Estimating mode effects from a sequential mixed-mode experiment using structural moment models

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Abstract

Until recently, the survey mode of the household panel study *Understanding Society* was mainly face-to-face interview, but it has now adopted a mixed-mode design where individuals can self complete the questionnaire via the web. As mode is known to affect survey data, a randomized mixed-mode experiment was implemented during the first year of the two-year Wave 8 fieldwork period to assess the impact of this change. The experiment involved a sequential design that permits the identification of mode effects in the presence of nonignorable nonrandom mode selection. While previous studies have used instrumental variables regression to estimate the effects of mode on the means of the survey variables, we set up a more general framework based on novel structural moment models to characterize the effect of mode on the distribution of the survey variables by its effect on the moments of the joint distribution. We adapt our estimation procedure to account for nonresponse and complex sampling designs,and to include suitable auxiliary data to improve inferences and relax key assumptions. Finally, we demonstrate how to estimate the effects of mode on the parameter estimates from generalized linear and other exponential family models when both outcomes and predictors are subject to mode effects. This framework is used to investigate the impact of the move to web mode on Wave 8 of *Understanding Society*.

Key Words: Causal inference; Generalized method of moments; Instrumental variable; Potential outcomes.

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1 Introduction

The term survey mode is used in survey research to refer to the method used to collect survey data. The survey mode of *Understanding Society*: the UK Household Longitudinal Study (UKHLS) and its predecessor, the British Household Panel Survey (BHPS), has predominantly been the face-to-face interview. However, UKHLS has now followed other major surveys by introducing a mixed-mode design where participants have the option of choosing web mode (ISER 2018, pp. 45-46).

A potential drawback of this change is that survey mode is known to affect the sample distribution of, and nonresponse pattern among, the survey variables. Whether the impact is positive or negative in terms of measurement error depends on the nature of the variable: web mode is hypothesized to reduce measurement error for questions where the presence of an interviewer could lead to social desirability, positivity or disclosure bias; but to increase it for complex questions where the absence of an interviewer could lead to bias due to 'satisficing' or presentation effects (Jäckle et al. 2010; D'Ardenne et al. 2017).

The effect of mode on the data collected is a special case of causal effect that contrasts the observed measurement with what would have been measured had the survey been administered using a different mode (Vannieuwenhuyze et al. 2010). In mixed-mode surveys, where the survey mode varies between participants, the difference between the group of participants who were administered the survey in one mode, and the group who were administered it in another, will generally be a biased estimate of the mode effect because of nonrandom selection, which arises if there are systematic differences between the characteristics of the two groups.

In this paper, we develop a method for estimating mode effects from mixed-mode surveys in the presence of nonignorable nonrandom selection and apply it to data from UKHLS Wave 8. Nonignorable selection is driven by unobserved confounding variables the effects of which cannot be adjusted for using standard covariate-adjustment methods. The method thus requires an instrumental variable (IV) to identify the mode effect (Angrist et al. 1996). An IV is available from the sequential mixed-mode experiment implemented at UKHLS Wave 8 in which households were initially randomized to face-to-face or web mode, but individual household members could noncomply with the household randomization. While this noncompliance could also lead to nonignorable selection, the initial randomization is a credible IV that can be used to identify mode effects.

IV regression based on two-stage least squares (e.g. Wooldridge 2010, ch. 6) has recently been used to estimate the effects of mode on survey variable *means* (Jäckle et al. 2017, p. 20). However, we have more general aims: **Aim A** to estimate the effect of mode on the *distribution* of the survey variables, not just the mean; and **Aim B** to estimate the effect of mode on the parameter estimates of statistical models such as linear and logistic regressions when both outcomes and predictors are subject to mode effects.

For Aim A, Vannieuwenhuyze (2015) suggested characterizing the overall effect of mode on the survey data by the effects of mode on the different moments of the survey variables' joint distribution. He devised estimators of the effects of mode on variances, covariances and correlations from data collected from a special case of sequential design in which those randomized to one of the modes cannot noncomply. We generalize this approach by extending the family of structural mean models (Robins 1994) to create a family of *structural moment models*, which include novel structural variance models, structural covariance models and models for categorical variables, to enable IV estimation of mode effects on arbitrary moments of a joint distribution. We show how all the mode effects within this framework can be estimated relatively simply using the generalized method of moments (GMM), and adapt GMM to adjust for nonresponse bias and the effects of complex sampling designs such as that used by UKHLS.

A further advantage of this framework is that it makes explicit the assumptions required to handle mode-effect heterogeneity, that is, where the effect of mode is different for each sample member. In the presence of heterogeneity, IVs can only identify bounds on mode effects in the absence of further assumptions or additional data (Hérnan and Robins 2020, ch. 16). The specification of structural moment models makes clear that mode-effect heterogeneity must be constrained by the *no effect modification* assumption to identify the required mode effects. As such, we further adapt GMM to incorporate auxiliary data from a single-mode survey of the same population to obviate the need for this assumption. We then use the single-mode data available from UKHLS Wave 8 in our application to assess the impact of no effect modification on our results.

Finally for Aim B, we consider the important problem of estimating the effect of mode on the estimates of parametric statistical models when both outcome and covariates are subject to mode effects. Park et al. (2016) devised a method based on *fractional imputation* to do this. We propose and apply an alternative approach, again based on generalizing an approach suggested by Vannieuwenhuyze (2015), in which estimates of mode effects on the sufficient statistics are combined to estimate mode effects on the parameter estimates from exponential family and generalized linear models.

2 Mode Effects

2.1 Survey measurement

Let $\mathbf{Y}_i$ represent the values of the survey variables recorded for sample member i. To represent the effect of mode on the *measurement* of these variables, we define the pair of potential mode outcomes $\mathbf{Y}_{0i}$ and $\mathbf{Y}_{1i}$ (Vannieuwenhuyze et al. 2010; Kolnenikov and Kennedy 2014). The first ($\mathbf{Y}_{0i}$) contains the values which will be recorded for individual i if the survey is administered in face-to-face mode, and the second ($\mathbf{Y}_{1i}$) contains the values which will be obtained if web mode is used. In practice, only one of the potential mode outcomes is observed: if $D_i \in \{0, 1\}$ indicates the mode chosen by individual i ($D_i = 0$ if i chooses face-to-face, $D_i = 1$ if web) then the observed data are $\mathbf{Y}_i = (1 - D_i)\mathbf{Y}_{0i} + D_i\mathbf{Y}_{1i}$.

The principal focus of this application is on the effect of introducing web mode to the study. For Aim A, this involves comparing the observed mixed-modes distribution of the survey variables, $F(\mathbf{Y})$, with the counterfactual distribution that would have been observed had only face-to-face mode been available, $F(\mathbf{Y}_0)$. Such a comparison hinges on those who chose web mode because $\mathbf{Y}_{0i}$ is unknown for them but observed for those who chose face-to-face mode. We note that it is also possible to compare $F(\mathbf{Y})$ with $F(\mathbf{Y}_1)$ (this simply involves recoding $0 = 1$ and $1 = 0$ for D_i in the development below), and the two counterfactual scenarios $F(\mathbf{Y}_0)$ and $F(\mathbf{Y}_1)$ (obtained by combining the two comparisons of $F(\mathbf{Y}_0)$ and $F(\mathbf{Y}_1)$ with $F(\mathbf{Y})$).

We follow Vannieuwenhuyze (2015) by characterizing the overall effect of mode by the set of *univariate* effects of mode on the means and variances of the survey variables, and the *bivariate* effects of mode on the covariances between pairs of survey variables. The simplest measure of the impact of mode on the mean of the distribution is the additive difference between the observed mean and the face-to-face mean

$$E(\mathbf{Y}_i - \mathbf{Y}_{0i}) = \pi \boldsymbol{\mu}_1, \tag{1}$$

where $\boldsymbol{\mu}_1 = E(\mathbf{Y}_{1i} - \mathbf{Y}_{0i} \mid D_i = 1)$ and $\pi = \Pr(D_i = 1)$ is the probability of a participant choosing web mode. The key parameter $\boldsymbol{\mu}_1$ is the *average* of $\mathbf{Y}_{1i} - \mathbf{Y}_{0i} \neq \boldsymbol{\mu}_1$ among those who choose web, that is, we assume there is between-individual heterogeneity in the effect of mode. Similarly, the univariate additive effect of mode on the covariance of survey variables X_i and Y_i is

$$\operatorname{cov}(X_i, Y_i) - \operatorname{cov}(X_{0i}, Y_{0i}) = \pi \left\{ \mu_{11}^{XY} - E(X_i)\mu_1^Y - E(Y_i)\mu_1^X + \pi \mu_1^X \mu_1^Y \right\},$$

where $\mu_1^X = E(X_i - X_{0i} \mid D_i = 1)$, $\mu_1^Y = E(Y_i - Y_{0i} \mid D_i = 1)$ and $\mu_{11}^{XY} = E(X_i Y_i - X_{0i} Y_{0i} \mid D_i = 1)$. Mode effects can also be measured in other ways, such as multiplicative mode effects on the variance, $\operatorname{var}(Y_i)/\operatorname{var}(Y_{0i})$. In Sections 3-6, we specify structural moment models for estimating the additive and multiplicative effects of mode on the means, variances, covariances and arbitrary moments of the survey-variable distribution.

For Aim B, the focus is on the difference between the observed parameter estimate $\hat{\boldsymbol{\theta}}$ and the estimate $\hat{\boldsymbol{\theta}}_0$ that would have been obtained if only face-to-face mode had been available. In Section 7, we set out how to do this by combining estimates of the mode effects on sufficient statistics of the model parameters obtained using structural moment models.

