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Abstract: I analyse the effects of a downstream merger in a differentiated oligopoly when there is bargaining between downstream firms and upstream agents (firms or unions). Bargaining outcomes can be observable or unobservable by rivals. When competition is in quantities, upstream agents are independent and bargaining is over a uniform input price, a merger between downstream firms may raise consumer surplus and overall welfare. However, when competition is in prices *or* the upstream agents are not independent *or* bargaining is over a two-part tariff *or* bargaining covers both the input price and the level of output, the standard welfare results are restored: a downstream merger always reduces consumer surplus and overall welfare.

Keywords: Mergers, bargaining, bilateral oligopoly, welfare.

JEL classification: D43, L13, J50.

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1. Introduction.

Unionised or vertically related industries have recently attracted considerable attention by policymakers, antitrust authorities and economists. Following the seminal work of Horn and Wolinsky (1988), several studies have shown that an increase in buyers' countervailing power or a downstream merger will reduce the prices charged by suppliers, although the welfare effects are less clear. When the downstream firms bargain with a single supplier, von Ungern-Sternberg (1996), Dobson and Waterson (1997) and Chen (2003) have found that countervailing power will benefit consumers only when downstream competition is strong.[1] Allowing for more than one upstream agent, Ziss (1995) has argued that a downstream merger between duopolists will raise output when upstream suppliers set two-part tariffs, while Lommerud et al. (2005, 2006) have shown that certain types of mergers between a subset of downstream firms when uniform input prices are set by upstream agents will reduce input prices and may increase welfare.[2]

The present paper brings together some existing results and also extends the literature in a number of ways. I examine the effects of downstream mergers but unlike previous studies I do not assume that input prices are necessary linear and/or set unilaterally by upstream agents. Instead, I allow for bargaining between

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[1] Fauli-Oller and Sandonis (2007) allow for an alternative source of supply at a cost that is independent of downstream market structure. In this context downstream mergers benefit consumers only if the alternative supplier is relatively inefficient.

[2] There is also a related literature on the effects of upstream mergers in vertically related industries. This again begins with Horn and Wolinsky (1988) and includes Ziss (1995), Chen and Ross (2003), O'Brien and Shaffer (2005), and Milliou and Petrakis (2007). Inderst and Wey (2003, 2007) and Inderst and Shaffer (2007) focus primarily on the effects of mergers in vertically related industries on innovation and product variety.

downstream firms and upstream agents (firm or unions). The importance of bargaining in unionised firms has long been recognised and its importance in vertically related industries is also increasingly being recognised due to rising levels of concentration in many downstream markets. Moreover, I analyse a range of bargaining structures, including bargaining over a uniform input price, over a two-part tariff and over both the input price and output. I also allow downstream competition to be over quantities or prices and I examine a variety of upstream market structures: independent, firm-specific and industry-wide upstream agents.[3] In this context, the bargaining outcome between a downstream firm and its upstream agent can be either observable or unobservable by rivals, and I examine both cases. Finally, I provide a comprehensive analysis of welfare results. A new and important result of the paper is that the welfare effects of a downstream merger are sensitive to the mode of downstream competition and to the bargaining structure, although they are not affected by whether bargaining outcomes are observable by rivals or not.

In the basic version of my model, two downstream firms (or divisions of a merged firm) sell a horizontally differentiated product. Prior to that, each of the two firms bargains with its upstream agent and the bargaining process is represented by the asymmetric Nash bargaining solution.[4] The structure of the

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[3] Lommerud et al. (2005) discuss some of these alternative scenarios, although they do not generally explore their welfare implications.

[4] A well-known and important property of the Nash bargaining solution is that it can be implemented as the outcome of a dynamic non-cooperative alternating-offers bargaining game (Binmore et al. 1986). The use of the Nash bargaining solution is common in models of bargaining between upstream agents and downstream firms. In contrast, Inderst and Wey (2003) apply an alternative

downstream market has been determined before the bargaining stage: a merger is a long-run decision, which is more difficult to change than a bargaining outcome. In common with most of the literature on bilateral oligopoly, I assume that a merger does not generate efficiency gains: this allows me to focus on the implications of the vertical relationships. Also, I do not consider the question of product choice: both varieties of the product are produced after the merger. For most of the paper, bargaining is decentralised, i.e. each downstream firm bargains separately with an upstream agent. This is an obvious modelling choice when the upstream agents are firms. Even in the case of unions, decentralised bargaining has long been predominant in several countries, and a trend toward decentralised bargaining structures has been observed in recent years in many other countries. Moreover, each downstream firm (or, in the case of a full merger, each division of the merged firm) and its upstream agent are locked into bilateral relations. This implies that a downstream firm cannot produce any output in the event of a breakdown in the negotiations with its upstream agent. This is discussed more extensively in the concluding section.

I begin with the case where the downstream firms set quantities, bargaining is over a uniform input price, bargaining outcomes are observable by all and the upstream agents remain independent after the downstream merger and each of them bargains with one division of the merged firm – which implies that the upstream agents are either independent firms or plant-specific unions. The independence of upstream agents is relevant for international mergers, since unions do not usually transcend national borders. It is also relevant for domestic

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bargaining procedure that gives rise to the Shapley value, while de Fonteney and Gans (2005) use a fully specified non-cooperative bargaining model.

mergers when each downstream firm becomes a plant of the merged firm and bargains with a plant-specific union (as may be the case when the products are differentiated) or an independent supplier (since there is no reason for suppliers to merge when buyers merge). Bargaining over a uniform input price is a common assumption in models of bilateral oligopoly and the literature on union-firm bargaining. In this context, consumer surplus and total welfare can be higher or lower under the merger. This result is not new, although my contribution lies in showing exactly how it depends on the degree of product differentiation and the distribution of bargaining power between the parties.

The rest of the paper considers four alternative scenarios: price competition, non-independent upstream agents, two-part tariffs, and bargaining over both input price and output. Furthermore, I check the robustness of the results to the assumption of observability of bargaining outcomes and I also extend the analysis to the case where there are three downstream firms, only two of which are involved in a merger. Price-setting is an obvious alternative mode of downstream competition. Firm-specific upstream agents are relevant for many domestic mergers when negotiations with unions take place at the level of the firm rather than separately for each plant. Industry-wide unions or single upstream sellers are also an institutional feature of many industries. The type of bargaining can also vary across industries: in fact, uniform price contracts are generally inefficient.[5] These changes are introduced one at a time and each has a significant impact on the results. In all cases the standard welfare results of oligopoly theory are

[5] The question of choice of bargaining structure is beyond the scope of this paper. See, for example, Dobson (1997), Petrakis and Vlassis (2000), Milliou et al. (2003).

restored: a downstream merger always reduces consumer surplus and social welfare.

A common and plausible assumption in the literature on union-firm wage bargaining under oligopoly is that bargaining outcomes are made public and are therefore observable by all before the product price or quantity setting-stage. On the other hand, in much of the literature on vertical oligopolies where the upstream agents are firms it is assumed instead that vertical contracts are secret and therefore unobservable by rivals. I examine both cases and I find that my results are the same whether bargaining outcomes are observable or not.

A somewhat restrictive feature of the basic model is that the downstream industry in the absence of a merger is a duopoly. Therefore a downstream merger is a merger to monopoly. This assumption is necessary to ensure that the model remains tractable while at the same time allowing for an arbitrary distribution of bargaining power between upstream agents and downstream firms. In practice, however, nearly all mergers involve only a subset of firms in an industry. To examine the welfare effects of such a merger I use the simplest possible setup: a three-firm downstream oligopoly where two of the firms merge. To ensure tractability of the model in this case I often need to assume a specific distribution of bargaining power between upstream agents and downstream firms. The results I derive are identical to those obtained for the duopoly case.

Some of the themes that I analyse here are also explored in a number of other papers. Ziss (1995) has found that under certain conditions a downstream merger will lead to higher output when upstream suppliers set two-part tariffs in a vertical duopoly. However, there is no bargaining in his model, no analysis of alternative bargaining structures and upstream market structures, and no analysis of the case of unobservable contracts. In fact, my results differ from those obtained by Ziss. Lommerud et al. (2005, 2006) find that a merger between a subset of downstream firms can lead to lower input prices and may increase welfare. However, they do not consider bargaining over input prices in their model and do not analyse two-part tariffs or unobservable input prices. My approach differs from theirs in several ways. I analyse a range of bargaining structures – and I find that the effects of a downstream merger depend on the bargaining structure. I allow for bargaining outcomes to be either observable or unobservable. Finally, I provide a more extensive and systematic analysis of welfare results and emphasise how the conventional welfare results of oligopoly theory are restored under a wide variety of setups for the bilateral industry.

Symeonidis (2008) is a companion paper to this one. However, there are several important differences. First, instead of examining the effect of a downstream merger, the companion paper analyses changes in the intensity of competition among independent downstream firms. Since any merger must occur before the bargaining stage, the downstream firms in the present paper cooperate at the bargaining stage when a merger occurs, while in the companion paper they always act independently. Hence the payoffs at the bargaining stage are different in the two papers, and so are many of the results. For instance, in the case of bargaining over two-part tariffs, the result of Symeonidis (2008) that a decrease in the intensity of downstream competition causes input prices to fall and welfare to increase is completely reversed: the present paper shows that input prices rise and welfare always decreases when downstream firms merge. Second, the companion paper only analyses a duopolistic industry and assumes observable bargaining outcomes, while the present paper relaxes both these assumptions. Third, the companion paper examines a much more restricted range of downstream competition modes, bargaining structures and upstream market structures: quantity-setting downstream firms and independent upstream agents. The main focus of the present paper, on the other hand, is the analysis of the alternative scenarios mentioned above. In all these alternative scenarios, welfare unambiguously decreases with a downstream merger. The conclusion is that the non-standard welfare results of the previous literature in similar settings appear in only one possible case among many. In all the other cases the standard welfare effects of mergers continue to hold when we add upstream firms or unions to the standard downstream duopoly. This conclusion contrasts sharply with most of the existing literature in this area, which has focused primarily on quantity competition and uniform input prices.[6]

The paper is structured as follows. Section 2 examines the benchmark case of two quantity-setting downstream firms, independent upstream agents and bargaining over a uniform input price with observable bargaining outcomes. In sections 3-6 alternative modes of downstream competition, bargaining structures and upstream market structures are analysed. Section 7 examines the case of unobservable bargaining outcomes. The case of a three-firm oligopoly is briefly discussed in the concluding remarks and analysed in detail in the Appendix.

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[6] Three other studies have examined the effects of mergers using a similar framework. Bárcena-Ruiz and Garzón (2000) focus on how a downstream merger affects the choice of organisational form for the merged firm and the unions. Ziss (2001) and González-Maestre and López-Cunat (2001) analyse mergers in a homogeneous Cournot model where each owner delegates output decisions to a manager, a setup that has similarities with a bilateral duopoly. However, they are mainly interested in merger profitability and do not explore the same issues as the present paper.

2. Quantity-setting downstream firms, independent upstream agents and bargaining over a uniform input price.

Consider an industry with two firms, each producing and selling to consumers one variety of a differentiated product. Preferences are described by a standard quadratic utility function of a representative consumer:

$$U = \alpha(x_1 + x_2) - \beta(x_1^2 + x_2^2) - \beta\sigma x_1 x_2 + M. \tag{1}$$

The x_i's are the quantities demanded of the different varieties of the product, while $M = Y - p_1 x_1 - p_2 x_2$ denotes expenditure on outside goods. The parameter σ, $\sigma \in (0,2)$, is an inverse measure of the degree of horizontal product differentiation: in the limit as $\sigma \to 0$ the goods become independent, while in the limit as $\sigma \to 2$ they become perfect substitutes. Finally, α and β are positive scale parameters.

The inverse demand function for variety i is given by

$$p_i = \alpha - 2\beta x_i - \beta\sigma x_j \tag{2}$$

in the region of quantity spaces where prices are positive, and the demand function is

$$x_i = \frac{2(\alpha - p_i) - \sigma(\alpha - p_j)}{\beta(2-\sigma)(2+\sigma)} \tag{3}$$

in the region of prices where quantities are positive. Let firm i have marginal cost of production w_i, where $w_i < \alpha$. In particular, assume that only one input, L, is used in the production of variety i and has a unit price equal to w_i. This input can be labour, in which case w_i is the wage rate; it can be an intermediate product sold by upstream suppliers to downstream manufacturers; or it can be the final product, in which case the downstream firms are distributors. There are constant returns to scale, so $x_i = L_i$.

Competition in the industry is described by a two-stage game as follows.[7] There are two downstream firms, which can be independent or merged; this is known at the beginning of the game. At stage 1, each downstream firm (or division) i forms a bargaining unit with an independent upstream agent and bargains over w_i. Although each bargain is independent, there is also interaction at this stage: the set of w_i that we obtain is the outcome of a non-cooperative Nash equilibrium between the two bargaining units. At stage 2, the downstream firms observe the outcomes of stage 1 and compete in quantities given the values of w_i from stage 1. In what follows I derive the pure strategy subgame-perfect equilibrium of this game.

