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Forecasting Extreme Volatility of FTSE-100 With Model Free VFTSE, Carr-Wu and Generalized Extreme Value (GEV) Option Implied Volatility Indices

Sheri M. Markose¹, Yue Peng* and Amadeo Alentorn*†

Abstract

Since its introduction in 2003, volatility indices such as the VIX based on the model-free implied volatility (MFIV) have become the industry standard for assessing equity market volatility. MFIV suffers from estimation bias which typically underestimates volatility during extreme market conditions due to sparse data for options traded at very high or very low strike prices, Jiang and Tian (2007). To address this problem, we propose modifications to the CBOE MFIV using Carr and Wu (2009) moneyness based interpolations and extrapolations of implied volatilities and so called GEV-IV derived from the Generalised Extreme Value (GEV) option pricing model of Markose and Alentorn (2011). GEV-IV gives the best forecasting performance when compared to the model-free VFTSE, Black-Scholes IV and the Carr-Wu case, for realised volatility of the FTSE-100, both during normal and extreme market conditions in 2008 when realised volatility peaked at 80%. The success of GEV-IV comes from the explicit modelling of the implied tail shape parameter and the time scaling of volatility in the risk neutral density which can rapidly and flexibly reflect extreme market sentiments present in traded option prices.

Keywords: Extreme Events;VFTSE; Model-Free Implied Volatility; Generalized Extreme Value; Implied Tail Index; Volatility Forecasting

JEL Classifications: C13, C16, G01

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1. Introduction

The recent financial crisis which started in 2007 has resulted in periods of extreme volatility in financial markets. This has prompted new studies on the behaviour of stock market volatility (see, Schwert, 2011, Mencía and Sentana, 2011, Andersen *et. al.* 2011 and Bollerslev and Todorov, 2011), especially in conjunction with option price implied volatility indexes such as the VIX. Since the introduction in 1993 by the Chicago Board of Trade (CBOE) of the equity market volatility index² based on the seminal work of Whaley (1993) on the implied volatility obtained from both call and put equity index options, the information content of the volatility index in forecasting future realized volatility has become an area of intense investigation. Whaley (2000) coined the term "investor fear gauge" to highlight the fact that the volatility index peaks when the underlying market index is at the lowest level and hence reflects investors' fear about market crashes. The volatility index also features in the pricing of volatility derivatives for hedging non-diversifiable market risk and is also cited in the management of systemic risk conditions.

In September 2003, the CBOE adopted the so called model-free method, for the construction of the VIX. Technically, VIX is the square root of the risk neutral expectation under a Q-measure of the integrated variance of the SP-500 over the next 30 calendar days reported on an annualized basis. The replication of this is independent of any model and involves only directly observed prices for out-of-the-money calls and out-of-the-money puts with the same maturity (see, Britten-Jones and Neuberger, 2000 and Carr and Madan,1998).³ The relationship between the implied volatility and historically realized volatility has particular significance as the measure of volatility risk premium. As risk averse investors buy index options to hedge their underlying equity positions, Carr and Wu (2009), Bollerslev, Tauchen and Zhou (2009) and others have

² The original CBOE volatility index, now referred to as VXO, was based on at the money Black-Scholes (1973) implied volatility. VXO is based on the SP-100 returns while the revamped VIX of 2003 has the SP-500 as the underlying.

³ Carr and Madan (1998) refined the model-free framework for option implied variance in the context of determining the variance swap rates. The VIX and volatility indexes adopted by the various exchanges for their stock indexes (such as VFTSE, VDAX, VX1 and VX6) are estimated as the square root of the model free implied variance. The Demeterfi *et. al.*(1999) fair value of future variance method has been shown by Jiang and Tian (2007) to be identical to the model free implied variance.

found that typically the spot VIX computed from option prices embeds volatility risk premium and exceeds expected realized volatility obtained under the P-measure. During turbulent market conditions, the value of traded option based volatility index goes up. However, the lack of robustness in the MFIV method for the VIX first identified by Jiang and Tian (2007), expecially under extreme market conditions, has implications for mispricing volatility derivatives such as VIX futures, options and variance swaps. Mencía and Sentana (2011) investigate the mispricing of volatility derivatives during the recent crisis. There is a large and growing literature on information from traded option implied distribution volatility indexes for their capacity (see, Giamouridis and Skiadopoulos, 2010, for a recent survey) to forecast future realized volatility and other statistics on the underlying asset. In recent conditions of severe market distress, Andersen *et. al.* (2011) have noted discrepancies in the intraday VIX in not showing a consistent inverse relationship with the underlying stock index, a condition that the 'fear guage' should satisfy especially during turbulent market conditions.

The objective of this paper is to use the extreme market volatility of about 30%-80% recorded in all the major stock index (daily) returns during the recent subprime financial crisis from mid 2007 to mid 2009 to test out the efficacy of differently constructed IV indexes to forecast realized volatility both in so called normal market conditions when volatility is no more than about 20% and during extreme market conditions. For this we analyse data on the FTSE-100 and its model free VFTSE volatility index from January 2000 to June 2009. The paper aims to test the MFIV using the VFTSE and to propose alternative IV models that can specifically deal with the interspersed nature of relatively calm periods with periods of extreme volatility analytically derived from a closed form option pricing result of Markose and Alentorn (2011) using the Generalized Extreme Value risk neutral density (GEV-RND) can overcome the well known problems of MFIV and other extant methods of dealing with time varying tail shape of RND and resulting normal and extreme implied volatilities. The issues involved here are briefly reviewed below.

Figure 1: FTSE 100 index level and FTSE-100 volatility index, VFTSE (January 2000-June 2009) Right hand side axis the VFTSE levels and the Left Hand side Axis the FTSE-100 levels



Figure 1 plots the FTSE 100 index level (blue) and its volatility index, VFTSE (green), from 4th January 2000 to 1st June 2009. It shows that in relatively calm periods, the VFTSE volatility index ranges between 10% to about 20%. However, there are also some spikes in the VFTSE series. On 11th September 2001 (9/11), VFTSE spiked at around 50%, and during the American invasion of Iraq in March 2003, VFTSE peaked at over 40%. The spike points of VFTSE during the crisis of autumn 2008 have been much higher than any recent market down turn. The recent crisis has manifested in extreme spikes in VFTSE at about 55% on the 15th September 2008 corresponding to Lehman Brother Bankruptcy and near 80% on the 28th October 2008. At about the latter spike of the VFTSE, the FTSE-100 records the first of its extreme minima followed by its all time low of this period in early March 2009.

Engle (2010) has stated that "this crisis involved 99% confidence set of events".

Model-free and non-parametric methods, in general, for option price implied statistics which rely on sparse data for options traded at very high or very low strike prices may not be able to capture extreme tail behaviour around the 99% confidence level of the underlying returns data. Hence, parametric models become unavoidable, replacing sampling error with model error, Markose and Alentorn (2011). The CBOE MFIV which has increasingly become industry standard globally has been called into question by Jiang and Tian (2007). They identify so called truncation and discretization errors in the CBOE procedure. Truncation errors arise from ignoring strike prices beyond the range of listed strike prices and discretization errors are ascribed to an *ad hoc* numerical integration scheme to fill in discrete data points for the strike prices. Using simulated option price data for listed strike prices on a typical trading day Jiang and Tian (2007) show that the CBOE model-free method for the VIX can lead to an underestimation of the true volatility by about 198 basis points and overestimation by 79 basis points.⁴ The worrisome point is that when the true volatility is high, the truncation errors kick in with large undestimation of implied volatility by the CBOE method. The fact that inadequate asset pricing models that failed to capture extreme market price drops contributed to chronic underpricing of credit risk and market risk that characterized the lead up to the recent financial crisis failure should add to the urgency of the Jiang and Tian (2007) agenda to improve accuracy of risk neutral pricing models for volatility.

Andersen and Bondarenko (2007) have aptly identified VIX and other MFIV as corridor implied volatility (CIV) measures with barriers set at the lowest (K_I) and the highest (K_H) strikes being used on a given day to compute the index. Andersen *et. al.* (2011) note that there is a lack of coherence in the VIX method on how return variation over the tail areas ([0; K_I] and [K_H; ∞]) is accounted for at different points in time.⁵ Andersen *et. al.* (2011) specify the use of a ratio statistic which indicates how far into the tail a given strike price K is and they propose a 'coherent' method for the selection of truncation strikes which is defined as the inverse function of a fixed percentile (typically [0.01, 0.99]

⁴ Jiang and Tian (2007) state that these translate to dollar values that range between - 1,980 and +£790 per contract.

⁵ The well known way of dealing with this is to take an effective range of moneyness with the highest and lowest strikes given as a ratio of at- the- money Black-Scholes implied volatilities, Figlewski (2002).

or [0.03,0.97]) of the ratio statistic. Recently, Extreme Value Theory (EVT) is being used to model the return variation in the tail areas under a Q risk neutral measure. In order to overcome *ad hoc* truncations and/or extrapolations into the tails of the option price implied risk neutral density (RND), Figelwski (2010) gives a parametric solution to an otherwise non-parametric model for the rest of the RND by using the Generalized Extreme Value (GEV) distribution for the tails of the RND. Bollerslev and Todarov (2011) specify a semi-parametric Lévy density function based on the Generalized Pareto Distribution to model the left and right tails of the returns distribution under both Q and P measures in the context of capturing extreme movements in the variance risk premium. In Bollerslev and Todarov (2011) the generalized method of moments is used to estimate the Q tail parameters from observed option prices with the log moneyness set at 0.9 and 1.1 for the left and right tails, respectively.

In this paper, in order to solve the problems encountered in the CBOE MFIV, we specify two methods. In the first method we modify the CBOE MFIV by following the steps taken by Carr and Wu (2009) (hence CW-IV) in the discretization needed for synthesizing the variance swap rate, based on interpolation and extrapolation in the implied volatilities at different moneyness levels of the options. We use the cubic spline interpolation recommended by Jiang and Tian (2007) and retain a fixed 8 times average implied volatility extrapolation scheme from Carr-Wu (2009). These adjustments can improve the accuracy of the IV calculation, especially in combating the problem of sparse data points at extreme tail regions. However, the limitation of this method is that it requires a pricing model such as Black (1976) to derive the implied volatilities for interpolation and extrapolation. This makes the approach no longer 'model-free' and runs the risk of model error. Further, there are issues relating to the construction of fixed 30 day horizon IV which follows the CBOE MFIV method of using options of only two maturities. The linear interpolation of the IVs of the closest and the second closest to maturity options that include the 30 day horizon assumes a linear term structure for implied variance in option maturity. Also there are roll over effects on IVs when near (and second) maturity month options are switched at about 7 days to maturity.

The second IV model we consider is based on the recently developed Markose and Alentorn (2011) closed form solution for option pricing using the Generalised Extreme Value (GEV) distribution for the risk neutral density. It is a parametric model which relies on the scale and tail shape parameters of the GEV distribution. Depending on the tail shape parameter which also controls the size and skew of the tails of the distribution, the GEV distribution subsumes the three classes of distributions. A zero value for the tail shape parameter yields the Gumbel class (which includes the normal, exponential and lognormal distributions) which have zero skew in the probability mass and symmetry in right and left tails. Positive value for the tail shape parameter yields the so called Fréchet class that is able to capture the fat tailed behaviour with the maxima of a stochastic variable. Negative value for the tail shape parameter yields the "reverse" Weibull class of distribution for the corresponding minima of the stochastic variable. Larger the non-zero values for the tail shape parameter lead to increased higher moments including the variance, skewness and kurtosis of the GEV distribution. As the selection of the tail shape parameter is not restricted apriori but is backed out from the traded option price data, the GEV model mitigates model error.

