
2. M-N -1.16 (-1.16)
3. C29-C30 -1.36 (-1.33)

HS CF

Industries most affected by the shock:

1. P -1.23 (-1.30)
2. J62-J63 -1.20 (-1.46)
3. Q -0.92 (-1.01)

Industries least affected by the shock:

1. L -0.21 (-0.07)
2. D-E -0.38 (-0.19)
3. F -0.44 (-0.29)

Industries most overestimated:

1. J62-J63 -1.20 (-1.46)
2. K -0.81 (-0.91)
3. J58-J60 -0.91 (-1.02)

Industries most underestimated:

1. L -0.21 (-0.07)
2. C19 -0.49 (-0.21)
3. D-E -0.38 (-0.19)

Industries least misestimated:

1. T -0.86 (-0.86)
2. R-S -0.81 (-0.82)
3. C13-C15 -0.62 (-0.61)

ES CF

Industries most affected by the shock:

1. T -2.79 (-2.79)
2. Q -2.30 (-2.42)
3. I -2.29 (-2.50)

Industries least affected by the shock:

1. L -0.58 (-0.22)
2. D-E -1.18 (-0.71)
3. J61 -1.51 (-1.19)

Industries most overestimated:

1. G46 -2.09 (-2.46)
2. G45 -2.27 (-2.57)
3. C13-C15 -2.22 (-2.47)

Industries most underestimated:

1. L -0.58 (-0.22)
2. C19 -1.57 (-0.87)
3. D-E -1.18 (-0.71)

Industries least misestimated:

1. T -2.79 (-2.79)
2. K -1.87 (-1.87)
3. O -1.82 (-1.82)

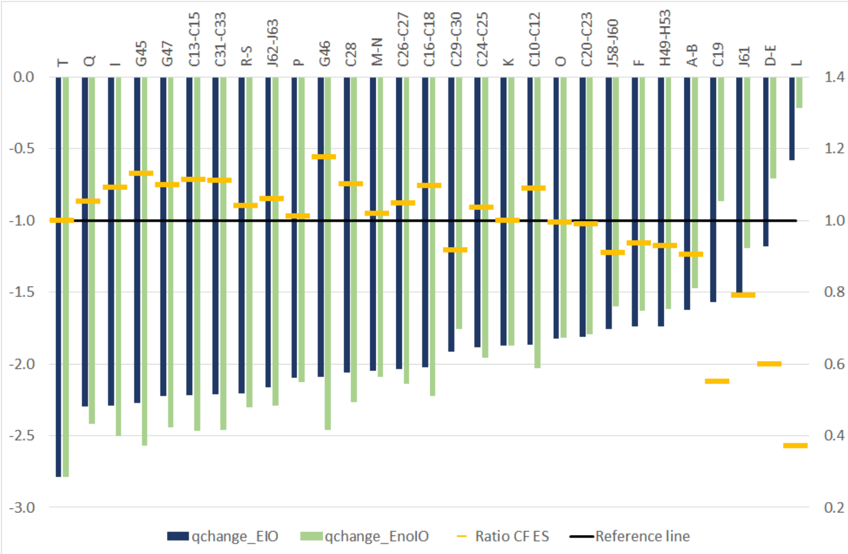
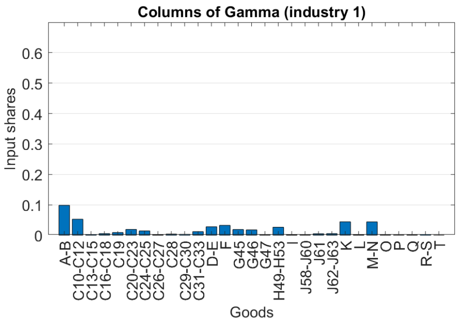


Figure 2.C.1: ESCF: IO and no-IO output changes (left) and ratios (right)

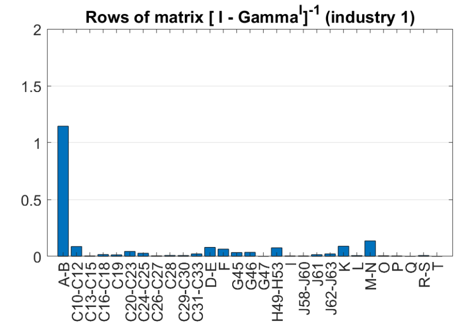
2.C.2 Specific industries

For completeness, I present here the plots for input use, row of the Leontief-inverse transposed, linkage weights and the shocks approximations for all industries.

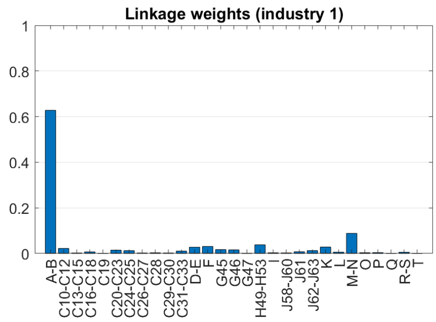
A-B: "Agriculture, Forestry and Fishing & Mining and quarrying"



(a) Nominal shares of purchases (γ_{ji})

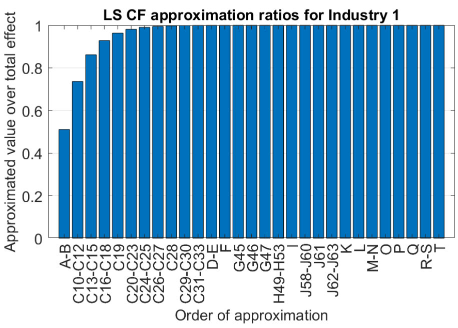


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

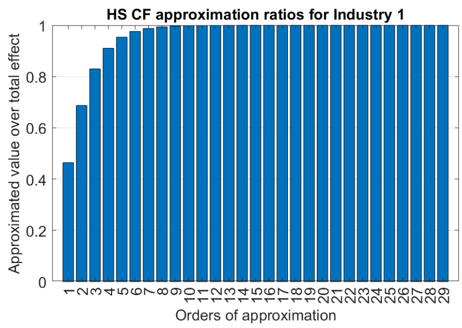


(c) Weights

Figure 2.C.2: Industry A-B's linkage effects



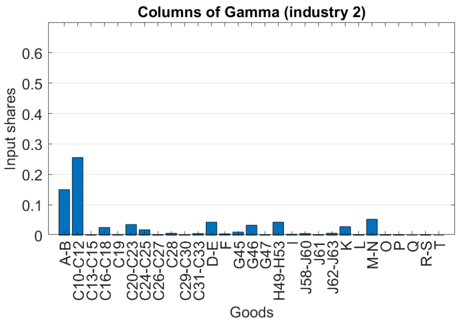
(a) Approximations LS CF shock



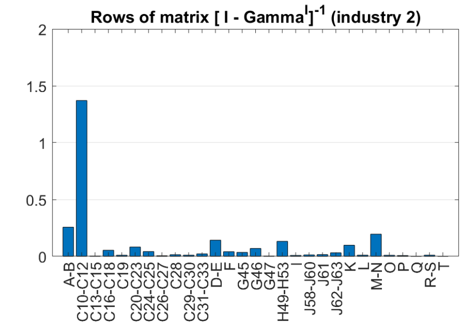
(b) Approximations HS CF shock

Figure 2.C.3: Industry A-B's approximations for the shocks

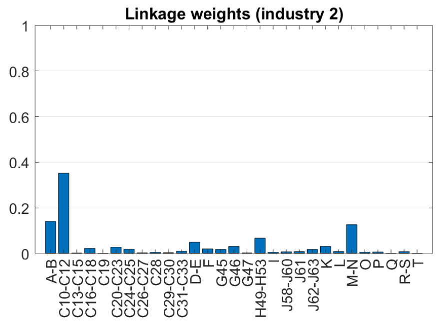
C10-C12: “Manufacture of food products, beverages and tobacco products”



(a) Nominal shares of purchases (γ_{ji})

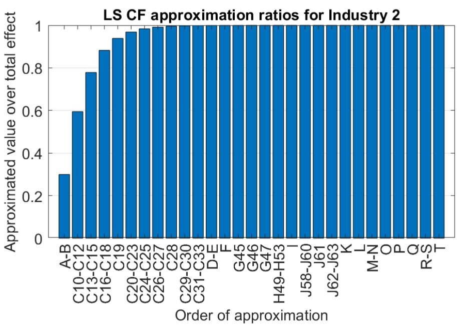


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

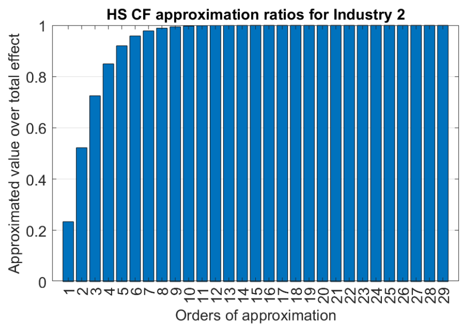


(c) Weights

Figure 2.C.4: Industry C10-C12’s linkage effects



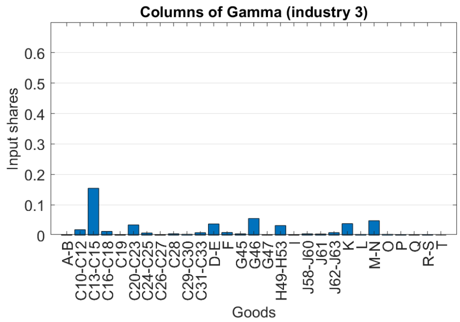
(a) Approximations LS CF shock



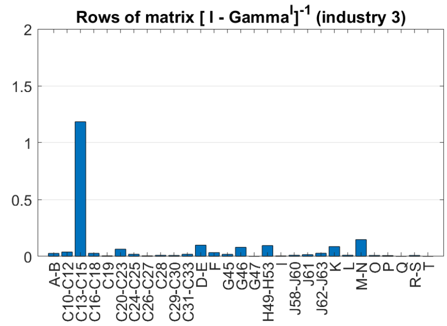
(b) Approximations HS CF shock

Figure 2.C.5: Industry C10-C12’s approximations for the shocks

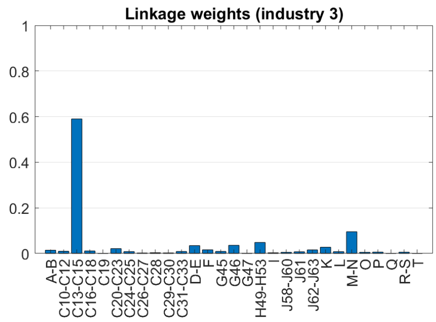
C13-C15: "Manufacture of textiles, wearing apparel and leather products"



(a) Nominal shares of purchases (γ_{ji})

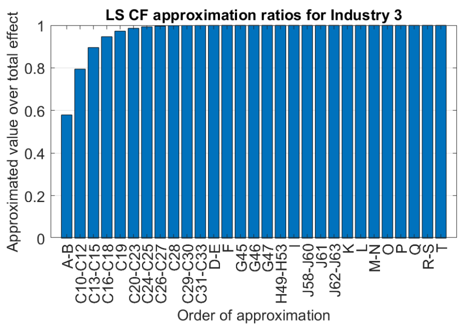


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

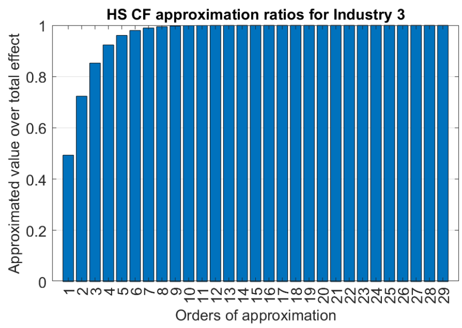


(c) Weights

Figure 2.C.6: Industry C13-C15's linkage effects



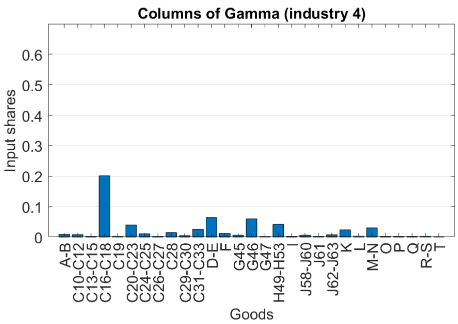
(a) Approximations LS CF shock



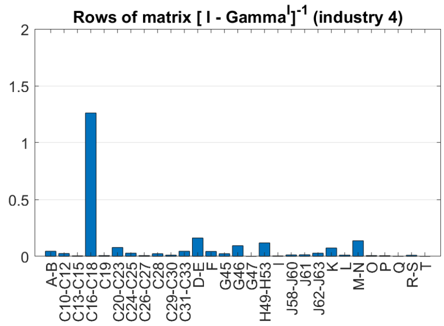
(b) Approximations HS CF shock

Figure 2.C.7: Industry C13-C15's approximations for the shocks

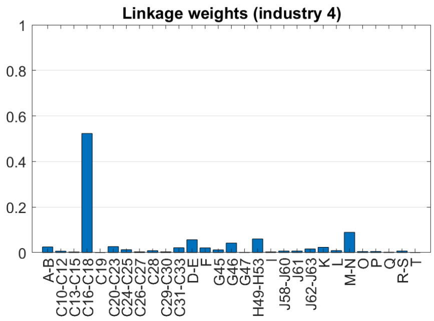
C16-C18: "Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials & Manufacture of paper and paper products & Printing and reproduction of recorded media"



(a) Nominal shares of purchases (γ_{ji})

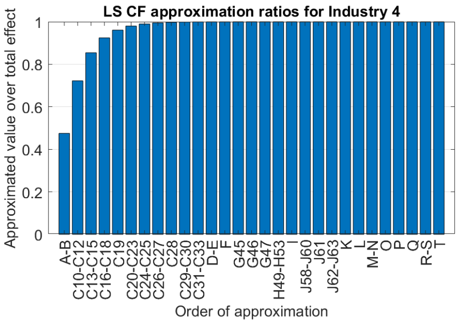


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

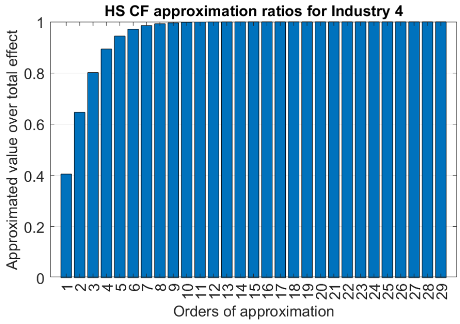


(c) Weights

Figure 2.C.8: Industry C16-C18's linkage effects



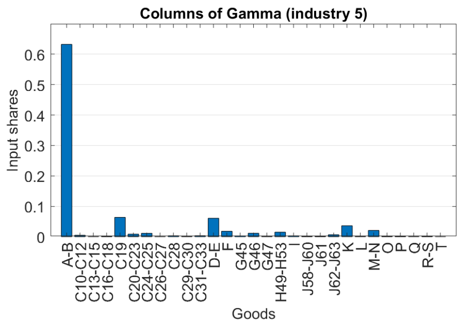
(a) Approximations LS CF shock



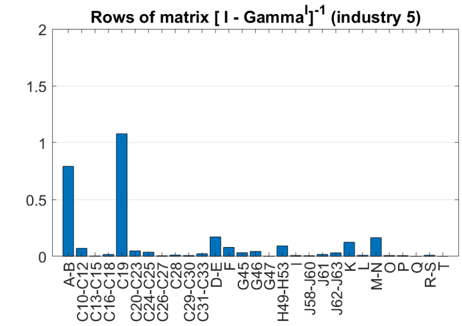
(b) Approximations HS CF shock

Figure 2.C.9: Industry C16-C18's approximations for the shocks

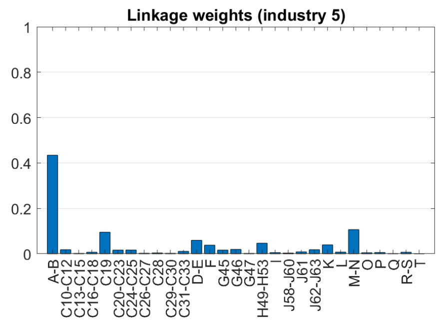
C19: "Manufacture of coke and refined petroleum products"



(a) Nominal shares of purchases (γ_{ji})

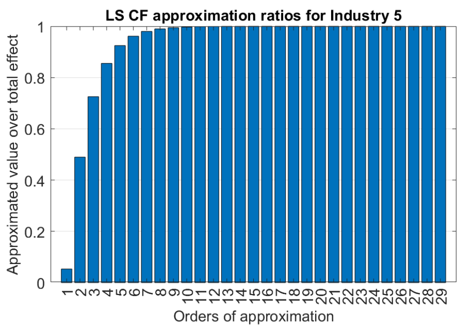


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

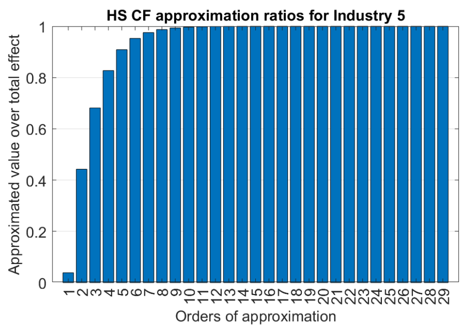


(c) Weights

Figure 2.C.10: Industry C19's linkage effects



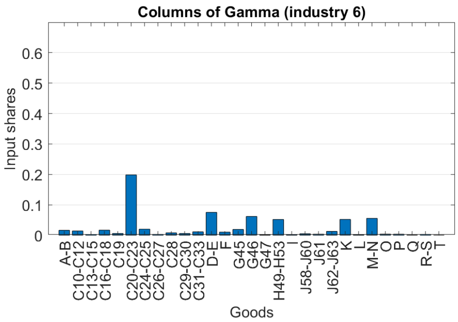
(a) Approximations LS CF shock



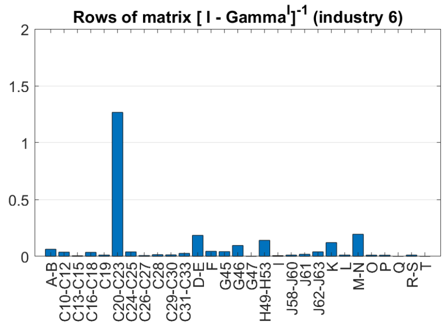
(b) Approximations HS CF shock

Figure 2.C.11: Industry C19's approximations for the shocks

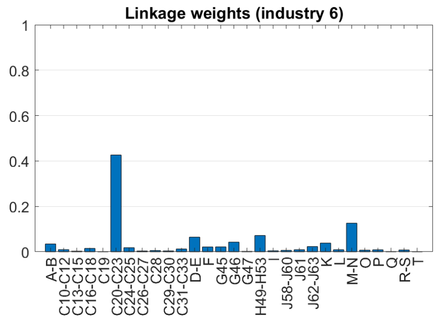
C20-C23: "Chemicals and chemical products & Rubber and plastics products, and other non-metallic mineral products"



(a) Nominal shares of purchases (γ_{ji})

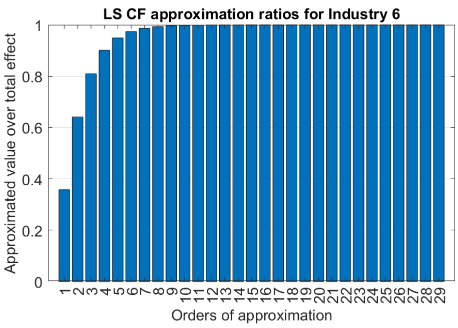


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

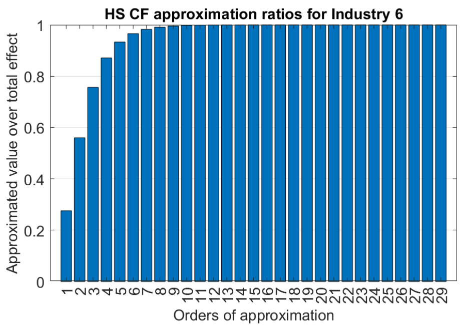


(c) Weights

Figure 2.C.12: Industry C20-C23's linkage effects



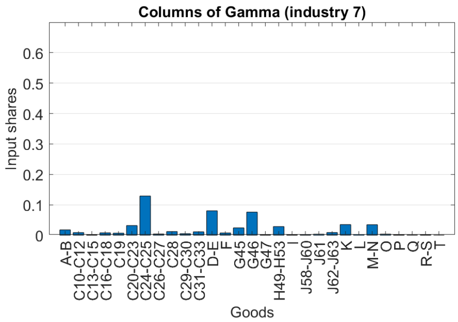
(a) Approximations LS CF shock



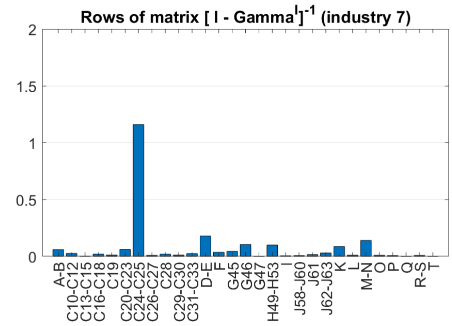
(b) Approximations HS CF shock

Figure 2.C.13: Industry C20-C23's approximations for the shocks

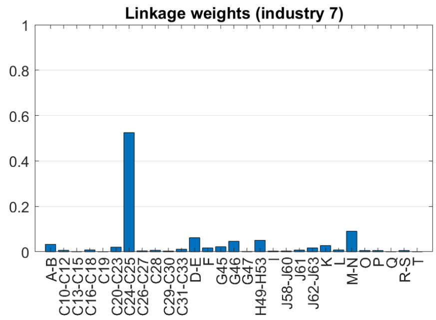
C24-C25: "Basic metals and fabricated metal products, except machinery and equipment"



(a) Nominal shares of purchases (γ_{ji})

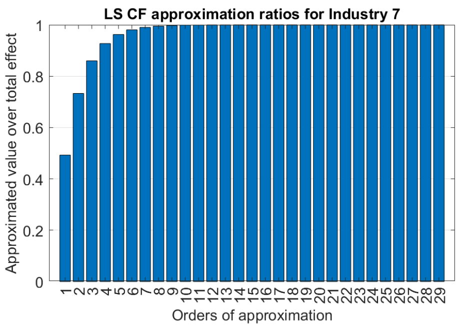


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

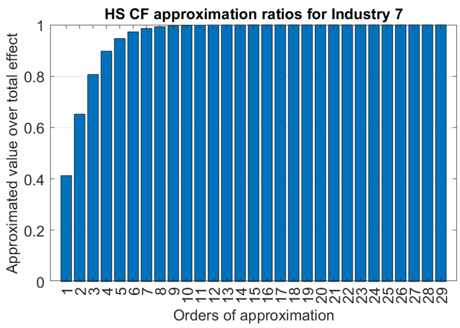


(c) Weights

Figure 2.C.14: Industry C24-C25's linkage effects



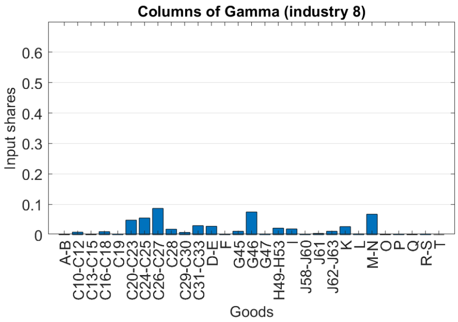
(a) Approximations LS CF shock



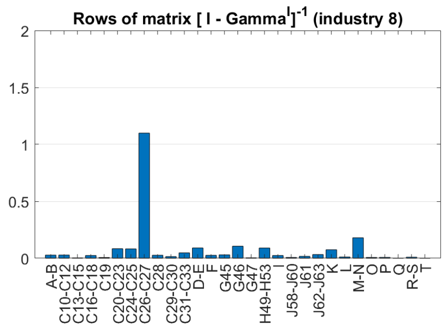
(b) Approximations HS CF shock

Figure 2.C.15: Industry C24-C25's approximations for the shocks

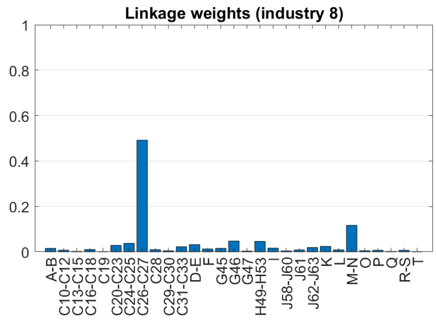
C26-C27: “Electrical and optical equipment”



(a) Nominal shares of purchases (γ_{ji})

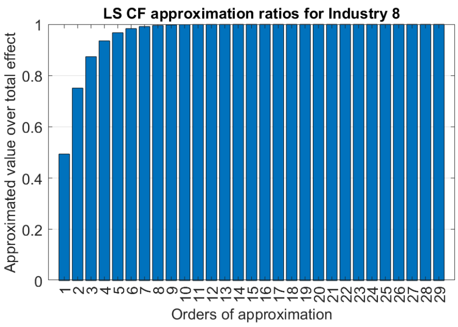


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

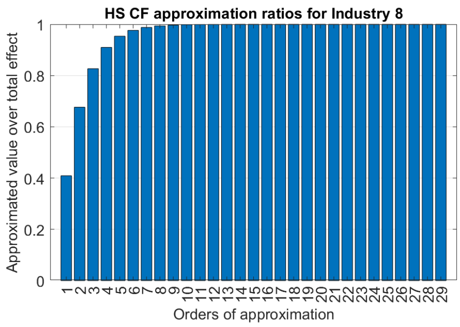


(c) Weights

Figure 2.C.16: Industry C26-C27’s linkage effects



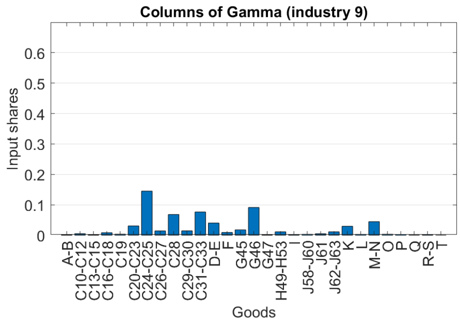
(a) Approximations LS CF shock



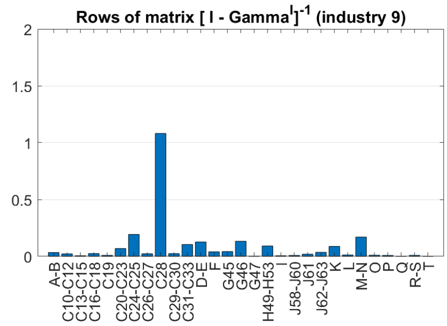
(b) Approximations HS CF shock

Figure 2.C.17: Industry C26-C27’s approximations for the shocks

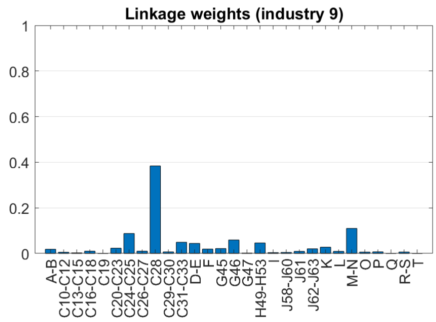
C28: “Machinery and equipment n.e.c.”



(a) Nominal shares of purchases (γ_{ji})

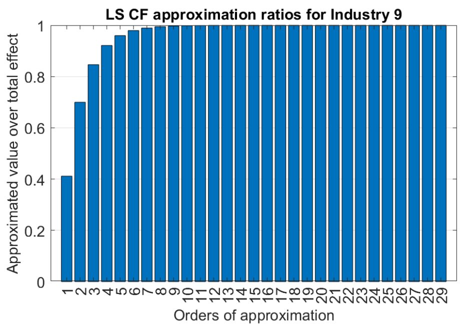


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

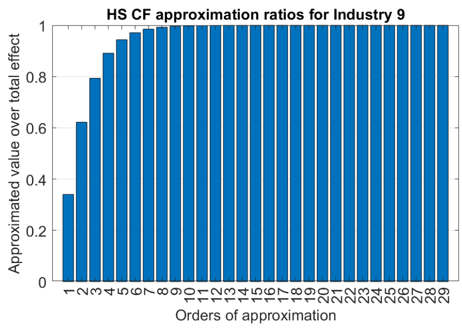


(c) Weights

Figure 2.C.18: Industry C28’s linkage effects



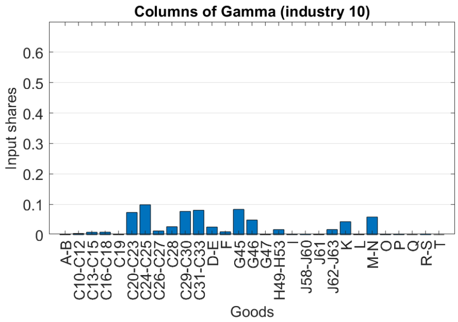
(a) Approximations LS CF shock



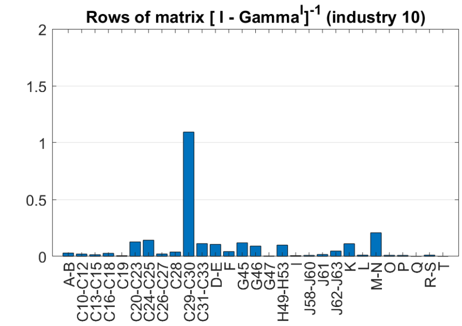
(b) Approximations HS CF shock

Figure 2.C.19: Industry C28’s approximations for the shocks

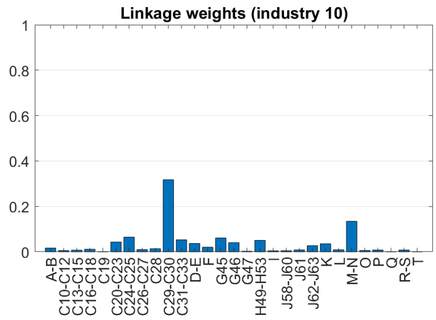
C29-C30: “Transport equipment”



(a) Nominal shares of purchases (γ_{ji})

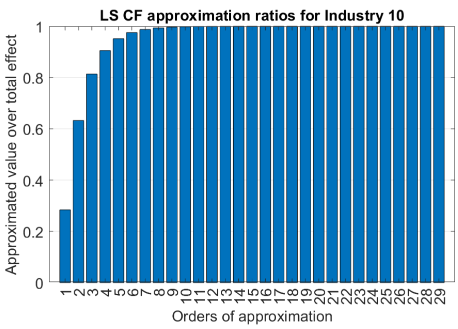


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

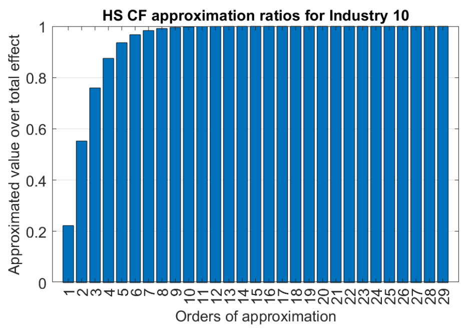


(c) Weights

Figure 2.C.20: Industry C29-C30’s linkage effects



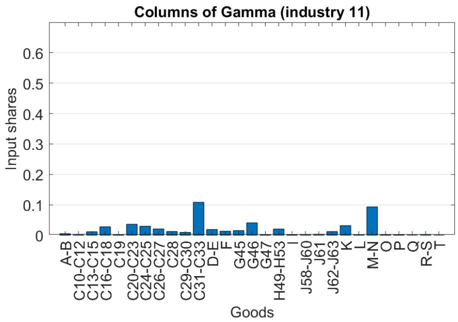
(a) Approximations LS CF shock



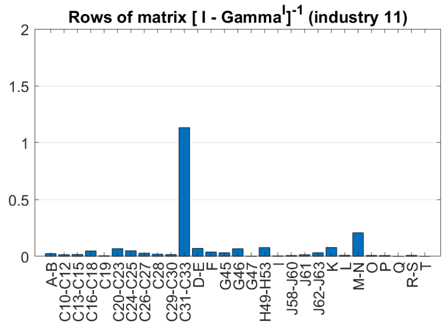
(b) Approximations HS CF shock

Figure 2.C.21: Industry C29-C30’s approximations for the shocks

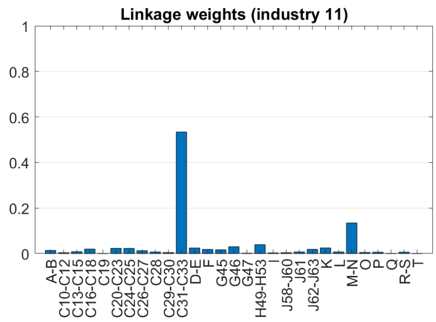
C31-C33: "Other manufacturing; repair and installation of machinery and equipment"



(a) Nominal shares of purchases (γ_{ji})

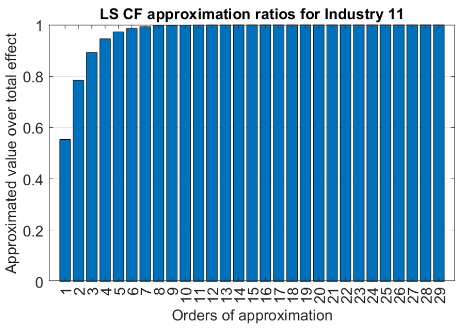


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

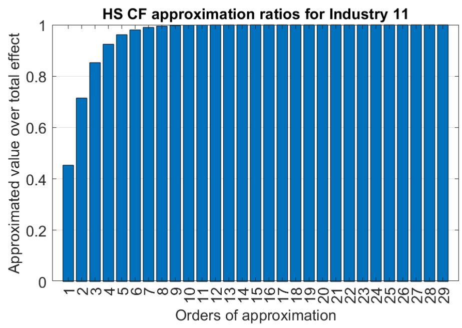


(c) Weights

Figure 2.C.22: Industry C31-C33's linkage effects



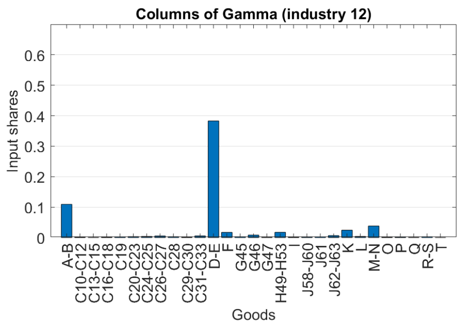
(a) Approximations LS CF shock



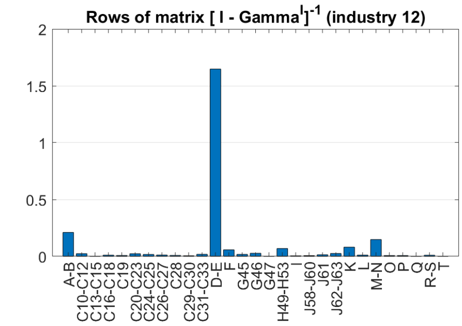
(b) Approximations HS CF shock

Figure 2.C.23: Industry C31-C33's approximations for the shocks

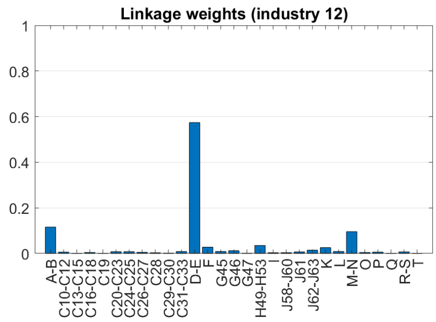
D-E: “Electricity, Gas and Water Supply”



(a) Nominal shares of purchases (γ_{ji})

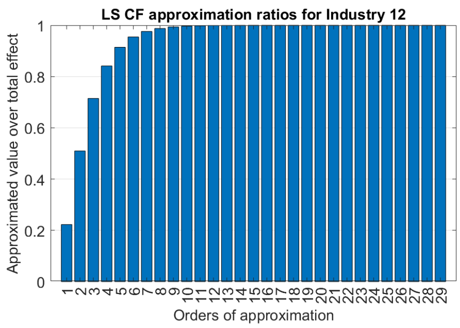


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

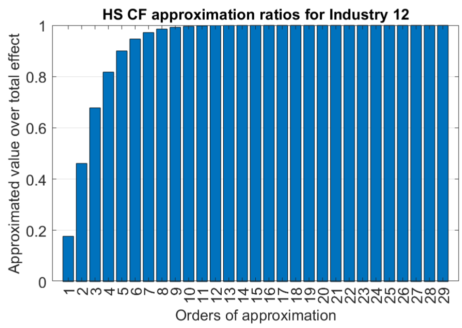


(c) Weights

Figure 2.C.24: Industry D-E’s linkage effects



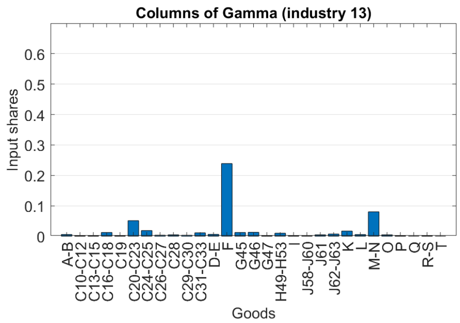
(a) Approximations LS CF shock



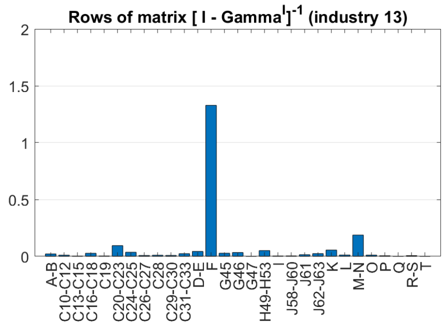
(b) Approximations HS CF shock

Figure 2.C.25: Industry D-E’s approximations for the shocks

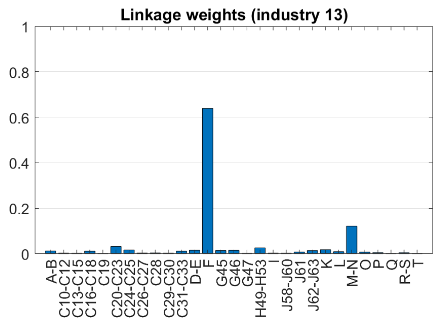
F: “Construction”



(a) Nominal shares of purchases (γ_{ji})

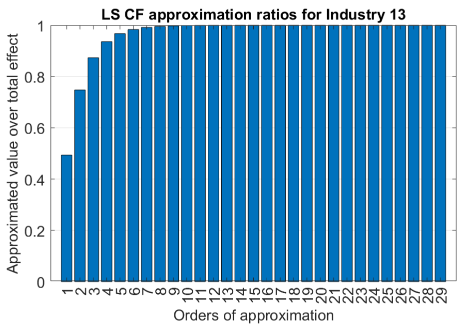


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

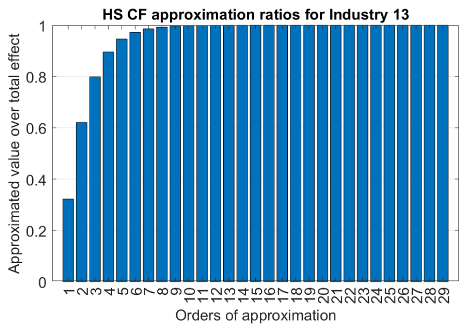


(c) Weights

Figure 2.C.26: Industry F’s linkage effects



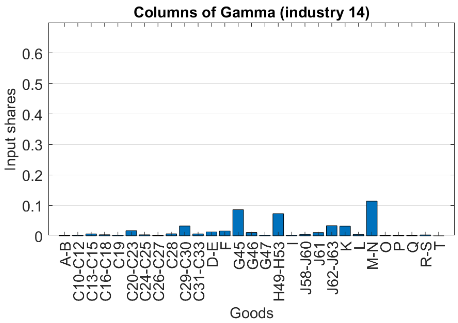
(a) Approximations LS CF shock



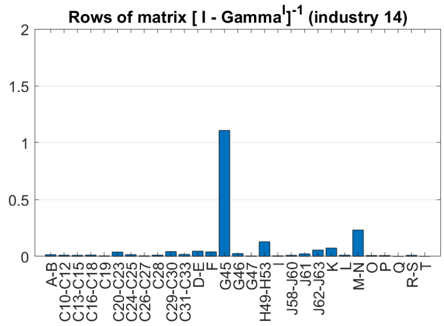
(b) Approximations HS CF shock

Figure 2.C.27: Industry F’s approximations for the shocks

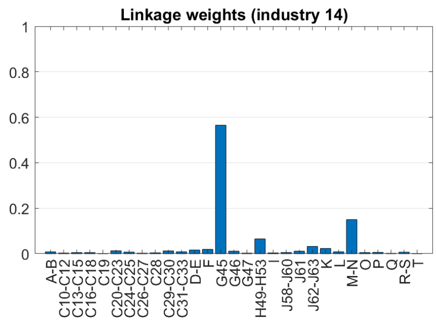
G45: “Wholesale and retail trade and repair of motor vehicles and motorcycles”



(a) Nominal shares of purchases (γ_{ji})