At the second-stage subgame, firm i chooses x_i to maximise the sum of its own profit and a fraction λ of the profit of its rival: $\Pi_i = \pi_i + \lambda\pi_j$, where

$$\pi_i = (p_i - w_i)x_i = (\alpha - 2\beta x_i - \beta\sigma x_j - w_i)x_i \,. \tag{4}$$

The parameter λ, $\lambda\in[0,1]$, is the degree of cross-ownership, with $\lambda = 0$ corresponding to the Cournot-Nash equilibrium and $\lambda = 1$ corresponding to a full merger. A positive value for λ could also result from a strategic alliance between the downstream firms. The equilibrium values of x_i and p_i in the second-stage subgame as functions of w_i and w_j are:

[7] See also Horn and Wolinsky (1988), Dowrick (1989), Petrakis and Vlassis (2000), Naylor (2002), Correa-Lopez and Naylor (2004), Correa-López (2007). These papers analyse models with a similar structure to the one presented here (i.e. multistage oligopoly games with a bargaining stage followed by a product market competition stage), but none of them examines the welfare effects of downstream mergers.

$$\hat{x}_i = \frac{4(\alpha - w_i) - \sigma(1+\lambda)(\alpha - w_j)}{\beta[4 - \sigma(1+\lambda)][4 + \sigma(1+\lambda)]}$$
$$\hat{p}_i = w_i + \frac{[8 - \sigma^2\lambda(1+\lambda)](\alpha - w_i) - 2\sigma(1-\lambda)(\alpha - w_j)}{[4 - \sigma(1+\lambda)][4 + \sigma(1+\lambda)]} \qquad (5)$$

in the region of w spaces where $\hat{x}_i \geq 0$, $\hat{p}_i \geq 0$, i = 1,2 (this is satisfied as long as w_i and w_j are not too dissimilar – but the case where w_i and w_j are too dissimilar is not relevant as a potential equilibrium of the two-stage game because a bargaining unit would not choose a level of w that would result in zero sales).

At stage 1 of the game, the downstream firm i and the upstream agent i form a bargaining unit and set w_i to maximise the Nash product

$$\Omega_i = [(w_i - w_0)\hat{x}_i]^{\varphi}[(\hat{p}_i - w_i)\hat{x}_i + \lambda(\hat{p}_j - w_j)\hat{x}_j - \lambda(\overline{p}_j - w_j)\overline{x}_j]^{1-\varphi} \quad . \qquad (6)$$

The parameter $\varphi \in [0,1]$ is a measure of the bargaining power of the upstream agent relative to that of the downstream firm. It depends on the relative degrees of impatience and risk aversion of the two parties, so it is taken here as exogenous. The value φ = 1 corresponds to the case where w_i is set by the upstream agent, while φ = 0 corresponds to the case where w_i is set by the downstream firm. The parameter w_o is either the wage that a union would obtain in a competitive labour market or the unit cost of an upstream firm. The utility of the upstream agent is given by $U_i = (w_i - w_0)x_i$. Recall that $x_i = L_i$. So when the upstream agent is a union, it aims to maximise the total rent – or the wage bill if $w_o = 0$. When the upstream agent is a firm, it aims to maximise its profit.

The upstream agent's payoff in the Nash product is its own utility, i.e. any degree of cross-ownership between downstream firms does not affect the independence of the upstream agents. The downstream firm i wishes to maximise

$\Pi_i = \pi_i + \lambda\pi_j$ minus its disagreement payoff. In particular, while $\hat{p}_i$, $\hat{x}_i$, $\hat{p}_j$ and $\hat{x}_j$ are given in equations (5), $\bar{p}_j = w_j + (\alpha - w_j)/2$ and $\bar{x}_j = (\alpha - w_j)/4\beta$ are the price and output of good *j* in case of a breakdown of negotiations between the downstream firm *i* and upstream agent *i*. Note that in case of disagreement within one bargaining unit, the other downstream firm acts as a monopolist and supplies the monopoly quantity.[8]

As pointed out earlier, the values of w_i and w_j that we obtain at stage 1 of the game are the outcome of a non-cooperative Nash equilibrium between the two bargaining units: w_i is the Nash solution to the bargaining problem between downstream firm *i* and its upstream agent given that both expect the input price w_j to be agreed between downstream firm *j* and its upstream agent. In the case of a merger, the merged firm bargains simultaneously and separately with the two upstream agents (see Davidson 1988): one bargain is over product *i*, the other over product *j*. Solving for the equilibrium we obtain:

$$w^* = w_0 + \frac{\varphi[4 - \sigma(1+\lambda)](\alpha - w_0)}{8 - \varphi\sigma(1+\lambda)} \tag{7}$$

and therefore

$$\frac{\partial w^*}{\partial \lambda} = \frac{-4\varphi\sigma(2-\varphi)(\alpha - w_0)}{[8 - \varphi\sigma(1+\lambda)]^2}, \tag{8}$$

which is negative for all $\sigma \in (0,2)$, $\varphi \in (0,1]$.[9]

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[8] While this is a plausible and common assumption, it is not the only one. Another possibility, not examined here, would be for the other firm to operate at the anticipated equilibrium level of production (Horn and Wolinsky 1988).

[9] The second-order condition for a maximum of the Nash product is always satisfied: $\partial^2\Omega_i/\partial w_i^2\,(w_i = w_j = w^*) = -\dfrac{2[8 - \varphi\sigma(1+\lambda)]^2}{\varphi(2-\varphi)[4 - \sigma(1+\lambda)]^2(\alpha - w_0)^2} < 0$.

Proposition 1. *When quantity-setting downstream firms and independent upstream agents bargain over a uniform input price and $\varphi \in (0,1]$, the input price decreases in the degree of cross-ownership λ. For $\varphi = 0$, the input price is independent of λ and equal to w_o.*

This is similar to the result obtained by Lommerud et al. (2005) and Symeonidis (2008) in a somewhat different context.[10] It holds for any values of $\varphi \in (0,1]$, i.e. as long as the upstream agents have some bargaining power. The intuition is as follows. First, output is lower for any given w the higher the value of λ, so upstream agents have less to gain from a unit rise in w. This is one reason why w^* falls as λ rises. Furthermore, an increase in the input price of one product shifts production to the other product and this effect is stronger the higher the value of λ. As a result, each upstream agent loses more output to the other upstream agent from a higher input price for its product the higher the value of λ. This effect is sometimes referred to as an increased level of rivalry between upstream agents, and it could also be described as a form of increased countervailing power of the downstream industry. It is a second reason why w^* falls as λ rises.

Equilibrium consumer surplus, aggregate downstream profit and aggregate upstream utility are, respectively, given as

$$CS^* = 2\alpha x^* - 2\beta(x^*)^2 - \beta\sigma(x^*)^2 - 2p^* x^* \quad (9)$$

$$\Pi^* = 2(p^* - w^*)x^* \quad (10)$$

[10] See also Correa-López and Naylor (2004) for a related argument, and Symeonidis (2000) for an analogous mechanism in the context of a vertical differentiation model.

and

$$U^* = 2(w^* - w_0)x^*, \tag{11}$$

where x^* and p^* are the equilibrium values of x and p in the two-stage game and are given by equations (5) after setting $w_i = w_j = w^*$.

The next result shows that consumer surplus and total welfare may be higher or lower under a downstream merger than when downstream firms are independent, while downstream profit always increases and upstream utility always decreases with the merger.

Proposition 2. *When quantity-setting downstream firms and independent upstream agents bargain over a uniform input price:*

(i) Consumer surplus is higher under a downstream merger than when downstream firms are independent if the products are close substitutes and upstream agents have significant bargaining power. Consumer surplus is lower under a downstream merger than when downstream firms are independent if the products are very differentiated or upstream agents have little bargaining power.

(ii) The aggregate downstream profit increases in the degree of cross-ownership λ *for all* $\lambda \in [0,1)$, $\varphi \in [0,1]$. *The aggregate upstream utility decreases in* λ *for* $\varphi \in (0,1]$. *When* $\varphi = 0$, *the upstream utility is independent of* λ *and equal to zero.*

(iii) Total welfare is higher under a downstream merger than when downstream firms are independent if the products are close substitutes and upstream agents have significant bargaining power. Total welfare is lower under a downstream merger than when downstream firms are independent if the products are very differentiated or upstream agents have little bargaining power.

Proof. See the Appendix.

The intuition for the first part of Proposition 2 is as follows. The total effect of a change in λ on consumer surplus is the sum of a negative direct effect on consumer surplus for any given level of w and an indirect effect working through the change in w. This latter effect is positive (or zero) since $\frac{\partial w^*}{\partial \lambda} \leq 0$ (Proposition 1) and $\frac{\partial CS}{\partial w^*} < 0$. Hence the total effect can be ambiguous. When the products are close substitutes (σ is close to 2) and the upstream agents have significant bargaining power (φ is large), the difference between $w^*(\lambda = 1)$ and $w^*(\lambda = 0)$ is larger and therefore the indirect positive effect of a downstream merger on consumer surplus is stronger. This effect then dominates the direct negative effect of the merger (which is also stronger the higher the value of σ).

A downstream merger has a direct and an indirect effect on downstream profit, with both effects working in the same direction. For any given w, downstream profit increases in λ for all $\lambda \in [0,1)$. Since the equilibrium input price decreases in λ for all $\varphi \in (0,1]$ (and is independent of λ when $\varphi = 0$) and a lower w raises downstream profit (a straightforward result from equations (5)), the indirect effect of λ on downstream profit will reinforce the direct effect (or be equal to zero).

Finally, the effect of a downstream merger on upstream utility can be decomposed into three different effects. First, there is the effect of a change in λ on the equilibrium input price w^*. This effect is negative – or, in a special case, zero. Second, there is the effect of a change in λ on the equilibrium level of output x^*. This is also negative, since output is lower the higher the value of λ for any given level of w. Third, the indirect effect of a change in λ on x^* that works through the change in w. Since we have $\frac{\partial w^*}{\partial \lambda} \leq 0$ and $\frac{\partial x^*}{\partial w^*}$, this is positive or zero. However, this third effect is a second-order one, and $\partial U^*/\partial\lambda$ is negative in the present model.

3. Price-setting downstream firms.

When the downstream firms set prices in the second-stage subgame, firm i chooses p_i to maximise $\Pi_i = \pi_i + \lambda\pi_j$, where

$$\pi_i = (p_i - w_i)x_i = \frac{(p_i - w_i)\left[2(\alpha - p_i) - \sigma(\alpha - p_j)\right]}{\beta(2-\sigma)(2+\sigma)}. \tag{12}$$

The equilibrium values of p_i and x_i in the second-stage subgame are:

$$\hat{p}_i = w_i + \frac{\left[8 - \sigma^2(1+\lambda)\right](\alpha - w_i) - 2\sigma(1-\lambda)(\alpha - w_j)}{\left[4 - \sigma(1+\lambda)\right]\left[4 + \sigma(1+\lambda)\right]}$$

$$\hat{x}_i = \frac{2\left[8 - \sigma^2(1+\lambda^2)\right](\alpha - w_i) - \sigma(4 - \sigma^2\lambda)(1+\lambda)(\alpha - w_j)}{\beta(2-\sigma)(2+\sigma)\left[4 - \sigma(1+\lambda)\right]\left[4 + \sigma(1+\lambda)\right]} \tag{13}$$

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in the region of w spaces where $\hat{x}_i \geq 0$, $\hat{p}_i \geq 0$, $i = 1,2$ (this is satisfied as long as w_i and w_j are not too dissimilar – but again this is the only relevant case).

At stage 1 of the game, the downstream firm i and the upstream agent i set w_i to maximise the Nash product given by equation (6), where $\hat{p}_i$, $\hat{x}_i$, $\hat{p}_j$ and $\hat{x}_j$ are now given in (13) and $\bar{p}_j$ and $\bar{x}_j$ are as previously defined. Once again w_i and w_j are the outcome of a non-cooperative Nash equilibrium between the two bargaining units. In the case of a merger, the merged firm bargains simultaneously and separately with the upstream agents. Solving for the equilibrium and evaluating the resulting expression for different values of λ we obtain:

$$w^P\Big|_{\lambda=0} = w_0 + \frac{\varphi(8 - 2\sigma - \sigma^2)(\alpha - w_0)}{2(8 - \varphi\sigma - \sigma^2)} \tag{14}$$

$$w^P\Big|_{\lambda=1} = w_0 + \frac{\varphi(2-\sigma)(\alpha - w_0)}{4-\varphi\sigma} \quad (15)$$

and

$$w^P\Big|_{\lambda=1} - w^P\Big|_{\lambda=0} = -\frac{(2-\varphi)\varphi\sigma(4-\sigma^2)(\alpha - w_0)}{2(4-\varphi\sigma)(8-\varphi\sigma-\sigma^2)}, \quad (16)$$

which is negative for all $\sigma\in(0,2)$, $\varphi\in(0,1]$.[11]

Proposition 3. *When price-setting downstream firms and independent upstream agents bargain over a uniform input price and $\varphi\in(0,1]$, the input price is lower under a downstream merger than when downstream firms are independent. For $\varphi = 0$, the input price is constant and equal to w_o.*

Although Proposition 3 mirrors Proposition 1, and the intuition is similar in the two cases, the welfare results are different.

Proposition 4. *When price-setting downstream firms and independent upstream agents bargain over a uniform input price:*

(i) Consumer surplus is lower under a downstream merger than when downstream firms are independent.

(ii) The aggregate downstream profit is higher under a downstream merger than when downstream firms are independent. The aggregate upstream utility is lower under a downstream merger than when downstream firms are independent for $\varphi\in(0,1]$. For $\varphi = 0$, the upstream utility is constant and equal to zero.

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[11] The second-order condition for a maximum of the Nash product is always satisfied:

$$\partial^2\Omega_i \big/ \partial w_i^2\,(\lambda = 0, w_i = w_j = w^*) = -\frac{8(8-\varphi\sigma-\sigma^2)^2}{\varphi(2-\varphi)(2-\sigma)^2(4+\sigma)^2(\alpha-w_0)^2} < 0,$$

$$\partial^2\Omega_i \big/ \partial w_i^2\,(\lambda = 1, w_i = w_j = w^*) = -\frac{2(4-\varphi\sigma)^2}{\varphi(2-\varphi)(2-\sigma)^2(\alpha-w_0)^2} < 0.$$

(iii) Total welfare is lower under a downstream merger than when downstream firms are independent.