Markose and Alentorn (2011) find that GEV-RND yields results that strongly challenge traditionally held views on tail behaviour of asset returns based on Gaussian distributions which predicate simultaneous existence of thin tails in both directions during all market conditions. The GEV RND which is governed by the option price implied tail shape parameter is found to switch tail shape with underlying market conditions. A non-zero value for the tail shape parameter results in significant skewness in the probability mass of the GEV density function during extreme market conditions which implies large one directional movements and truncation in the probability mass in the other direction. During extreme market turbulence, a positive value for the tail shape parameter of the GEV RND function for losses implies extreme price drops, while a large negative tail shape value signal large price increases. These switches in the implied tail shape parameter result in a much larger GEV-IV than can be obtained by other parametric and non-parametric methods during turbulent market conditions. Likewise, during normal market conditions, a close to zero or negative tail shape around -.3 for the GEV-RND

implies a much smaller GEV-IV than what is obtained by most other methods. To date, proposed option pricing models intended to deal with extreme and asymmetric volatility, fat tails and the skew in asset returns have failed to highlight the above characteristic features of the GEV-RND.⁶ Thus, as first demonstrated in Markose and Alentorn (2011), GEV-RND can capture both market perceptions of fat tailed behaviour, as well as, expectation of calmer periods consistent with the Gumbel class of distributions. Remarkably, this is achieved with none of the truncation and extrapolation exercises to capture extreme implied volatility that is encountered in model-free methods.

Markose and Alentorn (2008) also find that the GEV RND yields better estimates for high quantile extreme Value-at-Risk for a 10 day constant horizon than a number of parametric and non-parametric methods for this. The success in overcoming option maturity effects in order to report constant horizon implied GEV-RND, its quantile and other implied statistics such as volatility on a daily (or an intra daily) basis comes from using all maturities for traded options and explicitly backing out the term structure parameter to scale GEV volatility, Alentorn and Markose (2006). We will adopt this method to address problems of roll over effects of close to maturity option contracts and of assuming a linear scaling of the implied variance with time.

The suitably refurbished Carr and Wu (2009) modification for the CBOE method, CW-IV, the GEV-IV and the Black-Scholes IV (BS-IV) are constructed for a 30-day fixed horizon based on traded option price data on the FTSE-100. For realised volatility of the FTSE-100, we use the square root of the annualised 30 day daily squared returns (see, Siriopoulos and Fassas, 2008). We find that the volatility indices from the GEV model and the Carr-Wu method perform better than the Euronext VFTSE and the BS-IV for forecasting realised volatility, especially during the recent financial crisis of 2007-2008. Though we primarily investigate the forecasting performance of the

⁶ Other parametric option pricing models that aim to capture leptokurtosis, left skew and extreme volatility often start with specific fat tailed distributions for the RND such as skewed Student –t distribution, de Jong and Huisman (2000), or Weibull distribution, Savickas (2002) and hence run into model error. They also fail to have closed form solutions or end up over-fitting with far too many parameters. For example, the latter case is the mixture of two log-normals which has to estimate five parameters, Gemill and Saflekos (2000).

different IV models, the benchmark case of lagged realized volatility is also given. When the data is broken down into subsamples of normal and extreme market conditions, the GEV-IV yields the best forecast performance, in all cases, followed by the CW-IV. The GEV-IV performs best due to its explicit reliance on the implied tail shape parameter and implied volatility term structure parameter which can rapidly and flexibly reflect extreme and normal market sentiments being impounded in traded options.

The rest of this paper is organised as follows. Section 2 provides a literature review on forecasting future realised volatility with implied and historical volatility measures. Sections 3 and 4 introduce the two different methodologies (Carr-Wu and GEV) for the construction of the implied volatility index. Section 5 estimates the realized volatility and discusses the empirical results on the forecasting comparisons for VFTSE, Carr-Wu IV, BSIV and past realized volatility, RV. Both univariate and encompassing regressions are used to test the informational content of the IV and past RV models. Section 6 gives concluding remarks.

2. Review of Forecasting Future Realised Volatility with Implied Volatility

Discussions on traded option implied volatility and its efficacy in forecasting realized volatility has dominated the literature by far (see, Poon and Granger, 2003) though the role of option implied statistics for their capacity to incorporate market information is growing (see, Giamouridis and Skiadopoulis, 2010). The other main contender for volatility forecasting are times series based historical volatility, HV, models. Empirically, a regression setting is used to test whether the IV or HV measures are an unbiased and information efficient forecast of future realized volatility. Unbiasedness is typically assessed by examining the regression coefficients (intercept=0 and slope=1) of a univariate regression equation. Using encompassing regression equations, a forecast based on IV or HV measure is defined to be information efficient if it is not subsumed by other forecast variables. In all cases, efficiency of forecasts requires residual forecast errors to be white noise. Although the conclusions are somewhat contradictory and varied, most authors contend that option implied volatility measures. Within the

class of IV measures, while Jiang and Tian (2005) found the CBOE MFIV forecasts better than model based IV such as BS-IV, more recent papers find that modified model based IV measures including the Black (1976) IV model and the new Corridor Implied Volatility of Andersen and Bonderanko (2007) can out-perform MFIV constructed by the different authors and also the industry standard ones such as VIX.

Some of the early papers in this area find that the implied volatility is poor at forecasting future realised volatility (see, Canina and Figlewski (1993) and Lamoureux and Lastrapes (1993)). However, Christensen and Prabhala (1998) have countered their results by stating that the early studies are hampered by poor or insufficient data sets and lack a proper method for measuring realized volatility. They proceed to use non-overlapping data and a longer time series and show that the implied volatility outperforms historical time series based realised volatility such as ARCH and GARCH in forecasting future realised volatility.

Fleming, Ostdiek, and Whaley (1995) show that the VXO is strongly related to its future realised volatility and it forecasts realised volatility better than a first order autoregressive volatility model. However, the forecast result of the VXO has an upward bias. Blair, Poon, and Taylor (2001) show that the VXO provides almost all relevant information for forecasting index volatility from one to twenty days. Additionally, they find that the VXO gives more accurate forecast results compared with volatility constructed by Risk Metrics and GARCH type models. Giot (2003) investigates forecasting volatility and market risk with the VIX and VXN. The findings show that the volatility index has a higher information content than Risk Metrics and GARCH models at different time horizons.

There are also similar results for forecasting the future realised volatility with the volatility indices for stock markets in countries other than the US. For example, Franck, Patrick, and Christophe (1999) study the Marché des Options Négociables de Paris (MONEP) Market Volatility Index (VX1) and show that the VX1 is highly related with realised volatility and performs well in predicting future realised volatility with different

time horizons. The study by Siriopoulos and Fassas (2008) on the FTSE-100 volatility index VFTSE uses the same underlying as we do. They show that the VFTSE is a biased estimate of future realised volatility but that it includes more information on future realised volatility than historical volatility based methods.

There are also some opinions contrary to the above. Areal (2008) shows that the high frequency data based volatility of the FTSE-100 index gives a better forecast for the future realized volatility than the implied volatility indices constructed by the author. The latter were found to contain information on future volatility but they yield biased measures of future volatility. Becker, Clements, and White (2007) and Becker and Clements (2008) show that the model free VIX cannot offer additional information on volatility forecasts of the S&P 500 market compared with a combination of historical model based forecasts. However, no single historical model based forecasts is necessarily better than the VIX.

When looking exclusively at the forecasting performance of MFIV, the first study of this is that of Jiang and Tian (2005) on the SP-500 for the period 1988-1994. They find that the volatility forecast using MFIV outperforms both BS-IV and past RV. In the case of the MFIV constructed by the authors Cheng and Fung (2012) for the Hang Seng Index, they find that both the MFIV and the futures prices based Black (1976) IV outperform a number of time series based historical volatility (TS-HV) models in terms of their forecast power of realized volatility. Cheng and Fung (2012) find that futures prices based Black-IV subsumes the information content of both MFIV and TS-HV. Andersen and Bondarenko (2007) find similar results to Cheng and Fung (2012) that futures prices based Black-IV dominates MFIV and the VIX while their new Corridor Implied Volatility subsumes all others. However, as we discussed in the introduction, the robustness of the widely used industry standard MFIV is itself in question especially as the method exhibits truncation errors which leads to an underestimation of volatilty during market downturns, Jiang and Tian (2007), the focus of this paper is to contrast IV models that are specifically built to address this and those that are not. In the following, we introduce two alternative methods to construct implied volatility indices that can deal with extreme market conditions to see if they can improve on the model free method.

3. Carr-Wu Implied Volatitily With Spline Interpolation and Linear Extrapolation

As we intend to use the Carr-Wu (2009) discretization and truncation scheme to overcome problems in the CBOE MFIV, we will first briefly outline the CBOE construction. The generalized formula (see, CBOE VIX White Paper, 2003) used for the CBOE implied variance is:

$$\sigma^{2} = \frac{2e^{rT}}{T} \sum_{i} \frac{\Delta K_{i}}{K_{i}^{2}} Q(T, K_{i}) - \frac{1}{T} \left[\frac{F}{K_{0}} - 1 \right]^{2}.$$
 (1)

Here *T* is the time to maturity of the index option, *r* is the risk-free interest rate, *F* is the is the maturity matched forward index level, K_0 is the first strike below $F(K_0 \le F)$, $Q(T,K_i)$ is the midpoint of the bid-ask spread for the call and put options with strike price K_i where out-of-the-money call options are used if $K_i > K_0$, out-of-the money puts are used if $K_i < K_0$ and both calls and puts are used for $K_i=K_0$. Note, ΔK_i is the strike increment calculated as

$$\Delta K_{i} = \frac{K_{i+1} - K_{i-1}}{2} \; .$$

At the lowest or the highest strike prices, the strike price increment is simply the difference between the two two lowest and highest strike prices, respectively. The put-call parity condition is used to get the maturity matched forward index level at closest and second closest maturity dates j = 1,2

$$F_{t,T_j} = e^{\mathbf{r}(T_j - t)} \left(\prod_{t,T_j}^{C} (K *) - \prod_{t,T_j}^{p} (K *) \right) + K *.$$
(2)

Here K * is the strike price for which the call option price has the smallest difference with the put option price and $\prod_{t,T_j}^{c} (\prod_{t,T_j}^{p})$ are the call (put) option prices at $j = 1,2.^{7}$ The second term in (1) implements a correction for the discrepancy between the forward

 $[\]frac{1}{7}$ Closest to maturity options must have at least one week to expiration.

price and K_0 . To obtain a fixed 30 day horizon MFIV, the implied variance in (1) is evaluated at closest maturity date if this is greater than 30 days. If there are fewer than 30 days and next-term options have more than 30 days to expiration, the resulting MFIV reflects an interpolation of the implied variance in (1) for j = 1,2, viz. σ_1^2 and σ_2^2 . This will be defined in (8). Clearly, as the MFIV on the FTSE-100 is readily available in the form of VFTSE, we use this directly.

