Moneyness, Underlying Asset Volatility, and the Cross-Section of Option Returns*

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Abstract

We study the effect of an asset's volatility on the expected returns of European options on the asset. Deriving predictions from a stochastic discount factor model, we show that the effect depends on whether variations in the asset's volatility are driven by systematic or idiosyncratic volatility. While idiosyncratic-volatility-induced variations only affect the option elasticity, systematic-volatility-induced variations also oppositely affect the expected return of the asset. Since the expected asset return (elasticity) effect dominates for options with more linear (non-linear) payoffs, systematic volatility prices sufficiently in-the-money (out-of-the-money) options with the opposite (same) sign as idiosyncratic volatility. Using single-stock calls as test assets, double-sorted portfolios and Fama-MacBeth (1973) regressions broadly support the model's predictions.

Keywords: Asset pricing; option returns; moneyness; total, systematic, and idiosyncratic volatility.

JEL classification: G11, G12, G15.

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1. Introduction

A large literature in finance investigates the effect of an asset's volatility on the expected returns of levered positions in the asset. Looking into plain-vanilla European options,[1] Galai and Masulis (1976), Johnson (2004), Friewald, Wagner, and Zechner (2014), Lyle (2019), and Hu and Jacobs (2020), for example, offer theoretical and empirical evidence that the expected returns of calls (puts) decrease (increase) with underlying-asset volatility. Noteworthily, however, those studies implicitly assume that variations in underlying-asset volatility are exclusively driven by idiosyncratic volatility. As such, they are not informative about how underlying-asset volatility prices levered positions in periods in which variations in that volatility are more likely driven by systematic volatility, as, for example, in many recessions in which it is particularly important to understand risks and returns.[2]

To address that shortcoming in the literature, we offer a more comprehensive analysis of how variations in underlying asset volatility driven by either systematic or idiosyncratic volatility price the cross-section of option returns. Deriving predictions from a stochastic discount factor model, our main theoretical conclusion is that variations driven by systematic volatility are sometimes but not always differently priced compared to variations driven by idiosyncratic volatility. To be specific, while idiosyncratic-volatility-induced increases are, as in the other studies, unambiguously negatively priced in calls, systematic-volatility-induced increases are positively priced in in-the-money (ITM) and at-the-money (ATM) calls but negatively in sufficiently out-of-the-money (OTM) calls. Conversely, while idiosyncratic-volatility-induced increases are, again as in the other studies, unambiguously positively priced in puts, systematic-volatility-induced increases are positively priced in sufficiently OTM puts but negatively in ITM and ATM puts. An important upshot is that our model predicts that investments into ITM calls or OTM puts are risky in recessions in which total volatility is high due to systematic volatility, in opposition to the predictions made by the models in the other studies.

[1] Since we exclusively focus on plain-vanilla European options, we simply refer to them as "options" hereafter.

[2] In the global financial crisis, the mean and median adjusted R-squareds from stock-specific time-series estimations of the Fama-French (2016) six-factor model on twelve months of daily data, for example, rose from slightly more than 20% in 2007 to about 45% in 2010, indicating that cross-sectional variations in stock volatility were far more likely to be driven by systematic stock volatility over that period than over others.

While our theoretical work considers the holding-period return, the intuition behind our conclusions is perhaps easier to understand from instantaneous returns. Cox and Rubinstein (1985) show that the instantaneous expected excess call return, $E[\tilde{R}_c^{ins}] - R_f^{ins}$, can be written as:

$$E[\tilde{R}_c^{ins}] - R_f^{ins} = \Phi \times \left(E[\tilde{R}^{ins}] - R_f^{ins} \right), \tag{1}$$

where $E[\tilde{R}^{ins}] - R_f^{ins}$ is the instantaneous expected excess return of the underlying asset and Φ the call elasticity, defined as the partial derivative of the call's value with respect to the underlying asset's value multiplied by the ratio of underlying-asset value to call value. Taking the partial derivative with respect to systematic (σ_s^2) or idiosyncratic (σ_i^2) underlying asset variance, we obtain:

$$\frac{\partial E[\tilde{R}_c^{ins}] - R_f^{ins}}{\partial \sigma_q^2} = \underbrace{\frac{\partial \Phi}{\partial \sigma_q^2} \times \left(E[\tilde{R}^{ins}] - R_f^{ins} \right)}_{\text{call elasticity effect}} + \underbrace{\frac{\partial E[\tilde{R}^{ins}]}{\partial \sigma_q^2} \times \Phi}_{\text{underlying asset effect}}, \tag{2}$$

where $\sigma_q^2 \in \{\sigma_s^2, \sigma_i^2\}$. Assuming an underlying-asset-variance increase driven by idiosyncratic variance (i.e., $\sigma_q^2 = \sigma_i^2$), the higher variance does not affect the expected underlying-asset return ($\partial E[\tilde{R}^{ins}]/\partial \sigma_i^2 = 0$) but lowers the call's elasticity ($\partial \Phi / \partial \sigma_i^2 < 0$; see, e.g., Hu and Jacobs, 2020), inducing the expected call return to fall. Conversely, assuming an underlying-asset-variance increase driven by systematic variance (i.e., $\sigma_q^2 = \sigma_s^2$), the higher variance raises the expected underlying-asset return ($\partial E[\tilde{R}^{ins}]/\partial \sigma_s^2 > 0$) but, as before, lowers the call's elasticity ($\partial \Phi / \partial \sigma_s^2 < 0$), leaving the change in the expected call return to depend on which effect dominates. Even in the absence of our model, it is obvious that the underlying asset effect dominates for infinitely ITM calls (since they are identical to the underlying asset) but also that the elasticity effect becomes stronger relative to the underlying-asset effect as the call's moneyness falls. Our model adds to those more general insights by making sharp predictions about which of the two effects dominates in specific moneyness regions.[3]

In our theoretical derivations, we rely on the two-period continuous-variable stochastic discount

[3]We are indebted to Michael Brennan for suggesting the explanation of our results in this paragraph.

factor model studied in Rubinstein (1976) and Brennan (1979). The model assumes that the underlying-asset payoff and the stochastic discount factor realization are bivariate lognormal with a negative correlation coefficient, allowing us to derive the Black-Scholes (1973) option valuation formulas from it (see Rubinstein, 1976). In contrast to the other studies, we however linearly decompose the model's underlying asset variance parameter into a systematic variance (i.e., the variance of the optimal projection of the underlying asset payoff on the stochastic discount factor) and an idiosyncratic variance (i.e., the variance of the residual from the projection) parameter. While the model used by us is consistent with those used in other studies evaluating the pricing of volatility in options (see, e.g., Galai and Masulis, 1976; Friewald, Wagner, and Zechner, 2014; Hu and Jacobs, 2020), it makes extra assumptions allowing us to disentangle the separate effects of systematic and idiosyncratic volatility.[4]

We next conduct empirical tests of our model's moneyness and volatility predictions using American calls on zero-dividend stocks ("quasi-European calls") as test assets.[5] In doing so, we rely on the one-month call return from the start to the end of the calendar month before the month in which the call expires ("sold-before-maturity return") and the six-week call return from the same starting date to the call maturity date ("held-to-maturity return") minus the risk-free rate of return ("excess return"). We proxy for moneyness using the ratio of underlying stock price to call strike price at the start of the call return period. In our main tests, we calculate the systematic and idiosyncratic volatility of an underlying stock from stock-specific time-series estimations of the Fama-French (2016) six-factor model over the 24 months of daily data before the call return period, taking the square root of the variance of the fitted (residual) value as systematic (idiosyncratic) volatility estimate. In robustness tests, we however also use the CAPM, Fama-French-Carhart (1997), Hou-Mo-Xue-Zhang (2021) augmented q-theory, and Stambaugh and Yuan (2017) mispricing factor models. In further robustness tests, we

[4] While the underlying-asset payoff is, for example, lognormal both in the model used by us and in those used in the other studies, the lognormality property is directly assumed by the models used in the other studies but follows from the stronger assumption that the underlying-asset payoff and the stochastic discount factor realization are bivariate lognormal with a negative correlation coefficient in the model used in our study.

[5] As we explain in detail later, we rely on those test assets since option exchanges do not trade European single-stock options, yet Merton (1973) shows that it is never optimal to early exercise American calls written on zero-dividend assets, rendering them (but not other American options) equivalent to their European counterparts.

also run the time-series estimations over the three, twelve, and 60 months of daily data and 60 months of monthly data before the call return period. In yet other robustness tests, we finally also leave a one-month gap between call return and time-series regression estimation window.

Call portfolios double-sorted on moneyness and systematic or idiosyncratic volatility controlling for the other volatility component and Fama-MacBeth (FM; 1973) regressions broadly confirm our model's predictions. The portfolio sorts, for example, suggest that the mean excess sold-before-maturity return rises by 7% (t-statistic: 3.31) over the systematic volatility portfolios for ITM calls, but drops by 27% (t-statistic: –5.85) over those same portfolios for deep out-of-the-money (DOTM) calls. The spread is a significant 34% (t-statistic: 8.55). Conversely, the same return drops by 11% (t-statistic: –6.50) over the idiosyncratic volatility portfolios for ITM calls but by 26% (t-statistic: –4.76) for DOTM calls. The spread is a significant 15% (t-statistic: 3.09). In accordance, the FM regressions produce systematic and idiosyncratic volatility premiums in those same returns of 33% (t-statistic: 3.40) and –24% (t-statistic: –5.46) for ITM calls, but of –138% (t-statistic: –6.20) and –92% (t-statistic: –6.85) for DOTM calls, all respectively. Notwithstanding, while our ITM, OTM, and DOTM results strongly support our theory, we generally find the systematic volatility premium in ATM calls to be positive (which is the sign predicted by our theory) but insignificant, presumably due to a lack of statistical power.

Our empirical results are robust with respect to the factor model used to calculate systematic and idiosyncratic volatility, mostly because the excess market return, which is the first factor in all models, consistently explains the lion share of systematic volatility. Moreover, they are also robust with respect to leaving a one-month gap between the call return and volatility estimation windows; selecting one single call per underlying stock within each moneyness portfolio; using weighted least-squares (WLS) FM regressions relying on dollar call open interest as weighting variable; and adjusting for bid-ask transaction costs. They further survive controlling for characteristics known to price options, such as stock and option liquidity proxies (Cao and Han, 2013; Christoffersen et al., 2018); option mispricing proxies (Stein, 1989; Poteshman, 2001; Goyal and Saretto, 2009); the variance risk premium and implied risk-neutral moments (Bakshi and Kapadia, 2003); and firm characteristics known to price stocks (Cao et al., 2021). Conversely, our empirical results become weaker — without them however disappearing — with decreases in the length of the volatility estimation window, largely due to the estimations run over shorter windows yielding much higher standard errors.

Since a stock's systematic variance is linear in its squared factor exposures and cross exposures, we next also rerun the FM regressions replacing systematic volatility with the square root of the squared factor exposures (i.e., the absolute exposures). In line with the idea that the squared exposures, and not the cross exposures, drive our results, the regressions show that the absolute exposure premiums mostly also rise over the moneyness portfolios. Switching to the stocks underlying our sample calls, we finally illustrate that they have absolute exposure premiums similar to the ITM calls, consistent with the notion that ITM calls are close in nature to their underlying assets. Despite that, systematic volatility does not significantly price those stocks, in agreement with Frazzini and Pedersen's (2014) evidence but in conflict with our theory, while idiosyncratic volatility significantly negatively prices them, in agreement with Ang et al.'s (2006) evidence but again in conflict with our theory.

Our work adds to an emerging literature identifying variables pricing options. Using a general stochastic discount factor model, Coval and Shumway (2001) predict that the expected returns of both European calls and puts rise with the strike price. Relying on the Black-Scholes (1973) contingent claims setup, Galai and Masulis (1976), Friewald, Wagner, and Zechner (2014), and Hu and Jacobs (2020) claim that the expected European call (put) return falls (rises) with idiosyncratic underlying asset volatility. Using a stochastic discount factor model similar to ours, Johnson (2004) and Lyle (2019) confirm these predictions, while Hu and Jacobs (2020) use single-stock option data to offer empirical support for them. Simulating the expected put return using the Longstaff and Schwartz (2001) method, Aretz and Gazi (2020) demonstrate that American puts have higher (i.e., less negative) expected returns than equivalent European puts. Turning to delta-hedged option returns, Goyal and Saretto (2009) show that they rise with the realized-to-implied volatility of the underlying asset, Cao and Han (2013) report that they decrease with idiosyncratic underlying asset volatility, and Cao et al. (2021) reveal that they relate to well-known stock anomaly variables. We contribute to these studies by offering a refined and more comprehensive analysis of how underlying asset volatility prices options, paying close attention to the separate roles played by systematic and idiosyncratic volatility.

We also add to a large literature examining how idiosyncratic stock volatility prices stocks. While Ang et al. (2006, 2009) show that historical idiosyncratic volatility negatively prices stocks, Diavatopoulos, Doran, and Peterson (2008), Fu (2009), Chua et al. (2010), and Brockman, Schutte, and Yu (2012) report that option-implied, GARCH, or other *expected* idiosyncratic volatility estimates positively price those same stocks. Just like ours, the theoretical results of Johnson (2004) offer a potential explanation for why stock returns may decrease with idiosyncratic volatility, illustrating that financial leverage may be behind the negative relation in a stochastic discount factor model. While Song's (2008) and Chen, Chollete, and Ray's (2010) evidence that distress risk negatively conditions the idiosyncratic volatility premium supports that explanation, Ang et al.'s (2009) evidence that book leverage does not condition that premium does not support it. Interpreting single-stock calls as levered-up versions of their underlying stocks, we contribute to these studies by offering fresh evidence that a higher financial leverage does produce a more negative idiosyncratic volatility premium.

2. Theory

In this section, we use the stochastic discount factor model proposed in Rubinstein (1976) and Brennan (1979) to study how the volatility of an asset prices options written on that asset. We start with outlining the model's assumptions. We next derive the relations between several option and underlying asset characteristics (including underlying asset volatility) and expected call returns implied by the model. To conserve space, we derive the corresponding relations for puts in the Internet Appendix.

2.1 The Rubinstein (1976)-Brennan (1979) Model

Following Rubinstein (1976) and Brennan (1979), we consider a two-date continuous-variable securities market model with a single (primitive) asset and plain-vanilla European calls and puts written on that asset. Assuming the existence of a representative agent and that the agent's maximization problem can be solved, the two studies demonstrate that we can write the asset's price, p, as:

$$p = E[\tilde{M} \times \tilde{X}] = E[e^{\tilde{m}+\tilde{x}}], \tag{3}$$

where $E[.]$ is the expectation operator, $\tilde{M}$ is the realization of the stochastic discount factor on the second date, $\tilde{X}$ is the asset's payoff on that date, $\tilde{m} \equiv \ln \tilde{M}$, $\tilde{x} \equiv \ln \tilde{X}$, and a tilde indicates a random variable. They next assume that $\tilde{x}$ and $\tilde{m}$ are bivariate normal, with expectations μ_x and μ_m, variances σ_x^2 and σ_m^2, and correlation coefficient $\rho < 0$, respectively. Under those assumptions, Rubinstein (1976) shows that we are able to derive the Black-Scholes (1973) European option valuation formulas, while Brennan (1979) clarifies that a risk-neutral valuation relation exists only if the representative agent's preferences can be modelled using a constant relative risk aversion utility function.

Different from Rubinstein (1976) and Brennan (1979), we however next decompose the variance of the asset's log payoff, σ_x^2, into the sum of the variance of the optimal projection of the log asset payoff on the log stochastic discount factor, σ_s^2 ("systematic variance"), and the variance of the residual from that projection, σ_i^2 ("idiosyncratic variance"). To achieve that goal, we recognize that our distributional assumptions imply that the optimal projection, $\tilde{x}_s$, can be written as $a - b\tilde{m}$, where a and $b > 0$ are parameters. Thus, the systematic variance of the log asset payoff is $b^2\sigma_m^2$. Conversely, the residual, $\tilde{x}_i$, can be written as $\tilde{x} - \tilde{x}_s$, so that the idiosyncratic variance of the log asset payoff is $\sigma_x^2 - b^2\sigma_m^2$. Using those results, we can write the variance-covariance matrix of $\tilde{x}$ and $\tilde{m}$, $\sigma_{x,m}$, as:

$$\sigma_{x,m} \equiv \begin{bmatrix} \text{var}(\tilde{x}) & \text{cov}(\tilde{x},\tilde{m}) \\ \text{cov}(\tilde{x},\tilde{m}) & \text{var}(\tilde{m}) \end{bmatrix} = \begin{bmatrix} \sigma_x^2 = \sigma_s^2 + \sigma_i^2 & \kappa\sigma_s\sigma_m = -\sigma_s\sigma_m \\ \kappa\sigma_s\sigma_m = -\sigma_s\sigma_m & \sigma_m^2 \end{bmatrix}, \tag{4}$$

where var(.) and cov(.) are the variance and covariance operator, respectively, and κ is the correlation between the optimal projection of the log asset payoff on the log stochastic discount factor and the log discount factor. Since $\text{cov}(\tilde{x}_s, \tilde{m}) = -b\sigma_m^2$ and $\text{var}(\tilde{x}_s) = b^2\sigma_m^2$, we have $\kappa = -1$.[6]

[6]For simplicity, our main derivations assume that we can independently vary the expected log asset payoff μ_x, systematic variance σ_s^2, and idiosyncratic variance σ_i^2. Since variations in systematic variance must however come from variations in the slope coefficient b, this assumption requires $\mu_m = 0$ since $\mu_x = a - b\mu_m$. Thus, if $\mu_m \neq 0$, we must

2.2 Model Results

2.2.a The Expected Asset Return

Our model predicts that the expected asset return, $E[\tilde{R}]$, is:

$$E[\tilde{R}] = \frac{E[\tilde{X}]}{p} = \frac{E[\tilde{X}]}{E[\tilde{M} \times \tilde{X}]} = \frac{e^{\mu_x + \frac{1}{2}\sigma_x^2}}{e^{\mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 - 2\sigma_s\sigma_m + \sigma_m^2)}} = \frac{e^{\sigma_s\sigma_m}}{e^{\mu_m + \frac{1}{2}\sigma_m^2}} = R_f e^{\sigma_s\sigma_m}, \tag{5}$$

where $R_f \equiv 1/E[\tilde{M}] = e^{-(\mu_m + \frac{1}{2}\sigma_m^2)}$ is the gross risk-free rate of return. In line with our expectations, Equation (5) suggests that the expected asset return is positively related to systematic variance, σ_s^2, but unrelated to both the expected log asset payoff, μ_x, and idiosyncratic variance, σ_i^2.

2.2.b The Expected Returns of European Calls

Our model further predicts that the expected return of a European call on the asset, $E[\tilde{R}_c]$, is:

$$\begin{aligned} E[\tilde{R}_c] &= \frac{E[\tilde{X}_c]}{p_c} = \frac{E[\max(\tilde{X} - K, 0)]}{E[\tilde{M} \times \max(\tilde{X} - K, 0)]} \\ &= \frac{e^{\mu_x + \frac{1}{2}\sigma_x^2} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - KN\left[\frac{\mu_x - \ln K}{\sigma_x}\right]}{e^{\mu_m + \frac{1}{2}\sigma_m^2}\left[e^{\mu_x + \frac{1}{2}(\sigma_x^2 - 2\sigma_s\sigma_m)} N\left[\frac{\mu_x - \sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] - KN\left[\frac{\mu_x - \sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right]}, \end{aligned} \tag{6}$$

where K is the strike price of the call, max(.) the maximum operator, and $N[.]$ the cumulative standard normal density. While the closed-form solution for the expected call payoff, $E[\tilde{X}_c]$, can be derived from the formula for the expectation of a left-truncated lognormal variable (see Ingersoll, 1987), the closed-form solution for the call value, p_c, can be derived as in Rubinstein (1976).

Noticing that the asset value p is $e^{\mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 - 2\sigma_s\sigma_m + \sigma_m^2)}$, the gross risk-free rate of return R_f is $e^{-(\mu_m + \frac{1}{2}\sigma_m^2)}$, and $\mu_x - \sigma_s\sigma_m + \sigma_x^2 = \ln p + \ln R_f + \frac{1}{2}\sigma_x^2$, we could now derive the Black-Scholes (1973) European call valuation formula from the closed-formed solution for the call's value, p_c, showing that our

implicitly assume that the effects of variations in the slope coefficient b on the expected log asset payoff μ_x are offset by simultaneous variations in the constant a. Notwithstanding, part (d) of the proof of Proposition 1 in Section IA.1 in the Internet Appendix shows that our theoretical results are robust to allowing the expected log asset payoff μ_x to change with variations in systematic variance σ_s^2 induced through variations in the slope coefficient b.

model is consistent with the Black-Scholes (1973) model. Despite that, it is crucial to understand that the two models are not identical since the Black-Scholes (1973) model assumes that the asset value is exogenous, making it impossible to determine how that value depends on the expected log asset payoff, systematic variance, and idiosyncratic variance. In contrast, our model endogenously determines the asset value, explicitly showing how that value depends on the former characteristics and enabling us to study the separate pricing of systematic and idiosyncratic volatility in options.

The probability that the call ends up ITM and yields a positive payoff, π_c, is:

$$\pi_c = \text{Prob}(\tilde{X} > K) = \text{Prob}\left(\frac{\tilde{x} - \mu_x}{\sigma_x} > \frac{\ln K - \mu_x}{\sigma_x}\right) = N\left[\frac{\mu_x - \ln K}{\sigma_x}\right], \tag{7}$$

which shows that the spread between the expected log asset payoff and the log strike price, $\mu_x - \ln K$, is a sufficient statistic for the probability that the call ends up ITM on its maturity date. In accordance, we label those calls with a positive spread (and thus an above 50% chance of ending up ITM) "ITM calls," those with a zero spread (and thus a 50% chance) "ATM calls," and those with a negative spread (and thus a below 50% chance) "OTM calls." We further interpret $\mu_x - \ln K$ as a monotonic positive transformation of moneyness conditional on underlying-asset volatility, σ_x.[7]

Proposition 1 summarizes the relations between the expected call return, on one hand, and several asset and call characteristics, on the other, implied by our model:

PROPOSITION 1: *Assuming the existence of a representative agent, that the agent's maximization problem can be solved, and that the log asset payoff, $\tilde{x}$, and the log stochastic discount factor realization, $\tilde{m}$, are bivariate normal with a negative correlation between them, the expected return of a European call written on the asset and with strike price equal to K, $E[\tilde{R}_c]$,*

(a) decreases with the expected log asset payoff, μ_x.

(b) increases with the strike price specified by the call contract, K.

(c) decreases with moneyness, defined as the difference between μ_x and $\ln K$.

[7] Noticing that $\mu_x - \ln K = \ln(p/K) + \ln R_f - \frac{1}{2}\sigma_x^2 + \sigma_s \sigma_m$, our conditional moneyness measure is closely related to the most common moneyness proxy used in the empirical option literature, the asset-to-strike price ratio p/K.

(d) increases (decreases) with the asset's systematic variance, σ_s^2, if and only if:

$$(\sigma_x^2/\sigma_s^2)H'[c^*] - H'[\alpha - \sigma_x + \beta] - \left(\alpha - \sigma_m \frac{\sigma_i^2}{\sigma_x \sigma_s}\right) H[\alpha - \sigma_x + \beta][1 - H'[c^*]] > (<)\ 0,$$

where $H(x) \equiv n(x)/N(-x)$ is the hazard function of the normal random variable x, with $n(.)$ the standard normal density function, $H'(x)$ the first derivative of the hazard function with respect to x, $\alpha \equiv (\ln K - \mu_x)/\sigma_x$, $\beta \equiv \frac{\sigma_s \sigma_m}{\sigma_x}$, and $c^* \in (\alpha - \sigma_x + \beta, \alpha + \beta)$.

(e) decreases with the asset's idiosyncratic variance, σ_i^2.

Proof: See Section IA.1 in the Internet Appendix.

While part (b) of the proposition aligns with the conclusions of Coval and Shumway (2001), who study a more general stochastic discount factor model not making distributional assumptions except that the asset payoff and stochastic discount factor realization are negatively correlated, the other parts are new to the literature. Part (a) predicts that a higher expected log asset payoff lowers the expected call return. Combining expected log asset payoff and strike price into our definition of moneyness, part (c) predicts that a higher moneyness also lowers the expected call return. Part (d) opens up the possibility that the effect of systematic variance on the expected call return can vary across different types of calls. In accordance with that possibility, Corollary 1 suggests that the sign of the systematic variance premium in expected call returns differs across ITM, ATM, and OTM calls.

COROLLARY 1: Under the same assumptions as in Proposition 1, the sign of the relation between the expected call return, $E[\tilde{R}_c]$, and systematic asset variance, σ_s^2, is positive for ITM and ATM calls, but can be both positive or negative for OTM calls.

Proof: See Section IA.1 in the Internet Appendix.

To illustrate Corollary 1, Panel A of Figure 1 plots the partial derivative of the expected call return with respect to systematic variance against the strike price. The figure confirms that ITM and ATM calls produce a strictly positive relation between expected call return and systematic variance, while

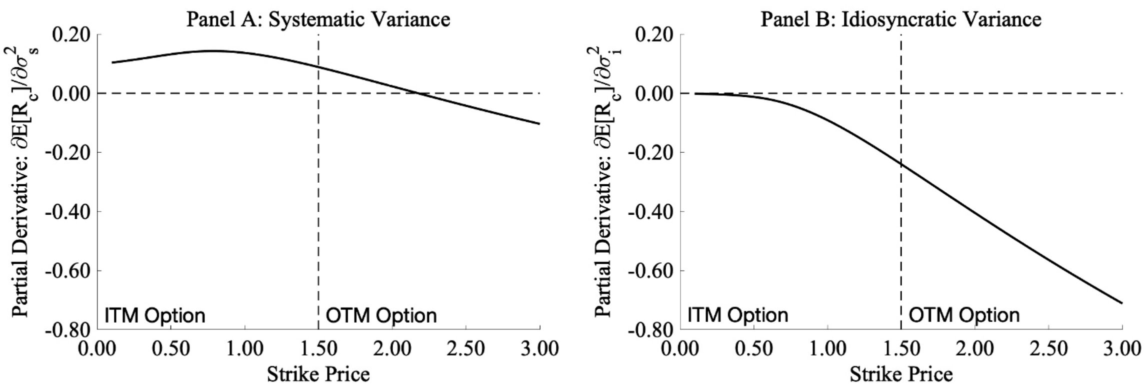

Figure 1: Partial Derivatives of Expected Call Return with Respect to Systematic and Idiosyncratic Variance The figure plots the partial derivatives of the expected call return with respect to systematic (Panel A) and idiosyncratic (Panel B) asset variance against the strike price. The basecase parameter values are: The expected log asset payoff (μ_x) and stochastic discount factor realization (μ_m) are 0.00 and –0.025, respectively. Systematic (σ_s^2) and idiosyncratic (σ_i^2) variance are 0.16 and 0.04, respectively. The log stochastic discount factor variance (σ_m^2) is 0.0225.

OTM calls produce a positive, zero, or negative relation. Noteworthily, the figure further suggests that a negative relation only occurs for sufficiently OTM calls. The intuition behind the ambiguous sign of the relation is that a higher systematic variance has two oppositely-signed effects on the expected call return, as already revealed by Equation (2). While the positive effect is that the higher systematic variance raises the expected asset return (recall Equation (5)), the negative effect is that it lowers the call's financial leverage as reflected in its elasticity. Since the financial leverage of ITM, ATM, and mildly OTM calls is (relatively) variance-insensitive, the asset effect dominates the leverage effect for such calls, yielding a positive systematic variance premium. Yet, as calls move deeper OTM, their financial leverage becomes more variance-sensitive, leading the leverage effect to ultimately dominate the asset effect and turning the systematic variance premium from positive to negative.

Part (e) of Proposition 1 suggests that idiosyncratic variance unambiguously negatively prices calls, echoing the results of studies using the Black-Scholes (1973) contingent claims model to study how variations in underlying-asset volatility driven by idiosyncratic volatility affect the expected call return. In line with the idea that infinitely ITM calls are equivalent to the underlying asset, Corollary 2 reveals that the idiosyncratic variance premium converges to zero with moneyness.