Proof. See the Appendix.

As in the quantity-setting case, a merger has a direct negative effect on consumer surplus and an indirect positive effect working through the reduction in w. However, downstream competition is relaxed more by a merger between duopolists under price-setting than under quantity-setting, so the direct negative effect on consumer surplus is stronger. This is one reason why the direct effect always dominates the indirect effect when the downstream firms set prices.

If this were the only reason, then the differences between the quantity-setting case and the price-setting case might be limited to the context of a downstream duopoly. However, as will be shown in the Appendix, this is not the case: these differences persist when two firms merge in a three-firm oligopoly. There is, in fact, also a second mechanism driving Proposition 6: the indirect positive effect of a merger on consumer surplus working through the fall in w is weaker when firms set prices (especially when σ is not close to 0 and φ is large) because the fall in w brought about by the merger is then smaller than in the quantity-setting case. To understand why this is so, note that for $\lambda = 1$ the equilibrium input price is independent of the short-term choice variable – price or quantity. When $\lambda = 0$, however, the input price is lower in the price-setting case than in the quantity-setting case: the incentive of a bargaining unit to set a low input price is stronger in the former case than in the latter because the anticipated downstream competition is tougher – and this is even more so the higher the values of σ and φ.[12]

A downstream merger has a direct and an indirect effect on downstream profit, with both effects working in the same direction. For any given input price, downstream profit increases after a merger. Moreover, because w decreases (or, if $\varphi = 0$, it does not change) and a lower w raises downstream profit (a straightforward result using equations (13)), the indirect effect is also positive (or zero).

The effect of a downstream merger on upstream utility can be decomposed into three different effects that work in opposite directions (see section 2). The negative direct effect of a merger on output for any given w is stronger than in the quantity-setting case because the difference between the Nash equilibrium and the monopoly output is larger under price-setting. On the other hand, the negative effect of a merger on the equilibrium input price is weaker than in the quantity-setting case and this also implies that the indirect effect of a merger on output that works through the change in w is also weaker. So the balance of these three different effects is not significantly changed when we replace quantity setting by

[12] Note that this result is not driven by strategic effects and the distinction between strategic substitutes and strategic complements. The strategic effects – an increase in the input price under price competition and a decrease in the input price under quantity competition relative to the case of no strategic effects – are present but they are relatively small: a comparison of the results of this section with those of section 7 below suggests that, as expected, the input price in lower when it is observable than when it is unobservable if downstream firms set quantities and the reverse is the case when downstream firms set prices. This also implies that the difference between the input price under quantity setting and that under price setting for $\lambda = 0$ is even greater when input prices are unobservable.

price setting, and it is therefore not very surprising that a downstream merger again reduces upstream utility – or, in a special case, does not affect it.

To summarise, although a downstream merger reduces the input price when downstream firms set prices, the direct effects on consumer surplus and total welfare are relatively strong and the indirect effects relatively weak. Therefore the former always dominate the latter – unlike the quantity-setting case – and the standard welfare results are restored: the merger always reduces consumer surplus and welfare.

4. Firm-specific or industry-wide upstream agents.

In this section I return to the case of quantity-setting firms in order to examine the implications of alternative upstream market structures: firm-specific or industry-wide upstream agents. I will focus on the former case because the latter is straightforward and has been examined in previous work on centralised bargaining. Thus it is known that the competitive regime facing downstream firms has no effect on the bargaining outcome under fairly general conditions when firms participate in centralised bargaining before competing in the downstream market (Dowrick 1989, Dhillon and Petrakis 2002). This is also the case here: the input price is the same whether the downstream firms merge or not. Since there are no indirect welfare effects of a downstream merger, the conventional welfare results apply.

When the upstream agents are firm-specific, they also merge when the downstream firms merge, a case that can be relevant for many domestic mergers when the upstream agents are unions. To assess the effects of a downstream merger in this case, the relevant comparison is between decentralised bargaining with downstream Cournot competition, on the one hand, and centralised bargaining with downstream monopoly, on the other. In the former case, the equilibrium input price and welfare expressions are the same as those obtained in section 2, after setting λ = 0. In the latter case, the second-stage equilibrium outcome is given by equations (5) after setting λ = 1, while at stage 1 the merged downstream firm and the single upstream agent set w_M so as to maximise the Nash product

$$\Omega = \left[(w_M - w_0)(\hat{x}_{iM} + \hat{x}_{jM})\right]^{\varphi}\left[(\hat{p}_{iM} - w_M)\hat{x}_{iM} + (\hat{p}_{jM} - w_M)\hat{x}_{jM}\right]^{1-\varphi} \quad . \tag{17}$$

Here $\hat{p}_{iM}$, $\hat{x}_{iM}$, $\hat{p}_{jM}$ and $\hat{x}_{jM}$ are the monopoly prices and quantities and are given in equations (5) for $\lambda = 1$.

The equilibrium input price is

$$w_M{}^* = w_0 + \frac{\varphi(\alpha - w_0)}{2} \tag{18}$$

and the difference between this and the input price in the absence of a merger is:

$$w_M{}^* - w^*\Big|_{\lambda=0} = \frac{(2-\varphi)\varphi\sigma(\alpha - w_0)}{2(8-\varphi\sigma)} > 0\,. \tag{19}$$

Proposition 5. *When quantity-setting downstream firms and firm-specific upstream agents bargain over a uniform input price and $\varphi\in(0,1]$, the input price is higher under a downstream merger than when downstream firms are independent. For $\varphi = 0$, the input price is independent of λ and equal to w_o.*

This is the opposite of the result obtained for the case of independent upstream agents, but it is not surprising (see also Lommerud et al. 2005). When the upstream agents also merge, the rivalry between them is eliminated and this causes the input price to rise.

The effect of a merger on consumer surplus is unambiguously negative: both the direct and the indirect effect on consumer surplus work in the same direction. The most interesting aspect of a merger with firm-specific upstream agents is its effect on downstream profit and upstream utility.[13]

Proposition 6. *When quantity-setting downstream firms and firm-specific upstream agents bargain over a uniform input price:*

(i) Consumer surplus is lower under a downstream merger than when downstream firms are independent.

(ii) The downstream profit and the upstream utility can be higher or lower under a downstream merger than when downstream firms are independent: if $\varphi <$ $\tilde{\varphi}_\Pi(\sigma) = \frac{8(4+\sigma-\sqrt{4+2\sigma})}{\sigma(4+\sigma)}$, *a downstream merger raises downstream profit and upstream utility; if* $\tilde{\varphi}_\Pi(\sigma) < \varphi < \tilde{\varphi}_U(\sigma) = \frac{8(4+\sigma)-4\sqrt{2(32+16\sigma-2\sigma^2-\sigma^3)}}{\sigma(4+\sigma)}$, *a downstream merger reduces downstream profit and raises upstream utility; and if* $\varphi > \tilde{\varphi}_U(\sigma)$, *a downstream merger reduces downstream profit and upstream utility.*

(iii) Total welfare is lower under a downstream merger than when downstream firms are independent.

Proof. See the Appendix.

The total effect of a merger on downstream profit can be decomposed into two effects. First, a direct effect: the merger restricts output and so raises downstream profit for any given value of *w*. Second, an indirect effect: the merger raises *w* and therefore reduces downstream profit. The higher the value of φ, i.e. the greater the bargaining power of the upstream agents, the greater the difference $w_M^* - w^*\big|_{\lambda=0}$, i.e.

[13] Lommerud et al. (2005) have also identified an ambiguous effect of a downstream merger on downstream profit when upstream agents are firm-specific. However, there

the more w rises following a merger, and therefore the more likely for the indirect effect to dominate and for downstream profit to fall.

The total effect of a merger on upstream utility can be decomposed into three effects. First, the merger restricts output and so reduces upstream utility for any given value of w. Second, it increases w and therefore increases upstream utility. Third, the increase in w has an indirect negative effect on output, and therefore also on upstream utility. An increase in φ strengthens these effects and also shifts the balance between them so that upstream utility is more likely to fall after the merger. Note that the positive direct effect of w on upstream utility helps explain why $\tilde{\varphi}_{\Pi}(\sigma) < \tilde{\varphi}_{U}(\sigma)$: downstream profit falls but upstream utility rises after a merger when φ takes intermediate values.

To summarise, a downstream merger increases the input price when upstream agents are firm-specific, and so it reduces consumer surplus and welfare. Downstream profit and upstream utility can be higher or lower after the merger: they will be higher if upstream agents have low bargaining power, lower if upstream agents have significant bargaining power, and there exists a range of intermediate values of φ for which the merger reduces downstream profit and raises upstream utility.

5. Bargaining over two-part tariffs.

The assumption that input prices are linear tariffs may be somewhat restrictive, especially when the upstream agents are firms, given that uniform price contracts are inefficient and upstream firms are less constrained than unions by institutional factors when specifying a contract with downstream firms. Of course, uniform

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is no bargaining in their model, so the mechanism I describe is different from theirs.

price contracts are often observed in practice.[14] Still, one would want to analyse how the results might change under non-linear price contracts between upstream agents and downstream firms. Although this analysis may be more relevant when the upstream agents are firms (especially when there are close relationships between downstream firms and upstream suppliers, which is the case examined in this paper), it is also possible to interpret this case as a union-firm bargain, where there is a lump-sum payment to the union or a non-monetary benefit such as an improvement in working conditions which has a monetary equivalent.[15]

In this section I modify the model of section 2 to allow for bargaining over two-part tariffs. Again I allow for different degrees of cross-ownership in the downstream market. The own profit of downstream firm i is now given by $\pi_i = (p_i - w_i)x_i - F_i$, where $F_i \geq 0$ is a lump sum transfer from downstream firm i to its upstream agent.[16] Stage 2 of the two-stage game is as in section 2, the only difference being that now the downstream firms compete in quantities given the unit input prices and fixed fees set at stage 1. At stage 1, each downstream firm i bargains over w_i and F_i with an independent upstream agent. If $\lambda = 1$, the merged

[14] See, for instance, Smith and Thanassoulis (2006). On the other hand, Villas-Boas (2007) and Bonnet et al. (2006) report evidence consistent with the use of non-linear contracts.

[15] There is one difficulty with the interpretation in terms of a lump-sum payment to union members: the derived equilibrium yields a wage lower than the reservation wage, which seems implausible. Note, however, that this is due to quantity-setting by downstream firms. I focus here on the quantity-setting case in order to identify in a clear way the effect of bargaining over two-part tariffs relative to the benchmark case of bargaining over linear tariffs examined in section 2.

firm bargains simultaneously with the two upstream agents. The values of w_i and F_i are chosen to maximise

$$\Omega_i = \left[(w_i - w_0)\hat{x}_i + F_i\right]^{\varphi} \times \left[(\hat{p}_i - w_i)\hat{x}_i - F_i + \lambda\left((\hat{p}_j - w_j)\hat{x}_j - F_j\right) - \lambda\left((\bar{p}_j - w_j)\bar{x}_j - F_j\right)\right]^{1-\varphi}, \tag{20}$$

taking as given the values of w_j and F_j – that is, w_i, w_j, F_i and F_j are the outcome of a non-cooperative Nash equilibrium between the bargaining units.

In this context, there are two instruments at the disposal of downstream firms and upstream agents. Hence w_i will be chosen to maximise the sum of the utility of the upstream agent *i* and the second-stage downstream profit $\Pi_i = \pi_i + \lambda\pi_j$ minus the disagreement payoff, while the fixed fee will be determined by the respective bargaining power of the parties. We obtain:

$$w^{**} = w_0 - \frac{\sigma^2(1-\lambda)(1+\lambda)^2(\alpha - w_0)}{16 + 4\sigma(1+\lambda) - \sigma^2(1-\lambda)(1+\lambda)^2} \tag{21}$$

$$F^{**} = \frac{2\left[16\varphi + 2\sigma^2(1-\lambda)(1+\lambda)^2 - \varphi\sigma^2(2-\lambda)(1+\lambda)^2\right](\alpha - w_0)^2}{\beta\left[16 + 4\sigma(1+\lambda) - \sigma^2(1-\lambda)(1+\lambda)^2\right]^2} \tag{22}$$

and

$$w^{**}\Big|_{\lambda=1} - w^{**}\Big|_{\lambda=0} = \frac{\sigma^2(\alpha - w_0)}{16 + 4\sigma - \sigma^2} > 0 \tag{23}$$

$$F^{**}\Big|_{\lambda=1} - F^{**}\Big|_{\lambda=0} = -\frac{\sigma\left[32\sigma(2+\sigma) + \varphi(256 + 96\sigma - 32\sigma^2 - 10\sigma^3 + \sigma^4)\right](\alpha - w_0)^2}{8\beta(2+\sigma)(16 + 4\sigma - \sigma^2)^2} < 0 \tag{24}$$

for all $\sigma \in (0,2)$ and $\varphi \in [0,1]$.

[16] There are similarities between the two-part tariff case examined here and the literature on managerial incentives in oligopoly (see Fershtman and Judd 1987, Sklivas 1987).