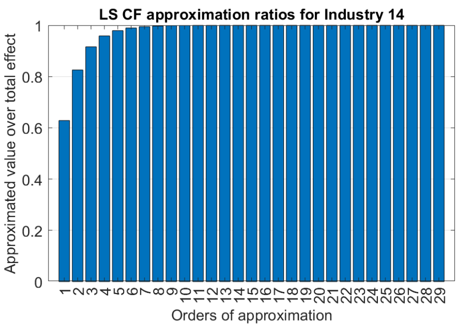


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

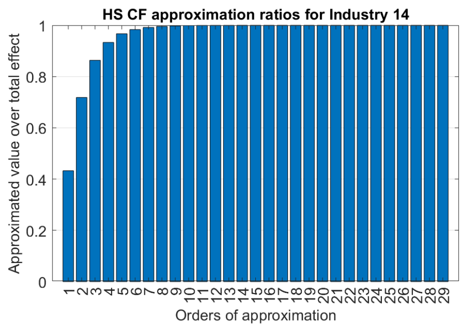


(c) Weights

Figure 2.C.28: Industry G45’s linkage effects



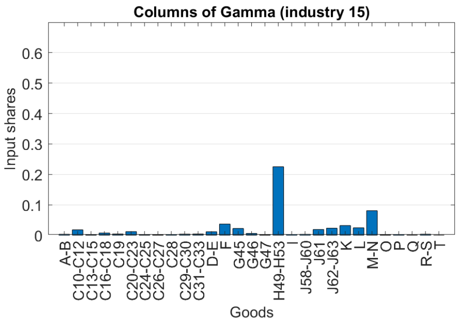
(a) Approximations LS CF shock



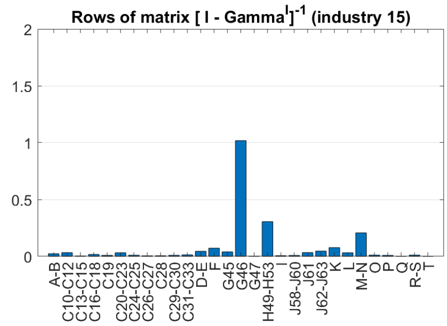
(b) Approximations HS CF shock

Figure 2.C.29: Industry G45’s approximations for the shocks

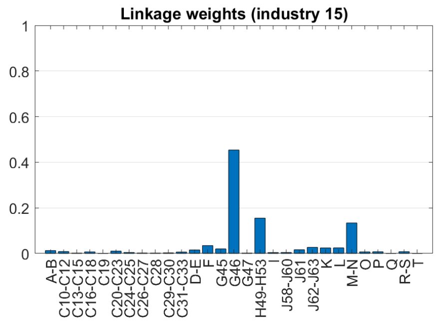
G46: "Wholesale trade, except of motor vehicles and motorcycles"



(a) Nominal shares of purchases (γ_{ji})

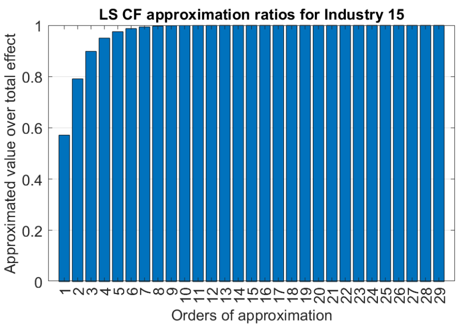


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

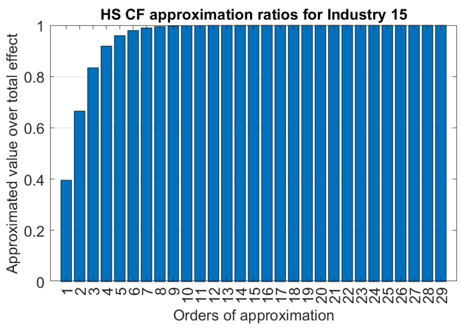


(c) Weights

Figure 2.C.30: Industry G46's linkage effects



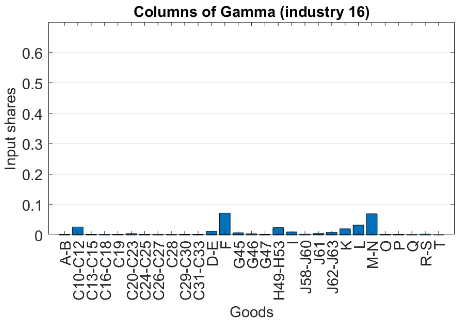
(a) Approximations LS CF shock



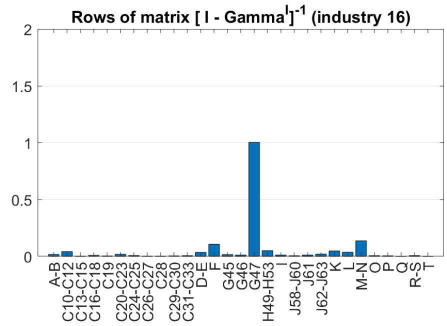
(b) Approximations HS CF shock

Figure 2.C.31: Industry G46's approximations for the shocks

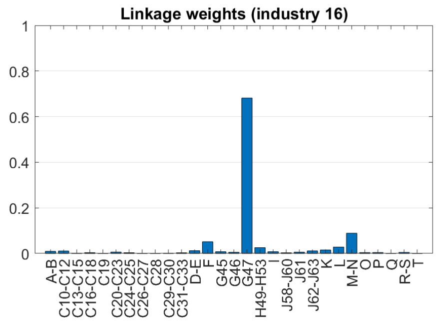
G47: “Retail trade, except of motor vehicles and motorcycles”



(a) Nominal shares of purchases (γ_{ji})

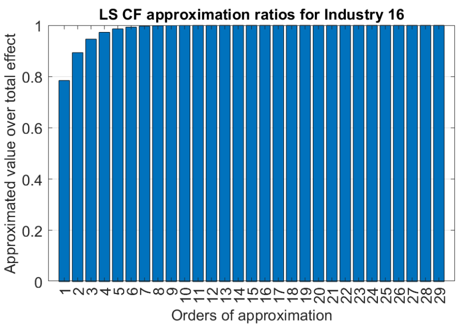


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

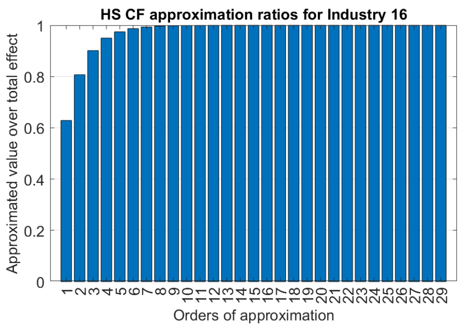


(c) Weights

Figure 2.C.32: Industry G47’s linkage effects



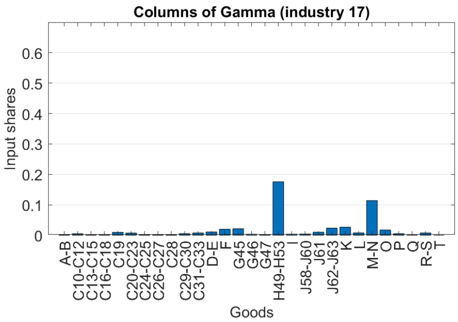
(a) Approximations LS CF shock



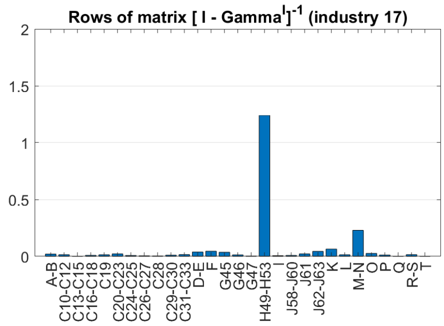
(b) Approximations HS CF shock

Figure 2.C.33: Industry G47’s approximations for the shocks

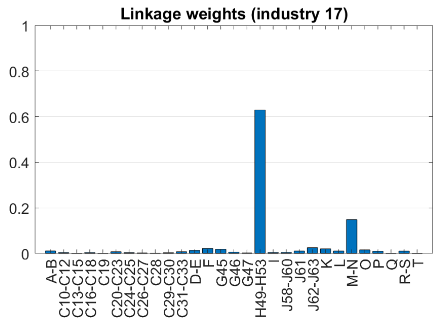
H49-H53: "Transport and storage & Postal and courier activities"



(a) Nominal shares of purchases (γ_{ji})

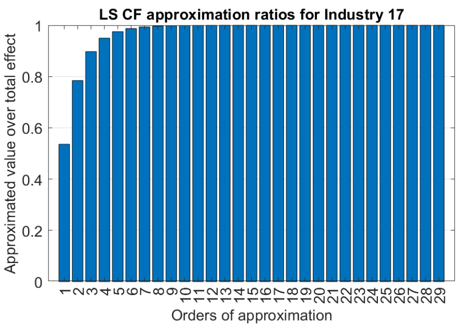


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

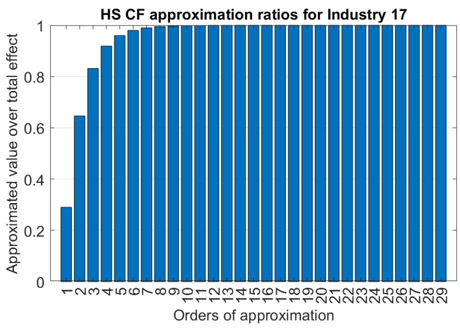


(c) Weights

Figure 2.C.34: Industry H49-H53's linkage effects



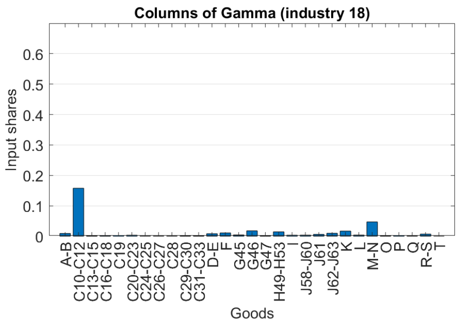
(a) Approximations LS CF shock



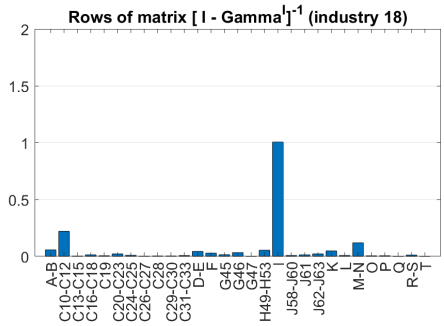
(b) Approximations HS CF shock

Figure 2.C.35: Industry H49-H53's approximations for the shocks

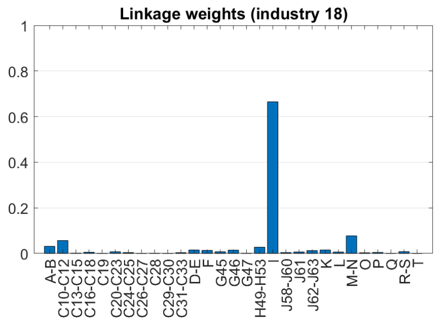
I: “Accommodation and Food Service Activities”



(a) Nominal shares of purchases (γ_{ji})

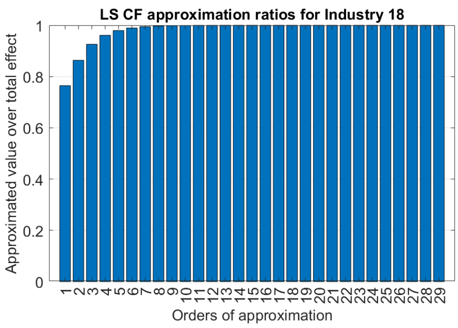


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

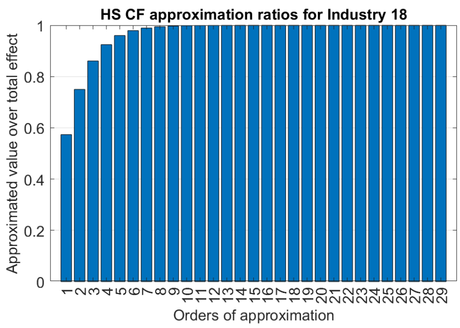


(c) Weights

Figure 2.C.36: Industry I’s linkage effects



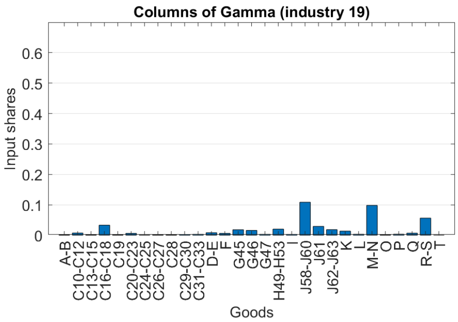
(a) Approximations LS CF shock



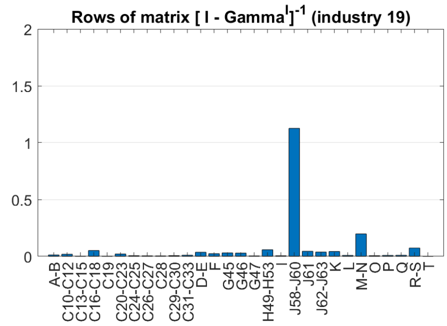
(b) Approximations HS CF shock

Figure 2.C.37: Industry I’s approximations for the shocks

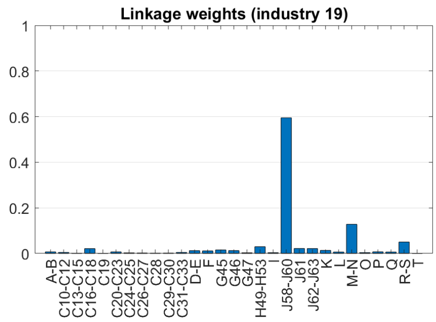
J58-J60: "Publishing, audiovisual and broadcasting activities"



(a) Nominal shares of purchases (γ_{ji})

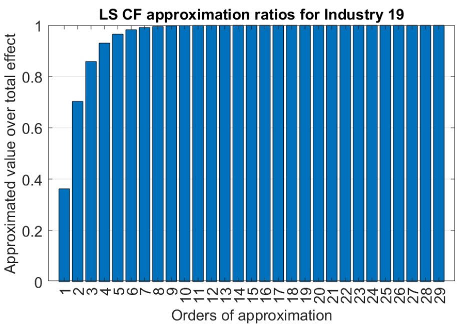


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

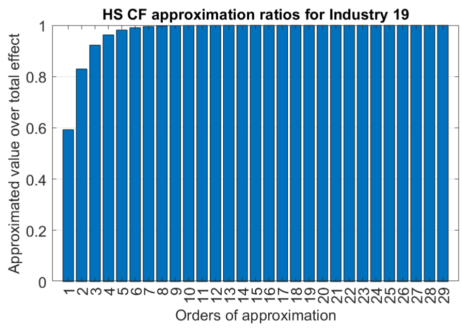


(c) Weights

Figure 2.C.38: Industry J58-J60's linkage effects



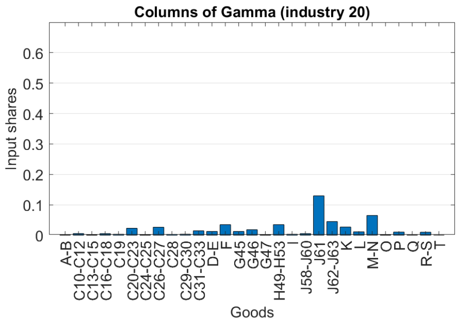
(a) Approximations LS CF shock



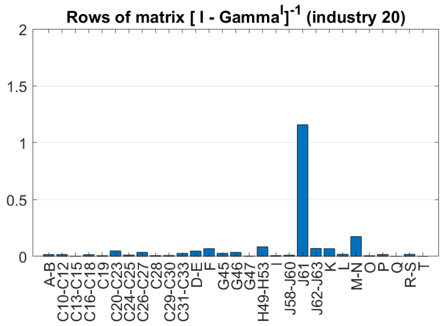
(b) Approximations HS CF shock

Figure 2.C.39: Industry J58-J60's approximations for the shocks

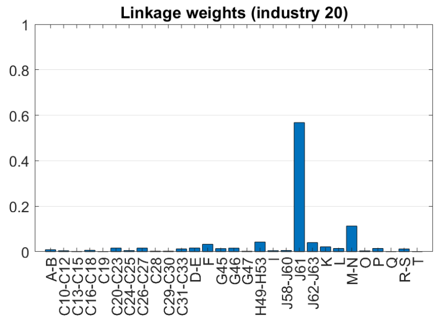
J61: “Telecommunications”



(a) Nominal shares of purchases (γ_{ji})

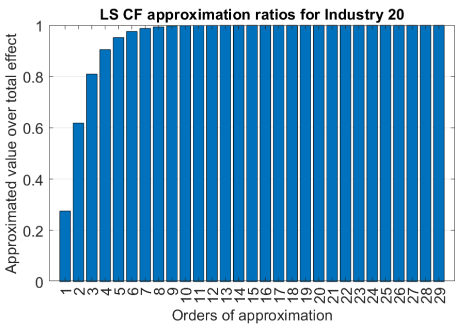


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

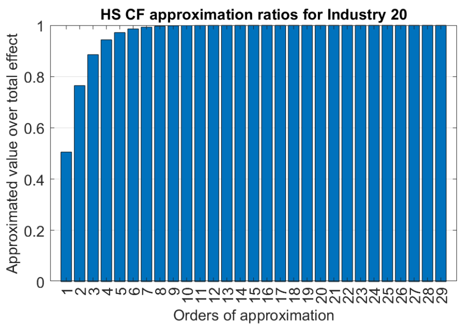


(c) Weights

Figure 2.C.40: Industry J61’s linkage effects



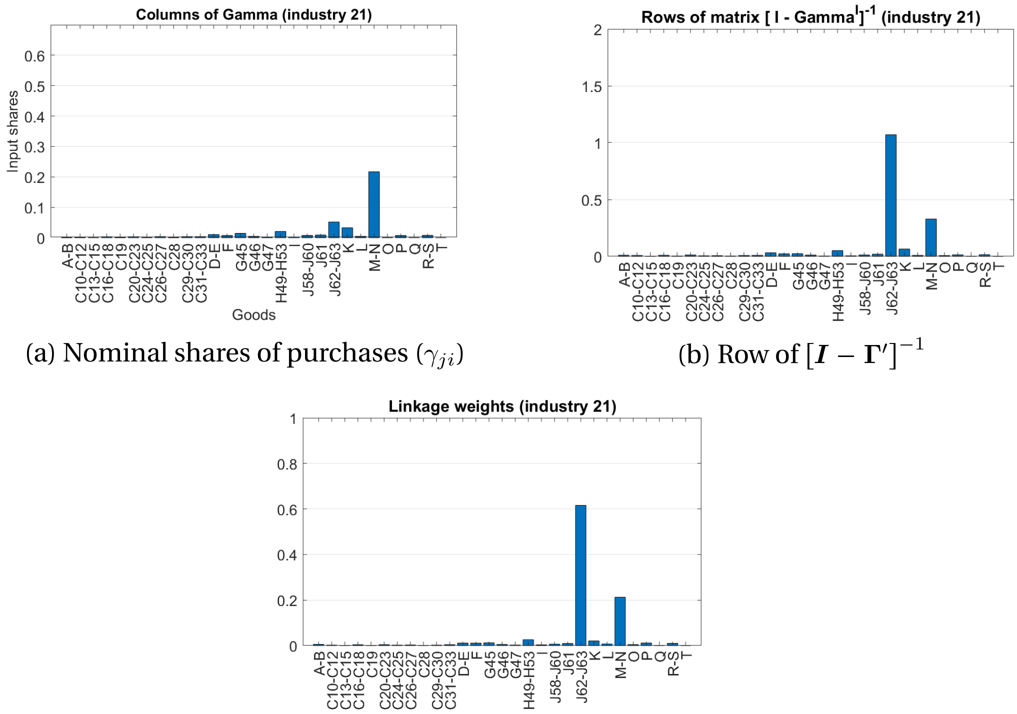
(a) Approximations LS CF shock



(b) Approximations HS CF shock

Figure 2.C.41: Industry J61’s approximations for the shocks

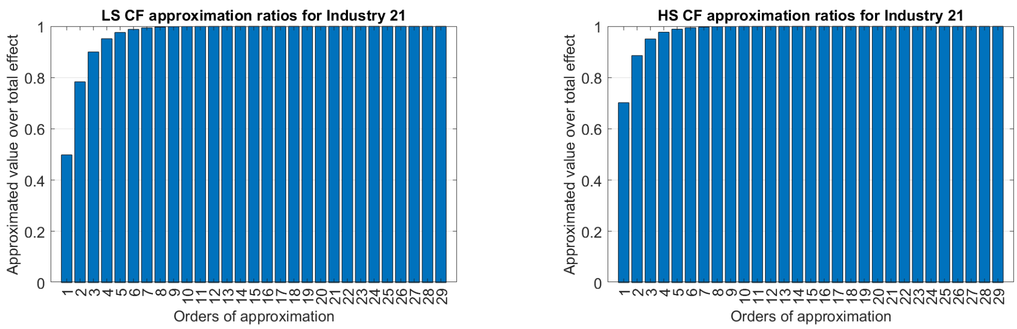
J62-J63: "IT and other information services"



(a) Nominal shares of purchases (γ_{ji}) (b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

(c) Weights

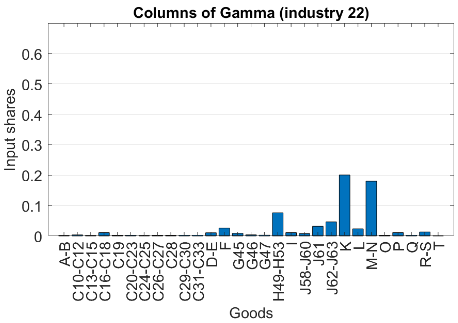
Figure 2.C.42: Industry J62-J63's linkage effects



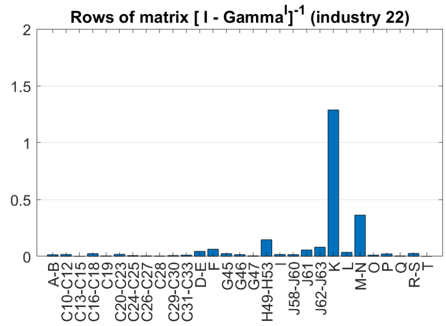
(a) Approximations LS CF shock (b) Approximations HS CF shock

Figure 2.C.43: Industry J62-J63's approximations for the shocks

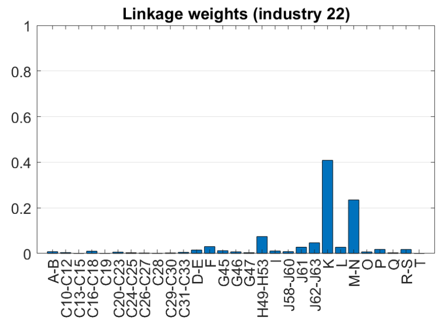
K: “Financial and insurance activities”



(a) Nominal shares of purchases (γ_{ji})

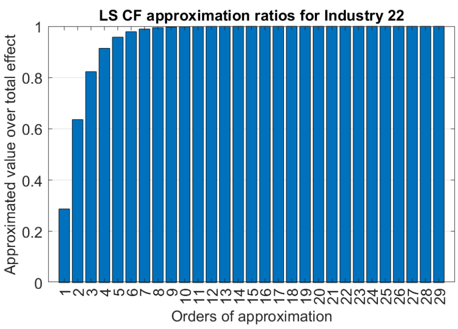


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

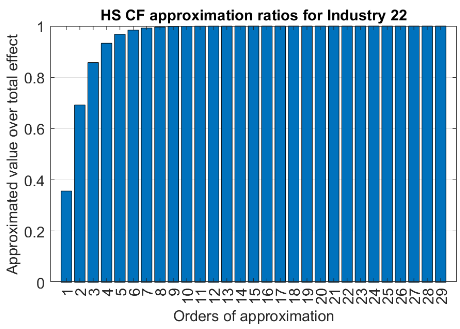


(c) Weights

Figure 2.C.44: Industry K’s linkage effects



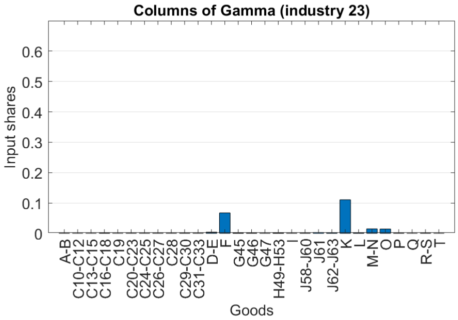
(a) Approximations LS CF shock



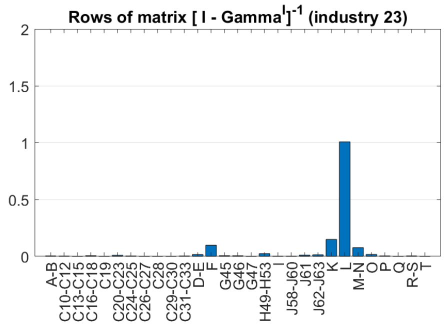
(b) Approximations HS CF shock

Figure 2.C.45: Industry K’s approximations for the shocks

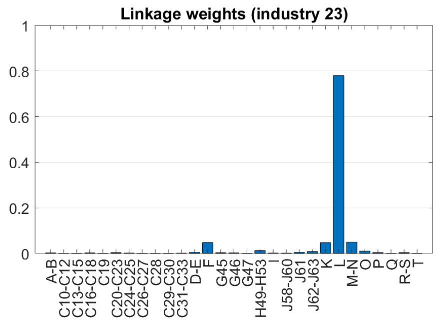
L: “Real estate activities”



(a) Nominal shares of purchases (γ_{ji})

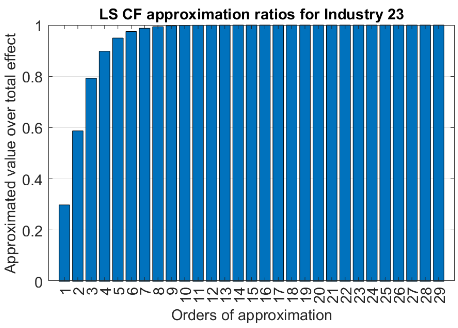


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

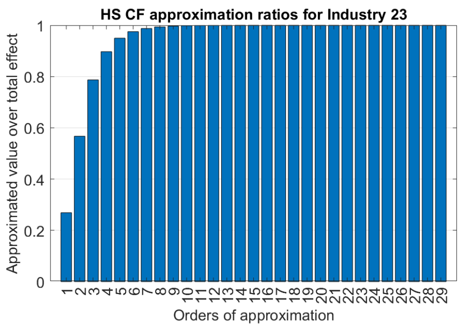


(c) Weights

Figure 2.C.46: Industry L’s linkage effects



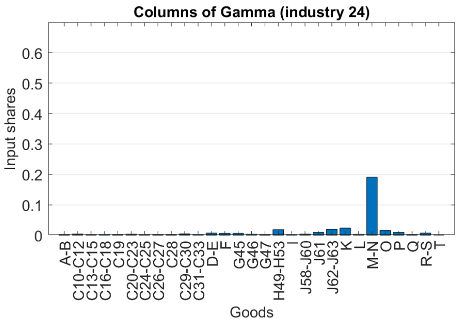
(a) Approximations LS CF shock



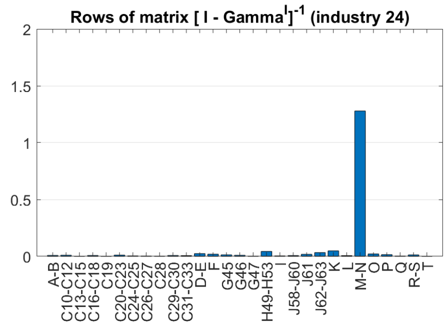
(b) Approximations HS CF shock

Figure 2.C.47: Industry L’s approximations for the shocks

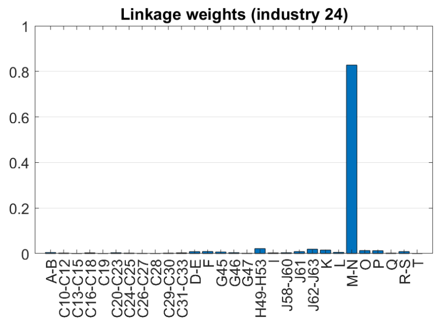
M-N: "Professional, scientific, technical, administrative and support service activities"



(a) Nominal shares of purchases (γ_{ji})

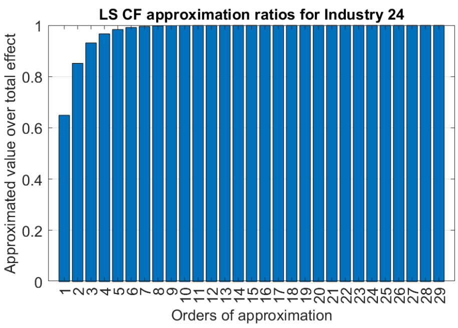


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

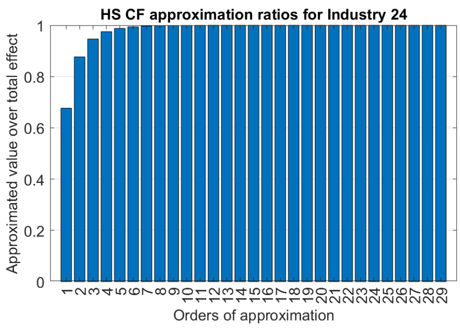


(c) Weights

Figure 2.C.48: Industry M-N's linkage effects



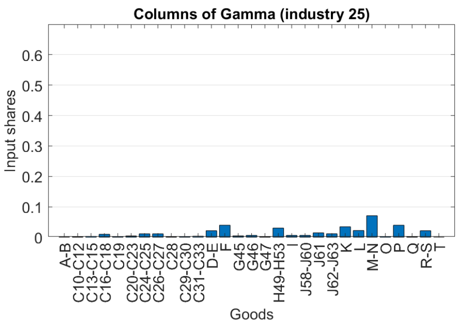
(a) Approximations LS CF shock



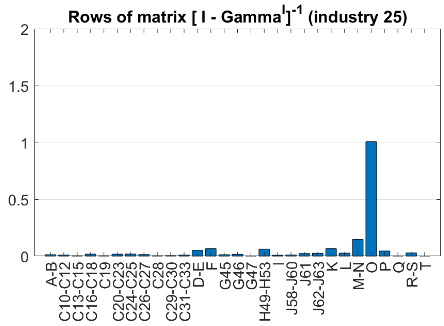
(b) Approximations HS CF shock

Figure 2.C.49: Industry M-N's approximations for the shocks

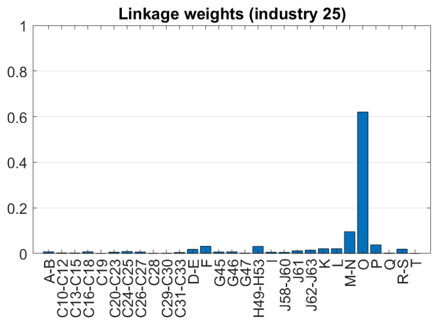
O: “Public administration and defence; compulsory social security”



(a) Nominal shares of purchases (γ_{ji})

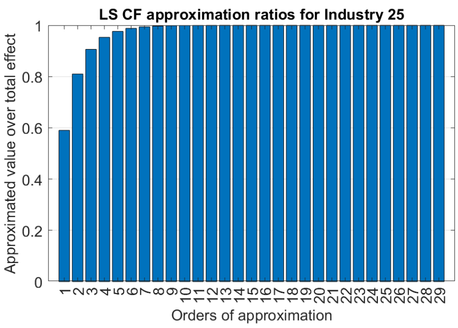


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

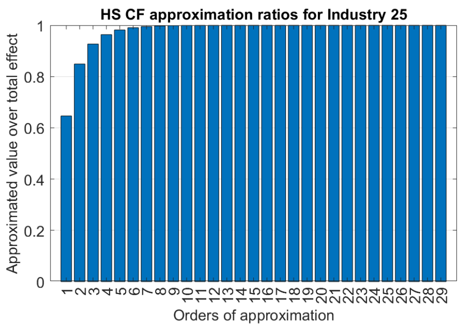


(c) Weights

Figure 2.C.50: Industry O’s linkage effects



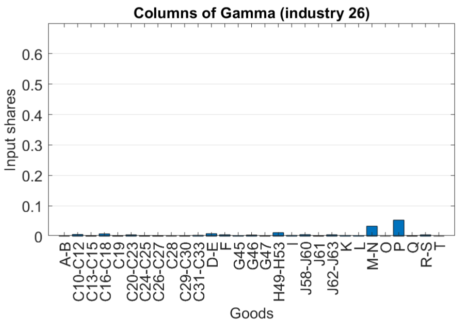
(a) Approximations LS CF shock



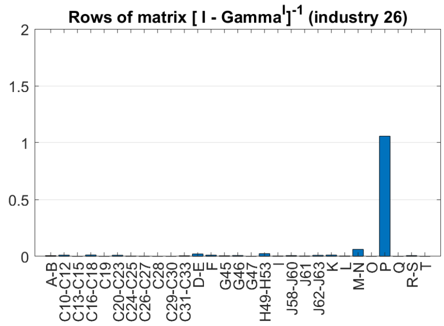
(b) Approximations HS CF shock

Figure 2.C.51: Industry O’s approximations for the shocks

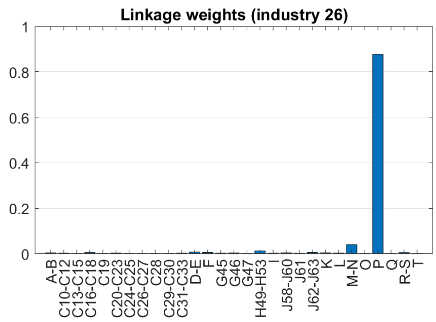
P: "Education"



(a) Nominal shares of purchases (γ_{ji})

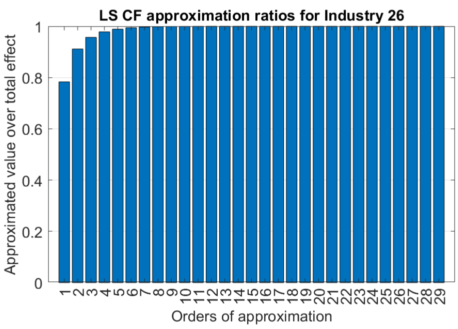


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

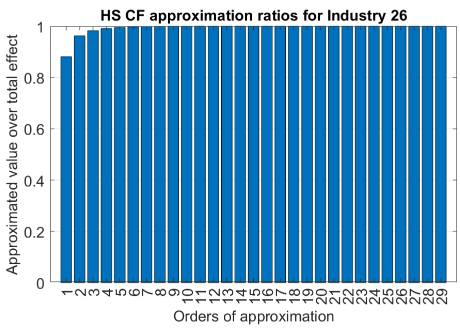


(c) Weights

Figure 2.C.52: Industry P's linkage effects



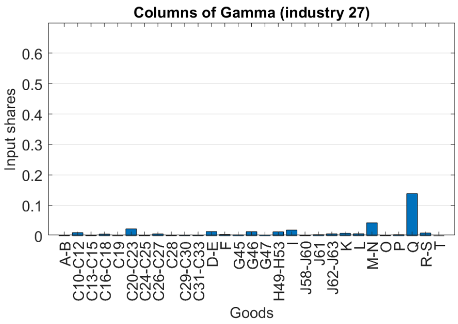
(a) Approximations LS CF shock



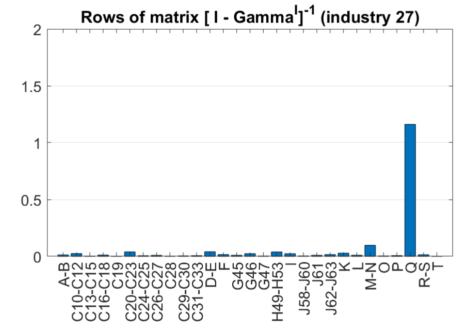
(b) Approximations HS CF shock

Figure 2.C.53: Industry P's approximations for the shocks

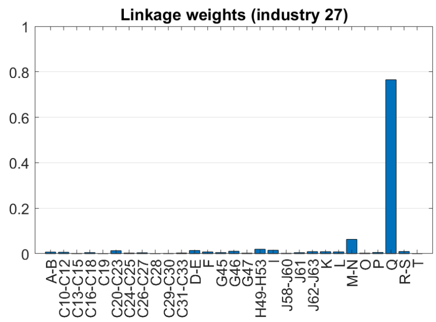
Q: "Health and social work"



(a) Nominal shares of purchases (γ_{ji})

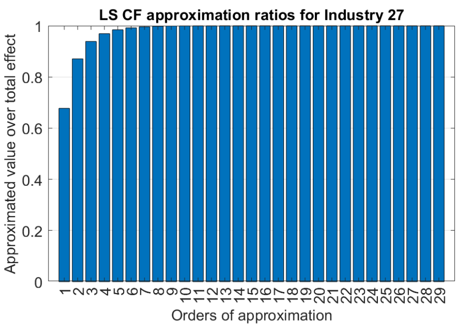


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

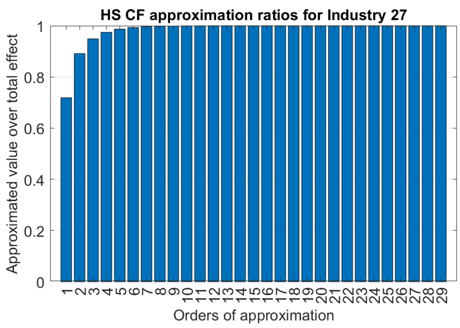


(c) Weights

Figure 2.C.54: Industry Q's linkage effects



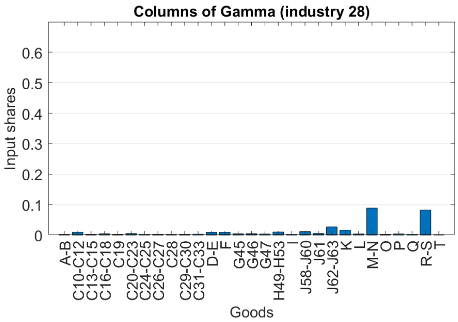
(a) Approximations LS CF shock



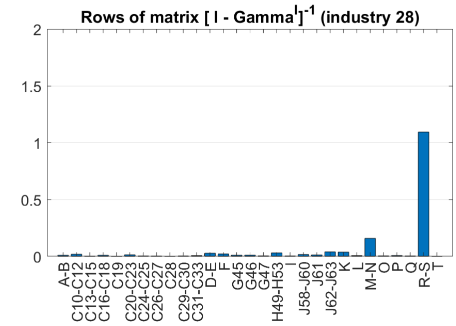
(b) Approximations HS CF shock

Figure 2.C.55: Industry Q's approximations for the shocks

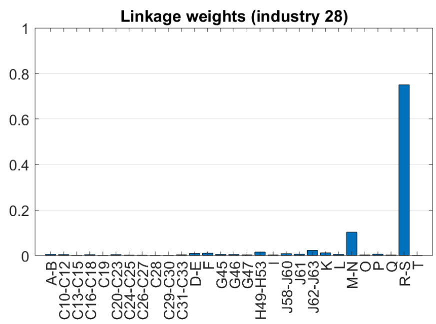
R-S: "Arts, entertainment, recreation and other service activities"



(a) Nominal shares of purchases (γ_{ji})

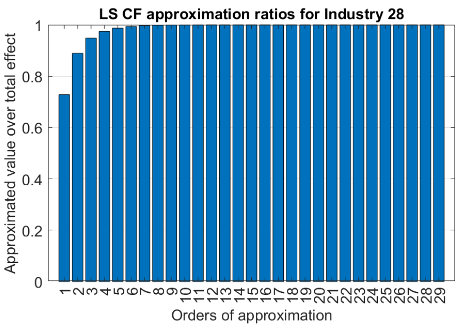


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

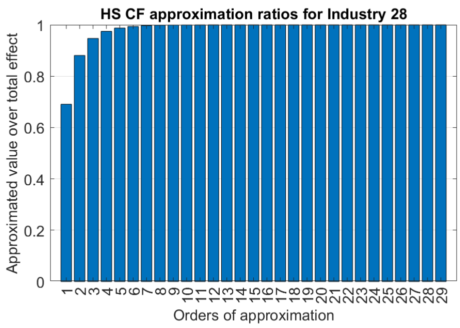


(c) Weights

Figure 2.C.56: Industry R-S's linkage effects



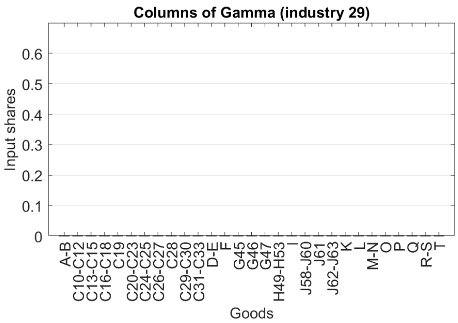
(a) Approximations LS CF shock



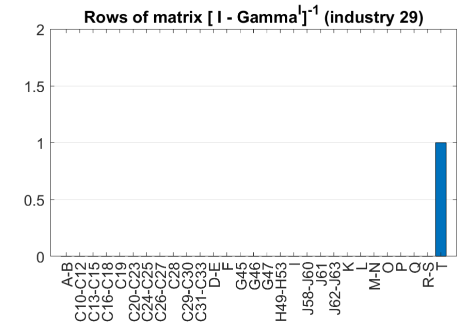
(b) Approximations HS CF shock

Figure 2.C.57: Industry R-S's approximations for the shocks

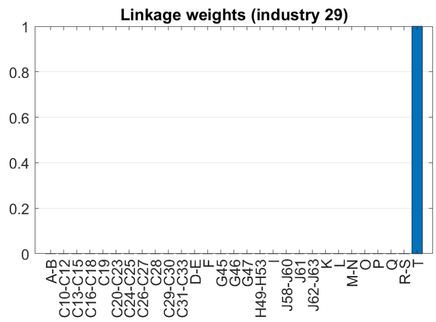
T: "Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use"



(a) Nominal shares of purchases (γ_{ji})

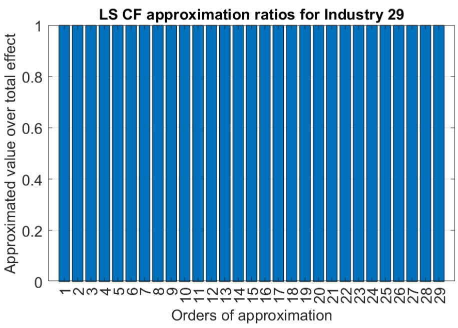


(b) Row of $[\boldsymbol{I} - \boldsymbol{\Gamma}']^{-1}$

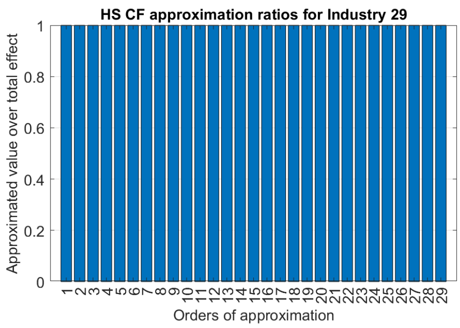


(c) Weights

Figure 2.C.58: Industry T's linkage effects



(a) Approximations LS CF shock



(b) Approximations HS CF shock

Figure 2.C.59: Industry T's approximations for the shocks

Chapter 3

Production network structures and micro effects of macro shocks

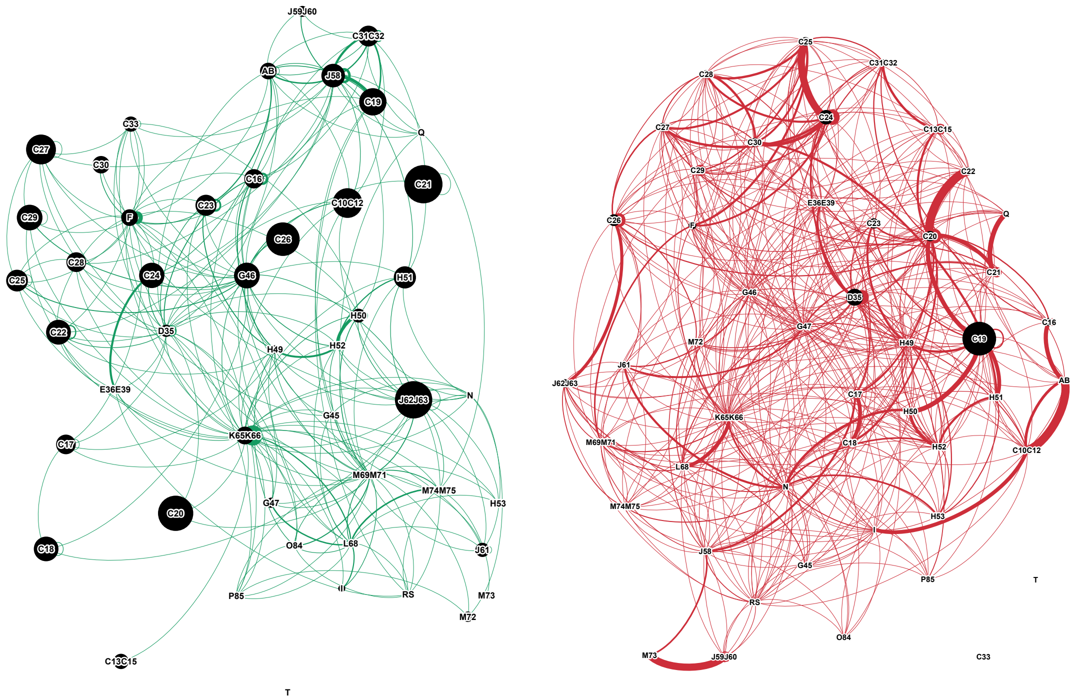
1 Introduction

The series of operations and transactions involved in the production of a good is vast, encompassing a network of domestic and foreign industries. Hardly any industry nowadays creates a product completely in-house and yet, input-output models are still neither mainstream nor fully explored in economics.