COROLLARY 2: *Under the same assumptions as in Proposition 1, the relation between the expected call return, $E[\tilde{R}_c]$, and idiosyncratic asset variance, σ_i^2, converges to zero (a negative value) as the call's moneyness increases to infinity (decreases to minus infinity).*

Proof: See Section IA.1 in the Internet Appendix.

Panel B of Figure 1 confirms that the idiosyncratic variance premium is unambiguously negative across ITM, ATM, and OTM calls and that it moves toward zero with moneyness.[8]

2.2.c The Expected Returns of European Puts

We also investigate the relations between the asset and option characteristics in Proposition 1 and the expected put return. Since we are however unable to empirically test those relations, we only discuss them in detail in Section IA.3 in the Internet Appendix. In a nutshell, Proposition IA.1 in that section predicts that the expected put return falls with the expected log asset payoff (μ_x); rises with the strike price (K); rises with moneyness defined as $\ln K - \mu_x$; and either rises or falls with both systematic (σ_s^2) and idiosyncratic (σ_i^2) variance. Further work then suggests that while the systematic variance premium is unambiguously negative in ITM and ATM puts ($\ln K - \mu_x \geq 0$), that same premium can be positive in OTM puts ($\ln K - \mu_x < 0$). Finally, Corollary IA.1 reveals that idiosyncratic variance virtually always positively prices puts, in agreement with Hu and Jacobs (2020).[9]

[8] In Section IA.2 of the Internet Appendix, we compare our stochastic discount factor model with an extended version of the contingent claims model used in Hu and Jacobs (2020) in which the asset drift depends on systematic volatility. While the section highlights that our model is theoretically preferable, it further demonstrates that the two models yield similar theoretical conclusions about the pricing of systematic and idiosyncratic variance in options.

[9] While the Rubinstein (1976)-Brennan (1979) model does not allow us to predict how option time-to-maturity affects our theoretical conclusions, we note that, if we assumed that the asset payoff $\tilde{X}$ and the stochastic discount factor realization $\tilde{M}$ followed two correlated Geometric Brownian motions, the two variables would still be bivariate-lognormally distributed on the second date, in line with the model. In that case, the drift and variance parameters would, however, all scale linearly with the option time-to-maturity, defined as the time gap between the two model dates. Given our theoretical conclusions hold for all parameter value sets under which asset payoff and stochastic discount factor realization are negatively correlated, they would then also hold over each option time-to-maturity.

3. Empirical Tests

In this section, we conduct empirical tests of the moneyness and volatility predictions of the Rubinstein (1976)-Brennan (1979) model for the cross-section of European call returns using American calls written on zero-dividend single stocks as test assets. Our main focus are the predictions about how moneyness conditions the pricing of systematic and idiosyncratic underlying-asset volatility in such calls. To be specific, these predictions are that systematic volatility positively prices ITM and ATM calls but that it can negatively price OTM calls; and that idiosyncratic volatility unambiguously negatively prices calls, with the idiosyncratic volatility premium however rising toward zero with moneyness. We also test the prediction that moneyness unambiguously negatively prices calls. We start the section with discussing our data sources and filters. We next elaborate on how we calculate the variables used in our empirical tests. We finally present our empirical results.

3.1 Data

We obtain data on American calls written on single stocks which do not pay out dividends over their remaining times-to-maturity from Optionmetrics. We only rely on those calls since European single-stock options are not traded in option exchanges, yet Merton (1973) shows that it is never optimal to early exercise American calls on zero-dividend assets, rendering them equivalent to their corresponding European calls. While other studies are often more relaxed about using American option data to test implications from European option theories (see, e.g., Carr and Wu, 2009; Martin and Wagner, 2019; Hu and Jacobs, 2020), Aretz and Gazi (2020) document that American single-stock put returns differ significantly from those of equivalent synthetic European puts, with moneyness and underlying-asset volatility conditioning the difference. To avoid systematic bias, we thus exclusively employ the above "quasi-European calls" in our empirical analysis.

We impose standard filters on our data. In particular, we only consider calls with a stock-to-strike price ratio (moneyness) between 0.80 and 1.20 at the start of the call return period.[10] We next exclude

[10] We only use calls with moneyness values within that range since the original cross-sectional moneyness distribution

calls violating well-known arbitrage restrictions, summarized by the condition that the arbitrage-free call price must lie above the maximum of zero and the equivalent long forward contract value but below the underlying stock's price. We further drop calls with a zero trading volume, a non-positive bid price, a negative bid-ask spread, and a below $\$\frac{1}{8}$ bid-ask midpoint price. We finally eliminate calls for which the current date is unequal to the date on which the call last traded. We winsorize the independent variables at the fifth and 95th percentiles by sample month. The exception is that, due to their extreme kurtosis, we winsorize the factor model exposures at the tenth and 90th percentiles by sample month when we use them as independent regression variables (i.e., in Table 7).

We obtain market and accounting data on the stocks underlying the sample calls from CRSP and Compustat, respectively. We collect the CAPM, Fama-French-Carhart (1997), Fama-French (2016) six-factor model, and risk-free rate of return data from Kenneth French's website. Conversely, we collect the Hou-Mo-Xue-Zhang (2021) augmented q-theory and Stambaugh and Yuan (2017) mispricing factor model data from Lu Zhang's and Robert Stambaugh's websites, respectively.[11] Our sample period starts in January 1996 and ends in June 2019. The only exception occurs when we rely on the mispricing factors since those factors are only available until December 2016.

3.2 Variable Construction

We use both the excess call return from the start to the end of the calendar month before the month in which the call expires ("sold-before-maturity return") and the excess call return from the same starting date to the call maturity date ("held-to-maturity return") as dependent variable.[12] In line with other studies in the options literature (see, e.g., Hu and Jacobs, 2020), we proxy for call moneyness using the stock price-to-call strike price ratio at the start of the call return period.

To derive proxies for systematic and idiosyncratic volatility, we need to impose some additional

is highly leptokurtic due to a few calls with extreme moneyness values. To wit, while the average cross-sectional kurtosis of the original distribution is about 38, our moneyness restriction decreases that average to about three.

[11] The URL addresses of those three websites are <https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>, <https://theinvestmentcapm.com/>, and <http://finance.wharton.upenn.edu/~stambaug/>, respectively.

[12] While studying held-to-maturity returns is more in line with our theory, Ni, Pearson, and Poteshman (2005) and Chiang (2014) discover "expiration-date biases" in those, motivating us to also study sold-before-maturity returns.

assumptions on the stochastic discount factor. To do so, we rely on Cochrane's (2001) insight that linear factor models implicitly assume that the stochastic discount factor, $\tilde{M}$, or its log value, $\tilde{m}$, is linear in a model's factors. Assuming, for example, that the CAPM holds, we can write:

$$\tilde{m} = c + d\tilde{r}^{em}, \tag{8}$$

where c and d are parameters, and $\tilde{r}^{em}$ is the excess market return. Regressing the log asset return, $\tilde{x} - \ln p$, onto the log stochastic discount factor, $\tilde{m}$, we obtain:

$$\begin{aligned} \tilde{x}_s - \ln p &= E[\tilde{x}|\tilde{m}] - \ln p = E[\tilde{x}] - \ln p + \frac{\text{cov}(\tilde{x}, \tilde{m})}{\text{var}(\tilde{m})}(\tilde{m} - E[\tilde{m}]) \\ &= E[\tilde{x}] - \ln p + \frac{\text{cov}(\tilde{x}, \tilde{r}^{em})}{\text{var}(\tilde{r}^{em})}(\tilde{r}^{em} - E[\tilde{r}^{em}]) = \alpha + \beta\tilde{r}^{em}, \end{aligned} \tag{9}$$

where $\alpha \equiv E[\tilde{x}] - \ln p - \frac{\text{cov}(\tilde{x},\tilde{r}^{em})}{\text{var}(\tilde{r}^{em})}E[\tilde{r}^{em}]$ and $\beta \equiv \frac{\text{cov}(\tilde{x},\tilde{r}^{em})}{\text{var}(\tilde{r}^{em})}$. As a result, $\sigma_s^2 = \beta^2\text{var}(\tilde{r}^{em})$, showing that we can use the square root of the variance of the fitted value from a time-series regression of an asset's return on the excess market return to estimate systematic volatility.

Under the assumption that a linear multifactor model holds, the optimal projection, $\tilde{x}_s - \ln p$, is similarly a linear combination of the model factors, implying that systematic variance, σ_s^2, is a linear combination of the variances and covariances of the model factors. In that case, however, a time-series regression of an asset's log return on the factors only generates estimates close but not identical to the true combination weights used to construct systematic variance. The reason is that while such a time-series regression gives us the explanatory power of the joint set of factors for an asset's log return, our theory is actually after the explanatory power of a linear combination of the factors. Despite that inconsistency, it is common to run time-series regressions to calculate systematic and idiosyncratic volatility from multifactor models in the empirical asset pricing literature (see, e.g., Ang et al., 2006; Boyer, Mitton, and Vorkink, 2010; Bali, Brown, and Caglayan, 2012; Cao and Han, 2013).

In line with the above, we calculate systematic (idiosyncratic) stock volatility as the square root of the annualized variance of the fitted (residual) value from a time-series regression of the stock's return on some linear factor model's factors estimated over some period ending before the start of the call return period. While we rely on the Fama-French (2016) six-factor model factors in our main tests, we use the CAPM, Fama-French-Carhart (1997), Hou-Mo-Xue-Zhang (2021) augmented q-theory, and Stambaugh and Yuan (2017) mispricing factor model factors in robustness tests.[13] Also, while we estimate the time-series regressions over 24 months of daily data in our main tests, we estimate them over three, 12, and 60 months of daily data and 60 months of monthly data in robustness tests. Finally, while we do not leave a gap between the call return period and the time-series regression estimation period in our main tests, we leave a one-month gap between them in robustness tests.

Since the variance of the fitted value from a time-series estimation of a linear factor model can be written as the sum over the squared factor exposures multiplied by the factor variances plus the sum over the cross factor exposures mutiplied by the factor covariances,[14] we also study whether our systematic volatility results are more strongly driven by the squared exposures or the cross exposures. To that end, we sometimes replace systematic volatility by the square roots of the squared exposures (i.e., by the absolute exposures) in our tests. We use the absolute exposures rather than the squared exposures in those tests to mitigate the effect of extreme outliers in the squared exposures.

[13]The single CAPM factor is the excess market return ("MKT"). The Fama-French-Carhart (1997) model adds the returns of spread portfolios long small and short large stocks ("SMB"), long high and short low book-to-market ratio stocks ("HML"), and long high and short low prior-year return stocks ("MOM"). The Fama-French (2016) model furthers adds the returns of spread portfolios long high and short low profitability stocks ("RMW") and long low and short high asset growth stocks ("CMA"). In addition to MKT, SMB (called "ME" in that model), and CMA (called "I/A") factors, the Hou-Mo-Xue-Zhang (2021) model adds the returns of spread portfolios long high and short low quarterly ROE stocks ("ROE") and long high and short low expected growth stocks ("EG"). In addition to MKT and SMB factors, the Stambaugh and Yuan (2017) model adds the returns of spread portfolios long high and short low management anomaly stocks ("MGMT") and long high and short low performance anomaly stocks ("PERF"). See Fama and French (1993, 2016), Carhart (1997), Stambaugh and Yuan (2017), and Hou et al. (2021) for details.

[14]Assuming that some linear factor model holds, we can write the systematic variance of stock i, $\sigma^2_{s,i}$, as:

$$\sigma^2_{s,i} = \mathrm{var}\left(\alpha_i + \sum_{k\in\Theta} \beta_{i,k} r_k\right) = \left(\sum_{k\in\Theta} \beta^2_{i,k}\mathrm{var}(r_k) + \sum_{k\in\Theta}\sum_{l\in\Theta,k\neq l} \beta_{i,k}\beta_{i,l}\mathrm{cov}(r_k, r_l)\right), \tag{10}$$

where var(.) and cov(.) are the variance and covariance operator, respectively, α_i is the constant, $\beta_{i,v}$ is the slope coefficient on factor v, r_v is the return of factor v, and Θ is a set containing the factors specified by the linear factor model. Assuming the Fama-French-Carhart (1997) model, we, for example, have $\Theta \in \{\mathrm{MKT}, \mathrm{SMB}, \mathrm{HML}, \mathrm{MOM}\}$.

3.3 Empirical Results

3.3.a Descriptive Statistics

Table 1 presents descriptive statistics on our sample calls (Panel A) and on the total, systematic, and idiosyncratic volatility estimates of their underlying stocks obtained from each of the five factor models estimated over the prior 24 months of daily data (Panel B). The table suggests that our data contain 336,487 call-month observations, translating into an average of 1,193 observations per sample month and representing 5,983 stocks. Panel A further documents that the mean sold-before-maturity and held-to-maturity returns are, respectively, 20% and 22%. Moreover, the standard deviation and the percentiles indicate that the held-to-maturity return is significantly more volatile and right-skewed than the sold-before-maturity return. Finally, the mean and median call are both close to ATM and have about 50 calendar days to maturity at the start of the call return period.

Turning to the volatility estimates, Panel B shows that the mean underlying stock has an annualized total volatility of 43%. Splitting total volatility into systematic and idiosyncratic volatility using each factor model, the annualized systematic volatility of that stock lies between 21% and 25%, while its annualized idiosyncratic volatility lies between 34% and 37%. In comparison to the CRSP universe over our sample period, our mean sample stock thus has about the same total volatility (about 42%), but a slightly higher systematic volatility (21% to 25% vs. 12% to 16%). Considering the standard deviations and percentiles of either the alternative systematic or idiosyncratic volatility estimates, we find them to be highly similar except for the 99th percentile, suggesting that the alternative estimates are close to one another. In accordance, the average cross-sectional correlations between the systematic (idiosyncratic) volatility estimates are never below 0.93 (0.99; not reported in table).

3.3.b Main Systematic and Idiosyncratic Volatility Pricing Results

We next use double-sorted portfolios and FM regressions to test the moneyness and volatility predictions of the Rubinstein (1976)-Brennan (1979) model on calls. Starting with the portfolio sorts, we follow An et al. (2014) in forming portfolios double-sorted on moneyness and either systematic or idiosyncratic volatility controlling for the other volatility component. At the end of each sample month $t-1$, we thus first sort our sample calls into portfolios according to the quartile breakpoints of the idiosyncratic (systematic) volatility distribution on that date. Within each portfolio, we next sort them into portfolios according to the same breakpoints of the moneyness distribution and, independently, according to the same breakpoints of the systematic (idiosyncratic) volatility distribution on that date, yielding $4 \times 4 \times 4$ portfolios. We equally-weight the 64 portfolios. We finally form equally-weighted portfolios of those portfolios with the same moneyness-systematic (idiosyncratic) volatility classification, averaging out the effect of idiosyncratic (systematic) volatility. We next form spread portfolios long the highest moneyness (volatility) portfolio and short the lowest within each volatility (moneyness) portfolio. We hold the 4×4 double-sorted plus spread portfolios either over month t (sold-before-maturity return) or from the same starting date to the call maturity date (held-to-maturity return).[15]

To adjust for risk, we regress the spread portfolio returns on the Fama-French (2016) six-factor model stock factors plus the Karakaya (2013) three-factor model option factors. While we use the Fama-French (2016) factors to capture the risk of the underlying stock, we rely on the Karakaya (2013) factors to capture the additional risk embedded in a call from leveraging up the underlying stock. The Karakaya (2013) factors are a level, slope, and value factor constructed from option data.[16]

Table 2 presents the portfolio sort results obtained from using the Fama-French (2016) six-factor model estimated over the 24 months of daily data before the call return period to calculate systematic and idiosyncratic volatility. While columns (1) to (4) show the moneyness-systematic volatility sorts controlling for idiosyncratic volatility, columns (5) to (8) show the moneyness-idiosyncratic volatility sorts controlling for systematic volatility. Given the moneyness values of the calls in the portfolios,

[15]We thus form our triple-sorted portfolios from an independent double-sort on moneyness and one of the two volatility components within a dependent univariate sort on the other volatility component. While we would have preferred to only rely on independent sorts, the two volatility components share an average cross-sectional correlation of about 0.60, implying that an independent triple sort produces many empty or at least ill-diversified portfolios.

[16]In particular, the level factor is an equally-weighted average of the returns of ATM option portfolios with different times-to-maturity, while the slope factor is an equally-weighted average of the returns of short time-to-maturity option portfolios with different moneyness values minus an equally-weighted average of the returns of the corresponding long time-to-maturity portfolios. Conversely, the value factor is an equally-weighted average of the returns of the three top historical-minus-implied Black-Scholes (1973) volatility option decile portfolios minus an equally-weighted average of the returns of the corresponding three bottom decile portfolios. See Section B.1 in Karakaya (2013) for details.

we label the top, second-to-top, second-to-bottom, and bottom moneyness portfolio the ITM, ATM, OTM, and DOTM portfolio, respectively.[17] The plain numbers in Panels A and B are, respectively, the two sets of mean excess portfolio returns and alphas (in decimals), while those in square brackets are Newey and West (1987) t-statistics calculated with a twelve-month lag length. The plain numbers in Panels C to E are, respectively, the time-series means of the equally-weighted cross-sectional means of moneyness and systematic and idiosyncratic volatility at the start of the call return period.

The portfolio sorts yield conclusions in broad agreement with our model. While controlling for idiosyncratic volatility mean excess returns fall with systematic volatility within the OTM portfolios (see columns (1) and (2)), they rise with that volatility within the ITM portfolio (column (4)). The mean excess sold-before-maturity returns in Panel A, for example, fall by 27% (t-statistic: –5.85) in the DOTM portfolio, but rise by 7% (t-statistic: 3.31) in the ITM portfolio. The difference is a significant 34% (t-statistic: 8.55; see “High–Low” row). Conversely, while controlling for systematic volatility mean excess returns unambiguously fall with idiosyncratic volatility, the magnitude of the drop becomes milder with moneyness (see columns (5) to (8)). The mean excess sold-before-maturity returns in Panel A, for example, fall by 26% (t-statistic: –4.76) in the DOTM portfolio but only by 11% (t-statistic: –6.50) in the ITM portfolio. The difference is a significant 15% (t-statistic: 3.09; see “High–Low” row). Further supporting our theory, mean excess returns also consistently fall with moneyness within each volatility portfolio, with t-statistics below minus five (unreported). The single non-supportive result is that while, in line with our theory, mean excess returns do rise with systematic volatility in the ATM portfolio, the rise is never significant (see column (3)).

Adjusting for risk, the Fama-French-Karakaya (FFK) alphas in Panels A and B suggest that the nine stock and option factors do not completely explain the pricing of systematic volatility within the

[17] Specifically, Panel C of Table 2 suggests that the calls in the top moneyness portfolio produce a mean cross-sectional average moneyness of 1.07, and only three percent of them have a moneyness value below unity. Conversely, the calls in the second-to-top portfolio produce a mean cross-sectional average moneyness of 1.00, with their moneyness values clustering around the unity value for the vast majority of cross-sections. Finally, the calls in the bottom and second-to-bottom portfolio produce a mean cross-sectional average moneyness of 0.90 and 0.96, respectively, with virtually all their moneyness values lying below unity. Using quartile rather than fixed moneyness breakpoints has the advantage that we obtain better diversified portfolios and avoid empty portfolios.

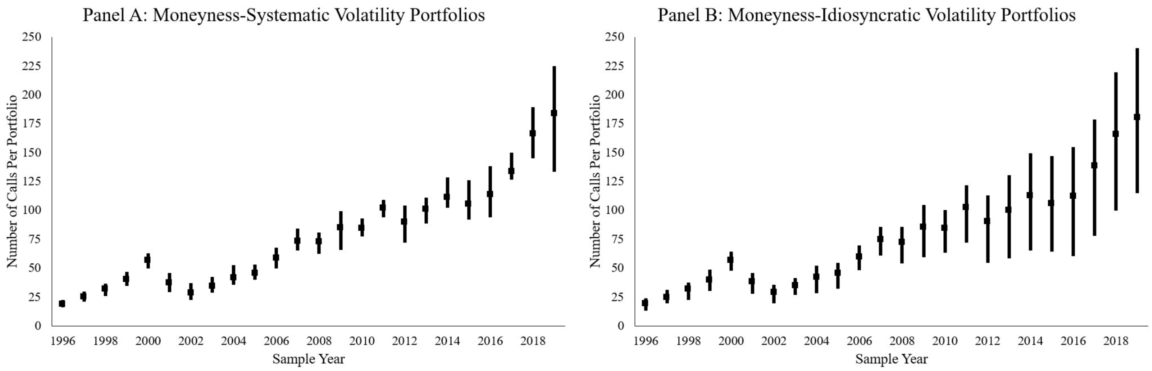

Figure 2: **Number of Calls Per Portfolio** The figure plots the minimum (lower end of line), median (dot), and maximum (upper end of line) number of calls for the double-sorted moneyness-systematic volatility (Panel A) and the moneyness-idiosyncratic volatility (Panel B) portfolios over our sample period, averaged by calendar year.

moneyness portfolios, with the systematic volatility spread portfolio alpha still being significantly positive (negative) for ITM (OTM and DOTM) calls and those alphas still significantly rising over the moneyness portfolios. Notably, however, those factors do largely explain the pricing of idiosyncratic volatility in the DOTM and OTM but not ATM and ITM portfolios, with the idiosyncratic volatility spread portfolio alpha for the OTM and DOTM calls often being insignificant. As a result, the factors also explain the conditional effect of moneyness on the idiosyncratic volatility premium, with the spread portfolio alphas no longer significantly rising over the moneyness portfolios.

Looking at the remaining panels, Panel C reveals that the mean moneyness values of the portfolios are consistent with our labels for the moneyness portfolios. More crucially, Panels D and E confirm that the portfolios in columns (1) to (4) ((5) to (8)) generate strong variations in systematic (idiosyncratic) volatility while keeping idiosyncratic (systematic) volatility almost constant, implying that our portfolio sorts do a good job in controlling for the volatility component not under consideration.

Motivated by concern that our double-sorted portfolios may often be ill-diversified, Figure 2 finally plots the minimum, median, and maximum number of calls within each portfolio over our sample period, averaged by calendar year to improve readability. In line with our expectations, the figure reveals that the number of calls within each portfolio greatly rises over that period, with the median for the portfolios sorted on moneyness and systematic (idiosyncratic) volatility increasing from 19

(19) to 184 (181). Notwithstanding, the figure further reveals that the portfolio with the fewest calls never attracts an annual average number of calls below 18 (15), with it, in fact, containing 15 or more in 97% (96%) of all sample months.[18] While we believe that our double sorted portfolios are sufficiently diversified for us to draw valid inferences, we acknowledge that the low numbers of calls in some portfolios in some of the earlier sample months are a limitation of our data.

In Table 3, we switch to FM regressions to test our model's predictions. To do so, we first split our sample into subsamples according to the quartile breakpoints of the moneyness distribution at the end of sample month $t-1$. We then conduct FM regressions of the excess sold-before-maturity (Panel A) or held-to-maturity (Panel B) return from start of month t on moneyness and systematic and idiosyncratic volatility measured until that date separately by subsample. We include moneyness as regressor to capture the effects of residual moneyness variations within the subsamples. Plain numbers are premium estimates (in decimals), while the numbers in square brackets are Newey and West (1987) t-statistics calculated using a lag length equal to twelve months.[19]

Our regression results completely agree with our portfolio sort results. While column (4) shows that the systematic volatility premium is positive in the ITM subsample, columns (1) and (2) document that it is negative in the OTM subsamples. Looking at the excess sold-before-maturity return in Panel A, the systematic volatility premium is, for example, 33% (t-statistic: 3.40) in the ITM subsample, while it is –138% (t-statistic: –6.20) in the DOTM subsample. While unreported, the difference is highly significant. Conversely, while the idiosyncratic volatility premium is negative for each subsample, its magnitude becomes less pronounced with moneyness. Again looking at the excess sold-before-maturity return in Panel A, the idiosyncratic volatility premium is, for example, –92% (t-statistic: –6.85) in

[18] Studying the number of unique stocks in the portfolio with the fewest calls does not greatly change our conclusions since there are only a tiny number of calls written on the same stocks in the double sorted portfolios over the earlier sample period. To be more specific, conditional on the moneyness-systematic (idiosyncratic) volatility portfolio with the fewest calls containing fewer than 30 calls, the number of calls in that portfolio drops by an average of only 2.8% (7.8%) upon us keeping only one call per stock, with the largest drop being equal to 14.8% (30.0%).

[19] Consistent with Vasquez (2017), Aretz and Gazi (2020), and Hu and Jacobs (2020), we also calculate 95% bootstrap confidence intervals for the t-statistic of each premium estimate in Table 3. In agreement with those studies, we however also find that the bootstrap confidence intervals are close to identical to the corresponding asymptotic confidence intervals. Given that, we do not report the bootstrap confidence intervals to conserve space.

the DOTM subsample but only –24% (t-statistic: –5.46) in the ITM subsample. While unreported, the difference is again highly significant. Also as before, while, in agreement with our theory, the systematic volatility premium in ATM calls is positive, it is again never significant.[20]

3.3.c Using Alternative Systematic and Idiosyncratic Volatility Estimates

While our empirical results in Section 3.3.b broadly support the moneyness and volatility predictions of the Rubinstein (1976)-Brennan (1979) model, we next evaluate their robustness to using alternative systematic and idiosyncratic volatility estimates. To that end, Table 4 first repeats the FM regressions in Table 3 using estimates obtained from alternative linear factor models, namely, the CAPM (Panel A), the Fama-French-Carhart (1997) model (Panel B), the Fama-French (2016) six-factor model (Panel C; repeated for comparison), the Hou-Mo-Xue-Zhang (2021) augmented q-theory model (Panel D), and the Stambaugh and Yuan (2017) mispricing factor model (Panel E). For the sake of brevity, the table only repeats the regressions using the sold-before-maturity return as dependent variable.

Remarkably, Table 4 suggests that our main FM regression estimates do not differ much across the alternative factor models, largely owing to the fact that the systematic or idiosyncratic volatility estimates are all highly cross-sectionally correlated, as we already noticed in Section 3.3.a. In turn, the high correlations arise since all models feature the excess market return as first factor, and the excess market return consistently captures the lion share of systematic volatility for the vast majority of stocks. Given our inability to perfectly calculate systematic and idiosyncratic volatility from multifactor models (recall our discussion in Section 3.2), we find it reassuring that the one-factor CAPM yields conclusions aligning with those from the multifactor models.[21]

[20] In Tables IA.1 and IA.2 in the Internet Appendix, we repeat the FM regressions in Table 3 using alternative data filters and methodologies. Specifically, Table IA.1 repeats those regressions keeping only one call per underlying stock within each moneyness portfolio, as is sometimes done in other studies. Consistent with the majority of stocks having no more than two to three calls written on them, leading the new restriction to only marginally decrease our sample size, the results in Table IA.1 are virtually identical to those in Table 3. Next, Table IA.2 relies on weighted rather than ordinary least-squares regressions, using call market capitalization calculated as mid-point price times open interest as weight. As before, the results in Table IA.2 are also virtually identical to those in Table 3.

[21] In Tables IA.3 and IA.4 in the Internet Appendix, we also repeat the portfolio sorts and the held-to-maturity return regressions using the systematic and idiosyncratic volatility estimates obtained from the alternative factor models. The evidence in those tables further corroborates the robustness of our main conclusions.

Table 5 next repeats our FM regressions in Table 3 using systematic and idiosyncratic volatility estimates obtained from the Fama-French (2016) six-factor model estimated over alternative data frequencies and estimation window lengths. In particular, we estimate the model over either the three (Panel A), twelve (Panel B), 24 (Panel C; repeated for comparison), or 60 months (Panel D) of daily data or the 60 months of monthly data (Panel E) before the start of the call return period. As before, the table only repeats the sold-before-maturity return regressions.[22] While the table suggests that our empirical results survive using either specification, the systematic volatility results are markedly weaker over the two shortest estimation window lengths, with the systematic volatility premium in ITM calls, for example, only being 14% (t-statistic: 1.74) and 24% (t-statistic: 2.73) over the three and twelve-month window, respectively. The reason is that the Fama-French (2016) six factor model parameters are far less well estimated over those windows, with their standard errors often three to four times higher than those obtained from longer windows. In turn, the higher standard errors induce more noise into the systematic volatility estimates, reducing their pricing ability.[23]

In Table 6, we finally repeat the FM regressions in Table 5 leaving a one-month gap between the call return and the volatility estimation windows. The table suggests that this modification only marginally affects our results, with the estimates in the table close to identical to those in Table 5.