Proposition 7. *When quantity-setting downstream firms and independent upstream agents bargain over two-part tariffs, the unit input price is higher and the fixed fee lower under a downstream merger than when downstream firms are independent.*

This is the opposite of the result obtained for the case of linear tariffs and also the opposite of the result obtained in Symeonidis (2008) for two-part tariffs in a context where a change in the intensity of downstream competition did not involve a merger. As already pointed out above, an important difference between the present model and Symeonidis (2008) is what downstream firms wish to maximise at the bargaining stage: in the companion paper, it is their own second-stage profit, π_i, while in the present model it is their overall second-stage profit, $\pi_i + \lambda\pi_j$, minus a disagreement payoff. Proposition 7 may seem counterintuitive: upstream agents are still independent here, so the mechanisms described earlier to provide intuition for Proposition 1 still apply. There is, however, an additional mechanism.[17]

First, note that under two-part tariffs the unit input price is set below w_0 when independent downstream firms set quantities: each upstream agent is effectively subsidising the downstream firm and using the fixed fee to compensate for this subsidy. When the downstream firms merge, the unit input price is set equal to w_0. It follows that a merger causes w to increase. Why is this not the case in Symeonidis (2008)? A decrease in w_i leads to a decrease in the output of product j. This implies a decrease in the subsidy provided by upstream agent j to the downstream firm j. Under a downstream merger this effect is internalised, but with independent downstream firms it is not (even when they may effectively

[17] See Milliou and Petrakis (2007) for an analogous argument in the context of upstream mergers in a bilateral duopoly.

collude in the final stage, as in Symeonidis 2008). So a division of the merged downstream firm is less keen to push for a reduction in the unit input price below w_0 than an independent downstream firm. It turns out that this effect, which is absent both in the linear tariff case and in Symeonidis (2008), dominates when the parties choose the level of w. Furthermore, since the fixed fee F is used to transfer profit from the downstream firm to the upstream agent, F is lower when w is higher and vice versa.

Consumer surplus, downstream profit and upstream utility are given as

$$CS^{**} = 2\alpha x^{**} - 2\beta(x^{**})^2 - \beta\sigma(x^{**})^2 - 2p^{**}x^{**} \tag{25}$$

$$\Pi^{**} = 2(p^{**} - w^{**})x^{**} - 2F^{**} \tag{26}$$

and

$$U^{**} = 2(w^{**} - w_0)x^{**} + 2F^{**}, \tag{27}$$

where p^{**} and x^{**} are given by equations (5) after setting $w_i = w_j = w^{**}$.

Note that consumer surplus and total welfare are independent of F^{**} and φ. This is because (i) changes in fixed costs have no effect on marginal costs or quantities produced at equilibrium, and (ii) marginal costs are independent of the relative bargaining power of upstream agents and downstream firms because the use of two-part tariffs leads to joint profit maximisation by each bargaining unit.

Since a merger between downstream firms increases the unit input price, it is not surprising that the effect on consumer surplus is standard. Downstream profit and upstream utility could move in either direction, since F falls while w rises, but these effects are also unambiguous in the present model.

Proposition 8. *When quantity-setting downstream firms and independent upstream agents bargain over two-part tariffs and $\varphi\in(0,1]$: (i) downstream profit increases* *in the degree of cross-ownership λ, and (ii) consumer surplus, upstream utility and total welfare decrease in λ.*

Proof. See the Appendix.

When $\varphi = 0$, U^{**} is independent of λ and all the other results are unchanged.

In summary, the welfare results are different under two-part tariffs than under linear tariffs when downstream firms set quantities. The main reason is that a downstream merger raises the bargained input price under a two-part tariff but reduces it under a linear tariff.[18]

6. Bargaining over the input price and the level of output.

I comment here briefly on the effects of a downstream merger when not only the input price but also output (or employment) is determined through bargaining. This setup is a plausible alternative to the model of section 2 when the upstream agents are unions, and it is also relevant for markets where downstream firms obtain their inputs from upstream suppliers under general non-linear contracts.

The profit functions are the same as in section 2, but the game is now a one-stage game between bargaining units: each unit i decides on w_i and x_i taking as given the values of w_j and x_j. As has been pointed out in previous work on 'efficient' bargaining, the bargaining units essentially compete by setting quantities with marginal costs equal to w_0 and w is set to share the surplus between the parties according to their respective bargaining power. This is also what happens in the

[18] When the downstream firms are independent and set prices and bargaining is over two-part tariffs, the unit input price is set above w_0. So a downstream merger reduces the bargained input price in this case (to w_0). However, the welfare effects of a downstream merger are again standard. As in section 3 above, this is due to strong direct effects and weak indirect effects of a merger on consumer surplus and welfare.

present model: the downstream merger reduces output, and for reasons similar to those described in section 2 it reduces w as well. But since output does not depend on w, there is no indirect effect on consumer surplus, so this unambiguously falls. The reduction in output and the fall in w cause downstream profit to rise and (in the absence of an indirect effect on output) upstream utility to fall. Total welfare falls.

7. Unobservable bargaining outcomes.

A common feature of all the variants of the model analysed in sections 2-5 is that bargaining outcomes are observable by rivals before the price or quantity setting-stage. This is a plausible assumption when the upstream agents are unions. On the other hand, when the upstream agents are firms, vertical contracts may be secret in some markets.[19] Observable contracts have a strategic commitment value: by agreeing on a low input price, for instance, the downstream firm commits to be tough. This strategic commitment is no longer possible under secret contracting. In this section I will assume that the bargaining outcome between downstream firm i and its upstream agent is not made public and is therefore not observable by

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[19] Much of the literature on secret vertical contracts examines settings where a single upstream manufacturer sells to many retailers (Hart and Tirole 1990, O'Brien and Shaffer 1992, McAfee and Schwartz 1993, Rey and Tirole 2007 – the last of these also discusses the case where more than one upstream firms sell to a single retailer). Fumagalli and Motta (2001) compare the effects of upstream and downstream mergers in a vertical duopoly with bargaining over two-part tariffs. Nocke and White (2007) examine the impact of vertical integration on upstream collusion in a setting where each of a number of upstream firms sells a product to many downstream firms and contracts can be observable or unobservable. In all these papers, the contracts offered in equilibrium depend on the nature of the downstream firms' out-of-equilibrium beliefs. Since in my model each upstream firm bargains with only one downstream firm, out-of-equilibrium beliefs do not play any role.

downstream firm j and its upstream agent. Of course, the secrecy of vertical contracts only applies when $\lambda \neq 1$; for $\lambda = 1$, the merged firm sets quantities or prices knowing both w_i and w_j.

I begin with the case of quantity-setting downstream firms, independent upstream agents and bargaining over a uniform input price. The structure of the game is the same as in section 2, except that bargaining outcomes are not made public. The inverse demand function for variety i is again given by (2) and the demand function by equation (3). To ensure tractability of the model we focus on a comparison of the Cournot-Nash case ($\lambda = 0$) with the full merger case ($\lambda = 1$). In the latter case, the equilibrium of the two-stage game is derived as in section 2. Under Cournot-Nash behaviour, the equilibrium is determined as follows (see also Rey and Stiglitz 1995, Irmen 1998). Quantities at the second-stage subgame respond only to changes in the own input price according to the downstream reaction functions

$$x_i = R_i(w_i, x_j) = \frac{\alpha - w_i - \beta\sigma x_j}{4\beta}. \tag{28}$$

Furthermore, the equilibrium input price w^* is the outcome of a non-cooperative Nash equilibrium between the two bargaining units, with w_i set to maximise

$$\Omega_i = \left[(w_i - w_0)R_i(w_i, x_j)\right]^{\varphi}\left[(p_i - w_i)R_i(w_i, x_j)\right]^{1-\varphi} \quad , \tag{29}$$

where $p_i = p_i\left[R_i(w_i, x_j), x_j\right]$ is given by the inverse demand function. The equilibrium quantities are then derived by solving the system of the two second-stage reaction functions given w^*.

The equilibrium input price is:

$$w^*\Big|_{\lambda=0} = w_0 + \frac{4\varphi(\alpha - w_0)}{8 + 2\sigma - \varphi\sigma} \tag{30}$$

and hence

$$w^*\Big|_{\lambda=1} - w^*\Big|_{\lambda=0} = -\frac{(2-\varphi)\varphi\sigma(2+\sigma)(\alpha - w_0)}{(4-\varphi\sigma)(8+2\sigma-\varphi\sigma)}. \tag{31}$$

Thus a downstream merger between firms *i* and *j* reduces *w**.

Consumer surplus, aggregate downstream profit and aggregate upstream utility are given by (9)-(11), where x_i^*, x_j^*, p_i^* and p_j^* are the new equilibrium values of *x* and *p* in the two-stage game. Straightforward calculations yield

$$CS^*\Big|_{\lambda=1} - CS^*\Big|_{\lambda=0} = \frac{(2-\varphi)^2\sigma(-2+\varphi+\varphi\sigma)(16+6\sigma-3\varphi\sigma-\varphi\sigma^2)(\alpha-w_0)^2}{\beta(2+\sigma)(4-\varphi\sigma)^2(8+2\sigma-\varphi\sigma)^2}, \tag{32}$$

which is positive when both σ and φ are large enough and negative when either σ or φ is small. The effect of a merger on downstream profit is always positive (since *w** falls) while its effect on upstream utility is always negative:

$$U^*\Big|_{\lambda=1} - U^*\Big|_{\lambda=0} = -\frac{(2-\varphi)\varphi\sigma\left[16(2-\varphi\sigma)+\varphi^2\sigma(6+5\sigma)+4(1-\varphi)(8+6\sigma+\sigma^2)\right](\alpha-w_0)^2}{\beta(2+\sigma)(4-\varphi\sigma)^2(8+2\sigma-\varphi\sigma)^2} < 0. \tag{33}$$

Finally, for total welfare we have:

$$\Delta W^* = W^*\Big|_{\lambda=1} - W^*\Big|_{\lambda=0} = \frac{(2-\varphi)(6+\varphi-2\varphi\sigma)(\alpha-w_0)^2}{\beta(2+\sigma)(4-\varphi\sigma)^2} - \frac{(2-\varphi)(12+2\sigma+2\varphi-\varphi\sigma)(\alpha-w_0)^2}{\beta(8+2\sigma-\varphi\sigma)^2}, \tag{34}$$

which is positive when both σ and φ are large enough and negative when either σ or φ is small.[20] To sum up, all the results are the same as in the case of observable contracts, and so are the mechanisms that drive the results.

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[20] It is easy to check that $\Delta W^*(\sigma = 2) = \frac{(-20+28\varphi+3\varphi^2)(\alpha-w_0)^2}{16\beta(6-\varphi)^2} > 0$ if and only if φ is large enough. $\Delta W^*(\sigma = 0) = 0$ and $\partial \Delta W^*/\partial\sigma$ at $\sigma = 0$ is given by $-\frac{(2-\varphi)(4-\varphi)^2(\alpha-w_0)^2}{128\beta} < 0.$

Next, I consider price-setting in the final stage. For λ = 1, the equilibrium of the two-stage game is derived as in section 3. Under Bertrand-Nash behaviour, prices at the second-stage subgame respond only to changes in the own input price according to the downstream reaction functions

$$p_i = R_i(w_i, p_j) = \frac{\alpha(2-\sigma) + 2w_i + \sigma p_j}{4}. \tag{35}$$

The equilibrium input price w^P is the outcome of a non-cooperative Nash equilibrium between the bargaining units, with w_i set to maximise

$$\Omega_i = \left[(w_i - w_0)x_i\right]^{\varphi}\left[R_i(w_i, p_j)x_i - w_i x_i\right]^{1-\varphi} \quad , \tag{36}$$

where $x_i = x_i\left[R_i(w_i, p_j), p_j\right]$ is given by the demand function. The equilibrium quantities are then derived by solving the system of the second-stage reaction functions given w^P.

The equilibrium input price is:

$$\left. w^P \right|_{\lambda=0} = w_0 + \frac{2\varphi(2-\sigma)(\alpha - w_0)}{8 - 2\sigma - \varphi\sigma} \tag{37}$$

and hence

$$\left. w^P \right|_{\lambda=1} - \left. w^P \right|_{\lambda=0} = -\frac{(2-\varphi)\varphi\sigma(2-\sigma)(\alpha - w_0)}{(4-\varphi\sigma)(8-2\sigma-\varphi\sigma)} < 0. \tag{38}$$

Consumer surplus, aggregate downstream profit and aggregate upstream utility are given by (9)-(11), where x_i*, x_j*, p_i* and p_j* are the new equilibrium values of x and p in the two-stage game with price-setting firms. The effect of a merger on downstream profit is always positive (since w^P falls). Furthermore, we obtain:

$$\left. CS^P \right|_{\lambda=1} - \left. CS^P \right|_{\lambda=0} = -\frac{(2-\varphi)^3\sigma(16-2\sigma-3\varphi\sigma)(\alpha - w_0)^2}{\beta(2+\sigma)(4-\varphi\sigma)^2(8-2\sigma-\varphi\sigma)^2} < 0 \tag{39}$$

$$U^P\Big|_{\lambda=1} - U^P\Big|_{\lambda=0} = -\frac{2(2-\varphi)^2\varphi\sigma(16-2\sigma-3\varphi\sigma)(\alpha-w_0)^2}{\beta(2+\sigma)(4-\varphi\sigma)^2(8-2\sigma-\varphi\sigma)^2} < 0 \tag{40}$$

and

$$W^P\Big|_{\lambda=1} - W^P\Big|_{\lambda=0} = -\frac{(2-\varphi)^2\sigma\left[(2-\sigma)(12+12\varphi(1-\varphi)-4\varphi\sigma-2\varphi^2\sigma)+8(1-\varphi)+2\varphi^2\right](\alpha-w_0)^2}{\beta(2+\sigma)(4-\varphi\sigma)^2(8-2\sigma-\varphi\sigma)^2} < 0. \tag{41}$$

Once again, all the results are the same as in the case of observable contracts.