2.2 Mixed-mode designs

If the sample members of a mixed-mode study are allowed to choose survey mode, the selection mechanism determining D_i will potentially be associated with the survey variables and even the mode effect itself. Plausible selection mechanisms for mixed-mode surveys are discussed by Vannieuwenhuyze et al. (2014) and in Supplementary Information Part A, S1. One way of handling potentially nonrandom selection bias is to adjust for any prechoice differences in control variables $\mathbf{C}_i$ between those who chose web and those who chose face-to-face mode. This can be done using survey weights, regression models, propensity scores or imputation (Jäackle et al. 2010; Lugtig et al. 2011; Kolnenikov and Kennedy 2014; Vannieuwenhuyze et al. 2014; Park et al. 2016; Buelens and Van den Brakel 2017). This approach depends on the ignorable selection assumption $D_i \perp\!\!\!\perp \mathbf{Y}_{0i}, \mathbf{Y}_{1i} \mid \mathbf{C}_i$, which is generally unrealistic because the range of control variables available is usually limited and mode selection is poorly understood Vannieuwenhuyze and Loosveldt (2013).

Randomized experimental designs offer a more robust way of investigating mode effects (Jäckle et al. 2010). Mixed-mode experiments typically involve sequential designs where the sample members are randomly allocated a mode but, if they decline to participate, are offered another mode according to a predetermined sequence until they either participate or there are no more options available. Sequential designs are thus subject to noncompliance: if M_i is the mode to which individual i is initially randomized, noncompliance arises if $D_i \neq M_i$. However, the initial randomization M_i can be used as an IV to identify the mode effect even if mode selection is nonignorable (see Section 3).

2.3 UKHLS Wave 8

A sequential experiment was implemented as part of the complex design of UKHLS Wave 8 (Carpenter 2018, sec. 1). We focus on the three major sample groups created by this design. The first two groups comprise the auxiliary data used in Sections 5.3 and 6.2. First, a randomly selected 20 percent of the sample households were assigned to the *ringfenced* group for which

UKHLS was carried out exactly as in previous years. A further 16 percent of the sample members were automatically assigned to face-to-face interview in the *low-propensity* group because their predicted probabilities of responding via the web were judged to be too low.

Finally, the remaining 64 percent of households were incorporated into the sequential mixed-mode experiment. The sequential experiment involved randomizing 60 percent of the remaining households to web and 40 percent to face-to-face interview. Each individual was initially invited to participate using the mode to which their household was randomized, and would either agree to take part (thus complying with the randomization) or not. Those sample members who did not wish to participate were offered the other mode, at which point they could agree to participate (as noncompliers) or nonrespond. Table 1 displays the initial (randomized) mode allocation by the final choice of mode in the sequential experiment, and the numbers in the ringfenced and low-propensity groups. We exclude those participants in the experimental group who were eventually interviewed by telephone, and those in the ringfenced and low-propensity groups who were eventually interviewed by web (515 in total), because the numbers are relatively small and their inclusion would complicate the subsequent development while making little difference to the final results.

Table 1: Mode Used to Administer Survey by Sample Group Membership in UKHS Wave 8

	Sequential Experiment				Ringfenced	Low-propensity
Randomization	F2F[1]	Web	Total	% Complied	F2F	F2F
F2F	4893	279	5172	94.6		
Web	2367	5278	7645	69.0		
Total	7260	5557	12817		4220	3921

Note: 1. F2F denotes face-to-face mode.

3 Instrumental Variables Estimation

3.1 The core conditions

IVs are widely used in economics and epidemiology for estimating causal effects in the presence of nonignorable nonrandom selection. In this study, we follow others by proposing to use randomization M_i as an IV (Vannieuwenhuyze et al. 2010). The core conditions under which M_i is a valid IV can be specified using potential outcomes notation (Angrist et al. 1996): let $\mathbf{Y}_{(md)i}$ be the potential outcomes of the survey variables should survey member i be randomized to mode m and choose mode d, so there are four potential outcomes of which only $Y_i = \sum_{m,d} I(M_i = m, D_i = d)Y_{(md)i}$ is observed. The IV core conditions can thus be stated as follows:

1. Independence: $M_i \perp\!\!\!\perp Y_{(00)i}, Y_{(10)i}, Y_{(01)i}, Y_{(11)i}$;

2. Exclusion restriction: $Y_{(md)i} = Y_{di}$ for all m, d;

3. Association: $M_i \not\perp\!\!\!\perp D_i$.

The exclusion restriction requires the survey measurements to be the same whether or not the respondents comply with their initial randomization. In clinical trials without double blinding, this assumption is questionable because there may be adverse impacts on patients after randomization (e.g. early-treatment side effects) which affect the study outcomes even if they choose another treatment. However, we take it to be highly unlikely that the choice of mode in a sequential mixed-modes experiment will lead to such effects, particularly as the design protocol is to offer unhappy respondents the option of completing the questionnaire using the other mode. Second, independence requires that M_i is independent of the characteristics measured by the survey so is plausibly taken to hold if M_i is randomized. Finally, that M_i and D_i are associated is straightforwardly verified from the observed data. (Note that the stable unit treatment value assumption (SUTVA), that an individual's potential outcomes do not depende on those of any other individual, is also taken to hold.)

3.2 Structural mean models

The approach we take is based on structural mean models (SMMs). A SMM is explicitly parameterized in terms of the causal effects of a treatment, or treatment regimen, among those who receive the treatment, and can be estimated using IVs (Robins 1994; Clarke and Windmeijer 2010; Vansteelandt and Joffe 2014). An example of an additive SMM for the causal effect of treatment $D_i \in \{0,1\}$ on outcome Y_i given baseline covariates $\mathbf{X}_i$ and instrumental variable Z_i is

$$E(Y_i - Y_{0i} \mid D_i, Z_i, \mathbf{X}_i) = \mu_1(\mathbf{X}_i)D_i,$$

where the analyst must specify a parametric model for treatment effect $\mu_1(\mathbf{X}_i)$, for example, a linear model $\mu_1(\mathbf{X}_i) = \boldsymbol{\theta}^T\mathbf{X}_i$. The parametric model for $\mu_1(\mathbf{X}_i)$ is explicitly specified to capture the dependence of the treatment effect on the baseline covariates.

In the context of the application in this paper, the 'treatment' is web mode, the 'control' is face-to-face mode, the IV is the mode to which each household member is initially randomized, M_i, and there are no baseline covariates. Thus, the SMM above simplifies as

$$E(Y_i - Y_{0i} \mid D_i, M_i) = \mu_1 D_i, \tag{2}$$

where $\mu_1 = E(Y_{1i} - Y_{0i} \mid D_i = 1)$ is the average mode effect among those who choose web discussed in Section 2.1.

Inferences about the parameters of SMMs are made using *g-estimation* (Robins 1994). G-estimation involves constructing estimating equations the solution to which is consistent and asymptotically normal under standard regularity conditions. The estimating equations for SMM (2) follow if M_i satisfies IV core conditions 1-2 above, which imply the conditional mean independence (CMI) restriction

$$E(Y_{0i} \mid M_i) = \mu_0, \tag{3}$$

where $\mu_0 = E(Y_{0i})$. In other words, the mean of the face-to-face responses does not depend on randomization. Then, by the law of iterated expectations, CMI (3) can be rewritten as

$$E(Y_{i0} - \mu_0 \mid M_i) = E\{E(Y_{0i} \mid D_i, M_i) - \mu_0 \mid M_i\} = E(U_i \mid M_i) = 0,$$

where $U_i = Y_i - \mu_0 - \mu_1 D_i$ is the SMM residual, and the last equality follows because $E(Y_{i0} \mid D_i, M_i) = E(Y_i \mid D_i, M_i) - \mu_1 D_i$ under (2).

The estimating equation is the sample analogue of $E\{\mathbf{a}(M_i)U_i\} = \mathbf{0}$, where the choice of $\mathbf{a}(M_i)$ does not affect consistency of the estimator but does affect its precision. The choice $\mathbf{a}^*(M_i) = (1, \pi_i)^T/\sigma_U^2$ leads to estimating equation

$$\frac{1}{\sigma_U^2} \sum_i \begin{pmatrix} 1 \\ \pi_i \end{pmatrix} U_i = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{4}$$

where $\pi_i = \Pr(D_i = 1 \mid M_i)$ is the predicted value of D_i given M_i that depends on M_i under core condition 3. The g-estimator $(\hat{\mu}_0, \hat{\mu}_1)$, the solution to (4), is semiparametrically efficient and satisfies

$$\sqrt{n} \begin{pmatrix} \hat{\mu}_0 - \mu_0 \\ \hat{\mu}_1 - \mu_1 \end{pmatrix} \sim N \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \frac{\sigma_U^2}{\text{var}(\pi_i)} \begin{pmatrix} E(\pi_i^2) & -\pi \\ -\pi & 1 \end{pmatrix} \right\}$$

as sample size $n \to \infty$ (Robins 1994).

3.3 The no effect modification assumption

The estimand of $\hat{\mu}_1 = \{E(Y_i \mid M_i = 1) - E(Y_i \mid M_i = 0)\}/\{\pi(1) - \pi(0)\}$, where $\pi(m) = \Pr(D_i = 1 \mid M_i = m)$, only equals μ_1 if the *no effect modification* (NEM) assumption that $E(Y_{1i} - Y_{i0} \mid D_i = 1, M_i = 0) = E(Y_{1i} - Y_{0i} \mid D_i = 1, M_i = 1)$ holds. In the presence of nonignorable selection, the IV core conditions only identify bounds on causal effects if there is heterogeneity such that $Y_{1i} - Y_{0i}$ varies between individuals; further assumptions are required to point identify causal effects (e.g. Hérnan and Robins 2020, ch. 16). There are two broad families of assumptions: the first is to assume that selection is *monotonic* to identify a *local average treatment effect* (Angrist et al. 1996); and the second is make assumptions that directly constrain causal-effect heterogeneity to identify effects like the *average treatment effect among the treated*.

The g-estimator $\hat{\mu}_1$ is identical to the classical two-stage least squares (2SLS) estimator of the effect of D_i from the linear regression of Y_i on D_i using M_i as an IV. However, the 2SLS estimand is ambiguously defined to be the *true* coefficient of the linear model for Y_i given D_i rather than a causal parameter. In contrast, SMM (2) makes explicit that NEM must be assumed to identify μ_1. For example, analysts who use 2SLS to make inferences about average treatment effects are implicitly making the assumption $Y_{1i} - Y_{0i} \perp\!\!\!\perp M_i, D_i$, which is stronger than NEM, and those who use 2SLS to estimate local average treatment effects are assuming monotonicity (see Clarke and Windmeijer (2010) and Supplementary Information Part A, S2).