However, there are further procedures in the CBOE VIX White Paper (page 6) in the selection of the calls and puts with respect to the strike prices needed for equation (1) which we give below having fully expanded $Q(T, K_i)^8$:

$$\sigma_{j}^{2} = \frac{2e^{rT_{j}}}{T_{j}} \sum_{K_{i} < K_{0}} \frac{\Delta K_{i}}{K_{i}^{2}} \Pi_{\tau_{j}}^{C}(K_{i}) + \frac{2e^{rT_{j}}}{T_{j}} \sum_{K_{i} > K_{0}} \frac{\Delta K_{i}}{K_{i}^{2}} \Pi_{\tau_{j}}^{P}(K_{i}) + \frac{2}{T_{j}} \frac{\Delta K_{0}}{K_{0}^{2}} e^{rT_{j}} \Pi_{\tau_{j}}^{C}(K_{0}) + \frac{2}{T_{j}} \frac{\Delta K_{0}}{K_{0}^{2}} e^{rT_{j}} \Pi_{\tau_{j}}^{P}(K_{0}) - \frac{1}{T_{j}} \left[\frac{F}{K_{0}} - 1 \right]^{2} .$$

$$(3)$$

We now apply the steps used by Carr-Wu (2009) to address the discretization and truncation problems in (3). For this, first we take both in-the-money and out-of-the-money option prices and then estimate the implied volatility $\sigma_{t,T}^{B}(K_i)$ with all available option prices $\Pi_t(K_i)$ corresponding to the strike prices K_i using the Black (1976) futures option pricing model expressed in moneyness levels:

$$\Pi_{T_i}^C(k_i) = F(N(d_1(k_i) - e^{-k_i}N(d_2(k_i)))) , \qquad (4.a)$$

$$\Pi_{\pi}^{P}(k_{i}) = -F(N(-d_{1}(k_{i}) + e^{-k_{i}}N(-d_{2}(k_{i})))$$
(4.b)

⁸This can be found at <u>www.cboe.com/micro/vix/vixwhite.pdf</u>. The bids of all the option prices must be non-zero. The CBOE method first sorts the call options with their strike prices ranked from low to high. Then it selects call options with strike prices greater than the at- the- money- strike price, K_0 . A similar method applies when selecting put option prices. Finally, at K_0 both call and puts are selected. The truncation of out- of- the- money put prices take the following form : if two puts with consecutive prices with zero bid prices are found then no puts with lower strikes are considered. Likewise for out- of- themoney calls, once two consecutive call options with zero bid prices are found, no calls with higher strikes are considered.

Here
$$d_1(k_i) = \frac{\ln k_i + \sigma^2 T_j / 2}{\sigma \sqrt{T_j}}$$
 and $d_2(k_i) = d_1(k_i) - \sigma \sqrt{T_j}$.

The moneyness level with all available strike prices K_i and the futures price F_{t,T_j} is defined as

$$k_i = \ln \frac{\kappa_i}{F_{t,T_j}}.$$
(5)

For each moneyness level k_i , there is a corresponding Black (1976) implied volatility $\sigma_{t,T_j}^{B}(k_i)$. Based on the moneyness level from low to high, we can use interpolation to generate very fine grids. We define the upper and lower bounds of the moneyness level grids as

$$up\tilde{k} = 8\sigma \# + \tilde{k} *, \tag{6.a}$$

and

$$low\tilde{k} = -8\sigma \# + \tilde{k} *, \tag{6.b}$$

where, σ # denotes the average of all available implied volatilities for options with the respective selected maturities, \tilde{k} denotes the generated artificial grids, and $\tilde{k} *= ln \frac{\tilde{k}_*}{F_{t,T_j}}$. The maximum and minimum available moneyness levels are denoted as k_{max} and k_{min} .⁹ Clearly extrapolations into the tails using (+/-)8 times Black (1979) average implied volatilities σ # in (6a,b) will reflect the market information for the time varying nature of the statistic on a daily basis and counter some of the criticism directed at the lack of 'coherence' that comes from the CBOE method (see, footnote 8) of moving into deep OTM tail regions. Andersen and Bondarenko (2011) state that OTM options that are excluded after two consecutive zero bid quotes are encountered "induces randomness" in the effective strike range.

We apply a cubic spline interpolation instead of linear interpolation used in Carr and Wu (2009) to calculate the implied variance $\tilde{v}_{t,\tau_j}^{CW}$ for each corresponding moneyness level

⁹Carr and Wu (2009) use 8 times the average implied volatility to make sure the strike price range is big enough. We also find 8 is a proper magnitude.

between k_{min} and k_{max} .¹⁰ For the moneyness level outside k_{min} and k_{max} , we apply a flat extrapolation using the range in (6a,b). For all moneyness levels lower than k_{min} , we use the implied volatility of k_{min} . Likewise, for all moneyness levels higher than k_{max} , we use the implied volatility of k_{max} . With this interpolation and extrapolation method we can have implied volatilities which cover a wide range and also have small strike price intervals and hence achieve a more precise approximation result for the CBOE replication equation (1). On substituting (4.a.b) into (3) and using (5) and (6a,b) we have the annualised implied variance, \tilde{v}_{t,T_j}^{CW} , with time to maturity from t to T_j :

$$\begin{split} \tilde{v}_{t,T_{j}}^{CW} &= \frac{2}{T_{j}-t} \sum_{\tilde{k}_{i} < k_{0}} \left(-e^{-\tilde{k}_{i}} N(-d_{1}(\tilde{k}_{i})) + N(-d_{2}(\tilde{k}_{i})) \right) \Delta \tilde{k}_{i} \\ &+ \frac{2}{T_{j}-t} \sum_{\tilde{k}_{i} > k_{0}} \left(e^{-\tilde{k}_{i}} N(d_{1}(\tilde{k}_{i})) - N(d_{2}(\tilde{k}_{i})) \right) \Delta \tilde{k}_{i} \\ &+ \frac{2}{T_{j}-t} \left(e^{-\tilde{k}_{0}} \left(N(d_{1}(\tilde{k}_{0})) - N(-d_{1}(\tilde{k}_{0})) \right) - N \left(d_{2}(\tilde{k}_{0}) \right) \right) \\ &+ N \left(-d_{2}(\tilde{k}_{0}) \right) \Delta \tilde{k}_{0} - \frac{1}{T_{j}-t} \left(\frac{1}{e^{\tilde{k}_{0}}} - 1 \right)^{2}. \end{split}$$
(7)

Here

$$\begin{split} d_1(\widetilde{k}_i) &= \frac{-\widetilde{k}_i + \left(\widetilde{\sigma}_{t,T_j}^B(\widetilde{k_i})\right)^2 (T_j - t)/2}{\widetilde{\sigma}_{t,T_j}^B(\widetilde{k_i})\sqrt{T_j - t}}, \\ d_2(\widetilde{k}_i) &= d_1(\widetilde{k}_i) - \widetilde{\sigma}_{t,T_j}^B(\widetilde{k}_i)\sqrt{T_j - t}, \end{split}$$

and \tilde{k}_0 denotes the moneyness level corresponding to at- the- money- strike price, K_0 .

In order to calculate the 30-day implied volatility, we follow the CBOE method of interpolating the implied variance \tilde{v}_{t,T_j}^{I} from the two closest maturities. If the closest maturity, T_I , is equal to or greater than 30 calendar days, $N_{T_1} \ge 30$, then we use the closest implied variance to the 30 calendar day implied variance of market. If $N_{T_1} < 30$ and $N_{T_2} > 30$, then a linear interpolation between these two is taken. The annualised 30-day implied volatility is the square root of the Carr-Wu implied variance \tilde{v}_{t,T_j}^{CW} in (7):

¹⁰The spline interpolation is also recommended by Jiang and Tian (2007) in order to avoid arbitrage opportunities.

$$IV_{t,t+30}^{CW} = \sqrt{((T_1 - t)\tilde{v}_{t,T_1}^{CW}\frac{N_{T_2} - 30}{N_{T_2} - N_{T_1}} + (T_2 - t)\tilde{v}_{t,T_2}^{CW}\frac{30 - N_{T_1}}{N_{T_2} - N_{T_1}})\frac{365}{30}}.$$
 (8)

4. Implied Volatility with Generalised Extreme Value (GEV) Option Pricing Model In this section we will derive an implied volatility index that is based on the GEV option pricing model which can respond flexibly to both extreme market as well as normal market conditions, Markose and Alentorn (2011). The GEV option pricing model offers a more accurate and empirically grounded approach to price derivatives, hedge volatility and estimate value at risk (VaR) under real world conditions as the log-normal distribution of returns on underlying financial assets has been found not to hold. The returns on financial assets are found to have fat-tails, skewness and other stylised facts regarding time varying volatility. Compared with BS model and other parametric models which make restrictive distributional assumptions, the GEV-RND function can capture the stylised facts on the non-normal skewness and kurtosis without any apriori restrictions on the class of distribution being implied for the underlying. Also using the Alentorn and Markose (2006) method described below, we estimate the constant horizon 30-day implied volatilities of the FTSE 100 from the implied RND term structure of the GEV model using both put and call options of all maturities on a daily basis. With an empirically obtained time varying term structure parameter in traded option maturity for the GEV RND, this model can capture the implied volatility for any time horizon without assuming the square root law where volatility scales with the square root of time.

4.1 GEV- RND and Option Pricing Model

Following the Harrison and Pliska (1981) arbitrage free option pricing result, under a risk neutral measure, Q, the call option at current time t and with maturity time T can be priced using a risk neutral density (RND) function $g(S_T)$:

$$\Pi_{t,T}^{C}(K) = e^{-r(T-t)} E_{t}^{Q} [\max(S_{T} - K, 0)] = e^{-r(T-t)} \int_{K}^{\infty} (S_{T} - K) g(S_{T}) dS_{T}$$
(9)

The corresponding put option pricing function is given as:

$$\Pi_{t,T}^{P}(K) = e^{-r(T-t)} E_{t}^{Q} [\max(S_{T} - K, 0)] = e^{-r(T-t)} \int_{0}^{K} (K - S_{T}) g(S_{T}) dS_{T}$$
(10)

Here, E_t^Q refers to the expectations operator under the risk neutral Q measure. In an arbitrage-free economy, the following martingale condition must also be satisfied:

$$S_{t} = e^{-r(T-t)} E_{t}^{Q}(S_{T}).$$
(11.a)

Using the price of index futures, $F_{t,T}$, which expire at the same date as the option, this condition yields

$$F_{t,T} = E_t^Q(S_T). \tag{11.b}$$

In keeping with the extreme value distribution modelling of economic losses¹¹, we assume negative returns defined as $L_T = -R_T = 1 - \frac{s_T}{s_t}$ follows the GEV density function for the tail shape parameter $\xi \neq 0$ (see, Reiss and Thomas, 2001, p. 16-17):

$$f(L_T) = \frac{1}{\varphi} \left(1 + \frac{\xi(L_T - \mu)}{\varphi} \right)^{-1 - 1/\xi} \exp \left(- \left(1 + \frac{\xi(L_T - \mu)}{\varphi} \right)^{-1/\xi} \right)$$
(12)

Note, the relationship between the density function for L_T and the RND function $g(S_T)$ for the underlying price S_T is given by the general formula (see, Baz and Chacko, 2004) $g(S_T) = f(L_T) |\frac{\partial L_T}{\partial S_T}| = f(L_T) \frac{1}{S_t}$. This yields the GEV RND function :

$$g(S_T) = \frac{1}{S_t \varphi} \left(1 + \frac{\xi(L_T - \mu)}{\varphi} \right)^{-1 - 1/\xi} \exp \left(- \left(1 + \frac{\xi(L_T - \mu)}{\varphi} \right)^{-1/\xi} \right) \quad (13.a)$$

With
$$1 + \frac{\xi}{\varphi}(L_T - \mu) = 1 + \frac{\xi}{\varphi} \left(1 - \frac{S_T}{S_t} - \mu \right) > 0$$
 for $\xi \neq 0$. (13.b)

¹¹ Since extreme economic losses are more probable than extreme economic gains, the fat tailed or Fréchet distribution is used to model extreme losses. To this end, we follow the practice of the insurance industry, Dowd [2002, p 272], and model returns as negative returns. This has also been discussed in Figlewski (2010).