When the upstream agents are firm-specific rather than independent and downstream firms set quantities, the analysis is the same as the quantity-setting case above for $\lambda = 0$, while for $\lambda = 1$ the results are those derived in section 4. It is straightforward to obtain:

$$w_M{*} - w{*}\Big|_{\lambda=0} = \frac{(2-\varphi)\varphi\sigma(\alpha-w_0)}{2[8+\sigma(2-\varphi)]} > 0. \tag{42}$$

Since the input price increases following a merger, consumer surplus falls. Moreover:

$$\Pi_M{*} - \Pi{*}\Big|_{\lambda=0} = \frac{(2-\varphi)^2\sigma\ \left[-16\varphi+\sigma(2-\varphi)^2\right]\ (\alpha-w_0)^2}{8\beta(2+\sigma)[8+\sigma(2-\varphi)]^2} \tag{43}$$

and

$$U_M{*} - U{*}\Big|_{\lambda=0} = \frac{(2-\varphi)\varphi\sigma\ \left[-16\varphi+\sigma(2-\varphi)^2\right]\ (\alpha-w_0)^2}{4\beta(2+\sigma)[8+\sigma(2-\varphi)]^2}. \tag{44}$$

In other words, a merger reduces aggregate downstream profit and upstream utility when φ is large enough, and increases them when φ is small. Finally:

$$W_M{*} - W{*}\Big|_{\lambda=0} = \frac{(2-\varphi)\sigma(2+\varphi)^2\left[-16-\sigma(2-\varphi)\right]\ (\alpha-w_0)^2}{16\beta(2+\sigma)[8+\sigma(2-\varphi)]^2} < 0. \tag{45}$$

In this case too, then, all the results – and the mechanisms that drive them – are the same as when contracts are observable.

The final case to examine is the case of bargaining over two-part tariffs between downstream firms and independent upstream agents (see also Fumagalli and Motta 2001). For $\lambda = 1$, the equilibrium of the two-stage game is derived as in section 5. Under Cournot-Nash behaviour, the equilibrium is determined as in the quantity-setting case with bargaining over a uniform secret input price except that two instruments, w_i and F_i, are now available to maximise the Nash product

$$\Omega_i = \left[(w_i - w_0)R_i(w_i, x_j) + F_i\right]^{\varphi}\left[(p_i - w_i)R_i(w_i, x_j) - F_i\right]^{1-\varphi} \quad , \tag{46}$$

where $p_i = p_i\left[R_i(w_i, x_j), x_j\right]$ is given by the inverse demand function. It is easy to show that

$$w^{**}\Big|_{\lambda=0} = w^{**}\Big|_{\lambda=1} = w_0 \tag{47}$$

$$F^{**}\Big|_{\lambda=1} - F^{**}\Big|_{\lambda=0} = -\frac{\varphi\sigma(16 + 6\sigma + \sigma^2)(\alpha - w_0)^2}{8\beta(2+\sigma)(4+\sigma)^2} < 0. \tag{48}$$

Thus a downstream merger has no effect on the input price, although it reduces the equilibrium fixed fee, as expected. The absence of an indirect effect of the merger on consumer surplus, downstream profit and upstream utility working through the input price implies that all the welfare results are standard and similar to those for observable contracts.

8. Concluding remarks.

I have analysed the welfare effects of a downstream merger when there is bargaining between downstream firms and upstream agents. There was no scope for innovation or efficiency gains in the model, so the focus was on static welfare results and the implications of the vertical relationships. I have first examined under what circumstances a downstream merger between duopolists may have unexpected welfare implications such as an increase in consumer surplus and welfare. I have then focused on showing how each of a number of changes in the benchmark model – in the mode of downstream competition, the bargaining structure or the upstream market structure – restores the standard negative welfare effects of mergers. Finally, I have shown that my results are robust to the bargaining outcomes being unobservable.

The results are also robust to the downstream market structure being a three-firm oligopoly where two of the firms merge, as I show in the Appendix. Modelling the downstream industry in the absence of a merger as a duopoly is a convenient simplification, necessary to ensure tractability of the model if one wants to allow φ to take any value between 0 and 1. However, mergers to monopoly are not likely to occur in practice. In the Appendix I relax this assumption. To ensure tractability of the model, I often focus on two special cases: when the bargaining power is equally distributed between upstream agents and downstream firms ($\varphi = ½$) and when upstream agents unilaterally set the input price or two-part tariff ($\varphi = 1$). The first of these cases is a natural benchmark, while the second is of particular interest since it was for high values of φ that the non-standard welfare results were obtained in section 2. I also assume that bargaining outcomes are observable by all downstream firms before the price-setting or quantity-setting stage. The results are identical to those of the basic model.

An important assumption of the model is that a downstream firm and its upstream agent are already locked into bilateral relations when they bargain. This assumption is uncontroversial when the upstream agents are unions (see the discussion in Horn and Wolinsky 1988). When the upstream agents are firms, the exclusive relationship between a buyer and a supplier can be due to the fact that, before bargaining on price, the two parties have already made relationship-specific investments that create lock-in effects and high switching costs. These investments would represent long-run decisions, while decisions about the input price are easier to reverse in the medium term.[21] If so, the structure of the game analysed in the present paper is valid whatever the identity of the upstream agent.

Despite the use of a specific structure and functional forms in the present model, many of the economic mechanisms than underlie the results are general. For instance, the fact that the input price is, under certain circumstances, lower when the downstream firms merge than when they are independent is not specific to the linear demand structure or even to the presence of bargaining. The fact that, following a reduction in the input price, the welfare results depend on the balance between a direct and an indirect effect of the merger is also quite general, as is the fact that this balance depends on a particular way on the mode of downstream competition. The mechanisms that cause the input price to rise under a downstream merger for particular bargaining and upstream market structures are also general.

Although the main focus of the present paper is on welfare results, a testable prediction of the model is that the effect of downstream mergers on wages and input prices will depend in specific ways on the bargaining structure and upstream market structure. The empirical evidence on the effects of downstream mergers on wages is mixed (see Lommerud et al., 2006), and this is consistent with the view that mergers may reduce wages in certain circumstances or in some industries and increase them in others.

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[21] Even when a basic input price is specified in a long-term contract between an upstream and a downstream firm before any relationship-specific investment is made, the contract needs to allow for some flexibility, so discounts and even the basic input price are likely to be subject to regular renegotiation.

There are circumstances where a downstream merger will be beneficial for consumers and for society as a whole and circumstances where it will be detrimental. The aim of the present paper was to shed more light on the conditions under which we may need to qualify the conventional economic wisdom on the welfare effects of mergers. Much of the previous literature in this area has tended to emphasise the possibility of welfare gains from downstream mergers (or less intense downstream competition) in the presence of upstream firms or unions. This paper shows that in the absence of efficiency gains, downstream mergers may increase consumer surplus and overall welfare in specific circumstances: when downstream firms set quantities and bargain with independent upstream agents over uniform prices, although this will also depend on the degree of product differentiation and the balance of bargaining power. However, in most cases – including price competition, non-independent upstream agents, bargaining over a two-part tariff and bargaining over both the input price and the level of output – downstream mergers will always reduce consumer surplus and welfare.

APPENDIX

Proof of Proposition 2

(i) From equation (9) and using (5) and (7):

$$CS^*\Big|_{\lambda=0} = \frac{16(2+\sigma)(2-\varphi)^2(\alpha-w_0)^2}{\beta(4+\sigma)^2(8-\sigma\varphi)^2} \tag{A1}$$

and $CS^*\Big|_{\lambda=1} = \dfrac{(2-\varphi)^2(\alpha-w_0)^2}{\beta(2+\sigma)(4-\sigma\varphi)^2}$. (A2)

Define $\Delta CS^* = CS^*\Big|_{\lambda=1} - CS^*\Big|_{\lambda=0}$. The sign of ΔCS^* can be positive or negative depending on the values of σ and φ. It is easy to check that $\Delta CS^*(\sigma = 2) = \dfrac{(-112+184\varphi-55\varphi^2)(\alpha-w_0)^2}{144\beta(4-\varphi)^2} > 0$ for all $\varphi\in(0.8,1]$. By continuity, $\Delta CS^* > 0$ when $\sigma \to 2$ and φ is large enough, while $\Delta CS^* < 0$ when $\sigma \to 2$ and φ is small. Moreover, $\Delta CS^*(\sigma = 0) = 0$ and $\partial\Delta CS^*/\partial\sigma$ at $\sigma = 0$ is given by $-\dfrac{(2-\varphi)^3(\alpha-w_0)^2}{128\beta}$, a negative expression. Hence $\Delta CS^* < 0$ for σ close to 0.

(ii) The result for aggregate downstream profit is straightforward (see the discussion in the main text). For aggregate upstream utility, we obtain from equation (11) and using (5) and (7):

$$\frac{\partial U^*}{\partial\lambda} = -\frac{16\varphi\sigma(2-\varphi)\left[16(1-\varphi)+16-4\varphi\sigma(1+\lambda)+\varphi\sigma^2(1+\lambda)^2\right](\alpha-w_0)}{\beta\left[4+\sigma(1+\lambda)\right]^2\left[8-\varphi\sigma(1+\lambda)\right]^3}, \tag{A3}$$

which is negative for $\varphi\in(0,1]$. When $\varphi = 0$, $w^* = w_o$ and $U^* = 0$.

(iii) From equations (9)-(11) and using (5) and (7) we obtain:

$$W^*\Big|_{\lambda=0} = \frac{8(2-\varphi)(24+4\sigma+4\varphi-2\varphi\sigma-\varphi\sigma^2)(\alpha-w_0)^2}{\beta(4+\sigma)^2(8-\varphi\sigma)^2} \tag{A4}$$

and $W^*\Big|_{\lambda=1} = \dfrac{(2-\varphi)(6+\varphi-2\varphi\sigma)(\alpha-w_0)^2}{\beta(2+\sigma)(4-\varphi\sigma)^2}$. (A5)

Define $\Delta W^* = W^*\Big|_{\lambda=1} - W^*\Big|_{\lambda=0}$. The sign of ΔW^* can be positive or negative depending on the values of σ and φ. It is easy to check that $\Delta W^*(\sigma = 2) = \dfrac{(-80+104\varphi-5\varphi^2)(\alpha-w_0)^2}{144\beta(4-\varphi)^2} > 0$ for all $\varphi \in (0.8,1]$. By continuity, $\Delta W^* > 0$ when $\sigma \to 2$ and φ is large enough, while $\Delta W^* < 0$ when $\sigma \to 2$ and φ is small. Moreover, $\Delta W^*(\sigma = 0) = 0$ and $\partial \Delta W^* / \partial \sigma$ at $\sigma = 0$ is given by $-\dfrac{(2-\varphi)(4-\varphi)^2(\alpha-w_0)^2}{128\beta}$, a negative expression. Hence $\Delta W^* < 0$ for σ close to 0. □

Proof of Proposition 4

(i) Consumer surplus, aggregate downstream profit and aggregate upstream utility are again given by equations (9)-(11), where x^* and p^* are now given by equations (13) after setting $w_i = w_j = w^P$. Consider first consumer surplus. We obtain:

$$CS^P\Big|_{\lambda=0} = \frac{(2-\varphi)^2(8-\sigma^2)^2(\alpha-w_0)^2}{\beta(4-\sigma)^2(2+\sigma)(8-\varphi\sigma-\sigma^2)^2} \quad \text{(A6)}$$

$$CS^P\Big|_{\lambda=1} = \frac{(2-\varphi)^2(\alpha-w_0)^2}{\beta(2+\sigma)(4-\varphi\sigma)^2} \quad \text{(A7)}$$

and

$$CS^P\Big|_{\lambda=1} - CS^P\Big|_{\lambda=0} = -\frac{(2-\varphi)^2\sigma\left[4-\varphi\sigma+(1-\varphi)(4-\sigma^2)\right] G(\alpha-w_0)^2}{\beta(4-\sigma)^2(2+\sigma)(4-\varphi\sigma)^2(8-\varphi\sigma-\sigma^2)^2}, \quad \text{(A8)}$$

where

$G = (2-\varphi\sigma)(20-4\sigma-\sigma^2)+(2-\sigma)(12+6\sigma+4\varphi\sigma)+\varphi\sigma^2+\sigma^3 > 0$.

(ii) The result for aggregate downstream profit is straightforward (see the discussion in the main text). For aggregate upstream utility, we have:

$$U^P\Big|_{\lambda=0} = \frac{(2-\varphi)\varphi(64-16\sigma-16\sigma^2+2\sigma^3+\sigma^4)^2(\alpha-w_0)^2}{\beta(4-\sigma)(2+\sigma)(8-\varphi\sigma-\sigma^2)^2} \tag{A9}$$

$$U^P\Big|_{\lambda=1} = \frac{2(2-\varphi)\varphi(2-\sigma)(\alpha-w_0)^2}{\beta(2+\sigma)(4-\varphi\sigma)^2} \tag{A10}$$

and

$$U^P\Big|_{\lambda=1} - U^P\Big|_{\lambda=0} = -\frac{(2-\varphi)\varphi\sigma(2-\sigma)H(\alpha-w_0)^2}{\beta(4-\sigma)(2+\sigma)(4-\varphi\sigma)^2(8-\varphi\sigma-\sigma^2)^2} \tag{A11}$$

where

$$H = 2(8-\sigma^2)\left[16-4\varphi(2+\sigma)+\sigma(2-\sigma)+2\sigma(1-\varphi)\right]+\varphi^2\sigma(24+10\sigma-4\sigma^2-\sigma^3)>0.$$

(iii) Finally, overall welfare is given by $W^P = CS^P + \Pi^P + U^P$. We obtain:

$$W^P\Big|_{\lambda=0} = \frac{(2-\varphi)(8-\sigma^2)\left[2(8-\sigma^2)(3-\sigma)+\varphi(8-8\sigma+\sigma^2)\right](\alpha-w_0)^2}{\beta(4-\sigma)^2(2+\sigma)(8-\varphi\sigma-\sigma^2)^2} \tag{A12}$$

$$W^P\Big|_{\lambda=1} = \frac{(2-\varphi)(6+\varphi-2\varphi\sigma)(\alpha-w_0)^2}{\beta(2+\sigma)(4-\varphi\sigma)^2} \tag{A13}$$

and

$$W^P\Big|_{\lambda=1} - W^P\Big|_{\lambda=0} = -\frac{(2-\varphi)\sigma\left[(2-\sigma)(2+\sigma-\varphi\sigma)+4(1-\varphi)+\varphi\sigma\right]\, J(\alpha-w_0)^2}{\beta(4-\sigma)^2(2+\sigma)(4-\varphi\sigma)^2(8-\varphi\sigma-\sigma^2)^2} \tag{A14}$$

where

$$J = 8(2-\sigma)(2-\varphi^2\sigma)+(4+\sigma-\sigma^2)(8-2\varphi\sigma^2+\varphi^2\sigma)+2(2-\sigma)^2(8+3\sigma)+ \\ \varphi(2-\sigma)\left[4-\sigma^2+4\sigma(2-\sigma)^2+4(7-10\sigma+4\sigma^2)\right]>0.$$

□

Proof of Proposition 6

(i) The result for consumer surplus is straightforward.