While NEM is typically taken to hold in conventional causal analysis, we argue that, for mode effects, it holds only under the generally implausible assumption that the mode effect is independent of the true value of Y (see Supplementary Information Part A, S1). However, we show in Section 4.3 how to extend our structural moment models to relax NEM, and in Sections 5.3 and 6.2 how to estimate these models using suitable auxiliary data.

4 Structural Moment Models

The specification and identification of a novel family of *structural moment models* is set out below, which generalizes SMM (2) to enable the estimation of mode effects on arbitrary moments (provided these exist) of the survey variables' joint distribution. Identification hinges crucially on M_i satisfying the IV core conditions: specifically, core conditions 1-2 ensure that any finite moment of the joint distribution of the face-to-face responses is mean independent of M_i; and core condition 3 ensures the resulting estimator will exist. We focus initially on *univariate* mode effects before proceeding to *bivariate* and *multivariate* effects in the following section.

The estimation of these models is discussed in Section 5.

4.1 Univariate mode effects

For univariate mode effects, we propose two types of structural moment model (SMoM) for estimating the mode effect on the distribution of single survey variable Y. The first of these is the additive SMoM

$$E(Y_i^k - Y_{0i}^k \mid D_i, M_i) = \mu_k D_i, \tag{5}$$

where $\mu_k = E(Y_{1i}^k - Y_{0i}^k \mid D_i = 1)$ if the attendant NEM assumption that $E(Y_{1i}^k - Y_{0i}^k \mid D_i = 1, M_i = 0) = E(Y_{1i}^k - Y_{0i}^k \mid D_i = M_i = 1)$ holds. This is simply SMM (2) but with Y replaced by Y^k.

The second, appropriate if $Y^k > 0$, is the multiplicative, or log-linear, SMoM

$$\log\{E(Y_i^k \mid D_i, M_i)\} - \log\{E(Y_{0i}^k \mid D_i, M_i)\} = \lambda_k D_i, \tag{6}$$

where $\exp(\lambda_k) = E(Y_{1i}^k \mid D_i = 1)/E(Y_{0i}^k \mid D_i = 1)$ if the NEM assumption that $E(Y_{1i}^k \mid D_i = 1, M_i)/E(Y_{0i}^k \mid D_i = 1, M_i)$ does not depend on M_i holds. Parameter $\exp(\lambda_k)$ is the ratio of moment k for the web data to that for the face-to-face data among those who choose web. Note that the log-linear SMoM for $k = 1$ is the log-linear SMM (Robins 1994).

Both μ_k and $\exp(\lambda_k)$ are valid measures of the effect of mode on moment $k \neq 0$ of the distribution of Y. The estimating equations for both SMoM (5) and (6) follow because core conditions 1-2 imply the CMI restriction

$$E(Y_{0i}^k \mid M_i) = \mu_{0k}, \tag{7}$$

where $\mu_{0k} = E(Y_{0i}^k)$ and $\mu_{01} = \mu_0$. Using exactly the same arguments as for SMM (2), it follows that the residual U_{ik} satisfying $E(U_{ik} \mid M_i) = 0$ for SMoM (5) is

$$U_{ik} = Y_i^k - \mu_{0k} - \mu_k D_i, \tag{8}$$

where $\hat{\mu}_k = \{\bar{m}_k(1) - \bar{m}_k(0)\}/\{\pi(1) - \pi(0)\}$ and $\bar{m}_k(m) = E(Y_i^k \mid M_i = m)$, and recalling that $\pi(m) = \Pr(D_i = 1 \mid M_i = m)$ for $m = 0, 1$. Using similar arguments, the residual for multiplicative SMoM (6) is

$$V_{ik} = \exp(-\lambda_k D_i) Y_i^k - \mu_{0k}, \tag{9}$$

which leads to estimator

$$\exp(\hat{\lambda}_k) = \frac{\bar{m}_k(0) - \bar{m}_k(1) + \{1-\pi(1)\}\bar{m}_k(1,0) - \{1-\pi(0)\}\bar{m}_k(0,0)}{\bar{m}_k(1) - \bar{m}_k(0) + \pi(0)\bar{m}_k(0,1) - \pi(1)\bar{m}_k(1,1)},$$

where $\bar{m}_k(m,d) = E(Y_i^k \mid M_i = m, D_i = d)$.

We now discuss two important special cases of univariate mode effect. The first is the multiplicative effect of mode on the variance, which we propose to estimate using the novel Structural Variance Model (SVM)

$$\log\left\{\frac{\text{var}(Y_i \mid D_i, M_i)}{\text{var}(Y_{0i} \mid D_i, M_i)}\right\} = \nu D_i. \tag{10}$$

NEM here holds if $\text{var}(Y_{1i} \mid D_i, M_i)/\text{var}(Y_{0i} \mid D_i, M_i)$ does not depend on M_i, in which case the SVM estimand is $\exp(\nu) = \text{var}(Y_{1i} \mid D_i = 1)/\text{var}(Y_{0i} \mid D_i = 1)$, that is, the ratio of the web and face-to-face variances among those who choose web. Core conditions 1-2 again lead to CMI (7) for $k = 1$ and 2 and residual

$$W_i = \exp(-\nu D_i)\epsilon_i^2 + \{\beta_0 + \beta_1 M_i + (\beta_2 - \mu_1)D_i + \beta_{12} M_i D_i\}^2 - \mu_{02}, \tag{11}$$

where ϵ_i is the residual of the linear 'association model' $Y_i = \beta_0 + \beta_1 M_i + \beta_2 D_i + \beta_{12} M_i D_i + \epsilon_i$. This depends on the parameter of additive SMM (2) and the parameters of the association model for Y_i as well as ν. The estimator $\hat{\nu}$ and its derivation are detailed in Supplementary Information Part A, S3.3; it can be viewed as an extension of Vannieuwenhuyze (2015) to unrestricted sequential designs.

The second special case is for binary and nominal categorical variables, where the mode effect is most straightforwardly characterized by the effects on the probabilities of the nonreference categories. If $Y \in \{0, 1, \ldots, L\}$ without loss of generality, these effects follow the multivariate linear SMM

$$E(\mathbf{Y}_i - \mathbf{Y}_{0i} \mid D_i, M_i) = \boldsymbol{\gamma}_1 D_i, \tag{12}$$

where $\mathbf{Y}_i = (I(Y_i = 1), \ldots, I(Y_i = L))^T$ is a vector of L dummy variables (that is, excluding reference category 0), $\mathbf{Y}_{0i}$ its face-to-face potential-outcome equivalent, $I(.)$ the indicator function equalling one if its argument is true or zero otherwise, $\boldsymbol{\gamma}_1 = (\gamma_{11}, \ldots, \gamma_{1L})^T$ and $\gamma_{1l} = \Pr(Y_{1i} = l \mid D_i = 1) - \Pr(Y_{0i} = l \mid D_i = 1)$ is the effect of mode on the mass point for $Y_i = l$ for $l = 1, \ldots, L$. Identification follows from L pairs of NEM and CMI (3) assumptions for each of the dummy variables (see Supplementary Information Part A, S3.2 and c.f. Imbens and Rubin (1997)).

4.2 Bivariate and multivariate mode effects

For the bivariate distribution of continuous X and Y, core conditions 1-2 ensure that the following CMI moment restriction holds:

$$E(X_{0i}^j Y_{0i}^k \mid M_i) = \mu_{0jk}^{XY}, \tag{13}$$

where $\mu_{0jk} = E(X_{0i}^j Y_{0i}^k)$ for any real-valued $j, k \neq 0$. The generalization of (13) to three or more variables is straightforward, but this CMI restriction identifies the additive SMoM

$$E(X_i^j Y_i^k - X_{0i}^j Y_{0i}^k \mid D_i, M_i) = \mu_{jk}^{XY} D_i \tag{14}$$

where $\mu_{jk}^{XY} = E(X_{1i}^j Y_{1i}^k - X_{0i}^j Y_{0i}^k \mid D_i = 1)$ if the attendant NEM assumption holds. Identification follows from solving (13) as before to obtain residual

$$U_{ijk}^{XY} = X_i^j Y_i^k - \mu_{0jk}^{XY} - \mu_{jk}^{XY} D_i, \tag{15}$$

which leads to estimator $\hat{\mu}_{jk}^{XY} = \{\bar{m}_{jk}^{XY}(1) - \bar{m}_{jk}^{XY}(0)\}/\{\pi(1) - \pi(0)\}$ where $\bar{m}_{jk}^{XY}(m) = E(X_i^j Y_i^k \mid M_i = m)$.

We now consider two special cases, the bivariate equivalents of the SVM and multivariate SMM defined above. The additive Structural Covariance Model (SCM) is

$$\text{cov}(X_i, Y_i \mid D_i, M_i) - \text{cov}(X_{0i}, Y_{0i} \mid D_i, M_i) = \sigma^{XY} D_i, \tag{16}$$

where $\sigma^{XY} = \text{cov}(X_{1i}, Y_{1i} \mid D_i = 1) - \text{cov}(X_{0i}, Y_{0i} \mid D_i = 1)$ under its attendant NEM assumption. This model is identified by CMI (3) for X and for Y and CMI (13) for $k = j = 1$, which lead to residual

$$W_i^{XY} = \epsilon_i^X \epsilon_i^Y + (U_i^X + \mu_0^X - \epsilon_i^X)(U_i^Y + \mu_0^Y - \epsilon_i^Y) - \sigma^{XY} D_i - \mu_{011} \tag{17}$$

satisfying $E(W_i^{XY} \mid M_i) = 0$, where ϵ_i^X and ϵ_i^Y are respectively the residuals of the association models for X_i and for Y_i, and U_i^X and U_i^Y are respectively the residuals for SMM (2) for X and for Y. Identification and derivation of estimator $\hat{\sigma}^{XY}$ is described in Supplementary Information Part A, S3.4. As with the SVM, the resulting estimator can be viewed as an extension of Vannieuwenhuyze (2015) to unrestricted sequential designs.