In fact, not only local and worldwide industrial interconnections are relevant to the economies, but they also vary considerably from country to country. Figure 3.1 presents the production networks of Ireland and South Korea. The sizes of the nodes are proportional to the import share in the total costs of each industry. The thickness of the edges between each pair of nodes depicts the share of the input transactions in the total costs of the buying industry.[1]

The node sizes make it clear at a glance that Irish goods depend heavily on foreign suppliers whilst the Korean economy is mostly self-reliant. The edges and their thickness, on the other hand, depict how the industries are highly interconnected in Korea and much less so in Ireland. In summary, the industries in these economies differ both in input and import use. While the European country relies heavily on foreign instead of domestic intermediate goods, the opposite is true in the Asian counterpart.

[1] Details on the data in Section 3.B.4. For each industry i, the diameter of the nodes is proportional to the total import shares σ_i while the thickness of the edges is given by the input shares γ_{ji} of good j.



(a) Ireland (b) Korea

Source: WIOD and author's calculations. Graphs plotted with the software package Gephi. Industry labels are described in Table 3.B.4.

Note: The diameters of the nodes represent the share of imports on the total costs of each industry. The thickness of the edges expresses the share of the input transactions between each pair of industries on the buying industry's total sales. Input shares smaller than 0.01 are discarded for readability of the graphs. The orientation of each input-output transaction between a pair of industries is in the clockwise direction. The location of the nodes indicates greater transactions among neighbours; industries placed in the central nodes are those interacting with a larger number of industries.

Figure 3.1: Production networks of Ireland and Korea in 2014

1.1 Contributions

> *Though ours is not the first paper showing that certain shocks spread through the network of input-output linkages [...], we still consider our paper as part of the early phase of this emerging literature documenting the empirical power of network-based propagation of shocks.*
>
> —Acemoglu, Akcigit, and Kerr (2016, p. 323).

In this chapter, I take Ireland and South Korea to identify the role of input linkages on the industry output effects of a labour supply and an import price shock. I show that the differences between the countries in terms of both input and import use play a key role in the transmission of the shocks across the industries and on their final effect.

My *first* contribution is to develop a mathematical framework to support the analysis. It is based on a neoclassical input-output model with international trade, which I extend to incorporate industry-specific factor intensities. Models are primarily useful to depict and analyse specific transmission mechanisms represented by relationships between variables; but even more powerfully they can provide an apparatus for controlled experiments.

After producing the general equilibrium solutions, I separately compute the output effect of a one-percent increase in the labour supply and in the price of an imported good. I also produce the predictions of auxiliary models without trade, without input-output linkages and without either trade or domestic linkages to perform comparisons with the full model. The theoretical toolkit is completed with an in-depth investigation of the first- and higher-order shock transmissions via input-output linkages and with the incorporation of measures of centrality and connectivity from network theory.

In the case of the labour shock, the effect on industry output in the full model comprises of five elements: (i) input-output linkages, represented by the *Leontief-inverse transposed* matrix presented in Definition 3.5; (ii) industry-specific intermediate input shares; (iii) industry-specific import shares; (iv) industry-specific labour shares; and (v) the aggregate labour share, reflecting the impact on GDP. In the case of the import price shock, the effect on industry output in the full model comprises of four elements: (i) the Leontief-inverse transposed; (ii) industry-specific import shares of the affected good; (iii) industry-specific total import shares; and (iv) the aggregate share of the imports of the affected good on consumption, which reflects the impact on GDP.

The Leontief-inverse transposed modulates the industry-specific shocks, giving more (less) weight to the industries j on which i relies more (less). In contrast with a model without input-output linkages, a labour-intensive (capital-intensive) industry will normally present a smaller (larger) output change since it trades with capital-intensive (labour-intensive) industries. The Leontief-inverse transposed plays a role in three aspects studied in this chapter: (i) the labour intensity of domestic intermediate input basket; (ii) the ratio of convergence of approximations of the shock effects; and (iii) the upstream centrality of the industries.

My *second* contribution is to bring the analytical tools to the World Input-Output Dataset (WIOD). I conduct a thorough study of the countries in the sample using both the model's framework and network measures in order to select a pair with very distinct profiles. I also present an extensive comparison of the countries selected with focus on the variables that may affect the counterfactual results.

I implement the full model in MATLAB, where I also perform the data calibration for all models. This allows for a straightforward computation of the counterfactual values for the endogenous variables, in particular for the output of the industries. Moreover, working on MATLAB provides the means to compute and visualise intricate calculations essential for the empirical part of this chapter. In particular, I effortlessly adapt a centrality measure from network theory to gauge the importance of each imported intermediate good to domestic industries.

My *third* and final contribution is to produce an empirical investigation of the role of direct and indirect input-output linkages and international trade in determining the industry output changes resulting from the counterfactual shocks. I show that the structure of the production network significantly affects the cross-industry effects.

In the analysis of the labour shock, I find that labour-intensive industries present diverging results in each country. In Ireland, much of the difference between the predictions of the models derive from trade while, in Korea, they come from input-output linkages.[2] I show that this happens because this group relies more on imports in the European country than their counterpart in the Asian economy.[3]

Particularly in Korea, labour-intensive industries, which are in general more reliant on inputs (greater upstream centralities), buy significantly from capital-intensive industries, which tend to be more relevant as suppliers (greater downstream centralities) in the production network.[4] Yet more interestingly in Korea, the approximations of the output impacts for capital-intensive industries present a slower rate of convergence in comparison to the labour-intensive industries,[5] even though they are less upstream connected than their peers. This occur because the shock approxima-

[2] As Figure 3.9b shows, Auxiliary NT's predictions are very close to Auxiliary 0's in Ireland while the full model's and Auxiliary NT's prediction are both far from Auxiliary 0's in Korea.

[3] See Figure 3.4d for the industry import shares in each country.

[4] See Figures 3.6 and 3.B.11 for the upstream and downstream Katz-Bonacich centrality, respectively, and Figure 3.10 for the labour intensity of the industry input baskets.

[5] See Figure 3.11 for the means of the ratios of the approximations of the impacts over the whole output change across industries per factor-intensity group.

tions refer specifically to the transactions of inputs according to their labour-intensity while the centrality measures are neutral in relation to the goods being transacted.

For the import price shock, I develop a measure of centrality for imported inputs and choose the good of industry A-B (mainly agricultural production and raw material activities) for the counterfactual exercise to get sizeable results in both countries.[6] Remarkably, good A-B is imported in significant amounts by only a few industries,[7] and yet it scores high in the centrality measures because these industries are relevant as suppliers of the production network. As a result, I find that many industries which do not import any quantity of good A-B end up with significant output changes due to their upstream input-output linkages. Indeed, their output impacts are the most underestimated by a model without this feature, even more so in Korea where the industries are largely connected.[8] The decomposition of shock transmission into the various steps of the supply chain draws an exceptional picture for Korea, with the initial effect (which corresponds to the model ignoring input-output linkages) having very little explanatory power for almost every industry.[9] In Ireland, however, the auxiliary model without domestic linkages does not perform as poorly; since the industries are not much connected domestically, most of the output effect comes from their isolated import use.

The differences between Ireland and Korea provide an exemplar case-study for tariff wars.[10] Assuming that rising an import duty is equivalent to increasing the price of the respective good and that government revenues are wasted, the findings of this chapter indicate that the consequences to the economy of such policy depend largely on its production network. In countries with disconnected industries, the negative effect is restricted to those that import the most whilst the fallout is widespread in highly interconnected economies. Therefore, models that incorporate domestic and international linkages along the supply chain like the one in this chapter are crucial in the assessment of trade policies aiming at changing tariffs.

[6]Section 3.B.6 discuss the selection of the import good.

[7]Notice the small number of colourful squares in the row number 1, corresponding to import good A-B, in Figure 3.5c and 3.5d.

[8]See Figure 3.14 for the ratios of the predictions of Auxiliary NIO over the full model's.

[9]See Figure 3.16 for the zeroth to the third order of approximation of the import price shock compared to the total industry output effect.

[10]I mean this from an empirical point of view only. Evidently, the model would have to be extended to incorporate strategic behaviour in a dynamic setting.

1.2 Literature

Production has become fragmented and dispersed across the globe, with intermediate goods[11] representing between 50 and 70 per cent of the world's imports (Mandras and Salotti, 2020). Nevertheless, Dhyne, Kikkawa, Mogstad, and Tintelnot (2021) and Bernard, Bradford Jensen, Redding, and Schott (2007) document that few firms trade directly, but are rather affected by international shocks via their domestic production networks.

There has been a surge of interest during the last decade in applying network concepts from the theory of graphs, mathematical sociology, and statistical physics to input-output analysis.[12] Most of this work focus on shock propagation (Contreras and Fagiolo, 2014; Acemoglu, Akcigit, and Kerr, 2016; Vandenbussche, Connell, and Simons, 2019) and industry systemic importance (Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2015; Aldasoro and Angeloni, 2015) having aggregate effects at its core.

Arguably, the frisson on the field was sparked by Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), who adapt the multi-sector[13] model of Long and Plosser (1983) and combine it with network theory to study the relevance of input-output structure on the propagation of idiosyncratic productivity shocks to the economy. Carvalho (2010) provides a comprehensive review of this literature. In contrast to their work, my research question goes in the opposite direction, asking how aggregate shocks affect individual micro entities when taking their interconnections into account. The reverse perspective naturally leads to differences in the findings. They, as well as Jones (2013), find that the sparseness of the production network does not affect their results regarding aggregate volatility and the economic multiplier, respectively. I, on the other hand, show that the input-output structure significantly influences the cross-industry effects resulting from aggregate shocks.

Within this literature, Baqaee (2015) is the first to point out that shocks travel either upstream or downstream throughout the network depending on their origin. Nevertheless, the presence of cycles or loops in the network —in which an initially purchasing industry eventually appear as a buyer within a single chain of transactions—

[11] The term 'goods' in this chapter refer to both physical goods and services.

[12] Important textbooks on social and economic networks have also been recently developed (Goyal, 2007; Vega-Redondo, 2007; Jackson, 2008).

[13] The terms 'industry' and 'sector' are often used interchangeably in input-output analysis and in macroeconomics. I opt for the former simply because this is the term used by the WIOD.

makes the distinction line blurry. Acemoglu, Akcigit, and Kerr (2016) better put it when they say that some [supply-side] shocks travel 'more powerfully' downstream than upstream, and vice-versa for other [demand-side] shocks. This understanding is at the core of the analytical results of this chapter.

I add to this literature in three ways. Firstly, I explicitly incorporate trade into the model in the form of imported intermediate inputs used by domestic producing industries. Secondly, I assess the effects of economic shocks not studied before, namely labour supply and the price of an import good. Lastly, I investigate how the structure of the production network affects the results.

The network-based approach to the analysis of input-output linkages includes several descriptive studies, assessing network measures of industries and countries (Blöchl, Theis, Vega-Redondo, and Fisher, 2011; McNerney, Fath, and Silverberg, 2013; Cerina, Zhu, Chessa, and Riccaboni, 2015). Attempts to identify and measure interindustry connections date as far back as the 1950s. Chipman (1950) first computes the output multiplier to measure how an autonomous disturbance in expenditure would affect each sector in particular and the economy in general. Rasmussen (1957) and Hirschman (1958) independently develop the first measures of backward and forward linkages. The network measures I use in this chapter are adapted from Carvalho (2014).

Studying the distributions of industry measures, Fadinger, Ghiglino, and Teteryatnikova (2016) and Bartelme and Gorodnichenko (2015) associate productivity and economic development with the sparsity of a country's input-output matrix. Luu, Fagiolo, Roventini, and Sgrignoli (2017) assess the inter-country strength of connectivity within the European Union (EU) by measuring the average number of links (connectivity) and of the weights of those links (intensity) between each pair of countries but do not study the intra-country connectivity. Applications abound on assessing the centrality of countries and industries within the global value networks (Koopman, Powers, Wang, and Wei, 2010; Cingolani, Panzarasa, and Tajoli, 2017; Mundt, 2021). Contreras and Fagiolo (2014) present the density measures for the EU countries using the Eurostat data. This chapter's contribution is to report the sparsity or connectivity of the production networks of the 43 countries in the WIOD sample.

With respect to the labour supply shock, recent literature investigates the effects of the COVID-19 pandemic using multi-sector models with linkages. Maria Del Rio-Chanona, Mealy, Pichler, Lafond, and Doyne Farmer (2020) develop a short-run input-output model with linkages between and within industries and occupations to predict the impact of the social distancing on economic units. They ascertain that in countries controlling the spread of the outbreak, social distancing would cause a labour loss of one order of magnitude larger than that caused by mortality and morbidity. Examining these last sources of labour loss, McKibbin and Fernando (2020) employs a global general equilibrium model with six sectors with linkages within and across countries to assess the impact of the population loss from COVID-19 on the GDP of the countries, but they do not disentangle the labour shock from other simultaneous shocks. Closely related to this chapter's work is Pichler and Farmer (2021), who consider higher-order effects of supply and demand shocks explicitly. However, to the best of my knowledge, no study has contributed to the analysis of the transmission of the labour shock specifically within the network as I do in this chapter.

In regard to the import price shock, very few studies analyse the effects on the economy using a multi-industry model with linkages. Maisonnave, Pycroft, Saveyn, and Ciscar (2012) study the impacts of oil price rises with a rich computable general equilibrium (CGE) model, but only report the results on aggregate variables. Zhang, Yang, Zhang, and Shackman (2017) present the effects of natural gas price hikes on the output of Chinese industries while Aydin and Acar (2011) produce a similar analysis for Turkey. I contribute to this literature by pinning down the mechanisms through which industries are affected by an import price shock, as these go beyond industry-specific characteristics and relates to the structure of a country's production network. Caraiani (2019) performs analogous investigation, but uses time-varying Bayesian VAR to track the effects of oil shocks on the GDP of OECD economies.

Insofar as an import price shock can be read as a tariff rise, our analysis can be related to the trade literature assessing the impacts of Brexit. Using a multi-country multi-sector network model, Vandenbussche, Connell, and Simons (2019) finds that the indirect effects account for more than 70% of the total effect of the trade shock for some sectors in the EU countries. Using centrality measures, Giammetti, Russo, and Gallegati (2020) identify the most systematically important industries within the European production network. They find that the UK as a whole to be the most affected by the tariff changes, but individually some EU sectors would suffer the most. My approach differs in considering a general equilibrium model and focusing on the propagation mechanism of the trade shock via the production network and imports.

1.3 Outline

The structure of this chapter is as follows. Section 2 presents the theoretical framework, which includes the full input-output model with trade, the predictions of the auxiliary models for the industry output changes, and key definitions and results supporting the analysis. Section 3 provides an overview of the production networks of the countries in the WIOD sample, outlines the data calibration and the implementation of the models in MATLAB, and describes the main parameters of Ireland and South Korea. Section 4 turns to the application, where I demonstrate how input-output linkages and trade patterns affect industries, making use of the comprehensive framework developed. Section 5 concludes. Algebraic derivations and proofs, data and computation details, and supplementary figures are available in the Appendices 3.A, 3.B and 3.C, respectively.

2 Theoretical analysis

2.1 An input-output model with trade

I develop here a general equilibrium small-open-economy model with input-output linkages and trade. It is an extension of the input-output model presented in Chapter 2 regarding the inclusion of the international dimension. But it is also a simplification with respect to the skill levels, as the WIOD do not have this breakdown and I had to accommodate for this data limitation. Additionally, compared to Fadinger, Ghiglino, and Teteryatnikova (2016), the model presented here includes industry-level factor intensities.

Each industry is taken as a single representative firm producing a homogeneous good. There are n industries in the economy. The output q_i of each industry i is given by the following constant returns to scale (CRS) Cobb-Douglas technology:[14]

$$q_i = A_i \left(k_i^{\alpha_i} e_i^{1-\alpha_i} \right)^{1-\gamma_i-\sigma_i} \prod_{j=1}^{n} d_{ji}^{\gamma_{ji}} \prod_{j=1}^{n} f_{ji}^{\sigma_{ji}} \tag{3.1}$$

Where the endogenous variables, besides q_i, are:

- k: capital
- e: labour employed
- d_{ji}: output of domestic industry j used in the production of i
- f_{ji}: quantity of imported good j used by industry i

I call capital and labour 'factors'. Intermediate inputs d_{ji} are also called 'materials' in related literature. Imports f_{ji} do not discriminate the country of origin.[15]

Regarding the technology parameters:

- A_i: industry total factor productivity
- α_i: factor income share of capital
- $1 - \alpha_i$: factor income share of labour
- γ_{ji}: share of good j in the production technology of firms in industry i
- $\gamma_i = \sum_{j=1}^{n} \gamma_{ji}$, i.e. the total input share of industry i
- σ_{ji}: share of import good j in the production technology of firms in industry i
- $\sigma_i = \sum_{j=1}^{n} \sigma_{ji}$, i.e. the total import share of industry i

Given the Cobb-Douglas technology in Equation (3.1) and competitive factor markets, the input shares γ_{ji} correspond to the entries of the input-output matrix $\mathbf{\Gamma}$, measuring the value of spending on input j per dollar of production of good i. The import shares σ_{ji} constitute the equivalent $\mathbf{\Sigma}$ matrix.

Output of industry i is used for final consumption, y_i, or as intermediate good, such that the market clearing condition for each good i holds, i.e.:

$$q_i = y_i + \sum_{j=1}^{n} d_{ij} \tag{3.2}$$

[14] If one were to follow strictly the premises of Leontief's input-output model, the Cobb-Douglas would have to present decreasing returns to scale. This is a consequence of the technical coefficients being fixed, i.e. input substitution not being possible when using the social accounting matrices. See Chapter 8 of ten Raa (2006) and references thereof for more details.

[15] To translate it into this chapter's notations, let g represent each of m countries. For each buying industry i and imported good j, it is true that $f_{ji} = \sum_{g=1}^{m} f_{ji}^{g}$.

The consumer side is synthesised by a final good aggregation. This can be interpreted as an entity which combines all the y_i left-overs from the industries and transforms them into the aggregate GDP. Consumer preferences are represented by good-specific final demand parameters $\beta_i \geqslant 0$, $\forall i$, where $\sum_{i=1}^{n} \beta_i = 1$.

$$Y = \prod_{i=1}^{n} y_i^{\beta_i} \tag{3.3}$$

Real GDP (Y) equals nominal GDP (PY) since the price deflator is chosen as the numeraire and its value is normalised to one $(P = 1)$. Production side equals expenditure side GDP, so that final output is equivalent to household's consumption and export, the latter two only considered in aggregate terms:

$$Y = C + X \tag{3.4}$$

Exports are assumed to balance out imports in a steady state equilibrium fashion.[16] Letting $\bar{p}_j$ represent the exogenous world price of good j, the balanced trade reads:

$$X = \sum_{i=1}^{n} \sum_{j=1}^{n} \bar{p}_j f_{ji} \tag{3.5}$$

Households supply the primary factors of production inelastically, receiving competitive market wages w_E and rental on capital w_K. Thus, households' income comprises of the aggregate value-added and is fully used to finance consumption:

$$C = w_K K + w_E E \tag{3.6}$$

Finally, factor markets are also assumed to clear, i.e. the sum of each factor employed in every industry equals the exogenous aggregate levels of physical capital K and labour E:

$$K = \sum_{i=1}^{n} k_i \tag{3.7}$$

$$E = \sum_{i=1}^{n} e_i \tag{3.8}$$

[16] Even in the short run, the balanced trade assumption is not so far-fetched. In the cross section of the 43 countries covered by the WIOD, exports and imports are not so far from each other in magnitude, as plotted in Figure 3.B.1 in Appendix.

2.2 General equilibrium

The competitive general equilibrium of this economy is trivially given by the profit maximisation of industries and final good aggregator taking prices as given. The formal definition of the equilibrium allocation follows:

Definition 3.1 (Equilibrium) *A competitive equilibrium consists of quantities regarding aggregate output Y, consumption C and exports X, the industry output $\{q_i\}$, intermediate input choices $\{d_{ji}\}$, import choices $\{f_{ji}\}$, final good demands $\{y_i\}$ and industry demand for factors $\{k_i\}$ and $\{e_i\}$, as well as prices of factors w_K and w_E and goods $\{p_i\}$ for every industry i, j in the economy such that:*

1. **Industry Problem:** *Given good prices $\{p_i\}$ and factor prices w_K, w_L and w_H the representative firm of each industry chooses factors, inputs and output to maximise its profit subject to the technology, such that:*

$$\max_{\{q_i,k_i,l_i,h_i,d_{ji}\}} p_i q_i - w_K k_i - w_L l_i - w_H h_i - \sum_{j=1}^{n} p_j d_{ji}$$

s.t. Equation (3.1).

2. **Final Good Problem:** *Given $\{p_i\}$ and P, the final good aggregator (entity) maximises profit subject to the available technology, such that:*

$$\max_{\{y_i,Y\}} PY - \sum_{i=1}^{n} p_i y_i$$

s.t. Equation (3.3).

3. *setting the the final good price as the numeraire, i.e. $P = 1$, prices clear the markets for all goods and factors, such that:*
   - *industry goods: Equation* (3.2)
   - *final good: Equation* (3.4)
   - *capital: Equation* (3.7)
   - *labour: Equation* (3.8)

First order conditions

The Cobb-Douglas combined with efficient markets yields the well-known first order conditions (FOCs) for the equilibrium in each industry i which states that the value of the each factor and input is a constant function of the value of the output. In terms of the nominal shares of capital (3.9) and labour (3.10), the intermediate input shares (3.11) and the imports share (3.12) in the total costs (sales) of an industry, respectively, these conditions read:

$$\frac{w_K k_i}{p_i q_i} = \alpha_i(1-\gamma_i-\sigma_i) \tag{3.9}$$

$$\frac{w_E e_i}{p_i q_i} = (1-\alpha_i)(1-\gamma_i-\sigma_i) \tag{3.10}$$

$$\frac{p_j d_{ji}}{p_i q_i} = \gamma_{ji} \tag{3.11}$$

$$\frac{\bar{p}_j f_{ji}}{p_i q_i} = \sigma_{ji} \tag{3.12}$$

Finally, the necessary equilibrium condition for the final good maximisation renders β_i as each industry's nominal sales share on GDP:

$$\frac{p_i y_i}{Y} = \beta_i \tag{3.13}$$

2.3 Special notes

Industry and aggregate factor shares

The nominal factor shares as given by Equations (3.9) and (3.10) appear often in the subsequent analysis of this chapter. For this reason, it is worth understanding their meaning precisely.

Consider each industry i value added, i.e. its total payment to factors, as given by:

$$va_i = w_K k_i + w_E e_i \tag{3.14}$$

Then, in equilibrium, having total costs equating total sales, every industry in the IO model must satisfy:

$$p_i q_i = va_i + \sum_{j=1}^{n} p_j d_{ji} \tag{3.15}$$

In possession of these references, I define the 'broad' as given by Equations (3.9) and (3.10) and the 'narrow' factor shares as below. The main difference between them is that while the former considers an industry's total costs, i.e. including the costs of the intermediate inputs, the latter are solely in terms of the total costs of factors.

Definition 3.2 (Broad factor shares) *The broad nominal shares for capital and labour for each industry i are respectively given by the equations below:*

$$\alpha_i(1-\gamma_i-\sigma_i) = \frac{w_K k_i}{p_i q_i} \tag{3.9}$$

$$(1-\alpha_i)(1-\gamma_i-\sigma_i) = \frac{w_E e_i}{p_i q_i} \tag{3.10}$$

Definition 3.3 (Narrow factor shares) *The narrow nominal shares for capital and labour for each industry i are respectively given by the equations below:*

$$\alpha_i = \frac{w_K k_i}{v_i} \tag{3.16}$$

$$(1-\alpha_i) = \frac{w_E e_i}{v_i} \tag{3.17}$$

Finally, to simplify the derivations and the analysis, I define α as the aggregate capital share in value added [Equation (3.6)], such that:

Definition 3.4 (Aggregate factor shares) *The aggregate nominal shares for capital and labour for each industry i are respectively given by the equations below:*

$$\alpha = \frac{w_K K}{C} \tag{3.18}$$

$$(1-\alpha) = \frac{w_E E}{C} \tag{3.19}$$

Input-output matrix

The input-output matrix represented by $\mathbf{\Gamma}$ mimics the array of domestic interindustry production and consumption of intermediate goods available in the data. The i^{th} column of matrix $\mathbf{\Gamma}$ consists of industry i's input shares for all goods j used in its production. By contrast, the elements of the j^{th} row of matrix $\mathbf{\Gamma}$ are the shares of inputs of industry j in the production technology of all other industries i.

$$\mathbf{\Gamma} = [\gamma_{ji}] = \underbrace{\begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nn} \end{bmatrix}}_{\text{input use}} \left.\vphantom{\begin{bmatrix} \gamma_{11} \\ \gamma_{21} \\ \vdots \\ \gamma_{n1} \end{bmatrix}}\right\} \text{input supply}$$

The chosen layout in form of 'origin-destination' is the standard one in input-output economics applying Leontief's activity analysis framework.[17] It is also more coherent with network theory, as shown in Section 2.6. However, readers of the macro literature incorporating intermediate goods in general equilibrium models may find it confusing since these models tend to reverse this order.[18]

Imports matrix

Matrix $\mathbf{\Sigma}$ contains the nominal shares σ_{ij} of the imported good i in the production technology of domestic industry j. The rows represent each rest-of-the-world (ROW) exporting industry while the columns represent the country-at-hand importing industries. In that way, once more I use the origin-destination order for the row-column indices of the matrix.

$$\mathbf{\Sigma} = [\sigma_{ji}] = \underbrace{\begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{bmatrix}}_{\text{import use}} \left.\vphantom{\begin{bmatrix} \sigma_{11} \\ \sigma_{21} \\ \vdots \\ \sigma_{n1} \end{bmatrix}}\right\} \text{foreign supply}$$

The key difference between matrices $\mathbf{\Sigma}$ and $\mathbf{\Gamma}$ is that while the latter has the same group of industries on both dimensions, the former has the columns representing the

[17] The input-output matrix is called the matrix of technical coefficients in this literature.

[18] See, for instance, the recent literature on production networks sparked by Carvalho (2010) and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and surveyed by Carvalho and Tahbaz-Salehi (2019).

domestic (importing) and the rows the foreign (exporting) industries. Therefore, the import matrix is not a proper input-output matrix and cannot be considered in the network analysis presented in Section 2.6.

Sales shares

An important parameter which aids significantly in the derivations and analysis is not present in the main formulae of the model. It is industry i's sales share on GDP, or Domar (Domar, 1961) weight, defined as i's nominal sales $p_i q_i$ over aggregate output.

$$\mu_i = \frac{p_i q_i}{Y} \tag{3.20}$$

Using the market clearing condition of Equation (3.2) in nominal terms and substituting in for Equations (3.20), (3.11) and (3.13) leads to an expression of μ_i solely in terms of parameters, i.e. $\mu_i = \sum_{j=1}^{n} \gamma_{ij}\mu_j + \beta_i$. Which in matrix form translates to:[19]

$$\mu = [\mathbf{I} - \mathbf{\Gamma}]^{-1}\beta \tag{3.21}$$

Matrix $[\mathbf{I} - \mathbf{\Gamma}]^{-1}$ is known as the *Leontief inverse*.[20] The main difference between the input-output matrix $\mathbf{\Gamma}$ and the Leontief inverse is that the former only considers the direct or first-order connections between each pair of industries while the latter takes into account all direct and indirect linkages.

Moreover, the 'origin-destination' orientation of the $\mathbf{\Gamma}$ matrix as presented in Section 2.3 results in the Leontief inverse capturing the transactions along the *downstream* chains, i.e. from the seller's perspective, on the rows and the *upstream* chains, i.e. from the buyer's perspective, on the columns.[21] Anticipating some of the discus-

[19] In full:

$$\begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{bmatrix} = \underbrace{\begin{bmatrix} \gamma_{11} & \gamma_{12} & \dots & \gamma_{1n} \\ \gamma_{21} & \gamma_{22} & \dots & \gamma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \dots & \gamma_{nn} \end{bmatrix} \cdot \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{bmatrix}}_{\text{Summing over destinations}} + \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix}$$

[20] Notice that having the input-output matrix with the input use along the rows and input supply along the columns, as in the literature on production networks surveyed by Carvalho and Tahbaz-Salehi (2019), would lead to the Leontief inverse being written as $[\mathbf{I} - \mathbf{\Gamma}']^{-1}$, where $\mathbf{\Gamma}'$ is the transpose of $\mathbf{\Gamma}$.

[21] A simple way of demonstrating this property, done by Fadinger, Ghiglino, and Teteryatnikova (2016), is to represent the Leontief-inverse as the sum of an infinite geometric series then post-multiply it by a vector of ones. Truncating the series at its first-order term leads to the vector of (weighted) out-degrees

sion presented in Section 2.6, the size of each industry i vis-à-vis the GDP given by μ_i is a downward centrality measure, reflecting the weight of its intermediate inputs on all other industries' final good sales.

2.4 Analytical results and tools

Equilibrium solutions

Despite the richness of the model, closed-form solutions are derivable for all endogenous variables. I present in the main text only the solution for the gross output of the industries, which is the most relevant for the subsequent analysis. All the remaining solutions and derivations are available in Appendix 3.A.

Proposition 3.1 (Industry output) *The output of each industry i in equilibrium reads:*

$$\begin{aligned}
\log q_i \quad = \quad & \log K\left[\sigma_i \alpha + (1-\gamma_i-\sigma_i)\alpha_i\right] \\
& + \log E\left[\sigma_i(1-\alpha) + (1-\gamma_i-\sigma_i)(1-\alpha_i)\right] \\
& + \log A_i + \log \mu_i - (1-\gamma_i)\log\left(1-\textstyle\sum_{i=1}^n \sigma_i\mu_i\right) \\
& -\sigma_i\left[\alpha\log\alpha + (1-\alpha)\log(1-\alpha)\right] \\
& +(1-\gamma_i-\sigma_i)\alpha_i(\log\alpha_i - \log\alpha) \\
& +(1-\gamma_i-\sigma_i)(1-\alpha_i)(\log(1-\alpha_i) - \log(1-\alpha)) \\
& +\sigma_i \tfrac{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)[\alpha_i\log\alpha_i + (1-\alpha_i)\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i)]}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)} \\
& +\sigma_i \tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \mu_i\log A_i + \sum_{i=1}^n \mu_i \sum_{j=1}^n \gamma_{ji}\log\gamma_{ji}\right] \\
& +\sigma_i \tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \mu_i \sum_{j=1}^n \sigma_{ji}\log\sigma_{ji}\right] \\
& +\sigma_i \tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \beta_i\log\beta_i - \sum_{i=1}^n \mu_i \sum_{j=1}^n \sigma_{ji}\log\bar{p}_j\right] \\
& +(1-\gamma_i-\sigma_i)\log(1-\gamma_i-\sigma_i) + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} + \sum_{j=1}^n \sigma_{ji}\log\sigma_{ji} \\
& -\textstyle\sum_{j=1}^n \sigma_{ji}\log\bar{p}_j - \sum_{j=1}^n \gamma_{ji}\log\mu_j + \sum_{j=1}^n \gamma_{ji}\log q_j
\end{aligned} \tag{3.22}$$

Analogously to the equilibrium industry output in a model without trade, as the one developed in Chapter 2, the appearance of the term $\sum_{j=1}^n \gamma_{ji}\log q_j$ on the right-hand side of this equation reveals how each industry's output is recursively related to all other's and makes explicit the model's supply-side characteristic. In this framework,

$\gamma^{\mathbf{out}}$, which is a measure of downstream influence (further presented in Section 2.6):

$$\begin{aligned}
[I-\Gamma]^{-1}\mathbf{1} \quad &= \quad \left(\textstyle\sum_{k=0}^{\infty}\Gamma^k\right)\mathbf{1} = \left(I + \Gamma + \Gamma^2 + \ldots\right)\mathbf{1} \\
&\approx \quad [I+\Gamma]\mathbf{1} = \mathbf{1} + \gamma^{\mathbf{out}}
\end{aligned}$$

The approximation is rather coarse and does not hold numerically, but it is helpful in the visualisation of the direction of the effects captured by the typical Leontief inverse premultiplying a vector.

goods are produced not responding to the demand but to the availability of resources: other industry's output j matters to the extend that it is used as input by industry i.

Corollary 3.1 (Vector of industry output) *The complete solution for the industry output can only be obtained in matrix form. Let* $\mathfrak{q}_i = \log q_i$ *and the* i^{th} *row of vector* $\mathbf{V}$ *be composed of all the constant terms of* (3.22) *—those prior to* $\sum_{j=1}^{n} \gamma_{ji} \log q_j$*— such that vector* $\mathfrak{q}$ *reads:*

$$\mathfrak{q} = \left[\mathbf{I} - \mathbf{\Gamma}'\right]^{-1} \mathbf{V} \tag{3.23}$$

Matrix $[\mathbf{I} - \mathbf{\Gamma}']^{-1}$ in Equation (3.23) summarises all the industries' interactions and is key in the study of the transmission of exogenous shocks. It is essential to the toolkit developed in this chapter and is discussed in the Section 2.4. The vector $\mathbf{V}$ contains all the exogenous variables and parameters other than the input shares.

Leontief-inverse transposed

As discussed in Chapter 2, matrix $[\mathbf{I} - \mathbf{\Gamma}']^{-1}$ in Equation (3.23) is identical to the Leontief inverse, except for having the input-output matrix transposed. But this change is not trivial, as it makes its rows capturing the input use instead of the input sales of each industry. In other words, having the input-output transpose matrix Γ' captures the *upstream* or buyers' chains on the rows. Algebraically, the Leontief-inverse transposed is trivially the transpose of the Leontief-inverse, i.e. $[I - \Gamma^T]^{-1} = [[I - \Gamma]^{-1}]^T$. For this reason, I define matrix $[\mathbf{I} - \mathbf{\Gamma}']^{-1}$ the Leontief-inverse transposed.

Definition 3.5 (Leontief-inverse transposed) *Matrix* $[\mathbf{I} - \mathbf{\Gamma}']^{-1}$ *is defined as the Leontief-inverse transposed. Each element* υ_{ij} *of it represents the overall relevance of the input of industry* j *in the production technology of industry* i*.*

In the Leontief-inverse transposed, the elements portray how much each industry relies on every other industry's inputs, while each element of the original Leontief inverse represents how important each industry is as a supplier to every other industry. In that way, υ_{ij} captures all the direct and indirect transactions of each input j over the supply chain of each good i.

Output effect of a labour supply shock

The full model's prediction for the effects on the output of an industry are given by the first derivative of its equilibrium solution with respect to the shocked variable. It follows from Corollary 3.1 that this differentiation is rather straightforward.

Corollary 3.2 (Output effect of a labour shock) *The vector of the effect of a labour supply shock on the real output of the industries in the full model is given by the product of the Leontief-inverse transposed and a 'shock vector'.*[22]

$$\begin{bmatrix} \frac{\mathrm{d}\log q_1}{\mathrm{d}\log E} \\ \frac{\mathrm{d}\log q_2}{\mathrm{d}\log E} \\ \vdots \\ \frac{\mathrm{d}\log q_n}{\mathrm{d}\log E} \end{bmatrix} = \underbrace{\left[\mathbf{I} - \mathbf{\Gamma}'\right]^{-1}}_{\textit{input-output linkages}} \cdot \underbrace{\begin{bmatrix} (1-\gamma_1-\sigma_1)(1-\alpha_1)+\sigma_1(1-\alpha) \\ (1-\gamma_2-\sigma_2)(1-\alpha_2)+\sigma_2(1-\alpha) \\ \vdots \\ (1-\gamma_n-\sigma_n)(1-\alpha_n)+\sigma_n(1-\alpha) \end{bmatrix}}_{\textit{industry-specific impacts}} \tag{full$_{ES}$}$$

The input-output linkages —in the form of the Leontief-inverse transposed— modulates how the industry-specific effects of every industry j affects each industry i. Each row of the shock vector represents industry j's isolated labour shock impact. It consists of its 'broad' labour share $(1-\gamma_j-\sigma_j)(1-\alpha_j)$ plus its total import share σ_j times the labour shock impact on GDP —the aggregate labour share $(1-\alpha)$.[23] Both terms highlight the supply-side nature of the model, in which output directly depends on the quantity of factor and inputs available in the economy.