(ii) Downstream profit and upstream utility in the absence of a merger are given by equations (9) and (10), using also (5) and (7) and setting $\lambda = 0$:

$$\Pi^*\Big|_{\lambda=0} = \frac{64(2-\varphi)^2(\alpha - w_0)^2}{\beta(4+\sigma)^2(8-\sigma\varphi)^2} \tag{A15}$$

$$U^*\Big|_{\lambda=0} = \frac{8\varphi(2-\varphi)(4-\sigma)(\alpha - w_0)^2}{\beta(4+\sigma)(8-\sigma\varphi)^2}. \tag{A16}$$

With a downstream merger, we obtain, using (9), (10), (18) and (5) and setting $\lambda = 1$:

$$\Pi_M^* = \frac{(2-\varphi)^2(\alpha - w_0)^2}{8\beta(2+\sigma)} \tag{A17}$$

$$U_M^* = \frac{\varphi(2-\varphi)(\alpha - w_0)^2}{4\beta(2+\sigma)}. \tag{A18}$$

Hence:

$$\Pi_M^* - \Pi^*\Big|_{\lambda=0} = \frac{\sigma(2-\varphi)^2(\alpha - w_0)^2 Z_\Pi}{8\beta(2+\sigma)(4+\sigma)^2(8-\varphi\sigma)^2}, \tag{A19}$$

where $Z_\Pi = 64\sigma - \varphi(4+\sigma)^2(16-\varphi\sigma)$, (A20)

$$U_M^* - U^*\Big|_{\lambda=0} = \frac{\varphi\sigma(2-\varphi)(\alpha - w_0)^2 Z_U}{4\beta(2+\sigma)(4+\sigma)(8-\varphi\sigma)^2}, \tag{A21}$$

where $Z_U = 32\sigma - \varphi(4+\sigma)(16-\varphi\sigma)$. (A22)

The sign of $\Pi_M^* - \Pi^*\Big|_{\lambda=0}$ is the same as the sign of Z_Π. Now $Z_\Pi(\varphi = 0) > 0$, $Z_\Pi(\varphi = 1) < 0$ and $\partial Z_\Pi / \partial\varphi < 0$. It follows that there exists $\tilde{\varphi}_\Pi(\sigma) \in (0,1)$ such that $\Pi_M^* - \Pi^*\Big|_{\lambda=0}$ is positive when $\varphi < \tilde{\varphi}_\Pi(\sigma)$ and negative when $\varphi > \tilde{\varphi}_\Pi(\sigma)$. Solving $Z_\Pi = 0$ we obtain $\tilde{\varphi}_\Pi(\sigma) = \dfrac{8(4+\sigma-\sqrt{4+2\sigma})}{\sigma(4+\sigma)}$. Also, the sign of $U_M^* - U^*\Big|_{\lambda=0}$ is the same as the sign of Z_U. We have $Z_U(\varphi = 0) > 0$, $Z_U(\varphi = 1) < 0$ and $\partial Z_U / \partial\varphi < 0$.

Hence there exists $\tilde{\varphi}_U(\sigma) \in (0,1)$ such that $U_M * - U *\Big|_{\lambda=0}$ is positive when $\varphi <$ $\tilde{\varphi}_U(\sigma)$ and negative when $\varphi > \tilde{\varphi}_U(\sigma)$. Solving $Z_U = 0$ we obtain $\tilde{\varphi}_U(\sigma) = \dfrac{8(4+\sigma) - 4\sqrt{2(32+16\sigma-2\sigma^2-\sigma^3)}}{\sigma(4+\sigma)}$. A comparison of Z_Π and Z_U shows that $Z_\Pi > 0 \Rightarrow Z_U > 0$, while the reverse is not necessarily true, which implies $\tilde{\varphi}_\Pi(\sigma) < \tilde{\varphi}_U(\sigma)$.

(iii) Overall welfare without a merger is

$$W *\Big|_{\lambda=0} = \frac{8(2-\varphi)(24+4\sigma+4\varphi-2\varphi\sigma-\varphi\sigma^2)(\alpha-w_0)^2}{\beta(4+\sigma)^2(8-\varphi\sigma)^2}, \tag{A23}$$

while with a merger it is

$$W_M * = \frac{(12-4\varphi-\varphi^2)^2(\alpha-w_0)^2}{16\beta(2+\sigma)}, \tag{A24}$$

which implies

$$W_M * - W *\Big|_{\lambda=0} = -\frac{(2-\varphi)\sigma[8+\varphi(4+\sigma)](128+16\sigma+64\varphi-4\sigma\varphi^2-6\varphi\sigma^2-\varphi^2\sigma^2)(\alpha-w_0)^2}{16\beta(2+\sigma)(4+\sigma)^2(8-\varphi\sigma)^2} < 0. \tag{A25}$$

□

Proof of Proposition 8

From equations (21), (23), (25)-(27) and (5), we obtain:

$$\frac{\partial CS **}{\partial \lambda} = \frac{-32\sigma(2+\sigma)\left[4-\sigma(1-2\lambda-3\lambda^2)\right](\alpha-w_0)^2}{\beta\left[16+4\sigma(1+\lambda)-\sigma^2(1-\lambda)(1+\lambda)^2\right]^3} < 0, \tag{A26}$$

$$\frac{\partial U^{**}}{\partial \lambda} = \frac{-4\varphi\sigma(\alpha - w_0)^2 \Phi}{\beta\left[16 + 4\sigma(1+\lambda) - \sigma^2(1-\lambda)(1+\lambda)^2\right]^3}, \tag{A27}$$

where $\Phi = 128 + 16\sigma(1 + 4\lambda + 3\lambda^2) - 4\sigma^2(1+\lambda)^3 + \sigma^3(1+\lambda)^3(1 - 8\lambda + 3\lambda^2) > 0$,

$$\frac{\partial \Pi^{**}}{\partial \lambda} = \frac{4\sigma(\alpha - w_0)^2 \Psi}{\beta\left[16 + 4\sigma(1+\lambda) - \sigma^2(1-\lambda)(1+\lambda)^2\right]^3}, \tag{A28}$$

where $\Psi = 2\sigma(1-\lambda)\left[4 + \sigma(1+\lambda)^2\right]\left[4 - \sigma(1+\lambda)(1-3\lambda)\right] + \varphi\Phi > 0$, and

$$\frac{\partial W^{**}}{\partial \lambda} = \frac{-8\sigma\left[4 - \sigma(1 - 2\lambda - 3\lambda^2)\right]\left[8 + \sigma\lambda(4-\sigma) - \sigma^2(1 - \lambda^2 - \lambda^3)\right](\alpha - w_0)^2}{\beta\left[16 + 4\sigma(1+\lambda) - \sigma^2(1-\lambda)(1+\lambda)^2\right]^3} < 0. \tag{A29}$$

□

Three-firm oligopoly

I analyse here the welfare effects of a merger between a subset of firms in an industry. I use the simplest possible setup: a three-firm downstream oligopoly where two of the firms merge. I also assume throughout that bargaining outcomes are observable by all before the price or quantity-setting stage. I begin with the case of quantity-setting downstream firms, independent upstream agents and bargaining over a uniform input price. The structure of the game is the same as in section 2, but there are now three downstream firms. The inverse demand function for variety *i* is given by

$$p_i = \alpha - 2\beta x_i - \beta\sigma(x_j + x_k) \tag{B1}$$

and the demand function is

$$x_i = \frac{(2+\sigma)(\alpha - p_i) - \sigma(2\alpha - p_j - p_k)}{\beta(2-\sigma)(2+2\sigma)} \tag{B2}$$

The Cournot-Nash equilibrium values of x_i and p_i in the second-stage subgame are:

$$\hat{x}_i = \frac{(4+\sigma)(\alpha - w_i) - \sigma(2\alpha - w_j - w_k)}{2\beta(4-\sigma)(2+\sigma)}$$

$$\hat{p}_i = w_i + \frac{2(4+\sigma)(\alpha - w_i) - 2\sigma(2\alpha - w_j - w_k)}{2(4-\sigma)(2+\sigma)} \tag{B3}$$

in the region of w spaces where $\hat{x}_i \geq 0$, $\hat{p}_i \geq 0$, i = 1,2,3. At stage 1 of the game, the downstream firm i and the upstream agent i form a bargaining unit and set w_i to maximise

$$\Omega_i = \left[(w_i - w_0)\hat{x}_i\right]^{\varphi}\left[(\hat{p}_i - w_i)\hat{x}_i\right]^{1-\varphi} \quad . \tag{B4}$$

Solving for the equilibrium we obtain:

$$w^*\Big|_{\lambda=0} = w_0 + \frac{\varphi(4-\sigma)(\alpha - w_0)}{2(4+\sigma-\varphi\sigma)} . \tag{B5}$$

Now consider a merger between two of the downstream firms, i and j. In the second-stage equilibrium we have:

$$\hat{x}_i = \frac{2(8-6\sigma+\sigma^2)\alpha - (16-\sigma^2)w_i + \sigma(8-\sigma)w_j + 2\sigma(2-\sigma)w_k}{4\beta(2-\sigma)(8+4\sigma-\sigma^2)}$$

$$\hat{p}_i = w_i + \frac{2(8+2\sigma-\sigma^2)\alpha - (16+8\sigma-\sigma^2)w_i - \sigma^2 w_j + 2\sigma(2+\sigma)w_k}{4(8+4\sigma-\sigma^2)}$$

$$\hat{x}_j = \frac{2(8-6\sigma+\sigma^2)\alpha - (16-\sigma^2)w_j + \sigma(8-\sigma)w_i + 2\sigma(2-\sigma)w_k}{4\beta(2-\sigma)(8+4\sigma-\sigma^2)}$$

$$\hat{p}_j = w_j + \frac{2(8+2\sigma-\sigma^2)\alpha - (16+8\sigma-\sigma^2)w_j - \sigma^2 w_i + 2\sigma(2+\sigma)w_k}{4(8+4\sigma-\sigma^2)}$$

$$\hat{x}_k = \frac{4\alpha - 2(2-\sigma)w_k + \sigma(w_i + w_j)}{2\beta(8+4\sigma-\sigma^2)}$$

$$\hat{p}_k = w_k + \frac{4\alpha - 2(2+\sigma)w_k + \sigma(w_i + w_j)}{8+4\sigma-\sigma^2}. \tag{B6}$$

At stage 1, the downstream firm k and its upstream agent set w_k to maximise

$$\Omega_k = \left[(w_k - w_0)\hat{x}_k\right]^{\varphi}\left[(\hat{p}_k - w_k)\hat{x}_k\right]^{1-\varphi} \quad , \tag{B7}$$

while the merged downstream firm bargains simultaneously and separately with the two upstream agents, *i* and *j*. The Nash product for the bargain over product *i* is:

$$\Omega_i = \left[(w_i - w_0)\hat{x}_i\right]^{\varphi}\left[(\hat{p}_i - w_i)\hat{x}_i + (\hat{p}_j - w_j)\hat{x}_j - (\bar{\bar{p}}_j - w_j)\bar{\bar{x}}_j\right]^{1-\varphi} \quad , \tag{B8}$$

where $\bar{\bar{p}}_j = w_j + \frac{8(\alpha - w_j) - 2\sigma(\alpha - w_k)}{(4-\sigma)(4+\sigma)}$, $\bar{\bar{x}}_j = \frac{4(\alpha - w_j) - \sigma(\alpha - w_k)}{\beta(4-\sigma)(4+\sigma)}$ are the price and output of good *j* in case of a breakdown of negotiations between the merged firm and upstream agent *i*: the downstream market structure becomes then a duopoly and two products are offered, *j* and *k*. The Nash product for the bargain over product *j* is similar.