Lastly, for two nominal categorical (or binary) variables, the mode effect is again most straightforwardly characterized by mode effects on the probabilities of the nonreference categories. If $X \in \{0, 1, \ldots, L_X\}$ and $Y \in \{0, 1, \ldots, L_Y\}$ without loss of generality, these effects follow the multivariate linear SMM

$$E(\mathbf{Y}_i \otimes \mathbf{X}_i - \mathbf{Y}_{0i} \otimes \mathbf{X}_{0i} \mid D_i, M_i) = \boldsymbol{\gamma}^{XY} D_i, \tag{18}$$

where $\mathbf{X}_{di} = (I(X_i = 0), \ldots, I(X_i = L_X))^T$ and $\mathbf{Y}_{di}$ is similarly defined, $\otimes$ is the Kronecker product, $\boldsymbol{\gamma}^{XY} = (\gamma_{00}^{XY}, \gamma_{10}^{XY}, \gamma_{20}^{XY}, \ldots, \gamma_{L_X L_Y}^{XY})^T$, and $\gamma_{lm}^{XY} = \Pr(X_{1i} = l, Y_{1i} = m \mid D_i = 1) - \Pr(X_{0i} = l, Y_{0i} = m \mid D_i = 1)$ is the effect of mode on the mass point for $X_i = l$ and $Y_i = m$.

4.3 Relaxing the NEM assumpion

All of the SMoMs (including the SVM and SCM) defined above rely on the NEM assumption to ensure the estimands of the g-estimators equal the target mode effects among those who choose web. To demonstrate how NEM is relaxed, we focus on additive SMoM (5) for univariate mode effects, but the same approach could be used for any member of the SMoM family. The approach is based on *extended* SMoMs with separate parameters for the mode effect for those randomized to face-to-face and those randomized to web mode. For the additive SMoM,

$$E(Y_{i1}^k - Y_{i0}^k \mid D_i, M_i) = \{\mu_k(0) + \Delta\mu_k M_i\} D_i, \tag{19}$$

where $\Delta\mu_k = \mu_k(1) - \mu_k(0)$ and $\mu_k(m) = E(Y_{1i}^k - Y_{0i}^k \mid D_i = 1, M_i = m)$ is the causal effect in group m. This model has two parameters but these cannot be identified by the single CMI restriction (7): further data constraints are required. We can hence assess whether NEM holds by testing $\Delta\mu_k = 0$. In Section 5.3, we show how auxiliary data from UKHLS Wave 8 can be used to estimate μ_{0k} and thus permit the identification of the extended SMoM, SVM and SCM.

5 Parameter Estimation

5.1 The generalized method of moments

In the previous section, it was explained how core conditions 1-3 (via CMI moment restrictions such as (7) and (13)) together with NEM identify the SMoM parameter, but that NEM can be relaxed if auxiliary data are available with which to estimate μ_{0k}. The g-estimator $\hat{\boldsymbol{\theta}}$ for one of the SMoMs introduced above is generally based on residual vector $\mathbf{r}_i = \mathbf{r}_i(\boldsymbol{\theta})$ constructed under the model to satisfy conditional moment restriction $E(\mathbf{r}_i \mid M_i) = \mathbf{0}$. The g-estimate is obtained by solving $n^{-1}\sum_i A_i\hat{\mathbf{r}}_i = \mathbf{0}$, where $\hat{\mathbf{r}}_i = \mathbf{r}_i(\hat{\boldsymbol{\theta}})$ and the choice of matrix $A_i = A_i(M_i)$ does not affect consistency but does affect precision because, under standard regularity conditions, $\sqrt{n}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \sim N\left\{\mathbf{0}, E(A_i\mathbf{r}_i\mathbf{r}_i^T A_i^T)^{-1}\right\}$ as $n \to \infty$, where the semiparametrically efficient choice is $A_i^* = E(-\partial\mathbf{r}_i^T/\partial\boldsymbol{\theta} \mid M_i)$ (see Robins (1994) and Supplementary Information Part A, S3).

Clarke et al. (2015) showed how g-estimation can be carried out using the generalized method of moments (GMM). We extend this to the SMoMs (including the SVM and SCM) specified above. In general, a GMM estimator $\hat{\boldsymbol{\theta}}$ is

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \left\{ n^{-1}\sum_i \mathbf{g}_i^T(\boldsymbol{\theta}) \right\} \hat{W}_n \left\{ n^{-1}\sum_i \mathbf{g}_i(\boldsymbol{\theta}) \right\}, \tag{20}$$

where $\hat{W}_n$ is a consistent estimator of symmetric weight matrix W. The GMM estimator is consistent and asymptotically normal under standard regularity conditions if $E\{\mathbf{g}_i(\boldsymbol{\theta})\} = \mathbf{0}$ (Hansen 1982). If the dimension of $\mathbf{g}_i(\boldsymbol{\theta})$ exceeds that of $\boldsymbol{\theta}$ then W can be chosen to minimize the asymptotic variance-covariance matrix. However, if the dimensions of $\mathbf{g}_i(\boldsymbol{\theta})$ and $\boldsymbol{\theta}$ are equal then $\hat{W}_n$ is redundant: $\hat{\boldsymbol{\theta}}$ is simply a method of moments estimator with asymptotic variance-covariance

$$\text{var}(\hat{\boldsymbol{\theta}}) = \frac{1}{n}\left[E\left\{\frac{\partial \mathbf{g}_i^T(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right\} E\left\{\mathbf{g}_i(\boldsymbol{\theta})\mathbf{g}_i^T(\boldsymbol{\theta})\right\}^{-1} E\left\{\frac{\partial \mathbf{g}_i(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^T}\right\}\right]^{-1}. \tag{21}$$

We focus on estimating equations of *IV models* of the form $\mathbf{g}_i(\boldsymbol{\theta}) = Z_i\mathbf{r}_i(\boldsymbol{\theta})$, where Z_i is now the *GMM instrument*, a matrix that can depend on parameters of the SMoM, and $\mathbf{r}_i(\boldsymbol{\theta})$ is the *GMM residual* satisfying $E\{\mathbf{r}_i(\boldsymbol{\theta}) \mid Z_i\} = \mathbf{0}$. Hansen (1982) showed that the GMM estimator is optimally efficient given the analyst's choice of GMM instrument Z_i, but in practice the analyst ideally seeks a feasible Z_i such that the estimator is either locally or semiparametrically efficient.

Clarke et al. (2015) showed that the form of the g-estimator and GMM estimator are the same if the SMM residual is constructed to have mean zero, and that the GMM estimator based on $Z_i = (1, M_i)^T$ is exactly the same as that based on $Z_i = A_i^*$, the efficient choice for g-estimators, if M_i and D_i are both binary and $\mathbf{r}_i$ is linear. The final condition obviously holds for additive SMoMs (5) and SCM (16), but also holds for multiplicative SMoM (6) and SVM (10). For example, the multiplicative SMoM residual (9) can be written $V_{ik} = (1 - D_i)Y_i^k - \mu_{0k} - \exp(-\lambda_k)D_i$ for binary D_i, that is, the residual of the linear regression of $(1 - D_i)Y_i^k$ on D_i.

We now outline how to set up the GMM estimator for some of the SMoMs introduced in Section 4. The *Stata* code for implementing these models can be found in Supplementary Information Part B, S10.

Example 1: Univariate SMoM ($k = 2$) and bivariate SMoM ($j = k = 1$). The GMM residuals for additive SMoM (5) and multiplicative SMoM (6) for $k = 2$ are respectively $\mathbf{r}_i(\mu_{01}, \mu_1) = U_i$ and $\mathbf{r}_i(\mu_{01}, \lambda_2) = V_i$. Likewise, the GMM residual for bivariate additive SMoM (14) is $\mathbf{r}_i(\mu_{0jk}, \mu_{jk}) = U_{ijk}^{XY}$. In all three cases, the IV instrument is $Z_i = \mathbf{Z}_i$ where $\mathbf{Z}_i = (1, M_i)^T$, that is, the GMM instrument is a vector.

Example 2: SVM. The GMM residual for SVM (10) is

$$\mathbf{r}_i(\boldsymbol{\theta}) = (\epsilon_i, U_i, W_i)^T, \tag{22}$$

where $\boldsymbol{\theta} = (\beta_0, \beta_1, \beta_2, \beta_{12}, \mu_{01}, \mu_1, \mu_{02}, \nu)^T$ and, we recall, $\epsilon_i = Y_i - \beta_0 - \beta_1 M_i - \beta_2 D_i - \beta_{12} M_i D_i$ is the residual of the linear association model. This residual includes that of the association model as well as those for SMM (2) and (10) so that all the parameters can be jointly estimated. The instrument combines $\mathbf{Z}_i$, the instrument for the additive and multiplicative SMMs, together with the predictors of the association model as follows:

$$Z_i = \begin{pmatrix} \mathbf{X}_i & \mathbf{0} & \mathbf{0} \\ & \mathbf{Z}_i & \mathbf{0} \\ & & \mathbf{Z}_i \end{pmatrix}, \tag{23}$$

that is, an 8×3 matrix where $\mathbf{X}_i = (1, M_i, D_i, M_i D_i)^T$ and $\mathbf{0}$ indicates a conformable vectors of zeros. The resulting GMM estimator is generally not semiparameterically efficient, but is locally efficient in the sense of being semiparametrically efficient if ϵ_i is homoscedastic (Tsiatis 2006, p. 94).