Rearranging terms to get $(1-\gamma_j)(1-\alpha_j)+\sigma_j[(1-\alpha)-(1-\alpha_j)]$ shows how the shock averages out across industries. The first term combined with the Leontief-inverse transposed results in the *linkage weights* defined in Chapter 2. These elements modulate the industry-specific shocks, giving more or less weight to the industries j on which i relies. Moreover, they explain how the output effect in each industry can be

[22]This result and the one in Corollary 3.5 are closely related to Equation (6) of Acemoglu, Akcigit, and Kerr (2016). Here too, the shocks are 'supply-side' and in a model with Cobb-Douglas production functions they present only downstream effects, i.e. they pass on from a supplier to its consuming industry and so on. For this reason, the Leontief inverse premultiplying the 'shock vector' presents a 'upstream orientation'. In other words, to capture how downstream effects eventually impact each industry one need an upstream measure that would capture how much each industry rely on the inputs of all others. This observation is analogue to Equation (13) of Acemoglu, Akcigit, and Kerr (2016).

[23]Algebraically, d_{ji} can be expressed in terms of q_j, but f_{ji} remains as a function of Y (see respectively Equations (3.48) and Equation (3.49) in the Appendix). This happens because of the exogeneity of international prices, which makes f_{ji} vary with GDP Y while $p_i q_i$ and $p_j d_{ji}$ varies with Y (see Equations (3.61) and (3.54) in the Appendix).

seen and a weighted average of all the labour shares of the supplier industries. Finally, the second term depicts the rebalancing of the aggregate impact transmitted via σ_j to labour-intensive $[(1-\alpha) < (1-\alpha_j)]$ and to capital-intensive $[(1-\alpha) > (1-\alpha_j)]$ industries.

In contrast, a model without either input-output linkages or imported inputs — which I call here Auxiliary 0 or Aux0 for short— predicts that each industry output would increase the equivalent of its share of labour.

Corollary 3.3 (Output effect Aux0 model) *In the Auxiliary 0 model, the effect of a labour supply shock on the real output of the industries is proportional to their affected 'narrow' labour shares, such that:*

$$\frac{\mathrm{d}\log q_i}{\mathrm{d}\log E} = (1-\alpha_i) \tag{Aux0$_{ES}$}$$

Finally, an input-output model ignoring international trade like the one on Chapter 2 —which I call here Auxiliary NT or AuxNT for short— would predict that the effect of the labour shock on the output of the industries is a combination of the input shares γ_i and the labour shares $(1-\alpha_i)$ as given below.

Corollary 3.4 (Output effect AuxNT model) *In the Auxiliary NT model, the vector of the effect of labour supply shock on the real output of the industries is given by the product of the Leontief-inverse transposed and a 'shock vector' comprised of the 'broad' labour shares of each industry i, such that:*

$$\begin{bmatrix} \frac{\mathrm{d}\log q_1}{\mathrm{d}\log E} \\ \frac{\mathrm{d}\log q_2}{\mathrm{d}\log E} \\ \vdots \\ \frac{\mathrm{d}\log q_n}{\mathrm{d}\log E} \end{bmatrix} = \left[\mathbf{I}-\mathbf{\Gamma}'\right]^{-\mathbf{1}} \cdot \begin{bmatrix} (1-\gamma_1)(1-\alpha_1) \\ (1-\gamma_2)(1-\alpha_2) \\ \vdots \\ (1-\gamma_n)(1-\alpha_n) \end{bmatrix} \tag{AuxNT$_{ES}$}$$

Output effect of an import price shock

The effect of a rise in the price of an import good z on the real output of the industries is a combination of the Leontief-inverse transposed and a shock vector comprised of aggregate and industry-specific effects.

Corollary 3.5 (Output effect of an import price shock) *The vector of the effect of an import price $\bar{p}_z$ shock on the real output of the industries q_i in the full model are given by the product of the Leontief-inverse transposed and a 'shock vector'.*

$$\begin{bmatrix} \frac{\mathrm{d}\log q_1}{\mathrm{d}\log \bar{p}_z} \\ \frac{\mathrm{d}\log q_2}{\mathrm{d}\log \bar{p}_z} \\ \vdots \\ \frac{\mathrm{d}\log q_n}{\mathrm{d}\log \bar{p}_z} \end{bmatrix} = \underbrace{\left[\mathbf{I} - \mathbf{\Gamma}'\right]^{-1}}_{\text{input-output linkages}} \cdot \underbrace{\begin{bmatrix} -\sigma_{z1} - \sigma_1 \frac{M_z}{C} \\ -\sigma_{z2} - \sigma_2 \frac{M_z}{C} \\ \vdots \\ -\sigma_{zn} - \sigma_n \frac{M_z}{C} \end{bmatrix}}_{\text{industry-specific impacts}} \qquad (\text{full}_{pz})$$

The first term of the shock vector σ_{zi} is simply the direct effect of the price increase, given by its respective intensity in each industry. The second term $-\sigma_i \frac{M_z}{C}$, however, reflects the aggregate impact on industry i of an increase in the price of the imported input z. The smaller aggregate level of imports affects every industry to the extent of their respective total share of imports σ_i. The model being strictly supply-side, where the reduced availability of inputs lowers every industry output even though good prices fall more than industry output.[24]

In contrast, a trade model without input-output linkages —which I call here Auxiliary NIO or AuxNIO for short— predicts that each industry would be affected by the correspondent line of the 'shock vector':

Corollary 3.6 (Output effect AuxNIO model) *In the Auxiliary NIO model, the effect of an import price shock on the real output of the industries is solely given by their direct and aggregate impact:*

$$\begin{bmatrix} \frac{\mathrm{d}\log q_1}{\mathrm{d}\log \bar{p}_z} \\ \frac{\mathrm{d}\log q_2}{\mathrm{d}\log \bar{p}_z} \\ \vdots \\ \frac{\mathrm{d}\log q_n}{\mathrm{d}\log \bar{p}_z} \end{bmatrix} = \begin{bmatrix} -\sigma_{z1} - \sigma_1 \frac{M_z}{C} \\ -\sigma_{z2} - \sigma_2 \frac{M_z}{C} \\ \vdots \\ -\sigma_{zn} - \sigma_n \frac{M_z}{C} \end{bmatrix} \qquad (\text{AuxNIO}_{pz})$$

Unlike in the industry output effect of labour shock, the vector of industry-specific impacts deriving from the import price shock has no terms deriving from domestic input use γ_{ji}. Therefore, the vector of industry output effect in the model without input-output linkages are identical to the shock vector resulting from the full model.

[24] Equation (3.68) in the Appendix shows how the production of intermediate good of industry j falls as much as its output q_j while Equation (3.70) shows that the price of each good.

2.5 Investigating the transmission via input-output linkages

The main takeaway of the theoretical analysis presented so far is that in the full model the effect of a labour or import price shock on the output of an industry comprises of the Leontief-inverse transposed post-multiplying a 'shock' vector of the isolated impact on each industry. Since the latter applies equally to every industry, what really determines the varying impact across industries in the full model is the former.

Matrix $[\mathbf{I} - \mathbf{\Gamma}']^{-1}$ not only contains the direct purchases from each industry to its supplier but also encloses all chains of transactions among the industries. I investigate how the first layers of the Leontief inverse relates to an industry's output impact by means of studying its domestic intermediate goods usage in Section 2.5. I then calculate and explore truncations of the shocks' approximation in Section 2.5 to capture the importance of higher order of interactions in the transmission across industries.

First-order linkage effects: direct input purchases

As discussed in Chapter 2, a model ignoring intraindustry connections tend to underestimate the output impact of a labour shock to more labour intense industries and overestimates it for less intense ones. I examine the intuitive idea that the averaging out of the impacts across industries may result from their direct transactions by constructing the weighted average of the factor intensity of the intermediate input basket of the industries.[25]

The intermediate input bundle of an industry is composed by goods with different factor intensity. I calculate the *network-adjusted*[26] of the the weighted average of the labour content of the input basket of a buying industry i by taking each producer j's labour intensity $(1 - \alpha_j)$ and weighting it by the relative amount of good j used by i $(p_j d_{ji} / \sum_j p_j d_{ji} = \gamma_{ji}/\gamma_i)$.

Definition 3.6 (Network-adjusted labour content) *The network-adjusted labour in-*

[25] In case one wonders, Corollary 3.2 posits that the labour-intensity of imports is not relevant for the impact of a labour supply shock on output because only the total shares σ_i matters for the effect on output given by Equation (full$_{ES}$).

[26] The term is borrowed from Baqaee (2015). Notice, however, that his measure considers all the levels of interactions while I am focusing here on the direct supplier-buyer transactions. The higher-orders are considered in this chapter by Equations (full$_{ES}$) and (full$_{pz}$), and in Section 2.5.

tensity of the input purchases of industry i is defined as:

$$LC_i = \sum_j (1 - \alpha_j) \left(\frac{\gamma_{ji}}{\gamma_i} \right) \tag{LC}$$

Likewise, I assess how much each industry i indirectly relies on a given import good z by computing the average import shares of every supplier industry j weighted by the intermediate input shares of buying industry i. In that way, I account of the relative importance of imported input z to all the direct suppliers of industry i.

Definition 3.7 (Import content) *The network-adjusted intensity of imports of good z in the input purchases of industry i is defined as:*

$$IC_{zi} = \sum_j \sigma_{zj} \left(\frac{\gamma_{ji}}{\gamma_i} \right) \tag{IC}$$

I show the results of this analysis in the empirical part. The labour content of the input basket of the industries is presented and discussed in Section 4.1, while the respective analysis for the import good is done in Section 4.2.

Higher-order linkage effects: shocks' approximations

As developed in Chapter 2, the power series approximation of the Leontief-inverse transposed is used to investigate the relevance of the different orders of transactions in explaining the total effect of the shocks.[27] Equation (3.24) depicts this relationship:

$$\left[\mathbf{I} - \mathbf{\Gamma}' \right]^{-1} = \sum_{k=0}^{\infty} \mathbf{\Gamma}'^k = \mathbf{I} + \mathbf{\Gamma}' + \mathbf{\Gamma}'^2 + \mathbf{\Gamma}'^3 + ... \tag{3.24}$$

Each order of approximation can be interpreted as the transactions occurring at the correspondent number of steps in the chain, i.e. from no intermediate input purchases (zeroth order), passing by direct transactions with input suppliers (first order), to those links more indirectly relating a pair of industries (higher orders). For instance, the second order of approximation includes all cases in which an industry g sells to j which then sell to an industry i.

[27] For more details about the power series approximation of the Leontief inverse, see Section 2.4 of Miller and Blair (2009).

To facilitate the analysis, I calculate the approximations for the total impact of the shocks on industry output. The terms associated with $\mathbf{I}$ stand for the 'initial' effects, matrix $\mathbf{\Gamma}'$ encompasses the 'direct' effects while the remaining terms of higher power are associated with the 'indirect' effects.[28]

The computations for the labour shock and the import price shock only differ by the 'shock' vector, i.e. the term post-multiplying the Leontief-inverse transposed in Equations (full_{ES}) and (full_{pz}), respectively. The equations below illustrate the first three orders of approximation for the import price shock:

- Zeroth-order approximation

$$\begin{bmatrix} \frac{\mathrm{d}\log q_1}{\mathrm{d}\log pz} \\ \frac{\mathrm{d}\log q_2}{\mathrm{d}\log pz} \\ \vdots \\ \frac{\mathrm{d}\log q_n}{\mathrm{d}\log pz} \end{bmatrix} = \begin{bmatrix} -\sigma_1 \frac{M_z}{C} - \sigma_{z1} \\ -\sigma_2 \frac{M_z}{C} - \sigma_{z2} \\ \vdots \\ -\sigma_n \frac{M_z}{C} - \sigma_{zn} \end{bmatrix} \tag{Appx_0}$$

- First-order approximation

$$\begin{bmatrix} \frac{\mathrm{d}\log q_1}{\mathrm{d}\log pz} \\ \frac{\mathrm{d}\log q_2}{\mathrm{d}\log pz} \\ \vdots \\ \frac{\mathrm{d}\log q_n}{\mathrm{d}\log pz} \end{bmatrix} = \left[I + \Gamma' \right] \cdot \begin{bmatrix} -\sigma_1 \frac{M_z}{C} - \sigma_{z1} \\ -\sigma_2 \frac{M_z}{C} - \sigma_{z2} \\ \vdots \\ -\sigma_n \frac{M_z}{C} - \sigma_{zn} \end{bmatrix} \tag{Appx_1}$$

- Second-order approximation

$$\begin{bmatrix} \frac{\mathrm{d}\log q_1}{\mathrm{d}\log pz} \\ \frac{\mathrm{d}\log q_2}{\mathrm{d}\log pz} \\ \vdots \\ \frac{\mathrm{d}\log q_n}{\mathrm{d}\log pz} \end{bmatrix} = \left[I + \Gamma' + \Gamma'^2 \right] \cdot \begin{bmatrix} -\sigma_1 \frac{M_z}{C} - \sigma_{z1} \\ -\sigma_2 \frac{M_z}{C} - \sigma_{z2} \\ \vdots \\ -\sigma_n \frac{M_z}{C} - \sigma_{zn} \end{bmatrix} \tag{Appx_2}$$

The zeroth order of approximation can be interpreted as an analogue to the impact predicted by the model without input-output linkages.[29] The first-order approximation includes the direct transactions only, from an industry to its suppliers. Each

[28] Terminology follows Miller and Blair (2009), Section 6.2.

[29] In the case of the labour shock, it differs by the total share of factors $(1 - \gamma_i)$ multiplying each industry's labour share in the 'shock' vector.

subsequent order includes a set of transactions one step farther than the preceding. Moreover, since the approximations are for the actual impact on output, the results will vary even though the matrices are the same in both cases.

In Sections 4.1 and 4.2 of the empirical analysis, I recast matrix $[\mathbf{I} - \mathbf{\Gamma}']^{-1}$ into the sum of powers of matrix $\mathbf{\Gamma}'$ times the respective shock vectors to investigate the relative importance to an industry of its indirect input transactions. The results confirm that industries' higher-order connections matter in explaining the input-output linkages, especially for industries which has little first-order transactions.

2.6 Networks: centrality and connectivity

I resort to concepts of network theory to compare the production structure of different countries and assess the relevance of each imported intermediate good. Before introducing the notions I use in this chapter, it is worth setting up the environment in which those apply.

In particular, the input-output matrix $\mathbf{\Gamma}$ is interpreted as the adjacent matrix of a directed weighted network, whose nodes are the n industries and the edges are directed from seller i to buyer j and weighted by their nominal shares on the buyer's total sales[30] γ_{ij}. Notice that this representation does not apply to the foreign matrix $\mathbf{\Sigma}$, as the nodes would overlay domestic and foreign industries rendering the direction of the edges meaningless.

Centrality measures of domestic industries

Industries are connected to others along *upstream* chains of input use and also via the *downstream* chains of input sales. The connections with suppliers are usually called *backward linkages* while those with clients are classified as *upward linkages* in input-output economics. Measures that quantify these 'economic connectivity' of the industries are used to assess their relative importance in the production network. Industries with larger upward linkages are said to be more 'beneficial' while those with larger downward linkages are more 'essential' to the economy.[31]

The simplest form of measuring the strength of backward linkages is the total input

[30]Under perfect competition and Pareto-efficiency, total costs equal total sales.
[31]See Section 12.2 of Miller and Blair (2009)'s textbook for a full characterization of these measures.

share γ_i of industry i featuring in Equation (3.1). In network analysis, this measure is called "weighted in-degrees". Being the sum of elements in the i^{th} column of matrix $\mathbf{\Gamma}$, it considers only the direct connections. Moreover, since each γ_i includes industry i's own consumption γ_{ii} —and these values tend to be significant in the dataset studied in this chapter—, this is not as much a measure of upstream connectivity as it is a measure of overall reliance on intermediate inputs. The same is true for the "weighted out-degrees", which is defined as the sum of intermediate inputs sold by industry i, i.e. $\gamma_i^{out} = \sum_{j=1}^{n} \gamma_{ij}$. This measure corresponds to the relative importance of industry i's output as an intermediate input in the production network. It is equal to the sum of elements in the j^{th} row of matrix $\mathbf{\Gamma}$.

To capture the indirect or higher order connections within the production network, a centrality measure has to incorporate the Leontief-inverse matrix. The vector μ given by Equation (3.21) portrays the terms known as the 'input-output multipliers' of each industry. It traces the output response of all industries to an shock in a particular industry.[32] Under certain assumptions, the input-output multipliers can be related to the Bonacich centrality measure (Bonacich, 1987).[33]

I take a slightly different approach to accommodate with the model's settings. The Katz–Bonacich centrality measure use as a reference rod for the industries comprises of an infinite power sum of the adjacent matrix g matrix (with origin in rows and destinations in columns), where an exponent l (also call length) corresponds to the l-order connection between nodes. It follows closely the network literature (Bloch, Jackson, and Tebaldi, 2016) and requires no assumptions.[34] This centrality measure is essentially "downstream" since it is based on the notion of that the "prestige" or "influence" of node is rendered as the number of walks emanating from it to its neigh-

[32]It is easy to see that having the real variables instead of the shares in Equation (3.21) leads to the solution for industry output in input-output activity analysis: $\mathbf{q} = [\mathbf{I} - \mathbf{\Gamma}]^{-1}\mathbf{y}$. It depicts how much each industry has to produce to generate a given vector of domestic final demand and exports. Thus, the Leontief inverse encompasses the full range of intermediate inputs embodied in the production of a final good.

[33]This analogy is first presented by Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012). In the paper, the output function attributes a weight α to the unique factor of production —labour— leaving $(1-\alpha)$ as the total share of intermediate inputs in the economy. Additionally, the final goods sales shares β_i are assumed identical —equal to $1/n$— across sectors, and the input-output matrix is transposed (row destinations, columns origins). Carvalho (2014) has other notations, but makes essentially the same assumptions as Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and calls the measure "Katz–Bonacich eigenvector centrality", even though these measures are not interchangeable. Given their assumptions, the resulting expression for the sales or "influence" vector looks different, but it is essentially the same as in Baqaee and Farhi (2019) and in Equation (3.21).

[34]In particular, there is no need for a discounting factor since the elements of the $\mathbf{\Gamma}$ matrix are significantly below one, so that the decay of the geometric series is very fast.

bours. The expression for the standard Katz–Bonacich centrality measure without discounting reads:

$$\mathbf{c}^{KB}(\mathbf{\Gamma}) = \sum_{l=1}^{\infty} \mathbf{\Gamma}^l \mathbf{1} = [\mathbf{I} - \mathbf{\Gamma}]^{-1} \mathbf{\Gamma} \mathbf{1} \tag{3.25}$$

Where $\mathbf{1}$ is the vector of ones. By using this concept one can easily transpose matrix $\mathbf{\Gamma}$ and get an upstream centrality measure, gauging which industries are more reliant on domestic intermediate inputs.

Definition 3.8 (Upstream Katz-Bonacich) *The upstream version of the Katz–Bonacich centrality measure of industry i is defined as:*

$$\mathbf{c}^{KBU}(\mathbf{\Gamma}') = \sum_{l=1}^{\infty} \mathbf{\Gamma}'^l \mathbf{1} = \left[\mathbf{I} - \mathbf{\Gamma}'\right]^{-1} \mathbf{\Gamma}' \mathbf{1} \tag{KBU}$$

Notice that $\mathbf{\Gamma 1}$ stands for the first term of the infinite sum and corresponds to the vector of the weighted out-degrees in the downstream version and to the vector of weighted in-degrees in the upstream one. Mathematically, the vector resulting of the downstream version with $\mathbf{\Gamma}$ has the summation of the input shares over destinations, i.e. $\sum_i \gamma_{ji}$ whilst the one of the upstream version with $\mathbf{\Gamma}'$ has the summation over origins, i.e. $\sum_j \gamma_{ji}$.

Centrality measures for imported goods

Summing the shares for each import j over all buying industries i, I get σ_j^{out}, a foreign equivalent to the outdegree centrality γ_j^{out} for domestic goods. It measures the overall relevance of each imported good for domestic industries, regardless of their size and position in the production network.

Definition 3.9 (Import outdegrees) *The outdegree centrality measure of an imported good j reads:*

$$\sigma_j^{out} = \sum_{i=1}^{n} \sigma_{ji} \tag{3.26}$$

Alternatively, the importance of each imported good can be determined by how central the industries which rely the most on it are. To construct this measure, I post-multiply the import shares given in matrix $\mathbf{\Sigma}$ by the downstream Katz-Bonacich centrality vector $\mathbf{c}^{KB}(\mathbf{\Gamma})$, as given by Equation 3.25.

Definition 3.10 (Imports centrality) *The centrality measure of imported goods reads:*[35]

$$\mathbf{c}^{M} = \mathbf{\Sigma} \cdot \mathbf{c}^{KB}(\mathbf{\Gamma}) \tag{3.27}$$

The downstream centrality measures how much each industry is important as a domestic seller of intermediate goods. By using this measure in this calculation I am harnessing the industries which transmit the import price shock the most through the production chain.[36]

Connectivity

Three measures of connectivity are chosen to evaluate the countries' production networks. These are used in Section 3.1 to assist in selecting the pair of economies the most and the least interconnected in the dataset.

The leading measure is *density*, as it captures the fraction of existing connections to all $n \times n$ possible connections. Weights are ignored and the threshold of $\gamma_{ij} > 0.01$ is chosen to stablish a connection, following Carvalho (2014).[37]

The *distance* or characteristic path length conveys the average smallest path between any two nodes.[38] A path is defined as the collection of edges from one node to another, where intermediate nodes appear only once. Departing from Carvalho (2014), the calculation of the characteristic path length here includes the edge weights.[39] The weighted measures of distance and diameter consider the length of two nodes as inversely related to their weights γ_{ij}. In other words, two industries that have a high volume of transactions are considered as closer than other pair whose volume is small. The distance matrix can be computed using the reciprocal or the negative logarithm function; I choose the latter for readability of the plots.

[35]In full:

$$\begin{bmatrix} c_1^M \\ c_2^M \\ \vdots \\ c_n^M \end{bmatrix} = \underbrace{\begin{bmatrix} \sigma_{11} & \sigma_{12} & \dots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \dots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \dots & \sigma_{nn} \end{bmatrix}}_{\text{Summing over destinations}} \cdot \begin{bmatrix} c_1^{KB} \\ c_2^{KB} \\ \vdots \\ c_n^{KB} \end{bmatrix}$$

[36]If I used the upstream centrality, I would be assessing how relevant each imported good weighting the importing industries by their direct and indirect importance as buyers of domestic input.

[37]The MATLAB coding was adapted from his, which is available at Carvalho (2019).

[38]Related to the distance, the *diameter* measure is a somewhat less comprehensive, as it singles out the largest smallest path connecting each pair of nodes. For completeness, I present the cross-countries values for this measure in Figure

[39]Theory and MATLAB functions from Sporns and Rubinov (2020).

3 Quantitative analysis

This section presents the main points regarding the methods supporting the empirical analysis. I start by presenting a cross-country assessment of the connectivity of production networks of WIOD in Section 3.1, which lays the ground for selecting Ireland and Korea as the pair with contrasting characteristics. Then, in Section 3.2, I highlight the fundamental steps in the data calibration and implementation of the models. Finally, in Section 3.3, I build up the profile of Ireland and Korea for the key parameters affecting the analysis and the production structure of their industries.

3.1 Cross-country production structures

I use the WIOD to survey the countries in terms of their production networks —in Section 3.1— and aggregate domestic input production and imported intermediate good use —in Section 3.1. The parameters, i.e. the nominal shares, are calculated with the values as available in the data. The WIOD and variables used are described in Appendix 3.B.[40]

To allow for a thorough understanding of the role of input-output linkages and imported input reliance in the shocks' transmission, the countries selected for the empirical analysis must have contrasting production networks. Ideally, to explore the full extent of the developed framework, the contrast would include differences in their use of imports.

Connectivity of production networks across countries

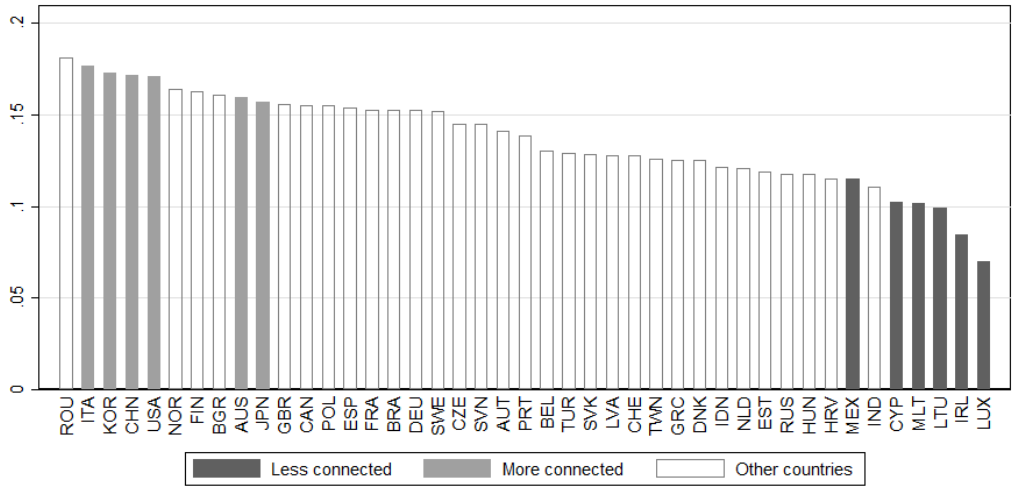
Using the domestic input-output matrix computed directly from the WIOD raw values to build the production networks of the countries —as described in Section 2.6— I calculate the measures of connectivity presented in Section 2.6.

As Figure 3.2 shows, these measures vary considerably for the countries of the WIOD sample. I highlight in the plots the economies which figure among the ten most (in light grey) and least interconnected (in dark grey) according to the rankings of both connectivity measures.[41] Only six countries appear in the top-10 least connected,

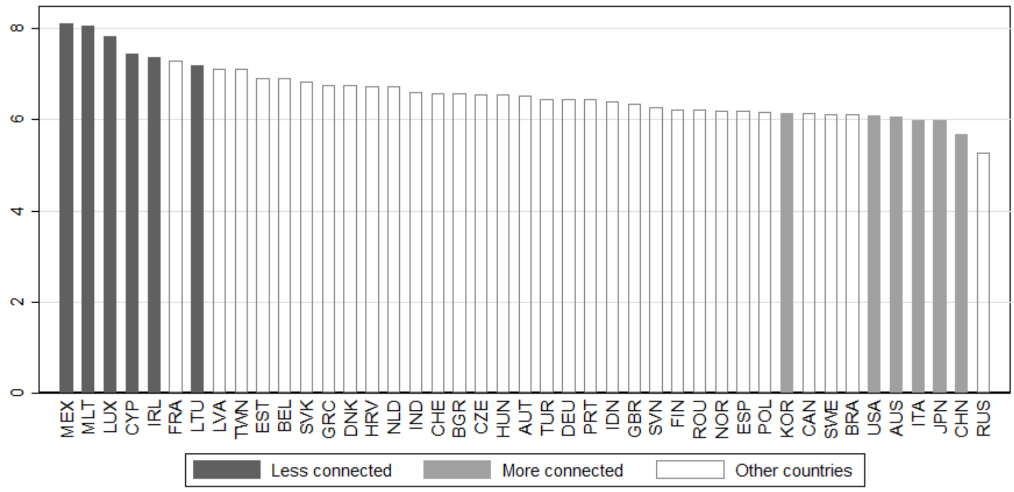
[40] Table 3.B.1 in the Appendix lists the countries acronyms and respective names.

[41] Table 3.B.3 ranks the ten most and least connected economies according to the density and distance

namely: Luxembourg, Ireland, Lithuania, Malta, Cyprus and Mexico. Coincidently, another six countries figure in the top-10 most connected, namely: Italy, South Korea, China, United States, Australia and Japan.



(a) Density: the number of non-zero ($\gamma_{ij} > 1\%$) connections over 56×56



(b) Distance: the average shortest path length in the network

The bars highlighted correspond to the countries figuring in the top-10 rankings of both density and distance. The measures are described in Section 2.6. The input-output matrix used to calculate these measures is constructed using the nominal values of domestic input use and gross output of each of the 56 industries of the 43 countries of the WIOD sample.

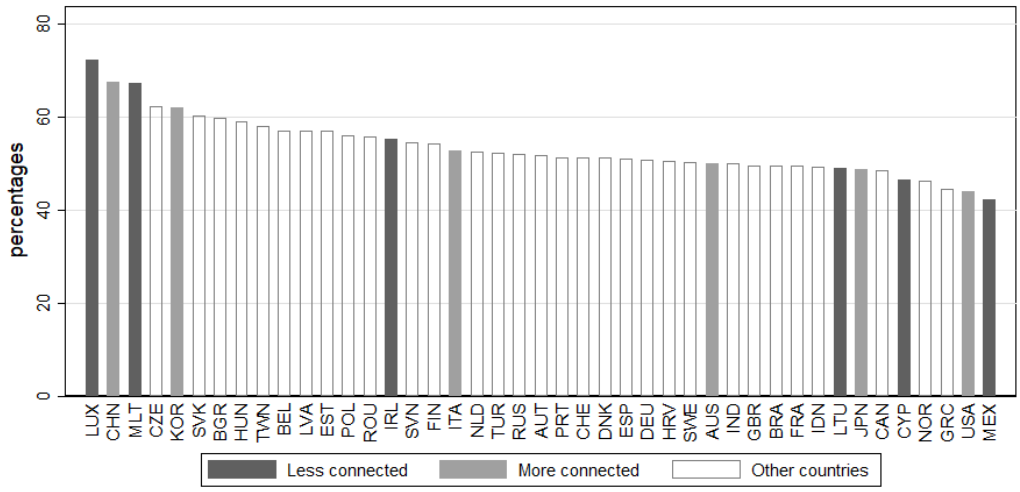
Figure 3.2: Network connectivity of the WIOD countries

Aggregate input and import shares

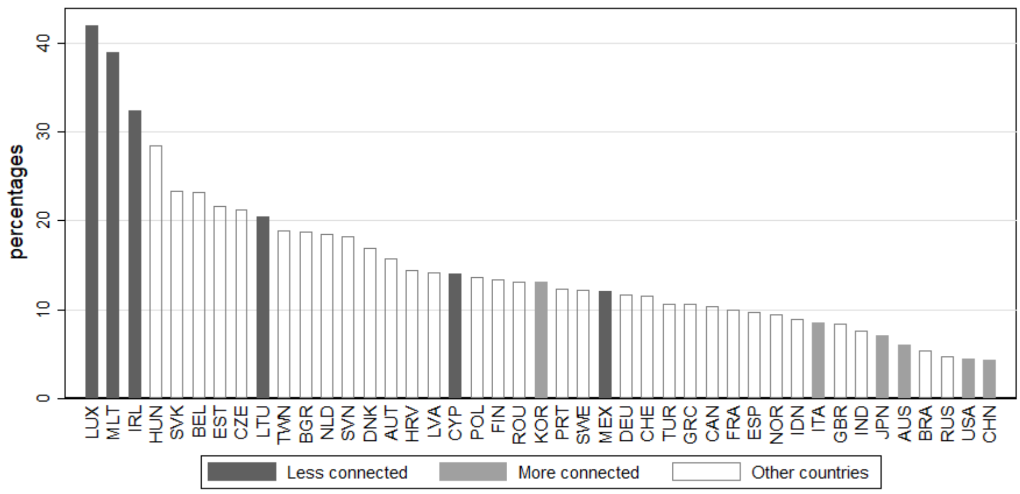
The aggregate domestic intermediate goods production varies roughly from 40% to 60% of the gross output for the WIOD countries. Interestingly, even though the decay

measures.

of the values plotted in Figure 3.3a has a shape similar to those of Figure 3.2, the position taken by the countries highlighted is remarkably dissimilar. One could expect that countries with more connected production networks would also present larger shares of intermediate inputs in gross output, but this is not the case.



(a) Intermediate input shares: domestic inputs over gross output



(b) Import shares: imported inputs over gross output

The aggregate shares of input use and intermediate imports are calculated using the nominal values as available in WIOD. Network connectivity derives from Table 3.B.3.

Figure 3.3: Input and import shares of WIOD countries versus network connectivity

The input shares surpass 60% in Luxembourg and Malta, for instance, even though these countries figure among those with the most sparse networks. The US is another extreme example, with an aggregate input share just above 40% and having one of the most connected domestic production networks. These findings reinforce the importance of using network measures to assess the relevance of input-output linkages in the propagation of shocks.

The relevance of aggregate imports in gross output, on the other hand, is more asymmetrical and rather correlated to network connectivity. As Figure 3.3b shows, the shares vary from below 5% to above 40% across countries.[42] Moreover, economies more reliant on imports seem to be those presenting more sparse domestic networks, in general, while the opposite is true for those with the smallest aggregate import shares.

Among the countries highlighted in Figure 3.2, Ireland and Korea stand out as interesting study cases since they not only have dissimilar network connectivity but also fairly contrasting total imports use (of 32% and 13%, respectively). Notwithstanding, the pair present reasonably large input shares, indicating that they possess considerable domestic input production.[43]

3.2 Calibration and implementation of the models

The data calibration is performed in MATLAB where the models are implemented.[44] All share parameters of the full model are computed following the formulae presented in Section 2. The industry productivity parameters A_i are calculated as the level required to match the output computed by the model to the data values in each industry.

For Auxiliary 0, I assume that all the input shares γ_{ji} and import shares σ_{ji} equal zero. This assumption is harmless to the analysis since I only study the industry output effect as given by Equation (Aux0_{ES}) and re-estimating the Aux0 would produce the same values of α_i. The reason for that is that the computation of the value added is unchanged across all the models used in this chapter. As Definition 3.3 for the narrow factor shares states, this is all that counts in the computation of these parameters.

Auxiliary NT distinguishes itself from the full model by taking the import shares σ_{ji}

[42] Appendix 3.B.2 includes the comparison of all countries': imports and exports; value-added; intermediate output production; gross output; input and factor usage; and import usage.

[43] Additionally, both economies can be considered "small" in international terms. As Figure 3.B.10 in the Appendix shows, Korean gross output (nominal US dollars 2014) accounts for only 2.5% of the total world value, while Ireland is even smaller, with a gross output of 0.4% of the total. Australia could also be a good candidate exemplar of a connected economy relying little on imports. However, for reasons discussed in Appendix 3.B.4, the setting of the model require the merging of industries, depending on the country's data. Korea and Ireland happen to impose the smallest industry loss while keeping the merging within the same broad groups of the ISIC, Rev. 4 classification.

[44] Appendix 3.B.4 presents the details on how the variables of the dataset are constructed.

as null while the Auxiliary NIO model sets the input shares γ_{ji} to zero. In the empirical analysis of Section 4, I choose for simplicity and consistency to use the remaining parameters as estimated under the premisses of the full model. This means that the values for γ_{ji} in Auxiliary NT and those for σ_{ji} in Auxiliary NIO are underestimated.

If I were to do a calibration exercise for the auxiliary models, I would have to re-estimate the parameters from scratch. This would produce smaller industry output for the auxiliary models.[45] This would occur because in the computation of the input γ_{ji} [Equation (3.11)] and import shares σ_{ji} [Equation (3.12)], I would consider smaller denominators. The greater the omitted variable corresponding to the nullified share, the larger would be the re-estimated parameter and, therefore, the greater the under-estimation produced by assuming it to be equal to that produced by the full model.

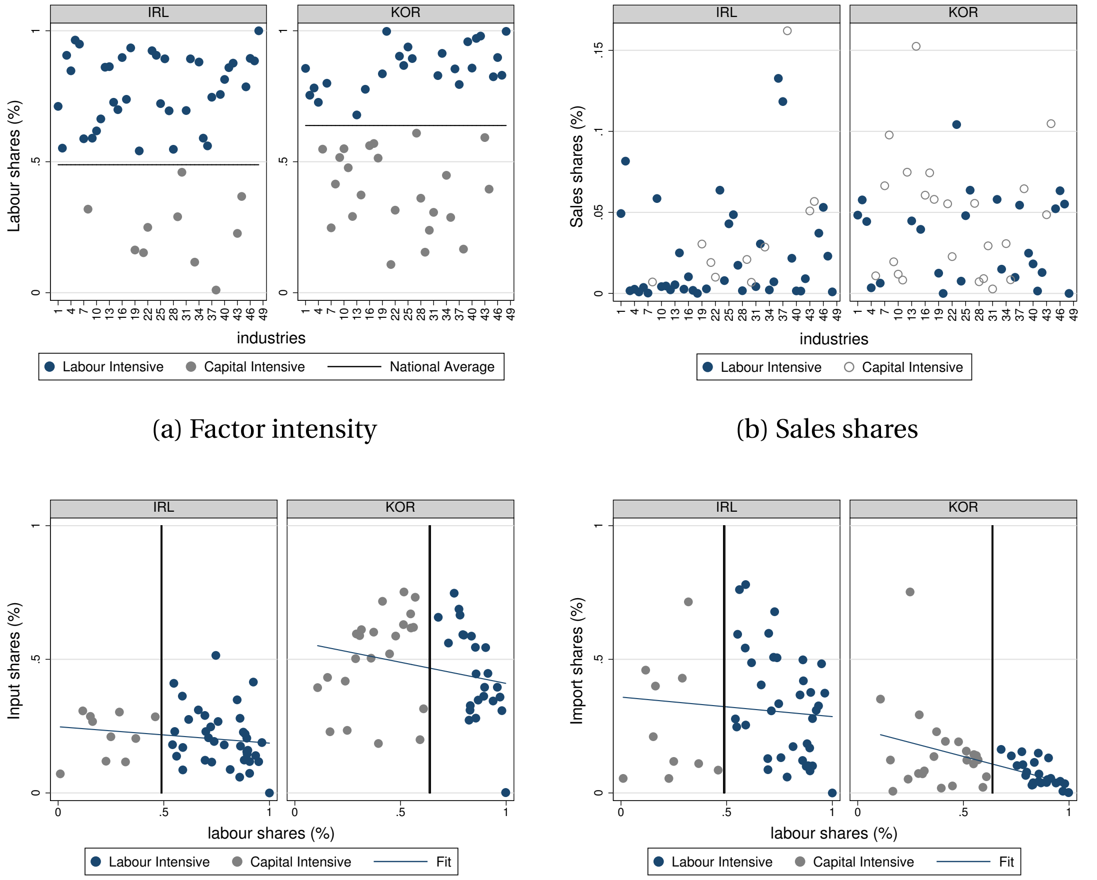
3.3 In-depth analysis: Ireland and Korea

Main parameters and input-output matrices

On average, South Korean industries are more labour-intensive than the Irish (Figure 3.4a), even though the number of capital-intensive industries is greater in Korea than in Ireland. In terms of representativity in final sales (Figure 3.4b), the European country has fewer industries with Domar shares above 5% than the Asian country.

The higher connectivity of the South Korean industry network is summarised by the higher values of total (domestic) intermediate usage, given by the γ_i shares (Figure 3.4c). Regarding the imports of inputs, labour-intensive industries present the highest σ_i shares in Ireland while the reverse is true in South Korea (Figure 3.4d).

[45] In the steps described in Appendix 3.B.4, the formulae for the industry gross output of the auxiliary models would be replaced by: (i) $p_i^g q_i^g = VA_i^g + II_i^{in,g}$ for AuxNT; and (ii) $p_i^g q_i^g = VA_i^g + M_i^g$ for AuxNIO.



(a) Factor intensity

(b) Sales shares

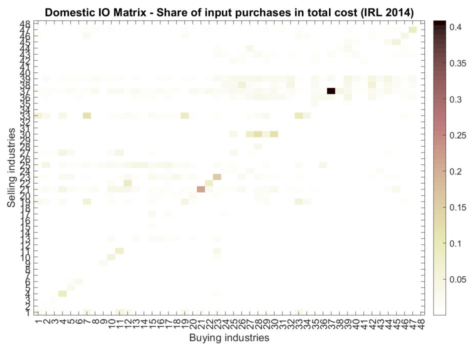
(c) Input shares

(d) Import shares

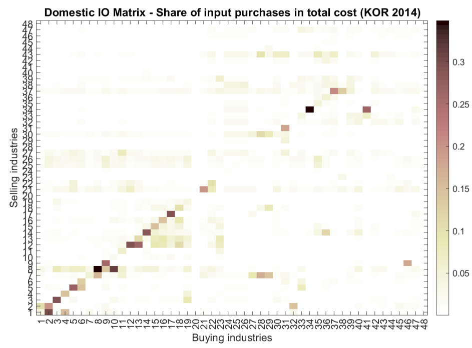
Note: vertical lines show national labour-intensity averages. Labour shares refer to the narrow factor shares α_i given by Equation (3.17). The sales shares μ_i are defined by Equation (3.20). Input shares γ_i and import shares σ_i are given by Equations (3.11) and (3.12), respectively. Values calculated using calibrated data as described in Section 3.2 and Appendix 3.B.4.

Figure 3.4: Ireland and Korea main variables

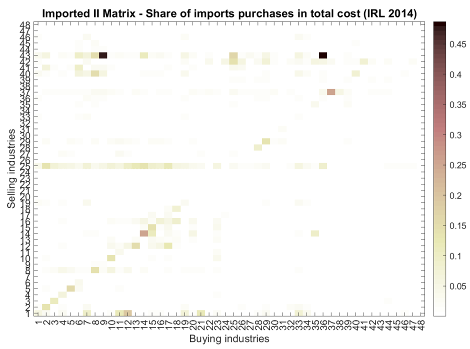
Figures 3.5a and 3.5b plot the domestic input-output structure of the pair of countries while Figures 3.5c and 3.5d show their matrix of imported inputs use. Noticeably, South Korea presents a much more connected production network than Ireland, which relies much more on imported inputs than Korea.