The values of w_i, w_j and w_k that we obtain at stage 1 of the game are the outcome of a non-cooperative Nash equilibrium between the three bargaining units. Solving the system of three first-order conditions we obtain, for φ = ½:

$$\left. w^*_{i,merger}\right|_{\varphi=1/2} = \left. w^*_{j,merger}\right|_{\varphi=1/2} = \\ w_0 + \frac{2}{\Theta}(4096 + 1280\sigma - 1536\sigma^2 - 320\sigma^3 + 96\sigma^4 + 6\sigma^5 + 9\sigma^6 - 2\sigma^7)(\alpha - w_0) \tag{B9}$$

$$\left. w^*_{k,merger}\right|_{\varphi=1/2} = \\ w_0 + \frac{1}{\Theta}(8192 + 4096\sigma - 1408\sigma^2 - 576\sigma^3 + 48\sigma^4 + 8\sigma^5 + 7\sigma^6 - \sigma^7)(\alpha - w_0), \tag{B10}$$

where $\Theta = 32768 + 28672\sigma + 1280\sigma^2 - 3584\sigma^3 - 640\sigma^4 + 32\sigma^5 + 38\sigma^6 + \sigma^7 > 0$. Hence

$$w^*_{i,merger}\Big|_{\varphi=1/2} - w^*\Big|_{\lambda=0,\varphi=1/2} = -\frac{\sigma(24576+20480\sigma+768\sigma^2-768\sigma^3+192\sigma^4-192\sigma^5-6\sigma^6+7\sigma^7)(\alpha-w_0)}{2(8+\sigma)\Theta} < 0 \quad \text{(B11)}$$

$$w^*_{k,merger}\Big|_{\varphi=1/2} - w^*\Big|_{\lambda=0,\varphi=1/2} = \frac{\sigma^2(9216+3584\sigma-1408\sigma^2-544\sigma^3+8\sigma^4+32\sigma^5-\sigma^6)(\alpha-w_0)}{2(8+\sigma)\Theta} > 0. \quad \text{(B12)}$$

For $\varphi = 1$:

$$w^*_{i,merger}\Big|_{\varphi=1} = w^*_{j,merger}\Big|_{\varphi=1} = w_0 + \frac{(16-2\sigma-5\sigma^2+\sigma^3)(\alpha-w_0)}{2(16+4\sigma-3\sigma^2)} \quad \text{(B13)}$$

$$w^*_{k,merger}\Big|_{\varphi=1} = w_0 + \frac{(32-7\sigma^2+\sigma^3)(\alpha-w_0)}{4(16+4\sigma-3\sigma^2)}. \quad \text{(B14)}$$

Hence

$$w^*_{i,merger}\Big|_{\varphi=1} - w^*\Big|_{\lambda=0,\varphi=1} = -\frac{\sigma(8+4\sigma-\sigma^2)(\alpha-w_0)}{8(16+4\sigma-3\sigma^2)} < 0 \quad \text{(B15)}$$

$$w^*_{k,merger}\Big|_{\varphi=1} - w^*\Big|_{\lambda=0,\varphi=1} = \frac{\sigma^2(2-\sigma)(\alpha-w_0)}{8(16+4\sigma-3\sigma^2)} > 0. \quad \text{(B16)}$$

Thus a downstream merger between firms *i* and *j* reduces w_i and w_j (as in Proposition 1) and increases w_k.

Consumer surplus, aggregate downstream profit and aggregate upstream utility are, respectively, given as

$$\begin{aligned} CS^* = {} & 2\alpha(x_i^*+x_j^*+x_k^*) - 2\beta\left[(x_i^*)^2+(x_j^*)^2+(x_k^*)^2\right] \\ & -\beta\sigma(x_i^*x_j^*+x_i^*x_k^*+x_j^*x_k^*) - 2(p_i^*x_i^*+p_j^*x_j^*+p_k^*x_k^*) \end{aligned} \quad \text{(B17)}$$

$$\Pi^* = (p_i^*-w_i^*)x_i^* + (p_j^*-w_j^*)x_j^* + (p_k^*-w_k^*)x_k^* \quad \text{(B18)}$$

and

$$U^* = (w_i^* - w_0)x_i^* + (w_j^* - w_0)x_{ji}^* + (w_k^* - w_0)x_k^*, \tag{B19}$$

where x_i*, x_j*, x_k*, p_i*, p_j* and p_k* are the equilibrium values of x and p in the two-stage game.

Straightforward calculations yield

$$\left.CS^*_{merger}\right|_{\varphi=1/2} - \left.CS^*\right|_{\lambda=0,\varphi=1/2} = -\frac{\Theta_1(\alpha - w_0)^2}{16\beta(2+\sigma)^2(8+\sigma)^2(8+4\sigma-\sigma^2)^2\Theta}, \tag{B20}$$

where Θ_1 is a positive function of σ for $\sigma\in(0,2)$: this expression is a higher-order polynomial in σ, but it is easy to check that it is positive for $\sigma = 0$ and for $\sigma = 2$ and that none of its roots is in the interval (0,2). Moreover,

$$\left.CS^*_{merger}\right|_{\varphi=1} - \left.CS^*\right|_{\lambda=0,\varphi=1} = -\frac{\sigma(\alpha - w_0)^2\Theta_2}{256\beta(2+\sigma)^2(16+4\sigma-3\sigma^2)^2(8+4\sigma-\sigma^2)^2}, \tag{B21}$$

where

$$\begin{aligned}\Theta_2 = {} & 131072 + 237568\sigma + 45056\sigma^2 - 124160\sigma^3 - 57344\sigma^4 + 13984\sigma^5 \\ & + 9168\sigma^6 - 456\sigma^7 - 444\sigma^8 + 3\sigma^9 + 7\sigma^{10}\end{aligned}$$

is positive for $\sigma = 0$, negative for $\sigma = 2$, and has a single root in the interval (0,2) ($\Theta_2 = 0$ for $\sigma \approx 1.6197$). In other words, a merger between two downstream firms always reduces consumer surplus when φ = ½. However, for φ = 1 a merger reduces consumer surplus only when σ is not too large. These results are similar to those in Proposition 2.

Simple but tedious calculations show that the effect of a merger on downstream profit is always positive and its effect on upstream utility is always negative, which is consistent with the results in section 2 (the details are omitted). Finally, I examine total welfare:

$$W^*\Big|_{\substack{merger\\ \varphi=1/2}} - W^*\Big|_{\lambda=0,\varphi=1/2} = -\frac{\Theta_3(\alpha-w_0)^2}{16\beta(2+\sigma)^2(8+\sigma)^2(8+4\sigma-\sigma^2)^2\Theta}, \tag{B22}$$

where Θ_3 is a positive higher-order polynomial in σ – it is positive for $\sigma = 0$ and for $\sigma = 2$ and none of its roots is in the interval (0,2). Moreover,

$$W^*\Big|_{\substack{merger\\ \varphi=1}} - W^*\Big|_{\lambda=0,\varphi=1} = -\frac{\sigma(\alpha-w_0)^2\Theta_4}{256\beta(2+\sigma)^2(16+4\sigma-3\sigma^2)^2(8+4\sigma-\sigma^2)^2}, \tag{B23}$$

where

$$\Theta_4 = 393216 + 352256\sigma - 225280\sigma^2 - 232192\sigma^3 + 38912\sigma^4 + 45920\sigma^5 - 5072\sigma^6 - 3384\sigma^7 + 476\sigma^8 + 45\sigma^9 - 7\sigma^{10}$$

is positive for $\sigma = 0$, negative for $\sigma = 2$, and has a single root in the interval (0,2) ($\Theta_4 = 0$ for $\sigma \approx 1.6204$). Thus a merger between two downstream firms always reduces total welfare when $\varphi = ½$. However, for $\varphi = 1$ a merger reduces welfare only when σ is not too large. These results are the same as Proposition 2.

Next, I consider price-setting in the final stage. In the absence of a merger, the Bertrand-Nash equilibrium values of x_i and p_i in the second-stage subgame are:

$$\begin{aligned} \hat{p}_i &= w_i + \frac{(8+6\sigma-\sigma^2)(\alpha-w_i)-\sigma(2+\sigma)(2\alpha-w_j-w_k)}{4(4+3\sigma)} \\ \hat{x}_i &= \frac{(2+\sigma)\left[(8+6\sigma-\sigma^2)(\alpha-w_i)-\sigma(2+\sigma)(2\alpha-w_j-w_k)\right]}{8\beta(2-\sigma)(1+\sigma)(4+3\sigma)} \end{aligned} \tag{B24}$$

in the region of w spaces where $\hat{x}_i \geq 0$, $\hat{p}_i \geq 0$, i = 1,2,3. At stage 1 of the game, the downstream firm i and the upstream agent i form a bargaining unit and set w_i to maximise the Nash product in equation (B4). Solving for the equilibrium we obtain:

$$w^P\Big|_{\lambda=0} = w_0 + \frac{\varphi(8+2\sigma-3\sigma^2)(\alpha-w_0)}{2(8+6\sigma-2\varphi\sigma-\sigma^2-\varphi\sigma^2)}. \tag{B25}$$

Now consider a merger between two of the downstream firms, *i* and *j*. In the second-stage equilibrium we have:

$$\hat{p}_i = \frac{2(8+2\sigma-3\sigma^2)\alpha + (16+8\sigma-\sigma^2)w_i + \sigma^2 w_j + 2\sigma(2+\sigma)w_k}{4(8+4\sigma-\sigma^2)}$$

$$\hat{x}_i = \frac{2(8+2\sigma-3\sigma^2)\alpha - (16+16\sigma+\sigma^2-\sigma^3)w_i + \sigma(8+5\sigma-\sigma^2)w_j + 2\sigma(2+\sigma)w_k}{4\beta(2-\sigma)(1+\sigma)(8+4\sigma-\sigma^2)}$$

$$\hat{p}_j = \frac{2(8+2\sigma-3\sigma^2)\alpha + (16+8\sigma-\sigma^2)w_j + \sigma^2 w_i + 2\sigma(2+\sigma)w_k}{4(8+4\sigma-\sigma^2)}$$

$$\hat{x}_j = \frac{2(8+2\sigma-3\sigma^2)\alpha - (16+16\sigma+\sigma^2-\sigma^3)w_j + \sigma(8+5\sigma-\sigma^2)w_i + 2\sigma(2+\sigma)w_k}{4\beta(2-\sigma)(1+\sigma)(8+4\sigma-\sigma^2)}$$

$$\hat{p}_k = \frac{(4-\sigma^2)\alpha + 2(2+\sigma)w_k + \sigma(w_i+w_j)}{8+4\sigma-\sigma^2}$$

$$\hat{x}_k = \frac{(2+\sigma)(4-\sigma^2)\alpha - (4+2\sigma-\sigma^2)w_k + \sigma(w_i+w_j)}{2\beta(2-\sigma)(1+\sigma)(8+4\sigma-\sigma^2)}. \qquad \text{(B26)}$$

At stage 1, the downstream firm *k* and its upstream agent set w_k to maximise Ω_k in (B7) while the merged downstream firm bargains simultaneously and separately with the two upstream agents, *i* and *j*. The Nash product for the bargain over product *i* is Ω_i in (B8), where $\bar{\bar{p}}_j = w_j + \dfrac{(8-\sigma^2)(\alpha-w_j) - 2\sigma(\alpha-w_k)}{(4-\sigma)(4+\sigma)}$, $\bar{\bar{x}}_j = \dfrac{2(8-\sigma^2)(\alpha-w_j) - 4\sigma(\alpha-w_k)}{\beta(2-\sigma)(2+\sigma)(4-\sigma)(4+\sigma)}$ are now the price and output of good *j* in a downstream price-setting duopoly with two products offered, *j* and *k*. The Nash product for the bargain over *j* is similar. The values of w_i, w_j and w_k that we obtain at stage 1 of the game are the outcome of a non-cooperative Nash equilibrium between the three bargaining units. The model is now tractable only for $\varphi = 1$. We obtain:

$$w^P_{i,merger}\Big|_{\varphi=1} = w^P_{j,merger}\Big|_{\varphi=1} = w_0 + \frac{(64+56\sigma-28\sigma^2-18\sigma^3+5\sigma^4)(\alpha-w_0)}{128+160\sigma-36\sigma^2+\sigma^4+\sigma^5} \quad \text{(B27)}$$

$$w^P_{k,merger}\Big|_{\varphi=1} = w_0 + \frac{(128+128\sigma-36\sigma^2-40\sigma^3+3\sigma^4+\sigma^5)(\alpha-w_0)}{2(128+160\sigma-36\sigma^2+\sigma^4+\sigma^5)}. \quad \text{(B28)}$$

Hence

$$w^P_{i,merger}\Big|_{\varphi=1} - w^P\Big|_{\lambda=0,\varphi=1} = -\frac{\sigma(2-\sigma)(1+\sigma)(8+4\sigma-\sigma^2)(8+4\sigma-3\sigma^2)(\alpha-w_0)}{2(4+2\sigma-\sigma^2)(128+160\sigma-36\sigma^2+\sigma^4+\sigma^5)} < 0 \quad \text{(B29)}$$

$$w^P_{k,merger}\Big|_{\varphi=1} - w^P\Big|_{\lambda=0,\varphi=1} = \frac{\sigma^2(2-\sigma)(1+\sigma)(16+16\sigma-\sigma^3)(\alpha-w_0)}{4(4+2\sigma-\sigma^2)(128+160\sigma-36\sigma^2+\sigma^4+\sigma^5)} > 0. \quad \text{(B30)}$$

Thus, for $\varphi = 1$ at least, a downstream merger between firms i and j reduces w_i and w_j (as in Proposition 3) and increases w_k.

Consumer surplus, aggregate downstream profit and aggregate upstream utility are given by (B17), (B18) and (B19), where x_i*, x_j*, x_k*, p_i*, p_j* and p_k* are now the equilibrium values of x and p in the two-stage game with price-setting firms. We obtain:

$$CS^P_{i,merger}\Big|_{\varphi=1} - CS^P\Big|_{\lambda=0,\varphi=1} = -\frac{\sigma(2-\sigma)(2+\sigma)\Theta_5(\alpha-w_0)^2}{1024\beta(1+\sigma)(8+4\sigma-\sigma^2)^2(4+2\sigma-\sigma^2)(128+160\sigma-36\sigma^2+\sigma^4+\sigma^5)^2}, \quad \text{(B31)}$$

where Θ_5 is a positive higher-order polynomial for $\sigma \in (0,2)$: it is positive for $\sigma = 0$ and for $\sigma = 2$ and none of its roots is in the interval (0,2). Hence, a merger between two downstream firms always reduces consumer surplus, at least when $\varphi = 1$, as in Proposition 4.