Example 3: SCM. The GMM residual for SCM (16) is

$$\mathbf{r}_i(\boldsymbol{\theta}) = (\epsilon_i^X, \epsilon_i^Y, U_i^X, U_i^Y, W_i^{XY})^T \tag{24}$$

where $\boldsymbol{\theta} = (\beta_0^X, ..., \beta_{12}^Y, \mu_{01}^X, \mu_1^X, \mu_{01}^Y, \mu_1^Y, \mu_{011}^{XY}, \sigma^{XY})^T$ and ϵ_i^X and ϵ_i^Y are respectively the residuals of the association models for X and Y, and U_i^X and U_i^Y are respectively the SMM residuals for X and Y. The GMM instrument is

$$Z_i = \begin{pmatrix} \mathbf{X}_i & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & \mathbf{X}_i & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & & \mathbf{Z_i} & \mathbf{0} & \mathbf{0} \\ & & & \mathbf{Z}_i & \mathbf{0} \\ & & & & \mathbf{Z}_i \end{pmatrix}. \tag{25}$$

Example 4: Multivariate SMM. The GMM residual for multivariate SMM (12) for categorical variable $Y \in \{0, 1, \ldots, L\}$ is

$$\mathbf{r}_i(\boldsymbol{\theta}) = (U_{1i}, \ldots, U_{Li})^T \tag{26}$$

where $U_{li} = I(Y_i = l) - \gamma_{0l} - \gamma_{1l} D_i$ and $\gamma_{0l} = \Pr(Y_{0i} = l)$. The GMM instrument is the $2L \times L$ matrix

$$Z_i = \begin{pmatrix} \mathbf{Z}_i & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \\ & \mathbf{Z}_i & \mathbf{0} & \ldots & \mathbf{0} \\ & & \vdots & & \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{Z}_i \end{pmatrix}. \tag{27}$$

The *Stata* code for fitting multivariate SMMs (12) for univariate effects and (18) for bivariate effects is respectfully presented in Supplementary Information Part B, S10.4 and S10.5.

5.2 Adjusting for nonresponse bias and complex sampling designs

In the context of a single wave from a panel study like UKHLS, *wave* nonresponse arises whenever the sample members cannot be contacted, or refuse to participate, in the current wave, and *item* nonresponse arises whenever a participating sample member does not respond to one or more questions. However it arises, nonresponse leads to incompletely observed data which must be analyzed carefully. If the nonresponse mechanism is associated with the characteristics measured by the survey variables, naive estimators of the analytical model parameters will be biased and inefficient. We treat nonresponse as a nuisance and seek to adjust the GMM estimating equations set out above by using the survey weights supplied with UKHLS (ISER 2018, pp. 40-44). These weights w_i incorporate a) design weights to adjust for unequal probabilities of selection, b) calibration weights to adjust the sample distribution to match the known population distributions of certain variables, and c) nonresponse weights to adjust for the dependence on wave nonresponse at Wave 8 on auxiliary variables and fully observed survey variables. Hence, for an analysis of the survey variables $\mathbf{Y}_i$ (or a subset thereof), we make the working assumption that

$$R_i^{cc} \perp\!\!\!\perp \mathbf{Y}_i \mid D_i, M_i, \mathbf{C}_i, \tag{28}$$

where $R_i^{cc} = I(\mathbf{Y}_i \text{ is observed})$ and $\mathbf{C}_i$ contains the variables used to construct the survey weights (noting that $\mathbf{C}_i$ enters the estimation procedure only through w_i). The limitations of this approach are discussed in Section 8.

Finally, as with many large-scale surveys, UKHLS uses a complex sampling design (in this case, a stratified multi-stage cluster sampling design) but the discussion of point and interval estimation so far has implicitly presumed that a simple random sampling design was used. We account for the potential effects of unequal selection probabilities, clustering and stratification on estimation by incorporating the survey weights into GMM estimation and using a *linearized* variance estimator rather than (21) to estimate the standard errors. Further details of the implementation are described in Supplementary Information Part A, S3.5, with the *Stata* code provided in Supplementary Information Part B, S10.2.

5.3 Incorporating auxiliary data

Up until this point, we have considered only estimation based on data from the survey units $\mathcal{S}$ involved in the sequential mixed-modes experiment. However, if suitable auxiliary data are available then these can be used to improve inferences about the mode effects in terms of both bias and precision.

Suppose that auxiliary sample $\mathcal{A}$ is drawn from the same population as $\mathcal{S}$ and measures the same survey variables $\mathbf{Y}_i$. Then define sample membership indicator S_i for all $i \in \mathcal{C}$, where $\mathcal{C} = \mathcal{S} \cup \mathcal{A}$ is the combined sample, such that $S_i = 1$ if $i \in \mathcal{S}$ and $S_i = 0$ if $i \in \mathcal{A}$. Then μ_{0k} can be estimated from the auxiliary data if

$$E(Y_i^k \mid S_i = 0) = E(Y_{0i}^k), \tag{29}$$

for any survey variable Y for target moment $k \neq 0$. It is thus clear that suitable auxiliary data must come from a survey administered using only face-to-face mode so that $\mathbf{Y}_i = \mathbf{Y}_{0i}$ for all $i \in \mathcal{A}$. The extension of this result to higher-order moments is obvious.

We can use these auxiliary data for two purposes: the first is **better inference**. From Table 1, it can be seen that M_i and D_i are strongly associated: the difference in the proportions choosing web among those randomized to web and face-to-face modes is $0.69-(1-0.95) = 0.64$. This corresponds to an F statistic of 8437 where the rule of thumb for identifying *weak* IVs, for which we can expect 2SLS to be severely biased, is a value less than 10 (Stock and Yogo 2005). However, the F-statistic rule is, strictly speaking, a guide that pertains only to 2SLS and the additive SMM.

We hence conduct a simulation study to assess the performance of our estimators for additive and multiplicative SMoMs and a SVM. The study design and results are described in

Supplementary Information Part A, S5, Table S5.1. The results show that, for sample sizes of order 10000 (as in the application in this paper), these estimators have small relative biases (for both coefficient and standard error estimates) but not for smaller sample sizes of 100 or 1000. Hence, the study also investigates whether estimating μ_{0k} using auxiliary data improves the performance of the GMM estimator for smaller sample sizes. Auxiliary data can be incorporated into GMM estimation as shown in the following example. The GMM estimator for SVM (10) is extended to estimate μ_{01} and μ_{02} as follows:

$$\mathbf{r}_i(\boldsymbol{\theta}) = (Y_i - \mu_{01}, Y_i^2 - \mu_{02}, \epsilon_i, U_i, W_i)^T, \tag{30}$$

and GMM instrument

$$Z_i = \begin{pmatrix} 1-S_i & 0 & 0 & 0 & 0 \\ & 1-S_i & 0 & 0 & 0 \\ & & S_i\mathbf{X}_i & \mathbf{0} & \mathbf{0} \\ & & & S_i\mathbf{Z}_i & \mathbf{0} \\ & & & & S_i\mathbf{Z}_i \end{pmatrix}. \tag{31}$$

The GMM instrument has 10 rows but there are only 8 parameters so the resulting estimator is a genuine *overidentified* GMM rather than a method of moments. Hence, we can either use *twostep* GMM, based on the asymptotically optimal choice of W (Hansen 1982), or replace $\mathbf{Z}_i$ with M_i in (31). The study results using both approaches show improved relative bias for smaller sample sizes.

Relaxing NEM: In a similar manner, the auxiliary data can also be used to fit extended SMoMs like (19). For example, consider estimating the extended SVM

$$\log\{\text{var}(Y_i \mid D_i, M_i)\} - \log\{\text{var}(Y_{0i} \mid D_i, M_i)\} = \{\nu(0) + \Delta\nu M_i\}D_i, \tag{32}$$

jointly with the extended SMoM (19) for $k = 1$. Each of these models has two parameters: the first ($\nu(0)$ and $\mu_1(0)$) correspond to the mode effects given $(M_i, D_i) = (0, 1)$, and the second ($\Delta\nu$ and $\Delta\mu_1$) to the differences between the mode effects for $(M_i, D_i) = (1, 1)$ and $(M_i, D_i) = (0, 1)$.

The extended residuals $\tilde{W}_i$ and $\tilde{U}_i$, which respectively replace W_i and U_i in GMM residual

(30), are

$$\tilde{W}_i = \epsilon_i^2 \exp[-\{\nu(0) + \Delta\nu M_i\}D_i] \\ + [\beta_0 + \beta_1 M_i + \{\beta_2 - \mu_1(0)\}D_i + (\beta_{12} - \Delta\mu_1)M_i D_i]^2 - \mu_{02}, \tag{33}$$

and

$$\tilde{U}_i = Y_i - \mu_{01} - \{\mu_1(0) + \Delta\mu_1 M_i\}D_i. \tag{34}$$

The GMM instrument is again (31). In the absence of auxiliary data, the GMM instrument would be (22) with only 8 rows for 10 parameters and so $\boldsymbol{\theta}$ would be inestimable.

The design of UKHLS Wave 8, analyzed in Section 6, is more complicated than the simple auxiliary-data scenario described above because the mixed-mode experiment is a selected sample and we have two separate sources of face-to-face auxiliary data available from the ringfenced and low-propensity groups (Section 2.3). It is thus necessary to extend the definition of the sample-membership indicator to be $S_i = 1$ if i is in the mixed-mode experiment, $S_i = 2$ for the low-propensity group, or $S_i = 3$ for the ringfenced group.

Now recall that the low-propensity group includes those sample members excluded from the experimental group because their estimated web-mode response probabilities were low, so that the mode effects being estimated from the UKHLS Wave 8 mixed-mode experiment in Section 6.1 are all implicitly conditional on $S_i = 1$. Hence, we wish to use the auxiliary data to estimate $\mu_{0k} = E(Y_{0i}^k|S_i = 1)$ but the ringfenced data only identify $E(Y_i^k \mid S_i = 3) = E(Y_{0i}^k)$. However, we also know that $E(Y_i^k \mid S_i = 2) = E(Y_{0i}^k \mid S_i = 2)$ from the low-propensity groups, and so

$$\hat{\mu}_{0k} = \frac{E(Y_i^k \mid S_i = 3) - p_{\text{low}} E(Y_i^k \mid S_i = 2)}{p_{\text{expt}}} \tag{35}$$

because $E(Y_{0i}^k) = p_{\text{low}} E(Y_{0i}^k \mid S_i = 2) + p_{\text{exp}} \mu_{0k}$ by the law of iterated expectations, where $p_{\text{low}} = \Pr(S_i = 2)/\Pr(S_i < 3)$ and $p_{\text{expt}} = \Pr(S_i = 1)/\Pr(S_i < 3)$ sum to 1. Note that Y_i^k can be replaced by the mean of $X_i^j Y_i^k$ in the above, and so on. The *Stata* code for incorporating the auxiliary data is given in Supplementary Information Part B, S10.6.