3.3.d Replacing Systematic Volatility By the Absolute Underlying Exposures

Given that cross-sectional variations in the multifactor-model systematic volatility estimates depend on both the squared factor exposures and the cross factor exposures (recall Equation (10)), we next find it interesting to ask whether our systematic volatility results are mostly driven by the squared exposures and, if so, whether specific squared exposures are most responsible for them. To answer those

[22] While we no longer report the portfolio sort and held-to-maturity return regression results from here on to conserve space, these always yield conclusions aligning with those from the sold-before-maturity return regressions.

[23] In unreported work, we contrast the standard errors of the exposure estimates obtained from all five linear factor models across the three, twelve, 24, and 60 month daily data estimation windows, first taking cross-sectional averages and then averaging over our sample period. Almost independent of the factor model and the exposure, the average standard error obtained from the three-month window is about four times larger than the average standard error obtained from the 60-month window. Conversely, the average standard error obtained from the 24-month window is only about 1.30 times larger than the average standard error obtained from the 60-month window.

questions, we replace the Fama-French (2016) six-factor model systematic volatility estimate used in the FM regressions in Table 3 with the square roots of the squared underlying factor exposures (i.e., with the absolute underlying factor exposures). The underlying exposures are the MKT, SMB, HML, MOM, RMW, and CMA exposures. As we said before, we use the absolute exposures instead of their squared values to mitigate the effect of extreme outliers in the squared exposures.

Columns (1) to (4) in Table 7 show that, with the exception of the MOM exposure, all absolute exposures rise with moneyness. The absolute CMA exposure, for example, rises from –0.04 (t-statistic: –1.75) for the DOTM calls to 0.02 (t-statistic: 2.78) for the ITM calls. Noteworthily, however, the absolute exposure premiums are generally more significant for OTM than for ITM calls, with, for example, four of them significant at the 90% confidence level for the DOTM but only one for the ITM calls. Adding the cross factor exposures as additional independent variables into the regressions, we generally do not find their premiums to be either significant or related to moneyness (unreported to conserve space). In summary, we thus conclude that the squared factor exposures, and not the cross factor exposures, drive our systematic volatility results in Table 3 and that most squared exposures, with the exception of the MOM exposure, contribute to those results.

3.3.e Rerunning the Regressions on the Stocks Underlying Our Calls

Spurred by the idea that stocks can be interpreted as infinitely ITM calls, our theory predicts them to produce an even more significantly positive systematic volatility premium and an even less significant idiosyncratic volatility premium than our sample ITM calls. To test those predictions, we thus finally repeat our FM regressions in Table 3 and in the first four columns of Table 7 on the stocks underlying our sample calls. To do so, we however exclude moneyness as independent variable since moneyness does not vary across stocks. In agreement with Ang et al.'s (2006) and Frazzini and Pedersen's (2014) evidence but in conflict with our predictions, column (5) of Table 7 suggests that while systematic volatility is only insignificantly priced in stocks (t-statistic: 1.58), idiosyncratic volatility is significantly negatively priced in them (t-statistic: –2.83). Notwithstanding, column (6) of the table reveals that the absolute exposure premiums in the stocks are most similar to those in the ITM calls, supporting the idea that stocks are closest to ITM calls. While it is surprising that our theory is more successful in explaining call rather than stock returns, we deem the reason for that divergence to be more related to specific features of stocks than calls and thus to be outside of the scope of our paper.

In summary, our evidence in this section broadly supports the moneyness and volatility predictions of the Rubinstein (1976)-Brennan (1979) model. While systematic volatility positively prices ITM calls, it negatively prices OTM calls. Moreover, while idiosyncratic volatility unambiguously negatively prices calls, its premium moves toward zero with moneyness. Finally, moneyness also unambiguously negatively prices calls. Our evidence is robust to using volatility estimates obtained from alternative linear factor models and to leaving a gap between call return and volatility estimation window, but becomes weaker, without however disappearing, using short volatility estimation windows. The results in this section not supporting our theory are that systematic volatility is only insignificantly positively priced in our ATM calls and in the stocks underlying our sample calls, whereas idiosyncratic volatility is significantly negatively priced in those same underlying stocks.

4. Controlling for Other Option Pricing Variables

In this section, we repeat our FM regressions in Table 3 controlling for other variables known to price options from the prior literature, reporting our results in Table 8. To wit, Panel A of Table 8 controls for call and stock liquidity proxies since Garleanu, Pedersen, and Poteshman (2009) argue that market makers charge higher prices for writing hard-to-delta-hedge options in net positive demand, lowering the options' expected returns. In line with Bollen and Whaley (2004), we use the call open interest-to-stock dollar trading volume ratio at the start of the call return period as proxy for call demand, while we use the call bid-ask spread-to-call midpoint price ratio at that time and the mean of a stock's absolute daily return-to-dollar trading volume ratio over the twelve months until that time ("Amihud (2002) stock illiquidity") as (inverse) proxies for the ability to delta hedge a call. Panel A suggests that adding those controls does not affect our main conclusions. Interestingly, it further suggests that while call open interest as well as stock illiquidity can positively or negatively price calls, the bid-ask spread unambiguously negatively prices them, in line with Cao and Han (2013).

In Panel B, we control for call mispricing proxies since Stein (1989), Poteshman (2001), and Goyal and Saretto (2009) establish that investors often overpay for options written on assets with recent increases in volatility or a high implied compared to historical volatility. Spurred by these studies, we thus add the change in an underlying stock's daily volatility over the month two months before the call return period to the month directly before it plus the ratio of daily underlying-stock volatility over the month before the call return period to Black-Scholes (1973) implied volatility at the start of that period as further independent variables into our regressions. In line with Cao and Han (2013), we finally also add the change in Black-Scholes (1973) implied volatility over the call return period to account for the potential correction of volatility-induced mispricing in calls. Panel B reveals that adding those controls does not affect our main conclusions. Consistent with other studies, it further shows that while the historical-to-implied volatility ratio and the change in implied volatility typically positively price calls, the change in stock volatility usually negatively prices them.

Panel C controls for higher-order moments estimates since Bakshi and Kapadia (2003) show that, in a stochastic volatility world with jumps, option returns contain components reflecting the underlying asset's variance and jump premiums. We follow Bali and Hovakimian (2009) and Carr and Wu (2009) in using the difference between realized variance until the start of the call return period and model-free implied variance at the start of that period as proxy for the variance risk premium (the second-order moment). Since downward jumps induce left skewness and excess kurtosis into an asset's return, we further use estimates of an underlying stock's third ("skewness") and fourth ("kurtosis") order moments obtained from the methodology of Bakshi, Kapadia, and Madan (2003) as proxies for the jump premium. See Section IA.5 in the Internet Appendix for more details about how we construct the higher-order moments estimates. Panel C reveals that adding those estimates does not affect our main conclusions. In line with Carr and Wu (2009) and Driessen, Maenhout, and Vilkov (2009), the variance risk premium is generally insignificant, while, in line with Bali and Murray (2013) and Boyer and Vorkink (2014), skewness (kurtosis) weakly negatively (positively) prices calls.[24]

In Panel D, we finally control for underlying-stock characteristics since, if the stock characteristics reflect the squared exposures on spread portfolios formed from them, their inclusion may drive out our systematic (but not our idiosyncratic) volatility results (see, e.g., Fama and French, 2015). We thus add a stock's market size, book-to-market ratio, past one-year ("momentum") return, asset growth, and profitability as further independent variables to our regressions.[25] Panel D suggests that adding the stock characteristics weakens but does not eliminate our systematic volatility results, whilst not affecting our other results. That the stock characteristics do not completely drive out our systematic volatility results is likely due to the non-MKT exposures often taking on negative values, inducing their squared values to only mildly correlate with the stock characteristics. Consistent with Cao et al. (2021), the stock characteristics can be differentially priced in calls and stocks. While market size, for example, often negatively prices stocks, it positively prices all but the DOTM calls.[26]

5. Adjusting for Bid-Ask Transaction Costs

Following other studies, we study call returns calculated from bid-ask midpoint prices in our main tests, implicitly arguing that they are more reflective of true option returns than those calculated from bid or ask prices. Notwithstanding, Santa-Clara and Saretto (2009) find that option bid-ask

[24]While some empirical studies (see, e.g., Vasquez, 2017) find a significant variance risk premium in single-stock options, they generally look into delta-hedged option or straddle returns, which are better suited to estimate that premium since they are effectively orthogonalized with respect to the underlying stock return.

[25]We follow the literature in calculating the stock characteristics. Market size is the log of the product of common shares outstanding and the share price at the end of June. The book-to-market ratio is the log of the ratio of the book value of equity at the end of the fiscal year in the prior calendar year to market value at the end of that calendar year. Asset growth is the log gross change in total assets from the end of the fiscal year in the two-year ago calendar year to the end of the fiscal year in the prior calendar year. Profitability is the ratio of sales net of costs of goods sold, selling, general, and administrative expenses, and interest expenses to the book value of equity at the end of the fiscal year in the prior calendar year, where the book value of equity is total assets minus total liabilities plus deferred taxes (zero if missing) minus preferred stock (zero if missing). We use the calculated values from July of the current calendar year to June of the next. In contrast, we calculate the momentum return as the compounded monthly return over the prior twelve months but excluding the most recent month. We use the calculated value over the next month only.

[26]In Table IA.5 in the Internet Appendix, we follow Hu and Jacobs (2020) in also controlling for the mean daily underlying-stock return over the six months before the call return period, despite it sharing an average cross-sectional correlation of 0.60 with the momentum return. The table shows that this modification does not change our conclusions.

spreads are generally much wider than those of stocks, while Goyal and Saretto (2009) and Cao and Han (2013) report that adjusting for bid-ask transaction costs greatly eats into the profitability of their option trading strategies. To study whether such transaction costs also affect our conclusions, we next repeat our regressions in Table 3 using call returns calculated as the ratio of call midpoint price minus S times call bid-ask spread at the end of the return period to call midpoint price plus S times call bid-ask spread at the start of that period. We set S equal to zero, 0.25, and 0.50, with $S = 0.50$ implying that option investors sell at the bid price and buy at the ask price.

Table 9 presents the results from the regressions adjusting for bid-ask transaction costs. Remarkably, those results more strongly support our theory than those derived from call returns not adjusting for bid-ask transaction costs. To be specific, while we still find that, independent of the bid-ask fraction S, systematic volatility positively (negatively) prices ITM (OTM) calls and idiosyncratic volatility negatively (more strongly negatively) prices those same calls, we now also find that systematic volatility not only positively but also significantly prices ATM calls. Assuming that $S = 0.50$, the systematic volatility premium in ATM calls is, for example, now 31% (t-statistic: 2.50).

6. Conclusion

We use the Rubinstein (1976)-Brennan (1979) stochastic discount factor model to study the effect of an asset's volatility on the expected returns of options written on that asset. We demonstrate that the effect depends crucially on both option moneyness and whether variations in volatility are driven by systematic or idiosyncratic volatility. While idiosyncratic volatility only affects an option's elasticity, systematic volatility also oppositely affects the expected underlying asset return. Whether systematic volatility variations produce a stronger elasticity or underlying asset effect depends on the convexity of the option's value in the underlying asset's value, with deeper ITM (OTM) options producing a stronger underlying asset (elasticity) effect. Using calls as example, systematic volatility positively prices ITM and ATM but negatively sufficiently OTM calls. Conversely, idiosyncratic volatility unambiguously negatively prices calls, with its effect however decreasing with moneyness.

We use calls written on zero-dividend single stocks to test the moneyness and volatility predictions of the Rubinstein (1976)-Brennan (1979) model. Double-sorted portfolios and FM regressions separately run on calls within different moneyness classes broadly support those predictions, both in case of sold-before-maturity and held-to-maturity returns. Our evidence is robust to using alternative linear factor models to estimate the volatility components, alternative volatility estimation windows, and alternative gaps between the call return and volatility estimation windows. Controlling for call and stock liquidity proxies, option mispricing proxies, higher-order moments estimates, and stock characteristics or adjusting for bid-ask transaction costs does not affect our conclusions.

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Table 1
Descriptive Statistics
In this table, we present descriptive statistics on the calls (Panel A) and the total, systematic, and idiosyncratic volatility of the single stocks underlying the calls (Panel B). The descriptive statistics are the mean, standard deviation, and the first, 25th, 50th, 75th, and 99th percentiles. We calculate the sold-before-maturity return from the start to the end of the month before the month in which the call expires, while we calculate the held-to-maturity return from the same starting date to the call maturity date. Moneyness and days-to-maturity are the ratio of the stock price to the call strike price and the number of calendar days until the call expiry date, respectively, both measured at the start of the call return period. We calculate an underlying stock's annualized total volatility using daily data over the prior 24 months. Conversely, we calculate its annualized systematic and idiosyncratic volatility using either the CAPM, the Fama-French-Carhart (1997) model, the Fama-French (2016) six-factor model, the Hou-Mo-Xue-Zhang (2021) augmented q-theory model, or the Stambaugh and Yuan (2017) mispricing factor model estimated using daily data over the prior 24 months. In the final rows of the table, we also report the total number of observations and the number of unique stocks.

		Standard	Percentiles				
	Mean	Deviation	1	25	50	75	99
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: Call Data							
Sold-Before-Maturity Return	0.20	1.63	−0.97	−0.68	−0.27	0.51	6.21
Held-To-Maturity Return	0.22	2.44	−1.00	−1.00	−0.82	0.66	9.22
Moneyness	0.98	0.07	0.82	0.94	0.98	1.02	1.16
Days-To-Maturity	48	5.45	29	47	50	51	53
Panel B: Total, Systematic, and Idiosyncratic Volatility							
Total Volatility	0.43	0.21	0.14	0.27	0.38	0.55	1.10
Systematic Volatility							
CAPM	0.21	0.12	0.05	0.13	0.18	0.25	0.64
Fama-French-Carhart Model	0.24	0.13	0.07	0.15	0.20	0.29	0.69
Fama-French Six-Factor Model	0.24	0.13	0.08	0.15	0.21	0.30	0.71
Augmented q-Theory Model	0.24	0.13	0.08	0.15	0.20	0.29	0.70
Mispricing Factor Model	0.25	0.14	0.08	0.16	0.22	0.31	0.74
Idiosyncratic Volatility							
CAPM	0.36	0.20	0.07	0.21	0.32	0.47	0.98
Fama-French-Carhart Model	0.35	0.19	0.03	0.21	0.31	0.46	0.95
Fama-French Six-Factor Model	0.34	0.19	0.03	0.20	0.30	0.45	0.93
Augmented q-Theory Model	0.35	0.19	0.03	0.21	0.31	0.45	0.95
Mispricing Factor Model	0.37	0.20	0.05	0.22	0.33	0.48	0.98
Number of Observations							336,487
Number of Unique Stocks							5,983

Table 2

Call Portfolios Double-Sorted on Moneyness and Systematic or Idiosyncratic Volatility

In this table, we present the results from call portfolios double-sorted on moneyness and systematic or idiosyncratic underlying-stock volatility controlling for the other volatility component. We form the portfolios double-sorted on moneyness and systematic volatility in columns (1) to (4) as follows. At the end of each sample month $t-1$, we first sort the calls into portfolios according to the quartile breakpoints of idiosyncratic volatility. Within each portfolio, we independently sort them into portfolios according to the quartile breakpoints of moneyness and systematic volatility. We equally-weight the resulting $4 \times 4 \times 4$ portfolios and form equally-weighted portfolios of the portfolios within the same moneyness-systematic volatility classification. We hold the resulting 4×4 portfolios from the start of month t. Within each moneyness (systematic volatility) portfolio, we finally form a spread portfolio long the highest and short the lowest systematic volatility (moneyness) portfolio ("High–Low" or "ITM–DOTM"). To adjust the spread portfolios for risk, we regress them on the six Fama-French (2016) stock factors plus the three Karakaya (2013) option factors and report the regression's intercept ("FFK Alpha"). To form the portfolios sorted on moneyness and idiosyncratic volatility in columns (5) to (8), we reverse the roles of systematic and idiosyncratic volatility. Moneyness is the ratio of stock price to strike price at the start of month t. We calculate systematic and idiosyncratic volatility using the Fama-French (2016) six-factor model estimated over the 24 months of daily data before the call return period. The plain numbers in Panels A and B are mean excess sold-before-maturity and mean excess held-to-maturity returns and their associated FFK alphas, respectively, while the numbers in square brackets are Newey and West (1987) t-statistics calculated using a twelve-month lag length. Panels C, D, and E report mean moneyness, systematic volatility, and idiosyncratic volatility at the start of the return period, respectively.

	Systematic Volatility Moneyness					Idiosyncratic Volatility Moneyness				
Volatility	DOTM	OTM	ATM	ITM	ITM–DOTM	DOTM	OTM	ATM	ITM	ITM–DOTM
	(1)	(2)	(3)	(4)	(4)–(1)	(5)	(6)	(7)	(8)	(8)–(5)
Panel A: Mean Excess Sold-Before-Maturity Return										
1 (Low)	0.61	0.22	−0.02	−0.17	−0.78	0.62	0.26	0.05	−0.06	−0.68
2	0.55	0.19	0.00	−0.11	−0.66	0.55	0.17	0.01	−0.11	−0.66
3	0.44	0.14	−0.02	−0.10	−0.54	0.46	0.14	−0.03	−0.14	−0.59
4 (High)	0.34	0.10	−0.02	−0.10	−0.44	0.36	0.07	−0.08	−0.17	−0.53
High–Low	−0.27	−0.12	0.00	0.07	0.34	−0.26	−0.19	−0.13	−0.11	0.15
	[−5.85]	[−4.26]	[0.14]	[3.31]	[8.55]	[−4.76]	[−5.26]	[−5.22]	[−6.50]	[3.09]
FFK Alpha	−0.36	−0.13	0.00	0.06	0.42	−0.13	−0.08	−0.08	−0.09	0.04
	[−5.24]	[−3.42]	[−0.09]	[2.25]	[6.20]	[−1.92]	[−2.26]	[−3.07]	[−5.25]	[0.59]
Panel B: Mean Excess Held-to-Maturity Return										
1 (Low)	0.62	0.23	0.01	−0.15	−0.77	0.66	0.33	0.12	−0.01	−0.67
2	0.63	0.23	0.05	−0.08	−0.71	0.68	0.23	0.09	−0.06	−0.74
3	0.45	0.18	0.05	−0.06	−0.51	0.45	0.18	0.01	−0.11	−0.56
4 (High)	0.41	0.18	0.06	−0.05	−0.46	0.36	0.09	−0.06	−0.16	−0.52

(continued on next page)

Table 2
Call Portfolios Double-Sorted on Moneyness and Systematic or Idiosyncratic Volatility (cont.)

	Systematic Volatility Moneyness					Idiosyncratic Volatility Moneyness				
Volatility	DOTM	OTM	ATM	ITM	ITM–DOTM	DOTM	OTM	ATM	ITM	ITM–DOTM
	(1)	(2)	(3)	(4)	(4)–(1)	(5)	(6)	(7)	(8)	(8)–(5)
Panel B: Mean Excess Held-to-Maturity Returns (cont.)										
High–Low	−0.21	−0.05	0.05	0.10	0.31	−0.30	−0.24	−0.18	−0.15	0.15
	[−2.63]	[−1.04]	[1.25]	[3.09]	[4.72]	[−4.04]	[−5.03]	[−5.84]	[−6.50]	[2.22]
FFK Alpha	−0.26	−0.06	0.02	0.07	0.33	−0.15	−0.10	−0.10	−0.10	0.04
	[−2.92]	[−1.40]	[0.45]	[2.24]	[4.28]	[−2.08]	[−2.20]	[−3.77]	[−3.93]	[0.58]
Panel C: Mean Moneyness										
1 (Low)	0.91	0.96	1.00	1.07	0.16	0.91	0.96	1.00	1.07	0.16
2	0.91	0.96	1.00	1.07	0.17	0.91	0.96	1.00	1.07	0.16
3	0.90	0.96	1.00	1.07	0.17	0.90	0.96	1.00	1.07	0.17
4 (High)	0.90	0.96	1.00	1.07	0.17	0.90	0.96	1.00	1.07	0.17
High–Low	−0.00	0.00	0.00	0.01		−0.01	−0.00	0.00	0.01	
Panel D: Mean Systematic Volatility										
1 (Low)	0.16	0.16	0.16	0.16	−0.00	0.26	0.25	0.25	0.25	−0.00
2	0.22	0.22	0.22	0.22	0.00	0.26	0.26	0.26	0.26	−0.00
3	0.28	0.28	0.28	0.28	0.00	0.27	0.26	0.26	0.26	0.00
4 (High)	0.39	0.38	0.38	0.38	−0.00	0.27	0.27	0.27	0.27	0.00
High–Low	0.23	0.22	0.22	0.23		0.02	0.02	0.02	0.02	
Panel E: Mean Idiosyncratic Volatility										
1 (Low)	0.39	0.38	0.38	0.39	−0.00	0.24	0.23	0.23	0.23	−0.01
2	0.40	0.39	0.39	0.39	−0.01	0.33	0.33	0.33	0.33	0.00
3	0.40	0.40	0.39	0.40	−0.01	0.42	0.42	0.42	0.42	−0.00
4 (High)	0.41	0.41	0.41	0.41	−0.00	0.61	0.60	0.60	0.61	−0.00
High–Low	0.03	0.03	0.03	0.03		0.37	0.37	0.38	0.38	

Table 3
Regressions of Call Returns on Systematic and Idiosyncratic Volatility
The table presents the results from Fama-MacBeth (1973) regressions of the excess call return calculated from start of month t to the end of that month ("sold-before-maturity," Panel A) or to the call maturity date ("held-to-maturity," Panel B) on moneyness and systematic and idiosyncratic underlying-stock volatility measured until end of month $t-1$. We run the regressions separately for calls with a start-of-month-t moneyness below the first quartile (column (1), "DOTM"), between the first and second (column (2), "OTM"), between the second and third (column (3), "ATM"), and above the third (column (4), "ITM"). Moneyness is the ratio of stock price to strike price at the start of month t. We calculate systematic and idiosyncratic volatility using the Fama-French (2016) six-factor model estimated over the 24 months of daily data before the call return period. Plain numbers are premium estimates (in decimals), while the numbers in square brackets are Newey and West (1987) t-statistics calculated using a twelve-month lag length.

	DOTM		OTM		ATM		ITM	
	(1)		(2)		(3)		(4)	
Panel A: Excess Sold-Before-Maturity Return								
Moneyness	−6.08	[−11.09]	−6.67	[−11.55]	−4.19	[−9.06]	−0.97	[−8.33]
Systematic Volatility	−1.38	[−6.20]	−0.75	[−4.52]	0.16	[1.24]	0.33	[3.40]
Idiosyncratic Volatility	−0.92	[−6.85]	−0.57	[−5.46]	−0.39	[−4.89]	−0.24	[−5.46]
Constant	6.66	[12.41]	6.90	[11.93]	4.22	[8.77]	0.88	[6.79]
Panel B: Excess Held-to-Maturity Return								
Moneyness	−5.01	[−6.27]	−6.57	[−7.18]	−4.70	[−7.12]	−1.19	[−6.80]
Systematic Volatility	−1.17	[−2.76]	−0.61	[−1.78]	0.34	[1.32]	0.46	[2.23]
Idiosyncratic Volatility	−1.15	[−5.16]	−0.79	[−4.82]	−0.53	[−4.89]	−0.39	[−5.18]
Constant	5.78	[7.21]	6.89	[7.36]	4.79	[6.96]	1.15	[5.87]

Table 4

Regressions of Call Returns on Systematic and Idiosyncratic Volatility Calculated from Alternative Linear Factor Models Proposed in the Recent Literature

The table presents the results from Fama-MacBeth (1973) regressions of the excess call return calculated from start of month t to its end on moneyness and systematic and idiosyncratic underlying-stock volatility measured until end of month $t-1$. We run the regressions separately for calls with a start-of-month-t moneyness below the first quartile (column (1), "DOTM"), between the first and second (column (2), "OTM"), between the second and third (column (3), "ATM"), and above the third (column (4), "ITM"). Moneyness is the ratio of stock price to strike price at the start of month t. We calculate systematic and idiosyncratic volatility using either the CAPM (Panel A), the Fama-French-Carhart (1997) model (Panel B), the Fama-French (2016) six-factor model (Panel C), the Hou-Mo-Xue-Zhang (2021) augmented q-theory model (Panel D), or the Stambaugh and Yuan (2017) mispricing factor model (Panel E) estimated over the 24 months of daily data before the call return period. Plain numbers are premium estimates (in decimals), while the numbers in square brackets are Newey and West (1987) t-statistics calculated using a twelve-month lag length.

	DOTM		OTM		ATM		ITM	
	(1)		(2)		(3)		(4)	
Panel A: CAPM Estimates								
Moneyness	−6.13	[−11.27]	−6.62	[−11.59]	−4.16	[−9.07]	−0.98	[−8.39]
Systematic Volatility	−1.36	[−5.11]	−0.84	[−4.00]	0.17	[1.08]	0.38	[3.64]
Idiosyncratic Volatility	−1.04	[−8.62]	−0.60	[−5.95]	−0.34	[−4.56]	−0.20	[−5.17]
Constant	6.72	[12.60]	6.87	[12.01]	4.19	[8.79]	0.88	[6.80]
Panel B: Fama-French-Carhart Model Estimates								
Moneyness	−6.11	[−11.19]	−6.66	[−11.60]	−4.18	[−9.07]	−0.97	[−8.33]
Systematic Volatility	−1.38	[−5.65]	−0.78	[−4.50]	0.17	[1.25]	0.36	[3.58]
Idiosyncratic Volatility	−0.96	[−7.12]	−0.58	[−5.46]	−0.38	[−4.82]	−0.24	[−5.48]
Constant	6.70	[12.49]	6.90	[12.00]	4.21	[8.78]	0.87	[6.75]
Panel C: Fama-French Six-Factor Model Estimates								
Moneyness	−6.08	[−11.09]	−6.67	[−11.55]	−4.19	[−9.06]	−0.97	[−8.33]
Systematic Volatility	−1.38	[−6.20]	−0.75	[−4.52]	0.16	[1.24]	0.33	[3.40]
Idiosyncratic Volatility	−0.92	[−6.85]	−0.57	[−5.46]	−0.39	[−4.89]	−0.24	[−5.46]
Constant	6.66	[12.41]	6.90	[11.93]	4.22	[8.77]	0.88	[6.79]
Panel D: Augmented q-Theory Model Estimates								
Moneyness	−6.10	[−11.11]	−6.65	[−11.55]	−4.18	[−9.02]	−0.98	[−8.43]
Systematic Volatility	−1.41	[−6.24]	−0.76	[−4.40]	0.17	[1.32]	0.35	[3.49]
Idiosyncratic Volatility	−0.94	[−7.13]	−0.57	[−5.46]	−0.38	[−4.79]	−0.24	[−5.38]
Constant	6.69	[12.42]	6.89	[11.92]	4.20	[8.73]	0.88	[6.86]
Panel E: Mispricing Factor Model Estimates								
Moneyness	−5.84	[−10.22]	−6.29	[−10.59]	−3.44	[−8.44]	−0.90	[−7.55]
Systematic Volatility	−1.36	[−5.67]	−0.71	[−4.05]	0.14	[1.08]	0.36	[3.57]
Idiosyncratic Volatility	−0.85	[−6.78]	−0.46	[−4.37]	−0.28	[−4.01]	−0.21	[−4.65]
Constant	6.38	[11.38]	6.47	[10.92]	3.44	[8.06]	0.79	[5.97]

Table 5

Regressions of Call Returns on Systematic and Idiosyncratic Volatility Calculated from Alternative Data Frequencies and Estimation Window Lengths

The table presents the results from Fama-MacBeth (1973) regressions of the excess call return calculated from start of month t to its end on moneyness and systematic and idiosyncratic underlying-stock volatility measured until end of month $t-1$. We run the regressions separately for calls with a start-of-month-t moneyness below the first quartile (column (1), "DOTM"), between the first and second (column (2), "OTM"), between the second and third (column (3), "ATM"), and above the third (column (4), "ITM"). Moneyness is the ratio of stock price to strike price at the start of month t. We calculate systematic and idiosyncratic volatility using the Fama-French (2016) six-factor model estimated over either the prior three (Panel A), twelve (Panel B), 24 (Panel C), or 60 (Panel D) months of daily data or the prior 60 months of monthly data (Panel E) before the call return period. Plain numbers are premium estimates (in decimals), while the numbers in square brackets are Newey and West (1987) t-statistics calculated using a twelve-month lag length.