Here, μ is location and φ is the scale parameter of the GEV distribution. Further, ξ is the tail shape parameter which governs whether the GEV distribution belongs to the Frechet ($\xi > 0$) class or the reverse Weibull class ($\xi < 0$). As discussed at length in Markose and Alentorn (2011), the implied GEV tail shaped parameter changes with market conditions. In distributions for which $\xi \neq 0$, the condition in equation (13.b) imposes a truncation on the probability mass and a distinct asymmetry in the right and left tails such that when the probability mass is high at one tail signifying non-negligible probability of an extreme event in that direction, there is an absolute maxima (or minima) in the other direction beyond which values of S_T have zero probability.¹² Only the case where the tail shape parameter $\xi=0$ yields thin tailed distributions belonging to the Gumbel class with the tail index $\alpha = \xi^{-1}$ being equal to infinity, implying that all moments of the distribution are either finite or zero. The Gumbel class has zero skew in the probability mass and displays symmetry in the right and left tails and there is no condition truncating the distribution in either direction for values of S_T.

On substituting (13.a) into (9) and using the constraint in (13.b) to get the upper limit of integration for the option price, a change of variable operation for the integration enables Markose and Alentorn (2011) to obtain a closed form solution for a GEV RND based option price using incomplete and generalized incomplete Gamma functions. Omitting the proof which can be found in Markose and Alentorn(2011), the GEV- RND call option price is given as

$$\Pi_{t,T}^{C}(K) = e^{-r(T-t)} \left\{ S_{t} \left((1 - \mu + \varphi/\xi) e^{-H^{-1/\xi}} - \frac{\varphi}{\xi} \Gamma(1 - \xi, H^{-1/\xi}) \right) - K e^{-H^{-1/\xi}} \right\}$$
(14)

¹² As shown in Reiss and Thomas (2001), kurtosis of the Fréchet distribution becomes infinite at $\xi > 0.25$ (the tail index, $\alpha < 4$), and all higher moments including kurtosis and the right skew become infinite at $\xi \ge 0.33$ (the tail index, $\alpha \le 3$). Even for small positive values of ξ , approximately at about $\xi = 0.1$, the rate of growth of skewness and kurtosis of the distribution, with both fast approaching infinite growth rates, result in a concentration of the probability density of the Fréchet distribution at the right tail. Markose and Alentorn (2011), and also see Figure 6 in this paper which reports the implied GEV tail shape parameter for the sample period analysed here, find that even during extreme market conditions, though the implied tail index results in fat tails for the GEV-RND – at all times the first four GEV-RND implied moments were finite.

Here, $\Gamma(1-\xi, H^{-1/\xi}) = \int_{H^{-1/\xi}}^{\infty} z^{-\xi} e^{-z} dz$ is the incomplete gamma function, and

 $H = 1 + \frac{\xi}{\varphi} \left(1 - \frac{K}{S_i} - \mu \right).$ The key to understanding the GEV option pricing formula lies

with the term, $e^{-H^{-1/\xi}} = e^{-\left(1+\frac{\xi}{\varphi}\left(1-\frac{K}{S_t}-\mu\right)\right)^{-1/\xi}}$. This term is the cumulative GEV distribution function given as the "standardized moneyness" or the percentage pay-off from the option defined as $(S_t - K)/S_t$. Hence, it corresponds to the risk neutral probability of the call option being in- the- money at maturity.¹³ For a given set of implied GEV parameters { μ, φ, ξ }we can work out the range of exercise prices *K* in relation to the given S_t which yield: $e^{-H^{-1/\xi}} = 1$ for deep in-the-money call options, $e^{-H^{-1/\xi}} = 0$ for deep out-of-the-money call options, and $0 < e^{-H^{-1/\xi}} < 1$ for all other cases.

The corresponding GEV- RND based put option price is given by

$$P_{t}(K) = e^{-r(T-t)} \left\{ K(e^{-h^{-1/\xi}} - e^{-H^{-1/\xi}}) - S_{t}\left((1 - \mu + \varphi/\xi) (e^{-H^{-1/\xi}} - e^{-h^{-1/\xi}}) - \frac{\varphi}{\xi} \Gamma(1 - \xi, h^{-1/\xi}, H^{-1/\xi}) \right) \right\}$$
(15)

where $h = 1 + \xi / \varphi (1 - \mu) > 0$ and $\Gamma (1 - \xi, h^{-1/\xi}, H^{-1/\xi}) = \int_{h^{-1/\xi}}^{H^{-1/\xi}} z^{-\xi} e^{-z} dz$ is the

generalized incomplete Gamma function (see, Markose and Alentorn, 2011).

Our next steps are analogous to the procedure in the Black-Scholes model where the mean of the distribution is replaced using the martingale condition in (11a,b) so that only the volatility parameter needs to be estimated. However, in the GEV case, neither the mean nor the volatility of the distribution directly correspond to the location, μ , and scale parameter, φ . As shown in Reiss and Thomas (2001), the mean of a GEV function can

¹³ Recall that in the case of the Black-Scholes model the probability of the option being in-the-money at maturity is given by N(d₂), where N() is the standard cumulative normal distribution function, and $d_2 = [\ln(S_t/K) + (r - \sigma^2/2)T]/\sigma\sqrt{T}$

$$m^{GEV} = \mu + (\Gamma(1-\xi)-1)/\xi)\varphi.$$
(16)

The GEV volatility for returns takes the form

$$\sigma^{\text{GEV}} = \left(\frac{\sqrt{\Gamma(1-2\xi) - \Gamma^2(1-\xi)}}{\xi}\right) \varphi .$$
 (17)

In order to construct constant horizon GEV RND and implied statistics, following Alentorn and Markose (2006), we propose that the GEV scale parameter φ is a function of time to maturity :

$$\varphi_{t,T} = \overline{\varphi}(T-t)^{b}, \qquad (18)$$

where $\overline{\varphi}$ is the annualised GEV scale parameter, and *b* is the parameter for the term structure of volatility that is implied by taking information from option prices for *all* maturities on a given day .¹⁴ Conventional literature assumes the square root of time rule for scaling volatility and it is widely used to scale up 1 day volatility to obtain volatility for N day returns. The square root scaling rule is only appropriate for time series that have Gaussian properties. In (18), the parameter b is not restricted to be 0.5. As will be seen in the next section the parameter b will be backed out of from traded options with all available maturities. Heuristically, b in (18) can be seen to be the fractal or Holder exponent (see, Calvet and Fisher, 2008) such that the expected variations under a risk neutral measure $E_t^Q/dS/ \approx (dt)^b$. Smaller values of *b*, *b*<0.5, as will be seen, coincide with abrupt changes in the volatility of market returns. Once the option implied term structure parameter b is obtained along with the other implied GEV parameters, the GEV

¹⁴See Alentorn and Markose (2006) for a full explanation of how the maturity effects are removed by using a term structure of RNDs which includes *all* strikes and maturities for traded options on a given day. Typically, as in the CBOE method discussed in Section 3, in order to report implied volatility on a daily basis for a fixed 30 day horizon, options prices with only two maturities that span the 30 day horizon are used. As shown in the next sub-section, we obtain a much simpler formula for a fixed horizon GEV implied volatility using (17) and (18) and having backed out the GEV RND implied parameters $\theta = (\xi, \overline{\varphi}, b)$ from (20).

implied volatility using (17) for any fixed time horizon can be obtained as a function of b by substituting (18) into (17). We will see that this GEV-RND based IV will be driven by the annualised GEV scale parameter $\overline{\varphi}$, the tail shape parameter ξ and b. Option traded implied values show $-0.3 \le \xi \le 0$ and value of $b \ge 0.5$ during normal market conditions. With the onset of market turbulence the absolute value of ξ values increases and b < 0.5.

Finally, we can use the martingale condition in (11a,b) and the definition in (16) of the GEV mean to yield the risk neutral expected value of the (negative) returns function,

$$E_t^Q(-R_T) = (1 - \frac{F_{t,T}}{S_t}) \text{ to express the GEV location parameter } \mu \text{ as follows:}$$
$$\mu = 1 - \frac{F_{t,T}}{S_t} - \left[\frac{\Gamma(1-\xi)-1}{\xi}\right]\varphi. \tag{19}$$

To satisfy a fixed horizon implied GEV mean free of maturity effects, φ in (19) is replaced by (18). This can now be substituted into the GEV option prices in (14) and (15) to eliminate the parameter μ .

4.2 30-Day GEV Implied Volatility

With the above closed form solutions for the call and put option prices, the parameters of the GEV option pricing model, $\theta = (\xi, \overline{\varphi}, b)$, are backed out from traded option prices. We constrain θ to be the same across all maturities and strikes for the available option contracts in any given day to remove maturity effects that normally prevail for option implied statistics (see, Alentorn and Markose, 2006). The sum of squared errors (SSE) between the analytical solution of GEV option prices and the observed traded option prices with all available strikes (N) and maturities (J)¹⁵ in the option markets is minimized with respect to θ :

¹⁵ This is subject to well known selection procedures of options contracts described in section 5.

$$SSE(t) = \min_{\theta} \left\{ \sum_{j=1}^{J} \sum_{i=1}^{N} \left(\prod_{t,T_{j}}^{GEV}(K_{i}) - \prod_{t,T_{j}}^{M}(K_{i}) \right)^{2} \right\}.$$
 (20)

Here, $\Pi_{t,T_j}^{GEV}(K_i)$ denotes the analytical GEV option price with maturity time T_j and strike price K_i , and $\Pi_{t,T_j}^M(K_i)$ denotes the observed traded option price with the same maturity time and strike price.¹⁶ From the above optimisation in (20), we obtain the implied GEV RND parameters $\hat{\theta} = (\hat{\xi}, \overline{\varphi}, \hat{b})$. The implied volatility for a time horizon given by T - t can be calculated using equation (17) and (19) :

$$IV_{t,T}^{GEV} = \left(\frac{\sqrt{\Gamma(1-2\hat{\xi}) - \Gamma^2(1-\hat{\xi})}}{\hat{\xi}}\right)\overline{\varphi}(T-t)^{\hat{b}}.$$
(21)

The prominent role of the GEV RND implied tail shape parameter ξ must be noted. As the ξ controls both shape and size of the tails of the GEV-RND and is time varying with the daily traded option data, we avoid the further calculations on how far into the tail to extrapolate or how to paste in the tail shape that model free and non-parametric option pricing models have to address mostly in an *ad hoc* way. Corresponding to the volatility indices launched for stock market indices, we construct a 30-day GEV-IV for the stock index by using the implied term structure parameter \hat{b} derived on each day to scale the required 30 day horizon given as a proportion of 365 days:

$$IV_{t,t+30}^{GEV} = \left(\frac{\sqrt{\Gamma(1-2\hat{\xi}) - \Gamma^2(1-\hat{\xi})}}{\hat{\xi}}\right)\overline{\varphi} \left(\frac{30}{365}\right)^{\hat{b}}.$$
 (22)

In order to compare it to the volatility index constructed with other methods, we multiply it with $\sqrt{\frac{365}{30}}$ to obtain an annualised 30-day implied volatility. Finally, the use of the implied term structure parameter \hat{b} for the GEV scale parameter φ obtained from the traded options of all available maturities, avoids issues of linear interpolation when exactly 30 day horizon contracts do not exist. Also there are no messy roll over effects

¹⁶This optimization is carried out with the non-linear least square algorithm of the interior-reflective Newton method described in Coleman and Li (1994) and Coleman and Li (1996).

when the closest maturity contract reaches 7 days to maturity.