6 Characterizing the Impact of Mode on UKHLS Wave 8

We now present estimates of the mode effects across a range of variables from UKHLS Wave 8. The variables were selected to be illustrative of the full set of UKHLS variables. An illustrative selection of these are described in Table 2, and the full set of variables we considered is listed in Supplementary Information Part B, S9. As set out in Section 2.1, the effect of mode on the joint distribution is characterized by the univariate effects of mode on the means and variances of the survey variables, and the bivariate effects on the covariances between pairs of survey variables. All estimates are weighted using the Wave 8 longitudinal weight `h_indinub_xw` to account for unequal selection probabilities and nonresponse (ISER 2018, pp. 40-44).

Table 2: Description of selected variables and summary statistics from sequential experiment

		Randomization			
		Face-to-face		Web mode	
Continuous or ordinal					
Label	Description	n	Mean (SD)	n	Mean (SD)
`age`	Age at interview	5220	49.7 (18)	7734	49.9 (18)
`scgh1`	Subjective wellbeing (GHQ: 0-36)	4780	10.9 (5)	7372	11.1 (5)
`sf12mcs`	SF-12 mental health component (0-100)	4760	49.4 (10)	7335	48.9 (10)
`sf12pcs`	SF-12 physical health component (0-100)	4760	50.0 (11)	7335	50.0 (11)
`fiyrdia`	Annual income from savings etc. (£100)	3776	1.71 (2.7)	5757	1.86 (2.7)
`workdis`	Distance from workplace (miles)	2697	10.1 (16)	3982	11.9(31)
`ncigs`	No. cigarettes smoked per day	637	12.0 (8)	977	12.5 (10)
`j2pay`	Monthly income from 2nd job (£100)	6766	20.9 (173)	5557	27.1 (261)
Categorical					
Label	Description (categories)	n	Prop.	n	Prop.
`paygwc`	Gross salary payment method	2332		3291	
	Hourly		0.01		0.02
	Weekly		0.11		0.10
	4 weeks		0.06		0.11
	Monthly		0.54		0.62
	Yearly		0.27		0.14
	Other		0.01		0.01
`huboss`	Household financial decisions made by	3443		5083	
	Respondent		0.13		0.10
	Spouse/Partner		0.12		0.10
	Both		0.74		0.79
	Other		0.01		0.00
`jbpl`	Workplace location	2767		3996	
	Home		0.03		0.03
	Employer		0.82		0.83
	Travelling		0.09		0.07
	Other		000		0.01

Note that the summary statistics are not weighted.

6.1 Analysis using only data from the sequential experiment

In this section, we estimate mode effects using data from the mixed-modes experiment implemented for UKHLS Wave 8. These results thus rely on the NEM assumption discussed at length in Sections 3-4, which we argued to be implausible in Section 4.3. However, for practice, the issue is less whether this assumption holds perfectly than whether its failure affects the results. Hence, in the next section, we compare the following results with those incorporating the auxiliary data, for which NEM has been relaxed.

In this spirit, we begin by focusing on univariate mode effects under NEM. For variables with interval, ratio and ordinal measurement scales, we summarize the univariate effect of mode on each variable by the mode effects on its mean μ_1 and variance ν; for dichotomous/binary variables, the effect of mode on its mean is sufficient to capture the effect of mode on its distribution; and, for nominal categorical variables, the mode effect is the set of mode effects on the mass points of the nonreference categories: if one (or more) of these effects is significant then there is evidence to support the presence of a univariate mode effect on this variable. Table 3 contains the results for a selection of variables. The GMM estimates are based on the relevant SMM (2), SVM (10) and multivariate SMM (12). Two sets of standard error estimates are presented: the first is based on the standard formula for *i.i.d.* observations; the second is based on the linearized estimator to account for the impact of stratification and clustering. All continuous variables were standardized to have mean zero and unit standard deviation.

Table 3: Estimated effects of mode on the mean (μ_1) and variance (ν) of selected variables

Variable	Par.	Est.	SE(*i.i.d.*)	SE(lin)
`j2pay`	μ_1	0.065	.029*	.037$^+$
($n_{cc} = 10432$)	ν	1.948	.581***	.712***
`age`	μ_1	0.017	.036	.073
($n_{cc} = 10432$)	ν	-0.024	.036	.056
`fiyrdia`	μ_1	0.091	.037**	.073
($n_{cc} = 7948$)	ν	0.050	.046	.076
`scghq1`	μ_1	0.103	.034***	.055$^+$
($n_{cc} = 10180$)	ν	0.048	.083	.132
`sf12mcs`	μ_1	-0.080	.035**	.057
($n_{cc} = 10141$)	ν	0.052	.069	.104
`sf12pcs`	μ_1	-0.006	.033	.057
($n_{cc} = 10141$)	ν	-0.010	.065	.116
`workdis`	μ_1	0.112	.037**	.048**
($n_{cc} = 5476$)	ν	1.486	.511***	.638**
`ncigs`	μ_1	1.190	.114$^+$	.139
($n_{cc} = 1279$)	ν	1.022	.590$^+$	.410**

Sig. level: *** < 0.01, **$0.01 - 0.025$, *$0.025 - 0.05$ and $^+0.05 - 0.1$
n_{cc} weighted size of complete-cases sample.

To validate these results, we look at the `age` variable because it is mode-invariant so we should infer a null mode effect. This is indeed the case: the estimates of μ_1 and ν for `age` are both smaller than the respective standard error estimates for each. Turning to the other variables, based on the *i.i.d.* standard error estimates, there is evidence (using a 0.05 significance cut-off) for mode effects on the means of `scghq1`, `sf12mcs`, `fiyrdia`, `workdis` and `j2pay` and evidence for mode effects on the variance of `workdis` and `j2pay`. However, the evidence based on the linearized standard error estimates is less strong: there is only evidence for an effect of mode on the variance of `j2pay` and for effects of mode on the mean and variance of `workdis`. This is because the impact of clustering generally increases the size of the standard error estimates; the exception to this is for `ncigs`, where the linearized estimate of the standard error of ν is slightly smaller so that some evidence emerges that there is a mode effect on the variance.

The effects on the variance are very large for `workdis` and `j2pay`. The variance of `workdis` measured using web mode is estimated to be $\exp(1.486) = 4.4$ times larger than it would have been if it had been measured using face-to-face mode, among those who chose web mode; and the ratio for `j2pay` is 7.0. This could indicate estimators which are biased or highly imprecise.

More widely, we found evidence for the presence of at least one univariate mode effect for 22 (13 percent) of 166 binary, continuous and ordinal variables using linearized standard error estimates. There was evidence for mode effects on the mean for 18 (11 percent of) variables and on the variance for 12 (7 percent); there were effects on both mean and variance for 8 (5 percent). The importance of accounting for the complex sampling design is demonstrated by noting that the respective figures based on the *i.i.d.* standard error estimates were 42 (25 percent), 37 (22 percent), 24 (14 percent) and 19 (11 percent).

Table 4 displays the estimates for three nominal categorical variables `paygwc`, `hubos` and `jbpl`. These are obtained using the multivariate SMM (12). The reference category is chosen either because it is the least substantively interesting or it contains the smallest frequency of individuals, and is dropped because it is redundant: the mode effects across all categories are constrained to sum to zero. All three variables are subject to significant mode effects but those on `paygwc` and `huboss` are particularly large. Most notably, the probability of those who chose web mode reporting they were paid annually is estimated to be 0.2 higher than it would have been had they been asked by an interviewer. Conversely, the probability that web users indicate household decisions are made jointly is estimated to be 0.09 *lower*. The former effect could be due to survey satisficing whereas the latter could be because of social desirability bias (D'Ardenne et al. 2017).

Table 5 displays estimates of the effect of mode on the pairwise covariances between the continuous variables. These effects were all estimated to be small with no evidence to reject the null hypothesis that there is no mode effect on the covariance. More widely among the 166 variables in the analysis, the 13695 pairwise estimates yielded 925 significant mode effects using the *i.i.d.* standard errors, but only 677 using the linearized standard errors: the latter figure is less than the 684 false positives one would have expected so there is little evidence for the presence of mode effects on the covariance.

Table 4: Estimated effects of mode on the distribution of selected nominal categorical variables

Variable	n_{cc}	Est.	SE(*i.i.d.*)	SE(lin)
`paygwc`	5548			
Hourly	78	0.025	0.01***	0.01*
Weekly	594	-0.014	0.02	0.02
4 weeks	508	0.090	0.01***	0.02***
Monthly	3251	0.096	0.02***	0.04**
Annually	1042	-0.195	0.02***	0.03***
Other (ref.)	75			
`huboss`	8354			
Respondent	957	-0.053	0.01***	0.02***
Spouse/Partner	918	-0.033	0.01***	0.02*
Equal say	6440	0.086	0.02***	0.03*
Other (ref.)	39			
`jbpl`	6677			
At home	210	0.004	0.01	0.01
Employer's premises	5506	0.017	0.02	0.03
Travelling	532	-0.045	0.01***	0.02**
Other place	398	0.020	0.01+	0.02
Spontaneous (ref.)	31			

Sig. level: *** < 0.01, **$0.01 - 0.025$, *$0.025 - 0.05$ and +$0.05 - 0.1$. Note: The estimates are the effects on the probability of being in that category, not contrasts with the reference category.

Table 5: Estimated effects of mode on the covariance (σ) between selected variables

	`age`	`j2pay`	`fiyrdia`	`scghq1`	`sf12mcs`	`sf12pcs`	`workdis`
`j2pay`	0.002						
	[n_{cc} = 10432]						
`fiyrdia`	0.027	0.043					
	[7948]	[7948]					
`scghq1`	-0.063	0.012	-0.033				
	[10180]	[10180]	[7766]				
`sf12mcs`	0.005	0.022	-0.004	-0.0448			
	[10141]	[10141]	[7755]	[10103]			
`sf12pcs`	-0.025	0.001	-0.029	0.034	0.028		
	[10141]	[10141]	[7755]	[10103]	[10141]		
`workdis`	0.000	-0.005	-0.031	0.044	-0.111	-0.003	
	[5476]	[5476]	[4455]	[5396]	[5397]	[5397]	
`ncigs`	0.035	0.065	-0.058	0.127	-0.170	-0.103	0.020
	[1279]	[1279]	[1108]	[1249]	[1245]	[1245]	[716]

6.2 Incorporating auxiliary data

In order to reduce bias and improve the accuracy of the IV estimators, we incorporated data from two auxiliary sources into the analysis as described in Sections 5.3. The first role of these data was to improve the accuracy of the GMM estimator while still taking the NEM assumption to hold: we refer to these as the *augmented* estimates. The second role was to enable us to estimate the extended models (from Section 4.3) to test NEM and estimate the target parameter without making the NEM assumption: we respectively refer to these as the *NEM test* and the *extended* estimates.