	DOTM		OTM		ATM		ITM	
	(1)		(2)		(3)		(4)	
Panel A: Daily Data Over Three Months								
Moneyness	−6.16	[−11.96]	−6.72	[−11.94]	−4.10	[−8.98]	−0.98	[−8.17]
Systematic Volatility	−1.40	[−6.87]	−0.84	[−5.44]	−0.07	[−0.61]	0.14	[1.74]
Idiosyncratic Volatility	−0.96	[−7.18]	−0.54	[−5.15]	−0.33	[−4.30]	−0.23	[−5.60]
Constant	6.74	[13.30]	6.95	[12.29]	4.15	[8.72]	0.91	[6.96]
Panel B: Daily Data Over Twelve Months								
Moneyness	−6.10	[−11.53]	−6.61	[−11.53]	−4.15	[−8.98]	−0.99	[−8.41]
Systematic Volatility	−1.30	[−5.81]	−0.79	[−4.90]	0.07	[0.55]	0.24	[2.73]
Idiosyncratic Volatility	−1.00	[−7.29]	−0.55	[−5.32]	−0.37	[−4.79]	−0.23	[−5.18]
Constant	6.68	[12.82]	6.85	[11.90]	4.18	[8.73]	0.91	[6.98]
Panel C: Daily Data Over 24 Months								
Moneyness	−6.08	[−11.09]	−6.67	[−11.55]	−4.19	[−9.06]	−0.97	[−8.33]
Systematic Volatility	−1.38	[−6.20]	−0.75	[−4.52]	0.16	[1.24]	0.33	[3.40]
Idiosyncratic Volatility	−0.92	[−6.85]	−0.57	[−5.46]	−0.39	[−4.89]	−0.24	[−5.46]
Constant	6.66	[12.41]	6.90	[11.93]	4.22	[8.77]	0.88	[6.79]
Panel D: Daily Data Over 60 Months								
Moneyness	−6.06	[−9.46]	−6.93	[−11.52]	−4.26	[−8.83]	−1.03	[−7.42]
Systematic Volatility	−1.23	[−4.72]	−0.71	[−3.23]	0.20	[1.16]	0.41	[3.11]
Idiosyncratic Volatility	−0.93	[−6.37]	−0.56	[−4.92]	−0.35	[−3.83]	−0.21	[−4.47]
Constant	6.66	[10.85]	7.16	[11.93]	4.29	[8.52]	0.92	[6.07]
Panel E: Monthly Data Over 60 Months								
Moneyness	−5.96	[−9.59]	−6.95	[−11.65]	−4.36	[−8.84]	−1.03	[−7.24]
Systematic Volatility	−0.71	[−3.80]	−0.43	[−3.18]	0.11	[0.98]	0.35	[4.13]
Idiosyncratic Volatility	−0.93	[−6.13]	−0.59	[−4.55]	−0.40	[−4.20]	−0.28	[−5.72]
Constant	6.43	[10.80]	7.11	[11.97]	4.39	[8.53]	0.94	[5.97]

Table 6
Regressions of Call Returns on Systematic and Idiosyncratic Volatility Calculated from Alternative Estimations: Leaving a Gap Between Return and Estimation Window
The table presents the results from Fama-MacBeth (1973) regressions of the excess call return calculated from start of month t to its end on moneyness at the end of month $t-1$ and systematic and idiosyncratic underlying-stock volatility measured until end of month $t-2$. We run the regressions separately for calls with a start-of-month-t moneyness below the first quartile (column (1), "DOTM"), between the first and second (column (2), "OTM"), between the second and third (column (3), "ATM"), and above the third (column (4), "ITM"). Moneyness is the ratio of stock price to strike price at the start of month t. We calculate systematic and idiosyncratic volatility using the Fama-French (2016) six-factor model estimated over either the prior three (Panel A), twelve (Panel B), 24 (Panel C), or 60 (Panel D) months of daily data or the prior 60 months of monthly data (Panel E) ending at the start of month $t-1$. Plain numbers are premium estimates (in decimals), while the numbers in square brackets are Newey and West (1987) t-statistics with a twelve-month lag length.

	DOTM		OTM		ATM		ITM	
	(1)		(2)		(3)		(4)	
Panel A: Daily Data Over Three Months								
Moneyness	−5.86	[−11.72]	−6.66	[−11.48]	−4.14	[−9.20]	−0.99	[−8.12]
Systematic Volatility	−1.22	[−6.31]	−0.78	[−5.44]	−0.04	[−0.37]	0.12	[1.70]
Idiosyncratic Volatility	−0.86	[−6.22]	−0.47	[−4.46]	−0.33	[−4.12]	−0.21	[−5.26]
Constant	6.39	[13.16]	6.86	[11.77]	4.18	[8.90]	0.92	[6.85]
Panel B: Daily Data Over Twelve Months								
Moneyness	−6.04	[−11.26]	−6.60	[−11.48]	−4.15	[−8.93]	−1.00	[−8.36]
Systematic Volatility	−1.17	[−5.91]	−0.68	[−4.52]	0.10	[0.86]	0.20	[2.55]
Idiosyncratic Volatility	−0.98	[−7.19]	−0.56	[−5.38]	−0.38	[−4.95]	−0.22	[−5.10]
Constant	6.59	[12.53]	6.81	[11.81]	4.18	[8.65]	0.91	[6.95]
Panel C: Daily Data Over 24 Months								
Moneyness	−6.03	[−10.93]	−6.63	[−11.41]	−4.19	[−8.99]	−0.98	[−8.24]
Systematic Volatility	−1.33	[−5.92]	−0.69	[−4.09]	0.18	[1.40]	0.35	[3.49]
Idiosyncratic Volatility	−0.90	[−6.58]	−0.57	[−5.33]	−0.38	[−4.80]	−0.24	[−5.27]
Constant	6.59	[12.25]	6.86	[11.78]	4.22	[8.70]	0.87	[6.67]
Panel D: Daily Data Over 60 Months								
Moneyness	−5.95	[−9.20]	−6.91	[−11.53]	−4.26	[−8.78]	−1.03	[−7.37]
Systematic Volatility	−1.17	[−4.52]	−0.70	[−3.12]	0.23	[1.27]	0.43	[3.23]
Idiosyncratic Volatility	−0.93	[−6.40]	−0.55	[−4.76]	−0.36	[−3.86]	−0.21	[−4.51]
Constant	6.55	[10.59]	7.13	[11.94]	4.28	[8.46]	0.91	[5.98]
Panel E: Monthly Data Over 60 Months								
Moneyness	−5.85	[−9.30]	−6.96	[−11.56]	−4.37	[−8.77]	−1.03	[−7.22]
Systematic Volatility	−0.63	[−3.37]	−0.46	[−3.25]	0.13	[1.09]	0.37	[4.20]
Idiosyncratic Volatility	−0.93	[−6.07]	−0.56	[−4.31]	−0.40	[−4.03]	−0.29	[−5.71]
Constant	6.32	[10.46]	7.12	[11.88]	4.39	[8.46]	0.93	[5.94]

Table 7

Regressions of Call or Stock Returns on Absolute Exposures and Idiosyncratic Volatility

The table presents the results from Fama-MacBeth (1973) regressions of the excess call (columns (1) to (4)) or underlying-stock (columns (5) and (6)) return calculated from start of month t to its end on various combinations of moneyness, the underlying-stock absolute Fama-French (2016) six-factor model exposures, and systematic and idiosyncratic underlying-stock volatility measured until end of month $t-1$. The exposures are the MKT, SMB, HML, MOM, RMW, and CMA exposures. We run the regressions separately for calls with a start-of-month t moneyness below the first quartile (column (1), "DOTM"), between the first and second (column (2), "OTM"), between the second and third (column (3), "ATM"), and above the third (column (4), "ITM"). Moneyness is the ratio of stock price to strike price at the start of month t. We calculate the absolute exposures and systematic and idiosyncratic volatility using the Fama-French (2016) six-factor model estimated over the 24 months of daily data before the call return period. Plain numbers are premium estimates (in decimals), while those in square brackets are Newey and West (1987) t-statistics calculated using a twelve-month lag length.

	Calls				Optionable Stocks	
	DOTM	OTM	ATM	ITM	All	All
	(1)	(2)	(3)	(4)	(5)	(6)
Moneyness	−5.79	−6.42	−3.99	−0.92		
	[−10.57]	[−11.92]	[−8.68]	[−8.00]		
Absolute MKT Exposure	−0.18	−0.14	−0.03	0.01		0.00
	[−3.70]	[−4.51]	[−1.42]	[0.90]		[0.27]
Absolute SMB Exposure	−0.03	0.00	0.01	0.01		0.00
	[−0.82]	[−0.05]	[0.48]	[0.62]		[−0.51]
Absolute HML Exposure	−0.05	0.00	−0.01	0.00		0.01
	[−1.77]	[0.21]	[−0.35]	[0.33]		[0.84]
Absolute MOM Exposure	0.06	0.04	0.04	0.03		0.00
	[1.25]	[1.30]	[1.63]	[1.61]		[0.34]
Absolute RMW Exposure	−0.07	−0.02	−0.01	−0.01		0.00
	[−2.65]	[−1.56]	[−0.78]	[−0.91]		[−0.76]
Absolute CMA Exposure	−0.04	−0.02	0.01	0.02		0.01
	[−1.75]	[−1.56]	[1.01]	[2.78]		[1.71]
Systematic Volatility					0.02	
					[1.58]	
Idiosyncratic Volatility	−1.01	−0.68	−0.36	−0.20	−0.02	−0.01
	[−6.86]	[−5.96]	[−4.54]	[−3.97]	[−2.83]	[−1.99]
Constant	6.38	6.71	4.06	0.84	−0.01	−0.01
	[12.20]	[12.46]	[8.48]	[6.63]	[−4.04]	[−3.36]

Table 8
Regressions of Call Returns on Systematic and Idiosyncratic Volatility and Controls
The table presents the results from Fama-MacBeth (1973) regressions of the excess call return calculated from start of month t to its end on moneyness, systematic and idiosyncratic underlying-stock volatility, and various sets of controls measured until end of month $t-1$. We use stock and option liquidity controls, option mispricing controls, higher moment controls, and stock characteristic controls in Panels A to D, respectively. We run the regressions separately for calls with a start-of-month-t moneyness below the first quartile (column (1), "DOTM"), between the first and second (column (2), "OTM"), between the second and third (column (3), "ATM"), and above the third (column (4), "ITM"). Moneyness is the ratio of stock price to strike price at the start of month t. We calculate systematic and idiosyncratic volatility using the Fama-French (2016) six-factor model estimated over the 24 months of daily data before the call return period. In Panel A, option open interest is the ratio of option open interest to stock dollar trading volume; option bid-ask spread is the ratio of option bid-ask spread to option midpoint price; and stock illiquidity is the ratio of absolute daily stock return to dollar trading volume averaged over the last twelve months. In Panel B, delta total volatility is the change in stock volatility from month $t-2$ to $t-1$; total-to-implied volatility is the ratio of stock volatility calculated from daily data over month $t-1$ to Black-Scholes (1973) implied volatility at the end of that month; and delta implied volatility is the change in Black-Scholes (1973) implied volatility over the call return period. In Panel C, variance risk premium is the difference between stock variance calculated from daily data over month $t-1$ and model-free implied volatility at the end of that month; implied skewness is risk-neutral skewness calculated at the end of month $t-1$; and implied kurtosis is risk-neutral kurtosis calculated at the end of month $t-1$. In Panel D, market size is a stock's log market capitalization from the most recent prior June; book-to-market is the log ratio of a stock's book value from the fiscal year ending in the prior calendar year to its market capitalization at the end of that calendar year from July of year t to June of year $t+1$; momentum is the stock return compounded over months $t-12$ to $t-2$; asset growth is the change in the log asset value from the fiscal year end in calendar year $t-2$ to the fiscal year end in calendar year $t-1$ from July of year t to June of year $t+1$; and profitability is the ratio of sales minus COGS, SG&A, and interest expenses to the book value of equity measured at the end of the fiscal year in calendar year $t-1$ from July of year t to June of year $t+1$. Plain numbers are premium estimates (in decimals), while the numbers in square brackets are Newey and West (1987) t-statistics calculated using a twelve-month lag length.

	DOTM		OTM		ATM		ITM	
	(1)		(2)		(3)		(4)	
Panel A: Stock and Option Liquidity Controls								
Moneyness	−5.83	[−10.52]	−6.47	[−10.97]	−4.10	[−8.97]	−0.92	[−7.66]
Systematic Volatility	−1.40	[−5.98]	−0.68	[−4.58]	0.18	[1.51]	0.29	[3.30]
Idiosyncratic Volatility	−0.97	[−7.32]	−0.41	[−4.42]	−0.18	[−2.76]	−0.02	[−0.38]
Option Open Interest	−2.45	[−2.65]	0.07	[0.07]	2.66	[3.51]	5.10	[9.22]
Option Bid Ask Spread	−0.10	[−4.45]	−0.07	[−5.35]	−0.04	[−4.10]	−0.03	[−3.35]
Stock Illiquidity	0.02	[1.62]	−0.01	[−1.12]	−0.02	[−3.61]	−0.03	[−7.58]
Constant	6.59	[13.24]	6.37	[11.32]	3.64	[8.52]	0.23	[1.81]

(continued on next page)

Table 8
Regressions of Call Returns on Systematic and Idiosyncratic Volatility and Controls (cont.)

	DOTM		OTM		ATM		ITM	
	(1)		(2)		(3)		(4)	
Panel B: Option Mispricing Controls								
Moneyness	−6.34	[−12.33]	−7.22	[−12.60]	−4.78	[−10.08]	−1.74	[−13.83]
Systematic Volatility	−1.38	[−6.98]	−0.67	[−4.78]	0.17	[1.45]	0.42	[4.99]
Idiosyncratic Volatility	−0.79	[−6.59]	−0.45	[−5.17]	−0.16	[−2.73]	−0.01	[−0.13]
Total-to-Implied Vol.	0.15	[1.77]	0.22	[3.42]	0.28	[5.84]	0.25	[7.38]
Delta Total Volatility	−0.04	[−0.64]	−0.16	[−4.74]	−0.16	[−6.24]	−0.16	[−8.55]
Delta Implied Volatility	2.49	[13.91]	2.24	[16.56]	1.90	[19.44]	1.53	[25.18]
Constant	6.84	[13.79]	7.36	[12.98]	4.69	[9.84]	1.52	[11.28]
Panel C: Higher Moment Controls								
Moneyness	−5.68	[−9.20]	−5.79	[−10.27]	−3.03	[−7.56]	−0.65	[−5.05]
Systematic Volatility	−0.94	[−4.08]	−0.41	[−2.72]	0.22	[1.69]	0.38	[3.65]
Idiosyncratic Volatility	−0.61	[−4.58]	−0.25	[−3.41]	−0.16	[−2.59]	−0.09	[−1.92]
Variance Risk Premium	−0.05	[−0.55]	−0.12	[−1.70]	0.03	[0.50]	0.04	[1.27]
Implied Skewness	−0.20	[−1.85]	−0.06	[−1.12]	−0.05	[−1.43]	−0.05	[−1.71]
Implied Kurtosis	0.34	[4.40]	0.16	[3.33]	0.04	[1.39]	0.06	[2.75]
Constant	4.86	[7.16]	5.32	[8.60]	2.82	[6.84]	0.25	[1.43]
Panel D: Firm Characteristic Controls								
Moneyness	−5.93	[−9.83]	−6.41	[−10.79]	−4.00	[−9.64]	−0.76	[−6.47]
Systematic Volatility	−1.45	[−5.57]	−0.91	[−4.78]	0.05	[0.30]	0.20	[1.71]
Idiosyncratic Volatility	−0.88	[−5.20]	−0.20	[−1.80]	−0.12	[−1.38]	0.01	[0.08]
Market Size	0.00	[−0.02]	0.04	[3.40]	0.02	[3.15]	0.03	[6.12]
Book-to-Market	0.05	[2.17]	0.03	[2.56]	0.01	[0.90]	0.00	[−0.14]
Momentum	−0.10	[−2.87]	−0.04	[−1.56]	−0.02	[−0.82]	0.03	[1.90]
Asset Growth	−0.09	[−2.07]	−0.06	[−2.34]	−0.07	[−3.32]	−0.03	[−2.24]
Profitability	0.08	[2.16]	0.03	[1.33]	0.01	[0.63]	−0.01	[−0.40]
Constant	6.57	[11.52]	6.04	[10.31]	3.63	[8.60]	0.07	[0.53]

Table 9

Regressions of Trading-Cost-Adjusted Call Returns on Systematic and Idiosyncratic Volatility

The table presents the results from Fama-MacBeth (1973) regressions of the transaction-cost-adjusted excess call return calculated from start of month t to its end on moneyness and systematic and idiosyncratic underlying-stock volatility measured until end of month $t-1$. We adjust the return by assuming investors buy (sell) at the midpoint price plus (minus) a zero (Panel A), 25% (Panel B), and 50% (Panel C) fraction of the call bid-ask spread. We run the regressions separately for calls with a start-of-month-t moneyness below the first quartile (column (1), "DOTM"), between the first and second (column (2), "OTM"), between the second and third (column (3), "ATM"), and above the third (column (4), "ITM"). Moneyness is the ratio of stock price to strike price at the start of month t. We calculate systematic and idiosyncratic volatility using the Fama-French (2016) six-factor model estimated over the 24 months of daily data before the call return period. Plain numbers are premium estimates (in decimals), while the numbers in square brackets are Newey and West (1987) t-statistics calculated using a twelve-month lag length.

	DOTM		OTM		ATM		ITM	
	(1)		(2)		(3)		(4)	
Panel A: Bid-Ask Fraction (S) = 0.00								
Moneyness	−6.08	[−11.09]	−6.67	[−11.55]	−4.19	[−9.06]	−0.97	[−8.33]
Systematic Volatility	−1.38	[−6.20]	−0.75	[−4.52]	0.16	[1.24]	0.33	[3.40]
Idiosyncratic Volatility	−0.92	[−6.85]	−0.57	[−5.46]	−0.39	[−4.89]	−0.24	[−5.46]
Constant	6.66	[12.41]	6.90	[11.93]	4.22	[8.77]	0.88	[6.79]
Panel B: Bid-Ask Fraction (S) = 0.25								
Moneyness	−4.61	[−8.85]	−5.59	[−10.37]	−3.51	[−8.38]	−0.69	[−6.10]
Systematic Volatility	−1.13	[−5.49]	−0.50	[−3.31]	0.25	[1.97]	0.37	[3.78]
Idiosyncratic Volatility	−0.77	[−6.24]	−0.48	[−5.16]	−0.36	[−4.69]	−0.27	[−6.05]
Constant	4.90	[9.54]	5.55	[10.43]	3.35	[7.79]	0.47	[3.73]
Panel C: Bid-Ask Fraction (S) = 0.50								
Moneyness	−3.68	[−7.56]	−4.86	[−9.64]	−3.05	[−7.77]	−0.55	[−5.12]
Systematic Volatility	−0.91	[−4.66]	−0.36	[−2.47]	0.31	[2.50]	0.40	[4.04]
Idiosyncratic Volatility	−0.71	[−6.12]	−0.48	[−5.32]	−0.39	[−5.22]	−0.31	[−7.07]
Constant	3.83	[7.96]	4.73	[9.52]	2.84	[7.03]	0.30	[2.49]

Internet Appendix:

Moneyness, Underlying Asset Volatility, and the Cross-Section of Option Returns

AUTHOR 1, AUTHOR 2, and AUTHOR 3

In this Internet Appendix, we present theoretical results and proofs plus supplementary and robustness test results not included in our main paper. In Section IA.1, we give the proofs of Proposition 1 and Corollaries 1 and 2 on the relations between call or underlying asset characteristics and the expected call return in our stochastic discount factor model, originally stated in Section 2 of the main paper. In Section IA.2, we compare those relations with the corresponding relations implied by an extended version of the Hu and Jacobs (2020) contingent claims model. Section IA.3 offers a proposition corresponding to Proposition 1 (Proposition IA.1) and a corollary corresponding to Corollary 2 (Corollary IA.1) for puts. In Section IA.4, we repeat our main portfolio sorts and FM regressions in Tables 2 and 3 of the main paper using alternative data filters, regression specifications, and volatility component estimates. Section IA.5 explains how we calculate the variance risk premium and the higher-order risk neutral moment variables in the regressions in Panel C of Table 8 in the main paper. In Section IA.6, we repeat our FM regressions in Panel D of Table 8 in the main paper also controlling for the historical average return, as also done in Hu and Jacobs (2020).

IA.1 Properties of the Expected Call Return

We start off with proving Proposition 1 in our main paper. While we keep in mind that the correlation between the optimal projection of the log asset payoff on the log stochastic discount factor, $\tilde{x}_s$, and the log stochastic discount factor, $\tilde{m}$, is minus one, we nonetheless denote that correlation by κ in our proofs, making some of our mathematical arguments easier to follow. For the sake of convenience, we first repeat Proposition 1, originally stated in Section 2.2.b of the main paper.

PROPOSITION 1: *Assuming the existence of a representative agent, that the agent's maximization problem can be solved, and that the log asset payoff, $\tilde{x}$, and the log stochastic discount factor realization, $\tilde{m}$, are bivariate normal with a negative correlation between them, the expected return of a European call written on the asset and with strike price equal to K, $E[\tilde{R}_c]$,*

(a) decreases with the expected log asset payoff, μ_x.

(b) increases with the strike price specified by the call contract, K.

(c) decreases with moneyness, defined as the difference between μ_x and $\ln K$.

(d) increases (decreases) with the asset's systematic variance, σ_s^2, if and only if:

$$(\sigma_x^2/\sigma_s^2)H'[c^*] - H'[\alpha - \sigma_x + \beta] - \left(\alpha - \sigma_m \frac{\sigma_i^2}{\sigma_x \sigma_s}\right) H[\alpha - \sigma_x + \beta]\,[1 - H'[c^*]] > (<)\ 0,$$

where $H(x) \equiv n(x)/N(-x)$ is the hazard function of the normal random variable x, with $n(.)$ the standard normal density function, $H'(x)$ the first derivative of the hazard function with respect to x, $\alpha \equiv (\ln K - \mu_x)/\sigma_x$, $\beta \equiv \frac{\sigma_s \sigma_m}{\sigma_x}$, and $c^* \in (\alpha - \sigma_x + \beta, \alpha + \beta)$.

(e) decreases with the asset's idiosyncratic variance, σ_i^2.

PROOF:

Proof of Part (a):

The partial derivative of the expected call return, $E[\tilde{R}_c]$, with respect to the expected log asset payoff, μ_x, is given by:

$$\frac{\partial E[\tilde{R}_c]}{\partial \mu_x} = \frac{\partial E[\tilde{X}_c]/p_c}{\partial \mu_x} = \frac{(\partial E[\tilde{X}_c]/\partial \mu_x)p_c - (\partial p_c/\partial \mu_x)E[\tilde{X}_c]}{p_c^2}. \tag{IA1}$$

The partial derivatives on the right-hand side of the second equality are given by:

$$\frac{\partial E[\tilde{X}_c]}{\partial \mu_x} = e^{\mu_x + \frac{1}{2}\sigma_x^2} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] > 0, \tag{IA2}$$

and

$$\frac{\partial p_c}{\partial \mu_x} = e^{\mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 + 2\kappa\sigma_s\sigma_m + \sigma_m^2)} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] > 0. \tag{IA3}$$

Defining $z_1 \equiv \mu_x + \frac{1}{2}\sigma_x^2$, $z_2 \equiv \mu_m + \frac{1}{2}\sigma_m^2$, and $z_3 \equiv \mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 + 2\kappa\sigma_s\sigma_m + \sigma_m^2)$ and substituting the numerator and denominator of the term on the right-hand side of the third equality in Equation (6) in the main paper, (IA2), and (IA3) into (IA1), we obtain:

$$\begin{aligned}
\frac{\partial E[\tilde{R}_c]}{\partial \mu_x} &= \frac{1}{p_c^2}\Bigg[e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right]\left(e^{z_3} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] - K e^{z_2} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right) \\
&\quad - \left(e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - K N\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\right) e^{z_3} N\left[\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\Bigg] \qquad \text{(IA4)} \\
&= -\frac{K e^{z_1 + z_2}}{p_c^2}\Bigg[N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right] \\
&\quad - e^{\kappa\sigma_s\sigma_m} N\left[\frac{\mu_x - \ln K}{\sigma_x}\right] N\left[\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\Bigg]. \qquad \text{(IA5)}
\end{aligned}$$

Because $e^{z_1+z_2} > 0$ and $p_c^2 > 0$, the sign of the partial derivative with respect to the expected log asset payoff depends on the sign of the term in the outer square parentheses in Equation (IA5). To obtain a negative relation between expected call return and that expected payoff, it must hold that:

$$N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right] > e^{\kappa\sigma_s\sigma_m} N\left[\frac{\mu_x - \ln K}{\sigma_x}\right] N\left[\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]. \tag{IA6}$$

Since $N[\cdot] > 0$, the last inequality is equivalent to:

$$\frac{N\left[\frac{\mu_x+\sigma_x^2-\ln K}{\sigma_x}\right]}{N\left[\frac{\mu_x-\ln K}{\sigma_x}\right]} \quad > \quad e^{\kappa\sigma_s\sigma_m}\frac{N\left[\frac{\mu_x+\sigma_x^2+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]}{N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]}. \tag{IA7}$$

We recognize that, if $\kappa = 0$, $\kappa\sigma_s\sigma_m = 0$, and Inequality (IA7) becomes an equality. Because only the right-hand side of the inequality depends on the correlation between the log asset payoff and the log stochastic discount factor realization, the inequality would hold if the right-hand side were monotonically decreasing with decreases in κ to minus one.

The natural logarithm of the right-hand side is:

$$\kappa\sigma_s\sigma_m + \ln\left(N\left[\frac{\mu_x+\sigma_x^2+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]\right) - \ln\left(N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]\right). \tag{IA8}$$

Taking the partial derivative of (IA8) with respect to κ and rearranging, we obtain:

$$\begin{aligned}
& \sigma_s\sigma_m\left[1-\left(\frac{n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]}{N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]}-\frac{n\left[\frac{\mu_x+\sigma_x^2+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]}{N\left[\frac{\mu_x+\sigma_x^2+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]}\right)/\sigma_x\right] && \text{(IA9)}\\
= \quad & \sigma_s\sigma_m\left[1-\left(\frac{n\left[-\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]}{N\left[-\left(-\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right)\right]}-\frac{n\left[-\frac{\mu_x+\sigma_x^2+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]}{N\left[-\left(-\frac{\mu_x+\sigma_x^2+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right)\right]}\right)/\sigma_x\right], && \text{(IA10)}
\end{aligned}$$

where the last equality follows from the symmetry of the normal distribution. Using the definition for the hazard function of the normally distributed random variable x, which is given by $H(x) = n(x)/N(-x)$, we are able to rewrite the right-hand side of (IA10) as:

$$\sigma_s\sigma_m\left[1-\frac{H\left[-\frac{\mu_x+\sigma_x^2+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}+\sigma_x\right]-H\left[-\frac{\mu_x+\sigma_x^2+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]}{\sigma_x}\right], \tag{IA11}$$

or, after applying the mean-value theorem:

$$\sigma_s\sigma_m\left(1-H'[x^*]\right), \tag{IA12}$$

where $x^* \in (-(\mu_x+\sigma_x^2+\kappa\sigma_s\sigma_m-\ln K)/\sigma_x, -(\mu_x+\sigma_x^2+\kappa\sigma_s\sigma_m-\ln K)/\sigma_x+\sigma_x)$. Freeman and Guermat (2006) theoretically prove that $H'[x] < 1$, implying that the right-hand side of Inequality (IA7) monotonically decreases with decreases in κ. Setting κ to minus one thus ensures that the term in the outer square parentheses in (IA5) is positive, in turn proving that the expected call return decreases with the expected log asset payoff, establishing part (a) of the proposition.