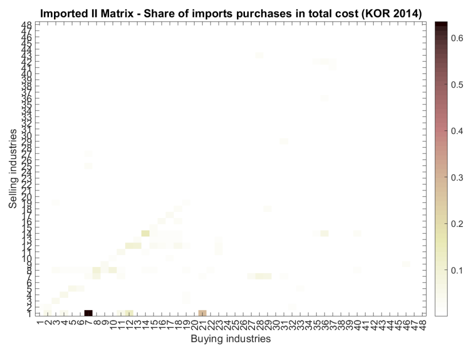
(a) $\mathbf{\Gamma}$ IRL



(b) $\mathbf{\Gamma}$ KOR



(c) $\mathbf{\Sigma}$ IRL



(d) $\mathbf{\Sigma}$ KOR

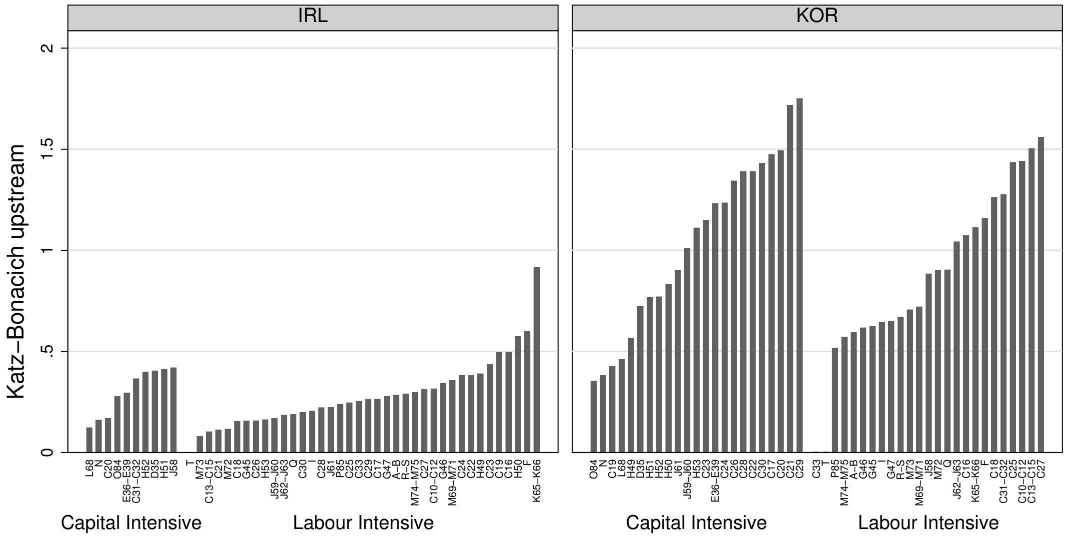
Note: the matrices are defined in Sections 2.3 and 2.3, respectively. The values are calculated using calibrated data. Plots 3.5a and 3.5b register that Korea has a much denser production network while plots 3.5c and 3.5d confirm that Ireland relies much more on imported inputs.

Figure 3.5: Domestic input-output linkages (matrix $\mathbf{\Gamma}$) and industry reliance on imported inputs (matrix $\mathbf{\Sigma}$) for Ireland and Korea

Centrality measures for domestic industries

The upstream Katz-Bonacich centrality measure (Definition 3.8) can be seen as an alternative way of assessing which industries would be the most impacted by the shocks studied in this chapter. This occurs because both solutions for the industry output changes following the labour shock (Corollary 3.2) and the import price shock (Corollary 3.5) have a form similar to the measure: the Leontief-inverse transposed premultiplying a vector.

Figure 3.6 plots the upstream centrality of the industries grouped by factor intensity. Notice that the values across countries are comparable since they are made up of the input shares, which have the same scale for any country. The higher connectivity of the Korean production network translates into higher centrality measures in general when compared to Ireland. Except for a few outliers, there is little difference between the capital- and labour-intensive groups in Ireland, while capital-intensive industries seem to be slightly more dependant on the domestic industry network than the labour-intensive ones.[46]



Note: the measures are calculated using calibrated data and the formula in Equation (KBU).

Figure 3.6: Domestic network: upstream centralities

Centrality measures for imported goods

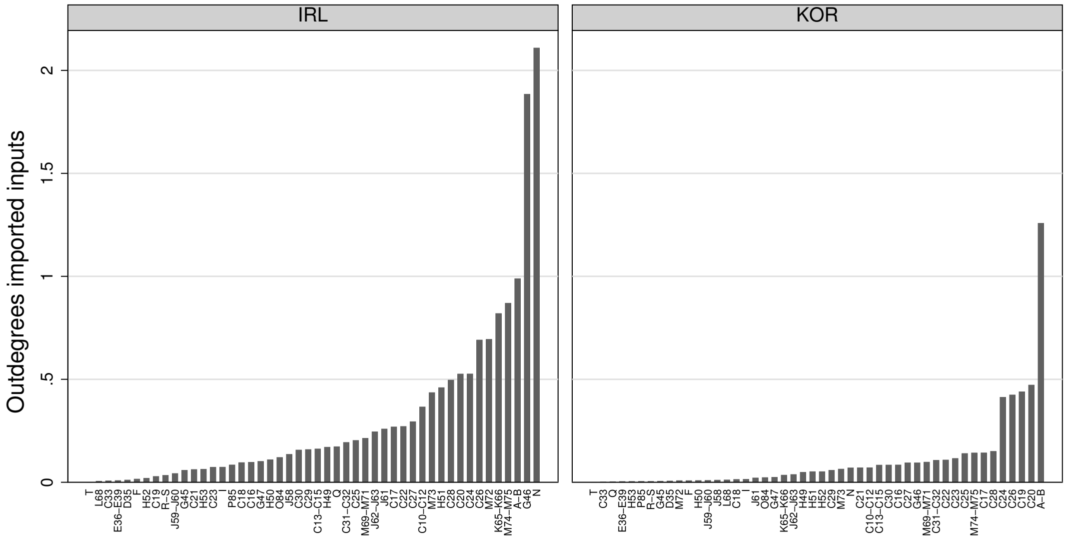
An encompassing way of studying the importance of an imported good to an economy is via the centrality measures presented in Section 2.6. Figure 3.7 plots the outdegree centrality measure σ_j^{out} for all imported goods in both countries.[47] Industry A-B is the single outlier in Korea, while also ranking high (third place) in Ireland.

Figure 3.8 plots the import goods ranked by their downstream importance to the production network.[48] Once more, industry A-B leads the ranking in Korea whilst occupies the forth place in Ireland.

[46]For completeness, the downstream Katz–Bonacich centralities are plotted in Figure 3.B.11 in the Appendix, as it is part of the centrality measure for imported goods (Definition 3.10). The plot shows a similar pattern to Figure 3.6. The outliers in Korea, however, are not the same as those given by the upstream measures, reinforcing that the most dependable industry to the network tends to be other than the most reliant on the other industries. In particular, capital-intensive industries are much more relevant as suppliers (average downstream centrality of 1.41 versus 0.78) while labour-intensive industries rely slightly more on domestic inputs (average upstream centrality of 0.98 versus 0.90).

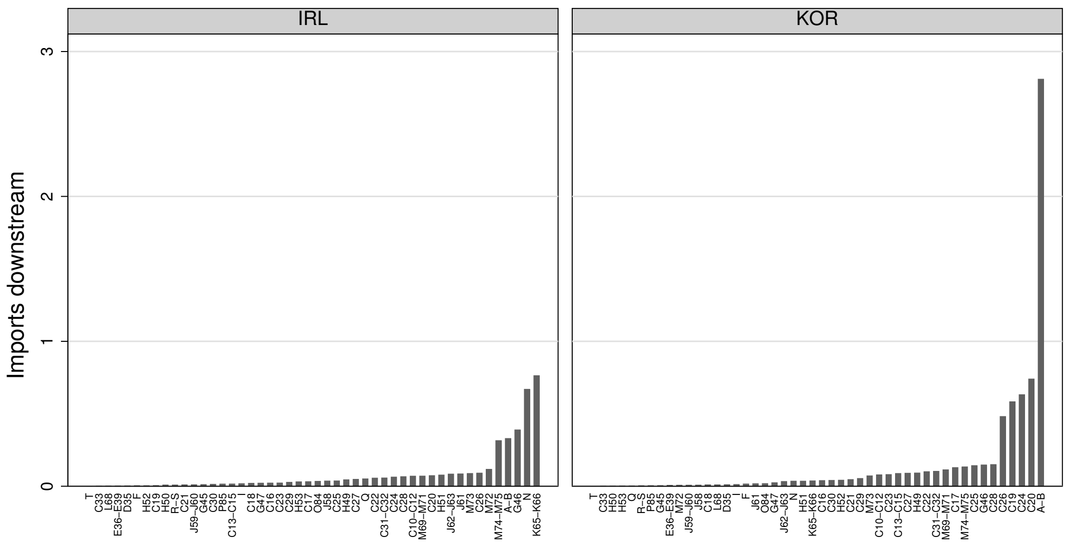
[47]The domestic outdegree centrality γ_j^{out} is plotted in Figure 3.B.12 of the Appendix for completeness.

[48]Figure 3.B.13 plots values of the version using the upstream Katz-Bonacich measure for completeness.



Note: the measures are calculated using calibrated data and the formula in Equation (3.26).

Figure 3.7: Outdegree centralities of imported goods



Note: the measures are calculated using calibrated data and the formula in Equation (3.27).

Figure 3.8: Downstream centralities of imported goods

In summary, the distribution of the values is not continuous in both countries, with five goods having a much higher relevance compared to all others. Imported intermediate inputs from the group of industries A-B is the only one appearing in the top-5 of both countries and, for this reason, it is the one selected to receive the one-percent positive shock in the counterfactual exercise.[49] In what follows $z = 1$ representing the group of industries A-B as presented in Table 3.B.4.

[49]Appendix 3.B.6 provides a broader assessment of how the imported good A-B was selected.

4 Empirical analysis

4.1 Counterfactual #1: a labour shock

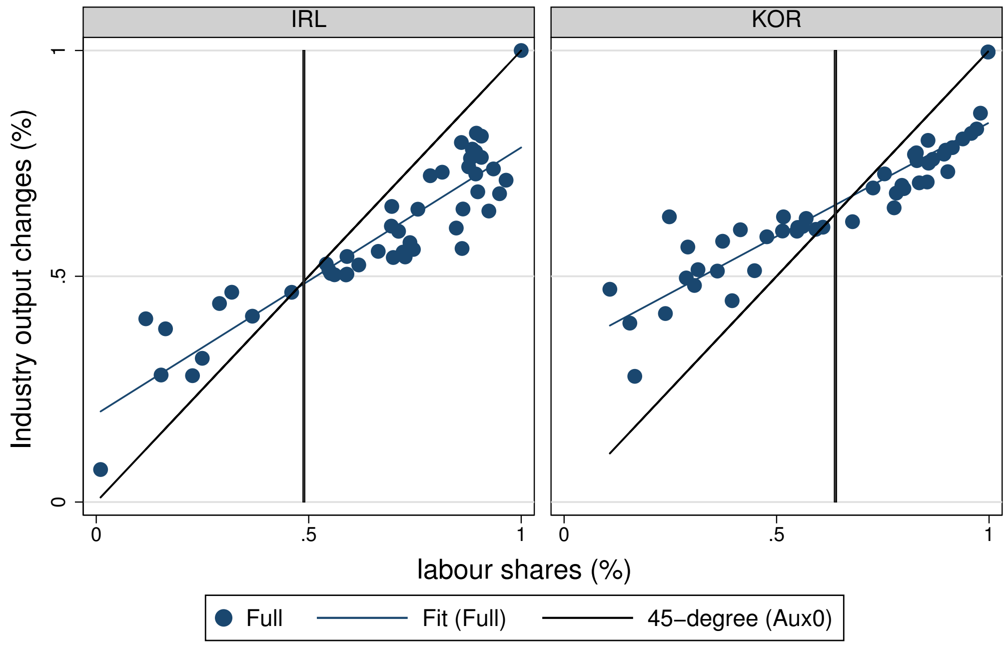
As developed in Chapter 2, the labour shares are not a sufficient statistic to predict the industry output changes following a labour supply shock. The use of inputs, which I now extend to imported goods, must be considered. Figure 3.9 plots the models' predictions versus the labour shares. In both countries, the expected pattern of angular scattering is seen: in the full model, more (less) labour-intensive industries grow less (more) than predicted by Aux0 and AuxNT. The more the features not present in the alternative models matter, the flatter is the fit line of the full model.

Given that the Irish production network is more sparse than the Korean, one would expect that ignoring linkages would matter less for the European than for the Asian country. But this is not the case; the full model's fit line for Ireland is not significantly steeper than the one for Korea (Figure 3.9a). However, the results change remarkably when ignoring trade. The fit line of AuxNT is quite close to the 45-degree line in Ireland (Figure 3.9b), indicating that ignoring domestic linkages would not matter much there since most of the dispersion revealed by the full model is caused by trade.

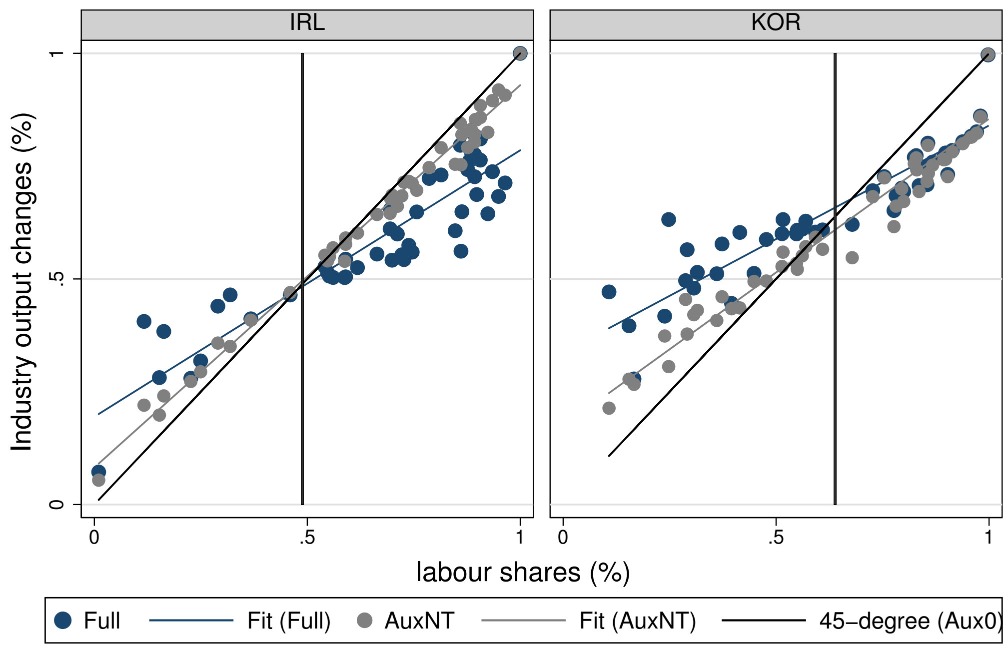
In Korea, on the other hand, there is a sharp difference in the patterns seen for capital-intensive vis-à-vis labour-intensive industries. While for the latter, the models with and without trade produce very similar results, for the former they are visibly different. This disparity comes from the distinct import patterns (Figure 3.4d), with labour-intensive industries relying very little on imports while capital-intensive industries present large import shares. Therefore, ignoring trade is innocuous regarding labour-intensive Korean industries while it matters for capital-intensive ones.

In summary, trade is important in general for Irish industries but only in part for Korean capital-intensive industries. For most labour-intensive industries in Korea, the non-trade model predicts an impact similar to the full model's since they import little. Additionally, removing only the trade component from the full model practically removes all the overestimation for this group, while it makes little difference for the remaining. This finding stresses that domestic input-output linkages are highly relevant for Korean labour-intensive industries.[50]

[50] Appendix 3.C.1 presents the ratios of the models' predictions.



(a) Full model versus Auxiliary 0



(b) Full model versus Auxiliary 0 and NT

Note: vertical lines show national averages. Figure 3.9a plots each industry's output change versus their labour shares, with the 45-degree line representing the auxiliary model without neither input-output nor trade linkages (Aux0). Figure 3.9b reproduces the previous plotting and adds the industry output growth predicted by the no-trade auxiliary model (grey dots). It allows for a distinction between the role of trade (gap between the grey dots and the blue dots) and the role of domestic linkages (distance between the 45-degree line and the gray dots) on the impact of the shock across industries.

Figure 3.9: Comparison of the labour shock effect across models

First-order linkage effects: direct input purchases

Figure 3.10 plots the labour intensity of the inputs basket of each industry, as given by Equation (LC), over their own labour intensities. The 45-degree line depicts the locus corresponding to industries having an input basket on average of the same labour intensity as their own.

Note: the labour content of each industry input basket is calculated via Equation (LC). Horizontal and vertical lines show national labour-intensity averages.

Figure 3.10: Labour intensity of domestic intermediate input purchases

In both countries, capital-intensive industries tend to domestically buy more from industries relatively more intense in labour (above the 45-degree line) and vice-versa for labour-intensive industries (below the 45-degree line). Moreover, some group of industries purchase patterns are reinforcing this redistribution process. If industries always bought relatively more from their peers, they would fall exclusively on quadrants I and III of the plots, which is not the case. In Ireland, most capital-intensive industries buy relatively more from labour-intensive industries, thus falling on quadrant II. In Korea, many labour-intensive industries buy relatively more from capital-intensive industries on quadrant IV. This finding helps to explain the relay of the impacts from more to less labour-intensive industries, albeit it only considers first-order interconnections.

Higher-order linkage effects: shock approximations

The ratios of the output impact of the labour shock over each order of approximation give the percentage explained by each truncation. Together, they indicate how fast the series of approximations converges to the full model's predictions.

Figure 3.11 depicts the means of these ratios within the factor-intensity groups. It shows that, in general, the capital-intensive industries present a slower rate of convergence, indicating that their upstream connectivity is particularly relevant along the chains of labour-intensive inputs. This finding is especially interesting in Korea, where capital-intensive industries are relatively less upstream connected (Figure 3.6) and do not purchase directly much from labour-intensive industries (Figure 3.10).

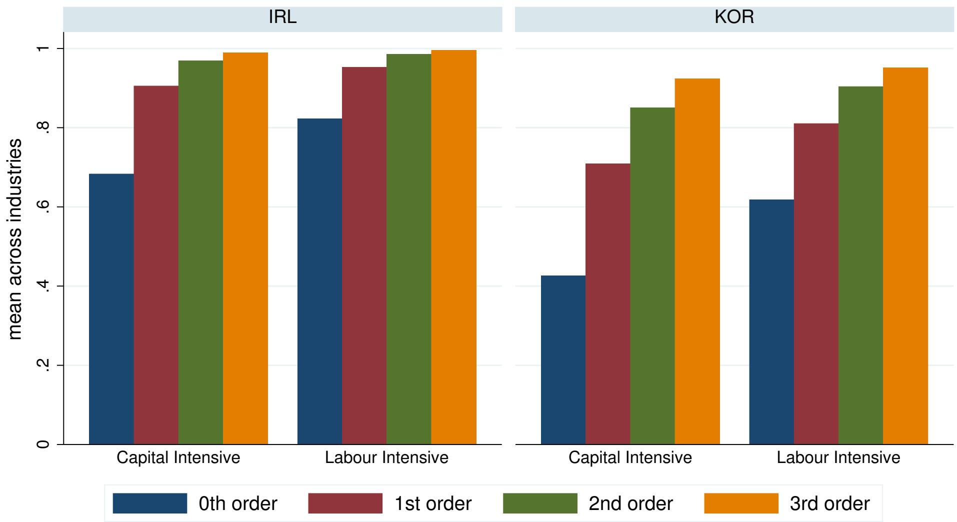
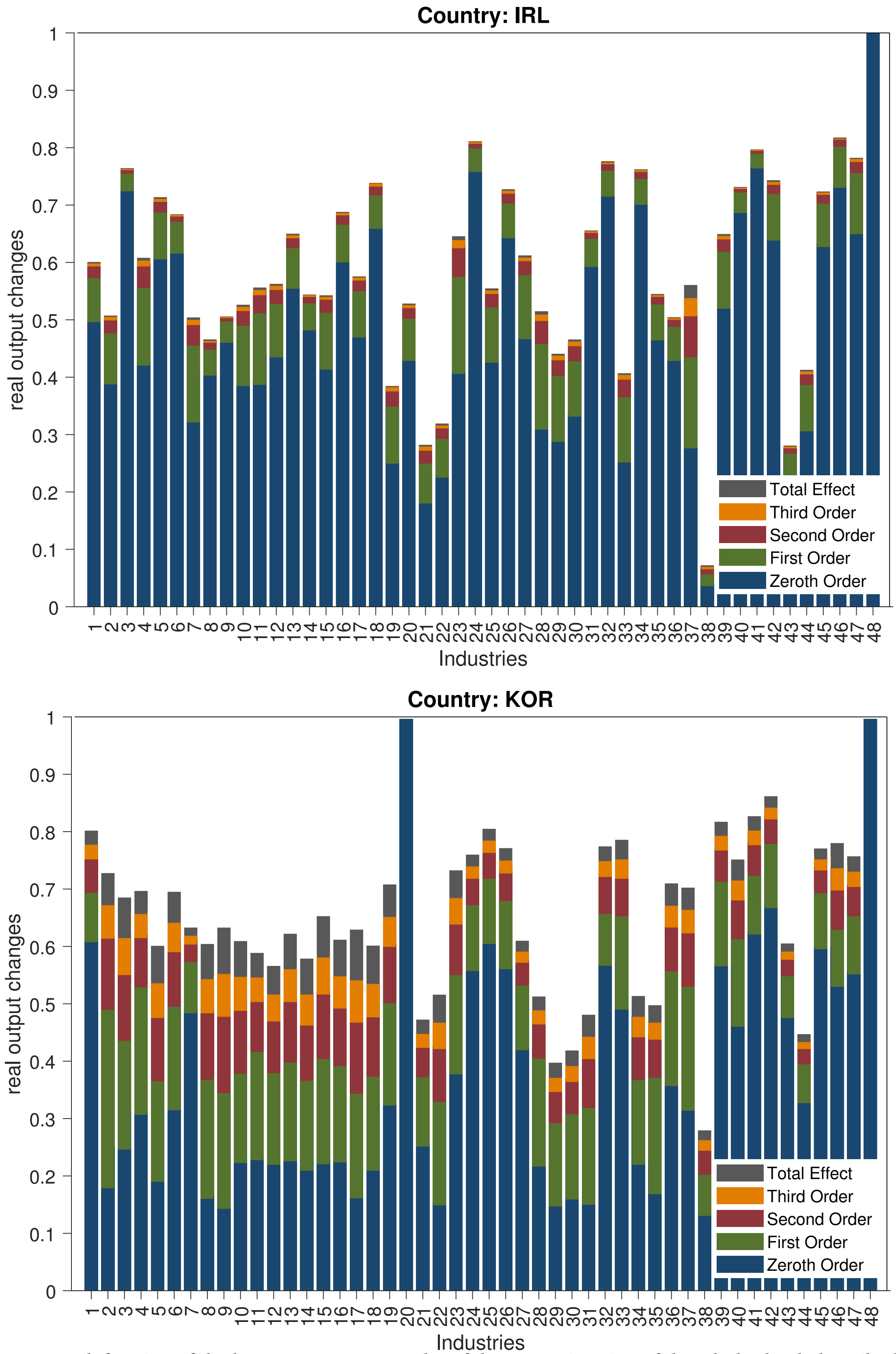


Figure 3.11: Means of the ratios of each order of approximation over full shock

Figure 3.12 plots the shocks' approximations from the zeroth to the third order for all industries. The bars are superimposed, each colour representing the portion of the total effect (in grey) explained by the corresponding approximation. In particular, the values of the zeroth order (in blue) convey how relevant is the shock vector. The little significance of input-output linkages in Ireland translates to a high performance of the shock vector, whereas in Korea it alone does not explain much. Overall, the rate of convergence in Korea is much slower, meaning that the indirect transactions are quite relevant within the production network.

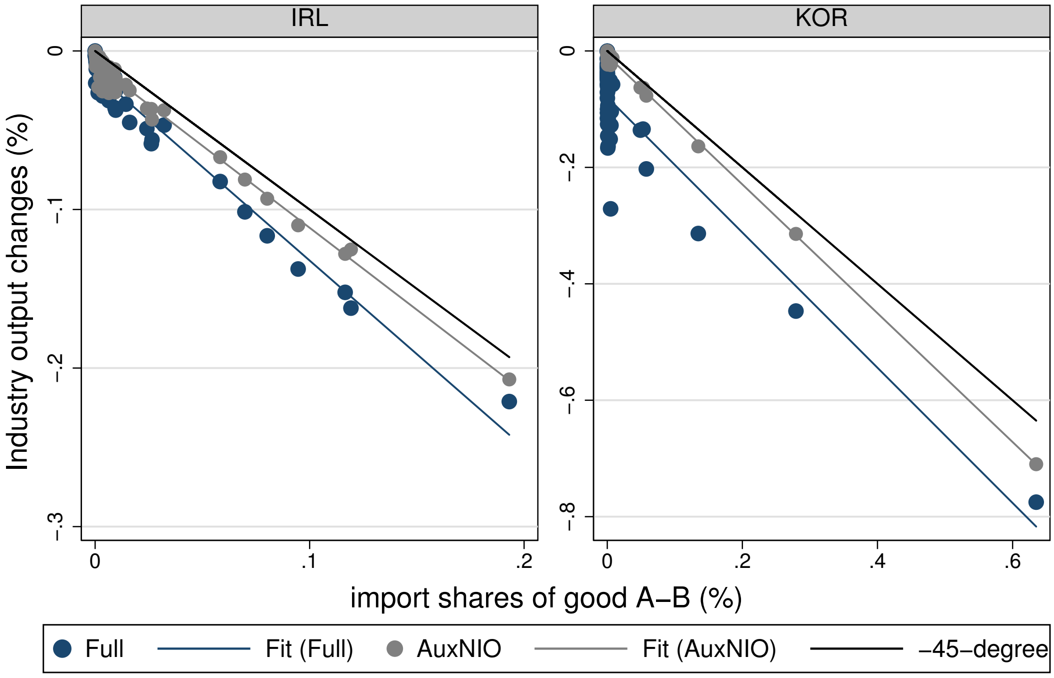


Note: each fraction of the bars represents an order of the approximation of the whole shock described in Section 2.5. The zeroth order is equivalent to ignoring the several stages of production as enclosed by the Leontief-inverse transposed. The total effect represents the predictions of the full model as given by Equation full$_{ES}$. In this way, the extent of the bars beyond the zeroth order depicts how much the higher-order transactions affect the impact of the labour supply shock on each industry.

Figure 3.12: Decomposition of the labour shock for Ireland and Korea

4.2 Counterfactual #2: an import price shock

In contrast with a trade model without input-output linkages, the industry output impact of the import price shock is not restricted to those directly importing the good. In fact, the domestic network boosts the output changes the most for the industries which rely less on the affected good.



Note: the blue dots depict the impact on the output of the industries predicted by the full model while the grey dots respresent the predictions of the Auxiliary NIO model. The 45-degree is a reference for the effect on output coming from the term $-\sigma_{zi}$ in Equation (full_{pz}).

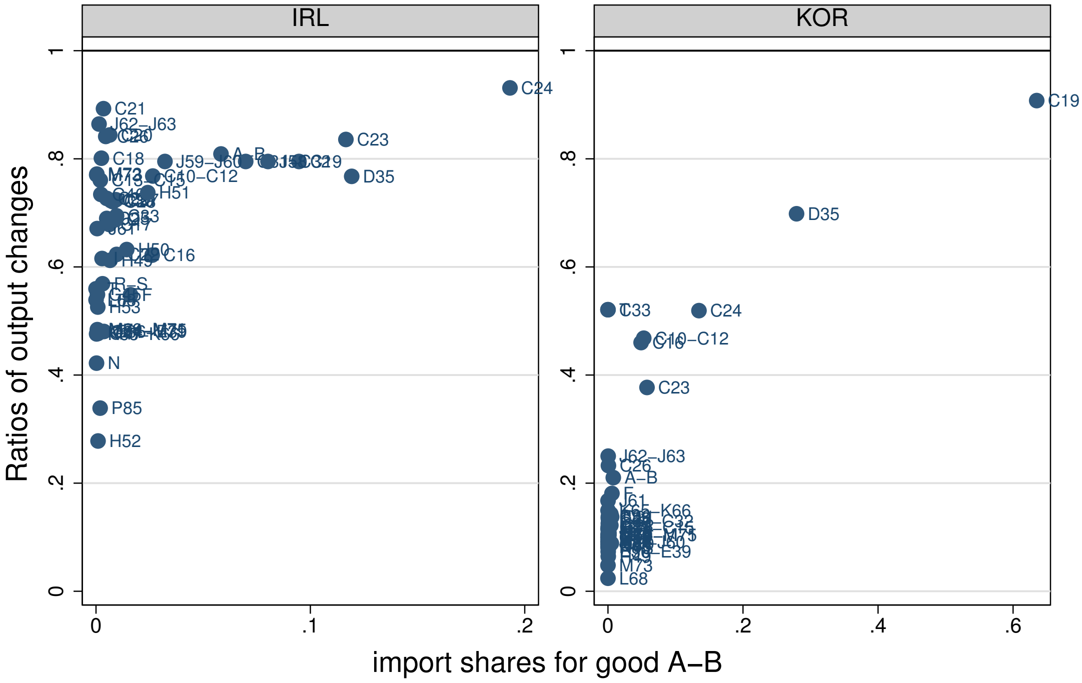
Figure 3.13: Comparison of the import price shock effect across models

In comparison to the reference line[51] of Figure 3.13, the model without input-output linkages produces more negative results since each industry is not only affected by the increase in its own import costs but also by the total imports and subsequent GDP drop [Equation (full_{pz})]. But since the transmission of the aggregate shock is minor, the fit line is found not far from the 45-degree one. In Ireland, in particular, Auxiliary NIO's fit line appears as a downward shit, reflecting that the overall higher values of total import shares are widespread in those industries.

Moreover, the differences between the full model and AuxNIO demonstrate that ignoring domestic linkages would result in large underestimations of the output changes

[51]Appendix 3.C.2 describes why import shares of good A-B are the appropriate measures to contrast with the industry output changes.

for the vast majority of Korean but also for several Irish industries. Naturally, the full model predicts an even larger industry output fall, as each industry is also affected by the negative impact undergone by other industries. Even those which do not import directly any quantity of good A-B may get significantly affected by the shock via input-output linkages.



Note: the ratios depict how much excluding domestic linkages results in an underestimation of the industry output changes. In a sparse domestic network like the Irish, the ratios are closer to one than in the case of the Korean well-connected network.

Figure 3.14: Ratio of output changes of Auxiliary NIO over Full model

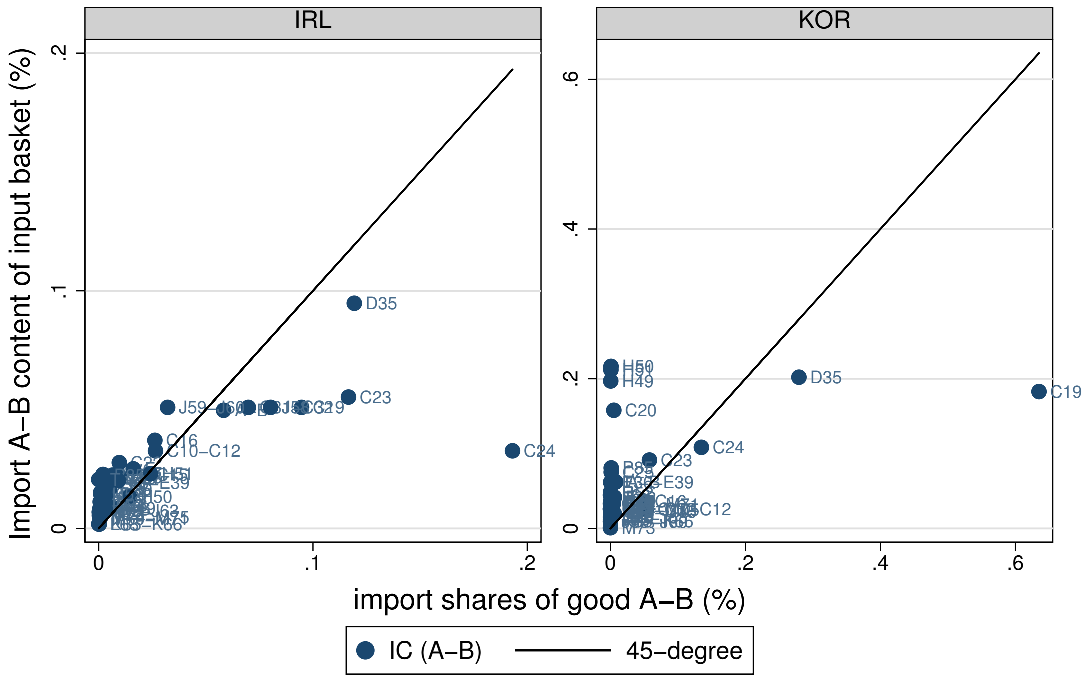
The ratios of the predictions of AuxNIO over those of the full model, plotted in Figure 3.14, portray how much ignoring domestic linkages would affect the calculated import price shock effect. The reference line at the value of one represents no underestimation of the AuxNIO model for the industry output change. The plots reinforce that the industries having the lowest import shares of good A-B are largely underestimated by the model without linkages.

But although these underestimations are minor, ranging from 0.2 to 1 of the predictions produced by the full model in Ireland, the errors are much larger in Korea, with the majority of the ratios lying between 0 and 0.2. These differences between the two countries clearly reflect their unequal production network, with the larger underestimations being produced for the most connected economy.

First-order linkage effects: direct input purchases

I evaluate here whether the direct input purchases of the industries can explain the relay of the shock across industries. Figure 3.15 plots the A-B content in the imported input purchases each industry, as described by Equation (IC).

The plot shows that the weighted average of A-B import shares is higher than one industry's own for the industries which import very little of that good. In particular, the most underestimated industries in Korea with zero import shares of good A-B have a relatively large content of that good in their input baskets. In Ireland, however, this measure does not fully explain the difference between the models since the most underestimated impacts are not among the highest values. On the other hand, as expected, the least underestimated industries which are also the biggest importers of good A-B naturally have little A-B import content on their inputs basket compared to their own AB import shares.



Note: the import content of each industry input basket is calculated via Equation (IC).

Figure 3.15: Intensity of A-B imports on domestic input basket

Higher-order linkage effects: shocks' approximations

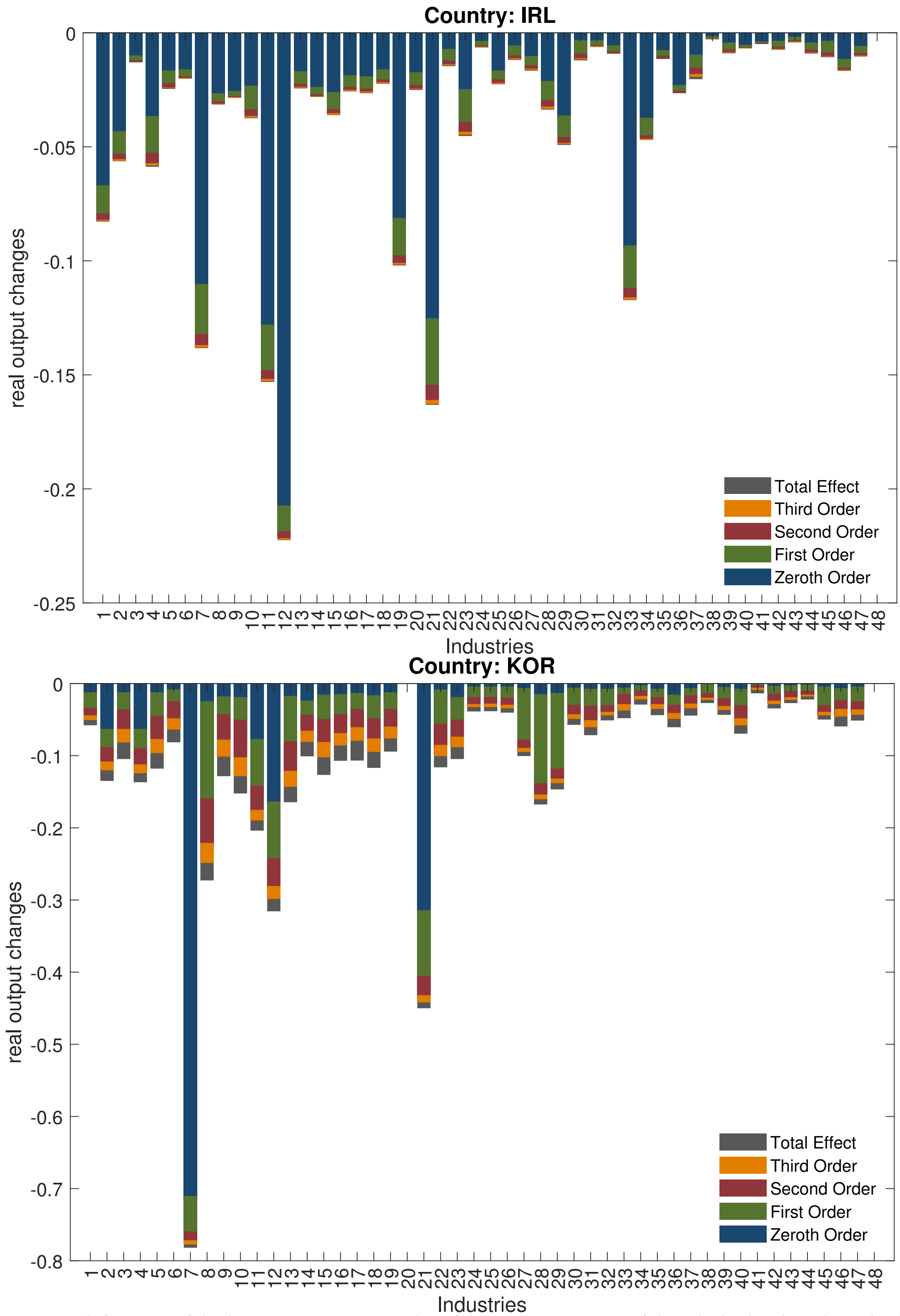
To assess whether other levels of interconnections explain the gaps left by the direct transactions, I explore the decomposition of the effects of the import price shock, as presented in Section 2.5.

Figure 3.16 plots the whole shock overlaid by the third to zeroth order of approximation. It is clear that in the case of the less connected Ireland, the shock vector alone explains most of the output effect of the import price shock, with the second and higher orders being almost irrelevant. In Korea, the well-connected production network translates into greater significance of the first to third orders, while still leaving some of the shock effects to be explained by the higher ranks of interactions.[52]

More remarkably, the zeroth-order approximation represents on average 67% of the whole shock in Ireland and astonishing 19% in Korea. The boosting of the industry effect of the import price shock caused by input-output linkages leaves no doubt that such feature in a model studying trade shocks. This is particularly true in economies with dense production networks, like Korea.[53]

[52] Regarding the industries, in particular, Figure 3.16 shows that the most underestimated industries, H52 (No 30) in Ireland and L68 (No 38) in Korea, are those for which the impact is the least explained by the shock vector alone (zeroth order). In the case of the former, it remains high in the raking (third place) of the least explained even by the third order. The latter gets most of the difference explained by the first-order approximation, when it jumps from the first to the 32nd place in the ranking. On the other hand, heavy A-B importers industries, for instance, C24 (No 12) in Ireland and C19 (No 7) in Korea, are not much upstream connected, repeatedly being the most explained industries (48th in the rankings) in all approximations. Moreover, as Figure 3.B.11 shows, although industry C24 is very little downstream connected in Ireland, other heavy A-B importers are, namely industries D35 and C25. In Korea, the two biggest A-B importers, i.e. industries C19 and D35, present high downstream centrality measures.

[53] With respect to the scales of the plots of Figure 3.16, one may worry that the impact of the shock is overall much larger for Korean industries and that this may be connected to the fact that, as Figure 3.8 shows, the Katz-Bonacich centrality of import good A-B is much bigger for that country (2.8) than for Ireland (0.3). It could be the case that the results are being biased by this discrepancy so, alternatively, I choose good C10-C12 for a test. Although it is much less significant, it has closer values in the pair of countries: 0.08 and 0.07, respectively. The results of the approximations plotted in Figure 3.C.3 in the Appendix are very close to those obtained when rising the price of good A-B. I conclude therefore that the choice of the shocked imported good does not affect the analysis.



Note: each fraction of the bars represents an order of the approximation of the whole shock as described in Section 2.5. The zeroth order is equivalent to the output changes predicted by the model ignoring domestic input-output linkages while the total effect represents the predictions of the full model. In this way, the extent of the bars beyond the zeroth order depicts how much the production network boosts the effect of the shock for each industry.

Figure 3.16: Decomposition of the A-B import price shock for Ireland and Korea

5 Conclusions

Aggregate labour and trade shocks can have varying effects across industries depending on the structure of the production network. In this chapter, I have developed a simple input-output model with an international dimension with focus on the analysis of the propagation of macro shocks.