5. Data Analysis and Empirical Construction of Implied Volatility for FTSE-100

The data period used in this paper is from 4th January 2000 to 1st June 2009.¹⁷ We use the spot price of FTSE-100 index level, annualised daily London Interbank Offered Rate (LIBOR), the FTSE-100 index future prices, and the daily settlement prices of the FTSE-100 index options. The futures and option data is from the London International Financial Futures and Options Exchange (LIFFE). There are four FTSE-100 futures contracts every year, which have maturity days on the third Friday of March, June, September and December. Options are all European style and their maturity days are the third Friday of their maturity months. The strike prices of the options have intervals of 50 or 100 points depending on the different time to maturity. The option price tick size (the minimum amount of the option prices can be changed) is 0.5 basis point. The money notion per basis point is £10. The time series for the Euronext model free VFTSE is available from Datastream.¹⁸ Note the FTSE 100 option prices used for calculating the Euronext VFTSE are mid-prices.¹⁹ For the calculation of the other volatility indices in this section the daily closing prices are used.

This study only uses traded option prices, viz. options that have non-zero traded volume on a given day. Also, options whose prices were quoted as zero, have less than one week to expiry, or more than 120 days to expiry were eliminated. Finally, option prices were checked for violations of the monotonicity condition.²⁰ A small number of option prices that did not satisfy this condition were removed from the sample.

The rest of this section is organised as follows: Subsection 5.1 gives the construction method and results of realised volatility for the FTSE-100 market. Subsection 5.2

¹⁷The VFTSE was launched on the 4th January 2000.

¹⁸A more detailed introduction to the VFTSE can be found on the website of the NYSE Euronext Volatility Indices (http://www.euronext.com/editorial/wide/editorial-3955-EN.html).

¹⁹See NYSE Euronext Volatility Indices methodology 2007

http://www.euronext.com/fic/000/035/208/352080.pdf).

²⁰ Monotonicity requires that the call (put) prices are strictly decreasing (increasing) with respect to the exercise price.

provides the results of four implied volatility indices for the FTSE-100. Subsection 5.3 shows the calculation and properties of volatility risk premium. Finally, subsection 5.4 gives the results of forecasting future realised volatility with the different constructed implied volatility indices.

5.1 Realised Volatility of FTSE 100

Realized volatility of stock index returns is a latent variable which in theory is the expost realized values of the expectation under the P measure, $E_{t,T}^{p}$, of the square root of the future quadratic variation of log prices over a time horizon, (t, T). As the sampling frequency increases, in the continous limit, following Andersen *et. al* (2003) and Barndorff-Nielsen and Shephard (2002), the quadratic variation is the integrated variance. Thus,

$$RV_{t,T} = E_{t,T}^{p} \sqrt{V_{t,T}} = E_{t,T}^{p} \sqrt{\int_{t}^{T} v_{s} ds} .$$
 (23)

Here $V_{t,T}$ is the quadratic variation of the log stock index from t to T and $\int_t^T v_s ds$ is the integrated variance. As we sample the observed price path over [t, T] on a daily basis, realized volatility defined as the square root of the quadratic variation of the log FTSE-100 spot index prices, involves taking the *ex post* cumulative sum of daily squared returns over [t, T]. The annualised 30 calendar days *ex post* realised volatility of the FTSE-100 can be calculated with a rolling window of the FTSE-100 index daily prices as

$$RV_{t,t+30} = \sqrt{\frac{365}{30} \sum_{i=1}^{30} (\log \frac{S_{t+i}}{S_{t+i-1}})^2} , \quad (24)$$

where S_t is the FTSE -100 index at time t.

Figure 2 shows the annualised *ex post* 30 day realised volatility of the FTSE-100 returns and Euronext VFTSE. These two time series have similar patterns. In most periods, especially during a boom, the value of realised volatility is lower than the VFTSE. However, exceptionally, there are also some periods where realised volatility can be higher than the VFTSE, for example, during the financial crisis at the end of 2008. The

sample statistics for RV and VFTSE are given in Table 2 along side those for the constructed IVs.



Figure 2: The realised volatility of FTSE-100 Returns and VFTSE

5.2 Construction of Implied Volatilities of FTSE 100

5.2.1 Carr-Wu IV Method

Following Section 3, we construct the CW- IV for the FTSE-100. We filter the option data using the CBOE VIX method. Table 1 gives the average number of different daily strikes with FTSE 100 call and put options for the nearest (1st) and the second nearest (2nd) maturities. Table A.1 in the Appendix reports the average annual call (Panel A) and put (Panel B) option prices and number of observations (in brackets) for six different categories of moneyness $(\frac{s}{k})$ and the two maturities. On average, the nearest maturity (1st) option prices are lower than the second nearest maturity (2nd) option prices for almost all moneyness categories for both calls and puts. For the CW-IV, we use all these option prices to get the Black implied volatilities, and interpolate and extrapolate to

get finely spaced implied volatility grids as explained in equations (4-6). We then proceed to construct the CW-IV for the FTSE 100 using equation (8).

Period	Average Number of Daily Strikes					
	1st Maturity	2nd Maturity				
2000	92	91				
2001	85	81				
2002	77	75				
2003	60	58				
2004	50	50				
2005	52	53				
2006	55	54				
2007	63	58				
2008	63	52				
2009 a	51	44				

Table 1: Yearly Average number of daily strikes

[a] The option data for 2009 only includes five months (2 January 2009 - 1 June 2009).

5.2.2 Implied Volatilities with GEV and BS (1973)

Following Section 4, for every trading day, the GEV implied parameters $\{\xi, \overline{\varphi}, b\}$ are derived from option prices using equation (20) and constructed to get the annualised 30-day GEV implied volatility with equation (22). In the latter, the constant 30-day horizon GEV-IV is scaled by using the implied term structure parameter b backed out in (20) from options with all maturities and strike prices available in the market, where their trading volumes are not zero. This is different from the VFTSE and the implied volatility model of Carr-Wu derived in section 3, which are calculated using at most only two options maturities that span the 30 day horizon. For the comparison, we also construct the standard IVs for the Black and Scholes (1973) model.

5.2.3 Statistical Analysis and Comparison

Figure 3 plots the time series of implied volatility indices for the FTSE 100 with the four different methods (VFTSE, Carr-Wu, GEV, and BS). These four indices show very

similar patterns with the most obvious visual detail being that the GEV-IV takes on the smallest values during normal market conditons and the biggest ones during turbulent conditions. From mid-2003 to mid-2007, the four volatility indices give corresponding low values, which are mostly below 20%. At the end of 2008, the GEV-IV and CW-IV peak at around 80% which is similar to the realized volatility in Figure 2. In contrast, VFTSE peaks at 75% and the BS-IV is the least volatile and peaks at 53%.

		Implied	Volatility		Realised Volatility
	VFTSE	BS-IV	GEV-IV	CW-IV	RV
Min	0.0910	0.0954	0.0566	0.0926	0.0512
Max	0.7554	0.5346	0.8229	0.8256	0.7973
Mean	0.2185	0.1982	0.2021	0.2178	0.1831
Std. Dev.	0.1021	0.0717	0.1016	0.1041	0.1155
Skewness	1.5379	1.1900	1.5888	1.6238	2.0594
Kurtosis	6.0321	4.6537	6.4667	6.3989	8.8822
ADF Test #	-3.7326	-3.0520	-3.8456	-3.7852	-2.2470

 Table 2: Summary Statistics of Implied and Realised Volatilities for FTSE 100

Augmented Dickey Fuller Test with critical value of -1.942 at 5% significance level (no constant and time trend)

Table 2 gives the sample statistics of the four implied volatility indices (VFTSE,

Carr-Wu, GEV, and BS) and of the realised volatility of the FTSE-100 The mean of the VFTSE is 0.2185, while that of the realised volatility is 0.1831. All the other constructed IV estimates share this property of having a mean greater than the mean of realised volatility. This is consistent with the findings (see, Anderson and Bondarenko, 2007, and Carr and Wu, 2009) on the negative volatility risk premum which is defined as the difference between the volatility derived under the P- measure and the volatility derived under the risk neutral Q-measure. The maximum values for the CW-IV and GEV-IV are greater than that for the RV while that for the VFTSE it is less at 0.755 and BS-IV peaks at only 0.53. Their minimum values are all less than 10%, with that for the GEV-IV at 5.6% being the closest to the minimum of the RV at 5.12%. BS-IV is much less volatile than the other three volatility indices, and its skewness and kurtosis are also the smallest of all other IV measures. This fully reflects the assumption of

log-normality of the model. The RV has kurtosis of 8.82 which is not matched by the kurtois of any of the IVs. GEV-IV is the closest at 6.46 while the VFTSE has a relatively low kurtosis of 6.03.

The augmented Dickey-Fuller (ADF) tests (without constant and time trend) in Table 2 show that the four volatility measures and RV all reject the null hypothesis of unit root at a 5% significance level. This implies that all of these volatility series are stationary. Also, we apply the Jarque-Bera test (not reported). All five volatility indices reject this at 5% significance level, indicating none of them is normally distributed. Siriopoulos and Fassas (2008) also applied the above statistical tests for the VFTSE levels, its changes and its log changes from February 2000 to May 2008, with similar results.²¹

Figure 3: The volatility indices of FTSE 100 with the four IV methods:VFTSE, Carr-Wu IV, GEV-IV, and BS-IV (Jan 2000- June 2009)



The following Figures 5 and 6, respectively, for the implied volatility term structure

²¹ We also do these tests for the changes and the log changes of the volatility series and find similar results.

parameter \hat{b} for the GEV-IV model and the time varying implied tail shape parameter, $\hat{\xi}$, highlight how GEV-IV produces the smallest values of all the four IVs during normal market conditions while it spikes up rapidly as conditions become turbulent. Using formula (22) for the 30 day horizon GEV-IV and as discussed in Section 4.1 relating to equations (17 and (18), in Figure 5 we see that the \hat{b} registers sudden plunges below 0.5 marking points of abrupt jumps in volatility in September 2001, June 2002, June 2006 and October 2008. The latter gives the lowest point for the implied volatility scale parameter \hat{b} at 0.223 and this coincides with the highest value attained by the GEV-IV. This indicates that the square root of time scaling rule which is true for Gaussian models and one that is used in almost all IV constructions, by permitting linear interpolations of implied variance obtained from option prices of the nearest and second nearest maturity contracts, has substantive implications that warrant closer scrutiny.