Table 6: Estimated effects of mode on the mean (μ_1) and variance (ν) of selected variables incorporating data from ringfenced and low-propensity groups

Variable	Par.	Previous (SE)	Augmented (SE)	NEM test[1]	Extended (SE)
j2pay_dv	μ_1	0.065 (.037)	-0.063 (.095)	0.42	-0.136 (.243)
	ν	1.948 (.712)	-0.790 (.906)	0.00	-1.262 (1.427)
age	μ_1	0.017 (.073)	-0.049 (.082)	.	.
	ν	-0.024 (.056)	0.083 (.076)	.	.
fiyrdia	μ_1	0.091 (.067)	-0.018 (.087)	0.45	-0.085 (.217)
	ν	0.050 (.076)	-0.071 (.077)	0.95	-0.132 (.170)
scghq1	μ_1	0.103 (.055)$^+$	0.127 (.062)*	0.83	0.141 (.157)
	ν	0.048 (.132)	0.057 (.155)	0.94	0.073 (.411)
sf12mcs	μ_1	-0.080 (.057)	-0.132 (.062)*	.	.
	ν	0.052 (.104)	0.104 (.126)	.	.
sf12pcs	μ_1	-0.006 (.057)	0.014 (.062)	0.84	0.025 (.151)
	ν	-0.010 (.116)	-0.020 (.130)	0.92	-0.021 (.320)
workdis	μ_1	0.112 (.048)**	0.119 (.056)*	.	.
	ν	1.486 (.638)**	3.534 (13.59)	.	.
ncigs	μ_1	0.190 (.139)	0.081 (.078)$^+$	0.08	0.196 (.108)$^+$
	ν	1.022 (1.190)	.	.	.

Note: [1]p-values; and . indicates the fitting procedure did not converge

The new estimates are displayed in Table 6. It can be seen that, in most cases, the augmented estimates are very similar to those obtained using only the experimental data. The potential for improved precision, in the sense of smaller standard errors, was not realized because the estimated standard errors were designed to reflect the sampling error of the adjustment term; simply treating the adjustment as true, on the other hand, would underestimate the standard errors and potentially lead us to conclude the presence of a mode effect. For `workdis` and `ncigs`, we were unable to obtain augmented estimates of the effect of mode on the variance due to nonconvergence: such nonconvergence with relatively small samples sizes is not uncommon for method-of-moments estimators. Testing for NEM, none of the variables for which the estimation procedure converged led to rejection of the null hypothesis that NEM holds; for the other variables, the estimation procedure did not converge. The extended estimates of the mode effects are again very similar to those displayed in Table 3 with inflated standard errors due to imprecision from estimating the Δ parameters of the extended SMoMs.

A similar picture emerges for the SCM results: there is little difference to the results obtained using the experimental data alone, with no evidence for failure of the NEM assumption, so we do not present these results.

7 Mode Effects on Parameter Estimates

The framework developed above has so far been used to estimate the effect of mode on the survey variables' joint distribution. The presence of mode effects involving the variables in a particular analysis would tell us whether the results obtained from fitting a statistical model were affected by the move to a mixed-mode design, but not the size of this effect. Estimating the difference between the observed estimate and that which would have been obtained had only face-to-face mode been used is not generally straightforward if both outcome and predictors are subject to mode effects (Park et al. 2016).

To illustrate the problem, we begin by considering the ordinary least squares estimator of θ from $Y_i = \theta X_i + e_i$, that is, the linear regression model of mean-centered Y_i on mean-centered X_i. The mode effect can be written $\hat{\theta} - \hat{\theta}_0$, that is, the difference between the observed estimator based on observed $\{X_i, Y_i\}$ and the counterfactual estimator based on $\{X_{0i}, Y_{0i}\}$. An intuitively appealing approach in this case would be to modify the *indicator method* in which D_i and its interaction with X_i are included in the model (Jäckle et al. 2010). The modification would be to use M_i as an IV to handle nonrandom selection of D_i as follows: Stage 1: Regress D_i on M_i to obtain $\hat{D}_i$; and Stage 2: Regress Y_i on X_i, $\hat{D}_i$ and interaction $\hat{D}_i.X_i$. The coefficient of the interaction term is an estimate of $\hat{\theta} - \hat{\theta}_0$. However, we show in Supplementary Information Part A, S6 that this estimator is only consistent if either X_i is mode-invariant or the mode effect on the mean of X does not depend on the true value of the characteristic measured by X, but we have already argued that this assumption is generally too strong.

Instead, we propose a more general, and robust, way to estimate the impact of mode on the maximum likelihood estimator (MLE) of parameter θ of parametric model $f(\mathbf{y}; \boldsymbol{\theta})$. This follows from inspecting the Taylor series expansions of the observed and counterfactual MLEs to show that

$$\hat{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}}_0 \approx \hat{\pi} V(\boldsymbol{\theta}^*) \left\{ \bar{\mathbf{s}}_1(\theta^*; Y) - \bar{\mathbf{s}}_1(\boldsymbol{\theta}^*; Y_0) \right\}, \tag{36}$$

where $\hat{\pi} = \sum_i D_i/n$, $\boldsymbol{\theta}^*$ is the probability limit of $\hat{\boldsymbol{\theta}}$,

$$\bar{\mathbf{s}}_1(\boldsymbol{\theta}^*; Y) = \sum_i D_i \mathbf{s}(\boldsymbol{\theta}^*; \mathbf{y}_i) / \sum_i D_i,$$

$$\bar{\mathbf{s}}_1(\boldsymbol{\theta}^*; Y_0) = \sum_i D_i \mathbf{s}(\boldsymbol{\theta}^*; \mathbf{y}_{0i}) / \sum_i D_i,$$

and $V(\boldsymbol{\theta}^*)$ is the inverse of the (single-observation) Fisher information matrix for $\boldsymbol{\theta}$ based on the observed data. This approximation relies on $V(\boldsymbol{\theta}^*) \approx V_0(\boldsymbol{\theta}^*)$, where V_0 is the inverse Fisher information based on the face-to-face data. Furthermore, for large samples,

$$\hat{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}}_0 \sim N \left\{ \pi V(\boldsymbol{\theta}^*) \Delta \mathbf{s}_1, V(\boldsymbol{\theta}^*) Q_1(\boldsymbol{\theta}^*) V(\boldsymbol{\theta}^*)/n \right\},$$

where $\pi = \Pr(D_i = 1)$ is the marginal probability of choosing web mode,

$$\Delta \mathbf{s}_1 = E \left\{ \mathbf{s}(\boldsymbol{\theta}^*; \mathbf{y}_i) - \mathbf{s}(\boldsymbol{\theta}^*; \mathbf{y}_{0i}) \mid D_i = 1 \right\},$$

and $Q_1(\boldsymbol{\theta}^*)$ is the covariance of this difference.

To estimate $\hat{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}}_0$ within the SMoM framework, the form of $f(\mathbf{y}_i; \boldsymbol{\theta})$ must allow the estimated mode effects to be straightforwardly plugged in to adjust the observed data score. For example, suppose that f is a member of the (curved) exponential family such that

$$f(\mathbf{y}_i; \boldsymbol{\theta}) = h(\mathbf{y}_i) \exp \{ \boldsymbol{\eta}^T(\boldsymbol{\theta}) \mathbf{T}(\mathbf{y}_i) - A(\boldsymbol{\eta}) \},$$

where $\mathbf{T}(\mathbf{y}_i)$ is the sufficient statistic for natural parameter $\boldsymbol{\eta}$, $\dim(\boldsymbol{\theta}) \leq \dim(\boldsymbol{\eta})$, and $A(\boldsymbol{\eta})$ is the normalization factor; then (36) reduces to

$$\hat{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}}_0 \approx \pi V(\hat{\boldsymbol{\theta}}) \frac{\partial \boldsymbol{\eta}^T}{\partial \boldsymbol{\theta}} \boldsymbol{\mu}_{1\mathbf{T}}, \tag{37}$$

where

$$\boldsymbol{\mu}_{1\mathbf{T}} = E\{\mathbf{T}(\mathbf{Y}_{1i}) - \mathbf{T}(\mathbf{Y}_{0i}) \mid D_i = 1\}$$

can be estimated using a multivariate linear SMoM for $\mathbf{T}(\mathbf{Y}_i)$. Alternatively, an exact solution, which does not require $V(\boldsymbol{\theta}^*) \approx V_0(\boldsymbol{\theta}^*)$, is given by the difference between $\hat{\boldsymbol{\theta}}$ and

$$\hat{\boldsymbol{\theta}}_0 = \left[\boldsymbol{\theta} : \frac{\partial \boldsymbol{\eta}^T}{\partial \boldsymbol{\theta}} \sum_{i=1}^{n} \left\{ \mathbf{T}(\mathbf{y}_i) - \frac{\partial A}{\partial \boldsymbol{\eta}} - D_i \hat{\boldsymbol{\mu}}_{1\mathbf{T}} \right\} = \mathbf{0} \right]. \tag{38}$$

The form of $\mathbf{T}$ is simple for a wide range of exponential family distributions. The derivations of (36-38) are set out in Supplementary Information Part A, S7.1.

To illustrate how to apply (37) and (38), we consider a simple multiple regression example involving variables from Table 2: $Y_i =$ `fiyrdia` on $X_{1i} =$ `age` and $X_{2i} =$ `scghq1` where all three variables are mean-centered. The analysis model is

$$Y_i = \theta_1 X_{1i} + \theta_2 X_{2i} + e_i,$$

where residual $e_i \sim N(0, \sigma_e^2)$. To simplify further, we ignore UKHLS's complex sampling design and estimate standard errors using the nonparametric bootstrap. However, it is generally possible to obtain bootstrap estimates of the standard errors for complex sampling designs (Field and Welsh 2007).