I have provided the means to analyse and empirically determine that input-output linkages and industry trade patterns matter in the study of economic shocks. In the model, trade buffers the labour shock and boosts the import price shock. The countries selected to illustrate the counterfactuals were Ireland and South Korea; the former has a relatively disconnected production network and relies heavily on imported inputs while the latter presents a dense network and much fewer imported inputs.

I have shown that the indirect flows of transactions may be relevant up to the third degree, which represents chains of interactions or paths having two intermediaries between a pair of consumer/supplier industries. Higher-order effects are the most significant for the industries more upstream-connected into the network, i.e. which relies the most on domestic inputs summing up across all stages of production.

Moreover, I have demonstrated that the relevance of indirect transactions varies across shocks reflecting the fact that the magnitude of the initial or isolated effect on each industry is a key part of the shock transmission to other industries. The rational is that what matters to the final output change of an industry is not only how much it relies on inputs overall but also how much the industries it directly and indirectly depends upon are initially affected by the shock.

Korea, having a densely connected production network with industries that import little inputs, is where the shocks are the most persistent, with much still to be explained after the third order of interactions. Besides, the isolated shock felt by each industry represents only about half of the full output change resulting from the labour supply shock. The results for the import price shock counterfactual are even more telling. Since only a couple of industries rely directly on the shocked good, the isolated effect is less than 20% for most industries,[54] with the whole industry output change resulting from domestic input transactions.

[54] Recall that the isolated effect of the import price shock given by the shock vector also includes the output change resulting from the GDP fall.

The results are strikingly different for Ireland. The sparse production network and heavily importing industries create a scenario where about 80% of the labour shock and 70% of the import price shock is fully explained by the isolated effects on the industries. Furthermore, the inclusion of the direct transactions makes up about 90% of the total industry output change, leaving little to be explained by the higher-order interactions.

Finally, contrasting the results of Ireland and Korea establishes the relevance of knowing the structures of the production network of the economies when studying the micro effects of macro shocks. In the case of the labour shock, ignoring the several stages of production leads to misestimations of the output changes 1.51 times larger in Korea than in Ireland on average. Regarding the import price shock, this ratio is even larger, of 3.52 times.[55]

5.1 Limitations and extensions

The use of the WIOD or any input-output data has the natural limitation of working with technical coefficients that are fixed by construction.[56] Clearly, Leontief production functions are in general unrealistic so that the adoption of Cobb-Douglas specifications became common practice in neoclassical input-output models. This functional form appeared as a suitable choice for its well-known equilibrium solutions which establishes that payments to inputs and factors are fixed shares of total sales therefore invariant to shocks' realisations. This allows for the direct use of the technical coefficients as given by the input-output matrix while still leaving room for some substitution between the resources used in the production.

Nevertheless, adopting a Cobb-Douglas technology imposes a unitary elasticity of substitution across inputs and factors. This property clashes with the empirical literature indicating varying degrees of complementarity and substitutability between labour, capital, domestic and imported intermediate goods.[57] Moreover, the use of

[55] I refer here to the ratios of the average proportion of the whole shock explained by the zeroth-order approximatation of the labour supply shock — of 0.79% in Ireland and 0.53% in Korea— and of the import price shock —of 67% in Ireland and 19% in Korea.

[56] See Section 2.2.2 of Miller and Blair (2009) for more details.

[57] Duffy, Papageorgiou, and Perez-Sebastian (2004) produce a comprehensive estimation of capital-labour elasticities for a large set of countries over more than two decades. The empirical literature on input substitution is usually restricted to particular commodities —for example, energy (Koetse, de Groot, and Florax, 2008) and information technology (Dewan and Min, 1997)— or particular sectors —for instance, microfinance institutions (Hartarska, Shen, and Mersland, 2013; Stiroh, 1999).

the Cobb-Douglas is not without consequences for the results regarding the propagation patterns via input-output linkages. As discussed by Carvalho and Tahbaz-Salehi (2019), this specification leads to the analytical solutions accounting solely for the shock transmission through the production network while richer structures accounting for substitutabilities and complementarities include the effects via the reallocation of resources

For these reasons, the use of the constant elasticity of substitution (CES) production functions has been proposed as an alternative to the Cobb-Douglas (Carvalho, Nirei, Saito, and Tahbaz-Salehi, 2016; Baqaee and Farhi, 2018). The estimation of the elasticities of substitutions across inputs and between those and factors of production, however, is not trivial even for closed-economy models. As discussed in Baqaee and Farhi (2018), the identification strategies require relative strong assumptions and the availability of data conveying industry-level exogenous shocks.

On a higher level, a limitation of the international dimension of the model developed in this chapter is the omission of industry exports. It does not compromise the results presented here since the shocks studied apply directly to the industries and, given the Cobb-Douglas specification, travel only upstream towards other domestic industries, whose output changes are the aim of the analysis. The study of demand-side shocks or the adoption of CES production functions would require a multi-country multi-sector network model, in the fashion of Vandenbussche, Connell, and Simons (2019) except that considering a general equilibrium closure. The implementation of a global input-output model would also allow for the study of shock propagations via geographic linkages, which are also attracting attention recently. Acemoglu, Akcigit, and Kerr (2016) find that input-output as well as geographic networks play a role in the amplification of micro-shocks and see the investigation of these linkages in the propagation of macro-shocks as an understudied area. Overall, the integration of domestic linkages, networks, and trade remains an interesting and underexplored field to which this chapter provides a useful contribution.

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3.A Model's derivations

I present here the detailed algebraic derivations of the model for completeness and verification.

3.A.1 Irrelevance of intermediate inputs in aggregate terms

Intermediate inputs are irrelevant in aggregate terms because they are only transfers between industries which are cancelled out in the total summation. Ignoring measurement errors, nominal value-added GDP (income) should equal final nominal demand (expenditure), i.e. $GDP = w_K K + w_E E = Y - X = C$.

Value added GDP:

$$w_K K + w_E E = \sum_{i=1}^{n} p_i q_i - \sum_{i=1}^{n}\sum_{j=1}^{n} p_j d_{ji} - \sum_{i=1}^{n}\sum_{j=1}^{n} \bar{p}_j f_{ji} \tag{3.28}$$

Expenditure GDP:

$$Y = C + X = \sum_{i=1}^{n} p_i q_i - \sum_{i=1}^{n}\sum_{j=1}^{n} p_i d_{ij} \tag{3.29}$$

Since $C = w_K K + w_E E$ and $X = M = \sum_{i=1}^{n}\sum_{j=1}^{n} \bar{p}_j f_{ji}$, it must be true that:

$$\underbrace{\sum_{i=1}^{n} \underbrace{\sum_{j=1}^{n} p_j d_{ji}}_{\text{total usage of inputs by } i}}_{\text{summing across destinations } i} = \underbrace{\sum_{i=1}^{n} \underbrace{\sum_{j=1}^{n} p_i d_{ij}}_{\text{total inputs produced by } i}}_{\text{summing across origins } i} \tag{3.30}$$

Which is equivalent to total intermediate inputs used being equal to total intermediate inputs produced:

$$\underbrace{\sum_{i=1}^{n} \underbrace{\sum_{j=1}^{n} \gamma_{ji} p_i q_i}_{\text{summing across origins } j}}_{\text{summing across destinations } i} = \underbrace{\sum_{i=1}^{n} \underbrace{\sum_{j=1}^{n} \gamma_{ij} p_j q_j}_{\text{summing across destinations } j}}_{\text{summing across origins } i} \tag{3.31}$$

3.A.2 Proofs

Proof of Proposition 3.1. Industry output [58]

To compute the equilibrium solution for the industry output, I first have to derive the solution for the wages w_E and for the GDP Y. Starting with the logs of the FOCs for the factors and inputs in each industry, as given by Equations (3.9), (3.10), (3.11) and (3.12):

$$\log k_i = \log \alpha_i + \log(1-\gamma_i-\sigma_i) + \log p_i + \log q_i - \log w_K \tag{3.32}$$

$$\log e_i = \log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i) + \log p_i + \log q_i - \log w_E \tag{3.33}$$

$$\log d_{ji} = \log \gamma_{ji} + \log p_i + \log q_i - \log p_j \tag{3.34}$$

$$\log f_{ji} = \log \sigma_{ji} + \log p_i + \log q_i - \log \bar{p}_j \tag{3.35}$$

Substitute them into Equation (3.1) in logs:

$$\begin{aligned}
\log q_i &= \log A_i + (1-\gamma_i-\sigma_i)\left[\alpha_i \log k_i + (1-\alpha_i)\log e_i\right] + \\
&\quad + \textstyle\sum_{j=1}^n \gamma_{ji}\log d_{ji} + \sum_{j=1}^n \sigma_{ji}\log f_{ji} \\
&= \log A_i + \\
&\quad +(1-\gamma_i-\sigma_i)\alpha_i[\log\alpha_i + \log(1-\gamma_i-\sigma_i) + \log p_i + \log q_i - \log w_K] + \\
&\quad +(1-\gamma_i-\sigma_i)(1-\alpha_i)[\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i) + \log p_i + \log q_i - \log w_E] + \\
&\quad + \textstyle\sum_{j=1}^n \gamma_{ji}(\log\gamma_{ji} + \log p_i + \log q_i - \log p_j) + \\
&\quad + \textstyle\sum_{j=1}^n \sigma_{ji}(\log\sigma_{ji} + \log p_i + \log q_i - \log \bar{p}_j) \\
0 &= \log A_i + \log p_i + (1-\gamma_i-\sigma_i)\log(1-\gamma_i-\sigma_i) + \\
&\quad +(1-\gamma_i-\sigma_i)\alpha_i[\log\alpha_i - \log w_K] + \\
&\quad +(1-\gamma_i-\sigma_i)(1-\alpha_i)[\log(1-\alpha_i) - \log w_E] + \\
&\quad + \textstyle\sum_{j=1}^n \gamma_{ji}(\log\gamma_{ji} - \log p_j) + \\
&\quad + \textstyle\sum_{j=1}^n \sigma_{ji}(\log\sigma_{ji} - \log \bar{p}_j) \\
0 &= \log A_i + \log p_i + (1-\gamma_i-\sigma_i)\log(1-\gamma_i-\sigma_i) + \\
&\quad +(1-\gamma_i-\sigma_i)\alpha_i\log\alpha_i - (1-\gamma_i-\sigma_i)\alpha_i\log w_K + \\
&\quad +(1-\gamma_i-\sigma_i)(1-\alpha_i)\log(1-\alpha_i) - (1-\gamma_i-\sigma_i)(1-\alpha_i)\log w_E + \\
&\quad + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^n \gamma_{ji}\log p_j + \\
&\quad + \textstyle\sum_{j=1}^n \sigma_{ji}\log\sigma_{ji} - \sum_{j=1}^n \sigma_{ji}\log \bar{p}_j
\end{aligned}$$

[58] Derivations adapted from Fadinger, Ghiglino, and Teteryatnikova (2016).

Solve for $(1-\alpha_i)\log w_E$:

$$\begin{aligned}(1-\alpha_i)\log w_E &= \alpha_i \log \alpha_i + (1-\alpha_i)\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i)\\ &-\alpha_i \log w_K\\ &+\tfrac{1}{1-\gamma_i-\sigma_i}\left[\log A_i + \log p_i + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^n \gamma_{ji}\log p_j\right]\\ &+\tfrac{1}{1-\gamma_i-\sigma_i}\left[\textstyle\sum_{j=1}^n \sigma_{ji}\log\sigma_{ji} - \sum_{j=1}^n \sigma_{ji}\log \bar{p}_j\right]\end{aligned}$$

Substitute for w_K using Equation (3.18) in logs:

$$\begin{aligned}(1-\alpha_i)\log w_E &= \alpha_i \log \alpha_i + (1-\alpha_i)\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i)\\ &-\alpha_i \log \alpha - \alpha_i \log C + \alpha_i \log K\\ &+\tfrac{1}{1-\gamma_i-\sigma_i}\left[\log A_i + \log p_i + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^n \gamma_{ji}\log p_j\right]\\ &+\tfrac{1}{1-\gamma_i-\sigma_i}\left[\textstyle\sum_{j=1}^n \sigma_{ji}\log\sigma_{ji} - \sum_{j=1}^n \sigma_{ji}\log \bar{p}_j\right]\end{aligned}$$

Solving Equation (3.19) for C and taking logs gives $\log C = \log w_E + \log E - \log(1-\alpha)$, which I sub in now to get:

$$\begin{aligned}(1-\alpha_i)\log w_E &= \alpha_i \log \alpha_i + (1-\alpha_i)\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i)\\ &-\alpha_i \log \alpha - \alpha_i \log w_E - \alpha_i \log E + \alpha_i \log(1-\alpha) + \alpha_i \log K\\ &+\tfrac{1}{1-\gamma_i-\sigma_i}\left[\log A_i + \log p_i + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^n \gamma_{ji}\log p_j\right]\\ &+\tfrac{1}{1-\gamma_i-\sigma_i}\left[\textstyle\sum_{j=1}^n \sigma_{ji}\log\sigma_{ji} - \sum_{j=1}^n \sigma_{ji}\log \bar{p}_j\right]\end{aligned}$$

Solving for $\log w_E$:

$$\begin{aligned}\log w_E &= \alpha_i \log \alpha_i + (1-\alpha_i)\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i)\\ &-\alpha_i \log \alpha - \alpha_i \log E + \alpha_i \log(1-\alpha) + \alpha_i \log K\\ &+\tfrac{1}{1-\gamma_i-\sigma_i}\left[\log A_i + \log p_i + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^n \gamma_{ji}\log p_j\right]\\ &+\tfrac{1}{1-\gamma_i-\sigma_i}\left[\textstyle\sum_{j=1}^n \sigma_{ji}\log\sigma_{ji} - \sum_{j=1}^n \sigma_{ji}\log \bar{p}_j\right]\end{aligned}$$

Now, consider the vector $\mu'\mathbf{Z}$ where μ is the $(nX1)$ vector of multipliers and $\mathbf{Z}$ is a diagonal matrix with $Z_{ii} = 1-\gamma_i-\sigma_i$ such that $\mu'\mathbf{Z} = \beta'[\mathbf{I}-\mathbf{\Gamma}']^{-1}\cdot\mathbf{Z}$. Take the i^{th}

element of this matrix and multiply by the equation above.

$$
\begin{aligned}
\mu_i(1-\gamma_i-\sigma_i)\log w_E &= \mu_i(1-\gamma_i-\sigma_i)\left[\alpha_i\log\alpha_i+(1-\alpha_i)\log(1-\alpha_i)+\log(1-\gamma_i-\sigma_i)\right] \\
&+\mu_i(1-\gamma_i-\sigma_i)\left[-\alpha_i\log\alpha-\alpha_i\log E+\alpha_i\log(1-\alpha)+\alpha_i\log K\right] \\
&+\mu_i\left[\log A_i+\log p_i+\textstyle\sum_{j=1}^n\gamma_{ji}\log\gamma_{ji}-\sum_{j=1}^n\gamma_{ji}\log p_j\right] \\
&+\mu_i\log A_i+\mu_i\log p_i+\mu_i\textstyle\sum_{j=1}^n\gamma_{ji}\log\gamma_{ji}-\mu_i\sum_{j=1}^n\gamma_{ji}\log p_j \\
&+\mu_i\textstyle\sum_{j=1}^n\sigma_{ji}\log\sigma_{ji}-\mu_i\sum_{j=1}^n\sigma_{ji}\log\bar{p}_j
\end{aligned}
$$

Summing over all industries i:

$$
\begin{aligned}
\textstyle\sum_{i=1}^n\mu_i(1-\gamma_i-\sigma_i)\log w_E &= \textstyle\sum_{i=1}^n\mu_i(1-\gamma_i-\sigma_i)\left[\alpha_i\log\alpha_i+(1-\alpha_i)\log(1-\alpha_i)\right] \\
&+\textstyle\sum_{i=1}^n\mu_i(1-\gamma_i-\sigma_i)\left[\log(1-\gamma_i-\sigma_i)-\alpha_i\log\alpha\right] \\
&+\textstyle\sum_{i=1}^n\mu_i(1-\gamma_i-\sigma_i)\left[\alpha_i\log(1-\alpha)\right] \\
&+\textstyle\sum_{i=1}^n\mu_i(1-\gamma_i-\sigma_i)\left[\alpha_i\log K-\alpha_i\log E\right] \\
&+\textstyle\sum_{i=1}^n\mu_i\log A_i+\sum_{i=1}^n\mu_i\sum_{j=1}^n\gamma_{ji}\log\gamma_{ji}+\sum_{i=1}^n\mu_i\sum_{j=1}^n\sigma_{ji}\log\sigma_{ji} \\
&+\textstyle\sum_{i=1}^n\mu_i\log p_i-\sum_{i=1}^n\mu_i\sum_{j=1}^n\gamma_{ji}\log p_j-\sum_{i=1}^n\mu_i\sum_{j=1}^n\sigma_{ji}\log\bar{p}_j
\end{aligned}
\tag{3.36}
$$

The only endogenous variables left now are the good prices. First, I combine the terms $\sum_{i=1}^n\mu_i\log p_i$ and $-\sum_{i=1}^n\mu_i\sum_{j=1}^n\gamma_{ji}\log p_j$ using Equation (3.21) to get an expression in terms of p_i:

$$
\begin{aligned}
\textstyle\sum_{i=1}^n\mu_i\log p_i-\sum_{i=1}^n\mu_i\sum_{j=1}^n\gamma_{ji}\log p_j &= \\
\textstyle\sum_{i=1}^n\mu_i\log p_i-\sum_{j=1}^n\log p_j\sum_{i=1}^n\mu_i\gamma_{ji} &= \\
\textstyle\sum_{i=1}^n\mu_i\log p_i-\sum_{j=1}^n\log p_j(\mu_j-\beta_j) &= \\
\textstyle\sum_{i=1}^n\mu_i\log p_i-\sum_{i=1}^n\log p_i(\mu_i-\beta_i) &= \textstyle\sum_{i=1}^n\beta_i\log p_i
\end{aligned}
$$

Then, I solve Equation (3.13) for y_i to get $y_i=\beta_iY/p_i$ which I substitute into Equation (3.3) to get an expression of p_i in terms of β_i as follows:

$$
\begin{aligned}
Y &= \textstyle\prod_{i=1}^n\left(\beta_i\frac{Y}{p_i}\right)^{\beta_i}= \\
&= Y\textstyle\prod_{i=1}^n\left(\frac{\beta_i}{p_i}\right)^{\beta_i}\Leftrightarrow \\
1 &= \textstyle\prod_{i=1}^n\left(\frac{\beta_i}{p_i}\right)^{\beta_i}= \\
&= \frac{\prod_{i=1}^n{\beta_i}^{\beta_i}}{\prod_{i=1}^n{p_i}^{\beta_i}}\Leftrightarrow \\
\textstyle\prod_{i=1}^n{p_i}^{\beta_i} &= \textstyle\prod_{i=1}^n{\beta_i}^{\beta_i}
\end{aligned}
$$

In logs:

$$\sum_{i=1}^{n} \beta_i \log p_i = \sum_{i=1}^{n} \beta_i \log \beta_i \tag{3.37}$$

Substituting back into Equation (3.36) and solving for $\log w_E$:

$$\begin{aligned}
\log w_E \quad = \quad & \left[\frac{\sum_i \mu_i(1-\gamma_i-\sigma_i)\alpha_i}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\right] \log K - \log \alpha \\
& + \left[\frac{\sum_i \mu_i(1-\gamma_i-\sigma_i)(-\alpha_i)}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\right] \log E - \log(1-\alpha) \\
& + \frac{\sum_i \mu_i(1-\gamma_i-\sigma_i)[\alpha_i \log \alpha_i + (1-\alpha_i)\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i)]}{\sum_i \mu_i(1-\gamma_i-\sigma_i)} \\
& + \left[\frac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\right] \textstyle\sum_i \mu_i \log A_i + \sum_i \mu_i \sum_{j=1}^n \gamma_{ji} \log \gamma_{ji} \\
& + \left[\frac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\right] \textstyle\sum_i \mu_i \sum_{j=1}^n \sigma_{ji} \log \sigma_{ji} \\
& + \left[\frac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\right] \textstyle\sum_i \beta_i \log \beta_i - \sum_i \mu_i \sum_{j=1}^n \sigma_{ji} \log \bar{p}_j
\end{aligned} \tag{3.38}$$

The ratios in square brackets can be further simplified by studying the model's parameters. Starting from the resource constraint or GDP by production:

$$\begin{aligned}
\sum_i p_i q_i - \sum_i \sum_j p_i d_{ij} &= \sum_i p_i y_i && \text{from Equation (3.2)} \\
\sum_i p_i q_i - \sum_i \sum_j \gamma_{ij} p_j q_j &= \sum_i p_i y_i && \text{sub in Equation (3.11)} \\
\sum_i p_i q_i - \sum_j p_j q_j \sum_i \gamma_{ij} &= \sum_i p_i y_i && \text{manipulating the summation} \\
\sum_i p_i q_i - \sum_{j=1}^{n} \gamma_j p_j q_j &= \sum_i p_i y_i && \text{since } \gamma_j = \sum_i \gamma_{ij} \\
\sum_i p_i q_i - \sum_i \gamma_i p_i q_i &= \sum_i p_i y_i && \text{replacing the indices} \\
\sum_i p_i q_i - \sum_i \gamma_i p_i q_i &= Y && \text{since } \sum_i p_i y_i = PY \text{ and } P = 1
\end{aligned}$$

The formula for the GDP by production is identical to that in the model without trade developed in Chapter 2. Therefore, it is also true that:

$$\begin{aligned}
\frac{\sum_i p_i q_i}{Y} - \frac{\sum_i \gamma_i p_i q_i}{Y} &= \frac{Y}{Y} && \text{dividing by } Y \\
\sum_i \mu_i - \sum_i \gamma_i \mu_i &= 1 && \text{sub in Equation (3.20)}
\end{aligned}$$

In other words, the sum of the sales shares minus intermediate inputs equals one:

$$\sum_i \mu_i(1-\gamma_i) = 1 \tag{3.39}$$

Now expanding the GDP by expenditure:

$$\begin{aligned}
Y &= C + X && \text{Equation (3.4)}\\
Y &= C + \sum_{i=1}^{n}\sum_{j=1}^{n} \bar{p}_j f_{ji} && \text{sub in Equation (3.5)}\\
Y &= C + \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ji} p_i q_i && \text{sub in Equation (3.12)}\\
Y &= C + \sum_{i=1}^{n} \sigma_i p_i q_i && \text{since } \sigma_i = \sum_j \sigma_{ji}
\end{aligned}$$

And combining the two derivations for the GDP:

$$\begin{aligned}
\sum_i p_i q_i - \sum_i \gamma_i p_i q_i &= C + \sum_{i=1}^{n} \sigma_i p_i q_i \\
\frac{\sum_i p_i q_i}{Y} - \frac{\sum_i \gamma_i p_i q_i}{Y} &= \frac{C}{Y} + \frac{\sum_{i=1}^{n} \sigma_i p_i q_i}{Y} && \text{dividing by } Y\\
\sum_i \mu_i - \sum_i \gamma_i \mu_i &= \frac{C}{Y} + \sum_i \sigma_i \mu_i && \text{sub in Equation (3.20)}
\end{aligned}$$

Which can be rewritten as:

$$\frac{C}{Y} = \sum_i \mu_i(1-\gamma_i-\sigma_i) \tag{3.40}$$

Also, since $\sum_i \mu_i \sigma_i$ represents the total value of imports (or exports) over GDP, using Equation (3.39):

$$\sum_i \mu_i(1-\gamma_i-\sigma_i) = 1 - \sum_i \sigma_i \mu_i \tag{3.41}$$

To finally get the first two ratios of Equation (3.38), I take the following steps:

$$
\begin{aligned}
\frac{\alpha C}{K} &= \frac{\alpha_i(1-\gamma_i-\sigma_i)p_i q_i}{k_i} && \text{equate Equations (3.18) and (3.9)} \\
k_i \alpha C &= \alpha_i(1-\gamma_i-\sigma_i)p_i q_i K && \text{multiply both sides by } k_i \text{ and } K \\
\sum_i k_i \alpha C &= \sum_i \alpha_i(1-\gamma_i-\sigma_i)p_i q_i K && \text{sum over the n industries} \\
K \alpha C &= \sum_i \alpha_i(1-\gamma_i-\sigma_i)p_i q_i K && \text{sub in Equation (3.7)} \\
\alpha C &= \sum_i \alpha_i(1-\gamma_i-\sigma_i)p_i q_i && \text{divide both sides by } K \\
\alpha &= \sum_i \alpha_i(1-\gamma_i-\sigma_i)\frac{p_i q_i}{C} && \text{solve for } \alpha \\
\alpha &= \sum_i \alpha_i(1-\gamma_i-\sigma_i)\frac{p_i q_i}{Y}\frac{Y}{C} && \text{multiply RHS by } \frac{Y}{Y} \\
\alpha &= \sum_i \alpha_i(1-\gamma_i-\sigma_i)\mu_i \frac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)} && \text{sub in Equations (3.20) and (3.40)}
\end{aligned}
$$

Which leads to the first two ratios of Equation (3.38):

$$
\alpha = \frac{\sum_i \mu_i(1-\gamma_i-\sigma_i)\alpha_i}{\sum_i \mu_i(1-\gamma_i-\sigma_i)} \tag{3.42}
$$

Then, substituting Equation (3.42) into Equation (3.38), I find the solution for the wages:

$$
\begin{aligned}
\log w_E \;=\;& \alpha[\log K - \log \alpha] + (-\alpha)[\log E - \log(1-\alpha)] \\
&+ \tfrac{\sum_i \mu_i(1-\gamma_i-\sigma_i)[\alpha_i \log \alpha_i + (1-\alpha_i)\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i)]}{\sum_i \mu_i(1-\gamma_i-\sigma_i)} \\
&+ \left[\tfrac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\right] \sum_i \mu_i \log A_i + \sum_i \mu_i \sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} \\
&+ \left[\tfrac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\right] \sum_i \mu_i \sum_{j=1}^n \sigma_{ji}\log\sigma_{ji} \\
&+ \left[\tfrac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\right] \sum_i \beta_i \log\beta_i - \sum_i \mu_i \sum_{j=1}^n \sigma_{ji}\log\bar{p}_j
\end{aligned} \tag{3.43}
$$

And analogously for rental on capital:

$$
\begin{aligned}
\log w_K \;=\;& (\alpha-1)(\log K - \log\alpha) + (1-\alpha)(\log E - \log(1-\alpha)) \\
&+ \tfrac{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)[\alpha_i \log \alpha_i + (1-\alpha_i)\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i)]}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)} \\
&+ \tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)} \left[\sum_{i=1}^n \mu_i \log A_i + \sum_{i=1}^n \mu_i \sum_{j=1}^n \gamma_{ji}\log\gamma_{ji}\right] \\
&+ \tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)} \left[\sum_{i=1}^n \mu_i \sum_{j=1}^n \sigma_{ji}\log\sigma_{ji}\right] \\
&+ \tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)} \left[\sum_{i=1}^n \beta_i \log\beta_i - \sum_{i=1}^n \mu_i \sum_{j=1}^n \sigma_{ji}\log\bar{p}_j\right]
\end{aligned} \tag{3.44}
$$

Which leads to the final step to derive the solution for the aggregate output:

$$
\begin{aligned}
Y &= \left(\frac{C}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\right) && \text{solving Equation (3.40) for } Y \\
&= \left(\frac{C}{1-\sum_i \sigma_i\mu_i}\right) && \text{sub in Equation (3.41)} \\
\log Y &= \log C - \log\left(1-\sum_i \sigma_i\mu_i\right) && \text{taking logs} \\
&= \log w_E + \log E - \log(1-\alpha) - \log\left(1-\sum_i \sigma_i\mu_i\right) && \text{sub in Equation (3.19)}
\end{aligned}
$$

Finally, subbing in Equation (3.43) into the expression above leads to the solution for the GDP:

$$
\begin{aligned}
\log Y \quad = \quad & \alpha(\log K - \log\alpha) + (1-\alpha)(\log E - \log(1-\alpha)) - \log(1-\textstyle\sum_i \sigma_i\mu_i) \\
& + \tfrac{\sum_i \mu_i(1-\gamma_i-\sigma_i)[\alpha_i\log\alpha_i + (1-\alpha_i)\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i)]}{\sum_i \mu_i(1-\gamma_i-\sigma_i)} \\
& + \tfrac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_i \mu_i \log A_i + \sum_i \mu_i \sum_j \gamma_{ji}\log\gamma_{ji}\right] \\
& + \tfrac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_i \mu_i \sum_j \sigma_{ji}\log\sigma_{ji}\right] \\
& + \tfrac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_i \beta_i\log\beta_i - \sum_i \mu_i\sum_j \sigma_{ji}\log\bar{p}_j\right]
\end{aligned}
\tag{3.45}
$$

Now, consider the industry output as given by Equation (3.1) in logs:

$$\begin{aligned}\log q_i &= \log A_i + (1-\gamma_i-\sigma_i)\left[\alpha_i \log k_i + (1-\alpha_i)\log e_i\right] \\ &+ \textstyle\sum_{j=1}^n \gamma_{ji}\log d_{ji} + \sum_{j=1}^n \sigma_{ji}\log f_{ji}\end{aligned} \tag{3.46}$$

First, simplify the terms within the square brackets by subbing in Equations (3.32) and (3.33) as follows:

$$\begin{aligned}&\alpha_i \log k_i + (1-\alpha_i)\log e_i = \\ &= \alpha_i\left[\log \alpha_i + \log(1-\gamma_i-\sigma_i) + \log \mu_i + \log Y - \log w_K\right] \\ &+(1-\alpha_i)\left[\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i) + \log \mu_i + \log Y - \log w_E\right] = \\ &= \alpha_i\left[\log \alpha_i - \log w_K\right] + (1-\alpha_i)\left[\log(1-\alpha_i) - \log w_E\right] \\ &+ \log(1-\gamma_i-\sigma_i) + \log \mu_i + \log Y\end{aligned}$$

Then, substitute back into Equation (3.46):

$$\begin{aligned}\log q_i &= \log A_i + (1-\gamma_i-\sigma_i)\left[\log(1-\gamma_i-\sigma_i) + \log \mu_i + \log Y\right] \\ &+(1-\gamma_i-\sigma_i)\left[\alpha_i(\log \alpha_i - \log w_K) + (1-\alpha_i)(\log(1-\alpha_i) - \log w_E)\right] \\ &+ \textstyle\sum_{j=1}^n \gamma_{ji}\log d_{ji} + \sum_{j=1}^n \sigma_{ji}\log f_{ji}\end{aligned} \tag{3.47}$$

Now take firm's i FOC for inputs as given by Equation (3.34) and manipulate as follow to express $\log d_{ji}$ in terms of $\log q_j$:

$$\begin{aligned}\log d_{ji} &= \log \gamma_{ji} + \log(p_i q_i) - \log p_j && \text{from Equation (3.34)} \\ &= \log \gamma_{ji} + \log \mu_i + \log Y - \log p_j && \text{sub in Equation (3.20) in logs} \\ &= \log \gamma_{ji} + \log \mu_i - \log \mu_j + \log q_j && \text{sub in Equation (3.20) in logs for good } j\end{aligned} \tag{3.48}$$

And take firm's i FOC for imports as given by Equation (3.35) and manipulate as follow to express $\log f_{ji}$ in terms of $\log Y$:

$$\begin{aligned}\log f_{ji} &= \log \sigma_{ji} + \log(p_i q_i) - \log \bar{p}_j && \text{from Equation (3.35)} \\ &= \log \sigma_{ji} + \log \mu_i + \log Y - \log \bar{p}_j && \text{sub in Equation (3.20) in logs}\end{aligned} \tag{3.49}$$

Sub in the expressions for $\log d_{ji}$ and $\log f_{ji}$ back into Equation (3.47):

$$\begin{aligned}
\log q_i \quad = \quad & \log A_i + (1-\gamma_i-\sigma_i)\left[\log(1-\gamma_i-\sigma_i) + \log\mu_i + \log Y\right] \\
& +(1-\gamma_i-\sigma_i)\left[\alpha_i(\log\alpha_i - \log w_K) + (1-\alpha_i)(\log(1-\alpha_i) - \log w_E)\right] \\
& + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} + \gamma_i\log\mu_i - \textstyle\sum_{j=1}^n \gamma_{ji}\log\mu_j + \textstyle\sum_{j=1}^n \gamma_{ji}\log q_j \\
& + \textstyle\sum_{j=1}^n \sigma_{ji}\log\sigma_{ji} + \sigma_i\log\mu_i + \sigma_i\log Y - \textstyle\sum_{j=1}^n \sigma_{ji}\log\bar{p}_j
\end{aligned}$$

Combining like terms:

$$
\begin{aligned}
\log q_i \quad = \quad & \log A_i + \log \mu_i + (1-\gamma_i)\log Y + (1-\gamma_i-\sigma_i)\log(1-\gamma_i-\sigma_i) \\
& +(1-\gamma_i-\sigma_i)\left[\alpha_i(\log\alpha_i - \log w_K) + (1-\alpha_i)(\log(1-\alpha_i) - \log w_E)\right] \\
& + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} + \sum_{j=1}^n \sigma_{ji}\log\sigma_{ji} \\
& - \textstyle\sum_{j=1}^n \sigma_{ji}\log\bar{p}_j - \sum_{j=1}^n \gamma_{ji}\log\mu_j + \sum_{j=1}^n \gamma_{ji}\log q_j
\end{aligned}
$$

Finally, sub in the equilibrium values of w_E, w_K and Y as respectively given by Equations (3.43), (3.44) and (3.45):

$$
\begin{aligned}
\log q_i \quad = \quad & \log A_i + \log \mu_i + (1-\gamma_i-\sigma_i)\log(1-\gamma_i-\sigma_i) \\
& +(1-\gamma_i-\sigma_i)\left[\alpha_i\log\alpha_i + (1-\alpha_i)\log(1-\alpha_i)\right] \\
& +(1-\gamma_i)\log Y \\
& -(1-\gamma_i-\sigma_i)\alpha_i\log w_K \\
& -(1-\gamma_i-\sigma_i)(1-\alpha_i)\log w_E \\
& + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} + \sum_{j=1}^n \sigma_{ji}\log\sigma_{ji} \\
& - \textstyle\sum_{j=1}^n \sigma_{ji}\log\bar{p}_j - \sum_{j=1}^n \gamma_{ji}\log\mu_j + \sum_{j=1}^n \gamma_{ji}\log q_j \\
= \quad & \log A_i + \log \mu_i + +(1-\gamma_i-\sigma_i)\log(1-\gamma_i-\sigma_i) \\
& +(1-\gamma_i-\sigma_i)\left[\alpha_i\log\alpha_i + (1-\alpha_i)\log(1-\alpha_i)\right] \\
& -(1-\gamma_i)\log\left(1-\textstyle\sum_{i=1}^n \sigma_i\mu_i\right) \\
& +(1-\gamma_i)\left[\alpha(\log K - \log\alpha) + (1-\alpha)(\log E - \log(1-\alpha))\right] \\
& -(1-\gamma_i-\sigma_i)\alpha_i\left[(\alpha-1)(\log K - \log\alpha) + (1-\alpha)(\log E - \log(1-\alpha))\right] \\
& -(1-\gamma_i-\sigma_i)(1-\alpha_i)\left[\alpha(\log K - \log\alpha) + (-\alpha)(\log E - \log(1-\alpha))\right] \\
& +\sigma_i\tfrac{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)[\alpha_i\log\alpha_i + (1-\alpha_i)\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i)]}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)} \\
& +\sigma_i\tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \mu_i\log A_i + \sum_{i=1}^n \mu_i\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji}\right] \\
& +\sigma_i\tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \mu_i\sum_{j=1}^n \sigma_{ji}\log\sigma_{ji}\right] \\
& +\sigma_i\tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \beta_i\log\beta_i - \sum_{i=1}^n \mu_i\sum_{j=1}^n \sigma_{ji}\log\bar{p}_j\right] \\
& + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} + \sum_{j=1}^n \sigma_{ji}\log\sigma_{ji} \\
& - \textstyle\sum_{j=1}^n \sigma_{ji}\log\bar{p}_j - \sum_{j=1}^n \gamma_{ji}\log\mu_j + \sum_{j=1}^n \gamma_{ji}\log q_j
\end{aligned}
$$

$$
\begin{aligned}
\log q_i \quad = \quad & \log A_i + \log \mu_i + \\
& (1-\gamma_i-\sigma_i)\left[\alpha_i \log \alpha_i + (1-\alpha_i)\log(1-\alpha_i)\right] \\
& -(1-\gamma_i)\log\left(1-\textstyle\sum_{i=1}^n \sigma_i \mu_i\right) \\
& +\sigma_i\left[\alpha(\log K - \log \alpha) + (1-\alpha)(\log E - \log(1-\alpha))\right] \\
& +(1-\gamma_i-\sigma_i)\alpha_i(\log K - \log \alpha) \\
& +(1-\gamma_i-\sigma_i)(1-\alpha_i)(\log E - \log(1-\alpha)) \\
& +\sigma_i \tfrac{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)\left[\alpha_i \log \alpha_i + (1-\alpha_i)\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i)\right]}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)} \\
& +\sigma_i \tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \mu_i \log A_i + \sum_{i=1}^n \mu_i \sum_{j=1}^n \gamma_{ji}\log \gamma_{ji}\right] \\
& +\sigma_i \tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \mu_i \sum_{j=1}^n \sigma_{ji}\log \sigma_{ji}\right] \\
& +\sigma_i \tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \beta_i \log \beta_i - \sum_{i=1}^n \mu_i \sum_{j=1}^n \sigma_{ji}\log \bar{p}_j\right] \\
& +(1-\gamma_i-\sigma_i)\log(1-\gamma_i-\sigma_i) + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} + \sum_{j=1}^n \sigma_{ji}\log \sigma_{ji} \\
& -\textstyle\sum_{j=1}^n \sigma_{ji}\log \bar{p}_j - \sum_{j=1}^n \gamma_{ji}\log \mu_j + \sum_{j=1}^n \gamma_{ji}\log q_j \\
= \quad & \log A_i + \log \mu_i + (1-\gamma_i-\sigma_i)\log(1-\gamma_i-\sigma_i) - (1-\gamma_i)\log\left(1-\textstyle\sum_{i=1}^n \sigma_i\mu_i\right) \\
& +\sigma_i\left[\alpha(\log K - \log \alpha) + (1-\alpha)(\log E - \log(1-\alpha))\right] \\
& +(1-\gamma_i-\sigma_i)\alpha_i(\log K - \log \alpha + \log \alpha_i) \\
& +(1-\gamma_i-\sigma_i)(1-\alpha_i)(\log E - \log(1-\alpha) + \log(1-\alpha_i)) \\
& +\sigma_i \tfrac{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)\left[\alpha_i \log \alpha_i + (1-\alpha_i)\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i)\right]}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)} \\
& +\sigma_i \tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \mu_i \log A_i + \sum_{i=1}^n \mu_i \sum_{j=1}^n \gamma_{ji}\log \gamma_{ji}\right] \\
& +\sigma_i \tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \mu_i \sum_{j=1}^n \sigma_{ji}\log \sigma_{ji}\right] \\
& +\sigma_i \tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \beta_i \log \beta_i - \sum_{i=1}^n \mu_i \sum_{j=1}^n \sigma_{ji}\log \bar{p}_j\right] \\
& +\textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} + \sum_{j=1}^n \sigma_{ji}\log \sigma_{ji} \\
& -\textstyle\sum_{j=1}^n \sigma_{ji}\log \bar{p}_j - \sum_{j=1}^n \gamma_{ji}\log \mu_j + \sum_{j=1}^n \gamma_{ji}\log q_j
\end{aligned}
$$

Combining the exogenous supply of factors leads to Equation (3.22). QED.

Proof of Corollary 3.1. Vector of industry output

Expressing Equation (3.22) in matrix form, with vector $\mathbf{V}$ being composed of all the constant terms prior to $\sum_{j=1}^{n} \gamma_{ji} \log q_j$:

$$\begin{bmatrix} \log q_1 \\ \log q_2 \\ \vdots \\ \log q_n \end{bmatrix} = V + \begin{bmatrix} \gamma_{11} & \gamma_{21} & \cdots & \gamma_{n1} \\ \gamma_{12} & \gamma_{22} & \cdots & \gamma_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{1n} & \gamma_{2n} & \cdots & \gamma_{nn} \end{bmatrix} \cdot \begin{bmatrix} \log q_1 \\ \log q_2 \\ \vdots \\ \log q_n \end{bmatrix}$$

Solving for the vector of the log of industry output leads to Equation (3.23). QED.

Proof of Corollary 3.2. Output effect of a labour shock

Rewriting (3.22) considering only the variables affected makes it easy to see the relevant elements of vector **V**:

$$\log q_i = \log E\left[(1 - \gamma_i - \sigma_i)(1 - \alpha_i) + \sigma_i(1 - \alpha)\right] + \sum_{j=1}^{n} \gamma_{ji} \log q_j$$

Deriving Equation (3.23) with respect to E leads to Equation (full$_{ES}$). QED.

Proof of Corollary 3.3. Output effect Aux0 model

Letting all γ_{ji} and σ_{ji} equal zero in Equation (full$_{ES}$) leads to Equation (Aux0$_{ES}$). QED.