Figure 5 Option Implied Term Structure Parameter,b, for Scaling GEV-IV Time To Maturity (Jan 2000- June 2009)



Implied Term Structure Parameter: b

The implied tail shape parameter, ξ_i for the GEV-IV model plotted in Figure 6 shows

that the maximum value is given by $0 < \hat{\xi} < 0.12$. This indicates that the $\hat{\xi} \approx 0.1$ region, large but finite varaince, skewness and kurtosis exists for the GEV RND based returns through out the sample period. Further, $\hat{\xi} > 0$, are by and large a rare events, with normal conditions with $-0.3 < \hat{\xi} < 0$ being more the norm. Figure 6 shows that $\hat{\xi} \approx 0.1$ during the June/July 2004 correction of the FTSE-100 and then in mid 2005. From January 2007 to after the Lehman debacle in September 2008, $\hat{\xi} > 0$, showing market expectations of great turbulence and also large price falls. During the 2007 period, $\hat{\xi} \approx 0.1$ in March and June. In the period after October 2008, $\hat{\xi} < 0$ and with a very large negative $\hat{\xi} = -0.4$ occuring in December 2008, marks market expectations of large upswings in the FTSE-100 in early 2009. Note, large non-zero values for $\hat{\xi}$ result in increased GEV-IV as skewness and kurtosis also grow.





5.3 Forecasting Realised Volatility with Implied Volatility Indices in FTSE 100 Market

The implied volatility is the expectation of future realised market volatility under the risk neutral measure Q and hence we have the following relationship (see, Carr and Lee, 2007, Bollerslev, Tauchen, and Zhou, 2008):

$$E_t^Q \left(RV_{t,T} \right) = IV_{t,T}.$$
(25)

We expect that the implied volatility contains information on *ex post* realised volatility defined in (24). In order to verify this, typically linear regressions are run with implied volatilities as the dependent variable and realised volatility as the independent variable. We will do this for both the whole sample period and sub-periods.

5.4.1 Estimation for the Whole Sample Period

Univariate Regression

We first run two variants of ordinary least square (OLS) regressions for realised volatility with the four different IV indices we constructed as respective dependent variables and also the lagged RV. The first set of regressions are in levels and the second set is in logarithms of the variables:

$$RV_{m,m+1} = \alpha + \beta B_{m,m+1} + \varepsilon_m \tag{26}$$

and

$$lnRV_{m,m+1} = \alpha + \beta lnB_{m,m+1} + \varepsilon_{m'}$$
(27)

where $RV_{m,m+1}$ is the realised volatility at month *m*. $B_{m,m+1}$ stands for the four volatility indices $IV_{m,m+1}$ and lagged realised volatility $RV_{m-1,m}$, respectively. Christensen and Prabhala (1998) show that overlapping data would exhibit a large amount of autocorrelation and result in estimation problems and incorrect results. Therefore, we use non-overlapping RV and IV which are selected from the data once every month. We choose the last trading day of every month as the observation day. The implied volatilities (realised volatility) on that day represents the risk neutral expected volatilities (realised volatility) for the next month. We also use 1 month

lagged realised volatility to forecast realised volatility to provide a comparison with the IV based forecasts. We run five OLS regressions one for each of the four implied volatilities and the fifth one for the lagged realised volatility.

	α	β		E togt	DW	
	(p-value)	(p-value)		F -test	DW	
	[std error]	[std error]	Adj.R ²			
	(Conf. II	nterval)		(p-value)	(p -value)	
B _m		Panel A: R	$V_m = \alpha + \beta B_n$	_n +e _m		
RV _{m-1}	0.053	0.706				
	(0.0004)	(0.0000)	0.495	109.95	1.959	
	[0.0014]	[0.0064]		(0.0000)	(0.7000)	
VFTSE _m	-0.013	0.909				
	(0.4945)	(0.0000)	0.528	125.39	1.484	
	[0.0018]	[0.0077]		(0.0000)	(0.0029)	
BS-IV _m	-0.047	1.170		102 15	1 255	
	(0.0539)	(0.0000)	0.477	102.15	1.355	
	[0.0023]	[0.0109]		(0.0000)	(0.0002)	
GEV-IV _m	-0.001	0.930		127 59	1 527	
	(0.9484)	(0.0000)	0.552	137.30	1.327	
	[0.0016]	[0.0075]		(0.0000)	(0.0062)	
CW-IV _m	-0.009	0.893		131 14	1 527	
	(0.6364)	(0.0000)	0.539	131.11	1.521	
	[0.0017]	[0.0074]		(0.0000)	(0.0062)	

Table 4: The basic OLS regression estimation

lnB _m		Panel B: Inl	RV _m =α+βl	nB _m +e _m	
lnRV _{m-1}	-0.443	0.762		153 40	2 184
	(0.0003)	(0.0000)	0.578	155.40	2.104
	[0.0112]	[0.0058]		(0.0000)	(0.4154)
InVFTSE _m	-0.098	1.084		203.279	1.732
	(0.4426)	(0.0000)	0.645		
	[0.0120]	[0.0072]		(0.0000)	(0.1057)
lnBS-IV _m	0.296	1.274		179.20	1 560
	(0.0742) (0.0000) (0.615	178.39	1.309
	[0.0155]	[0.0090]		(0.0000)	(0.012)
lnGEV-IV _m	-0.127	1.003		230.38	1.784
	(0.2807)	(0.0000)	0.674		
	[0.0111]	[0.0062]		(0.0000)	(0.1813)
lnCW-IV _m	-0.122	1.064		204.45	1 770
	(0.3321)	(0.0000)	0.647	204.45	1.770
	[0.0118]	[0.0070]		(0.0000)	(0.1577)

The *F*-test is for the joint hypothesis of $\alpha = 0$ and $\beta = 1$, and DW is the Durbin-Watson test for the null hypothesis of zero serial correlation. We provide their corresponding *p*-values in brackets in the relevant columns of Table 4.

Table 4 gives the results for the two sets of regressions in Panel A and B, respectively. Panel A shows that the regressions for the four IV series do not pass the Durbin-Watson (DW) test, which indicates that there are autocorrelations in the residuals. In contrast, in Panel B for the logarithmic case, the DW test shows that except for the lnBS-IVcase, the other lnIVs and the lagged lnRV regressions pass the DW test and give efficient estimations. Hence, we will focus our analysis on Panel B on the logarithmic regressions as do other studies such as Cheng and Fung (2012). All β coefficients are significantly different from zero, indicating that both IVs and the lagged RV contain

information on future realised volatility. While both the lagged RV and ln BS-IV show a statistically significant negative coefficient α , for lnVFTSE, lnGEV-IV and lnCW-IV, the null hypothesis of $\alpha = 0$ cannot be rejected. However, the *F*-test shows all the regressions reject the joint null hypothesis of $\beta = 1$ and $\alpha = 0$, indicating that all these forecasts are biased to a greater or lesser extent. Finally, we find that the GEV-IV performs best with an adjusted R^2 of 67.4%, followed by that for the CW-IV at 64.7% and that for the VFTSE at 64.5%. The lagged lnRV regression has the lowest adjusted R^2 of 57.8%. Moreover, the GEV-IV has the best *F*-test value which indicates that it provides the least biased estimates.

Encompassing OLS Regressions

We also run the encompassing OLS regression for each of the implied volatility indices along with the lagged RV as follows:

$$lnRV_{m,m+1} = \alpha + \beta lnIV_{m,m+1} + \gamma lnRV_{m-1,m} + e_m.$$
(28)

Table 5 gives the results. The intercept coefficients, α , taken individually are all not significantly different from zero. The β coefficients of all implied volatilities are significantly different from zero, but the coefficients (γ) of past realised volatility are a lot smaller and in the VFTSE, GEV-IV and CW-IV cases γ is not significantly different from zero. This indicates that all implied volatilities have a much better explanatory power than the lagged realised volatility. For the BS model, the coefficient of BS-IV is the smallest and the coefficient on the lagged RV is the biggest and statistically significant at 90% confidence level. Of all the IV regressions, the β coefficient for the GEV IV is the highest. This confirms that the GEV-IV model has the best explanatory power and the BS model has the worst. But again the *F*-test is rejected in all cases, which shows that all the estimations are biased. The encompassing regressions show better DW test results with zero serial correlation for the error term than the simple regressions in Table 5 Panel B.

Table 5: Encompassing OLS regression estimation

	α (p-value)	β (p-value)	γ (p-value)	Adj \mathbf{P}^2	F-test	DW						
	[std error]	[std error]	[std error]	Auj.N	(p-value)	(p -value)						
lnB _m	$lnRV_m = \alpha + \beta lnB_m + \gamma lnRV_{(m-1)} + e_m$											
InVFTSE _m	-0.118 (0.3642)	0.934 (0.0000)	0.120 (0.4253)	0.645	101.628	1.847						
	[0.0123]	[0.0191]	[0.0142]		(0.0000)	(0.3057)						
lnBS-IV _m	0.120 (0.5181)	0.867 (0.0002)	0.276 (0.0514)	0.625	93.471	1.880						
lnGEV-IV _m	-0.131 (0.2718)	0.953 (0.0000)	0.044 (0.7492)	0.671	114.300	1.823						
lnCW-IV _m	[0.0112] -0.138 (0.2807)	[0.0159] 0.925 (0.0000)	0.113 (0.4528)	0.646	(0.0000)	(0.2571) 1.871 (0.2720)						
	[0.0120]	[0.0187]	[0.0142]		(0.0000)	(0.3729)						

The *F*-test is the joint hypothesis of $\alpha = 0$, $\gamma = 0$ and $\beta = 1$, and DW is the Durbin-Watson test. We provide the test statistics of these two tests and and their corresponding *p*-values in brackets.

As a robustness check, we apply the instrumental variable estimation suggested by Christensen and Prabhala (1998). This test accounts for Error-In-Variables problems in the implied volatility time series. We regress the implied volatility in the following regressions to obtain the instrumental variables for each of the four implied volatilities:

$$lnIV'_{im,m+1} = \alpha' + \beta' lnIV_{im,m-1} + \gamma' lnRV_{m-1,m} + e'_{m}, \quad i=1...,4.$$
(29)

We use the estimated log implied volatility in (29), $lnIV'_{im,m+1}$, to replace the log implied volatility in equation (26). The final results of the instrumental variable regressions are summarised in Table 6 (see Appendix, Table A.2 for the first stage of this regression). Again, the estimation result for BS-IV is not efficient as the DW test is failed and there is significant bias from a non-zero intercept term. Though the adjusted R^2 is now close for all the IV models, the GEV-IV gives the best results, and the CW-IV

	α (p-value)	β (p-value)		F-test	DW
	[std error]	[std error]	Adj.R ²	(p-value)	(p -value)
lnB′ _m		lnRV _m	$=\alpha +\beta \ln B'_{m}$	+e _m	
InVFTSE _m	-0.099 (0.4984)	1.082	0.581	153.317	1.769
	[0.0138]	[0.0083]		(0.0000)	(0.1572)
lnBS-IV _m	0.307 (0.0919)	1.280 (0.0000)	0.573	148.742	1.597
	[0.0172]	[0.0100]		(0.0000)	(0.0188)
lnGEV-IV _m	-0.139 (0.3272)	0.995 (0.0000)	0.585	155.959	1.804
	[0.0134]	[0.0076]		(0.0000)	(0.2201)
InCW-IV _m	-0.116 (0.4203)	1.066 (0.0000)	0.583	154.660	1.799
	[0.0137]	[0.0081]		(0.0000)	(0.2106)

 Table 6: The final result (second stage) for instrumental variable regression

 estimation

comes second best also in terms of the DW and F tests.

The *F*-test is the joint hypothesis of $\alpha = 0$ and $\beta = 1$, and DW is the Durbin-Watson test. We provide the test statistics of these two tests and and their corresponding *p*-values in brackets.