We fit a multivariate linear SMoM for the sufficient statistics for η, namely, Y_i^2, X_{1i}^2, X_{2i}^2, $Y_i X_{1i}$, $Y_i X_{2i}$ and $X_{1i} X_{2i}$. The conditional maximum likelihood estimate $\hat{\theta}$ and its estimated variance-covariance matrix $\hat{V}$ are obtained using `mlexp` in *Stata* (noting that $n\hat{V}$ is an estimate of $V(\hat{\boldsymbol{\theta}})$ above). The coefficient estimates are $\hat{\theta}_1 = 0.048$ (0.002) and $\hat{\theta}_2 = -0.040$ (0.004) (estimated standard errors in parentheses). The impact of mode on these estimates is estimated by plugging in estimates of the mode effects on the sufficient statistics to (37). The estimates of the mode effects are as follows: $Y^2 = 0.463$ (0.34); $X_1^2 = -1.186$ (10.5); $X_2^2 = 0.912$ (2.22); $YX_1 = 2.357$ (1.48); $YX_2 = 0.226$ (0.44); and $X_1 X_2 = -2.884$ (3.07). Finally, the effects of mode on the coefficient estimates are

$$\hat{\theta}_1 - \hat{\theta}_{01} = 0.003 \ (0.002) \text{ approx}; = 0.004 \ (0.002) \text{ exact};$$
$$\hat{\theta}_2 - \hat{\theta}_{02} = 0.005 \ (0.005) \text{ approx}; = 0.006 \ (0.006) \text{ exact}.$$

In other words, the estimates of the coefficients would have been smaller by around 7 percent and 12-15 percent, respectively, had only face-to-face mode been available. However, the estimated standard errors indicate that these effects are not statistically significant. The *Stata* code for this example is given in Supplementary Information Part B, S10.7.

An important application of this idea for practice is for generalized linear models when both outcome and predictors are subject to mode effects. In short, these models are specified by link function $g\{E(Y_i \mid \mathbf{X}_i = \mathbf{x}_i\}\} = \mathbf{x}_i^T\boldsymbol{\beta}$ and density

$$f(y_i \mid \mathbf{x}_i; \boldsymbol{\beta}, \phi) = h(y_i, \phi) \exp\left\{\frac{\eta_i T(y_i) - A(\eta_i)}{\phi}\right\}$$

from the (overdispersed) exponential family with overdispersion parameter ϕ and nuisance parameter η_i now varying between individuals because of its dependence on $\mathbf{x}_i$. For simple canonical models where $\eta_i = \mathbf{x}_i^T\boldsymbol{\beta}$, $T(y_i) = y_i$ and ϕ is a known constant, the resulting score for $\boldsymbol{\beta}$ is

$$\mathbf{s}(\boldsymbol{\beta}) = \phi^{-1} \sum_i \mathbf{X}_i Y_i - \mathbf{X}_i g^{-1}(\mathbf{X}_i^T\boldsymbol{\beta}),$$

which must be solved iteratively if g is nonlinear. The exact solution for $\hat{\boldsymbol{\beta}} - \hat{\boldsymbol{\beta}}_0$ is the difference between the solution to $\mathbf{s}(\boldsymbol{\beta}) = 0$ and $\mathbf{s}_0(\boldsymbol{\beta}) = 0$, where $\mathbf{s}_0(\boldsymbol{\beta})$ is the mode-effect-adjusted score function.

For example, the logistic regression model for Bernoulli-distributed Y_i has link $g(p) = \text{logit}(p) = \log\{p/(1-p)\}$, $E(Y_i \mid \mathbf{X}_i = \mathbf{x}_i) = g^{-1}(\mathbf{x}_i^T\boldsymbol{\beta}) = 1/\{1 + \exp(-\mathbf{x}_i^T\boldsymbol{\beta})\}$, $T(y_i) = y_i$, $\eta_i = \mathbf{x}_i^T\boldsymbol{\beta}$, $A(\eta_i) = \log\{1 + \exp(\eta_i)\}$ and $\phi = 1$. The iterative estimation procedure is as follows: choose starting value $\boldsymbol{\beta}^0$ and set $j = 1$; then apply g-estimation to the SMoM

$$E\left\{\mathbf{X}_i g^{-1}(\mathbf{X}_i\boldsymbol{\beta}^{j-1}) - \mathbf{X}_{0i} g^{-1}(\mathbf{X}_{0i}\boldsymbol{\beta}^{j-1}) \mid D_i, M_i\right\} = \boldsymbol{\tau}_{Xg}^j D_i$$

to obtain τ_{Xg}^j; the updated estimate $\boldsymbol{\beta}^j$ is the solution to

$$\mathbf{s}_0(\boldsymbol{\beta}) = \sum_i \mathbf{X}_i Y_i - \mathbf{X}_i g^{-1}(\mathbf{X}_i^T\boldsymbol{\beta}) - (\hat{\boldsymbol{\psi}}_{XY} - \hat{\boldsymbol{\tau}}_{Xg}^j) D_i,$$

where $\hat{\boldsymbol{\psi}}_{XY}$ is the g-estimate based on a linear SMoM for $\mathbf{X}_i Y_i$. Finally, increment j and repeat until convergence is achieved. The second stage relies on treating $\boldsymbol{\beta}^{j-1}$ as 'known' so that $\mathbf{X}_i g^{-1}(\mathbf{X}_i\boldsymbol{\beta}^{j-1})$ is simply a random variable.

Note that if g is the identity link then

$$\hat{\boldsymbol{\beta}}_0 = \left(\sum_i \mathbf{X}_i \mathbf{X}_i^T - \hat{\boldsymbol{\tau}}_{XX} D_i \right)^{-1} \left(\sum_i \mathbf{X}_i Y_i - \hat{\boldsymbol{\psi}}_{XY} D_i \right),$$

where $\boldsymbol{\tau}_{XX}$ is the mode effect on $\mathbf{X}_i \mathbf{X}_i^T$ defined using an appropriate linear SMoM. For the normal linear regression example above, the results obtained using this approach are identical to the exact solution described above.

To illustrate this, we use the same variables as before but derive the following binary outcome from the semicontinuous $Y_i =$ `fiyrdia`: $Y_i^{bin} = I(Y_i > 0)$, which equals 0 if $Y_i = 0$ or 1 if $Y_i > 0$. The estimated coefficients of $\text{logit}\{\Pr(Y_i^{bin} = 1 \mid X_{1i}, X_{2i})\} = \theta_0 + \theta_1 X_{1i} + \theta_2 X_{2i}$ based on the mixed-mode data are $\hat{\theta}_0 = -1.716$ (0.09), $\hat{\theta}_1 = 0.0315$ (0.001) and $\hat{\theta}_2 = -0.0333$ (0.004). Taking these estimates as starting values, the method took six iterations to converge (as determined by a successive relative difference smaller than 1×10^{-7}) to $\hat{\theta}_{00} = -1.708$ (0.13), $\hat{\theta}_{01} = 0.0305$ (0.002) and $\hat{\theta}_{02} = -0.0407$ (0.007). The mode effects are thus both small and non-significant: $\hat{\theta}_0 - \hat{\theta}_{00} = -0.008$ (0.10), $\hat{\theta}_1 - \hat{\theta}_{01} = +0.0010$ (0.002) and $\hat{\theta}_2 - \hat{\theta}_{20} = 0.0068$ (0.005). The *Stata* code for this example can be found in Supplementary Information Part B, S10.8.

Supplementary Information Part A, S7.2 reports some results from a simulation study to assess the performance of this method for normal linear and logistic regression. The exact methods were more likely not to converge than the approximate method, but almost always converged for sample sizes 1000 or larger. In terms of the relative bias of the coefficient estimates, this could be quite large even for sample sizes of order 1000, in line with the extent to which the mode-effect estimates are biased, but the relative bias was small for sample sizes of order 10000 as in this application.

8 Discussion

We have developed a very general framework for efficiently using IVs to estimate the effects of mode (or causal effects) on the survey variable distribution of UKHLS Wave 8. The mode effects were identified because the sequential experiment provided us with an instrumental variable, in the form of the initial randomization of households to either face-to-face or web mode. While we found evidence for some univariate mode effects (especially for some categorical variables), there was little evidence for the pairwise mode effects that could potentially affect multivariate analyses of the survey data. This finding was robust to the no effect modification assumption we would have had to make had auxiliary data been unavailable. The importance of accounting for the multistage stratified sampling design of UKHLS was apparent here because failing to account for it would have led to attenuated standard error estimates that overstated the impact of mode.

The generalized method of moments (GMM) proved to be a relatively simple but flexible estimation procedure that was straightforwardly adapted to produce linearized estimates of the standard errors and incorporate auxiliary data from UKHLS Wave 8. Our approach for estimating mode effects on parameter estimates can be viewed as a complement to the parametric fractional imputation approach developed by Park et al. (2016). Their method requires the analyst to specify jointly an analytical model and a measurement model, and can be adapted for IV estimation, but we argue that our method is more flexible and simpler to implement.

In terms of handling nonresponse, we used the survey weights available from UKHLS. These weights incorporate inverse probability weights to adjust for nonresponse. This approach is not fully efficient but robust in that it will correct for the effect of nonresponse on any analysis provided that assumption (28) holds. In practice, this assumption is unlikely to hold perfectly but the weights will reduce the impact of nonresponse bias. However, one point worth noting is that, if the SMoM depends on $\mathbf{C}_i$ used to create these weights such that, for example, $E(Y^k_{1i} - Y^k_{0i} \mid D_i, M_i, \mathbf{C}_i) = \mu_{1k}(\mathbf{C}_i)D_i$, then the induced marginal model (5) (obtained by integrating over $\mathbf{C}_i$) cannot satisfy NEM because of IV core condition 3. An alternative set of weights that will theoretically overcome this problem for additive SMoMs is proposed in Supplementary Information Part A, S8, but the assessment of these weights is left for future work.

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