Proof of Corollary 3.4. Output effect AuxNT model

Setting all σ_{ji} to zero in Equation (full$_{ES}$) leads to Equation (AuxNT$_{ES}$). QED.

Proof of Corollary 3.5. Output effect of an import price shock

Rewriting (3.22) considering only the variables affected makes it easy to see the relevant elements of vector **V**:

$$\log q_i = -\sigma_i \frac{\sum_i \mu_i \sigma_{zi}}{\sum_i \mu_i (1 - \gamma_i - \sigma_i)} \log \bar{p}_z - \sigma_{zi} \log \bar{p}_z + \sum_j \gamma_{ji} \log q_j$$

Since $\sum_i \mu_i(1 - \gamma_i - \sigma_i) = C/Y$ [Equation (3.40)] and $\sum_i \mu_i \sigma_i = M/Y$ represents the share of total imports over GDP [Equation (3.41)], by extension, $\sum_i \mu_i \sigma_{zi}$ represents the aggregate share of good z imports. Thus, $\frac{\sum_i \mu_i \sigma_{zi}}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}$ stands for $\frac{M_z/Y}{C/Y} = M_z/C$, a constant share given by parameters.

Deriving Equation (3.23) with respect to p_z leads to Equation (full$_{pz}$). QED.

Proof of Corollary 3.6. Output effect AuxNIO model

Letting all γ_{ji} equal zero in Equation (full$_{pz}$) leads to Equation (AuxNIO$_{pz}$). QED.

3.A.3 General equilibrium solutions

Equilibrium Y

The equilibrium aggregate output is given by:

$$
\begin{aligned}
\log Y \quad = \quad & \alpha(\log K - \log \alpha) + (1-\alpha)(\log E - \log(1-\alpha)) - \log(1 - \textstyle\sum_i \sigma_i \mu_i) \\
& + \frac{\sum_i \mu_i(1-\gamma_i-\sigma_i)[\alpha_i \log \alpha_i + (1-\alpha_i)\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i)]}{\sum_i \mu_i(1-\gamma_i-\sigma_i)} \\
& + \frac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_i \mu_i \log A_i + \sum_i \mu_i \sum_j \gamma_{ji} \log \gamma_{ji}\right] \\
& + \frac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_i \mu_i \sum_j \sigma_{ji} \log \sigma_{ji}\right] \\
& + \frac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_i \beta_i \log \beta_i - \sum_i \mu_i \sum_j \sigma_{ji} \log \bar{p}_j\right]
\end{aligned} \tag{3.45}
$$

The solution clearly shows how the exogenous factor supplies K and E relate to the GDP through their respective nominal shares. Notice, however, that the Hulten (1978)'s theorem does not hold for this model with the non-domestic dimension. In other words, the effect on aggregate output Y of a one-percent change to the productivity of industry i does not equal solely the the industrial multiplier μ_i but is magnified by $\sum_i \mu_i(1-\gamma_i-\sigma_i)$, which is equivalent to the share of domestic consumption over GDP, i.e. excluding international trade.[59]

[59]Mathematically,

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log A_i} = \frac{\mu_i}{\sum_i \mu_i(1-\gamma_i-\sigma_i)} = \mu_i \frac{Y}{C} \tag{3.50}$$

Full derivation of Y including the share C/Y is available in Appendix 3.A.2.

Equilibrium w_K

$$\begin{aligned}
\log w_K \quad = \quad & (\alpha-1)(\log K-\log\alpha)+(1-\alpha)(\log E-\log(1-\alpha)) \\
& +\tfrac{\sum_i \mu_i(1-\gamma_i-\sigma_i)[\alpha_i\log\alpha_i+(1-\alpha_i)\log(1-\alpha_i)+\log(1-\gamma_i-\sigma_i)]}{\sum_i \mu_i(1-\gamma_i-\sigma_i)} \\
& +\tfrac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\left[\sum_i \mu_i\log A_i+\sum_i \mu_i\sum_j \gamma_{ji}\log\gamma_{ji}\right] \\
& +\tfrac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\left[\sum_i \mu_i\sum_j \sigma_{ji}\log\sigma_{ji}\right] \\
& +\tfrac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\left[\sum_i \beta_i\log\beta_i-\sum_i \mu_i\sum_j \sigma_{ji}\log\bar{p}_j\right]
\end{aligned} \tag{3.44}$$

Equilibrium w_E

$$\begin{aligned}
\log w_E \quad = \quad & \alpha(\log K-\log\alpha)+[(1-\alpha)-1](\log E-\log(1-\alpha)) \\
& +\tfrac{\sum_i \mu_i(1-\gamma_i-\sigma_i)[\alpha_i\log\alpha_i+(1-\alpha_i)\log(1-\alpha_i)+\log(1-\gamma_i-\sigma_i)]}{\sum_i \mu_i(1-\gamma_i-\sigma_i)} \\
& +\tfrac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\left[\sum_i \mu_i\log A_i+\sum_i \mu_i\sum_j \gamma_{ji}\log\gamma_{ji}\right] \\
& +\tfrac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\left[\sum_i \mu_i\sum_j \sigma_{ji}\log\sigma_{ji}\right] \\
& +\tfrac{1}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}\left[\sum_i \beta_i\log\beta_i-\sum_i \mu_i\sum_j \sigma_{ji}\log\bar{p}_j\right]
\end{aligned} \tag{3.43}$$

Factors usage

From the first order conditions of firm i's profit maximisation given by Equation (3.32):

$$\log k_i = \log\alpha_i+\log(1-\gamma_i-\sigma_i)+\log\mu_i+\log Y-\log w_K \tag{3.51}$$

Likewise, from Equation (3.33):

$$\log e_i = \log(1-\alpha_i)+\log(1-\gamma_i-\sigma_i)+\log\mu_i+\log Y-\log w_E \tag{3.52}$$

Demand for intermediate inputs and imports

From the first order conditions of firm i's profit maximisation given by Equation (3.34):

$$\log d_{ji} = \log\gamma_{ji}+\log\mu_i+\log Y-\log p_j \tag{3.48}$$

Likewise, from Equation (3.35):

$$\log f_{ji} = \log\sigma_{ji}+\log\mu_i+\log Y-\log\bar{p}_j \tag{3.49}$$

Real output by industry

$$\begin{aligned}
\log q_i \quad = \quad & \log K\left[\sigma_i\alpha + (1-\gamma_i-\sigma_i)\alpha_i\right] \\
& + \log E\left[\sigma_i(1-\alpha) + (1-\gamma_i-\sigma_i)(1-\alpha_i)\right] \\
& + \log A_i + \log\mu_i - (1-\gamma_i)\log\left(1-\textstyle\sum_{i=1}^n \sigma_i\mu_i\right) \\
& -\sigma_i\left[\alpha\log\alpha + (1-\alpha)\log(1-\alpha)\right] \\
& +(1-\gamma_i-\sigma_i)\alpha_i(\log\alpha_i - \log\alpha) \\
& +(1-\gamma_i-\sigma_i)(1-\alpha_i)(\log(1-\alpha_i) - \log(1-\alpha)) \\
& +\sigma_i\tfrac{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)[\alpha_i\log\alpha_i + (1-\alpha_i)\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i)]}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)} \\
& +\sigma_i\tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \mu_i\log A_i + \sum_{i=1}^n \mu_i\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji}\right] \\
& +\sigma_i\tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \mu_i\sum_{j=1}^n \sigma_{ji}\log\sigma_{ji}\right] \\
& +\sigma_i\tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \beta_i\log\beta_i - \sum_{i=1}^n \mu_i\sum_{j=1}^n \sigma_{ji}\log\bar{p}_j\right] \\
& +(1-\gamma_i-\sigma_i)\log(1-\gamma_i-\sigma_i) + \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} + \sum_{j=1}^n \sigma_{ji}\log\sigma_{ji} \\
& - \textstyle\sum_{j=1}^n \sigma_{ji}\log\bar{p}_j - \sum_{j=1}^n \gamma_{ji}\log\mu_j + \sum_{j=1}^n \gamma_{ji}\log q_j
\end{aligned} \tag{3.22}$$

Prices of goods/inputs

$$\begin{aligned}
\log p_i \quad = \quad & -\log A_i \\
& + \log K(1-\gamma_i-\sigma_i)(\alpha-\alpha_i) \\
& + \log E(1-\gamma_i-\sigma_i)\left[(1-\alpha)-(1-\alpha_i)\right] \\
& - \log\alpha(1-\gamma_i-\sigma_i)(\alpha-\alpha_i) \\
& - \log(1-\alpha)(1-\gamma_i-\sigma_i)\left[(1-\alpha)-(1-\alpha_i)\right] \\
& -(1-\gamma_i-\sigma_i)\alpha_i\log\alpha_i - (1-\gamma_i-\sigma_i)(1-\alpha_i)\log(1-\alpha_i) \\
& +(1-\gamma_i-\sigma_i)\tfrac{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)[\alpha_i\log\alpha_i + (1-\alpha_i)\log(1-\alpha_i) + \log(1-\gamma_i-\sigma_i)]}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)} \\
& +(1-\gamma_i-\sigma_i)\tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \mu_i\log A_i + \sum_{i=1}^n \mu_i\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji}\right] \\
& +(1-\gamma_i-\sigma_i)\tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \mu_i\sum_{j=1}^n \sigma_{ji}\log\sigma_{ji}\right] \\
& +(1-\gamma_i-\sigma_i)\tfrac{1}{\sum_{i=1}^n \mu_i(1-\gamma_i-\sigma_i)}\left[\textstyle\sum_{i=1}^n \beta_i\log\beta_i - \sum_{i=1}^n \mu_i\sum_{j=1}^n \sigma_{ji}\log\bar{p}_j\right] \\
& -(1-\gamma_i-\sigma_i)\log(1-\gamma_i-\sigma_i) - \textstyle\sum_{j=1}^n \gamma_{ji}\log\gamma_{ji} - \sum_{j=1}^n \sigma_{ji}\log\sigma_{ji} \\
& + \textstyle\sum_{j=1}^n \sigma_{ji}\log\bar{p}_j + \sum_{j=1}^n \gamma_{ji}\log p_j
\end{aligned} \tag{3.53}$$

3.A.4 General equilibrium effects of a change in E

Nominal and real aggregate output

From Equation (3.45), the effect on real output is given by aggregate labour income share:

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log E} = (1-\alpha) \tag{3.54}$$

Factor quantities and prices

Total differentiating Equation (3.51):

$$\frac{\mathrm{d}\log k_i}{\mathrm{d}\log E} = \frac{\mathrm{d}\log Y}{\mathrm{d}\log E} - \frac{\mathrm{d}\log w_K}{\mathrm{d}\log E} = (1-\alpha) - (1-\alpha) = 0 \tag{3.55}$$

Likewise for Equation (3.52):

$$\frac{\mathrm{d}\log e_i}{\mathrm{d}\log E} = \frac{\mathrm{d}\log Y}{\mathrm{d}\log E} - \frac{\mathrm{d}\log w_E}{\mathrm{d}\log E} = (1-\alpha) - (-\alpha) = 1 \tag{3.56}$$

From Equation (3.44)

$$\frac{\mathrm{d}\log w_K}{\mathrm{d}\log E} = (1-\alpha) \tag{3.57}$$

From Equation (3.43)

$$\frac{\mathrm{d}\log w_E}{\mathrm{d}\log E} = (-\alpha) \tag{3.58}$$

Total imports and exports

Since the ratio C/Y is given by parameters [Equation (3.40)], C grows by the same rate as Y, i.e. $(1-\alpha)$. As $Y = C + X$, X also increases by $(1-\alpha)$. Same follows for M since trade is balanced by assumption.

$$\frac{\mathrm{d}\log X}{\mathrm{d}\log E} = \frac{\mathrm{d}\log M}{\mathrm{d}\log E} = (1-\alpha) \tag{3.59}$$

Intermediate inputs and imports

From (3.48):

$$\frac{\mathrm{d}\log d_{ji}}{\mathrm{d}\log E} = \frac{\mathrm{d}\log q_j}{\mathrm{d}\log E} \tag{3.60}$$

From (3.49):

$$\frac{\mathrm{d}\log(f_{ji})}{\mathrm{d}\log E} = (1-\alpha) \tag{3.61}$$

The first order conditions of firm i's profit maximisation establishes that the value of each imported input $p_j f_{ji} = \sigma_{ji} p_i q_i$ will change as much as nominal output $p_i q_i$ since σ_{ji} is constant. Nominal output $p_i q_i$ varies symmetrically for every industry i by the same rate of Y [Equation (3.20)]. Since international prices $\bar{p}_j$ are exogenous, therefore not affected by E, real imports f_{ji} varies with GDP.

Real industry output

$$\begin{bmatrix} \frac{\mathrm{d}\log q_1}{\mathrm{d}\log E} \\ \frac{\mathrm{d}\log q_2}{\mathrm{d}\log E} \\ \vdots \\ \frac{\mathrm{d}\log q_n}{\mathrm{d}\log E} \end{bmatrix} = \underbrace{\left[\mathbf{I}-\mathbf{\Gamma}'\right]^{-1}}_{\text{input-output linkages}} \cdot \underbrace{\begin{bmatrix} (1-\gamma_1-\sigma_1)(1-\alpha_1)+\sigma_1(1-\alpha) \\ (1-\gamma_2-\sigma_2)(1-\alpha_2)+\sigma_2(1-\alpha) \\ \vdots \\ (1-\gamma_n-\sigma_n)(1-\alpha_n)+\sigma_n(1-\alpha) \end{bmatrix}}_{\text{industry-specific impacts}} \tag{full$_{ES}$}$$

Goods prices

$$\begin{bmatrix} \frac{\mathrm{d}\log p_1}{\mathrm{d}\log E} \\ \frac{\mathrm{d}\log p_2}{\mathrm{d}\log E} \\ \vdots \\ \frac{\mathrm{d}\log p_n}{\mathrm{d}\log E} \end{bmatrix} = \left[I - \begin{pmatrix} \gamma_{11} & \gamma_{21} & \dots & \gamma_{n1} \\ \gamma_{12} & \gamma_{22} & \dots & \gamma_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{1n} & \gamma_{2n} & \dots & \gamma_{nn} \end{pmatrix} \right]^{-1} \cdot \begin{bmatrix} (1-\gamma_1-\sigma_1)\left((1-\alpha)-(1-\alpha_1)\right) \\ (1-\gamma_2-\sigma_2)\left((1-\alpha)-(1-\alpha_2)\right) \\ \vdots \\ (1-\gamma_n-\sigma_n)\left((1-\alpha)-(1-\alpha_n)\right) \end{bmatrix} \tag{3.62}$$

3.A.5 General equilibrium effects of a change in $\bar{p}_z$

Nominal and real aggregate output

Rewriting equilibrium $\log Y$ given by Equation (3.45) with only the affected variables:

$$\log Y = \frac{1}{\sum_{i=1}^{n} \mu_i(1-\gamma_i-\sigma_i)} \left[-\sum_{i=1}^{n} \mu_i \sigma_{zi} \log \bar{p}_z \right] \tag{3.63}$$

Since $\sum_i \mu_i(1-\gamma_i-\sigma_i) = C/Y$ [Equation (3.40)] and $\sum_i \mu_i \sigma_i = M/Y$ represents the share of total imports over GDP [Equation (3.41)], by extension, $\sum_i \mu_i \sigma_{zi}$ represents the aggregate share of good z imports. Thus, $\frac{\sum_i \mu_i \sigma_{zi}}{\sum_i \mu_i(1-\gamma_i-\sigma_i)}$ stands for $\frac{M_z/Y}{C/Y} = M_z/C$, a constant share given by parameters.

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log \bar{p}_z} = -\frac{M_z}{C} \tag{3.64}$$

Factor quantities and prices

From (3.51) and (3.52):

$$\frac{\mathrm{d}\log w_K}{\mathrm{d}\log \bar{p}_z} = \frac{\mathrm{d}\log w_E}{\mathrm{d}\log \bar{p}_z} = \frac{\mathrm{d}\log Y}{\mathrm{d}\log \bar{p}_z} = -\frac{M_z}{C} \tag{3.65}$$

Total differentiating (3.51) and (3.52):

$$\frac{\mathrm{d}\log k_i}{\mathrm{d}\log \bar{p}_z} = \frac{\mathrm{d}\log e_i}{\mathrm{d}\log \bar{p}_z} = \frac{\mathrm{d}\log Y}{\mathrm{d}\log \bar{p}_z} - \frac{\mathrm{d}\log w_K}{\mathrm{d}\log \bar{p}_z} = 0 \tag{3.66}$$

Total exports and imports

$$\frac{\mathrm{d}\log X}{\mathrm{d}\log \bar{p}_z} = \frac{\mathrm{d}\log M}{\mathrm{d}\log \bar{p}_z} = \frac{\mathrm{d}\log Y}{\mathrm{d}\log \bar{p}_z} = -\frac{M_z}{C} \tag{3.67}$$

Intermediate inputs

From (3.48):

$$\frac{\mathrm{d}\log d_{ji}}{\mathrm{d}\log \bar{p}_z} = \frac{\mathrm{d}\log q_j}{\mathrm{d}\log \bar{p}_z} \tag{3.68}$$

So the intermediate inputs demand vary as much as the total output of the producing industry (derived below).

Imports

For the imported good z whose price was shocked, deriving (3.49) with respect to p_z:

$$\frac{\mathrm{d}\log f_{zi}}{\mathrm{d}\log \bar{p}_z} = -\frac{M_z}{C} - 1 \tag{3.69}$$

The other imports are affected in the same way as GDP Y.

Real industry output

$$\begin{bmatrix} \frac{\mathrm{d}\log q_1}{\mathrm{d}\log \bar{p}_z} \\ \frac{\mathrm{d}\log q_2}{\mathrm{d}\log \bar{p}_z} \\ \vdots \\ \frac{\mathrm{d}\log q_n}{\mathrm{d}\log \bar{p}_z} \end{bmatrix} = \underbrace{\left[\mathbf{I} - \mathbf{\Gamma}'\right]^{-1}}_{\text{input-output linkages}} \cdot \underbrace{\begin{bmatrix} -\sigma_{z1} - \sigma_1 \frac{M_z}{C} \\ -\sigma_{z2} - \sigma_2 \frac{M_z}{C} \\ \vdots \\ -\sigma_{zn} - \sigma_n \frac{M_z}{C} \end{bmatrix}}_{\text{industry-specific impacts}} \qquad (\text{full}_{pz})$$

Goods prices

$$\begin{bmatrix} \frac{\mathrm{d}\log p_1}{\mathrm{d}\log \bar{p}_z} \\ \frac{\mathrm{d}\log p_2}{\mathrm{d}\log \bar{p}_z} \\ \vdots \\ \frac{\mathrm{d}\log p_n}{\mathrm{d}\log \bar{p}_z} \end{bmatrix} = \left[I - \begin{pmatrix} \gamma_{11} & \gamma_{21} & \cdots & \gamma_{n1} \\ \gamma_{12} & \gamma_{22} & \cdots & \gamma_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{1n} & \gamma_{2n} & \cdots & \gamma_{nn} \end{pmatrix} \right]^{-1} \cdot \begin{bmatrix} -(1-\gamma_1-\sigma_1)\frac{M_z}{C} - \sigma_{z1} \\ -(1-\gamma_2-\sigma_2)\frac{M_z}{C} - \sigma_{z2} \\ \vdots \\ -(1-\gamma_n-\sigma_n)\frac{M_z}{C} - \sigma_{zn} \end{bmatrix} \qquad (3.70)$$

3.B Data and computations

3.B.1 WIOD

World Input-Output Database (WIOD), Release 2016 (Timmer, Dietzenbacher, Los, Stehrer, and de Vries, 2015) covers 43 countries and a model for the rest of the world for the period 2000-2014. It includes 56 industries, classified according to the International Standard industry Classification of All Economic Activities, Revision 4 (ISIC, Rev. 4). Table 3.B.1 presents the countries while Table 3.B.2 lists all the industries.

WIOT

The World Input-Output Tables consist in all the intra-industry trade within and between each pair the countries in the dataset. Given the setting of the model, I extract the domestic and foreign use of intermediate goods, and the production of intermediate inputs sold domestically by each industry.

SEA

The Socio-Economic Accounts (WIOD-SEA-16) are available for all 43 countries and 56 industries covered by the WIOT. The November 2016 release was last updated in

14-Feb-2018. All nominal values in millions of national currency. The series extracted were:

- EMP: Number of persons engaged (thousands)
- LAB: Labour compensation
- K: Nominal capital stock
- CAP: Capital compensation
- II: Intermediate inputs at current purchasers' prices
- II_PI: Price levels of intermediate inputs (2010=100)
- GO: Gross output by industry at current basic prices
- GO_PI: Price levels of gross output (2010=100)

Values were converted to dollars using the exchange rate provided by WIOD. Notice that value added VA is given by the total payments to factors $LAB + CAP$ and gross output GO equals value added VA plus intermediate input production II.

WIOD's country abbreviations

Abbreviation	Country	Abbreviation	Country
AUS	Australia	IRL	Ireland
AUT	Austria	ITA	Italy
BEL	Belgium	JPN	Japan
BGR	Bulgaria	KOR	Republic of Korea
BRA	Brazil	LTU	Lithuania
CAN	Canada	LUX	Luxembourg
CHE	Switzerland	LVA	Latvia
CHN	China	MEX	Mexico
CYP	Cyprus	MLT	Malta
CZE	Czech Republic	NLD	Netherlands
DEU	Germany	NOR	Norway
DNK	Denmark	POL	Poland
ESP	Spain	PRT	Portugal
EST	Estonia	ROU	Romania
FIN	Finland	RUS	Russian Federation
FRA	France	SVK	Slovakia
GBR	United Kingdom	SVN	Slovenia
GRC	Greece	SWE	Sweden
HRV	Croatia	TUR	Turkey
HUN	Hungary	TWN	Taiwan
IDN	Indonesia	USA	United States
IND	India		

Table 3.B.1: WIOD's country abbreviations

WIOD's industries and codes

Table 3.B.2: WIOD's industry codes and description

No	Code	Description
1	A01	Crop and animal production, hunting and related service activities
2	A02	Forestry and logging

No	Code	Description
3	A03	Fishing and aquaculture
4	B	Mining and quarrying
5	C10-C12	Manufacture of food products, beverages and tobacco products
6	C13-C15	Manufacture of textiles, wearing apparel and leather products
7	C16	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials
8	C17	Manufacture of paper and paper products
9	C18	Printing and reproduction of recorded media
10	C19	Manufacture of coke and refined petroleum products
11	C20	Manufacture of chemicals and chemical products
12	C21	Manufacture of basic pharmaceutical products and pharmaceutical preparations
13	C22	Manufacture of rubber and plastic products
14	C23	Manufacture of other non-metallic mineral products
15	C24	Manufacture of basic metals
16	C25	Manufacture of fabricated metal products, except machinery and equipment
17	C26	Manufacture of computer, electronic and optical products
18	C27	Manufacture of electrical equipment
19	C28	Manufacture of machinery and equipment n.e.c.
20	C29	Manufacture of motor vehicles, trailers and semi-trailers

No	Code	Description
21	C30	Manufacture of other transport equipment
22	C31_C32	Manufacture of furniture; other manufacturing
23	C33	Repair and installation of machinery and equipment
24	D35	Electricity, gas, steam and air conditioning supply
25	E36	Water collection, treatment and supply
26	E37-E39	Sewerage; waste collection, treatment and disposal activities; materials recovery; remediation activities and other waste management services
27	F	Construction
28	G45	Wholesale and retail trade and repair of motor vehicles and motorcycles
29	G46	Wholesale trade, except of motor vehicles and motorcycles
30	G47	Retail trade, except of motor vehicles and motorcycles
31	H49	Land transport and transport via pipelines
32	H50	Water transport
33	H51	Air transport
34	H52	Warehousing and support activities for transportation
35	H53	Postal and courier activities
36	I	Accommodation and food service activities
37	J58	Publishing activities

No	Code	Description
38	J59_J60	Motion picture, video and television programme production, sound recording and music publishing activities; programming and broadcasting activities
39	J61	Telecommunications
40	J62_J63	Computer programming, consultancy and related activities; information service activities
41	K64	Financial service activities, except insurance and pension funding
42	K65	Insurance, reinsurance and pension funding, except compulsory social security
43	K66	Activities auxiliary to financial services and insurance activities
44	L68	Real estate activities
45	M69_M70	Legal and accounting activities; activities of head offices; management consultancy activities
46	M71	Architectural and engineering activities; technical testing and analysis
47	M72	Scientific research and development
48	M73	Advertising and market research
49	M74_M75	Other professional, scientific and technical activities; veterinary activities
50	N	Administrative and support service activities
51	O84	Public administration and defence; compulsory social security
52	P85	Education

No	Code	Description
53	Q	Human health and social work activities
54	R_S	Other service activities
55	T	Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use
56	U	Activities of extraterritorial organizations and bodies

3.B.2 Plots from raw data

I use the raw data to survey the the countries in terms of the main variables conveyed in the model. The parameters, i.e. the nominal shares, are calculated with the values as available in the data.

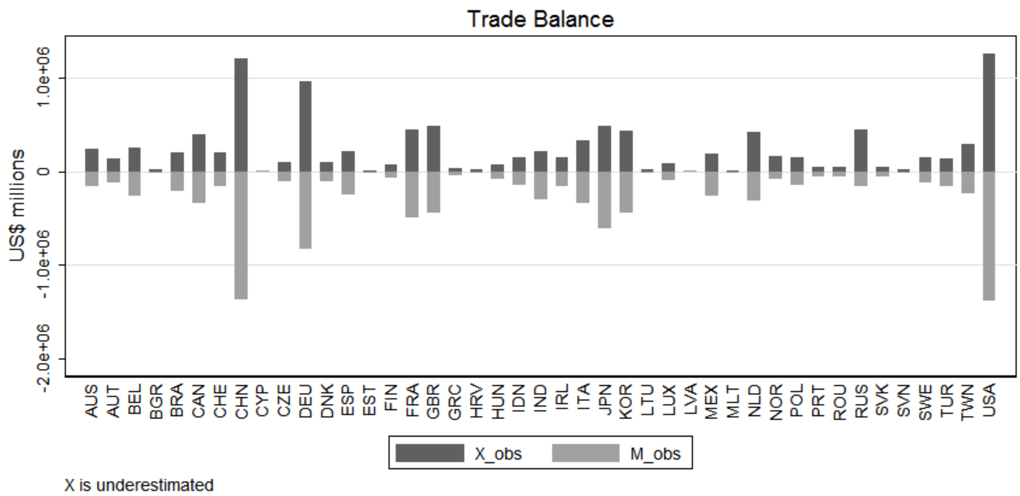


Figure 3.B.1: Exports and imports

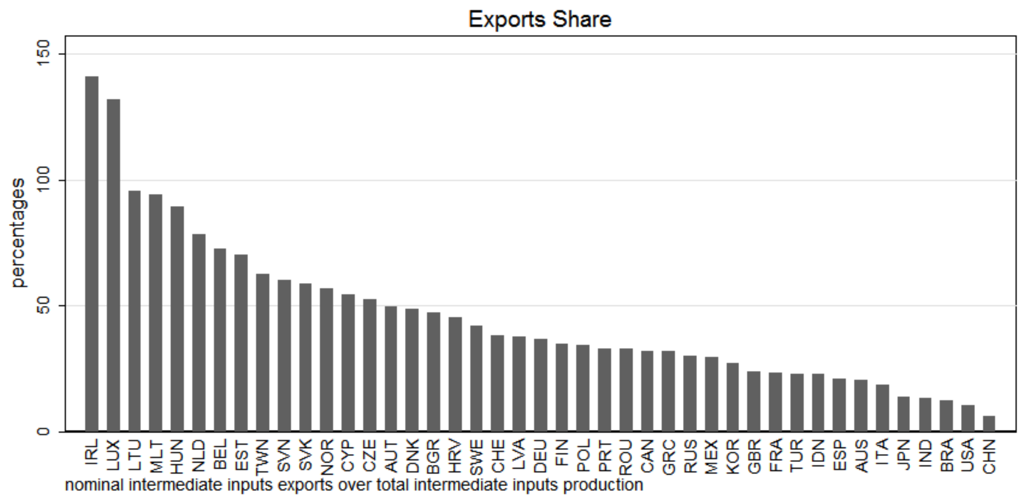


Figure 3.B.2: Ratio of intermediate input exports over intermediate input production

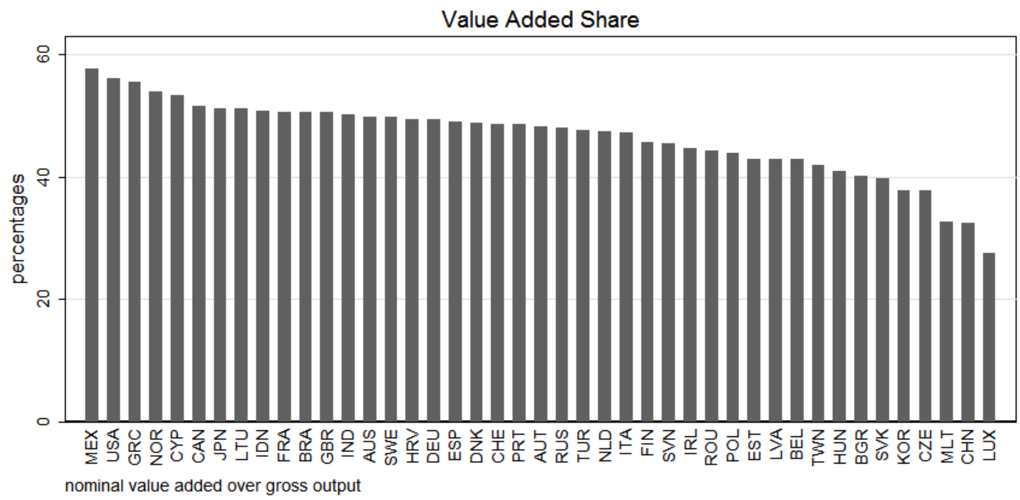


Figure 3.B.3: Aggregate share of value added on gross output

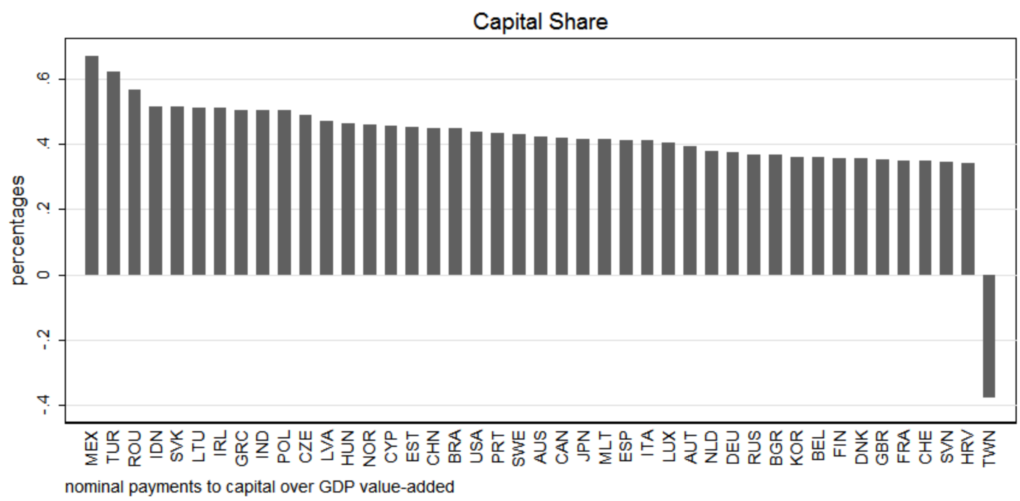


Figure 3.B.4: Aggregate share of capital on value added

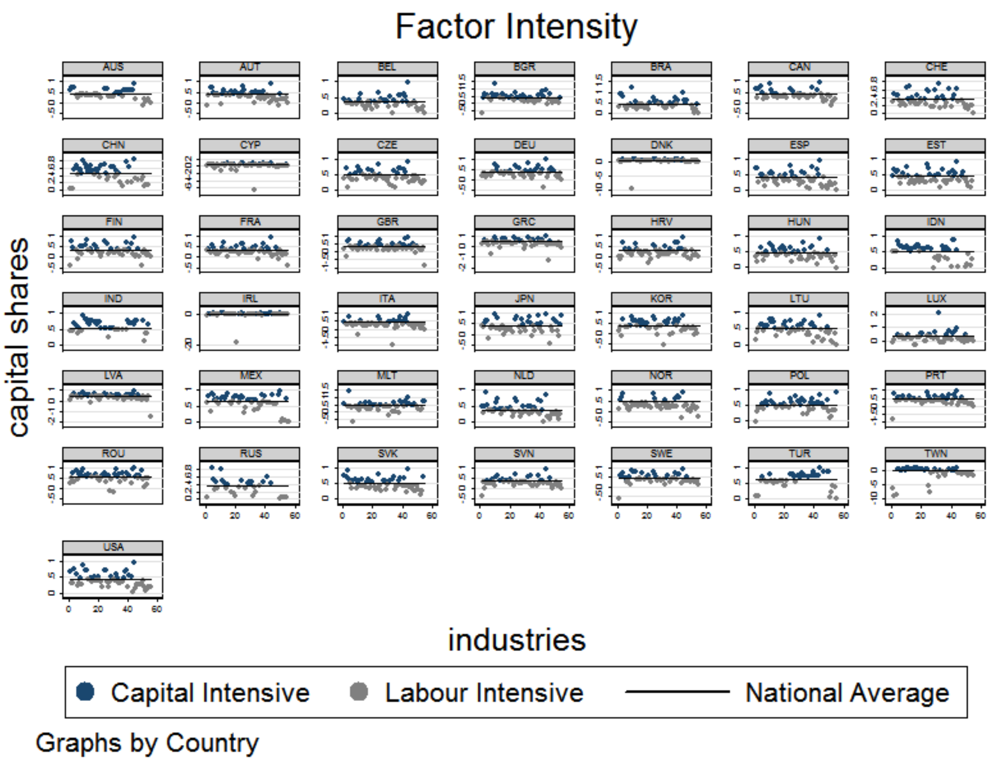


Figure 3.B.5: Industry capital shares (α_i)

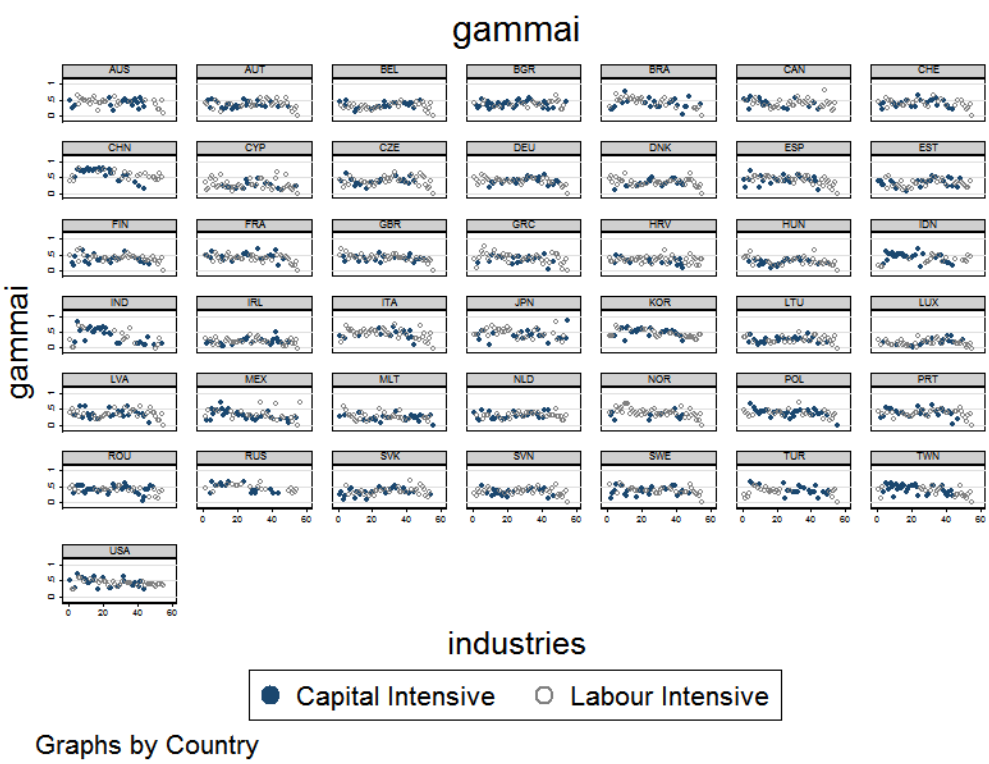


Figure 3.B.6: Industry input shares (γ_i)

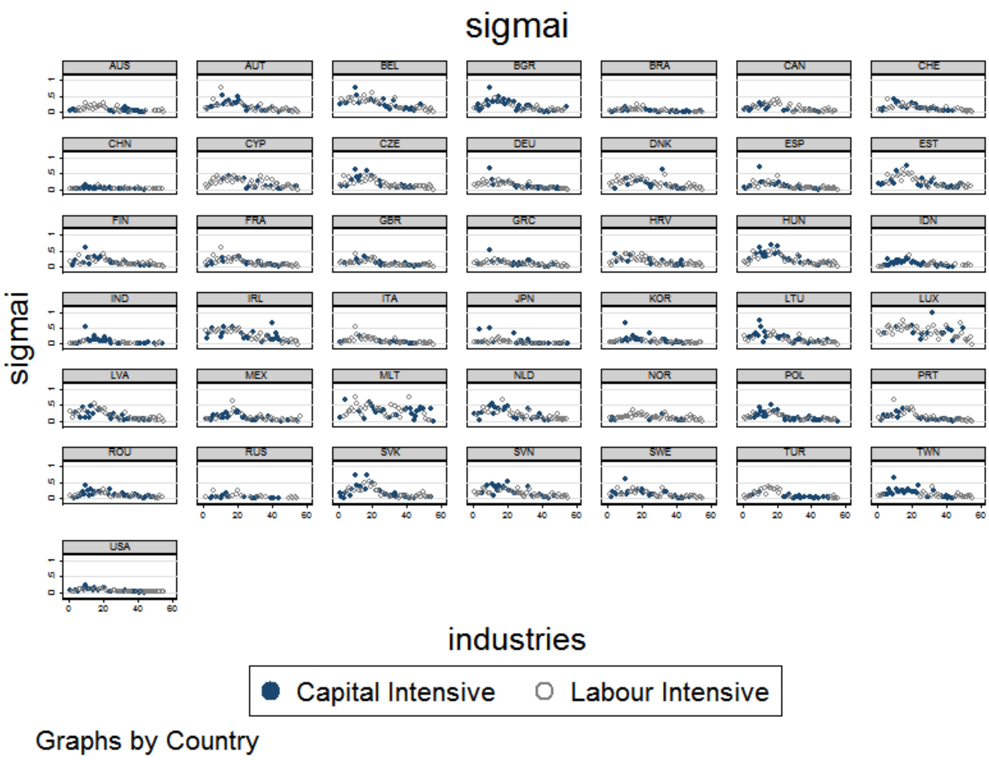
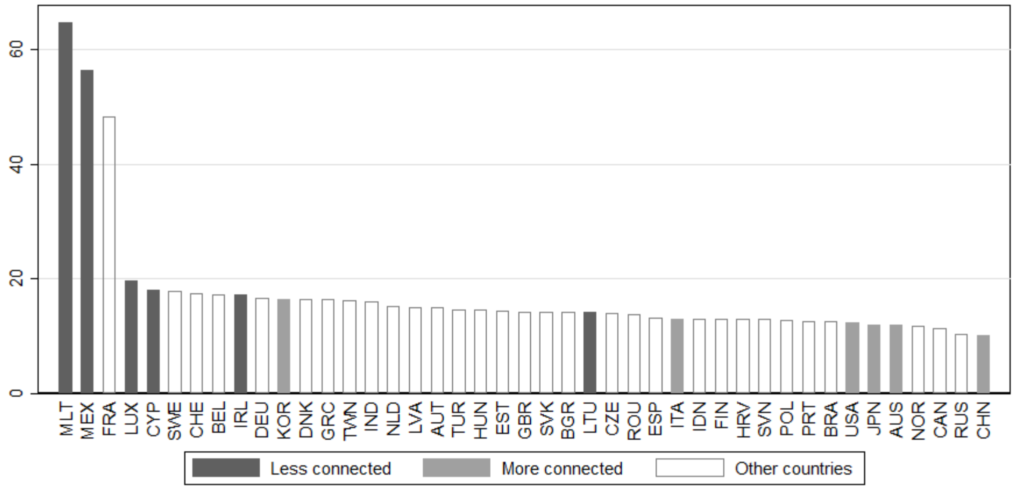


Figure 3.B.7: Industry import shares (σ_i)

3.B.3 Choosing the pair of countries



Note: the measure gives the largest shortest distance between each pair of nodes in the network

Figure 3.B.8: Connectivity measure of WIOD countries: Diameter

Ranking	Density		Distance		Diameter	
1	LUX	0.07	MEX	8.1	MLT	64.5
2	**IRL**	0.08	MLT	8.1	MEX	56.4
3	LTU	0.10	LUX	7.8	FRA	48.2
4	MLT	0.10	CYP	7.4	LUX	19.6
5	CYP	0.10	**IRL**	7.4	CYP	18.1
6	IND	0.11	FRA	7.3	SWE	17.8
7	MEX	0.11	LTU	7.2	CHE	17.5
8	HRV	0.12	LVA	7.1	BEL	17.3
9	HUN	0.12	TWN	7.1	**IRL**	17.2
10	RUS	0.12	EST	6.9	DEU	16.6
⋮	⋮	⋮	⋮	⋮	⋮	⋮
34	JPN	0.16	**KOR**	6.1	POL	12.9
35	AUS	0.16	CAN	6.1	PRT	12.7
36	BGR	0.16	SWE	6.1	BRA	12.6
37	FIN	0.16	BRA	6.1	USA	12.3
38	NOR	0.16	USA	6.1	JPN	12.1
39	USA	0.17	AUS	6.1	AUS	12.0
40	CHN	0.17	ITA	6.0	NOR	11.8
41	**KOR**	0.17	JPN	6.0	CAN	11.3
42	ITA	0.18	CHN	5.7	RUS	10.4
43	ROU	0.18	RUS	5.3	CHN	10.2

Table 3.B.3: Top-10 least and most connected production networks

3.B.4 Calibration

Value added

The capital stock and employment of each industry i in each country g, k_i^g and e_i^g and their respective summation over each country, the national aggregate supply of factors K^g and E^g are taken directly from the data; the latter pair is given by K and EMP, respectively, among the SEA variables described in Section 3.B.1. The average factor prices w_K^g and w_E^g are then computed by dividing the total payment to factors CAP and LAB in each country by their total quantity K^g and E^g, respectively. Following the model, I assume this price is adopted by every industry within a country. Finally, I construct the dataset's value-added using the formula: $VA_i^g = w_K^g \cdot k_i^g + w_E^g \cdot e_i^g$.