In the final encompassing regression we have lnRV as the dependent variable and all constructed IVs independent variables, along with the lagged lnRV_{m-1}. Due to multi-collinearity, lnVFTSE is modelled with a lag. The only two dependent variables that are significant at 10% confidence level are lnGEV-IV and ln CW-IV with the β coefficient on lnGEV-IV being 0.78 and dominating the one for the lnCW-IV at 0.508. The composite regression gives a marked improvement in R² and appears to be both efficient and unbiased with regard to the coefficients. The joint F test for the null hypothesis that the sum of beta coefficiants equals 1 and the constant equals 0 cannot be rejected.

Table 7: The final multiple variable encompassing regression

	lnRV
lnVFTSE _{m-1}	0.035
	[0.864)
	(0.201)
lnBS-IV _m	0.105
	[0.861]
	(0.596)
lnGEV-IV _m	0.781
	[0.094]
	(0.455)+
lnCW-IV _m	0.508
	[0.090]
	(0.292)+
lnRV _{m-1}	-0.149
	[0.344]
	(0.155)
Constant	0.505
	$\begin{bmatrix} 0.044 \end{bmatrix}$
P squared	0.9152
Stendard errors in brockets [] and n subject ()	0.9132
+ significant at 10%; * significant at 5%	
Joint Test	
Null Hypothesis: Sum Beta's=1 and alpha=0	
(We do not reject the Null hypothesis)	
F(2, 38)	2.17
Prob > F	0.1282
Tests show there is NO SERIAL CORRELATION	
Durbin-Watson Statistic	1.135987
Durbin's Alternative Test (p-value)	0.4611
Breusch-Godfrey LM test (p-value)	0.425

5.4.2 Estimation with Sub-periods

We divide the whole sample period into three sub-periods. The first one is from 4 January 2000 to 30 April 2003. This period accounts for the recession in the early 2000s and the

average realized volatility was 21.27%. The second one is from 1st May 2003 to 30th June 2007. In this period, the market recovers and shows characteristics of a boom economy and an average realized volatility of 10.85%. The third period is from 1st July 2007 to 1st June 2009. This period covers the recent subprime crisis when the market experienced extreme negative movements and the average realized volatility was 29.05%.

α	β			DUV								
(p-value)	(p-value)		F-Statistic	DW								
[std error]	[std error]	Adj.R ²	(p-value)	(p -value)								
$\ln RV_m = \alpha + \beta \ln B_m + e_m$												
InB _m Panel A: 4 January 2000 - 30 April 2003, 39 Observations												
-0.746	0.542		15 123	1 0/15								
(0.003)	(0.000)	0.276	13.123	1.745								
[0.0378]	[0.0226]		(0.0004)	(0.6578)								
-0.426	0.857		21.048	1 782								
(0.120)	(0.000)	0.351	21.040	1.702								
[0.0433]	[0.0303]		(0.0001)	(0.3231)								
-0.038	1.059		15.513	1.568								
(0.9257)	(0.0004)	0.282	(0.0001)	(0.0925)								
[0.0660]	[0.0436]											
-0.369	0.847		24.063	1.830								
(0.166)	(0.0000)	0.384	(0.0000)	(0.40217)								
[0.0424]	[0.0280]											
-0.420	0.851		21.539	1.797								
(0.1216)	(0.0000)	0.357										
[0.0430]	[0.0297]		(0.0000)	(0.3481)								
Panel B	8:1 May 2003 -	31 June 20	007, 50 Observ	ations								
-1.547	0.326	0.097	6.173	2.103								
	α (p-value) [std error] Panel A: 4 -0.746 (0.003) [0.0378] -0.426 (0.120) [0.0433] -0.038 (0.9257) [0.0660] -0.369 (0.166) [0.0424] -0.420 (0.1216) [0.0430] Panel E -1.547	α β (p-value) (p-value) [std error] [std error] [std error] [std error] [std error] [std error] Panel A: 4 January 2000 [] -0.746 0.542 (0.003) (0.000) [0.0378] [] [] -0.426 0.857 (0.120) (0.000) [] [] [] (0.120) (] [] (0.120) (] [] (] [] [] [] (] [] [] [] (] [] [] [] (] [] [] [] (] [] [] [] (] [] [] [] (] [] [] [] [] [] [] [] (] [] [] [] [] [] [] []	α β (p-value) (p-value) [std error] [std error] Adj.R ² [std error] InRV _m =α+βlnB _m + em Panel A: 4 January 2000 - 30 April 2 -0.746 0.542 (0.003) (0.000) 0.276 [0.0378] [0.0226] -0.426 0.857 (0.120) (0.000) 0.351 [0.0433] [0.0303]	α β F-Statistic (p-value) (p-value) F-Statistic [std error] [std error] Adj.R ² (p-value) [std error] InRV _m =α+βlnB _m + e _m (p-value) (p-value) -0.746 0.542 15.123 (0.003) (0.000) 0.276 0.0004) -0.746 0.857 21.048 (0.120) (0.000) 0.351 21.048 (0.120) (0.0004) 0.282 (0.0001) -0.038 1.059 15.513 (0.9257) (0.0004) 0.282 (0.0001) [0.0660] [0.0436] 24.063 (0.166) (0.0000) 0.384 (0.0000) [0.0424] [0.0280] 21.539 (0.1216) (0.0000) 0.357 21.539 (0.1216) (0.0000) 0.357 (0.0000) [0.0430] [0.0297] (0.0000) 21.539 (0.1216) (0.0003 0.357 (0.0000) [0.0430] [0.02								

(0.0166)

(0.000)

Table 8: The basic OLS regression estimation for three sub-periods

	[0.0432]	[0.0188]		(0.017)	(0.869)
lnVFTSE _{m-1}	-1.001	0.660	0 211	13.804	2.079
	[0.0499]	[0.0254]	0.211	(0.001)	(0.978)
lnBS-IV _m	-0.844	0.733		11.458	1.933
	(0.055)	(0.0014)	0.179		
	[0.0613]	[0.0309]		(0.001)	(0.613)
lnGEV-IV _m	-0.978	0.627		20.597	2.186
	(0.0016)	(0.000)	0.289		
	[0.0416]	[0.0197]		(0.000)	(0.675)
lnCW-IV _m	-1.039	0.636		12.931	2.067
	(0.0047)	(0.0008)	0.199		
	[0.0500]	[0.0253]		(0.001)	(0.995)

	Panel (C: 1 July 2007 - 1 June 2009, 24 Observations						
lnRV _{m-1}	-0.656	0.515		7 171	1 902			
	(0.023)	(0.0145)	0.227	/.1/1	1.802			
	[0.0567]	[0.0410]		(0.015)	(0.393)			
lnVFTSE _{m-1}	-0.302	0.821		10 507	1 417			
	(0.3666)	(0.004)	0.313	10.587	1.41/			
	[0.0696]	[0.0538]		(0.004)	(0.058)			
lnBS-IV _m	-0.135	0.870		7.680	1.414			
	(0.763)	(0.012)	0.241					
	[0.0939]	[0.0670]		(0.012)	(0.055)			
lnGEV-IV _m	-0.264	0.813		11.791	1.434			
	(0.4192)	(0.0026)	0.339					
	[0.0683]	[0.0505]		(0.003)	(0.064)			
lnCW-IV _m	-0.353	0.789		10.696	1.458			
	(0.267)	(0.0038)	0.316					
	[0.0660]	[0.0515]		(0.004)	(0.074)			

The *F*-Statistic for the test of the joint hypothesis of $\alpha = 0$ and $\beta = 1$, and DW is the Durbin-Watson test. We provide their corresponding *p*-values in brackets.

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Table 8 gives the OLS regression results for the three periods. It shows that the results for the three periods are consistent with the whole sample period estimation and the GEV-IV gives the highest adjusted \mathbb{R}^2 , followed by CW-IV except for the period of low volatility (Panel B in Table 8). The latter is telling as the CW-IV is inflexibly 'hard-wired' to extrapolate using the 8 times average volatility rule into the extreme volatility surface and hence will not have sufficient flexibility to reflect regime switches. The DW statistics for the last sample period are quite low for all implied volatilities. This is due to the limited observations for the last sample period. In conclusion, Table 8 implies that in both bull and bear markets, the GEV implied volatility can forecast future realised volatility better than other implied volatilities.²²

6. Conclusion

The period marking the run up to the 2007 financial crisis had very low volatility of stock market index returns, while the 2007-2009 period encompassed extreme tail events which have been difficult to model using Gaussian models. The clear message from the paper, is that to cope with the stochastic, time varying and extreme movements in the realized volatility of stock returns, implied volatility models, which purport to be the expected value of realized volatility under a risk neutral Q-measure, have to incorporate information from the tails of the risk neutral density function of option pricing models. This paper has higlighted fundamental problems in extant methods in modelling implied volatility that can flexibly incorporate traded option implied market expectations of extreme returns volatility.

The industry standard CBOE model free method based on observable option prices for constructing implied volatilities, MFIV, runs into problems first highlighted by Jian and Tian (2005, 2007). The downward bias of MFIV during periods of high realized volatility is of particular concern. Further, the reliance on deep out of the money options to model tail behaviour is compounded by VIX White paper truncation rule that

²²The encompassing regressions and instrumental variable regressions for sub-periods also give similar results.

has aptly been termed 'incoherent' by Andersen *et. al.* (2011). It is somewhat arbitrary as to how much of tail behaviour is captured on a certain day (see, footnote 8).

A recent sophisticated approach towards capturing the time-variation in implied volatility has been to directly involve the time variation in the shape of the tails of the risk neutral density function. This was proposed by Figelwski (2010) where he fits a GEV tail to replace extrapolations of the kind described above. Bollerslev and Todorov (2011), specifically in the context of estimating P and Q measure volatility during the recent period of extreme volatility spikes, have recommended the use the Generalized Pareto Distribution to model jump tail events. The main contribution of this paper is to develop an implied volatility index derived from a closed form option pricing model for a GEV based risk neutral density function developed by Markose and Alentorn (2011). This paper was the first to show how a fixed horizon traded option implied GEV tail shape parameter can be obtained to flexibly capture expected market conditions that reflect Fréchet, Gumbel or reverse Weibull RND and hence implied higher moments for the equity returns. In particular, the GEV-IV model given in equation (22) dispenses with the need for extrapolations and interpolations that feature in most extant model free and semi-parametric Q-measure volatility models in order for them to capture extreme tail movements. There is also remarkable economy in the GEV-IV modelling which is based on 3 parameters $\theta = (\xi, \overline{\varphi}, b)$ (respectively, the GEV tail shape parameter, the annualized GEV scale parameter and the time scaling or term structure parameter) backed out in from the function in (20) minimizing sum of squared errors between the traded option prices and the GEV closed form call and put option prices given in (14) and (15).

In all Q-measure estimates of volatility where a fixed horizon (30 days for VIX/VFTSE) is required and option maturity effects have to be removed, the square root to time scaling of volatility is assumed as implied variance is linearly interpolated between two adjacent maturities for options. As this reflects Guassian asumptions, it is not in keeping with recent attempts to incorporate non-Gaussian fat tailed behaviour of stock returns.²³ We

²³ Note, in Markose and Alentorn (2011) the square root law for time scaling was assumed and the full non-Gaussian time scaling implied from traded options of *all* available maturities derived here was not used.