Proof of Part (b):

The partial derivative of the expected call return with respect to the strike price is:

$$\frac{\partial E[\tilde{R}_c]}{\partial K} = \frac{\partial E[\tilde{X}_c]/p_c}{\partial K} = \frac{(\partial E[\tilde{X}_c]/\partial K)p_c - (\partial p_c/\partial K)E[\tilde{X}_c]}{p_c^2}. \tag{IA13}$$

The partial derivatives on the right-hand side of the second equality are given by:

$$\frac{\partial E[\tilde{X}_c]}{\partial K} = -N\left[\frac{\mu_x - \ln K}{\sigma_x}\right] < 0, \tag{IA14}$$

and

$$\frac{\partial p_c}{\partial K} = -e^{\mu_m+\frac{1}{2}\sigma_m^2} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right] < 0. \tag{IA15}$$

Defining $z_1 \equiv \mu_x+\frac{1}{2}\sigma_x^2$, $z_2 \equiv \mu_m+\frac{1}{2}\sigma_m^2$, and $z_3 \equiv \mu_x+\mu_m+\frac{1}{2}(\sigma_x^2+2\kappa\sigma_s\sigma_m+\sigma_m^2)$ and substituting the numerator and denominator of the term on the right-hand side of the third equality in Equation (6) in the main paper, (IA14), and (IA15) into (IA13), we obtain:

$$\begin{aligned}
\frac{\partial E[\tilde{R}_c]}{\partial K} &= \frac{1}{p_c^2}\Bigg[-N\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\left(e^{z_3}N\left[\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right] - Ke^{z_2}N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right) \\
&\quad -\left(-e^{z_2}N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right)\left(e^{z_1}N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - KN\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\right)\Bigg] \qquad \text{(IA16)} \\
&= \frac{e^{z_1+z_2}}{p_c^2}\Bigg[N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right]N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right] - \\
&\quad e^{\kappa\sigma_s\sigma_m}N\left[\frac{\mu_x - \ln K}{\sigma_x}\right]N\left[\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\Bigg], \qquad \text{(IA17)}
\end{aligned}$$

or, alternatively, $-\frac{1}{K}\frac{\partial E[\tilde{R}_c]}{\partial \mu_x}$. Thus, the partial derivative of the expected call return with respect to the strike price must have the opposite sign of the partial derivative of that return with respect to the expected log asset payoff. It follows from part (a) of Proposition 1 that the expected call return and the strike price are positively related, establishing part (b) of the proposition.

Proof of Part (c):

Defining moneyness as the expected log asset payoff minus the strike price, $\psi(\mu_x, K) \equiv \mu_x - \ln K$, the total derivative of moneyness with respect to that expected payoff and the strike price is:

$$d\psi(\mu_x, K) = \frac{\partial \psi(\mu_x, K)}{\partial \mu_x} d\mu_x + \frac{\partial \psi(\mu_x, K)}{\partial K} dK = d\mu_x - dK/K. \tag{IA18}$$

The total derivative of the expected call return with respect to the expected log asset payoff and the strike price is given by:

$$\begin{aligned} dE[\tilde{R}_c] &= \frac{\partial E[\tilde{R}_c]}{\partial \mu_x} d\mu_x + \frac{\partial E[\tilde{R}_c]}{\partial K} dK & \text{(IA19)} \\ &= \frac{\partial E[\tilde{R}_c]}{\partial \mu_x} d\mu_x - \frac{1}{K} \frac{\partial E[\tilde{R}_c]}{\partial \mu_x} dK & \text{(IA20)} \\ &= \frac{\partial E[\tilde{R}_c]}{\partial \mu_x} (d\mu_x - dK/K), & \text{(IA21)} \end{aligned}$$

where we use $\frac{\partial E[\tilde{R}_c]}{\partial \mu_x} = -\frac{1}{K} \frac{\partial E[\tilde{R}_c]}{\partial K}$ in the last equality (see the proof of part (b) of the proposition). As part (a) of the proposition states that $\partial E[\tilde{R}_c]/\partial \mu_x < 0$, a higher moneyness as defined above decreases the expected call return, establishing part (c) of Proposition 1.

Proof of Part (d):

Since the partial derivative of the expected call return, $E[\tilde{R}_c]$, with respect to the systematic variance of the asset, σ_s^2, has the same sign as that partial derivative with respect to the systematic volatility of the asset, σ_s, we focus on the latter for convenience. The latter partial derivative is given by:

$$\frac{\partial E[\tilde{R}_c]}{\partial \sigma_s} = \frac{\partial E[\tilde{X}_c]/p_c}{\partial \sigma_s} = \frac{(\partial E[\tilde{X}_c]/\partial \sigma_s)p_c - (\partial p_c/\partial \sigma_s)E[\tilde{X}_c]}{p_c^2}. \tag{IA22}$$

The partial derivatives on the right-hand side of the second equality are given by:

$$\frac{\partial E[\tilde{X}_c]}{\partial \sigma_s} = \sigma_s e^{\mu_x+\frac{1}{2}\sigma_x^2} N\left[\tfrac{\mu_x+\sigma_x^2-\ln K}{\sigma_x}\right] + \tfrac{\sigma_s}{\sigma_x} K n\left[\tfrac{\mu_x-\ln K}{\sigma_x}\right] > 0, \tag{IA23}$$

and

$$\frac{\partial p_c}{\partial \sigma_s} = e^{\mu_m+\frac{1}{2}\sigma_m^2}\left[(\sigma_s+\kappa\sigma_m)e^{\mu_x+\frac{1}{2}[\sigma_x^2+2\kappa\sigma_s\sigma_m]} N\left[\tfrac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right] + \tfrac{\sigma_s}{\sigma_x} K n\left[\tfrac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]\right]. \tag{IA24}$$

Substituting the numerator and denominator of the term on the right-hand side of the third equality in Equation (6) in the main paper, (IA23), and (IA24) into (IA22) and using $z_1 \equiv \mu_x + \frac{1}{2}\sigma_x^2$, $z_2 \equiv \mu_m + \frac{1}{2}\sigma_m^2$, and $z_3 \equiv \mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 + 2\kappa\sigma_s\sigma_m + \sigma_m^2)$ to simplify the notation, we obtain:

$$\begin{aligned}\frac{\partial E[\tilde{R}_c]}{\partial \sigma_s} = \; & \frac{1}{p_c^2}\Bigg[\Bigg(\sigma_s e^{z_1} N\left[\frac{\mu_x+\sigma_x^2-\ln K}{\sigma_x}\right] + \frac{\sigma_s}{\sigma_x} K n\left[\frac{\mu_x-\ln K}{\sigma_x}\right]\Bigg) \\ & \times\Bigg(e^{z_3} N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right] - K e^{z_2} N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]\Bigg) - \\ & \Bigg(\Bigg(e^{z_1} N\left[\frac{\mu_x+\sigma_x^2-\ln K}{\sigma_x}\right] - K N\left[\frac{\mu_x-\ln K}{\sigma_x}\right]\Bigg)\Bigg((\sigma_s+\kappa\sigma_m)e^{z_3} \\ & \times N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right] + \frac{\sigma_s}{\sigma_x} K e^{z_2} n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]\Bigg)\Bigg)\Bigg].\end{aligned} \tag{IA25}$$

The sign of the partial derivative is positive if and only if:

$$\begin{aligned}&\Bigg(\sigma_x e^{z_1} N\left[\frac{\mu_x+\sigma_x^2-\ln K}{\sigma_x}\right] + K n\left[\frac{\mu_x-\ln K}{\sigma_x}\right]\Bigg)\Bigg(e^{z_3} N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right] \\ & - K e^{z_2} N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]\Bigg) > \Bigg(e^{z_1} N\left[\frac{\mu_x+\sigma_x^2-\ln K}{\sigma_x}\right] - K N\left[\frac{\mu_x-\ln K}{\sigma_x}\right]\Bigg) \\ & \Bigg((\sigma_x + \kappa\frac{\sigma_x\sigma_m}{\sigma_s})e^{z_3} N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right] + K e^{z_2} n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]\Bigg),\end{aligned} \tag{IA26}$$

and negative if and only if the inequality holds with the opposite inequality sign.

Because $E[\tilde{X}_c] \equiv e^{z_1} N\left[\frac{\mu_x+\sigma_x^2-\ln K}{\sigma_x}\right] - K N\left[\frac{\mu_x-\ln K}{\sigma_x}\right] > 0$ and $p_c \equiv e^{z_3} N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right] - K e^{z_2} N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right] > 0$, we can divide the inequality first by the right term in the product on the left-hand side of the inequality and then, second, by the left term in the product on the right-hand side, without changing the inequality sign. The result is:

$$\frac{\sigma_x e^{z_1} N\left[\frac{\mu_x+\sigma_x^2-\ln K}{\sigma_x}\right]+Kn\left[\frac{\mu_x-\ln K}{\sigma_x}\right]}{e^{z_1}N\left[\frac{\mu_x+\sigma_x^2-\ln K}{\sigma_x}\right]-KN\left[\frac{\mu_x-\ln K}{\sigma_x}\right]} > \frac{(\sigma_x+\kappa\frac{\sigma_x\sigma_m}{\sigma_s})e^{z_1+\kappa\sigma_s\sigma_m}N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]+Kn\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]}{e^{z_1+\kappa\sigma_s\sigma_m}N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]-KN\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]} \tag{IA27}$$

Dividing the numerator and the denominator of the right-hand side of Inequality (IA27) by $e^{z_1+\kappa\sigma_s\sigma_m}$ $N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]$, the right-hand side of the former inequality becomes:

$$\frac{(\sigma_x+\kappa\frac{\sigma_x\sigma_m}{\sigma_s})+e^{-(z_1+\kappa\sigma_s\sigma_m-\ln K)}n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]/N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]}{1-e^{-(z_1+\kappa\sigma_s\sigma_m-\ln K)}N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]/N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]}. \tag{IA28}$$

We now recognize that $e^{-(z_1+\kappa\sigma_s\sigma_m-\ln K)}n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]$ can be rewritten as:

$$\begin{aligned} & e^{-(\mu_x+\frac{1}{2}\sigma_x^2+\kappa\sigma_s\sigma_m-\ln K)}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\frac{(\mu_x+\kappa\sigma_s\sigma_m-\ln K)^2}{\sigma_x^2}} \\ = \quad & \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\frac{(\mu_x+\kappa\sigma_s\sigma_m-\ln K)^2+2(\mu_x+\kappa\sigma_s\sigma_m-\ln K)\sigma_x^2+\sigma_x^4}{\sigma_x^2}} = n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]. \end{aligned} \tag{IA29}$$

Using (IA29), we can write (IA28) as:

$$\frac{(\sigma_x+\kappa\frac{\sigma_x\sigma_m}{\sigma_s})+n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]/N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]}{1-\left(n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]\right)/\left(n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]\right)}. \tag{IA30}$$

Using the definition for the hazard function of the normally distributed random variable x, which is $H(x) = n(x)/N(-x)$, we can write the right-hand side of Inequality (IA27) as:

$$\frac{(\sigma_x+\kappa\frac{\sigma_x\sigma_m}{\sigma_s})+H\left[-\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]}{1-H\left[-\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]/H\left[-\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]}. \tag{IA31}$$

We now recognize that, if $\kappa = 0$, then $\kappa\sigma_s\sigma_m = 0$, and Inequality (IA27) becomes an equality. As only the right-hand side of the inequality depends on the correlation between the log asset payoff and the log stochastic discount factor realization, the inequality would hold for $\kappa = -1$ if the right-hand side were monotonically increasing with increases in κ. Conversely, the inequality with the opposite sign would hold if the right-hand side were monotonically decreasing with increases in κ. Defining $\alpha \equiv (\ln K - \mu_x)/\sigma_x$ and $\beta \equiv \frac{\sigma_s \sigma_m}{\sigma_x}$, we write the right-hand side of (IA27) (which is (IA31)) as:

$$\frac{(\sigma_x + \kappa \frac{\sigma_x \sigma_m}{\sigma_s}) + H\left[\alpha - \sigma_x - \beta\kappa\right]}{1 - H\left[\alpha - \sigma_x - \beta\kappa\right]/H\left[\alpha - \beta\kappa\right]}. \tag{IA32}$$

The partial derivative of (IA32) with respect to κ is proportional to:[1]

$$\begin{aligned} & \beta\Big[(\sigma_x^2/\sigma_s^2) - H'[\alpha - \sigma_x - \beta\kappa]\Big]\big[1 - H[\alpha - \sigma_x - \beta\kappa]/H[\alpha - \beta\kappa]\big] + \beta\Big[\sigma_x + \kappa\frac{\sigma_x\sigma_m}{\sigma_s} + H\left[\alpha - \sigma_x - \beta\kappa\right]\Big] \\ \times \quad & \big[-H'[\alpha - \sigma_x - \beta\kappa]/H[\alpha - \beta\kappa] + H[\alpha - \sigma_x - \beta\kappa]H'[\alpha - \beta\kappa]/H[\alpha - \beta\kappa]^2\big]. \end{aligned} \tag{IA33}$$

Multiplying by $\frac{1}{\beta} > 0$ and $H[\alpha - \beta\kappa] > 0$, adding and subtracting α and $\beta\kappa$ inside the third main expression, using the relationship $H'[x] = H[x][H[x] - x]$, and rearranging yields:

$$\begin{aligned} & \left[\frac{\sigma_x^2}{\sigma_s^2} - H'[\alpha - \sigma_x - \beta\kappa]\right]\big[H[\alpha - \beta\kappa] - H[\alpha - \sigma_x - \beta\kappa]\big] + H[\alpha - \sigma_x - \beta\kappa]\Big[H\left[\alpha - \sigma_x - \beta\kappa\right] \\ & -(\alpha - \sigma_x - \beta\kappa) + \alpha + \kappa\sigma_m\frac{\sigma_i^2}{\sigma_x\sigma_s}\Big]\Big[-\Big(H[\alpha - \sigma_x - \beta\kappa] - (\alpha - \sigma_x - \beta\kappa)\Big) + \Big(H[\alpha - \beta\kappa] - (\alpha - \beta\kappa)\Big)\Big] \quad \text{(IA34)} \\ = \quad & \left[\frac{\sigma_x^2}{\sigma_s^2} - H'[\alpha - \sigma_x - \beta\kappa]\right]\big[H[\alpha - \beta\kappa] - H[\alpha - \sigma_x - \beta\kappa]\big] + H[\alpha - \sigma_x - \beta\kappa]\Big[H\left[\alpha - \sigma_x - \beta\kappa\right] \\ & -(\alpha - \sigma_x - \beta\kappa) + \alpha + \kappa\sigma_m\frac{\sigma_i^2}{\sigma_x\sigma_s}\Big]\big[H[\alpha - \beta\kappa] - H[\alpha - \sigma_x - \beta\kappa] - \sigma_x\big]. \quad \text{(IA35)} \end{aligned}$$

Dividing by $\sigma_x > 0$, using the mean-value theorem, and $H'[x] = H[x][H[x] - x]$ gives:

$$\begin{aligned} & \left[\frac{\sigma_x^2}{\sigma_s^2} - H'[\alpha - \sigma_x - \beta\kappa]\right]H'[c^*] + H'[\alpha - \sigma_x - \beta\kappa]\big[H'[c^*] - 1\big] + \left(\alpha + \kappa\sigma_m\frac{\sigma_i^2}{\sigma_x\sigma_s}\right)H[\alpha - \sigma_x - \beta\kappa]\big[H'[c^*] - 1\big] \\ = \quad & \frac{\sigma_x^2}{\sigma_s^2}H'[c^*] - H'[\alpha - \sigma_x - \beta\kappa] - \left(\alpha + \kappa\sigma_m\frac{\sigma_i^2}{\sigma_x\sigma_s}\right)H[\alpha - \sigma_x - \beta\kappa]\big[1 - H'[c^*]\big], \end{aligned} \tag{IA36}$$

where $c^* \in (\alpha - \sigma_x - \beta\kappa, \alpha - \beta\kappa)$. Since $\kappa = -1$, a positive (negative) value for the right-hand side

[1] The partial derivative of (IA32) with respect to κ is (IA33) divided by $(1 - H\left[\alpha - \sigma_x - \beta\kappa\right]/H\left[\alpha - \beta\kappa\right])^2$.

of Equality (IA36) implies that the expected call return increases (decreases) with the systematic volatility (or variance) of the log asset payoff, establishing part (d) of the proposition.

In deriving the right-hand side of Equality (IA36), we assume that we can raise systematic asset volatility, σ_s, without changing the expected log asset payoff, μ_x. As we, however, say in footnote 6 of the main paper, that assumption is only strictly valid if $\mu_m = 0$. If we instead vary σ_s through varying b whilst accounting for the effect of b on μ_x, the right-hand side of (IA36) becomes:

$$\frac{\sigma_x^2}{\sigma_s^2} H'[c^*] - H'[\alpha - \sigma_x - \beta\kappa] - \left(\alpha + \kappa\sigma_m \frac{\sigma_i^2}{\sigma_x\sigma_s} - \frac{\mu_m\sigma_x}{\sigma_s\sigma_m}\right) H[\alpha - \sigma_x - \beta\kappa][1 - H'[c^*]]. \quad \text{(IA37)}$$

Since $\frac{\mu_m\sigma_x}{\sigma_s\sigma_m}$ must have the same sign as μ_m, the term in (IA37) can be smaller or larger than the right-hand side of Equality (IA36) depending on the sign of μ_m, implying that the effect of systematic volatility on the expected call return is positive either more or less often.[2]

Proof of Part (e):

Since the partial derivative of the expected call return, $E[\tilde{R}_c]$, with respect to the idiosyncratic variance of the asset, σ_i^2, has the same sign as that partial derivative with respect to idiosyncratic asset volatility, σ_i, we focus on the latter for convenience. The latter derivative is:

$$\frac{\partial E[\tilde{R}_c]}{\partial \sigma_i} = \frac{\partial E[\tilde{X}_c]/p_c}{\partial \sigma_i} = \frac{(\partial E[\tilde{X}_c]/\partial\sigma_i)p_c - (\partial p_c/\partial\sigma_i)E[\tilde{X}_c]}{p_c^2}. \quad \text{(IA38)}$$

The partial derivatives on the right-hand side of the second equality are given by:

$$\frac{\partial E[\tilde{X}_c]}{\partial \sigma_i} = \sigma_i e^{\mu_x + \frac{1}{2}\sigma_x^2} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] + \frac{\sigma_i}{\sigma_x} K n\left[\frac{\mu_x - \ln K}{\sigma_x}\right] > 0, \quad \text{(IA39)}$$

and

$$\frac{\partial p_c}{\partial \sigma_i} = e^{\mu_m + \frac{1}{2}\sigma_m^2}\left[\sigma_i e^{\mu_x + \frac{1}{2}[\sigma_x^2 + 2\kappa\sigma_s\sigma_m]} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] + \frac{\sigma_i}{\sigma_x} K n\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right]. \quad \text{(IA40)}$$

[2]The detailed derivation of (IA37) is available from the authors upon request.

Substituting the numerator and denominator of the term on the right-hand side of the third equality in Equation (6) in the main paper, (IA39), and (IA40) into (IA38) and using $z_1 \equiv \mu_x + \frac{1}{2}\sigma_x^2$, $z_2 \equiv \mu_m + \frac{1}{2}\sigma_m^2$, and $z_3 \equiv \mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 + 2\kappa\sigma_s\sigma_m + \sigma_m^2)$ to simplify the notation, we obtain:

$$\begin{aligned}\frac{\partial E[\tilde{R}_c]}{\partial \sigma_i} \quad = \quad & \frac{1}{p_c^2}\Bigg[\Bigg(\sigma_i e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] + \frac{\sigma_i}{\sigma_x} K n\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\Bigg) \\ & \times\Bigg(e^{z_3} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] - K e^{z_2} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\Bigg) - \\ & \Bigg(\Bigg(e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - K N\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\Bigg)\Bigg(\sigma_i e^{z_3} \\ & \times N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] + \frac{\sigma_i}{\sigma_x} K e^{z_2} n\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\Bigg)\Bigg)\Bigg]. \quad \text{(IA41)}\end{aligned}$$

The sign of the partial derivative is negative if and only if:

$$\begin{aligned}&\Bigg(\sigma_x e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] + K n\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\Bigg)\Bigg(e^{z_3} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] \\ & - K e^{z_2} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\Bigg) < \Bigg(e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - K N\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\Bigg) \\ & \Bigg(\sigma_x e^{z_3} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] + K e^{z_2} n\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\Bigg). \qquad \text{(IA42)}\end{aligned}$$

Because $E[\tilde{X}_c] \equiv e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - K N\left[\frac{\mu_x - \ln K}{\sigma_x}\right] > 0$ and $p_c \equiv e^{z_3} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] - K e^{z_2} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right] > 0$, we are again able to first divide by the right term in the product on the left-hand side of the inequality and then, second, by the left term in the product on the right-hand side, without changing the sign of the inequality. The result is:

$$\frac{\sigma_x e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] + K n\left[\frac{\mu_x - \ln K}{\sigma_x}\right]}{e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - K N\left[\frac{\mu_x - \ln K}{\sigma_x}\right]} < \frac{\sigma_x e^{z_1 + \kappa\sigma_s\sigma_m} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] + K n\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]}{e^{z_1 + \kappa\sigma_s\sigma_m} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] - K N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]}. \quad \text{(IA43)}$$

Dividing the numerator and the denominator of the right-hand side of inequality (IA43) by $e^{z_1 + \kappa\sigma_s\sigma_m}$

$N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]$, the right-hand side becomes:

$$\frac{\sigma_x + e^{-(z_1+\kappa\sigma_s\sigma_m-\ln K)} n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]/N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]}{1 - e^{-(z_1+\kappa\sigma_s\sigma_m-\ln K)} N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]/N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]}. \tag{IA44}$$

Using (IA29), we can write (IA44) as:

$$\frac{\sigma_x + n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]/N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]}{1-\left(n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]\right)/\left(n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]\right)}. \tag{IA45}$$

Using the definition for the hazard function of the normally distributed random variable x, which is $H(x) = n(x)/N(-x)$, we can write the right-hand side of Inequality (IA43) as:

$$\frac{\sigma_x + H\left[-\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]}{1 - H\left[-\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]/H\left[-\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]}. \tag{IA46}$$

We again recognize that, if $\kappa = 0$, then $\kappa\sigma_s\sigma_m = 0$, and Inequality (IA43) becomes an equality. As only the right-hand side of the inequality depends on the correlation between the log asset payoff and the log stochastic discount factor realization, the inequality would hold for $\kappa = -1$ if the right-hand side were monotonically decreasing with increases in κ. Defining $\alpha \equiv (\ln K - \mu_x)/\sigma_x$ and $\beta \equiv \frac{\sigma_s\sigma_m}{\sigma_x}$, we can write the right-hand side of (IA43) as:

$$\frac{\sigma_x + H\left[\alpha - \sigma_x - \beta\kappa\right]}{1 - H\left[\alpha - \sigma_x - \beta\kappa\right]/H\left[\alpha - \beta\kappa\right]}. \tag{IA47}$$

The partial derivative of (IA47) with respect to κ is proportional to:[3]

$$\begin{aligned} &-\beta H'[\alpha-\sigma_x-\beta\kappa]\big[1 - H[\alpha-\sigma_x-\beta\kappa]/H[\alpha-\beta\kappa]\big] + \beta\Big[\sigma_x + H\left[\alpha-\sigma_x-\beta\kappa\right]\Big] \\ \times\quad &\big[-H'[\alpha-\sigma_x-\beta\kappa]/H[\alpha-\beta\kappa] + H[\alpha-\sigma_x-\beta\kappa]H'[\alpha-\beta\kappa]/H[\alpha-\beta\kappa]^2\big]. \end{aligned} \tag{IA48}$$

[3]The partial derivative of (IA47) with respect to κ is (IA48) divided by $(1 - H\left[\alpha - \sigma_x - \beta\kappa\right]/H\left[\alpha - \beta\kappa\right])^2$.

Multiplying by $\frac{1}{\beta} > 0$ and $H[\alpha - \beta\kappa] > 0$, adding and subtracting α and $\beta\kappa$ inside the third main expression, using the relationship $H'[x] = H[x][H[x] - x]$, and rearranging yields:

$$
\begin{aligned}
& -H'[\alpha-\sigma_x-\beta\kappa]\big[H[\alpha-\beta\kappa]-H[\alpha-\sigma_x-\beta\kappa]\big]+H[\alpha-\sigma_x-\beta\kappa]\Big[H\left[\alpha-\sigma_x-\beta\kappa\right] \\
& -(\alpha-\sigma_x-\beta\kappa)+\alpha-\beta\kappa\Big]\big[-\big(H[\alpha-\sigma_x-\beta\kappa]-(\alpha-\sigma_x-\beta\kappa)\big)+\big(H[\alpha-\beta\kappa]-(\alpha-\beta\kappa)\big)\big] \quad \text{(IA49)} \\
= \; & -H'[\alpha-\sigma_x-\beta\kappa]\big[H[\alpha-\beta\kappa]-H[\alpha-\sigma_x-\beta\kappa]\big]+H[\alpha-\sigma_x-\beta\kappa]\Big[H\left[\alpha-\sigma_x-\beta\kappa\right] \\
& -(\alpha-\sigma_x-\beta\kappa)+\alpha-\beta\kappa\Big]\big[H[\alpha-\beta\kappa]-H[\alpha-\sigma_x-\beta\kappa]-\sigma_x\big]. \quad \text{(IA50)}
\end{aligned}
$$

Dividing by $\sigma_x > 0$, using the mean-value theorem, and $H'[x] = H[x][H[x] - x]$ gives:

$$
\begin{aligned}
& -H'[\alpha-\sigma_x-\beta\kappa]H'[c^*]+H'[\alpha-\sigma_x-\beta\kappa]\big[H'[c^*]-1\big]+(\alpha-\beta\kappa)\,H[\alpha-\sigma_x-\beta\kappa]\big[H'[c^*]-1\big] \\
= \; & -H'[\alpha-\sigma_x-\beta\kappa]-(\alpha-\beta\kappa)\,H[\alpha-\sigma_x-\beta\kappa]\big[1-H'[c^*]\big], \quad \text{(IA51)}
\end{aligned}
$$

where $c^* \in (\alpha - \sigma_x - \beta\kappa, \alpha - \beta\kappa)$. If $(\alpha - \beta\kappa) \geq 0$, then (IA51) is negative since $H[.] > 0$, $H'[.] > 0$, and $(1 - H'[.]) > 0$. If $(\alpha - \beta\kappa) < 0$, we use $H'[x] = H[x][H[x] - x]$ to write:

$$
\begin{aligned}
& -H'[\alpha-\sigma_x-\beta\kappa]-(\alpha-\beta\kappa)\,H[\alpha-\sigma_x-\beta\kappa]\big[1-H'[c^*]\big] \quad \text{(IA52)} \\
= \; & H[\alpha-\sigma_x-\beta\kappa]\left(-(H[\alpha-\sigma_x-\beta\kappa]-(\alpha-\sigma_x-\beta\kappa))-(\alpha-\beta\kappa)\left[1-H'[c^*]\right]\right), \quad \text{(IA53)}
\end{aligned}
$$

which has the same sign as:

$$
\begin{aligned}
& -(H[\alpha-\sigma_x-\beta\kappa]-(\alpha-\sigma_x-\beta\kappa))-(\alpha-\beta\kappa)\Big[1-H'[c^*]\Big] \quad \text{(IA54)} \\
< \; & -H[\alpha-\sigma_x-\beta\kappa]+\alpha-\sigma_x-\beta\kappa-\alpha+\beta\kappa \quad \text{(IA55)} \\
= \; & -H[\alpha-\sigma_x-\beta\kappa]-\sigma_x < 0, \quad \text{(IA56)}
\end{aligned}
$$

where the first inequality follows from Freeman and Guermat's (2006) theoretical result that $(1-H'[.])$ is bounded by zero and one. As a result, the expected call return unambiguously decreases with the idiosyncratic volatility of the log asset payoff, establishing part (e) of Proposition 1.

■

We next prove Corollary 1 in our main paper. We again start with repeating that corollary.