It is straightforward to show that the effect of a merger on downstream profit is always positive and its effect on upstream utility is always negative, which is consistent with the results in section 3. Finally, for total welfare we have:

$$W^P_{i,merger}\Big|_{\varphi=1} - W^P\Big|_{\lambda=0,\varphi=1} = \tag{B32}$$
$$-\frac{\sigma(2-\sigma)\Theta_6(\alpha-w_0)^2}{1024\beta(1+\sigma)(8+4\sigma-\sigma^2)^2(4+2\sigma-\sigma^2)(128+160\sigma-36\sigma^2+\sigma^4+\sigma^5)^2},$$

where Θ_6, a higher-order polynomial in σ, is positive for $\sigma\in(0,2)$: it is positive for $\sigma = 0$ and for $\sigma = 2$ and none of its roots is in the interval (0,2). Thus a merger between two downstream firms always reduces total welfare, at least for $\varphi = 1$, as in Proposition 4.

When the upstream agents are firm-specific rather than independent and downstream firms set quantities, the analysis of the three-firm case is the same as above and the equilibrium input price is given in equation (B5). If two of the downstream firms, i and j, merge, the second-stage equilibrium is given by equations (B6). At stage 1, the downstream firm k and its upstream agent set w_k to maximise Ω_k in (B7) while the merged downstream firm bargains with a single upstream agent over w_i and w_j. The Nash product is

$$\Omega_{i,j} = \left[(w_i - w_0)\hat{x}_i + (w_j - w_0)\hat{x}_j\right]^{\varphi}\left[(\hat{p}_i - w_i)\hat{x}_i + (\hat{p}_j - w_j)\hat{x}_j\right]^{1-\varphi} \quad . \tag{B33}$$

The values of w_i, w_j and w_k that we obtain at stage 1 are the outcome of a non-cooperative Nash equilibrium between the two bargaining units. Solving the system of three first-order conditions we obtain, for $\varphi = ½$:

$$w^*_{i,M}\Big|_{\varphi=1/2} = w^*_{j,M}\Big|_{\varphi=1/2} = w_0 + \frac{2(16+5\sigma-2\sigma^2)(\alpha-w_0)}{128+64\sigma-\sigma^2} \tag{B34}$$

$$w^*_{k,M}\Big|_{\varphi=1/2} = w_0 + \frac{(32+4\sigma-\sigma^2)(\alpha-w_0)}{128+64\sigma-\sigma^2}. \tag{B35}$$

Hence

$$w^*_{i,M}\Big|_{\varphi=1/2} - w^*\Big|_{\lambda=0,\varphi=1/2} = \frac{3\sigma(32+8\sigma-3\sigma^2)(\alpha-w_0)}{2(8+\sigma)(128+64\sigma-\sigma^2)} > 0 \tag{B36}$$

$$\left. w^*_{k,M} \right|_{\varphi=1/2} - \left. w^* \right|_{\lambda=0,\varphi=1/2} = \frac{3\sigma^2(20-\sigma)(\alpha-w_0)}{2(8+\sigma)(128+64\sigma-\sigma^2)} > 0. \tag{B37}$$

For $\varphi = 1$:

$$\left. w^*_{i,M} \right|_{\varphi=1} = \left. w^*_{j,M} \right|_{\varphi=1} = w_0 + \frac{2(8+3\sigma-\sigma^2)(\alpha-w_0)}{32+16\sigma-\sigma^2} \tag{B38}$$

$$\left. w^*_{k,M} \right|_{\varphi=1} = w_0 + \frac{(16+4\sigma-\sigma^2)(\alpha-w_0)}{32+16\sigma-\sigma^2}. \tag{B39}$$

Hence

$$\left. w^*_{i,M} \right|_{\varphi=1} - \left. w^* \right|_{\lambda=0,\varphi=1} = \frac{\sigma(16+4\sigma-\sigma^2)(\alpha-w_0)}{8(32+16\sigma-\sigma^2)} < 0 \tag{B40}$$

$$\left. w^*_{k,M} \right|_{\varphi=1/2} - \left. w^* \right|_{\lambda=0,\varphi=1/2} = \frac{\sigma^2(12-\sigma)(\alpha-w_0)}{8(32+16\sigma-\sigma^2)} > 0. \tag{B41}$$

Thus a downstream merger between firms *i* and *j* increases w_i and w_j when upstream agents also merge (as in Proposition 5) and also increases w_k.

Since all input prices increase following a merger, it follows that consumer surplus falls. For aggregate downstream profit we have:

$$\left. \Pi^*_{M} \right|_{\varphi=1/2} - \left. \Pi^* \right|_{\lambda=0,\varphi=1/2} = \frac{9\sigma(\alpha-w_0)^2\Theta_7}{8\beta(2+\sigma)^2(8+\sigma)^2(128+64\sigma-\sigma^2)^2(8+4\sigma-\sigma^2)^2}, \tag{B42}$$

where

$$\Theta_7 = -4194304 - 3276800\sigma + 2916352\sigma^2 + 3525632\sigma^3 + 827904\sigma^4 \\ -120768\sigma^5 - 41024\sigma^6 + 5648\sigma^7 + 1664\sigma^8 + 13\sigma^9$$

is negative for $\sigma = 0$, positive for $\sigma = 2$, and has a single root in the interval (0,2) ($\Theta_7 = 0$ for $\sigma \approx 1.02$). Moreover,

$$\left. \Pi^*_{M} \right|_{\varphi=1} - \left. \Pi^* \right|_{\lambda=0,\varphi=1} = \frac{\sigma(\alpha-w_0)^2\Theta_8}{128\beta(2+\sigma)^2(32+16\sigma-\sigma^2)(8+4\sigma-\sigma^2)^2}, \tag{B43}$$

where

$$\Theta_8 = -524288 - 819200\sigma - 311296\sigma^2 + 87040\sigma^3 + 67072\sigma^4 + 2368\sigma^5 - 3392\sigma^6 - 176\sigma^7 + 96\sigma^8 - 3\sigma^9$$

is negative for $\sigma = 0$ and for $\sigma = 2$ and has no root in the interval (0,2). In other words, a merger between two downstream firms always reduces aggregate downstream profit when upstream agents also merge and $\varphi = 1$. However, for $\varphi = ½$ a merger reduces downstream profit only if σ is small. These results are similar to those obtained in section 4. In particular, Proposition 6 can be rephrased, once we fix φ to a certain value, to describe how downstream profit depends on the value of σ.

For aggregate upstream utility we have:

$$U^*_M\Big|_{\varphi=1/2} - U^*\Big|_{\lambda=0,\varphi=1/2} = \frac{3\sigma(\alpha - w_0)^2\Theta_9}{8\beta(2+\sigma)(8+\sigma)^2(128+64\sigma-\sigma^2)^2(8+4\sigma-\sigma^2)}, \tag{B44}$$

where

$$\Theta_9 = -524288 + 114688\sigma + 577536\sigma^2 + 178560\sigma^3 - 25920\sigma^4 - 6936\sigma^5 + 1004\sigma^6 + 5\sigma^7$$

is negative for $\sigma = 0$, positive for $\sigma = 2$, and has a single root in the interval (0,2) ($\Theta_9 = 0$ for $\sigma \approx 0.79$). Moreover,

$$U^*_M\Big|_{\varphi=1} - U^*\Big|_{\lambda=0,\varphi=1} = \frac{\sigma(\alpha - w_0)^2\Theta_{10}}{128\beta(2+\sigma)(32+16\sigma-\sigma^2)(8+4\sigma-\sigma^2)}, \tag{B45}$$

where

$$\Theta_{10} = -65536 - 36864\sigma + 26624\sigma^2 + 14976\sigma^3 - 1728\sigma^4 - 760\sigma^5 + 108\sigma^6 - 3\sigma^7$$

is negative for $\sigma = 0$, positive for $\sigma = 2$, and has a single root in the interval (0,2) ($\Theta_{10} = 0$ for $\sigma \approx 1.70$). In other words, both for $\varphi = 1$ and for $\varphi = ½$, a merger between two downstream firms with firm-specific upstream agents reduces aggregate upstream utility if σ is small and increases it if σ is large. Also, upstream utility is more likely to decrease when $\varphi = 1$ than when $\varphi = ½$. These results are again similar to those obtained in section 4.

Finally, it can be easily verified that overall welfare always falls as a result of a merger when upstream agents are firm-specific, as in Proposition 6.

The final case to examine is the case of bargaining over two-part tariffs between quantity-setting downstream firms and independent upstream agents. The Cournot-Nash equilibrium values of x_i and p_i in the second-stage subgame are again given in (B3). At stage 1, each downstream firm i bargains over w_i and F_i with an independent upstream agent. The Nash product is

$$\Omega_i = \left[(w_i - w_0)\hat{x}_i + F_i\right]^{\varphi}\left[(\hat{p}_i - w_i)\hat{x}_i - F_i\right]^{1-\varphi}. \quad \text{(B46)}$$

Solving for the equilibrium we obtain:

$$w^{**}\Big|_{\lambda=0} = w_0 - \frac{\sigma^2(\alpha - w_0)}{2(4+3\sigma)} \quad \text{(B47)}$$

$$F^{**}\Big|_{\lambda=0} = \frac{(4+\sigma)\left[\sigma^2 + \varphi(4+\sigma-\sigma^2)\right]\,(\alpha - w_0)^2}{8\beta(4+3\sigma)^2} \quad \text{(B48)}$$

Note that φ can take any value in the interval [0,1]. Now consider a merger between two of the downstream firms, i and j. The second-stage equilibrium is given in (B6). At stage 1, the downstream firm k and its upstream agent set w_k and F_k to maximise

$$\Omega_k = \left[(w_k - w_0)\hat{x}_k + F_k\right]^{\varphi}\left[(\hat{p}_k - w_k)\hat{x}_k - F_k\right]^{1-\varphi} \quad , \quad \text{(B49)}$$

while the merged downstream firm bargains simultaneously and separately with the two upstream agents, i and j. The Nash product for the bargain over i is:

$$\Omega_i = \left[(w_i - w_0)\hat{x}_i + F_i\right]^{\varphi}\left[(\hat{p}_i - w_i)\hat{x}_i + F_i + (\hat{p}_j - w_j)\hat{x}_j + F_j - (\overline{\overline{p}}_j - w_j)\overline{\overline{x}}_j - F_j\right]^{1-\varphi}. \quad \text{(B50)}$$

Solving for the equilibrium we obtain:

$$\underset{i,merger}{w^{**}} = \underset{j,merger}{w^{**}} = w_0 - \frac{\sigma^2(8-2\sigma-3\sigma^2+\sigma^3)(\alpha-w_0)}{64+64\sigma-12\sigma^2-12\sigma^3+3\sigma^4} \quad \text{(B51)}$$

$$\underset{k,merger}{w^{**}} = w_0 - \frac{\sigma^2(16-3\sigma^2+\sigma^3)(\alpha-w_0)}{2(64+64\sigma-12\sigma^2-12\sigma^3+3\sigma^4)}. \quad \text{(B52)}$$

Hence

$$\left.\underset{i,merger}{w^{**}} - w^{**}\right|_{\lambda=0} = \frac{\sigma^3(2+\sigma)(16+4\sigma-3\sigma^2)(\alpha-w_0)}{2(4+3\sigma)(64+64\sigma-12\sigma^2-12\sigma^3+3\sigma^4)} > 0 \quad \text{(B53)}$$

$$\left.\underset{k,merger}{w^{**}} - w^{**}\right|_{\lambda=0} = \frac{\sigma^3(16-7\sigma^2)(\alpha-w_0)}{2(4+3\sigma)(64+64\sigma-12\sigma^2-12\sigma^3+3\sigma^4)}. \quad \text{(B54)}$$

A downstream merger between firms *i* and *j* increases w_i and w_j (as in Proposition 7) and can increase or decrease w_k, depending on the value of σ. It is also easy to verify that the merger decreases F_i and F_j and can increase or decrease F_k.

For consumer surplus, straightforward calculations yield

$$\left.\underset{merger}{CS^{**}} - CS^{**}\right|_{\lambda=0} = \frac{\sigma(\alpha-w_0)^2\Theta_{11}}{16\beta(4+3\sigma)^2(64+64\sigma-12\sigma^2-12\sigma^3+3\sigma^4)^2}, \quad \text{(B55)}$$

where

$$\Theta_{11} = -65536 - 172032\sigma - 139264\sigma^2 - 11776\sigma^3 + 31744\sigma^4 + 10784\sigma^5$$
$$- 928\sigma^6 - 1258\sigma^7 - 295\sigma^8 + 57\sigma^9 + 18\sigma^{10}$$

is negative: it is negative for $\sigma = 0$ and for $\sigma = 2$ and none of its roots is in the interval (0,2). Thus, consumer surplus is reduced by a merger, which again confirms the results of section 5. Furthermore, downstream profit always increases but upstream utility can increase or decrease following a merger (the details are omitted). Finally, for total welfare we have:

$$\left.\underset{merger}{W^{**}} - W^{**}\right|_{\lambda=0} = \frac{\sigma(\alpha-w_0)^2\Theta_{12}}{16\beta(4+3\sigma)^2(64+64\sigma-12\sigma^2-12\sigma^3+3\sigma^4)^2}, \quad \text{(B56)}$$

where

$$\Theta_{12} = -65536 - 122880\sigma - 40960\sigma^2 + 36352\sigma^3 + 16896\sigma^4 - 4256\sigma^5 \\ -1760\sigma^6 + 154\sigma^7 + 151\sigma^8 + 51\sigma^9 - 18\sigma^{10}$$

is negative: it is negative for $\sigma = 0$ and for $\sigma = 2$ and none of its roots is in the interval (0,2). Thus, total welfare is reduced by a merger, as in Proposition 8.

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