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have argued that time scaling exponents for volatility can themselves be backed out of option pricing data and abrupt decreases in this parameter signal increased volatility as has been noted in Calvet and Fisher (2008). We have incorporated this non-Guassian implied time scaling parameter in the derivation of the GEV-IV model in equations (21) and (22).

We analyse data on the FTSE-100 and its model free VFTSE volatility index from January 2000 to June 2009. This period incorporates both extremely low and high volatilities and provides an ideal test bed to see the the efficacy of differently constructed IV indexes to forecast realized volatility. We retain the CBOE framework as closely as possible and apply cubic spline interpolations and extrapolations in the Black (1976) implied volatility space as in Carr and Wu (2009) and Jiang and Tian (2007) to address the well known problems in the CBOE MFIV methodology. The Carr and Wu (2009) rule of 8 times average implied volatilities obtained from observed (and reliable) traded option prices appears to be robust in capturing extreme high volatility and performs better than the VFTSE with the exception of the sub-period (1 May 2003 - 31 June 2007) of very low volatility. Quite comprehensively, the results show that the GEV-IV gives the best explanatory power for predicting future realised volatility for both whole data sample periods and sub-sample periods which include both boom and bust market conditons. In low volatility periods, GEV-IV gave the least values of the four IV measures we compared, while GEV-IV was best capable of matching the extreme highs of realized volatilty (see, Table 2 and Figure 3). The VFTSE came third after the GEV-IV and CW-IV and only these two IV measures remained statistically significant in the encompassing regression equation with all other contenders (see, Table 7). The standard BS–IV simply does not have the capacity to track the extreme realized volatility and lagged realized volatility also does not fare well.

In conclusion, the recent financial crisis has shown the inadequacy of extant risk neutral pricing models to flexibly reflect to extreme market sentiments impounded in traded options in terms of implied volatility and higher implied moments of stock returns. Our findings raise a number of issues for future research related to the use and construction

of implied volatility measures. The square root of time scaling widely used in model free and parametric IV models does not sit well with the recent proposed uses of extreme value theory in this area. The use of a time varying implied scaling law for time to maturity backed out from traded options in the style of Calvet and Fisher (2008) needs further investigation than was feasible here. It is our view that the traded option implied switches of the GEV tail shape parameter to about 0.1 and less than -0.3 are both key to the capacity of the GEV-IV model to capture extreme Q-measure volatility. Finally, it is well known that flawed Q-measure equity return volatility contributes to inaccuracies in the volatility risk premia and the pricing of volatility derivatives. Future research will investigate the volatility risk premia obtained from GEV-IV and compare this with those obtained by other IV models including the industry standard ones and others that explicitly model tail jumps.

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Appendix

Table A.1: Yearly Average Prices for FTSE-100 Index Options For DifferentMoneyness Categories And For Nearest (1st) and Second Nearest (2nd) Maturities(Number of observations in brackets)

Dowind	Moturity		Panel A: Call option prices OTM ATM									ITM			
I cilou	Maturity	<u>s</u> x	<	0.94	0.94	- 0.97	0.9	7-1	1 -	1.03	1.03	- 1.06	> :	1.06	
2000	1st		4.25	(1769)	30.36	(1024)	88.20	(972)	189.32	(906)	322.38	(862)	1375.53	(9942)	
-000	2nd		11.30	(2367)	72.70	(687)	147.61	(656)	250.77	(616)	374.90	(583)	1372.35	(6624)	
2001	1st		4.63	(1959)	31.05	(897)	80.85	(860)	169.29	(802)	286.32	(758)	1104.41	(7755)	
	2nd		10.64	(2301)	67.03	(626)	131.71	(577)	220.58	(551)	329.13	(524)	1106.18	(5262)	
2002	1st		5.95	(2159)	38.19	(756)	83.38	(696)	157.81	(666)	253.57	(627)	1040.58	(7639)	
	2nd		12.24	(2430)	75.71	(508)	132.10	(472)	207.21	(440)	297.02	(435)	1059.12	(5213)	
2003	1st		3.24	(1427)	21.01	(674)	55.12	(642)	119.19	(599)	205.84	(567)	918.25	(6979)	
	2nd		7.36	(1777)	46.50	(464)	91.73	(435)	157.57	(408)	236.82	(378)	871.04	(4429)	
2004	1st		0.85	(457)	4.33	(713)	27.53	(723)	98.58	(669)	205.80	(645)	881.07	(6434)	
	2nd		1.61	(675)	12.67	(524)	49.83	(487)	123.58	(455)	223.29	(437)	892.36	(4397)	
2005	1st		0.63	(178)	2.30	(691)	21.13	(821)	101.30	(774)	229.25	(737)	1006.12	(7353)	
	2nd		0.97	(445)	7.05	(587)	39.27	(572)	121.91	(528)	240.29	(509)	1011.46	(5114)	
2006	1st		1.18	(463)	6.59	(905)	39.47	(935)	134.33	(878)	272.60	(835)	1083.84	(7644)	
	2nd		2.18	(925)	18.32	(691)	68.83	(649)	166.32	(613)	293.49	(571)	1061.18	(5152)	
2007	1st		2.81	(1194)	16.07	(1212)	63.48	(1280)	169.67	(1208)	313.37	(1038)	1157.52	(7916)	
	2nd		5.23	(1451)	37.43	(810)	104.08	(787)	212.50	(724)	347.35	(650)	1180.89	(5141)	
2008	1st		9.43	(3106)	58.48	(1132)	117.37	(1288)	203.00	(1253)	310.73	(1017)	854.51	(4687)	
	2nd		21.16	(2698)	107.33	(628)	179.07	(594)	270.05	(574)	373.13	(527)	888.35	(2918)	
2009	1st		8.19	(1618)	42.33	(630)	86.62	(784)	149.31	(766)	232.18	(558)	669.93	(2827)	
	2nd		19.96	(1370)	81.46	(379)	135.68	(361)	202.23	(317)	278.74	(280)	684.39	(1745)	

Period M	Madauida			IJ	Panel B: Put option prices ITM ATM					ОТМ				
	Maturity	<u>5</u> x	< (0.94	0.94	- 0.97	0.9	7-1	1 -	1.03	1.03	- 1.06	>	1.06
2000	1st		1303.25	(8827)	331.19	(1026)	186.01	(972)	96.89	(906)	50.31	(862)	7.54	(5685)

	2nd	1296.89	(5910)	373.17	(687)	246.27	(656)	159.21	(616)	103.20	(583)	19.03	(5718)
2001	1st	1354.76	(9897)	294.75	(901)	166.95	(860)	88.56	(802)	48.24	(758)	9.27	(4942)
	2nd	1256.49	(6112)	328.18	(626)	216.12	(577)	140.77	(551)	91.39	(524)	19.18	(4750)
2002	1st	1158.76	(8545)	255.44	(757)	153.91	(696)	91.42	(666)	56.62	(627)	10.99	(5223)
	2nd	1102.85	(5390)	292.37	(508)	202.66	(472)	142.08	(440)	101.46	(435)	22.98	(4575)
2003	1st	838.23	(5897)	212.59	(675)	117.39	(642)	60.57	(599)	32.75	(567)	5.51	(4150)
	2nd	836.03	(4020)	237.02	(464)	153.63	(435)	99.20	(408)	66.02	(378)	13.24	(4028)
2004	1st	626.59	(3494)	217.66	(765)	96.63	(723)	33.27	(669)	13.07	(645)	2.55	(3596)
	2nd	632.57	(2380)	227.59	(524)	119.57	(487)	58.91	(455)	30.49	(437)	6.12	(3396)
2005	1st	589.88	(2717)	245.16	(859)	101.32	(821)	27.22	(774)	8.67	(737)	1.79	(2914)
	2nd	606.83	(2003)	249.72	(590)	118.90	(572)	47.73	(528)	20.57	(509)	3.60	(3168)
2006	1st	640.10	(2683)	286.40	(1000)	130.33	(935)	48.58	(878)	19.45	(835)	3.26	(3279)
	2nd	633.15	(1823)	297.46	(691)	159.94	(649)	80.62	(613)	41.66	(571)	7.15	(3796)
2007	1st	722.09	(3358)	318.06	(1271)	161.54	(1280)	78.06	(1208)	40.80	(1038)	8.05	(4710)
	2nd	728.49	(2105)	338.32	(810)	201.14	(787)	121.07	(724)	75.19	(650)	17.62	(4038)
2008	1st	1089.11	(6863)	308.81	(1135)	195.96	(1288)	126.18	(1253)	82.41	(1017)	25.94	(4060)
	2nd	1018.22	(3988)	352.19	(628)	255.17	(594)	186.02	(574)	137.23	(527)	49.33	(2893)
2009	1st	1069.71	(3099)	251.36	(487)	165.20	(763)	106.27	(790)	67.51	(634)	17.59	(3126)
	2nd	1144.94	(2011)	307.33	(288)	227.77	(333)	166.79	(317)	122.95	(316)	40.42	(2206)

This table gives the average prices and the corresponding number of observations in brackets with FTSE 100 call and put options for the nearby (1st) and the second nearby (2nd) maturities. We divide the option prices into six categories based on the value of $\frac{\pi}{x}$ (see see Bakshi, Cao, and Chen (1997)). For call options, it is out-of-money (OTM) when $\frac{\pi}{x} < 0.97$ and far out-of-money when $\frac{\pi}{x} < 0.94$; it is at-the-money (ATM) when $0.97 < \frac{\pi}{x} < 1.03$; it is in-the-money (ITM) when $\frac{\pi}{x} > 1.03$ and deep in-the-money when $\frac{\pi}{x} > 1.06$. Vice versa for the put options.

	α	$\alpha_0 = 0$	β	$\beta_0 = 0$	Y	$\gamma_0 = 0$		F-test	DW
							Adj. R ²		
	[std]		[std]		[std]				(p
		(p-value)		(p-value)		(p-value)		(p-value)	-value)
lnB _m	$\ln B_m = \alpha' + \beta' \ln B_{m-1} + \gamma' \ln R V_{m-1} + \epsilon'_m$								
			0.3870		0.4547				1.8780
lnVFTSE_	-0.1515	(0.0022)		(0.0000)		(0.0000)	0.9083	545.8256	
	[0.0046]		[0.0046]		[0.0034]			(0.0000)	(0.3976)
lnBS_			0.5188		0.3086				1.9310
	-0.2407	(0.0000)		(0.0000)		(0.0000)	0.9288	718.8800	
	[0.0042]		[0.0039]		[0.0024]			(0.0000)	(0.5650)
lnGEV_			0.3743		0.4995				2.1059
	-0.1528	(0.0087)		(0.0000)		(0.0000)	0.8876	435.5328	
	[0.0054]		[0.0053]		[0.0044]			(0.0000)	(0.7127)
InCW_			0.3612		0.4800				1.9611
	-0.1516	(0.0033)		(0.0000)		(0.0000)	0.9018	505.8124	
	50.00.105		50.00.465		F0.000.00			(0.0000)	(0.0001)
	[0.0048]		[0.0048]		[0.0036]			(0.0000)	(0.6881)

Table A.2: The first stage for instrumental variable regression estimation

The column of $\alpha_0 = 0$ gives the *p*-value of the null hypothesis of $\alpha = 0$, the column of $\beta_0 = 0$ gives the *p*-value of the null hypothesis of $\beta = 0$. The *F*-test is the joint hypothesis of $\alpha = 0$ and $\beta = 1$, and DW is the Durbin-Watson test. We provide the test statistics of these two tests and and their corresponding *p*-values in brackets.