COROLLARY 1: Under the same assumptions as in Proposition 1, the sign of the relation between the expected call return, $E[\tilde{R}_c]$*, and systematic asset variance,* σ_s^2*, is positive for ITM and ATM calls, but can be both positive or negative for OTM calls.*

PROOF:

Recallig part (d) of Proposition 1, the sign of the relation between the expected call return and the systematic variance (or volatility) of the log asset payoff is determined by the sign of the sum:

$$\frac{\sigma_x^2}{\sigma_s^2} H'[c^*] - H'[\alpha - \sigma_x - \beta\kappa] - \left(\alpha + \kappa\sigma_m \frac{\sigma_i^2}{\sigma_x \sigma_s}\right) H[\alpha - \sigma_x - \beta\kappa]\Big[1 - H'[c^*]\Big], \qquad \text{(IA57)}$$

with a positive (negative) sign for the sum revealing a positive (negative) relation.

Given that we define a call's moneyness as $(\mu_x - \ln K)$, $\alpha \equiv \frac{\ln K - \mu_x}{\sigma_x}$ decreases with moneyness, and ITM (ATM) [OTM] calls have an α value below (equal to) [above] zero. Noticing that $\sigma_x^2 \geq \sigma_s^2$ and $H'[c^*] > H'[\alpha - \sigma_x - \beta\kappa]$ (since $c^* > (\alpha - \sigma_x - \beta\kappa)$ and $H[.]$ is convex), the sum of the first two terms in (IA57) is positive. Thus, if $\left(\alpha + \kappa\sigma_m \frac{\sigma_i^2}{\sigma_x\sigma_s}\right) \leq 0$, as is the case for ITM ($\alpha < 0$), ATM ($\alpha = 0$), and slightly OTM calls ($0 < \alpha < -\kappa\sigma_m \frac{\sigma_i^2}{\sigma_x\sigma_s}$), the sum in (IA57) is positive, and the expected call return increases with the systematic variance (or volatility) of the log asset payoff. That the expected return of sufficiently OTM calls can decrease with the systematic variance of that payoff can be shown using a numerical example (see, e.g., Figure 1 in the main paper).

■

We finally prove Corollary 2 in our main paper. We again start with repeating that corollary.

COROLLARY 2: *Under the same assumptions as in Proposition 1, the relation between the expected call return,* $E[\tilde{R}_c]$*, and idiosyncratic asset variance,* σ_i^2*, converges to zero (a negative value) as the* *call's moneyness increases to infinity (decreases to minus infinity).*

PROOF:

Recalling part (e) of Proposition 1, the sign of the relation between the expected call return and the idiosyncratic variance (or volatility) of the log asset payoff is determined by the sign of the sum:

$$-H'[\alpha - \sigma_x - \beta\kappa] - (\alpha - \beta\kappa)\, H[\alpha - \sigma_x - \beta\kappa]\Big[1 - H'[c^*]\Big], \tag{IA58}$$

with a negative (zero) sign for the sum revealing a negative (zero) relation.

Given that we define a call's moneyness as $(\mu_x - \ln K)$, $\alpha \equiv \frac{\ln K - \mu_x}{\sigma_x}$ decreases with moneyness, and ITM (ATM) [OTM] calls have an α value below (equal to) [above] zero. Letting α go to minus infinity, the call moves perfectly ITM, while $H[.]$ and $H'[.]$ converge to zero, $(1 - H'[.])$ to one, and $-(\alpha - \beta\kappa)$ to plus infinity. Thus, the first term in the sum in (IA58) converges to zero. In principle, the second term could converge to any number between zero and plus infinity. Since the relation between the expected call return and idiosyncratic volatility is, however, unambiguously negative, as stated by part (e) of Proposition 1, it must converge to zero. Thus, the expected returns of perfectly ITM calls are unrelated to the idiosyncratic variance (or volatility) of the log asset payoff.

Letting α go to plus infinity, the call moves perfectly OTM, while $H[.]$ converges to plus infinity, $H'[.]$ to one, $(1 - H'[.])$ to zero, and $-(\alpha - \beta\kappa)$ to minus infinity. Thus, the first term in (IA58) converges to minus one, while the second term converges to a number between zero and minus infinity. The upshot is that the expected returns of perfectly OTM calls are negatively related to the idiosyncratic variance (or volatility) of the log asset payoff.

■

IA.2 A Comparison with Hu and Jacobs (2020)

In a related paper, Hu and Jacobs (2020) use the Black-Scholes (1973) contingent claims framework to study the effect of the *total* variance of an asset on the expected return of a European call or put written on the asset. In particular, they start from the assumption that the value of the asset, S_t, in their notation, is exogenous and evolves according to Geometric Brownian motion:

$$dS_t = \mu S_t dt + \sigma S_t dB_t, \tag{IA59}$$

where μ and σ are, respectively, the (constant) annualized expected return and return volatility of the asset, and dB_t is the differential of a Brownian motion. In agreement with the first equality in Equation (6) in our main paper, they next define the expected call return, R_{call}, as:

$$R_{call} = \frac{E_t[\max(S_T - K, 0)]}{C_t(t, T, S_t, \sigma, K, r)}, \tag{IA60}$$

where $E_t[.]$ is the expectation conditional on time-t information, $\max(.)$ the maximum operator, S_T the asset value on the call maturity date T, K the call's strike price, $C_t(.)$ the arbitrage-free value of the call on date t, and r the (constant) risk-free rate of return. Also relying on other assumptions made by Black and Scholes (1973), such as no transaction costs, continuous trading, etc., Hu and Jacobs (2020) show that the closed-form solution for the expected call return is:

$$R_{call} = \frac{e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} N(d_2^*)]}{S_t N(d_1) - e^{-r\tau} K N(d_2)}, \tag{IA61}$$

where $\tau \equiv T - t$ is the time-to-maturity, $N(.)$ the cumulative normal density, and:

$$d_1 = \frac{\ln(S_t/K) + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \quad , \quad d_2 = \frac{\ln(S_t/K) + (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}, \tag{IA62}$$

$$d_1^* = \frac{\ln(S_t/K) + (\mu + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \quad , \quad d_2^* = \frac{\ln(S_t/K) + (\mu - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}. \tag{IA63}$$

They finally establish that $\partial R_{call}/\partial\sigma < 0$, suggesting that, in the Black-Scholes (1973) framework, the expected call return is unambiguously negatively related to total asset volatility.[4]

[4] While we exclusively look into Hu and Jacobs' (2020) theoretical call results in this section, we note that they also derive corresponding theoretical results for puts. See their paper for more details.

Hu and Jacobs' (2020) result that $\partial R_{call}/\partial\sigma < 0$ conflicts with Proposition 1 in Section 2.2.b of our main paper, which suggests that a higher total asset volatility can raise the expected call return if the call is sufficiently ITM and the increase in volatility is driven by systematic volatility. The reason for this divergence is that Hu and Jacobs (2020) implicitly assume that changes in asset volatility neither affect the expected asset return (i.e., $\partial\mu/\partial\sigma = 0$) nor the asset's value (i.e., $\partial S_t/\partial\sigma = 0$). In other words, Hu and Jacobs (2020) exclusively focus on changes in *idiosyncratic* asset volatility. In accordance with their results, our Proposition 1 confirms that changes in idiosyncratic asset volatility also have an unambiguously negative effect on the expected call return in our stochastic discount factor model in which expected asset return and asset value are endogenously determined.

Notwithstanding, we may be able to modify the Black-Scholes (1973) framework in such a way that it allows us to meaningfully study the effect of systematic asset variance on expected option returns. We could, for example, write the expected asset return, μ, as $r + \gamma\sigma_s$ (with γ a positive scaling factor[5] and σ_s systematic asset volatility) and total asset variance, σ^2, as $\sigma_s^2 + \sigma_i^2$ (with σ_i idiosyncratic asset volatility) and then investigate how systematic asset volatility, σ_s, affects the expected call return, R_{call}, in Equations (IA61) to (IA63). Doing so, however, generates a model admitting arbitrage opportunities. To see that, consider two identical assets, with identical model input parameters and, importantly, asset values. Now raise the systematic volatility of the first asset without changing its univariate payoff moments through lowering the correlation between that asset's payoff and the stochastic discount factor. According to the first fundamental theorem of asset pricing, which states that, in the absence of arbitrage opportunities, any asset's value, p, is: $E[\tilde{X}]E[\tilde{M}] + \text{cov}(\tilde{X}, \tilde{M})$ (with $\tilde{X}$ the asset payoff and $\tilde{M}$ the stochastic discount factor), the first asset's value must now fall. Alas, in the extended contingent claims model, that value does not fall since it is exogenous, generating an arbitrage opportunity between the two assets.

While it is never ideal to derive any asset pricing predictions from a model admitting arbitrage opportunities,[6] the predictions derived from the stochastic discount factor model and the extended

[5]In the stochastic discount factor model, γ is equal to the volatility of the stochastic discount factor, σ_m.

[6]Shreve (2004) is rather explicit about that recommendation. In his textbook, he advocates: "One should never offer

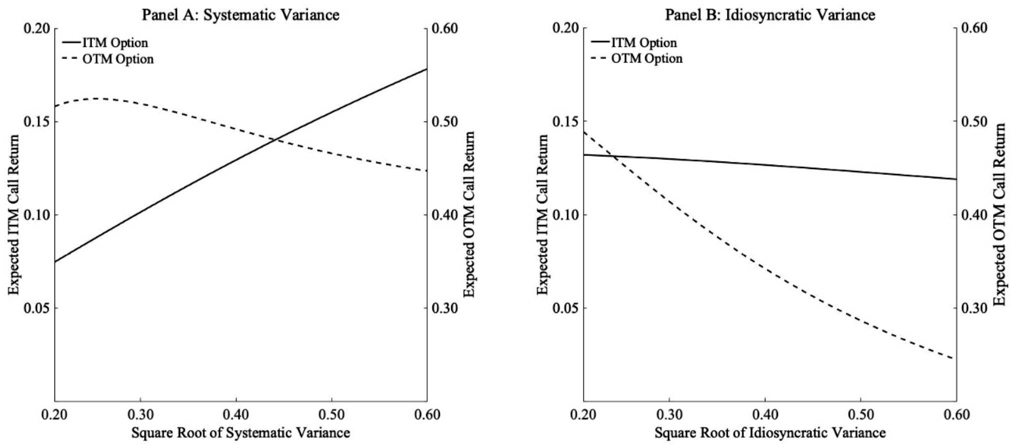

Figure IA1: The Relation Between Expected Call Return and Systematic and Idiosyncratic Variance According to the Extended Contingent Claims Model The figure plots the relations between expected call return and systematic (Panel A) and idiosyncratic (Panel B) underlying asset variance as implied by the extended contingent claims model. The basecase parameters are as follows. The expectations of the log asset payoff and the log stochastic discount factor realization are 0.00 and –0.025, respectively. Systematic and idiosyncratic variance are 0.16 and 0.04, respectively, while the variance of the log stochastic discount factor is 0.0225. The strike prices of the in-the-money (ITM) and out-of-the-money (OTM) calls are 0.50 and 2.50, respectively. The value of the underlying asset is fixed at the stochastic discount factor model closed-form solution evaluated at the basecase parameters.

contingent claims model are nonetheless qualitatively identical. We can easily see that from Figure IA1, which plots the expected return of an ITM and OTM call derived from the extended contingent claims model against systematic (Panel A) and idiosyncratic (Panel B) volatility. While the basecase parameter values are as those in Figure 1 in the main paper, the underlying asset's value is now fixed at its stochastic discount factor model closed-form solution evaluated at the basecase values. Figure IA1 suggests that the stochastic discount factor and extended contingent claims models produce similar relations between the expected call return and systematic or idiosyncratic volatility.

prices derived from a model that admits arbitrage, and the First Fundamental Theorem provides a simple condition one can apply to check that the model one is using does not have this *fatal flaw*" (p.231; our emphasis).

IA.3 Properties of the Expected Put Return

Our main paper focuses on the theoretical properties of the expected call return since we are unable to empirically test the corresponding properties of the expected put return (recall the discussion in Section 3.1 of the main paper). In this section of the Internet Appendix, we now however turn to the theoretical properties of the expected put return implied by our stochastic discount factor model. In dealing with expected put returns, it is helpful to recall Coval and Shumway's (2001) result that the expected put return lies below the risk-free rate of return and that, as a result, a positive partial derivative of that return with respect to some put or asset characteristic implies that the return moves upward toward the risk-free rate of return (and thus, in most circumstances, becomes less negative). The expected put return in our model, $E[\tilde{R}_p]$, is given by:

$$\begin{aligned} E[\tilde{R}_p] &= \frac{E[\tilde{X}_p]}{p_p} = \frac{E[\max(K-\tilde{X},0)]}{E[\tilde{M}\times\max(K-\tilde{X},0)]} \\ &= \frac{KN\left[\frac{\ln K-\mu_x}{\sigma_x}\right]-e^{\mu_x+\frac{1}{2}\sigma_x^2}N\left[\frac{\ln K-\mu_x-\sigma_x^2}{\sigma_x}\right]}{e^{\mu_m+\frac{1}{2}\sigma_m^2}\left[KN\left[\frac{\ln K-\mu_x+\sigma_s\sigma_m}{\sigma_x}\right]-e^{\mu_x+\frac{1}{2}(\sigma_x^2-2\sigma_s\sigma_m)}N\left[\frac{\ln K-\mu_x+\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]\right]}, \end{aligned} \quad \text{(IA64)}$$

where $\tilde{X}_p$ is the put payoff and p_p the put value. The probability that the put ends up ITM, π_p, is:

$$\pi_p = \text{Prob}(K>\tilde{X}) = \text{Prob}\left(\frac{\ln K-\mu_x}{\sigma_x} > \frac{\tilde{x}-\mu_x}{\sigma_x}\right) = N\left[\frac{\ln\ K-\mu_x}{\sigma_x}\right]. \quad \text{(IA65)}$$

so that the spread between log strike price and expected log asset payoff, $\ln K-\mu_x$, is a sufficient statistic for the probability that the put ends up ITM. Thus, we label puts with a positive spread (and thus an above 50% probability of the put ending up ITM) "ITM puts," those with a zero spread (and thus a 50% probability) "ATM puts," and those with a negative spread (and thus a below 50% probability) "OTM puts." In addition, we also interpret $\ln K-\mu_x$ as a monotonic positive transformation of put moneyness conditional on underlying-asset volatility, σ_x.

Proposition IA.1 states the relations between the expected put return and the put and asset characteristics in our stochastic discount factor model:

PROPOSITION IA.1: *Assuming the existence of a representative agent, that the agent's maximization problem can be solved, and that the log asset payoff, $\tilde{x}$, and the log stochastic discount factor realization, $\tilde{m}$, are bivariate normal with a negative correlation between them, the expected return of a European put written on the asset and with strike price equal to K, $E[\tilde{R}_p]$,*

(a) decreases with the expected log asset payoff, μ_x.

(b) increases with the strike price specified by the put contract, K.

(c) increases with moneyness, now defined as the difference between $\ln K$ and μ_x.

(d) increases (decreases) with the asset's systematic variance, σ_s^2, if and only if:

$$-(\sigma_x^2/\sigma_s^2)H'[c^*] + H'[\sigma_x - \beta - \alpha] - \left(\alpha - \sigma_m \frac{\sigma_i^2}{\sigma_x \sigma_s}\right) H[\sigma_x - \beta - \alpha]\left[1 - H'[c^*]\right] > (<)\ 0,$$

where $\alpha \equiv (\ln K - \mu_x)/\sigma_x$, $\beta \equiv \frac{\sigma_s \sigma_m}{\sigma_x}$, and $c^* \in (-\beta - \alpha, \sigma_x - \beta - \alpha)$.

(e) increases (decreases) with the asset's idiosyncratic variance, σ_i^2, if and only if:

$$H'[\sigma_x - \beta - \alpha] - (\alpha + \beta)\, H[\sigma_x - \beta - \alpha]\left[1 - H'[c^*]\right] > (<)\ 0.$$

PROOF:

Proof of Part (a):

The partial derivative of the expected put return, $E[\tilde{R}_p]$, with respect to the expected log asset payoff, μ_x, is given by:

$$\frac{\partial E[\tilde{R}_p]}{\partial \mu_x} = \frac{\partial E[\tilde{X}_p]/p_p}{\partial \mu_x} = \frac{(\partial E[\tilde{X}_p]/\partial \mu_x)p_p - (\partial p_p/\partial \mu_x)E[\tilde{X}_p]}{p_p^2}. \tag{IA66}$$

The partial derivatives on the right-hand side of the second equality are given by:

$$\frac{\partial E[\tilde{X}_p]}{\partial \mu_x} = -e^{\mu_x + \frac{1}{2}\sigma_x^2} N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right], \tag{IA67}$$

and

$$\frac{\partial p_p}{\partial \mu_x} = -e^{\mu_x+\mu_m+\frac{1}{2}(\sigma_x^2+2\kappa\sigma_s\sigma_m+\sigma_m^2)} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right], \tag{IA68}$$

where we, as before, keep in mind that κ is minus one. Defining $z_1 \equiv \mu_x + \frac{1}{2}\sigma_x^2$, $z_2 \equiv \mu_m + \frac{1}{2}\sigma_m^2$, and $z_3 \equiv \mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 + 2\kappa\sigma_s\sigma_m + \sigma_m^2)$ and substituting the numerator and denominator of the term on the right-hand side of the third equality in (IA64), (IA67), and (IA68) into (IA66), we obtain:

$$\begin{aligned}
\frac{\partial E[\tilde{R}_p]}{\partial \mu_x} &= \frac{1}{p_p^2}\left[-e^{z_1}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\left(e^{z_2}KN\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - e^{z_3}N\left[\frac{\ln K - \mu_x - \sigma_x^2 - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\right)\right. \\
&\quad \left. + e^{z_3}N\left[\frac{\ln K - \mu_x - \sigma_x^2 - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\left(KN\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - e^{z_1}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right)\right] \qquad \text{(IA69)} \\
&= \frac{Ke^{z_1+z_2}}{p_p^2}\left(-N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\right. \\
&\quad \left. + e^{\kappa\sigma_s\sigma_m}N\left[\frac{\ln K - \mu_x}{\sigma_x}\right]N\left[\frac{\ln K - \mu_x - \sigma_x^2 - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\right). \qquad \text{(IA70)}
\end{aligned}$$

Because $e^{z_1+z_2} > 0$ and $p_p^2 > 0$, the sign of the partial derivative with respect to the expected log asset payoff depends on the sign of the term in the outer round parentheses in (IA70). Using steps identical to those in the proof of part (a) of Proposition 1, it can be shown that:

$$N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] \quad > \quad e^{\kappa\sigma_s\sigma_m}N\left[\frac{\ln K - \mu_x}{\sigma_x}\right]N\left[\frac{\ln K - \mu_x - \sigma_x^2 - \kappa\sigma_s\sigma_m}{\sigma_x}\right], \tag{IA71}$$

implying that the term in the outer round parentheses is negative and that the expected put return decreases with the expected log asset payoff, establishing part (a) of the proposition.

Proof of Part (b):

The partial derivative of the expected put return with respect to the strike price is:

$$\frac{\partial E[\tilde{R}_p]}{\partial K} = \frac{\partial E[\tilde{X}_p]/p_p}{\partial K} = \frac{(\partial E[\tilde{X}_p]/\partial K)p_p - (\partial p_p/\partial K)E[\tilde{X}_p]}{p_p^2}. \tag{IA72}$$

The partial derivatives on the right-hand side of the second equality are given by:

$$\frac{\partial E[\tilde{X}_p]}{\partial K} = N\left[\frac{\ln K - \mu_x}{\sigma_x}\right], \tag{IA73}$$

and

$$\frac{\partial p_p}{\partial K} = e^{\mu_m + \frac{1}{2}\sigma_m^2} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right]. \tag{IA74}$$

Defining $z_1 \equiv \mu_x + \frac{1}{2}\sigma_x^2$, $z_2 \equiv \mu_m + \frac{1}{2}\sigma_m^2$, and $z_3 \equiv \mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 + 2\kappa\sigma_s\sigma_m + \sigma_m^2)$ and substituting the numerator and denominator of the term on the right-hand side of the third equality in (IA64), (IA73), and (IA74) into (IA72), we obtain the following result:

$$\begin{aligned}
\frac{\partial E[\tilde{R}_p]}{\partial K} &= \frac{1}{p_p^2}\left[N\left[\frac{\ln K - \mu_x}{\sigma_x}\right]\left(e^{z_2} K N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - e^{z_3} N\left[\frac{\ln K - \mu_x - \sigma_x^2 - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\right)\right. \\
&\quad \left. - e^{z_2} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\left(K N\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - e^{z_1} N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right)\right] \qquad \text{(IA75)} \\
&= \frac{e^{z_1+z_2}}{p_p^2}\left(N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right] N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\right. \\
&\quad \left. - e^{\kappa\sigma_s\sigma_m} N\left[\frac{\ln K - \mu_x}{\sigma_x}\right] N\left[\frac{\ln K - \mu_x - \sigma_x^2 - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\right) \qquad \text{(IA76)} \\
&= -\frac{1}{K}\frac{\partial E[\tilde{R}_p]}{\partial \mu_x}, \qquad \text{(IA77)}
\end{aligned}$$

which, since $\frac{\partial E[\tilde{R}_p]}{\partial \mu_x} < 0$ (see part (a) of Proposition IA.1), implies a positive relation between expected put return and strike price, establishing part (b) of the proposition.

Proof of Part (c):

Defining a put's moneyness as the difference between the log strike price and the expected log asset payoff, $\Upsilon(\mu_x, K) \equiv \ln K - \mu_x$, the total differential of moneyness with respect to its input arguments, the expected log asset payoff and the strike price, is given by:

$$d\Upsilon(\mu_x, K) = \frac{\partial(\ln K - \mu_x)}{\partial \mu_x} d\mu_x + \frac{\partial(\ln K - \mu_x)}{\partial K} dK = \frac{1}{K} dK - d\mu_x, \tag{IA78}$$

Conversely, the total differential of the expected put return, $E[\tilde{R}_p]$, with respect to the expected log asset payoff, μ_x, and the strike price, K, is given by:

$$dE[\tilde{R}_p] = \frac{\partial E[\tilde{R}_p]}{\partial \mu_x} d\mu_x + \frac{\partial E[\tilde{R}_p]}{\partial K} dK \tag{IA79}$$

$$= \frac{\partial E[\tilde{R}_p]}{\partial \mu_x} d\mu_x - \frac{1}{K} \frac{\partial E[\tilde{R}_p]}{\partial \mu_x} dK \tag{IA80}$$

$$= -\frac{\partial E[\tilde{R}_p]}{\partial \mu_x} \left(\frac{1}{K} dK - d\mu_x \right), \tag{IA81}$$

where use $\frac{\partial E[\tilde{R}_p]}{\partial K} = -\frac{1}{K} \frac{\partial E[\tilde{R}_p]}{\partial \mu_x}$ in the second equality, as we established in the proof of part (b) of Proposition IA.1. Since $\frac{\partial E[\tilde{R}_p]}{\partial \mu_x} < 0$ (see part (a) of the same proposition), the expected put return increases with moneyness, establishing part (c) of Proposition IA.1.

Proof of Part (d):

Since the partial derivative of the expected put return, $E[\tilde{R}_p]$, with respect to the systematic variance of the asset, σ_s^2, has the same sign as that partial derivative with respect to the systematic volatility of the asset, σ_s, we focus on the latter for convenience. The latter partial derivative is given by:

$$\frac{\partial E[\tilde{R}_p]}{\partial \sigma_s} = \frac{\partial E[\tilde{X}_p]/p_p}{\partial \sigma_s} = \frac{(\partial E[\tilde{X}_p]/\partial \sigma_s)p_p - (\partial p_p/\partial \sigma_s)E[\tilde{X}_p]}{p_p^2}. \tag{IA82}$$

The partial derivatives on the right-hand side of the second equality are given by:

$$\frac{\partial E[\tilde{X}_p]}{\partial \sigma_s} = \frac{\sigma_s}{\sigma_x} K n \left[\frac{\ln K - \mu_x}{\sigma_x} \right] - \sigma_s e^{\mu_x + \frac{1}{2}\sigma_x^2} N \left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x} \right], \tag{IA83}$$

and

$$\frac{\partial p_p}{\partial \sigma_s} = e^{\mu_m + \frac{1}{2}\sigma_m^2} \left[\frac{\sigma_s}{\sigma_x} K n \left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x} \right] - (\sigma_s + \kappa\sigma_m) e^{\mu_x + \frac{1}{2}[\sigma_x^2 + 2\kappa\sigma_s\sigma_m]} N \left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x} \right] \right] \tag{IA84}$$

Substituting the numerator and denominator of the term on the right-hand side of the third equality in (IA64), (IA83), and (IA84) into (IA82) and using $z_1 \equiv \mu_x + \frac{1}{2}\sigma_x^2$, $z_2 \equiv \mu_m + \frac{1}{2}\sigma_m^2$, and

$z_3 \equiv \mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 + 2\kappa\sigma_s\sigma_m + \sigma_m^2)$ to simplify the notation, we obtain:

$$
\begin{aligned}
\frac{\partial E[\tilde{R}_p]}{\partial \sigma_s} \quad = \quad & \frac{1}{p_p^2}\Bigg[\Bigg(\left(\frac{\sigma_s}{\sigma_x}Kn\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - \sigma_s e^{z_1} N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right) \\
& \times\left(Ke^{z_2}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - e^{z_3}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]\right) - \\
& \Bigg(\left(KN\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - e^{z_1}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right) \times \Bigg(\frac{\sigma_s}{\sigma_x}Ke^{z_2}n\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - \\
& (\sigma_s + \kappa\sigma_m)e^{z_3}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]\Bigg)\Bigg)\Bigg].
\end{aligned} \tag{IA85}
$$

The sign of the partial derivative is positive if and only if:

$$
\begin{aligned}
& \left(Kn\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - \sigma_x e^{z_1}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right)\Bigg(Ke^{z_2}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] \\
& -e^{z_3}N\left[\frac{\ln K - \mu_x - \kappa\sigma_x\sigma_m - \sigma_x^2}{\sigma_x}\right]\Bigg) > \left(KN\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - e^{z_1}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right) \\
& \left(Ke^{z_2}n\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - (\sigma_x + \kappa\frac{\sigma_x\sigma_m}{\sigma_s})e^{z_3}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]\right),
\end{aligned} \tag{IA86}
$$

and negative if and only if the inequality holds with the opposite sign.