Use of domestic and imported intermediate inputs

For each country, the National Input-Output Tables is created by simply dropping the variables and values of the remaining countries whilst the quantity imported of each good equals the vertical sum of all foreign countries for each industry. In nominal values, the domestic intermediate inputs used by industry i equals $II_i^{in,g} = \sum_{j=1}^{n} ii_{ji}^g$; the imported intermediate inputs used by industry i equals $M_i^g = \sum_{j=1}^{n} \sum_{\substack{z=1 \\ z \neq g}}^{m} ii_{ji}^{zg}$; and, finally, the consistent measure of output of each industry i is constructed as $p_i^g q_i^g = VA_i^g + II_i^{in,g} + M_i^g$.

Inputs production, final goods consumption and exports

I simply sum the intermediate inputs production from i used domestically to get $II_i^{out,g} = \sum_{j=1}^{n} p_i^g d_{ij}^g$. Following the model, exports and final goods consumption are considered as a single final demand variable y_i^g. I obtain its nominal values by residual using: $p_i^g y_i^g = p_i^g q_i^g - II_i^{out,g}$. Finally, total GDP equals $Y^g = \sum_{i=1}^{n} p_i^g y_i^g$. Notice that, following Equation (3.6), consumption equals value added, so that aggregate exports are given by $X^g = Y^g - VA_i^g$ which by construction equals aggregate imports M_i^g.

Merging industries

The model's condition of perfect competition in the factor markets —in practice, same factor prices being paid across industries— is far from observed in the data. Following the steps above leads to a few industries presenting negative values for final demand $p_i^g y_i^g$. The chosen solution is to merge such industries with similar ones, i.e. those nearby in the code classification.

South Korea presents four industries with negative values: (i) B, which has more economical proximity with the industries of group A and whose high py value requires the merging of four industries; (ii) E37-E39, which can be naturally merged to E36; K64, which is absorbed by industries K65 and K66; and M71, which is easily incorporated by M69-M70. In the case of Ireland, only two industries end up with negative final demand values, K66 and M69-M70, which are also vanished when combining the industries as required by South Korea's values. For clarity, the merged industries and the resulting groups are listed below:

- A01, A02, A03 and B
  - A01 Crop and animal production, hunting and related service activities
  - A02 Forestry and logging
  - A03 Fishing and aquaculture
  - B Mining and quarrying
- E36 and E37-E39
  - E36 Water collection, treatment and supply
  - E37-E39 Sewerage; waste collection, treatment and disposal activities; materials recovery; remediation activities and other waste management services
- K64, K65 and K66
  - K64 Financial service activities, except insurance and pension funding
  - K65 Insurance, reinsurance and pension funding, except compulsory social security
  - K66 Activities auxiliary to financial services and insurance activities
- M69-M70 and M71
  - M69-M70 Legal and accounting activities; activities of head offices; management consultancy activities

– M71 Architectural and engineering; technical testing and analysis

Finally, industry U was excluded due to all values being null. The resulting 48 industries are listed in Table 3.B.4.

#	Code	Description
1	A-B	Crop and animal production, hunting and related service activities; Forestry and logging; Fishing and aquaculture; Mining and quarrying
2	C10-C12	Manufacture of food products, beverages and tobacco products
3	C13-C15	Manufacture of textiles, wearing apparel and leather products
4	C16	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials
5	C17	Manufacture of paper and paper products
6	C18	Printing and reproduction of recorded media
7	C19	Manufacture of coke and refined petroleum products
8	C20	Manufacture of chemicals and chemical products
9	C21	Manufacture of basic pharmaceutical products and pharmaceutical preparations
10	C22	Manufacture of rubber and plastic products
11	C23	Manufacture of other non-metallic mineral products
12	C24	Manufacture of basic metals
13	C25	Manufacture of fabricated metal products, except machinery and equipment
14	C26	Manufacture of computer, electronic and optical products
15	C27	Manufacture of electrical equipment
16	C28	Manufacture of machinery and equipment n.e.c.
17	C29	Manufacture of motor vehicles, trailers and semi-trailers
18	C30	Manufacture of other transport equipment
19	C31-C32	Manufacture of furniture; other manufacturing
20	C33	Repair and installation of machinery and equipment
21	D35	Electricity, gas, steam and air conditioning supply
22	E36-E39	Water collection, treatment and supply; Sewerage; waste collection, treatment and disposal activities; materials recovery; remediation activities and other waste management services
23	F	Construction
24	G45	Wholesale and retail trade and repair of motor vehicles and motorcycles
25	G46	Wholesale trade, except of motor vehicles and motorcycles
26	G47	Retail trade, except of motor vehicles and motorcycles
27	H49	Land transport and transport via pipelines

#	Code	Description
28	H50	Water transport
29	H51	Air transport
30	H52	Warehousing and support activities for transportation
31	H53	Postal and courier activities
32	I	Accommodation and food service activities
33	J58	Publishing activities
34	J59-J60	Motion picture, video and television programme production, sound recording and music publishing activities; programming and broadcasting activities
35	J61	Telecommunications
36	J62-J63	Computer programming, consultancy and related activities; information service activities
37	K64-K66	Financial service activities, except insurance and pension funding; Insurance, reinsurance and pension funding, except compulsory social security; Activities auxiliary to financial services and insurance activities
38	L68	Real estate activities
39	M69-M71	Legal and accounting activities; activities of head offices; management consultancy activities; Architectural and engineering activities; technical testing and analysis
40	M72	Scientific research and development
41	M73	Advertising and market research
42	M74-M75	Other professional, scientific and technical activities; veterinary activities
43	N	Administrative and support service activities
44	O84	Public administration and defence; compulsory social security
45	P85	Education
46	Q	Human health and social work activities
47	R-S	Other service activities
48	T	Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use

Table 3.B.4: Dataset's industry codes and description

Prices and real values

After merging the industries, domestic prices for each good i in each country p_i^g is extracted from the ratio of nominal GO over real gross output. But since the variable GO_QI is an index, I first compute the gross output in currency by dividing GO by the price index GO_PI, which allows me to combine the values of the merged industries.[60] The price index is then normalized to equal one in aggregate terms ($P^g = 1$). Finally, I compute the real values of the variables by dividing the corresponding nominal values by their respective price. For example, the real domestic intermediate inputs used by industry i: $d^g_{ji} = ii^g_{ji}/p^g_j$.

From each country's perspective the imported prices p^z_j are exogenous. Moreover, the imports enter the model as the sum originated from all foreign countries $f^g_{ji} = \sum_{\substack{z=1 \\ z\neq g}}^m f^{zg}_{ji}$, i.e. there is no differentiation among the countries of origin of the imports. In this context, I choose for simplicity to use the world average for each product p^W_j.

To calculate the world average I use the nominal intermediate inputs variable II and generate real intermediate inputs values using the intermediate inputs price variable II_PI. I construct the volume in national currency values by dividing the nominal values II its respective price index II adjusted to the unitary base. The average world prices are then constructed by summing sum all the nominal and real values of all countries by industry code and dividing the former by the latter. Then, the real imported intermediate inputs used by industry i is given by: $f^g_{ji} = \sum_{\substack{z=1 \\ z\neq g}}^m ii^{zg}_{ji}/p^W_j$.

Figure 3.B.9 plots the domestic prices in Ireland and South Korea and the calculated world average for the resulting 48 industries. It shows that the latter tends to have domestic prices closer to international ones than the former, suggesting that Korea may be more open to trade than Ireland. Moreover, this finding may relate to the countries' characteristics, as discussed in Section 3.3. In particular, Ireland may present less efficient industries, which may explain the greater reliance on imported inputs.

Nevertheless, both economies can be considered "small" in international terms. Even though among the 43 countries in WIOD, Korean ranks 10th in gross output (nominal US dollars 2014) accounts for only 2.5% of the total. As Figure 3.B.10 shows, the top-2 countries, China and US, account for 23.4% and 22.8%, respectively. Ireland, however, is even smaller, taking the 28th position in the ranking with a gross output of 0.4% of the total.

[60] Notice that the data values GO_PI cannot be directly used when merging industries since one cannot sum the industry values of an index. Without merging, the method above leads to the same values as GO_PI variable.

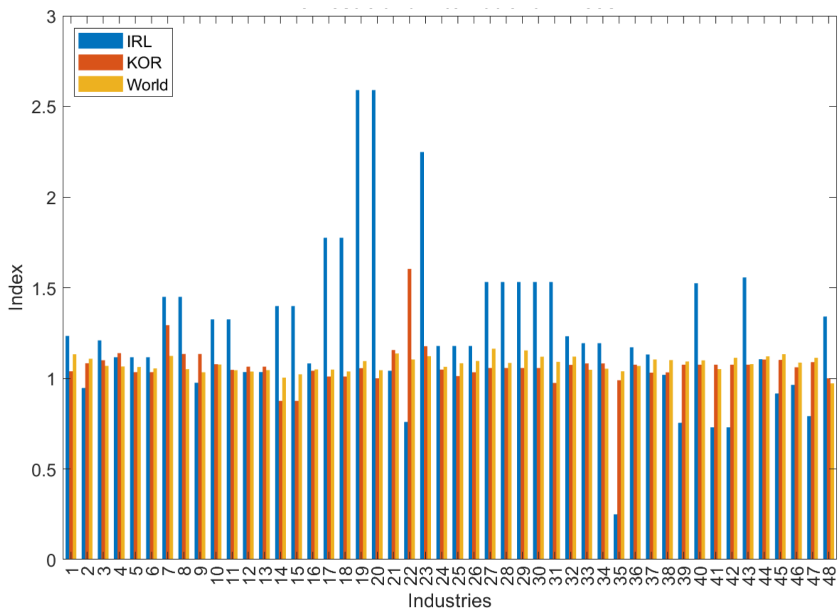


Figure 3.B.9: Price comparison - domestic and international prices

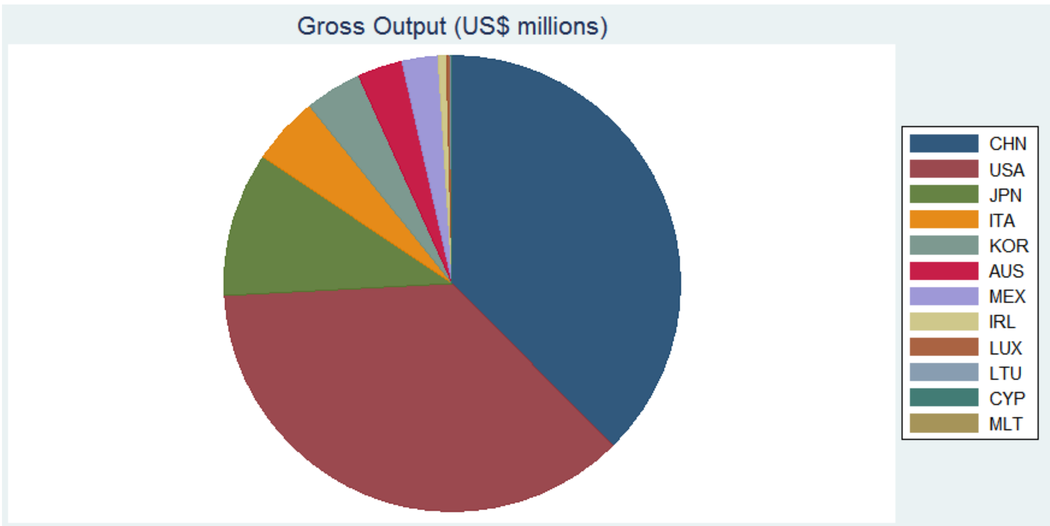


Figure 3.B.10: Gross Output - selected countries

3.B.5 Complementary centrality measures

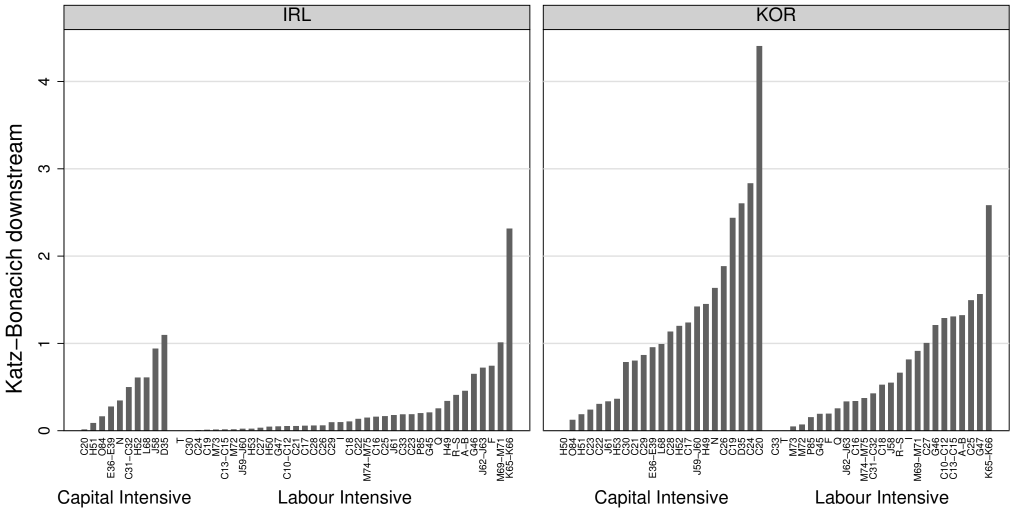


Figure 3.B.11: Domestic network: downstream centralities

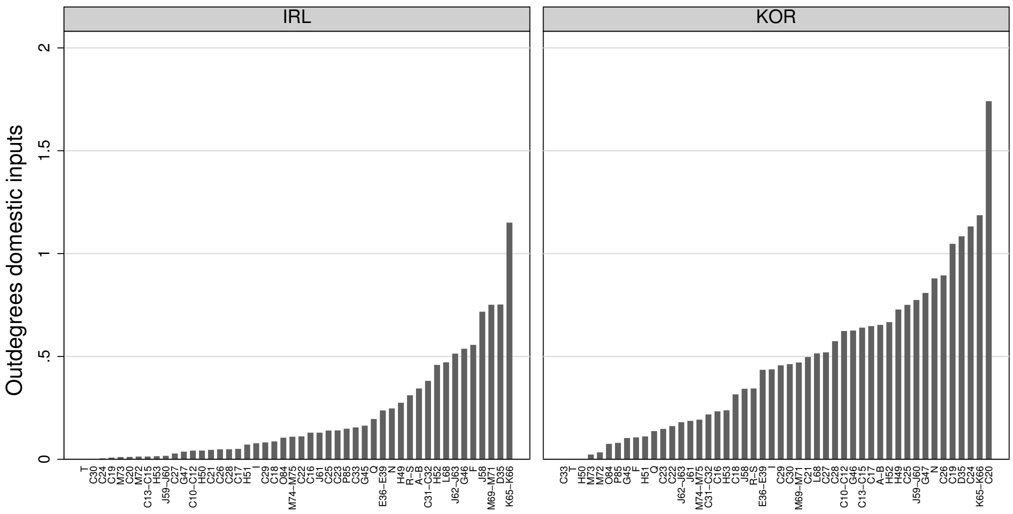


Figure 3.B.12: Outdegree centralities of domestic intermediate inputs

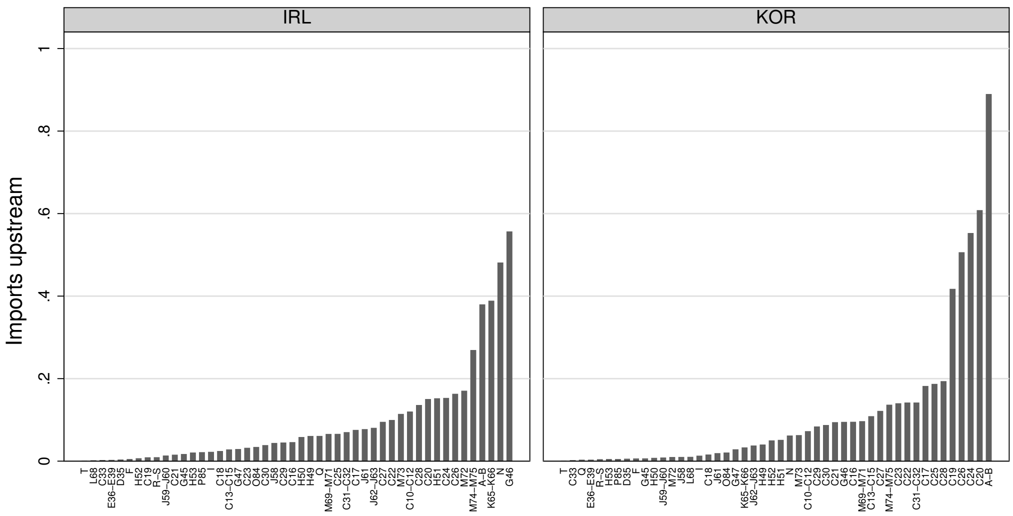


Figure 3.B.13: Upstream centralities of imported goods

3.B.6 Choosing good z for the price shock

The idea is to evaluate which imported intermediate goods are more relevant to the production network of a country so that the results of the import good price shock may be sizeable.

Corollary 3.5 states that the effect of an import price change on the real output of each industry is given by the product of the Leontief-inverse transposed $[\mathbf{I} - \mathbf{\Gamma}']^{-1}$ and an industry-specific shock vector. The former comprises exclusively of domestic input shares and therefore only explain how the shock is relayed among the industries.

The shock vector, on the other hand, summarises the industry-specific impact of an rise in the price of an import. It comprises of the sum of the negative of each industry i's import share of the affected good z $(-\sigma_{zi})$ and the effect of the aggregate shock $\left(-\sigma_i \frac{M_z}{C}\right)$ to industry i.

Matrix $\mathbf{\Sigma}$, plotted in Figures 3.5c for Ireland and 3.5d for Korea, contains the nominal shares σ_{ij} of the imported good i in the production technology of domestic industry j. The rows represent each rest-of-the-world (ROW) exporting industry while the columns represent the country-at-hand importing industries. Korea, being less reliant on imports, has fewer foreign inputs significant to Korean industries, worthy of note only that of industry A-B (#1).[61] Ireland, on the other hand, has several imported goods standing out in terms of domestic use, including those sold by industry A-B.

The constant ratio M_z/C given by the aggregate share of imported good z over total consumption varies significantly across goods. Figure 3.B.14 plots these values for every import. Industry A-B turns out to be the most relevant in aggregate terms for Korea while it is also of high importance for Ireland, ranking fifth among the imported goods.

[61] See Table 3.B.4 for the whole description of this industry.

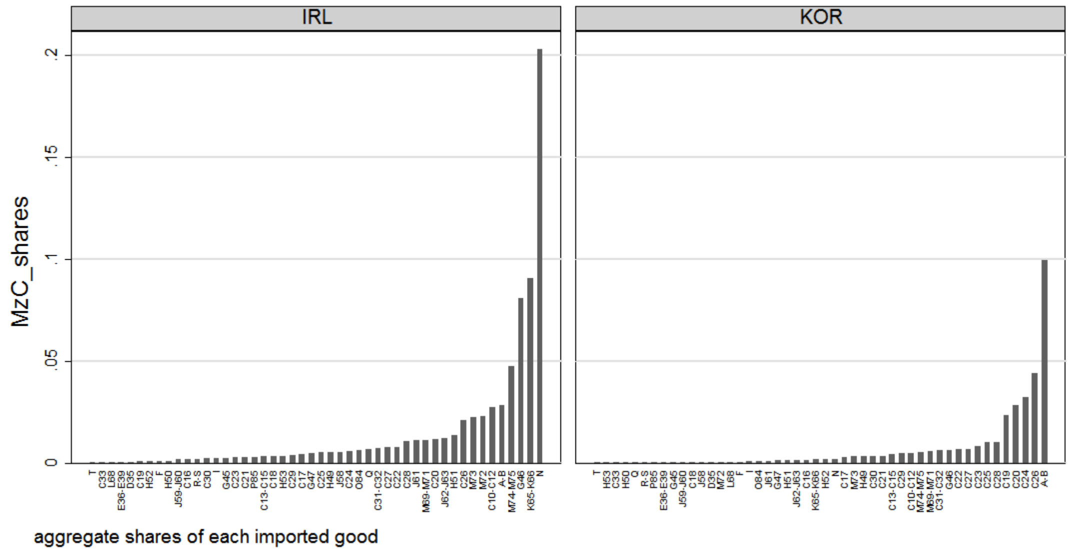


Figure 3.B.14: Good z total imports over consumption (M_z/C)

3.B.7 Assessing the accuracy of first-order approximations

I compute the model directly with Matlab, instead of relying on the first-order approximations. This allows for exact numerical solutions, obtained straight from the data. The rich dataset conceived as a closed-system warrants the implementation of the general equilibrium model. In the examples below, underscore-'obs' refers to values calibrated as explained in Section 3.B.4 while underscore-'cf' represents the values computed in the counterfactual scenarios.

Labour shock

As seen in Chapter 2, the first order derivation given by the model is a very good approximation for small counterfactual shocks. The calculations below compare the model's predictions with the numerical results produced by MATLAB for the GDP Y variable. The proximity is of the same order of magnitude for the other variables.

For a one-percent increase in the total supply of labour, such that $E_{cf} = 1.01 \cdot E_{obs}$:

- Ireland
  - Model's predictions: $\frac{\mathrm{d}\log Y}{\mathrm{d}\log E} = (1-\alpha) = 0.4885\%$
  - Results: $\frac{Y_{cf}-Y_{obs}}{Y_{obs}} = 0.4872\%$
- Korea
  - Model's predictions: $\frac{\mathrm{d}\log Y}{\mathrm{d}\log E} = (1-\alpha) = 0.5632\%$
  - Results: $\frac{Y_{cf}-Y_{obs}}{Y_{obs}} = 0.5620\%$

Import price shock

Even closer results are seen in the case of the import price shock. For a one-percent increase in the price of imported good of industry $z = 1$, such that $\bar{p}_{1cf} = 1.01 \cdot \bar{p}_{1obs}$:

- Ireland
  - Model's predictions: $\frac{\mathrm{d}\log Y}{\mathrm{d}\log \bar{p}_1} = \frac{M_1}{C} = -0.0093\%$
  - Results: $\frac{Y_{cf} - Y_{obs}}{Y_{obs}} = -0.0093\%$
- Korea
  - Model's predictions: $\frac{\mathrm{d}\log Y}{\mathrm{d}\log \bar{p}_1} = \frac{M_1}{C} = -0.0011\%$
  - Results: $\frac{Y_{cf} - Y_{obs}}{Y_{obs}} = -0.0011\%$

3.C Supporting analysis

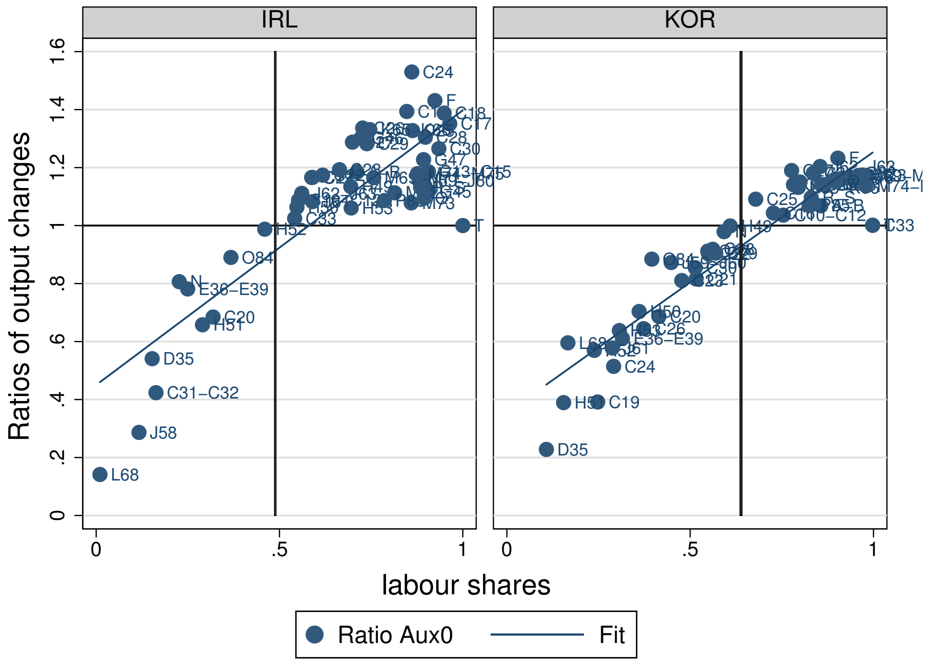
3.C.1 Counterfactual #1: a labour shock

Ratios of models' predictions

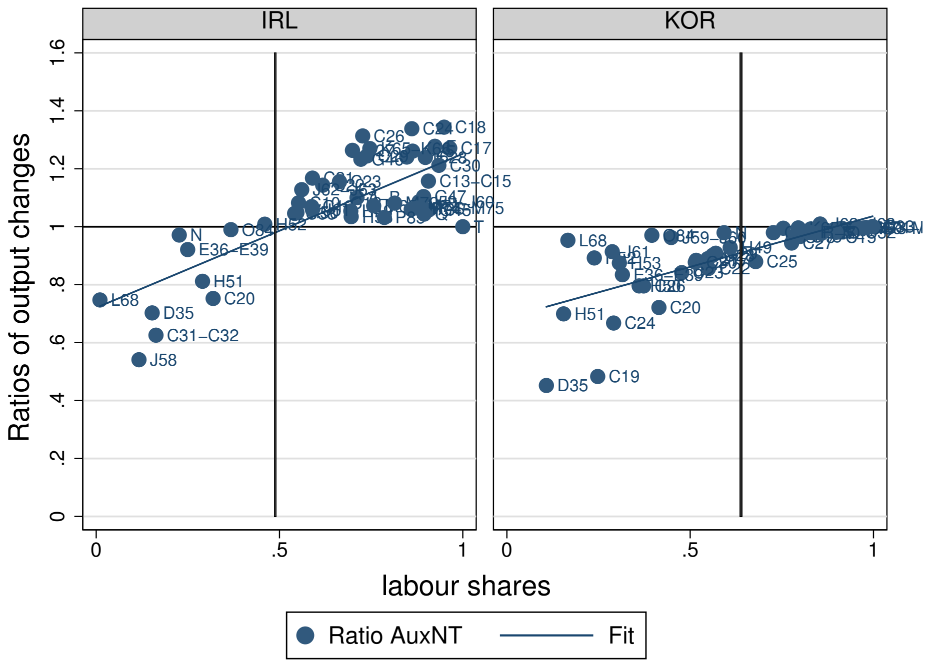
The auxiliary models underestimate the impact of a counterfactual shock if their calculated values are smaller than in the full model, and reversely for overestimations. In Figure 3.C.1, I plot the ratios of the impacts predicted by each auxiliary model over the full model. Points below or above the horizontal reference line on those figures correspond respectively to the under and overestimation of the auxiliary model.

Plot 3.C.1a shows that a model without any linkages underestimates the impact of a labour supply shock on capital-intensive industries about the same in each country, while the labour-intensive industries are more overestimated in Ireland. The most underestimated industries in Ireland (L68 Real estates) and Korea (D35 Electricity and gas) both have their predicted output change in the Auxiliary 0 model about a fifth of that in the full model. Among the opposite group, industry C24 (Manufacture of basic metals) in Ireland is overestimated about twice as much (ratio of 1.53) than the most overestimated industry (F Construction) in Korea (ratio of 1.23). This finding indicates that labour-intense industries in Ireland are more reliant on trade and domestic linkages combined than their correspondent group in Korea.

Plot 3.C.1b reinforces how important trade is in general for Irish industries and only in part for South Korean capital-intensive industries. On contrary, for most labour-intensive industries in Korea the non-trade model predicts a very similar impact to the full model since imports are of very little relevance for them. Contrasting the top with the bottom plot reveals that removing only the trade component from the full model reduces considerably the underestimation for capital-intensive industries in Ireland and practically removes the overestimation for labour-intensive industries in Korea. This finding stresses how domestic input-output linkages are relevant for these industries.



(a) Auxiliary 0 over Full model



(b) Auxiliary NT over Full model

Note: vertical lines show national averages. Horizontal line at one is a reference for auxiliary model performing as well as full model.

Figure 3.C.1: Ratios of output changes predicted by auxiliaries over full model

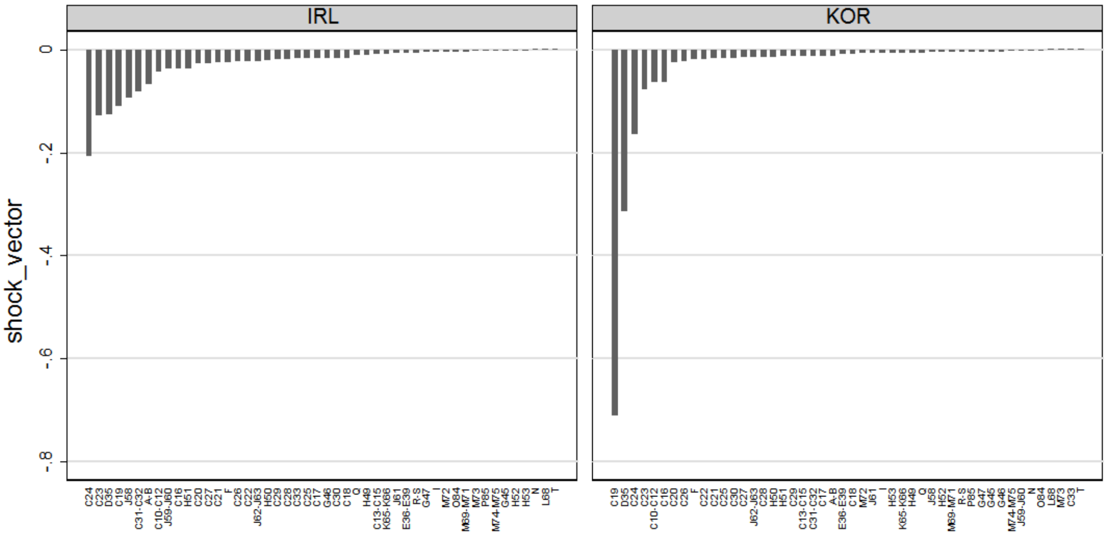
3.C.2 Counterfactual #2: an import price shock

Establishing a base of comparison

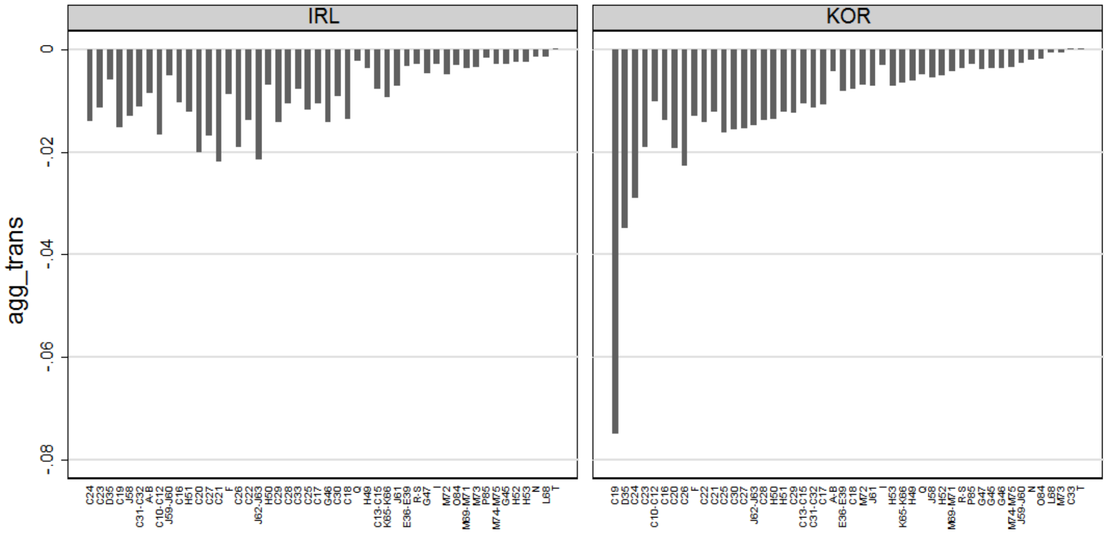
Comparing the effect on industry output with labour shares emerged as a natural appraisal of how the shock affected the industries in the case of the labour shock. In the assessment of the import shock, I chose the comparison with the import shares of the affected good for the reasons I present now.

Among the three main components of Equation (full$_{pz}$), the Leontief-inverse transposed $[\mathbf{I} - \mathbf{\Gamma}']^{-1}$ has no elements related to the import price itself and is also not present in the model without input-output linkages, Therefore it is not suitable as a base of comparison. Within the shock vector, there is $\sigma_i \frac{M_z}{C}$ coming from the aggregate shock and σ_{zi} related to the direct effect of the increase in the import price of good z.

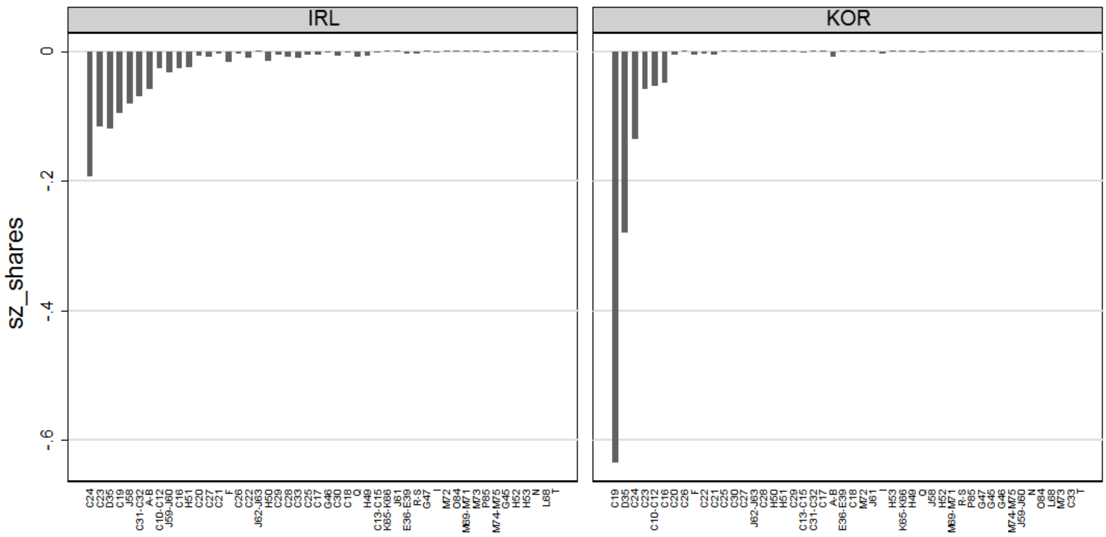
I plot the shock vector itself and its two constituent vectors in Figures 3.C.2a, 3.C.2b and 3.C.2c across industries for the output effect of a one-percent rise of import good A-B. All values are ranked by those of the shock vector. It becomes clear that the relevance of the aggregate shock is very small, being of an order of magnitude smaller than each industry's import share of good A-B. This indicates that the last term, σ_{zi} is an appropriate choice to contrast with the output changes of the counterfactual exercises.



(a) shock$_{v}ector$



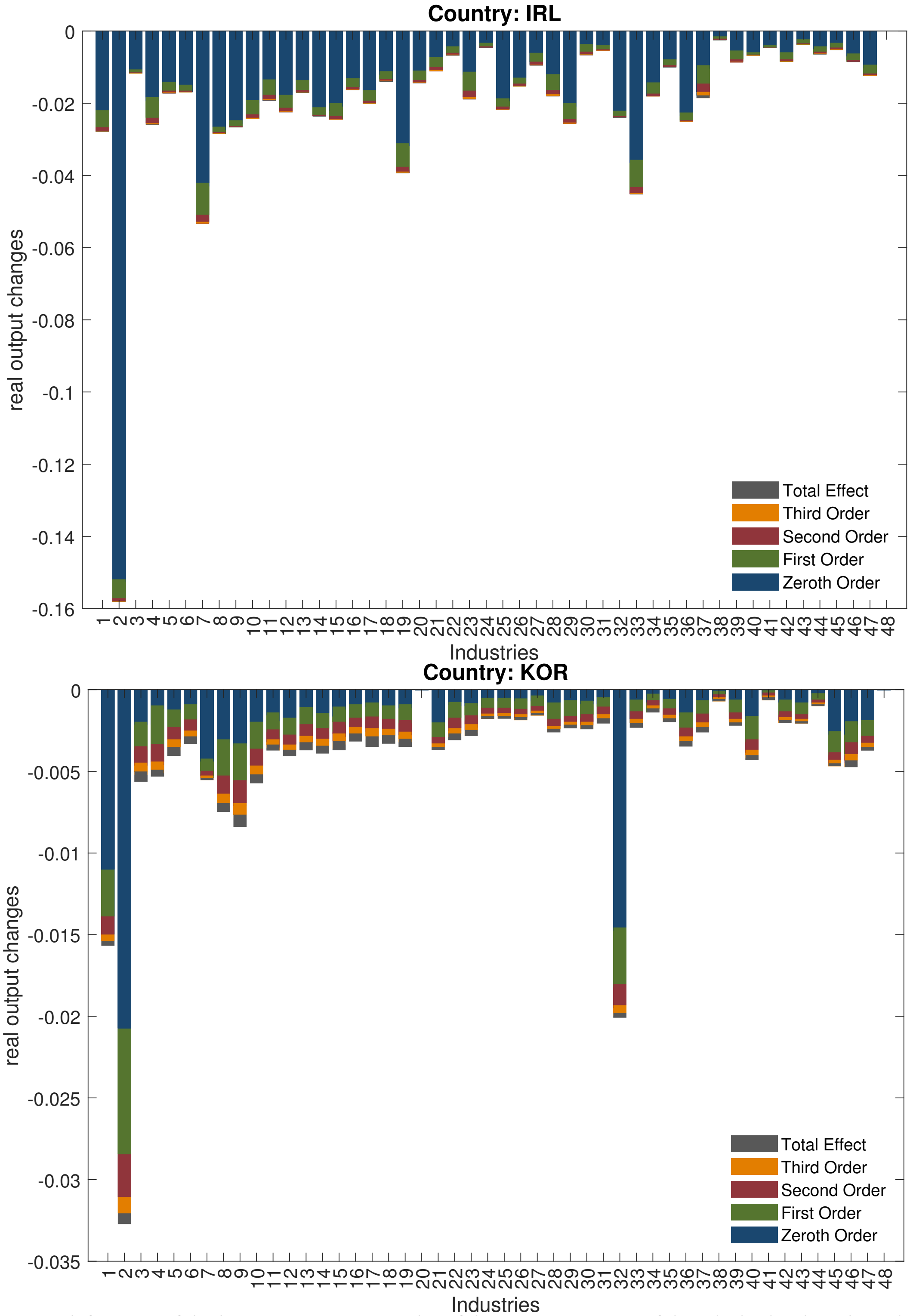
(b) agg$_{t}rans$



(c) sz$_{s}hares$

Figure 3.C.2: Decomposition of the shock vector for A-B's import price rise

Sensitivity test: Price shock on imported good C10-C12



Note: each fraction of the bars represents an order of the approximation of the whole shock as described in Section 2.5. The zeroth order is equivalent to the output changes predicted by the model ignoring domestic input-output linkages while the total effect represents the predictions of the full model. In this way, the extent of the bars beyond the zeroth order depicts how much the production network boosts the effect of the shock for each industry.

Figure 3.C.3: Decomposition of the C10-C12 import price shock for Ireland and Korea