As in the proof of part (d) of Proposition 1, we can first divide the inequality by the right term in the product on the left-hand side of the inequality and then, second, by the left term in the product on the right-hand side, without changing the sign of the inequality. The result is:

$$
\frac{Kn\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - \sigma_x e^{z_1}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]}{KN\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - e^{z_1}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]} > \frac{Kn\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - (\sigma_x + \kappa\frac{\sigma_x\sigma_m}{\sigma_s})e^{z_1 + \kappa\sigma_s\sigma_m}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]}{KN\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - e^{z_1 + \kappa\sigma_s\sigma_m}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]} \tag{IA87}
$$

Dividing the numerator and the denominator of the right-hand side of inequality (IA87) by $e^{z_1 + \kappa\sigma_s\sigma_m}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]$, the right-hand side becomes:

$$
\frac{e^{-(z_1 + \kappa\sigma_s\sigma_m - \ln K)}n\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right]/N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right] - (\sigma_x + \kappa\frac{\sigma_x\sigma_m}{\sigma_s})}{e^{-(z_1 + \kappa\sigma_s\sigma_m - \ln K)}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right]/N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right] - 1}. \tag{IA88}
$$

We now recognize that $e^{-(z_1+\kappa\sigma_x\sigma_m-\ln K)}n\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m}{\sigma_x}\right]$ can be rewritten as:

$$\begin{aligned}
& e^{-(\mu_x+\frac{1}{2}\sigma_x^2+\kappa\sigma_s\sigma_m-\ln K)}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\frac{(\ln K-\mu_x-\kappa\sigma_s\sigma_m)^2}{\sigma_x^2}} \\
= \quad & \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\frac{(\ln K-\mu_x-\kappa\sigma_s\sigma_m)^2-2(\ln K-\mu_x-\kappa\sigma_s\sigma_m)\sigma_x^2+\sigma_x^4}{\sigma_x^2}} = n\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right].
\end{aligned} \tag{IA89}$$

Using (IA89), we can write (IA88) as:

$$\frac{n\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]/N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]-(\sigma_x+\kappa\frac{\sigma_x\sigma_m}{\sigma_s})}{\left(n\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m}{\sigma_x}\right]\right)/\left(n\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m}{\sigma_x}\right]N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]\right)-1}. \tag{IA90}$$

Using the definition for the hazard function of the normally distributed random variable x, which is $H(x) = n(x)/N(-x)$, we can write the right-hand side of Inequality (IA87) as:

$$\frac{H\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]-(\sigma_x+\kappa\frac{\sigma_x\sigma_m}{\sigma_s})}{H\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]/H\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]-1}. \tag{IA91}$$

We now recognize that, if $\kappa = 0$, $\kappa\sigma_s\sigma_m = 0$, and Inequality (IA87) becomes an equality. As only the right-hand side of that inequality depends on the correlation between the log asset payoff and the log stochastic discount factor realization, the inequality would hold for $\kappa = -1$ if the right-hand side were monotonically increasing with increases in κ. Conversely, the inequality with the opposite sign would hold if the right-hand side were monotonically decreasing with increases in κ. Defining $\alpha \equiv (\ln K - \mu_x)/\sigma_x$ and $\beta \equiv \frac{\sigma_s\sigma_m}{\sigma_x}$, we write the right-hand side of (IA87) (which is (IA91)) as:

$$\frac{H\left[\sigma_x+\beta\kappa-\alpha\right]-(\sigma_x+\kappa\frac{\sigma_x\sigma_m}{\sigma_s})}{H\left[\sigma_x+\beta\kappa-\alpha\right]/H\left[\beta\kappa-\alpha\right]-1}. \tag{IA92}$$

The partial derivative of (IA92) with respect to κ is proportional to:[7]

$$\begin{aligned} & \beta\Big[H'[\sigma_x+\beta\kappa-\alpha]-(\sigma_x^2/\sigma_s^2)\Big]\Big[H[\sigma_x+\beta\kappa-\alpha]/H[\beta\kappa-\alpha]-1\Big]+\beta\Big[H\left[\sigma_x+\beta\kappa-\alpha\right]-(\sigma_x+\kappa\frac{\sigma_x\sigma_m}{\sigma_s})\Big] \\ \times & \Big[-H'[\sigma_x+\beta\kappa-\alpha]/H[\beta\kappa-\alpha]+H[\sigma_x+\beta\kappa-\alpha]H'[\beta\kappa-\alpha]/H[\beta\kappa-\alpha]^2\Big]. \end{aligned} \tag{IA93}$$

Multiplying by $1/\beta > 0$ and $H[\beta\kappa-\alpha] > 0$, adding and subtracting $\beta\kappa$ and α inside the third main expression, using the relationship $H'[x] = H[x][H[x]-x]$, and rearranging yields:

$$\begin{aligned} & \left[H'[\sigma_x+\beta\kappa-\alpha]-\frac{\sigma_x^2}{\sigma_s^2}\right]\Big[H[\sigma_x+\beta\kappa-\alpha]-H[\beta\kappa-\alpha]\Big]+H[\sigma_x+\beta\kappa-\alpha]\Big[H\left[\sigma_x+\beta\kappa-\alpha\right] \\ & -(\sigma_x+\beta\kappa-\alpha)-\alpha-\kappa\sigma_m\frac{\sigma_i^2}{\sigma_x\sigma_s}\Big]\Big[-\Big(H[\sigma_x+\beta\kappa-\alpha]-(\sigma_x+\beta\kappa-\alpha)\Big)+\Big(H[\beta\kappa-\alpha]-(\beta\kappa-\alpha)\Big)\Big] \quad \text{(IA94)} \\ = & \left[H'[\sigma_x+\beta\kappa-\alpha]-\frac{\sigma_x^2}{\sigma_s^2}\right]\Big[H[\sigma_x+\beta\kappa-\alpha]-H[\beta\kappa-\alpha]\Big]+H[\sigma_x+\beta\kappa-\alpha]\Big[H\left[\sigma_x+\beta\kappa-\alpha\right] \\ & -(\sigma_x+\beta\kappa-\alpha)-\alpha-\kappa\sigma_m\frac{\sigma_i^2}{\sigma_x\sigma_s}\Big]\Big[-H[\sigma_x+\beta\kappa-\alpha]+H[\beta\kappa-\alpha]+\sigma_x\Big]. \quad \text{(IA95)} \end{aligned}$$

Dividing by $\sigma_x > 0$, using the mean-value theorem, and $H'[x] = H[x][H[x]-x]$ gives:

$$\begin{aligned} & \left[H'[\sigma_x+\beta\kappa-\alpha]-\frac{\sigma_x^2}{\sigma_x^2}\right]H'[c^*]+H'[\sigma_x+\beta\kappa-\alpha]\Big[-H'[c^*]+1\Big]-\left(\alpha+\kappa\sigma_m\frac{\sigma_i^2}{\sigma_x\sigma_s}\right)H[\sigma_x+\beta\kappa-\alpha]\Big[-H'[c^*]+1\Big] \\ = & -\frac{\sigma_x^2}{\sigma_s^2}H'[c^*]+H'[\sigma_x+\beta\kappa-\alpha]-\left(\alpha+\kappa\sigma_m\frac{\sigma_i^2}{\sigma_x\sigma_s}\right)H[\sigma_x+\beta\kappa-\alpha]\Big[1-H'[c^*]\Big], \end{aligned} \tag{IA96}$$

where $c^* \in (\beta\kappa-\alpha, \sigma_x+\beta\kappa-\alpha)$. Given that $\kappa = -1$, a positive (negative) value of the sum in (IA96) implies that the expected put return increases (decreases) with the systematic variance (or volatility) of the log asset payoff, establishing part (d) of the proposition.

Proof of Part (e):

Since the partial derivative of the expected put return, $E[\tilde{R}_p]$, with respect to the idiosyncratic variance of the asset, σ_i^2, has the same sign as that partial derivative with respect to the idiosyn-

[7] The partial derivative of (IA92) with respect to κ is (IA93) divided by $\left(H\left[\sigma_x+\beta\kappa-\alpha\right]/H\left[\beta\kappa-\alpha\right]-1\right)^2$.

cratic volatility of the asset, σ_i, we focus on the latter for convenience. That partial derivative is:

$$\frac{\partial E[\tilde{R}_p]}{\partial \sigma_i} = \frac{\partial E[\tilde{X}_p]/p_p}{\partial \sigma_i} = \frac{(\partial E[\tilde{X}_p]/\partial \sigma_i)p_p - (\partial p_p/\partial \sigma_i)E[\tilde{X}_p]}{p_p^2}. \tag{IA97}$$

The partial derivatives on the right-hand side of the second equality are given by:

$$\frac{\partial E[\tilde{X}_p]}{\partial \sigma_i} = \tfrac{\sigma_i}{\sigma_x} Kn\left[\tfrac{\ln K - \mu_x}{\sigma_x}\right] - \sigma_i e^{\mu_x + \frac{1}{2}\sigma_x^2} N\left[\tfrac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right], \tag{IA98}$$

and

$$\frac{\partial p_p}{\partial \sigma_i} = e^{\mu_m + \frac{1}{2}\sigma_m^2}\left[\tfrac{\sigma_i}{\sigma_x} Kn\left[\tfrac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - \sigma_i e^{\mu_x + \frac{1}{2}[\sigma_x^2 + 2\kappa\sigma_s\sigma_m]} N\left[\tfrac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]\right]. \tag{IA99}$$

Substituting the numerator and denominator of the term on the right-hand side of the third equality in (IA64), (IA98), and (IA99) into (IA97) and using $z_1 \equiv \mu_x + \frac{1}{2}\sigma_x^2$, $z_2 \equiv \mu_m + \frac{1}{2}\sigma_m^2$, and $z_3 \equiv \mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 + 2\kappa\sigma_s\sigma_m + \sigma_m^2)$ to simplify the notation, we obtain:

$$\begin{aligned}\frac{\partial E[\tilde{R}_p]}{\partial \sigma_i} = \; & \frac{1}{p_p^2}\Bigg[\left(\frac{\sigma_i}{\sigma_x} Kn\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - \sigma_i e^{z_1} N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right) \\ & \times\left(Ke^{z_2} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - e^{z_3} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2 - \ln K}{\sigma_x}\right]\right) - \\ & \left(\left(KN\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - e^{z_1} N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right) \times \left(\frac{\sigma_i}{\sigma_x} Ke^{z_2} n\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - \right.\right. \\ & \left.\left.\sigma_i e^{z_3} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]\right)\right)\Bigg].\end{aligned} \tag{IA100}$$

The sign of the partial derivative is positive if and only if:

$$\begin{aligned}&\left(Kn\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - \sigma_x e^{z_1} N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right)\left(Ke^{z_2} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\right. \\ &\left. - e^{z_3} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]\right) > \left(KN\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - e^{z_1} N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right) \\ &\left(Ke^{z_2} n\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - \sigma_x e^{z_3} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]\right),\end{aligned} \tag{IA101}$$

and negative if and only if the inequality holds with the opposite sign.

As in the proof of part (e) of Proposition 1, we again first divide the inequality by the right term in the product on the left-hand side of the inequality and then, second, by the left term in the product on the right-hand side, without changing the sign of the inequality. The result is:

$$\frac{Kn\left[\frac{\ln K-\mu_x}{\sigma_x}\right]-\sigma_x e^{z_1}N\left[\frac{\ln K-\mu_x-\sigma_x^2}{\sigma_x}\right]}{KN\left[\frac{\ln K-\mu_x}{\sigma_x}\right]-e^{z_1}N\left[\frac{\ln K-\mu_x-\sigma_x^2}{\sigma_x}\right]}>\frac{Kn\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m}{\sigma_x}\right]-\sigma_x e^{z_1+\kappa\sigma_s\sigma_m}N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]}{KN\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m}{\sigma_x}\right]-e^{z_1+\kappa\sigma_s\sigma_m}N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]}. \tag{IA102}$$

Dividing the numerator and the denominator of the right-hand side of inequality (IA102) by $e^{z_1+\kappa\sigma_s\sigma_m}$ $N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]$, the right-hand side becomes:

$$\frac{e^{-(z_1+\kappa\sigma_s\sigma_m-\ln K)}n\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m}{\sigma_x}\right]/N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]-\sigma_x}{e^{-(z_1+\kappa\sigma_s\sigma_m-\ln K)}N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m}{\sigma_x}\right]/N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]-1}. \tag{IA103}$$

Using (IA89), we can write (IA103) as:

$$\frac{n\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]/N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]-\sigma_x}{\left(n\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m}{\sigma_x}\right]\right)/\left(n\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m}{\sigma_x}\right]N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]\right)-1}. \tag{IA104}$$

Using the definition for the hazard function of the normally distributed random variable x, which is $H(x)=n(x)/N(-x)$, we can write the right-hand side of Inequality (IA102) as:

$$\frac{H\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]-\sigma_x}{H\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]/H\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]-1}. \tag{IA105}$$

We now recognize that, if $\kappa=0$, $\kappa\sigma_s\sigma_m=0$, and Inequality (IA102) becomes an equality. As only the right-hand side of the inequality depends on the correlation between the log asset payoff and the log stochastic discount factor realization, the inequality would hold for $\kappa=-1$ if the right-hand side were monotonically increasing with increases in κ. Conversely, the inequality with the opposite sign would hold if the right-hand side were monotonically decreasing with increases in κ. Defining

$\alpha \equiv (\ln K - \mu_x)/\sigma_x$ and $\beta \equiv \frac{\sigma_s \sigma_m}{\sigma_x}$, we write the right-hand side of (IA102) (which is (IA105)) as:

$$\frac{H\left[\sigma_x + \beta\kappa - \alpha\right] - \sigma_x}{H\left[\sigma_x + \beta\kappa - \alpha\right]/H\left[\beta\kappa - \alpha\right] - 1}. \tag{IA106}$$

The partial derivative of (IA106) with respect to κ is proportional to:[8]

$$\begin{aligned} &\beta H'[\sigma_x + \beta\kappa - \alpha]\big[H[\sigma_x + \beta\kappa - \alpha]/H[\beta\kappa - \alpha] - 1\big] - \beta\big[H\left[\sigma_x + \beta\kappa - \alpha\right] - \sigma_x\big] \\ \times\quad &\big[H'[\sigma_x + \beta\kappa - \alpha]/H[\beta\kappa - \alpha] - H[\sigma_x + \beta\kappa - \alpha]H'[\beta\kappa - \alpha]/H[\beta\kappa - \alpha]^2\big]. \end{aligned} \tag{IA107}$$

Multiplying by $1/\beta > 0$ and $H[\beta\kappa - \alpha] > 0$, adding and subtracting $\beta\kappa$ and α inside the third main expression, using the relationship $H'[x] = H[x][H[x] - x]$, and rearranging yields:

$$\begin{aligned} & H'[\sigma_x + \beta\kappa - \alpha]\big[H[\sigma_x + \beta\kappa - \alpha] - H[\beta\kappa - \alpha]\big] - H[\sigma_x + \beta\kappa - \alpha]\big[H\left[\sigma_x + \beta\kappa - \alpha\right] \\ & -(\sigma_x + \beta\kappa - \alpha) - \alpha + \beta\kappa\big]\big[\big(H[\sigma_x + \beta\kappa - \alpha] - (\sigma_x + \beta\kappa - \alpha)\big) - \big(H[\beta\kappa - \alpha] - (\beta\kappa - \alpha)\big)\big] \end{aligned} \tag{IA108}$$

$$\begin{aligned} = \quad & H'[\sigma_x + \beta\kappa - \alpha]\big[H[\sigma_x + \beta\kappa - \alpha] - H[\beta\kappa - \alpha]\big] - H[\sigma_x + \beta\kappa - \alpha]\big[H\left[\sigma_x + \beta\kappa - \alpha\right] \\ & -(\sigma_x + \beta\kappa - \alpha) - \alpha + \beta\kappa\big]\big[H[\sigma_x + \beta\kappa - \alpha] - H[\beta\kappa - \alpha] - \sigma_x\big]. \end{aligned} \tag{IA109}$$

Dividing by $\sigma_x > 0$, using the mean-value theorem, and $H'[x] = H[x][H[x] - x]$ gives:

$$\begin{aligned} & H'[\sigma_x + \beta\kappa - \alpha]H'[c^*] - H'[\sigma_x + \beta\kappa - \alpha]\big[H'[c^*] - 1\big] + (\beta\kappa - \alpha)\, H[\sigma_x + \beta\kappa - \alpha]\big[1 - H'[c^*]\big] \\ = \quad & H'[\sigma_x + \beta\kappa - \alpha] - (\alpha - \beta\kappa)\, H[\sigma_x + \beta\kappa - \alpha]\big[1 - H'[c^*]\big], \end{aligned} \tag{IA110}$$

where $c^* \in (\beta\kappa - \alpha, \sigma_x + \beta\kappa - \alpha)$. As $\kappa = -1$, a positive (negative) value for the sum on the right-hand side of Equation (IA110) implies the expected put return increases (decreases) with the idiosyncratic variance (or volatility) of the log asset payoff, establishing part (e) of the proposition.

$\blacksquare$

[8]The partial derivative of (IA106) with respect to κ is (IA107) divided by $\left(H\left[\sigma_x + \beta\kappa - \alpha\right]/H\left[\beta\kappa - \alpha\right] - 1\right)^2$.

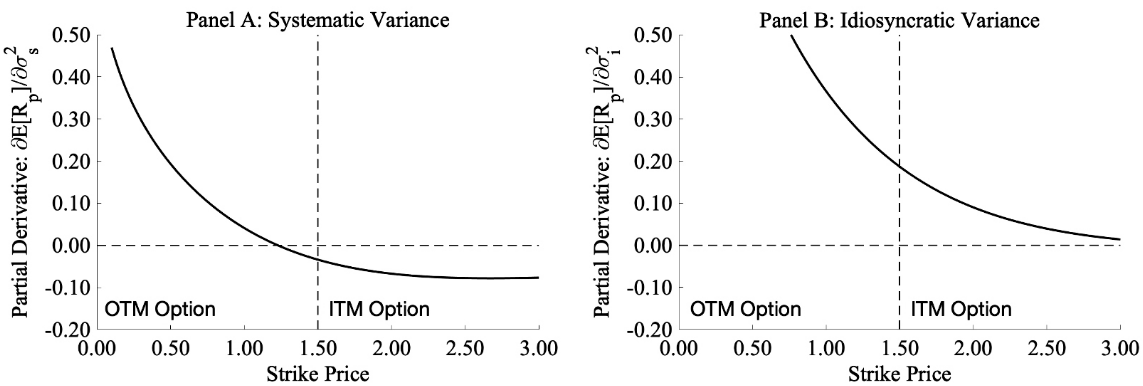

Figure IA2: Partial Derivatives of Expected Put Return with Respect to Systematic and Idiosyncratic Variance The figure plots the partial derivatives of the expected put return with respect to the systematic (Panel A) and idiosyncratic (Panel B) variance of the log asset payoff against the strike price. The basecase parameter values are the same as those used in Figure 1 in the main paper.

As in case of part (b) of Proposition 1, part (b) of Proposition IA.1 follows from the more general analysis of Coval and Shumway (2001). The other parts of Proposition IA.1 are, however, again new to the literature. Part (a) reveals that a higher expected log asset payoff decreases the expected put return, while part (c) indicates that a higher moneyness increases that return irrespective of whether the higher moneyness is due to a higher strike price or a lower expected log asset payoff. Part (d) again opens up the possibility that the relation between expected put return and systematic log asset payoff variance is ambiguous. In case of puts, we, however, find it impossible to analytically determine the sign of the relation. The put case only allows us to prove that, if idiosyncratic variance is zero ($\sigma_i^2 = 0$), the sign of the relation is negative for ATM and ITM puts, but can be positive, zero, or negative for OTM puts.[9] Notwithstanding, plotting the partial derivative of the expected put return with respect to systematic variance against the strike price, Panel A of Figure IA2 suggests that, even if idiosyncratic asset variance is positive ($\sigma_i^2 > 0$), systematic asset variance exerts a negative (positive, zero, or negative) effect on the expected put return for ITM (OTM) puts.

[9]The problem is the sum of the first two terms in the inequality stated in part (d) of Proposition IA.1, which is equal to $-(\sigma_x^2/\sigma_s^2)H'[c^*] + H'[\sigma_x - \beta - \alpha]$. While $-H'[c^*] + H'[\sigma_x - \beta - \alpha] > 0$, $(\sigma_x^2/\sigma_s^2) > 1$, raising the value of the negative summand and rendering it impossible to sign the sum. In contrast, in case of calls in part (d) of Proposition 1 in our main paper, $(\sigma_x^2/\sigma_s^2)H'[c^*] - H'[\alpha - \sigma_x + \beta] > 0$, because $(\sigma_x^2/\sigma_s^2) > 1$ and $H'[c^*] - H'[\alpha - \sigma_x + \beta] > 0$.

Interestingly, part (e) of Proposition IA.1 opens up the possibility that the idiosyncratic variance of the log asset payoff is also ambiguously related to the expected put return. Consistent with this possibility, Corollary IA.1 shows that, identical to the sign of the systematic variance effect, the sign of the idiosyncratic variance effect also depends on put moneyness.

COROLLARY IA.1: *Under the assumptions in Proposition IA.1, the relation between the expected put return, $E[\tilde{R}_p]$, and the idiosyncratic variance of the log asset payoff, σ_i^2, is positive for sufficiently OTM puts, but can be positive, zero, or negative for ITM and ATM puts.*

PROOF:

Recalling part (e) of Proposition IA.1, the sign of the relation between the expected put return and the idiosyncratic variance of the log asset payoff is determined by the sign of:

$$H'[\sigma_x + \beta\kappa - \alpha] - (\alpha - \beta\kappa)\, H[\sigma_x + \beta\kappa - \alpha]\Big[1 - H'[c^*]\Big], \tag{IA111}$$

with the relation being positive (negative) [zero] if the sum in (IA111) is positive (negative) [zero].

Given we define a put's moneyness as $(\ln K - \mu_x)$, $\alpha \equiv \frac{\ln K - \mu_x}{\sigma_x}$ increases with moneyness, and ITM (ATM) [OTM] puts have an α value above (equal to) [below] zero. Since $H'[.] > 0$, sufficiently OTM $((\alpha - \beta\kappa) < 0)$ puts thus produce a positive relation between the expected put return and the idiosyncratic variance of the log asset payoff. Numerical examples establish that ITM and ATM puts can produce a weakly negative relation under some parameter value combinations.

■

While Corollary IA.1 conflicts with Hu and Jacobs' (2020) claim that, in a Black and Scholes (1973) world, the expected put return relates unambiguously positively to the idiosyncratic variance of the log asset payoff, we only ever came across numerical examples generating an extremely weakly negative relation for deep ITM puts. To be more specific, we never encountered examples in which the partial derivative of the expected put return with respect to idiosyncratic variance was below –0.0001. While the mildly negative relation is impossible to see from Panel B of Figure IA2, the panel confirms that the effect of idiosyncratic variance on the expected put return converges to a value *somewhere close to zero* as put moneyness converges to infinity. Overall, part (e) of Proposition IA.1 is thus in broad agreement with the conclusions of Hu and Jacobs (2020).

IA.4 Alternative Portfolio Sort and Regression Specifications

In this section, we repeat our main portfolio sorts and FM regressions in Tables 2 and 3 in the main paper using alternative data filters, regression specifications, and volatility estimates. We first repeat the regressions keeping only one call per underlying stock in each moneyness subsample per sample month. We then repeat them using weighted least-squares (WLS) cross-sectional regressions using call market capitalization as weighting variable. We finally reproduce the portfolio sorts in Table 2 as well as the held-to-maturity return regressions in Table 3 using the systematic and idiosyncratic volatility estimates obtained from the alternative factor models considered in the main paper.

IA.4.1. Using Only One Call Per Stock Within a Moneyness Subsample

In our main paper, we conduct our FM regressions on moneyness subsamples potentially containing more than one single call per underlying stock in a sample month. Because the returns of calls on the same underlying stock are likely to be almost perfectly correlated, it is conceivable that subsamples containing such calls yield more volatile cross-sectional regression estimates, in turn lowering the absolute t-statistics of the FM premium estimates. Notwithstanding, since we do not see a reason for why the cross-sectional estimates obtained from such subsamples should be less normally distributed than those obtained from subsamples only keeping one call per underlying stock, we do not expect that those subsamples produce less valid inferences than the others. Moreover, as the great majority of stocks has no more than two to three calls written on them in a sample month, we also do not expect those subsamples to produce much lower inference levels. Given that any strategy to select a single call per underlying stock and moneyness subsample must necessarily be ad hoc, we thus do not impose such a restriction in the FM regressions conducted in our main paper.

In Table IA.1, we change our strategy, repeating the FM regressions in Table 3 of our main paper on moneyness subsamples keeping only one call per underlying stock. To do so, we only keep that call with the highest moneyness in the ITM subsample; that call with a moneyness closest to unity in the ATM subsample; and those calls with the lowest moneyness in the OTM and DOTM subsamples. The table suggests that this modification yields conclusions close to identical to those reported in our main paper. While the systematic volatility premium in sold-before-maturity ITM call returns is, for example, 33% (t-statistic: 3.40) when keeping all calls on an underlying stock in the ITM subsample (see Table 3 in the main paper), the same premium is 30% (t-statistic: 3.32) when only keeping the call with the highest moneyness. Similarly, while the idiosyncratic volatility premium in sold-before-maturity DOTM call returns is –92% (t-statistic: –6.85) when keeping all calls (again see Table 3 in the main paper), the same premium is –82% (t-statistic: –6.72) when only the keeping the call with the lowest moneyness. In line with our arguments above, the highly similar estimates and inferences come from the majority of stocks only having a few calls written on them, leading the one-call-per-stock restriction to only mildly decrease our sample size.

IA.4.2. Using Weighted Regressions Based on Call Market Capitalization

While we calculate call returns from midpoint prices in most tests run in our main paper, ensuring that upward bias in mean returns originating from bid-ask bounce effects should not invalidate our conclusions, Asparouhova, Bessembinder, and Kalcheva (2010; 2013) discuss other reasons for why true asset values could deviate from observed asset prices, generating an equivalent bias. To see whether such a bias distorts our conclusions, we next repeat our FM regressions in Table 3 of the main paper using cross-sectional regressions weighting each observation by a call's market capitalization, calculated as call open interest multiplied by call midpoint price. Doing so mitigates the bias since calls with temporarily inflated (deflated) prices at the start of the call return period likely also have an inflated (deflated) market capitalization at the same time, leading their subsequent returns (which are, in expectation, too low (high)) to be overweighted (underweighted) in the regressions.

In Table IA.2, we report the results from repeating the FM regressions in Table 3 of the main paper using cross-sectional regressions weighting each observation by call market capitalization. The table suggests that this modification only helps to strengthen our results. To be more specific, while the weighted sold-before-maturity return regression results broadly align with the corresponding results in our main paper (see Table 3 in the main paper), the weighted held-to-maturity return regression results are slightly better than their corresponding results. To see that, note that the systematic volatility premium in held-to-maturity ITM calls is 126% (t-statistic: 3.59) according to the weighted regression results, compared to only 46% (t-statistic: 2.23) according to the main results. Despite that improvement, the weighted regressions however also produce somewhat less significant idiosyncratic volatility premiums across the moneyness subsamples than the standard regressions, without those premiums however ever coming close to turning statistically insignificant.

IA.4.3. Repeating the Portfolio Sorts and Held-to-Maturity Return Regressions Using the Alternative Factor Model Volatility Estimates

In Table 4 in Section 3.3.c of the main paper, we repeat our main asset pricing tests using systematic and idiosyncratic volatility estimates obtained from alternative linear factor models. To conserve space, we however only repeat the sold-before-maturity regressions in that table, and not the portfolio sorts or the held-to-maturity return regressions. To further corroborate that our empirical results are robust to the factor model used to estimate the volatility components, Table IA.3 now also reports the mean returns ("High–Low") and Fama-French-Karakaya ("FFK") alphas of the systematic (columns (1) to (4)) and idiosyncratic (columns (5) to (8)) volatility spread portfolios within each moneyness portfolio plus their spreads over the moneyness portfolios ("ITM–DOTM") obtained from using the CAPM, the Fama-French-Carhart (1997) model, the Fama-French (2016) six-factor model (for comparison), the Hou-Mo-Xue-Zhang (2021) augmented q-theory model, and the Stambaugh and Yuan (2017) mispricing factor model to estimate the volatility components in Panels A to E, respectively. Comparing mean returns and alphas across the panels, the table shows that our portfolio sort results are extremely robust with respect to our choice of factor model. Looking into the excess held-to-maturity return in Panel B, the systematic volatility spread portfolio within the ITM portfolio, for example, yields a highly similar mean return between 9% and 11% and a highly similar t-statistic between 3.07 and 3.28 across the five factor models (see column (4)).

In Table IA.4, we next repeat the FM regressions in Table 4 of the main paper relying on held-to-maturity rather than sold-before-maturity returns as dependent variable. In Panels A to E of that table, we employ the CAPM, the Fama-French-Carhart (1997) model, the Fama-French (2016) six-factor model (for comparison), the Hou-Mo-Xue-Zhang (2021) augmented q-theory model, and the Stambaugh and Yuan (2017) mispricing factor model to estimate systematic and idiosyncratic volatility, respectively. The table suggests that our held-to-maturity return regression results are also extremely robust with respect to our choice of factor model. While the regressions, for example, produce a similar systematic volatility premium in ITM calls between 46% and 58% with a t-statistic between 2.17 and 2.81, they further produce a similar idiosyncratic volatility premium in DOTM calls between –114% and –126% with a t-statistic between –4.85 and –6.37.

Overall, this section illustrates that our empirical results are robust to: (i) selecting either one or all available calls per underlying stock for each moneyness portfolio; (ii) running ordinary regressions or weighted regressions using call market capitalization as weighting variable; and (iii) running our portfolio sorts and sold-before-maturity and held-to-maturity return regressions using volatility component estimates obtained from alternative popular linear factor models.

IA.5 Calculation of Implied Volatility and Higher Moments

In Section 4 of the main paper, we repeat our main FM regressions in Table 3 of that paper controlling for the variance risk premium and the third and fourth higher-order risk-neutral moments of the underlying stock. In this section of the Internet Appendix, we offer details on how we estimate those control variables. In line with Bali and Hovakimian (2009) and Carr and Wu (2009), we estimate the variance risk premium of stock i at the end of month $t-1$, $\text{VRP}_{i,t-1}$, as:

$$\text{VRP}_{i,t-1} = \text{RealizedVariance}_{i,t-1} - \text{ImpliedVariance}_{i,t-1}, \qquad \text{(IA112)}$$

where $\text{RealizedVariance}_{i,t-1}$ is the sum of stock i's squared daily log returns over month $t-1$ times twelve, and $\text{ImpliedVariance}_{i,t-1}$ is Britten-Jones and Neuberger's (2000) model-free estimate of stock i's annualized implied variance at the end of month $t-1$, given by:

$$\text{ImpliedVariance}_{i,t-1} = \int_{K=0}^{F} \frac{2P(K)}{K^2} dK + \int_{K=F}^{\infty} \frac{2C(K)}{K^2} dK, \qquad \text{(IA113)}$$

where $C(.)$ and $P(.)$ are the prices of calls and puts, respectively, K the strike price, and F the stock's forward price, with all derivative contracts sharing the same maturity date.

Using Bakshi, Kapadia, and Madan's (2003) methodology, we calculate stock i's third and fourth higher-order moments at the end of month $t-1$, $\text{RNS}_{i,t-1}$ and $\text{RNK}_{i,t-1}$, respectively, as:

$$\text{RNS}_{i,t-1} = \frac{e^r W_{i,t-1} - 3\mu_{i,t-1} e^r V_{i,t-1} + 2\mu_{i,t-1}^3}{[e^r V_{i,t-1} - \mu_{i,t-1}^2]^{3/2}}, \qquad \text{(IA114)}$$

and

$$\text{RNK}_{i,t-1} = \frac{e^r X_{i,t-1} - 4\mu_{i,t-1} e^r W_{i,t-1} + 6 e^r \mu_{i,t-1}^2 V_{i,t-1} - 3\mu_{i,t-1}^4}{[e^r V_{i,t-1} - \mu_{i,t-1}^2]^2}, \qquad \text{(IA115)}$$

where r is the risk-free rate of return over the time-to-maturity, $\mu_{i,t-1} = e^r - 1 - \frac{e^r}{2} V_{i,t-1} - \frac{e^r}{6} W_{i,t-1} - \frac{e^r}{24} X_{i,t-1}$, and $V_{i,t-1}$ $(W_{i,t-1})$ $[X_{i,t-1}]$ the value of a volatility (cubic) [quartic] contract, paying out the squared (cubic) [quartic] log return on the maturity date. In line with others, we refer to $\text{RNS}_{i,t-1}$ as risk-neutral skewness and to $\text{RNK}_{i,t-1}$ as risk-neutral kurtosis. Bakshi and Madan (2000) and Bakshi, Kapadia, and Madan (2003) establish that $V_{i,t-1}$, $W_{i,t-1}$, and $X_{i,t-1}$ can be calculated using:

$$V_{i,t-1} = \int_{S_{i,t-1}}^{\infty} \frac{2(1 - \ln(K/S_{i,t-1}))}{K^2} C(K) dK + \int_{0}^{S_{i,t-1}} \frac{2(1 + \ln(S_{i,t-1}/K))}{K^2} P(K) dK, \qquad \text{(IA116)}$$

$$W_{i,t-1} = \int_{S_{i,t-1}}^{\infty} \frac{6\ln(K/S_{i,t-1}) - 3(\ln(K/S_{i,t-1}))^2}{K^2} C(K)dK - \int_{0}^{S_{i,t-1}} \frac{6\ln(S_{i,t-1}/K) + 3(\ln(S_{i,t-1}/K))^2}{K^2} P(K)dK, \quad \text{(IA117)}$$

and

$$X_{i,t-1} = \int_{S_{i,t-1}}^{\infty} \frac{12(\ln(K/S_{i,t-1}))^2 - 4(\ln(K/S_{i,t-1}))^3}{K^2} C(K)dK + \int_{0}^{S_{i,t-1}} \frac{12(\ln(S_{i,t-1}/K))^2 + 4(\ln(S_{i,t-1}/K))^3}{K^2} P(K)dK, \quad \text{(IA118)}$$

where $S_{i,t-1}$ is stock i's price at the end of month $t-1$.

We approximate the integrals in (IA113), (IA116), (IA117), and (IA118) as follows. We estimate a stock-specific cubic regression model of Black-Scholes (1973) implied volatility on strike price and time-to-maturity using data from the last trading day of each sample month. We next use the estimates from that regression model to calculate 1,000 interpolated implied volatility estimates, with a strike-to-stock price ratio from 0.0001 to three (in equal increments) and a one-month time-to-maturity. We then plug the interpolated implied volatility estimates into the Black-Scholes (1973) call and put formulas to obtain $C(K)$ and $P(K)$. We finally employ the trapezoidal approximation together with $C(K)$ and $P(K)$ to calculate $\text{ImpliedVariance}_{i,t-1}$, $V_{i,t-1}$, $W_{i,t-1}$, and $X_{i,t-1}$.

Following other studies, we use American option data to approximate the integrals. Moreover, we only calculate the integrals for stocks with at least two traded calls with a delta above 0.50 and two traded puts with a delta below –0.50 at the end of each sample month.

IA.6 Controlling for the Historical Stock Return

Studying the empirical effect of total stock variance (rather than the separate effects of systematic and idiosyncratic variance) on option returns, Hu and Jacobs (2020) also run tests controlling for the expected underlying-stock return, similar in spirit to the FM regressions in Panel D of Table 8 of our main paper controlling for stock characteristics. In contrast to us, their initial tests along those lines however aim to achieve that goal by simply controlling for a stock's mean daily return over the six months of data before the option return period. While Hu and Jacobs (2020) are themselves rather critical about their tests controlling for the mean daily stock return, saying that "historical averages are notoriously imprecise" (p.14) and reporting results only in their Internet Appendix, for the sake of completeness, we now also add that return as additional control variable to the other firm characteristics in the FM regressions in Panel D of Table 8 in the main paper.[10]

In Table IA.5, we offer the results from repeating the FM regressions in Panel D of Table 8 of our main paper also including the mean daily return as control variable. The table suggests that adding that return has almost no effect on the systematic and idiosyncratic volatility premium estimates or their inference levels across the moneyness subsamples. Moreover, the historical mean return is only strongly significantly priced in ITM calls, with a positive premium.

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[10] We do not add the mean daily return to the FM regressions in Panel D of Table 8 of our main paper since it shares an average cross-sectional correlation of about 0.60 with the momentum past return, potentially creating severe multicollinearity problems in those regressions (recall our discussion in footnote 26 in the main paper).

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Table IA.1

Regressions of Call Returns on Systematic and Idiosyncratic Volatility: Selecting One Call Per Underlying Stock Within Each Moneyness Subsample

The table presents the results from Fama-MacBeth (1973) regressions of the excess call return calculated from start of month t to the end of that month ("sold-before-maturity," Panel A) or to the call maturity date ("held-to-maturity," Panel B) on moneyness and systematic and idiosyncratic underlying-stock volatility measured until end of month $t-1$. We run the regressions separately for calls with a start-of-month-t moneyness below the first quartile (column (1), "DOTM"), between the first and second (column (2), "OTM"), between the second and third (column (3), "ATM"), and above the third (column (4), "ITM"). Within each moneyness classification, we however only keep one call per stock. In particular, we only keep the call with the highest (closest to one) [lowest] moneyness in the ITM (ATM) [OTM or DOTM] classification. Moneyness is the ratio of stock price to strike price at the start of month t. We calculate systematic and idiosyncratic volatility using the Fama-French (2016) six-factor model estimated over the 24 months of daily data before the call return period. Plain numbers are premium estimates (in decimals), while the numbers in square brackets are Newey and West (1987) t-statistics calculated using a twelve-month lag length.

	DOTM		OTM		ATM		ITM	
	(1)		(2)		(3)		(4)	
Panel A: Excess Sold-Before-Maturity Return								
Moneyness	−7.55	[−12.27]	−7.67	[−12.29]	−4.30	[−9.29]	−1.56	[−10.22]
Systematic Volatility	−1.40	[−6.89]	−0.70	[−4.65]	0.12	[1.03]	0.30	[3.32]
Idiosyncratic Volatility	−0.82	[−6.72]	−0.46	[−5.30]	−0.32	[−5.13]	−0.23	[−5.74]
Constant	7.92	[13.59]	7.78	[12.56]	4.32	[9.02]	1.51	[9.17]
Panel B: Excess Held-to-Maturity Return								
Moneyness	−6.79	[−7.31]	−7.69	[−7.80]	−4.85	[−6.83]	−1.82	[−8.05]
Systematic Volatility	−1.29	[−3.08]	−0.57	[−1.73]	0.27	[1.08]	0.42	[2.14]
Idiosyncratic Volatility	−1.01	[−4.79]	−0.68	[−4.49]	−0.48	[−4.65]	−0.39	[−5.37]
Constant	7.33	[8.17]	7.89	[7.92]	4.95	[6.73]	1.85	[7.42]

Table IA.2

Weighted Regressions of Call Returns on Systematic and Idiosyncratic Volatility

The table presents the results from weighted least-squares Fama-MacBeth (1973) regressions of the excess call return calculated from start of month t to the end of that month ("sold-before-maturity," Panel A) or to the call maturity date ("held-to-maturity," Panel B) on moneyness and systematic and idiosyncratic underlying-stock volatility measured until end of month $t-1$. We use dollar call open interest as weight in the regressions. We run the regressions separately for calls with a start-of-month-t moneyness below the first quartile (column (1), "DOTM"), between the first and second (column (2), "OTM"), between the second and third (column (3), "ATM"), and above the third (column (4), "ITM"). Moneyness is the ratio of stock price to strike price at the start of month t. We calculate systematic and idiosyncratic volatility using the Fama-French (2016) six-factor model estimated over the 24 months of daily data before the call return period. Plain numbers are premium estimates (in decimals), while the numbers in square brackets are Newey and West (1987) t-statistics calculated using a twelve-month lag length.

	DOTM		OTM		ATM		ITM	
	(1)		(2)		(3)		(4)	
Panel A: Excess Sold-Before-Maturity Return								
Moneyness	−2.39	[−2.96]	−2.98	[−2.96]	−2.52	[−3.85]	−0.70	[−3.41]
Systematic Volatility	−0.89	[−2.43]	−0.39	[−1.25]	0.26	[1.03]	0.61	[3.66]
Idiosyncratic Volatility	−0.60	[−3.50]	−0.33	[−2.48]	−0.29	[−2.53]	−0.29	[−3.84]
Constant	2.88	[3.72]	3.19	[3.28]	2.53	[3.80]	0.62	[2.79]
Panel B: Excess Held-to-Maturity Return								
Moneyness	−2.18	[−2.70]	−3.03	[−3.06]	−2.43	[−3.89]	−0.66	[−3.27]
Systematic Volatility	−1.66	[−2.42]	−0.85	[−1.34]	0.57	[1.12]	1.26	[3.59]
Idiosyncratic Volatility	−0.71	[−3.61]	−0.51	[−2.73]	−0.44	[−2.54]	−0.44	[−3.99]
Constant	2.46	[3.28]	3.15	[3.32]	2.42	[3.83]	0.61	[2.75]

Table IA.3

Call Portfolios Double-Sorted on Moneyness and Systematic or Idiosyncratic Volatility Calculated from Alternative Linear Factor Models in the Recent Literature

In this table, we present the results from call spread portfolios constructed from portfolios double-sorted on moneyness and systematic or idiosyncratic underlying-stock volatility controlling for the other volatility component. We form the portfolios double-sorted on moneyness and systematic volatility used in columns (1) to (4) as follows. At the end of each sample month $t-1$, we first sort the calls into portfolios according to the quartile breakpoints of idiosyncratic volatility. Within each portfolio, we independently sort them into portfolios according to the quartile breakpoints of moneyness and systematic volatility. We equally-weight the resulting $4 \times 4 \times 4$ portfolios and form equally-weighted portfolios of the portfolios within the same moneyness-systematic volatility classification. We hold the resulting 4×4 portfolios from the start of month t. Within each moneyness portfolio, we finally form a spread portfolio long the highest and short the lowest systematic volatility portfolio ("High–Low"). To adjust the spread portfolios for risk, we regress them on the six Fama-French (2016) stock factors plus the three Karakaya (2013) option factors and report the regression's intercept ("FFK Alpha"). To form the portfolios sorted on moneyness and idiosyncratic volatility used in columns (5) to (8), we reverse the roles of systematic and idiosyncratic volatility. The plain numbers in Panels A and B are mean excess sold-before-maturity and mean excess held-to-maturity returns and their associated FFK alphas, respectively, while the numbers in square brackets are Newey and West (1987) t-statistics calculated using a twelve-month lag length. We calculate systematic and idiosyncratic volatility using either the CAPM (Subpanel 1), the Fama-French-Carhart (1997) model (Subpanel 2), the Fama-French (2016) six-factor model (Subpanel 3), the Hou-Mo-Xue-Zhang (2021) augmented q-theory model (Subpanel 4), and the Stambaugh and Yuan (2017) mispricing factor model (Subpanel 5) estimated over the 24 months of daily data before the call return period.

	Systematic Volatility Moneyness					Idiosyncratic Volatility Moneyness				
Volatility	DOTM	OTM	ATM	ITM	ITM–DOTM	DOTM	OTM	ATM	ITM	ITM–DOTM
	(1)	(2)	(3)	(4)	(4)–(1)	(5)	(6)	(7)	(8)	(8)–(5)
Panel A: Mean Excess Sold-Before-Maturity Return										
Panel A.1: CAPM Estimates										
High–Low	−0.24	−0.13	−0.01	0.06	0.30	−0.29	−0.20	−0.11	−0.09	0.20
	[−5.48]	[−4.56]	[−0.60]	[3.08]	[7.86]	[−5.72]	[−5.79]	[−4.52]	[−5.88]	[4.33]
FFK Alpha	−0.28	−0.13	−0.03	0.05	0.33	−0.22	−0.12	−0.05	−0.07	0.15
	[−5.10]	[−3.79]	[−0.88]	[1.93]	[6.05]	[−3.05]	[−3.51]	[−2.15]	[−4.32]	[2.06]
Panel A.2: Fama-French-Carhart Model Estimates										
High–Low	−0.29	−0.13	−0.01	0.07	0.36	−0.27	−0.20	−0.12	−0.10	0.17
	[−6.06]	[−4.66]	[−0.42]	[3.33]	[8.62]	[−5.15]	[−5.66]	[−4.99]	[−6.40]	[3.52]
FFK Alpha	−0.34	−0.12	−0.02	0.06	0.40	−0.15	−0.11	−0.07	−0.08	0.07
	[−5.06]	[−3.38]	[−0.62]	[2.27]	[5.82]	[−2.26]	[−3.10]	[−2.98]	[−4.87]	[1.01]

(continued on next page)

Table IA.3
Call Portfolios Double-Sorted on Moneyness and Systematic or Idiosyncratic Volatility Calculated from Alternative Linear Factor Models in the Recent Literature (cont.)

	Systematic Volatility Moneyness					Idiosyncratic Volatility Moneyness				
Volatility	DOTM	OTM	ATM	ITM	ITM–DOTM	DOTM	OTM	ATM	ITM	ITM–DOTM
	(1)	(2)	(3)	(4)	(4)–(1)	(5)	(6)	(7)	(8)	(8)–(5)
Panel A: Mean Excess Sold-Before-Maturity Return (cont.)										
Panel A.3: Fama-French Six-Factor Model Estimates										
High–Low	−0.27	−0.12	0.00	0.07	0.34	−0.26	−0.19	−0.13	−0.11	0.15
	[−5.85]	[−4.26]	[0.14]	[3.31]	[8.55]	[−4.76]	[−5.26]	[−5.22]	[−6.50]	[3.09]
FFK Alpha	−0.36	−0.13	0.00	0.06	0.42	−0.13	−0.08	−0.08	−0.09	0.04
	[−5.24]	[−3.42]	[−0.09]	[2.25]	[6.20]	[−1.92]	[−2.26]	[−3.07]	[−5.25]	[0.59]
Panel A.4: Augmented q-Theory Model Estimates										
High–Low	−0.28	−0.13	0.01	0.06	0.34	−0.27	−0.18	−0.13	−0.10	0.16
	[−5.87]	[−4.35]	[0.33]	[3.18]	[8.51]	[−5.26]	[−4.97]	[−5.16]	[−6.10]	[3.60]
FFK Alpha	−0.33	−0.12	0.00	0.06	0.39	−0.15	−0.09	−0.07	−0.08	0.06
	[−4.79]	[−2.98]	[−0.05]	[2.34]	[5.62]	[−2.45]	[−2.47]	[−2.92]	[−4.47]	[1.03]
Panel A.5: Mispricing Factor Model Estimates										
High–Low	−0.25	−0.12	0.01	0.07	0.32	−0.26	−0.15	−0.11	−0.10	0.16
	[−5.19]	[−3.93]	[0.52]	[3.30]	[7.90]	[−4.58]	[−4.87]	[−4.50]	[−6.02]	[3.06]
FFK Alpha	−0.32	−0.12	0.00	0.06	0.37	−0.13	−0.08	−0.06	−0.09	0.04
	[−4.21]	[−2.69]	[−0.13]	[1.66]	[5.05]	[−1.74]	[−2.11]	[−2.27]	[−4.51]	[0.55]
Panel B: Mean Excess Held-to-Maturity Return										
Panel B.1: CAPM Estimates										
High–Low	−0.18	−0.05	0.03	0.10	0.27	−0.37	−0.25	−0.16	−0.14	0.23
	[−2.31]	[−1.01]	[0.96]	[3.28]	[4.23]	[−4.37]	[−5.63]	[−4.98]	[−6.37]	[2.92]
FFK Alpha	−0.19	−0.07	0.01	0.07	0.27	−0.27	−0.14	−0.07	−0.10	0.16
	[−2.48]	[−1.37]	[0.41]	[2.32]	[3.98]	[−2.60]	[−3.37]	[−2.59]	[−3.94]	[1.70]

(continued on next page)

Table IA.3
Call Portfolios Double-Sorted on Moneyness and Systematic or Idiosyncratic Volatility Calculated from Alternative Linear Factor Models in the Recent Literature (cont.)

	Systematic Volatility Moneyness					Idiosyncratic Volatility Moneyness				
Volatility	DOTM	OTM	ATM	ITM	ITM–DOTM	DOTM	OTM	ATM	ITM	ITM–DOTM
	(1)	(2)	(3)	(4)	(4)–(1)	(5)	(6)	(7)	(8)	(8)–(5)
Panel B: Mean Excess Held-to-Maturity Return (cont.)										
Panel B.2: Fama-French-Carhart Model Estimates										
High–Low	−0.20	−0.06	0.04	0.09	0.29	−0.39	−0.25	−0.17	−0.15	0.24
	[−2.54]	[−1.37]	[1.14]	[3.07]	[4.50]	[−4.40]	[−5.49]	[−5.32]	[−6.60]	[2.95]
FFK Alpha	−0.22	−0.08	0.02	0.07	0.29	−0.29	−0.13	−0.08	−0.10	0.19
	[−2.78]	[−1.69]	[0.51]	[2.02]	[4.05]	[−2.62]	[−3.38]	[−3.22]	[−3.98]	[1.82]
Panel B.3: Fama-French Six-Factor Model Estimates										
High–Low	−0.21	−0.05	0.05	0.10	0.31	−0.30	−0.24	−0.18	−0.15	0.15
	[−2.63]	[−1.04]	[1.25]	[3.09]	[4.72]	[−4.04]	[−5.03]	[−5.84]	[−6.50]	[2.22]
FFK Alpha	−0.26	−0.06	0.02	0.07	0.33	−0.15	−0.10	−0.10	−0.10	0.04
	[−2.92]	[−1.40]	[0.45]	[2.24]	[4.28]	[−2.08]	[−2.20]	[−3.77]	[−3.93]	[0.58]
Panel B.4: Augmented q-Theory Model Estimates										
High–Low	−0.20	−0.07	0.05	0.09	0.30	−0.35	−0.23	−0.19	−0.15	0.21
	[−2.53]	[−1.52]	[1.43]	[3.10]	[4.47]	[−4.30]	[−4.83]	[−5.78]	[−6.30]	[2.75]
FFK Alpha	−0.23	−0.07	0.03	0.07	0.30	−0.24	−0.11	−0.10	−0.10	0.15
	[−2.57]	[−1.61]	[0.91]	[2.20]	[3.93]	[−2.71]	[−2.35]	[−3.75]	[−3.49]	[1.68]
Panel B.5: Mispricing Factor Model Estimates										
High–Low	−0.15	−0.05	0.07	0.11	0.26	−0.43	−0.22	−0.17	−0.15	0.27
	[−1.86]	[−1.02]	[1.67]	[3.18]	[3.90]	[−4.09]	[−4.57]	[−5.15]	[−6.44]	[2.82]
FFK Alpha	−0.22	−0.06	0.05	0.08	0.30	−0.29	−0.11	−0.09	−0.11	0.18
	[−2.44]	[−1.08]	[1.09]	[2.23]	[4.19]	[−2.50]	[−2.52]	[−3.28]	[−4.39]	[1.57]

Table IA.4

Regressions of Held-to-Maturity Call Returns on Systematic and Idiosyncratic Volatility Calculated from Alternative Linear Factor Models Proposed in the Recent Literature

The table presents the results from Fama-MacBeth (1973) regressions of the excess call return calculated from start of month t to the call maturity date on moneyness and systematic and idiosyncratic underlying-stock volatility measured until end of month $t-1$. We run the regressions separately for calls with a start-of-month-t moneyness below the first quartile (column (1), "DOTM"), between the first and second (column (2), "OTM"), between the second and third (column (3), "ATM"), and above the third (column (4), "ITM"). Moneyness is the ratio of stock price to strike price at the start of month t. We calculate systematic and idiosyncratic volatility using either the CAPM (Panel A), the Fama-French-Carhart (1997) model (Panel B), the Fama-French (2016) six-factor model (Panel C), the Hou-Mo-Xue-Zhang (2021) augmented q-theory model (Panel D), or the Stambaugh and Yuan (2017) mispricing factor model (Panel E) estimated over the 24 months of daily data before the call return period. Plain numbers are premium estimates (in decimals), while the numbers in square brackets are Newey and West (1987) t-statistics calculated using a twelve-month lag length.

	DOTM		OTM		ATM		ITM	
	(1)		(2)		(3)		(4)	
Panel A: CAPM Estimates								
Moneyness	−5.04	[−6.31]	−6.51	[−7.11]	−4.65	[−7.11]	−1.20	[−6.84]
Systematic Volatility	−1.03	[−2.22]	−0.62	[−1.63]	0.46	[1.52]	0.58	[2.81]
Idiosyncratic Volatility	−1.26	[−6.37]	−0.81	[−5.28]	−0.49	[−4.75]	−0.35	[−5.41]
Constant	5.81	[7.21]	6.83	[7.28]	4.73	[6.96]	1.15	[5.85]
Panel B: Fama-French-Carhart Model Estimates								
Moneyness	−5.04	[−6.30]	−6.56	[−7.16]	−4.69	[−7.10]	−1.19	[−6.79]
Systematic Volatility	−1.09	[−2.51]	−0.63	[−1.79]	0.39	[1.42]	0.50	[2.36]
Idiosyncratic Volatility	−1.20	[−5.55]	−0.79	[−4.84]	−0.53	[−4.89]	−0.38	[−5.37]
Constant	5.81	[7.23]	6.89	[7.34]	4.78	[6.95]	1.15	[5.85]
Panel C: Fama-French Six-Factor Model Estimates								
Moneyness	−5.01	[−6.27]	−6.57	[−7.18]	−4.70	[−7.12]	−1.19	[−6.80]
Systematic Volatility	−1.17	[−2.76]	−0.61	[−1.78]	0.34	[1.32]	0.46	[2.23]
Idiosyncratic Volatility	−1.15	[−5.16]	−0.79	[−4.82]	−0.53	[−4.89]	−0.39	[−5.18]
Constant	5.78	[7.21]	6.89	[7.36]	4.80	[6.96]	1.15	[5.88]
Panel D: Augmented q-Theory Model Estimates								
Moneyness	−5.01	[−6.30]	−6.54	[−7.15]	−4.67	[−7.11]	−1.20	[−6.88]
Systematic Volatility	−1.11	[−2.59]	−0.61	[−1.72]	0.40	[1.45]	0.48	[2.30]
Idiosyncratic Volatility	−1.17	[−5.50]	−0.79	[−4.87]	−0.54	[−4.82]	−0.38	[−5.11]
Constant	5.77	[7.23]	6.87	[7.32]	4.75	[6.94]	1.16	[5.92]
Panel E: Mispricing Factor Model Estimates								
Moneyness	−4.82	[−5.83]	−6.15	[−6.15]	−4.24	[−6.01]	−1.19	[−6.31]
Systematic Volatility	−1.05	[−2.23]	−0.53	[−1.36]	0.34	[1.19]	0.50	[2.17]
Idiosyncratic Volatility	−1.14	[−4.85]	−0.72	[−4.01]	−0.45	[−3.90]	−0.36	[−4.59]
Constant	5.56	[6.63]	6.43	[6.29]	4.31	[5.87]	1.13	[5.37]

Table IA.5

Regressions of Call Returns on Systematic and Idiosyncratic Volatility and Stock Characteristic Controls Including the Historical Average Stock Return

The table presents the results from Fama-MacBeth (1973) regressions of the excess call return calculated from start of month t to its end on moneyness, systematic and idiosyncratic underlying-stock volatility, and stock characteristic controls including the historical average daily underlying-stock return measured until end of month $t-1$. We run the regressions separately for calls with a start-of-month-t moneyness below the first quartile (column (1), "DOTM"), between the first and second (column (2), "OTM"), between the second and third (column (3), "ATM"), and above the third (column (4), "ITM"). Moneyness is the ratio of stock price to strike price at the start of month t. We calculate systematic and idiosyncratic volatility using the Fama-French (2016) six-factor model estimated over the 24 months of daily data before the call return period. Market size is a stock's log market capitalization from the most recent prior June; book-to-market is the log ratio of a stock's book value from the fiscal year ending in the prior calendar year to its market capitalization at the end of that calendar year from July of year t to June of year $t+1$; momentum is the stock return compounded over months $t-12$ to $t-2$; historical return is the average daily stock return calculated over months $t-6$ to $t-1$; asset growth is the change in the log asset value from the fiscal year end in calendar year $t-2$ to the fiscal year end in calendar year $t-1$ from July of year t to June of year $t+1$; and profitability is the ratio of sales minus COGS, SG&A, and interest expenses to the book value of equity measured at the end of the fiscal year in calendar year $t-1$ from July of year t to June of year $t+1$. Plain numbers are premium estimates (in decimals), while the numbers in square brackets are Newey and West (1987) t-statistics calculated using a twelve-month lag length.

	DOTM		OTM		ATM		ITM	
	(1)		(2)		(3)		(4)	
Moneyness	−5.90	[−9.71]	−6.40	[−10.76]	−4.04	[−9.56]	−0.75	[−6.46]
Systematic Volatility	−1.52	[−5.72]	−0.95	[−5.07]	0.02	[0.13]	0.19	[1.70]
Idiosyncratic Volatility	−0.89	[−5.22]	−0.22	[−1.98]	−0.13	[−1.50]	−0.01	[−0.17]
Market Size	0.00	[−0.03]	0.04	[3.42]	0.02	[3.32]	0.03	[6.04]
Book-to-Market	0.04	[1.83]	0.03	[2.53]	0.01	[0.90]	0.00	[−0.28]
Historical Return	0.04	[1.60]	0.03	[1.87]	0.02	[1.19]	0.04	[3.27]
Momentum	−0.15	[−4.25]	−0.07	[−2.56]	−0.04	[−1.58]	0.00	[−0.22]
Asset Growth	−0.08	[−1.79]	−0.06	[−2.25]	−0.07	[−3.26]	−0.03	[−2.00]
Profitability	0.08	[2.32]	0.04	[1.54]	0.01	[0.59]	0.00	[−0.37]
Constant	6.56	[11.43]	6.03	[10.28]	3.65	[8.51]	0.06	[0.39]